

Distributed Systems

ECE428

Lecture 10

Adopted from Spring 2021

Today's agenda

- Mutual Exclusion
 - Chapter 15.2
- Leader Election
 - Chapter 15.3

Problem statement for mutual exclusion

- *Critical Section Problem*:
 - Piece of code (at all processes) for which we need to ensure there is at most one process executing it at any point of time.
- Each process can call three functions
 - **enter()** to enter the critical section (CS)
 - **AccessResource()** to run the critical section code
 - **exit()** to exit the critical section

Mutual exclusion in distributed systems

- Processes communicating by passing messages.
- Cannot share variables like semaphores!
- *How do we support mutual exclusion in a distributed system?*

Mutual exclusion in distributed systems

- Our focus today: Classical algorithms for mutual exclusion in distributed systems.
 - Central server algorithm
 - Ring-based algorithm
 - Ricart-Agrawala Algorithm
 - Maekawa Algorithm

System Model

- Each pair of processes is connected **by reliable channels** (such as TCP).
- Messages sent on a channel are eventually delivered to a recipient, and **in FIFO order**.
- Processes do not fail.
 - Fault-tolerant variants exist in literature.

Analysis of Central Algorithm

- Safety – at most one process in CS
 - Exactly one token
- Liveness – every request for CS granted eventually
 - With N processes in system, queue has at most N processes
 - If each process exits CS eventually and no failures, liveness guaranteed
- Ordering:
 - FIFO ordering guaranteed in order of requests received at leader
 - Not in the order in which requests were sent or the order in which processes enter CS!

Analyzing Performance

Three metrics:

- **Bandwidth:** the total number of messages sent in each *enter* and *exit* operation.
- **Client delay:** delay incurred by a process at each enter and exit operation (when *no* other process is in CS, or waiting)
 - We will *focus on the client delay for the enter operation.*
- **Synchronization delay:** the time interval between one process exiting the critical section and the next process entering it (when there is *only one* process waiting).
Measure of the *throughput* of the system.

Analysis of Central Algorithm

- **Bandwidth**: the total number of messages sent in each *enter* and *exit* operation.
 - 2 messages for enter
 - 1 message for exit
- **Client delay**: delay incurred by a process at each enter and exit operation (when *no* other process is in, or waiting)
 - 2 message latencies (1 round-trip, request + grant) on enter.
- **Synchronization delay**: the time interval between one process exiting the critical section and the next process entering it (when there is *only one* process waiting)
 - 2 message latencies (release + grant)

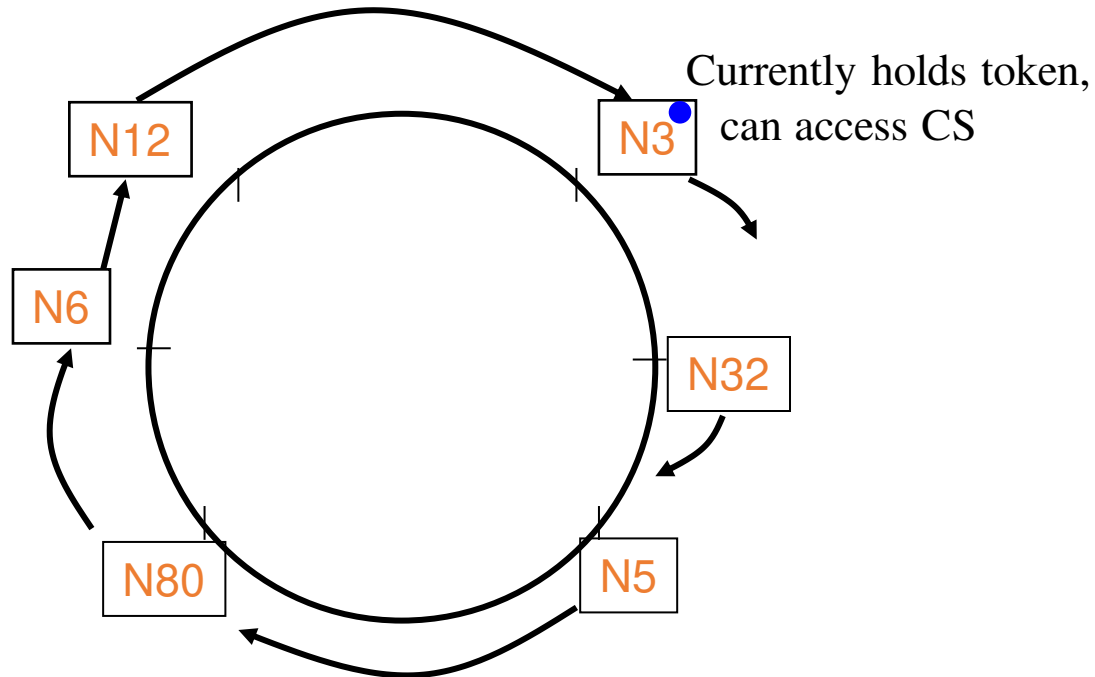
Limitations of Central Algorithm

- The leader is the performance bottleneck and single point of failure.

Mutual exclusion in distributed systems

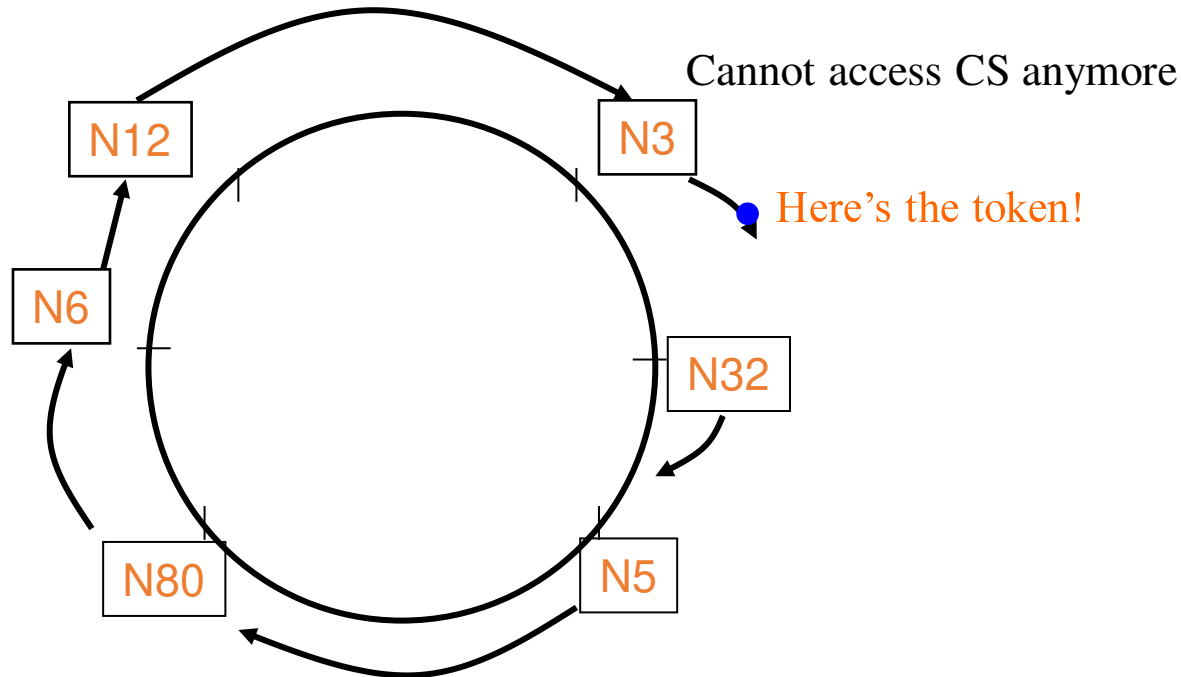
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Ring-based Mutual Exclusion



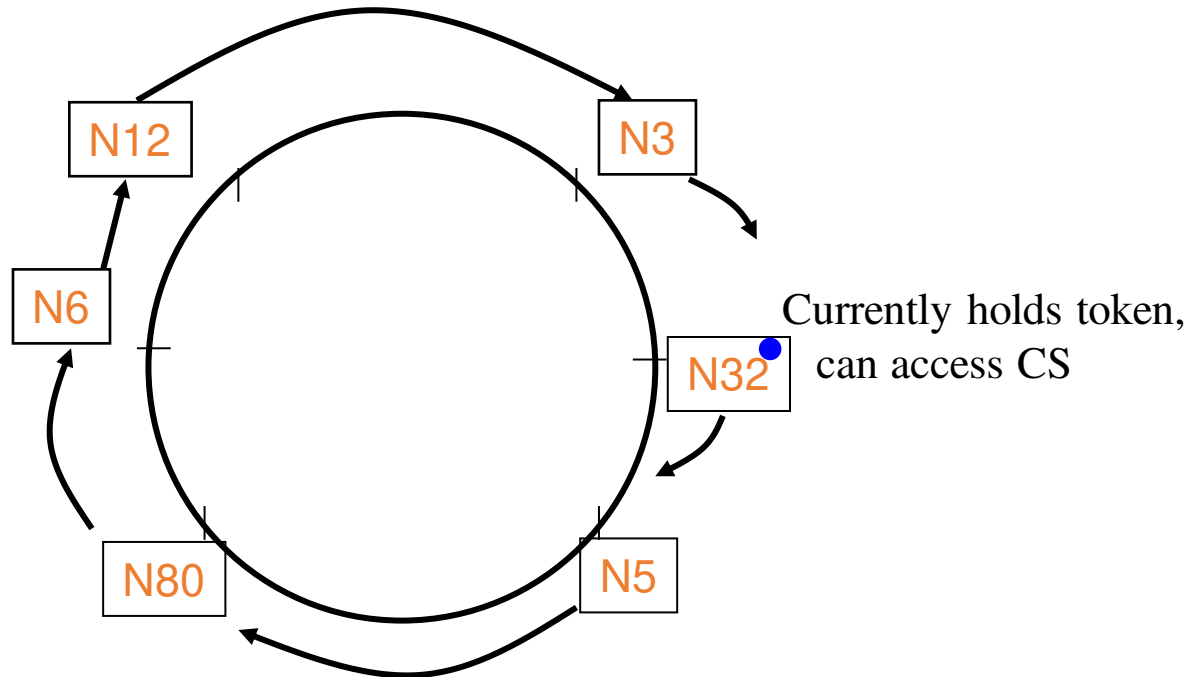
Token: ●

Ring-based Mutual Exclusion



Token: ●

Token: ●



Ring-based Mutual Exclusion

- N Processes organized in a virtual ring
- Processes send message to their successor in ring
- Exactly 1 token
- **enter()**
 - Wait until you get token
- **exit()** // already have token
 - Pass on token to ring successor
- If receive token, and not currently in enter(), just pass on token to ring successor

Analysis of Ring-based algorithm

- Safety
 - Exactly one token
- Liveness
 - Token eventually loops around ring and reaches requesting process (we assume no failures)
- Ordering
 - Token not always obtained in order of enter events.

Analysis of ring-based algorithm

- Bandwidth
 - Per enter, 1 message at requesting process but up to N messages throughout system.
 - 1 message sent per exit.
 - *Constantly consumes bandwidth even when no process requires entry to the critical section (except when a process is executing critical section).*

Analysis of ring-based algorithm

- Client delay:
 - Best case: just received token
 - Worst case: just sent token to neighbor
 - 0 to N message transmissions after entering enter()
- Synchronization delay between one process' exit() from the CS and the next process' enter():
 - Best case: process in enter() is successor of process in exit()
 - Worst case: process in enter() is predecessor of process in exit()
 - Between 1 and $(N-1)$ message transmissions.

Can we improve upon $O(n)$ client and synchronization delays?

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Ricart-Agrawala's Algorithm

- Classical algorithm from 1981
- Invented by Glenn Ricart (NIH) and Ashok Agrawala (U. Maryland)
- No token.
- Uses the notion of causality and multicast.
- Has lower waiting time to enter CS than Ring-Based approach.

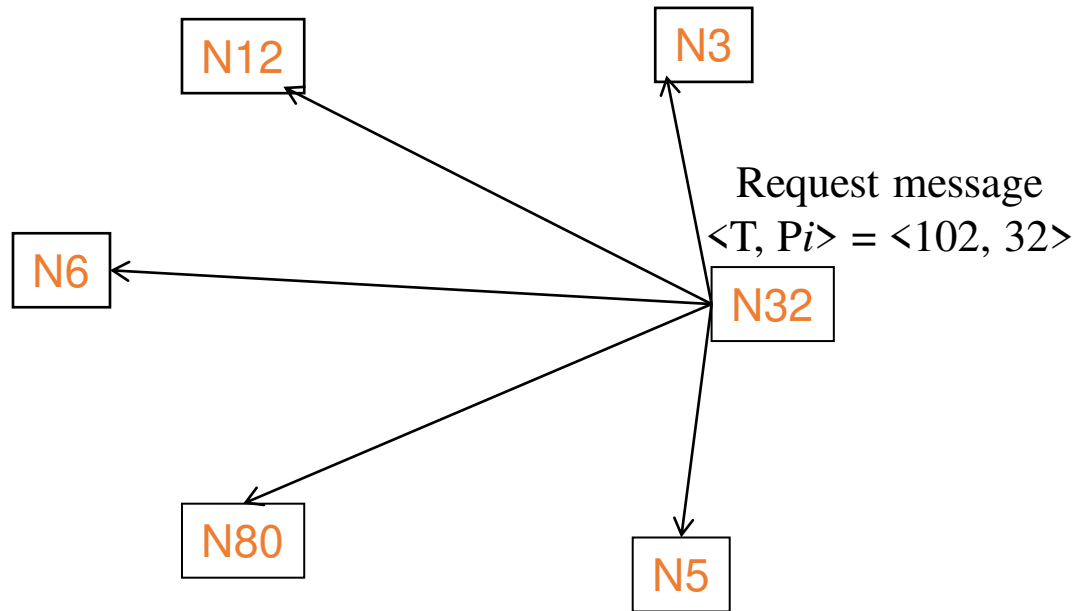
Key Idea: Ricart-Agrawala Algorithm

- **enter()** at process P_i
 - **multicast** a request to all processes
 - Request: $\langle T, P_i \rangle$, where T = current Lamport timestamp at P_i
 - Wait until **all** other processes have responded positively to request
- Requests are granted in order of causality.
- $\langle T, P_i \rangle$ is used lexicographically: P_i in request $\langle T, P_i \rangle$ is used to break ties (since Lamport timestamps are not unique for concurrent events).

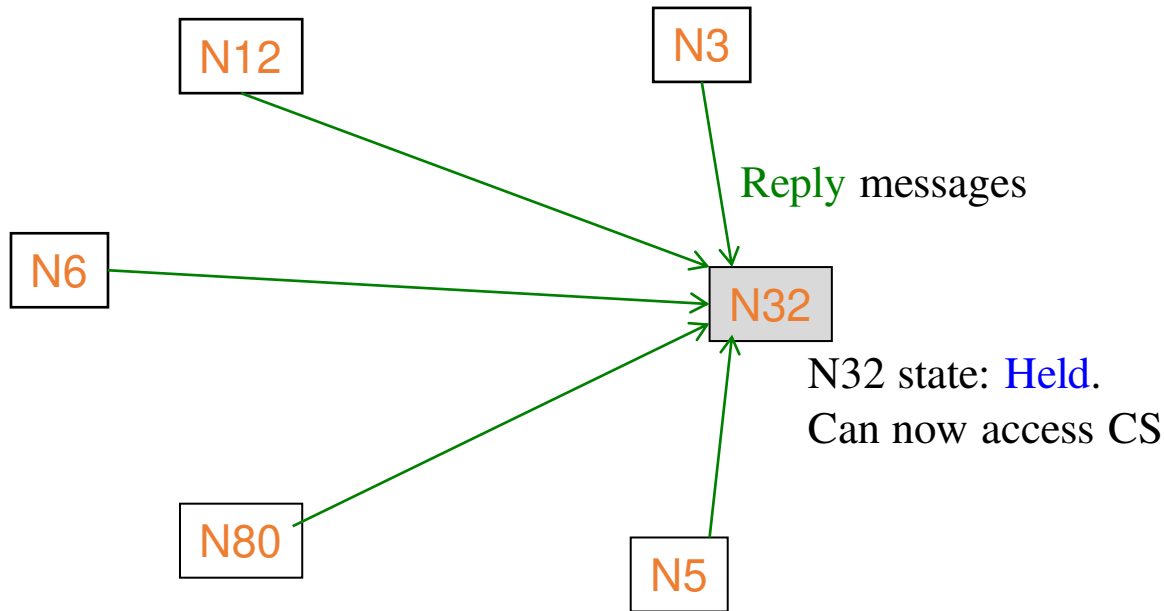
Messages in RA Algorithm

- **enter()** at process P_i
 - set state to Wanted
 - multicast “Request” $\langle T_i, P_i \rangle$ to all other processes, where T_i = current Lamport timestamp at P_i
 - wait until all other processes send back “Reply”
 - change state to Held and enter the CS
- On receipt of a Request $\langle T_j, j \rangle$ at P_i ($i \neq j$):
 - if (state = Held) or (state = Wanted & $(T_i, i) < (T_j, j)$)
// lexicographic ordering in (T_j, j) , T_i is Lamport timestamp of P_i 's request
add request to local queue (of waiting requests)
else send “Reply” to P_j
- **exit()** at process P_i
 - change state to Released and “Reply” to all queued requests.

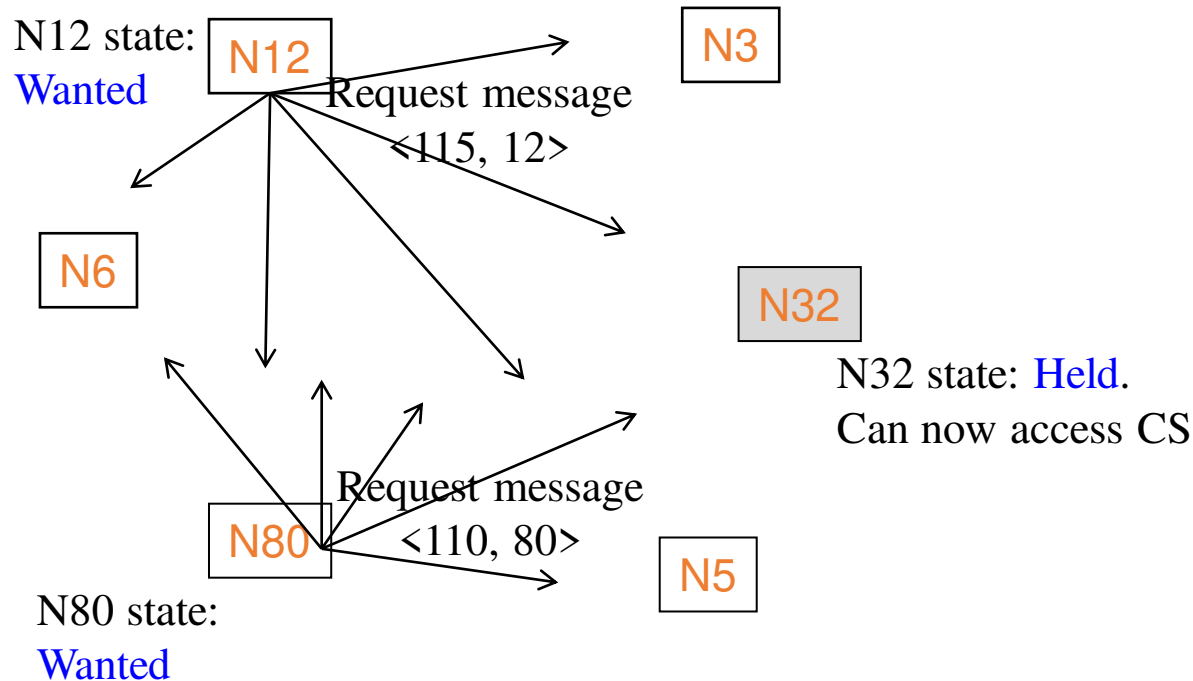
Example: Ricart-Agrawala Algorithm



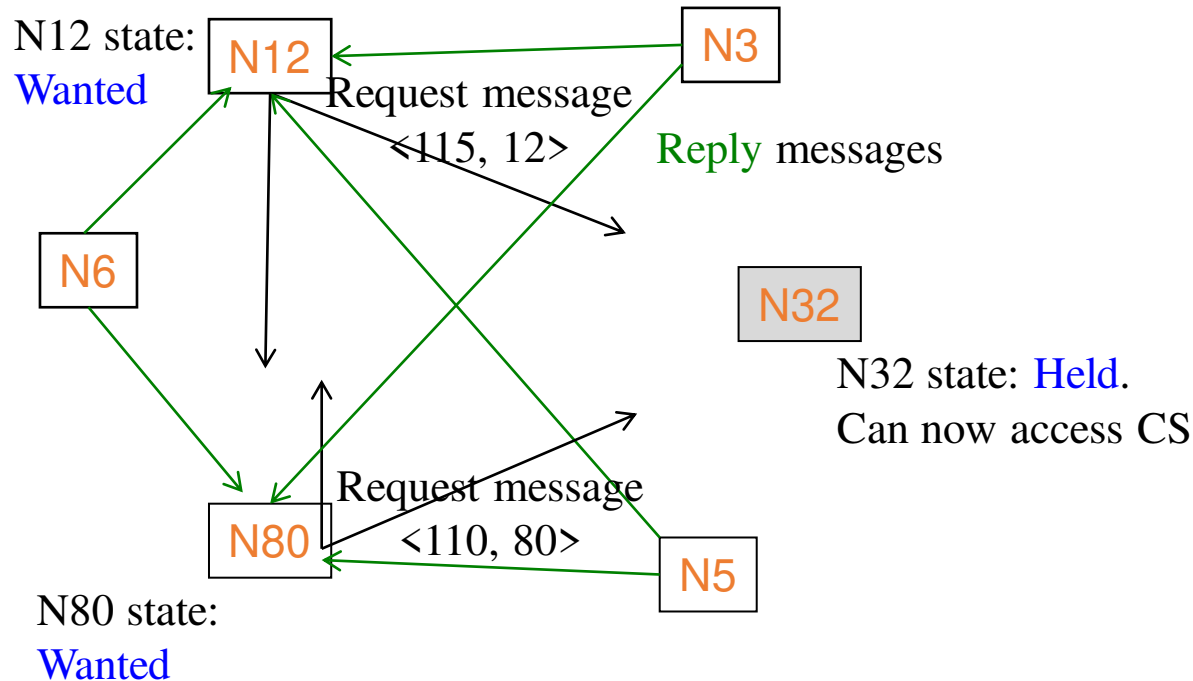
Example: Ricart-Agrawala Algorithm



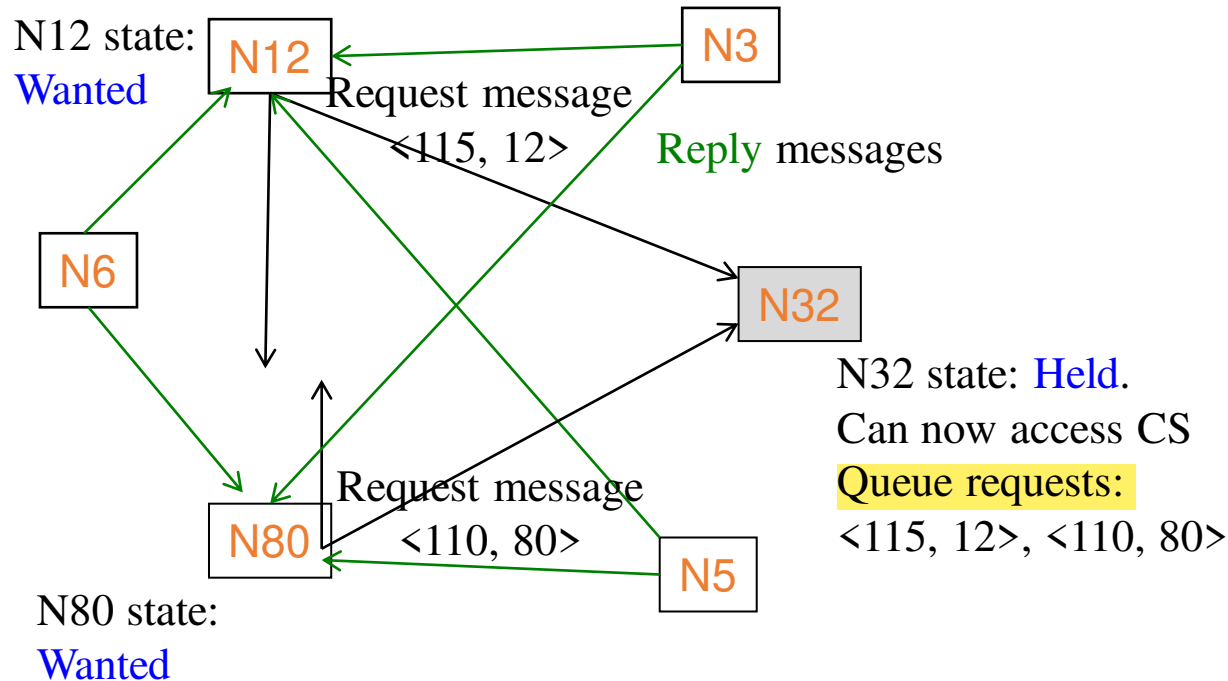
Example: Ricart-Agrawala Algorithm



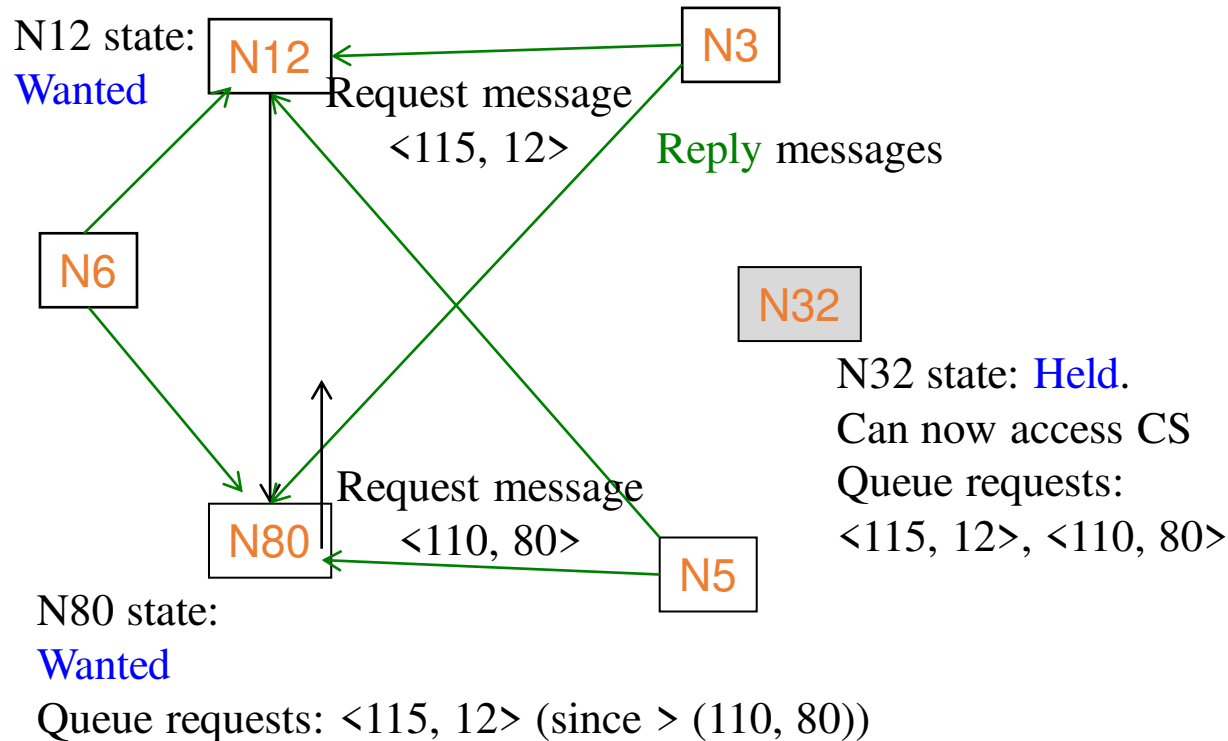
Example: Ricart-Agrawala Algorithm



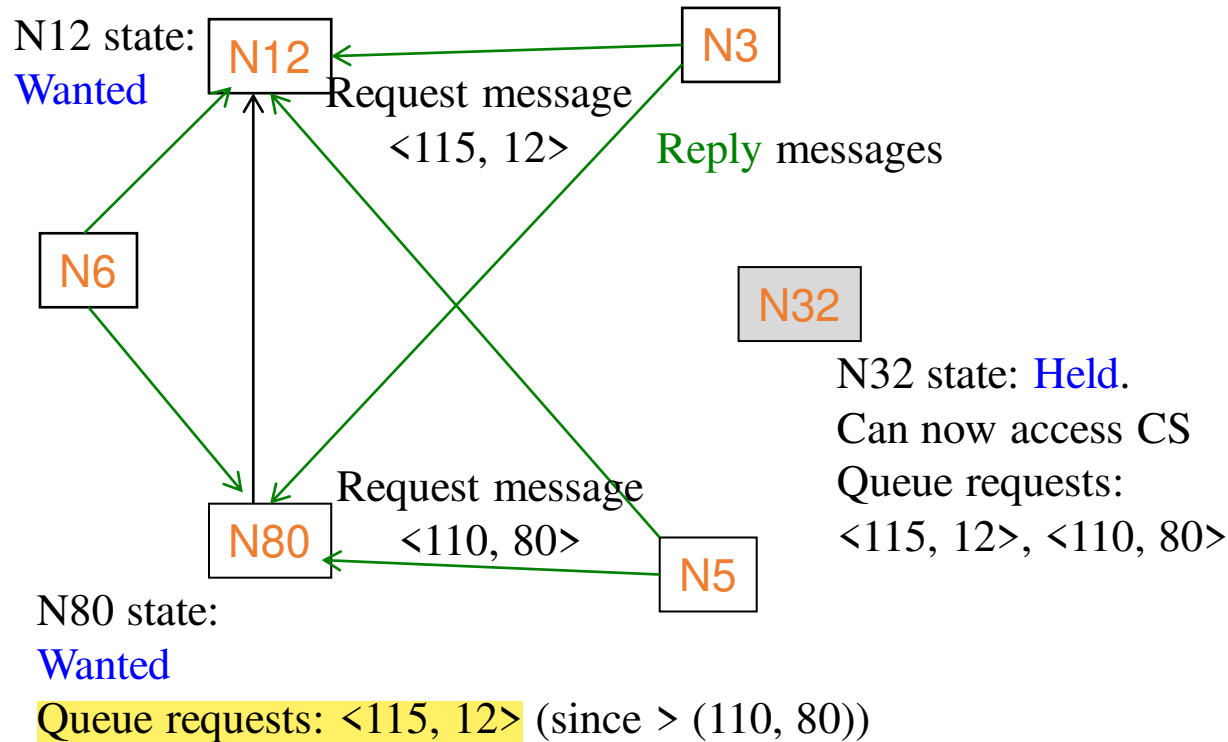
Example: Ricart-Agrawala Algorithm



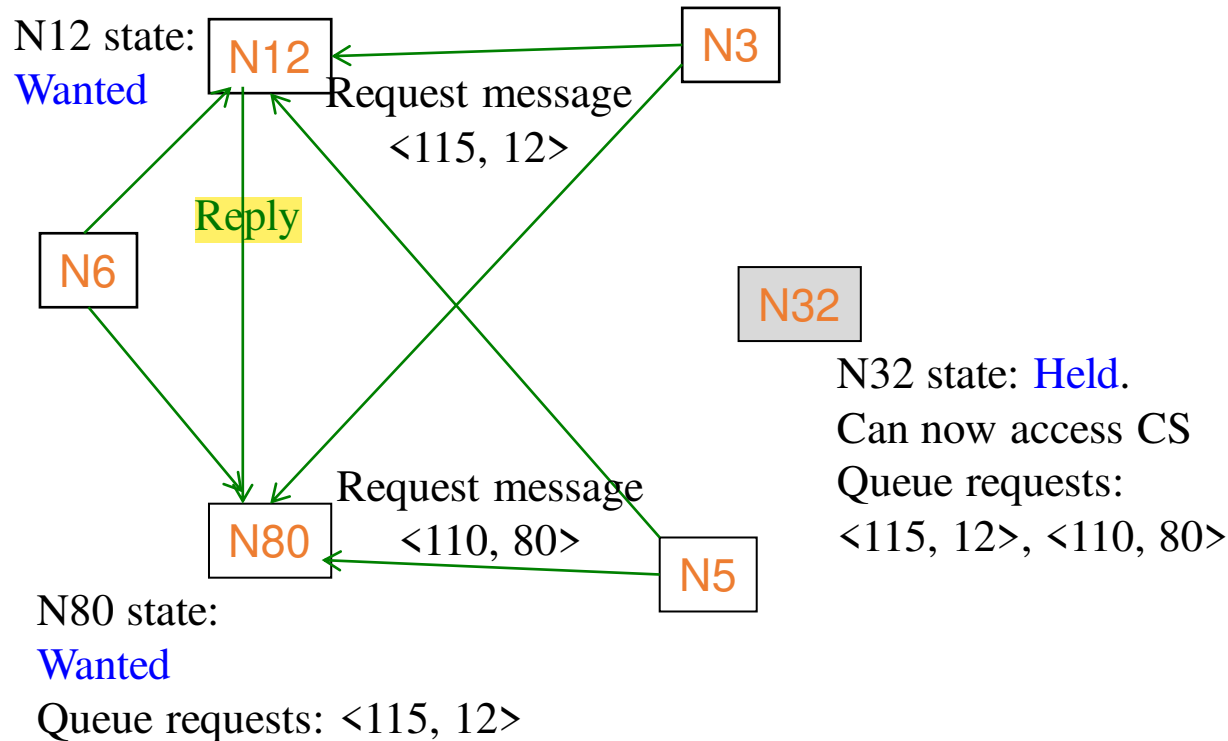
Example: Ricart-Agrawala Algorithm



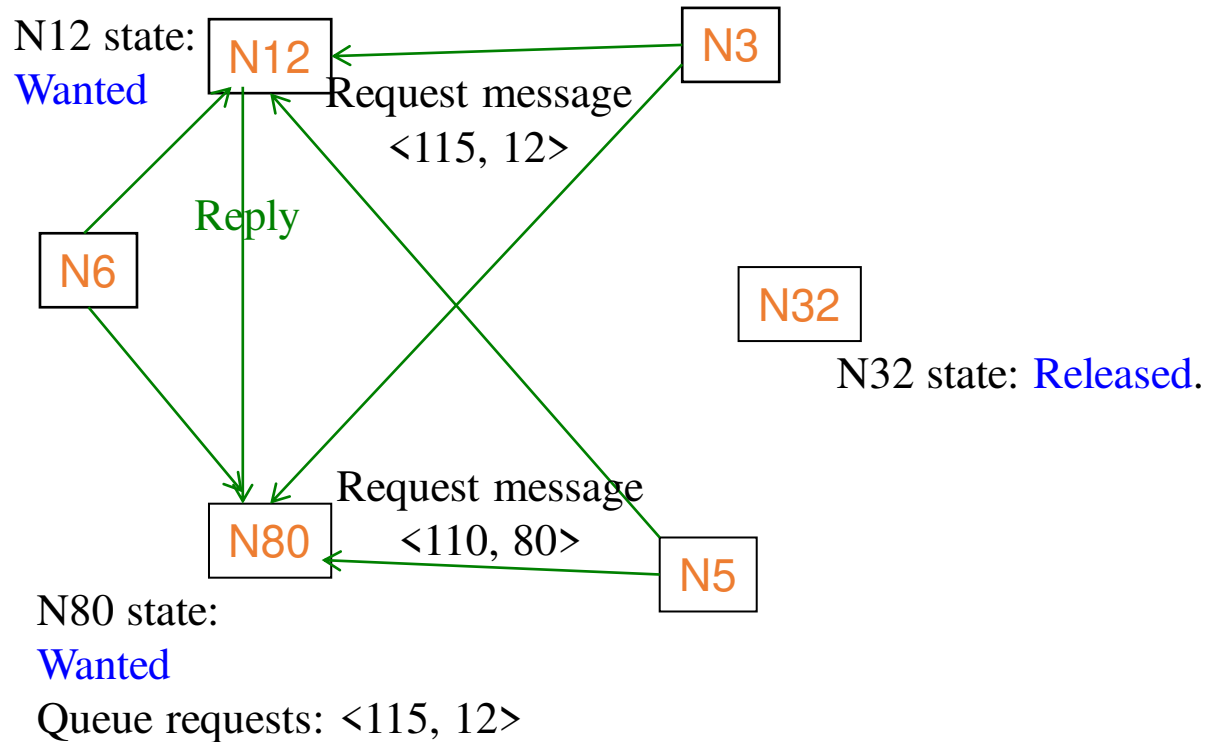
Example: Ricart-Agrawala Algorithm



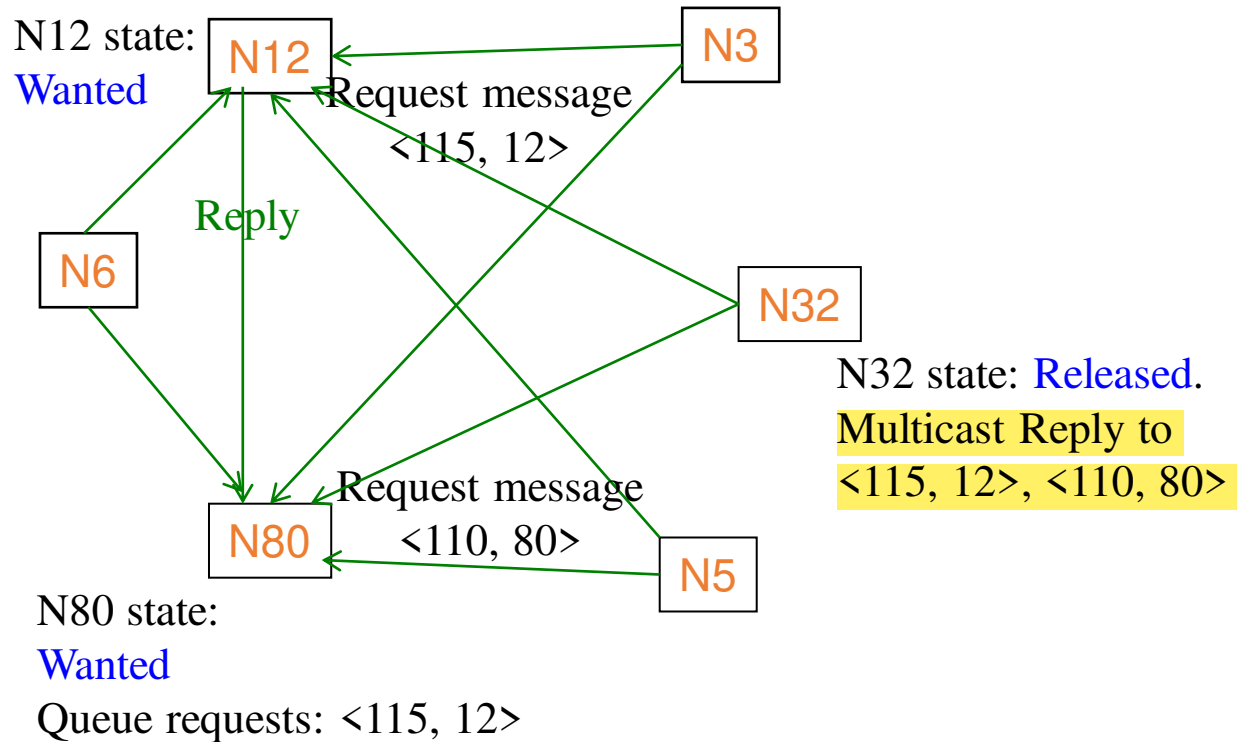
Example: Ricart-Agrawala Algorithm



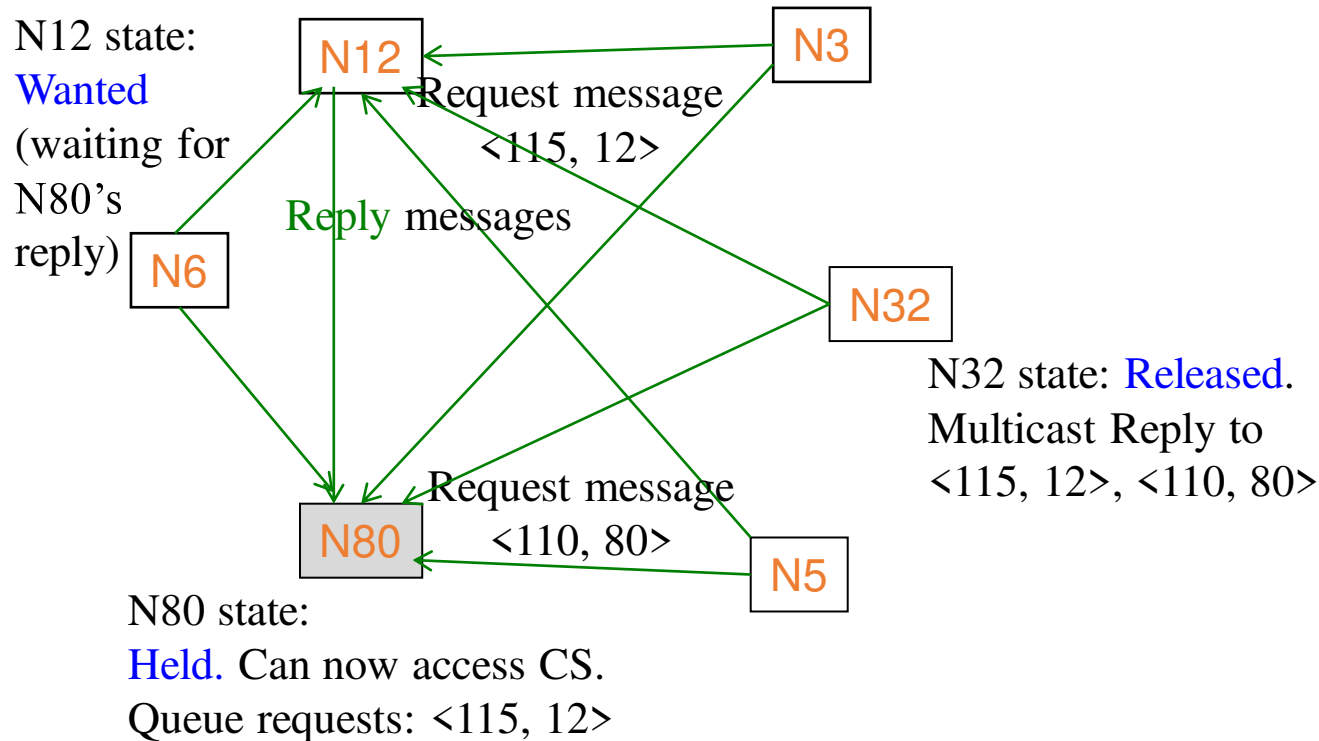
Example: Ricart-Agrawala Algorithm



Example: Ricart-Agrawala Algorithm



Example: Ricart-Agrawala Algorithm



Analysis: Ricart-Agrawala Algorithm

- Safety

- Two processes P_i and P_j cannot both have access to CS
 - If they did, then both would have sent Reply to each other.
 - Thus, $(T_i, i) < (T_j, j)$ and $(T_j, j) < (T_i, i)$, which are together not possible.
 - What if $(T_i, i) < (T_j, j)$ and P_i replied to P_j 's request before it created its own request?
 - But then, causality and Lamport timestamps at P_i implies that $T_i > T_j$, which is a contradiction.
 - So this situation cannot arise.

Analysis: Ricart-Agrawala Algorithm

- Safety

- Two processes P_i and P_j cannot both have access to CS.

- Liveness

- Worst-case: wait for all other $(N-1)$ processes to send Reply.

- Ordering

- Requests with lower Lamport timestamps are granted earlier.

Analysis: Ricart-Agrawala Algorithm

- Bandwidth:
 - $2*(N-1)$ messages per enter operation
 - $N-1$ unicasts for the multicast request + $N-1$ replies
 - Maybe fewer depending on the multicast mechanism.
 - $N-1$ unicasts for the multicast release per exit operation
 - Maybe fewer depending on the multicast mechanism.
- Client delay:
 - one round-trip time
- Synchronization delay:
 - one message transmission time
- *Client and synchronization delays have gone down to $O(1)$.*
- *Bandwidth usage is still high. Can we bring it down further?*

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Maekawa's Algorithm: Key Idea

- Ricart-Agrawala requires replies from *all* processes in group.
- Instead, get replies from only *some* processes in group.
- But ensure that only one process is given access to CS (Critical Section) at a time.

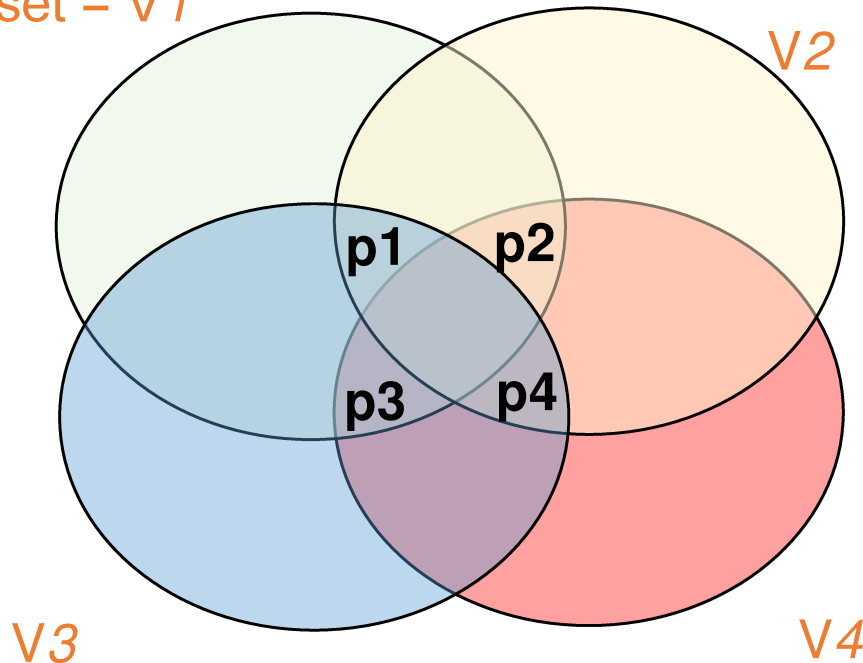
Maekawa's Voting Sets

- Each process P_i is associated with a voting set V_i (subset of processes).
- Each process belongs to its own voting set.
- *The intersection of any two voting sets must be non-empty.*

A way to construct voting sets

One way of doing this is to put N processes in a \sqrt{N} by \sqrt{N} matrix and for each P_i , its voting set V_i = row containing P_i + column containing P_i .
Size of voting set = $2\sqrt{N}-1$.

P_1 's voting set = V_1



p_1	p_2
p_3	p_4

Maekawa: Key Differences From Ricart-Agrawala

- Each process requests permission from only its voting set members.
 - Not from all
- Each process (in a voting set) gives permission to at most one process at a time.
 - Not to all

Actions

- state = Released, voted = false
- enter() at process P_i :
 - state = Wanted
 - Multicast **Request** message to all processes in V_i
 - Wait for **Reply (vote)** messages from all processes in V_i (including vote from self)
 - state = Held
- exit() at process P_i :
 - state = Released
 - Multicast **Release** to all processes in V_i

Actions (contd.)

- When P_i receives a **Request** from P_j :
 - if (state == Held OR **voted** = true)
 - queue **Request**
 - else
 - send **Reply** to P_j and set **voted** = true
- When P_i receives a **Release** from P_j :
 - if (queue empty)
 - voted** = false
 - else
 - dequeue head of queue, say P_k
 - Send **Reply** *only* to P_k
 - voted** = true

Size of Voting Sets

- Each voting set is of size K .
- Each process belongs to M other voting sets.
- Maekawa showed that $K=M=\text{approx. } \sqrt{N}$ works best.

Optional self-study: Why \sqrt{N} ?

- Let each voting set be of size K and each process belongs to M other voting sets.
- Total number of voting set members (processes may be repeated) = $K*N$
- But since each process is in M voting sets
 - $K*N = M*N \Rightarrow K = M$ (1)
- Consider a process P_i
 - Total number of voting sets = members present in P_i 's voting set and all their voting sets = $(M-1)*K + 1$
 - All processes in group must be in above
 - To minimize the overhead at each process (K), need each of the above members to be unique, i.e.,
 - $N = (M-1)*K + 1$
 - $N = (K-1)*K + 1$ (due to (1))
 - $K \sim \sqrt{N}$

Size of Voting Sets

- Each voting set is of size K .
- Each process belongs to M other voting sets.
- Maekawa showed that $K=M=approx. \sqrt{N}$ works best.
- Matrix technique gives a voting set size of $2*\sqrt{N}-1 = O(\sqrt{N})$.

Performance: Maekawa Algorithm

- Bandwidth
 - $2K = 2\sqrt{N}$ messages per enter
 - $K = \sqrt{N}$ messages per exit
 - Better than Ricart and Agrawala's ($2*(N-1)$ and $N-1$ messages)
 - \sqrt{N} quite small. $N \sim 1$ million $\Rightarrow \sqrt{N} = 1K$
- Client delay:
 - One round trip time
- Synchronization delay:
 - 2 message transmission times

Safety

- When a process P_i receives replies from all its voting set V_i members, no other process P_j could have received replies from all its voting set members V_j .
 - V_i and V_j intersect in at least one process say P_k .
 - But P_k sends only one Reply (vote) at a time, so it could not have voted for both P_i and P_j .

Liveness

- Does not guarantee liveness, since can have a *deadlock*.
- System of 6 processes $\{0, 1, 2, 3, 4, 5\}$. 0, 1, 2 want to enter critical section:
 - $V_0 = \{0, 1, 2\}$:
 - 0, 2 send **reply** to 0, but 1 sends **reply** to 1;
 - $V_1 = \{1, 3, 5\}$:
 - 1, 3 send **reply** to 1, but 5 sends **reply** to 2;
 - $V_2 = \{2, 4, 5\}$:
 - 4, 5 send **reply** to 2, but 2 sends **reply** to 0;
- Now, 0 waits for 1's reply, 1 waits for 5's reply (5 waits for 2 to send a release), and 2 waits for 0 to send a release. Hence, deadlock!

Analysis: Maekawa Algorithm

- Safety:

- When a process P_i receives replies from all its voting set V_i members, no other process P_j could have received replies from all its voting set members V_j .

- Liveness

- Not satisfied. Can have deadlock!

- Ordering:

- Not satisfied.

Next Class

- How can we extend Maekawa's algorithm to break deadlock?