CS570

Analysis of Algorithms

Summer 2005

Midterm Exam

Name: ______________________

Student ID: __________________

Problem No.	Max. Points	Received
1	20	
2	15	
3	20	
4	15	
5	15	
6	15	
Total	100	

1) 20 pts
Consider the following two problems:
In P1 we are given as input a set of n squares (specified by their corner points) in the plane, and a number k. the problem is to determine whether there is any point in the plane that is covered by k or more squares.
In P2 we are given as input an n-vertex graph, and a number k; the problem is to determine whether there is a set of k mutually adjacent vertices. (E.g. for $k = 3$ we are looking for a triangle in the graph.)

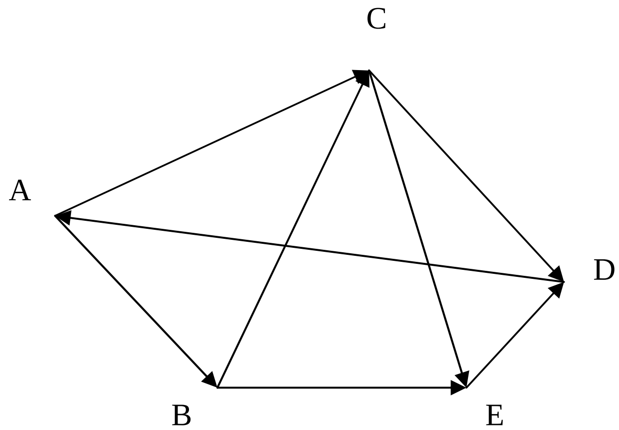
(a) Are P1 and P2 in NP?

(b) Show a reduction between P1 and P2.

(c) If P1 is NP-complete, would your reduction in (b) (and answer to (a)) imply that P2 is NP-complete?

(d) If P2 is NP-complete, would your reduction in (b) (and answer to (a)) imply that P1 is NP-complete?

2) 15 pts
Is there a feasible circulation in G?
- If yes, show the flow on each edge.
You need to show all your work.
- If no, which edge capacity (capacities) need to be increased and by how much before a feasible circulation can be achieved?



Graph G

Directed Edges	Lower Bound	Capacity
AC	5	15
AB	10	15
DA	5	25
BC	5	15
BE	0	10
CE	5	10
CD	0	15
ED	5	20

	Demand
A	10
B	20
C	-5
D	-15
E	-10

3) 20 pts
We are given an undirected unweighted graph G = (V,E), and specified two vertices s, t Є V . The length of a path in this graph is defined to be the number of edges in the path. Recall that the shortest path between two vertices is a path connecting them that has minimum length.

a- Create a linear programming solution to the problem of finding the shortest path from s to t.

b- Prove that the optimal solution of the linear program gives the optimal value of the shortest path.

c- Describe how to construct the shortest path using the solution to the linear program.

4) 15 pts

Let G=(V,E) be a flow network with source s, sink t, and integer capacities, Suppose that we are given a maximum flow in G.

a- suppose that the capacity of a single edge (u,v) Є E is increased by 1. Give an O(V+E)-time algorithm to update the maximum flow.

b- Suppose that the capacity of a single edge $(u,v) \in E$ is decreased by 1. Give an $O(V+E)$-time algorithm to update the maximum flow.

5) 15 pts

In a daring burglary, someone attempted to steal all the candy bars for the CS570 final. Luckily, he was quickly detected, and now, the course staff and students will have to keep him from escaping from campus. In order to do so, they can be deployed to monitor strategic routes.

More formally, we can think of the USC campus as a graph, in which the nodes are locations, and edges are pathways or corridors. One of the nodes (the instructor's office) is the burglar's starting point, and several nodes (the USC gates) are the escape points — if the burglar reaches any one of those, the candy bars will be gone forever. Students and staff can be placed to monitor the edges. As it is hard to hide that many candy bars, the burglar cannot pass by a monitored edge undetected.

Give an algorithm to compute the minimum number of students/staff needed to ensure that the burglar cannot reach any escape points undetected (you don't need to output the corresponding assignment for students — the number is enough). As input, the algorithm takes the graph $G = (V,E)$ representing the USC campus, the starting point s, and a set of escape points $P \subseteq V$. Prove that your algorithm is correct, and runs in polynomial time.

6) 15 pts

Let G = (V,E) be an undirected graph in which the vertices represent small towns and the edges represent roads between those towns. Each edge e has a positive integer weight d(e) associated with it, indicating the length of that road. The distance between two vertices (towns) in a graph is defined to be the length of the shortest weighted path between those two vertices.

Each vertex v also has a positive integer c(v) associated with it, indicating the cost to build a fire station in that town.

In addition, we are given two positive integer parameters D and C. Our objective is to determine whether there is a way to build fire stations such that the total cost of building the fire stations does not exceed C and the distance from any town to a fire station does not exceed D. This problem is known as the Rural Fire Station (RFS) Problem.

Your company has been hired by the American League of Rural Fire Departments to study this problem. After spending months trying unsuccessfully to find an efficient algorithm for the problem, your boss has a hunch that the problem is NP-complete. Prove that RFS is NP-complete.