

**Lecture 23:**

# **Domain-Specific Programming on Graphs**

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**Parallel Computer Architecture and Programming  
CMU 15-418/15-618, Spring 2020**

# Last time: Increasing acceptance of domain-specific programming systems

- **Challenge to programmers: modern computers are **parallel, heterogeneous machines****
  - (Architects striving for high area and power efficiency)
- **Programming systems trend: **give up generality** in what types of programs can be expressed in exchange for achieving **high productivity** and **high performance****
- **“Performance portability” is a key goal: programs should execute efficiently **on a variety of parallel platforms****
  - Good implementations of same program for different systems required different data structures, algorithms, and approaches to parallelization — not just differences in low-level code generation (e.g., not just a matter of generating SSE vs. AVX vs ARM Neon vs. NVIDIA PTX instructions)

# Today's topic: analyzing big graphs

- **Many modern applications:**
  - Web search results, recommender systems, influence determination, advertising, anomaly detection, etc.
- **Public dataset examples:**

**Twitter social graph, Wikipedia term occurrences, IMDB actors, Netflix, Amazon communities, G+**



**Thought experiment: if we wanted to design a programming system for computing on graphs, where might we begin?**

**What abstractions do we need?**

# Whenever I'm trying to assess the importance of a new programming system, I ask two questions:

- “What tasks/problems does the system take off the hands of the programmer? (are these problems challenging or tedious enough that I feel the system is adding sufficient value for me to want to use it?)”
- “What problems does the system leave as the responsibility for the programmer?” (likely because the programmer is better at these tasks)

## Liszt (recall last class): Programmer's responsibility:

- Describe mesh connectivity and fields defined on mesh
- Describe operations on mesh structure and fields

## Liszt system's responsibility:

- Parallelize operations without violating dependencies or creating data races (uses different algorithms to parallelize application on different platforms)
- Choose graph data structure / layout, partition graph across parallel machine, manage low-level communication (MPI send), allocate ghost cells, etc.

## Halide (recall last class): Programmer's responsibility:

- Describing image processing algorithm as pipeline of operations on images
- Describing the schedule for executing the pipeline (e.g., “block this loop, “parallelize this loop”, “fuse these stages”)

## Halide system's responsibility:

- Implementing the schedule using mechanisms available on the target machine (spawning pthreads, allocating temp buffers, emitting vector instructions, loop indexing code)

# Programming system design questions:

- What are the **fundamental operations** we want to be easy to express and efficient to execute?
- What are the **key optimizations** performed by the best implementations of these operations?
  - high-level abstractions should not prevent these
  - maybe even allow system to perform them for the application



# Example graph computation: Page Rank

## Page Rank: iterative graph algorithm

- Devised by Larry Page & Sergey Brinn, 1996

Graph nodes = web pages

Graph edges = links between pages

$$R[i] = \frac{1 - \alpha}{N} + \alpha \sum_{j \text{ links to } i} \frac{R[j]}{\text{Outlinks}[j]}$$

Diagram illustrating the Page Rank formula with annotations:

- Rank of page  $i$** : Points to  $R[i]$
- discount**: Points to  $\alpha$
- Weighted combination of rank of pages that link to it**: Points to the summation term  $\sum_{j \text{ links to } i} \frac{R[j]}{\text{Outlinks}[j]}$

- A system for describing **iterative computations** on graphs
- History:
  - 2009 Prof Carlos Guestrin at CMU, then at U Washington
  - 2013 Commercialized as Turi
  - 2016 Acquired by Apple
- Implemented as a C++ runtime
- Runs on shared memory machines or distributed across clusters
  - GraphLab runtime takes responsibility for scheduling work in parallel, partitioning graphs across **clusters of machines**, communication between master, etc.



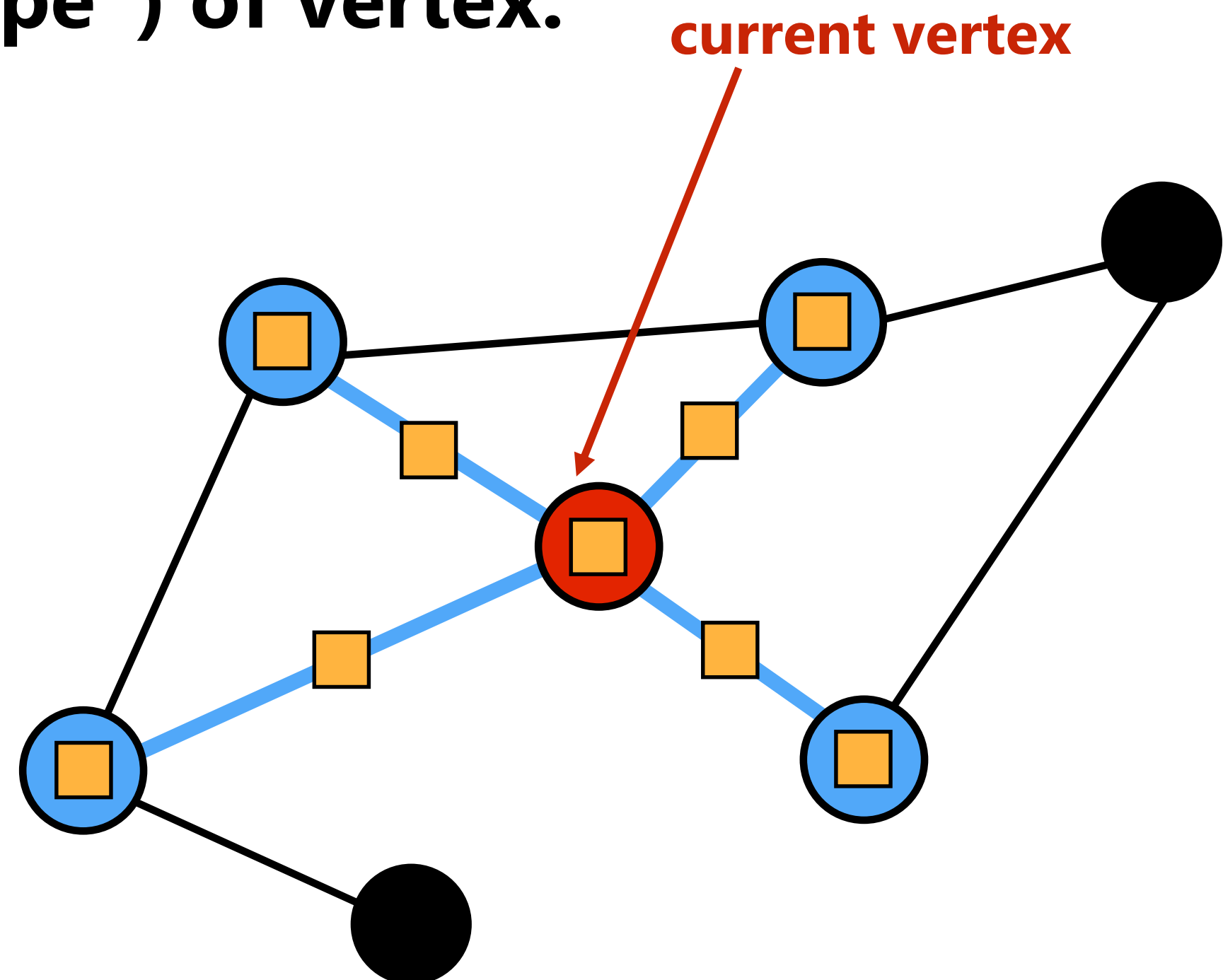
# GraphLab programs: state

- The graph:  $G = (V, E)$ 
  - Application defines **data blocks** on each vertex and directed edge
  - $D_v$  = data associated with **vertex**  $v$
  - $D_{u \rightarrow v}$  = data associated with **directed edge**  $u \rightarrow v$
- **Read-only global data**
  - Can think of this as per-graph data, rather than per vertex or per-edge data)

**Notice: I always first describe  
program **state**  
And then describe what operations are  
available to manipulate this state**

# GraphLab operations: the vertex program

- Defines **per-vertex operations** on the vertex's local **neighborhood**
- **Neighborhood** (aka "scope") of vertex:
  - The current vertex
  - Adjacent edges
  - Adjacent vertices



 = vertex or edge data "in scope" of red vertex  
(graph data that can be accessed when  
executing a vertex program at the current (red)  
vertex)

# Simple example: PageRank \*

$$R[i] = \frac{1 - \alpha}{N} + \alpha \sum_{j \text{ links to } i} \frac{R[j]}{\text{Outlinks}[j]}$$

```
PageRank_vertex_program(vertex i) {
```



```
// (Gather phase) compute the sum of my neighbors rank
```

```
double sum = 0;
```

```
foreach(vertex j : in_neighbors(i)) {
```

```
    sum = sum + j.rank / num_out_neighbors(j);
```

```
}
```

```
// (Apply phase) Update my rank (i)
```

```
i.rank = (1-0.85)/num_graph_vertices() + 0.85*sum;
```

(Shown for  $\alpha = 0.85$ )

```
}
```

**Programming in GraphLab amounts to defining how to update graph state at each vertex. The system takes responsibility for scheduling and parallelization.**

\* This is made up syntax for slide simplicity: actual syntax is C++, as we'll see on the next slide

# GraphLab: data access

- The application's **vertex program** **executes per-vertex**
- The **vertex program** defines:
  - What adjacent edges are inputs to the computation
  - What computation to perform per edge
  - How to update the vertex's value
  - What adjacent edges are modified by the computation
  - How to update these output edge values
- Note how GraphLab requires the program to tell it **all data that will be accessed**, and **whether it is read or write** access

# GraphLab-generated vertex program (C++ code)

```
struct web_page {  
    std::string pagename;  
    double pagerank;  
    web_page(): pagerank(0.0) { }  
}
```

```
typedef graphlab::distributed_graph<web_page, graphlab::empty> graph_type;
```

Graph has record  
of type web\_page  
per vertex, and  
no data on edges

```
class pagerank_program:  
    public graphlab::ivertex_program<graph_type, double>,  
    public graphlab::IS_POD_TYPE {  
public:
```

```
// we are going to gather on all the in-edges  
edge_dir_type gather_edges(icontext_type& context,  
                            const vertex_type& vertex) const {  
    return graphlab::IN_EDGES;  
}
```

Define edges to  
gather over in  
"gather phase"

```
// for each in-edge gather the weighted sum of the edge.  
double gather(icontext_type& context, const vertex_type& vertex,  
              edge_type& edge) const {  
    return edge.source().data().pagerank / edge.source().num_out_edges();  
}
```

Compute  
value to  
accumulate  
for each edge

```
// Use the total rank of adjacent pages to update this page  
void apply(icontext_type& context, vertex_type& vertex,  
           const gather_type& total) {  
    double newval = total * 0.85 + 0.15;  
    vertex.data().pagerank = newval;  
}
```

Update vertex  
rank

```
// No scatter needed. Return NO_EDGES  
edge_dir_type scatter_edges(icontext_type& context,  
                             const vertex_type& vertex) const {  
    return graphlab::NO_EDGES;  
}  
};
```

PageRank  
example  
performs no  
scatter

# Running the program

```
graphlab::omni_engine<pagerank_program> engine(dc, graph, "sync");  
engine.signal_all();  
engine.start();
```

GraphLab runtime provides “**engines**” that **manage scheduling** of vertex programs

`engine.signal_all()` marks all vertices for execution

You can think of the **GraphLab runtime** as a **work queue scheduler**.

And invoking a **vertex program** on a vertex as a **task** that is placed in the work queue.

So it's reasonable to read the code above as: “**place all vertices into the work queue**”

Or as: “**foreach vertex**” run the vertex program.



# Vertex signaling: GraphLab's mechanism for generating new work

$$R[i] = \frac{1 - \alpha}{N} + \alpha \sum_{j \text{ links to } i} \frac{R[j]}{\text{Outlinks}[j]}$$

Iterate update of all  $R[i]$ 's 10 times

Uses generic “**signal**” primitive (could also wrap code on previous slide in a for loop)

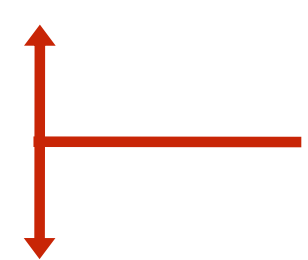
```
struct web_page {  
    std::string pagename;  
    double      pagerank;  
    int         counter;  
    web_page(): pagerank(0.0), counter(0) { }  
}
```

Per-vertex “counter”



```
// Use the total rank of adjacent pages to update this page  
void apply(context_type& context, vertex_type& vertex,  
           const gather_type& total) {  
    double newval = total * 0.85 + 0.15;  
    vertex.data().pagerank = newval;  
    vertex.data().counter++;  
    if (vertex.data().counter < 10)  
        vertex.signal();  
}
```

If counter < 10, signal to scheduler to run the vertex program on the vertex again at some point in the future



# Signal: general primitive for scheduling work

Parts of graph may converge at different rates  
(iterate PageRank **until convergence**, but **only for vertices that need it**)

```
class pagerank_program:
    public graphlab::ivertex_program<graph_type, double>,
    public graphlab::IS_POD_TYPE {
private:
    bool perform_scatter;
public:
```

Private variable set during  
apply phase, used during  
scatter phase

```
    // Use the total rank of adjacent pages to update this page
    void apply(icontext_type& context, vertex_type& vertex,
               const gather_type& total) {
        double newval = total * 0.85 + 0.15;
        double oldval = vertex.data().pagerank;
        vertex.data().pagerank = newval;
        perform_scatter = (std::fabs(oldval - newval) > 1E-3);
    }
```

Check for  
convergence

```
    // Scatter now needed if algorithm has not converged
    edge_dir_type scatter_edges(icontext_type& context,
                                const vertex_type& vertex) const {
        if (perform_scatter) return graphlab::OUT_EDGES;
        else return graphlab::NO_EDGES;
    }
```

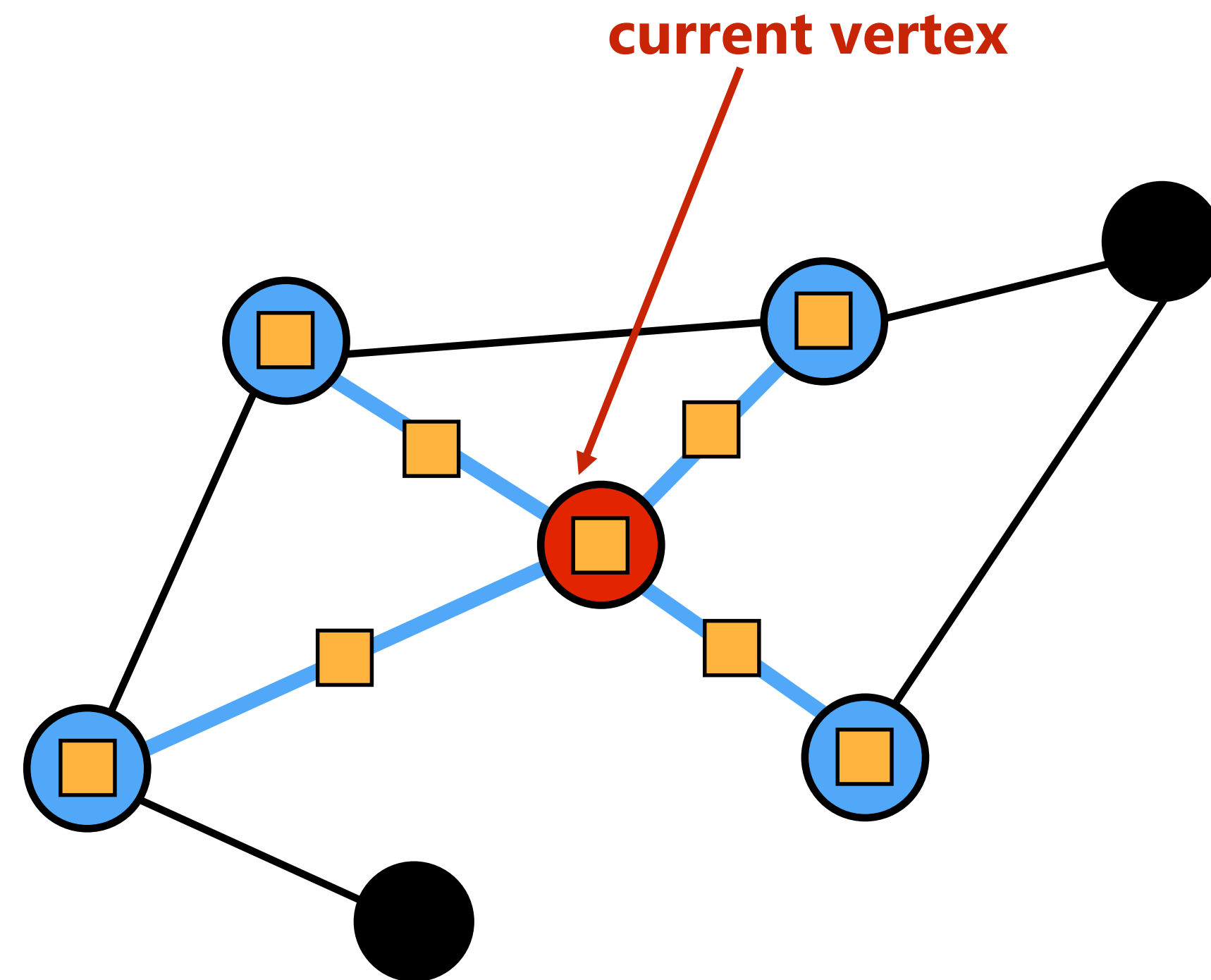


```
    // Make sure surrounding vertices are scheduled
    void scatter(icontext_type& context, const vertex_type& vertex,
                 edge_type& edge) const {
        context.signal(edge.target());
    }
};
```

Schedule update  
of neighbor  
vertices

# Synchronizing parallel execution

Local neighborhood of vertex (vertex's "scope") can be read and written to by a vertex program



 = vertex or edge data in scope of red vertex

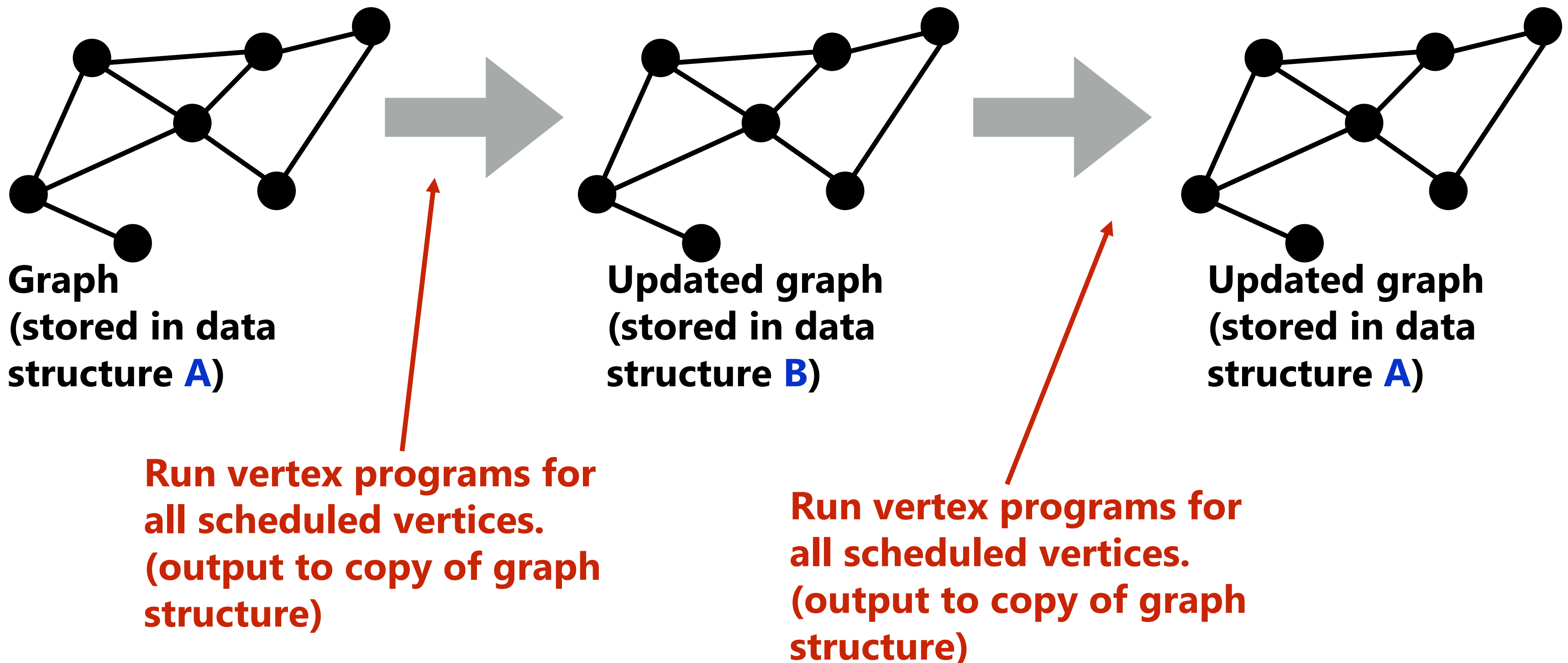
Programs specify what **granularity of atomicity ("consistency")** they want GraphLab runtime to provide: this determines amount of available parallelism

- "**Full consistency**": implementation ensures no other execution reads or writes to data in scope of  $v$  when vertex program for  $v$  is running.
- "**Edge consistency**": no other execution reads or writes any data in  $v$  or in edges adjacent to  $v$
- "**Vertex consistency**": no other execution reads or writes to data in  $v$  ...

# GraphLab: job scheduling order

GraphLab implements several work scheduling policies

- **Synchronous**: update all scheduled vertices “simultaneously” (vertex programs observe no updates from programs run on other vertices in same “round”)



# GraphLab: job scheduling order

- GraphLab implements several work scheduling policies
  - **Synchronous**: update all vertices simultaneously (vertex programs observe no updates from programs run on other vertices in same “round”)
  - **Round-robin**: vertex programs observe most recent updates
  - **Graph coloring**: Avoid simultaneous updates by adjacent vertices
  - **Dynamic**: based on new work created by signal
    - Several implementations: fifo, priority-based, “splash” ...
- Application developer has flexibility for choosing consistency guarantee and scheduling policy
  - Implication: choice of schedule impacts program’s correctness/output
  - Our opinion: this seems like a weird design at first glance, but this is common (and necessary) in the design of efficient graph algorithms

# Summary: GraphLab concepts

- **Program state:** data on **graph vertices and edges** + globals
- **Operations:** **per-vertex update** programs and global reduction functions (reductions not discussed today)
  - Simple, intuitive description of work (follows mathematical formulation)
  - Graph restricts data access in vertex program to local neighborhood
  - Asynchronous execution model: **application creates work dynamically by “signaling vertices”** (enable **lazy execution**, work efficiency on real graphs)
- **Choice of scheduler and consistency implementation**
  - In this domain, **the order** in which nodes are processed can be critical property for both performance and quality of result
  - Application responsible for choosing right scheduler for its needs



# **Elements of good domain-specific programming system design**

# #1: good systems identify the **most important cases**, and provide most benefit in these situations

- Structure of code should **mimic natural structure of problems in the domain**
  - e.g., graph processing algorithms are designed in terms of per-vertex operations
- Efficient expression: **common operations are easy and intuitive** to express
- Efficient implementation: the **most important optimizations** in the domain are **performed by the system** for the programmer
  - Our experience: a parallel programming system with “convenient” abstractions that precludes best-known implementation strategies will almost always fail

# #2: good systems are usually **simple** systems

- They have a **small number of key primitives and operations**
  - GraphLab: run computation per vertex, trigger new work by signaling
    - But GraphLab's scheduling design gets messy...
  - Halide: only a few scheduling primitives
  - Hadoop: map + reduce
- **Allows compiler/runtime to focus on optimizing these primitives**
  - Provide parallel implementations, utilize appropriate hardware
- **Common question that good architects ask: “do we really need that?” Or can we reuse an existing primitive?**
  - For every domain-specific primitive in the system: there better be a strong performance or expressivity justification for its existence

# #3: good primitives **compose**

- **Composition of primitives allows for wide application scope, even if scope remains limited to a domain**
  - e.g., frameworks discussed today support a wide variety of graph algorithms
- **Composition often allows for generalizable optimization**
- **Sign of a good design:**
  - System ultimately is used for applications original designers never anticipated
- **Sign that a new feature should not be added (or added in a better way):**
  - The new feature does not compose with all existing features in the system

# **Optimizing graph computations**

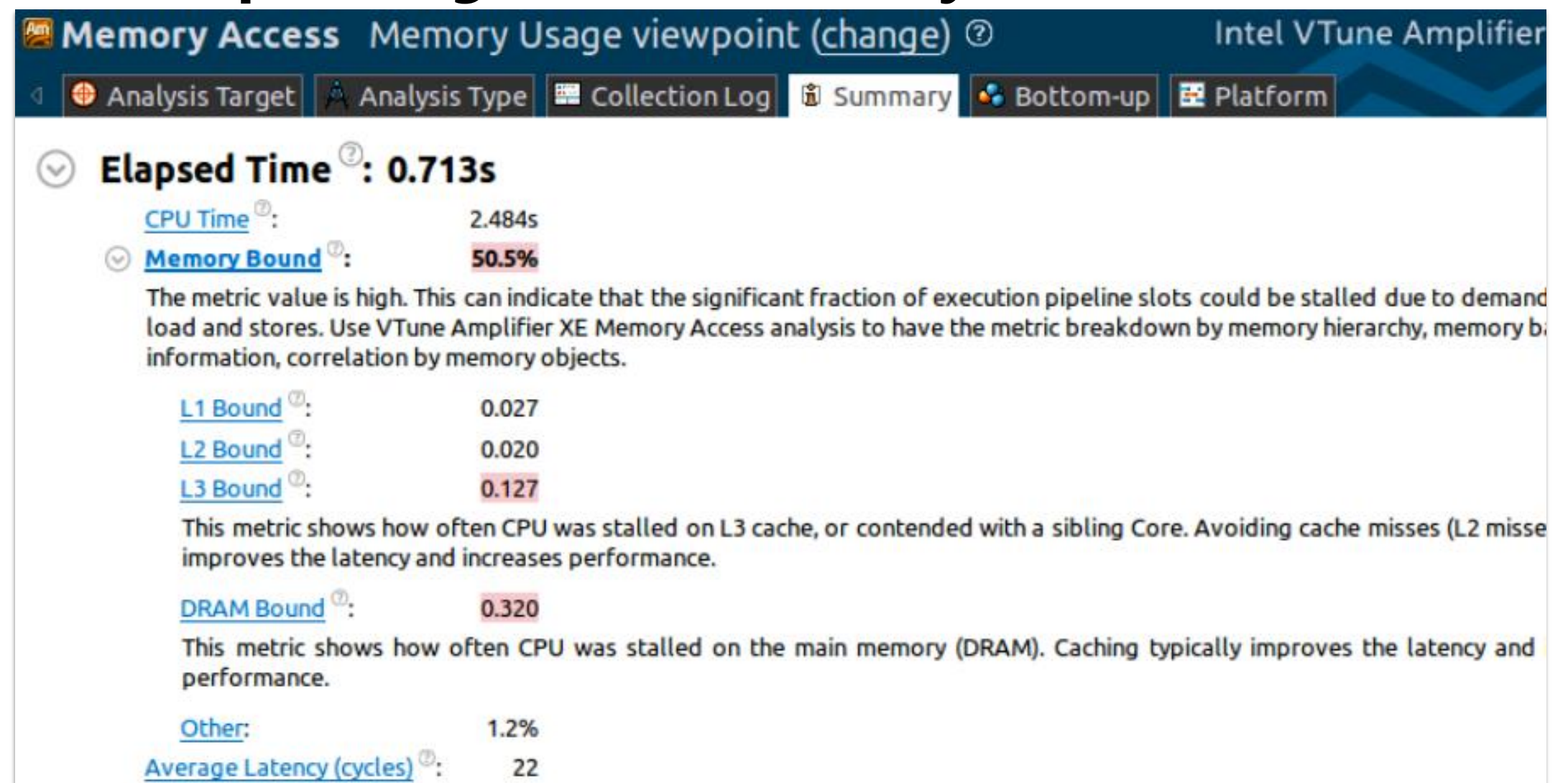
## **(now we are talking about implementation)**

# Wait a minute...

- So far in this lecture, we've discussed issues such as parallelism, synchronization ...
- But graph processing typically has **low arithmetic intensity**

Walking over edges  
accesses information  
from “random” graph  
vertices

VTune profiling results: Memory bandwidth bound!



Or just consider **PageRank**: ~ 1 multiply-accumulate per iteration of summation loop

$$R[i] = \frac{1 - \alpha}{N} + \alpha \sum_{j \text{ links to } i} \frac{R[j]}{\text{Outlinks}[j]}$$



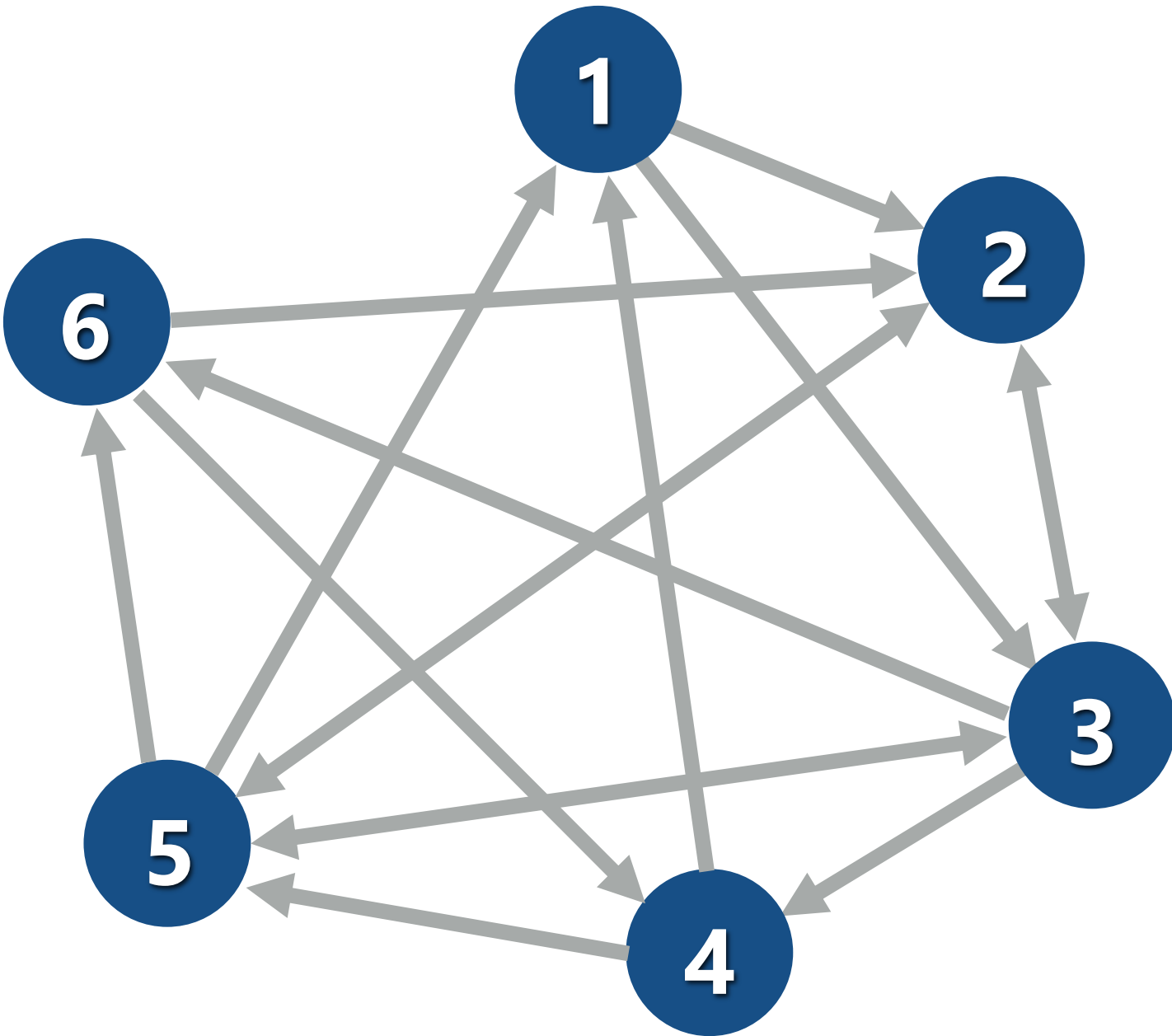
# Two ideas to increase the performance of operations on large graphs \*

1. Reorganize graph structure to **increase locality**
2. **Compress** the graph




\* Both optimizations might be performed by a framework without application knowledge

# Directed graph representation

Vertex Id Outgoing Edges	1	2	3					4	5					6
	2 3	3 5	2 4 5 6					1 5	1 2 3 6					2 4
Vertex Id Incoming Edges	1	2					3	4	5					6
	4 5	1 3 5 6					1 2 5	3 6	2 3 4					3 6

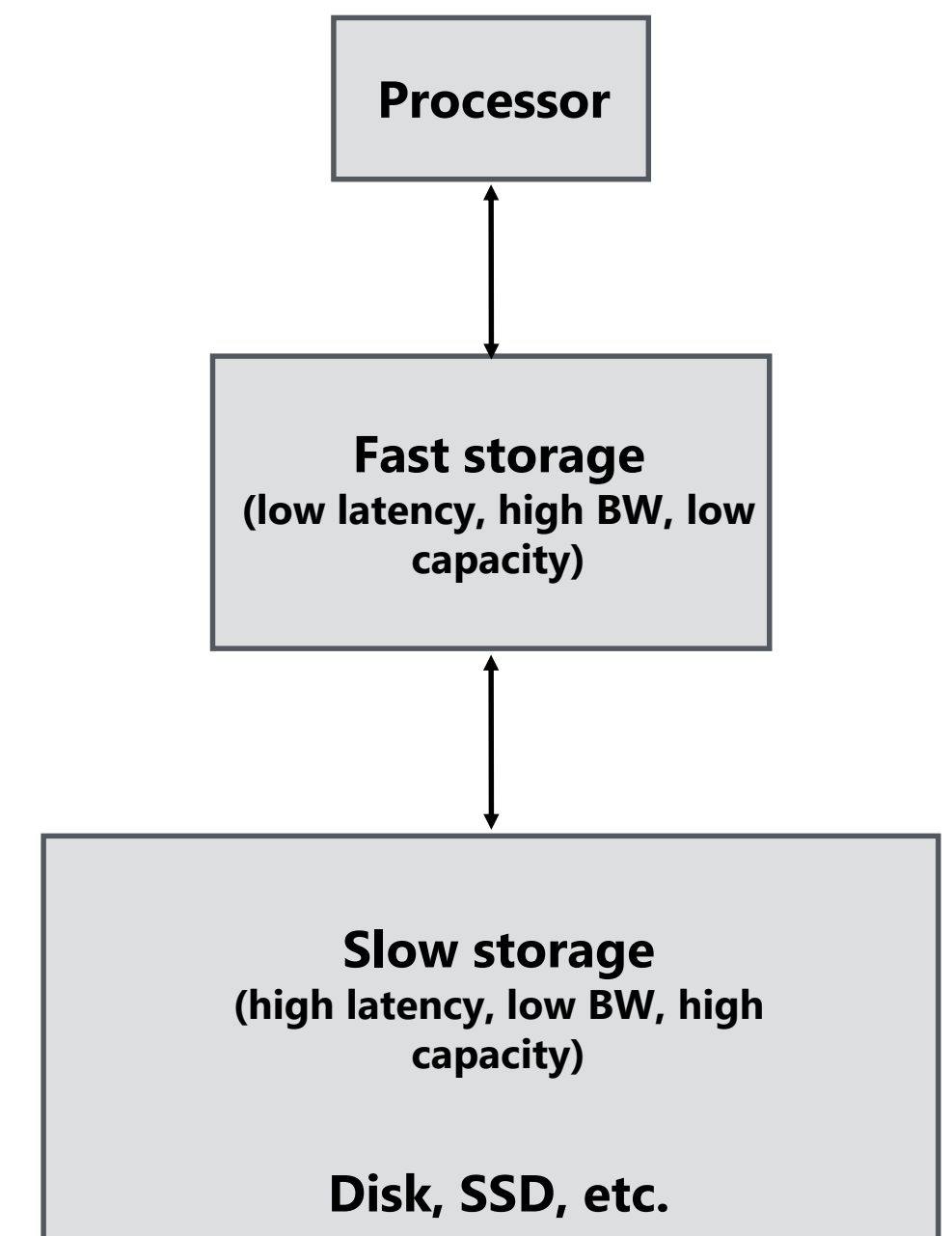


# Memory footprint challenge of large graphs

- **Challenge:** cannot fit all edges in memory for large graphs (but graph vertices may fit)
  - From example graph representation:
    - Each edge represented twice in graph structure (as incoming/outgoing edge) 
    - 8 bytes per edge to represent adjacency
  - May also need to store per-edge values (e.g., 4 bytes for a per-edge weight)
  - 1 billion edges (modest): ~12 GB of memory for edge information
  - Algorithm may need multiple copies of per-edge structures (current, prev data, etc.)
- Could employ cluster of machines to store graph in memory
  - Rather than store graph on disk 
- Would prefer to process large graphs on a single machine
  - Managing clusters of machines is difficult
  - Partitioning graphs is expensive (also needs a lot of memory) and difficult 

# “Streaming” graph computations

- Graph operations make “**random**” accesses to graph data (edges adjacent to vertex  $v$  may distributed arbitrarily throughout storage)
  - Single pass over graph’s edges might make billions of fine-grained accesses to disk
- **Streaming data access pattern**
  - Make large, predictable data accesses to slow storage (achieve high bandwidth data transfer)
  - Load data from slow storage into fast storage\*, then reuse it as much as possible before discarding it (achieve high arithmetic intensity)
  - Can we restructure graph data structure so that data access requires only a small number of efficient bulk loads/stores from slow storage?



\* By fast storage, in this context I mean DRAM. However, techniques for streaming from disk into memory would also apply to streaming from memory into a processor’s cache

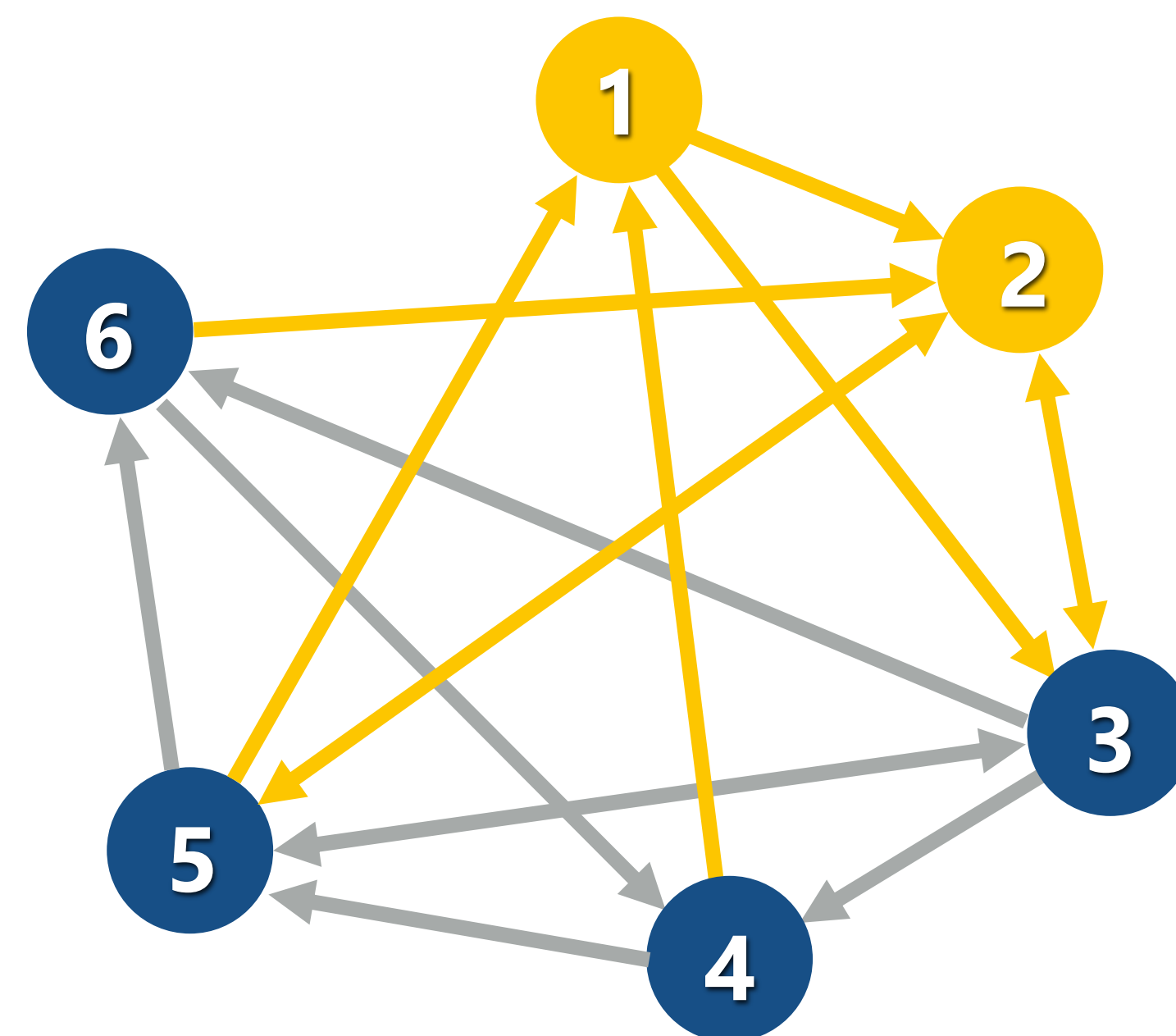
# Sharded graph representation

GraphChi: Large-scale graph  
computation on just a PC  
[Kryola et al. 2013]

- **Partition** graph vertices into intervals (sized so that subgraph for interval fits in memory)
- Vertices and (only) incoming edges to these vertices are stored together in a shard
- **Sort** edges in a shard by source vertex id

Shard 1: vertices (1-2)			Shard 2: vertices (3-4)			Shard 3: vertices (5-6)		
src	dst	value	src	dst	value	src	dst	value
1	2	0.3	1	3	0.4	2	5	0.6
3	2	0.2	2	3	0.9	3	5	0.9
4	1	0.8	3	4	0.15	3	6	0.85
5	1	0.25	5	3	0.2	4	5	0.3
5	2	0.6	6	4	0.9	5	6	0.2
6	2	0.1						

Yellow = data required to process  
subgraph containing vertices in  
shard 1



**Notice: to construct subgraph containing vertices in shard 1 and their incoming and outgoing edges, only need to load contiguous information from other P-1 shards**

**Writes to updated outgoing edges require P-1 bulk writes**

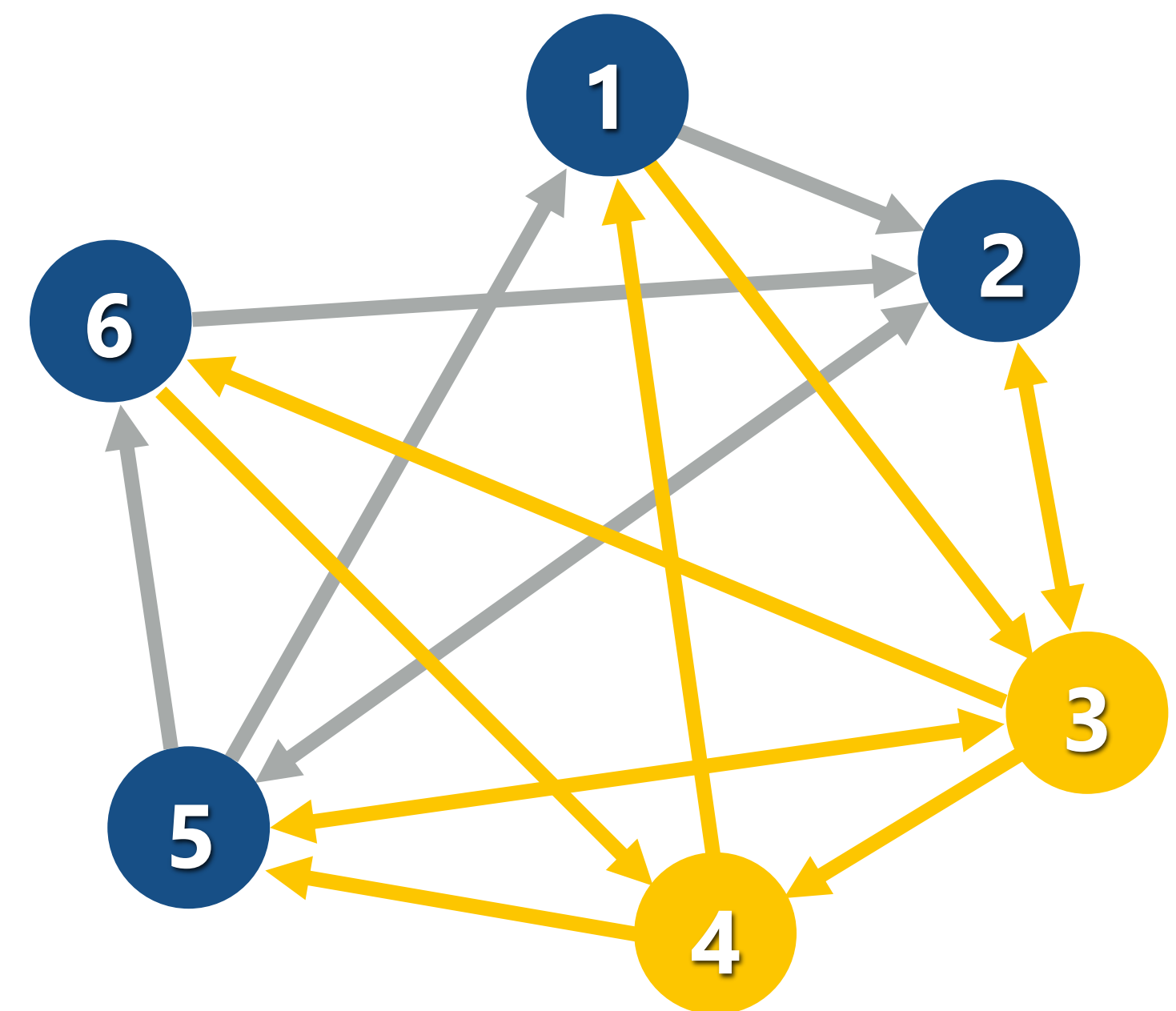
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3	2	0.2	2	3	0.9	3	5	0.9
4	1	0.8	3	4	0.15	3	6	0.85
5	1	0.25	5	3	0.2	4	5	0.3
5	2	0.6	6	4	0.9	5	6	0.2
6	2	0.1						

Yellow = data required to process  
subgraph containing vertices in  
shard 2





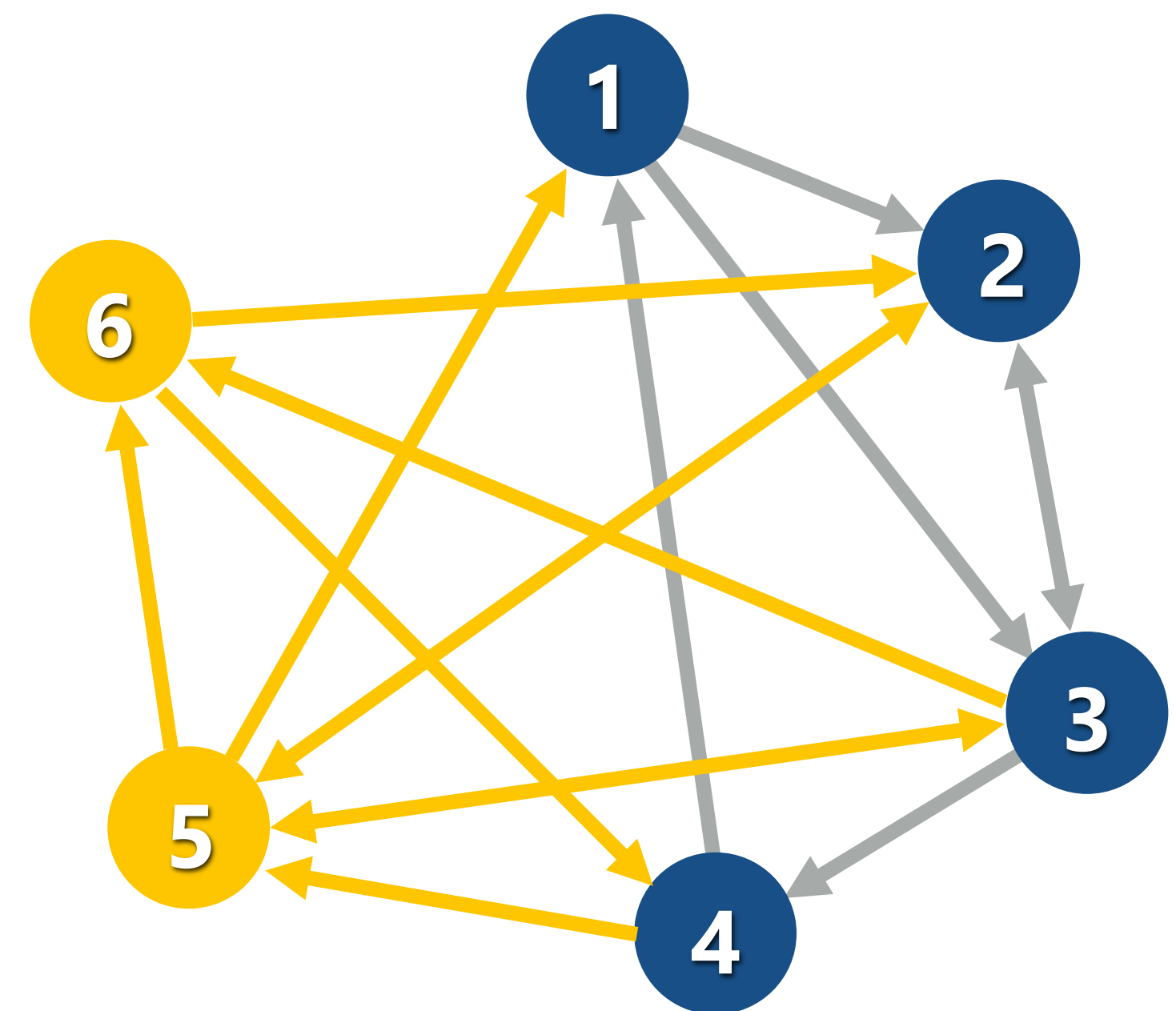
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src	dst	value	src	dst	value	src	dst	value
1	2	0.3	1	3	0.4	2	5	0.6
3	2	0.2	2	3	0.9	3	5	0.9
4	1	0.8	3	4	0.15	3	6	0.85
5	1	0.25	5	3	0.2	4	5	0.3
5	2	0.6	6	4	0.9	5	6	0.2
6	2	0.1						


Yellow = data required to process  
subgraph containing vertices in  
shard 3



**Observe: due to sort of incoming edges, iterating over all intervals results in contiguous sliding window over the shards**

# Putting it all together: looping over all graph edges

For each **partition  $i$**  of vertices:

- Load shard  $i$  (contains all incoming edges)
- For each other shard  $s$ 
  - Load section of  $s$  containing data for edges leaving  $i$  and entering  $s$
- Construct **subgraph** in memory
- **Do processing on subgraph** 

**Note: a good implementation could hide disk I/O by prefetching data for next iteration of loop**

# PageRank in GraphChi

**GraphChi** is a system that implements the **out-of-core sliding window approach**

## PageRank in GraphChi:

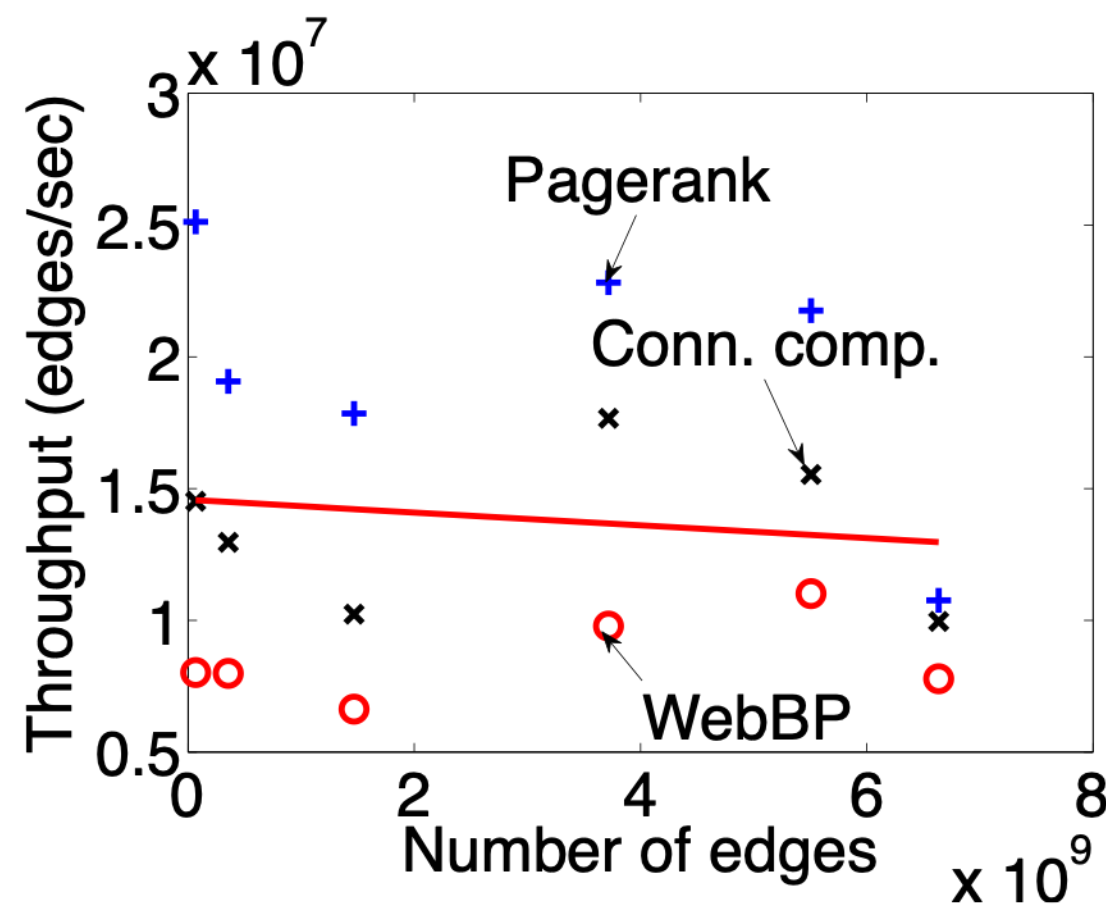
```
1 typedef: VertexType float
2 Update(vertex) begin
3     var sum ← 0
4     for e in vertex.inEdges() do
5         | sum += e.weight * neighborRank(e)
6     end
7     vertex.setValue(0.15 + 0.85 * sum)
8     broadcast(vertex)
9 end
```

Take per-vertex rank and distribute to all outbound edges (memory inefficient: replicates per-vertex rank to all edges)

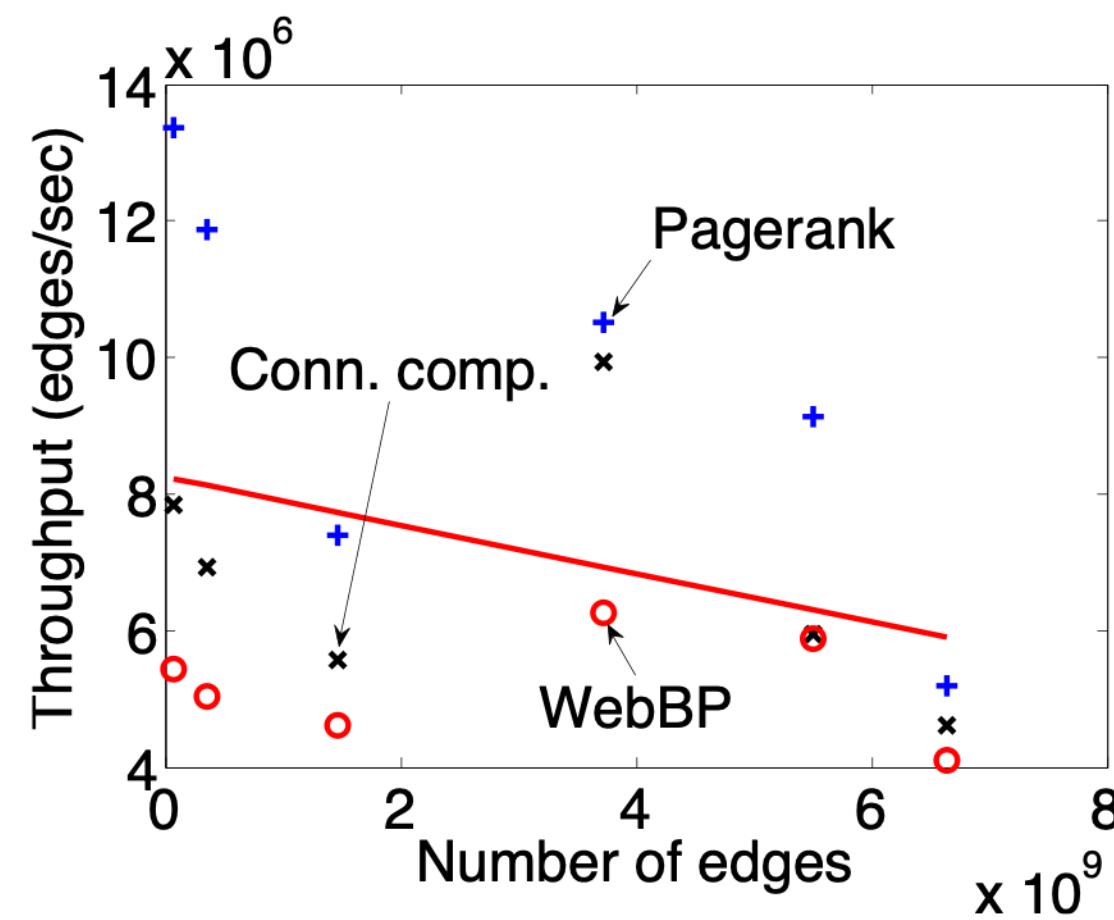
**Alternative model:** assume vertex data can be kept in memory and redefine **neighborRank()** function

```
1 typedef: EdgeType { float weight; }
2 float[] in_mem_vert
3 neighborRank(edge) begin
4     | return edge.weight * in_mem_vert[edge.vertex_id]
5 end
```

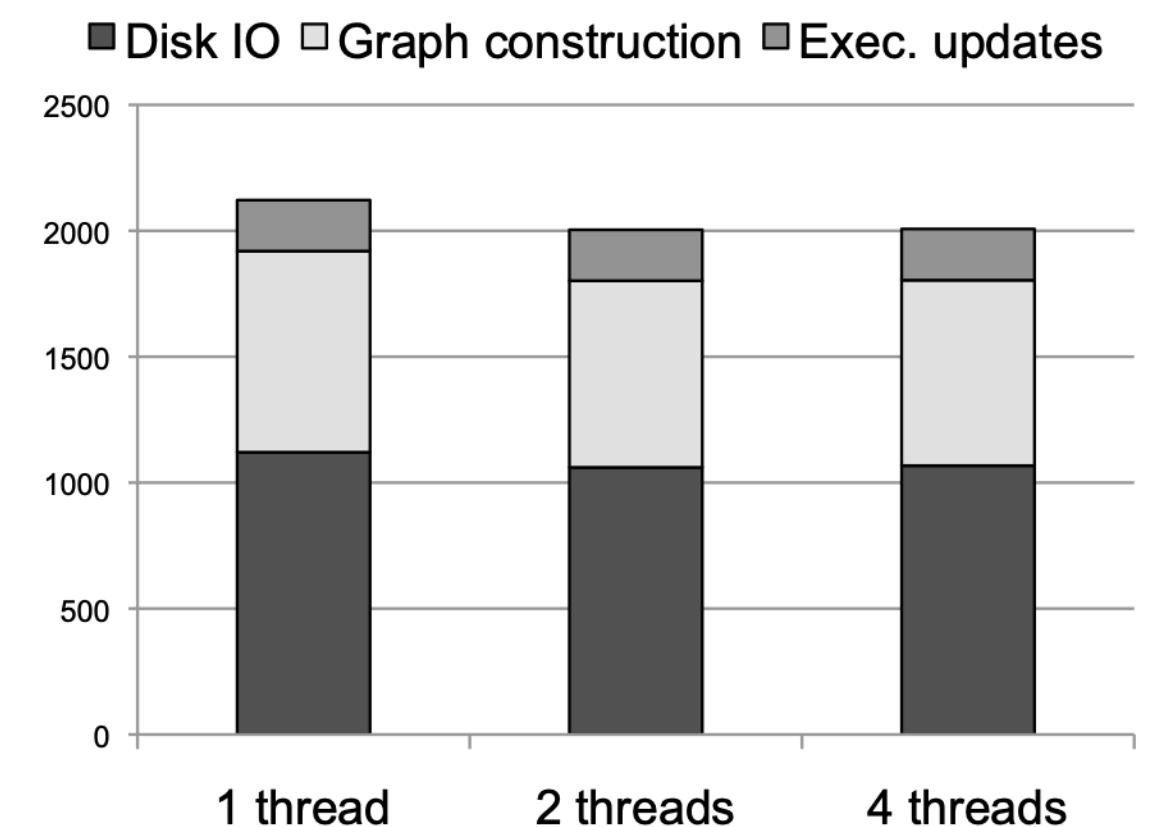
# Performance on a Mac mini (8 GB RAM)



(a) Performance: SSD



(b) Performance : Hard drive



(c) Runtime breakdown

**Throughput (edges/sec) remains stable as graph size is increased**

- **Desirable property: throughput largely invariant of dataset size**

# Graph compression

- Recall: graph operations are often **BW-bound**
- Implication: using CPU instructions to reduce BW requirements can benefit overall performance (the processor is waiting on memory anyway!)
- Idea: **store graph** **compressed in memory**, **decompress on-the-fly** when operation wants to read data

# Compressing an edge list

Vertex Id	32
Outgoing Edges	1001 10 5 30 6 1025 200000 1010 1024 100000 1030 275000

## 1. Sort edges for each vertex

5 6 10 30 1001 1010 1024 1025 1030 100000 200000 275000

## 2. Compute differences

5 6 10 30 1001 1010 1024 1025 1030 100000 200000 275000  
0 1 4 20 971 9 14 1 5 98070 100000 75000

## 3. Group into sections requiring same number of bytes

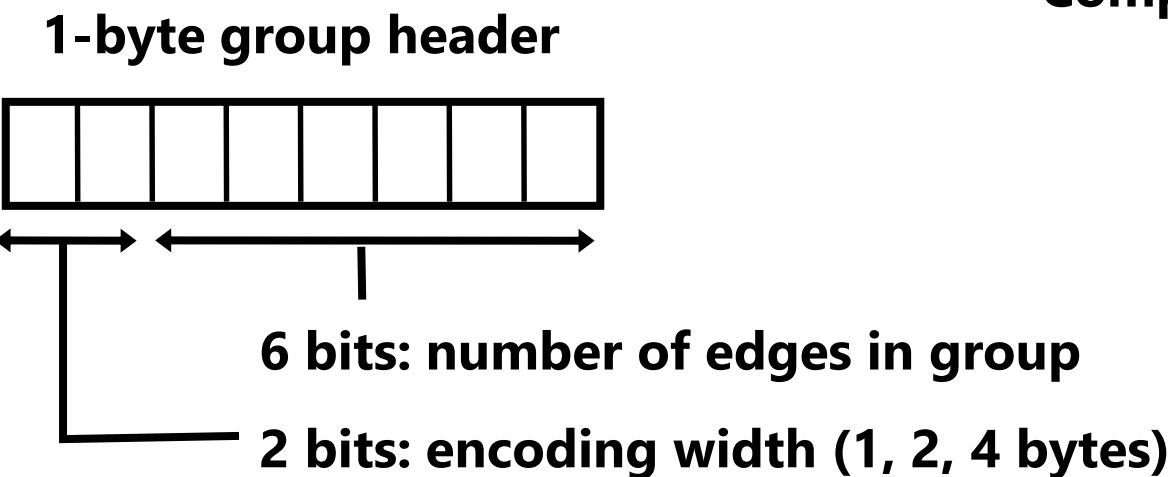
relative to vertex index

5 6 10 30 1001 1010 1024 1025 1030 100000 200000 275000  
-27 1 4 20 971 9 14 1 5 98070 100000 75000

1 byte 2 bytes 1 byte 4 bytes

## 4. Encode deltas

Uncompressed encoding: 12 x 4 bytes = 48 bytes  
Compressed encoding: 26 bytes

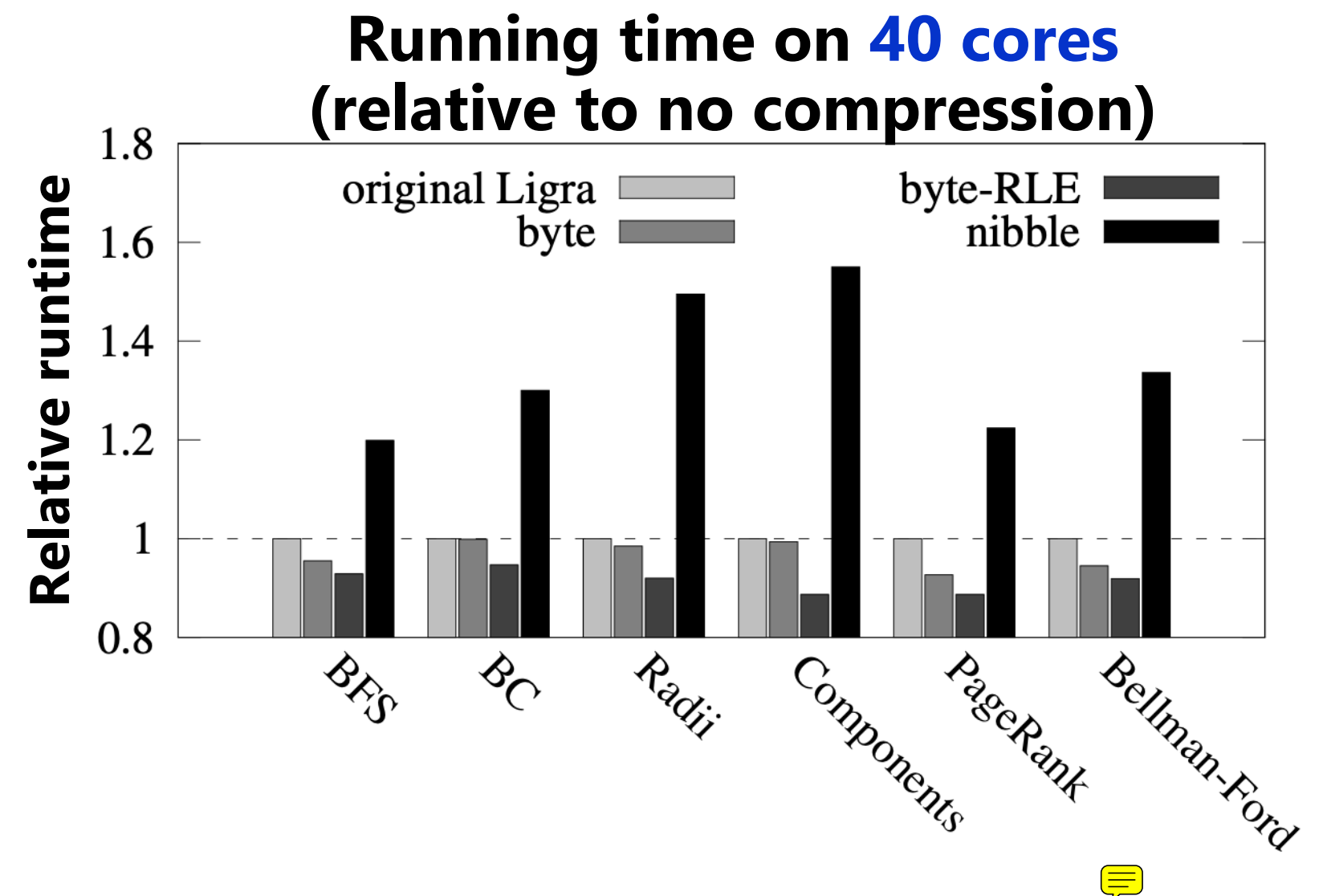
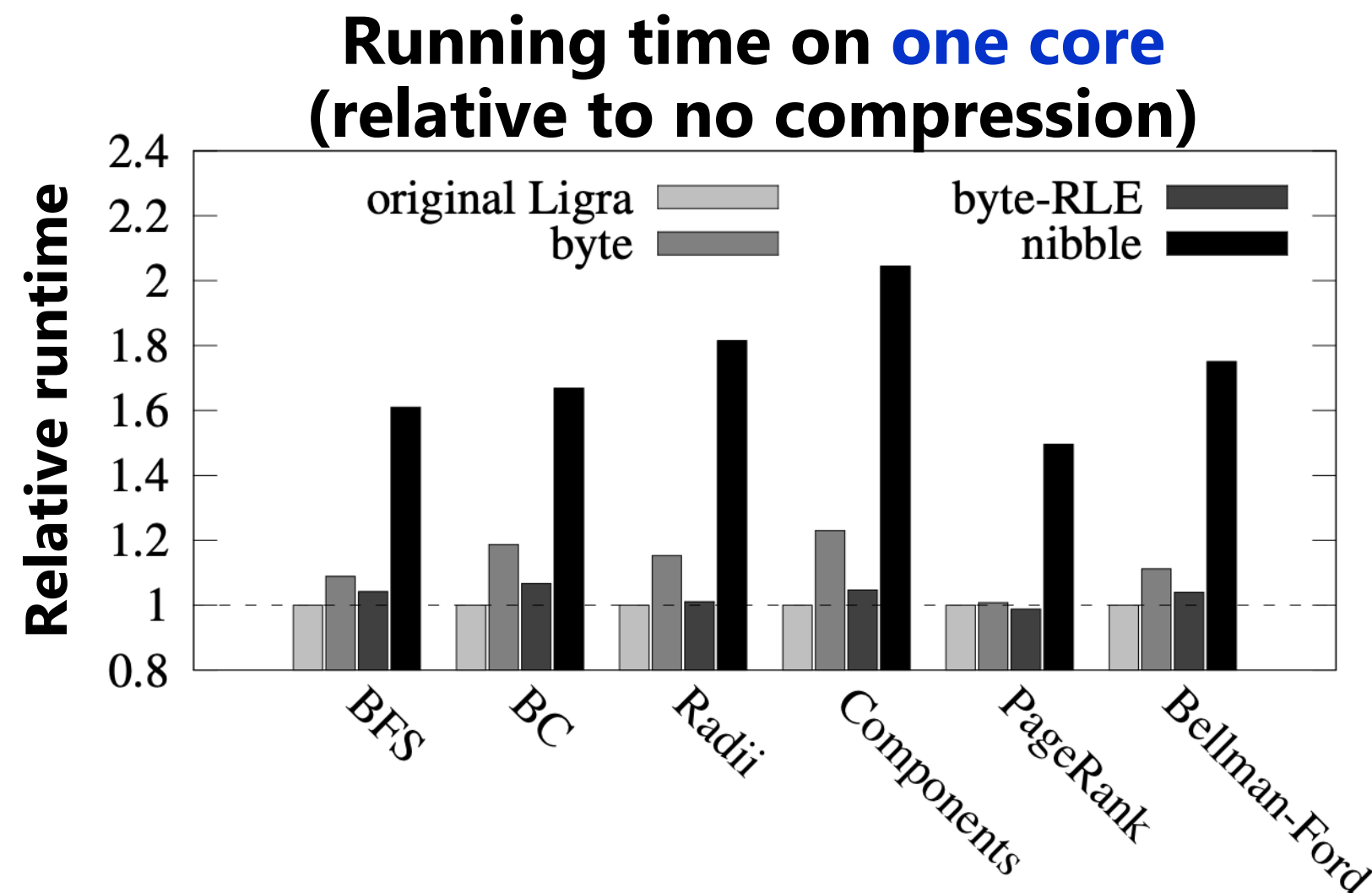


[ONE\_BYTE, 4], -27, 1, 4, 20 (5 bytes)  
[TWO\_BYTE, 1], 971 (3 bytes)  
[ONE\_BYTE, 4], 9, 14, 1, 5 (5 bytes)  
[FOUR\_BYTE, 3], 98070, 100000, 75000 (13 bytes)



# Performance impact of graph compression

[Shun et al. DCC 2015]



- **Benefit of graph compression increases with higher core count, since computation is increasingly bandwidth bound**
- **Performance improves even if graphs already fit in memory**
  - Added benefit is that compression enables larger graphs to fit in memory

\* Different data points on graphs are different compression schemes (byte-RLE is the scheme on the previous slide)

# Summary

- **Today there is significant interest in high performance computation on large graphs**
- **Graph processing frameworks abstract details of efficient graph processing from application developer**
  - **handle parallelism and synchronization for the application developer**
  - **handle graph distribution (across a cluster)**
  - **may also handle graph compression and efficient iteration order (e.g., to efficiently stream off slow storage)**
- **Great example of domain-specific programming frameworks**
  - **for more, see: GraphLab, GraphX, Pregel, Ligra/Ligra+**