
Question1 (3 points)

Let $\mathcal{V}$ be the space $\mathcal{C}[-1,1]$ with the inner product

$$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)\,dx$$

(a) (1 point) Find the orthogonal projection of e^x onto the subspace

$$\mathcal{H} = \text{span}\left\{1, x, x^2\right\}$$

(b) (1 point) Find the orthogonal projection of e^x onto the subspace

$$\mathcal{W} = \text{span}\left\{1, \cos x, \sin x\right\}$$

(c) (1 point) Explain why both of the orthogonal projections are the best.

Question2 (1 points)

Using the Fourier series to find the steady-state current $i(t)$ in a simple RLC-circuit,

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dE}{dt}$$

where $R = 10\Omega$, $L = 1H$, $C = 10^{-1}F$ and

$$E(t) = \begin{cases} -50t^2 & \text{if} \quad -\pi < t < 0 \\ 50t^2 & \text{if} \quad 0 < t < \pi \end{cases} \qquad E(t+2\pi) = E(t)$$

Question3 (1 points)

Using the Fourier series to find the steady-state current $i(t)$ in a simple RLC-circuit,

$$L\frac{di}{dt} + Ri = E(t), \quad \text{where} \quad E(t) = \begin{cases} 1 & \text{if} \quad 0 < t < 1 \\ 0 & \text{if} \quad 1 < t < 2 \end{cases}, \qquad E(t+2) = E(t)$$

Question4 (0 points)

(a) (1 point (bonus)) Show $\langle \mathbf{x}, \mathbf{y}\rangle = \mathbf{x}^{\mathrm{T}} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{y}$ is an inner product on $\mathbb{R}^2$ if and only if

$$a > 0, \qquad b = c \qquad \text{and} \qquad ad - bc > 0.$$

(b) (1 point (bonus)) Find the solution to the following problem

$$4\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad \text{for} \qquad t > 0, \quad x \in [0, 2]$$

$$\text{with the conditions } u(0,x) = \begin{cases} 0 & \text{if} \quad x \in [0, \frac{2}{3}), \\ 5 & \text{if} \quad x \in [\frac{2}{3}, \frac{4}{3}], \\ 0 & \text{if} \quad x \in (\frac{4}{3}, 2]. \end{cases} \quad u(t,0) = 0 \quad \text{and} \quad u(t,2) = 0.$$