Formula Sheet

- Cylindrical

$$\vec{e}_r = \frac{1}{\left|\frac{\partial \vec{r}}{\partial r}\right|}\frac{\partial \vec{r}}{\partial r} = \begin{bmatrix}\cos\theta\\ \sin\theta\\ 0\end{bmatrix}, \quad \vec{e}_\theta = \frac{1}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|}\frac{\partial \vec{r}}{\partial \theta} = \begin{bmatrix}-\sin\theta\\ \cos\theta\\ 0\end{bmatrix}, \quad \vec{e}_z = \frac{1}{\left|\frac{\partial \vec{r}}{\partial z}\right|}\frac{\partial \vec{r}}{\partial z} = \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{e}_z$$

$$\nabla \cdot \vec{F} = \frac{\partial F_r}{\partial r} + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z} + \frac{F_r}{r}$$

$$\nabla \times \vec{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial \theta} - \frac{\partial F_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\vec{e}_\theta + \left(\frac{\partial F_\theta}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial \theta} + \frac{F_\theta}{r}\right)\vec{e}_z$$

- Spherical

$$\vec{e}_\rho = \frac{1}{\left|\frac{\partial \vec{r}}{\partial \rho}\right|}\frac{\partial \vec{r}}{\partial \rho} = \frac{\partial \vec{r}}{\partial \rho}, \qquad \vec{e}_\theta = \frac{1}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|}\frac{\partial \vec{r}}{\partial \theta} = \frac{1}{\rho\sin\phi}\frac{\partial \vec{r}}{\partial \theta}, \qquad \vec{e}_\phi = \frac{1}{\left|\frac{\partial \vec{r}}{\partial \phi}\right|}\frac{\partial \vec{r}}{\partial \phi} = \frac{1}{\rho}\frac{\partial \vec{r}}{\partial \phi}$$

$$= \begin{bmatrix}\cos\theta\sin\phi\\ \sin\theta\sin\phi\\ \cos\phi\end{bmatrix} \qquad = \begin{bmatrix}-\sin\theta\\ \cos\theta\\ 0\end{bmatrix} \qquad = \begin{bmatrix}\cos\theta\cos\phi\\ \sin\theta\cos\phi\\ -\sin\phi\end{bmatrix}$$

$$\nabla f = \frac{\partial f}{\partial \rho}\vec{e}_\rho + \frac{1}{\rho\sin\phi}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\vec{e}_\phi$$

$$\nabla \cdot \vec{F} = \frac{\partial F_\rho}{\partial \rho} + \frac{1}{\rho\sin\phi}\frac{\partial F_\theta}{\partial \theta} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{2F_\rho}{\rho} + \frac{F_\phi}{\rho\tan\phi}$$

$$\nabla \times \vec{F} = \left(\frac{1}{\rho}\frac{\partial F_\theta}{\partial \phi} - \frac{1}{\rho\sin\phi}\frac{\partial F_\phi}{\partial \theta} + \frac{F_\theta}{\rho\tan\phi}\right)\vec{e}_\rho$$

$$+ \left(\frac{\partial F_\phi}{\partial \rho} - \frac{1}{\rho}\frac{\partial F_\rho}{\partial \phi} + \frac{F_\phi}{\rho}\right)\vec{e}_\theta + \left(\frac{1}{\rho\sin\phi}\frac{\partial F_\rho}{\partial \theta} - \frac{\partial F_\theta}{\partial \rho} - \frac{F_\theta}{\rho}\right)\vec{e}_\phi$$