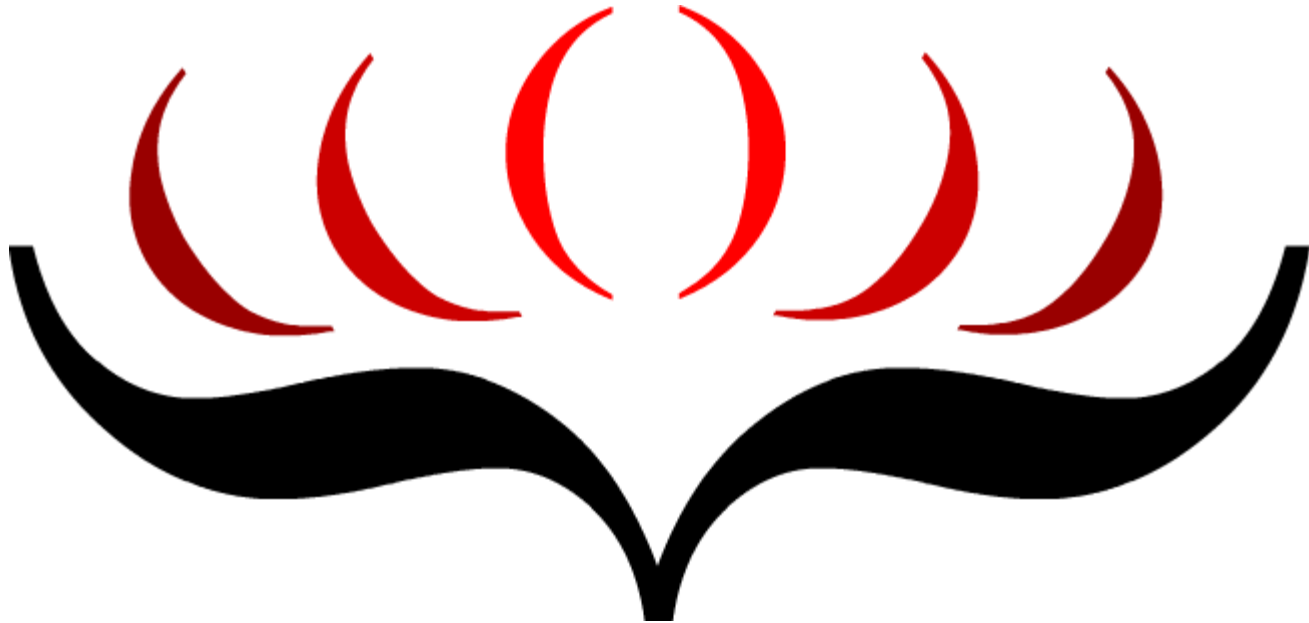


Intro to monads



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Agenda

- What is a monad?
- Functions: ($>>==$), ($>>$), **return** and **fail**
- Monad laws
- **do** notation
- Monads: **Maybe**, **list**

Haskell

It is an advanced
purely-functional
programming language.

Haskell

It is an advanced
purely-functional
programming language.

Purity

That means you **cannot mutate** variables or introduce „**side effects**“ anywhere in your program (short of I/O).

This is **good** and **bad**.

Laboratory is cool, but...



Real life it isn't a laboratory!



Impure computation

- May do file or terminal **input/output**.
- May raise **exceptions**.
- May read or write **shared state** (global or local).
- May sometimes fail to produce any results.
- ...

„Pure” pill or „impure” pill?



„Pure” pill or „impure” pill?

- Pure + Pure = Pure
- Pure + Impure = Impure
- Impure + Impure = Impure

Pure

```
val = 10 :: Integer  
resDouble = sqrt (fromIntegral val)  
resInteger = round (sqrt (fromIntegral val))
```

Pure

```
val = 10 :: Integer  
resDouble = sqrt $ fromIntegral val  
resInteger = round $ sqrt $ fromIntegral val
```


Pure

```
val = 10 :: Integer  
resDouble = sqrt ($) fromIntegral val  
resInteger = round ($) sqrt ($) fromIntegral val
```

Pure

Function composition

```
sqrtInteger = round . sqrt . fromIntegral  
resInteger' = sqrtInteger val
```

Pure

Function composition

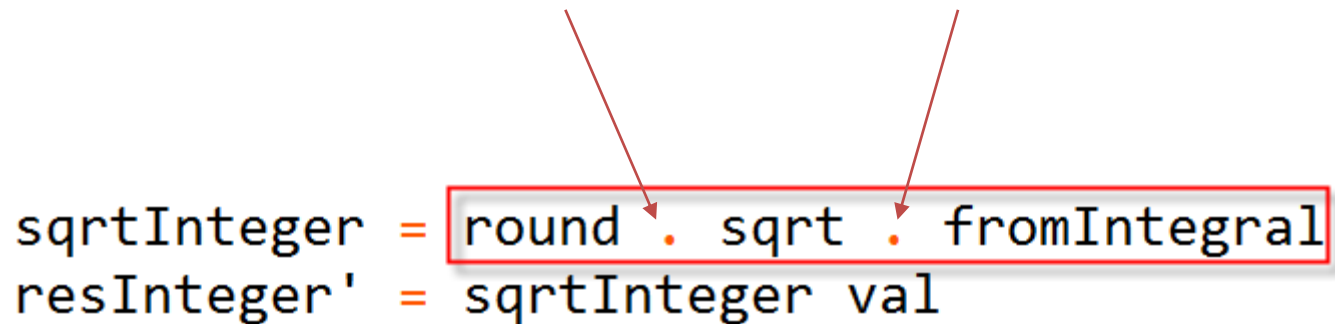
```
sqrtInteger = round . sqrt . fromIntegral  
resInteger' = sqrtInteger val
```

Pure

Function composition

$(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$

`sqrtInteger = round . sqrt . fromIntegral`
`resInteger' = sqrtInteger val`



Pure

Function composition

Integer

- **fromIntegral** :: (Integral a, Num b) => a -> b

Double

- **sqrt** :: Floating a => a -> a

Integer

- **round** :: (Integral b, RealFrac a) => a -> b

Impure

Monadic functions, monadic values

$f :: a \rightarrow [something\ else] b$

- $f :: a \rightarrow [IO] b$
- $f :: a \rightarrow [error] b$
- $f :: a \rightarrow [State\ s] b$
- $f :: a \rightarrow [Maybe] b$

Impure

Monadic functions, monadic values

$f :: a \text{ } \textit{IO} \rightarrow b$

or

$f :: a \rightarrow \textit{IO} \ b \quad \longleftarrow \text{ Haskell}$

Monadic functions

They're represented as
pure functions with a *funky*
„monadic return type“.

Monadic functions

$x :: a$

x has type a

$f\ x :: m\ b$

$f\ x$ is a „monadic value“

$g :: a \rightarrow () \rightarrow a$

$g\ x\ () = x$

$h = g\ 10$

$h\ ()$ -- this will evaluate to 10

The unit type and value are both written as `()` in Haskell.

So what is **$g (f x)$** ?

$f x :: m b$

$g :: a \rightarrow () \rightarrow a$

$g (f x) :: () \rightarrow m b$

It's a function which takes a unit value as its argument and returns a monadic value.

IO

- **getLine** :: () -> *String* -- not in Haskell
- **putStrLn** :: *String* -> () -- not in Haskell

If we removed the ()s from the type signatures

- **getLine** :: *String*
- **putStrLn** :: *String*

IO

getLine :: *IO String*

It is a monadic value.

putStrLn :: *String -> IO ()*

It is just a monadic function which happens to be in the IO monad.

Two monadic functions

$f :: a \rightarrow m\ b$

$g :: b \rightarrow m\ c$

$f :: a \rightarrow IO\ b$

$g :: b \rightarrow IO\ c$

$h :: a \rightarrow IO\ c$

Compose monadic functions

```
f :: a -> IO b  
g :: b -> IO c  
h :: a -> IO c
```



It doesn't work!

$h = g \cdot f$

$(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$

We are looking for solution

$\text{val} :: a$

$\text{fVal} = \text{f val} :: \text{IO } b$

$\text{extVal} = \text{extract fVal} :: b$

$\text{g extVal} :: c$

But it's not **IO c**.



We are looking for solution



val :: *a*

fVal = **f** **val** :: *IO b*

mapply **fVal** **g** :: *IO c*

Nice 😊

mapply :: *m b -> (b -> m c) -> m c*

Bind

$(\gg=) :: m\ a \rightarrow (a \rightarrow m\ b) \rightarrow m\ b$

$(\gg=) :: \text{Maybe}\ a \rightarrow (a \rightarrow \text{Maybe}\ b) \rightarrow \text{Maybe}\ b$

$(\gg=) :: \text{IO}\ a \rightarrow (a \rightarrow \text{IO}\ b) \rightarrow \text{IO}\ b$

$(\gg=) :: \text{State}\ s\ a \rightarrow (a \rightarrow \text{State}\ s\ b) \rightarrow \text{State}\ s\ b$

Compose monadic functions

$$f :: a \rightarrow IO\ b$$
$$g :: b \rightarrow IO\ c$$
$$h :: a \rightarrow IO\ c$$
$$h\ x = f\ x\ \>\>=\ g$$
$$h = \backslash x \rightarrow f\ x\ \>\>=\ g$$

$(\>\>=) :: Monad\ m \Rightarrow m\ a \rightarrow (a \rightarrow m\ b) \rightarrow m\ b$

bind & compose monadic functions

$(\gg) :: m\ a \rightarrow m\ b \rightarrow m\ b$

$(\gg) f\ g = f\ x\ \gg = g$

Compose monadic functions

$$f :: a \rightarrow IO\ b$$
$$g :: b \rightarrow IO\ c$$
$$h :: a \rightarrow IO\ c$$
$$h = g \gg f$$
$$(>>) :: Monad\ m \Rightarrow m\ a \rightarrow m\ b \rightarrow m\ b$$

Value apply to monadic function

$f :: a \rightarrow m\ b$
 $g :: b \rightarrow c$

$g \gg f$

It doesn't work!

return $g \gg f$


$\text{return} :: a \rightarrow m\ c$

It works 😊.

Monad

```
class Monad m where
    (>>=) :: m a -> (a -> m b) -> m b
    (>>)  :: m a -> m b -> m b
    return :: a -> m a
    fail   :: String -> m a
```


Monad

```
class Monad m where
  ( $\gg=$ )  :: m a -> (a -> m b) -> m b
  ( $\gg$ )  :: m a -> m b -> m b
  return :: a -> m a
  fail   :: String -> m a
```

Monad laws

- Left identity
- Right identity
- Associativity

Monad laws

The nice version

$\text{return } \Rightarrow f == f$

$f \Rightarrow \text{return} == f$

$(f \Rightarrow g) \Rightarrow h == f \Rightarrow (g \Rightarrow h)$

$(\Rightarrow) :: (a \rightarrow m b) \rightarrow (b \rightarrow m c) \rightarrow (a \rightarrow m c)$

Monad laws

The ugly (formal) version

$$\text{return } x \gg= f == f x$$

$$m \gg= \text{return} == m$$

$$(m \gg= f) \gg= g == m \gg= (\backslash x \rightarrow (f x \gg= g))$$

$$(\gg=) :: m a \rightarrow (a \rightarrow m b) \rightarrow m b$$

The Maybe monad

```
data Maybe a = Nothing | Just a
```

```
data Maybe Int = Nothing | Just Int
```

Attempt to define Maybe

```
instance Monad Maybe where  
  (>>=) = {- definition of >>= for Maybe -}  
  return = {- definition of return for Maybe -}
```

The complete definition of the Maybe

```
instance Monad Maybe where
    return x = Just x

    Nothing >>= f = Nothing
    Just x >>= f = f x
```

do notation

```
main :: IO ()  
main = do  
    putStrLn "What's your name?"  
    name <- getLine  
    putStrLn $ "Hello, " ++ name ++ "!"
```


do notation

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main :: IO ()  
main = do  
    putStrLn "What's your name?"  
    name <- getLine  
    putStrLn $ "Hello, " ++ name ++ "!"
```



```
main :: IO ()  
main =  
    putStrLn "What's your name?" >>  
    getLine >>=  
    \name -> putStrLn $ "Hello, " ++ name ++ "!"
```

do notation

```
main :: IO ()  
main = do  
    putStrLn "What's your name?"  
    name <- getLine  
    putStrLn $ "Hello, " ++ name ++ "!"
```



```
main :: IO ()  
main =  
    putStrLn "What's your name?" >>  
    getLine >>=  
    \name -> putStrLn $ "Hello, " ++ name ++ "!"
```

The list monad

f :: a -> [b]

f :: a -> m b

data List a = Nil / Cons a (List a)

f :: a -> List b

Using do-notation

```
run :: [(Integer, Integer)]
run =
  do
    x <- [1..6]
    y <- [1..6]
    if (x + y) == 7
      then return(x, y)
      else []
-- Result: [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]
```

Without the do-notation

```
run1 :: [(Integer, Integer)]
run1 =
    [1..6] >>= \x ->
        [1..6] >>= \y ->
            if (x+y) == 7
            then return (x, y)
            else []
-- Result: [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]
```

List comprehension

```
run2 :: [(Integer, Integer)]
run2 = [(x, y) | x <- [1..6], y <- [1..6], x + y == 7]
-- Result: [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]
```

Resources

- <http://mvanier.livejournal.com>
- <https://www.fpccomplete.com/school/starting-with-haskell/basics-of-haskell>
- <http://adit.io>
- <http://learnyouahaskell.com>