CS570

Analysis of Algorithms

Spring 2005

Midterm Exam

Name: _____

Student ID: _____

1) 12 pts
Considering an undirected graph G=(V, E), are the following statements true or false? Provide a brief explanation. The space provided should suffice.

A- Suppose all edge weights are different. Then the shortest edge must be in the minimum spanning tree.

B- Suppose all edge weights are different. Then the longest edge cannot be in the minimum spanning tree.

C- Suppose all edge weights are different. Then the minimum spanning tree is unique.

D- Suppose all edge weights are different. Then the shortest path from A to B is unique.

2) 9 pts
Considering amortized and actual cost analysis of heap operations {insert, extract-min, union, decrease-key, delete-key} on heap data structures we have studied. are the following statements true or false? Provide a brief explanation. The space provided should suffice.

A- **Actual** cost of any operation in a **Fibonacci** heap data structure takes at most O(log n) time.

B- **Actual** cost of any operation in a **binomial** heap data structure takes at most O(log n) time.

C- **Actual** cost of any operation in a **binary** heap data structure takes at most O(log n) time.

3) 20 pts

A- Give an example of an undirected graph **G** with positive edge weights and a starting vertex **s** in **G** for which Dijktra's and Prim's algorithms visit nodes in different orders.

B- Show the order in which nodes in **G** (the graph you presented in part A) are visited in both algorithms.

C- Show the minimum spanning tree resulting from the execution of Prim's algorithm on your graph.

4) 20 pts

Consider the following Change Problem. The input to this problem is an integer *L*. The output should be the minimum cardinality collection of coins required to make *L* shillings of change (that is, you want to use as few coins as possible). The coins are worth 1, 5, 10, 20, 25, 50 shillings. Assume that you have an unlimited number of coins of each type.

Does the following greedy algorithm correctly solve the problem?
Take as many coins as possible from the highest denominations.
If it does, prove the accuracy of the solution. If it does not, give a counter example.

5) 20 pts
For the Change problem defined in the previous question, what other solution technique (other than greedy) seems appropriate?

A- Present your solution using an alternate technique.

B- Analyze the running time of the algorithm presented above.

6) 20 pts

Using a straightforward approach, finding both the minimum and the maximum elements of a set of **n** numbers can take **2*(n-1)** comparisons, i.e. finding the minimum element takes **n-1** comparisons and then to find the maximum element takes another **n-1** comparisons. We seek to reduce the number of comparisons using a divide and conquer approach.

A- Can the number of comparisons be improved by any method to be lower than **O(n)**? If yes, demonstrate how, if no prove why not.

B- Can we improve on the constant ***a*** involved in the ***a*n+*b*** number of comparisons? In other words can we reduce **2*n-2** to ***a*n+*b*** where ***a*** is less than **2**.

Additional space--continuation of problem ____