Q3. Singular Value Decomposition (SVD):

Perform Singular Value Decomposition on the matrix A obtained in Question 2. Separate and print matrices U, Σ, and .Verify that A equals the product of U, Σ, and VV.Additionally, find the rank 2 and rank 3 approximations of matrix A

In [2]:
```
import numpy as np
import sympy as sp
matrix_A = np.array([[3,8,5,1,2],
                     [2,8,8,2,1],
                     [5,5,4,5,7],
                     [4,4,1,3,9],
                     [5,6,5,8,2]])
print("Matrix A:")
sp.Matrix(matrix_A)
```

```
Matrix A:
```

Out[2]:
$$\begin{bmatrix} 3 & 8 & 5 & 1 & 2 \\ 2 & 8 & 8 & 2 & 1 \\ 5 & 5 & 4 & 5 & 7 \\ 4 & 4 & 1 & 3 & 9 \\ 5 & 6 & 5 & 8 & 2 \end{bmatrix}$$

In [3]:
```
U, Sigma, Vt = np.linalg.svd(matrix_A)
Sigma_matrix = np.diag(Sigma)
rank2approximation = U[:, :2] @ Sigma_matrix[:2, :2] @ Vt[:2, :]
rank3approximation = U[:, :3] @ Sigma_matrix[:3, :3] @ Vt[:3, :]
print("\nRank 2 Approximation of Matrix A:")
sp.Matrix(rank2approximation)
```

```
Rank 2 Approximation of Matrix A:
```

Out[3]:
$$\begin{bmatrix} 2.85031050591369 & 6.64577586599108 & 5.70352697545123 & 3.05266335687381 \\ 2.76601093442301 & 8.03048882046051 & 7.24801813048877 & 3.0399869968941 \\ 4.86095139137802 & 5.55725750174494 & 3.46845279276121 & 4.92253665867825 \\ 4.55959670350939 & 3.1876027856745 & 1.05935508641506 & 4.51797125854986 \\ 4.13775460922921 & 7.15304781173591 & 5.5771027646645 & 4.30907678480227 \end{bmatrix}$$

In [4]:
```
print("\nRank 3 Approximation of Matrix A:")
sp.Matrix(rank3approximation)
```

```
Rank 3 Approximation of Matrix A:
```

Out[4]:

$$\begin{bmatrix} 2.42501317141526 & 7.38668619311697 & 5.83536006755488 & 1.14159643116996 \\ 2.50517211626757 & 8.48489602786658 & 7.32887259429266 & 1.86791184409676 \\ 4.90223781524851 & 5.48533243818483 & 3.45565488297143 & 5.10805655908518 \\ 4.19839446005451 & 3.81685307870157 & 1.17132006895038 & 2.89491462272411 \\ 4.95461255804467 & 5.72999988507534 & 5.32389424331604 & 7.97961521133762 \end{bmatrix}$$

In [ ]:
```

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