Analysis of Algorithms

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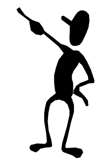
Discussion 2 | University of Southern California

Runtime Complexity

Graph Traversals

Topological Sort

Computational Complexity



Useful notation to discuss growth rates

For any monotonic functions f, g from the positive integers to the positive integers, we say

$$f(n) = O(g(n))$$

if

g(n) eventually dominates f(n)

[Formally: there exists a constant c such that for all sufficiently large n: $f(n) \le c*g(n)$]

Another useful notation: Ω

For any monotonic functions f, g from the positive integers to the positive integers, we say

$$f(n) = \Omega(g(n))$$

if:

f(n) eventually dominates g(n)

[Formally: there exists a constant c such that for all sufficiently large n: $f(n) \ge c*g(n)$]

More useful notation: Θ

For any monotonic functions f, g from the positive integers to the positive integers, we say

$$f(n) = \Theta(g(n))$$

if:

$f(n) = O(g(n))$ <u>and</u> $f(n) = \Omega(g(n))$

T(n) = n log n + 1024 n log (log n)

Circle **ALL** answers that apply

$O(n^2)$	O(n log n)	O(n log log n)	O(n)	O(log n)

$T(n) = n \log n + 1024\, n \log (\log n)$

Circle **ALL** answers that apply

$\Omega(n^2)$	$\Omega(n \log n)$	$\Omega(n \log \log n)$	$\Omega(n)$	$\Omega(\log n)$

$T(n) = n \log n + 1024\, n \log (\log n)$

Circle **ALL** answers that apply

$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n \log \log n)$	$\Theta(n)$	$\Theta(\log n)$

What is the Big-O runtime complexity of the following function?

```
int bigOh1(int n):
    int value = 0;
    for i=0 to n
       for j=0 to length(binary(n))
          value = i * j;
    return value;
```

O(n log n)

What is the Big-O runtime complexity of the following function?

```
string bigOh2(int n):
   string tmp = "";
   for k=0 to n
        tmp =tmp + toString(k);
     return tmp;
```

O(n²)

What is the Big-O runtime complexity of the following function?

```
void bigOh3(int n):
  for i=1 to n
     j=1
     while j < n
         j = j*2;
```

O(n log n)

What is the Big-O runtime complexity of the following function?

HARD - !!!

```
void bigOh4(int[] L, int n)
   while (n > 0)
      find_max(L, n); //finds the max in L[0…n-1]
      n = n/4;
```

$$n + n/4 + n/16 + \ldots + 1 =$$

$$= n\,(1 + 1/4 + 1/16 + \ldots) = O(n)$$

What is the Big-O runtime complexity of the following function?

HARD - !!!

```
string bigOh5(int n)
  if(n == 0) return "a";
  string str = bigOh5(n-1);
  return str + str;
```

bigOh5(0) → "a"
bigOh5(1) → "aa"
bigOh5(2) → "aaaa"
bigOh5(3) → "aaaaaaaa"

$$1 + 2 + 4 + 8 + 16 + \ldots = O(2^n)$$

What is the Big-O runtime complexity of the following function?

```
void bigOh6(int n)
  for i = 1 to n
     for j = i to n
        sum = 0;
        for k = i to j
           sum += A[k];
        B[i, j] = sum;
```

Sum[1, {i,1,n}, {j,i,n}, {k,i,j}] =

$$n(n+1)(n+2)/6 = O(n^3)$$

See the next slide

Triple summation in bigOh6

$$\sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=i}^{j}1=\sum_{i=1}^{n}\sum_{j=i}^{n}(j-i+1)=\sum_{i=1}^{n}\left(\sum_{j=i}^{n}j-\sum_{j=i}^{n}(i-1)\right)=$$

$$\sum_{i=1}^{n}\left(\frac{(n+1-i)(n+2-i)}{2}\right)=\frac{n(n+1)(n+2)}{6}$$

BFS and DFS

Graph Traversals

Depth-First Search (DFS)
Breadth-First Search (BFS)

DFS uses a stack for bookkeeping.

BFS uses a queue for bookkeeping.

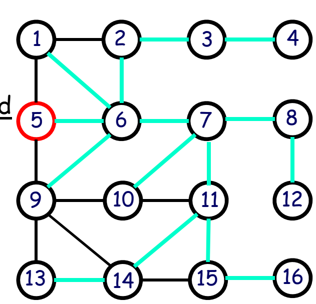
Properties of Traversals

Property 1

They visit all the vertices in the connected component

Property 2

The result of traversal is a spanning tree of the connected component



Exercise

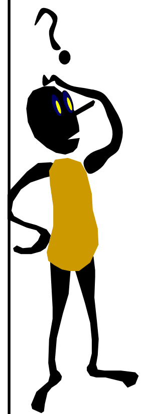
The complete graph on n vertices, denoted K_n, is a simple graph in which there is an edge between every pair of distinct vertices.

What is the height of the DFS tree for the complete graph K_n?

n-1

What is the height of the BFS tree for the complete graph K_n?

1



Exercise

What is the visual difference between BFS and DFS trees?

DFS trees is tall and skinny.

BFS trees is short and bushy.

Problem 2

Mathematicians often keep track of a statistic called their Erdős Number, after the great 20th century mathematician. Paul Erdős himself has a number of zero. Anyone who wrote a mathematical paper with him has a number of one, anyone who wrote a paper with someone who wrote a paper with him has a number of two, and so forth and so on. Supposing that we have a database of all mathematical papers ever written along with their authors:

a. Explain how to represent this data as a graph.

b. Explain how we would compute the Erdős number for a particular researcher.

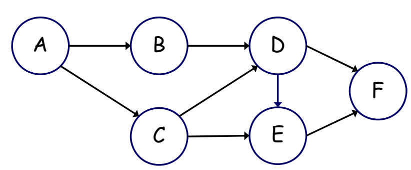
c. Explain how we would determine all researchers with Erdős number at most two.

Topological Sort

Definition

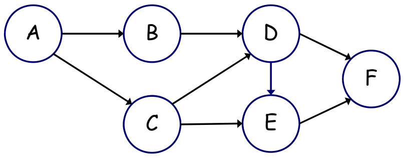
Suppose each vertex represents a task that must be completed, and an edge (u, v) indicates that task u depends on task v. That is v must be completed before u. The topological ordering of the vertices is a valid order in which you can complete the tasks.

Algorithm



- Select a vertex that has in-degree zero.
- Add the vertex to the output.
- Delete the vertex and all its outgoing edges.
- Repeat.

Better Algorithm

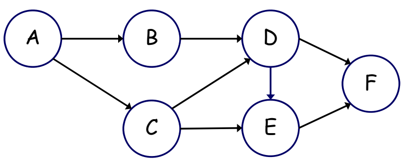


- Select a vertex that has in-degree zero.
- Run DFS and return vertex that has no undiscovered leaving edges

Problem 3

In class we discussed finding the shortest path between two vertices in a graph. Suppose instead we are interested in finding the longest simple path in a directed acyclic graph. In particular, we are interested in finding a path (if one exists) that visits all vertices (also known as a Hamiltonian Path). Given a DAG, give a linear time algorithm to determine if there is a simple path that visits all vertices.

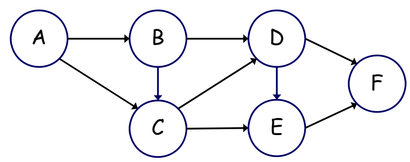
Solution 3



Topological Orderings: A B C D E F or A C B D E F

There is no simple path that visits every node

Solution 3



Topological Ordering: A B C D E F

The ordering is a valid path in the DAG