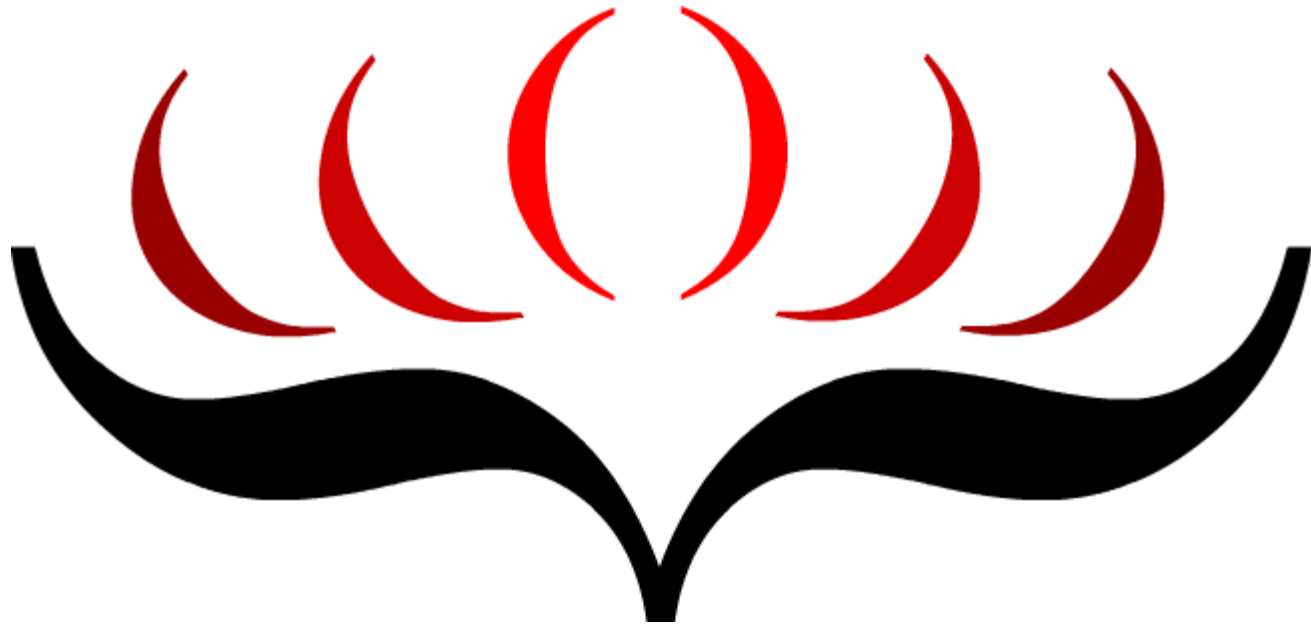


Lazy evaluation and currying in Haskell



Agenda

- Lazy programming
 - Lazy evaluation
 - Infinite lists
 - Infinite loops
- Currying and uncurrying

Lazy evaluation





Lazy evaluation



Questions



Lazy evaluation

- No expression is evaluated until its value is needed.
- No shared expression is evaluated more than **once**; if the expression is ever evaluated then the result is shared between all those places in which it is used.

Non-strict & strict

- Lazy functions are also called **non-strict** and evaluate their arguments **lazily** or **by need**.
- C# and Java methods are **strict** and evaluate their arguments eagerly.

Passing arguments to a function

- By value
- By reference
- By name
- By need

listFrom

```
{- infinity list
--
-- listFrom 10
-- 10 : listFrom 11
-- 10 : 11 : listFrom 12
-- ...
-}
listFrom :: Integer -> [Integer]
listFrom n = n : (listFrom (n + 1))
```

repeat

```
{- repeat value
--
-- repeat 1
-- 1 : repeat 1
-- 1 : 1 : repeat 1
-- ...
-}
repeat    :: a -> [a]
repeat e = ls where ls = e : ls
```

cycle

```
{- cycle value
--
-- cycle "123"
-- "123" : cycle "123"
-- "123" : "123" : cycle "123"
-}
cycle    :: [a] -> [a]
cycle xs = ls where ls = xs ++ ls
```

sumFirst

```
{- sum of list
--
-- sumFirst 0 [1, 2, 3]
-- 0
--
-- sumFirst 3 [1, 2, 3]
-- 1 + sumFirst (3-1) [2, 3]
-- 1 + 2 + sumFirst (3-1-1) [3]
-- 1 + 2 + 3 + sumFirst 0 []
-- 6
-}
sumFirst :: Integer -> [Integer] -> Integer
sumFirst 0 _ = 0
sumFirst n (x:xs) = x + sumFirst (n-1) xs
```


iterate

```
{-  
--  
-- iterate (+1) 2  
-- 2 : iterate (+1) ((+1) 2)  
-- 2 : 3 : iterate (+1) ((+1) 3)  
-- ...  
-}  
iterate :: (a -> a) -> a -> [a]  
iterate f e = e : iterate f (f e)
```

diff

```
-- take 10 (diff [x*x | x <- [1..]])  
-- [3,5,7,9,11,13,15,17,19,21]  
  
-- take 10 (filter (\x -> mod x 5 == 0) (diff [x*x | x <- [1..]]))  
-- [5,15,25,35,45,55,65,75,85,95]  
  
-- take 10 (filter (\x -> mod x 115 == 0) (diff [x*x | x <- [1..]]))  
-- [115,345,575,805,1035,1265,1495,1725,1955,2185]  
diff :: (Num a) => [a] -> [a]  
diff (x:xs@(y:ys)) = y-x : diff xs
```

Sieve of Eratosthenes

| | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

Prime numbers

primes

```
{- prime numbers
--
-- primes
-- sieve [2..]
-- 2 : (sieve [x | x <- [3..], mod x 2 /= 0])
-}
primes :: [Integer]
primes = sieve [2..] where
    sieve (e:ls) = e : (sieve [x | x <- ls, mod x e /= 0])
```

pairs

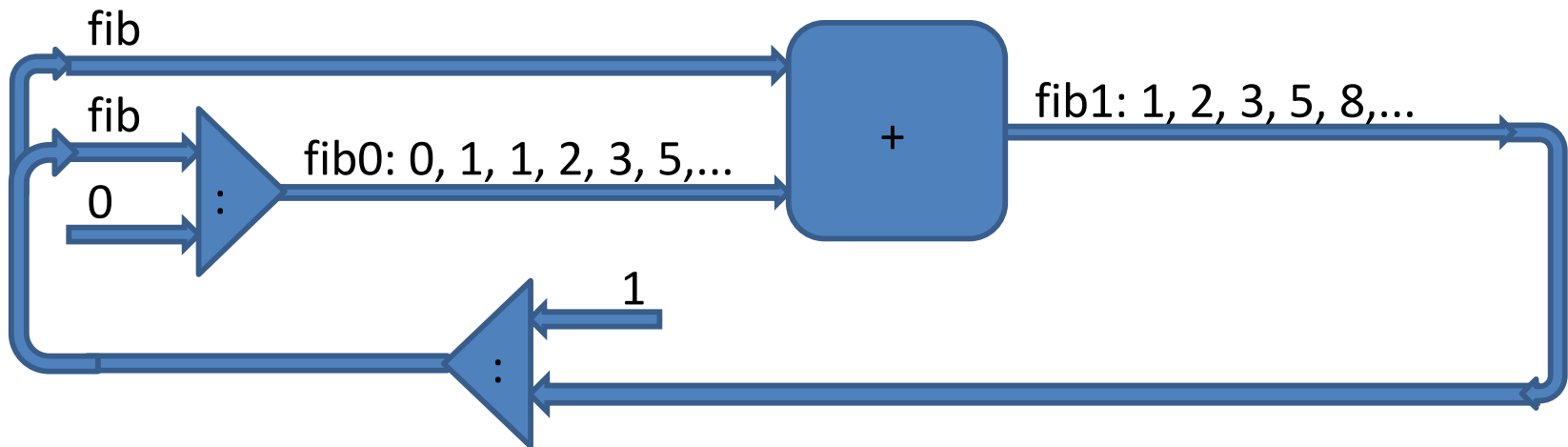
```
{-  
--  
-- take 3 (pairs primes)  
-- [(2,3),(3,5),(5,7)]  
-}  
pairs :: [Integer] -> [(Integer, Integer)]  
pairs (x:xs@(y:ys)) = (x,y):pairs xs
```


twins

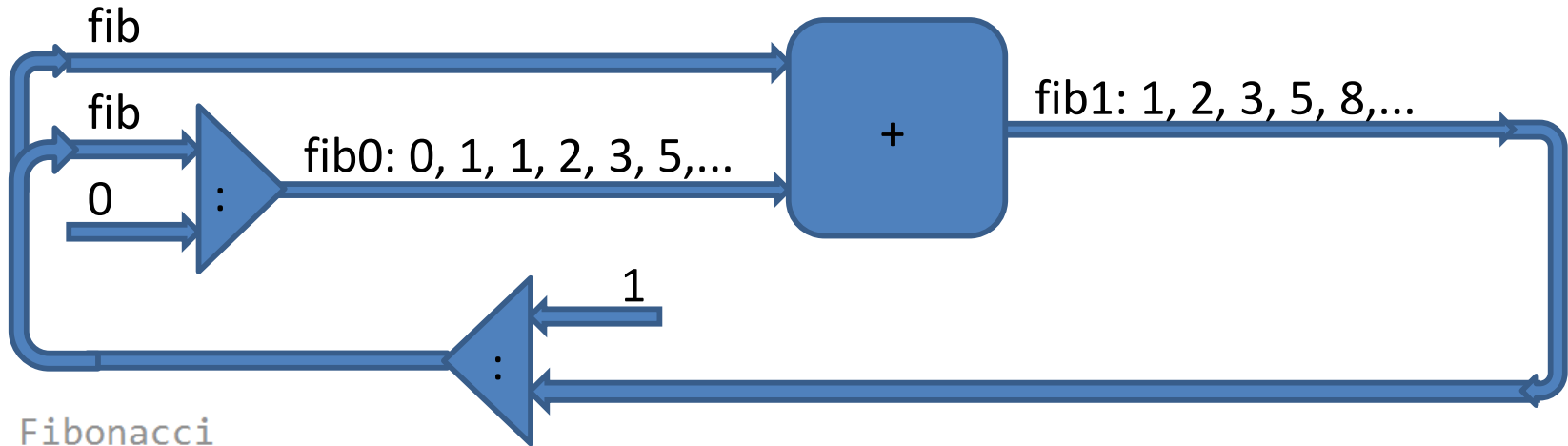
```
{-  
--  
-- take 5 twins  
-- [(3,5),(5,7),(11,13),(17,19),(29,31)]  
-}  
twins :: [(Integer, Integer)]  
twins = [(x,y) | (x,y) <- pairs primes, y - x == 2]
```

Fibonacci

fib: 1, 1, 2, 3, 5, 8,...



Fibonacci



```
{- Fibonacci
--
-- fib
-- 1 : fib1
-- 1 : zipWith (+) fib fib0
-- 1 : zipWith (+) (1:fib1) (0:fib)
-- 1 : zipWith (+) (1:(zipWith (+) fib fib0)) (0:1:fib1)
-- 1 : zipWith (+) (1:(zipWith (+) fib fib0)) (0:1:(zipWith (+) fib fib0))
-}
fib0 = 0:fib
fib1 = zipWith (+) fib fib0
fib  = 1:fib1
```

Currying

It is the process of transforming a function that takes multiple arguments into a function that takes just a single argument and returns another function if any arguments are still needed.

raiseList

```
{-  
-- raiseList [1, 2, 3]  
-- [2, 3, 4]  
--  
-}  
raiseList :: (Num a) => [a] -> [a]  
raiseList ls = map (+1) ls
```


raiseList

```
{-  
-- raiseList [1, 2, 3]  
-- [2, 3, 4]  
--  
-}  
raiseList :: (Num a) => [a] -> [a]  
raiseList is = map (+1) is
```

raiseList

```
{-  
-- raiseList [1, 2, 3]  
-- [2, 3, 4]  
-- map (+1) is -- \ls -> map (+1) ls  
-}  
raiseList :: (Num a) => [a] -> [a]  
raiseList    = map (+1)
```

plus

```
plus      :: Integer -> Integer -> Integer  
plus x y = x + y
```

```
plus      :: Integer -> Integer -> Integer  
plus x y = \x y -> x + y
```

```
plus      :: Integer -> (Integer -> Integer)  
plus x = \y -> x + y
```

mult & plus1

```
{-  
-- mult 3 5  
-- 15  
-}  
mult :: Integer -> Integer -> Integer  
mult  x y = x * y
```

```
{-  
-- plus1 3  
-- 4  
-}  
plus1 :: Integer -> Integer  
plus1 x = 1 + x
```

```
{-  
-- plus1AndMult 3 5  
-- 16  
-}  
plus1AndMult      :: Integer -> Integer -> Integer  
plus1AndMult x y = plus1 (mult x y)  
  
plus1AndMultShort :: Integer -> Integer -> Integer  
plus1AndMultShort x = plus1 . (mult x)  
--plus1AndMultShort x = \y -> plus1 (mult x y)
```


Composition

```
-- | Function composition.  
-- Make sure it has TWO args only on the left, so that it inlines  
-- when applied to two functions, even if there is no final argument  
(.) :: (b -> c) -> (a -> b) -> a -> c  
(.) f g = \x -> f (g x)  
  
-- f . g = \y -> f (g y)  
-- plus1 . (mult x) = \y -> plus1 (mult x y)
```

curried & uncurried

```
{-  
-- is the curried form  
-}  
plusFirst      :: Integer -> Integer -> Integer  
plusFirst x y = x + y  
  
{-  
-- is the uncurried form  
-}  
plusSecond     :: (Integer -> Integer) -> Integer  
plusSecond (x, y) = x + y  
  
curry plusSecond == plusFirst  
uncurry plusFirst == plusSecond
```

Partial application

```
{- Partial application
-}
(+) :: (Num a) => a -> a -> a
(+) 2 3 -> 5
(+) 2 -> \x -> 2 + x
(5 +)

{-
-- (+1) 1 :: map (+1) [2, 3]
-- 2 :: (+1) 2 :: map (+1) [3]
-}
map (+1) [1, 2, 3]
```

checkedList

```
{-  
-- map (== 2) [1, 2]  
-- 1 == 2 : map (==2) [2]  
-- 1 == 2 :: 2 == 2  
-- [False, True]  
-}  
checkedList :: (Eq a) => a -> [a] -> [Bool]  
checkedList e ls = map (== e) ls
```

anyTrue

```
{-  
-- foldr (||) False [False, True]  
-- False || (foldr (||) False [True])  
-- False || (True || False)  
-- False || True  
-- True  
-}  
anyTrue :: [Bool] -> Bool  
anyTrue ls = foldr (||) False ls
```

searchList

```
searchList :: (Eq a) => a -> [a] -> Bool
searchList e ls = anyTrue (checkedList e ls)
```

```
searchList2 :: (Eq a) => a -> [a] -> Bool
searchList2 e ls = foldr (||) False (map (== e) ls)
```

```
{-
-- searchList3 3 [1, 2, 3, 4, 5]
-- (foldr (||) False) . (map (== 3) [1, 2, 3, 4, 5])
-- foldr (||) False [1 == 3, 2 == 3, 3 == 3, 4 == 3, 5 == 3]
-- foldr (||) False [False, False, True, False, False]
-- True
-}
```

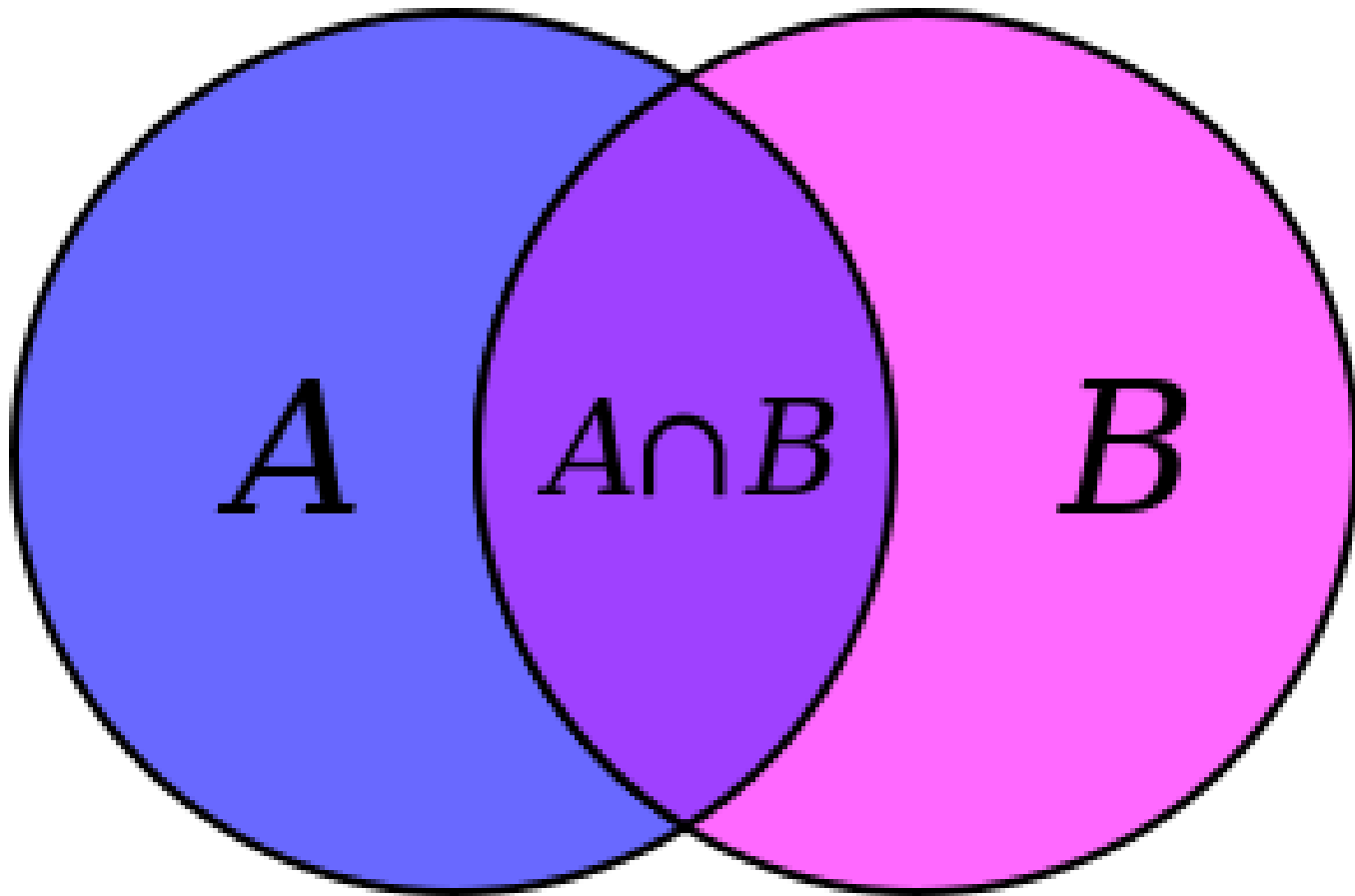
```
searchList3 :: (Eq a) => a -> [a] -> Bool
searchList3 e = (foldr (||) False) . (map (== e))
```

Partial application

```
{- Partial application
-}
(+) :: (Num a) => a -> a -> a
(+) 2 3 -> 15
(+) 2 -> \x -> 2 + x
(5 +)

{-
-- (+1) 1 :: map (+1) [2, 3]
-- 2 :: (+1) 2 :: map (+1) [3]
-}
map (+1) [1, 2, 3]
```

The math set



IntSet & empty

```
type IntSet = (Integer -> Bool)
empty      :: IntSet
empty e = False
```

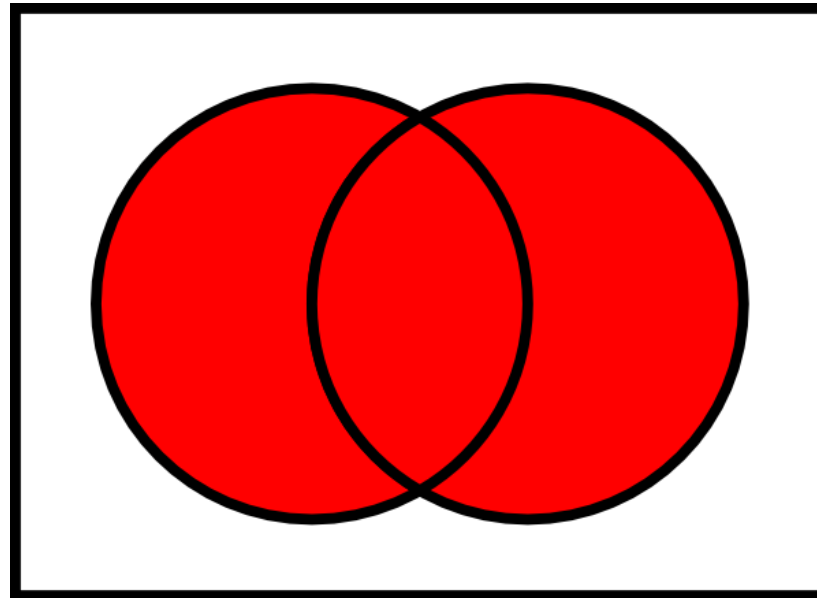
from2to100

```
{- (e > 1) && (e < 101)
--
-- from2to100 1
-- False
--
-- from2to100 100
-- True
-}
from2to100 e = (e >= 2) && (e <= 100)
```

odds

```
{- The set of odd numbers
--
-- odds 39129
-- True
--
-- odds 3290
-- False
-}
odds    :: IntSet
odds e = mod e 2 == 1
```

Unions

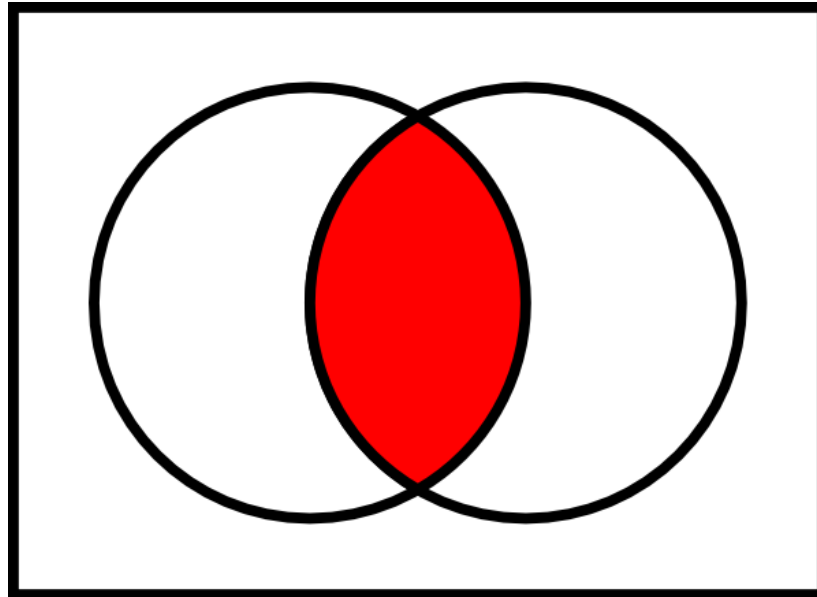


The **union** of A and B ,
denoted $A \cup B$.

union

```
{- Unions
-- Two sets can be "added" together.
-- The union of A and B, denoted by  $A \cup B$ ,
-- is the set of all things that are members of either A or B.
-}
union      :: IntSet -> IntSet -> IntSet
union s0 s1 e = (s0 e) || (s1 e)
```

Intersections

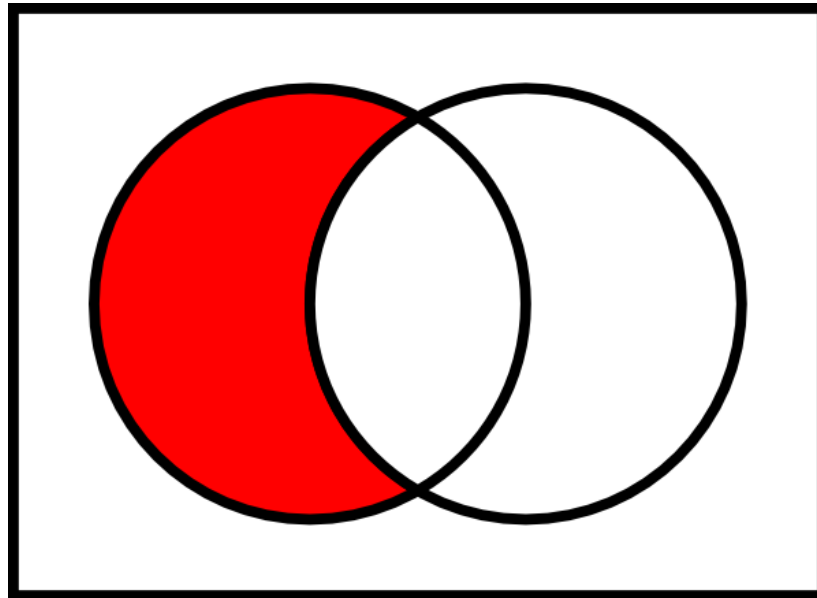


The **intersection** of A
and B , denoted $A \cap B$.

intersec

```
{- Intersections
--
-- A new set can also be constructed by determining
-- The intersection of A and B, denoted by  $A \cap B$ , :
--
-- intersec empty empty is empty
-- intersec odds from2to100 is [3,5..99]
-}
intersec :: IntSet -> IntSet -> IntSet
(intersec s0 s1) e = (s0 e) && (s1 e)
```

Complements



The **relative complement** of B in A .

compl

```
{- Complements
--
-- Two sets can also be "subtracted".
-- The relative complement of B in A
-- denoted by  $A \setminus B$  (or  $A - B$ ),
-- is the set of all elements that are members of A but not members of B.
--
-- compl odds from2to100 is [1, 101, 103, 105 ...]
-}
compl :: IntSet -> IntSet -> IntSet
(compl s0 s1) e = (s0 e) && False == (s1 e)
```

addElem

```
{- add new element
--
-- addElem 1 empty
-- \e -> (1 == e) || (empty e)
-- \e -> (1 == e) || False
-}
addElem :: Integer -> IntSet -> IntSet
addElem new s e = (new == e) || (s e)
```

remElem

```
{- remove element
--
-- remElem 1 empty
-- \e -> (1 /= e) && (empty e)
-- \e -> (1 /= e) && False
--
-- remElem 1 (addElem 1 empty)
-- \e -> (1 /= e) && (addElem 1 empty)
-- \e -> (1 /= e) && ((1 == e) || (empty e))
-- \e -> (1 /= e) && ((1 == e) || False)
--
-}
remElem :: Integer -> IntSet -> IntSet
remElem a s e = (e /= a) && (s e)
```

Resources

- **Real World Haskell** by *Bryan O'Sullivan, Don Stewart, John Goerzen*.
- **Learn You a Haskell for Great Good!** by *Miran Lipovača*.
- **Haskell: The Craft of Functional Programming** by *Simon Thompson*.