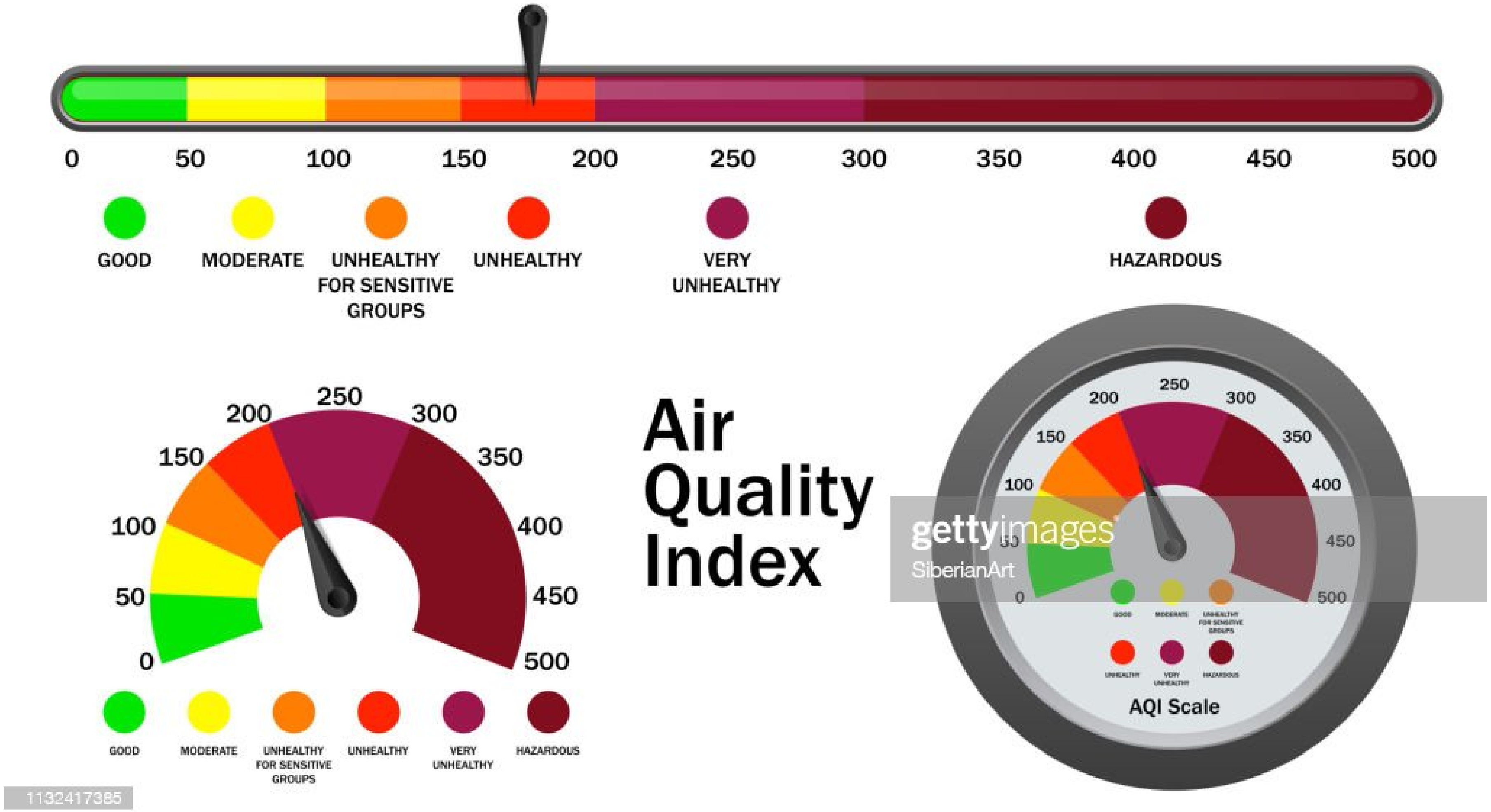


In [1]:

```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
plt.rcParams.update({'figure.figsize':(9,7), 'figure.dpi':120})
```

In [2]:

```
data= pd.read_excel('AirQualityUCI/AirQualityUCI.xlsx', parse_dates=[['Date', 'Time']])
data.head()
```

Out[2]:

	Date_Time	CO(GT)	PT08.S1(CO)	NMHC(GT)	C6H6(GT)	PT08.S2(NMHC)	NOx(GT)	PT08.S3(NOx)	NO2(GT)	PT08.S4(NO2)	PT08.S5(O3)	T	
0	2004-03-10 18:00:00	2.6	1360.00	150	11.881723	1045.50	166.0	1056.25	113.0	1692.00	1267.50	13.60	48.
1	2004-03-10 19:00:00	2.0	1292.25	112	9.397165	954.75	103.0	1173.75	92.0	1558.75	972.25	13.30	47.
2	2004-03-10 20:00:00	2.2	1402.00	88	8.997817	939.25	131.0	1140.00	114.0	1554.50	1074.00	11.90	53.
3	2004-03-10 21:00:00	2.2	1375.50	80	9.228796	948.25	172.0	1092.00	122.0	1583.75	1203.25	11.00	60.
4	2004-03-10 22:00:00	1.6	1272.25	51	6.518224	835.50	131.0	1205.00	116.0	1490.00	1110.00	11.15	59.

In [3]:

```
data.shape
```

Out[3]:

```
(9357, 14)
```

In [4]:

```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 9357 entries, 0 to 9356
Data columns (total 14 columns):
 #   Column         Non-Null Count  Dtype         
---  ------         --------------  -----         
 0   Date_Time      9357 non-null   datetime64[ns]
 1   CO(GT)         9357 non-null   float64       
 2   PT08.S1(CO)    9357 non-null   float64       
 3   NMHC(GT)       9357 non-null   int64         
 4   C6H6(GT)       9357 non-null   float64       
 5   PT08.S2(NMHC)  9357 non-null   float64       
 6   NOx(GT)        9357 non-null   float64       
 7   PT08.S3(NOx)   9357 non-null   float64       
 8   NO2(GT)        9357 non-null   float64       
 9   PT08.S4(NO2)   9357 non-null   float64       
 10  PT08.S5(O3)    9357 non-null   float64       
 11  T              9357 non-null   float64       
 12  RH             9357 non-null   float64       
 13  AH             9357 non-null   float64       
dtypes: datetime64[ns](1), float64(12), int64(1)
memory usage: 1023.5 KB
```

In [5]:
```
data.rename(columns = {'PT08.S4(NO2)':'PT08_S4_NO2'}, inplace = True)
```

In [6]:
```
data.corr()['PT08_S4_NO2']
```

Out[6]:
```
CO(GT)          -0.073721
PT08.S1(CO)      0.845133
NMHC(GT)         0.162689
C6H6(GT)         0.774649
PT08.S2(NMHC)    0.874761
NOx(GT)          0.035580
PT08.S3(NOx)     0.122672
NO2(GT)         -0.022092
PT08_S4_NO2      1.000000
PT08.S5(O3)      0.723670
T                0.755053
RH               0.640685
AH               0.691889
Name: PT08_S4_NO2, dtype: float64
```

The matrix of regression variables X will contain two variables:

- Temperature T
- Absolute Humidity AH

STEP 1: Prepare the data

In [7]:
```
type(data['Date_Time'])
```

Out[7]:
```
pandas.core.series.Series
```

In [8]:
```
data['DateTimeIndex']= pd.to_datetime(data['Date_Time'])
data = data.set_index(keys=['DateTimeIndex'])
```

Set the frequency attribute of the index to Hourly. This will create several empty rows corresponding to the missing hourly measurements in the original data set. Fill up all the empty data cells with the mean of the corresponding column.

In [9]:
```
ata = data.asfreq('H')
data = data.fillna(data.mean(numeric_only=True))
data.shape
```

Out[9]:
```
(9357, 14)
```

In [10]:
```
data.isin([np.nan, np.inf, -np.inf]).sum()
```

Out[10]:
```
Date_Time        0
CO(GT)           0
PT08.S1(CO)      0
NMHC(GT)         0
C6H6(GT)         0
PT08.S2(NMHC)    0
NOx(GT)          0
PT08.S3(NOx)     0
NO2(GT)          0
PT08_S4_NO2      0
PT08.S5(O3)      0
T                0
RH               0
AH               0
dtype: int64
```

In [11]:
```
dataset_len = len(data)
split_index = round(dataset_len*0.9)
train_set_end_date = data.index[split_index]
```

In [12]:
```
df_train = data.loc[data.index <= train_set_end_date].copy()
df_test = data.loc[data.index > train_set_end_date].copy()
```

STEP 2: Create a Linear Regression model

In [13]:
```
from patsy import dmatrices
expr = 'PT08_S4_NO2 ~ T + AH'
```

In [14]:
```
y_train, X_train = dmatrices(expr, df_train, return_type='dataframe')
y_test, X_test = dmatrices(expr, df_test, return_type='dataframe')
```

Ordinary Least Squares Linear Regression model

In [15]:
```
from statsmodels.regression import linear_model

olsr = linear_model.OLS(y_train, X_train).fit()
```

In [16]:
```
olsr.summary()
```

Out[16]:

OLS Regression Results

Dep. Variable:	PT08_S4_NO2	**R-squared:**	0.654
Model:	OLS	**Adj. R-squared:**	0.654
Method:	Least Squares	**F-statistic:**	7969.
Date:	Tue, 31 May 2022	**Prob (F-statistic):**	0.00
Time:	17:23:49	**Log-Likelihood:**	-59426.
No. Observations:	8422	**AIC:**	1.189e+05
Df Residuals:	8419	**BIC:**	1.189e+05
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1113.6844	7.218	154.302	0.000	1099.536	1127.833
T	20.5513	0.365	56.237	0.000	19.835	21.268
AH	-13.9624	0.405	-34.444	0.000	-14.757	-13.168

Omnibus:	904.777	**Durbin-Watson:**	0.286
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1410.196
Skew:	0.785	**Prob(JB):**	6.02e-307
Kurtosis:	4.247	**Cond. No.**	144.

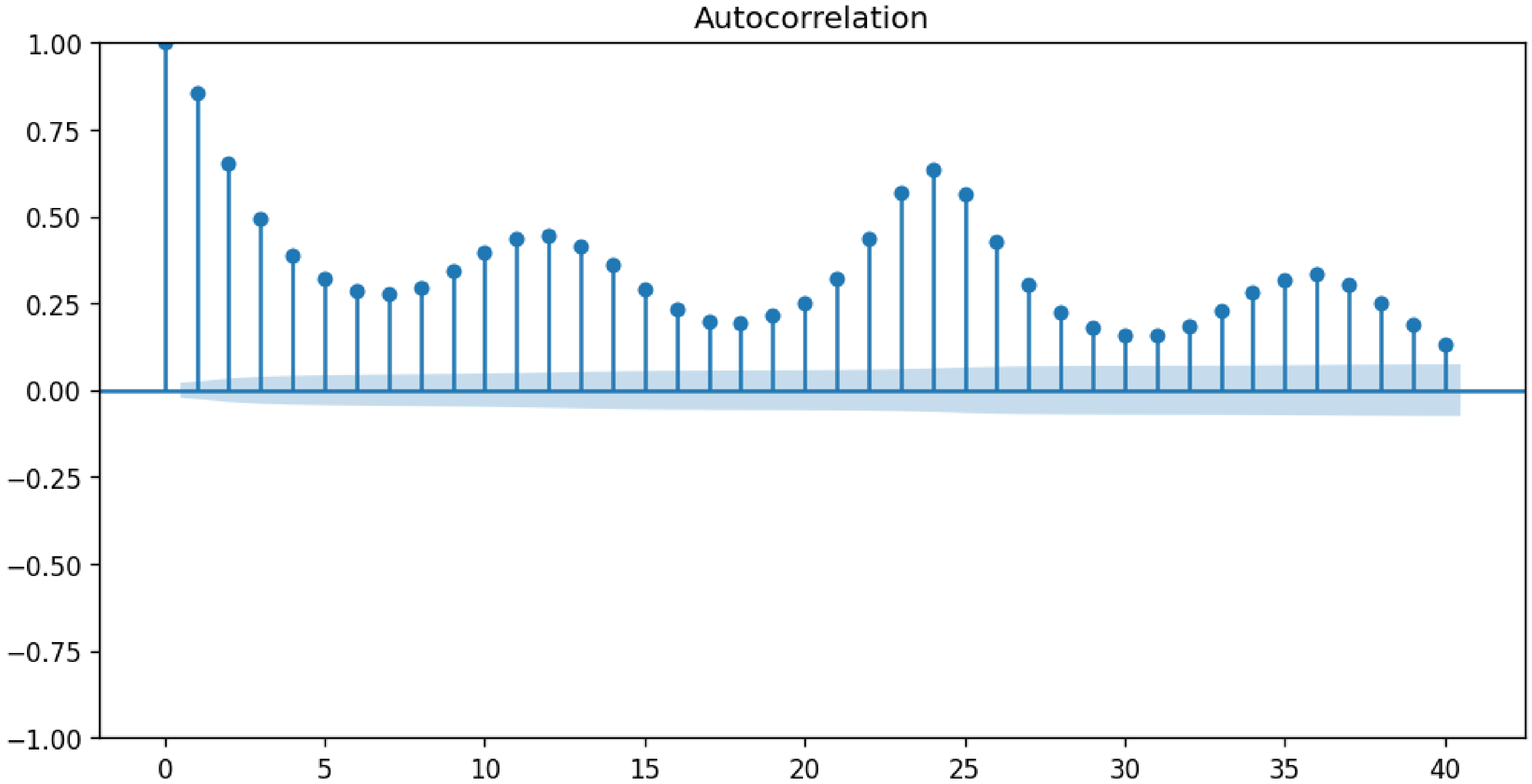
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Regression coefficients of both regression variables T and AH are significant at a 99.99% confidence level as indicated by their P values (P > |t| column) which are essentially 0.

The second thing to note in these results is the output of the Durbin-Watson test which measures the degree of LAG-1 auto-correlation in the residual errors of regression. A value of 2 implies no LAG-1 auto-correlation. A value closer to 0 implies strong positive auto-correlation while a value close to 4 implies a strong negative auto-correlation at LAG-1 among the residuals errors ε.

In the above output, we see that the DW test statistic is 0.28 indicating a strong positive auto-correlation among the residual errors of regression at LAG-1. This was completely expected since the underlying data is a time series and the linear regression model has failed to explain the auto-correlation in the dependent variable. The DW test statistic just confirms it.

In [17]:
```
import statsmodels.graphics.tsaplots as tsa

plt.rcParams.update({'figure.figsize':(10,5), 'figure.dpi':120})

tsa.plot_acf(olsr.resid, alpha=0.05)
plt.show()
```



The ACF tells us three things:

- There are strong auto-correlations extending out to multiple lags indicating that the residual errors time series has a trend. We'll need to de-trend this time series by using one or possibly 2 orders of differencing. Thus, the parameter d is likely to be 1, or possibly 2.
- The wavelike pattern in the ACF evidences a seasonal variation in the data.
- The peak at LAG = 24 indicates that the seasonal period is likely to be 24 hours. i.e. m is likely to be 24. This seems reasonable for data containing vehicular pollution measurements. We'll soon verify this guess using the time series decomposition plot.

Before we estimate the rest of the (S)ARIMA parameters, let's difference the time series once i.e. d=1:

In [18]:
```
olsr.resid.head(2)
```

Out[18]:
```
DateTimeIndex
2004-03-10 18:00:00    309.398174
2004-03-10 19:00:00    181.863043
dtype: float64
```

In [19]:
```
olsr_resid_diff_1 = olsr.resid.diff()
olsr_resid_diff_1.head(2)
```

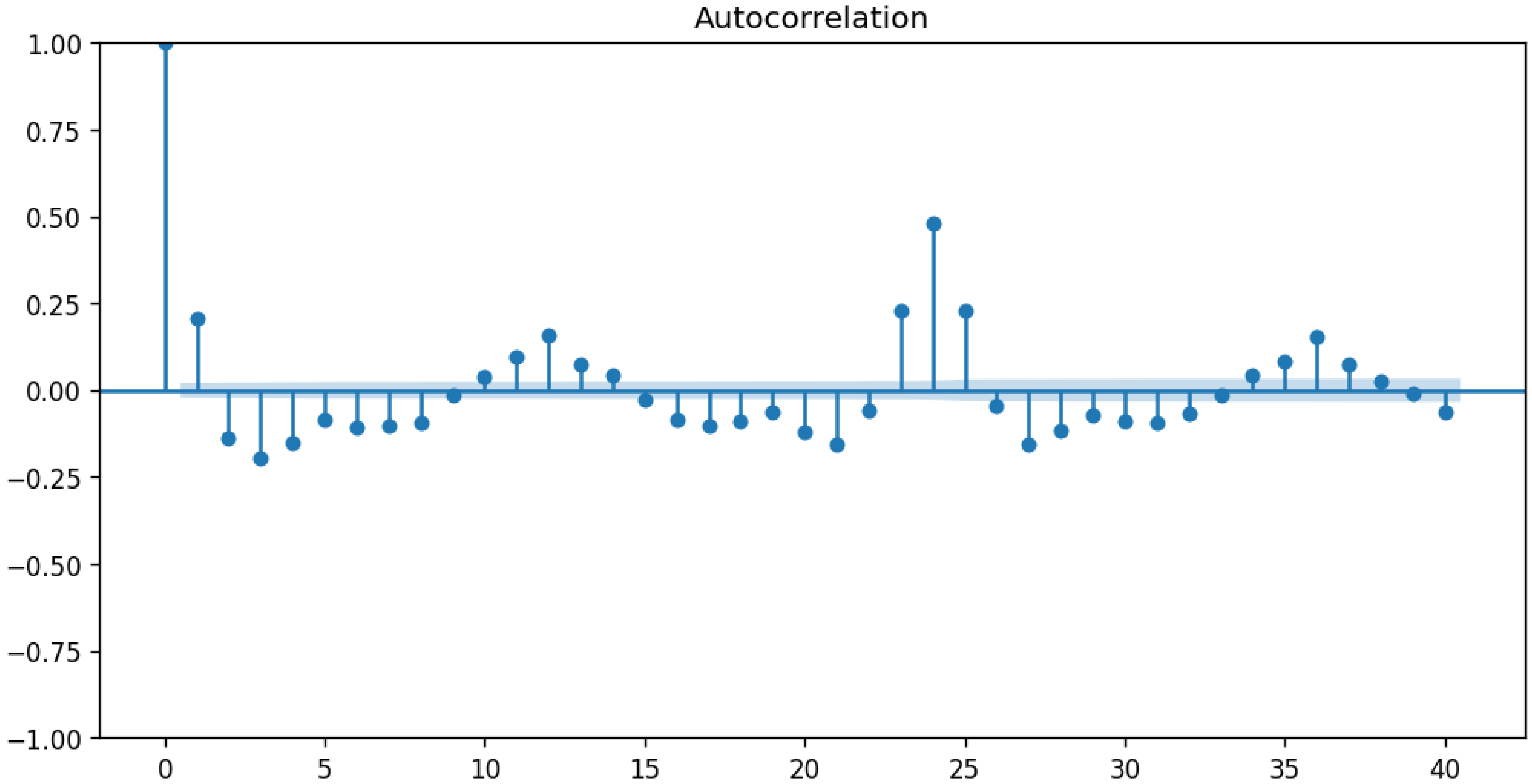
Out[19]:
```
DateTimeIndex
2004-03-10 18:00:00           NaN
2004-03-10 19:00:00   -127.535131
dtype: float64
```

In [20]:
```
olsr_resid_diff_1.isnull().sum()
```

Out[20]:
```
1
```

In [21]:
```
olsr_resid_diff_1 = olsr_resid_diff_1.dropna()
```

In [22]:
```
plt.rcParams.update({'figure.figsize':(10,5), 'figure.dpi':120})

tsa.plot_acf(olsr_resid_diff_1, alpha=0.05)
plt.show()
```

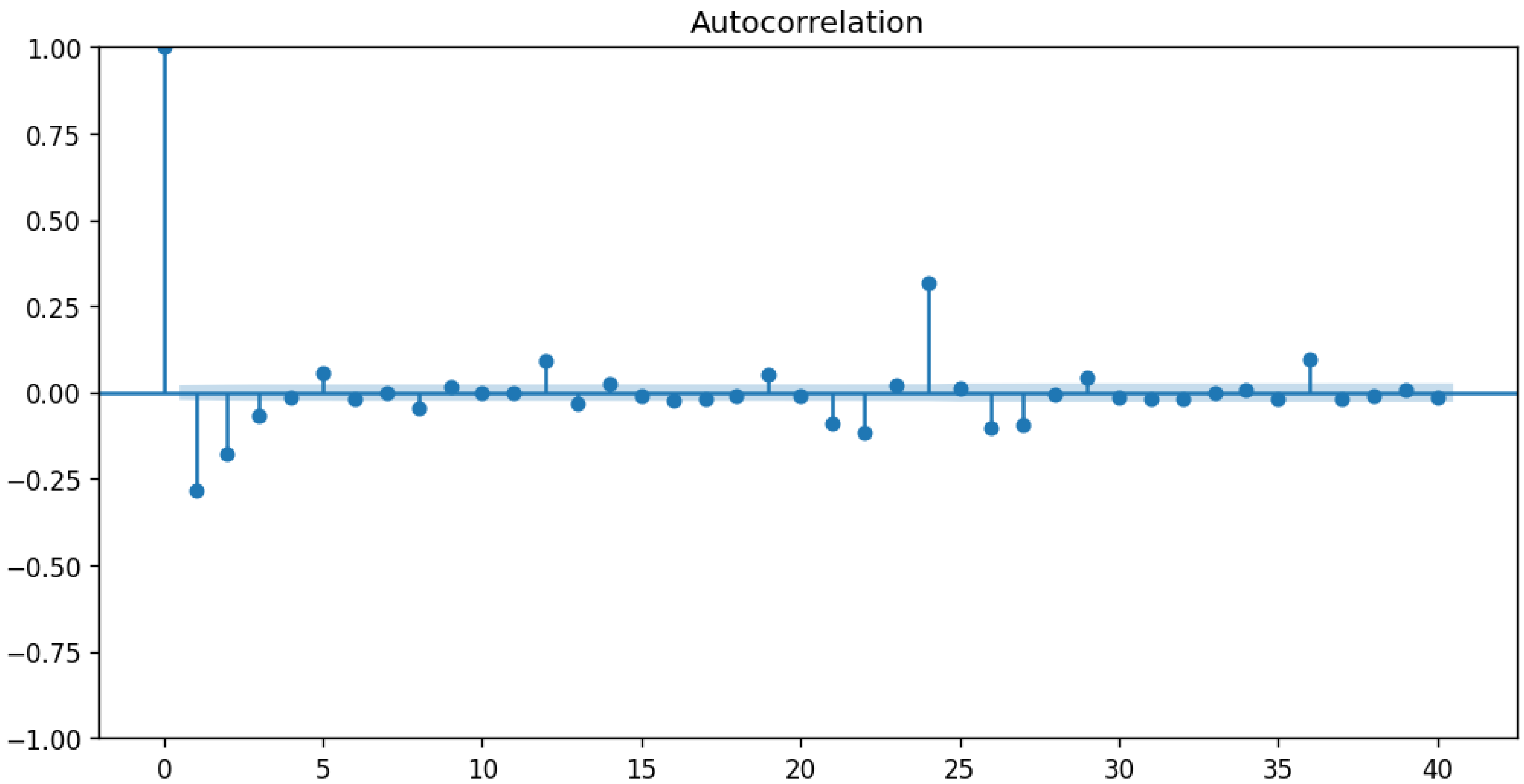


We now see a very different picture in the ACF. The auto-correlations are significantly reduced at all lags. The wavelike pattern still exists but that's because we did nothing to remove the possible seasonal variation. The LAG-24 auto-correlation is once again especially prominent.

We see that there is still a significant auto-correlation at LAG-1 in the differenced time series. We could try extinguishing it by taking one more difference, i.e. d=2 and plotting the resulting time series' ACF:

In [23]:
```
olsr_resid_diff_2 = olsr_resid_diff_1.diff()
olsr_resid_diff_2 = olsr_resid_diff_2.dropna()

plt.rcParams.update({'figure.figsize':(10,5), 'figure.dpi':120})
```

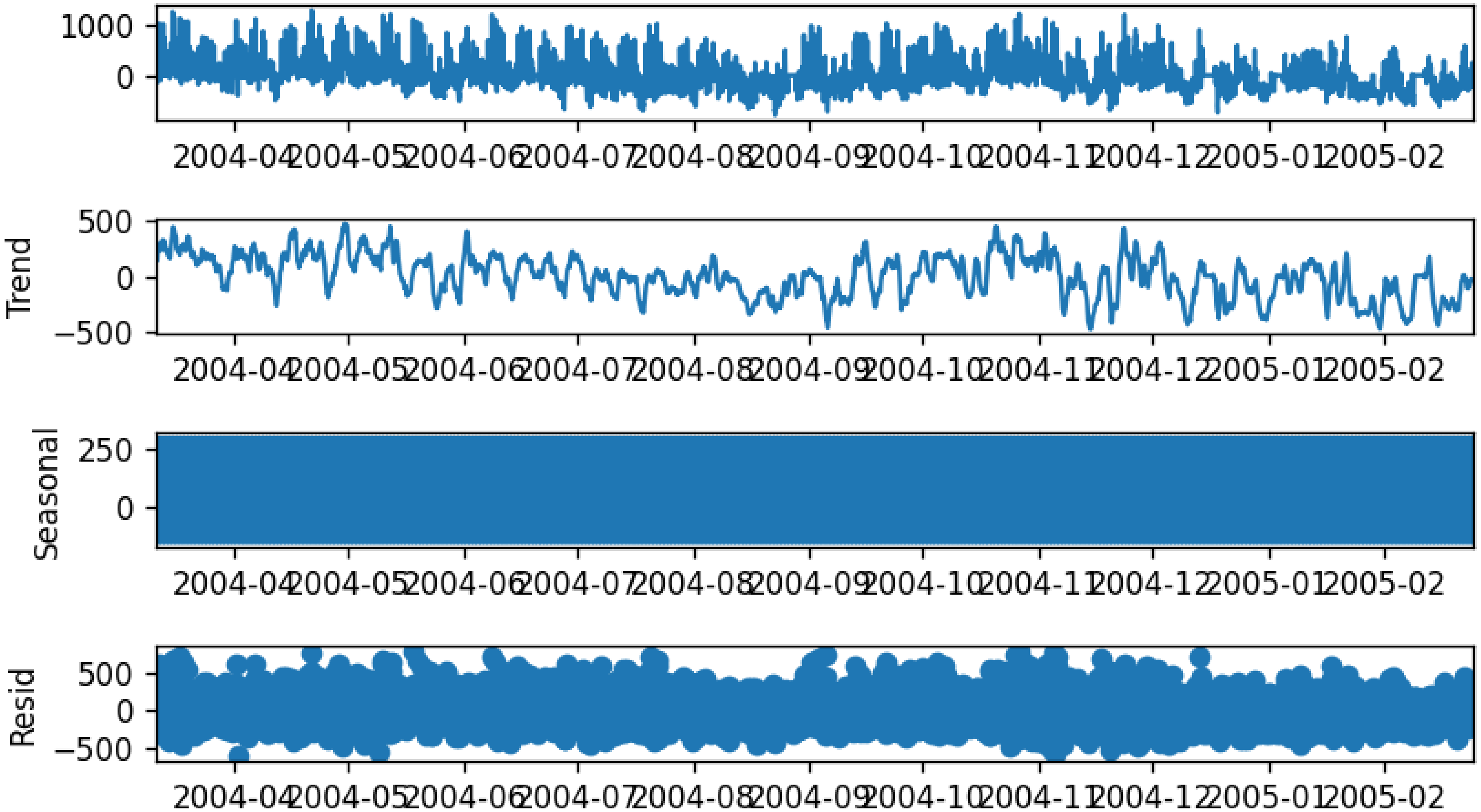
```
tsa.plot_acf(olsr_resid_diff_2, alpha=0.05)
plt.show()
```

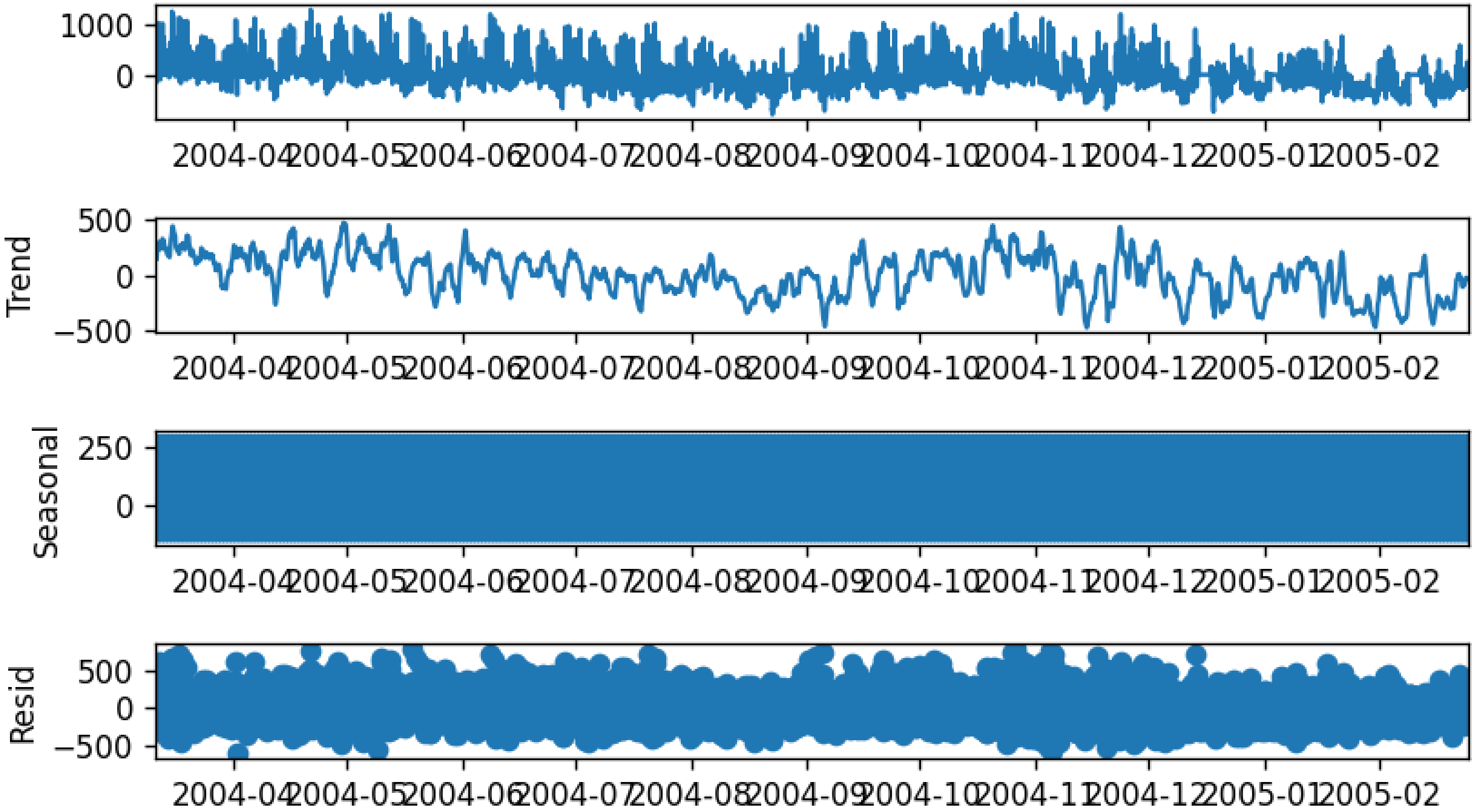


Differing the time series a second time has produced a heavy negative auto-correlation at LAG-1. This is bad sign. We seem to have over-done the differencing. We should stick with d=1.

In [24]:
```
from statsmodels.tsa.seasonal import seasonal_decompose
components = seasonal_decompose(olsr.resid)

plt.rcParams.update({'figure.figsize':(7,4), 'figure.dpi':120})
components.plot()
```

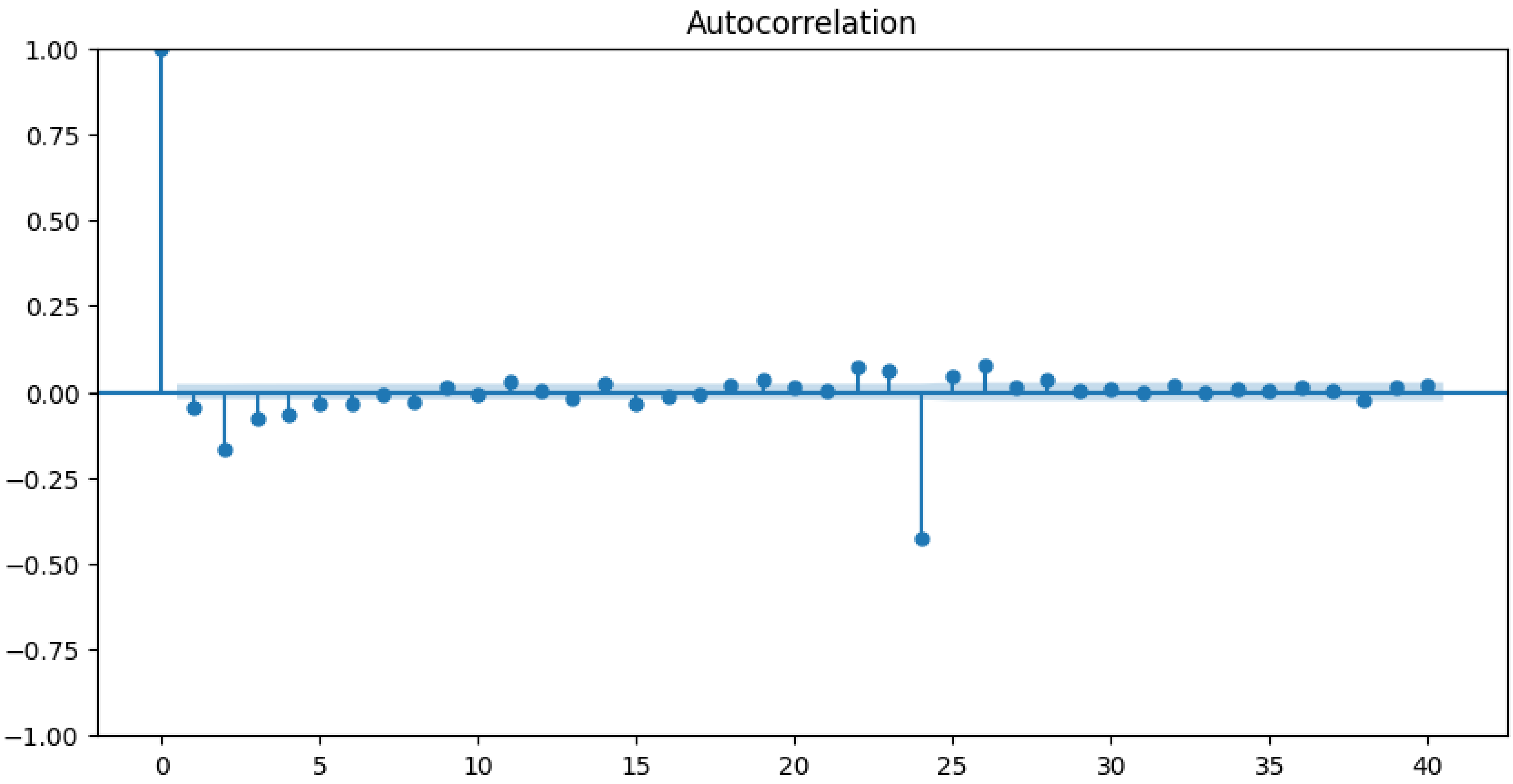
Out[24]:


1000
0
2004-04 2004-05 2004-06 2004-07 2004-08 2004-09 2004-10 2004-11 2004-12 2005-01 2005-02
Trend
500
0
−500
Seasonal
250
0
Resid
500
0
−500



In [25]:

```
olsr_resid_diff_1_24 = olsr_resid_diff_1.diff(periods=24)
olsr_resid_diff_1_24 = olsr_resid_diff_1_24.dropna()

plt.rcParams.update({'figure.figsize':(10,5), 'figure.dpi':100})

tsa.plot_acf(olsr_resid_diff_1_24, alpha=0.05)
plt.show()
```



The strong negative correlation at LAG-24 indicates a Seasonal MA (SMA) signature with order 1. i.e. Q=1. Moreover, an absence of positive correlation at LAG-1, indicates an absence of a Seasonal AR component. i.e. P=0.

We have fixed P=0, D=1 and Q=1, and m=24 hours

We have managed to estimate all 7 params of the SARIMA model as follows: p=1, d=1, q=0, P=0, D=1, Q=1 and m=24 i.e. SARIMAX(1,1,0)(0,1,1)24

STEP 4: Build and fit the Regression Model with Seasonal ARIMA errors

In [26]:
```
X_train_minus_intercept = X_train.drop('Intercept', axis=1)
X_train_minus_intercept = X_train_minus_intercept.asfreq('H')
```

In [27]:
```
y_train = y_train.asfreq('H')
```

In [28]:
```
from statsmodels.tsa.arima.model import ARIMA
sarimax_model = ARIMA(endog=y_train,
                      exog=X_train_minus_intercept,
                      order=(1,1,0),
                      seasonal_order=(0,1,1,24))
```

In [29]:
```
sarimax_results = sarimax_model.fit()
```

In [30]:
```
sarimax_results.summary()
```

Out[30]:

SARIMAX Results

Dep. Variable:	PT08_S4_NO2	**No. Observations:**	8422
Model:	ARIMA(1, 1, 0)x(0, 1, [1], 24)	**Log Likelihood**	-51815.958
Date:	Tue, 31 May 2022	**AIC**	103641.916
Time:	17:24:53	**BIC**	103677.094
Sample:	03-10-2004	**HQIC**	103653.928
	- 02-24-2005		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
T	-4.6351	0.628	-7.383	0.000	-5.865	-3.405
AH	13.3304	0.673	19.808	0.000	12.011	14.649
ar.L1	-0.0041	0.008	-0.519	0.604	-0.020	0.011
ma.S.L24	-0.9184	0.003	-281.217	0.000	-0.925	-0.912
sigma2	1.334e+04	124.277	107.318	0.000	1.31e+04	1.36e+04

Ljung-Box (L1) (Q):	0.01	**Jarque-Bera (JB):**	5180.05
Prob(Q):	0.93	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.66	**Skew:**	-0.10
Prob(H) (two-sided):	0.00	**Kurtosis:**	6.84

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [31]:

```
sarimax_model = ARIMA(endog=y_train,
                      exog=X_train_minus_intercept,
                      order=(1,1,0),
                      seasonal_order=(0,1,0,24))
```

```
sarimax_results = sarimax_model.fit()
sarimax_results.summary()
```

Out[31]:

SARIMAX Results

Dep. Variable:	PT08_S4_NO2	**No. Observations:**	8422
Model:	ARIMA(1, 1, 0)x(0, 1, 0, 24)	**Log Likelihood**	-53887.658
Date:	Tue, 31 May 2022	**AIC**	107783.315
Time:	17:24:58	**BIC**	107811.458
Sample:	03-10-2004	**HQIC**	107792.925
	- 02-24-2005		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
T	-4.9003	0.688	-7.123	0.000	-6.249	-3.552
AH	13.6175	0.738	18.442	0.000	12.170	15.065
ar.L1	-0.0519	0.008	-6.482	0.000	-0.068	-0.036
sigma2	2.195e+04	208.195	105.422	0.000	2.15e+04	2.24e+04

Ljung-Box (L1) (Q):	0.67	**Jarque-Bera (JB):**	4288.52
Prob(Q):	0.41	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.69	**Skew:**	0.06
Prob(H) (two-sided):	0.00	**Kurtosis:**	6.50

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

STEP 5: Prediction

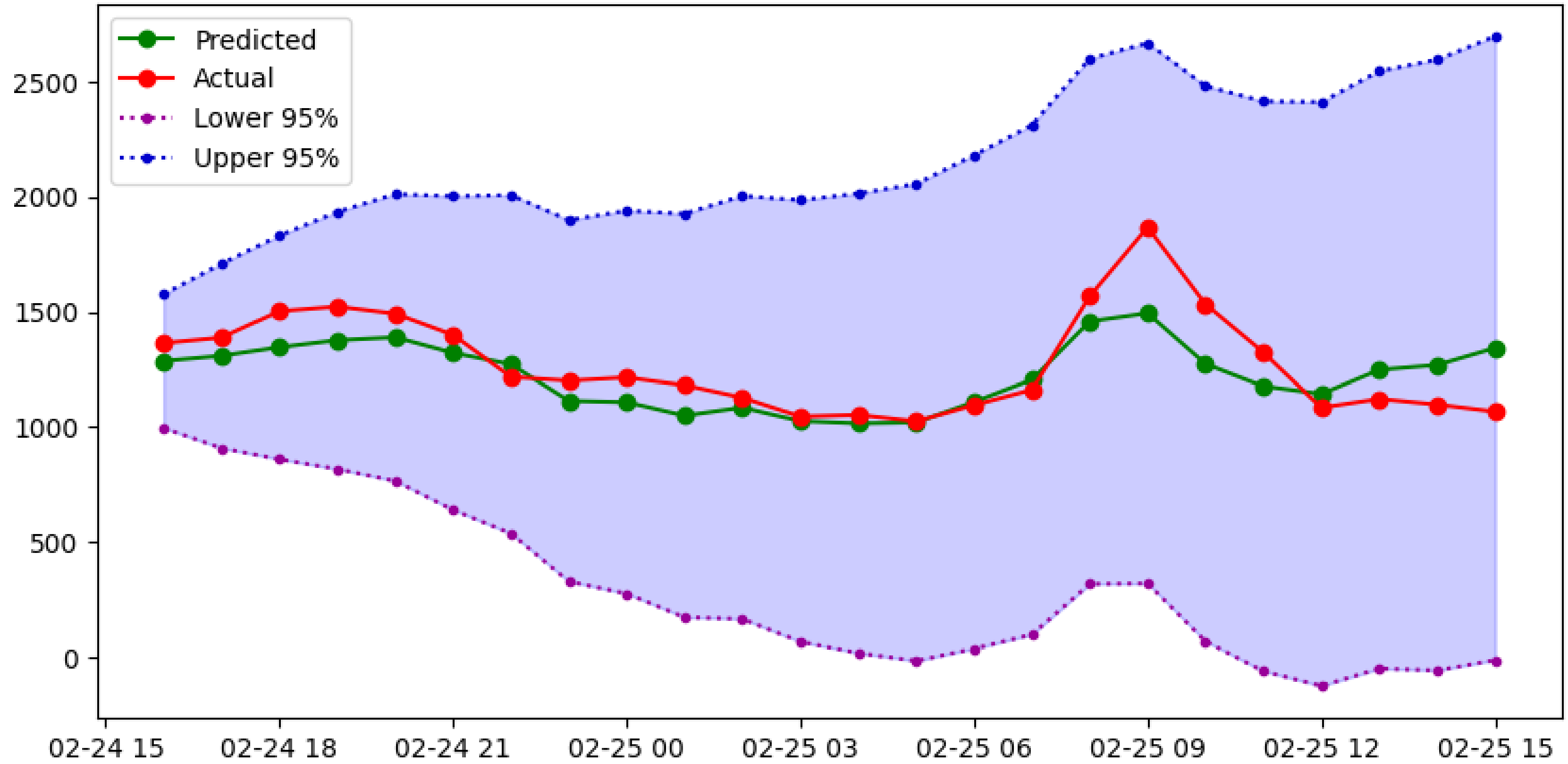
In [32]:
```
X_test_minus_intercept = X_test.drop('Intercept', axis=1)
X_test_minus_intercept = X_test_minus_intercept.asfreq('H')
y_test = y_test.asfreq('H')
```

In [33]:
```
predictions = sarimax_results.get_forecast(steps=24, exog=X_test_minus_intercept[:24])
predictions.summary_frame().head()
```

Out[33]:

PT08_S4_NO2	mean	mean_se	mean_ci_lower	mean_ci_upper
2005-02-24 16:00:00	1286.066400	148.149772	995.698183	1576.434617
2005-02-24 17:00:00	1308.454485	204.153164	908.321635	1708.587335
2005-02-24 18:00:00	1345.693115	248.033957	859.555492	1831.830738
2005-02-24 19:00:00	1376.087728	285.232092	817.043100	1935.132356
2005-02-24 20:00:00	1388.823150	318.110281	765.338456	2012.307845

In [34]:
```
plt.rcParams.update({'figure.figsize':(10,5), 'figure.dpi':100})

predicted, = plt.plot(X_test_minus_intercept[:24].index, predictions.summary_frame()['mean'], 'go-', label='Predicted')

actual,  = plt.plot( y_test[:24], 'ro-', label='Actual')

lower, = plt.plot(X_test_minus_intercept[:24].index, predictions.summary_frame()['mean_ci_lower'], color='#990099', marker='.', l

upper, = plt.plot(X_test_minus_intercept[:24].index, predictions.summary_frame()['mean_ci_upper'], color='#0000cc', marker='.', l

plt.fill_between(X_test_minus_intercept[:24].index, predictions.summary_frame()['mean_ci_lower'], predictions.summary_frame()['me
plt.legend(handles=[predicted, actual, lower, upper])
plt.show()
```



ARIMA model

In [35]:

```
df = pd.read_csv('https://raw.githubusercontent.com/selva86/datasets/master/wwwusage.csv', names=['value'], header=0)
```

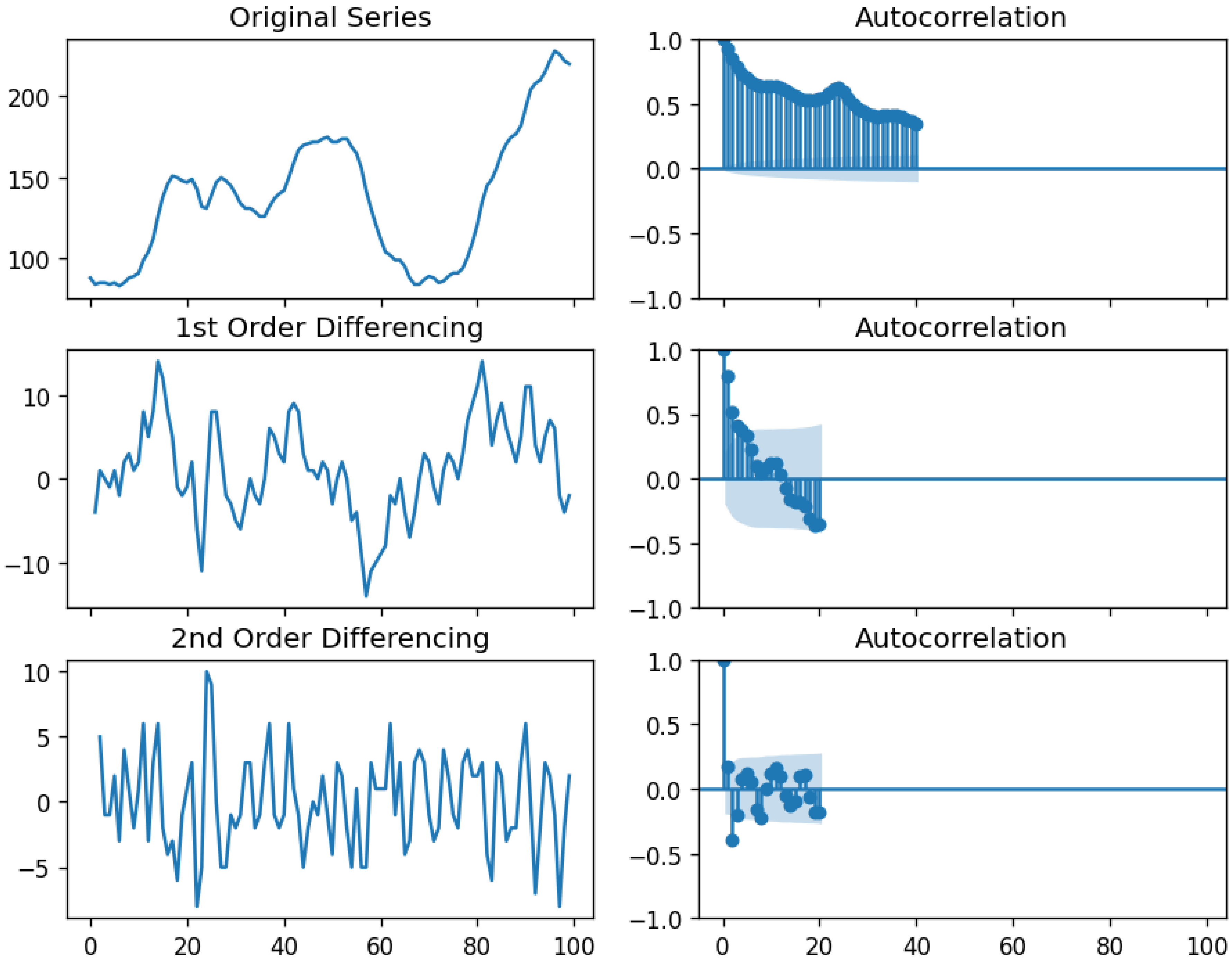
Find the order of differencing (d) in ARIMA model

In [36]:

```
from statsmodels.tsa.stattools import adfuller
from numpy import log
result = adfuller(df)
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
```

```
ADF Statistic: -2.464240
p-value: 0.124419
```

In [37]:
```
import numpy as np, pandas as pd
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
import matplotlib.pyplot as plt
plt.rcParams.update({'figure.figsize':(9,7), 'figure.dpi':120})

# Original Series
fig, axes = plt.subplots(3, 2, sharex=True)
axes[0, 0].plot(df.value); axes[0, 0].set_title('Original Series')
plot_acf(data[['PT08_S4_NO2']], ax=axes[0, 1])

# 1st Differencing
axes[1, 0].plot(df.value.diff()); axes[1, 0].set_title('1st Order Differencing')
plot_acf(df.value.diff().dropna(), ax=axes[1, 1])

# 2nd Differencing
axes[2, 0].plot(df.value.diff().diff()); axes[2, 0].set_title('2nd Order Differencing')
plot_acf(df.value.diff().diff().dropna(), ax=axes[2, 1])

plt.show()
```

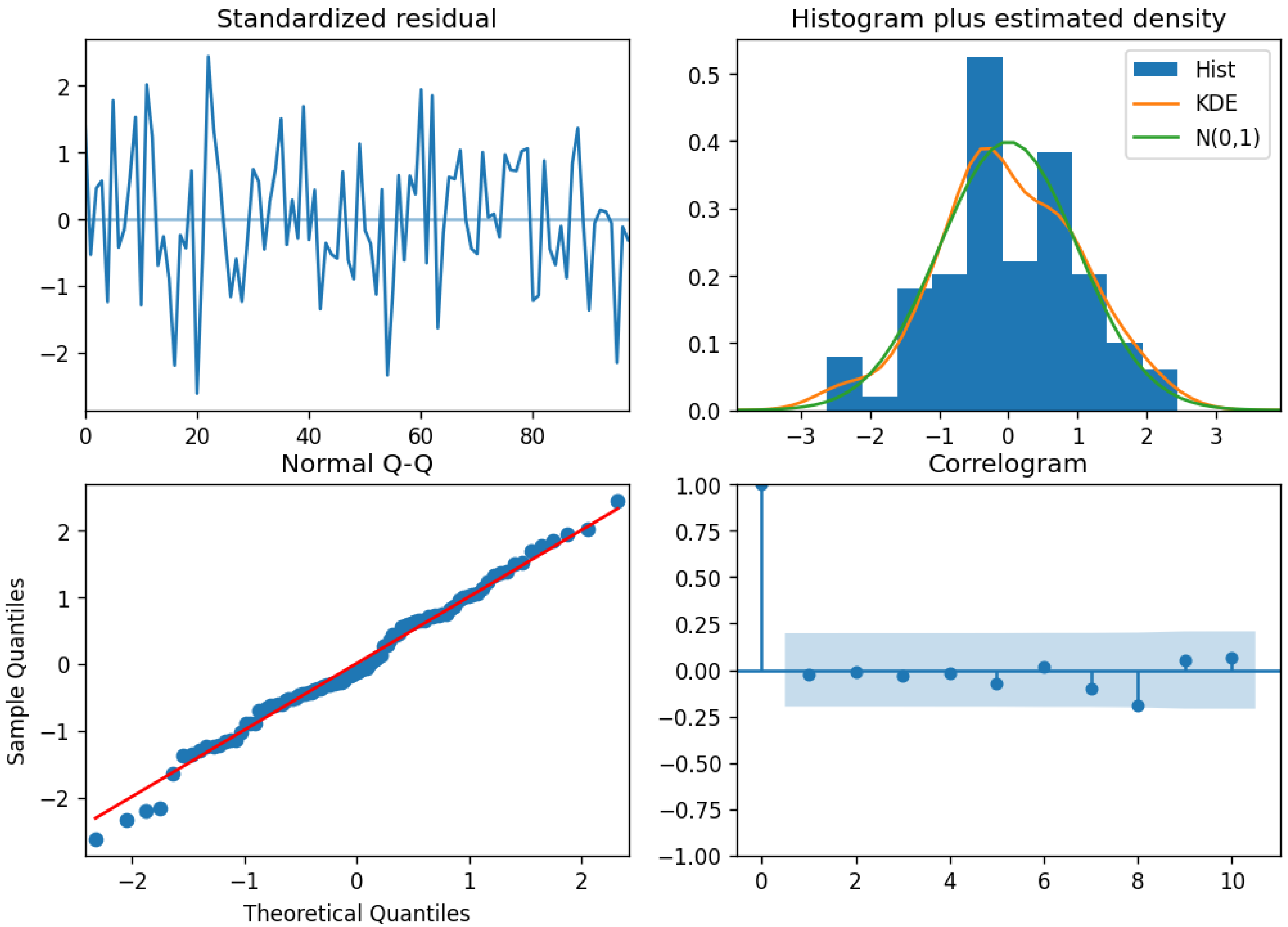

Original Series
Autocorrelation
1st Order Differencing
Autocorrelation
2nd Order Differencing
Autocorrelation
200
150
100
10
0
−10
5
−5
1.0
0.5
0.0
−0.5
−1.0
0
20
40
60
80
100

Auto Arima Forecast

In [38]:
```
from statsmodels.tsa.arima_model import ARIMA
import pmdarima as pm
model = pm.auto_arima(df.value, start_p=1, start_q=1,
                      test='adf',       # use adftest to find optimal 'd'
                      max_p=3, max_q=3, # maximum p and q
                      m=1,              # frequency of series
                      d=None,           # let model determine 'd'
                      seasonal=False,   # No Seasonality
                      start_P=0,
                      D=0,
                      trace=True,
                      error_action='ignore',
                      suppress_warnings=True,
                      stepwise=True)

print(model.summary())
```

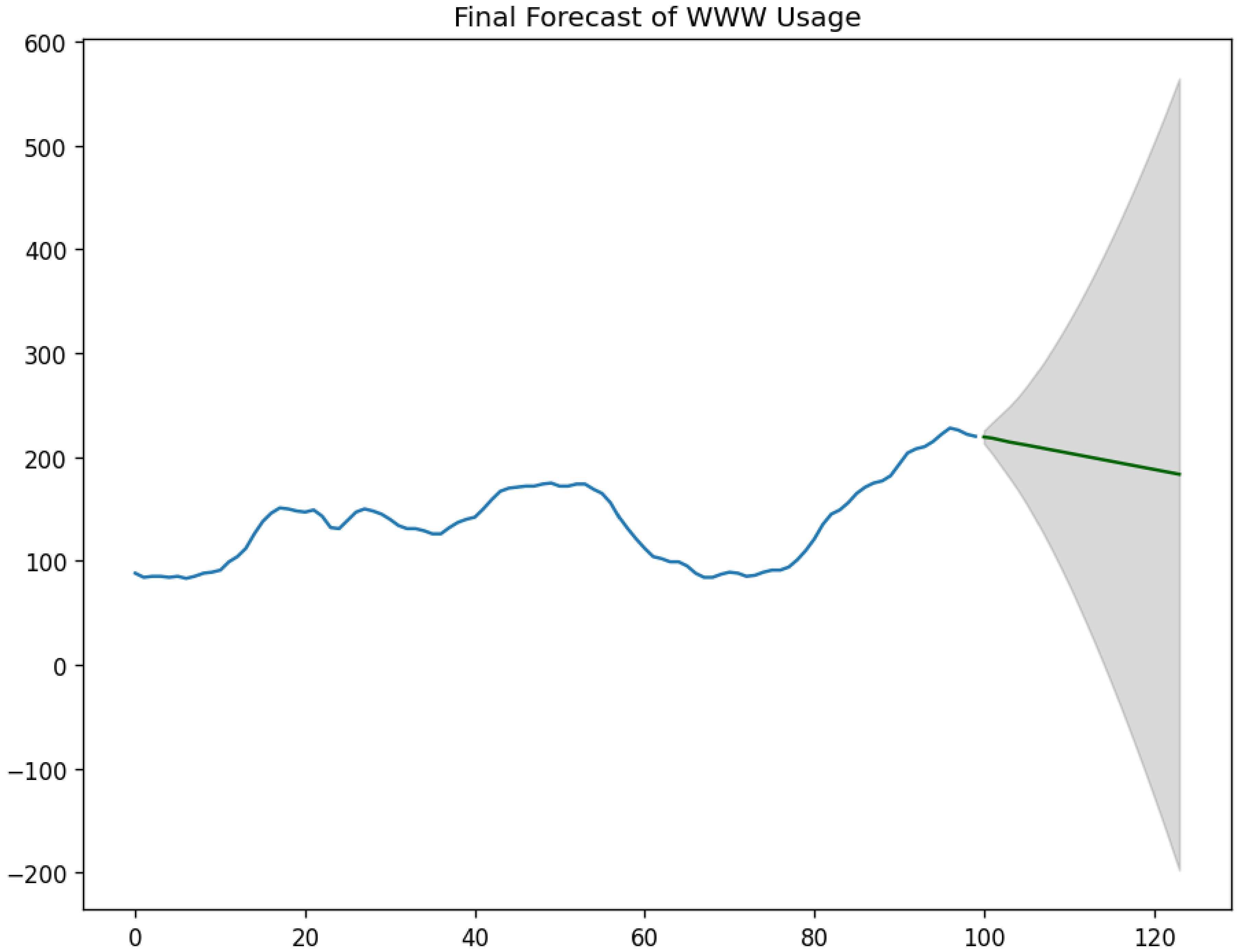
```
Performing stepwise search to minimize aic
 ARIMA(1,2,1)(0,0,0)[0] intercept   : AIC=525.587, Time=0.03 sec
 ARIMA(0,2,0)(0,0,0)[0] intercept   : AIC=533.474, Time=0.01 sec
 ARIMA(1,2,0)(0,0,0)[0] intercept   : AIC=532.437, Time=0.02 sec
 ARIMA(0,2,1)(0,0,0)[0] intercept   : AIC=525.893, Time=0.03 sec
 ARIMA(0,2,0)(0,0,0)[0]             : AIC=531.477, Time=0.01 sec
 ARIMA(2,2,1)(0,0,0)[0] intercept   : AIC=515.248, Time=0.05 sec
 ARIMA(2,2,0)(0,0,0)[0] intercept   : AIC=513.459, Time=0.03 sec
 ARIMA(3,2,0)(0,0,0)[0] intercept   : AIC=515.284, Time=0.05 sec
 ARIMA(3,2,1)(0,0,0)[0] intercept   : AIC=inf, Time=0.22 sec
 ARIMA(2,2,0)(0,0,0)[0]             : AIC=511.465, Time=0.03 sec
 ARIMA(1,2,0)(0,0,0)[0]             : AIC=530.444, Time=0.00 sec
 ARIMA(3,2,0)(0,0,0)[0]             : AIC=513.291, Time=0.03 sec
 ARIMA(2,2,1)(0,0,0)[0]             : AIC=513.256, Time=0.03 sec
 ARIMA(1,2,1)(0,0,0)[0]             : AIC=523.592, Time=0.02 sec
 ARIMA(3,2,1)(0,0,0)[0]             : AIC=inf, Time=0.16 sec

Best model:  ARIMA(2,2,0)(0,0,0)[0]
Total fit time: 0.714 seconds
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  100
Model:               SARIMAX(2, 2, 0)   Log Likelihood                -252.732
Date:                Tue, 31 May 2022   AIC                            511.465
Time:                        17:25:01   BIC                            519.220
Sample:                             0   HQIC                           514.601
                                - 100
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.2579      0.103      2.510      0.012       0.056       0.459
ar.L2         -0.4407      0.087     -5.093      0.000      -0.610      -0.271
sigma2        10.1268      1.519      6.668      0.000       7.150      13.103
===================================================================================
Ljung-Box (L1) (Q):                   0.05   Jarque-Bera (JB):                 0.10
Prob(Q):                              0.82   Prob(JB):                         0.95
Heteroskedasticity (H):               0.49   Skew:                            -0.07
Prob(H) (two-sided):                  0.05   Kurtosis:                         2.92
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

In [39]:
```
model.plot_diagnostics(figsize=(10,7))
plt.show()
```



In [40]:
```
# Forecast
```

```
n_periods = 24
fc, confint = model.predict(n_periods=n_periods, return_conf_int=True)
index_of_fc = np.arange(len(df.value), len(df.value)+n_periods)

# make series for plotting purpose
fc_series = pd.Series(fc, index=index_of_fc)
lower_series = pd.Series(confint[:, 0], index=index_of_fc)
upper_series = pd.Series(confint[:, 1], index=index_of_fc)

# Plot
plt.plot(df.value)
plt.plot(fc_series, color='darkgreen')
plt.fill_between(lower_series.index,
                 lower_series,
                 upper_series,
                 color='k', alpha=.15)
plt.title("Final Forecast of WWW Usage")
plt.show()
```


Final Forecast of WWW Usage
600
500
400
300
200
100
0
−100
−200
0
20
40
60
80
100
120

In [ ]: