CS570

Analysis of Algorithms

Summer 2006

Exam 1

Name: _____________________

Student ID: _________________

	Maximum	Received
Problem 1	20	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Problem 5	10	
Problem 6	20	
Problem 7	20	

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
The running time of an algorithm is $\theta(g(n))$ if and only if its best-case running time is $\Omega(g(n))$ and its average-case running time is $O(g(n))$.

[TRUE/FALSE]
A dynamic programming algorithm tames the complexity by making sure that no subproblem is solved more than once.

[TRUE/FALSE]
The memoization approach in dynamic programming has the disadvantage that sometimes one may solve subproblems that are not really needed.

[TRUE/FALSE]
A greedy algorithm finds an optimal solution by making a sequence of choices and at each decision point in the algorithm, the choice that seems best at the moment is chosen.

[TRUE/FALSE]
If a problem can be solved correctly using the greedy strategy, there will only be one greedy choice (such as "choose the object with highest value to weight ratio") for that problem that leads to the optimal solution.

[TRUE/FALSE]
Whereas there could be many optimal solutions to a combinatorial optimization problem, the value associated with them will be unique.

[TRUE/FALSE]
Consider an undirected graph G=(V, E). Suppose all edge weights are different. Then the longest edge cannot be in the minimum spanning tree.

[TRUE/FALSE]
Consider an undirected graph G=(V, E). Suppose all edge weights are different. Then the shortest edge must be in the minimum spanning tree.

[TRUE/FALSE]
Consider an undirected graph G=(V, E). Suppose all edge weights are different. Then the shortest path from A to B is unique.

[TRUE/FALSE]
Bellman Ford's algorithms of finding the shorting s-t path is more suitable for parallel processing than Dijkstra's.

2) 10 pts

Indicate for each pair of expressions (A,B) in the table below, whether A is **O**, **Ω**, or **Θ** of B. Assume that k and c are positive constants. You can mark each box with Y (yes) and N (no).

A	B	**O**	**Ω**	**Θ**
$(\lg n)^k$	Cn			
2^n	$2^{(n+1)}$			
$2^n n^k$	$2^n n^{2k}$			

3) 10 pts

Is an array that is in sorted order a min-heap? Explain your answer

4) 10 pts

Given the following nested loop

```
X = 0
For I=1 to n
        For J= 1 to O(n)
                X = X +1
        Endfor
Endfor
```

a- Can we conclude that at loop termination $X=O(n^2)$? If yes, explain why. If no, give counter example.

b- Can we conclude that at loop termination $O(n^2)$ is a tight upper bound on X? If yes, explain why. If no, give counter example.

5) 10 pts
Give an input instance where Dijsktra's algorithm will not solve the single source shortest path problem but the Bellman-Ford algorithm will.

6) 20 pts
   Design an efficeint algorithm to find a spanning tree for a connected weighted undirected graph $G=(V,E)$ such that the weight of the maximum-weight edge in the spanning tree is minimized. Prove its correctness and analyze its complexity.

7) 20 pts

The Subset Addition problem has inputs that include a target value M and weights W[1…n] for n objects. The problem asks whether there exists a subset S of objects with $S \subseteq \{1, 2, \ldots, n\}$, such that the sum over $j \in S$ of the weights W[j] is equal to exactly M. The output should be either true (if the answer is yes) or false (if it is no). Provide an O(nM)-time algorithm that solves this Subset Addition problem.

Additional Space

Additional Space