For part iii), we will formulate a solution by considering each case separately.

Case 1: Four singles

This is simply part i): we choose four different numbers from a total of ten with order unimportant:

$$\binom{10}{4}.$$

Case 2: A pair and two singles

We want the numbers which form the pair and both singles to all be distinct, so we want to choose three different numbers. Firstly, we may choose a number to form the pair: $\binom{10}{1}$. We have nine numbers remaining and we want to choose two for the remaining singles: $\binom{9}{2}$. So, overall,

$$\binom{10}{1} \times \binom{9}{2}.$$

Alternatively[2], we can choose three different numbers: $\binom{10}{3}$. Then, choose one of these three to be the pair: $\binom{3}{1}$, giving

$$\binom{10}{3} \times \binom{3}{1}.$$

A small amount of algebra (or direct evaluation) will reveal that these are both equivalent.

Case 3: Two pairs

We want two different numbers, each of which will form a pair, so we can simply choose two numbers from ten:

$$\binom{10}{2}.$$

Case 4: A triple and a single

Much as in case 2, we first choose one digit to form the triple – $\binom{10}{1}$ – and then choose one of the remaining nine to make up the single – $\binom{9}{1}$, giving

$$\binom{10}{1} \times \binom{9}{1}.$$

Or we can choose two numbers – $\binom{10}{2}$ – and pick one to form the triple – $\binom{2}{1}$ – giving

$$\binom{10}{2} \times \binom{2}{1}.$$

Case 5: A quad

We simply need to choose one number to comprise the quad, which can be done in

$$\binom{10}{1}$$

ways.

So, overall, there are

$$\underbrace{\binom{10}{4}}_{\text{Case 1}} + \underbrace{\binom{10}{1} \times \binom{9}{2}}_{\text{Case 2}} + \underbrace{\binom{10}{2}}_{\text{Case 3}} + \underbrace{\binom{10}{1} \times \binom{9}{1}}_{\text{Case 4}} + \underbrace{\binom{10}{1}}_{\text{Case 5}} = 715 \text{ ways.}$$

[2]Thanks, Carlos!