CS570

Analysis of Algorithms

Summer 2005

Midterm Exam

Name: _____

Student ID: _____

1) 20 pts
Binomial min-heap H1 contains elements {12, 7, 25, 15, 28, 41, 33}, and binomial min-heap H2 contains elements {18, 3, 37, 6, 8, 30, 45, 55, 32, 23, 24, 22, 29, 48, 50, 31, 10, 17, 44}.
a) Show heaps H1 and H2 (Note: there could be more than one possible correct construction of the heap)

b) Merge H1 and H2 into a new Binomial min-heap H. Show all you work.

2) 20 pts

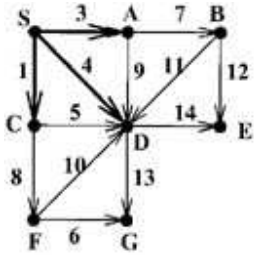
Suppose that you are playing a very simple arcade game. You start at the left end of the screen (position 0), and are supposed to make it to the right end (position k). In each step, you can jump between 1- 9 positions to the right (but never to the left). The problem is that some of those positions contain harmful things, like fires, monsters, rotating knives, CS 570 midterms, etc. When you touch such a position, you lose some of your “life energy".

More formally, when you touch position i (with $0 \leq i \leq k$), you lose $x_i \geq 0$ points of energy (notice that you don't lose any energy if you jump over a position). Now you are supposed to find a sequence of jumps that gets you from 0 to k while losing the smallest total amount of energy.

Give an algorithm to compute, in polynomial time, the smallest total amount of energy you can lose going from 0 to k. Analyze the running time of your algorithm.

3) 20 pts
We are running one of these three algorithms on the graph below, where the algorithm has already "processed" the bold-face edges (SA, SD, and SC). (Ignore the directions on the edges for Prim's and Kruskal's algorithms.)

- Prim's for the minimum spanning tree, starting from S.
- Kruskal's for the minimum spanning tree.
- Dijkstra's for shortest path from S.



a) Which two edges would be added next in Prim's algorithm? Be sure to indicate the order in which they are added.

b) Which two edges would be added next in Kruskal's algorithm? Be sure to indicate the order in which they are added.

c) At this point in the running of Dijkstra's algorithm, S has been taken off the top of the heap and marked. Which four vertices would be marked next in Dijkstra's algorithm, i.e. deleted from the top of the heap? Be sure to indicate the order in which they are deleted. Which final edges would Dijkstra's algorithm choose as part of the shortest path to these vertices (i.e. which edge connects to this vertex as part of the shortest path from S)?

4) 20 pts

Consider the Change Problem in Binaryland The input to this problem is an integer L. The output should be the minimum cardinality collection of coins required to make L nibbles of change (that is, you want to use as few coins as possible).

In Binaryland the coins are worth $1, 2, 2^2, 2^3, \ldots, 2^{1000}$ nibbles. Assume that you have an unlimited number of coins of each type. Prove or disprove that the greedy algorithm (that takes as many coins of the highest value as possible) solves the change problem in Binaryland.

5) 20 pts

You have successfully implemented an algorithm using divide and conquer to find the k^{th} smallest term in an unsorted array of n elements. Your algorithm has a divide step that takes O(n) time during which it divides the problem into 4 equal pieces. It then forms 3 sub-problems that it solves recursively. The sub-problems are then combined in O(lg n).

Determine if this algorithm has a better complexity than the straight-forward method of first sorting the array and then finding the k^{th} smallest term in the sorted array. Show all your work.

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