CSCI 570 Fall 2016 Discussion 6

1. There are 2 sorted arrays A and B of size n each. Design a D&C algorithm to find the median of the array obtained after merging the above 2 arrays (i.e. array of length 2n). Discuss its runtime complexity.

2. A tromino is a figure composed of three 1x1 squares in the shape of an L. Given a $2^n x 2^n$ checkerboard with 1 missing square, tile it with trominoes. Design a D&C algorithm and discuss its runtime complexity.

3. The standard multiplication of two n-digit integers involves n^2 single digit multiplications. Design a D&C algorithm to multiply two n-digit integers. Discuss its runtime complexity.

4. You are given an unsorted ~~array~~ list of ALL integers in the range $[0,\ldots, 2^k-1]$ except for one integer, denoted the missing number by M. Describe a divide-and-conquer to find the missing number *M*, and discuss its the worst-case runtime complexity in terms of $n = 2^k$.

(1)

$A = [1, 3, 5, 16, 18, 21, 30]$

$B = [2, 13, 17, 20, 23, 29, 35]$

obvious solution, w/o using D&C:
merge the two arrays in $O(n)$
find the median of the merged array in $O(1)$
This Takes $O(n)$ time

D&C sol.

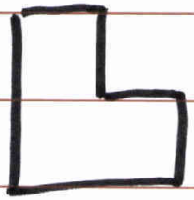
$A = [1, 3, 5, \textcircled{16}, 18, 21, 30]$

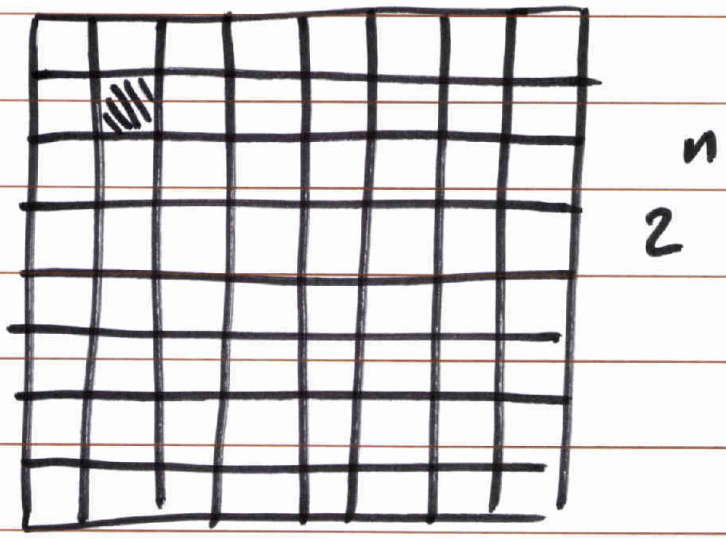
$B = [2, 13, 17, \textcircled{20}, 23, 29, 35]$

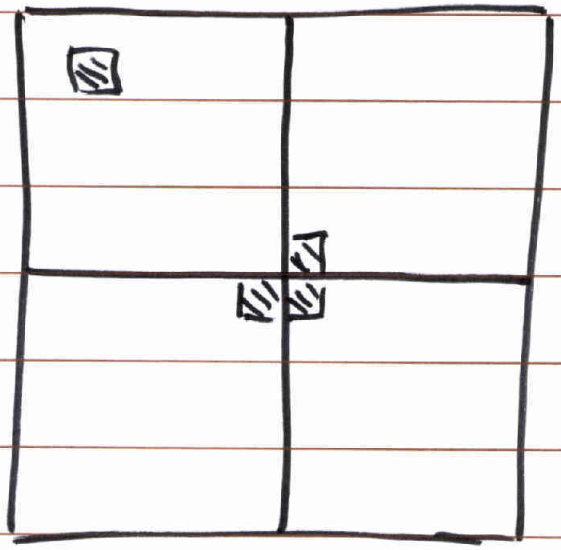
$T(n) = T(n/2) + c$

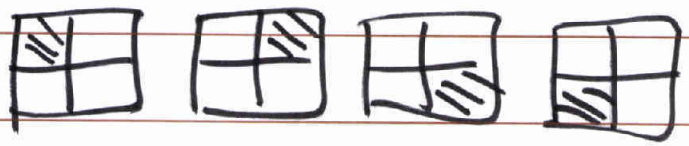
$\rightarrow T(n) = \Theta(\lg n)$

(2)









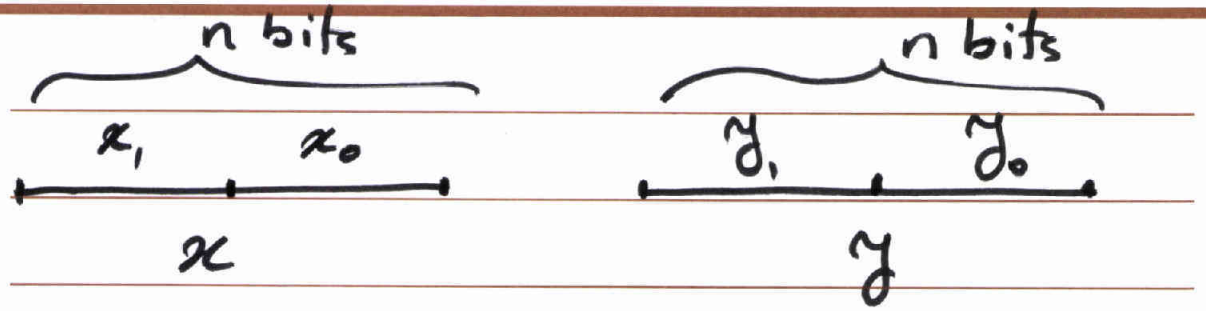
$T(n) = 4T(n/2) + c \Rightarrow T(n) = \Theta(n^2)$

(3)

n bits

12	1100
x 13	x 1101
36	1100
12	0000
156	1100
	1100
	10011100

→ $\Theta(n^2)$



$$xy = x_1 y_1 \cdot 2^{n} + (x_0 y_1 + x_1 y_0) \cdot 2^{n/2} + x_0 y_0$$

$$T(n) = 4T(n/2) + cn$$

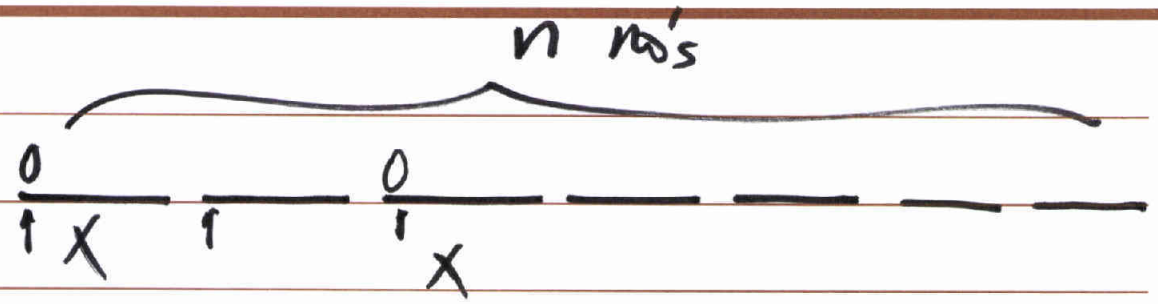
$$T(n) = \Theta(n^2)$$

$$(x_0 + x_1)(y_0 + y_1) = x_0 y_0 + x_0 y_1 + x_1 y_0 + x_1 y_1$$

$$T(n) = 3T(n/2) + cn$$

$$\Rightarrow T(n) = \theta(n^{\log_2 3}) = \theta(n^{1.59})$$

(4)



0 to $2^{k}-1$ except for 1 are in the list

The function returns the bit in a given position for a given no.

Use this function $O(n)$ times

000
001
010
011
100
101
110
111

1st level 2n calls
2nd " n "
1
1
4n = $\Theta(n)$