Analysis of Algorithms

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Discussion 10 University of Southern California

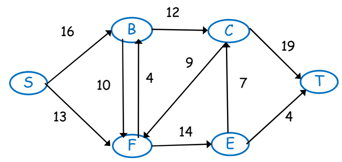
Network Flow

Problem 1

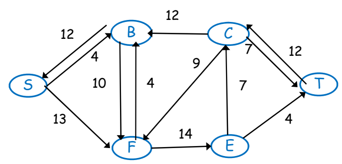
You have successfully computed a maximum s-t flow f for a network G = (V, E) with integer edge capacities. Your boss now gives you another network G′ that is identical to G except that the capacity of exactly <u>one edge is decreased by one</u>. You are also explicitly given the edge whose capacity was changed. Describe how you can compute a maximum flow for G′ in O(|V| + |E|) time.

Solution

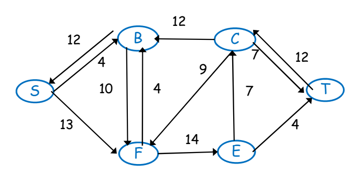
Start off finding the maximum flow



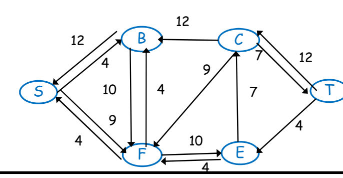
SBCT



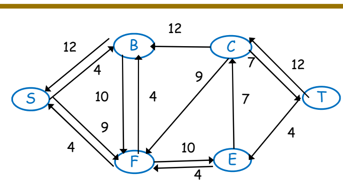
Solution



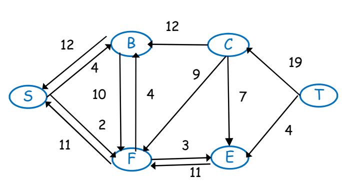
SFET



Solution



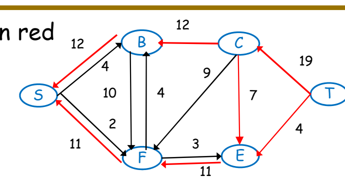
SFECT



Solution

Flow is in red

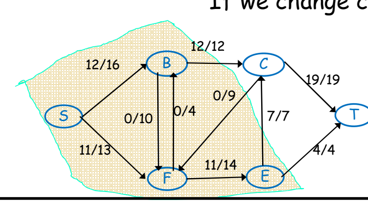
G_f



Min cut-?

If we change c(B,C) to 11 -?

G



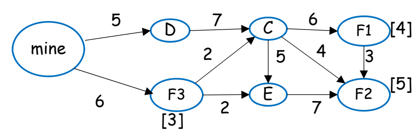
Solution

If the edge e=(u,v) we decreased *was* saturated (as in this example). Find a path in the graph following only edges with flow on them from s to u; decrease each edge's flow by one. Do the same from v to t. Lastly, attempt to find a path from s to t in the residual graph.

If the edge e=(u,v) we decreased was *not* saturated, nothing changes.

Problem 2

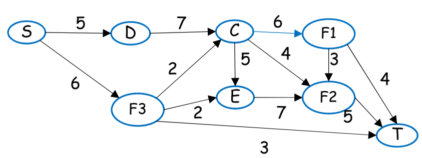
You need to transport iron-ore from a mine to factories F_k. You are given the road and factory capacities (equal to their processing per minute.) We would like to determine how much ore we can transport.



Solution

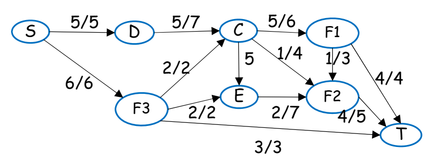
We can model this as a max flow problem with the iron-ore mine being the source S. The sink T will be a new node that is connected to all of the factories.

The capacity of these new edges will be the factory capacity.



Solution

Run a max flow algorithm, such as Ford-Fulkerson ($O(E\ |f|)$) on the new graph.



This shows us we are able to deliver 11 pieces of iron-ore from the mine to the factories each minute.

Problem 3

In a daring burglary, someone attempted to steal all the candy bars from the CS department. Luckily, he was quickly detected, and now, the course staff and students will have to keep him from escaping from campus. In order to do so, they can be deployed to monitor strategic routes. More formally, we can think of the USC campus as a graph, in which the nodes are locations, and edges are pathways or corridors. One of the nodes is the burglar's starting point, and several nodes (the USC gates) are the escape points. Students and staff can be placed to monitor the edges. Compute the minimum number of students/staff needed.

Solution

For each edge in the graph, give a capacity of 1.

Let the source s be the burglar's starting point

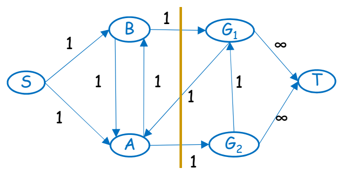
Create a sink t with edges from every escape point with a capacity of infinity.

Compute a minimum s-t cut.

Note that every edge cut is between s and the escape points, as the infinite capacity edges will keep those edges from being on a cut.

Since the edge capacities in the cut are all 1, the cut capacity will define the min number of students.

Solution



Minimum cut in the graph has capacity=2