
Question1 (1 points)
Find the point on the parabola

$$x = t, \qquad y = t^2 \qquad \text{for} \quad -\infty < t < \infty$$

closest to the point $(2, 1/2)$.

Question2 (1 points)
Find the volume swept out by revolving the region bounded by the x-axis and one arch of the curve

$$x = t - \sin t, \qquad y = 1 - \cos t$$

about the x-axis.

Question3 (1 points)
Find the length of the curve

$$x = 8\cos t + 8t\sin t, \qquad y = 8\sin t - 8t\cos t, \qquad 0 \le t \le \pi/2$$

Question4 (1 points)
Find the coordinates of the centroid of the curve

$$x = \cos t, \qquad y = t + \sin t, \qquad 0 \le t \le \pi$$

Question5 (1 points)
Find the area of the region bounded by the spiral

$$r = \theta \qquad \text{for} \quad 0 \le \theta \le \pi$$

Question6 (1 points)
Find the area of the region shared by two curves defined by polar equations

$$r = 2 \qquad \text{and} \qquad r = 2(1 - \cos\theta).$$

Question7 (3 points)
Evaluate the integral. Show all your workings.

(a) (1 point) $\displaystyle\int_0^\infty \frac{dv}{(1+v^2)(1+\arctan v)}$

(b) (1 point) $\displaystyle\int_{-1}^4 \frac{dx}{\sqrt{|x|}}$

(c) (1 point) $\displaystyle\int_{-1}^\infty \frac{dx}{x^2+5x+6}$

Question8 (2 points)
Testing for Convergence

(a) (1 point)

$$\int_0^{\pi/2} \tan\theta \, d\theta$$

(b) (1 point)

$$\int_1^\infty \frac{dx}{x\left(\sqrt{\ln(x)}+\ln^2(x)\right)}$$

Question9 (1 points)
Suppose the following improper integral is convergent for all $x > 0$.

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt, \qquad x > 0$$

Prove that if n is a natural number, then

$$\Gamma(n+1) = n!$$

Question10 (1 points)
Find the values of p for which each integral converges.

$$\int_1^2 \frac{dx}{x(\ln x)^p}$$

Question11 (1 points)
Let

$$S(x) = \int_0^x |\cos t|\ dt$$

Find

$$\lim_{x\to+\infty}\frac{S(x)}{x}$$

Question12 (1 points)
Find

$$\int_0^\infty \frac{dx}{(1+x^2)^n}$$

where n is a positive integer.

Question13 (1 points)
Let

$$x = 0.9999\ldots$$

Determine whether $x < 1$, $x = 1$ or $x > 1$. Justify your answer.

Question14 (3 points)

(a) (1 point) Determine whether the series with partial sum $s_n = \dfrac{n}{3n-1}$ is convergent.

(b) (1 point) Determine whether the series $\displaystyle\sum_n^\infty \frac{n}{3n-1}$ is convergent.

(c) (1 point) Find all values of x for which the series converges, and what it converges to.

$$1 - e^{-x} + e^{-2x} - e^{-3x} + e^{-4x} - e^{-5x} + e^{-6x} - \cdots$$

Question15 (2 points)

Determine whether the series converges. If so, find the sum.

(a) (1 point) $\ln\dfrac{1}{2}+\ln\dfrac{2}{3}+\ln\dfrac{3}{4}+\cdots+\ln\dfrac{k}{k+1}+\cdots$

(b) (1 point) $\ln\left(1-\dfrac{1}{4}\right)+\ln\left(1-\dfrac{1}{9}\right)+\ln\left(1-\dfrac{1}{16}\right)+\cdots+\ln\left(1-\dfrac{1}{(k+1)^2}\right)+\cdots$

Question16 (6 points)

Use appropriate tests or theorems to determine convergence or divergence.

(a) (1 point) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{e^{n/3}}$

(b) (1 point) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{\ln n}$

(c) (1 point) $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{n+3}{n^4+4}}$

(d) (1 point) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2(n+2)!}{n!3^{2n}}$

(e) (1 point) $\displaystyle\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}$

(f) (1 point) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

Question17 (3 points)

Prove the Ratio test.

$$\text{If}\quad \lim_{n\to\infty}\sqrt[n]{|a_n|}=L<1 \implies \text{Absolutely convergent} \implies \text{Convergent}$$

$$\text{If}\quad \lim_{n\to\infty}\sqrt[n]{|a_n|}=L>1 \implies \text{divergent}$$

$$\text{If}\quad \lim_{n\to\infty}\sqrt[n]{|a_n|}=1 \implies \text{Inconclusive}$$

Question18 (0 points)

(a) (1 point (bonus)) Prove the following improper integral is convergent for all $x>0$.

$$\Gamma(x)=\int_0^{\infty}t^{x-1}e^{-t}dt$$