VG441 Final Exam

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THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

Signature: 潘崇聃

1 Problem 1

Task 1

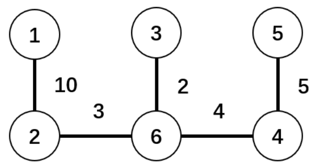

Figure 1: MST

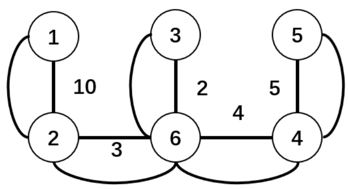

Figure 2: Double Edge MST

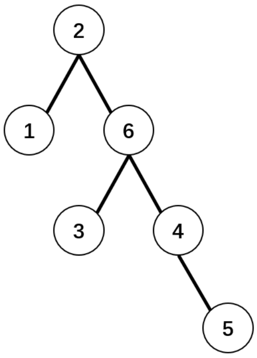

Figure 3: Eulerian Path

Before short cutting, our path is $2 \rightarrow 6 \rightarrow 4 \rightarrow 5 \rightarrow 4 \rightarrow 6 \rightarrow 3 \rightarrow 6 \rightarrow 2 \rightarrow 1 \rightarrow 2$
After short cutting our path is $2 \rightarrow 6 \rightarrow 4 \rightarrow 5 \rightarrow 3 \rightarrow 1 \rightarrow 2$ with cost $3 + 4 + 5 + 86 + 100 + 10$

Task 2
The odd degree set is $\{1, 3, 6, 5\}$

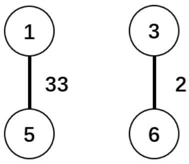

Figure 4: Min-weight-matching K

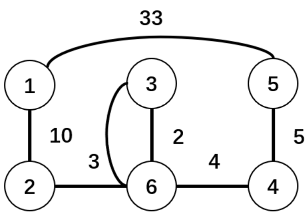

Figure 5: Add K to M

Before short cutting, our path is $1 \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 6 \rightarrow 4 \rightarrow 5 \rightarrow 1$
After short cutting, our path is $1 \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 1$ with cost=10+3+2+6+5+33=59
From eyeballing solution, it's same as $1 \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 1$ with cost=59

2 Problem 2

Task 1
$n = 4, v_{max} = v_3 = 6$

$T[1,0]=0, T[1,4]=3$
$T[2,0]=0, T[2,4]=3, T[2,8]=6$
$T[3,0]=0, T[3,4]=3, T[3,6]=8, T[3,8]=6, T[3,10]=11, T[3,14]=14$
$T[4,0]=0, T[4,4]=3, T[4,5]=5, T[4,6]=8, T[4,8]=6, T[4,9]=8, T[4,10]=11$
$T[4,11]=13, T[4,13]=11, T[4,14]=14, T[4,15]=16, T[4,19]=19$
Since $T[4,9]=8$, the maximum value we can put is 9

Task 2
After ranking, we get $\frac{v_1}{s_1}=\frac{v_2}{s_2}>\frac{v_4}{s_4}>\frac{v_3}{s_3}$
So we put i_1 and i_2 in the bag and the total value is 8 by occupying 6 of 8 big size, which is smaller than 9

3 Problem 3

Task 1 At first $\frac{C_1}{|S_1|}=1.2$, $\frac{C_2}{|S_2|}=3$, $\frac{C_3}{|S_3|}=1$
So we prefer to use S_3, and the remaining elements are $\{e_1,e_2,e_3,e_6,e_7\}$
Then $\frac{C_1}{3}=2$, $\frac{C_2}{|S_2|}=3$
So we prefer to use S_1 and the remaining elements are $\{e_6,e_7\}$
At last, we have to choose S_3 to cover the remaining elements.
In total, the cost is 6+7+15=28

Task 2
The greedy algorithm is not optimal, since we can cover all elements with S_2 and S_3 and total cost is 7+15=22

4 Bonus Problem

Task 1
Through Lagrangian Relaxation:

$$\max_{\alpha_j,\beta_{ij}} \sum_{j\in D}\alpha_j$$

$$\text{s.t. } \sum_{j\in D}\beta_{ij}\le f_i, \forall i\in F$$

$$\alpha_j-\beta_{ij}\le d_{ij}, \forall i\in F, j\in D$$

$$\alpha_j\ge 0, \forall j\in D$$

$$\beta_{ij}\ge 0, \forall i\in F, j\in D$$

Task 2
Obviously, α_j and β_{ij} can't be less than 0, since the demand won't contribute a negative cost. For the object, we want to maximize the amount of money demand j will to contribute, when it's max, it's same to we spend least cost, since it's covered by demand.

The sum of β_{ij} smaller than f_i means the amount of money contributed from demands towards a facility should be less than the cost of the corresponding facilities.

$\alpha_j-\beta_{ij}$ represents the remaining cost after paying for the facilities, which should contribute for the demand d_{ij}, so it's should be smaller than d_{ij}