Now, examining either $\triangle AEG$ or $\triangle CFH$, the angle sum of a triangle requires that

$$\alpha + \beta + 45^\circ = 180^\circ \iff \alpha + \beta = 135^\circ.$$

Thus, taking the tan of both sides,

$$\begin{aligned} \tan(\alpha+\beta) &= \tan(135^\circ) \\ &= -1. \end{aligned} \tag{4}$$

Substituting this result into equation (3), we see that

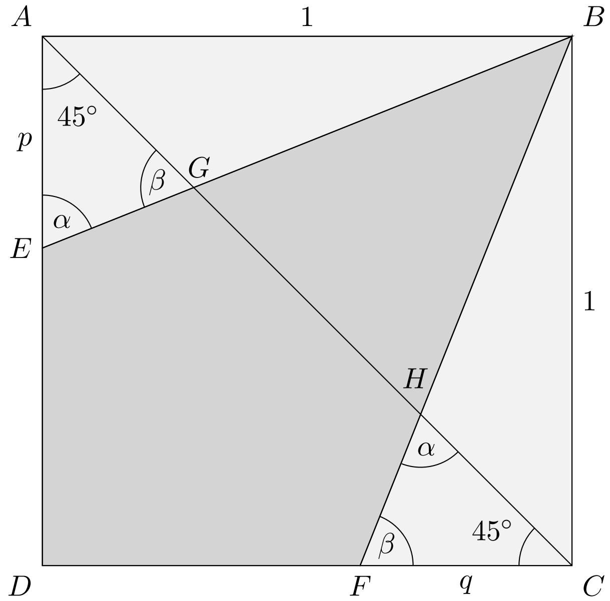
$$\begin{aligned} \frac{p+q}{1-pq} &= -\tan(\alpha+\beta) \\ &= -(-1) \\ &= 1. \\ \iff p+q &= 1-pq \end{aligned} \tag{5}$$

as required.

iii) We are required to show that the area of the quadrilateral $EBFD$ is given by

$$1 - \frac{p}{2} + \frac{p-1}{2(p+1)}.$$

We refer back to our diagram.



It is clear that the area of the quadrilateral $EBFD$ (the darker area) is the area of the square less the areas of triangles AEB and CFH (the lighter areas). Let A be the area of the quadrilateral required, and A_s, A_p and A_q be the areas of the square and the two triangles hitherto referenced respectively. Then,

$$A = A_s - A_q - A_q.$$

Now, we are given that the square $ABCD$ is a unit square, so $A_s = 1$. Recalling that the area of a triangle is given by $A = \frac{1}{2}bh$ where b and h are the base length and perpendicular height respectively, we see that

$$A_p = \frac{p}{2} \qquad\qquad A_q = \frac{q}{2}.$$

So,

$$A = 1 - \frac{p}{2} - \frac{q}{2}. \tag{6}$$