Probabilistic Methods in Engineering

VE401

Assignment VI

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Team number: 20

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Exercise 6.1

1) There's no enough evidence that the success rate is greater for longer tears.

   Large-sample test for differences in proportions will be used.

$$H_0: \ p_1 - p_2 = 0$$

   based on the statistic

$$Z = \frac{\hat{p_1} - \hat{p_2}}{\sqrt{\frac{\hat{p_1}(1-\hat{p_1})}{n_1} + \frac{\hat{p_2}(1-\hat{p_2})}{n_2}}}.$$

   Here we obtain that

$$\begin{aligned} \hat{p_1} &= 78\%, \\ \hat{p_2} &= 73\%, \\ Z &= \frac{0.78 - 0.73}{\sqrt{\frac{0.78(1-0.78)}{18} + \frac{0.73(1-0.73)}{30}}} = 0.39. \end{aligned}$$

   According to the standard normal distribution, $z_{0.05} = 1.645$. Hence $Z = 0.39 < z_{0.05}$, which means we have no sufficient evidence to support $p_1 > p_2$.

2) The standard deviation is

$$\sqrt{\frac{0.78(1-0.78)}{18} + \frac{0.73(1-0.73)}{30}} = 0.127.$$

   The 95% confidence interval is

$$p_1 - p_2 > 0 + 1.645 \times 0.127 = 0.209.$$

   Thus

$$p_1 - p_2 > 0.209.$$

Exercise 6.2

1) The standard deviation is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 0.95.$$

$$\bar{x_1} - \bar{x_2} - z_{0.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{x_1} - \bar{x_2} + z_{0.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
$$-6 - 1.96 \times 0.95 \leq \mu_1 - \mu_2 \leq -6 + 1.96 \times 0.95$$

Hence,

$$\mu_1 - \mu_2 = (-6.00 \pm 1.86) cm/s.$$

2)

$$Z_0 = \frac{\bar{x_1} - \bar{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{-6}{0.95} = -6.32 < -1.96 = -z_{0.025}.$$

Hence, we reject H_0 at a 5% level of significance.

3) We know that

$$\beta = 1 - p = 0.10, \quad , \alpha = 0.05, \quad \sigma = 3cm/s, \quad \delta = 14cm/s.$$

Hence,

$$n \approx \frac{2(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{2(1.96 + 1.28)^2 \times 3^2}{14^2} = 0.96.$$

Exercise 6.3

$$H_0 : \ \sigma_1^2 = \sigma_2^2 \quad H_1 : \ |\sigma_1^2 - \sigma_2^2| \geq 0.5$$

Thus,

$$F_{n_1-1,n_2-1} = F_{9,15} = \frac{s_1^2}{s_2^2} = \frac{4.7^2}{5.8^2} = 0.66 < f_{0.025,9,15} = 3.1227$$
$$F_{n_2-1,n_1-1} = F_{15,9} = \frac{s_2^2}{s_1^2} = \frac{5.8^2}{4.7^2} = 1.52 < f_{0.025,15,9} = 3.7694$$

We claim that $\sigma_1^2 = \sigma_2^2$ at a 5% level of significance.

Hence, there's no sufficient evidence to conclude that the two population variances differ by at least 0.5 grams per liter.

Exercise 6.4

We set the hypotheses

$$H_0 : \sigma_1^2 = \sigma_2^2, \quad H_1 : \sigma_1^2 \neq \sigma_2^2$$

We know that

$$s_1^2 = 0.0015, \quad s_2^2 = 0.0120.$$

Then using F test, we obtain

$$F_{n_1-1,n_2-1} = F_{9,9} = \frac{0.0015}{0.0120} = 0.1234.$$

Hence,

$$F_{9,9} = 0.1234 < f_{0.025,9,9} = 4.0260$$
$$F_{9,9} = 0.1234 < f_{0.975,9,9} = 0.2484$$

Thus, we claim $\sigma_1^2 \neq \sigma_2^2$ at 5% level of significance.

Since $0.1234 \approx f_{0.9977,9,9}$, then the P-value is $0.0023 \times 2 = 0.0046$.

Exercise 6.5

1) We set the hypothesis

$$H_0 : \ \mu_1 - \mu_2 = 0$$

We know form the table that

$$(\bar{x}_1 - \bar{x}_2) = 15.32 - 15.38 = -0.06.$$

Thus we use pooled T-Test

$$S_p^2 = \frac{(9-1) \times 0.0124 + (9-1) \times 0.0323}{9+9-2} = 0.0223.$$
$$T_{9+9-2} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{0.0223(9^{-1} + 9^{-1})}} = \frac{-0.06}{0.0704} = -0.852.$$

$|T_{16}| = 0.852 < t_{0.025,16} = 2.1199.$

Hence, we claim that $\mu_1 = \mu_2$ at 5% level of significance. The temperature does not affect the mean reading of concentration.

2) We set the hypothesis

$$H_0 : \ \mu_1 - \mu_2 = 0$$

We know form the table that

$$(\bar{x}_1 - \bar{x}_2) = 0.430444 - 1.65844 = -1.228.$$

Thus we use pooled T-Test

$$\begin{aligned} S_p^2 &= \frac{(9-1)\times 0.0420 + (9-1)\times 0.0660}{9+9-2} = 0.054. \\ T_{9+9-2} &= \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{0.054(9^{-1}+9^{-1})}} = \frac{-1.228}{0.110} = -11.21. \\ |T_{16}| &= 11.21 > t_{0.025,16} = 2.1199. \end{aligned}$$

Hence, we claim that $\mu_1 \neq \mu_2$ at 5% level of significance. The temperature affects the mean reading of concentration.

Exercise 6.6

1)

$$H_0 : \ \sigma_1^2 = \sigma_2^2$$

We apply F test.

$$\begin{aligned} F_{9,15} &= \frac{s_1^2}{s_2^2} = \frac{9}{7.29} = 1.23 \\ f_{0.1,9,15} &= 2.0862 \\ f_{0.9,9,15} &= 0.4274 \end{aligned}$$

Since $F_{9,15} < f_{0.1,9,15}$ and $F_{9,15} > f_{0.9,9,15}$, then we claim that $\sigma_1^2 = \sigma_2^2$ at 0.2 level of significance.

2)

$$s_p^2 = \frac{(10-1)\times 9 + (16-1)\times 7.9}{10+16-2} = 8.3125.$$

3) The statistic is

$$T_{24} = \frac{\mu_1 - \mu_2 - (64.95 - 57.06)}{\sqrt{8.3125(10^{-1} + 16^{-1})}} = \frac{\mu_1 - \mu_2 - 7.89}{1.16}$$

Then, the 99% confidence interval on $\mu_1 - \mu_2$ is

$$\mu_1 - \mu_2 = 7.89 \pm t_{0.005,24} \cdot 1.16$$
$$\mu_1 - \mu_2 = 7.89 \pm 3.25.$$

4) We claim the there's a difference between μ_1 and μ_2 because we are 99% sure that $\mu_1 - \mu_2 > 4.64$.

Exercise 6.7

We set the hypothesis

$$H_0: \ \mu_1 = \mu_2 \quad H_1: \ \mu_1 < \mu_2,$$

which is a one-sided hypothesis.

We know that

$$\bar{x}_1 = 0.210, \quad s_1^2 = 0.001116.$$
$$\bar{x}_2 = 0.225, \quad s_2^2 = 0.001099.$$

Then,

$$\begin{aligned} T_{9+9-2} &= \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{s_p^2(9^{-1} + 9^{-1})}} \\ &= \frac{0.210 - 0.225}{\sqrt{(8 \times 0.001116 + 8 \times 0.001099)/16 \times 0.22}} \\ &= \frac{-0.015}{0.0157} \\ &= -0.955. \end{aligned}$$

Since $t_{0.05,16} = 1.7459$, then $T_{16} > -t_{0.05,16}$.

Hence, we claim that we have no sufficient evidence to support $\mu_1 < \mu_2$ at 5% level of significance.

Exercise 6.8

$$H_0: \ \mu_1 = \mu_2, \quad H_1: \mu_1 > \mu_2$$

We denote $D = X_2 - X_1$, so that

$$\overline{D} = -16.93, \quad s_D = 33.7.$$

$$T_{15} = \frac{-16.93}{33.7/\sqrt{15}} = -1.945.$$

Since $t_{0.05,15} = 1.7531$, then $T_{15} < -t_{0.05,15}$. Hence we reject $\mu_1 = \mu_2$ and claim that $\mu_1 > \mu_2$ at 5% level of significance.

Using Wilcoxon Rank-Sum Test, we set

$$H_0: \ M_D = 0 \quad H_1: \ M_D < 0.$$

D	Rank
-4	-1.5
4	1.5
9	3
-11	-4.5
11	4.5
14	6
-15	-7.5
15	7.5
-33	-9.5
33	9.5
-37	-11
-46	-12
-48	-13
-56	-14
-90	-15

Thus, $W_+ = 32$, $W_- = 88$, $W = \min(W_-, W_+) = 32$. However, for $n = 15$ at 5% level of significance, we have a critical value $n = 30$ for one-sided test. $W > 30$.

Thus, we claim $M_1 = M_2$ at 5% level of significance.