CS570
Analysis of Algorithms
Fall 2014
Exam I

Name: _____

Student ID: _____

Wednesday Evening Section

	Maximum	Received
Problem 1	20	
Problem 2	16	
Problem 3	16	
Problem 4	16	
Problem 5	16	
Problem 6	16	
Total	100	

2 hr exam
Close book and notes

If a description to an algorithm is required please limit your description to within 150 words, anything beyond 150 words will not be considered.

1) 20 pts
   Mark the following statements as **TRUE** or **FALSE**. No need to provide any
   justification.

   [TRUE/FALSE]
   Let T be a complete binary tree with n nodes. Finding a path from the root of T to a
   given vertex $v \in T$ using breadth-first search takes $O(logn)$.

   [TRUE/FALSE]
   Dijkstra's algorithm works correctly on a directed acyclic graph even when there are
   negative-weight edges.

   [TRUE/FALSE]
   If the edge e is not part of any MST of G, then it must be the maximum weight edge
   on some cycle in G.

   [TRUE/FALSE]
   If $f(n) = O(g(n))$ and $g(n) = O(f(n))$ then $f(n) = g(n)$.

   [TRUE/FALSE]
   The following array is a max heap: [10; 3; 5; 1; 4; 2].

   [TRUE/FALSE]
   There are at least 2 distinct solutions to the stable matching problem: one that is
   preferred by men and one that is preferred by women.

   [TRUE/FALSE]
   In a binary max-heap with n elements, the time complexity of finding the second
   largest element is O(1).

   [TRUE/FALSE]
   Given a binary max-heap with n elements, the time complexity of finding the smallest
   element is O(lg n).

   [TRUE/FALSE]
   Kruskal's algorithm can fail in the presence of negative cost edges.

   [TRUE/FALSE]
   If a weighted undirected graph has two MSTs, then its vertex set can be partitioned
   into two, such that the minimum weight edge crossing the partition is not unique.

2) 16 pts

The diameter of a graph is the maximum of the shortest paths' lengths between all pairs of nodes in graph G. Design an algorithm which computes the diameter of a connected, undirected, unweighted graph in $O(mn)$ time, and explain why it has that runtime.

3) 16 pts

Consider a stable marriage problem where the set of men is given by $M = \{m_1, m_2, ..., m_N\}$ and the set of women is $W = \{w_1, w_2, ..., w_N\}$. Consider their preference lists to have the following properties:

$$\forall w_i \in W : w_i \text{ prefers } m_i \text{ over } m_j, \forall j > i$$
$$\forall m_i \in M : m_i \text{ prefers } w_i \text{ over } w_j, \forall j > i$$

Prove that a unique stable matching exists for this problem.

Note: symbol $\forall$ means "for all".

4) 16 pts

Ivan is a businessman and he has several large containers of fruits to ship. As he has only one ship he can transport only one container at a time and also it takes certain fixed amount of time per trip. As there are several varieties of fruits in the containers, the cost and depreciation factor associated with each container is different. He has n containers, a_1, a_2, ... , a_n. Initial value of each container is v_1, v_2, ... , v_n and depreciation factor of each container is d_1, d_2, ... , d_n. So, if container a_i happens to be the jth shipment, its value will be $v_i / (d_i \times j)$. Can you help Ivan maximize profit (value) by providing an efficient algorithm to ship the containers? Provide proof of correctness and state the complexity of your algorithm.

5) 16 pts

Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G. Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$. Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

6) 16 pts
Mark all the correct statements in the following:

a. Consider these two statements about a connected undirected graph with V vertices and E edges:
I. $O(V) = O(E)$
II. $O(E) = O(V^2)$

(a) I and II are both false.
(b) Only I is true.
(c) Only II is true.
(d) I and II are both true.

b. Suppose the shortest path from node i to node j goes through node k and that the cost of the subpath from i to k is D_{ik}. Consider these two statements:
I. Every shortest path from i to j must go through k.
II. Every shortest path from i to k has cost D_{ik}.

(a) I and II are both true.
(b) Only I is true.
(c) Only II is true.
(d) I and II are both false.

c. Consider the execution times of two algorithms I and II:
I. $O(n \log n)$
II. $O(\log(n^n))$

(a) Only I is polynomial.
(b) Only II is polynomial.
(c) I and II are both polynomial.
(d) Neither I nor II is polynomial.

d. Suppose $f(n) = 3n^3 + 2n^2 + n$. Consider these statements:
I. $f(n) = O(n^3)$
II. $f(n) = O(n^4)$

(a) Only I is true.
(b) Only II is true.
(c) I and II are both true.
(d) I and II are both false.

Additional Space