ECE408/CS483/CSE408 Fall 2022

Applied Parallel Programming

Lecture 12: Computation in Deep Neural Networks

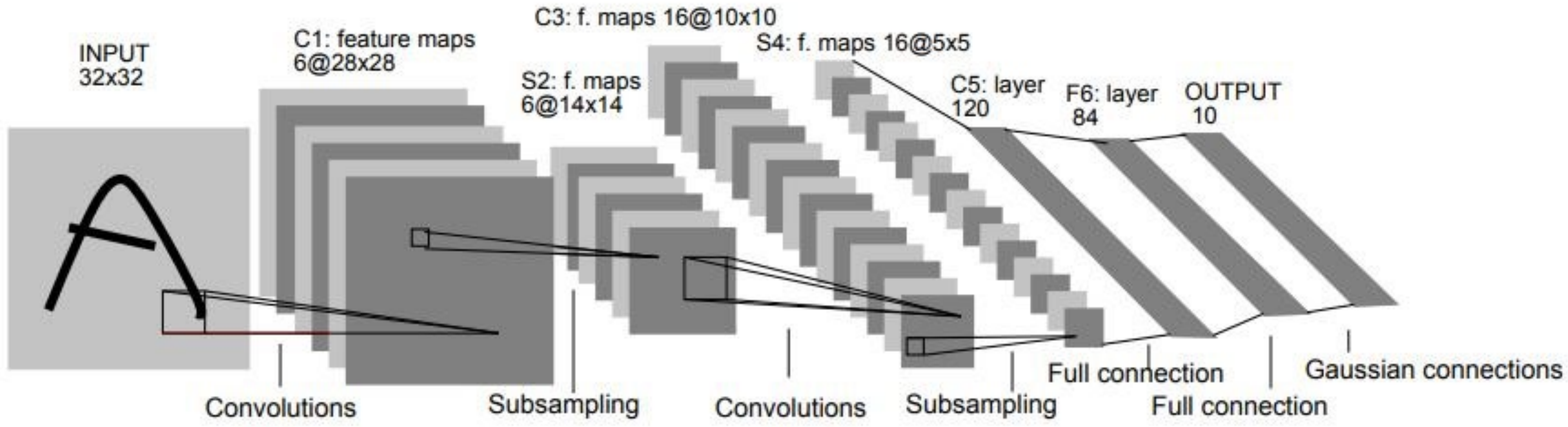
Course Reminders

- Lab 1-4 grades will be posted on Canvas by the end of this week
- Midterm 1 is on Tuesday, October 11th
  - On-line, everybody will be taking it at the same time
    - Tuesday, Oct. 11th 7:00pm-8:20pm US Central time
    - Wednesday, Oct. 12th 8:00am-9:20am Beijing time
  - Includes materials from Lecture 1 through Lecture 10
  - If you have a conflict, please let us know by Friday Sept 30th
- Project Milestone 1: Baseline CPU implementation is due Friday October 14th
  - Project details to be posted this week

Objective

- To learn to implement the different types of layers in a Convolutional Neural Network (CNN)

LeNet-5:CNN for hand-written digit recognition



Anatomy of a Convolution Layer

Input features

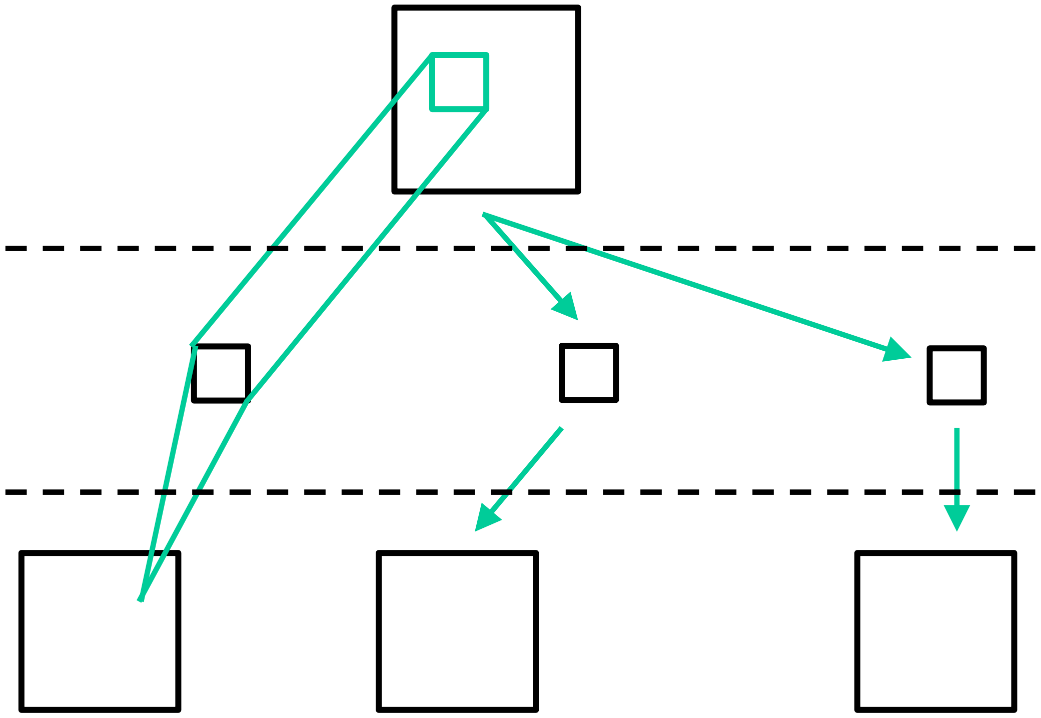
- A inputs each $N_1 \times N_2$

Convolution Layer

- B convolution kernels each $K_1 \times K_2$

Output Features (total of B)

- A × B outputs each $(N_1 - K_1 + 1) \times (N_2 - K_2 + 1)$



Notion of a Channel in Input Layer

Some Set of Input Features are Related

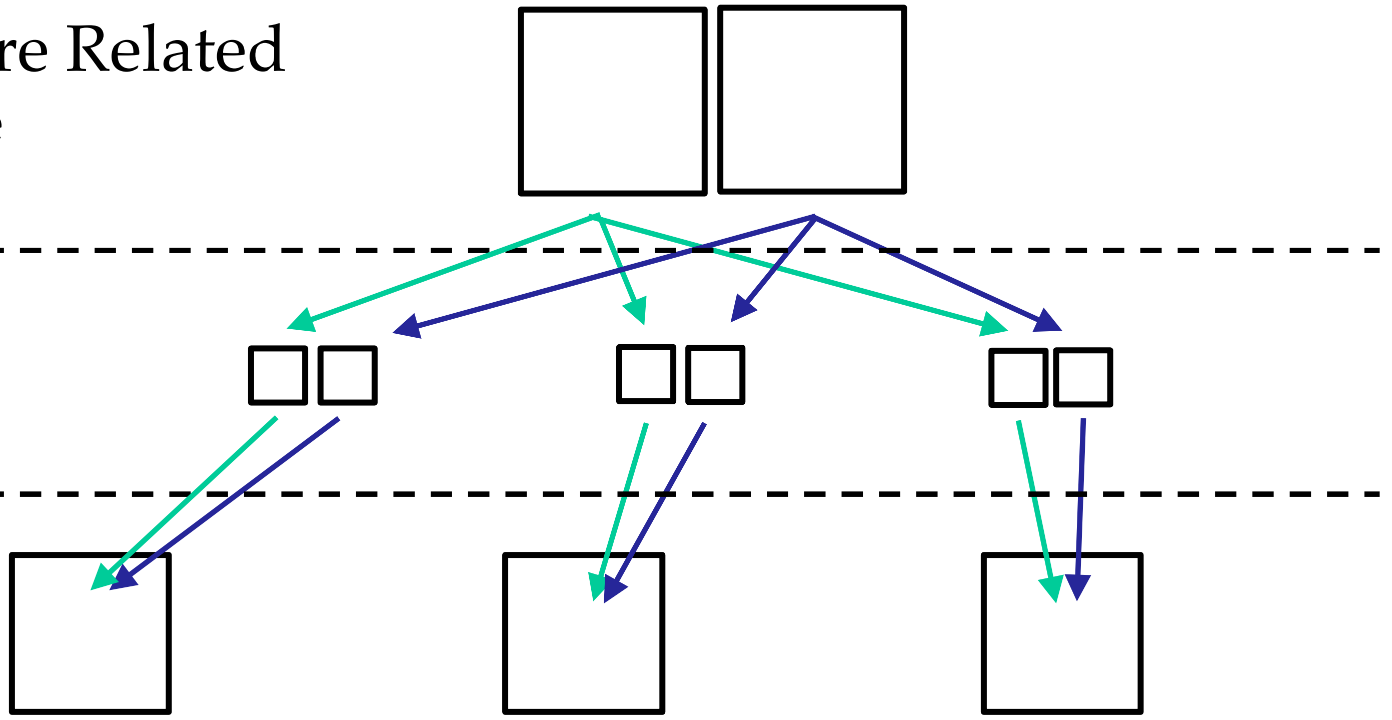
- For example: Red, Green, Blue

Convolution Layer

- Different kernels per channel

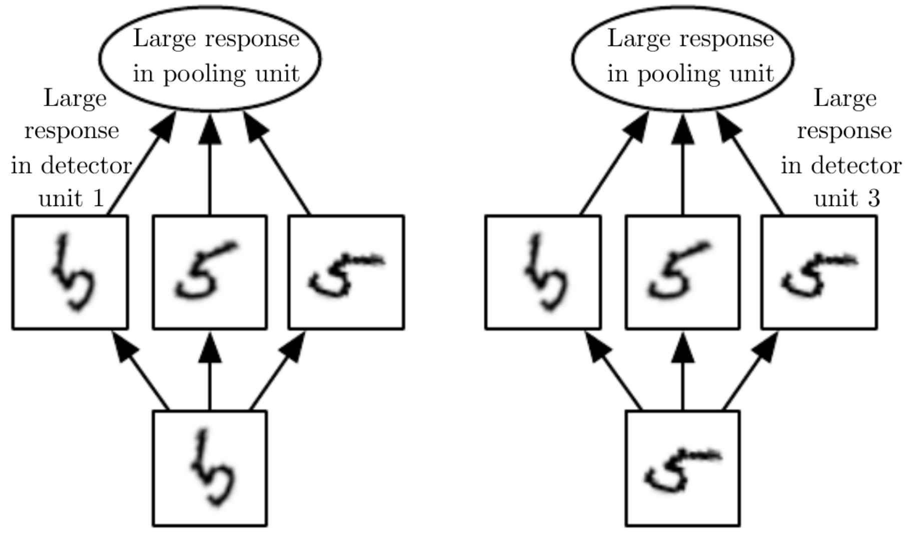
Output Features

- Channels combine per output feature



2-D Pooling (Subsampling)

- A subsampling layer
  – Sometimes with bias and non-linearity built in
- Common types
  – max, average, L^2 norm, weighted average
- Helps make representation invariant to size scaling and small translations in the input

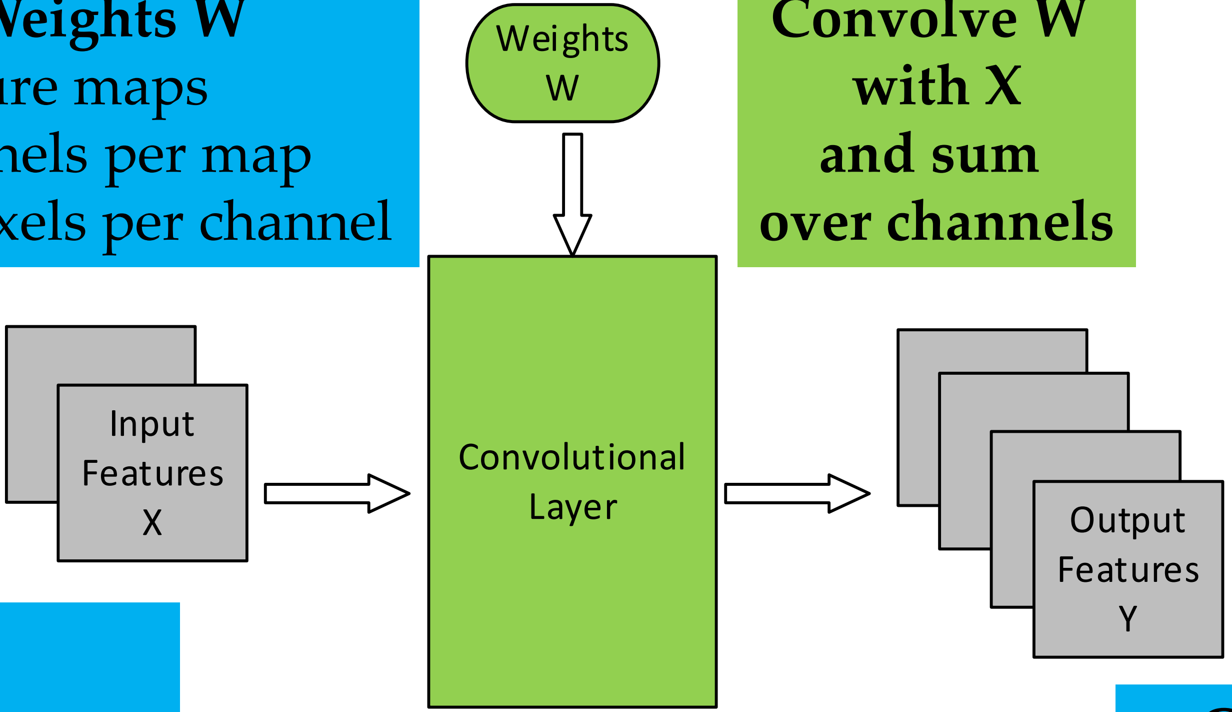

Large response in pooling unit
Large response in detector unit 1
Large response in pooling unit
Large response in detector unit 3

Forward Propagation

Output Size
$H_{out} = H - K + 1$
$W_{out} = W - K + 1$

Weights W
M feature maps
C channels per map
K×K pixels per channel

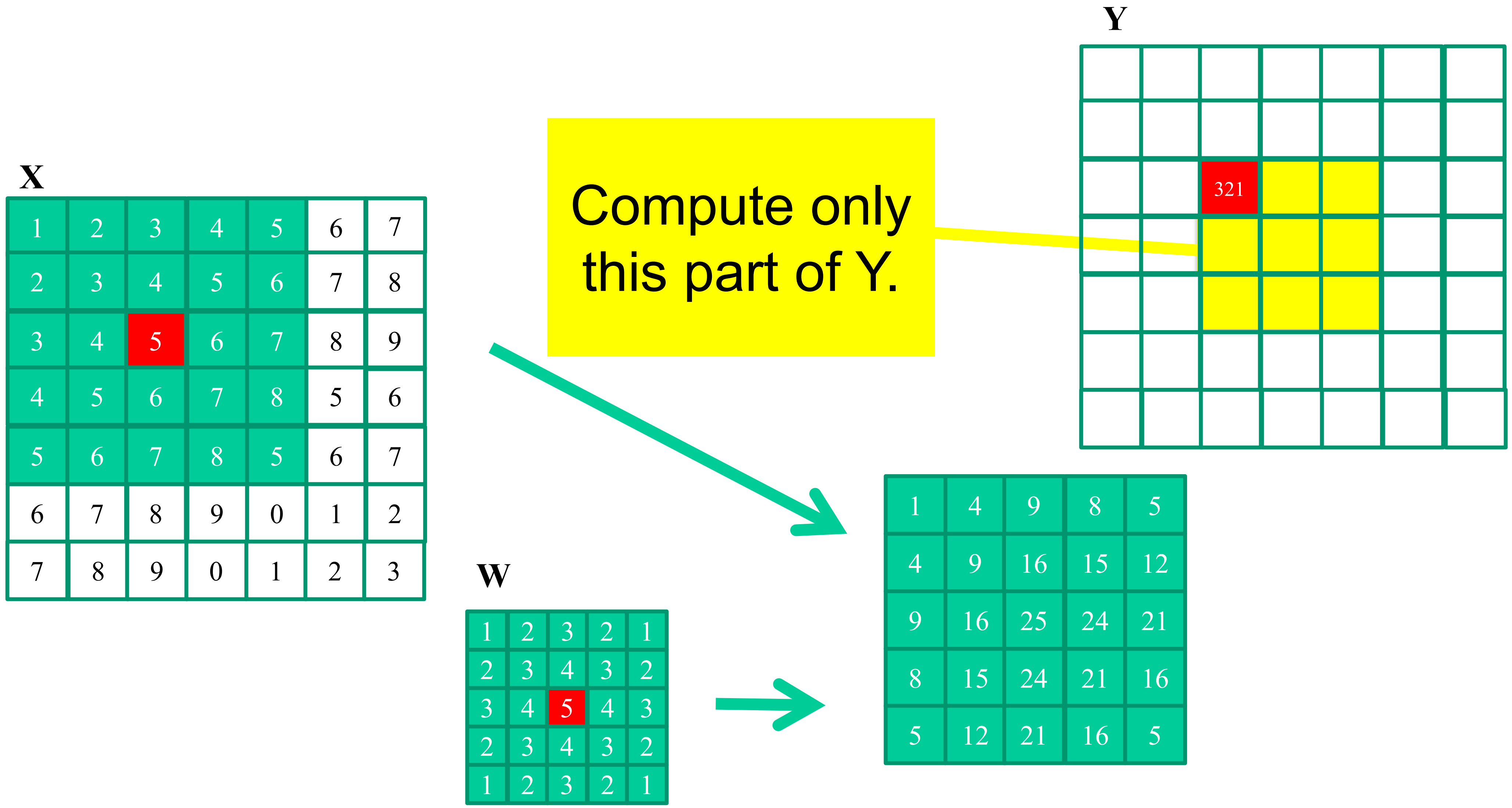
Convolve W with X and sum over channels



Input X
B images
C channels per image
H×W pixels per channel

Convolution Output Y
B images
M features per image
$H_{out} \times W_{out}$ values per feature

Outputs Must Use Full Mask/Kernel

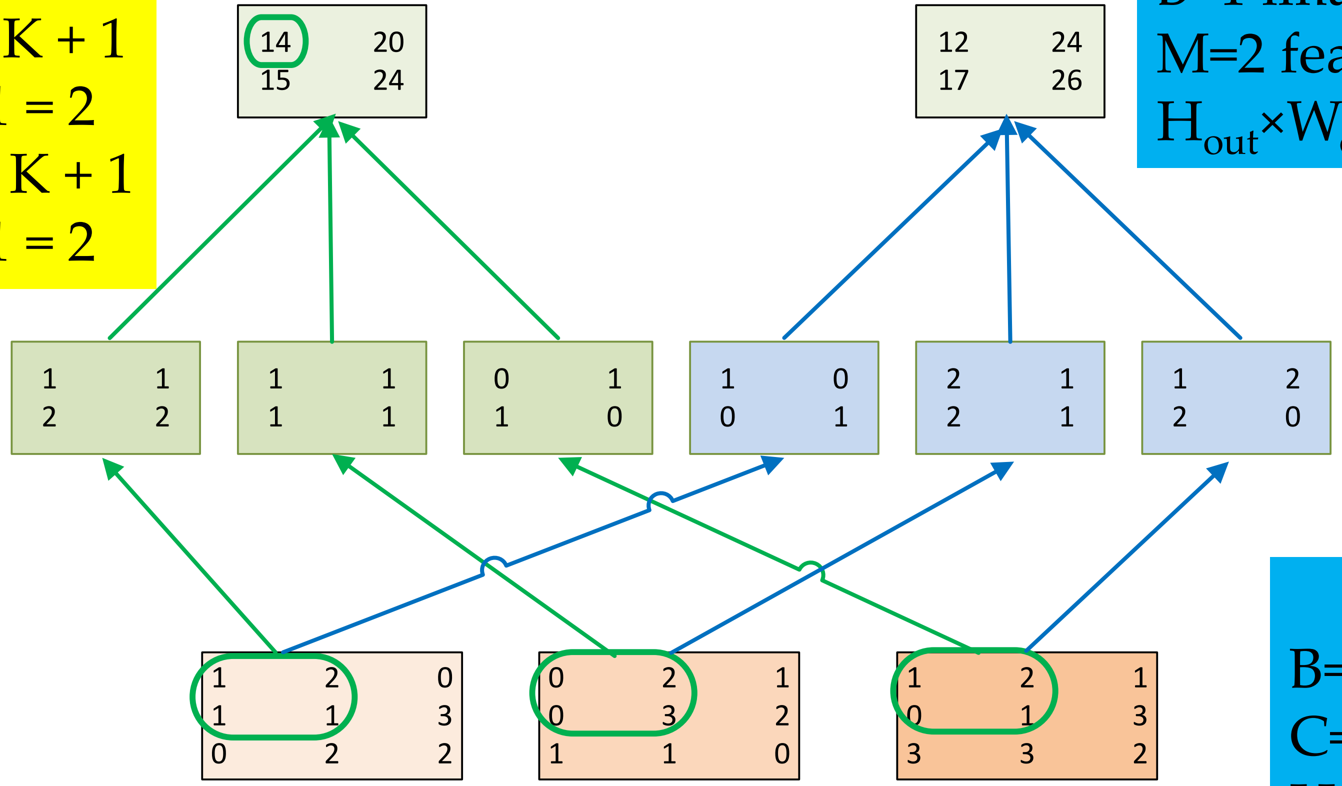


Compute only this part of Y.

Example of the Forward Path of a Convolution Layer

Output Size

$H_{out} = H - K + 1$
$= 3 - 2 + 1 = 2$
$W_{out} = W - K + 1$
$= 3 - 2 + 1 = 2$



Convolution Output Y

B=1 image

M=2 features per image

$H_{out} \times W_{out} = 2 \times 2$ values per feature

Weights W

M=2 feature maps

C=3 channels per map

K×K=2×2 pixels per channel

Input X

B=1 image

C=3 channels

H×W=3×3 pixels per channel

Sequential Code: Forward Convolutional Layer

```
void convLayer_forward(int B, int M, int C, int H, int W, int K, float* X, float* W, float* Y) {
  int H_out = H – K + 1;                              // calculate H_out, W_out
  int W_out = W – K + 1;

  for (int b = 0; b < B; ++b)                         // for each image
    for(int m = 0;  m < M;  m++)                      // for each output feature map
      for(int h = 0; h < H_out; h++)                  // for each output value (two loops)
        for(int w = 0; w < W_out; w++) {
          Y[b, m, h, w] = 0.0f;                       // initialize sum to 0
          for(int c = 0; c < C; c++)                  // sum over all input channels
            for(int p = 0; p < K; p++)                  // KxK filter
              for(int q = 0; q < K; q++)
                Y[b, m, h, w] += X[b, c, h + p, w + q] * W[m, c, p, q];
       }
}
```

A Small Convolution Layer Example

Image *b* in mini batch

input channel

X[b, 1,_,_]

W[0,1,_, _]

Y[b, 0,_, _]

output map

X[b,0,_, _]

1	2	0	1
1	1	3	2
0	2	2	0
2	1	0	3

W[0,0,_, _]

1	1	1
2	2	3
2	1	0

X[b,1,_, _]

0	2	1	0
0	3	2	1
1	1	0	2
2	1	0	3

W[0,1,_, _]

1	2	3
1	1	0
3	0	1

Y[b,0,_, _]

0	?
?	?

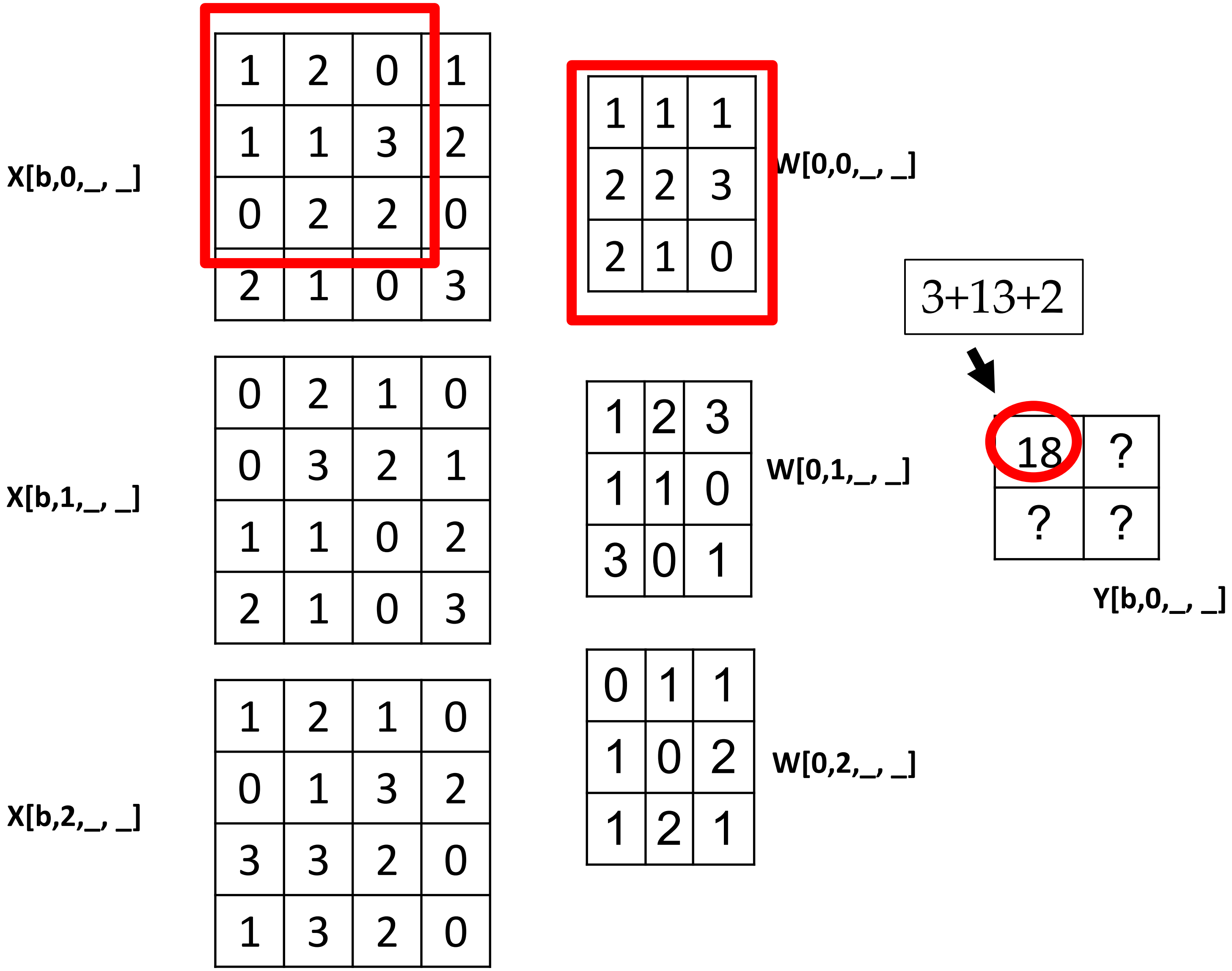
X[b,2,_, _]

1	2	1	0
0	1	3	2
3	3	2	0
1	3	2	0

W[0,2,_, _]

0	1	1
1	0	2
1	2	1

A Small Convolution Layer Example c = 0



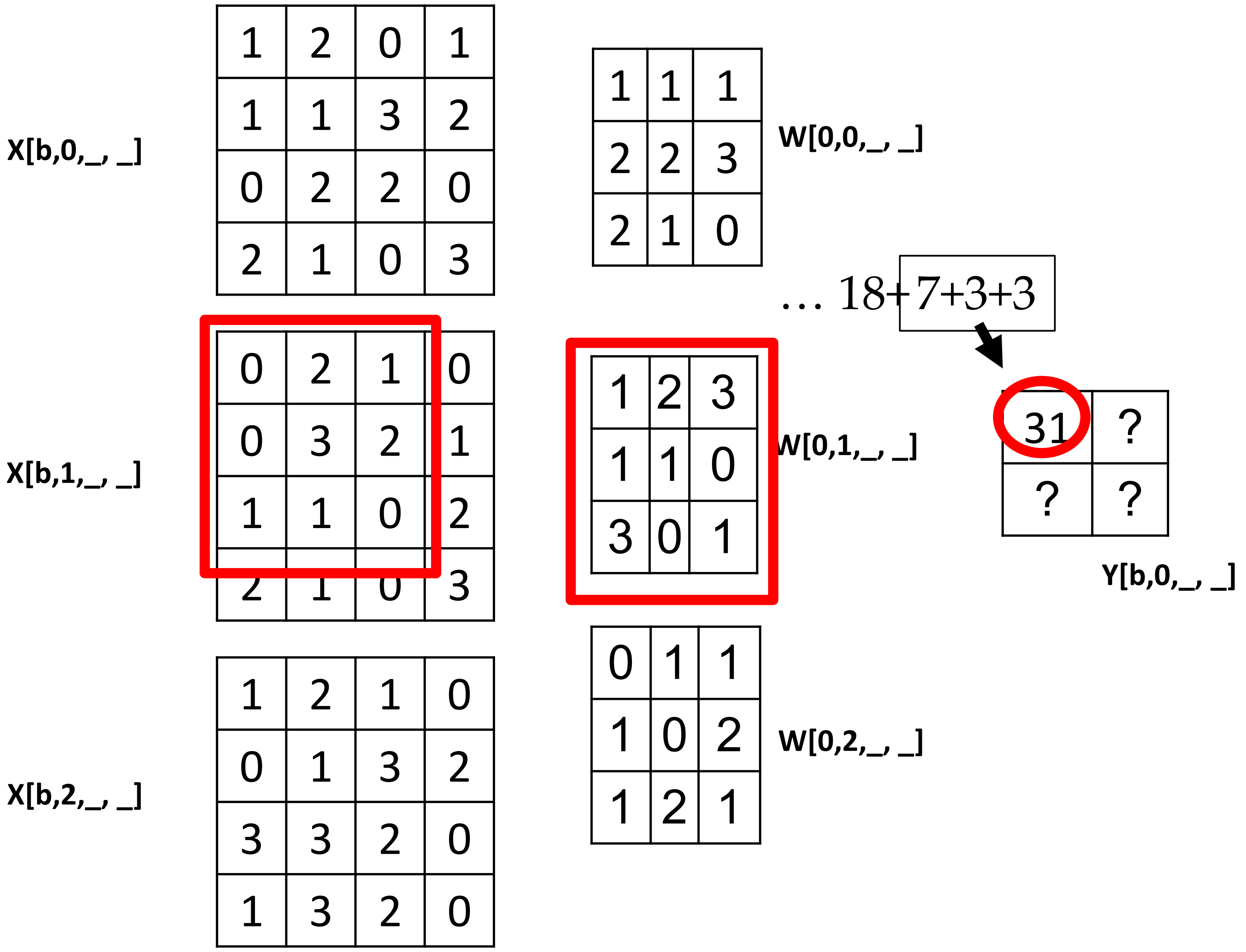
$$X[b,0,\_,\_] = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 0 & 1 \\ \hline 1 & 1 & 3 & 2 \\ \hline 0 & 2 & 2 & 0 \\ \hline 2 & 1 & 0 & 3 \\ \hline\end{array} \quad W[0,0,\_,\_] = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 2 & 2 & 3 \\ \hline 2 & 1 & 0 \\ \hline\end{array}$$

$$X[b,1,\_,\_] = \begin{array}{|c|c|c|c|}\hline 0 & 2 & 1 & 0 \\ \hline 0 & 3 & 2 & 1 \\ \hline 1 & 1 & 0 & 2 \\ \hline 2 & 1 & 0 & 3 \\ \hline\end{array} \quad W[0,1,\_,\_] = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 1 & 1 & 0 \\ \hline 3 & 0 & 1 \\ \hline\end{array}$$

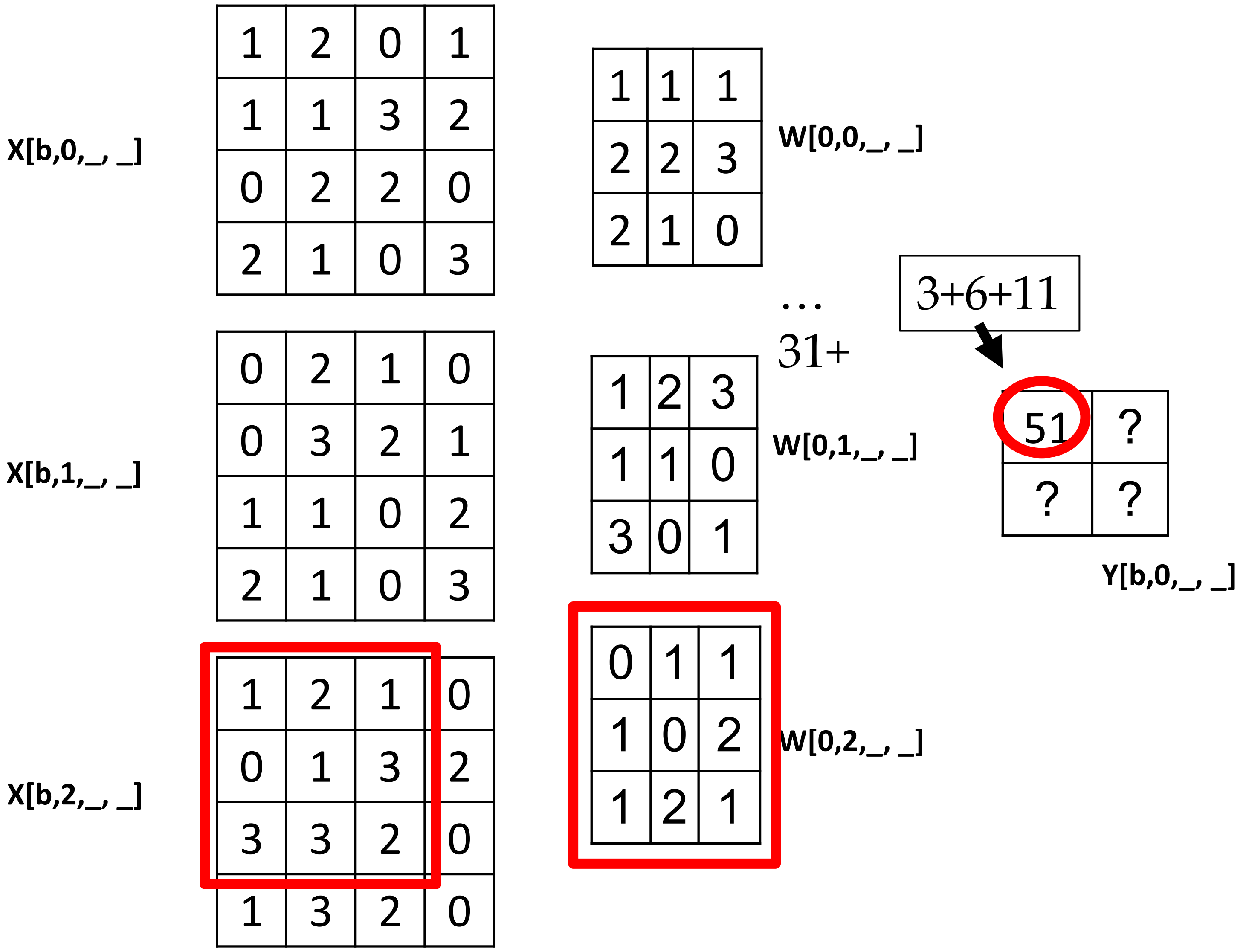
$$X[b,2,\_,\_] = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 1 & 0 \\ \hline 0 & 1 & 3 & 2 \\ \hline 3 & 3 & 2 & 0 \\ \hline 1 & 3 & 2 & 0 \\ \hline\end{array} \quad W[0,2,\_,\_] = \begin{array}{|c|c|c|}\hline 0 & 1 & 1 \\ \hline 1 & 0 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

$$3+13+2 \rightarrow Y[b,0,\_,\_] = \begin{array}{|c|c|}\hline 18 & ? \\ \hline ? & ? \\ \hline\end{array}$$

A Small Convolution Layer Example c = 1



A Small Convolution Layer Example c = 2



Parallelism in a Convolution Layer

Output feature maps can be calculated in parallel

- Usually a small number, not sufficient to fully utilize a GPU

All **output** feature map **pixels** can be calculated in parallel

- All rows can be done in parallel
- All pixels in each row can be done in parallel
- Large number but diminishes as we go into deeper layers

All **input feature maps** can be processed in parallel,
but need atomic operation or tree reduction (we'll learn later)

Different layers may demand different strategies.

Subsampling (Pooling) by Scale N

Convolution Output Y
B images
M features per image
$H_{out} \times W_{out}$ values per feature

Average over N×N blocks, then calculate sigmoid

Subsampling/Pooling Output S
B images
M features per image
$H_{S(N)} \times W_{S(N)}$ values per feature

Output Size
$H_{S(N)} = \text{floor}\ (H_{out} / N)$
$W_{S(N)} = \text{floor}\ (W_{out} / N)$

Sequential Code: Forward Pooling Layer

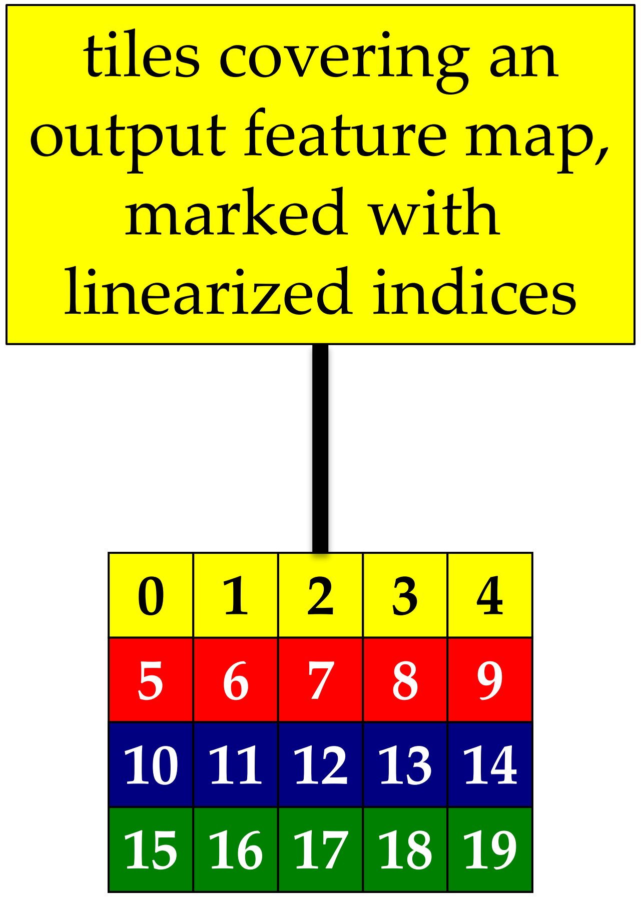
```
void poolingLayer_forward(int B, int M, int H_out, int W_out, int N, float* Y, float* S)
{
  for (int b = 0; b < B; ++b)                 // for each image
    for (int m = 0;  m < M; ++m)               // for each output feature map
      for (int x = 0; x < H_out/N; ++x)         // for each output value (two loops)
        for (int y = 0; y < W_out/N; ++y) {
          float acc = 0.0f                         // initialize sum to 0
          for (int p = 0; p < N; ++p)               // loop over NxN block of Y (two loops)
             for (int q = 0; q < N; ++q)
                acc += Y[b, m, N*x + p, N*y + q];
          acc /= N * N;                               // calculate average over block
          S[b, m, x, y] = sigmoid(acc + bias[m])  // bias, non-linearity
        }
}
```

Kernel Implementation of Subsampling Layer

- Straightforward mapping from grid to subsampled output feature map pixels
- in GPU kernel,
  - need to manipulate index mapping
  - for accessing the output feature map pixels
  - of the previous convolution layer.
- Often merged into the previous convolution layer to save memory bandwidth

Design of a Basic Kernel

- Each block computes
  - a tile of output pixels for one feature
  - TILE_WIDTH pixels in each dimension
- Grid's X dimension maps to M output feature maps
- Grid's Y dimension maps to the tiles in the output feature maps (linearized order).
- (Grid's Z dimension is used for images in batch, which we omit from slides.)


tiles covering an output feature map, marked with linearized indices
0 1 2 3 4
5 6 7 8 9
10 11 12 13 14
15 16 17 18 19

A Small Example

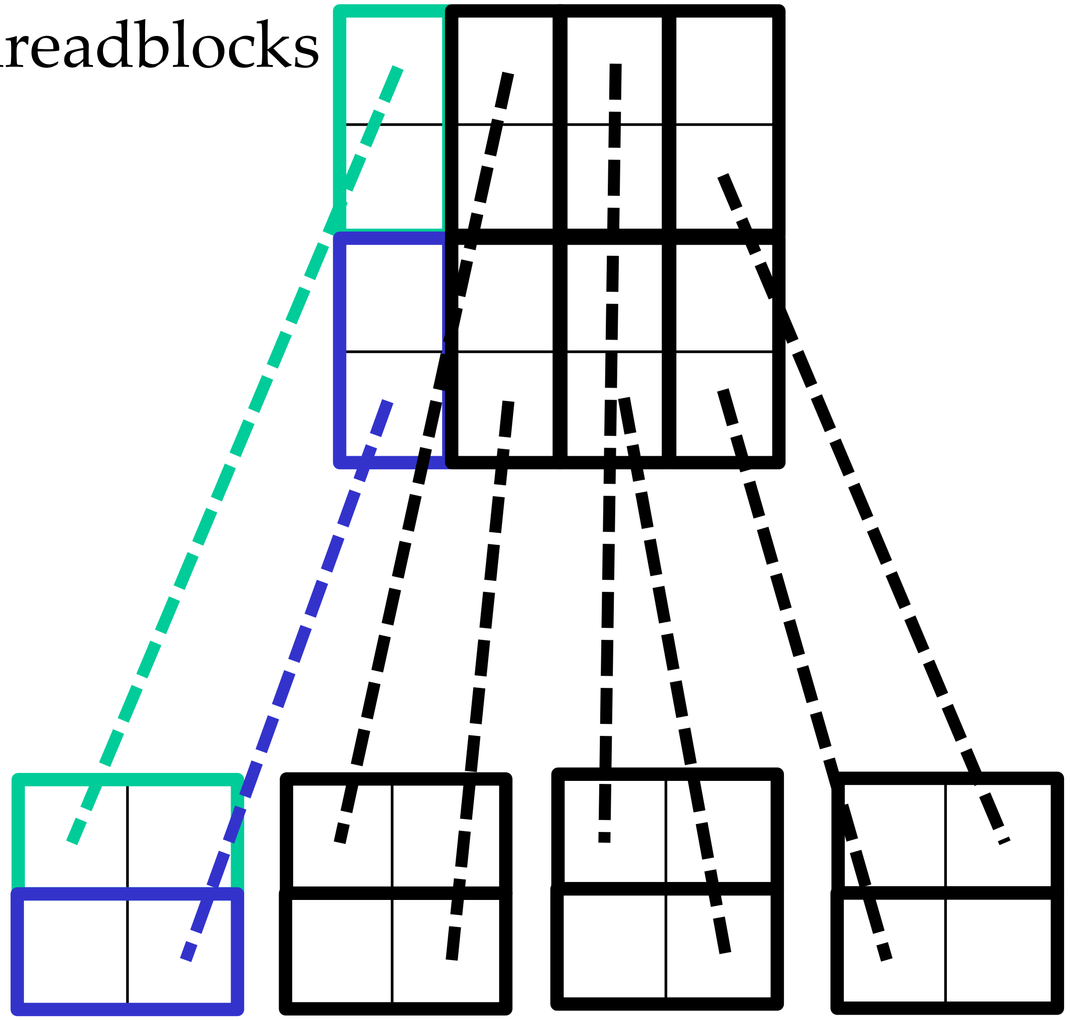
Assume

- **M = 4** (4 output feature maps),
- thus 4 blocks in the X dimension, and
- **W_out = H_out = 8** (8x8 output features).

If **TILE_WIDTH = 4**,
we also need 4 blocks in the Y dimension:

- for each output feature,
- top two blocks in each column calculates the top row of tiles, and
- bottom two calculate the bottom row.

CUDA Grid and Threadblocks

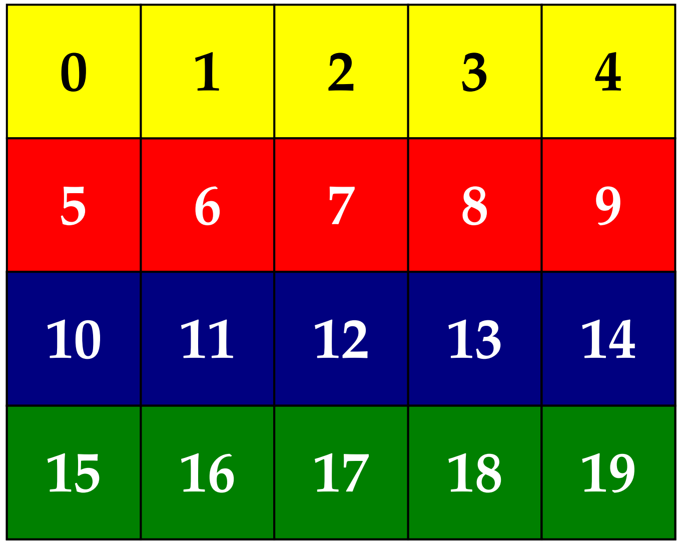


Output Feature Maps and Tiles

Host Code for a Basic Kernel: CUDA Grid

Consider an output feature map:

- width is **W_out**, and
- height is **H_out**.
- Assume these are multiples of **TILE_WIDTH**.



Let **X_grid** be the number of blocks needed in X dim (5 above).

Let **Y_grid** be the number of blocks needed in Y dim (4 above).

Host Code for a Basic Kernel: CUDA Grid

(Assuming W_out and H_out are multiples of TILE_WIDTH.)

```
#define TILE_WIDTH 16        // We will use 4 for small examples.
W_grid = W_out/TILE_WIDTH;   // number of horizontal tiles per output map
H_grid = H_out/TILE_WIDTH;   // number of vertical tiles per output map
Y = H_grid * W_grid;

dim3 blockDim(TILE_WIDTH, TILE_WIDTH, 1); // output tile for untiled code
dim3 gridDim(M, Y, 1);

ConvLayerForward_Kernel<<< gridDim, blockDim >>>(…);
```

Partial Kernel Code for a Convolution Layer

```
__global__ void ConvLayerForward_Basic_Kernel
    (int C, int W_grid, int K, float* X, float* W, float* Y)
{
    int m = blockIdx.x;
    int h = (blockIdx.y / W_grid) * TILE_WIDTH + threadIdx.y;
    int w = (blockIdx.y % W_grid) * TILE_WIDTH + threadIdx.x;
    float acc = 0.0f;
    for (int c = 0;  c < C; c++) {          // sum over all input channels
       for (int p = 0; p < K; p++)          // loop over KxK filter
          for (int q = 0; q < K; q++)
             acc += X[c, h + p, w + q] * W[m, c, p, q];
    }
    Y[m, h, w] = acc;
}
```

Some Observations

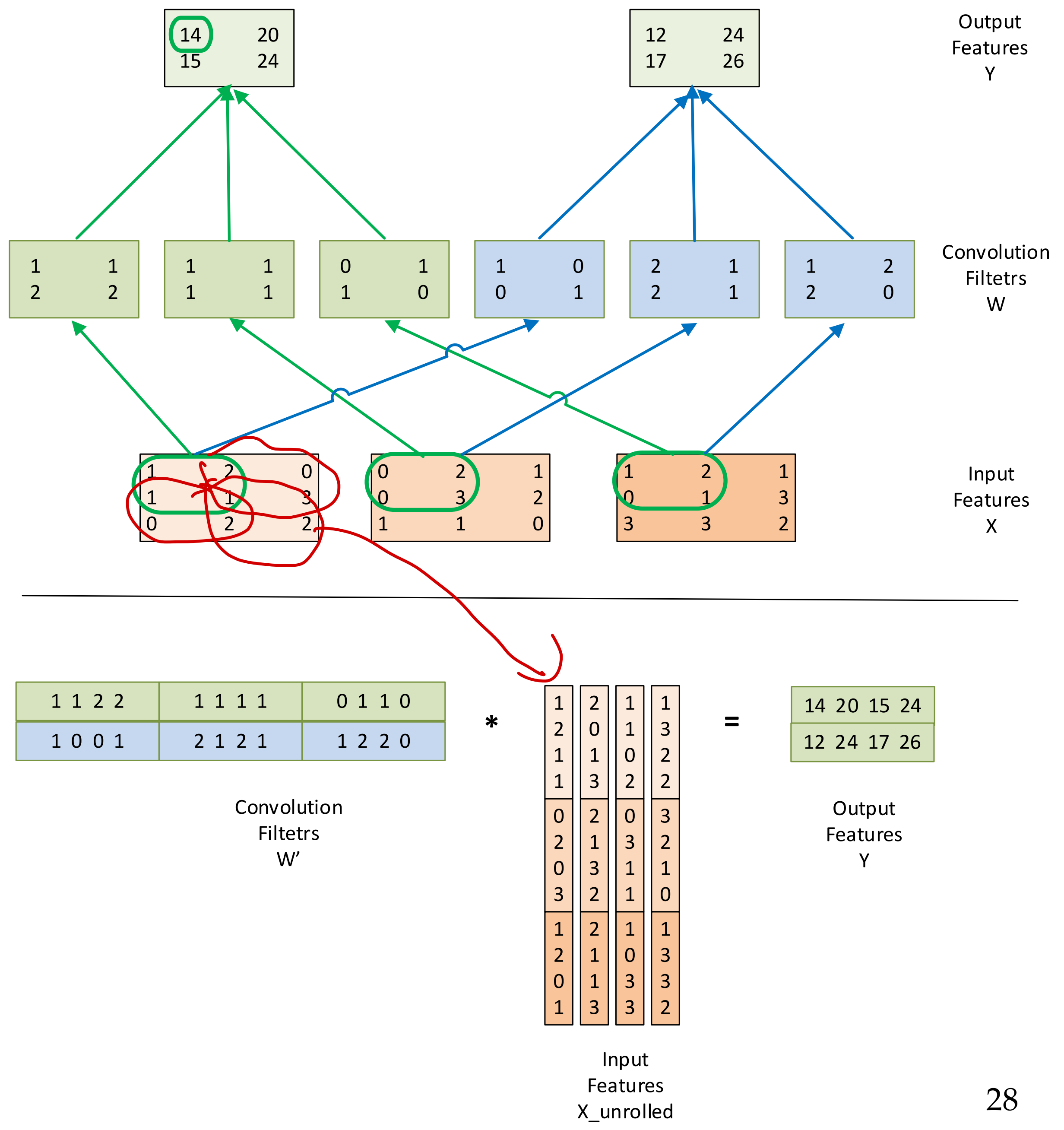
Enough parallelism

- if the total number of pixels
- across all output feature maps is large
- (often the case for CNN layers)

Each input tile

- loaded M times (number of output features), so
- **not efficient in global memory bandwidth,**
- but block scheduling in X dimension should give cache benefits.

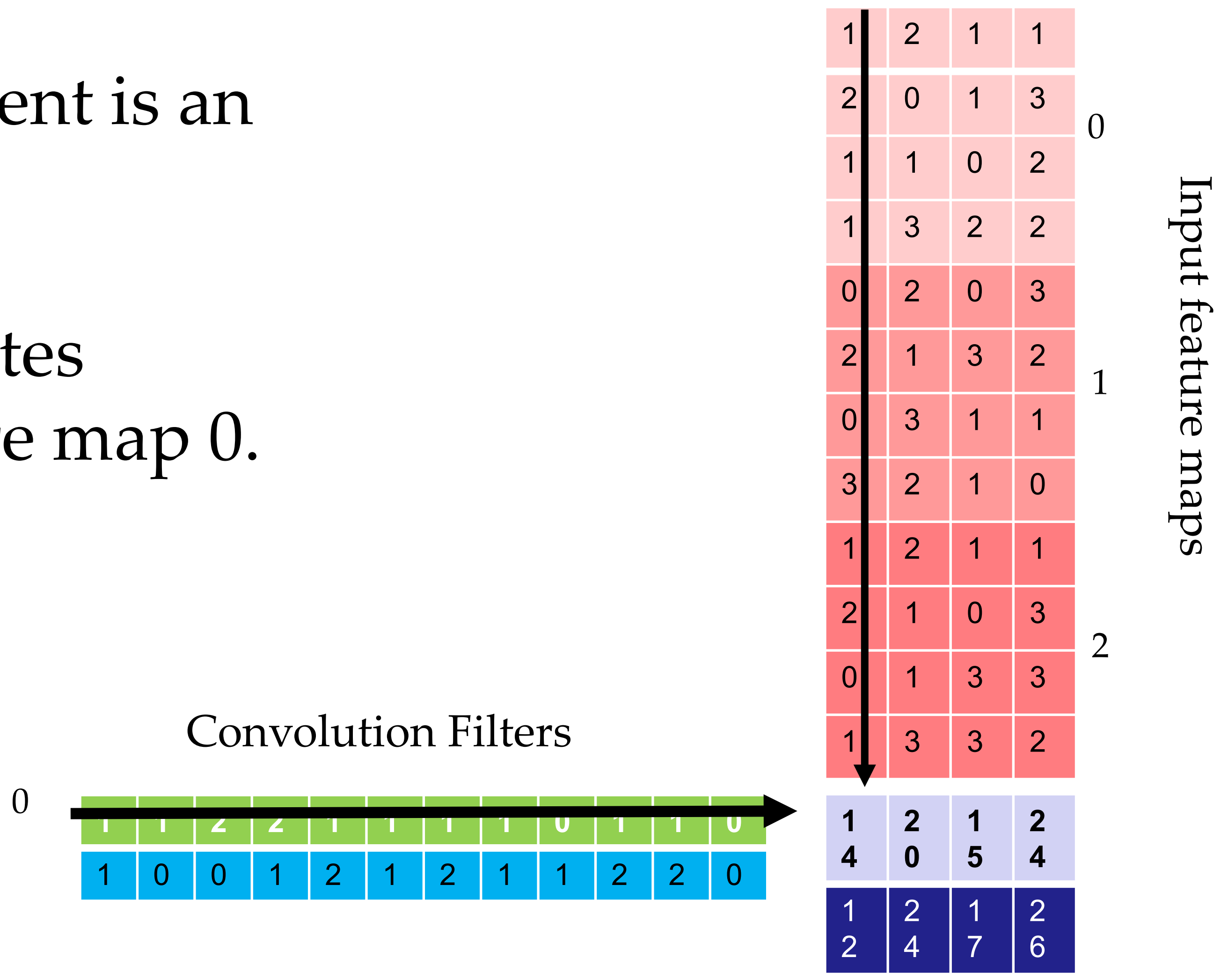
Implementing a Convolution Layer with Matrix Multiplication



Simple Matrix Multiplication

Each product matrix element is an output feature map pixel.

This inner product generates element 0 of output feature map 0.

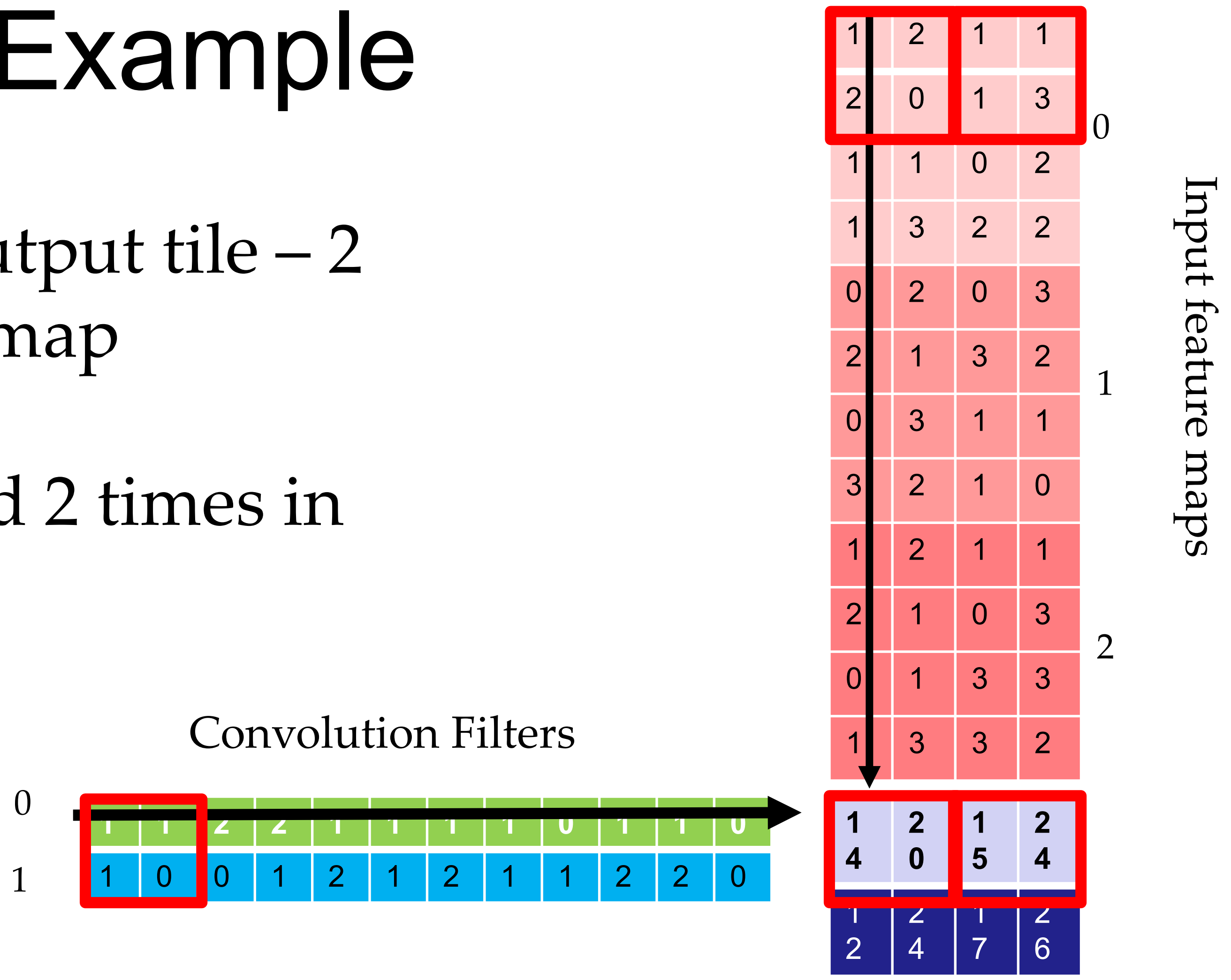


Tiled Matrix Multiplication

2x2 Example

Each block calculates one output tile – 2 elements from each output map

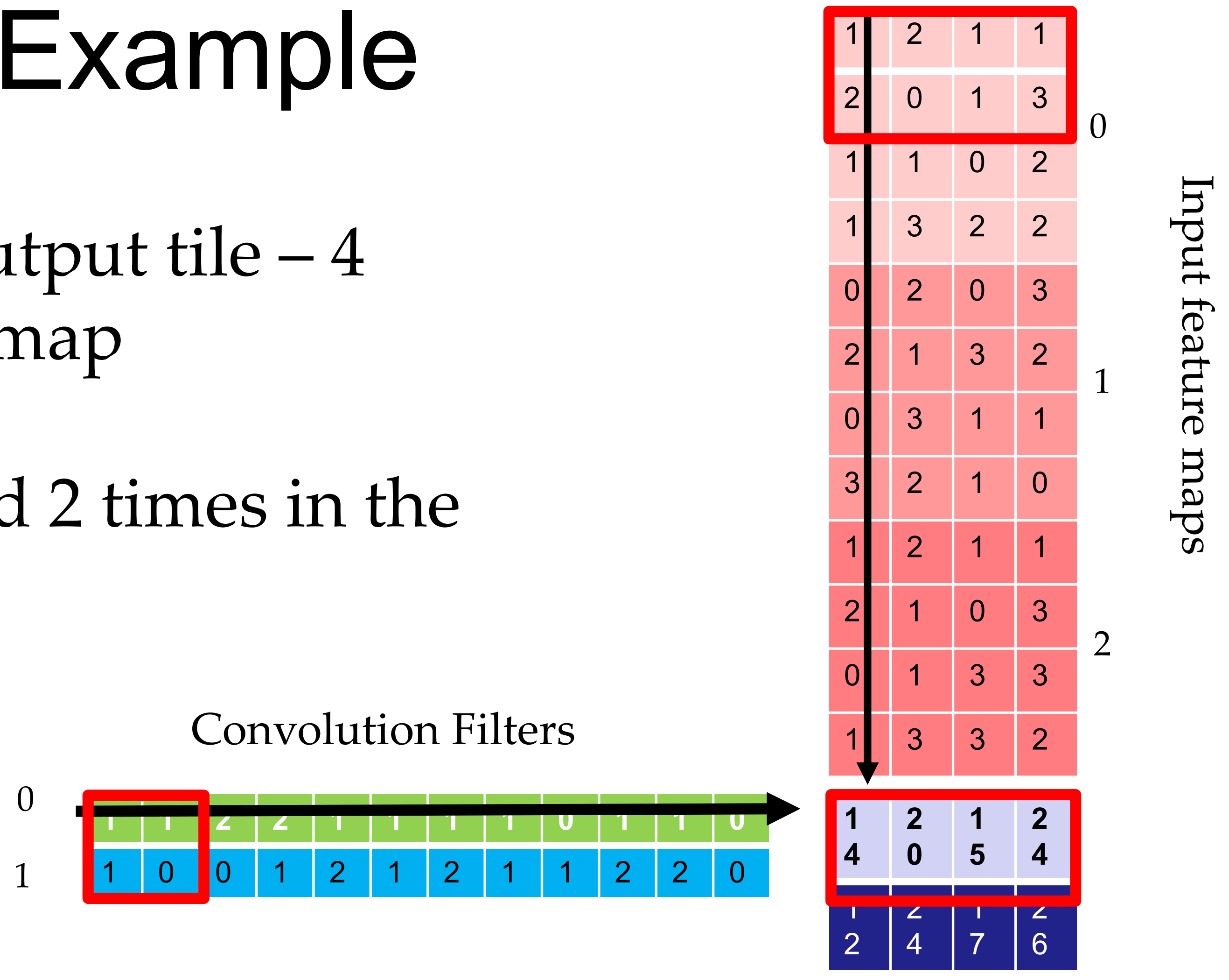
Each input element is reused 2 times in the shared memory



Tiled Matrix Multiplication 2x4 Example

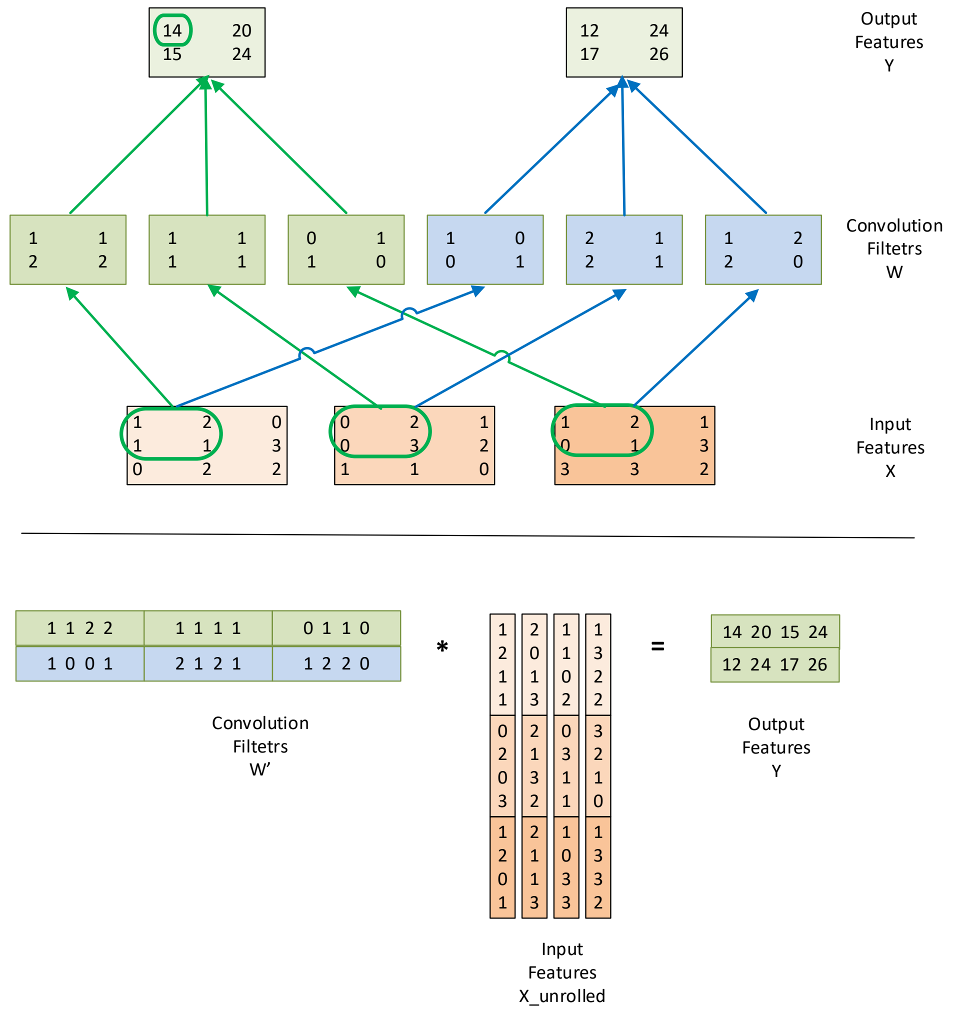
Each block calculates one output tile – 4 elements from each output map

Each input element is reused 2 times in the shared memory



Efficiency Analysis: Total Input Replication

- Replicated input features are shared among output maps
  - There are H_out * W_out output feature map elements
  - Each requires K*K elements from the input feature maps
  - So, the total number of input element after replication is H_out*W_out*K*K times for each input feature map
  - The total number of elements in each original input feature map is (H_out+K-1) * (W*out+K-1)



Analysis of a Small Example

H_out = 2

W_out = 2

K = 2

There are 3 input maps (channels)

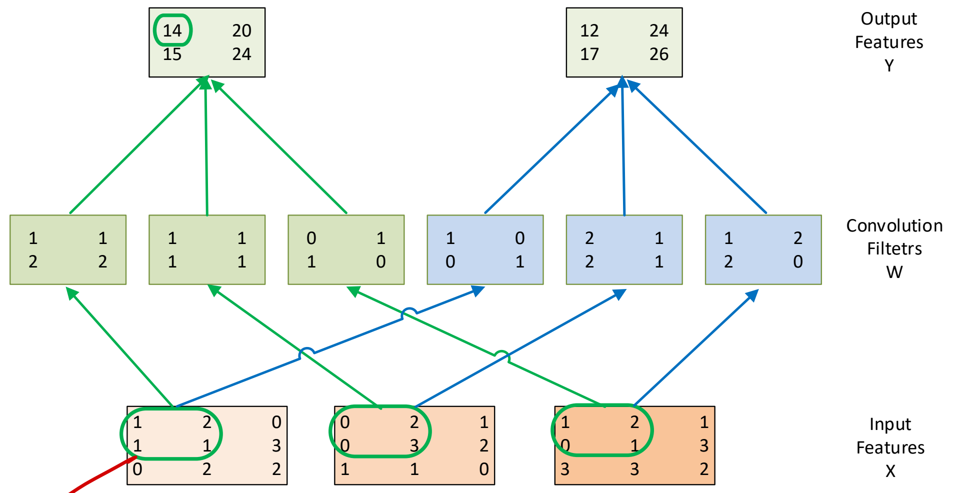
The total number of input elements in the replicated (“unrolled”) input matrix is 3*2*2*2*2

The replicating factor is

(3*2*2*2*2)/(3*3*3) = 1.78

$C \cdot K \cdot K \cdot H\_out \cdot W\_out$

$C \cdot H\_in \cdot W\_in$



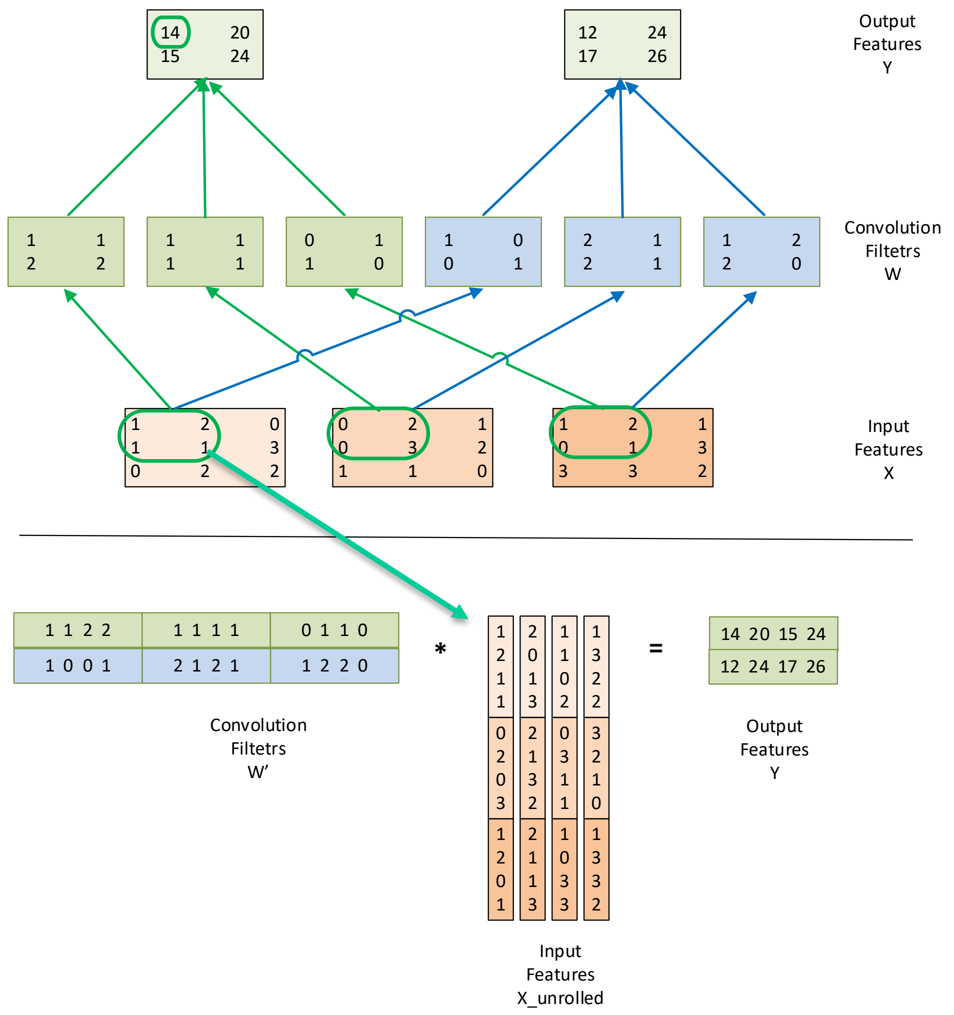
$$
\underbrace{\begin{bmatrix} 1&1&2&2&1&1&1&1&0&1&1&0 \\ 1&0&0&1&2&1&2&1&1&2&2&0 \end{bmatrix}}_{\text{Convolution Filtetrs } W'}
*
\underbrace{\begin{bmatrix} 1&2&1&1 \\ 2&0&1&3 \\ 1&1&0&2 \\ 1&3&2&2 \\ 0&2&0&3 \\ 2&1&3&2 \\ 0&3&1&1 \\ 3&2&1&0 \\ 1&2&1&1 \\ 2&1&0&3 \\ 0&1&3&3 \\ 1&3&3&2 \end{bmatrix}}_{\text{Input Features } X\_unrolled}
=
\underbrace{\begin{bmatrix} 14&20&15&24 \\ 12&24&17&26 \end{bmatrix}}_{\text{Output Features } Y}
$$

Memory Access Efficiency of Original Convolution Algorithm

- Assume that we use tiled 2D convolution
- For input elements
  - Each output tile has $TILE\_WIDTH^2$ elements
  - Each input tile has $(TILE\_WIDTH+K-1)^2$
  - The total number of input feature map element accesses was $TILE\_WIDTH^2*K^2$
  - The reduction factor of the tiled algorithm is $K^2*TILE\_WIDTH^2/(TILE\_WIDTH+K-1)^2$
- The convolution filter weight elements are reused within each output tile

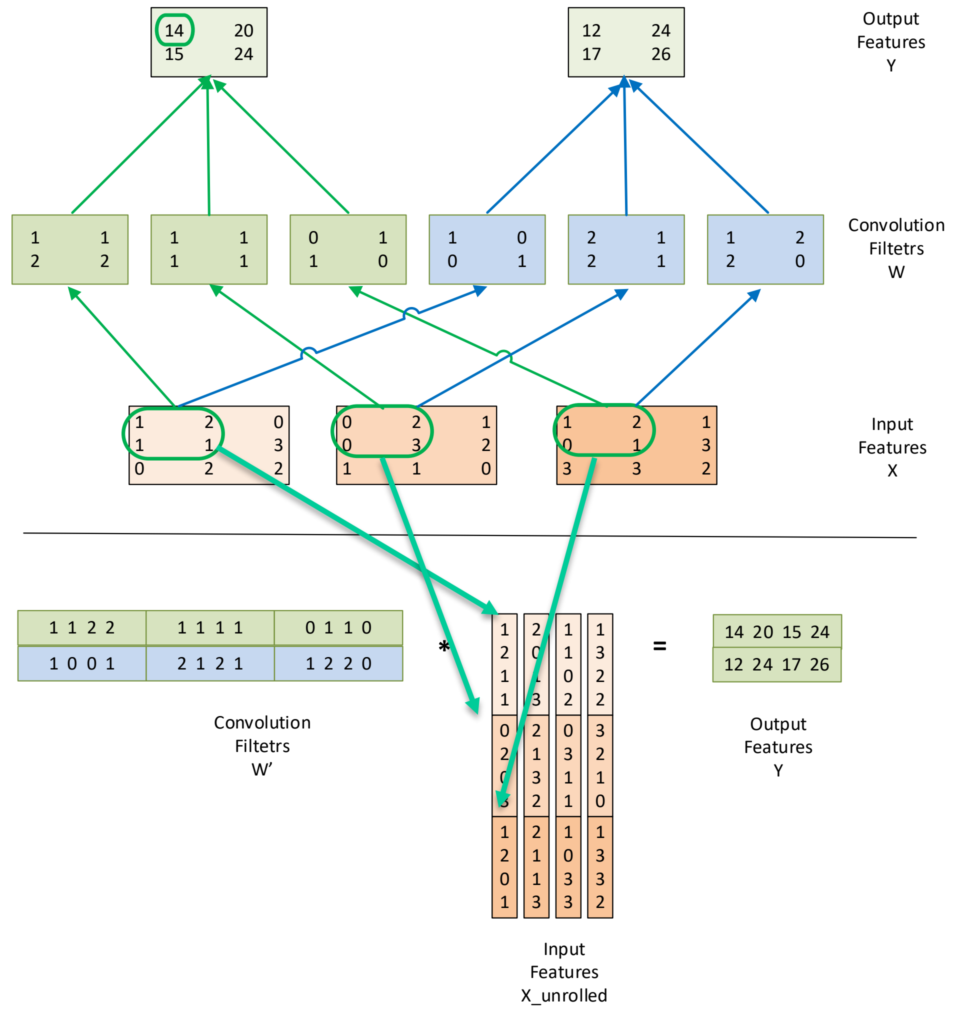
Properties of the Unrolled Matrix

- Each unrolled column corresponds to an output feature map element
- For an output feature element (h,w), the index for the unrolled column is h*W_out+w (linearized index of the output feature map element)



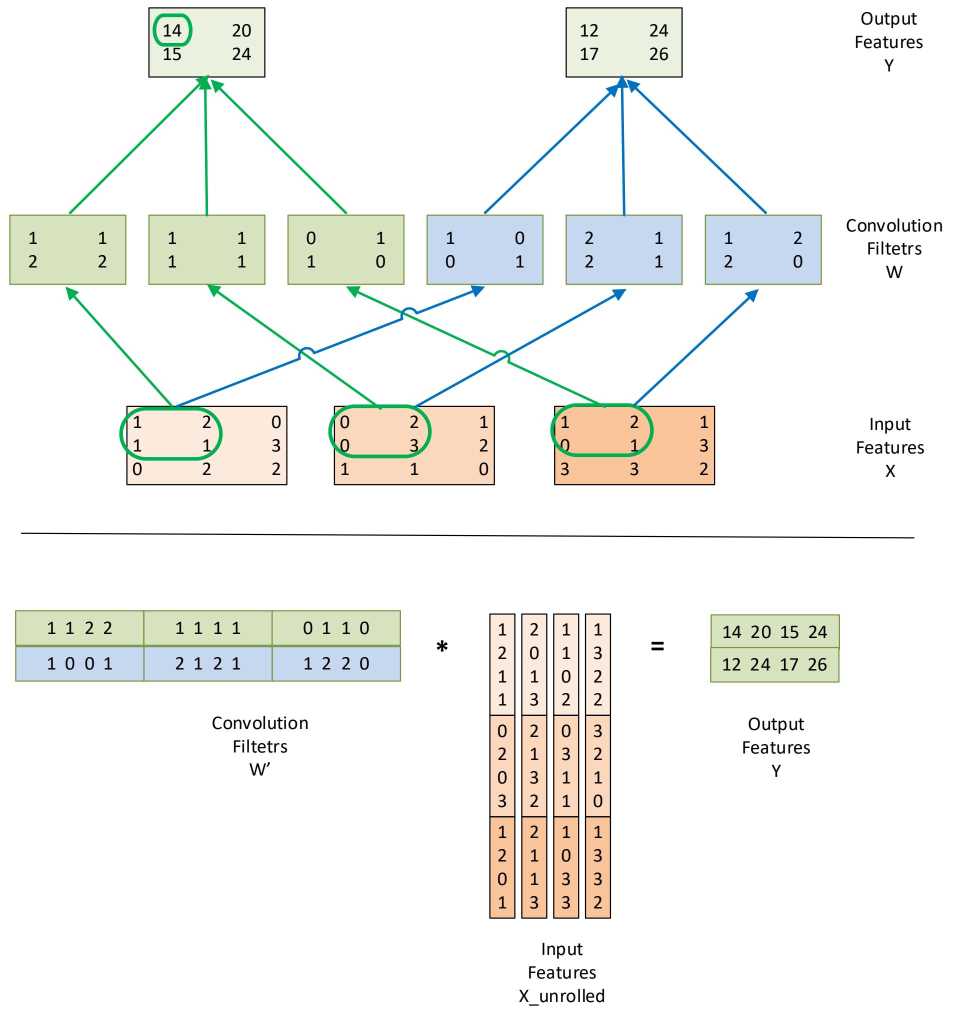
Properties of the Unrolled Matrix (cont.)

- Each section of the unrolled column corresponds to an input feature map
- Each section of the unrolled column has k*k elements (convolution mask size)
- For an input feature map c, the vertical index of its section in the unrolled column is c*k*k (linearized index of the output feature map element)



To Find the Input Elements

- For output element (h,w), the base index for the upper left corner of the input feature map c is (c, h, w)
- The input element index for multiplication with the convolution mask element (p, q) is (c, h+p, w+q)



Input to Unrolled Matrix Mapping

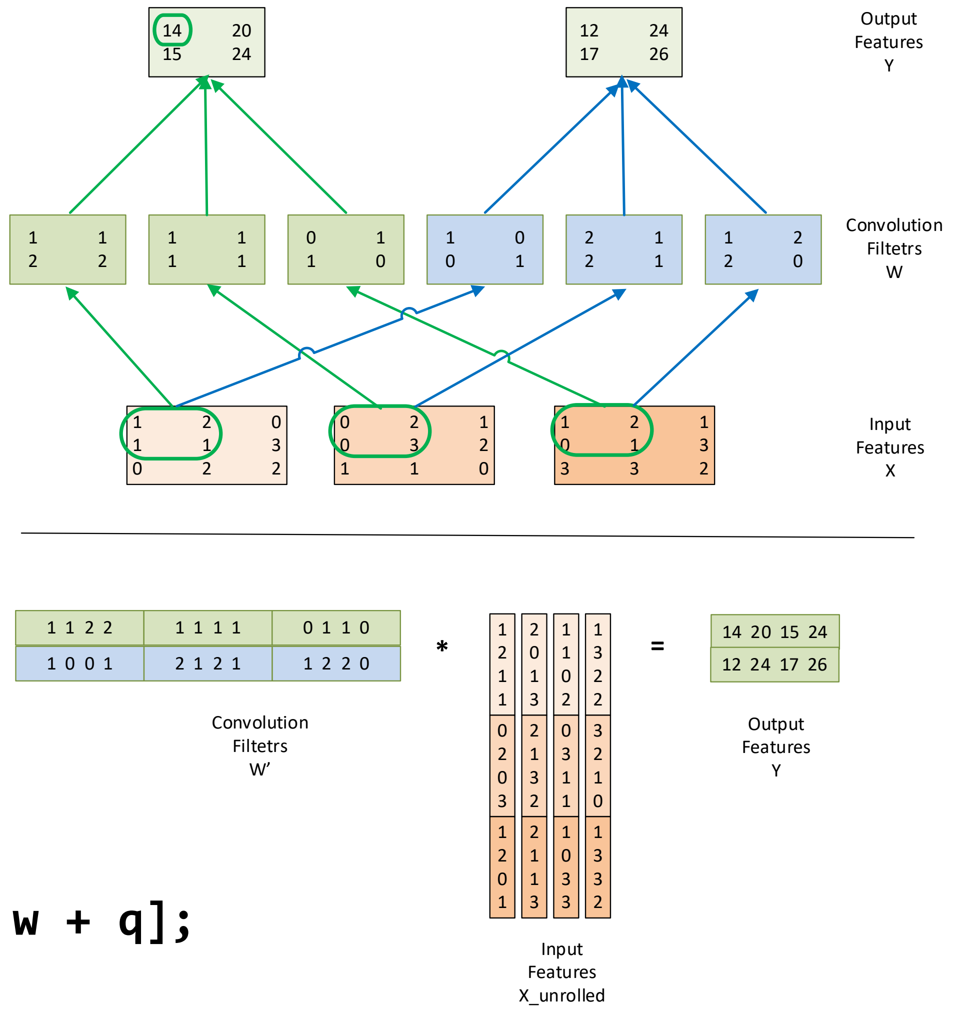
```
Output element (h, w)
Mask element (p, q)
Input feature map c

// calculate the horizontal matrix index
int w_unroll = h * W_out + w;

// find the beginning of the unrolled
int w_base = c * (K*K);

// calculate the vertical matrix index
int h_unroll = w_base + p * K + q;

X_unroll[b, h_unroll, w_unroll] = X[b, c, h + p, w + q];
```



Function to generate “unrolled” X

```
void unroll(int B, int C, int H, int W, int K, float* X, float* X_unroll)
{
  int H_out = H – K + 1;                          // calculate H_out, W_out
  int W_out = W – K + 1;
  for (int b = 0; b < B; ++b)                     // for each image
    for (int c = 0; c < C; ++c) {                 // for each input channel
      int w_base = c * (K*K);                     // per-channel offset for smallest X_unroll index
      for (int p = 0; p < K; ++p)                 // for each element of KxK filter (two loops)
        for (int q = 0; q < K; ++q) {
          for (int h = 0; h <  H_out; ++h)        // for each thread (each output value, two loops)
            for (int w = 0; w < W_out; ++w) {
              int h_unroll = w_base + p * K + q;   // data needed by one thread
              int w_unroll = h * W_out + w;       // smallest index--across threads (output values)
              X_unroll[b, h_unroll, w_unroll] = X[b, c, h + p, w + q];     // copy input pixels
            }
    }
}
```

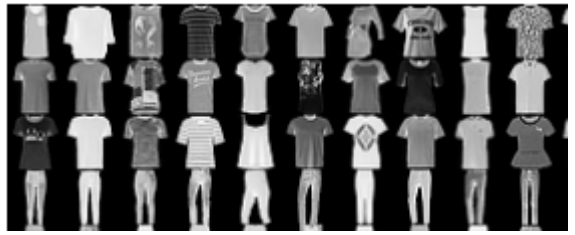
Implementation Strategies for a Convolution Layer

- Baseline
  - Tiled 2D convolution implementation, use constant memory for convolution masks
- Matrix-Multiplication Baseline
  - Input feature map unrolling kernel, constant memory for convolution masks as an optimization
  - Tiled matrix multiplication kernel
- Matrix-Multiplication with built-in unrolling
  - Perform unrolling only when loading a tile for matrix multiplication
  - The unrolled matrix is only conceptual
  - When loading a tile element of the conceptual unrolled matrix into the shared memory, use the properties in the lecture to load from the input feature map
- More advanced Matrix-Multiplication
  - Use joint register-shared memory tiling

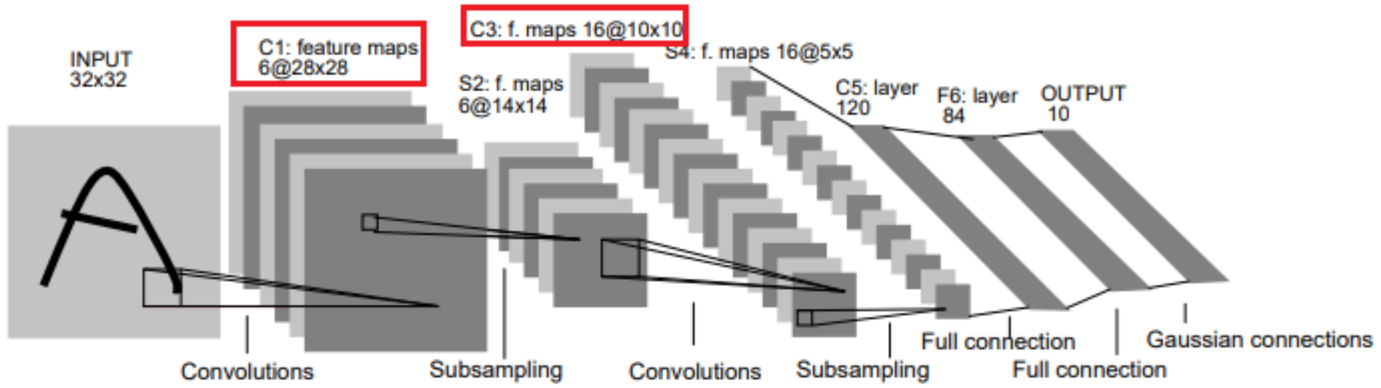
ANY MORE QUESTIONS?
READ CHAPTER 16

Project Overview

- Optimize the forward pass of the convolutional layers in a modified LeNet-5 CNN using CUDA. (CNN implemented using Mini-DNN, a C++ framework)
- The network will be classifying Fashion MNIST dataset
- Some network parameters to be aware of
  - Input Size: 86x86 pixels, batch of 10k images
  - Input Channels: 1
  - Convolutional kernel size: 7x7
  - Number of kernels: Variable (your code should support this)



https://github.com/zalandoresearch/fashion-mnist



http://yann.lecun.com/exdb/publis/pdf/lecun-01a.pdf

Project Timeline

- **All milestones are due on Fridays at 8 pm Central Time**
- Everyone must individually submit all Milestones.
  - **No sharing of code is allowed**

- October 14th: Project milestone 1:
  - CPU Convolution, profiling
- November 4th: Project milestone 2:
  - Baseline GPU Convolution Kernel
- December 2nd: Project milestone 3:
  - GPU Convolution Kernel Optimizations

Project Release

- Project is released now (only PM1 for now)
  - Check the course wiki page for the link to the github repository
  - https://github.com/aschuh703/ECE408/tree/main/Project
- The readme in the repository contains all the instructions and details to complete the project.
- The github repo will be updated with additional code and instructions for PM2 & PM3