IE6400 Fall 2023 Assignment 3

Problem 1 (20 points)
From a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. Find the probability distribution for the number of green balls.

Problem 2 (20 points)
The time to failure in hours of an important piece of electronic equipment used in a manufactured DVD player has the density function:

$$f(x) = \frac{1}{2000}\exp(-x/2000), x \geq 0$$
$$f(x) = 0, x < 0$$

(a) Find F(x)
(b) Determine the probability that the component (and thus the DVD player) lasts more than 800 hours before the component needs to be replaced
(c) Determine the probability that the component fails before 2000 hours

Problem 3 (20 points)
A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

$$f(x,y) = 24xy, 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1$$
$$f(x,y) = 0, \text{ elsewhere}$$

(a) Find the probability that in a given box the cordials account for more than ½ of the weight
(b) Find the marginal density for the weight of the creams.
(c) Find the probability that the weight of the toffees in the box is less than ⅛ of a kilogram if it is known that creams constitute ¾ of the weight

Problem 4 (20 points) 4.64
Suppose that X and Y are independent random variables with probability densities expressed below. Find the expected value of Z = XY.

$$g(x) = \begin{cases} \frac{8}{x^3}, & x > 2, \\ 0, & \text{elsewhere}, \end{cases}$$

and

$$h(y) = \begin{cases} 2y, & 0 < y < 1, \\ 0, & \text{elsewhere}. \end{cases}$$

Problem 5 (20 points)
Random variables X and Y follow a joint distribution:

$$f(x, y) = \begin{cases} 2, & 0 < x \leq y < 1, \\ 0, & \text{otherwise}. \end{cases}$$

Determine the correlation coefficient between X and Y

Submission Format

1. Submit solutions in .docx, .ipynb, or handwritten format (scan and upload)
2. All the formulas used for numerical solutions **must be included**
    a. If using .ipynb for formulas, use Text cells to write formulas before computing
3. Submit by 12 PM PT on Wednesday, October 25