**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS : **B**

**let time taken for service transmission= T**

**T is normally distributed with *μ* = 45 minutes and standard deviation *σ* = 8 minutes.**

**Time delay= 10 minutes**

**Time available to finish the work= 60-10=50 minutes.**

**Therefore from the equation Z=(T-µ)/ *σ***

**P(T≤50)=p(Z≤(50-45)/8)=p(Z≤0.625)= 0.7324(using z table)**

**Therefore p(T>50)=1-p(≤50)= 1-0.7324= 0.2676**

**(Or)**

**Using R-function : [1-pnorm(50,45,8)]=**

|  |
| --- |
| **> 1-pnorm(50,45,8)**  **[1] 0.2659855** |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: **False.**

**• 68% of the data falls within one standard deviation of the mean (µ+*σ)*.**

**Here µ=38, *σ* =6**

**Then, µ+*σ= 38+*6=44**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans) True**

**Z=(X-µ)/ *σ***

**P(X≤30)=p(Z≤(30-38)/6)=p(Z≤-1.33)= 0.0918(using z table)**

**Expected count=0.0918\*400= 36.72**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans: 2 is simply a larger scale version of the random variable *X1.* If is normally distributed then 2X1 is also normally distributed.**

***X*1 and *X*2 are normal distributed, the associated sums and random samples are exactly (and not just approximately) normal, with the appropriate parameters.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans D**

**Here we need range of 99% data which lies between 3rd standard deviation of the mean.**

**Here µ=100, *σ* =20**

**From empirical rule, µ±3*σ= 100±3\*20=>(100-60, 100+60)=>(40,160).***

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans : let, X is the sum of two random variables having normal distribution.**

**E[X]= E[45\*(profit 1+profit 2)]= 45\*(5+7)=540 million rupees**

**SD[X]= SD[profit 1 +profit 2]=> 45\*()**

**= 45\*= 225 million rupees.**

**Therefore, X~ N(540,)**

1. **From the empirical rule, Approximately 95% of the data falls within two standard deviation of the mean.**

**μ ± 2σ = 540±2\*225=> (540-450, 540+450)=> (90,990)**

**B)**

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**From the above normal distribution we can say that to find 5th percentile from the left side we can use the formula,**

**μ - 1.5σ => 540-(1.5\*225) =>202.5 million rupees.**

**c) this question concerns the original profit distributions.**

**For division1= Z score for a profit of zero: Z=(X-µ)/ *σ =>*  (0-5)/3 => -1.66=0.0485**

**(or)**

**> pnorm(0,5,3)**

**[1] 0.04779035**

**For division2= Z score for a profit of zero: Z=(X-µ)/ *σ*  =(0-7)/4 => -1.75= .0401**

**> pnorm(0,7,4)**

**[1] 0.04005916**

**Division2 has a higher probability of making a loss.**