ECE408/CS483/CSE408 Fall 2022

Applied Parallel Programming

Lecture 17
Parallel Computation Patterns – Parallel Scan (Prefix Sum) – Part 2

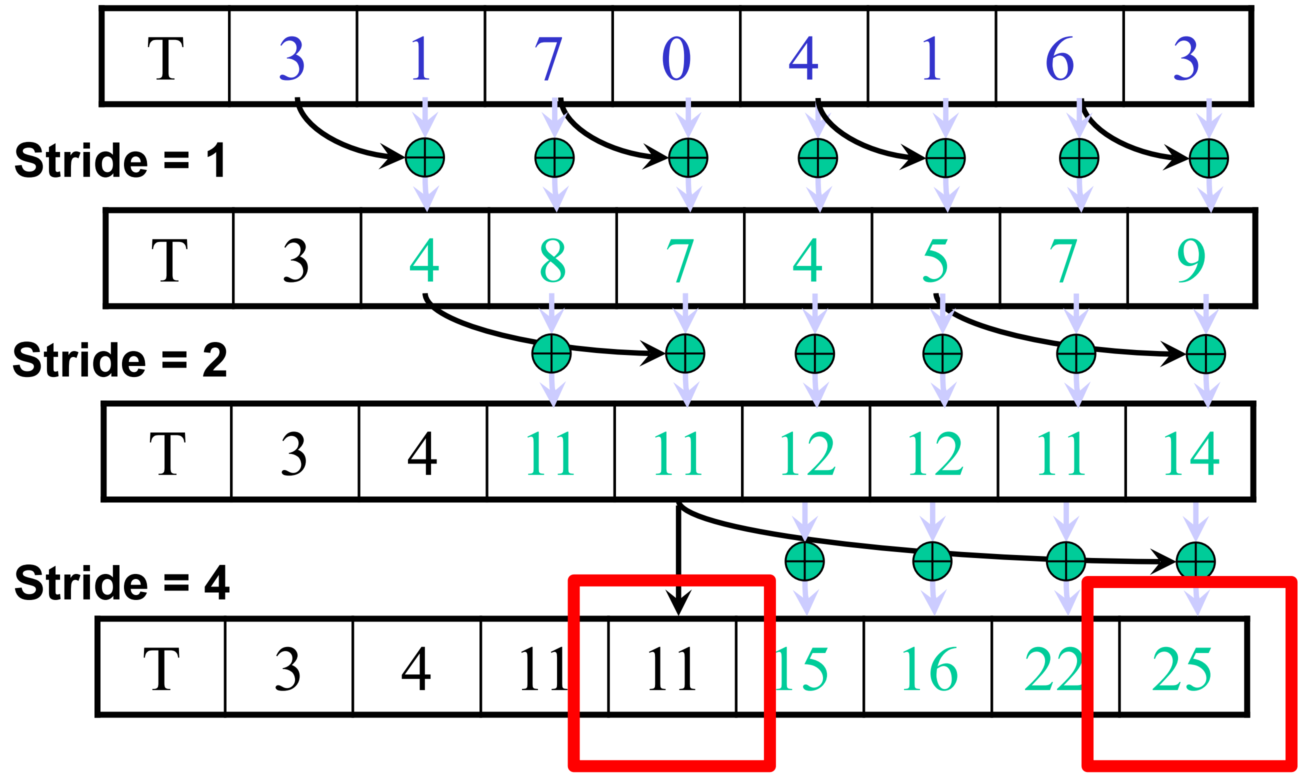
Course Reminders

- MP5.1 & MP5.2
  - MP5.1: Implement a kernel and associated host code that performs reduction of a 1D list stored in a C array. The reduction should give the sum of the list. You should implement the improved kernel discussed in the lecture. Your kernel should be able to handle input lists of arbitrary length.
  - MP5.2: Implement one or more kernels and their associated host code to perform parallel scan on a 1D list. The scan operator used will be addition. You should implement the work- efficient kernel discussed in lecture. Your kernel should be able to handle input lists of arbitrary length. However, for simplicity, you can assume that the input list will be at most 2,048 * 2,048 elements.
- Project Milestone 2: Baseline Convolution Kernel
  - Due November 4th

Objective

- To master parallel scan (prefix sum) algorithms
  - Work-efficiency vs. latency
  - Brent-Kung Tree Algorithm
  - Hierarchical algorithms

A Kogge-Stone Parallel Scan Algorithm



Work Efficiency Analysis

- A Kogge-Stone scan kernel executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) add operations each
  - Total # of add operations: n * log(n) - (n-1) → O(n*log(n)) work

- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 1,000,000 elements!
  - Typically used within each block, where n ≤ 1,024

- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

Improving Efficiency

- A common parallel algorithm pattern:

  Balanced Trees

  – Build a balanced binary tree on the input data and sweep it to and from the root

  – Tree is not an actual data structure, but a conceptual pattern

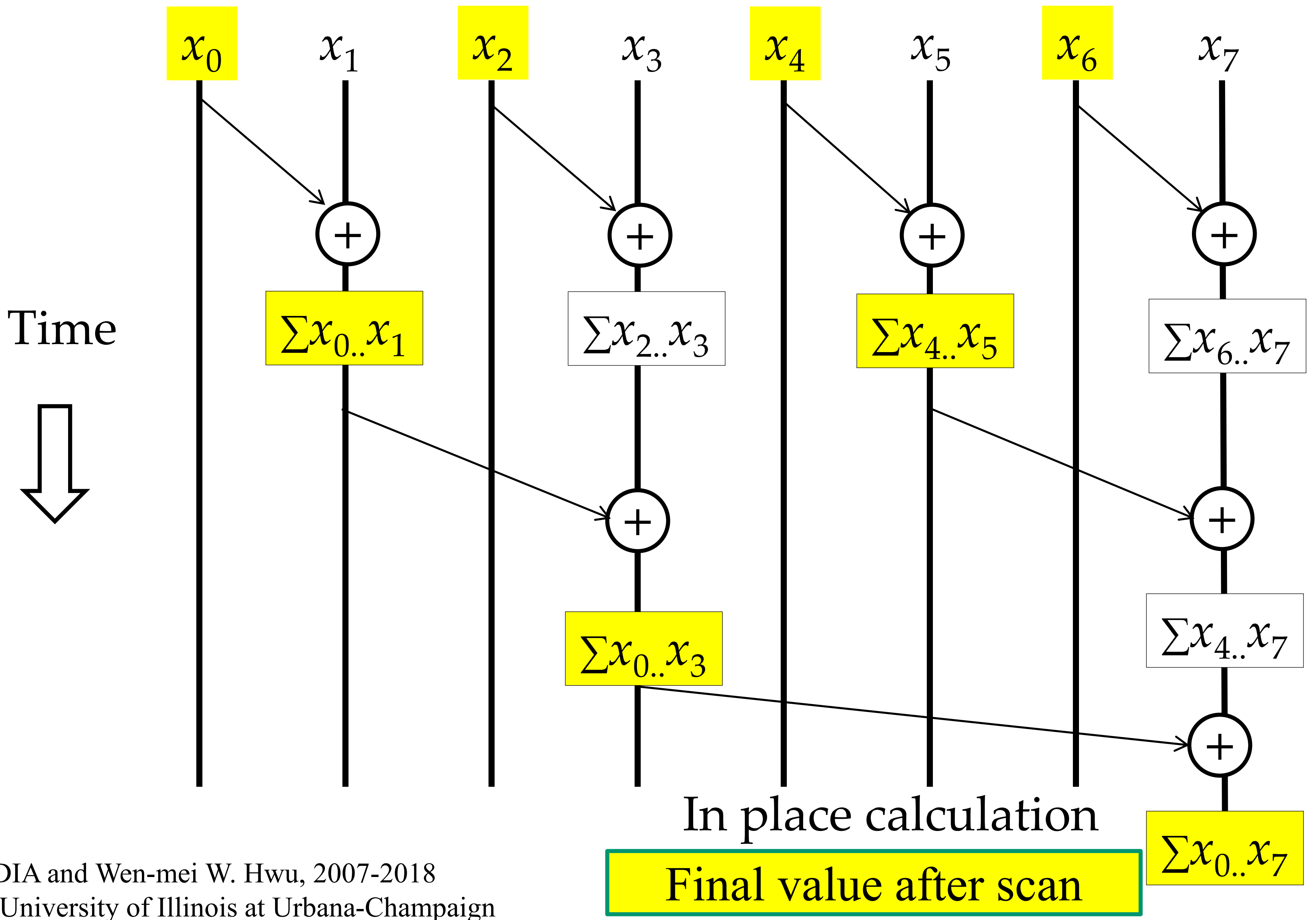
- For scan:

  – Traverse down from leaves to root building partial sums at internal nodes in the tree

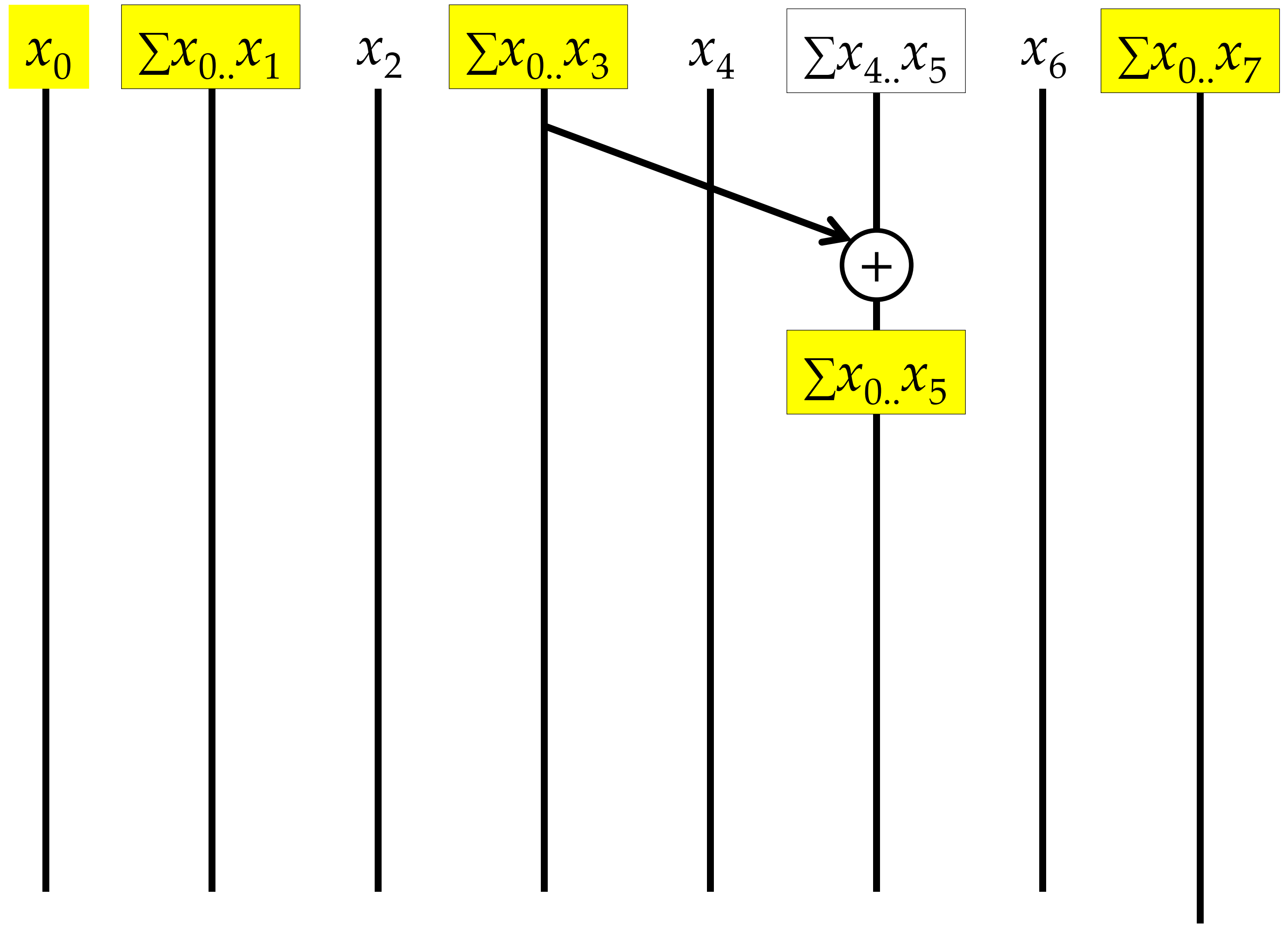
    - Root holds sum of all leaves

  – Traverse back up the tree building the scan from the partial sums

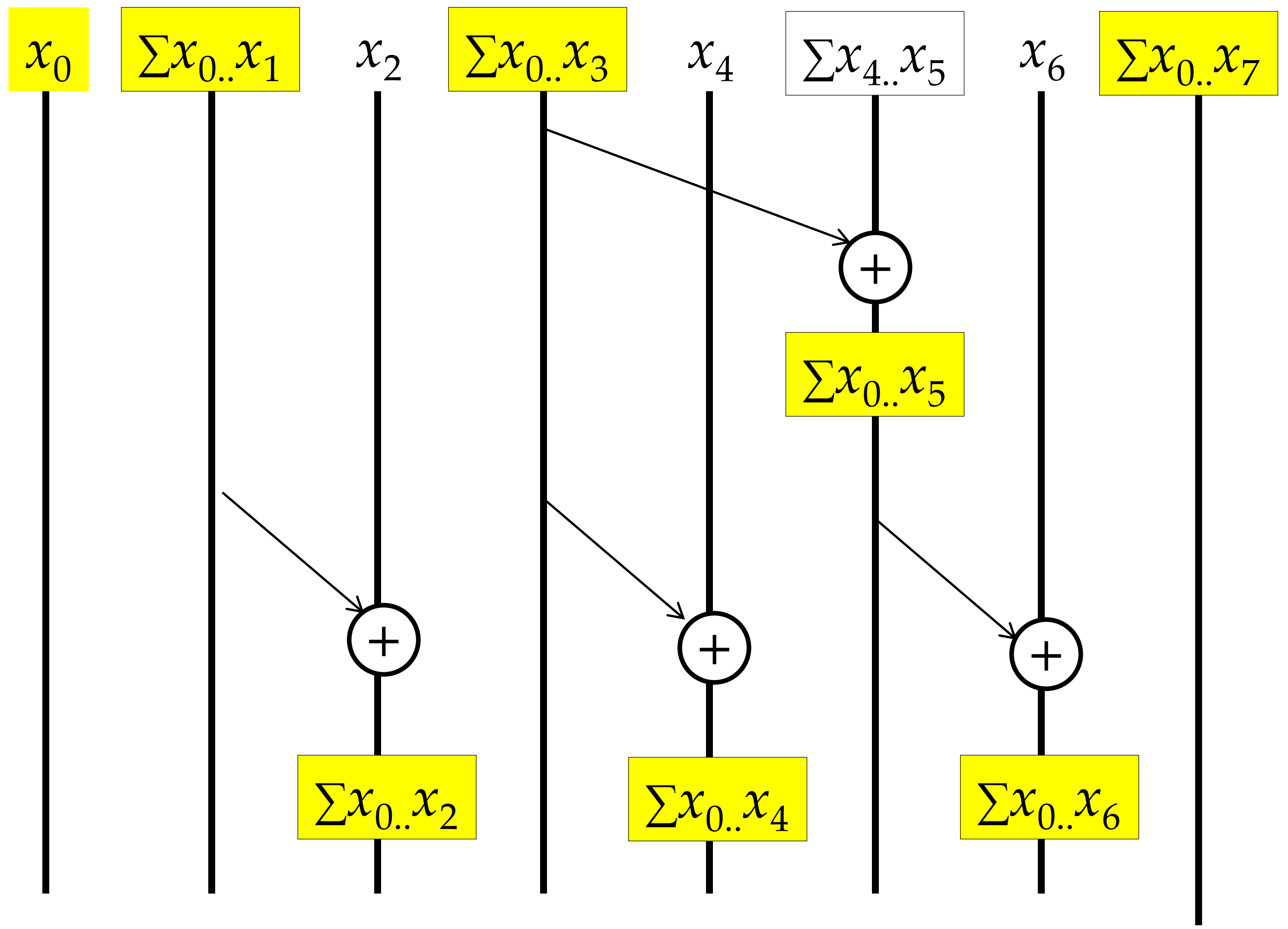
Brent-Kung Parallel Scan Step



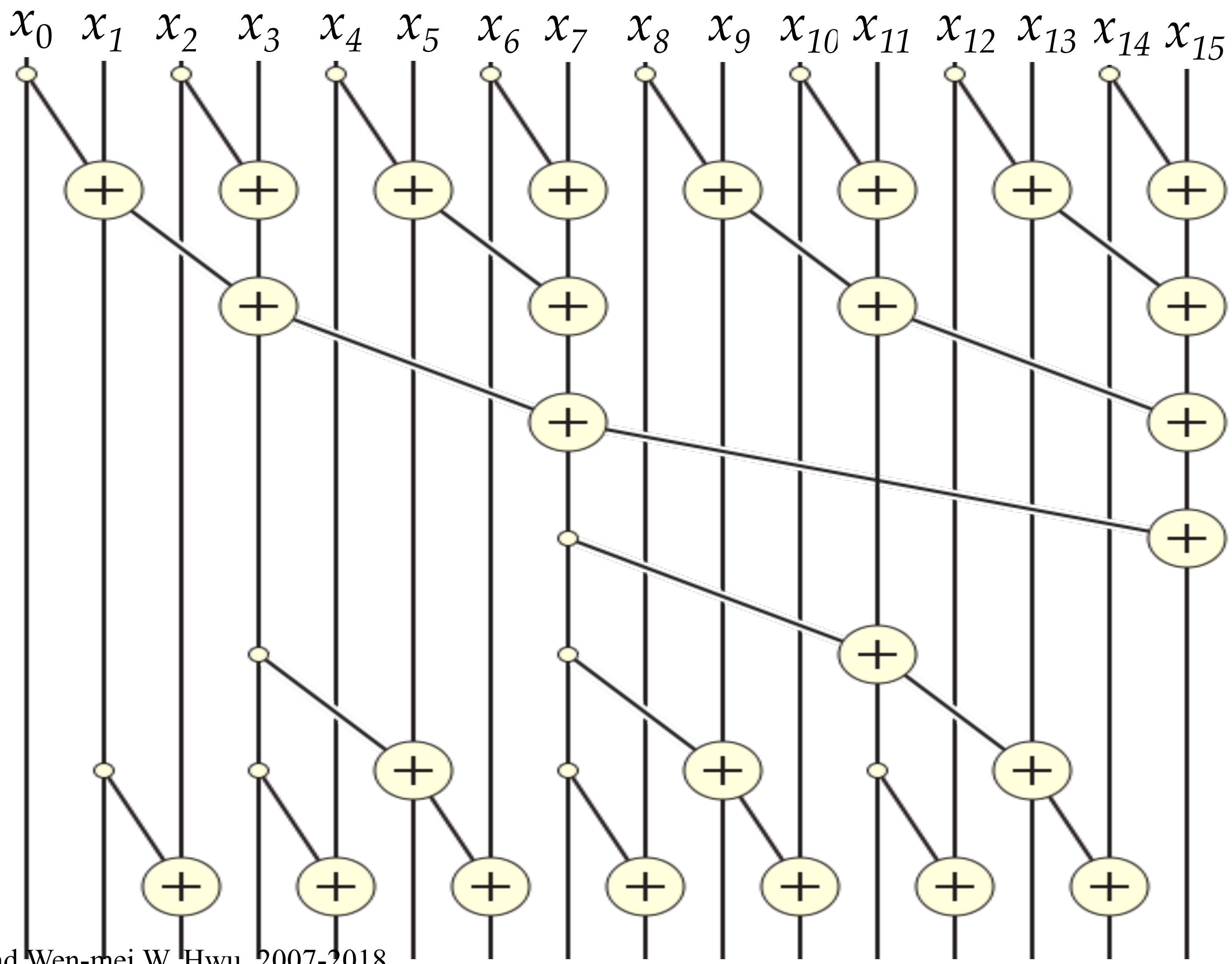
Inclusive Post-Scan Step



Inclusive Post Scan Step



Putting it Together (Data View)


x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12} x_{13} x_{14} x_{15}

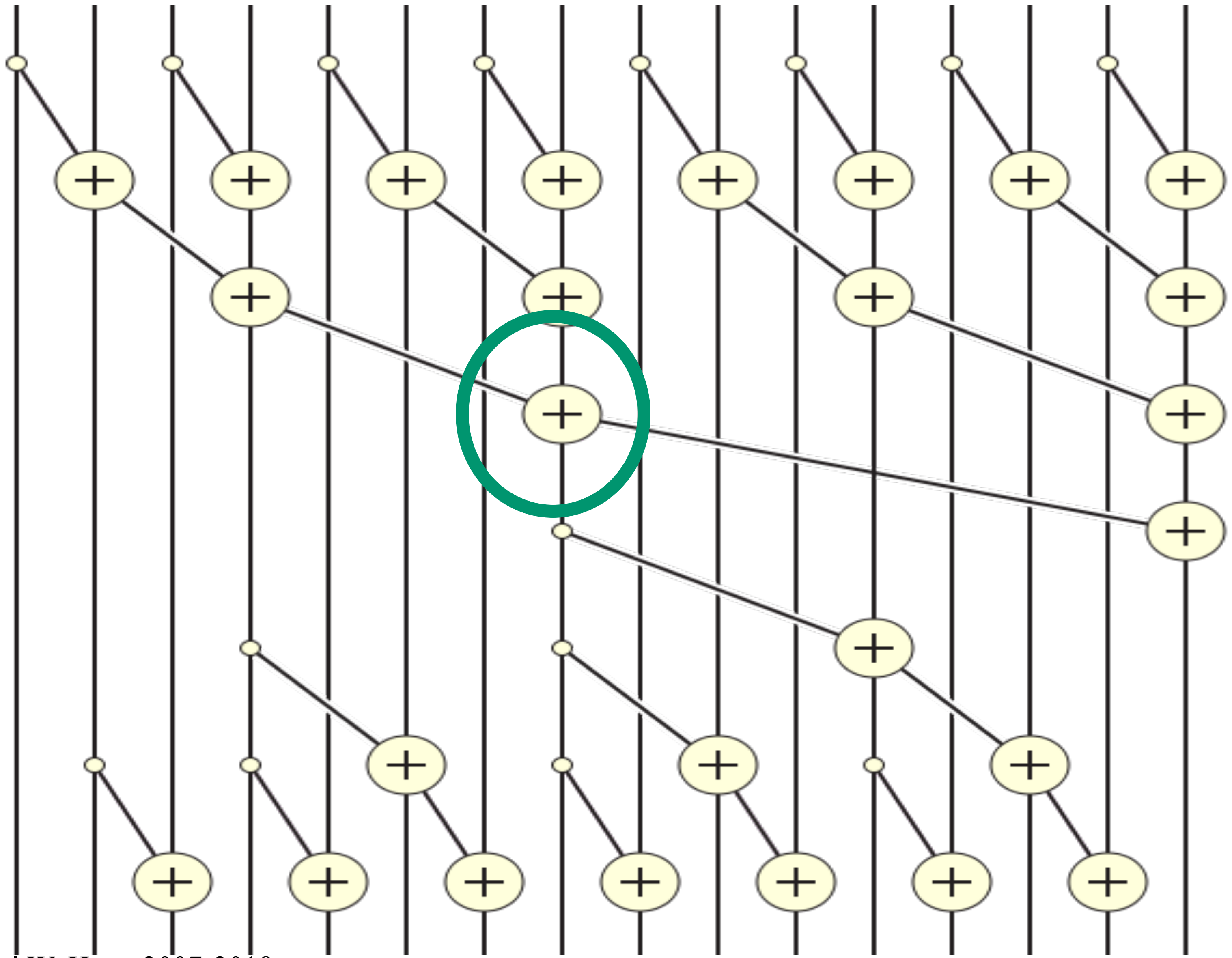
Reduction Step Kernel Code

```
// float T[2*BLOCK_SIZE] is in shared memory
// for previous slide, BLOCK_SIZE is 8

int stride = 1;
while(stride < 2*BLOCK_SIZE) {
    __syncthreads();
    int index = (threadIdx.x+1)*stride*2 - 1;
    if(index < 2*BLOCK_SIZE && (index-stride) >= 0)
        T[index] += T[index-stride];
    stride = stride*2;
}
```

```
// In our example,
// threadIdx.x+1    = 1, 2, 3, 4, 5, 6, 7, 8
// stride = 1, index = 1, 3, 5, 7, 9, 11, 13, 15
```

Putting it Together



Post Scan Step (Distribution Tree)

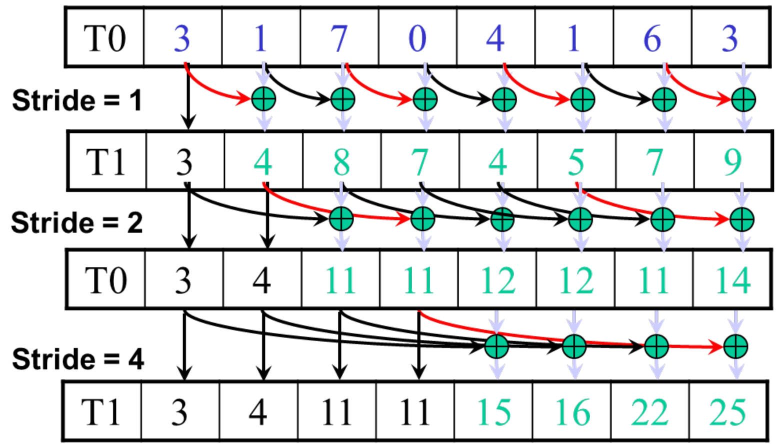
```
int stride = BLOCK_SIZE/2;
while(stride > 0) {
   __syncthreads();
   int index = (threadIdx.x+1)*stride*2 - 1;
   if ((index+stride) < 2*BLOCK_SIZE)
       T[index+stride] += T[index];
   stride = stride / 2;
 }
```

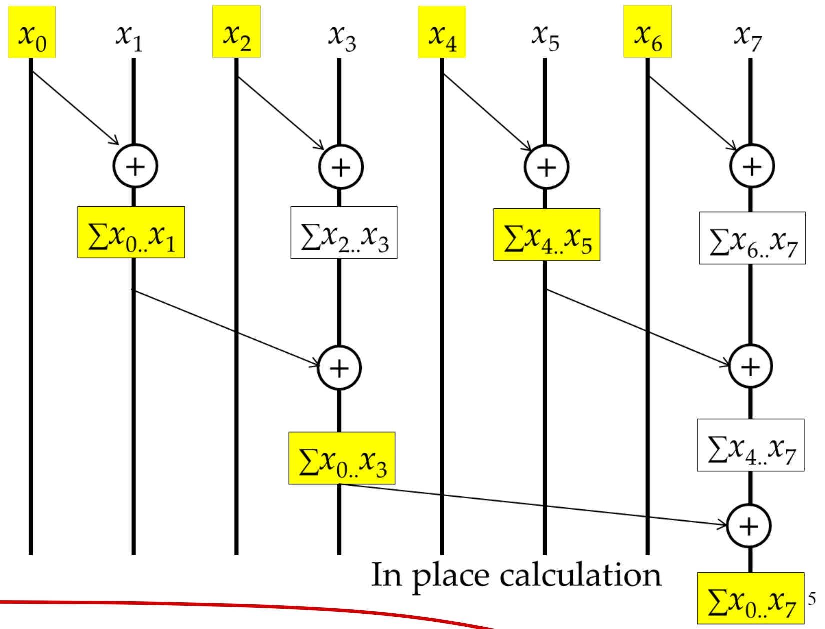
```
// In our example,
// BLOCK_SIZE=8 stride=4, 2, 1
// for first iteration, active thread = 0 index = 7, stride = 11
```

Work Analysis

- The parallel Scan executes 2* log(n) parallel iterations
  - log(n) in reduction and log(n) in post scan
  - The iterations do n/2, n/4,..1, (2-1), …., (n/4-1), (n/2-1) useful adds
  - In our example, n = 16, the number of useful adds is 16/2 + 16/4 + 16/8 + 16/16 + (16/8-1) + (16/4-1) + (16/2-1)
  - Total adds: $\boxed{(n-1) + (n-2) – (\log(n) -1)}$ = 2*(n-1) – log(n)→ O(n) work
- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

Kogge-Stone vs. Brent-Kung

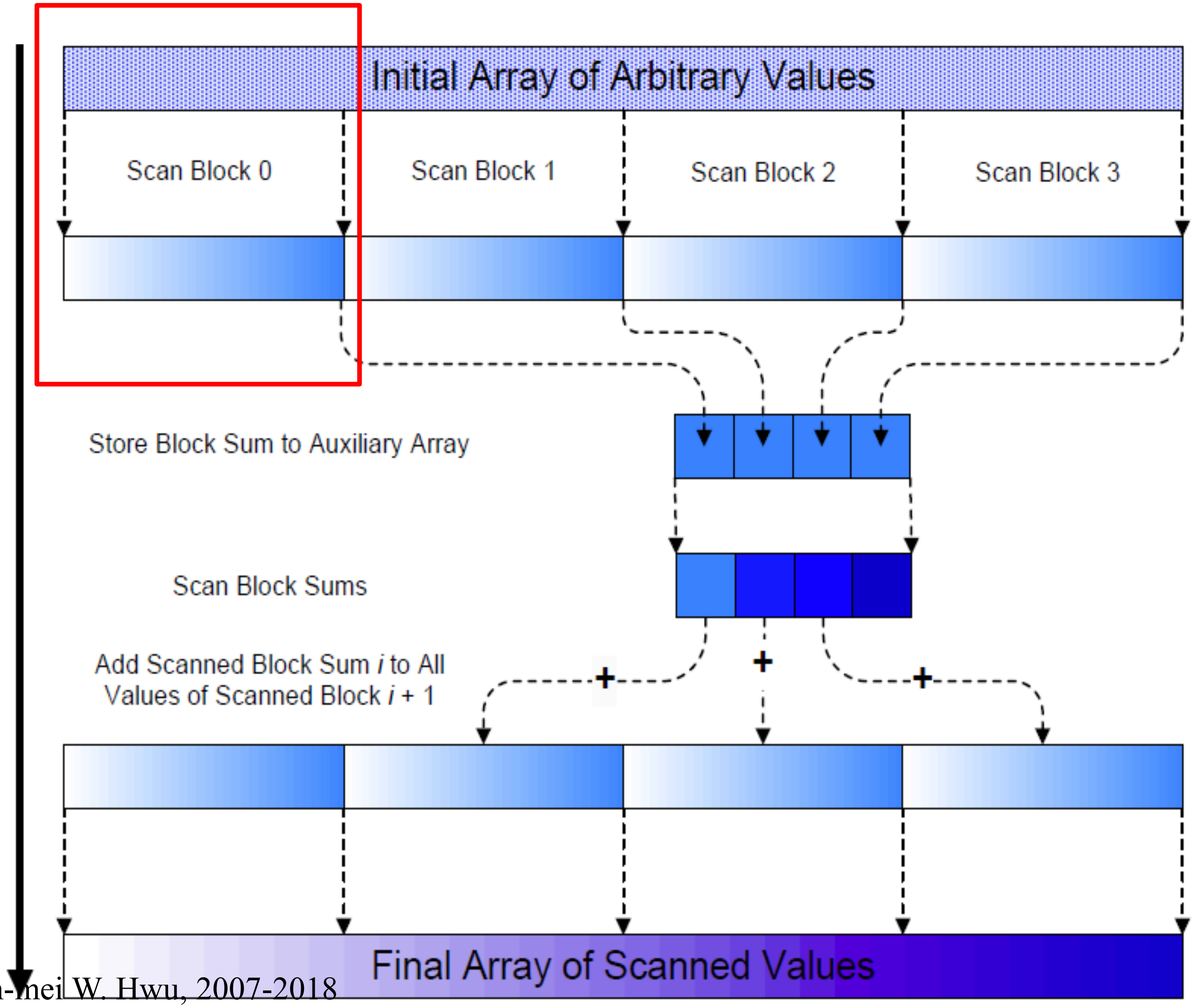




- Brent-Kung uses half the number of threads compared to Kogge-Stone
  - Each thread should load two elements into the shared memory
- Brent-Kung takes twice the number of steps compared to Kogge-Stone
  - Kogge-Stone is more popular for parallel scan with blocks in GPUs

Overall Flow of Complete Scan A Hierarchical Approach

Scan kernel



Initial Array of Arbitrary Values
Scan Block 0
Scan Block 1
Scan Block 2
Scan Block 3
Store Block Sum to Auxiliary Array
Scan Block Sums
Add Scanned Block Sum *i* to All Values of Scanned Block *i* + 1
+
+
+
Final Array of Scanned Values

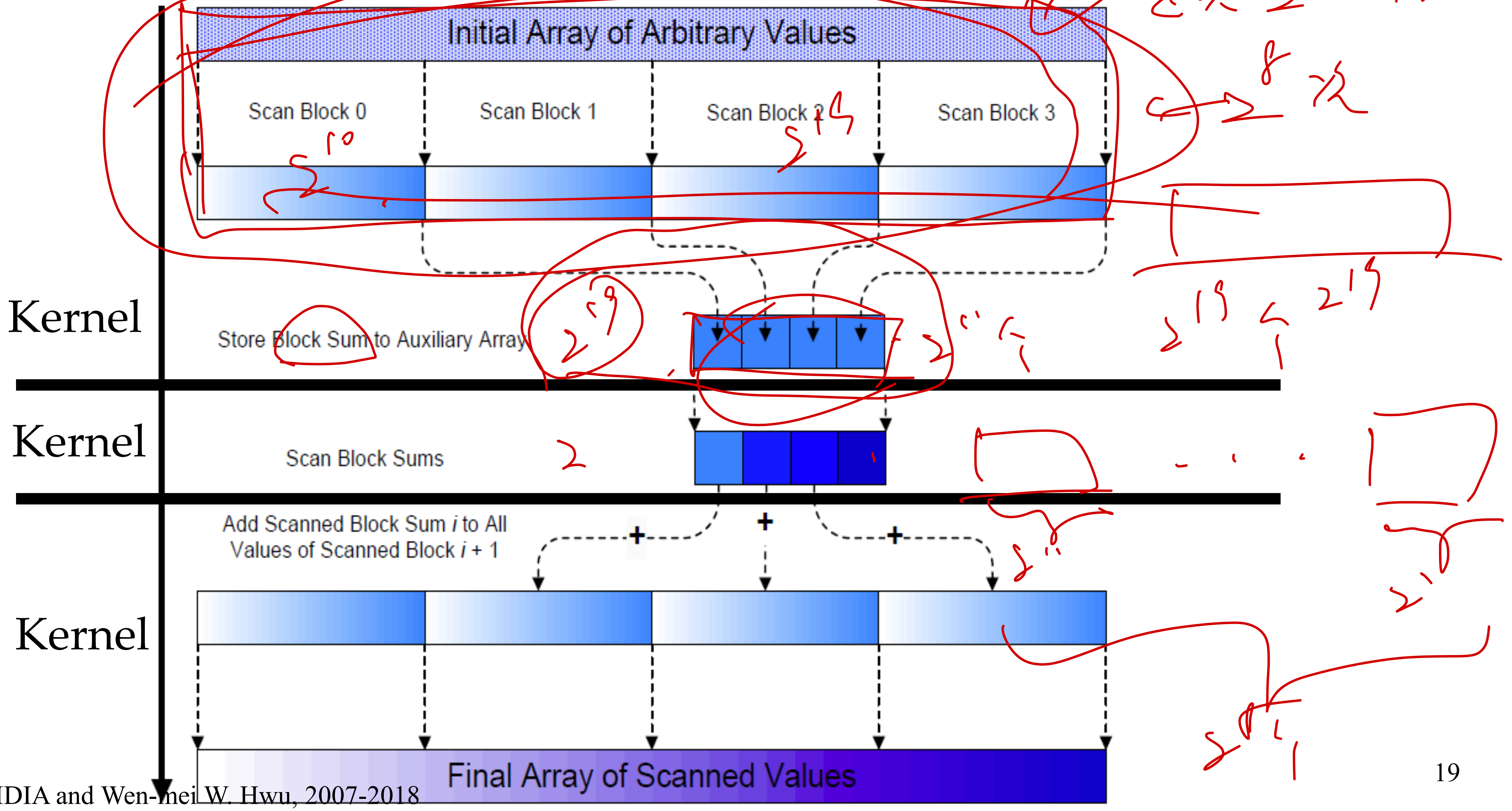
Using Global Memory Contents in CUDA

- Data in registers and shared memory of one thread block are not visible to other blocks
- To make data visible, the data has to be written into global memory
- However, any data written to the global memory are not visible until a memory fence. This is typically done by terminating the kernel execution
- Launch another kernel to continue the execution. The global memory writes done by the terminated kernels are visible to all thread blocks.

Scan of Arbitrary Length Input

- Build on the scan kernel that handles up to 2*blockDim.x elements from Brent-Kung
  - For Kogge-Stone, have each section of blockDim.x elements assigned to a block
- Have each block write the sum of its section into a Sum array using its blockIdx.x as index
- Run parallel scan on the Sum array
  - May need to break down Sum into multiple sections if it is too big for a block
- Add the scanned Sum array values to the elements of corresponding sections

Overall Flow of Complete Scan
A Hierarchical Approach



Initial Array of Arbitrary Values
Scan Block 0
Scan Block 1
Scan Block 2
Scan Block 3
Kernel
Store Block Sum to Auxiliary Array
Kernel
Scan Block Sums
Kernel
Add Scanned Block Sum *i* to All Values of Scanned Block *i* + 1
+
Final Array of Scanned Values
scan grid
1 thread
8
2^{10}
2^{19}
2^{15}

(Exclusive) Scan Definition

Definition: *The exclusive* scan *operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the exclusive scan operation on

[3 1 7 0 4 1 6 3],

would return [0 3 4 11 11 15 16 22].

Why Exclusive Scan

- To find the beginning address of allocated buffers
- Inclusive and Exclusive scans can be easily derived from each other; it is a matter of convenience

[3 1 7 0 4 1 6 3]

Exclusive [0 3 4 11 11 15 16 22]

Inclusive [3 4 11 11 15 16 22 25]

A simple exclusive scan kernel

- Adapt an inclusive, Kogge-Stone scan kernel
  - Block 0:
    - Thread 0 loads 0 into (shared) XY[0]
    - Other threads load (global) X[threadIdx.x-1] into XY[threadIdx.x]
  - All other blocks:
    - All thread load X[blockIdx.x*blockDim.x+threadIdx.x-1] into XY[threadIdex.x]

- Similar adaption for Brent-Kung kernel but pay attention that each thread loads two elements
  - Only one zero should be loaded
  - All elements should be shifted by only one position

ANY MORE QUESTIONS? READ CHAPTER 8