Q1. Prove, without using the product formula for $\varphi(n)$, that if p is prime, then

$$\varphi(p^k) = p^k - p^{k-1}$$

(3 marks)

Q2. Prove that for all $n \in \mathbb{N}\backslash\{0\}$, $n^3 + 2n$ and $n^4 + 3n^2 + 1$ are relatively prime.
(2 marks)

Q3. Prove that every subgroup of a cyclic group is cyclic.
(2 marks)

Q4. Show that if $a, b, c \in \mathbb{N}$ with $a^2 + b^2 = c^2$, then $3 \mid ab$.
(3 marks)

Q5. Find a generator of the group $((\mathbb{Z}/11\mathbb{Z})^*, \otimes_{11})$.
(2 marks)

Q6. Find the inverse of $[12]_{89}$ in the group $((\mathbb{Z}/89\mathbb{Z})^*, \otimes_{89})$.
(2 marks)

Q7. What is the order of $[27]_{56}$ in the group $((\mathbb{Z}/56\mathbb{Z})^*, \otimes_{56})$?
(2 marks)

Q8. Is $((\mathbb{Z}/14\mathbb{Z})^*, \otimes_{14})$ cyclic? Justify your answer.
(2 marks)

Q9. Draw a Cayley Table for the group $((\mathbb{Z}/9\mathbb{Z})^*, \otimes_9)$. Is $((\mathbb{Z}/9\mathbb{Z})^*, \otimes_9)$ cyclic?
(3 marks)