UM-SJTU JOINT INSTITUTE
DISCRETE MATHEMATICS
(VE203)

ASSIGNMENT 1

Name: Pan Chongdan
ID: 516370910121

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1 Q1

A	B	¬(A∧B)	(¬A∨¬B)	¬(A∧B)⇔(¬A∨¬B)	¬(A∨B)	(¬A∧¬B)	¬(A∨B)⇔(¬A∧¬B)
T	T	F	F	T	F	F	T
T	F	T	T	T	F	F	T
F	T	T	T	T	F	F	T
F	F	T	T	T	T	T	T

Table 1: Truth Table

2 Q2

1.
$$A \cap B \Leftrightarrow \{x|x \in A \wedge x \in B\}$$
$$M \setminus (A \cap B) \Leftrightarrow \{x|x \in M \wedge (x \notin (A \cap B)\}$$

$$M \setminus A \Leftrightarrow \{x|x \in M \wedge x \notin A\}$$
$$M \setminus B \Leftrightarrow \{x|x \in M \wedge x \notin B\}$$
$$(M \setminus A) \cup (M \setminus B) \Leftrightarrow \{x|(x \in M \wedge x \notin A) \vee (x \in M \wedge x \notin B)\}$$
$$\Leftrightarrow \{x|x \in M \wedge (x \notin A \vee x \notin B)\} \Leftrightarrow \{x|x \in M \wedge (x \notin (A \cap B)\}$$
$$\therefore M \setminus (A \cap B) = (M \setminus A) \cup (M \setminus B)$$

2.
$$A \cup B \Leftrightarrow \{x|x \in A \vee x \in B\}$$
$$M \setminus (A \cup B) \Leftrightarrow \{x|x \in M \wedge x \notin A \wedge x \notin B\}$$

$$M \setminus A \Leftrightarrow \{x|x \in M \wedge x \notin A\}$$
$$M \setminus B \Leftrightarrow \{x|x \in M \wedge x \notin B\}$$
$$(M \setminus A) \cap (M \setminus B) \Leftrightarrow \{x|(x \in M \wedge x \notin A) \wedge (x \in M \wedge x \notin B\}$$
$$\Leftrightarrow \{x|x \in M \wedge x \notin A \wedge x \notin B\}$$
$$\therefore M \setminus (A \cup B) = (M \setminus A) \cap (M \setminus B)$$

3 Q3

(i) $(A \Rightarrow (B \Rightarrow C)) \Rightarrow (B \Rightarrow (A \Rightarrow C))$ is a tautology.

A	B	C	A⇒(B⇒C)	B⇒(A⇒C)	(A⇒(B⇒C))⇒(B⇒(A⇒C))
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Table 2: Truth Table

(ii) $((A \vee B) \wedge (A \vee C)) \Rightarrow (B \vee C)$ is not a tautology.

A	B	C	(A∨B)∧(A∨C)	B∨C	((A∨B)∧(A∨C))⇒(B∨C)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	F	T

Table 3: Truth Table

(iii) $A \Rightarrow (\neg B) \Rightarrow B \Rightarrow (\neg A)$ is a tautology.

A	B	A⇒(¬B)	B⇒(¬A)	A⇒(¬B)⇒B⇒(¬A)
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

Table 4: Truth Table

4 Q4

Since we need to take the disjunction of conjunctions of the variables or their negations, there must at least two variables. If the disjunctive normal form of the first two variables is true, then we don't need to consider the remaining part. The proposition can be

$$(A_1 \wedge A_2) \vee (\neg A_1 \wedge \neg A_2) \vee (\neg A_1 \wedge A_2) \vee (A_1 \wedge \neg A_2) \vee \cdots$$

It is always true no matter what the remaining part is.

5 Q5

1. $$A \wedge B \Leftrightarrow \neg((\neg A) \vee (\neg B))$$

2. $$A \Rightarrow B \Leftrightarrow (\neg A) \vee B$$

3. $$A \Leftrightarrow B \Leftrightarrow (\neg((\neg A) \vee (\neg B))) \vee (\neg(A \vee B))$$

6 Q6

(i)

$$X \triangle Y = (X \cup Y) \setminus (X \cap Y)$$

$$(X \cup Y) \setminus (X \cap Y) = (X \setminus (X \cap Y)) \cup (Y \setminus (X \cap Y))$$

$$X \setminus (X \cap Y) \Leftrightarrow \{x|x \in X \wedge \neg(x \in X \wedge x \in Y)\} \Leftrightarrow \{x|x \in X \wedge (x \notin X \vee x \notin Y)\}$$

$$\Leftrightarrow \{x|(x \in X \wedge x \notin X) \vee (x \in X \wedge x \notin Y)\} \Leftrightarrow \{x|(x \in X \wedge x \notin Y)\} \Leftrightarrow X \setminus Y$$

Similarly,

$$Y \setminus (X \cap Y) = Y \setminus X$$

$$(X \setminus (X \cap Y)) \cup (Y \setminus (X \cap Y)) \Leftrightarrow (X \setminus Y) \cup (Y \setminus X)$$

$$\therefore X \triangle Y = (X \setminus Y) \cup (Y \setminus X)$$

(ii)

$$(M \setminus X) \triangle (M \setminus Y) = ((M \setminus X) \setminus (M \setminus Y)) \cup ((M \setminus Y) \setminus (M \setminus X))$$

$$(M \setminus X) \setminus (M \setminus Y) \Leftrightarrow \{x|(x \in M \wedge \neg x \in X) \wedge \neg(x \in M \wedge \neg x \in Y)\}$$

$$\Leftrightarrow \{x|(x \in M \wedge \neg x \in X) \wedge (\neg x \in M \vee x \in Y)\} \Leftrightarrow \{x|x \in M \wedge x \in Y \wedge \neg x \in X\} \Leftrightarrow Y \setminus X$$

Similarly,

$$(M \setminus Y) \setminus (M \setminus X) = X \setminus Y$$

$$\therefore (M \setminus X) \triangle (M \setminus Y) = (Y \setminus X) \cup (X \setminus Y) = X \triangle Y$$

(iii)

$$X \setminus Y \Leftrightarrow \{x|x \in X \wedge \neg x \in Y\} \Leftrightarrow X \cap Y^c$$

Similarly

$$Y \setminus X \Leftrightarrow Y \cap X^c$$

$$X \triangle Y = (X \setminus Y) \cup (Y \setminus X) = (X \cap Y^c) \cup (Y \cap X^c) = (X \cup Y) \cap (X^c \cup Y^c)$$

$$(X \triangle Y) \triangle Z = ((X \triangle Y) \cup Z) \cap ((X \triangle Y)^c \cup Z^c)$$

$$= (((X \cup Y) \cap (X^c \cup Y^c)) \cup Z) \cap (((X^c \cup Y) \cap (X \cup Y^c)) \cup Z^c)$$

$$= (X \cup Y \cup Z) \cap (X^c \cup Y \cup Z) \cap (X \cup Y^c \cup Z) \cap (X \cup Y \cup Z^c)$$

Since its symmetric about X, Y, Z,,similarly,

$$X \triangle (Y \triangle Z) = (X \cup Y \cup Z) \cap (X^c \cup Y \cup Z) \cap (X \cup Y^c \cup Z) \cap (X \cup Y \cup Z^c)$$

$$\therefore (X \triangle Y) \triangle Z = X \triangle (Y \triangle Z)$$

(iv)

$$X \cap (Y \triangle Z) = X \cap ((Y \setminus Z) \cup (Z \setminus Y)) = (X \cap (Y \setminus Z)) \cup (X \cap (Z \setminus Y))$$

$$= ((X \cap Y) \setminus (X \cap Z)) \cup ((X \cap Z) \setminus (X \cap Y))$$

$$(X \cap Y) \triangle (X \cap Z) = ((X \cap Y) \setminus (X \cap Z)) \cup ((X \cap Z) \setminus (X \cap Y))$$

$$\therefore X \cap (Y \triangle Z) = (X \cap Y) \triangle (X \cap Z)$$

7 Q7

(i)

$$x \in X \triangle Y = \{x|x \in (A \setminus B) \cup (B \setminus A)\} \Leftrightarrow \{x|(x \in A \wedge \neg x \in B) \vee (x \in B \wedge \neg x \in A)\}$$

$A(x)$	$B(x)$	$A(x) \wedge \neg B(x)$	$B(x) \wedge \neg A(x)$	$A(x) \oplus B(x)$	$x \in X \triangle Y$
T	T	F	F	F	F
T	F	T	F	T	T
F	F	F	F	F	F
F	T	F	T	T	T

Table 5: Truth Table

$$\therefore x \in X \triangle Y \Leftrightarrow A(x) \oplus B(x)$$

(ii)

$$\neg((A(x) \cap B(x)) \cup (\neg A(x) \cap \neg B(x))) \Leftrightarrow A(x) \oplus B(x)$$

(iii) It's a valid argument because $(A(x) \oplus B(x)) \wedge (B(x) \oplus C(x)) \Rightarrow \neg(A(x) \oplus C(x))$ is always true according the following truth table.

A	B	C	$A(x) \oplus B(x)$	$B(x) \oplus C(x)$	$(A(x) \oplus B(x)) \wedge (B(x) \oplus C(x))$	$\neg(A(x) \oplus C(x))$
T	T	T	F	F	F	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	F	F
F	F	F	F	F	F	T

Table 6: Truth Table

8 Q8

$$\begin{aligned}
\exists x(P(x) \Rightarrow Q(x)) &\Leftrightarrow \exists x(\neg P(x) \vee Q(x)) \Leftrightarrow \exists x \neg P(x) \vee \exists x Q(x) \\
&\Leftrightarrow (\neg \forall x P(x)) \vee (\exists x Q(x)) \Leftrightarrow (\forall x P(x)) \Rightarrow (\exists x Q(x)) \\
&\therefore \exists x(P(x) \Rightarrow Q(x)) \Leftrightarrow (\forall x P(x)) \Rightarrow (\exists x Q(x))
\end{aligned}$$

9 Q9

$$\begin{aligned}
T = (A \cap B) \cup ((M \setminus A) \cap (M \setminus B)) = (A \cap B) \cup (M \setminus (A \cup B)) \\
M \setminus T = (M \setminus (A \cap B)) \cap (A \cup B) \supseteq (M \setminus (A \cap B)) \cap B \\
= ((M \setminus A) \cup (M \setminus B)) \cap B \supseteq (M \setminus A) \cap B \\
\therefore (M \setminus A) \cap B \subseteq M \setminus T
\end{aligned}$$

10 Q10

(i) The truth tables:

A	B	$A \vert B$	$A \downarrow B$
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T

Table 7: Truth Table

(ii)

$$A \wedge B \equiv (A|B)|(A|B)$$
$$A \vee B \equiv (A|A)|(B|B)$$
$$A \Rightarrow B \equiv A|(A|B)$$
$$A \Leftrightarrow B \equiv [A|(A|B)]|[B|(B|A)]|[A|(A|B)]|[B|(B|A)]$$

(iii)

$$A \wedge B \equiv (A \downarrow A) \downarrow (B \downarrow B)$$
$$A \vee B \equiv (A \downarrow B) \downarrow (A \downarrow B)$$
$$A \Rightarrow B \equiv [(A \downarrow B) \downarrow B] \downarrow [(A \downarrow B) \downarrow B]$$
$$A \Leftrightarrow B \equiv [(A \downarrow B) \downarrow B] \downarrow [(B \downarrow A) \downarrow A]$$

(iv) No

A	B	C	$A \downarrow (B \downarrow C)$	$(A \downarrow B) \downarrow C$
T	T	T	F	F
T	T	F	F	T
T	F	T	F	F
T	F	F	F	T
F	T	T	T	F
F	T	F	T	T
F	F	T	T	F
F	F	F	F	F

Table 8: Truth Table

(v) I can prove it through truth tables

A	B	C	$(\neg A) \downarrow B$	$A|C$	$(\neg A) \downarrow B \wedge A|C$	$B \downarrow C$
T	T	T	F	F	F	F
T	T	F	F	T	F	F
T	F	T	T	F	F	F
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	T	F	F	T	F	F
F	F	T	F	T	F	F
F	F	F	F	T	F	T

Table 9: Truth Table

$((\neg A) \downarrow B \wedge A|C) \Rightarrow B \downarrow C$ is true, so,it's valid