Vv256
Dr Jing Liu

JOINT INSTITUTE
交大密西根学院

Assignment 0
Due: Sept. 21, 2017

Question1 (4 points)

Compute the followings, where $\mathrm{i} = \sqrt{-1}$, and express in the rectangular form

$$x + \mathrm{i}y, \qquad \text{where } x \text{ and } y \text{ are real.}$$

(a) (1 point) $(6 - \mathrm{i}\sqrt{2})(2 + \mathrm{i}4)$

(b) (1 point) $\dfrac{1+\mathrm{i}}{1-\mathrm{i}}$

(c) (1 point) $|4 - \mathrm{i}5|$

(d) (1 point) $\mathrm{Re}(4 - \mathrm{i}5) + \mathrm{i}\,\mathrm{Im}(6 + \mathrm{i}2)$
where $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ denote the real and imaginary part of z, respectively.

Question2 (2 points)

Compute the followings, where $\mathrm{i} = \sqrt{-1}$, and express in the polar form

$$r(\cos\theta + \mathrm{i}\sin\theta), \qquad \text{where } r \geq 0 \text{ and } 0 \leq \theta < 2\pi.$$

(a) (1 point) $(1 + \mathrm{i})^2$

(b) (1 point) $(1 + \mathrm{i})^8$

Question3 (1 points)

Find all complex solutions of the following equation.

$$x^2 + 2x + 5 = 0$$

Question4 (1 points)

Prove that, for any complex numbers z and w,

$$|z + w|^2 + |z - w|^2 = 2(|z|^2 + |w|^2)$$

Question5 (0 points)

(a) (1 point (bonus)) Let a, b, c and d be real numbers. Find the conditions under which the quadratic equation

$$z^2 + (a + ib)z + (c + id) = 0$$

has at least one real root.

(b) (1 point (bonus)) Show that the diameter of a circle subtends a right angle at the circumference using complex numbers.

(c) (1 point (bonus)) Study the definition and properties of the exponential function of a complex number. Then write the following number in the rectangular form.

$$e^{\mathrm{i}\pi + \ln 2}$$

(d) (1 point (bonus)) The nth order Chebyshev polynomial of the first kind is defined by

$$T_n(x) = \cos\left(n \cos^{-1} x\right), \qquad \text{where } n \text{ is a positive integer, and } -1 \leq x \leq 1.$$

Using De Moivre's theorem, show that $T_n(x)$ has the formal polynomial representation

$$T_n(x) = \frac{1}{2}\left\{\left[x + (x^2 - 1)^{1/2}\right]^n + \left[x - (x^2 - 1)^{1/2}\right]^n\right\}$$

Hint: One needs not be concerned with the occurrence of fractional powers of $x^2 - 1$.