



Now that we have the angle BFC , we can write

$$\begin{aligned}\tan(\beta) &= \frac{1}{q} \\ \Rightarrow \beta &= \tan^{-1}\left(\frac{1}{q}\right).\end{aligned}$$

So, overall, we have

$$\tan(\alpha) = \frac{1}{p} \qquad \tan(\beta) = \frac{1}{q}$$

or

$$\alpha = \tan^{-1}\left(\frac{1}{p}\right) \qquad \beta = \tan^{-1}\left(\frac{1}{q}\right).$$

ii) We are required to prove

$$p + q = 1 - pq.$$

From the previous part, we have that

$$\begin{aligned}p + q &= \frac{1}{\tan(\alpha)} + \frac{1}{\tan(\beta)} \\ &= \frac{\tan(\alpha) + \tan(\beta)}{\tan(\alpha)\tan(\beta)}.\end{aligned} \tag{1}$$

Additionally,

$$\begin{aligned}1 - pq &= 1 - \frac{1}{\tan(\alpha)\tan(\beta)} \\ &= \frac{\tan(\alpha)\tan(\beta) - 1}{\tan(\alpha)\tan(\beta)}.\end{aligned} \tag{2}$$

Dividing equation (1) by (2), we obtain

$$\begin{aligned}\frac{p + q}{1 - pq} &= \frac{\tan(\alpha) + \tan(\beta)}{\tan(\alpha)\tan(\beta)} \cdot \frac{\tan(\alpha)\tan(\beta)}{\tan(\alpha)\tan(\beta) - 1} \\ &= \frac{\tan(\alpha) + \tan(\beta)}{\tan(\alpha)\tan(\beta) - 1} \\ &= -\tan(\alpha + \beta).\end{aligned} \tag{3}$$