**Q1.** Consider
$$A = \{4k + 3 \mid k \in \mathbb{N}\}$$
Prove that $A$ contains infinitely many prime numbers.
**(2 marks)**

**Q2.** Let $m \in \mathbb{N}$ with $m > 1$. Prove that if $ac \equiv bc \pmod{m}$, then
$$a \equiv b \left(\text{mod } \frac{m}{\gcd(c, m)}\right)$$
**(2 marks)**

**Q3.** Find a solution to the linear congruence $36x \equiv 75 \pmod{1309}$.
**(2 marks)**

**Q4.** Find all solutions to the system:
$$\begin{aligned} 2x &\equiv 4 \pmod{5} \\ 3x &\equiv 5 \pmod{7} \\ 7x &\equiv 2 \pmod{13} \end{aligned}$$
**(2 marks)**

**Q5.** Find all, if any, solutions to the system:
$$\begin{aligned} x &\equiv 5 \pmod{6} \\ x &\equiv 3 \pmod{10} \\ x &\equiv 8 \pmod{15} \end{aligned}$$
**(2 marks)**

**Q6.** Find all, if any, solutions to the system:
$$\begin{aligned} x &\equiv 5 \pmod{5} \\ x &\equiv 3 \pmod{7} \\ x &\equiv 8 \pmod{11} \\ x &\equiv 2 \pmod{17} \end{aligned}$$
**(2 marks)**

**Q7.** Show that $n \log_2(n)$ is not $O(\log_2(n))$.
**(1 mark)**

**Q8.** Show that $\log_2(n)$, $\log_{10}(n)$ and $\ln(n)$ all have the same order.
**(1 mark)**

**Q9.** For all $n \in \mathbb{N}\backslash\{0\}$ define

$$H(n) = \sum_{k=0}^{n-1} \frac{1}{n-k}$$

(i) Show that

$$\sum_{j=2}^{n} \frac{1}{j} < \int_{1}^{n} \frac{1}{x} dx$$

(ii) Prove that $H(n)$ is $O(\ln(n))$.

**(4 marks)**

**Q10.** Show that $\lfloor x^3 - 4 \rfloor$ is order $x^3$.
**(1 mark)**

**Q11.** Let $k > 1$. Do $n^n$ and $n^{n-k}$ have the same order?
**(2 marks)**