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#### **HCF / GCF(Highest/Greatest Common Factor)**

• HCF of two or more numbers is the greatest / largest / highest/biggest number which can divide those two or more numbers exactly.

Factors of 6: 1, 2, 3, 6

Factors of 8: 1, 2, 4, 8

#### Common 1 & 2 Highest & Common 2

- LCM(Least Common Multiple)
- The LCM of two or more numbers is the smallest / lowest / least number which is exactly divisible by those two or more numbers.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54,...

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64....

Common 24, 48, .... Lowest & common 24



# **HCF (Factorization method)**

• Eg. HCF for 136, 144, 168

2	136	144	<u> 168</u>			
2	68	72	84			
2	34	36	42			
	17	18	21			
	NO FURTHER COMMON FACTOR					

So HCF = 
$$2 \times 2 \times 2 = 8$$

Note: HCF is always <= the smallest of given numbers

### **HCF (Factorization method) - (Assignment)**

• HCF of 54,72,126 (factorization method)

A. 21

B. 18

C. 36

D. 54

Ans: B



# **HCF (Difference Method)**

• Find HCF of 203,319

Keep smaller here

- (203, 319)
- (116,203)
- (87,116)
- (29,87)
- (29,58)
- (29,29)

HCF = 29



# HCF (Difference Method) - (Assignment)

• HCF of 161,253 (difference method)

A. 27

B. 18

C. 23

D. 17

Ans: C



# **HCF (Difference Method)**

#### Q. Find HCF of 84,125

- (84,125)
- (41,84)
- (41,43)
- (2,41)
- (2,39)

 If nothing is common then HCF = 1 and numbers are said to be co prime numbers.



Q. Find the greatest number which can divide 284, 698 & 1618 leaving the same remainder 8 in each case?

A. 36 B. 46

C. 56

D. 43.

Soln-

Remainder 8  $\rightarrow$  (numbers – 8) would be exactly divisible.

 $\rightarrow$  284-8 = 276

 $\rightarrow$ 698-8 = 690

 $\rightarrow$ 1618-8 = 1610

 $\rightarrow$  Greatest number dividing above 3 = HCF(276, 690, 1610) (difference method)

 $\rightarrow$ HCF = 46

Ans: B

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

A. 35 B. 46

C. 56

D. 43.

#### Soln:-

If two numbers a & b are divisible by a number n then

Their difference (a-b) is also divisible by n.

 $\rightarrow$ 132-62 = 70

 $\rightarrow$  237-132 = 105

 $\rightarrow$  237-62 = 175

 $\rightarrow$  Greatest number dividing above 3 = HCF(70, 105, 175)

 $\rightarrow$ HCF = 35

Ans: A

Q. Find the largest number such that 43,65,108 are divisible by that number and we get the remainder as 1,2,3 respectively in each case?

A. 21

B. 27

C.42

D. 63

#### Soln:

→ (numbers – remainder) would be exactly divisible.

$$\rightarrow$$
 43 – 1 = 42

$$\rightarrow$$
 65 - 2 = 63

$$\rightarrow$$
 108 – 3 = 105

HCF(42,63,105)=21

Ans: A

Q. A teacher has 25 books, 73 pens & 97 erasers. She wants to distribute them equally to maximum number of students so that after distribution she has equal number of books, pens & erasers left. What is the maximum number of students for such a distribution?

A. 32

B. 21

C. 12

D. 24

Soln:-

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

**→**73-25 = 48

**→**97-73 = 24

**→**97-25 = 72

 $\rightarrow$  Greatest number dividing above 3 = HCF(72, 48, 24)

 $\rightarrow$ HCF = 24

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

A. 35

B. 46

C. 56

D. 43.

Ans: A



Q. Find largest number such that if 45,68 and 113 are divided by that number we get the remainder as 1,2 and 3 respectively.

A. 21

B. 22

C. 26

D. 24

Ans: B



Q. Find the greatest number which can divide 41, 131 & 77 leaving the same remainder in each case?

A. 28

B. 18

C. 36

D. 24

Ans: B



• Eg. LCM for 18, 28, 108, 105

_	2	18	28	108	105
	2	9	14	54	105
	3	9	7	27	105
_	3	3	7	9	35
	3	1	7	3	35
_	5	1	7	1 3	35
	7	1	7	1	7
Till all quotients are 1		1	1	1	1

So LCM =  $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$ 

Note: LCM is always >= the greatest of given nos



Q. LCM for 12,24,20

A. 210

B. 180

C. 120

D. 144

Ans: C



Q. Find LCM of 72,125

A. 9000

B. 1200

C. 1000

D. 800

Ans: A



### **Rules to Remember**

Product of two given numbers is equal to the product of their HCF & LCM

$$A \times B = HCF(A,B) \times LCM(A,B)$$

• If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

```
n = (LCM of a, b, c) + r e.g 3,4,5 & rem 1
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#### Q. Find LCM of 147 & 231

#### Soln:-

- As we know,
- HCF X LCM = product
- Find HCF by difference method
- Put in the formula,
- $21 \times LCM = (147 \times 231)$
- 1617



#### Q. Find LCM of 84 and 125

#### Soln:-

- As they are co-prime numbers the product is the LCM because HCF =1 (for co-primes)
- HCF x LCM = product
- 1 x LCM = 84 x 125
- LCM = 10500



# Q. Find the least number which when divided by 12,15,24 leaves a remainder of 5 in each case

- Soln:
- Find LCM(12,15,24) = ?

If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (LCM of a, b, c) + r$$
 e.g 3,4,5 & rem 1

- LCM = 120
- In an LCM problem, if remainder is common then,

$$= 120+5 = 125$$



Q. Find the smallest number which when divided by 20,36,45 leaves a remainder 15,31 and 40 respectively.

- Soln:
- Find LCM(20,36,45)
- In LCM problem, if difference is common(constant) then,
- Result = LCM Common difference

- 20
  36
  45
  40
- Result = 180 5 = 175



Q. Four numbers are in the ratio of 10: 12: 15: 18. If their HCF is 3, then find their LCM.

A. 420

B. 540

C. 620

D. 680

Ans: B



Q. Find the least number which when divided by 5,6,7 and 8 leaves a reminder of 3 but when divided by 9 leaves no remainder.

A. 1677

B. 2523

C. 3363

D. 1683

Ans: D



### **HCF/LCM with Decimal point**

- Find HCF of 1.08, 0.36 and 0.9
- Soln:
- 1. Convert each of the decimals into like decimals.
- 1.08, 0.36 and 0.90
- 2. Write each number without decimal point.

HCF(108,36,90) = 18

3. Put decimal point after the numbers which are in like decimals.

Here it is after 2 numbers(digits)

HCF (1.08, 0.36 and 0.90) = 0.18



# **HCF(Assignment)**

Q. In a school of 437 boys & 342 girls it was decided to divide the girls & boys into separate classes. However it was required that each class consist of the same number of students. What would be the number of classrooms required?

A. 41 classrooms B. 14 classrooms C. 17 classrooms D. 26 classrooms

#### Ans: A

Same Class Size = HCF (Boys, Girls)

- $\rightarrow$  HCF (437,342) = 19
- $\rightarrow$  Boys Classes = 437/19 = 23
- $\rightarrow$  Girls Classes = 342/19 = 18
- $\rightarrow$  Total Classes = 23 + 18 = 41



Q. Find the least number which when divided by 12,15,40 leaves a remainder of 5 in each case

A. 120

B. 125

C. 130

D. 140

Ans: B



Q. If the product of two numbers is 324 and their HCF is 3, then their LCM will be = ?

A. 972

B. 327

C. 321

D. 108

Ans: D



Q. Three number are in the ratio of 3:4:5 and their L.C.M. is 2400. Their H.C.F. is:

A. 40

B. 80

C. 120

D. 200

Ans: A



Q. Find the least number which when divided by 16,18,20 and 25 leaves a reminder of 4 but when divided by 7 leaves no remainder.

A. 17004

B. 18000

C. 18002

D. 18004

Ans: D



Q. The HCF of two numbers is 8. Which one of the following can never be their LCM?

A. 24

B. 48

C. 56

D. 60

Ans: D

If HCF = 8 then LCM should have a factor of 8 Going by options 60 does not have a factor 8. So never be their LCM.



Q. The LCM of three different numbers is 120. Which of the following cannot be their HCF?

A. 8

B. 12

C. 24

D. 35

Ans: D



Q. HCF of 204,1190,1445

A. 17

B. 18

C. 19

D. 21

Ans: A



Q. LCM of 22,54,108,135 and 198 is -

A. 330

B. 1980

C. 5940

D. 11880

Ans: C



Q. Find HCF of 36 and 84

A. 4

B. 6

C. 12

D. 18

Ans: C



# **Numbers(Assignment)**

Q. The number nearest to 43582 divisible by each of 25, 50 and 75 is?

A. 43500

B. 43550

C. 43600

D. 43650

Ans: D



## **Numbers(Assignment)**

Q. What is the smallest 5 digits number which is divisible by 12, 15, and 18?

A.10010

B. 10015

C.10020

D. 10080

Ans: D



### **Rules to Remember**

#### • Fractions:

**LCM** = **LCM** of **Numerators** / **HCF** of **Denominators** 

**HCF = HCF of Numerators / LCM of Denominators** 

LCM of 25/12 & 35/18

LCM = 175/6

HCF of 25/12 & 35/18

HCF = 5/36



# **HCF & LCM Fractions(Assignment)**

- Find HCF & LCM of 5/9 and 25/36
- Ans : HCF = 5/36 and LCM = 25/9



# Number system(Assignment)

Q. Find a positive number x, such that the difference between the square of this number and 21 is the same as the product of 4 times the number?

A. 9

B. 27

C. 7

D. 13



#### • Arithmetic Progression:

- If quantities increase or decrease by a common difference then they are said to be in AP e.g. 3, 5, 7, 9,11,....
- If a is first term, d is the common difference, I is the last term then
- General form: a, a+d, a+2d, a+3d,...,a+(n-1)d
- $n^{th}$  term Tn = a + (n-1)d, n = 1, 2, ...
- Sum of n terms Sn =  $\frac{n}{2}$  [2a + (n-1)d]

$$Sn = \frac{n}{2}(a + I)$$



- Prove that the sum Sn of n terms of an Arithmetic Progress (A.P.) whose first term 'a' and common difference 'd' is
- S = n/2[2a + (n 1)d]
- Or, S = n/2[a + l], where l = last term = a + (n 1)d
- Proof:
- a, a+d, a+2d, a+3d,...., a(n-2)d, a(n-1)d, as I = last term
- a, a+d, a+2d, a+3d,...., I-d, I
- $S = a + a + d + a + 2d + a + 3d + \dots + I d + I \dots + I d + I$
- Writing equation 1 in reverse order(sum remains same even if we write in reverse order)
- S = I + I-d + I-2d + I-3d + ...... + a+d + a-----2
- Adding equation 1 and 2
- 2S = (a + I) + (a + I) + (a + I) + ----- + (a + I) + (a + I)
- So for n terms,
- 2S = n(a + I)
- $S = \frac{n}{2} (a + 1)$



Q. The sum of all two digit numbers divisible by 3 is

A. 550

B. 1550

C. 1665

D. 1680

Soln

Two digit numbers divisible by 3 are:

12, 15, 18, 21, ....., 96, 99.

This is an A.P. with a = 12, d = 3, l = 99

Let n be the number of terms.

Last term = a + (n-1)d

99 = 12 + (n-1)x3

3n = 90 , n = 30

Sum = n/2 (a + I) = 30/2 x (12+99)

= 1665



#### Q. Find the sum of all natural numbers between 10 and 200 which are divisible by 7

OR

A. 2835

B. 2865

C. 2678

D. 2646

#### Soln:

Two digit numbers divisible by 7 are:

14, 21, 28, 35, ....., , 196.

This is an A.P. with a = 14, d = 7, l=196

Last term = a + (n-1)d

196 = 14 + (n-1)x7

196-14 = (n-1)x7

n-1 = 26

n = 27

Sum = n/2 (a + I)

 $= 27/2 \times (14+196)$ 

 $= 27 \times 210 / 2$ 

 $= 27 \times 105$ 

= 2835

$$n = \frac{LastTerm - FirstTerm}{d} + 1$$

## **Progression(Assignment)**

Q. Find the sum of the series 3,8,13,18, ......,93

A. 912

B. 925

C. 998

D. 936



#### • Geometric Progression :

- If quantities increase or decrease by a constant factor then they are said to be in GP e.g. 4, 8, 16, 32, .....
- If a is first term, r is the common ratio, then
- General form: a, ar, ar<sup>2</sup>, ar<sup>3</sup>,...., ar<sup>n-1</sup>
- $n^{th}$  term  $Tn = ar^{(n-1)}$
- Sum of n terms  $\mathbf{Sn} = \frac{\mathbf{a}(\mathbf{r}^{n}-1)}{(\mathbf{r}-1)}$



## **Geometric Progression of n terms:**

- To prove that the sum of first n terms of the Geometric Progression whose first term 'a' and common ratio 'r' is given by-
- $S = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$  ------
- Multiply both sides of this equation by r
- $Sr = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} + ar^n$  ----- 2
- Eq 2 Eq 1
- $Sr S = ar^n a$
- $S(r-1) = a(r^n 1)$
- $S = \frac{a(r^{n}-1)}{(r-1)}$



## **Geometric Progression**

Q. Find the 10<sup>th</sup> term of the series: 4,16, 64, 256, 1024, ....

A. 4<sup>10</sup>

B. 48

**C**. 49

D. 1022480

#### Soln:

The given series is in geometric progression

Where a = 4, r = 4

So T10 = 
$$a \times r^{(10-1)}$$
  
=  $4 \times 4^{(10-1)}$   
=  $4^{10}$ 

- What is the difference between arithmetic progression and geometric progression?
- A sequence is a set of numbers, called terms, arranged in some particular order. An arithmetic sequence is a sequence with the difference between two consecutive terms constant. The difference is called the common difference. A geometric sequence is a sequence with the ratio between two consecutive terms constant.



#### Simple Average :

- An average of a set of values is the sum of values divided by the total number of values.
- Average of 'n' values = (Sum of the 'n' values)/n
- This is also called as Arithmetic Mean.
- Average (A) = Sum (S)/ Number(n)
- $S = A \times n$
- Weighted Average :
- When all values whose average we want to find do not have uniform occurrences we calculate the weighted average.
- If values y1, y2, y3...occur w1, w2, w3... times then
- Weighted Avg = (w1y1+w2y2+w3y3+ ..)(w1+w2+w3..)



Q. In a class of 50 students, 24 secured 60 in Physics, 16 secured 70 marks and the rest secured 80. What is the average score for Physics in the class?

A.64.8

B. 65.4

C. 67.2

D. 66.7

Soln:-

Students 24 16

Marks

60

70

80

10.

Average

= 24x60 + 16x70 + 10x80

24 + 16 + 10

= 3360/50

= 67.2



- Only For Consecutive Numbers -
- Whenever, we have consecutive numbers or consecutive odd numbers or consecutive even numbers, then always remember the middle number is the Average.
- Examples-
- A.  $5,6(7)8,9 \rightarrow Avg = 7$
- B. 5,6,7,8  $\rightarrow$  Avg =6.5
- C. 1,3(5)7,9  $\rightarrow$  Avg =5
- D. 21,23,25,27  $\rightarrow$  Avg =24

Q. The average age of a class of 22 students is 21 years. The average increased by 1 when the teacher's age also included. What is the age of the teacher?

A. 48

B. 45

C. 43

D. 44

Ans: D



Q. The average age of a class of 22 students is 21 years. The average increased by 1 when the teacher's age also included. What is the age of the teacher?

#### Solution 1:-

- Before teacher, total age of students = 22x21
- After teacher is added,

Total age of all students + Age of the teacher =  $23 \times 22$ 

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• Age of the teacher =23\times22-22\times21
=22(23-21)
=22\times2
```

=44 years



 The average age of a class of 22 students is 21 years. The average increased by 1 when the teacher's age also included. What is the age of the teacher?

- Solution 2:-
- New value = old avg + (n + 1)(diff)
- Where, n = total no. of students
- New value = 21 + (22+1)(1) = 21 + 23 = 44 years

+ if member added

- If member removed

difference = | Old avg – new avg |

Q. There are 50 students in a class. Their average weight is 45 kg. When one student leaves the class the average weight reduces by 100 g. What is the weight of the student who left the class?

A. 45 kg.

B. 47.9 kg.

C. 49.9 kg.

D. 50.1 kg.

Soln:

New value = old avg + (n + 1)(diff)

=45+(50-1)(0.1)

=45 + 49(0.1)

= 45 + 4.9

= 49.9 kg

( as we convert 100g into kg = 
$$\frac{100}{1000}$$
 = 0.1 kg )

Q. There are 50 students in a class. Their average weight is 45 kg. When one student leaves the class the average weight reduces by 100 g. What is the weight of the student who left the class?

A. 45 kg.

B. 47.9 kg.

C. 49.9 kg.

D. 50.1 kg.

#### Soln:

Total weight of 50 students =  $(45 \times 50)$  kg = 2250 kg

Average weight of 49 students = 45 kg - 100 g = 44.9 kg

So, total weight of 49 students =  $(44.9 \times 49)$ kg = 2200.1kg

Weight of the students who left the class = 2250 - 2200.1 = 49.9 kg



Q. The average age of 16 men increases by 3 years when a person 27 years old is replaced by another. How old is the new person?

A.75

B. 30

C. 48

D. 64

#### Soln:-

Number of men = 16

Let average age be a

→ Total age of 16 men = 16a (Old total)

New average = a+3

→ New total age of 16 men = 16 (a+3) = 16a + 48

New Total - Old Total = 48

 $\rightarrow$  Age of new man = 27 + 48 = 75



Q. The average age of 16 men increases by 3 years when a person 27 years old is replaced by another. How old is the new person?

A.75

B. 30

C. 48

D. 64

#### Soln:-

- Average of 16 men increases by 3 years means,
- total age increases by  $16 \times 3 = 48$
- If the age of new person same as replaced person then there would have been no change in average.
- But average age of 16 men increased by 3 years
- So, total age of the person replacing another person = 27 + 48 = 75years

Q. The average age of 8 men is decreased by 2 years when two of them, whose ages are 22 and 28, are replaced by two new men.. What is the average age of two men?

A. 34years

B. 30years

C. 15years

D. 17years

#### Soln:

- Average of 8 men reduce by 2 years means total age reduces by 16 if two men leave.
- So, the total age of the new men replacing the old men = 22+28–16=34
- => Average = 34/2 = 17 years.

#### <u>OR</u>

- Total age decreased= (8 \* 2) years = 16 years.
- Sum of ages of two new men = (22 + 28 16) years = 34 years
- Average age of two new men = (34/2) years = 17 years.
- · Ans: D



Q. The average age of students is 7 years and average age of 10 teachers is 50 years. If average age of group of all teachers and students is 8 years. Find the number of students?

A. 420

B. 250

C. 300

D. 270

#### Soln:

We know, Total = avg x n

S

No. 10

50 Avg

= (student) x avg + (teacher) x avg (student + teacher)x avg

 $(z + 10) \times 8$ 

8z+80 = 7z + 500

Z=420 students

 $\times 7 + (10)$ 

Q. Find average of all the numbers between 6 and 34 which are divisible by 5.

A. 18

B. 20

C. 34

D. 3

Ans: B



Q. The average weight of 16 boys in a class is 50.25 kg and that of the remaining 8 boys is 45.15 kg. Find the average weights of all the boys in the class.

A. 47.55 kg

B. 48 kg

C. 48.55 kg

D. 49.25 kg

$$= 50.25 \times 16 + 45.15 \times 8$$

$$= (804+361.2)/24$$

$$= 1165.2 / 24$$

$$=48.55$$



Q. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.

A. 20 years

B. 25 years

C. 30 years

D. 27 years

Ans: B



Q. The average marks of a class of 87 students is 56. When a new student was added and average becomes 56.5. Find marks of new student.

A. 56

B. 44

C. 100

D. 90



Q. Find the average of first 97 natural numbers.

A. 47

B. 37

C. 48

D. 49

E. 49.5

Ans: D



Q. The average age of a class of 30 students is 9years. When teacher's age is also added, the average becomes 10. What is the age of the teacher?

A. 41 years

B. 40 years

C. 39 years

D. 42 years

Ans: B



Q. The average of 50 numbers is 30. If two numbers, 35 and 40 are discarded, then the average of the remaining numbers is nearly:

A. 28.32

B. 29.68

C. 28.78

D. 29.27

Ans: B



Q. The average age 8 men is increased by 2 years when two of them whose ages are 21 years and 23 years are replaced by two new men. The average age of the two new men is?

A. 22 years

B. 24 years

C. 28 years

D. 30 years

Ans: D



Q. The average weight of the students of a class is 60 kg. If eight new students of average weight 64 kg join the class, the average weight of the entire class becomes 62 kg. How many students were there in the class initially?

A. 8 students

B. 16 students C.10 students

D. 12 students



Q. The average of ten numbers is 8. If the average of first nine numbers is 7. Find the 10<sup>th</sup> number?

A. 17

B. 16

C.15

D. 12



Q. The average marks obtained by 150 students is 30. If the average marks of passed candidates was 40 and that of failed candidates was 20. Find the number of candidates who passed the exam?

A. 25

B. 85

C.75

D. 45



Q. The average expenditure of a man for the first five months is Rs. 3600 and for next seven months is Rs. 3900, if he saves Rs.8700 during the year, his average income per month is ?

A. Rs.4500

B. Rs.8500

C. Rs.7500

D. Rs.5400



Q. The average of first five multiples of 3 is:

A. 9

B. 10

C. 8

D. 11



Q. Find the average of first 100 positive numbers

A. 49.5

Ans: B

B. 50.5

C. 51

D. 100



Q. The average expenditure of a man for the first five months of a year is Rs. 5000 and for next seven months is Rs. 5400, if he saves Rs.2300 during the year, his average income per month is ?

A. Rs.5425

B. Rs.5446

C. Rs.5500

D. Rs.5600





