# CSCI 570 Fall 2016 Discussion 5

1. Suppose we have two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, along with $T_1$ which is a MST of $G_1$ and $T_2$ which is a MST of $G_2$. Now consider a new graph $G = (V, E)$ such that $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$ where $E_3$ is a new set of edges that all cross the cut $(V_1, V_2)$.

Consider the following algorithm, which is intended to find a MST of G.

```
Maybe-MST(T₁, T₂, E₃)
      e_min = a minimum weight edge in E₃
      T = T₁ U T₂ U { e_min}
      return T
```

Does this algorithm correctly find a MST of G? Either prove it does or prove it does not.

2. You are given a graph representing the several career paths available in industry. Each node represents a position and there is an edge from node *v* to node *u* if and only if *v* is a prerequisite for *u*. Top positions are the ones which are not prerequisites for any positions. The cost of an edge *(v, u)* is the effort required to go from one position *v* to position *u*. Ivan wants to start a career and achieve a top position with minimum effort. Using the given graph can you provide an algorithm with the same run time complexity as Dijkstra's? You may assume the graph is a DAG.

3: (a): Suppose we are given an instance of the Minimum Spanning Tree problem on a graph G. Assume that all edges costs are distinct. Let T be a minimum spanning tree for this instance. Now suppose that we replace each edge cost $c_e$ by its square, $c_e^2$ thereby creating a new instance of the problem with the same graph but different costs.

Prove or disprove: T is still a MST for this new instance.

(b): Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G. Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$. Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

4. Solve the following recurrences using the Master Method:
   a. $A(n) = 3A(n/3) + 15$
   b. $B(n) = 4B(n/2) + n^3$
   c. $C(n) = 4C(n/2) + n^2$
   d. $D(n) = 4D(n/2) + n$