Ve401 Probabilistic Methods in Engineering

Sample Exercises for the Second Midterm

The following exercises have been compiled from past second midterm exams of Ve401. A Second midterm will usually consist of 4-5 such exercises to be completed in 100 minutes. In the actual exam, necessary tables of values of distributions will be provided. You may use all tables in Appendix A of the textbook as well as the uploaded OC curves to solve the sample exercises.

Exercise 1. Let X be a continuous random variable with density

$$f_\theta(x) = \begin{cases} \dfrac{\theta+1}{x^{\theta+2}} & \text{for } x > 1, \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is a parameter. Find the method-of-moments estimator for the parameter θ.
(3 Marks)

Exercise 2. Let (X, f_X) be a continuous random variable following a uniform distribution on the interval $[0, \theta]$, $\theta > 0$, i.e.,

$$f_X(x) = \begin{cases} 1/\theta & 0 \le x \le \theta \\ 0 & \text{otherwise.} \end{cases}$$

i) Find the method-of-moments estimator for θ.

ii) Find the maximum-likelihood estimator for θ.

(2+3 Marks)

Exercise 3. A certain type of harddrive is known to have a lifetime given by an exponential distribution with (unknown) parameter $\beta > 0$. To estimate β, n identical and independent hard drives are tested and their times of failure recorded. After time $T > 0$ the test is stopped and $n - m$ hard drives are found to be still working.

Find the maximum likelihood estimator for β.
(4 Marks)

Exercise 4. The diameter of steel rods manufactured on two different extrusion machines is being investigated. Two random samples of $n_1 = 15$ and $n_2 = 18$ are selected, and the sample means and sample variances are $\bar{x}_1 = 8.73$, $s_1^2 = 0.34$, $\bar{x}_2 = 8.68$, and $s_2^2 = 0.30$, respectively.

i) Test $\sigma_1^2 = \sigma_2^2$ at $\alpha = 0.10$.

ii) Assuming that $\sigma_1^2 = \sigma_2^2$, test $\mu_1 = \mu_2$ using a pooled T-test.

iii) Assuming that $\sigma_1^2 = \sigma_2^2$, construct a 95% two-sided confidence interval on the difference in mean rod diameter.

(2+2+2 Marks)

Exercise 5. A soft drink bottler is studying the internal pressure strength of 1-liter glass non-returnable bottles. A random sample of 16 bottles is tested and the pressure strengths obtained. The data (in units of psi) are shown below.

226.16	202.20	219.54	193.73	208.15	195.45	193.71	200.81
211.14	203.62	188.12	224.39	221.31	204.55	202.21	201.63

i) Draw a histogram for these data.

ii) Create a boxplot, clearly identifying any near and far outliers.

iii) Do you believe the data is normally distributed? Why or why not?

iv) Assuming normality, find 95% confidence intervals for the mean pressure strength μ and the variance σ^2.

(2+2+1+2 Marks)

Exercise 6. The manufacturer of a power supply is interested in the variability of output voltage. He has tested 16 units, chosen at random, with the following results:

5.34	5.00	5.07	5.25	5.65	5.55	5.35	5.35
4.96	5.54	5.54	4.61	5.28	5.93	5.38	5.47

i) It is thought that the output voltage is normally distributed. Create a stem-and-leaf diagram for the data. Does it support this assumption?

ii) Create a histogram for the data.

iii) Create a boxplot for the data.

iv) Assuming that $\sigma^2 = 0.1$, give a 95% confidence interval for the mean.

v) Using the sample variance, use the T-distribution to give a 95% confidence interval for the mean.

vi) Test the hypothesis that $\sigma^2 = 0.1$ at the $\alpha = 1\%$ level of significance (you should formulate H_0 and H_1 explicitly).

((2 + 1) + 2 + 2 + 3 + 2 + 2 Marks)

Exercise 7. A manufacturer of precision measuring instruments claims that the standard deviation in the use of an instrument is not more than 0.00002 inch. An analyst, who is unaware if the claim, uses the instrument eight times and obtains a sample standard deviation of 0.00005 inch.

i) Using $\alpha = 0.01$, is the manufacturer's claim justified?

ii) What is the power of the test if the true standard deviation equals 0.00004 inch?

iii) What is the smallest sample size that can be used to detect a true standard deviation of 0.00004 inch or more with a probability of at least 0.95? Use $\alpha = 0.01$.

(2+1+1 Marks)

Exercise 8. The diameters of bolts are known to have a standard deviation of 0.0001 inch. A random sampe of 10 bolts yields an average diameter of 0.2546 inch.

i) Test the hypothesis that the true mean diameter of bolts equals 0.255 inch, using $\alpha = 0.05$.

ii) What size sample would be necessary to detect a true mean bolt diameter of 0.2552 inch or more with a probability of at least 0.90, assuming $\alpha = 0.05$?

(2+2 Marks)

Exercise 9. A company wants to test whether a new assembly line procedure increases the physical stress on its workers. It selects eleven workers to work for one day using each of the assembly line procedures. At the end of each day, their pulse frequency is measured:

Procedure 1	X	63	65	71	75	72	75	68	74	62	73	72
Procedure 2	Y	80	78	96	87	88	96	82	83	77	79	71

It is thought that the median pulse frequency is higher in Procedure 2 than in Procedure 1.

i) Formulate H_0 and H_1.

ii) Use the Wilcoxon signed rank test at the 5% level of significance to determine whether you can reject H_0.

iii) Use a paired T-test (formally; the sample size is actually to small for it to give meaningful results) at the 5% level of significance to determine whether you can reject H_0.

(2 + 2 + 2 Marks)

Exercise 10. In a hardness test, a steel ball is pressed into the material being tested at a standard load. The diameter of the indentation is measured, which is related to the hardness. Two types of steel balls are available, and their performance is compared on 10 randomly selected specimens. The hypothesis that the two steel balls give the same expected hardness measurement is to be tested at a significance level of $\alpha = 0.05$. Each specimen is tested twice, once with each ball. The results are given below:

Specimen	1	2	3	4	5	6	7	8	9	10
Ball x	75	46	57	43	58	38	61	56	64	65
Ball y	52	41	43	47	32	49	52	44	57	60

Use each of the following methods to test the hypothesis

i) A pooled T-test (assume that the variances are unequal).

ii) A Wilcoxon signed rank test.

iii) A paired T-test.

Compare the results obtained by each of the above tests. What assumptions are necessary for the validity of each test? What is your final conclusion regarding the hypothesis?
(2+2+2+3 Marks)

Exercise 11. A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order.

i) The two sample average drying times are $\overline{x}_1 = 121$ minutes and $\overline{x}_2 = 112$ minutes. Perform a significance test to judge the effectiveness of the new ingredient. What is the P-value of the test? What conclusions can you draw about the effectiveness of the new ingredient?

ii) If the true difference in mean drying times is as much as 10 minutes, find the sample sizes required to detect this difference with probability at least 0.90, assuming the hypothesis test is conducted with $\alpha = 0.01$.

(3+3 Marks)