CSCI 570 Fall 2016: Discussion 1 Problems

1.8 For this problem, we will explore the issue of *truthfulness* in the Stable Matching Problem and specifically in the Gale-Shapley algorithm. The basic question is: Can a man or a woman end up better off by lying about his or her preferences? More concretely, we suppose each participant has a true preference order. Now consider a woman *w*. Suppose *w* prefers man *m* to *m'*, but both *m* and *m'* are low on her list of preferences. Can it be the case that by switching the order of *m* and *m'* on her list of preferences and running the algorithm with this false preference list, *w* will end up with a man *m''* that she truly prefers to both *m* and *m'* ?

Resolve this question by doing one of the following two things:

* Give a proof that, for any set of preference lists, switching the order of a pair on the list cannot improve a woman's partner in the Gale-Shapley algorithm; or

* Give an example of a set of preference lists for which there is a switch that would improve the partner of a woman who switched preferences.

2. Given *n* men and *n* women along with their preference lists, a *consensus-optimal* stable matching is a matching which simultaneously pairs every man with his best **valid** partner and pairs every woman with her best **valid** partner. Recall that a valid partnership must be a matched pair in some stable matching solution. Give an algorithm to determine whether a consensus-optimal stable matching exists for a given set of preference lists; the running time of your algorithm should not be slower than a constant times the running time of a single call to the Gale-Shapley algorithm.

3. In a connected bipartite graph, is the bipartition unique? Justify your answer.