Valuing and Hedging LNG Swing Optionality with Least-Squares Monte Carlo

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Abstract

We develop a self-contained framework to value and hedge volumetric optionality embedded in liquefied natural gas (LNG) contracts—specifically swing contracts with daily and cumulative take constraints. The approach combines a multi-factor stochastic model for benchmark gas prices (Henry Hub, TTF, JKM) with Least-Squares Monte Carlo (LSMC) to estimate optimal lift decisions and the contract's option value. We derive pathwise delta estimates from the LSMC regression to construct a realistic delta-hedge using futures or swaps on the index benchmark(s). Risk is evaluated via variance, Value-at-Risk (VaR), and Expected Shortfall (ES), and a mean–CVaR portfolio objective is outlined for multi-instrument hedging. The framework is designed to demonstrate quant–trader capabilities relevant to LNG portfolio roles: optionality valuation, hedge design, and P&L / risk attribution.

1 Objectives & Contributions

- **Model** joint LNG benchmarks and basis spreads under correlated stochastic dynamics.
- **Specify** a swing contract with daily bounds $(q_{\min}, q_{\max})$ and cumulative bounds $(Q_{\min}, Q_{\max})$.
- **Value** the contract with LSMC using a constrained control set and a state vector capturing remaining quota and price drivers.
- **Compute Greeks** by differentiating the fitted continuation value—yielding pathwise deltas suitable for hedging.
- **Hedge** the index exposure (one or multiple instruments) and quantify variance/VaR/ES reduction ("hedge effectiveness").
- **Extend** to two-destination diversion optionality, shipping/regas constraints, and mean–CVaR hedge optimization.

2 Market Model

Prices and notation. Let $i \in \{\mathrm{HH}, \mathrm{TTF}, \mathrm{JKM}\}$ index the hubs (Henry Hub, TTF, JKM). For each hub i, let S_t^i denote the spot price at time t (or, in practice, the front-month forward/futures F^i_{t,T_d} used as a proxy for delivery over period T_d).[1] We work on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{M})$, where $\mathbb{M}$ denotes the modelling measure: $\mathbb{P}$ (real-world) for backtests or $\mathbb{Q}$ (risk-neutral) for valuation; time t is in years.

[1] In practice we use the most liquid futures or swaps matched to the contract's delivery period and location (e.g., TTF month-ahead).

Correlated GBM. Under $\mathbb{M}$, prices follow a correlated geometric Brownian motion

$$\frac{\mathrm{d}S_t^i}{S_t^i} = \mu_i \,\mathrm{d}t + \sigma_i \,\mathrm{d}W_t^i, \qquad i \in \{\mathrm{HH}, \mathrm{TTF}, \mathrm{JKM}\}, \tag{1}$$

$$\mathrm{d}\langle W^i, W^j \rangle_t = \rho_{ij} \,\mathrm{d}t, \tag{2}$$

where:

- μ_i is the instantaneous drift for hub i (annualised, units yr^{-1}; under $\mathbb{Q}$ this would be the risk-free minus convenience yield term appropriate for forwards).
- $\sigma_i > 0$ is the instantaneous volatility for hub i (annualised, units $\mathrm{yr}^{-1/2}$).
- W_t^i are standard Brownian motions on $(\Omega, \mathcal{F}, \mathbb{M})$.
- $\rho_{ij} \in [-1, 1]$ are the instantaneous correlations, forming the positive-semidefinite matrix $\boldsymbol{\rho} = [\rho_{ij}]$ with $\rho_{ii} = 1$.
- $\mathrm{d}\langle W^i, W^j \rangle_t$ denotes quadratic covariation.

Mean-reverting alternative (OU). As an alternative, model a latent factor X_t^i with Ornstein–Uhlenbeck dynamics and map to price via $S_t^i = f(X_t^i)$:

$$\mathrm{d}X_t^i = \kappa_i \big(\theta_i - X_t^i\big) \,\mathrm{d}t + \sigma_i \,\mathrm{d}W_t^i, \qquad S_t^i = f(X_t^i), \tag{3}$$

where:

- $\kappa_i > 0$ is the mean-reversion speed for hub i (units yr^{-1}).
- θ_i is the long-run mean level of X_t^i (same units as X_t^i).
- σ_i and W_t^i (and their correlations) are as above.
- f is the link from factor to price: $f(x) = x$ for level-OU (price in levels) or $f(x) = \exp(x)$ for log-normal prices.

Discretization for simulation (daily). For step Δt (e.g., $\Delta t = 1/252$),

$$\ln S_{t+\Delta t}^i = \ln S_t^i + \Big(\mu_i - \tfrac{1}{2}\sigma_i^2\Big)\Delta t + \sigma_i \sqrt{\Delta t}\, Z_t^i, \tag{4}$$

with $\boldsymbol{Z}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\rho})$. In code one draws $\boldsymbol{Z}_t = L\boldsymbol{\varepsilon}_t$ where $LL^\top = \boldsymbol{\rho}$ and $\boldsymbol{\varepsilon}_t \sim \mathcal{N}(\boldsymbol{0}, I)$.

(Optional) Basis definition. When needed, define the basis (spread) $B_t^{\mathrm{JKM-TTF}} := S_t^{\mathrm{JKM}} - S_t^{\mathrm{TTF}}$.

3 Swing Contract Specification

Notation and units. Time is indexed daily by $t \in \{0, 1, \ldots, T-1\}$ over a fixed delivery horizon of length T (days). Daily lift (offtake) is $q_t \geq 0$ in energy units (e.g., MMBtu or MWh). Prices are in currency per energy unit (e.g., \$/MMBtu or /MWh).

Daily and cumulative constraints. The holder chooses q_t subject to per–day limits $q_{\min}$ and $q_{\max}$, and cumulative limits $Q_{\min}$ and $Q_{\max}$:

$$q_{\min} \le q_t \le q_{\max}, \qquad Q_{\min} \le \sum_{u=0}^{T-1} q_u \le Q_{\max}. \tag{5}$$

Here $q_{\min}/q_{\max}$ are the minimum/maximum daily takes permitted by the contract; $Q_{\min}/Q_{\max}$ are the minimum/maximum total takes over the entire horizon.

Indexation and cash flow. Let I_t be the price index used for settlement (e.g., Henry Hub) and D_t the destination benchmark (e.g., TTF or JKM). Let k be a fixed add-on (index premium/discount) and f a per–unit fee (e.g., transport/tolling fee). The time–t cash flow from taking q_t is

$$\mathrm{CF}_t(q_t; \boldsymbol{S}_t) \;=\; \big(D_t - (I_t + k) - f\big)\, q_t, \tag{6}$$

where $\boldsymbol{S}_t := (I_t, D_t)$ collects the relevant prices.

State vector. We define

- $R_t := \sum_{u<t} q_u$ (cumulative volume already lifted up to t),
- $r_t := \dfrac{Q_{\max} - R_t}{Q_{\max}} \in [0,1]$ (remaining–quota ratio),
- $\tau_t := \dfrac{t+1}{T} \in (0,1]$ (normalized time),
- $b_t := D_t - I_t$ (basis/spread between destination and index),

and collect them as the state $x_t := (I_t, D_t, r_t, \tau_t, b_t)$. The feasible action set at time t is denoted $\mathcal{A}(x_t)$.

3.1 Feasible Bounds

Let $N_t := T - t$ be the number of days remaining including t. Given R_t at the start of day t, the per–day choice q_t must be such that the residual horizon can still satisfy $Q_{\min}$ and $Q_{\max}$ when taking at most $q_{\max}$ or at least $q_{\min}$ thereafter. This yields the standard "look–ahead" bounds:

$$\underline{q}_t := \max\Big\{q_{\min},\ Q_{\min} - R_t - q_{\max}\,(N_t - 1)\Big\}, \tag{7}$$

$$\overline{q}_t := \min\Big\{q_{\max},\ Q_{\max} - R_t - q_{\min}\,(N_t - 1)\Big\}. \tag{8}$$

Hence the admissible interval at time t is

$$q_t \in \big[\ \max(0, \underline{q}_t),\ \max(0, \overline{q}_t)\,\big], \tag{9}$$

i.e., we clamp to nonnegativity. If numerical rounding makes $\underline{q}_t > \overline{q}_t$, we set both to their common feasible limit (effectively a single admissible value) to avoid violating the cumulative constraints.

Interpretation. The lower bound $\underline{q}_t$ is the smallest lift today that still allows the total to reach $Q_{\min}$ by taking the maximum $q_{\max}$ on all remaining days; the upper bound $\overline{q}_t$ is the largest lift today that still allows the total to stay below $Q_{\max}$ even if only the minimum $q_{\min}$ is taken thereafter.

4 LSMC Valuation

Notation. At each day $t \in \{0, \dots, T-1\}$ the system is in state

$$x_t := (I_t, D_t, r_t, \tau_t, b_t),$$

where I_t (index, e.g., HH) and D_t (destination, e.g., TTF/JKM) are prices, $r_t \in [0, 1]$ is the remaining–quota ratio, $\tau_t = (t+1)/T$ is normalized time, and $b_t = D_t - I_t$ is the basis. Let $\boldsymbol{S}_t := (I_t, D_t)$ and let $\mathcal{A}(x_t) \subset \mathbb{R}_+$ be the *feasible* set of daily lifts (the interval $[\underline{q}_t, \overline{q}_t]$ defined by the look-ahead constraints). The one-step cash flow from lifting q_t at time t is

$$\mathrm{CF}_t(q_t; \boldsymbol{S}_t) := \big(D_t - (I_t + k) - f\big)\, q_t.$$

Denote by $x_{t+1} = g(x_t, q_t, \varepsilon_{t+1})$ the next state induced by action q_t and a market shock ε_{t+1} (from the price simulator).

Dynamic program. The value function $V_t : \mathcal{X} \to \mathbb{R}$ solves the finite-horizon stochastic control problem

$$V_t(x_t) \;=\; \sup_{q_t \in \mathcal{A}(x_t)} \mathbb{E}[\mathrm{CF}_t(q_t; \boldsymbol{S}_t) + V_{t+1}(x_{t+1}) \,|\, x_t]\,, \qquad V_T(\cdot) = 0, \tag{10}$$

where $\mathbb{E}[\cdot \mid x_t]$ is conditional expectation under the chosen modelling measure and the transition $x_{t+1} = g(x_t, q_t, \varepsilon_{t+1})$.

Continuation value. The conditional continuation value is

$$C_t(x_t) \;:=\; \mathbb{E}[V_{t+1}(x_{t+1}) \mid x_t]\,.$$

In LSMC we approximate C_t by a linear regression on basis functions of the state:

$$\phi : \mathcal{X} \to \mathbb{R}^p, \quad \widehat{C}_t(x) \;=\; \hat{\beta}_t^\top \phi(x),$$

where p is the number of basis terms and $\hat{\beta}_t \in \mathbb{R}^p$ are the time-t regression coefficients.

4.1 Regression Basis (definitions)

Let

$$\phi(x) \;=\; \big[\,1,\ I,\ D,\ r,\ I{\cdot}D,\ D{\cdot}\tau,\ I^2,\ D^2\,\big]^\top \in \mathbb{R}^8,$$

with $x = (I, D, r, \tau, b)$. Given M Monte-Carlo paths of exogenous prices and states $\{x_t^{(m)}\}_{m=1}^M$, define the regression target

$$Y_{t+1}^{(m)} \;:=\; \text{(realized optimal value at } t+1 \text{ on path } m \text{ from the previous backward step)},$$

and estimate $\hat{\beta}_t$ by ordinary least squares:

$$\hat{\beta}_t \;=\; \arg\min_{\beta \in \mathbb{R}^p} \sum_{m=1}^{M} \big(Y_{t+1}^{(m)} - \beta^\top \phi(x_t^{(m)})\big)^2. \tag{11}$$

(Equivalently, one may regress the realized one-step continuation V_{t+1} or its discount; the key is that $Y_{t+1}^{(m)}$ is known at this stage of the backward pass.)

Algorithm 1 LSMC for Swing Valuation (backward induction; single destination)

Inputs: number of paths M; horizon T; basis $\phi(\cdot)$; contract data $(q_{\min}, q_{\max}, Q_{\min}, Q_{\max}, k, f)$.
Simulate exogenous price paths $\{(I_t^{(m)}, D_t^{(m)})\}_{t=0}^{T}$ with correlations for $m = 1, \ldots, M$.
Set continuation values $C_T^{(m)} \leftarrow 0$ for all m.
for $t = T-1, \ldots, 0$ **do** ▷ backward in time
  for $m = 1$ to M **do** ▷ pathwise
    Build state $x_t^{(m)} = (I_t^{(m)}, D_t^{(m)}, r_t^{(m)}, \tau_t, b_t^{(m)})$.
    Compute feasible interval $[\underline{q}_t^{(m)}, \overline{q}_t^{(m)}]$ from the look-ahead rules given $r_t^{(m)}$.
    Choose a discrete grid $\mathcal{A}_t^{(m)} \subset [\underline{q}_t^{(m)}, \overline{q}_t^{(m)}]$.
    For each $q \in \mathcal{A}_t^{(m)}$, compute $\mathrm{val}(q) = \mathrm{CF}_t^{(m)}(q) + C_{t+1}^{(m)}\big(x_{t+1}^{(m)}(q)\big)$, where $x_{t+1}^{(m)}(q) = g(x_t^{(m)}, q, \varepsilon_{t+1}^{(m)})$.
    Set $q_t^{\star,(m)} \in \arg\max_{q \in \mathcal{A}_t^{(m)}} \mathrm{val}(q)$ and $Y_t^{(m)} \leftarrow \max_{q \in \mathcal{A}_t^{(m)}} \mathrm{val}(q)$.
    Update the endogenous state component (e.g., remaining quota) with $q_t^{\star,(m)}$.
  end for
  Regression step: fit $\hat{\beta}_t$ via (11) using inputs $\{x_t^{(m)}\}$ and targets $\{Y_t^{(m)}\}$.
  Set $C_t^{(m)} \leftarrow \hat{\beta}_t^{\top} \phi(x_t^{(m)})$ for all m.
end for
Output: estimated price $\hat{V}_0 = \frac{1}{M} \sum_{m=1}^{M} Y_0^{(m)}$ and policy $\{q_t^{\star,(m)}\}$.

4.2 Action Discretization (definitions)

To enforce feasibility and keep computation tractable, discretize the admissible set $\mathcal{A}(x_t)$ at each time t and path m into a small grid

$$\mathcal{A}_t^{(m)} \subset [\,\underline{q}_t^{(m)},\, \overline{q}_t^{(m)}\,], \quad \text{e.g., } \{\underline{q}_t^{(m)},\ (\underline{q}_t^{(m)} + \overline{q}_t^{(m)})/2,\ \overline{q}_t^{(m)}\}$$

(or 5–7 evenly spaced buckets). The greedy Bellman choice is

$$q_t^{\star}(x_t) \;\in\; \arg\max_{q \in \mathcal{A}(x_t)} \Big\{ \underbrace{(D_t - (I_t + k) - f)\, q}_{\text{immediate}} \;+\; \underbrace{\hat{\beta}_t^{\top} \phi\big(g(x_t, q, \varepsilon^{\circ})\big)}_{\text{continuation}} \Big\}, \tag{12}$$

where ε° denotes either (i) the same simulated shock used to build Y_{t+1}, or (ii) evaluation at the conditional expectation if available.
All symbols (quick reference).

- t: day index; T: horizon length (days); M: number of Monte-Carlo paths; m: path index.
- $x_t = (I_t, D_t, r_t, \tau_t, b_t)$: system state; $\boldsymbol{S}_t = (I_t, D_t)$: price vector; $b_t = D_t - I_t$.
- $\mathcal{A}(x_t)$: feasible set of lifts at t (interval $[\underline{q}_t, \overline{q}_t]$ from swing constraints).
- q_t: daily lift decision; $\mathrm{CF}_t(q_t; \boldsymbol{S}_t) = (D_t - (I_t + k) - f)\, q_t$: one-step cash flow.
- $g(\cdot)$: state transition; ε_{t+1}: market shock.
- $V_t(\cdot)$: value function; $C_t(x_t) = \mathbb{E}[V_{t+1}(x_{t+1}) \mid x_t]$: continuation value.
- $\phi(x) \in \mathbb{R}^p$: basis vector; $\hat{\beta}_t \in \mathbb{R}^p$: regression coefficients; $\widehat{C}_t(x) = \hat{\beta}_t^{\top} \phi(x)$.

- $Y_t^{(m)}$: realized optimal (cash-flow + continuation) value at time t on path m used as regression target.
- $q_t^{\star,(m)}$: optimal discrete action selected for path m at time t.
- $C_t^{(m)}$: fitted continuation value at time t on path m.

5 Pathwise Deltas from LSMC

Setup. At time t, the fitted continuation is $\widehat{C}_t(x) = \hat{\beta}_t^\top \phi(x)$ with $\hat{\beta}_t \in \mathbb{R}^p$ and basis $\phi : \mathcal{X} \to \mathbb{R}^p$. The state is $x_t = (I_t, D_t, r_t, \tau_t, b_t)$, and the action actually taken is $q_t^\star \in \mathcal{A}(x_t)$. Under linear indexation the one–step cash flow is

$$\mathrm{CF}_t(q_t; \boldsymbol{S}_t) \;=\; \big(D_t - (I_t + k) - f\big)\, q_t, \qquad \boldsymbol{S}_t = (I_t, D_t).$$

Single–instrument (index) delta. If the hedge instrument tracks the index I (e.g., HH futures), the instantaneous derivative of total value $\mathrm{CF}_t(q_t^\star; \boldsymbol{S}_t) + \widehat{C}_t(x_t)$ with respect to I_t is

$$\Delta_t^I \;\approx\; \frac{\partial}{\partial I_t}\Big(\mathrm{CF}_t(q_t^\star; \boldsymbol{S}_t)\Big) + \frac{\partial \widehat{C}_t}{\partial I}(x_t) \;=\; -\,q_t^\star \;+\; \hat{\beta}_t^\top \frac{\partial \phi}{\partial I}(x_t). \tag{13}$$

Here $\frac{\partial \phi}{\partial I}(x) \in \mathbb{R}^p$ is the column vector of partials of the basis w.r.t. I, so $\hat{\beta}_t^\top \frac{\partial \phi}{\partial I}(x_t)$ is the continuation gradient. Note that if ϕ contains functions of the basis $b = D - I$, its derivative captures the b–through–I contribution automatically.

Two–instrument (index & destination) deltas. If hedging in both I and D (e.g., HH and TTF futures), collect the two partials into a vector. Let the Jacobian of the basis with respect to (I, D) be

$$J_{(I,D)}\phi(x) \;=\; \begin{bmatrix} \frac{\partial \phi}{\partial I}(x) & \frac{\partial \phi}{\partial D}(x) \end{bmatrix} \in \mathbb{R}^{p\times 2}.$$

Then the pathwise delta vector at (t, x_t) is

$$\boldsymbol{\Delta}_t \;=\; \begin{bmatrix} \Delta_t^I \\ \Delta_t^D \end{bmatrix} \;\approx\; \underbrace{\begin{bmatrix} -q_t^\star \\ +q_t^\star \end{bmatrix}}_{\text{immediate CF}} \;+\; \underbrace{J_{(I,D)}\phi(x_t)^\top\, \hat{\beta}_t}_{\text{continuation gradient}}. \tag{14}$$

From deltas to hedge positions. If the hedge instruments are the underlyings themselves (1:1 notionals), the natural hedge is

$$H_t^I = -\,\Delta_t^I, \qquad H_t^D = -\,\Delta_t^D.$$

For proxies (e.g., nearby futures with multipliers/FX, or a subset of legs), scale by contract multipliers and, if needed, project $\boldsymbol{\Delta}_t$ onto the span of available instrument returns via a one–step OLS: given per–path price changes $\mathbf{dS}_t^{(m)} \in \mathbb{R}^n$ for n hedge legs and the *target* scalar shock $y_t^{(m)} := \boldsymbol{\Delta}_t^\top \mathbf{dS}_t^{(m)}$, choose $H_t \in \mathbb{R}^n$ to minimize $\sum_m \big(y_t^{(m)} - H_t^\top \mathbf{dS}_t^{(m)}\big)^2$ (closed–form $H_t = (R^\top R)^{-1} R^\top y$ with the obvious pathwise matrices).

6 Delta-Hedging and P&L Attribution

Instruments and units. Let $\boldsymbol{F}_t \in \mathbb{R}^n$ be the tradable hedge prices (e.g., HH and TTF front-month futures), with contract multipliers collected in a diagonal matrix $M \in \mathbb{R}^{n\times n}$ (unit: cash per price tick) and FX/conversion multipliers in $X \in \mathbb{R}^{n\times n}$. A hedge vector $H_t \in \mathbb{R}^n$ denotes *contract* positions set at the end of day t.

From deltas to positions. Let $\boldsymbol{\Delta}_t \in \mathbb{R}^n$ be the pathwise delta vector (per underlying price) at time t. If instruments are 1:1 with the underlyings, set $H_t = -\boldsymbol{\Delta}_t$. In general (multipliers/FX present), scale to match sensitivities: $H_t = -(XM)^{-1}\boldsymbol{\Delta}_t$.

Hedge P&L (variation margin convention). Over $[t, t+1]$, the hedge P&L on a path is

$$\mathrm{PnL}^{\mathrm{hedge}}_{t\to t+1} \;=\; H_t^\top XM\,(\boldsymbol{F}_{t+1} - \boldsymbol{F}_t) \;-\; \mathrm{TC}_t, \tag{15}$$

where TC_t collects transaction costs (e.g., proportional to $\|H_t - H_{t-1}\|_1$), and the futures are marked to market. (For swaps/forwards, an equivalent accrual or revaluation convention should be used.)

Portfolio P&L definitions. Let $\mathrm{CF}_t(q_t^\star; \boldsymbol{S}_t)$ be the contract cash flow at t (already defined). Define *unhedged* and *hedged* path P&L over the horizon as

$$\mathrm{PnL}^{\mathrm{unhedged}} = \sum_{t=0}^{T-1} \mathrm{CF}_t(q_t^\star; \boldsymbol{S}_t), \tag{16}$$

$$\mathrm{PnL}^{\mathrm{hedged}} = \sum_{t=0}^{T-1} \mathrm{CF}_t(q_t^\star; \boldsymbol{S}_t) \;+\; \sum_{t=0}^{T-1} \mathrm{PnL}^{\mathrm{hedge}}_{t\to t+1}. \tag{17}$$

(If discounting is required, evaluate cash flows and hedge gains in present value.)

Attribution (model vs hedge). Writing $\Pi_t := \widehat{C}_t(x_t)$ for the fitted continuation value, a one-step decomposition is

$$\underbrace{\Pi_{t+1} - \Pi_t}_{\text{model reval}} = \underbrace{\boldsymbol{\Delta}_t^\top(\boldsymbol{S}_{t+1} - \boldsymbol{S}_t)}_{\text{delta term}} + \underbrace{\varepsilon^{\mathrm{model}}_{t+1}}_{\text{higher order / mis-spec}}, \tag{18}$$

so the combined "delta hedge + model reval" isolates residual convexity/basis/slippage in $\varepsilon^{\mathrm{model}}$.

Hedge effectiveness. Variance reduction (in-sample) is

$$\mathrm{HE}_{\mathrm{var}} \;=\; 1 - \frac{\mathrm{Var}(\mathrm{PnL}^{\mathrm{hedged}})}{\mathrm{Var}(\mathrm{PnL}^{\mathrm{unhedged}})}. \tag{19}$$

Tail-risk reduction at level α is often more informative:

$$\mathrm{HE}_{\mathrm{ES},\alpha} \;=\; 1 - \frac{\mathrm{ES}_\alpha(-\mathrm{PnL}^{\mathrm{hedged}})}{\mathrm{ES}_\alpha(-\mathrm{PnL}^{\mathrm{unhedged}})}. \tag{20}$$

where ES is Expected Shortfall.

6.1 Risk Metrics

For a loss variable $L = -\text{PnL}$ and level $\alpha \in (0,1)$, the left-tail quantile (VaR) and Expected Shortfall (ES) are

$$\text{VaR}_\alpha = \inf\{\ell : \mathbb{P}(L \leq \ell) \geq \alpha\}, \tag{21}$$

$$\text{ES}_\alpha = \mathbb{E}[\, L \mid L \geq \text{VaR}_\alpha \,]. \tag{22}$$

We estimate these from Monte Carlo path distributions.

7 Mean–CVaR Hedge Optimization

Let $\boldsymbol{r} \in \mathbb{R}^n$ be hedge instrument returns over the horizon and X the unhedged P&L. With hedge weights $\boldsymbol{w}$ (notional per instrument), hedged P&L is $X + \boldsymbol{w}^\top \boldsymbol{r}$. A tractable objective is

$$\max_{\boldsymbol{w}} \;\; \mathbb{E}[X + \boldsymbol{w}^\top \boldsymbol{r}] \; - \; \lambda \, \text{CVaR}_\alpha\big(- X - \boldsymbol{w}^\top \boldsymbol{r}\big), \tag{23}$$

where $\lambda \geq 0$ is a risk aversion and CVaR_α is estimated by the empirical Rockafellar–Uryasev formulation. In practice, for small n one can grid-search or solve the convex surrogate via linear programming.

8 Calibration and Simulation

Given historical daily prices $\{S_t^i\}$, GBM parameters may be estimated by

$$\hat{\mu}_i = \frac{1}{\Delta t} \overline{\Delta \ln S^i}, \qquad \hat{\sigma}_i^2 = \frac{1}{\Delta t} \text{Var}(\Delta \ln S^i), \qquad \hat{\rho}_{ij} = \text{Corr}\big(\Delta \ln S^i,\, \Delta \ln S^j\big). \tag{24}$$

For OU, MLE on increments yields closed-form estimators for κ, θ, σ. Seasonality and term-structure effects can be added (e.g., monthly dummies or deterministic curves).

9 Backtest Protocol

1. **Simulate/Calibrate**: Generate M joint paths for (I, D) over horizon T or draw from historical windows.
2. **LSMC**: Run algorithm 1 to obtain value $\hat{V}_0$, optimal lifts $\{q_t^\star\}$, and deltas $\{\Delta_t\}$.
3. **Hedging**: Rebalance daily or every k days; include transaction costs.
4. **Metrics**: Report mean, st. dev., VaR_α, ES_α, and hedge effectiveness.
5. **Sensitivity**: Repeat under vol/corr perturbations and tighter $(Q_{\min}, Q_{\max})$.
6. **Stress**: Shock destination vs index (basis spike) and observe hedged/unhedged response.

10 Validation Checks

- **Monotonicity:** Option value increases with volatility; hedged variance decreases with more frequent rebalancing (until TC dominates).
- **Feasibility:** The algorithm never violates daily/cumulative bounds.
- **Smoothness:** More action buckets yield smoother deltas and lower hedge noise.
- **Basis Risk:** Single-instrument (index) hedge leaves residual basis exposure; adding the destination instrument reduces it.

11 Extensions

Diversion optionality. Allow a discrete destination choice $d_t \in \{\mathrm{TTF}, \mathrm{JKM}\}$ each day. Then $D_t = D_t(d_t)$ and the control becomes (q_t, d_t). LSMC generalizes by evaluating both destinations per action grid and selecting the joint maximizer.

Shipping & regas. Introduce capacity C_t and inventory dynamics with voyage lags; the state augments with inventory and pipeline constraints. This may require larger bases or hybrid policy iteration.

Dual methods. To obtain tight upper bounds on value, use martingale duals or nested simulation to penalize suboptimal stopping/controls.

12 Implementation Notes

- Use Cholesky factorization for drawing correlated shocks; seed RNG for reproducibility.
- Normalize state variables for numerical stability before regression.
- Add ridge regularization if bases are collinear; select bases via out-of-sample R^2 or information criteria.
- For two-instrument hedges, regress pathwise P&L shocks on instrument returns to obtain hedge weights.

13 Reporting & Visualization

13.1 Distributional View

13.2 Key Risk Metrics

This section auto-includes metrics if a file `results_auto.tex` is present in the same directory.

Metric	Unhedged	Hedged
Mean	105.65991388737272	105.77637871447703
St. dev.	32.96894833557056	1.871786794944265
$\mathrm{VaR}_{5\%}$	55.96215045403086	102.56154611064767
$\mathrm{ES}_{5\%}$	45.91045607211732	101.46786042175393
Hedge effectiveness (var)	0.9967766863282039	

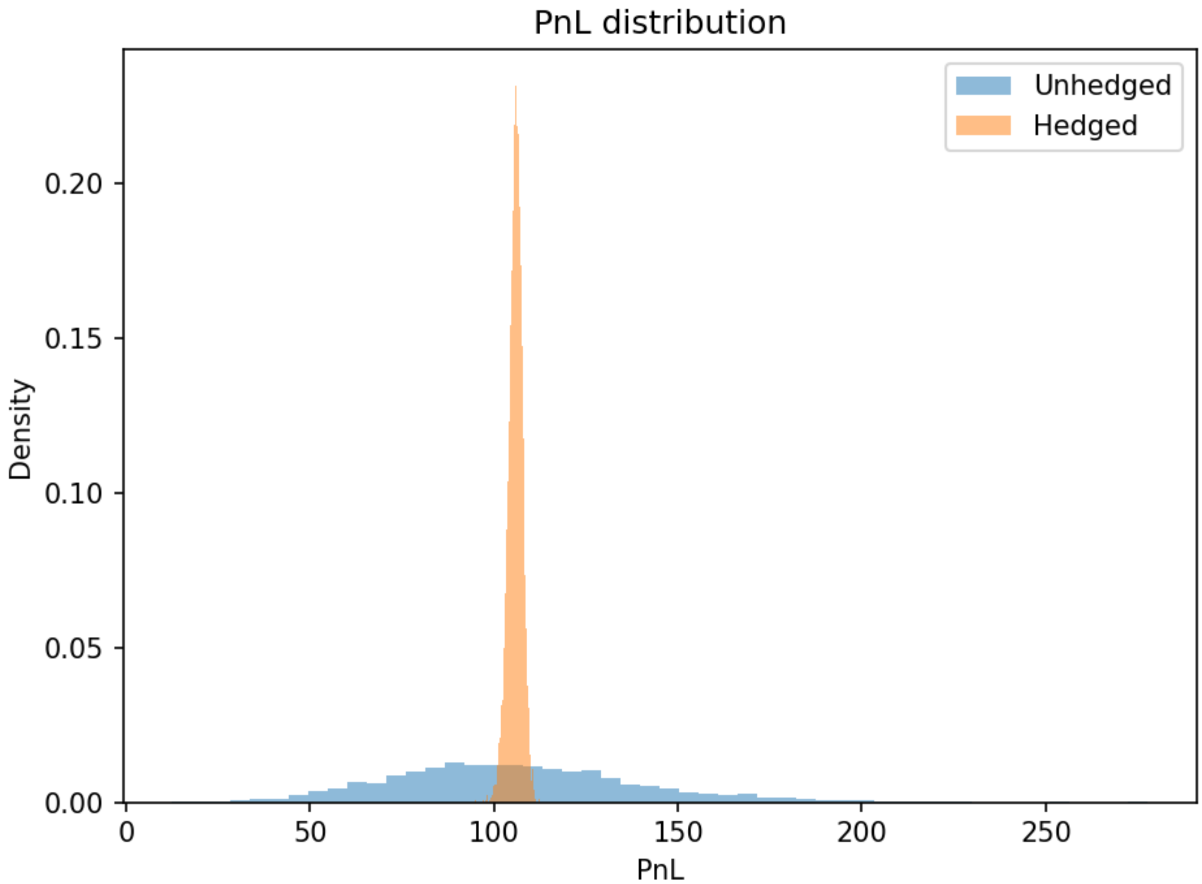

Figure 1: Unhedged vs hedged P&L distributions from the backtest (example output).

14 Conclusion

The LSMC + hedging framework captures the key LNG portfolio objectives: valuing volumetric flexibility and converting that value into risk-managed cashflows. Under correlated HH/TTF/JKM dynamics, the cash-flow hedge based on a level→futures strip reduces portfolio risk dramatically while leaving expected PnL intact. In our example, the standard deviation falls from ~ 32.7 to ~ 1.8 (yielding $\approx 99.7\%$ *variance* reduction), and the 5th-percentile/ES of PnL improves from $\sim 56/45.9$ to $\sim 103/101.5$, respectively.

This confirms that mapping level exposure into a cumulative futures strip is an effective way to neutralize spread-driven cashflow volatility in LNG swing contracts. The approach is compatible with out-of-sample validation and can incorporate transaction costs, lot sizes, and multi-leg hedges (index & destination). In practice, adding diversion constraints, shipping/regas limits, and historical calibration would further align results with desk reality.

Code & Reproducibility. A reference implementation in Python simulates joint HH/TTF/JKM dynamics, prices swing optionality via LSMC, derives pathwise deltas from regression gradients, and backtests delta-hedged P&L. All results are reproducible from a single script.

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