pro2

May 10, 2023

0.1 Zadanie 5

[1]:
```
from scipy.spatial.distance import hamming

v1 = [1, 2, 0, 1]
v2 = [0, 0, 0, 1]
print(hamming(v1, v2))
```

```
0.5
```

[2]:
```
v = [
    [1, 2, 1, 2, 0],
    [1, 1, 1, 1, 1],
    [0, 0, 2, 1, 1],
    [2, 2, 2, 1, 0]
]

search_ = [None, None, float("inf")]
for v1 in v:
    for v2 in v:
        if v1 != v2 and hamming(v1, v2) < search_[-1]:
            search_ = [v1, v2, hamming(v1, v2)]

print(search_[:2])
```

```
[[1, 2, 1, 2, 0], [1, 1, 1, 1, 1]]
```

0.2 Zadanie 6

[3]:
```
import numpy as np

np.random.seed(123)

def generateAllVectors(B, k):
    ready = False
    multiplyers = np.zeros(len(B), dtype=np.uint8)
    words = set()

    while not ready:
```

```python
        words.add(tuple(multiplyers.dot(B) % k))

        i = len(B) - 1
        multiplyers[i] += 1
        while not ready and multiplyers[i] == k:
            multiplyers[i] = 0
            multiplyers[i - 1] += 1
            i -= 1

            if i == -1:
                ready = True
    return sorted(words)
```

[4]:
```python
B = np.array([
    [1, 0, 0, 2, 4],
    [0, 1, 0, 1, 0],
    [0, 0, 1, 5, 6]
], dtype=np.uint8)

words = generateAllVectors(B, 7)
```

[5]:
```python
# dla przejzystosci po 3 w jednej kolumnie

line = []
for word in words:
    if len(line) == 3:
        print("\t".join(line))
        line = []
    line.append(str(word))
print("\t".join(line))
```

```
(0, 0, 0, 0, 0) (0, 0, 1, 5, 6) (0, 0, 2, 3, 5)
(0, 0, 3, 1, 4) (0, 0, 4, 6, 3) (0, 0, 5, 4, 2)
(0, 0, 6, 2, 1) (0, 1, 0, 1, 0) (0, 1, 1, 6, 6)
(0, 1, 2, 4, 5) (0, 1, 3, 2, 4) (0, 1, 4, 0, 3)
(0, 1, 5, 5, 2) (0, 1, 6, 3, 1) (0, 2, 0, 2, 0)
(0, 2, 1, 0, 6) (0, 2, 2, 5, 5) (0, 2, 3, 3, 4)
(0, 2, 4, 1, 3) (0, 2, 5, 6, 2) (0, 2, 6, 4, 1)
(0, 3, 0, 3, 0) (0, 3, 1, 1, 6) (0, 3, 2, 6, 5)
(0, 3, 3, 4, 4) (0, 3, 4, 2, 3) (0, 3, 5, 0, 2)
(0, 3, 6, 5, 1) (0, 4, 0, 4, 0) (0, 4, 1, 2, 6)
(0, 4, 2, 0, 5) (0, 4, 3, 5, 4) (0, 4, 4, 3, 3)
(0, 4, 5, 1, 2) (0, 4, 6, 6, 1) (0, 5, 0, 5, 0)
(0, 5, 1, 3, 6) (0, 5, 2, 1, 5) (0, 5, 3, 6, 4)
(0, 5, 4, 4, 3) (0, 5, 5, 2, 2) (0, 5, 6, 0, 1)
(0, 6, 0, 6, 0) (0, 6, 1, 4, 6) (0, 6, 2, 2, 5)
(0, 6, 3, 0, 4) (0, 6, 4, 5, 3) (0, 6, 5, 3, 2)
(0, 6, 6, 1, 1) (1, 0, 0, 2, 4) (1, 0, 1, 0, 3)
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(1, 0, 2, 5, 2) (1, 0, 3, 3, 1) (1, 0, 4, 1, 0)
(1, 0, 5, 6, 6) (1, 0, 6, 4, 5) (1, 1, 0, 3, 4)
(1, 1, 1, 1, 3) (1, 1, 2, 6, 2) (1, 1, 3, 4, 1)
(1, 1, 4, 2, 0) (1, 1, 5, 0, 6) (1, 1, 6, 5, 5)
(1, 2, 0, 4, 4) (1, 2, 1, 2, 3) (1, 2, 2, 0, 2)
(1, 2, 3, 5, 1) (1, 2, 4, 3, 0) (1, 2, 5, 1, 6)
(1, 2, 6, 6, 5) (1, 3, 0, 5, 4) (1, 3, 1, 3, 3)
(1, 3, 2, 1, 2) (1, 3, 3, 6, 1) (1, 3, 4, 4, 0)
(1, 3, 5, 2, 6) (1, 3, 6, 0, 5) (1, 4, 0, 6, 4)
(1, 4, 1, 4, 3) (1, 4, 2, 2, 2) (1, 4, 3, 0, 1)
(1, 4, 4, 5, 0) (1, 4, 5, 3, 6) (1, 4, 6, 1, 5)
(1, 5, 0, 0, 4) (1, 5, 1, 5, 3) (1, 5, 2, 3, 2)
(1, 5, 3, 1, 1) (1, 5, 4, 6, 0) (1, 5, 5, 4, 6)
(1, 5, 6, 2, 5) (1, 6, 0, 1, 4) (1, 6, 1, 6, 3)
(1, 6, 2, 4, 2) (1, 6, 3, 2, 1) (1, 6, 4, 0, 0)
(1, 6, 5, 5, 6) (1, 6, 6, 3, 5) (2, 0, 0, 4, 1)
(2, 0, 1, 2, 0) (2, 0, 2, 0, 6) (2, 0, 3, 5, 5)
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(2, 5, 2, 5, 6) (2, 5, 3, 3, 5) (2, 5, 4, 1, 4)
(2, 5, 5, 6, 3) (2, 5, 6, 4, 2) (2, 6, 0, 3, 1)
(2, 6, 1, 1, 0) (2, 6, 2, 6, 6) (2, 6, 3, 4, 5)
(2, 6, 4, 2, 4) (2, 6, 5, 0, 3) (2, 6, 6, 5, 2)
(3, 0, 0, 6, 5) (3, 0, 1, 4, 4) (3, 0, 2, 2, 3)
(3, 0, 3, 0, 2) (3, 0, 4, 5, 1) (3, 0, 5, 3, 0)
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(3, 2, 4, 0, 1) (3, 2, 5, 5, 0) (3, 2, 6, 3, 6)
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(3, 4, 5, 0, 0) (3, 4, 6, 5, 6) (3, 5, 0, 4, 5)
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(3, 6, 6, 0, 6) (4, 0, 0, 1, 2) (4, 0, 1, 6, 1)
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(4, 2, 6, 5, 3) (4, 3, 0, 4, 2) (4, 3, 1, 2, 1)
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(4, 6, 5, 4, 4) (4, 6, 6, 2, 3) (5, 0, 0, 3, 6)
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(5, 0, 4, 2, 2) (5, 0, 5, 0, 1) (5, 0, 6, 5, 0)
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(5, 1, 6, 6, 0) (5, 2, 0, 5, 6) (5, 2, 1, 3, 5)
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(5, 2, 5, 2, 1) (5, 2, 6, 0, 0) (5, 3, 0, 6, 6)
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(5, 3, 4, 5, 2) (5, 3, 5, 3, 1) (5, 3, 6, 1, 0)
(5, 4, 0, 0, 6) (5, 4, 1, 5, 5) (5, 4, 2, 3, 4)
(5, 4, 3, 1, 3) (5, 4, 4, 6, 2) (5, 4, 5, 4, 1)
(5, 4, 6, 2, 0) (5, 5, 0, 1, 6) (5, 5, 1, 6, 5)
(5, 5, 2, 4, 4) (5, 5, 3, 2, 3) (5, 5, 4, 0, 2)
(5, 5, 5, 5, 1) (5, 5, 6, 3, 0) (5, 6, 0, 2, 6)
(5, 6, 1, 0, 5) (5, 6, 2, 5, 4) (5, 6, 3, 3, 3)
(5, 6, 4, 1, 2) (5, 6, 5, 6, 1) (5, 6, 6, 4, 0)
(6, 0, 0, 5, 3) (6, 0, 1, 3, 2) (6, 0, 2, 1, 1)
(6, 0, 3, 6, 0) (6, 0, 4, 4, 6) (6, 0, 5, 2, 5)
(6, 0, 6, 0, 4) (6, 1, 0, 6, 3) (6, 1, 1, 4, 2)
(6, 1, 2, 2, 1) (6, 1, 3, 0, 0) (6, 1, 4, 5, 6)
(6, 1, 5, 3, 5) (6, 1, 6, 1, 4) (6, 2, 0, 0, 3)
(6, 2, 1, 5, 2) (6, 2, 2, 3, 1) (6, 2, 3, 1, 0)
(6, 2, 4, 6, 6) (6, 2, 5, 4, 5) (6, 2, 6, 2, 4)
(6, 3, 0, 1, 3) (6, 3, 1, 6, 2) (6, 3, 2, 4, 1)
(6, 3, 3, 2, 0) (6, 3, 4, 0, 6) (6, 3, 5, 5, 5)
(6, 3, 6, 3, 4) (6, 4, 0, 2, 3) (6, 4, 1, 0, 2)
(6, 4, 2, 5, 1) (6, 4, 3, 3, 0) (6, 4, 4, 1, 6)
(6, 4, 5, 6, 5) (6, 4, 6, 4, 4) (6, 5, 0, 3, 3)
(6, 5, 1, 1, 2) (6, 5, 2, 6, 1) (6, 5, 3, 4, 0)
(6, 5, 4, 2, 6) (6, 5, 5, 0, 5) (6, 5, 6, 5, 4)
(6, 6, 0, 4, 3) (6, 6, 1, 2, 2) (6, 6, 2, 0, 1)

```
(6, 6, 3, 5, 0) (6, 6, 4, 3, 6) (6, 6, 5, 1, 5)
(6, 6, 6, 6, 4)
```

0.3 Zadanie 7

[6]:
```python
C = np.array(words, dtype=np.uint8)
```

[7]:
```python
def minimizeHammingDistance(C, B, v):
    dst = np.array([hamming(w, v) for w in C], dtype=np.float64)
    m = np.min(dst)
    L = C[np.where(dst == m)]
    w = L[np.random.randint(0, len(L))]

    # korzystajac ze znamy z gory B dla zadania 7
    return [w[0], w[1], w[2]]
```

[8]:
```python
minimizeHammingDistance(C, B, [1, 2, 3, 4, 5])
```

[8]:
```
[1, 2, 0]
```

0.4 Zadanie 8

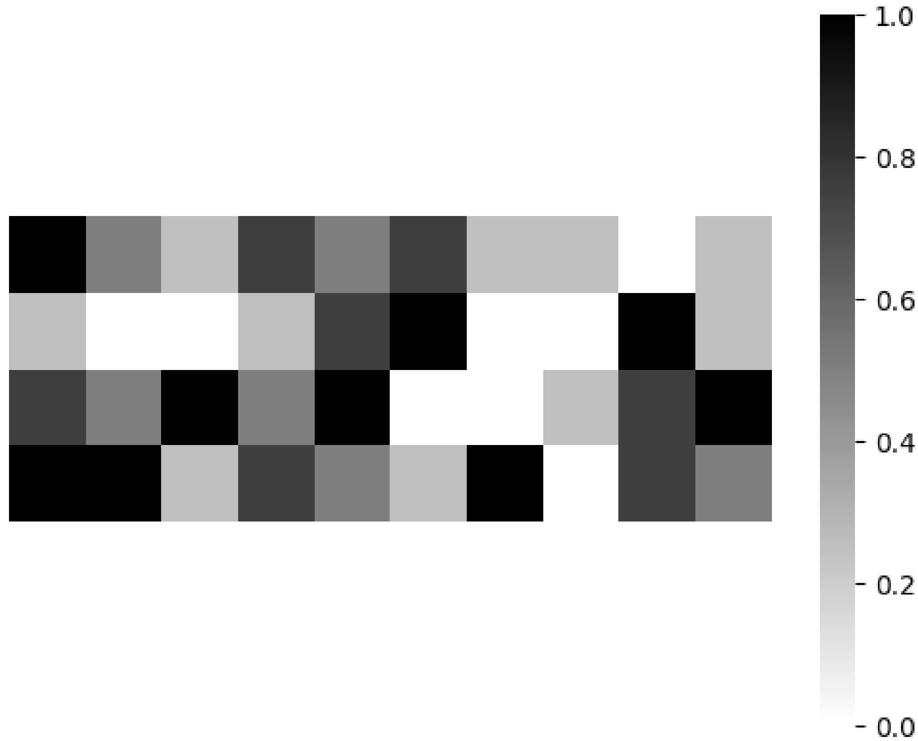
a)

[9]:
```python
M = np.random.randint(5, size=(4, 10))
```

b)

[10]:
```python
import matplotlib.pyplot as plt
import seaborn as sns

M_norm = M / 4.
sns.heatmap(M_norm, cmap="binary", square=True)
plt.axis("off")
plt.show()
```



c)

Tu nic nie trzeba robić, na podstawie pierwszysch 4 pozycji w kazdym wektorze od razu widać, ze są one liniowo niezalezne, a zatem G.T (wektory z G) jest bazą 4 wymiarowej podprzestrzeni przestrzeni 11 wymiarowej nad ciałem Z_5. A zatem jest macierzą generującą kodu (11, 4) liniowego nad tym ciałem.

[11]:
```
G = np.array([
    [1, 0, 0, 0, 0, 4, 4, 2, 0, 1, 1],
    [0, 1, 0, 0, 0, 3, 0, 2, 2, 1, 0],
    [0, 0, 1, 0, 0, 2, 0, 1, 1, 1, 1],
    [0, 0, 0, 1, 1, 0, 0, 0, 4, 3, 0]
], dtype=np.uint8)
```

d)

[12]:
```
M_coded = np.array([
    (w.T.dot(G)).T for w in M.T
])
```

[13]:
```
print(M.T[0], " - zakodowany to: ", M_coded[0])
```

```
[4 1 3 4]  - zakodowany to:  [ 4  1  3  4  4 25 16 13 21 20  7]
```

e)

[14]:
```
probabilities_mask = np.random.random(size=M_coded.shape)
channel_mask = np.vectorize(lambda x: 0 if x < 0.95 else 3)(probabilities_mask)
M_send = M_coded + channel_mask
```

f)

[15]:
```
def minimizeHammingDistance(C, B, v):
    dst = np.array([hamming(w, v) for w in C], dtype=np.float64)
    m = np.min(dst)
    L = C[np.where(dst == m)]
    w = L[np.random.randint(0, len(L))]

    # korzystajac ze znamy z gory B dla zadania 8
    return [w[0], w[1], w[2], w[3]]
```

[16]:
```
C = np.array(generateAllVectors(G, 5), dtype=np.uint8)

M_encoded = np.array([minimizeHammingDistance(C, G, v) for v in M_send]).T
```

h)

[17]:
```
np.sum(np.all(M_encoded == M, axis=0))
```

[17]: 10

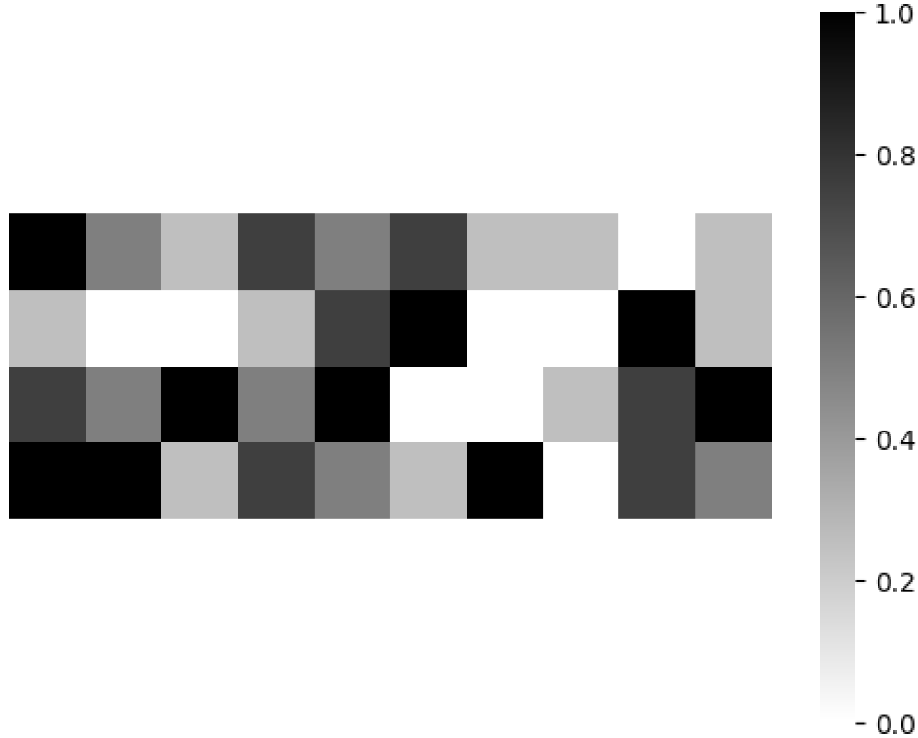
[18]:
```
# zatem wszystkie - na potwierdzenie:
np.all(M_encoded == M)
```

[18]: True

i)

[19]:
```
M_encoded_norm = M_encoded / 4.

sns.heatmap(M_encoded_norm, cmap="binary", square=True)
plt.axis("off")
plt.show()
```


1.0
0.8
0.6
0.4
0.2
0.0