Assignment 4.

I] given :- Mean $\mu = 30$
S.D. $\sigma = 2$

① longer than 31.7 cm ?

formula $z = \frac{x-\mu}{\sigma}$

$z = \frac{31.7-30}{2}$

$\boxed{z = 0.85}$

$\therefore P(X > 31.7) = P(Z > 0.85)$
$= 1 - P(Z \leq 0.85)$
$= 1 - 0.8023$
$= 0.1977 = \underline{\underline{19.77\%}}$

② between 29.3 & 33.5 cm ?

formula $z = \frac{x_1-\mu}{\sigma}$ $\qquad z = \frac{x_2-\mu}{\sigma}$

$z = \frac{29.3-30}{2}$ $\qquad z = \frac{33.5-30}{2}$

$\boxed{z = -0.35}$ $\qquad \boxed{z = 1.75}$

$\therefore P(29.3 < X < 33.5) = P(-0.35 < Z < 1.75)$
$= P(Z < 1.75) - P(Z < -0.35)$
$= 0.9599 - 0.3632$
$= 0.5967 = \underline{\underline{59.67\%}}$

③ shorter than 25.5 cm ?

formula $z = \frac{x-\mu}{\sigma}$

$z = \frac{25.5-30}{2}$

$\boxed{z = -2.25}$

$\therefore P(X < 25.5) = P(Z < -2.25)$
$= 0.0122$
$= \underline{\underline{1.22\%}}$

2] a) Let 'X' represent the no. of tosses to get 3 heads   P(third head on 7^{th} flip)

$K = 3$.
$x = 7$
$p = \frac{1}{2}, q = \frac{1}{2}$

Using Binomial Distribution,

formula $P(X = x) = b(x; K, p) \quad \forall x = K, K+1, K+2, \ldots$

$= \binom{x-1}{K-1} p^K q^{x-K}$

$P(X = 7) = b(7, 3, \frac{1}{2})$

$= ({}^6C_2) \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^4$

$= \frac{6!}{(6-2)!\, 2!} \left(\frac{1}{8}\right)\left(\frac{1}{16}\right)$

$= \frac{6 \times 5 \times 4!}{4! \times 2!} \left(\frac{1}{8}\right)\left(\frac{1}{16}\right)$

$\therefore P(X = 7) = 0.1172$

$\therefore$ Probability of getting third head on seventh flip is $\underline{\underline{0.1172}}$

b) P(first head on fourth flip)
Let 'x' represent the no. of tosses to get 1 head

$K = 1$

$x = 4$

$p = \frac{1}{2}$, $q = \frac{1}{2}$

Using Binomial Distribution,

formula $P(X=x) = b(x; K, p) \quad \forall x = K, K+1, K+2 \ldots$

$= \binom{x-1}{K-1} p^K q^{x-K}$

$P(X=4) = b(4, 1, \frac{1}{2})$

$= (3C_0) \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3$

$= \frac{3!}{(3-0)!\,0!} \left(\frac{1}{2}\right)\left(\frac{1}{8}\right)$

$= \frac{\cancel{3!}}{\cancel{3!}.1} \left(\frac{1}{16}\right)$

$\therefore P(X=4) = 0.0625$

$\therefore$ Probability of getting first head on fourth flip is $\underline{0.0625}$

3] a) Let 'X' represent the no. of defective items among 3 tested items.
The box will be shipped if there is no defective item.
So, we need to find $P(X=0)$.
we have, total items $N = 25$

random sample $n = 3$

$K = 3$ (success)

$N-K = 25-3 = 22$ (failure)

$x = 0$

'X' is a hypergeometric random variable.
$\therefore$ Probability Distribution of hypergeometric random variable X is,

formula $h(x; N, n, K) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

$$h(0; 25, 3, 3) = \frac{(3C_0)(22C_3)}{(25C_3)}$$

$$= \frac{\frac{3!}{3!\,0!}\ \frac{22!}{19!\,3!}}{\frac{25!}{22!\,3!}}$$

$$= \frac{22!}{19!} \times \frac{22!}{25!}$$

$$= \frac{22 \times 21 \times 20 \times \cancel{19!}}{\cancel{19!}} \times \frac{\cancel{22!}}{25 \times 24 \times 23 \times \cancel{22!}}$$

$$= \frac{9240}{13800}$$

$$= \underline{\underline{0.6696}}$$

b) We need to find $P(X=1)$

we have, $N = 25$

$n = 3$

$K = 1$

$x = 1$

Probability Distribution of hypergeometric random variable X is,

formula $h(x; N, n, K) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

$$h(1; 25, 3, 1) = \frac{{}^{1}C_{1}\ {}^{24}C_{2}}{{}^{25}C_{3}}$$

$$= \frac{\frac{24!}{22!\,2!}}{\frac{25!}{22!\,3!}}$$

$$= \frac{24!}{22!\,2!} \times \frac{22!\,\times 3!}{25!}$$

$$= \frac{24! \times 3}{25 \times 24!}$$

$$= \frac{3}{25}$$

$$= \underline{\underline{0.12}}$$

4] given :- No. of vehicles $n = 9$

$q = 0.75$

$p = 1 - q = 1 - 0.75 = 0.25$

$x = 4$

$P(X < 4) = P(X \leq 3)$

Using Binomial Distribution

formula $P(X = x) = \sum {}^{n}C_x \, p^x q^{n-x}$

$= \sum_{x=0}^{3} {}^{n}C_x \, p^x q^{n-x}$

$= \sum_{i=0}^{3} P(X = i)$

$= \sum_{i=0}^{3} b(i; 9, 0.25) \longrightarrow (1)$

Using Table of Binomial Probability Sums,

$P(X<4) = \sum_{i=0}^{3} b(i; 9, 0.25)$

$= \underline{\underline{0.8343}}$

5] Let 'X' follows a continuous uniform distribution from 1 to 5

$\therefore$ The density function of X is,

$$f(x) = \begin{cases} \frac{1}{5-1} = \frac{1}{4} ; & 1 \leq x \leq 5 \\ 0 , & \text{otherwise} \end{cases}$$

Now, let's find the conditional probability

$f_{x|y}(x|y) = f(x,y)/f_y(y)$

formula $P(X > 2.5 \mid X \leq 4) = \dfrac{P(2.5 < X \leq 4)}{P(X \leq 4)}$

$$P(2.5 < X < 4) = \int_{2.5}^{4} f(x)\,dx$$

$$= \int_{2.5}^{4} \left(\frac{1}{4}\right) dx$$

$$= \frac{1}{4}[x]_{2.5}^{4}$$

$$= \frac{1}{4}[4 - 2.5]$$

$$= \underline{\underline{0.375}}$$

$$P(X \leq 4) = \int_{1}^{4} \left(\frac{1}{4}\right) dx$$

$$= \frac{1}{4}[x]_{1}^{4}$$

$$= \frac{1}{4}[4 - 1]$$

$$= \underline{\underline{0.75}}$$

$$\therefore P(X > 2.5 \mid X \leq 4) = \frac{0.375}{0.75}$$

$$= \underline{\underline{0.5}}$$