CSCI 570 Fall 2016: Discussion 1 Solutions

1. Yes, it's possible. Consider the following instance with *n*=3 men and women. Woman w3 will lie in this instance; the first six columns are true preference lists, and the final one is w3's false, but stated, preference list. For the sake of this example, let's assume that G-S breaks ties by using the lowest-numbered unmatched man to ask; similar examples exist for other tiebreakers.

m1	m2	m3	w1	w2	w3	w3'
w3	w1	w3	m1	m1	m2	m2
w1	w3	w1	m2	m2	m1	**m3**
w2	w2	w2	m3	m3	m3	**m1**

Initially, with the listed tie-breaker, Gale-Shapley will produce the pairs (m1,w3) (m2, w1) and (m3,w2). However, if w3's false preference list is used (and the other five remain truthful), we are left with (m1,w1), (m2,w3), and (m3,w2) -- leaving w3 with her truly first choice.

2. Recall that Gale-Shapley matches each man with his top-preference among valid partners, and each woman with her bottom-preference. For a consensus-optimal matching to exist, each woman would also have to be matched with her top-preference. As such, the stable matching must be unique (since, for each woman, her top valid choice and bottom valid choice must be the same).

Run Gale-Shapley with the input and store the solution. Then run it again, but with the genders reversed (women ask men, or reverse the inputs). If the solutions are the same, we have a consensus-optimal stable matching. If they aren't, we don't.

3. Yes, unless you consider switching the two colors to be a different bipartition (it shouldn't be). Because the graph is connected, changing any one vertex's color would require each of its neighbors to be changed, and so on in this fashion, until we end up with just a pallet swap.

Note that if the graph *isn't* connected, swapping colors in one or more connected components, and leaving one or more unchanged, *does* result in a different bipartition.