UM-SJTU JOINT INSTITUTE
DISCRETE MATHEMATICS
(VE203)

ASSIGNMENT 2

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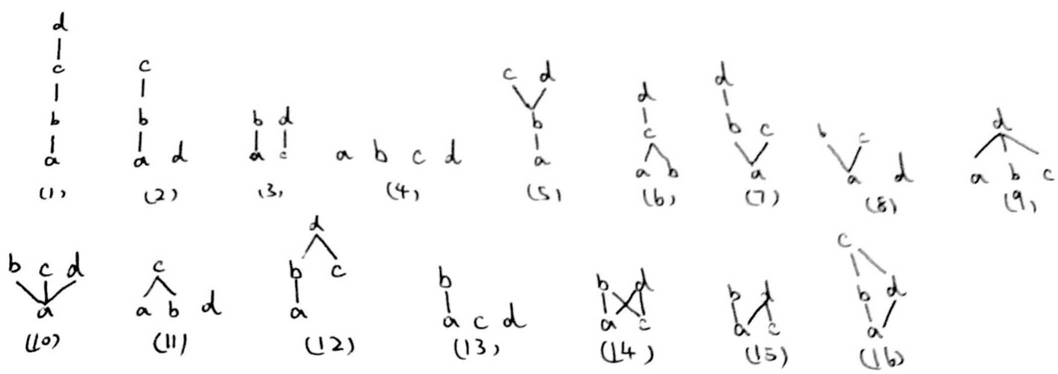
1 Q1



These are 16 possible diagrams of possible partial order. They're all chain complete. The first poset is a linear order and a well order. The 1, 16 posets are lattices and complete lattices.

2 Q2

(i) If the complete lattice has many maximal elements, then the set of these maximal elements doesn't have a least upper bound, so it can't be a complete lattice.

(ii)

$$(R, \leq)$$

(iii) $\forall x \in [a, b], x \leq x \Rightarrow (\mathbb{R}, \leq)$ is reflexive.
$\forall x, y \in [a, b]$,if $x \leq y$ since $x \neq y \Rightarrow y \nleq x$,
$\therefore (\mathbb{R}, \leq)$ is antisymmetric.
$\forall x, y, z \in [a, b]$,$(x \leq y) \wedge (y \leq z) \Rightarrow (x \leq z)$,
$\therefore (\mathbb{R}, \leq)$ is transitive.
$\therefore (\mathbb{R}, \leq)$ is a poset.

$\forall M \subseteq$ if M has one biggest element then it is the least upper bound.
if M doesn't have the biggest element and it's an open interval (c,d) $(d \leq b)$, then d is the least upper bound.
if M neither has a biggest element and be a interval, then b is its least upper bound.
$\therefore M$ must has a least upper bound, similarly it also has a greatest lower bound.
$\therefore (\mathbb{R}, \leq)$ is a complete lattice.

(iv)

$$(\mathbb{N}, \geq)$$

3 Q3

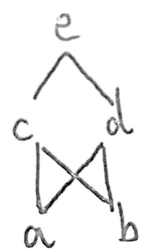


Figure 1: Example of Q3

4 Q4

$\forall (a,b),(c,d) \in (\mathbb{N} \times \mathbb{N})$ if$(a,b) \not\preceq (c,d)$then$((a = c) \wedge (b > d)) \vee (a > c)$
$\Rightarrow c < a$ or $(c = a$ and $d \leq b) \Rightarrow (c,d) \preceq (a,b)$
$\therefore (\mathbb{N} \times \mathbb{N}, \preceq)$ is a linear order.

$(\mathbb{N} \times \mathbb{N}, \preceq)$ is well order.
$\forall A \subseteq \mathbb{N} \times \mathbb{N}$,assume $A = \mathbb{X} \times \mathbb{Y}$. Since $(\mathbb{N}, \leq)$ is a well-order ,then there must exist $x_0 \in \mathbb{X}$ and $y_0 \in \mathbb{Y}$ such that only $x_0 \leq x_0$ and $y_0 \leq y_0$.
$\forall [(x,y) \in A] \wedge [(x,y) \neq (x_0, y_0)] \Rightarrow (x_0 \leq x) \wedge (y_0 \leq y)$
if$(x > x_0)$ then $(x,y) \not\preceq (x_0, y_0)$
if$(x = x_0) \wedge (y > y_0)$ then $(x,y) \not\preceq (x_0, y_0)$
$\therefore$ only $(x_0, y_0) \preceq (x_0, y_0)$ and $(\mathbb{N} \times \mathbb{N}, \preceq)$ is a well order.

5 Q5

$\forall (a,b) \in (\mathbb{N} \times \mathbb{N})(a,b) \preceq (a,b)$
$\therefore$ it is reflective.
if$(a,b) \preceq (c,d)$ then $(a \leq c) \wedge (b \leq d)$
Since $(a,b) \neq (c,d) \therefore (c,d) \not\preceq (a,b)$ and it's antisymmetric.
if$(c,d) \preceq (e,f)$ then $(c \leq e) \wedge (d \leq f)$
$\Rightarrow (a \leq e) \wedge (b \leq f) \Rightarrow (a,b) \preceq (e,f)$ and it's transitive. $\therefore (\mathbb{N} \times \mathbb{N}, \preceq)$ is a poset.

It's not a linear order because $(1,3) \not\preceq (2,2)$ and $(2,2) \not\preceq (1,3)$

$\forall (a,b),(c,d) \in (\mathbb{N} \times \mathbb{N}), (max(a,c), max(b,d))$ is the l.u.b and $(min(a,c), min(b,d))$ is the g.l.b. $\therefore$ It's a lattice.

6 Q6

For $(3,3,3),(1,4,4),(2,2,2) \in (\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}), (3,3,3) \preceq (1,4,4)$ because $3 \leq 4$, $(1,4,4 \preceq (2,2,2)$ because $1 \leq 2$ but $(3,3,3) \not\preceq (1,4,4), \therefore$ it's not transitive and it's not a poset.

7 Q7

$\sim$ is equivalence relation
$f(x) = f(x) \Rightarrow x \sim x \Rightarrow \sim$ is reflexive
$x \sim y \Rightarrow f(x) = f(y) \Rightarrow f(y) = f(x) \Rightarrow y \sim x \Rightarrow \sim$ is symmetric
$(x \sim y) \wedge (y \sim z) \Rightarrow (f(x) = f(y)) \wedge (f(y) = f(z))$
$\Rightarrow f(x) = f(z) \Rightarrow x \sim z \Rightarrow \sim$ is transitive
$\therefore \sim$ is equivalence relation

Since $f : A \to B$ is function, $\forall x_0 \in A$, there only exists one $f(x_0)$.
$g([x_0]_\sim) = f(x_0)$
$\therefore g$ is a function because f is a function, only $([x_0]_\sim, f(x_0)) \in g$.

$\forall f(x_0) \in B, [x_0]_\sim = \{x | (x \in A) \wedge (f(x) = f(x_0))\}$
If g is not injective, then there must exist $[x_1]_\sim \neq [x_0]_\sim$ such that $g([x_1]_\sim) = f(x_0)$
$g([x_1]_\sim) = f(x_1) \Rightarrow f(x_0) = f(x_1) \Rightarrow x_0 \sim x_1 \Rightarrow [x_0]_\sim = [x_1]_\sim$ It's a contradiction. $\therefore g$ is injective.

$f(x) = x^3 - 4x^2 - 15x + 18 = 0 \Rightarrow x_1 = 6, x_2 = 1, x_3 = -3$
$\therefore g^{-1}(0) = \{6, 1, -3\}$