VE472 Lecture 3

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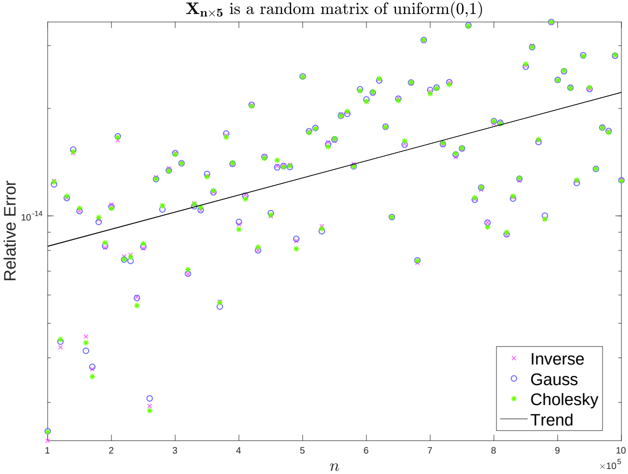
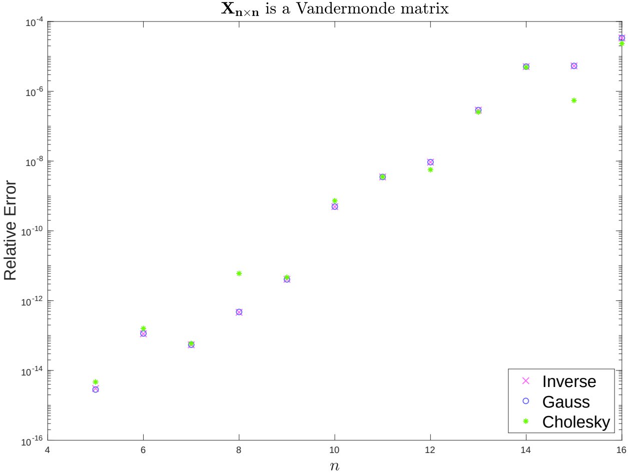
UM-SJTU Joint Institute

Summer

Random full rank matrix

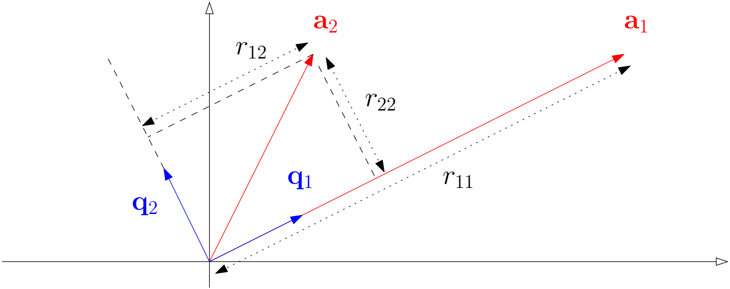


Vandermonde matrix



Theorem 0.1

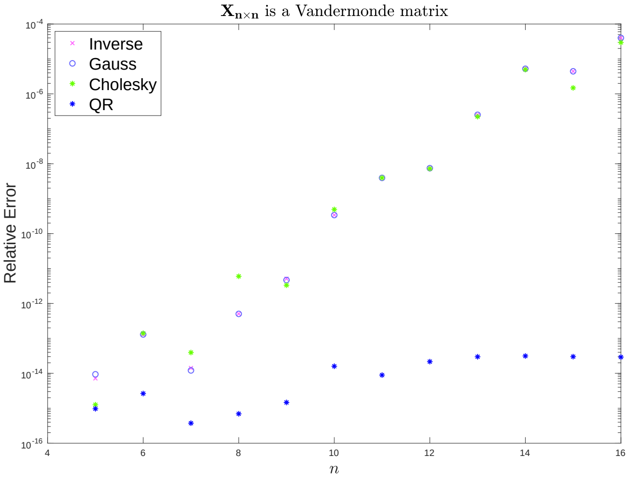
Let $\mathbf{A}$ be a matrix of $m \times n$ with $m \geq n$. Suppose $\mathbf{A}$ is full rank. Then there is a matrix $\mathbf{Q}$ of $m \times n$ with orthonormal columns, $\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{I}$, and an upper triangular matrix $\mathbf{R}$ of $n \times n$ with positive diagonals $r_{ii} > 0$ such that $\mathbf{A} = \mathbf{QR}$.



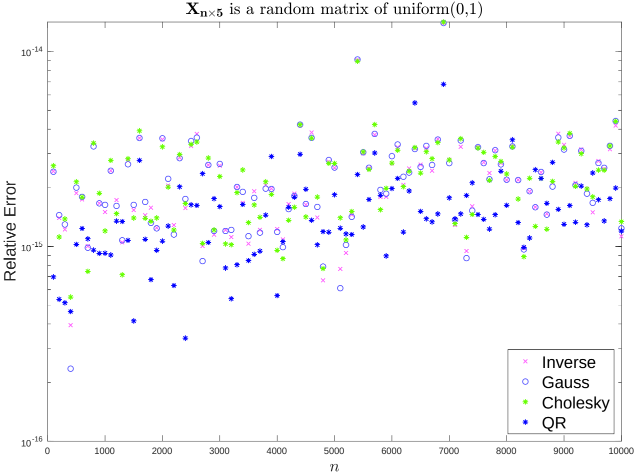
In general, the theorem replies on the idea of projection.

Q: Why is this theorem useful in terms of dealing with a big data matrix $\mathbf{X}$?

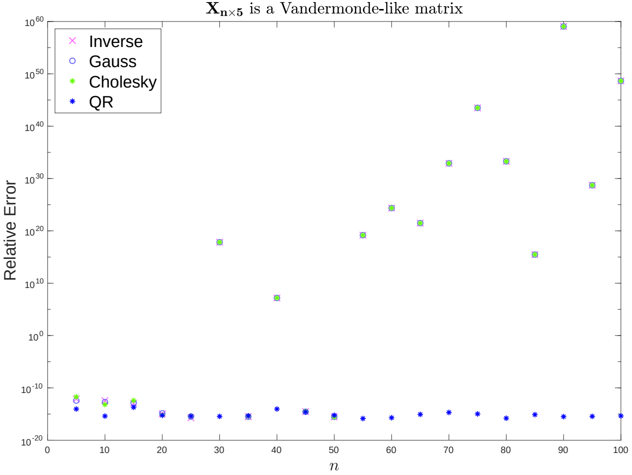
Vandermonde matrix



Random full rank matrix



Vandermonde-like matrix



- QR is no question the slowest!

```
>> clear all
>> n = 1000; X = rand(n, 100); y = randn(n, 1);
tic; for i = 1:10000
    XX = transpose(X)*X; yy = transpose(X)*y;
    [L, U] = lu(XX); bhat = U\(L\yy);
end; toc;
tic; for i = 1:10000
    XX = transpose(X)*X; yy = transpose(X)*y;
    C = chol(XX, 'lower');
    bhat =  transpose(C)\(C\yy);
end; toc;
tic ;for i = 1:10000
    [Q, R] = qr(X); bhat =  R\(transpose(Q)*y);
end; toc;
```

```
Elapsed time is 3.266948 seconds.
Elapsed time is 2.457167 seconds.
Elapsed time is 44.321296 seconds.
```