Roll No. 22EGJCS049

GLOBAL INSTITUTE OF TECHNOLOGY

B. Tech. I Semester, II Midterm Exam 2022

1FY2-01 / ENGINEERING MATHEMATICS-I /

3/3/2023/ FRIDAY

Time: 3 Hours **Maximum Marks: 70**

Attempt all questions

Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. **no supplementary sheet shall be issued in any case.**

Part A(Answer should be given up to 25 words only)

All questions are compulsory

Q.1 The directional derivative of the scalar function $f(x,y,z) = \sin x + e^y + z^2$ at the point $P(\pi/2, 0, 1)$ in the direction of the vector $(2,3,-1)$ is (CO2)

Q.2 Let $f(x,y) = x^y + y^x$, what is the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$(CO1)

Q.3 Define stationary point and saddle point for a function. (CO1)

Q.4 Find the extremum for the function $u = x^2y^2 - 5x^2 - 8xy - 5y^2$.(CO4)

Q.5State the stoke'stheorem.(CO2)

Q.6 In the Taylor's series expansion of $\cos x$about x=1, the coefficient of $(x-1)^2$ is ... (CO2)

Q.7 Find the curl of vector $V = (4xy - z^3)\hat{\imath} + (2x^2)\hat{\jmath} - 3xz^2\hat{k}$, at $x = y = z = 1$ (CO2)

Q.8 If V is a three dimensional region bounded by planes $x \geq 0, y \geq 0, z \geq 0 \& x + y + z \leq 1$ then $\iiint_V x^{p-1}y^{m-1}z^{n-1}dx\,dy\,dz =$ (CO4)

Q.9Evaluate $\int_C \vec{F}.dr$,where $\vec{F} = x^2y^2\,\hat{\imath} + y\,\hat{\jmath}$, and C is the curve $y^2 = 4x$ in the xy-plane from (0,0) to (4,4). (CO1)

Q.10 If $f(x,y) = \frac{x^3+y^2}{2y^3+x}$, when $(x,y) \neq (0,0)$and f(0,0) = 0.

Discuss the continuity of $f(x,y)$ at the origin. (CO2)

10x 2 = 20

Part B Analytical/Problem solving questions

Attempt all questions (word Limit 100)

Q.1 Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (CO4)

Q.2 If $u = \sec^{-1}\frac{x^3+y^3}{x+y}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$. (CO2)

Q.3Find the work done in moving a particle once round a square C formed by lines y ±1, x±1 in the xy- plane if the force field is given by $\vec{F} = (x^2 + xy + z)\hat{\imath} + (x^2 + y^2 - z)\hat{\jmath} + xy\hat{k}$ (CO1)

Q.4If $u = x f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = 0$ (CO2)

Q.5Test the convergence or divergence of the following series if $x > 0$.

$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$ (CO3)

5 x 4 =20

Part C(Descriptive/Analytical/Problem Solving/Design Question)

Attempt all questions

Q.1 If $\vec{F} = 4xz\,\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$, Evaluate $\iint \vec{F}.\hat{n}\,ds$ over S, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$ (CO4)

Q.2State the Green's theorem and verify this, in the plane for $\oint(x^2 - 2xy)dx + (x^2y + 3)dy$, where C is theboundary of the region defined by $y^2 = 8x$and x =2 . (CO1)

Q.3Find the maximum and minimum values of $\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$, where $px + my + nz = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (CO4)

3x 10 =30