Case 5: A quad

There are $\binom{10}{1}$ ways of choosing the digits. There is one quad (four repeats), so there are $\dfrac{4!}{4!}$ ways of arranging them. Thus, we have

$$\binom{10}{1} \times \frac{4!}{4!}$$

arrangements.

This gives us a total of

$$\binom{10}{4} \times \frac{4!}{1!1!1!1!} + \binom{10}{1} \times \binom{9}{2} \times \frac{4!}{2!1!1!} + \binom{10}{2} \times \frac{4!}{2!2!} + \binom{10}{1} \times \binom{9}{1} \times \frac{4!}{3!1!} + \binom{10}{1} \times \frac{4!}{4!} = 10000 \text{ ways}$$

by direct evaluation.

Alternatively, we may simply note that we are finding the number of ways of arranging four digits out of a total of ten with order important and repetition permitted, which is simply $10 \times 10 \times 10 \times 10 = 10000$ ways.

Five digit serial numbers are identical:

1) Five singles:
   (a) $\binom{10}{5}$ choices;
   (b) $\dfrac{5!}{1!1!1!1!1!}$ arrangements per choice;
   (c) $\binom{10}{5} \times \dfrac{5!}{1!1!1!1!1!}$ total arrangements;
2) A pair and three singles:
   (a) $\binom{10}{1} \times \binom{9}{3}$ choices;
   (b) $\dfrac{5!}{2!1!1!1!}$ arrangements per choice;
   (c) $\binom{10}{1} \times \binom{9}{3} \times \dfrac{5!}{2!1!1!1!}$ total arrangements;
3) Two pairs and a single:
   (a) $\binom{10}{2} \times \binom{8}{1}$ choices;
   (b) $\dfrac{5!}{2!2!1!}$ arrangements per choice;
   (c) $\binom{10}{2} \times \binom{8}{1} \times \dfrac{5!}{2!2!1!}$ total arrangements;
4) A triple and two singles:
   (a) $\binom{10}{1} \times \binom{9}{2}$ choices;
   (b) $\dfrac{5!}{3!1!1!}$ arrangements per choice;
   (c) $\binom{10}{1} \times \binom{9}{2} \times \dfrac{5!}{3!1!1!}$ total arrangements;
5) A triple and a pair:
   (a) $\binom{10}{1} \times \binom{9}{1}$ choices;
   (b) $\dfrac{5!}{3!2!}$ arrangements per choice;
   (c) $\binom{10}{1} \times \binom{9}{1} \times \dfrac{5!}{3!2!}$ total arrangements;