

*Computer Vision & Multimedia Analysis Course — A.A. 2019/2020*

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# Lab 3: Tracking

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# What's up today (and tomorrow)

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- ❖ Good Features to Track + Lucas Kanade optical flow
- ❖ Meanshift algorithm
- ❖ Kalman filter



# Good Features to Track

- ❖ For each candidate point, compute:

$$Z = \begin{bmatrix} \sum_W J_x^2 & \sum_W J_x J_y \\ \sum_W J_y J_x & \sum_W J_y^2 \end{bmatrix}$$

- ❖  $J_x$  and  $J_y$  are the gradients evaluated on the point in  $x$  and  $y$  direction within  $W$  ( $n \times n$  window)
- ❖ A good feature point is where the smallest eigenvalue of  $Z$  is larger than a specified threshold
- ❖ In practice, it highlights corner points and textures



# Lucas-Kanade optical flow estimation

- ❖ Two-frame differential method for optical flow estimation developed by Bruce D. Lucas and Takeo Kanade (1981)
- ❖ Consider  $u=[u_x, u_y]$  in frame I and  $v=[v_x, v_y]$  in frame J
- ❖ The goal is to find  $\mathbf{d}$  that satisfies  $\mathbf{v}=\mathbf{u}+\mathbf{d}$  such as I and J are similar (translational model)
- ❖ Because of the aperture problem, **similarity** must be defined in 2D
- ❖  $\mathbf{d}$  is the vector that minimizes

$$\epsilon(d) = \epsilon(d_x, d_y) = \sum_{x=u_x-\omega_x}^{u_x+\omega_x} \sum_{y=u_y-\omega_y}^{u_y+\omega_y} (I(x, y) - J(x + d_x, y + d_y))^2$$

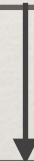
- ❖  $\omega$  is the integration window





# GFF+LK tracking

Use GFF to detect and select good Features



Track detected feature using LK optical flow





# Exercise

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## Part 1

- ❖ Track features in the environment using
- ❖ `calcOpticalFlowPyrLK(previous_frame, current_frame, previous_keypoints, current_keypoints, status, err);`





# Exercise

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## Part 2

- ❖ How to avoid losing features after some time?
- ❖ Re-detect features using GFF





# Meanshift algorithm

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- ❖ Inside the Virtual Machine (or in your programming environment)
- ❖ Go to this link and download the file
- ❖ [https://github.com/nick1392/mean-shift\\_demo](https://github.com/nick1392/mean-shift_demo)





# Meanshift algorithm

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- ❖ RGB to HSV image conversion
- ❖ Manually select Region Of Interest (ROI)
- ❖ Calculate histogram of ROI
- ❖ Back projection of the histogram
- ❖ Tracking





# Camshift algorithm

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- ❖ Finds an object center using `meanShift()`
- ❖ Adjusts the window size and finds the optimal rotation.

## Exercise

- ❖ `camShift` algorithm instead of `MeanShift`
- ❖ Check documentation on the website
- ❖ Display the window using the ellipse function
- ❖ `void ellipse(Mat& img, const RotatedRect& box, const Scalar& color, int thickness=1, int lineType=8)`





# Kalman filter

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- ❖ Inside the Virtual Machine (or in your programming environment)
- ❖ Go to this link and download the file
- ❖ <https://github.com/nick1392/kalman>



# Kalman filter

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$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{w}_{k-1}$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

- ❖  $\mathbf{x}_k$  is the current state
- ❖  $\mathbf{x}_{k-1}$  is the previous state
- ❖  $\mathbf{A}_k$  is the state transition matrix
- ❖  $\mathbf{w}_k$  is the process noise
- ❖  $\mathbf{z}_k$  is the actual measurement
- ❖  $\mathbf{H}_k$  is the measurement matrix
- ❖  $\mathbf{v}_k$  is the measurement noise



# Kalman filter

$$\hat{\mathbf{x}}_k^- = \mathbf{A}_k \hat{\mathbf{x}}_{k-1}$$

$$\mathbf{P}_k^- = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{Q}_{k-1}$$

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-$$



# Kalman filter applied on mouse motion

❖ Motion equation:  $P_t = P_0 + V * t$

$$x_k = A_k x_{k-1} + w_{k-1}$$

$$z_k = H_k x_k + v_k$$

- ❖  $x_k$  is the current state —> a vector with the position and velocity
- ❖  $A_k$  is the state transition matrix —> matrix that describe the system, in our case the motion equation
- ❖  $H_k$  is the measurement matrix —> determined by the current measured position of the mouse
- ❖  $z_k$  is the actual measurement —> used to compute the “posteriori”



# Transition matrix

$$\diamond X = [x, y, v\_x, v\_y]^t$$

$$\diamond x_{t+1} = x_t + v\_x_t \longrightarrow [1, 0, 1, 0]$$

$$\diamond y_{t+1} = y_t + v\_y_t$$

$$\diamond v\_x_{t+1} = v\_x_t \longrightarrow [0, 0, 1, 0]$$

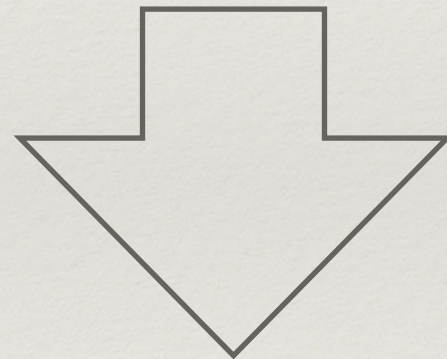
$$\diamond v\_y_{t+1} = v\_y_t$$



# Exercise

- ❖ Insert acceleration in the transition matrix of the Kalman filter

$$x_t = x_0 + v_x * t$$



$$x_t = x_0 + v_x * t + \frac{1}{2} a_x * t^2$$



# Transition matrix

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$$\diamond X = [x, y, v\_x, v\_y, a\_x, a\_y]^t$$

$$\diamond x_{t+1} = x_t + v\_x_t + 0.5 a\_x_t$$

$$\diamond y_{t+1} = y_t + v\_y_t + 0.5 a\_y_t$$

$$\diamond v\_x_{t+1} = v\_x_t + a\_x_t$$

$$\diamond v\_y_{t+1} = v\_y_t + a\_y_t$$

$$\diamond a\_x_{t+1} = a\_x_t$$

$$\diamond a\_y_{t+1} = a\_y_t$$