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Growth Curves

SimpleAI.ch

LOGISTIC

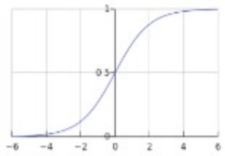
$$f(x) = \frac{L}{1 + e^{-k(x-x_0)}},$$

where

x_0, the x value of the sigmoid's midpoint;

L, the curve's maximum value;

k, the logistic growth rate or steepness of the curve.[1]



Standard logistic function where $L = 1, k = 1, x_0 = 0$

NOTES

- **values** *X ∈ (-inf, +inf), Y ∈ (0, L)*
- **Shape,** *the S-curve shown on the left is obtained, with the graph of f(x) approaching L, as the x approaches +inf, and approaching zero, as x approaches -inf. A generalization of the logistic function is the hyperbolastic function of type I.*
- **Sigmoid function:** *The standard logistic function, **L=1, k=1, x0=0**. It is also sometimes called the **expit**, being the inverse of the logit, and it used in logistic regression*

APPLICATIONS

- **Medicine:** Tumor Growth:
- **Agronomy**: Crop response to fertilizers, treatments etc..
- **Ecology**: self-limiting growth of a biological population.
- **Logistic regression:**
- **Neural Networks:** to clamp signals to within a specified interval, or to introduce non-linearity, or as activation function.
- **Economy:** Diffusion of innovations - a theory that seeks to explain how, why, and at what rate new ideas and technology spread from innovators, to early adopters, late adopters etc…

EXPOTENTIAL *(Growth or Decay)*

f(x) = aˣ

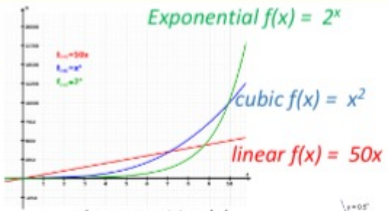
Most known

y = eˣ

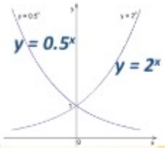
examples

$2^{-1} = ½^1 = ½$

$2^{-2} = ½^2 = ¼$



- always positive (+)
- Horizontal asymptote; y=0
- a in y=aˣ called "base"

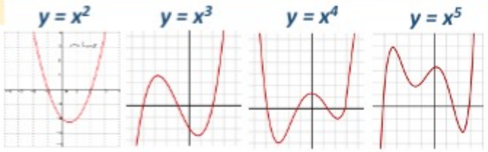


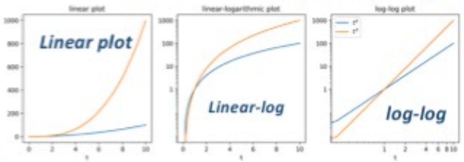
- **exponential decay**; *the constant of proportionality is negative = the quantity decreases over time*
- **geometric growth**; *exponential growth/decay applied at equal intervals on discrete values. The function values form a geometric progression.*
- ***DO NOT CALL IT rapid***; *although it may increase the value "fast", change rate may be slow at the beginning,*
- ***Logistic growth***; *many processes, have exponential growth phase, but it is later limited with recourses of by environments, and follows logistic growth curve*
- **Exponential growth bias;** Studies show that human beings have difficulty understanding exponential growth. Exponential growth bias is the tendency to underestimate compound growth processes.

- **Biology:** Cell of bacteria growth,
- **Epidemiology**: to model virus spread, with constant rate r, of infections
- **Physics:** Nuclear chain reaction, positive feedback, (exponential growth of the amplified signal)
- **Economics**: increase in GDP, assumed it has no end, :p
- **Finance**: ***(i) Compound interest*** (ie, constant interest rate provides exponential growth of the capital), (ii) Ponzi Schemes,
- **CS**: computational complexity, eg in grid search,
- **Social networks**: exponential spread of the viral videos, eg Gangnam Style,

POLYNOMIAL

f(x) = xᵃ





- **Polynomial function:** a function with **only non-negative integer powers or positive integer exponents of a variable.** Eg: quadratic equation, cubic equation, etc.
- **2x+5** is a polynomial that has exponent equal to 1

Machine Learning & CS

- **Polynomial approximations:** *every differentiable function locally looks like a polynomial function (Taylors theorem),*
- **Polynomial expansion:** *any curved line can be fitted with the polynomial of an arbitrary degree*
- **polynomial time;** *the time it takes to complete an algorithm is bounded by a polynomial function of some variable, such as the size of the input.*

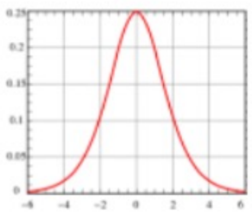
Encoding information: *Polynomials are frequently used to encode information about some other object.*

- *The characteristic polynomial of a matrix or linear operator contains information about the operator's eigenvalues.*
- *The minimal polynomial of an algebraic element records the simplest algebraic relation satisfied by that element.*

HUBBERT CURVE

an approximation of the production rate of a resource over time

$$x = \frac{e^{-t}}{(1+e^{-t})^2} = \frac{1}{2+2\cosh t} = \frac{1}{4}\operatorname{sech}^2\frac{t}{2}.$$



The shape: often confused with the "normal" gaussian function.

- **symmetric logistic** distribution curve
- The density of a Hubbert curve approaches zero more slowly than a gaussian
  - a gradual rise from zero resource production that then increases quickly
  - a "Hubbert peak", representing the maximum production level
  - a drop from the peak that then follows a steep production decline.
- **Real world Hubbert curves are often not symmetrical**, because of additional factors affecting the production of recourses eg: development of enhanced production techniques, availability of competing resources, and government regulations

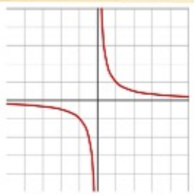
IMPORTANT: *many researchers made attempts to create Hubbert curve for other processes, eg. natural gas, cola, copper, water, whaling, et…, it often failed, because of lack of data, or strong influence of regulations or new techniques on the processes*

- **Peak oil;** *Using the curve, Hubbert modeled the rate of petroleum production for several regions, determined by the rate of new oil well discovery, and extrapolated a world production curve.*
  - ***The relative steepness of decline*** - *a steep drop in the production implies that the world will not have enough time to develop new energy sources*
  - ***Problems***; *After the predicted early-1970s peak of oil production in the U.S., production declined the following a pattern matching the Hubbert curve. However, new extraction methods developed in ~2000 allowed reaching the highest-ever oil production levels in ~2017. From that reason, the Hubbert curve has to be calculated separately for different oil provinces, whose exploration has started at a different time, and oil extracted by new techniques*

HYPERBOLIC

Asymptotic lines that grows toward singularity (infinity)

*Eg: reciprocal function **1/x** has a hyperbola as a graph, and has a singularity at 0, meaning that the limit as x→0 and at 0 it is infinite: any similar graph is said to **exhibit hyperbolic growth.***



Comparisons with other growth; *Like exponential growth and logistic growth, hyperbolic growth is highly nonlinear, but differs in important respects. These functions can be confused, as exponential growth, hyperbolic growth, and the first half of logistic growth are convex functions; however their asymptotic behavior (behavior as input gets large) differs dramatically:*

- **logistic growth is constrained** *(has a finite limit, even as time goes to infinity),*
- **Exp. growth grows to inf. as time goes to inf.** *(but is always finite for finite time),*
- **hyperbolic growth has a singularity in finite time** *(grows to infinity at a finite time).*

- **CS; Quening theory:** *the average waiting time of randomly arriving customers grows hyperbolically as a function of the average load ratio of the server. The singularity in this case occurs when the average amount of work arriving to the server equals the server's processing capacity.*
- *Enzyme kinetics*
- *Population growth*

Figures and some text from Wikipedia