# Дифференциалы высших порядков

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$$d^2 f(x,y) = \frac{d^2 f}{dx^2} dx^2 + 2\frac{d^2 f}{dx\,dy} dx\,dy + \frac{d^2 f}{dy^2} dy^2$$

$$d^3 f(x,y) = \frac{d^3 f}{dx^3} dx^3 + 3\frac{d^3 f}{dx^2 dy} dx^2 dy + 3\frac{d^3 f}{dx\,dy^2} dx\,dy^2 + \frac{d^3 f}{dy^3} dy^3$$

1) Дано:

$f(x,y) = x^2 y^2$

найти $d^2 f$

$u'_x = 2x y^2$

$u'_y = 2y x^2$

$u''_{xx} = 2y^2$

$u''_{xy} = 2y\, 2x \qquad u''_{yx} = 2x 2y$

$u''_{yy} = 2x^2$

$$d^2 f = 2y^2 dx^2 + 4x\cancel{x}y\, dx\,dy + 2x^2 dy^2$$

Неявные функции

1) $F(x,y) \Rightarrow y = y(x)$

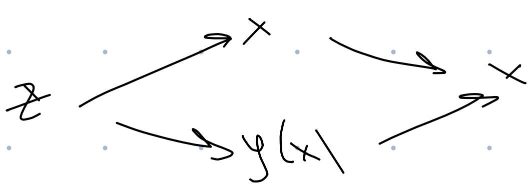
$$y' = -\frac{F'_x}{F'_y}$$

2) $F(x,y,z)=0$, где $z(x,y)$

$$z'_x = -\frac{F'_x}{F'_z}; \quad z'_y = -\frac{F'_y}{F'_z}$$

Сложные функции

1) $z = f(x,y)$, $y(x)$



$$\frac{dz}{dx} = \frac{\partial f}{\partial x}\cdot\cancel{\frac{dx}{dx}} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dx}$$

б) Дано:

$$z = f(x,y) = x^2+10x+y^3x\sqrt{1-y^2-x^3}$$

Найти: $\frac{dz}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dx}$

$$\frac{\partial f}{\partial x} = z'_x = 2x+10+y^3\cdot\left(\sqrt{1-y^2-x^3} + x\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{\ }}\cdot(-3x^2)\right)$$

$$\frac{\partial f}{\partial y} = z'_y = x\left(3y^2\sqrt{1-y^2-x^3} + y^3\frac{1}{2\sqrt{1-y^2-x^3}}(-2y)\right)$$

$$\frac{dy}{dx} = \cos^2 x - 2x\cos x\cdot\sin x$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

6) Дано: $z = f(x,y) = x^2 + y^2 - x\sin(2y)$

$x(t) = t^2 + \operatorname{arcctg} t$

$y(t) = \dfrac{t^3 + t^2 - t - 1}{t^2 + 2}$

$z(u,v) \quad u(x,y), \quad v(x,y)$

$$\begin{tikzcd}
 & u \arrow[r] \arrow[ddr] & x \\
z \arrow[ur] \arrow[dr] & & \\
 & v \arrow[uur] \arrow[r] & y
\end{tikzcd}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial z}{\partial y}$$

8) Дано: $z = \sqrt[3]{\sin uv + u\cos^2 v}$

$u = x^2 - y^2$

$v = x + \operatorname{tg} y$

$$\frac{dz}{du} = z'_y = \frac{1}{1}\left(\sin uv + u\cos^2 v\right)^{-\frac{2}{3}} \left(\cos uv + \cos^2 v\right)$$