Parametric higher-order shelving filters

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ABSTRACT

The main characteristic of shelving filters, as commonly used in audio equalization, is to amplify or attenuate a certain frequency band by a given gain. For parametric equalizers, a filter structure is desirable that allows independent adjustment of the width and center frequency of the band, and the gain. In this paper, we present a design for arbitrary-order shelving filters and a suitable parametric digital filter structure. A low-shelving filter design based on Butterworth filters is decomposed such that the gain can be easily adjusted. Transformation to the digital domain is performed, keeping gain and denormalized cut-off frequency independently controllable. Finally, we obtain band- and high-shelving filters using a simple manipulation, providing the desired parametric filter structure.

1. INTRODUCTION

For audio equalization, recursive shelving filters are commonly employed. Their function is to amplify or attenuate a specific frequency band, while leaving signal components outside that band mainly unchanged. For equalizers which can be tuned during operation, special parametric filter structures are desirable which allow for easy and independent control of the respective band's width and center frequency and the gain in that band.

For first-order low- and high-shelving filters and second-order band-shelving filters, such designs based on an all-pass decompositions were presented in [1, 2]. In [3] a decomposition of second-order low- and high-shelving filters and fourth-order band-shelving filters was presented that allows the gain to be adjusted without recomputation of the other filter coefficients, but where the band limits cannot be easily adjusted.

In the following, we will first derive arbitrary order low-shelving filters in the continuous-time domain based on a Butterworth design. These designs will then be transformed to the discrete-time domain and a parametric realization will be presented. Finally, by low-pass/high-pass and low-pass/band-pass transformations, high-shelving and band-shelving filters will be obtained.

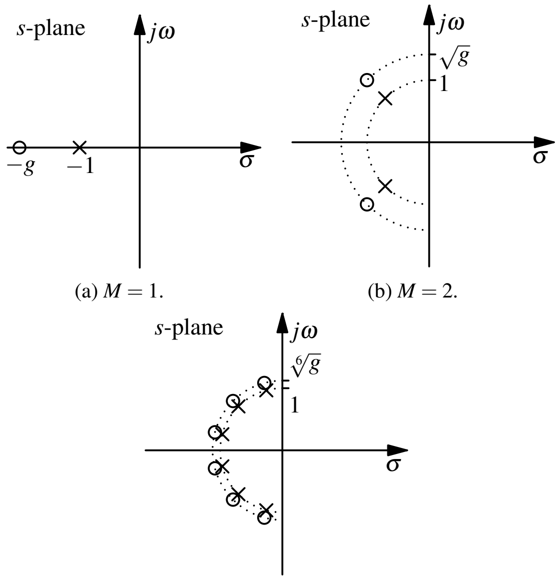
2. CONTINUOUS-TIME LOW-SHELVING FILTERS

In [4] the transfer function of frequency-normalized first- and second-order low-shelving filters with gain g are given by

$$H_{\mathrm{LS},1}(s) = \frac{s+g}{s+1} \tag{1}$$

and

$$H_{\mathrm{LS},2}(s) = \frac{s^2+\sqrt{2g}s+g}{s^2+\sqrt{2}s+1}, \tag{2}$$



(a) $M=1$. (b) $M=2$. (c) $M=6$.

Figure 1: Pole/zero locations in the s-plane of low-shelving filters with $g=2$ for different filter orders M.

respectively. The pole and zero locations of these filters are shown in Fig. 1 (a) and (b). These can be generalized to filters of order M as

$$H_{\mathrm{LS},M}(s) = \prod_{m=1}^{M} \frac{s+\sqrt[M]{g}e^{j\alpha_m}}{s+e^{j\alpha_m}}, \quad \alpha_m = \left(\frac{1}{2}-\frac{2m-1}{2M}\right)\pi \tag{3}$$

of which (1) and (2) are special cases. Fig. 1 (c) gives an example of the resulting pole and zero locations for $M=6$.

2.1 Filter properties

The magnitude response of the filters according to (3) can be found to be

$$\left|H_{\mathrm{LS},M}(j\omega)\right|^2 = \frac{\omega^{2M}+g^2}{\omega^{2M}+1},$$

of which examples are depicted in Fig. 2. As can be easily seen, $\left|H_{\mathrm{LS},M}(0)\right| = g$ and $\left|H_{\mathrm{LS},M}(\infty)\right| = 1$, with a transitional region around $\omega = 1$. As desired, the slope in the transitional region increases with the filter order M. At the normalized cut-off frequency $\omega_c = 1$, the magnitude response is

$$\left|H_{\mathrm{LS},M}(j\omega_c)\right|^2 = \frac{g^2+1}{2},$$

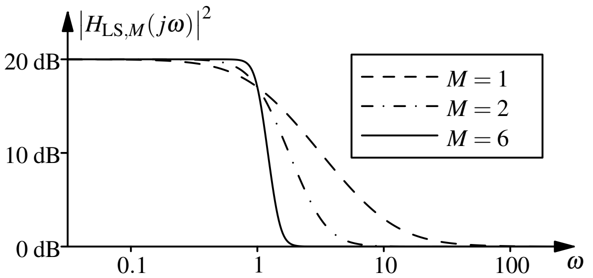


Figure 2: Magnitude responses for continuous-time low-shelving filters of different orders M with gain $g = 10$ (20 dB).

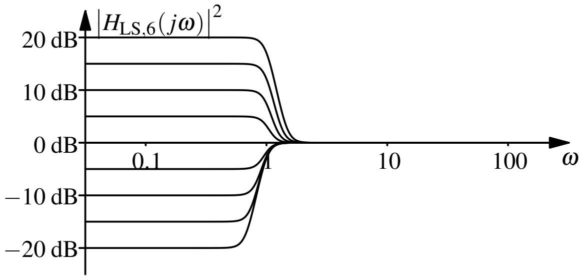


Figure 3: Magnitude responses for continuous-time low-shelving filters of order $M = 6$ and varying gain g.

i.e. approximately 3 dB below the maximum. It should be noted that this yields an asymmetry between amplification ($g > 1$) and attenuation ($g < 1$), see Fig. 3.

By construction, the filters have minimum-phase behavior. This can be exploited in several equalization applications, for example when input-to-output latency is a concern.

2.2 Parametric representation

To develop a filter structure later on where the parameters gain, center-frequency and bandwidth are decoupled and can be adjusted independently, we will first develop a representation of the low-shelving filters where the gain can be easily manipulated. We start by rewriting the complex-valued first-order sections of (3),

$$H_{\mathrm{LS},M}^{(m)}(s) = \frac{s + \sqrt[M]{g}e^{j\alpha_m}}{s + e^{j\alpha_m}},$$

as

$$H_{\mathrm{LS},M}^{(m)}(s) = 1 + V\frac{e^{j\alpha_m}}{s + e^{j\alpha_m}},\ V = \sqrt[M]{g} - 1.$$

Combining $H_{\mathrm{LS},M}^{(m)}$ and $H_{\mathrm{LS},M}^{(M+1-m)}$, having complex conjugate poles and zeros as $e^{j\alpha_m} = e^{-j\alpha_{M+1-m}}$, yields the parametric real-valued second-order section

$$\begin{aligned}\bar{H}_{\mathrm{LS},M}^{(m)}(s) &= H_{\mathrm{LS},M}^{(m)}(s) \cdot H_{\mathrm{LS},M}^{(M+1-m)}(s) \\ &= 1 + 2V\frac{1 + c_m s}{s^2 + 2c_m s + 1} + V^2\frac{1}{s^2 + 2c_m s + 1}\end{aligned} \tag{4}$$

with $c_m = \cos(\alpha_m)$. If M is odd, (3) furthermore has one real-valued first-order section

$$H_{\mathrm{LS},M}^{(\frac{M+1}{2})}(s) = \frac{s + \sqrt[M]{g}}{s+1} = 1 + V\frac{1}{s+1}. \tag{5}$$

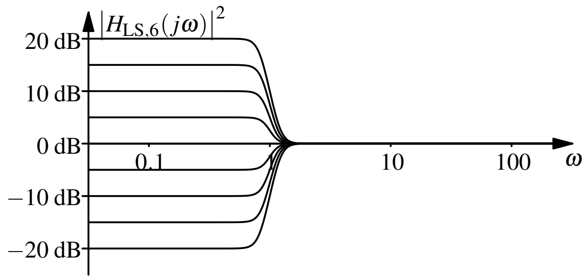


Figure 4: Magnitude responses for continuous-time low-shelving filters of order $M = 6$ and varying gain g, alternative design for gain symmetry.

Thus, the low-shelving filter of (3) can be rewritten in terms of parametric real-valued first- and second-order sections according to (5) and (4) as

$$H_{\mathrm{LS},M} = \underbrace{H_{\mathrm{LS},M}^{(\frac{M+1}{2})}(s)}_{\text{for odd } M \text{ only}} \cdot \prod_{m=1}^{\lfloor \frac{M}{2} \rfloor} \bar{H}_{\mathrm{LS},M}^{(m)}(s).$$

2.3 Design alternatives

The filter defined by (3) can in fact be understood as the combination of a Butterworth low-pass with normalized cut-off frequency 1 and the inverse of a Butterworth low-pass with cut-off frequency $\sqrt[M]{g}$. In a similar manner, shelving filters based on Chebyshev and elliptic filters can be designed. As expected, their magnitude responses have steeper roll-off, but oscillation inside and/or outside the amplified/attenuated band [5].

The asymmetry of the magnitude responses between amplification and attenuation (see Fig. 3) can be avoided by using

$$H_{\hat{\mathrm{LS}},M}(s) = \prod_{m=1}^{M} \frac{s + \sqrt[2M]{g}e^{j\alpha_m}}{s + \frac{1}{\sqrt[2M]{g}}e^{j\alpha_m}}$$

instead of (3), yielding the magnitude responses of Fig. 4. In fact, for this design, taking the reciprocal gain is equivalent to inverting the filter. However, this approach does not lend itself to the decoupling as performed above, so we stay with the design of (3).

3. DISCRETE-TIME REALIZATION

To obtain discrete-time filters, we simply apply a bilinear transformation [6]. In the same step, we also denormalize the cut-off frequency, so that we arrive at the transformation

$$s = \frac{1}{K}\frac{1 - z^{-1}}{1 + z^{-1}}, \quad K = \tan\left(\frac{\Omega_B}{2}\right), \tag{6}$$

where Ω_B denotes the filter bandwidth in radians per sample.

Substituting (6) in (4) and (5) gives

$$\begin{aligned}&\bar{H}_{\mathrm{LS},M}^{(m)}(z) = \\ &\quad 1 + 2VK\frac{K + c_m + 2Kz^{-1} + (K - c_m)z^{-2}}{1 + 2Kc_m + K^2 + (2K^2 - 2)z^{-1} + (1 - 2Kc_m + K^2)z^{-2}} \\ &\quad + V^2K^2\frac{1 + 2z^{-1} + z^{-2}}{1 + 2Kc_m + K^2 + (2K^2 - 2)z^{-1} + (1 - 2Kc_m + K^2)z^{-2}}\end{aligned} \tag{7}$$

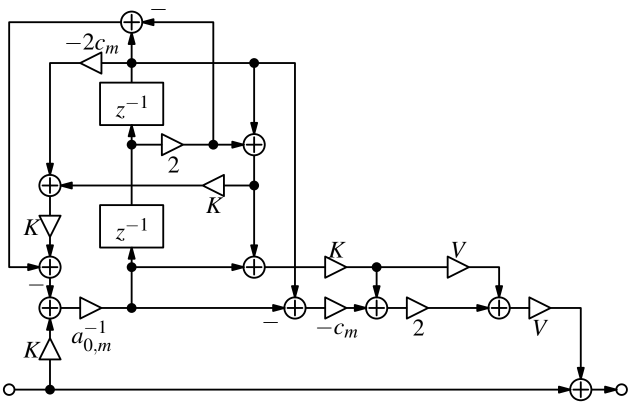


Figure 5: Parametric realization of a second-order section $\tilde{H}_{\mathrm{LS},M}^{(m)}(z)$.

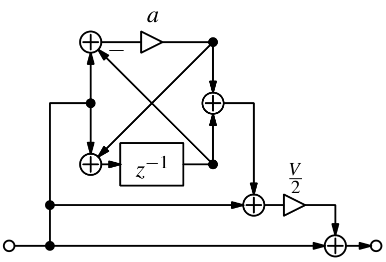


Figure 6: Parametric realization of a first-order section $H_{\mathrm{LS},M}^{\left(\frac{M+1}{2}\right)}(z)$.

and

$$H_{\mathrm{LS},M}^{\left(\frac{M+1}{2}\right)}(z) = 1 + VK\frac{1+z^{-1}}{1+K+(K-1)z^{-1}}, \tag{8}$$

respectively.

Realization of (7) with two parallel second-order systems, such that these sub-systems only depend on K and hence the cut-off frequency and are added to a direct-path using weights that only depend on V and hence the gain, is straight-forward. But as the two non-trivial summands of (7) have the same denominator, they can share the feed-back part of the filter structure and only require distinct feed-forward parts. By further modifying the filter structure to reduce the number of operations required, the realization of Fig. 5 can be derived, where

$$a_{0,m}^{-1} = \frac{1}{1+2Kc_m+K^2}.$$

The first-order section of (8) actually is a standard low-shelving filter and can be realized with any of the known approaches. For convenience, we reproduce the design presented in [2] which uses the all-pass decomposition

$$H_{\mathrm{LS},M}^{\left(\frac{M+1}{2}\right)}(z) = 1 + \frac{V}{2}\cdot\left(1+\frac{a+z^{-1}}{1+az^{-1}}\right), \quad a = \frac{K-1}{K+1}$$

yielding the realization shown in Fig. 6.

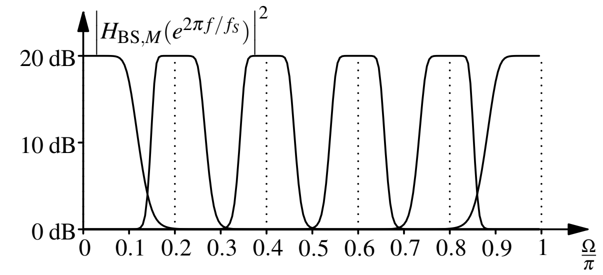


Figure 7: Magnitude responses of band-shelving filters with order $M = 6$, gain $g = 10$, bandwidth $\Omega_B = 0.1\pi$ and center frequencies $\Omega_0 = [0, 0.2\pi, 0.4\pi, 0.6\pi, 0.8\pi, \pi]$.

4. HIGH- AND BAND-SHELVING FILTERS

From the low-shelving filters, high- and band-shelving filters can easily be obtained as

$$H_{\mathrm{HS},M}(z) = H_{\mathrm{LS},M}(-z)$$

and

$$H_{\mathrm{BS},M}(z) = H_{\mathrm{LS},M}\left(z\frac{c_0-z}{1-c_0z}\right), \quad c_0 = \cos(\Omega_0),$$

where Ω_0 is the desired center-frequency of the band-shelving filter [7]. The resulting magnitude response is given by

$$\left|H_{\mathrm{BS},M}\left(e^{j\Omega}\right)\right|^2 = \frac{(c_0-\cos\Omega)^{2M}+(K\sin\Omega)^{2M}g^2}{(c_0-\cos\Omega)^{2M}+(K\sin\Omega)^{2M}},$$

as depicted in Fig. 7 for various center frequencies.

The center frequency here specifies the frequency at which the maximum gain is reached. This generally is not the center of the filter's active band in the sense of being the arithmetic or geometric mean of the band edges. Instead,

$$\tan^2\left(\frac{\Omega_0}{2}\right) = \tan\left(\frac{\Omega_{c,1}}{2}\right)\tan\left(\frac{\Omega_{c,2}}{2}\right)$$

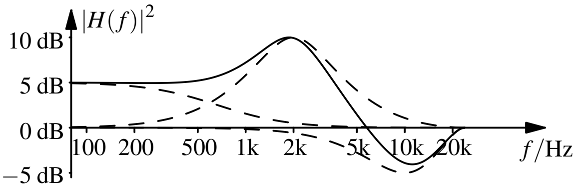
where $\Omega_{c,i}$ are the band edges where $\left|H_{\mathrm{BS},M}\left(e^{j\Omega_{c,i}}\right)\right|^2 = \frac{g^2+1}{2}$.

Especially, for $\Omega_0 \to 0$, the band-shelving filter becomes a low-shelving filter, and thus Ω_0 is at the lower end of the active band, and likewise for $\Omega_0 \to \pi$, the filter becomes a high-shelving filter with Ω_0 at the upper end of the active band.

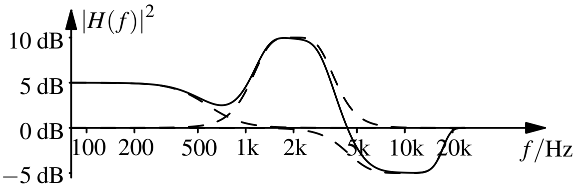
To retain the decoupling of the filter parameters, no new coefficients depending on Ω_0 are computed, but instead, the unit delays of Fig. 5 and 6 are replaced with an all-pass as

$$z^{-1} \leftarrow A(z) = z^{-1}\frac{c_0-z^{-1}}{1-c_0z^{-1}}. \tag{9}$$

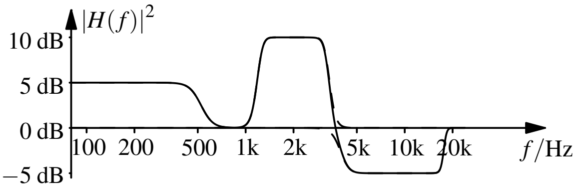
Note that (9) has the proper limits, i.e. $A(z) = z^{-1}$ for $\Omega_0 = 0$ and $A(z) = -z^{-1}$ for $\Omega_0 = \pi$, so that the same implementation may be used for low-, band- and high-shelving filters, provided the realization of the frequency shifting all-pass is numerically stable for $|c_0| = 1$.



(a) $M = 1$.



(b) $M = 2$.



(c) $M = 6$.

Figure 8: Design example results for different filter orders.

5. DESIGN EXAMPLE

Let us consider a digital parametric three-band equalizer operating at a sampling rate of 48 kHz. The equalizer is realized as a cascade of shelving filters as presented in this paper. The parameters of the three bands are set to

$$\begin{aligned} f_{0,1} &= 0\,\text{Hz}, & f_{B,1} &= 500\,\text{Hz}, & G_1 &= 5\,\text{dB}, \\ f_{0,2} &= 2\,\text{kHz}, & f_{B,2} &= 2\,\text{kHz}, & G_2 &= 10\,\text{dB}, \\ f_{0,3} &= 10\,\text{kHz}, & f_{B,3} &= 14\,\text{kHz}, & G_3 &= -5\,\text{dB}. \end{aligned}$$

We shall compare realizations utilizing filter orders $M = 1, 2$ and 6. (Due the frequency shifting all-pass, the effective filter order of course is $2M$.) The frequency-dependent coefficients execpt for the $a_{0,m}^{-1}$ are the same across filter orders and can be computed to be

$$\begin{aligned} K_1 &= 0.032737, & c_{0,1} &= 1.00000, \\ K_2 &= 0.131652, & c_{0,2} &= 0.96593, \\ K_3 &= 1.303225, & c_{0,3} &= 0.25882. \end{aligned}$$

Depending on the filter order, we furthermore find the gain-dependent $V_{band,M}$ as

$$\begin{aligned} V_{1,1} &= 0.77828, & V_{1,2} &= 0.33352, & V_{1,6} &= 0.10069, \\ V_{2,1} &= 2.16228, & V_{2,2} &= 0.77828, & V_{2,6} &= 0.21153, \\ V_{3,1} &= -0.43766, & V_{3,2} &= -0.25011, & V_{3,6} &= -0.09148. \end{aligned}$$

For the sake of brevity, the resulting values for the $a_{0,m}^{-1}$ are omitted.

In Fig. 8, the resulting magnitude responses of the individual shelving filters (dashed) and of their cascade (solid line) are depicted. As can clearly be seen, the 0 dB-valley between 500 Hz and 1 kHz can only be achieved for the higher-order filters. For $M = 1$, i.e. the traditional biquad case, in the third band around 10 kHz, the desired gain of −5 dB is not reached at all. This shows that higher-order shelving filters with their steep band-edges are a necessity for equalizers that are used to model a specific magnitude response.

6. COMPUTATIONAL PERFORMANCE

While the parametric realization of the first-order section in Fig. 6 has the minimum number of two multipliers for two coefficients, the realization of the second-order section in Fig. 5 needs nine multipliers (not counting multiplication by 2) — considerable more than a direct-form implementation would need.

So the proposed structure is beneficial only if the filter parameters are changed frequently, as recomputation of the coefficients is relatively cheap thanks to the decoupled design. In particular, changing the bandwidth Ω_B requires one trigonometric function evaluation for the complete filter, which is unavoidable when the filter is constructed using the bilinear transform, and M divisions to determine the $a_{0,m}^{-1}$. Changing the gain requires one exponentiation to determine the gain per section, which also seems unavoidable.

The frequency shift to obtain band-shelving filters can be efficiently realized with all-passes in one-multiplier form. This also needs only one trigonometric function evaluation for the complete filter whenever the center-frequency is changed.

7. CONCLUSIONS

We have presented an approach to design minimum-phase shelving filters of arbitrary order. For this design, we have derived a parametric filter structure in which center-frequency, bandwidth and gain of the shelving filter can be adjusted independently and cheaply. This realization is suitable for several equalization applications where the filter parameters have to be updated frequently.

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