Introduction to Numerical Optimization - Project
Solar Microgrid

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$$
\begin{aligned}
\min_{C_{PV}, E_B} \quad & \pi^{PV} C_{PV} + \pi^B E_B + N \sum_t \left(\left(\pi^{G+} P_t^{G+} - \pi^{G-} P_t^{G-} \right) \Delta t \right) \\
\text{s.t.} \quad & P_t^{PV} \leq \eta^{PV} \, i_t \, C_{PV}, \\
& P_t^{PV} + P_t^{G+} + P_t^{dis} - P_t^{ch} - P_t^{G-} = P_t^C, \\
& E_t^{soc} = E_{t-1}^{soc} + \eta^B P_t^{ch} \Delta t - \frac{1}{\eta^B} P_t^{dis} \Delta t, \\
& 0 \leq E_t^{soc} \leq E_B, \\
& E_1^{soc} = E_{T+1}^{soc}, \\
& C_{PV}, E_B, P_t^{G+}, P_t^{G-}, P_t^{ch}, P_t^{dis}, P_t^{PV} \geq 0.
\end{aligned}
$$

Explanation of variables (those not included in the statement):

P_t^{ch} - battery charging power during the hour t [W]
P_t^{dis} - battery discharging power during the hour t [W]
E_t^{soc} - battery state of charge level during the hour t [Wh]
Δt - 1 hour
N - number of days

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MICROGRID – COST MINIMIZATION RESULTS	
Simulation period	1 year
Status	OPTIMAL
PV capacity (C_{PV})	0.000 kWp
Battery capacity (E_B)	0.000 kWh
Objective value	345.29
Simulation period	5 years
Status	OPTIMAL
PV capacity (C_{PV})	0.501 kWp
Battery capacity (E_B)	0.000 kWh
Objective value	1713.80

Table 1: Microgrid cost minimization summary - task 2

One year is too short a period to make it worth buying solar panels and batteries; it is more profitable to buy electricity from the grid. However, for 5 years, it is already worth buying a small panel that supplies electricity for the current needs of the household.

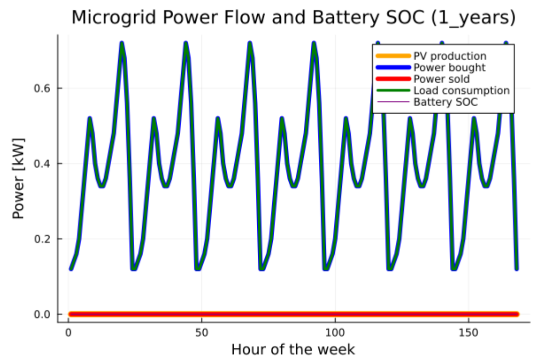

Figure 1: Production, consumption, state of charge for 1 year - task 2

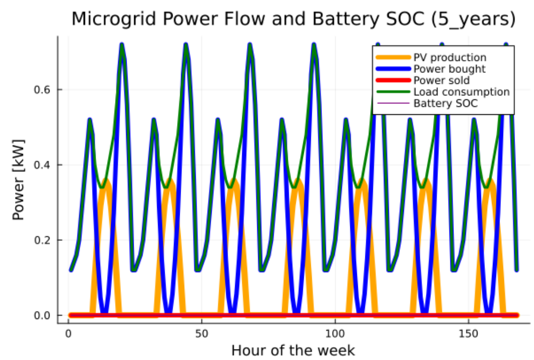

Figure 2: Production, consumption, state of charge for 5 years - task 2

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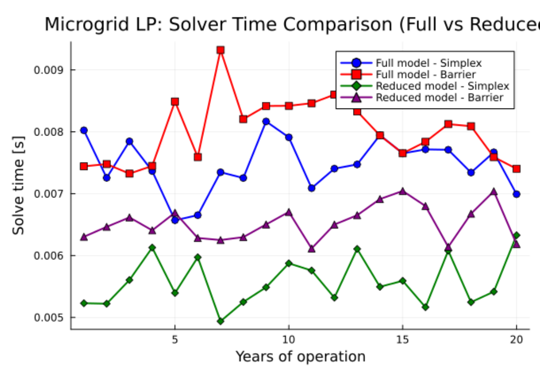

Figure 3: Time comparison for all mentioned methods - task 3 and 4

Why the Simplex Method Is Faster than the Barrier Method?

- Simplex method moves along the edges of the feasible polyhedron from one vertex to another. In each step algorithm updates locally inverted **basis matrix**. Each iteration exploits the sparse structure of the constraint matrix (computing cost function is cheap due to it).
- In contrast each Barrier method iteration requires solving a dense linear system derived from the KKT conditions. And even if A is sparse, after multiplying matrices and Cholesky or LU decomposition, the sparsity is partly lost due to fill-in effect. Memory usage increases significantly, each iteration becomes numerically heavy.

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Cumulated energy gain function: $S_t = E_0^{soc} + \sum_{k=1}^{t} \left(\eta^B P_k^{ch} - \frac{1}{\eta^B} P_k^{dis} \right) \Delta t$

$$
\begin{aligned}
\min_{C_{PV}, E_B} \quad & \pi^{PV} C_{PV} + \pi^B E_B + N \sum_{t=1}^{T} \left(\pi^{G+} P_t^{G+} - \pi^{G-} P_t^{G-} \right) \Delta t \\
\text{s.t.} \quad & P_t^{PV} \le \eta^{PV}\, i_t\, C_{PV}, \\
& P_t^{PV} + P_t^{G+} + P_t^{dis} - P_t^{ch} - P_t^{G-} = P_t^C, \\
& 0 \le E_0^{soc} + \sum_{k=1}^{t} \left(\eta^B P_k^{ch} - \frac{1}{\eta^B} P_k^{dis} \right) \Delta t \le E_B \\
& \sum_{k=1}^{T} \left(\eta^B P_k^{ch} - \frac{1}{\eta^B} P_k^{dis} \right) \Delta t = 0 \\
& 0 \le E_0^{soc} \le E_B, \\
& C_{PV}, E_B, P_t^{G+}, P_t^{G-}, P_t^{ch}, P_t^{dis}, P_t^{PV} \ge 0.
\end{aligned}
$$

Why the reduced model (without the SOC variable) is faster? (figure 3 on previous page)

The recursive SOC constraint was transformed into one cumulative equation that depends only on charging and discharging powers. We have one initial variable instead of the whole array (fewer variables), then:

- In Simplex algorithm each pivot operation becomes cheaper
- In Barrier method we reduce the dimension of the computed KKT equation.

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LINEAR EQUALITY CONSTRAINTS	CORRESPONDING DUAL VARIABLES
The hourly power balance	λ_t (power balance in hour t)
The battery State of Charge (SOC) dynamics	μ_t (SOC transition between t and $t+1$)
The SOC periodicity condition	ξ (SOC periodicity)

Table 2: linear quality constraints and dual variables - task 5

Power balance dual: λ_t

$$\lambda_t = \frac{\partial z^*}{\partial P_t^C}$$

The dual variable λ_t represents the *marginal cost of supplying one additional kWh of demand* in hour t. In practice, λ_t can be interpreted as the *internal price of energy* for the microgrid in hour t.

SOC dynamics dual: μ_t The dual variable μ_t measures the *value of shifting 1 kWh of energy from hour t to hour $t+1$* (the opportunity value of storage). When efficiency of charging is 1, one typically has the relation:

$$\mu_t \approx \lambda_{t+1} - \lambda_t.$$

Numerical example (2-hour case)

Assume PV produces 5 kW in hour 1, demand equals $[2, 5]$ kW, battery capacity is 1 kWh, and grid prices are 0.10 €/kWh (purchase) and 0.02 €/kWh (sale). The optimal operation is:

- Hour 1: charge the battery with 1 kWh and sell the remaining 2 kWh,
- Hour 2: discharge 1 kWh and purchase 4 kWh from the grid.

The resulting dual variables are:

$$\lambda_1 = 0.02 \text{ €/kWh}, \qquad \lambda_2 = 0.10 \text{ €/kWh}, \qquad \mu_1 = 0.08 \text{ €/kWh}.$$

Interpretation An additional 1 kWh of demand in hour 1 costs the system 0.02 €, because it reduces PV export revenue. An additional 1 kWh of demand in hour 2 costs 0.10 €, because it must be purchased from the grid. The storage value of 0.08 € is the difference in shadow prices:

$$\mu_1 = \lambda_2 - \lambda_1,$$

showing how much the system gains by shifting energy from a surplus hour to a deficit hour.

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π_B SENSITIVITY ANALYSIS (1 YEAR)

Duration	1 year
Optimal π_B stability interval	[0.00, ∞)
Current π_B	500.00
PV capacity(C^*_{PV})	0.000 kWh
Battery capacity (E^*_B)	0.000 kWh
Objective value (z^*)	345.29

For any $\Delta\pi_B \in [-500.00, \infty)$: $\quad z(\Delta\pi_B) = z^* + E^*_B \times \Delta\pi_B = 345.29 + 0.000 \times \Delta\pi_B$
In this interval, the solution (C_{PV}, E_B) remains unchanged. This means that regardless of the price per unit of battery capacity, it is not profitable to set up a microgrid for one year.

Example for $\pi_B = 0.0$ (in the interval):

PV capacity (C_{PV})	0.000 kWp
Battery capacity (E_B)	0.000 kWh
Objective value	345.29

Table 3: π_B sensitivity analysis (1 year) - task 6

π_B SENSITIVITY ANALYSIS (5 YEARS)

Duration	5 years
Optimal π_B stability interval	[20.43, ∞)
Current π_B	500.00
PV capacity(C^*_{PV})	0.501 kWh
Battery capacity (E^*_B)	0.000 kWh
Objective value (z^*)	1713.80

For any $\Delta\pi_B \in [-479.57, \infty)$: $\quad z(\Delta\pi_B) = z^* + E^*_B \times \Delta\pi_B = 1713.80 + 0.000 \times \Delta\pi_B$
In this interval, the solution (C_{PV}, E_B) remains unchanged. If the price per 1 kWh of battery capacity falls below 20.43, purchasing batteries, and therefore also solar panels, could be profitable.

Example for $\pi_B = 20.0$:

PV capacity (C_{PV})	0.504 kWp
Battery capacity (E_B)	0.002 kWh
Objective value	1713.80

Table 4: π_B sensitivity analysis (5 years) - task 6

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$$\begin{aligned}
\min_{C_{PV},E_B} \quad & \theta^{PV} C_{PV} + \theta^B E_B + N \sum_t \left(\theta^G P_t^{G+} \Delta t\right) \\
\text{s.t.} \quad & P_t^{PV} \leq \eta^{PV}\, i_t\, C_{PV}, \\
& P_t^{PV} + P_t^{G+} + P_t^{dis} - P_t^{ch} - P_t^{G-} = P_t^C, \\
& E_t^{soc} = E_{t-1}^{soc} + \eta^B P_t^{ch} \Delta t - \frac{1}{\eta^B} P_t^{dis} \Delta t, \\
& 0 \leq E_t^{soc} \leq E_B, \\
& E_1^{soc} = E_{T+1}^{soc}, \\
& C_{PV}, E_B, P_t^{G+}, P_t^{G-}, P_t^{ch}, P_t^{dis}, P_t^{PV} \geq 0.
\end{aligned}$$

θ^G SENSITIVITY ANALYSIS (1 YEAR)

Duration	1 year
Optimal θ^G stability interval	[0.0, 0.61]
Current θ^G	0.1 $kgCO_2/kWh$
Optimal C_{PV}^*	0.0 kWp
Optimal E_B^*	0.0 kWh
Objective value	345.290

For any $\Delta\theta^G \in [-0.1, 0.51]$: $z(\Delta\theta^G) = z^* + \left(\frac{\partial z}{\partial \theta^G}\right) \Delta\theta^G = 345.29 + (3452.9) \times \Delta\theta^G$
where $\frac{\partial z}{\partial \theta^G} = N_{\text{days}} \times \sum_t P_t^{G+} = 3452.9$

Example for $\theta_G = 0.62$:

PV capacity (C_{PV})	0.501 kWp
Battery capacity (E_B)	0.0 kWh
Objective value	2129.122

Table 5: θ^G sensitivity analysis (1 year) - task 7

θ^G SENSITIVITY ANALYSIS (5 YEARS)

Duration	5 years
Optimal θ^G stability interval	[0.0, 0.13]
Current θ^G	0.1 $kgCO_2/kWh$
Optimal C_{PV}^*	0.0 kWp
Optimal E_B^*	0.0 kWh
Objective value	1726.450

For any $\Delta\theta^G \in [-0.1, 0.03]$: $z(\Delta\theta^G) = z^* + \left(\frac{\partial z}{\partial \theta^G}\right) \Delta\theta^G = 1726.45 + (17264.5) \times \Delta\theta^G$
where $\frac{\partial z}{\partial \theta^G} = N_{\text{days}} \times \sum_t P_t^{G+} = 17264.5$

Example for $\theta^G = 0.14$:

PV capacity (C_{PV})	0.504 kWp
Battery capacity (E_B)	0.002 kWh
Objective value	2339.082

Table 6: θ^G sensitivity analysis (5 years) - task 7

For a microgrid operating for 1 year and one operating for 5 years, the range in which the values of C_{PV} and E_B do not change (for θ^G) is [0.0, 0.61] for 1 year and [0.0, 0.13] for five. This means that if the amount of CO_2 production (in kg) of an energy company per 1 kW of power is greater than 0.61 or 0.13 per hour, it would be sustainable to invest in a microgrid.

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$$cost_s = \pi^{PV} C_{PV} + \pi^B E_B + \pi^W C_W + N \sum_{t=1}^{T} \left(\pi^{G+} P_{t,s}^{G+} - \pi^{G-} P_{t,s}^{G-} \right) \Delta t$$

$$\begin{aligned}
&\min_{C_{PV}, E_B, C_W} \left(\sum_{s=1}^{3} (cost_s) + \max_{s \in 1,2,3} cost_s \right) \\
\text{s.t.} \quad & P_{s,t}^{PV} \leq \eta^{PV}\, i_t\, C_{PV}, \\
& P_{s,t}^{W} = w_{s,t} C_W \\
& P_{s,t}^{PV} + P_{s,t}^{W} + P_{s,t}^{G+} + P_{s,t}^{dis} - P_{s,t}^{ch} - P_{s,t}^{G-} = P_{s,t}^{C}, \\
& E_{s,t}^{soc} = E_{s,t-1}^{soc} + \eta^B P_{s,t}^{ch} \Delta t - \frac{1}{\eta^B} P_{s,t}^{dis} \Delta t, \\
& 0 \leq E_{s,t}^{soc} \leq E_B, \\
& E_{s,1}^{soc} = E_{s,T+1}^{soc}, \\
& C_{PV}, C_W, E_B, P_{s,t}^{G+}, P_{s,t}^{G-}, P_{s,t}^{ch}, P_{s,t}^{dis}, P_{s,t}^{PV} \geq 0.
\end{aligned}$$

θ^G MICROGRID WITH WIND TURBINE– COST MINIMIZATION RESULTS (1 YEAR)

Simulation period	1 year
Solver status	OPTIMAL
PV capacity (C_{PV})	0.000 kWp
Battery capacity (E_B)	0.000 kWh
Wind turbine capacity (C_W)	0.000 kWp
Scenario 1 total, operating cost	345.29, 345.29
Scenario 2 total, operating cost	345.29, 345.29
Scenario 3 total, operating cost	345.29, 345.29
Worst-case (z)	345.29
Objective value (sum + worst)	1381.16

Table 7: Microgrid cost minimization summary for 1 year - task 8

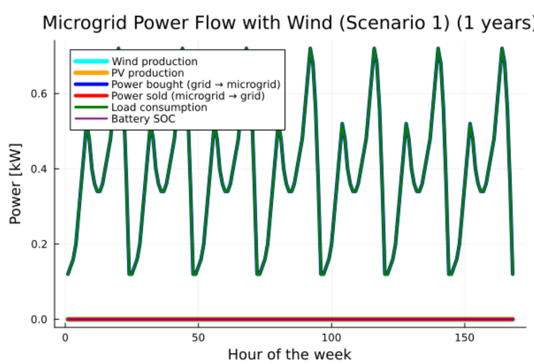

Figure 4: Production, consumption, state of charge for 1 year with wind turbine - task 8

Over a one-year horizon, the optimal solution is not to invest in PV, batteries or wind turbines. One year is too short a period to recoup the high capital cost of renewable energy sources, and the savings from purchasing less energy from the grid are too small. As a result, relying solely on the grid proves to be the most economically advantageous, and the identical costs in all three scenarios remove the need for risk reduction.

θ^G MICROGRID WITH WIND TURBINE– COST MINIMIZATION RESULTS (5 YEARS)

Simulation period	5 years
Solver status	OPTIMAL
PV capacity (C_{PV})	0.000 kWp
Battery capacity (E_B)	0.000 kWh
Wind turbine capacity (C_W)	0.642 kWp
Scenario 1 total, operating cost	1529.549, 566.705
Scenario 2 total, operating cost	1396.277, 433.434
Scenario 3 total, operating cost	1529.549, 566.705
Worst-case (z)	1529.549
Objective value (sum + worst)	5984.923

Table 8: Microgrid cost minimization summary for 5 years - task 8

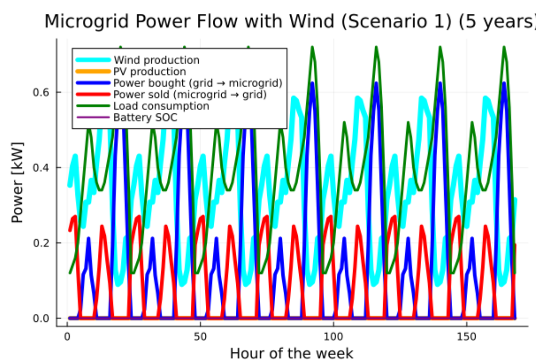

Figure 5: Production, consumption, state of charge for 5 years with wind turbine - task 8

After extending the period to five years, investing in a small wind turbine (0.642 kWp) becomes profitable - the longer horizon allows the investment costs to be offset by lower energy purchases from the grid. Scenarios 1 and 3, despite differences in wind profile, give the same cost value, which can be explained by different times of high production (e.g. one model produces more energy at night but, due to the absence of battery storage, must sell it immediately, which generates income, while the other model produces more energy during the day, which it consumes and thus saves). This, in practice, may lead to similar final costs. PV remains unprofitable due to poor sunlight profile and lack of attractive energy selling prices, and the battery is not chosen due to small price differences and lack of energy surpluses that it could store.