UM-SJTU JOINT INSTITUTE
DISCRETE MATHEMATICS
(VE203)

ASSIGNMENT 9

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1 Q1

(i) The sequence is (0,4,9,27,$\cdots$)
So the formula

$$g(n) = \begin{cases} 0, n = 1 \\ 4, n = 2 \\ 3^{n-1}, n \geq 3 \end{cases} \tag{1}$$

(ii) The sequence is (0,0,3,0,$\frac{9}{2}$, $\cdots$)
So the formula

$$g(n) = \begin{cases} 0, n = 1 \vee n \quad \text{is} \quad \text{even} \\ \frac{3^{n-2}}{(n-2)!}, n = 2k+1, k \in \mathbb{Z}^+ \end{cases} \tag{2}$$

(iii) The sequence is (0,1,-1,0,$\cdots$)
So the formula

$$g(n) = \begin{cases} 0, n = 3k \\ 1, n = 3k+1 \\ -1, n = 3k+2, k \in \mathbb{Z} \end{cases} \tag{3}$$

2 Q2

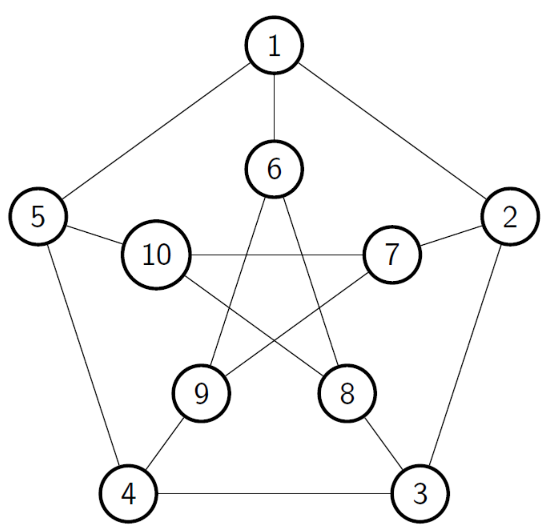


Figure 1: Petersen Graph

Since the degree of all vertex is 3, and we can only get in and out of one vertex once, so every vertex must have exactly two edges in the Hamilton circuit, assume there exists a Hamilton circuit E_H includes all the edges of the Hamilton circuit.

1. If $\{1, 2\} \notin E_H \Rightarrow \{1, 5\}, \{1, 6\}, \{2, 7\}, \{2, 3\} \in E_H$
   (a) Then if $\{3, 8\} \in E_H \Rightarrow \{4, 9\}, \{5, 4\} \in E_H \Rightarrow \{10, 8\}, \{10, 7\} \in E_H \Rightarrow \{9, 6\} \in E_H$. So the circuit become $\{1 \to 5 \to 4 \to 9 \to 6 \to 1\}$, which doesn't pass every vertex.
   (b) If $\{3, 4\} \in E_H \Rightarrow \{8, 10\}, \{8, 6\} \in E_H \Rightarrow \{9, 4\}, \{9, 7\} \in E_H \Rightarrow \{10, 5\} \in E_H$. So the circuit become $\{1 \to 5 \to 10 \to 8 \to 6 \to 1\}$, which also doesn't pass every vertex.
2. If $\{1, 6\} \notin E_H \Rightarrow \{1, 5\}, \{1, 2\}, \{6, 9\}, \{6, 8\} \in E_H$
   (a) Then if $\{5, 4\} \in E_H \Rightarrow \{10, 8\}, \{10, 7\} \in E_H \Rightarrow \{3, 2\}, \{3, 4\} \in E_H \Rightarrow \{3, 6\} \in E_H$. So the circuit become $\{1 \to 5 \to 4 \to 3 \to 2 \to 1\}$, which doesn't pass every vertex.
   (b) If $\{5, 10\} \in E_H \Rightarrow \{4, 9\}, \{4, 3\} \in E_H \Rightarrow \{7, 10\}, \{7, 2\} \in E_H$. So the circuit become $\{1 \to 5 \to 10 \to 7 \to 2 \to 1\}$, which also doesn't pass every vertex.

So we can never find a Hamilton circuit passing through vertex 1.

If we cut vertex 1 from the Graph then the circuit can be $\{2 \to 3 \to 4 \to 5 \to 10 \to 8 \to 6 \to 9 \to 7 \to 2\}$

If we cut vertex 6 from the Graph then the circuit can be $\{1 \to 2 \to 7 \to 9 \to 4 \to 3 \to 8 \to 10 \to 5 \to 1\}$

3 Q3

Yes

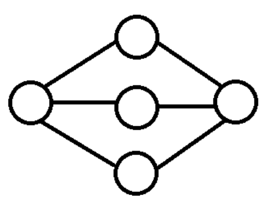
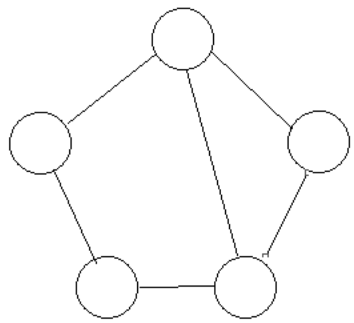


Figure 2: The graph doesn't have a Hamilton circuit

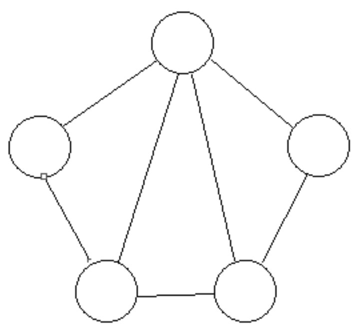
4 Q4

(i) Yes



(ii) No, since there are two vertexes whose degrees are 4, then there can't exist a vertex with degree=1

(iii) Yes



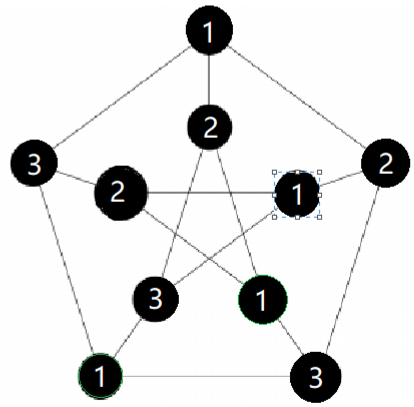
(iv) No, because the sum must be even

5 Q5

It's clear that for a graph whose order is n and only has one connected components, it's minimal size is $e \geq n$. If we break it into to connected two components, we must cut off at least one wedge $\therefore e_1 \geq e - 1 \geq n - 1$. Similarly, if we break it into n connected components, we must cut off at least k wedge $\therefore e_k \geq e - k \geq n - k \Rightarrow k \geq n - e_k$

6 Q6

The least number is 3



7 Q7

(i) Assume $V = \{v_1, v_2 \cdots, v_n\}$ and $\forall e = \{v_a, v_b\} \in E$.
Let X be the set of all sets containing two elements of $\mathbb{N}$

For example, if $G = (V, E), V = \{1, 2, 3, 4\}, E = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{3, 4\}\}$ $X = \{\{1, 2\}, \{1, 3\}, \{1, 4\} \cdots \{3, 4\} \cdots\}$. According to G, we know that vertex 1 is connected to vertex 2,3,4 ,so we use $\{\{1, 2\}, \{1, 3\}, \{1, 4\}\}$ to donate the bijection of vertex 1. Similarly $\{\{1, 2\}, \{2, 3\}\}$ can donate vertex 2. All these donation can form χ which is a subset of $P(X)$ Then $\{x, y\} = \{\{\{1, 2\}, \{1, 3\}, \{1, 4\}\}, \{\{1, 2\}, \{2, 3\}\}\} \in E_\chi$ can donate the wedge. $\therefore H \cong G_\chi$

(ii) In the last example $|X| = 3 + 2 + 1 = 6, n = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$

8 Q8

(i)

$$f(n)a_n = g(n)a_{n-1} + h(n)$$

$$Q(n)f(n)a_n = b_{n-1} + Q(n)h(n)$$

$$Q(n)f(n) = \frac{f(1)f(2)\cdots f(n)}{g(1)g(2)\cdots g(n+1)}g(n+1) = g(n+1)Q(n+1)$$

$$\therefore b_n = b_{n+1} + Q(n)h(n)$$

(ii)

$$b_0 = g(1)Q(1)C \Rightarrow b_n = g(1)Q(1)C + \sum_{i=1}^{n} Q(i)h(i)$$

$$a_n = \frac{b_n}{g(n+1)Q(n+1)}, Q(1) = \frac{1}{g(1)}$$

$$\therefore a_n = \frac{C + \sum_{i=1}^{n} Q(i)h(i)}{g(n+1)Q(n+1)}$$

(iii)

$$C = 1, f(n) = 1, h(n) = n, g(n) = n + 3$$

$$Q(n) = \frac{6}{(n+3)!}$$

$$a_n = \frac{1 + \sum_{i=1}^{n} \frac{6n}{(n+3)!}}{\frac{6(n+4)}{(n+4)!}} = \frac{(n+3)! + (n+3)! \sum_{i=1}^{n} \frac{6n}{(n+3)!}}{6}$$