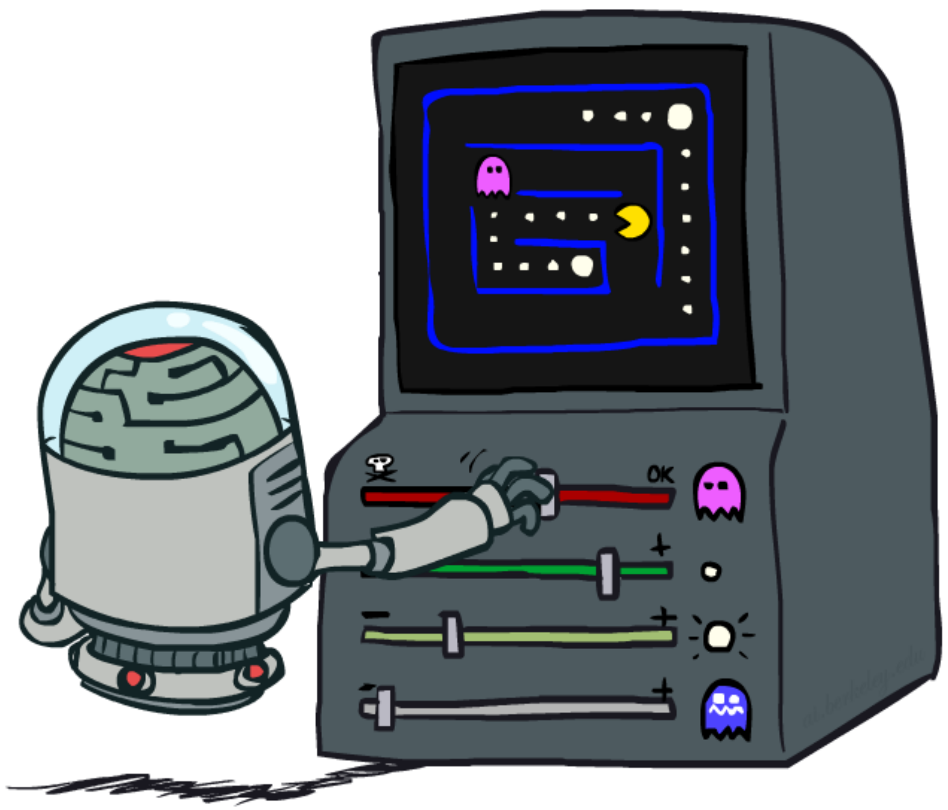
Ve492: Introduction to Artificial Intelligence
Reinforcement Learning II



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Slides adapted from http://ai.berkeley.edu, AIMA, UM, CMU

Reinforcement Learning

- We still assume an MDP:
  - A set of states s ∈ S
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Still looking for a policy π(s)

- New twist: don't know T or R, so must try out actions
- Big idea: Compute all averages over T using sample outcomes

The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

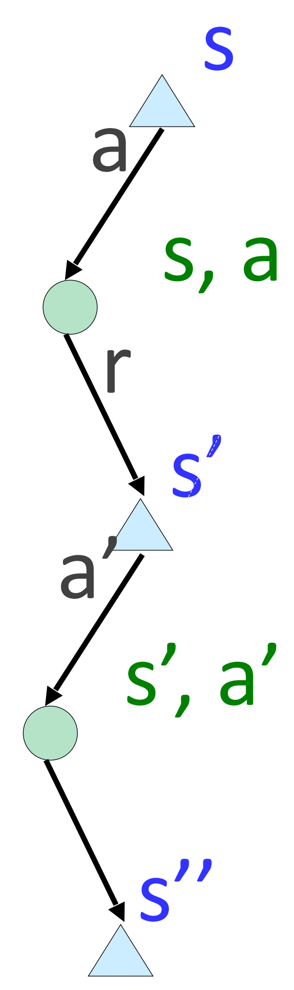
Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	Value Learning

Model-Free Learning

- Model-free (temporal difference) learning
  - Experience world through episodes
    $$(s, a, r, s', a', r', s'', a'', r'', s'''' \ldots)$$
  - Update estimates each transition (s, a, r, s')
  - Over time, updates will mimic Bellman updates



Q-Learning

- We'd like to do Q-value updates to each Q-state:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

  - But can't compute this update without knowing T, R
- Instead, compute average as we go
  - Receive a sample transition (s,a,r,s')
  - This sample suggests

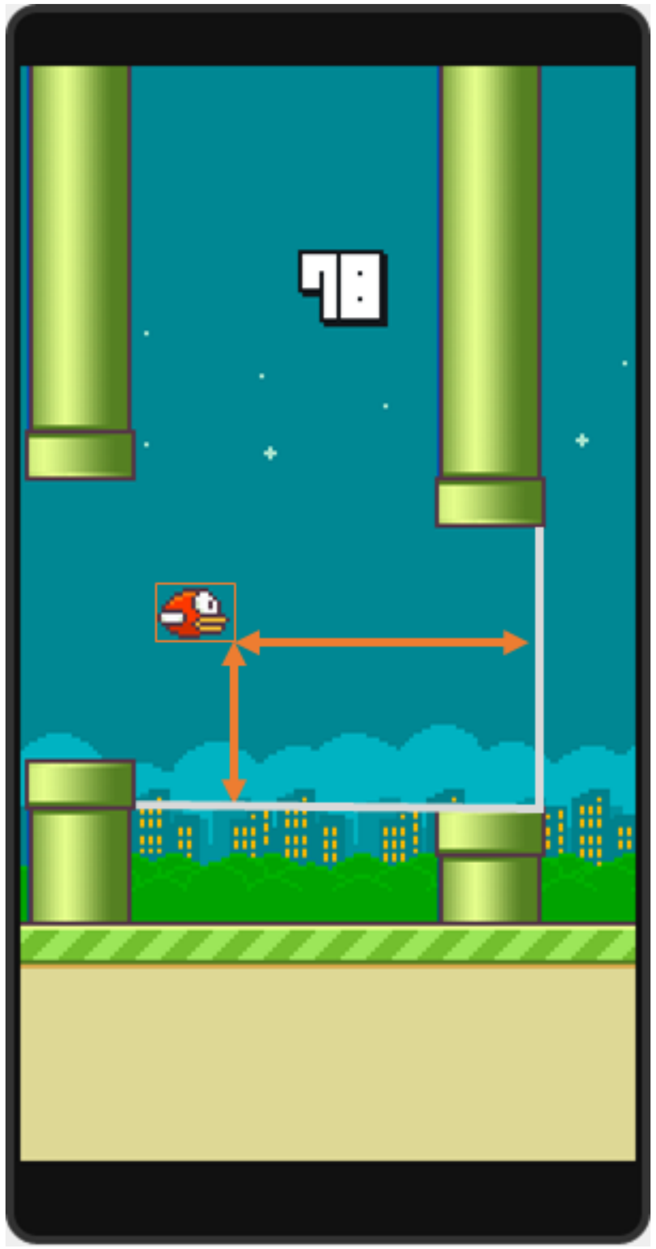
$$Q(s,a) \approx r + \gamma \max_{a'} Q(s',a')$$

  - But we want to average over results from (s,a) (Why?)
  - So keep a running average

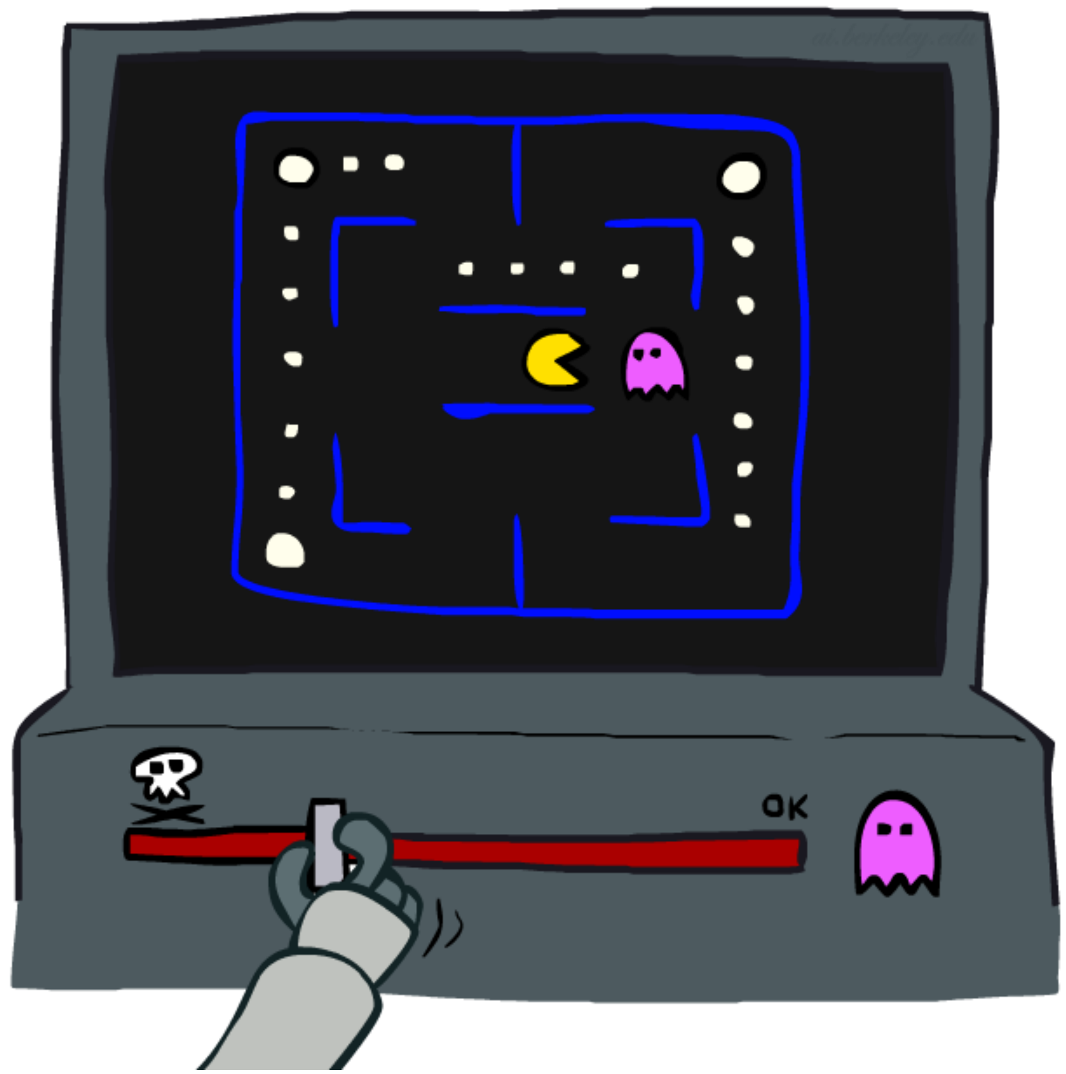
$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a')\right]$$

Example: Flappy Bird RL

- State space
  - Discretized vertical distance from lower pipe
  - Discretized horizontal distance from next pair of pipes
  - Life: Dead or Living
- Actions
  - Click
  - Do nothing
- Rewards
  - +1 if Flappy Bird still alive
  - -1000 if Flappy Bird is dead
- 6-7 hours of Q-learning

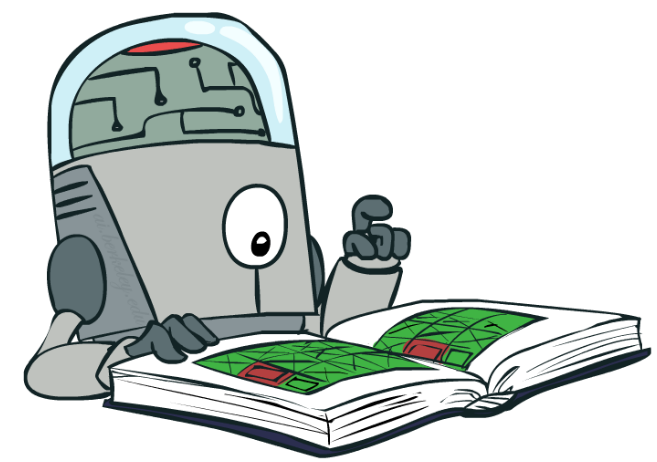


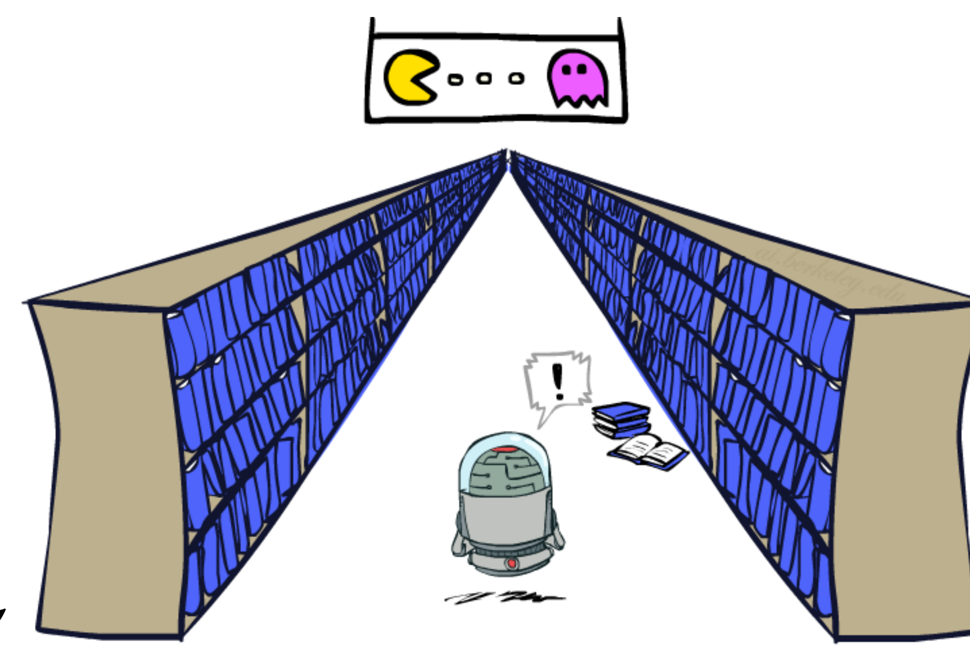
Approximate Q-Learning



Generalizing Across States

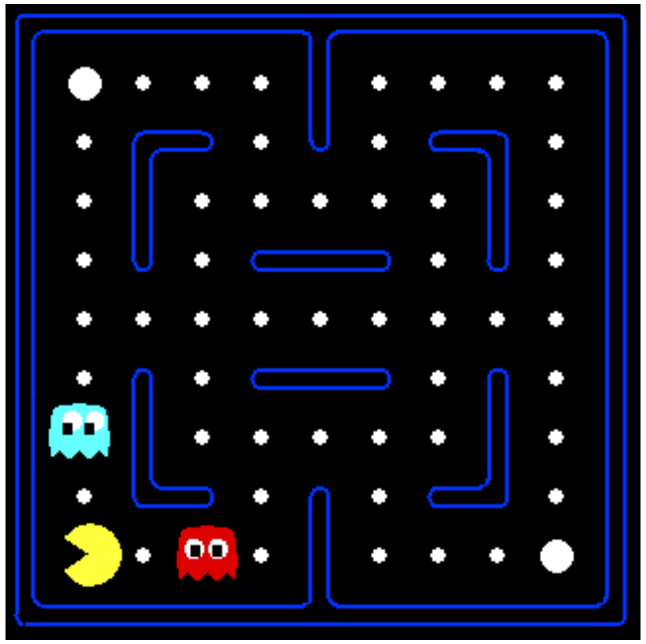
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again



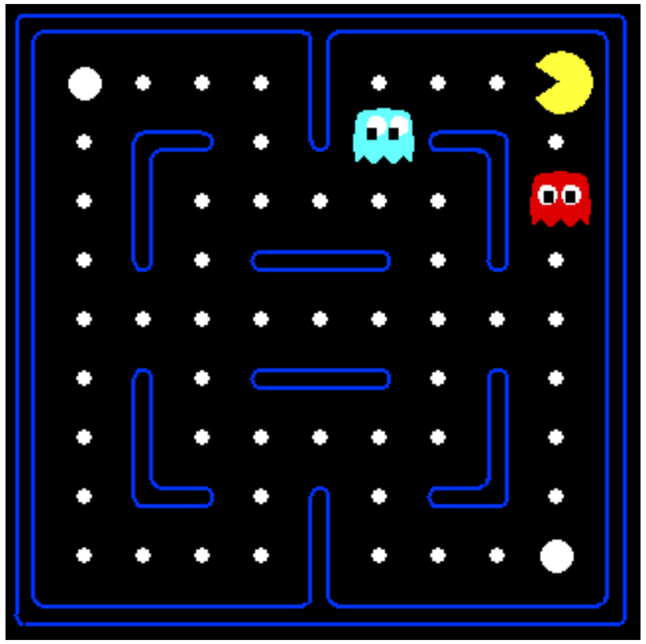


Example: Pacman

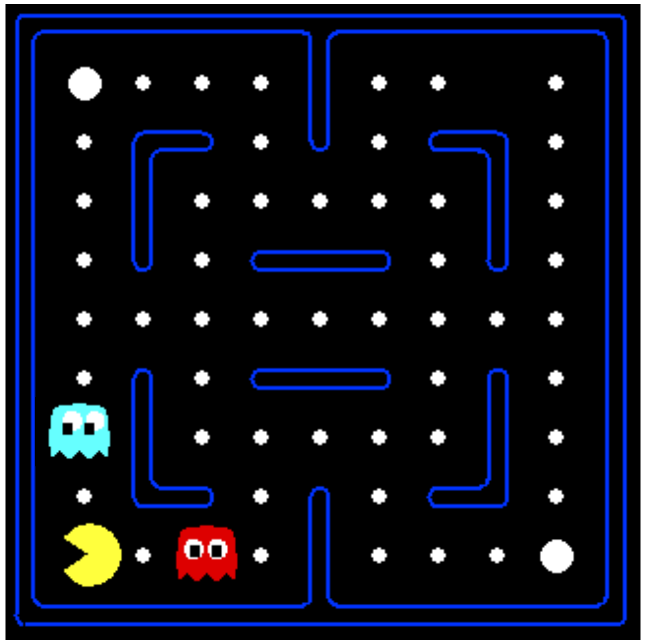
Let's say we discover through experience that this state is bad:



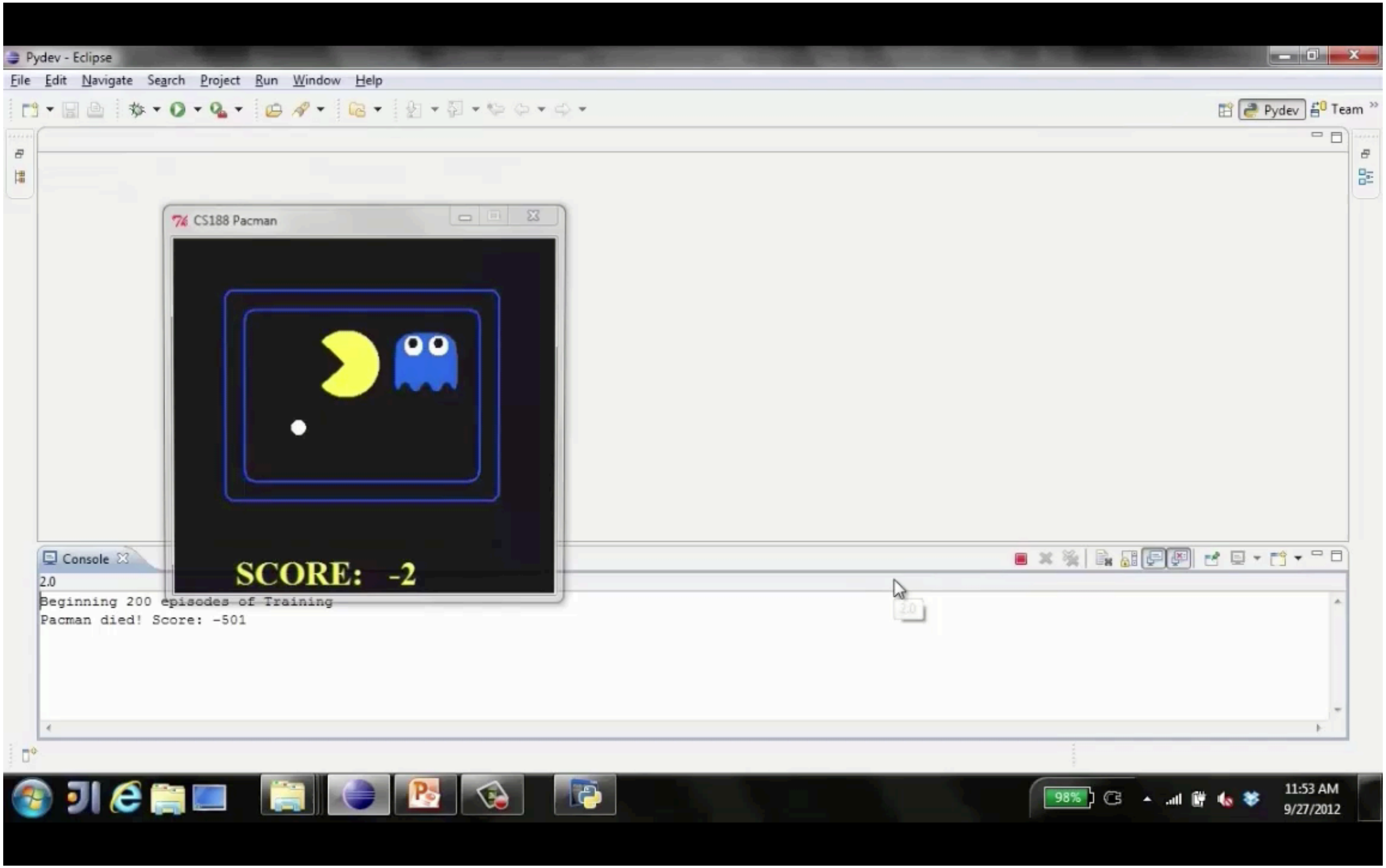
In naïve q-learning, we know nothing about this state:



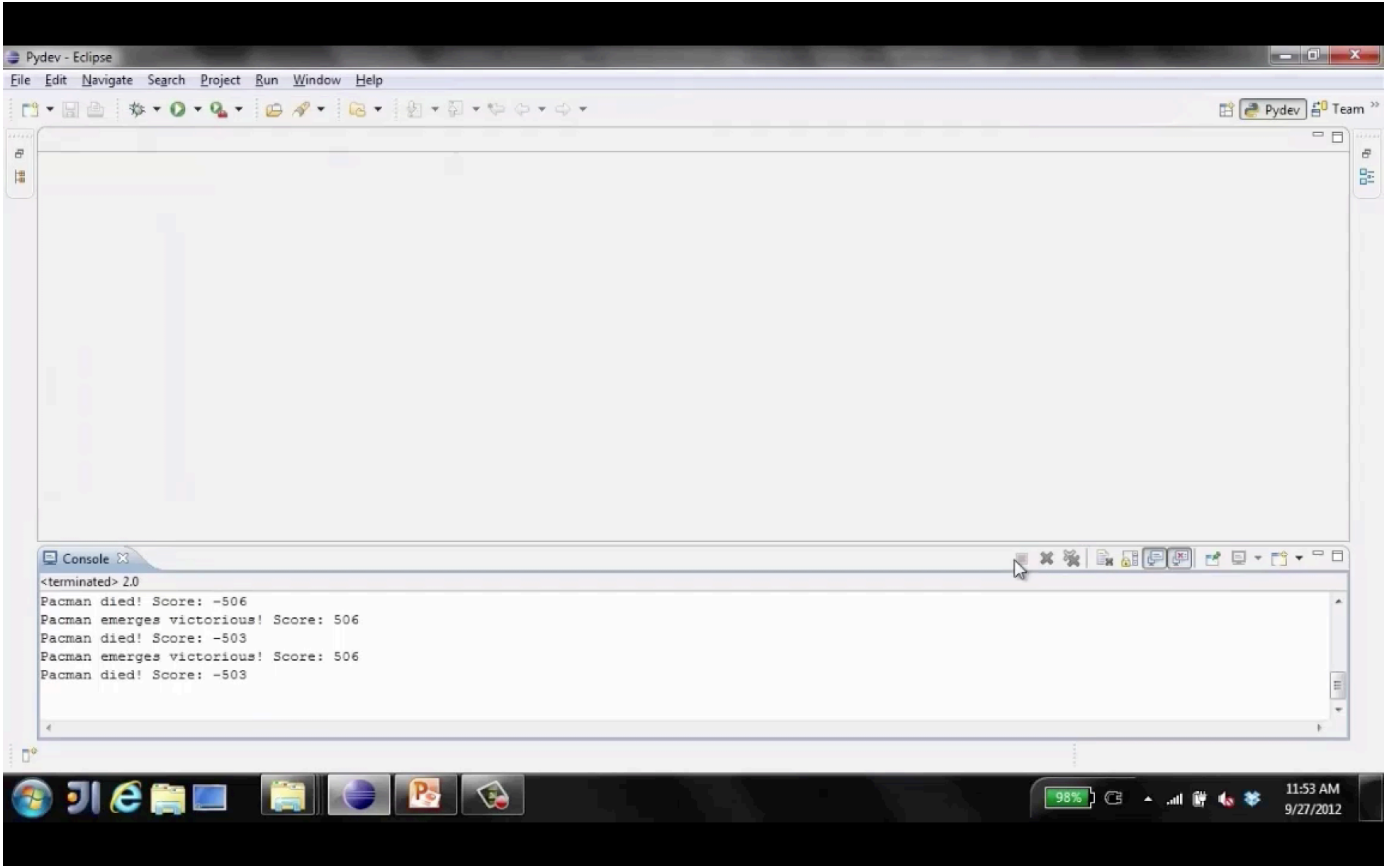
Or even this one!



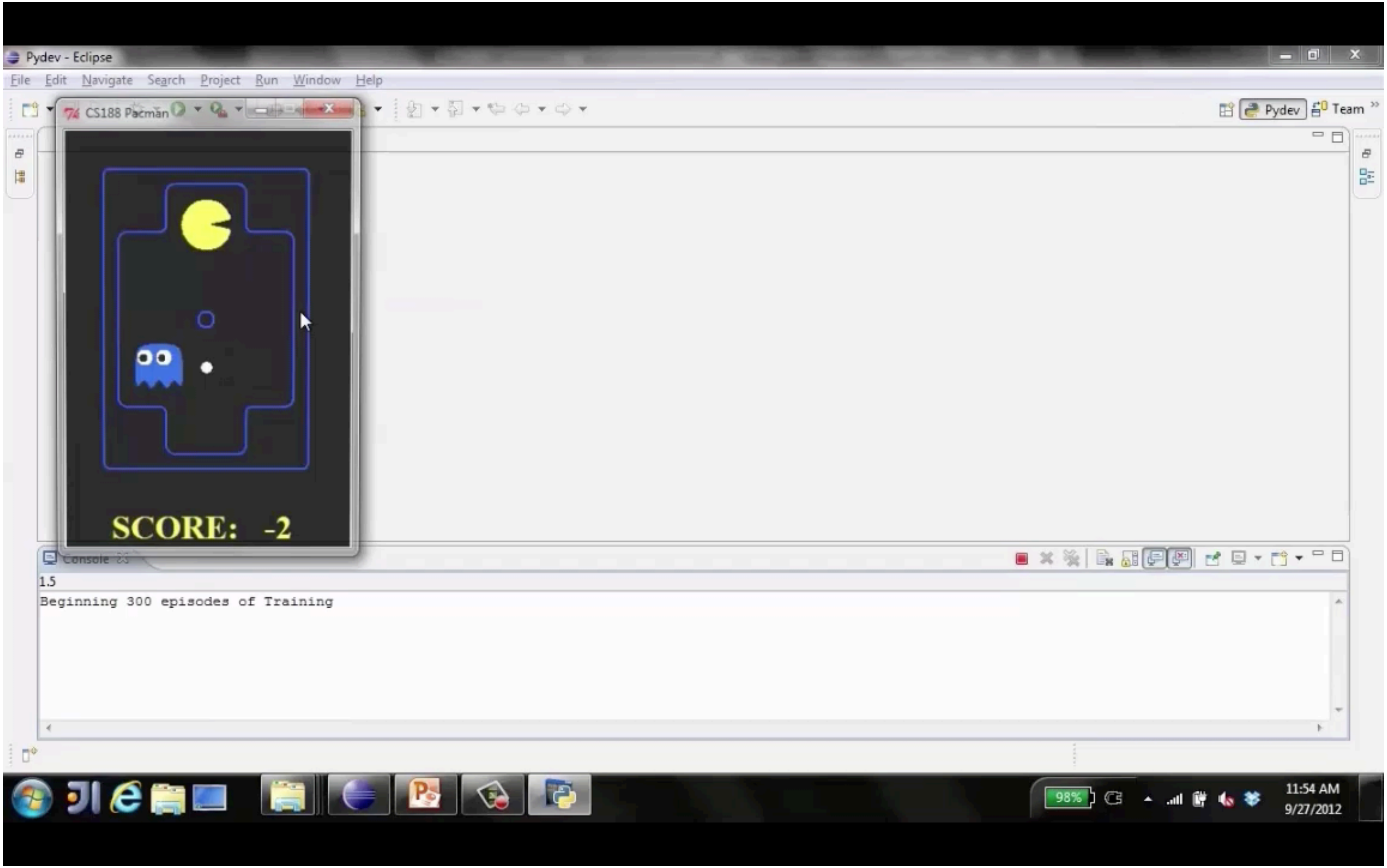
Video of Demo Q-Learning Pacman – Tiny – Watch All



Video of Demo Q-Learning Pacman – Tiny – Silent Train

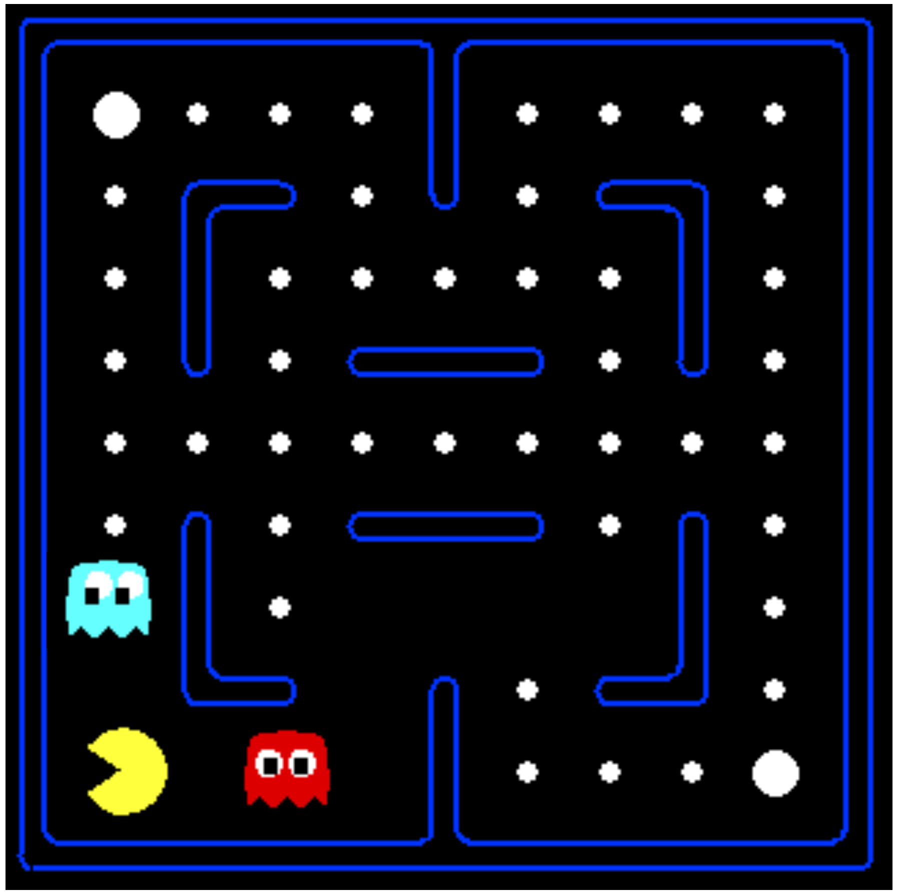


Video of Demo Q-Learning Pacman – Tricky – Watch All



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

- Q-learning with linear Q-functions:

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

$$\text{transition } = (s,a,r,s')$$

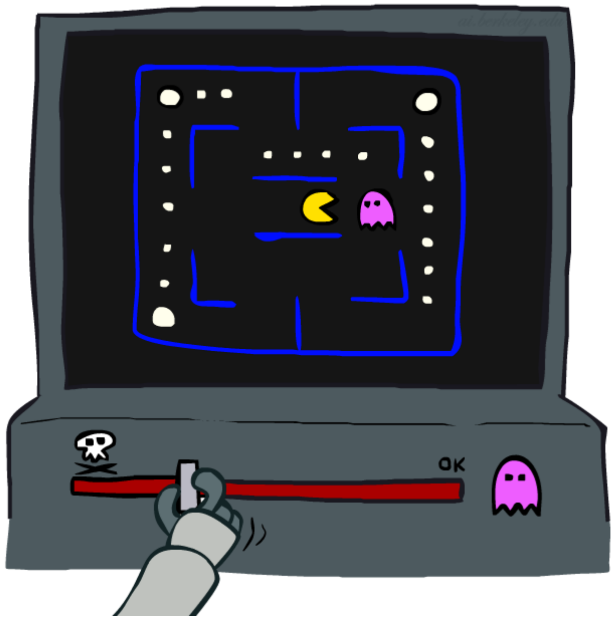
$$\text{difference} = \left[r + \gamma \max_{a'} Q(s',a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}]$$

Exact Q's

$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a)$$

Approximate Q's



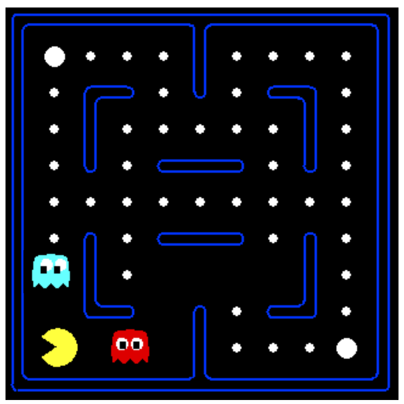
- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

- Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s

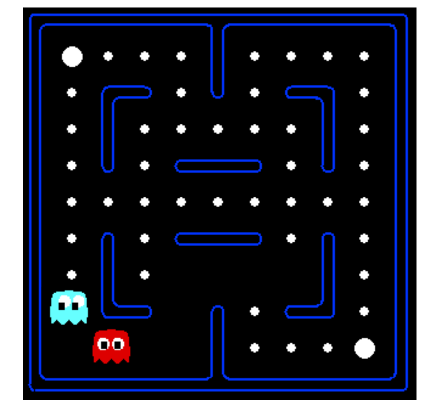


$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$a = \text{NORTH}$
$r = -500$

s'



$Q(s, \text{NORTH}) = +1$

$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

$Q(s', \cdot) = 0$

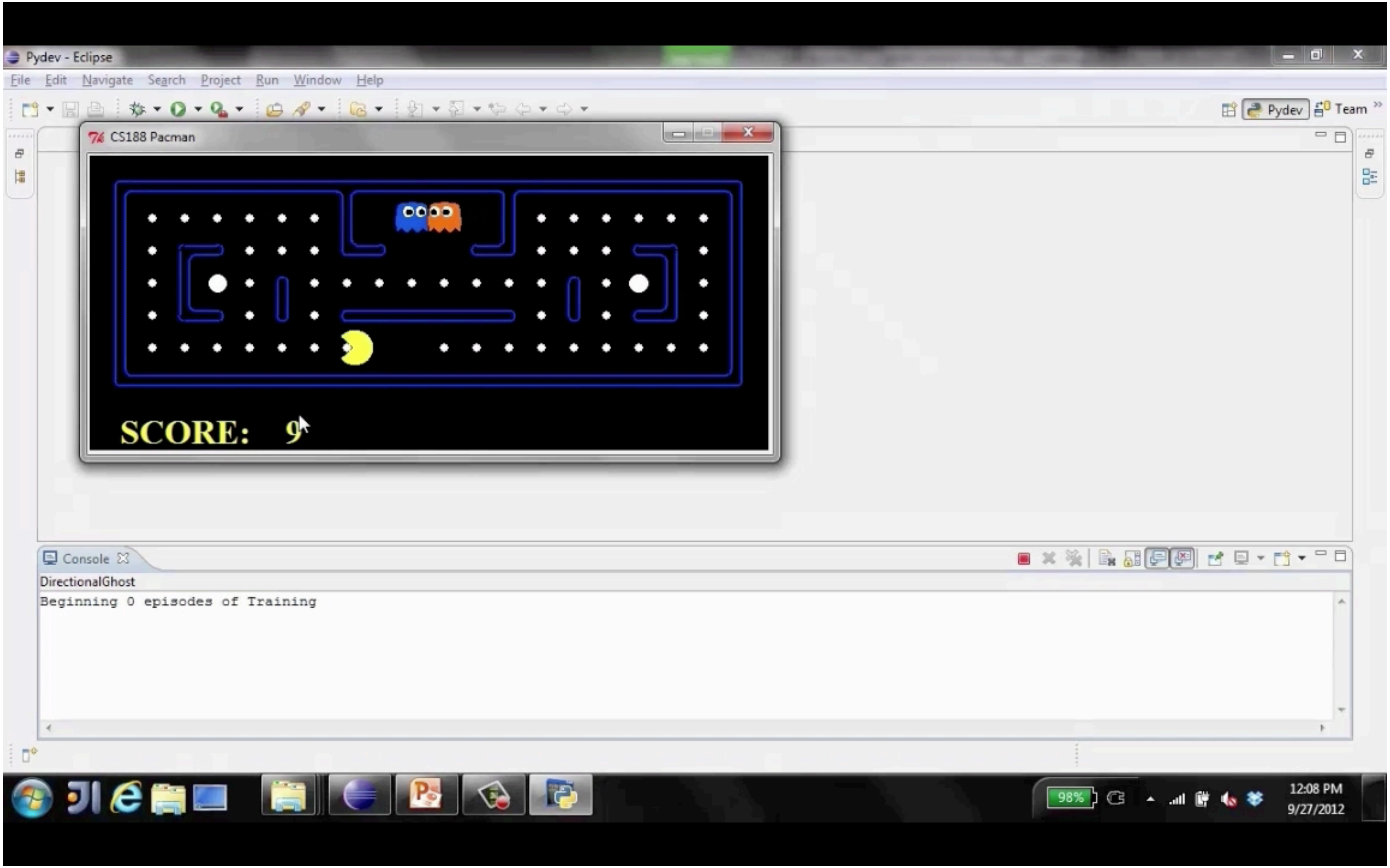
difference $= -501$

$w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5$

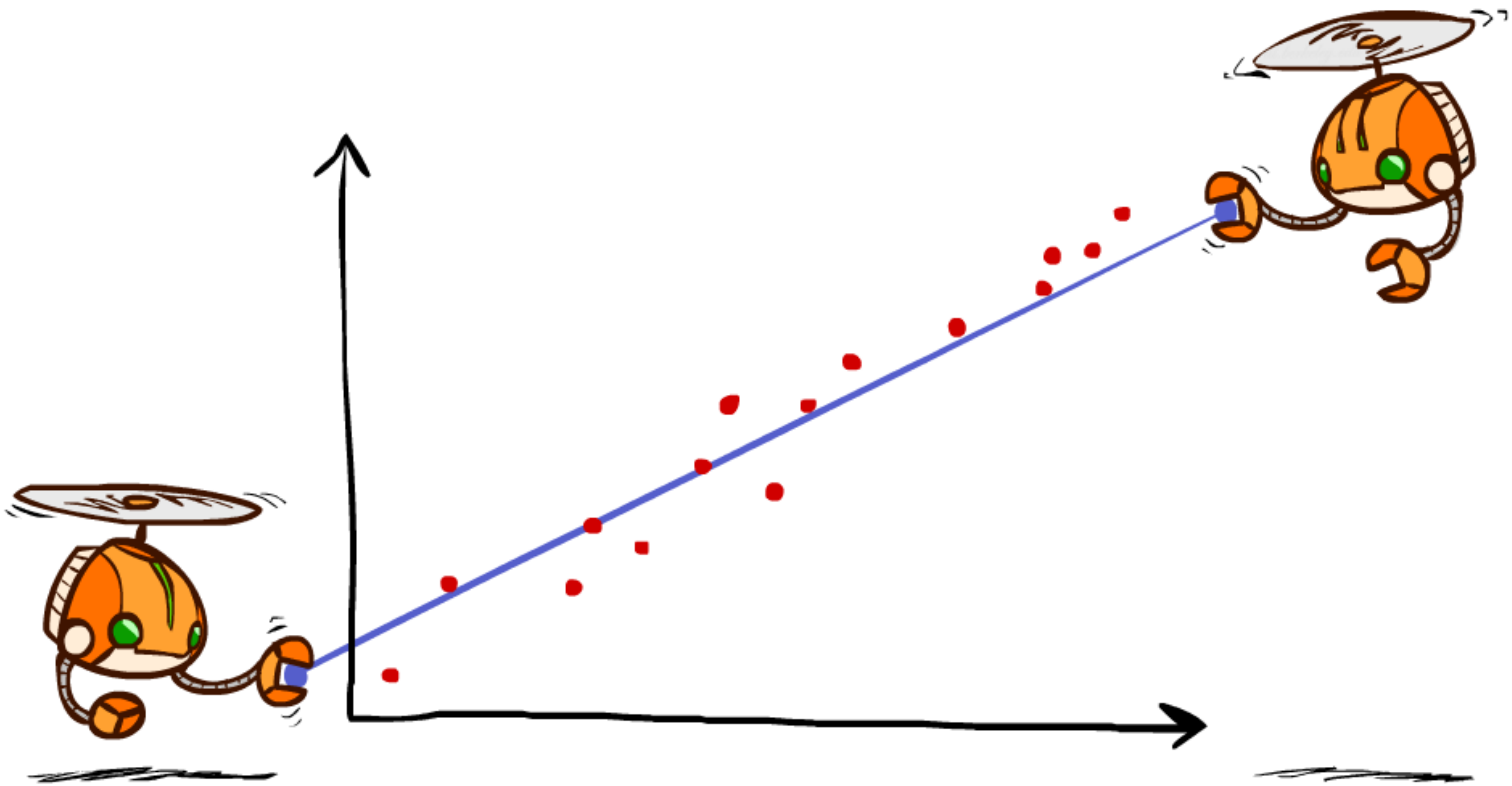
$w_{GST} \leftarrow -1.0 + \alpha\,[-501]\,1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

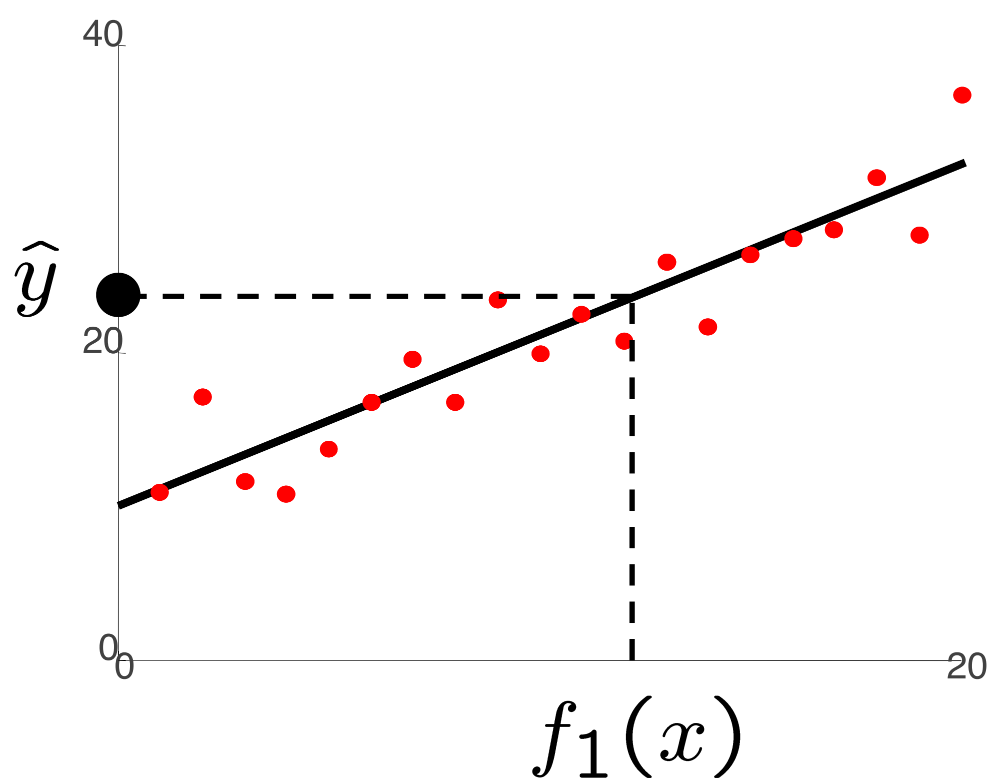
Video of Demo Approximate Q-Learning -- Pacman



Q-Learning and Least Squares

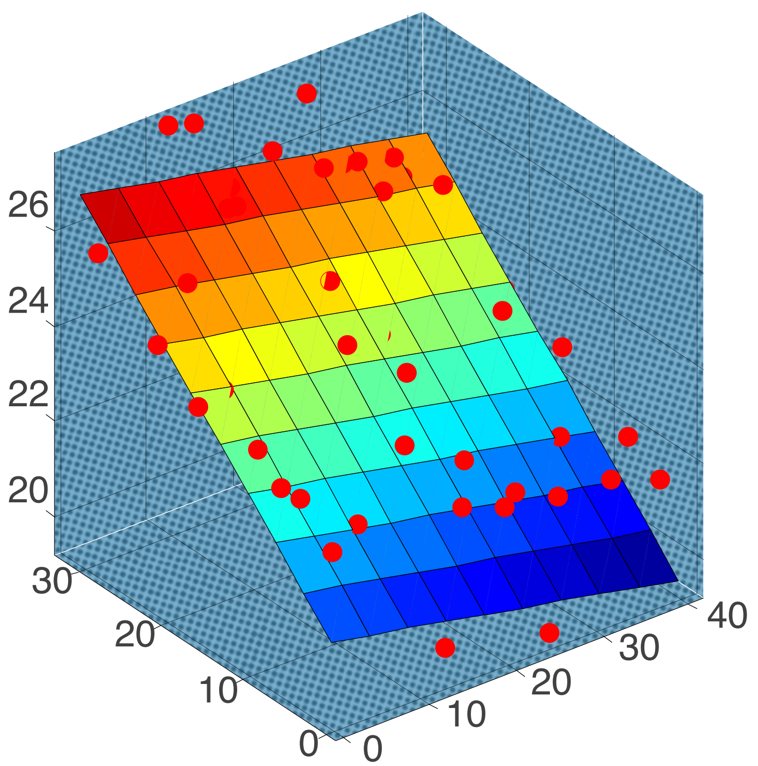


Linear Approximation: Regression*



Prediction:

$$\widehat{y} = w_0 + w_1 f_1(x)$$

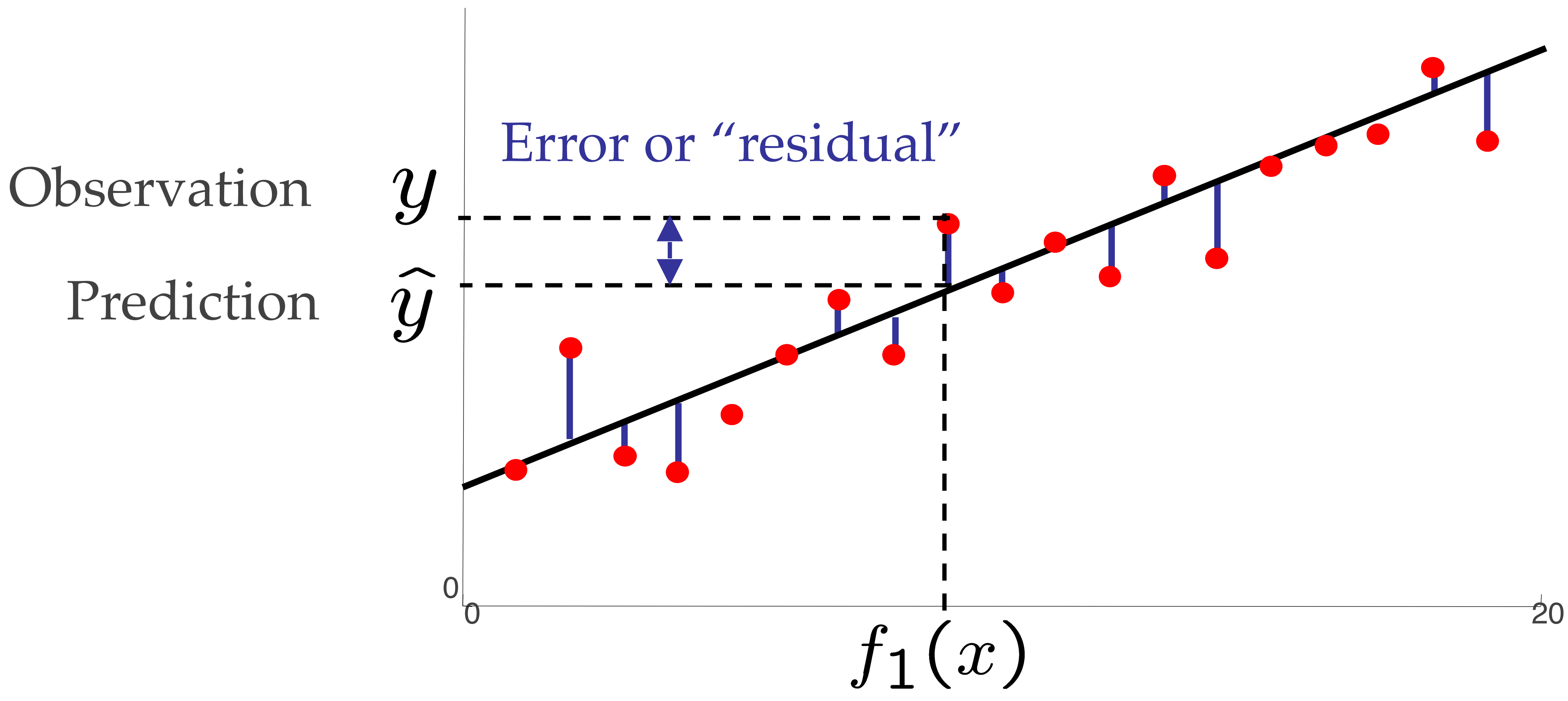


Prediction:

$$\widehat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares*

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



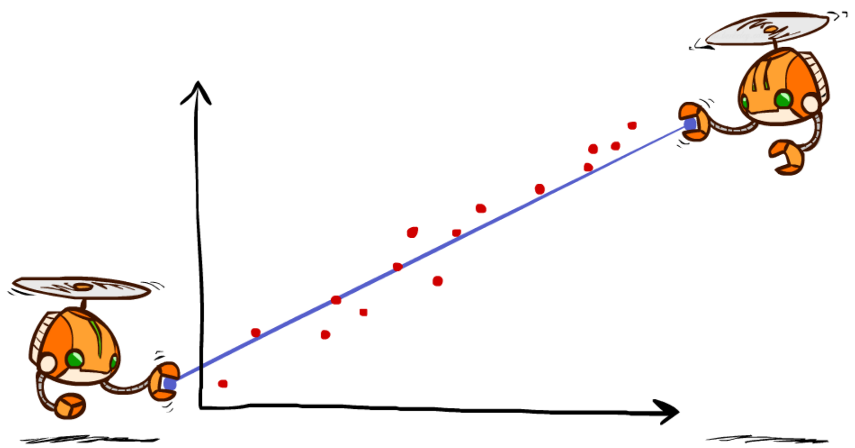
Minimizing Error*

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$



Approximate q update explained:

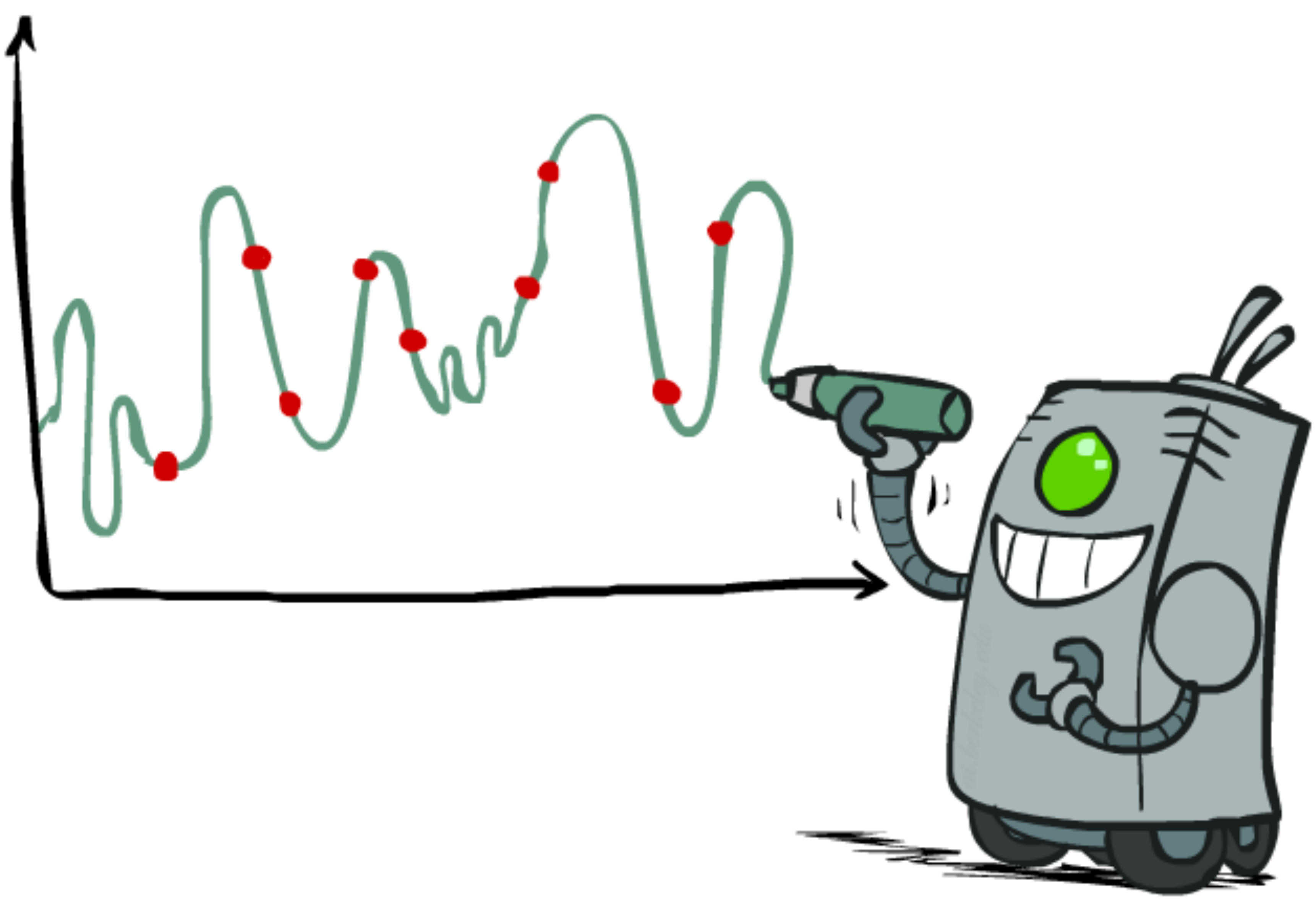
$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a)\right] f_m(s, a)$$

"target" "prediction"

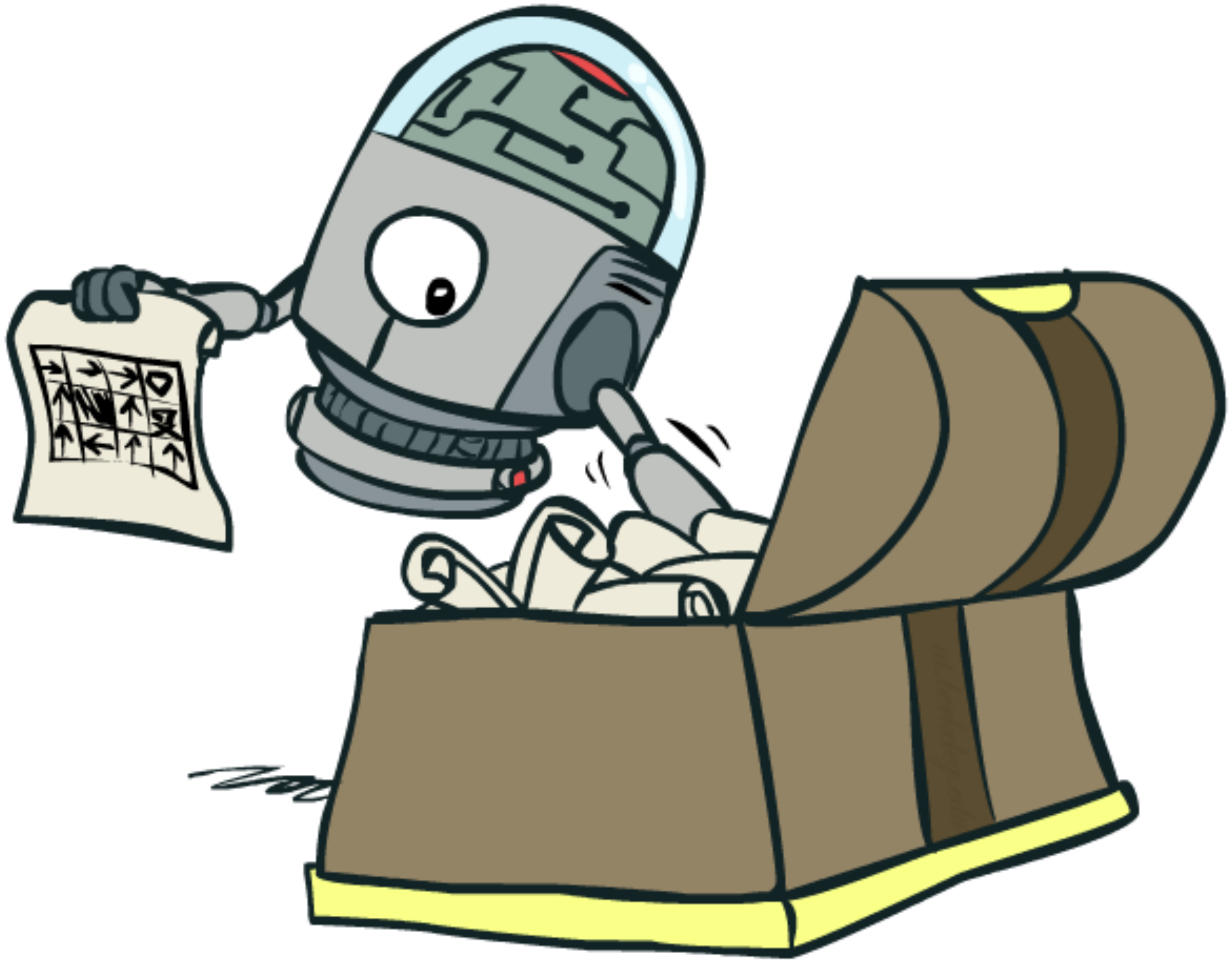
More Powerful Function Approximation

- Linear:
  - $Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$
- Polynomial:
  - $Q(s,a) = w_{11} f_1(s,a) + w_{12} f_1^2(s,a) + w_{13} f_1^3(s,a) + \ldots$
- Neural Network:
  - $Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$
  - where f_i's are also learned
- $w_m \leftarrow w_m + \alpha(r + \gamma \max_a Q(s',a') - Q(s,a)) \dfrac{\partial Q}{\partial w_m}(s,a)$

Overfitting: Why Limiting Capacity Can Help*



Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)
  - We'll see this distinction between modeling and prediction again later in the course

- Solution: learn policies that maximize rewards, not the values that predict them

- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before

- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

- Better methods exploit lookahead structure, sample wisely, change multiple parameters…

Policy Search

[Andrew Ng]