Part iv) asks us to repeat this process, but for five digit serial numbers. The general idea is identical, one simply needs to take care in counting cases. In particular, there are **seven** unique cases:

1) Five (distinct) singles (e.g. 12345);
2) A pair and three singles (e.g. 11234);
3) Two pairs and a single (e.g. 11223);
4) A triple and two singles (e.g. 11123);
5) A triple and a pair (e.g 11122);
6) A quad and a single (e.g. 11112);
7) Five of a kind ("quint" seems a bit too far) (e.g 11111).

Case 1: Five singles

We are simply choosing five distinct numbers from a total of ten with order unimportant, so

$$\binom{10}{5}.$$

Case 2: A pair and three singles

First, we can choose a number to form the pair: $\binom{10}{1}$. We have nine numbers remaining, and we wish to choose three to form the remaining singles: $\binom{9}{3}$. So,

$$\binom{10}{1} \times \binom{9}{3}.$$

Case 3: Two pairs and a single

Initially, we want to populate two pairs with two different numbers. The order in which we assign numbers to the pairs is not important, so we can simply choose two from ten: $\binom{10}{2}$. Then, we have eight remaining numbers to assign to the single: $\binom{8}{1}$. This gives us

$$\binom{10}{2} \times \binom{8}{1}.$$

Case 4: A triple and two singles

We can choose the triple first – $\binom{10}{1}$ – and then the two singles – $\binom{9}{2}$ – giving us

$$\binom{10}{1} \times \binom{9}{2}.$$

Alternatively, we note that this case is actually identical to case 3.

Case 5: A triple and a pair

We can choose the triple first – $\binom{10}{1}$ – and then the pair – $\binom{9}{1}$. Thus, we obtain

$$\binom{10}{1} \times \binom{9}{1}.$$

Case 6: A quad and a single

This is identical to case 5:

$$\binom{10}{1} \times \binom{9}{1}.$$