MSE 312 Mechatronics Design II

Truss Arm Report

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SFU SCHOOL OF MECHATRONIC SYSTEMS ENGINEERING

Contents

List of Figures

List of Tables

1 Introduction

For the course project for MSE 312 Mechatronics Design II, we were required to design a truss along with the required electronics to transport a 10g mass along a pre-specified arc. The objective of the project is to pick up the mass and move it from point to point in the shortest time possible. In the first half of the semester we were required to design, analyze, and construct the truss to have minimum defection at the end effector of the truss as well as minimum moment of inertia.

2 Design Approach

The design approach of the project is to brainstorm different truss designs, and then designed plus optimized through SolidWorks or other simulations software. The optimization process consists of lowering moment inertia and deflection. After the first step, a decision matrix would be used to compare the different models to decide the most suitable design for the project. After the final design is decided, the truss would be built and tested for quality.

- The design would have less mass further away from the pivot point (where the motor is mounted) to minimize moment of inertia
- The design would have minimal joints and utilize 7.5cm links to simplify the design and building process

2.1 Objectives

The objective of the mechanical component of the project is to minimize moment inertia for higher rotational acceleration and to optimize truss design for minimizing deflection, this will allow faster and accurate response time of the system.

2.2 Constraints

- Span of 30cm from motor mount plate to electromagnet mount plate
- The single link should be no longer than 7.5cm
- Truss should have at least 1 redundant member
- Max width of 10cm, max height of 5cm
- Overhang limit is 5cm
- Max clearance between electromagnet surface and object is 2mm
- The motor mount will have 3 threaded holes of 1/16”
- The electromagnet mount will have 1 threaded hole of 1/16”

3 Quantitative Analysis of Design Concepts

Table 1: Data From Solidworks Simulation

	Truss Design 1	Truss Design 2	Truss Design 3
Mass (g)	154.66	157.42	184.28
Moment inertia at the plate along the y-axis (gcm^4)	33833	37840	35474
Highest Stress(N/m^2),Input of 9.81N	13900000	13900000	7860000
Maximum Deflection (mm), Input of 9.81N	0.24mm	0.48mm	0.16mm

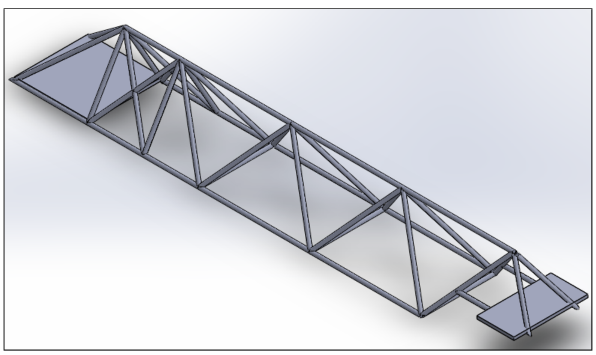


Figure 1: Truss 1

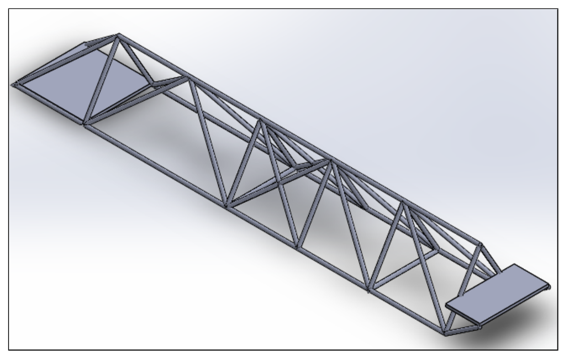


Figure 2: Truss 2

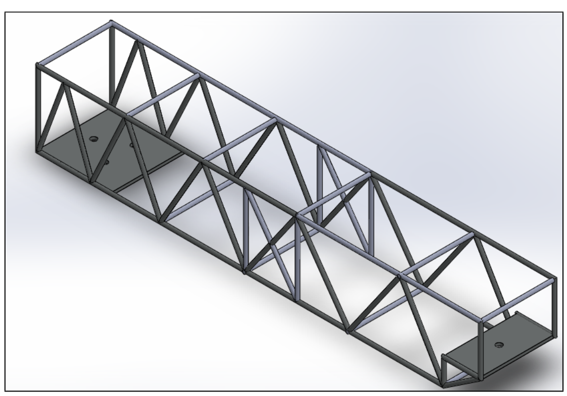


Figure 3: Truss 3

4 Evaluation of Designs

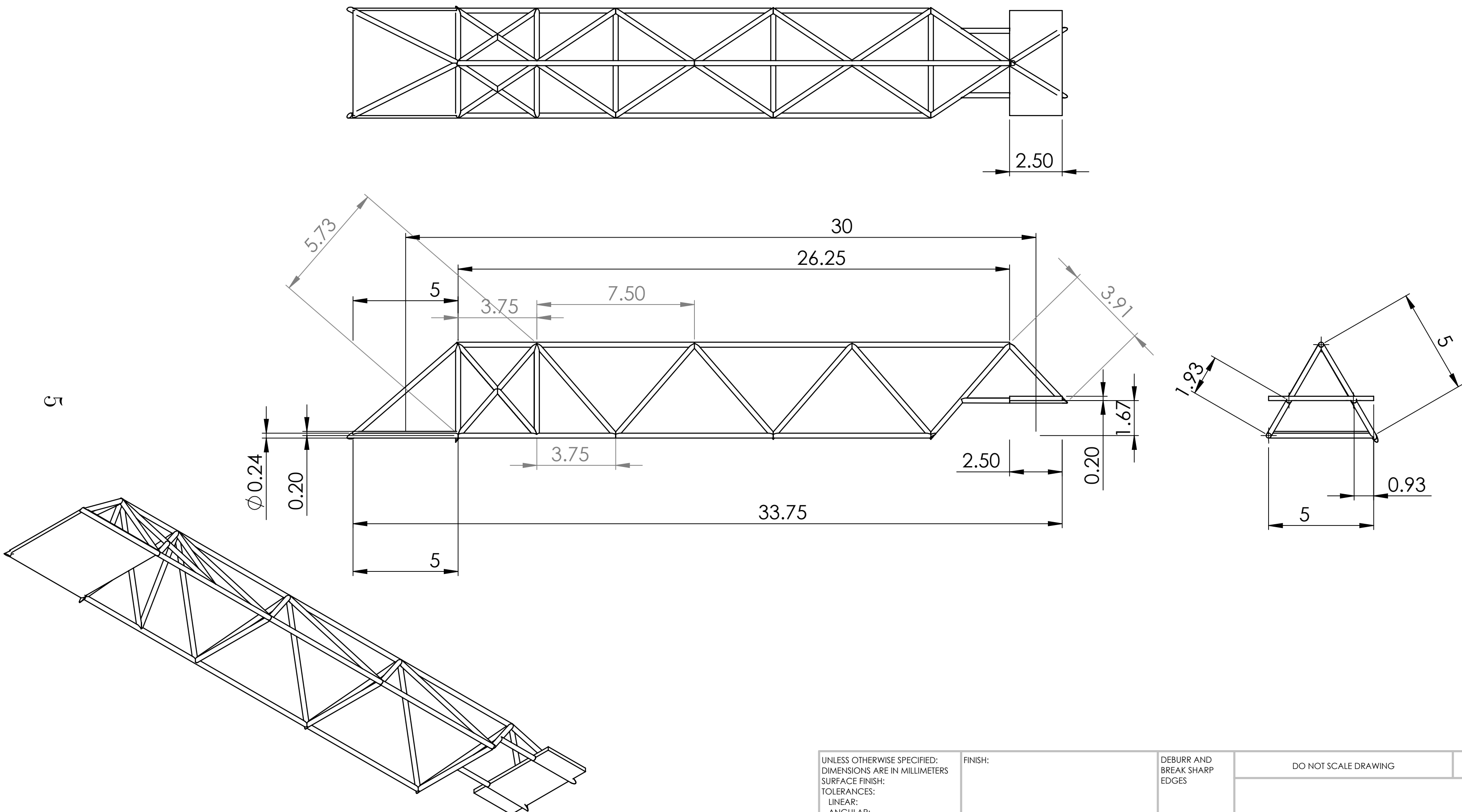
We ideally wish to have a design with least moment of inertia while having a high strength to weight ratio. This would ensure a faster response of the system while having reasonable strength. Furthermore , we compare the number of joints. Having less number of joints signifies less points of failure. Tension and compression forces were also compared to evaluate the maximum load the structure can withhold. Lastly, uniqueness and simplicity were accounted for project aesthetics

Table 2: Decision Matrix

		Truss 1		Truss 2		Truss 3	
Selection Criteria	Weight	Rating	Score	Rating	Score	Rating	Score
S/W ratio	30%	0.8	0.24	0.6	0.18	0.4	0.12
Weight	30%	0.7	0.21	0.6	0.18	0.3	0.09
Number of Joints	10%	0.8	0.08	0.7	0.21	0.4	0.04
High Compression	10%	0.7	0.07	0.5	0.05	0.8	0.08
High Tension	10%	0.8	0.08	0.6	0.06	0.5	0.05
Uniqueness	5%	0.7	0.035	0.6	0.03	0.4	0.02
Simplicity	5%	0.7	0.03	0.5	0.025	0.8	0.04
Total		0.75			0.73		0.43

From table 2, Even though truss 1 and truss 2 scored very closely, we decided to build truss 1 due to its relatively low forces in member and higher strength to weight ratio.

5 Engineerng Drawing



UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH:
TOLERANCES:
LINEAR:
ANGULAR:

FINISH:

DEBURR AND BREAK SHARP EDGES

DO NOT SCALE DRAWING

REVISION

	NAME	SIGNATURE	DATE
DRAWN			
CHK'D			
APPV'D			
MFG			
Q.A			

TITLE:

MATERIAL:

WEIGHT:

DWG NO.
Final Truss Design

A3

SCALE:1:2

SHEET 1 OF 1

6 Hand Calculations

To determine the force in the redundant member was done by the method of deflection using virtual work. Firstly, we separating the total systems in P0 system and P1 system as shown in 4. To simplify the calculations, we assumed that the length of the member where the force is applied is negligible and hence the load is concentrated at Joint 7. Furthermore, P0 system, member forces were calculated by applying 1N force at joint 7. Then we accounted for 60° inclination of the xy-plane by a factoring sin(60). Then we accounted for having two truss structures and scaled the member force by actual applied force F N as shown in 3.

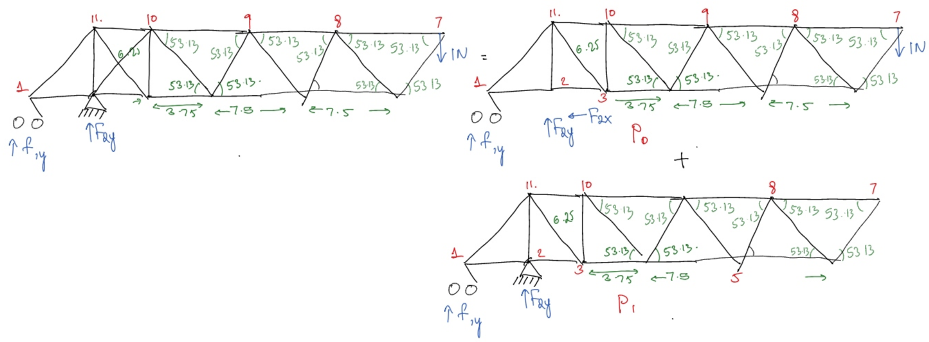


Figure 4: Truss Redundant Member Decomposition

Table 3: Scaled Force

Scaling Force	9.81
Angle Between Planes	60
Force in Y $(\sin\theta)$(Force)	8.49

Once we calculated virtual work in P0 system and P1 system, we calculated the force in the redundant member and the deformation of the redundant member as follows:

Table 4: Data for P_0 System

P_0 system						
Member	Length(m)	T_{e-i} (1N)	T_{e-i} (N) scaled	δ_i	T_{v-i}	W_i
1 2	0.05	-5.25	-22.3012	-1.11506	0	0
2 3	0.0375	-5.25	-22.3012	-0.8363	-0.6	0.501778
3 4	0.0375	-4.5	-19.1153	-0.71683	0	0
4 5	0.075	-3	-12.7436	-0.95577	0	0
5 6	0.075	-1.5	-6.37178	-0.47788	0	0
6 7	0.0625	-1.25	-5.30982	-0.33186	0	0
7 8	0.075	0.75	3.185891	0.238942	0	0
8 9	0.075	2.25	9.557673	0.716825	0	0
9 10	0.075	3.75	15.92945	1.194709	0	0
1011	0.0375	4.5	19.11535	0.716825	-0.6	-0.4301
1 11	0.0707	7.42	31.51908	2.228399	0	0
2 11	0.05	-6.25	-26.5491	-1.32745	-0.8	1.061964
3 11	0.0625	1.25	5.309818	0.331864	1	0.331864
3 10	0.05	-1	-4.24785	-0.21239	-0.8	0.169914
4 9	0.0625	-1.25	-5.30982	-0.33186	0	0
4 10	0.0625	1.25	5.309818	0.331864	0	0
5 8	0.0625	-1.25	-5.30982	-0.33186	0	0
5 9	0.0625	1.25	5.309818	0.331864	0	0
6 8	0.0625	1.25	5.309818	0.331864	0	0
2 10	0.0625	0	0	0	0	0
					Total	1.633542

Table 5: Data for P_1 System

P1 system						
Member	Length(mm)	Length(cm)	T_{e-i} (N)	δ_i	T_{v-i}(N)	W_i
1 2	5	0.05	0	0	0	0
2 3	3.75	0.0375	-0.6	-0.023	-0.6	0.014
3 4	3.75	0.0375	0	0	0	0
4 5	7.5	0.075	0	0	0	0
5 6	7.5	0.075	0	0	0	0
6 7	6.25	0.0625	0	0	0	0
7 8	7.5	0.075	0	0	0	0
8 9	7.5	0.075	0	0	0	0
9 10	7.5	0.075	0	0	0	0
10 11	3.75	0.0375	-0.6	-0.023	-0.6	0.014
1 11	7.07	0.0707	0	0	0	0
2 11	5	0.05	-0.8	-0.04	-0.8	0.032
3 11	6.25	0.0625	1	0.0625	1	0.063
3 10	5	0.05	-0.8	-0.04	-0.8	0.032
4 9	6.25	0.0625	0	0	0	0
4 10	6.25	0.0625	0	0	0	0
5 8	6.25	0.0625	0	0	0	0
5 9	6.25	0.0625	0	0	0	0
6 8	6.25	0.0625	0	0	0	0
2 10	6.25	0.0625	1	0.0625	1	0.063
					Total	0.216

$$
\begin{aligned}
W_e &= W_i \\
1 \times \delta_{p0} &= \frac{3.27}{AE}
\end{aligned} \tag{1}
$$

Similarly, we have:

$$
1 \times \delta_{p1} = \frac{0.216P}{AE} \tag{2}
$$

Equating 1 and 2 we can determine P:

$$
P = \frac{3.27}{0.216} = 15.14N \tag{3}
$$

$$
\delta = 7.34 \times 10^{-6} m \tag{4}
$$

From the total system data in table 7 we get:

$$
\delta_{system} = \sum W_i = 0.2578mm \tag{5}
$$

Table 6: System Summary

Force in Redundant Member	15.14281503
Displacement in Redundant Member	7.34447E-06
Displacement in Node 7	1.86294E-06
E	1E+11

Table 7: Total Data

Total system						
Member	Length(cm)	T_{e-i} (N)	δ_i	T_{v-i}(N)	W_i	Stress(N/m^2)
12	0.05	-44.60	-5E-06	-5.25	2.63E-05	-1.00E+07
23	0.0375	-53.69	-4.5E-06	-5.25	2.37E-05	-1.21E+07
34	0.0375	-38.23	-3.2E-06	-4.5	1.45E-05	-8.58E+06
45	0.075	-25.49	-4.3E-06	-3	1.29E-05	-5.72E+06
56	0.075	-12.74	-2.1E-06	-1.5	3.22E-06	-2.86E+06
67	0.0625	-10.62	-1.5E-06	-1.25	1.86E-06	-2.38E+06
78	0.075	6.37	1.07E-06	0.75	8.05E-07	1.43E+06
89	0.075	19.12	3.22E-06	2.25	7.24E-06	4.29E+06
910	0.075	31.86	5.37E-06	3.75	2.01E-05	7.15E+06
1011	0.0375	29.15	2.45E-06	4.5	1.1E-05	6.54E+06
111	0.0707	63.04	1E-05	7.42	7.43E-05	1.42E+07
211	0.05	-65.21	-7.3E-06	-6.25	4.58E-05	-1.46E+07
311	0.0625	25.76	3.62E-06	1.25	4.52E-06	5.78E+06
310	0.05	-20.61	-2.3E-06	-1	2.31E-06	-4.63E+06
49	0.0625	-10.62	-1.5E-06	-1.25	1.86E-06	-2.38E+06
410	0.0625	10.62	1.49E-06	1.25	1.86E-06	2.38E+06
58	0.0625	-10.62	-1.5E-06	-1.25	1.86E-06	-2.38E+06
59	0.0625	10.62	1.49E-06	1.25	1.86E-06	2.38E+06
68	0.0625	10.62	1.49E-06	1.25	1.86E-06	2.38E+06
210	0.0625	15.14	2.13E-06	0	0	3.40E+06
				Total	0.000258	

7 Buckling Analysis

When a beam is under compressive loading it might deflect sideways. The force required to cause buckling is given by equation 6.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{6}$$

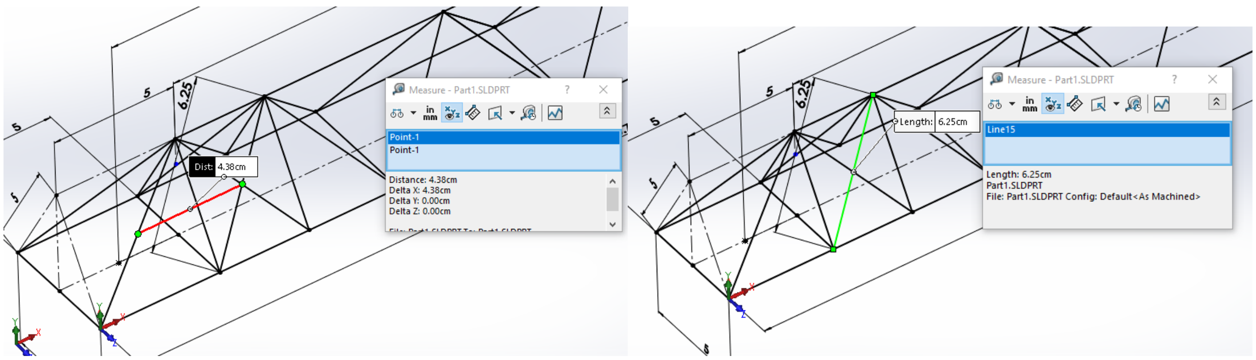
As the equation shows, the buckling force is dependent on the length/inertia of the beam and the Youngs's modulus of the material. The critical buckling force for all beams under compression can be seen in table 8.

Table 8: Critical Buckling Force

Member	Length	Te-I (N)	Volume	Mass(kg)	Inertia	Pcr(N)
1 2	0.05	-4.46E+01	8.9E-07	0.00778	2.5E-11	9969.41
2 3	0.0375	-5.37E+01	6.7E-07	0.00583		17723.4
3 4	0.0375	-3.82E+01	6.7E-07	0.00583		17723.4
4 5	0.075	-2.55E+01	1.3E-06	0.01166		4430.85
5 6	0.075	-1.27E+01	1.3E-06	0.01166		4430.85
6 7	0.0625	-1.06E+01	1.1E-06	0.00972		6380.42
7 8	0.075	6.37E+00	1.3E-06	0.01166		
8 9	0.075	1.91E+01	1.3E-06	0.01166		
9 10	0.075	3.19E+01	1.3E-06	0.01166		
10 11	0.0375	2.91E+01	6.7E-07	0.00583		
1 11	0.0707	6.30E+01	1.3E-06	0.011		
2 11	0.05	-6.52E+01	8.9E-07	0.00778		9969.41
3 11	0.0625	2.58E+01	1.1E-06	0.00972		
3 10	0.05	-2.06E+01	8.9E-07	0.00778		9969.41
4 9	0.0625	-1.06E+01	1.1E-06	0.00972		6380.42
4 10	0.0625	1.06E+01	1.1E-06	0.00972		
5 8	0.0625	-1.06E+01	1.1E-06	0.00972		6380.42
5 9	0.0625	1.06E+01	1.1E-06	0.00972		
6 8	0.0625	1.06E+01	1.1E-06	0.00972		
2 10	0.0625	1.51E+01	1.1E-06	0.00972		

8 Moment of Inertia

Moment of inertia is calculated on Excel because it can auto-generate values based on the given equations. To show the process of calculation, on of the truss beam is selected for demonstration.



(a) (b)

Figure 5: Length of Beams

$$I = md^2 + \frac{1}{2}L^2 sin^2\theta \tag{7}$$

$$m = \rho V = \rho \pi r^2 h \tag{8}$$

$$\begin{aligned} I &= md^2 + \frac{1}{12}L^2 sin\theta^2 \\ &= 0.00235 \times 4.38^2 + \frac{1}{12}\frac{6.25}{100}^2 sin(43.85)^2 \\ &= 1.01658 \times 10^{-5} \end{aligned}$$

Table 9: Inertia of System

Mass(m)	Distance from Base(cm)	Length(mm)	Angle(θ	Inertia
0.009430768	12.56	25.12	90	4.95913E-05
0.001329018	1.25	3.54	90	6.92197E-08
0.002654281	0	7.07	37.76	1.54236E-06
0.001877143	2.5	5	30	0.00020455
0.002346429	4.38	6.25	43.85	1.01658E-05
0.002346429	4.38	6.25	43.85	1.01658E-05
0.001877143	6.25	5	30	0.000210709
0.002346429	8.13	6.25	43.85	2.11734E-05
0.002346429	11.88	6.25	43.85	3.87805E-05
0.002346429	15.63	6.25	43.85	6.29868E-05
0.002346429	19.38	6.25	43.85	9.37925E-05
0.002346429	23.13	6.25	43.85	0.000131198
0.002346429	26.88	6.25	43.85	0.000175202
0.001543012	28.75	4.11	65.99	0.000127578
0.001370315	30	3.65	90	4.11094E-05
0.000938572	28.75	2.5	46.78	8.34992E-05
0.001659395	30	4.42	36.95	0.000224845
0.009855003	13.125	26.25	90	5.65893E-05
0.001877143	10	5	90	0.000185278
0.001877143	17.5	5	90	0.000223994
0.001877143	25	5	90	0.000283828
0.04215	2.5	5	0	1.75625E-05
0.021075	30	2.5	0	0.001905531
			total sum	0.004159739

9 Comparison of Data Acquired

The deflection FEA from SolidWork and hand calculations had no more than 8% difference for both the 0.5kg and 1kg. However, the theoretical values differed from the experimental values by 400% for the 1kg weight test. The discrepancies for the deflection value is due to imperfect soldiering on the joints, and human error such as measuring accuracy when aligning the brass rod. The experimental measurement may also be off to increase the discrepancies. Although the 1kg experiment had a 0.75mm difference from the theoretical value, the error is minimal from the equipment and resource given.

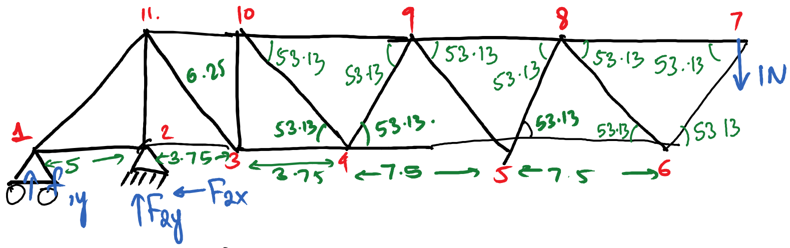
Table 10: Comparison of Data

	0.5kg Weight	1kg Weight
SolidWorks FEA Deflection (mm)	0.12	0.24
Hand Calculation Deflection (mm)	0.13	0.25
Experimental Deflection (mm)	0	1

10 Conclusion

The truss has shown great result both in simulation as well as actual testing. With the weight of 158g and applying 1 kg of weight that the end of the truss, the deflection of 1 mm is the result. The chosen design is the less heavy among the truss design that we have. Decision matrix indicated that the selected design can achieve the greatest overall result. With the other two design, although having lesser value on deflection, is not a wholesome design due to heavier weight, thus require more material. Triangular truss design has the advantage of obtaining lesser material thus lighter weight when compared with rectangular shaped design. One significant disadvantage of triangular design is the complexity in weldment procedure. The Triangular truss requires more angular weldments which are difficult to achieve. Hand welding in the lab setting is not ideal for achieving great weldment result due to limitation of tools and space. The lack of precision in cuts is also an issue that may cause error in experimental result. Nevertheless, our design outcomes in great performance for mechanical portion of the project.

11 Appendix



$A = 4.453 \times 10^{-6}$
$E = 120 \times 10^{9}$

Reaction forces

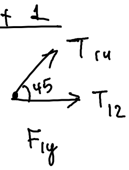
$\circlearrowleft + \sum M_1 = 0$
$\therefore F_{2y}(5) = 1(31.25)$
$\boxed{\therefore F_{2y} = 6.25N}$

$+\uparrow \sum F_y = 0$
$\therefore F_{1y} + F_{2y} = 1N$
$\therefore F_{1y} = 1 - 6.25$
$\boxed{F_{1y} = -5.25N}$

$\xrightarrow{+} \sum F_x = 0$
$\boxed{\therefore F_{2x} = 0N}$

Joint Method for truss Analysis

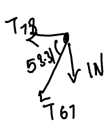
Joint 1



$T_{11} \sin 45 = F_{1y}$
$\therefore T_{11} = \frac{F_{1y}}{\sin 45}$
$\boxed{T_{11} = 7.42N}$

$T_{12} = -T_{11} \cos 45$
$\boxed{T_{12} = -5.25N}$

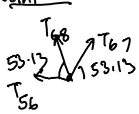
Joint 7



$T_{67} \sin 53.13 = -1$
$\therefore T_{67} = \frac{-1}{\sin 53.13}$
$\boxed{T_{67} = -1.25N}$

$T_{78} = -T_{67} \cos 53.13$
$\therefore T_{78} = 1.25 \cos 53.13$
$\boxed{\therefore T_{78} = 0.75N}$

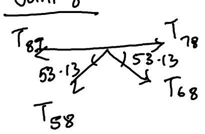
Joint 6



$T_{67} \sin 53.13 + T_{68} \sin 53.13 = 0$
$\therefore T_{67} = -T_{68}$
$\therefore \boxed{T_{68} = 1.25N}$

$T_{67} \cos 53.13 - T_{68} \cos 53.13 - T_{56} = 0$
$\therefore T_{56} = (\cos 53.13)(T_{67} - T_{68})$
$\therefore T_{56} = (\cos 53.13)(-1.25 - 1.25)$
$\boxed{\therefore T_{56} = -1.5N}$

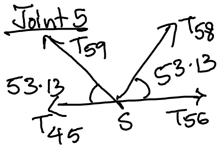
Joint 8



$T_{58} \sin 53.13 = -T_{68} \sin 53.13$
$\therefore T_{58} = -T_{68}$
$\boxed{\therefore T_{58} = -1.25N}$

$-T_{89} - T_{58} \cos 53.13 + T_{68} \cos 53.13 + T_{78} = 0$
$T_{89} = -T_{58} \cos 53.13 + T_{68} \cos 53.13 + T_{78}$
$= 1.25 \cos 53.13 + 1.25 \cos 53.13 + 0.75$
$\boxed{T_{89} = 2.25N}$

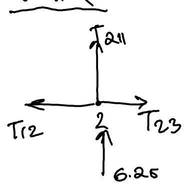
Joint 5



$T_{59} = -T_{58}$
$\boxed{\therefore T_{59} = 1.25N}$

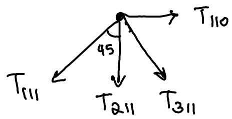
$T_{45} = -T_{59} \cos 53.13 + T_{58} \cos 53.1 + T_{56}$
$= -1.25 \cos 53.13 - 1.25 \cos 53.13 - 1.5$
$\boxed{T_{45} = -3N}$

Joint 2



$\boxed{T_{211} = -6.25N}$
$\boxed{T_{23} = -5.25N}$

Joint 11



$-T_{211} - T_{111}\cos 45 - T_{311}\sin 53.13 = 0$

$\therefore 6.25 - 7.42\cos 45 - T_{311}\sin 53.13 = 0$

$\therefore T_{311}\sin 53.13 = 6.25 - 7.42\cos 45$

$T_{311} = \dfrac{1}{\sin 53.13}$

$\boxed{T_{311} = 1.25N}$

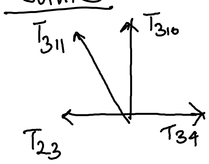
$-T_{111}\cos 45 + T_{1110} + T_{311}\cos 53.13 = 0$

$\therefore T_{1110} = -T_{311}\cos 53.13 + T_{111}\cos 45$

$= -1.25\cos 53.13 + 7.42\cos 45$

$\boxed{T_{1110} = 4.5N}$

Joint 3



$T_{310} = -T_{311}\sin 53.13$

$\boxed{\therefore T_{310} = -1N}$

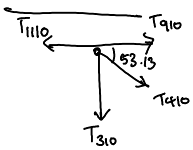
$-T_{23} + T_{34} - T_{311}\cos 53.13 = 0$

$\therefore T_{34} = T_{23} + T_{311}\cos 53.13$

$\therefore T_{34} = -5.25 + 1.25\cos 53.13$

$\boxed{\therefore T_{34} = -4.5N}$

Joint 10



$T_{410} = -\dfrac{T_{310}}{\sin 53.13}$

$\boxed{T_{410} = 1.25N}$

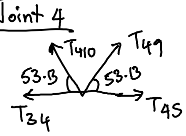
$T_{910} - T_{1110} + T_{410}\cos 53.13 = 0$

$\therefore T_{910} = T_{1110} - T_{410}\cos 53.13$

$T_{910} = 4.5 - 1.25\cos 53.13$

$\boxed{T_{910} = 3.75N}$

Joint 4



$T_{49} = -T_{410}$

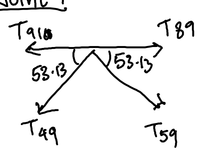
$\boxed{\therefore T_{49} = -1.25N}$

$T_{49}\cos 53.13 + T_{45} - T_{34} - T_{410}\cos 53.13 = 0$

$\therefore -1.25\cos 53.13 - 3 + 4.5 - 1.25\cos 53.13$

$= 0$ ← check

Joint 9

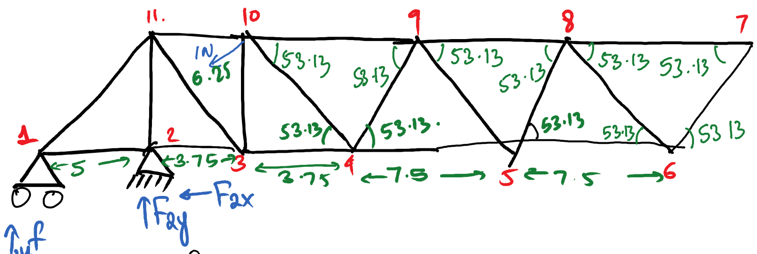


$T_{89} - T_{910} - T_{49}\cos 53.13 + T_{59}\cos 53.13 = 0$

$\therefore T_{89} = T_{910} + T_{49}\cos 53.13 - T_{59}\cos 53.13$

$\therefore T_{89} = 3.75 - 1.25\cos 53.13 - 1.25\cos 53.13$

$\boxed{T_{89} = 2.25N}$ ← check



Reaction forces

$\circlearrowleft \sum M_1 = 0$

$\therefore F_{2y}(5) + (\cos 53.13)(5) - \sin 53.13\,(8.75) = 0$

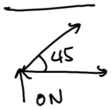
$\boxed{F_{2y} = 0.8\text{ N}}$

$F_{2y} + F_{1y} = 0.8\text{ N}$

$\boxed{\therefore F_{1y} = 0\text{ N}}$

$\boxed{F_{2x} = 0.6\text{ N}}$

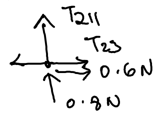
Joint 1



$\boxed{\therefore T_{11} = 0}$

$\boxed{T_{12} = 0}$

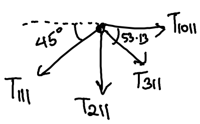
Joint 2



$\boxed{T_{211} = -0.8\text{N}}$

$\boxed{T_{23} = -0.6\text{N}}$

Joint 11



$-T_{211} - T_{311}\sin 53.13 = 0$

$\therefore T_{211} = -T_{311}\sin 53.13$

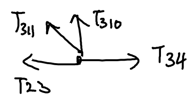
$\therefore T_{311} = -\dfrac{T_{211}}{\sin 53.13}$

$\boxed{T_{311} = 1\text{N}}$

$-T_{311}\cos 53.13 = T_{1011}$

$\boxed{\therefore T_{1011} = -0.6\text{ N}}$

Joint 3



$T_{310} = -T_{311}\sin 53.13$

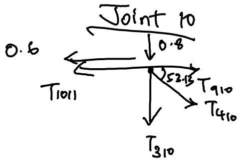
$\boxed{\therefore T_{310} = -0.8\text{N}}$

$T_{34} = T_{311}\cos 53.13 + T_{23}$

$= 0.6 + (-0.6)$

$\boxed{T_{34} = 0\text{ N}}$

Joint 10



$-0.8 - T_{410}\sin 53.13 - T_{310} = 0$

$T_{410}\sin 53.13 = -0.8 + 0.8$

$\boxed{T_{410} = 0\text{ N}}$

$T_{910} = T_{1011} + 0.6$

$\boxed{T_{910} = 0\text{N}}$

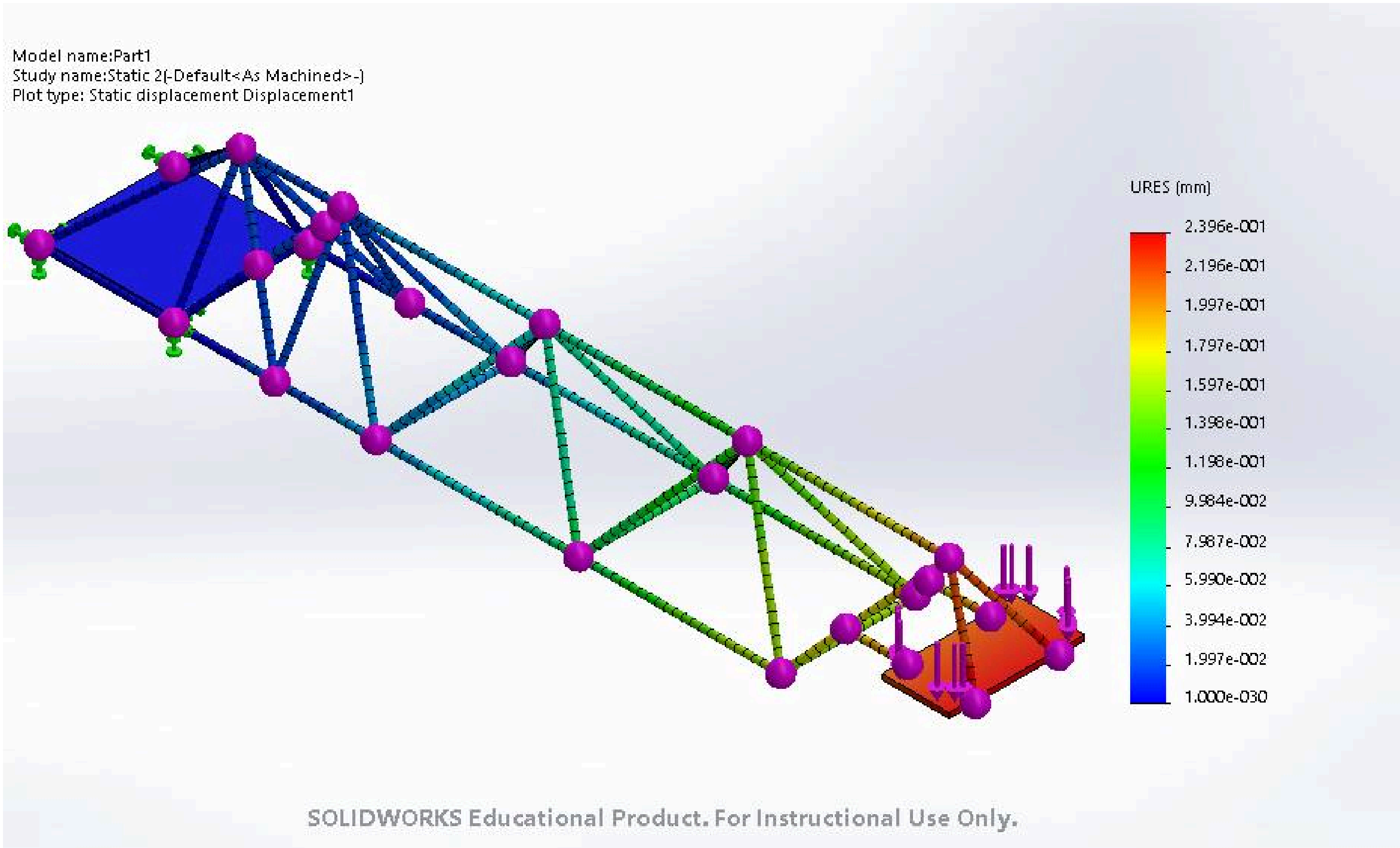


Figure 6: Deflection of Final Design

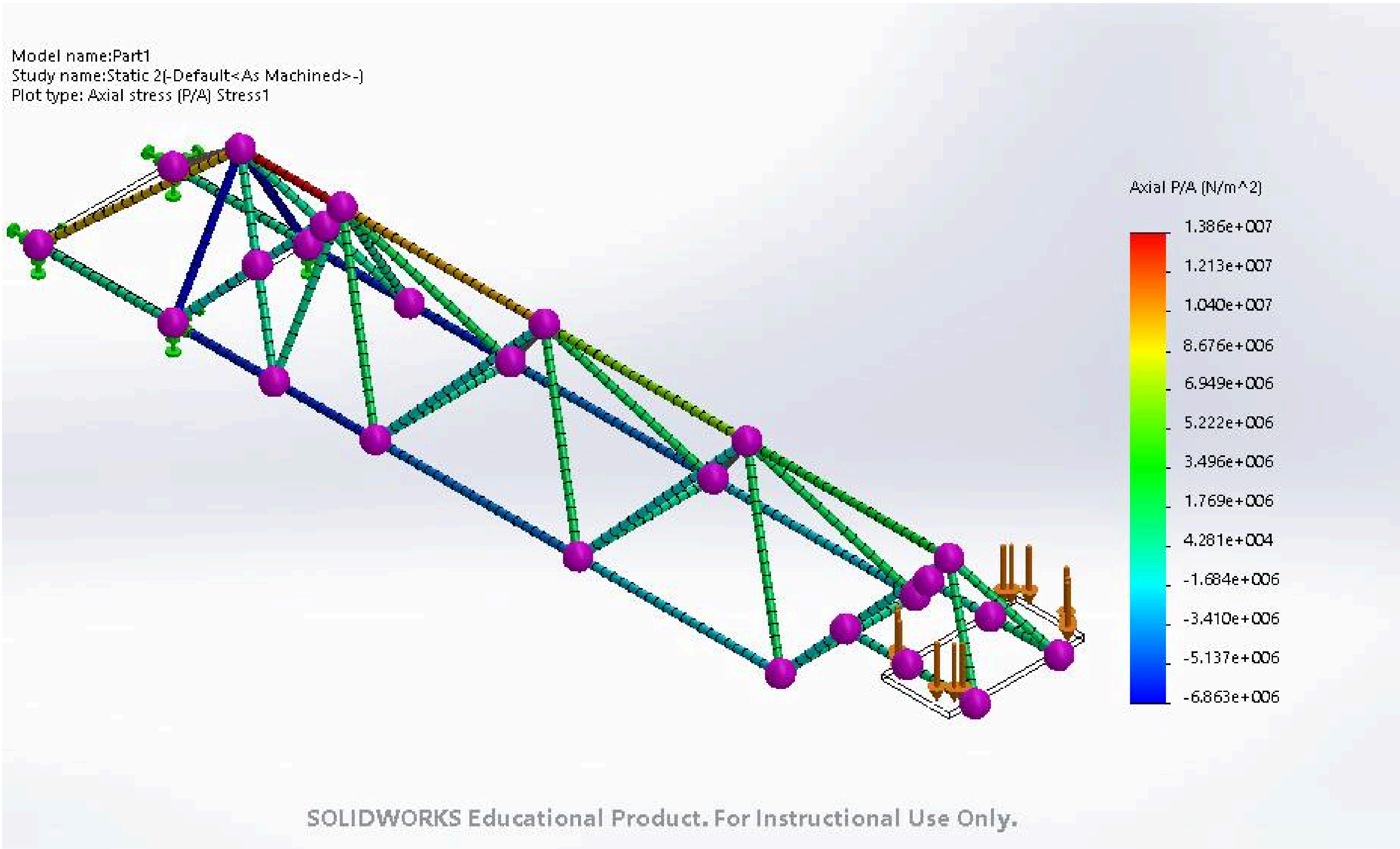


Figure 7: Stress of Final Design