Introduction to Linear Algebra

Review Class 2

Wang Tianyu[1]

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Outline

What does a real symmetric matrix represent?

In Linear Algebra, a real symmetric matrix represent a **self-adjoint operator** *over a real inner product space.*

Basic properties of symmetric matrices

1. *Given symmetric matrices A and B, then AB is symmetric* **if and only if** $AB = BA$;
2. *If A is symmetric, then every integer power of A is also symmetric;*
3. *An $n \times n$ matrix A is defined to be* **symmetrizable** *if there exists an invertible diagonal matrix D and symmetric matrix S such that $A = DS$.*

Basic properties of skew-symmetric matrices

1. *The elements on the diagonal of a skew-symmetric matrix are zero;*
2. *If A is a* **real** *skew-symmetric matrix and λ is a* **real** *eigenvalue, then $\lambda = 0$;*
3. *If A is a* **real** *skew-symmetric matrix, then $I + A$ is invertible, where I is the identity matrix.*

Interpretation of determinant

1. *Multiply the n pivots (times 1 or -1);*
2. *Add up $n!$ terms (times 1 or -1);*
3. *Combine n smaller determinants (times 1 or -1).*

Several properties of determinant

1. *Interchanging rows result in the change of sign in determinant;*
2. *Multiplying a row result in the same scaling of determinant;*
3. *Adding a multiple of a row to another does not change determinant;*
4. *If A and B are $n \times n$ matrices, det(AB)=det(A)det(B);*
5. *If A is square and det(A)$\neq 0$, then $A^{-1} = \frac{1}{\det(A)}\mathrm{adj}(A)$;*
6. *An $n \times n$ matrix is singular* **if and only** *if det(A)=0;*
7. *Cramer's rule (not commonly used in practice).*

Reasons to introduce LU decomposition

1. *Previous ways of calculating determinant can sometimes be very tedious when the size becomes large;*
2. *The calculation of a triangular matrixs determinant is simple.(product of diagonal entries);*
3. *Transforming a matrix into triangular form is simple (Gauss Elimination);*
4. *Estimating the influence of trasformation on the determinant is simple(there are only elementary row operations).*

Interesting facts

1. *When a row of A* **starts with** *zeros, so does that row of L; When a column of A* **starts with** *zeros, so does that column of U;*
2. *U has the pivots on its diagonal; L has all the 1's on its diagonal;*
3. *The multipliers $l_{i,j}$ are below the diagonal of L.*

Better balance for this decomposition ...

$A = LU$ is "unsymmetric" because U has the pivots on its diagonals where L has 1's. Therefore, we may prefer more the decomposition $A = LDU'$, where D is the diagonal matrix with pivots. Each row of U' is that of the original U divided by its pivot.

Solving $A\mathbf{x} = \mathbf{b}$ with LU decomposition: first solve $L\mathbf{c} = \mathbf{b}$, then solve $U\mathbf{x} = \mathbf{c}$.