Some things that I thought were interesting about this question

When dealing with counting problems, generally there are two major factors that will determine the type of approach one takes: whether order is important or unimportant and whether repetitions are permitted or not. Given k objects from a total of n, we can summarise the standard approaches taught as follows:

	order important	order unimportant
repetitions allowed	n^k	$\binom{n}{k}$
repetitions not allowed	${}^nP_k$	–

That fourth case is not really addressed in school in general; this question is slightly more difficult because it falls squarely into that category. As it happens, however, there is a good theory around dealing with problems of this form (it's pretty slick, actually). The final result is as follows: given a total of n (distinct) objects, the number of ways of making unordered selections of k objects with repetitions permitted is given by

$$\binom{n+k-1}{n-1}.$$[3]

In our case, we have ten numbers and we are making selections of either four or five, which gives us

$$\text{4 numbers: } \binom{10+4-1}{10-1} = 715 \qquad \text{5 numbers: } \binom{10+5-1}{10-1} = 2002$$

ways respectively, in line with our previous calculations.

Another thing that I thought was interesting was calculating the number of cases for each serial number length. Recall, we had five unique cases for four digit numbers and seven unique cases for five digit numbers (I was tempted to do six or seven, rather than five, but it started to get tedious counting all the cases out). As it happens, for a serial number of length n, the number of unique cases is given by the number of unique ways of writing n as a sum of positive integers. For example, consider $n = 4$. We can write it as 4, $3+1$, $2+2$, $2+1+1$ or $1+1+1+1$. Each decomposition can be associated with a case where the numbers correspond to the sizes of the groups present. For example, we can write $5 = 2+2+1$, which can be associated with the case consisting of two pairs and a single; we could also write $5 = 3+2$, which can be associated with the case consisting of a triple and a pair.

If we make this association, then, when calculating the number of permutations for each case in the second half of the question, the decomposition gives the number of repetitions that one needs to divide $n!$ by to give the total number of arrangements per choice. For example, we can write $4 = 3+1$; we associate that with the case consisting of a triple and a single; there are $\frac{4!}{3!1!}$ arrangements per choice of digits.

When calculating the number of choices for each arrangement, I have deliberately not simplified the combinations, because I think it makes precisely what is being counted more clear and lends itself more to generalisation. When assigning numbers to groups of the same size, it does not matter in what order we do so: for instance, 1122 and 2211 are considered the same arrangement (since order is not important). In general, when assigning numbers to a set of r groups of size k out of a total of n (remaining) numbers, there are $\binom{n}{r}$ choices. A simple example is this; consider the $n = 5$ case without repetitions. Then, we have five groups of size one and ten values left to choose from, so we have $\binom{10}{5}$ choices. Note as well that the size of the groups is not actually relevant; simply the number of groups of the same size present and the number of digits to choose from. We can effectively break each case down into calculating the number of ways of choosing digits across groups of the same size, for all different sizes of groups that appear.

[3] I have conspicuously omitted a proof of this result; if people are interested, I will put one up in a separate post.