

UM-SJTU JOINT INSTITUTE  
Data Structures and Algorithms  
(VP281)

Programming Assignment

Programming Assignment Two  
Linear-Time Selection

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Date: October 20, 2018

# 1 Introduction

The programming assignment asks me to implement two linear time selection algorithms including random selection and deterministic algorithm. The goal is clear, to gain experience in implementing these two sorting algorithms to find the n-th small number in a given random array.

Second, the assignment asks me to study the performance of these 2 studies by studying their time efficiency. In lecture slides 5, professor gives a summary for the time required to complete the selection.

## Average Runtime Analysis

$$\begin{aligned} E[\text{runtime}] &\leq E \left[ \sum_j X_j \cdot c \cdot \left(\frac{3}{4}\right)^j n \right] \\ &= cn \sum_j \left(\frac{3}{4}\right)^j E[X_j] \leq 2cn \sum_j \left(\frac{3}{4}\right)^j \leq 2cn \frac{1}{1 - \frac{3}{4}} \\ &= 8cn = O(n) \end{aligned}$$

Figure 1: Random Selection

## Proof $T(n) = O(n)$

- Claim: suppose there exists a positive constant  $c$  such that
  1.  $T(1) \leq c$
  2.  $T(n) \leq cn + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right)$Then  $T(n) \leq 10cn$
- Proof by induction
  - Base case:  $T(1) \leq 10c$
  - Inductive step: inductive hypothesis  $T(k) \leq 10ck, \forall k < n$ .  
Then
$$T(n) \leq cn + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) \leq cn + 2cn + 7cn = 10cn$$

Dselect runs in linear time

Figure 2: Deterministic Selection

However, we need to test the algorithms' time efficiency by ourselves, so I wrote a cpp file to print out the time required for each algorithm, which is included in appendix part.

# 2 Result

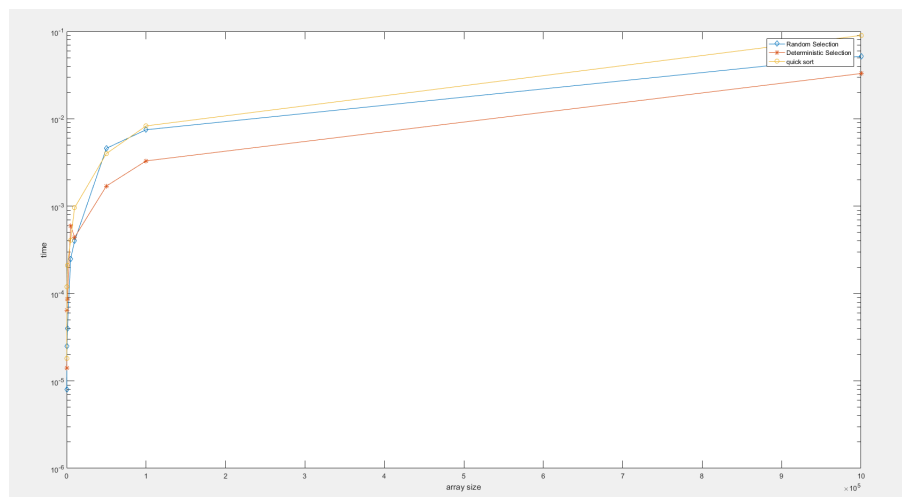
To test time efficiency, I used clock() function to get the time required. To get rid of other disturbing factor, I used to variable start and stop to get the time right before and

after the sorting complete, then their difference is the time used. In my analysis, I get 10 set of data ,from which the sorted array' size is 100, 500, 1000, 5000, 10000, 50000, 100000, 1000000.

Data Size	100	500	1000	5000	10000	50000	100000	1000000
Random Selection	8e-6	2.5e-5	4e-5	2.5e-4	4e-4	4.6e-3	7.5e-3	5.2e-2
Deterministic Selection	1.4e-5	6.4e-4	8.6e-4	6e-4	4.4e-4	1.7e-3	3.3e-3	3.3e-1
Quick Sort (in place)	1.8e-5	1.2e-4	2.1e-4	4.1e-4	9.6e-4	4.0e-3	8.3e-3	9e-2

### 3 Conclusion

This chart is plot by matlab, showing the time efficiency of each algorithms compared to each other, and it has following characteristics.



1. The average time required for each algorithm grows as the array grows bigger.
2. These three algorithm runs in a linear time when the array is big
3. As shown in the graph and lecture slides, deterministic selection requires the more time than the random selection and they're all quicker than quick sort when the array is big. Also these three ways are all Big O complexity.

### 4 Appendix

The appendix shows the cpp code for main project and time comparison.

```

1  #include <iostream>
2  #include <cstdlib>
3  #include <ctime>
4  using namespace std;
5  int partition(int*n,int left,int right){
6      int a,i=left,j=0,k=0,p;
7      p=rand()%(right-left+1)+left;
8      if(p>left){
9          n[left]=n[left]+n[p];
10         n[p]=n[left]-n[p];
11         n[left]=n[left]-n[p];
12     }
13     i=left+1;j=right;
14     while(1)
15     {
16         while(n[i]<n[left]&&i<right)i++;
17         while(n[j]>=n[left]&&j>left)j--;
18         if(j>i){
19             n[i]=n[i]+n[j];
20             n[j]=n[i]-n[j];
21             n[i]=n[i]-n[j];
22         }
23         else {
24             if(j>left){
25                 n[left]=n[left]+n[j];
26                 n[j]=n[left]-n[j];
27                 n[left]=n[left]-n[j];
28             }
29             break;
30         }
31     }
32     p=j;
33     return p;
34 }

```

```

35 int insertion(int n[],int N,int order){
36     int i,j,k,t;
37     for(int j=1;j<N;j++){
38         t=n[j];
39         for(i=0;i<=j-1;i++){
40             if(n[i]>t){
41                 for(k=j;k>i;k--)n[k]=n[k-1];
42                 n[i]=t;
43                 break;
44             }
45         }
46     }
47     return n[order];
48 }
49 int RanSel(int*n,int N,int order){
50     int pivot,i;
51     if(N==1)return n[0];
52     pivot=partition(n,0,N-1);
53     if(pivot==order) return n[pivot];
54     if(pivot>order) return RanSel(n,pivot,order);
55     else return RanSel(n+pivot+1,N-pivot-1,order-pivot-1);
56 }
57 int DelSel(int*n,int N,int order){
58     int k=N/5,j=N%5,p,i=0,left=0,right=N-1,answer;
59     if(j!=0)k++;
60     if(N==1)return n[0];
61     int c[k];
62     for(p=0;p<k;p++){
63         if((i+4)>=N)c[p]=insertion(n+i,j,j/2);
64         else c[p]=insertion(n+i,5,2);
65         i+=5;
66     }
67     p=DelSel(c,k,N/10);
68     for(i=0;i<N;i++)if(n[i]==p)break;
69     p=i;
70     if(p>left){
71         n[left]=n[left]+n[p];
72         n[p]=n[left]-n[p];
73         n[left]=n[left]-n[p];
74     }
75     i=left+1;j=right;
76     while(1)
77     {
78         while(n[i]<n[left]&&i<right)i++;

```

```

79     while(n[j]>=n[left]&& j>left)j--;
80     if(j>i){
81         n[i]=n[i]+n[j];
82         n[j]=n[i]-n[j];
83         n[i]=n[i]-n[j];
84     }
85     else {
86         if(j>left){
87             n[left]=n[left]+n[j];
88             n[j]=n[left]-n[j];
89             n[left]=n[left]-n[j];
90         }
91         break;
92     }
93 }
94 p=j;
95 if(p==order) return n[p];
96 if(p>order) return DelSel(n,p,order);
97 else return DelSel(n+p+1,N-p-1,order-p-1);
98 }
99 }
100
101
102
103 int main(){
104     srand(time(NULL));
105     int type,N,order;
106     cin>>type>>N>>order; //Algorithm typer and N numbers
107     int *n=new int[N],i,j,k,t;
108     for(i=0;i<N;i++)cin>>n[i];
109     if(type==0)cout<<"The order-"<<order<<" item is "<<RanSel(n,N,order);
110     else cout<<"The order-"<<order<<" item is "<<DelSel(n,N,order);
111     delete[] n;
112     return 0;
113 }

```

Then the appendix shows the cpp code for each sort algorithm

```

1  #include <iostream>
2  #include <cstdlib>
3  #include <ctime>
4  using namespace std;
5  int insertion(int n[],int N,int order){
6      int i,j,k,t;
7      for(int j=1;j<N;j++){
8          t=n[j];
9          for(i=0;i<=j-1;i++){
10             if(n[i]>t){
11                 for(k=j;k>i;k--)n[k]=n[k-1];
12                 n[i]=t;
13                 break;
14             }
15         }
16     }
17     return n[order];
18 }
19 int partition(int*n,int left,int right){
20     int a,i=left,j=0,k=0,p;
21     p=rand()%(right-left+1)+left;
22     if(p>left){
23         n[left]=n[left]+n[p];
24         n[p]=n[left]-n[p];
25         n[left]=n[left]-n[p];
26     }
27     i=left+1;j=right;
28     while(1)
29     {
30         while(n[i]<n[left]&&i<right)i++;
31         while(n[j]>=n[left]&&j>left)j--;
32         if(j>i){
33             n[i]=n[i]+n[j];
34             n[j]=n[i]-n[j];
35             n[i]=n[i]-n[j];
36         }
37         else {
38             if(j>left){
39                 n[left]=n[left]+n[j];
40                 n[j]=n[left]-n[j];
41                 n[left]=n[left]-n[j];
42             }
43             break;
44         }
45     }

```

```

46     p=j;
47     return p;
48 }
49 void quicksort(int*n,int left,int right){
50     int pivotat;
51     if(left>=right)return;
52     pivotat=partition(n,left,right);
53     quicksort(n,left,pivotat-1);
54     quicksort(n,pivotat+1,right);
55 }
56 int RanSel(int*n,int N,int order){
57     int pivot,i;
58     if(N==1)return n[0];
59     pivot=partition(n,0,N-1);
60     if(pivot==order) return n[pivot];
61     if(pivot>order) return RanSel(n,pivot,order);
62     else return RanSel(n+pivot+1,N-pivot-1,order-pivot-1);
63 }
64 int DelSel(int*n,int N,int order){
65     int k=N/5,j=N%5,p,i=0,left=0,right=N-1,answer;
66     if(j!=0)k++;
67     if(N==1)return n[0];
68     int c[k];
69     for(p=0;p<k;p++){
70         if((i+4)>=N)c[p]=insertion(n+i,j,j/2);
71         else c[p]=insertion(n+i,5,2);
72         i+=5;
73     }
74     p=DelSel(c,k,N/10);
75     for(i=0;i<N;i++)if(n[i]==p)break;
76     p=i;
77     if(p>left){
78         n[left]=n[left]+n[p];
79         n[p]=n[left]-n[p];
80         n[left]=n[left]-n[p];
81     }
82     i=left+1;j=right;
83     while(1)
84     {
85         while(n[i]<n[left]&&i<right)i++;
86         while(n[j]>=n[left]&&j>left)j--;
87         if(j>i){
88             n[i]=n[i]+n[j];
89             n[j]=n[i]-n[j];
90             n[i]=n[i]-n[j];

```



```

46     p=j;
47     return p;
48 }
49 void quicksort(int*n,int left,int right){
50     int pivotat;
51     if(left>=right)return;
52     pivotat=partition(n,left,right);
53     quicksort(n,left,pivotat-1);
54     quicksort(n,pivotat+1,right);
55 }
56 int RanSel(int*n,int N,int order){
57     int pivot,i;
58     if(N==1)return n[0];
59     pivot=partition(n,0,N-1);
60     if(pivot==order) return n[pivot];
61     if(pivot>order) return RanSel(n,pivot,order);
62     else return RanSel(n+pivot+1,N-pivot-1,order-pivot-1);
63 }
64 int DelSel(int*n,int N,int order){
65     int k=N/5,j=N%5,p,i=0,left=0,right=N-1,answer;
66     if(j!=0)k++;
67     if(N==1)return n[0];
68     int c[k];
69     for(p=0;p<k;p++){
70         if((i+4)>=N)c[p]=insertion(n+i,j,j/2);
71         else c[p]=insertion(n+i,5,2);
72         i+=5;
73     }
74     p=DelSel(c,k,N/10);
75     for(i=0;i<N;i++)if(n[i]==p)break;
76     p=i;
77     if(p>left){
78         n[left]=n[left]+n[p];
79         n[p]=n[left]-n[p];
80         n[left]=n[left]-n[p];
81     }
82     i=left+1;j=right;
83     while(1)
84     {
85         while(n[i]<n[left]&&i<right)i++;
86         while(n[j]>=n[left]&&j>left)j--;
87         if(j>i){
88             n[i]=n[i]+n[j];
89             n[j]=n[i]-n[j];
90             n[i]=n[i]-n[j];

```

```

136         cout<<"type= "<<type<<"average time="<<T[type]/k<<endl;
137     }
138     for(k=0;k<50;k++)if(a0[k]!=a1[k]||a1[k]!=a2[k]||a0[k]!=a2[k]){
139         cout<<"wrong k="<<k<<" a0[k]= "<<a0[k]<<" a1[k]= "<<a1[k]<<" a2[k]= "<<a2[k]<<endl;
140         cout<<"lasta0="<<a0[k-1]<<" next="<<a0[k+1]<<endl;
141         cout<<"lasta1="<<a1[k-1]<<" next="<<a1[k+1]<<endl;
142         cout<<"lasta2="<<a2[k-1]<<" next="<<a2[k+1]<<endl;
143     }
144     delete[] n;
145     delete[] b;
146     return 0;
147 }

```