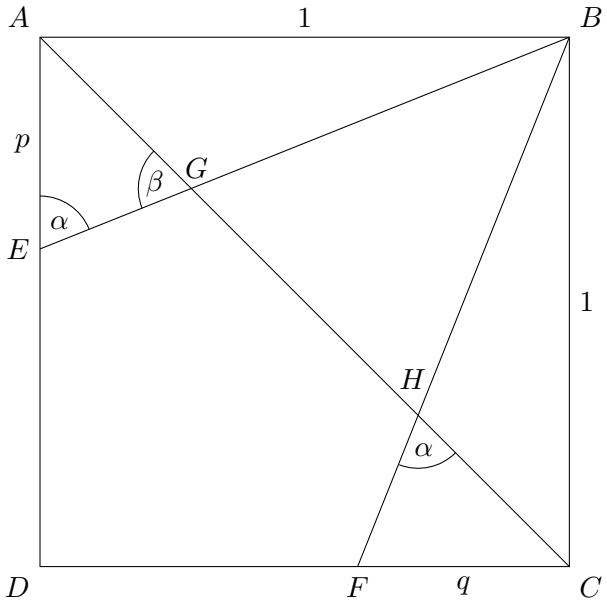
Question Breakdown - Set 1, Question 8



In the diagram, $ABCD$ is a unit square. Points E and F are chosen on AD and DC respectively, such that $\angle AEG = \angle FHC$, where G and H are the points at which BE and BF respectively cut the diagonal AC.

Let $AE = p$, $FC = q$, $\angle AEG = \alpha$ and $\angle ABE = \beta$.

i) Express α in terms of p and β in terms of q. (2 marks)

ii) Prove that $p + q = 1 - pq$. (2 marks)

iii) Show that the area of the quadrilateral $EBFD$ is given by

$$1 - \frac{p}{2} + \frac{p-1}{2(p+1)}. \quad \text{(1 mark)}$$

iv) What is the maximum value of the area of $EBFD$? (2 marks)

This question is a more difficult application of plane geometry and right-angled trigonometry. The first two parts of the question are probably the most difficult, but the question is constructed such that one can at least do parts iii and iv if parts i and ii prove too difficult.

Remark. This is quite common. More difficult questions will often be written such that subsequent parts depend on previous parts, but if one is unable to do the previous part, the result can still be used, and some marks can still be attained. This is particularly common in questions such as this one where one is required to prove a particular result. Even if the proof is not completed, one can take the result as given for the rest of the question.

The best place to start in a trigonometric or geometric problem such as this is with a diagram. It's usually a good idea to copy down the diagram given in the question (if one is given). In particular, ensure that your diagram

- is **large** (about half a page is sufficient);
- **contains all the information given** in the question (in this case, the angles at A, B and C);
- **contains what you are required to find** (the bearing of B from Uluru, which will involve the angle between B and North);