Question 1: 2001 Q6 b.

Consider the point $P(2at, at^2)$ on the parabola $x^2 = 4ay$.

i) Prove that the equation of the normal at P is $x + ty = 2at + at^3$.

ii) Find the coordinates of the point Q on the parabola such that the normal at Q is perpendicular to the normal at P.

iii) Show that the two normals of part ii) intersect at the point R with coordinates

$$x = a\left(t - \frac{1}{t}\right) \qquad y = a\left(t^2 + 1 + \frac{1}{t^2}\right).$$

iv) Find the Cartesian form of the equation of the locus of the point R.

Question 2: 2014 Q13 c.

The point $P(2at, at^2)$ lies on the parabola $x^2 = 4ay$ with focus S. The point Q divides the interval PS internally in the ratio $t^2 : 1$.

i) Show that the coordinates of the point Q are given by

$$x = \frac{2at}{1+t^2} \qquad y = \frac{2at^2}{1+t^2}.$$

ii) Express the slope of OQ in terms of t.

iii) Using the result from part ii), or otherwise, show that Q lies on a fixed circle of radius a.