Dynamic modeling

The quadcopter model is illustrated in Figure 1. The quadcopter has 6 degrees of freedom, three related to positional coordinates (x, y, z) and three related to angular positions (θ, ϕ, ψ). In this study, the degrees of freedom pertaining to positional coordinates relative to the ground are considered (x_E, y_E, z_E). The Newton-Euler equations of motion for the 6 degrees of freedom have been utilized to describe the system's dynamics[1];

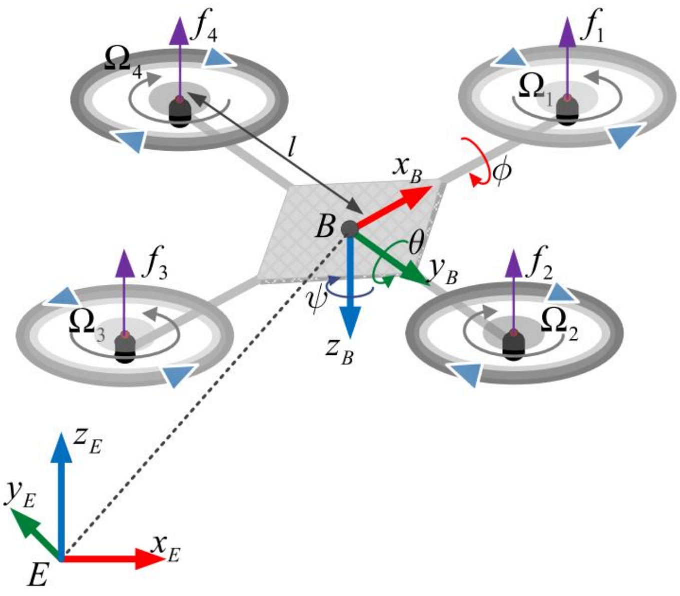

Figure 1. Schematic of the quadcopter model.[HA LE NHU NGOC THANH 2018]

Parameters	Discreption
I_{xx}, I_{yy}, I_{zz} (kgm^2)	The moments of inertia about the body-fixed x, y, and z axes
m (kg)	Total mass of the quadcopter
l (m)	Length of quadcopter's arm
b (Ns^2)	Lift coefficient of propellers
d (Nms^2)	Drag coefficient of propellers
J_r (kgm^2)	The moments of inertia of each propellers

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \\ \ddot{z} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} + \dot{\theta}\frac{J_r}{I_{xx}}\Omega_r + \frac{l}{I_{xx}}U_2 \\ \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\phi}\dot{\psi} - \dot{\phi}\frac{J_r}{I_{yy}}\Omega_r + \frac{l}{I_{yy}}U_3 \\ \frac{I_{xx}-I_{yy}}{I_{zz}}\dot{\phi}\dot{\theta} + \frac{l}{I_{zz}}U_4 \\ g - \frac{\cos(\phi)\cos(\theta)}{m}U_1 \\ \frac{U_1}{m}\left(\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)\right) \\ \frac{U_1}{m}\left(\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi)\right) \end{bmatrix} + \begin{bmatrix} \xi_\phi(t) \\ \xi_\theta(t) \\ \xi_\psi(t) \\ \xi_h(t) \\ 0 \\ 0 \end{bmatrix}$$

Where $\xi_\phi(t)$, $\xi_\theta(t)$, $\xi_\psi(t)$ and $\xi_h(t)$ correspond, respectively, to disturbances and uncertainties in the roll, pitch, yaw, and altitude angles. The value of Ω_r is obtained from the following equation where Ω_i is the rotational speed of each motor. g represents the acceleration due to gravity and is considered to be 9.8 m/s².

$$\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4$$

When all four motors spin at the same speed, a force is exerted on the quadcopter in the positive direction of the z_B axis (U_1). If the rotational speed of motors 1 and 3 is the same while the speed of motor 4 exceeds that of motor 2, the quadcopter will rotate about the angle ϕ (U_2). Similarly, if the rotational speed of motors 2 and 4 is the same while the speed of motor 1 exceeds that of motor 3, the quadcopter will rotate about the angle θ (U_3). When the sum of the squares of the speeds of motors 2 and 4 surpasses that of motors 1 and 3, the quadcopter will rotate about the angle ψ (U_4). The mathematical expression for the relationships between the speeds and the mentioned forces is given by:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ b\left(-\Omega_2^2 + \Omega_4^2\right) \\ b\left(\Omega_1^2 - \Omega_3^2\right) \\ d\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \end{bmatrix}$$

Controller

The control system diagram is shown in Figure 2. The system states (12 states) include 6 degrees of freedom in terms of position and angles, as well as their derivatives (linear and angular velocities). The rotational speeds of the rotors are also sensed and recorded by sensors on the quadcopter system. The controller consists of two parts: the altitude controller and the attitude controller. The input to the altitude controller is the quadcopter's position and the reference position, and its output is three control commands: u_x, u_y, and u_z. These values are transformed into three outputs by the conversion block, to form the reference angles θ and ϕ and first control command (U_1). The reference angles θ_d and ϕ_d are then sent to the attitude controller. The attitude controller takes the reference values for angles θ and φ from the conversion block and for ψ from the user as inputs, and generates the control commands U_2, U_3, and U_4.

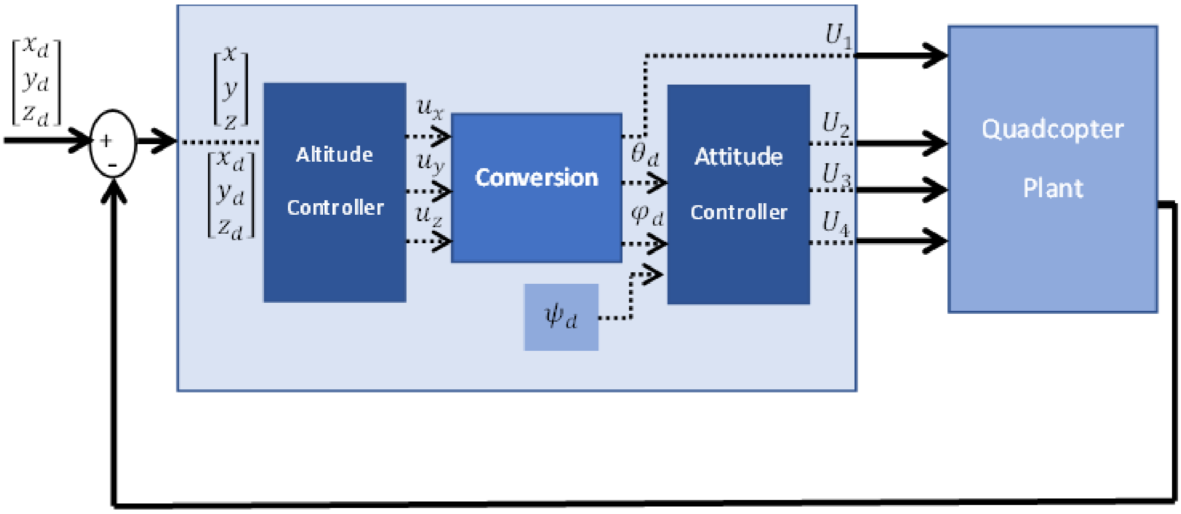

Figure 2. Control system diagram.

Altitude Controller

A linear controller has been used to determine the control commands ux, uy, and uz. It is assumed that the positional error dynamics are represented by the following linear relationship.

$$\begin{bmatrix} (\ddot{x}_d - \ddot{x}) + k_{d\_x}(\dot{x}_d - \dot{x}) + k_{p\_x}(x_d - x) \\ (\ddot{y}_d - \ddot{y}) + k_{d\_y}(\dot{y}_d - \dot{y}) + k_{p\_y}(y_d - y) \\ (\ddot{z}_d - \ddot{z}) + k_{d\_z}(\dot{z}_d - \dot{z}) + k_{p\_z}(z_d - z) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \ddot{e}_x + k_{d\_x}\dot{e}_x + k_{p\_x}e_x \\ \ddot{e}_y + k_{d\_y}\dot{e}_y + k_{p\_y}e_y \\ \ddot{e}_z + k_{d\_z}\dot{e}_z + k_{p\_z}e_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Conversion Block

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \ddot{x}_d + k_{d\_x}\dot{e}_x + k_{p\_x}e_x \\ \ddot{y}_d + k_{d\_y}\dot{e}_y + k_{p\_y}e_y \\ \ddot{z}_d + k_{d\_z}\dot{e}_z + k_{p\_z}e_z \end{bmatrix} = \begin{bmatrix} \frac{U_1}{m}\left(\cos(\phi)\sin(\theta)\cos(\psi)+\sin(\phi)\sin(\psi)\right) \\ \frac{U_1}{m}\left(\cos(\phi)\sin(\theta)\sin(\psi)-\sin(\phi)\cos(\psi)\right) \\ g-\frac{\cos(\phi)\cos(\theta)}{m}U_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \theta_d \\ U_1 \\ \phi_d \end{bmatrix} = \begin{bmatrix} \tan^{-1}(\frac{\ddot{x}}{\ddot{z}+g}\cos(\psi_d)+\frac{\ddot{y}}{\ddot{z}+g}\sin(\psi_d)) \\ \sqrt{\ddot{x}^2+\ddot{y}^2+(\ddot{z}+g)^2} \\ \tan^{-1}(\frac{\ddot{x}\times\sin(\psi_d)+\ddot{y}\times\cos(\psi_d)}{U_1}) \end{bmatrix}$$

Attitude Controller

The sliding surface is described by a nonlinear relationship as expressed as:

$$s = e+\beta\,|\,\dot{e}\,|^{\lambda}\ sign(\dot{e}) \quad , \quad \beta>0, \quad 1<\lambda<2$$

To achieve the sliding surface, the following control law must be satisfied using the control command:

$$\dot{s} = -k_1 s - k_2\times|\,s\,|^{\rho}\ sign(s)$$

Lyapunov theory is utilized to demonstrate stability. The Lyapunov function is defined as $V=\frac{s^2}{2}$. Equations for the control command U_2 are written, and U_3 and U_4 are obtained in a similar manner.

$$\dot{V} = s\dot{s} \xrightarrow{3.6} \dot{V} = s\left(\dot{e}+\beta\lambda\,|\,\dot{e}\,|^{\lambda-1}(\ddot{e})\right)$$

$$\xrightarrow{2.1} \dot{V} = s\left(\dot{e}+\beta\lambda\,|\,\dot{e}\,|^{\lambda-1}[\ddot{\phi}_d-(\frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi}+\dot{\theta}\frac{J_r}{I_{xx}}\Omega_d+\frac{l}{I_{xx}}U_2)]\right)$$

$$\xrightarrow{3.7} U_2 = \frac{I_{xx}}{l}(-\dot{\theta}\frac{J_r}{I_{xx}}\Omega_d-\frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi}+\ddot{\phi}_d)\times\frac{-\dot{e}}{\beta\lambda\,|\,\dot{e}\,|^{\lambda-1}}-\frac{I_{xx}}{l}(k_1 s+k_2\times|\,s\,|^{\rho}\ sign(s))$$

Hence, the control laws are obtained as following equation:

$$U_2 = \frac{I_x}{l}(\ddot{\phi}_d - a_1\dot{\theta}\dot{\psi} - a_2\dot{\theta}\Omega_d - \frac{\dot{e}}{\beta\lambda\,|\,\dot{e}\,|^{\lambda-1}} - k_{1\phi}s_\phi - k_{2\phi}\times|\,s_\phi\,|^{\rho}\ sign(s_\phi))$$

$$U_3 = \frac{I_y}{l}(\ddot{\theta}_d - a_3\dot{\phi}\dot{\psi} - a_4\dot{\phi}\Omega_d - \frac{\dot{e}}{\beta\lambda\,|\,\dot{e}\,|^{\lambda-1}} - k_{1\theta}s_\theta - k_{2\theta}\times|\,s_\theta\,|^{\rho}\ sign(s_\theta))$$

$$U_3 = \frac{I_z}{l}(\ddot{\psi}_d - a_5\dot{\phi}\dot{\theta} - \frac{\dot{e}}{\beta\lambda\,|\,\dot{e}\,|^{\lambda-1}} - k_{1\psi}s_\psi - k_{2\psi}\times|\,s_\psi\,|^{\rho}\ sign(s_\psi))$$

Fuzzy inference system

A Sugeno fuzzy system is employed to adjust the parameters k_1 and k_2, which determine the controller gains. In cases where the level of disturbance and uncertainty in the system is high, larger values for k1 and k2 are chosen. The fuzzy system as presented in Figure 3 has four inputs: θ, ϕ, and two additional parameters representing the distance from the sliding surface and the rate of change of this distance.

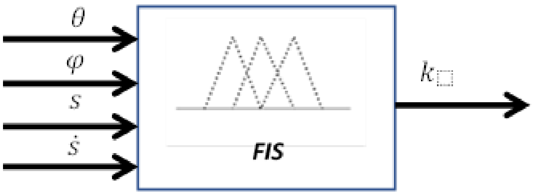

Figure 3. Inputs of the adaptive fuzzy controller tuning system.

Results

The quadcopter control results have been evaluated for two scenarios: one without disturbances and another with disturbances.

Without input disturbances

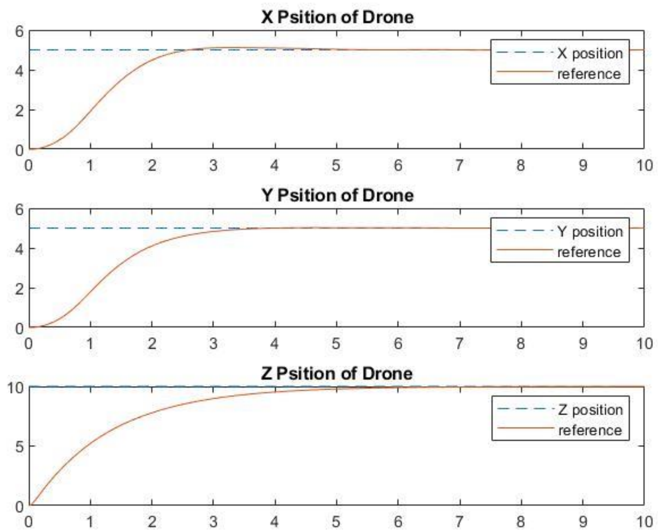

Figure 4. Degrees of freedom of position and their reference values.

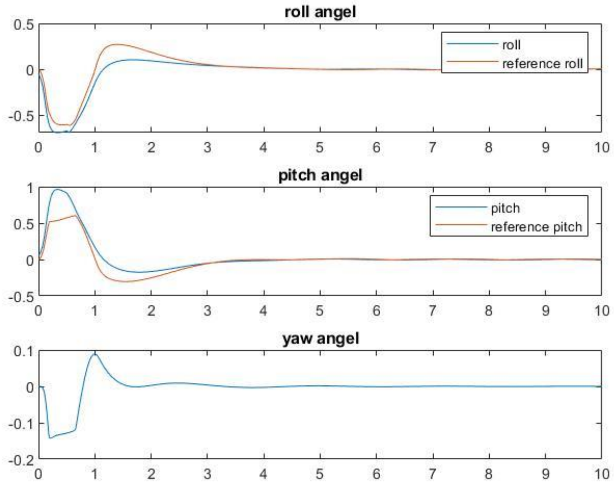

Figure 5. Angular degrees of freedom and their reference values.

And for sinusoidal reference

$$x_{ref} = 5\sin(t)$$
$$y_{ref} = 5\cos(t)$$
$$z_{ref} = 0.25t$$

, trajectories are:

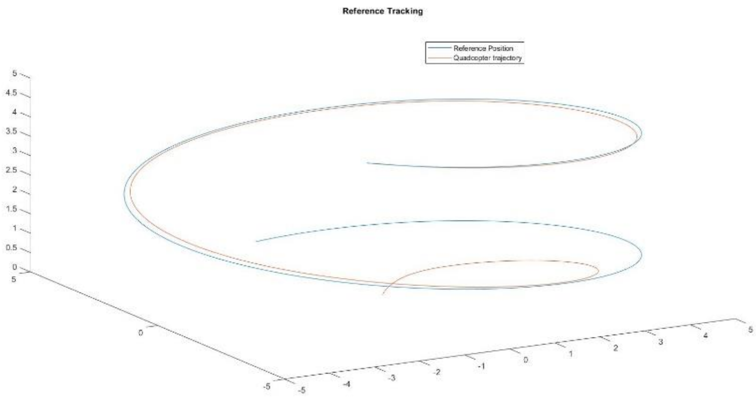

Figure 6. Quadcopter's positional coordinates along with the reference trajectory for tracking.

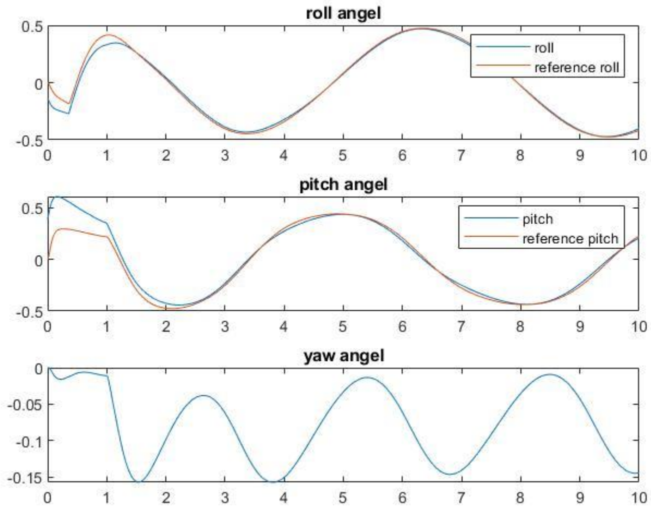

Figure 7. Angular degrees of freedom and their reference values.

With input disturbances

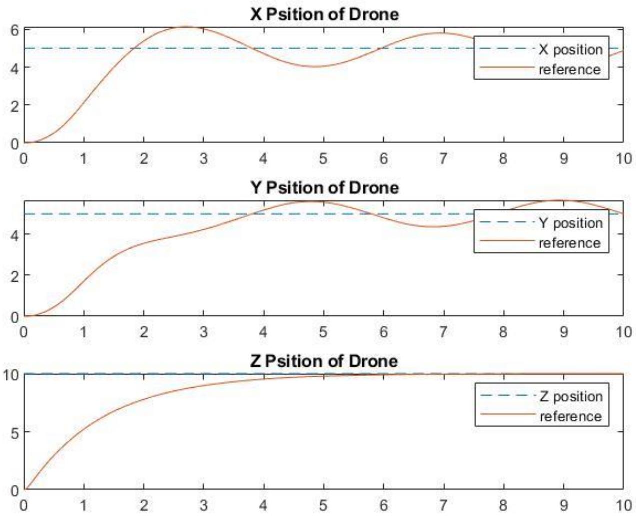

Figure 8. Degrees of freedom of position and their reference values.

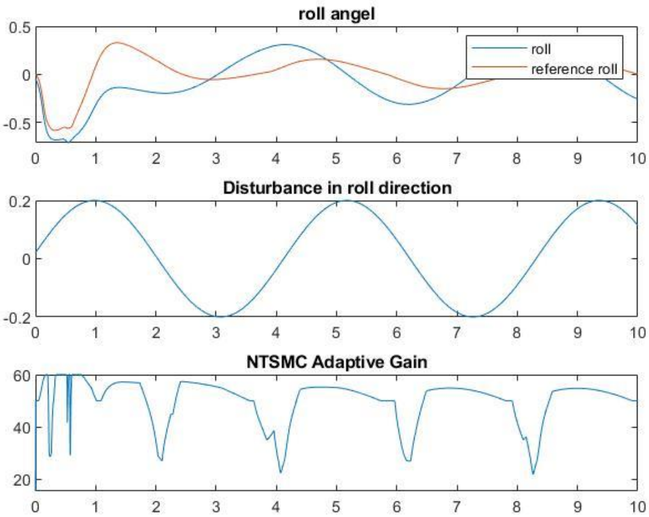

Figure 9. Roll angle and values of disturbance and adaptive gain.

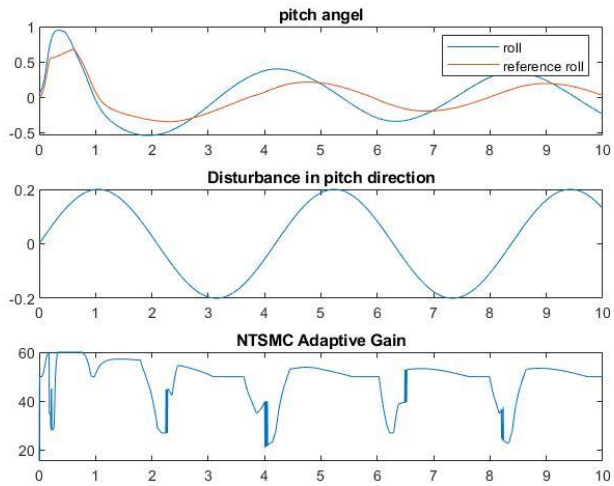

Figure 10. Pitch angle and values of disturbance and adaptive gain.