Secret Hitler

mghaf073

June 2025

1 Introduction to the Game

There are 10 players, participants are split into two hidden teams: Liberals (majority) and Fascists, with one player secretly assigned the special role of Hitler. The game is over when **Liberal team aims to pass 5 liberal policies or assassinate Hitler, or Fascists, including Hitler, win by passing 6 fascist policies or, after three fascist policies, by electing Hitler as Chancellor, or Hitler's identity is known to fellow fascists, but Hitler doesn't know who they are in larger games.**

2 Gameplay

Each round players elect a President and Chancellor. The President draws three policy cards, discards one, and gives two to the Chancellor; the Chancellor selects one to enact. If the government enacts a fascist policy, executive powers activate—like investigations, special elections, or executions—as the fascist track advances. Failed elections (three in a row) result in a policy being drawn randomly into power.

Players use discussion, deception, voting, and deduction to expose fascists or hide Hitler. Bluffing and reading others' intentions are key skills for both sides.

3 Technical Details

Unbiased Role and Hitler Assignment

Let N be the number of players, $N_L < N$ denotes the number of liberals in the game, and $N_F = N - N_L$ denotes the number of fascists. As in classical game, numbers are set in a way so $N_F < N_L$ (proportions of parties is parameter to decide). Each player's role is decided by partial measurements of the following state

$$|\Psi_{\text{role}}\rangle = \frac{1}{\sqrt{\binom{N}{N_L}}} \sum_{\substack{(r_1,\ldots,r_N)\in\{0,1\}^N \\ \#\{i:r_i=0\}=N_L}} \bigotimes_{i=1}^{N} |r_i\rangle \,. \tag{1}$$

Here, we assign 0 to the role of a liberal, and 1 to the role of a fascist. So, each r_i takes either 0 or 1. i'th player is assigned to be a liberal if the partial measurement of i'th qubit yielded 0, and the player is assigned to fascists in either case, $i \in [N]$.

When the roles are distributed, there's external system which informs each fascist who are the other fascists, so they know each other in the beginning. However, there appears a difference with the original game.

In the original game, Hitler knows who he is, fascists know each other and know which player is the Hitler, but Hitler does not know the other fascist players. In our game, the Hitler's role is distributed in the beginning uniformly between all the fascists, but during the gameplay the chance of particular fascist to be the Hitler increases depending on the choices player does.

So then initial Hitler's role distribution is given by the state

$$|\Psi_{\text{Hitler}}\rangle = \otimes^{N-N_F} |0\rangle \otimes \frac{1}{\sqrt{N_F}} \sum_{j=1}^{N_F} |e_j\rangle \,, \tag{2}$$

where $e_1, \ldots e_{N_F}$ is the canonical basis on the space of N_F qubit vectors. Here, we assume, for simplicity, that first N_L players are assigned as liberals.

Elections

There are two important roles in the game - the president and the chancellor. They are responsible for law system, which is similar to the one in classical game, but with a zest of randomness.

President Elections

Role of the president circles around, it is out of players' power. At first, role of the president is assigned randomly, then it goes clockwise or in any order the players prefer.

Chancellor Elections

Chancellor elections are determined by the quantum voting which is organized as follows.

As in the original game, the president picks the chancellor, then the others vote for or against the candidate. However, we define new, quantumly fair, voting procedure.

Define the initial decision state

$$|\psi_0\rangle = |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}. \tag{3}$$

The next two operators represent quantum bulletin

$$S_0 = R_y(\phi), \quad S_1 = R_y(-\phi), \quad R_y(\phi) = e^{-i\phi Y/2}, \tag{4}$$

which are unitary and commute. Each player will apply to the decision state S_0 or S_1, whether they agree with the choice of the candidate or not.

We have

$$\Pr(0 \mid S_0) = \left|\langle 0| S_0 |+\rangle\right|^2 = \frac{1+\sin\phi}{2} = \frac{1}{2} + \delta, \tag{5}$$

$$\Pr(1 \mid S_1) = \left|\langle 1| S_1 |+\rangle\right|^2 = \frac{1+\sin\phi}{2} = \frac{1}{2} + \delta, \tag{6}$$

with

$$\delta = \frac{\sin\phi}{2}, \quad \phi = \arcsin(2\delta). \tag{7}$$

To keep the individual bias small in an $N-1$-voter election, choose

$$\delta = \frac{c}{N-1}, \tag{8}$$

with a fixed constant $0 < c < \frac{1}{2}$ so that $2\delta < 1$.

Procedure

1. Begin with $|\psi_0\rangle = |+\rangle$.
2. Each voter applies either S_0 to slightly favor outcome 0 or S_1 to slightly favor 1.
3. After all voters made their choice, the final state is measured in $\{|0\rangle, |1\rangle\}$ basis.
4. If 1 is observed, the chancellor is elected. Else, we follow the standard procedure from the game.

Laws

In the original game, the law system allows liberals win, after accepting 5 liberal laws, and it allows win fascists if 6 fascist laws are accepted. Or, if 3 fascist laws are accepted and Hitler person is elected to be the chancellor, the fascists also win.

We keep this rule but slightly modify the law selecting procedure. In the original game there's shuffled deck of laws with 6 liberal policy cards and 11 fascists policy cards. Accordingly, the initial state corresponding to the law which will be chosen is given by

$$|\psi_0^p\rangle = \sqrt{\frac{6}{17}}\,|0\rangle + \sqrt{\frac{11}{17}}\,|1\rangle\,.$$

The president applies $S_L^P = R_y(\phi)$ if he wishes to favor the measurement of liberal policy, and the president applies $S_F^P = R_y(-\phi/2)$ in other case.

If the i'th player who is the president applies S_F^P, automatically acts S_i^H which increases the i'th player chance to be the Hitler. In fact, $S_1^H, \ldots, S_N^H$ are non-commutative, the the initial choice of the president impacts the change of $|\Psi_{\text{Hitler}}\rangle$. This is deliberate design choice which makes the game even more unpredictable. Nothing happens to the $|\Psi_{\text{Hitler}}\rangle$ in case if the president used S_L^P.

After the president applied the matrix, the chancellor does the same with $S_L^C = R_y(\phi)$ and $S_F^C = R_y(-\phi/2)$. Same modification of $|\Psi_{\text{Hitler}}\rangle$ happens if the chancellor wishes to use S_F^C.

Choose ϕ such that if S_L^P and S_L^C are both applied, the chance of measuring 0 is 0.8.

S_i^H has form:

$$S_i^H = \exp\left(-i\chi(|\Psi_{\text{Hitler}}\rangle \langle e_i| + |e_i\rangle \langle \Psi_{\text{Hitler}}|)\right), \quad i \in [N]. \tag{9}$$

We set

$$\chi = \sqrt{\frac{p^* - \frac{1}{N_F}}{1 - \frac{1}{N_F}}}$$

for some $\frac{1}{N_F} \leq p^* \leq 1$.

Once the midgame is reached, the probabilities of fascists to be the Hitler stop changing.

Middle Game

If the players arrived to the point when there are three fascist policies accepted, the state vector $|\Psi_{\text{Hitler}}\rangle$ starts playing role. After each successful election of the chancellor and assigning its role to the player j, we do the partial measurement to determine if the person is Hitler or not. If the person is, i.e. the measurement yielded 1, the game is over and the fascists win. In other case, the game continues and the winning conditions of each party as identical to ones in the original game.

Quantum Bullet Mechanic

In the original game, when the fourth and fifth fascists policies are accepted, the current president obtains the right to kill the player of their choice. However, we change this rule and let the randomness decide the destination of the bullet.

In our modification, the president chooses to use the killing right or not. If they choose to use it, they pick a player who will be shot with 0.8 chance. However, this means that there's possibility of other person being shot, including the president itself.

This mechanic could be implemented via acting on

$$|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |e_j\rangle \,, \tag{10}$$

where $\{|e_j\rangle\}_{j=1}^{N}$ is the computational basis (player j gets the bullet).

Biased State Toward Player i

$$|\Psi_i\rangle = \sqrt{0.8}\ |e_i\rangle \ + \ \sqrt{\frac{0.2}{N-1}} \sum_{\substack{j=1 \\ j\neq i}}^{N} |e_j\rangle \,, \tag{11}$$

so that $\Pr(i) = 0.8$ and $\Pr(j \neq i) = 0.2$.

Unitary S_i Mapping $|\Psi_0\rangle \to |\Psi_i\rangle$

$$|u\rangle \ = \ |\Psi_0\rangle \,, \qquad |v_i\rangle = \frac{|e_i\rangle - \langle u|e_i|\,\rangle\,|u\rangle}{\big\||e_i\rangle - \langle u|e_i|\,\rangle\,|u\rangle\big\|}, \tag{12}$$

perform Gram–Schmidt on $\{|u\rangle\,, |v_i\rangle\}$ to get $|v_{\perp,i}\rangle$ orthogonal to $|\Psi_i\rangle$. Then

$$S_i = |\Psi_i\rangle\langle u| \ + \ |v_{\perp,i}\rangle\langle v_i| \ + \sum_{w\,\perp\,\{u,v_i\}} |w\rangle\langle w| \,, \tag{13}$$

which is unitary and satisfies $S_i\,|\Psi_0\rangle = |\Psi_i\rangle$.

Killing-right Mechanic

When the president chooses to use their killing right:

1. They apply S_i to the shared "death-target" register to bias toward their chosen victim i.
2. They then measure in the $\{|e_j\rangle\}$ basis.
3. With probability 0.8 player i is shot; with probability $0.2/(N-1)$ any other player (including the president) is shot.