Online Sources of Information

External Class Website:

- Contact informations
- Syllabus
- office hours
- exam dates

www-bcf.usc.edu/~adamchik/csci570.html

DEN Website:

- lecture videos
- lecture notes
- HW assignments
- any other documents

Roles and Responsibilities

- Instructors	lectures & discussions
- TAs	HWs & Exams
- Course producers	?
- Graders	HWs
- CS Dept advisers	registration issues
- DEN	DEN issues

Text books

- Algorithm Design by Jon Kleinberg & Eva Tardos
- Supplemental textbook: Introduction to Algorithms, 3rd edition by Cormen

Your Responsibilities

- Attend lectures and discussion sessions
- study the material from textbook
- Do HW problems
- Do as many other problems from the textbook as possible

Your grade

Exam 1	30%	Sept 30
Exam 2	30%	Nov 4
Exam 3	40%	Dec 2

Prerequisites

- Discrete Math - Mathematical Induction
- Asymptotic notation
- Sorting methods
- Basic data structures: Arrays, Stacks, queues, linked lists
- Basics of graphs: Trees, cycles, DAG, adjacency list, adjacency matrix, etc.
- Graph search alg's: BFS, DFS

1- An alg. is a set of instructions in machine language.

Kharazmi 780-850

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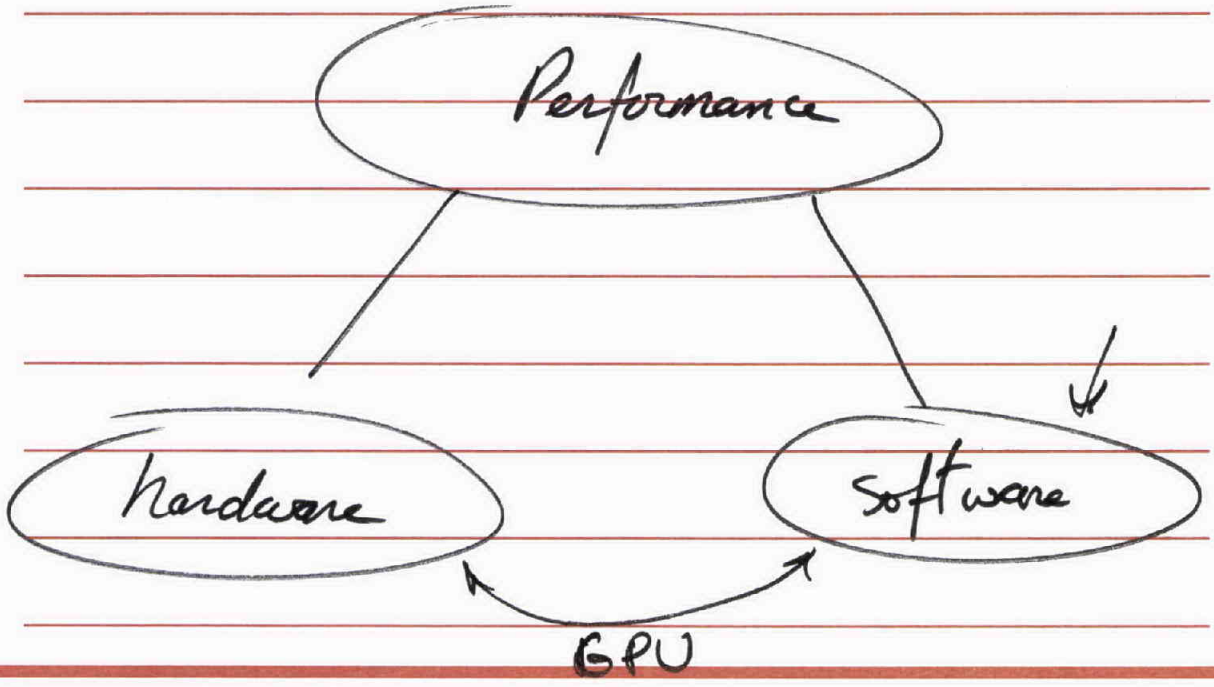
Algorithm

2- Alg. science advanced on Wall St.

3- Invite 6 million Alg's for a listen

1- Correctness

2- Performance



GPU

SMP

DMP

1- Come up w/ problem statement

2- Present a solution

3- Proof of Correctness

4- Analyze complexity

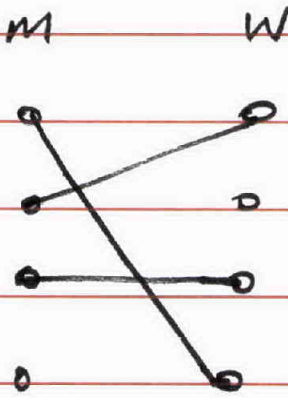
Stable Matching

Problem We are interested in matching n men with n women so that they could stay happily married ever after.

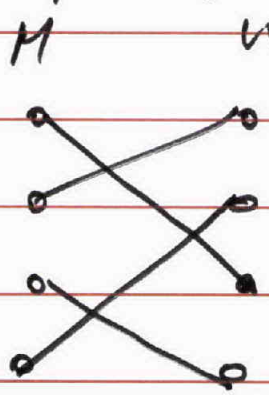
We have a set of n men $M = \{m_1, \ldots m_n\}$

" " " " n women $W = \{w_1 - \ldots w_n\}$

Def. A *matching* S is a set of ordered pairs.



Def. A *perfect matching* S' is a matching with the property that each member of M and each member of W appear in exactly one pair in S'



Add notion of preferences

Each man $m \in M$ ranks all women

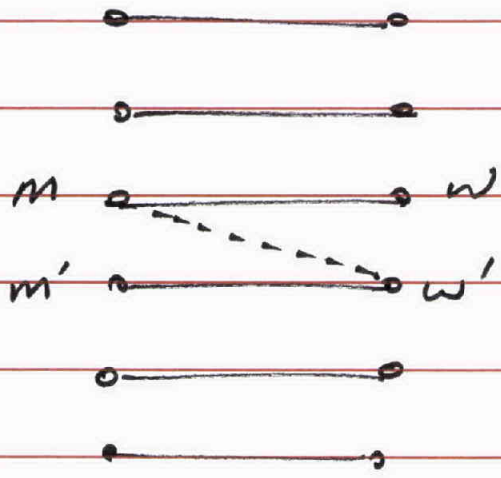
- m prefers w to w' if m ranks w higher than w'.
- ordered ranking of m is his preference list

$$P_{m_i} = \{ w_{i_1}, w_{i_2}, \ldots w_{i_n} \}$$

Same for women, i.e. each woman $w \in W$ ranks all men ...

Stable Matching

S



Def. such a pair (m, w') is an instability WRT S.

Def. Matching S is stable if

1- It is perfect

2- There are no instabilities WRT S

Step 1: Input: Preference lists for a set of n men & n women.

Output: Set of n marriages w/ no instabilities

Step 2 Gale-Shapley Alg.

Step 3 Proof of Correctness

(1) From the woman's perspective, she starts single, and once she gets engaged she can only get into better engagements

(2) From the man's perspective, he starts single, gets engaged, and might be dropped repeatedly only to settle for a woman w/ lower rankings

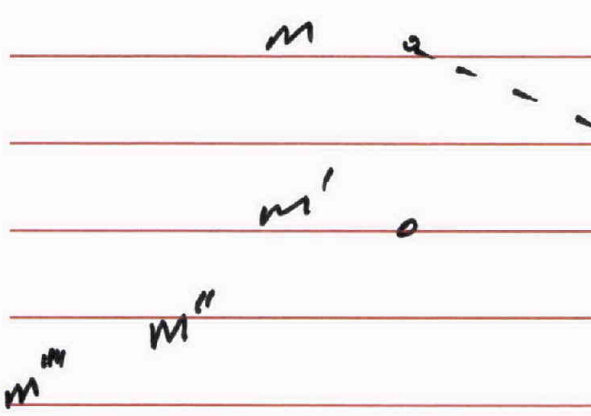
(3) Alg. terminates after n^2 iterations

(4) Solution is a perfect matching

(5) Solution is a stable matching

Proof by contradiction

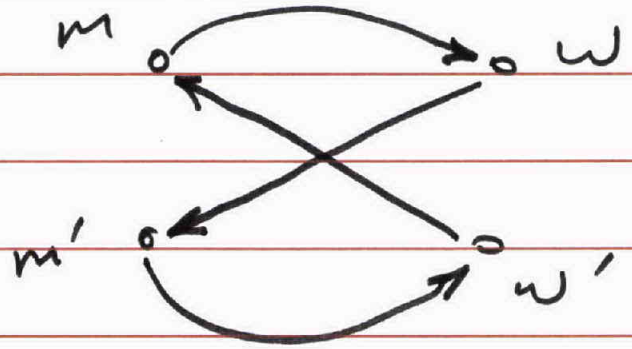
Assume instability exists in our solution involving two pairs $(m, w), (m', w')$



Q. Did $\underline{m}$ propose to w' at some point in the execution?

If no, then w must be higher than w' on his list $\Rightarrow$ Contradiction!

If yes, he must have been rejected in favor of m'' and due to ① either $m'' = m'$ or m' is better than m'' $\Rightarrow$ Contradiction!



$(m, w), (m', w')$

$(m, w'), (m', w)$

Step 4 Complexity Analysis

1- Identify a free man. O(1)

2- For a man m, identify the highest ranked woman to whom he has not yet proposed.

3- For a woman w, decide if w is engaged, and if so to whom.

4- For a woman w and two men m & m', decide which man is preferred by w.

5- Place a man back in the list of free men. O(1)

3- Keep an array called Current[1..n]

Current[w] is Null if w is not engaged, and set to m if she is engaged to m.

$O(1)$

4-

$manPref_i$ = | 3 | 8 | (1) | 22 | . | . | . .

Create a Ranking array where Ranking[w, m] contains the rank of man m based on w's preference.

This construction takes $O(n^2)$

1 - Could use an array
or a linked list
" " stack
" " queue
steps 1 & 5 will take $O(1)$

2 - Keep an array Next[1..n]
Next[m] points to the position of the next woman he will be proposing to on his ranked list

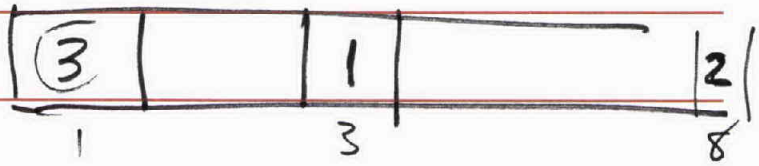
ManPref[1..n, 1..n]

ManPref[m, i] denotes the i^{th} woman on man m's pref. list

w = ManPref(m, Next[m])

takes $O(1)$

Rank list



takes $O(n^2)$ to create this array

Overall cost of Gale-Shapley

$\underbrace{O(n^2)}_{\text{preparation}} + O(n^2) = O(n^2)$