535240192
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Memo No. ....
Date 10 / 9 / 2025

1. Diketahui $u = (2, -2, 1)$
$v = (2, 1, 2)$

a. $u \cdot v = x_1 \cdot y_1 + y_2 \cdot y_2 + x_3 \cdot y_3$
$= 2 \cdot 2 + (-2) \cdot 1 + 1 \cdot 2$
$= 4 - 2 + 2 = 4$ //

b. $\cos\theta = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \cdot \|\bar{v}\|} = \frac{4}{9}$ //
$\|\bar{u}\| = \sqrt{2^2 + (-2)^2 + 1^2} = 3$
$\|\bar{v}\| = \sqrt{2^2 + 1^2 + 2^2} = 3$

c. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{0^2 + 3^2 + 1^2}$
$= \sqrt{10}$ //

d. $u \times v = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$
$= \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix}, \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -5, -2, 6$ //

e. $\|u \times v\| = \sqrt{(-5)^2 + (-2)^2 + 6^2} = \sqrt{25 + 4 + 36} = \sqrt{65}$ //

2. $a = (1, k, k^2)$
$b = (-2, -1, 1)$

a. $a \cdot b = 0$
$1 \cdot (-2) + k(-1) + k^2 \cdot 1 = 0$
$k^2 - k - 2 = 0$
$(k - 2)(k + 1) = 0$
$k = 2 \lor k = 1$ //

b. $k = 2 = a = (1, 2, 4)$
$b = (-2, -1, 1)$

$a \times b = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$
$= \begin{vmatrix} 2 & 4 \\ -1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 4 \\ -2 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$ A
$= 6, 9, 3$

$\|a \times b\| = \sqrt{6^2 + 9^2 + 3^2} = \sqrt{36 + 81 + 9} = \sqrt{126}$ //

Dipindai dengan CamScanner

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(3.) $U = (k+1, k+1, 1)$

$V = (-k-1, -k-1, k)$

$\cos\theta = \cancel{\frac{U \cdot V}{\|U\|\|V\|}} \quad \frac{U \cdot V}{\|U\|\|V\|} \qquad \cos 180 = -1 = \frac{U \cdot V}{\|U\|\|V\|}$

$U \cdot V = (k+1)(-k-1) + (k+1)(-k-1) + 1 \cdot k$
$= -k^2 - 2k - 1 - k^2 - 2k - 1 + k$
$= -2k^2 - 3k - 2$

$\|U\| = \sqrt{(k+1)^2 + (k+1)^2 + 1^2}$
$= \sqrt{k^2 + 2k + 1 + k^2 + 2k + 1 + 1}$
$= \sqrt{2k^2 + 4k + 3}$

$\|V\| = \sqrt{(-k-1)^2 + (-k-1)^2 + k^2}$
$= \sqrt{k^2 + 2k + 1 + k^2 + 2k + 1 + k^2}$
$= \sqrt{3k^2 + 4k + 2}$

$-1 = \frac{-2k^2 - 3k - 2}{\sqrt{(2k^2 + 4k + 3)(3k^2 + 4k + 2)}}$

$-1 = \frac{-2k^2 - 3k - 2}{\sqrt{6k^4 + 20k^3 + 29k^2 + 20k + 6}}$

$1 = \frac{4k^4 + 12k^3 + 17k^2 + 12k + 4}{6k^4 + 20k^3 + 29k^2 + 20k + 6}$

$6k^4 + 20k^3 + 29k^2 + 20k + 6 = 4k^4 + 12k^3 + 17k^2 + 12k + 4$

$6k^4 - 4k^4 + 20k^3 - 12k^3 - 12k^3 + 29k^2 - 17k^2 + 20k - 12k + 6 - 4 = 0$

$2k^4 + 8k^3 + 12k^2 + 8k + 2 = 0$

$k = -1$ //

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Transformasi Liniar

1. Diketahui $f = R^2 \to R^3$

$f(x,y) = (y, -5x+13y, 7x+16x)$

a. Diketahui $v = (1,2)$ dan $V(4,5)$ dan $k = 3$

b. Cek $T(u+v) = T(u) + T(v)$

$T(u+v) = T(u) + T(v)$

$T(1+4, 2+5) = T(5,7) = (7, -25+13(7), -35+16(7))$

$= 7, 66, 77)$

$T(1,2) + T(4,5) = (2, 21, 25) + (5, 45, 52)$

$= (7, 66, 77)$

c. Cek Syarat kedua $T(kv) = kT(v)$

$T(kv) = T[3(1,2)) = T((3,6)) = (6, 63, 75)$

$kT(v) = 3T(1,2) = 3((2,21,25)) = (6, 63, 75)$

$= f(x,y) = (y, -5x+13y, -7x+16y)$

adalah transformasi liniar

2. Diketahui $f(x,y) = (y, -5x+13y, -7x+16y)$

$P_1(0,1) \to (1, 13, 16)$

$P_2(2,1) \to (1, 3, 2)$

$P_3(1,3) \to (3, 34, 41)$

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Pencerminan

$P_1 = (1, 13, 16) \xrightarrow[xz]{\text{Pencerminan}} x' = x$

$y' = -y$

$z' = z$

$= (1, -13, 16)$

$P_2 = (1, 3, 2) \xrightarrow[xz]{\text{Pencerminan}} x' = x$

$y' = -y$

$z' = z$

$= (1, -3, 2)$

$P_3 = (3, 34, 41) \xrightarrow[xz]{\text{Pencerminan}} x' = x$

$y' = -y$

$z' = z$

$= (3, -34, 41)$

- Rotasi 45

$\cos 45^\circ = \frac{\sqrt{2}}{2} = \sin 45^\circ$

$P_1 (45^\circ) \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \times \begin{bmatrix} 1 \\ 13 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{17\sqrt{2}}{2} \\ -13 \\ \frac{15\sqrt{2}}{2} \end{bmatrix} = \left(\frac{17\sqrt{2}}{2}, -13, \frac{15\sqrt{2}}{2} \right)$

$P_2 (45^\circ) \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \times \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{2}}{2} \\ -3 \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \left(\frac{3\sqrt{2}}{2}, -3, \frac{\sqrt{2}}{2} \right)$

$P_3 (45^\circ) \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \times \begin{bmatrix} 3 \\ -34 \\ 41 \end{bmatrix} = \begin{bmatrix} \frac{44\sqrt{2}}{2} \\ -34 \\ \frac{38\sqrt{2}}{2} \end{bmatrix} = (22\sqrt{2}, -34, 19\sqrt{2})$