VE281

Data Structures and Algorithms

Recitation Class

Oct.22 2018

VE281 TA Group

Bloom Filter Implementation: Components

- An array of n **bits**. Each bit 0 or 1
  - $n = b|S|$, where b is small real number. For example, $b \approx 8$ for 32-bit IP address (That's why it is space efficient)

- k hash functions $h_1, \ldots, h_k$, each mapping inside $\{0, 1, \ldots, n-1\}$.
  - k usually small.

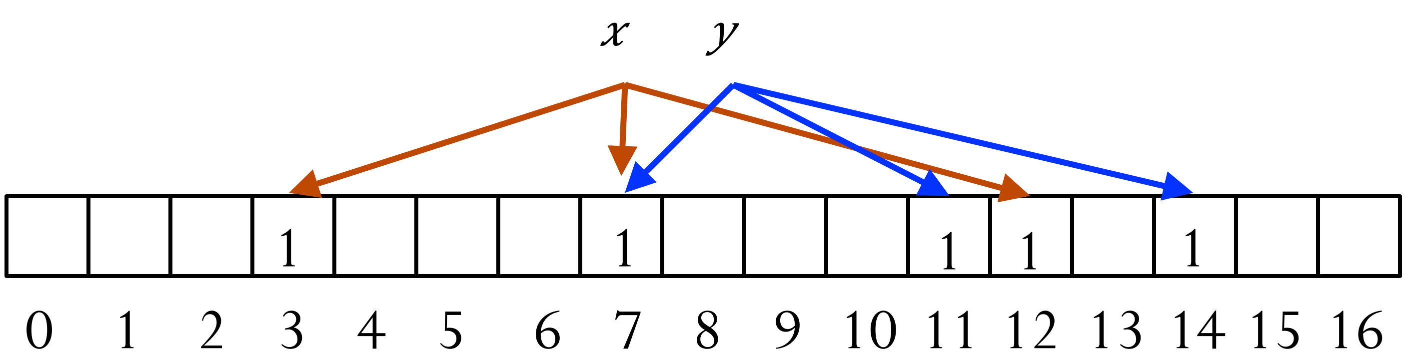
Bloom Filter Insert

- Initially, the array is all-zero.
- Insert x: For $i=1,2,\ldots,k$, set $A[h_i(x)]=1$
  - No matter whether the bit is 0 or 1 before

<u>Example</u>: $n=17$, 3 hash functions

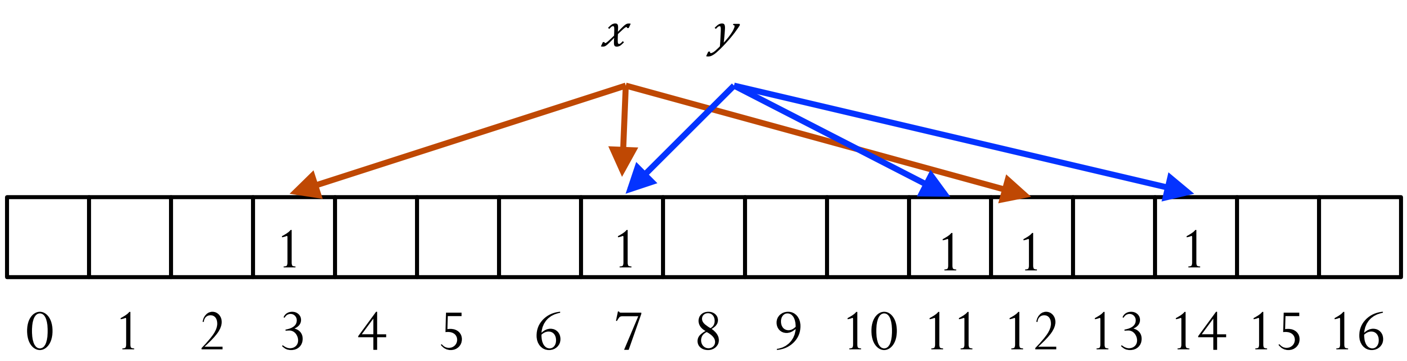
$h_1(x)=7, h_2(x)=3, h_3(x)=12$

$h_1(y)=11, h_2(y)=14, h_3(y)=7$



Bloom Filter Find

- Find x: return true if and only if $A[h_i(x)]=1, \forall i=1,\ldots,k$



Suppose $h_1(x)=7, h_2(x)=3, h_3(x)=12$. Find x? Yes!

Suppose $h_1(z)=3, h_2(z)=11, h_3(z)=5$. Find z? No!

- No false negative: if x was inserted, find(x) guaranteed to return true
- False positive possible: consider $h_1(w)=11, h_2(w)=12, h_3(w)=7$ in the above example

Heuristic Analysis of Error Probability

- Intuition: should be a trade-off between space (array size) and false positive probability
  - Array size decreases, more reuse of bits, false positive probability increases
- Goal: analyze the false positive probability
- Setup: Insert data set S into the Bloom filter, use k hash functions, array has n bits
- Assumption: All k hash functions map keys uniformly random and these hash functions are independent

Probability of a Slot Being 1

- For an arbitrary slot j in the array, what's the probability that the slot is 1?

- Consider when slot j is 0
  - Happens when $h_i(x) \neq j$ for all $i = 1, \ldots, k$ and $x \in S$
  - $\Pr(h_i(x) \neq j) = 1 - 1/n$
  - $\Pr(A[j] = 0) = (1 - 1/n)^{k|S|} \approx e^{-k|S|/n} = e^{-k/b}$
    - $b = n/|S|$ denotes # of bits per object
- $\Pr(A[j] = 1) \approx 1 - e^{-k/b}$

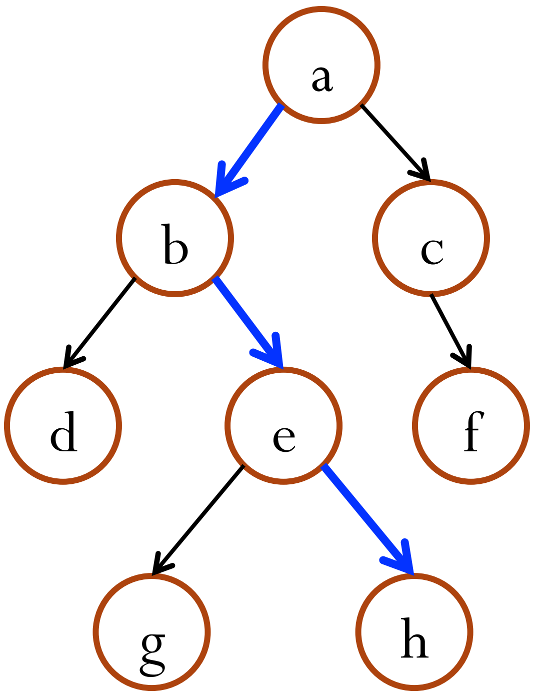
False Positive Probability

- For x not in S, the false positive probability happens when all $A[h_i(x)]=1$ for all $i=1,\ldots,k$
  - The probability is $\epsilon \approx (1-e^{-k/b})^k$
- For a fixed b, ϵ is minimized when $k=(\ln 2)\cdot b$
- The minimal error probability is $\epsilon \approx (1/2)^{\ln 2 \cdot b} \approx 0.6185^b$
  - Error probability decreases exponentially with b
- Example: $b=8$, could choose k as 5 or 6. Min error probability ≈ 2%

Depth, Level, and Height of a Node

- The **depth** or **level of a node** is the length of the unique path from the **root** to the node.
  - E.g., depth(b)=1, depth(a)=0.

- The **height of a node** is the length of the **longest** path from the node to a **leaf**.
  - E.g., height(b)=2, height(a)=3.
  - All leaves have height zero.



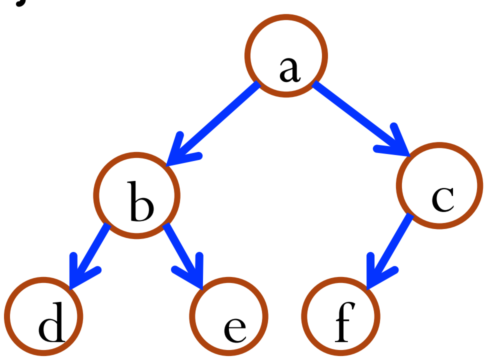
Binary Tree Traversal Methods

- Depth-first traversal
  - Pre-order
    - Center - Left -Right
  - Post-order
    - Left – Right -Center
  - In-order
    - Left-Center-Right
- Level-order traversal
  - Use the queue structure

Level-Order Traversal

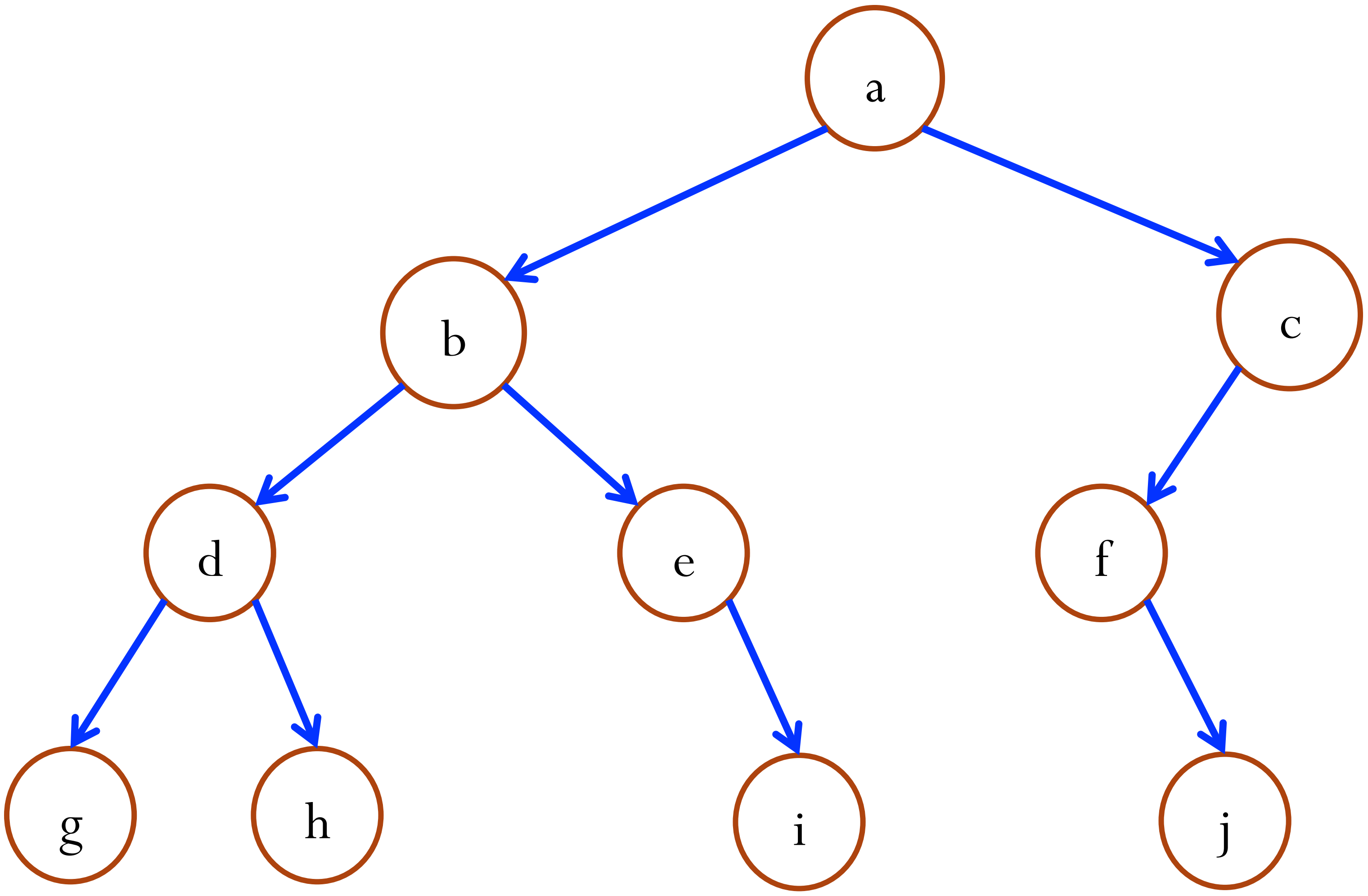
Code and Example

```
void levelOrder(node *root) {
  queue q; // Empty queue
  q.enqueue(root);
  while(!q.isEmpty()) {
    node *n = q.dequeue();
    visit(n);
    if(n->left) q.enqueue(n->left);
    if(n->right) q.enqueue(n->right);
  }
}
```



Queue: | a | b | c | d | e | f |

Output: a b c d e f


a
b
c
d
e
f
g
h
i
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