## A harder combinatorics question: inspired by my morning commute

It is well known by frequent users of public transport that every carriage on a Sydney Trains train has a four digit serial number comprised of digits between 0 and 9 (inclusive!). The "Train Game" (or whatever students call it now) has the aim of combining the digits of the carriage serial number, using standard operations, to make ten[1]. Traditionally, it is stipulated that no number can be used more than once and all numbers must be used in the order in which they appear. For example, 1345: $(1 - 3 + 4) \times 5$.

I'm bad at basic arithmetic and my morning train ride isn't that long, so I prefer to rearrange the numbers into whatever order is convenient. Thus, suppose the order of the digits is NOT important (i.e. 1234 and 3421 are equivalent).

i) If serial numbers have no repetitions, how many different serial numbers are possible?

Serial numbers often DO have repeats, however. For example, a three digit serial number could comprise a triple (e.g. 111), a pair and a single (e.g. 112) or three singles (e.g. 123); there are three unique cases to consider.

ii) For a four digit serial number, how many unique cases are there? List them, with an example.

iii) How many different arrangements are possible, assuming order is unimportant (i.e. 1234 and 3421 are considered the same, as they both comprise 1, 2, 3 and 4) and repetitions are permitted?

iv) Repeat parts i), ii) and iii) for five digit serial numbers.

Suppose we insist that we use the numbers in order. That is, order is important (i.e. 1234 and 3421 are NOT equivalent).

v) Repeat the question assuming order IS important.

[1]Sometimes this is very easy; other times this is particularly difficult and sometimes it is impossible! An interesting problem to consider is this: what conditions must be imposed on a serial number in order to guarantee that there exists at least one way to make ten? This is an extremely non-trivial problem!