Case 7: Five of a kind

We are simply choosing one number from ten, so

$$\binom{10}{1}.$$

So, overall, we have

$$\underbrace{\binom{10}{5}}_{\text{Case 1}}+\underbrace{\binom{10}{1}\times\binom{9}{3}}_{\text{Case 2}}+\underbrace{\binom{10}{2}\times\binom{8}{1}}_{\text{Case 3}}+\underbrace{\binom{10}{1}\times\binom{9}{2}}_{\text{Case 4}}+\underbrace{\binom{10}{1}\times\binom{9}{1}}_{\text{Case 5}}+\underbrace{\binom{10}{1}\times\binom{9}{1}}_{\text{Case 6}}+\underbrace{\binom{10}{1}}_{\text{Case 7}}=2002\text{ ways.}$$

The final part of the question mandates that we recalculate considering order important. The cases remain the same, we simply have to take into account the number of possible permutations of the digits chosen. The previous parts of the question gave us the number of ways of choosing the digits, so, now that they have been chosen, we must determine the number of ways in which they can be arranged. This is identical to determining the number of "words" possible using a certain set of letters.

For four digit numbers, we have the same five cases from before.

Case 1: Four singles (also, part i)

There are $\binom{10}{4}$ ways of choosing the digits. Additionally, we have four digits and no repeats, so there are $\dfrac{4!}{1!1!1!1!}$ ways of arranging them. So, overall, there are

$$\binom{10}{4}\times\frac{4!}{1!1!1!1!}$$

arrangements.

Case 2: A pair and two singles

There are $\binom{10}{1}\times\binom{9}{2}$ ways of choosing the digits. There are four digits with one pair, so there are $\dfrac{4!}{2!1!1!}$ ways of arranging them. So, we have

$$\binom{10}{1}\times\binom{9}{2}\times\frac{4!}{2!1!1!}$$

arrangements.

Case 3: Two pairs

There are $\binom{10}{2}$ choices of digits and two sets of pairs, giving $\dfrac{4!}{2!2!}$ ways of arranging them. So, we have

$$\binom{10}{2}\times\frac{4!}{2!2!}$$

arrangements.

Case 4: A triple and a single

There are $\binom{10}{1}\times\binom{9}{1}$ ways of choosing the digits. We have one set of three, so there are $\dfrac{4!}{3!1!}$ ways of arranging them, giving us

$$\binom{10}{1}\times\binom{9}{1}\times\frac{4!}{3!1!}$$

total arrangements.