Vv156 Lecture 17

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- Because FTC links the concept of the derivative of a function with the concept of the integral, so for definite integration as well as indefinite integration it is essential to find the antiderivative. However, a table of derivatives doesn't tell us how to evaluate every integrals, for example

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$$

- The following technique, can often be used to transform complicated intgrals into simpler ones.

Substitution

If $g'(x)$ is continuous on the interval $[a, b]$ and $f(u)$ is continuous on the range of

$$u = g(x),$$

then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Proof

- Let F denote any antiderivative of f, then

$$\frac{d}{dx}F(g(x)) = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x)$$

- Using integral notation, the above is equivalent to

$$F(g(x)) = \int f(g(x)) \cdot g'(x)\, dx + C$$

- Now if we consider the definite integral over $[a, b]$, FTC states

$$\begin{aligned}\int_a^b f(g(x)) \cdot g'(x)\, dx &= \Big[F(g(x))\Big]_{x=a}^{x=b} \\ &= F(g(b)) - F(g(a)) \\ &= \Big[F(u)\Big]_{u=g(a)}^{u=g(b)} = \int_{g(a)}^{g(b)} f(u)\, du \quad \square\end{aligned}$$

Exercise

(a) Evaluate

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$$

(b) Find

$$\int x\sqrt{2x+1}\,dx$$

(c) Evaluate

$$\int_{0}^{9} \sqrt{4-\sqrt{x}}\,dx$$

(d) Find

$$\int \frac{dx}{e^x+e^{-x}}$$

Q: Can we apply FTC directly to the following

$$\frac{d}{dx}\int_0^{x^2} \cos(t)\, dt$$

In other words, is the following true?

$$\frac{d}{dx}\int_0^{x^2} \cos(t)\, dt = \cos(x^2)$$

Exercise

Find the following derivative.

$$\frac{d}{dx}\int_{\sqrt{x}}^{x^2-3x} \tan(t)\, dt$$