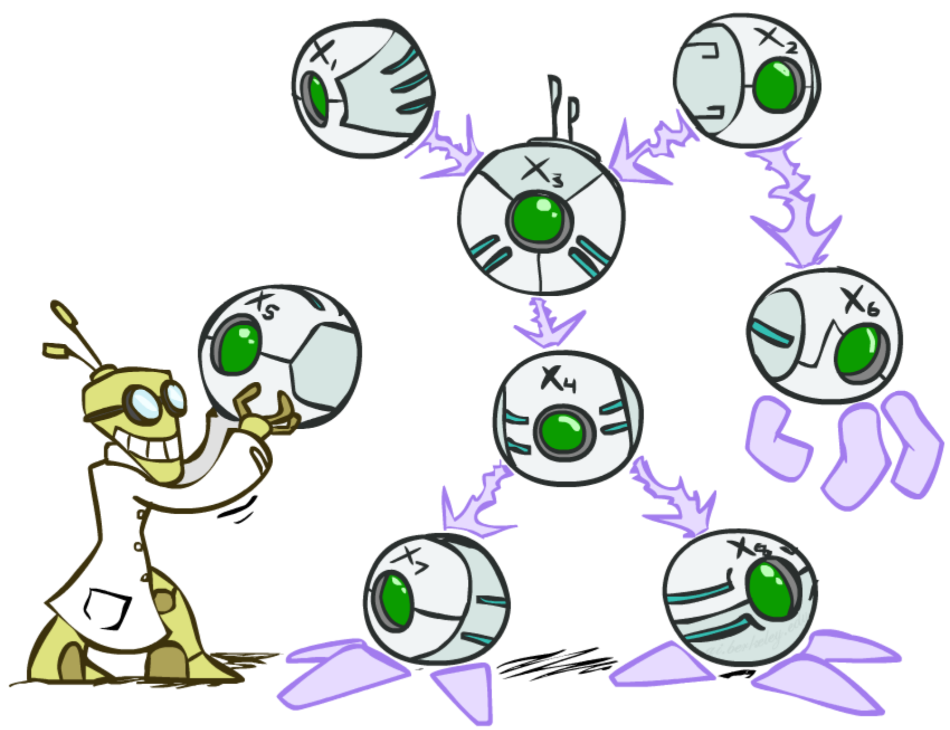
# Ve492: Introduction to Artificial Intelligence

## Bayesian Networks: Representation





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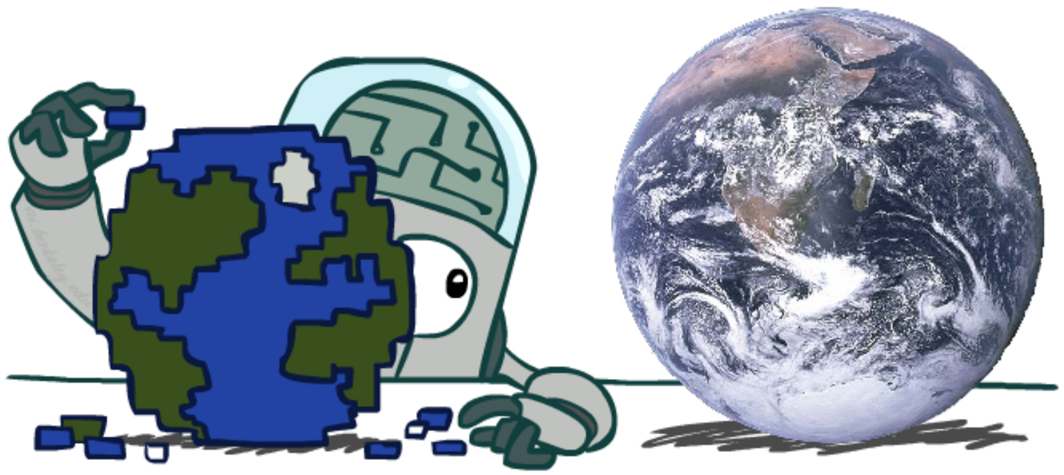
Slides adapted from http://ai.berkeley.edu, CMU, AIMA, UM

# Probabilistic Models

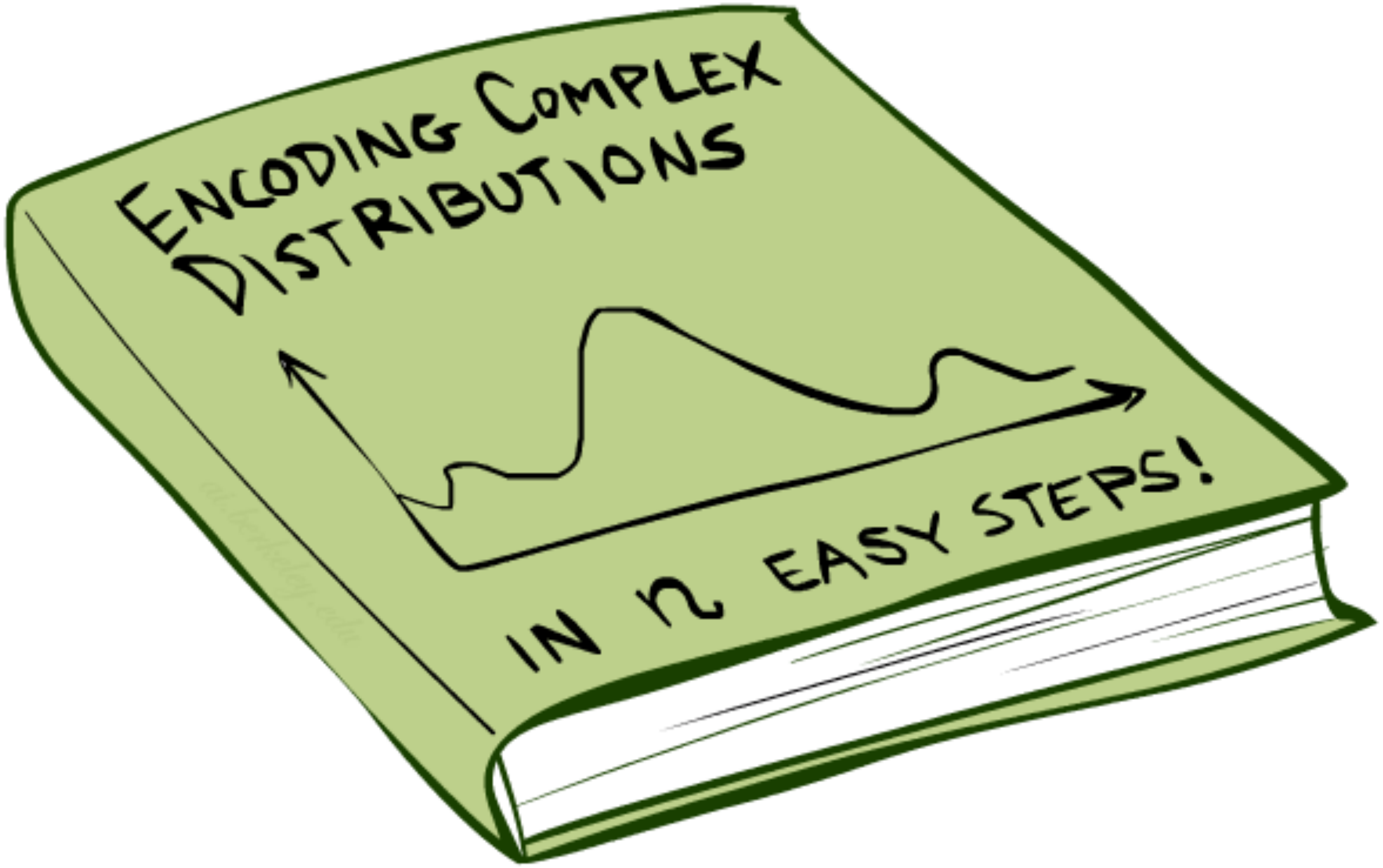
- Models describe how (a portion of) the world works

- Models are always simplifications
  - May not account for every variable
  - May not account for all interactions between variables
  - "All models are wrong; but some are useful."
    – George E. P. Box



- What do we do with probabilistic models?
  - We (or our agents) need to reason about unknown variables, given evidence
  - Example: explanation (diagnostic reasoning)
  - Example: prediction (causal reasoning)

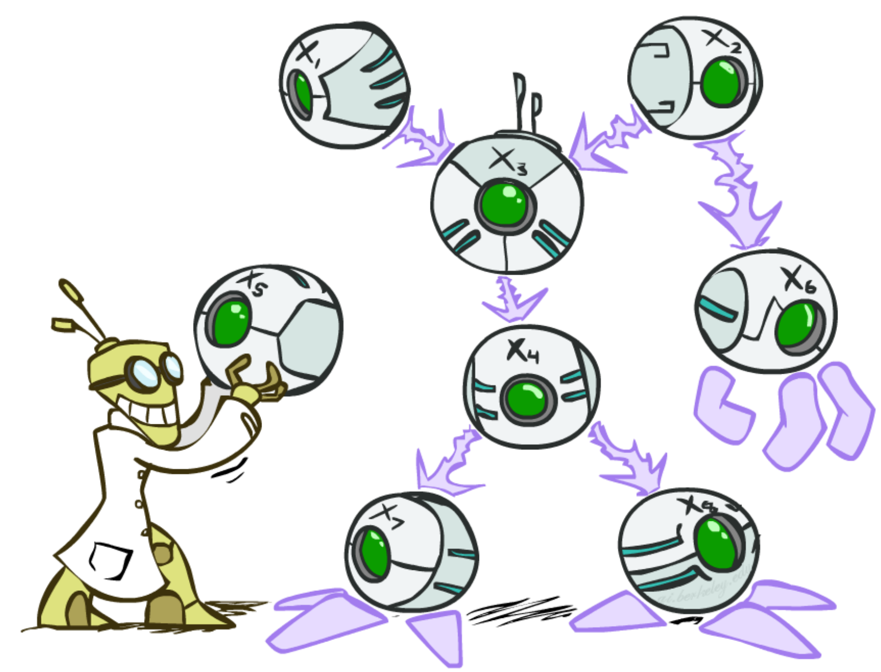
# Bayes' Nets: Big Picture



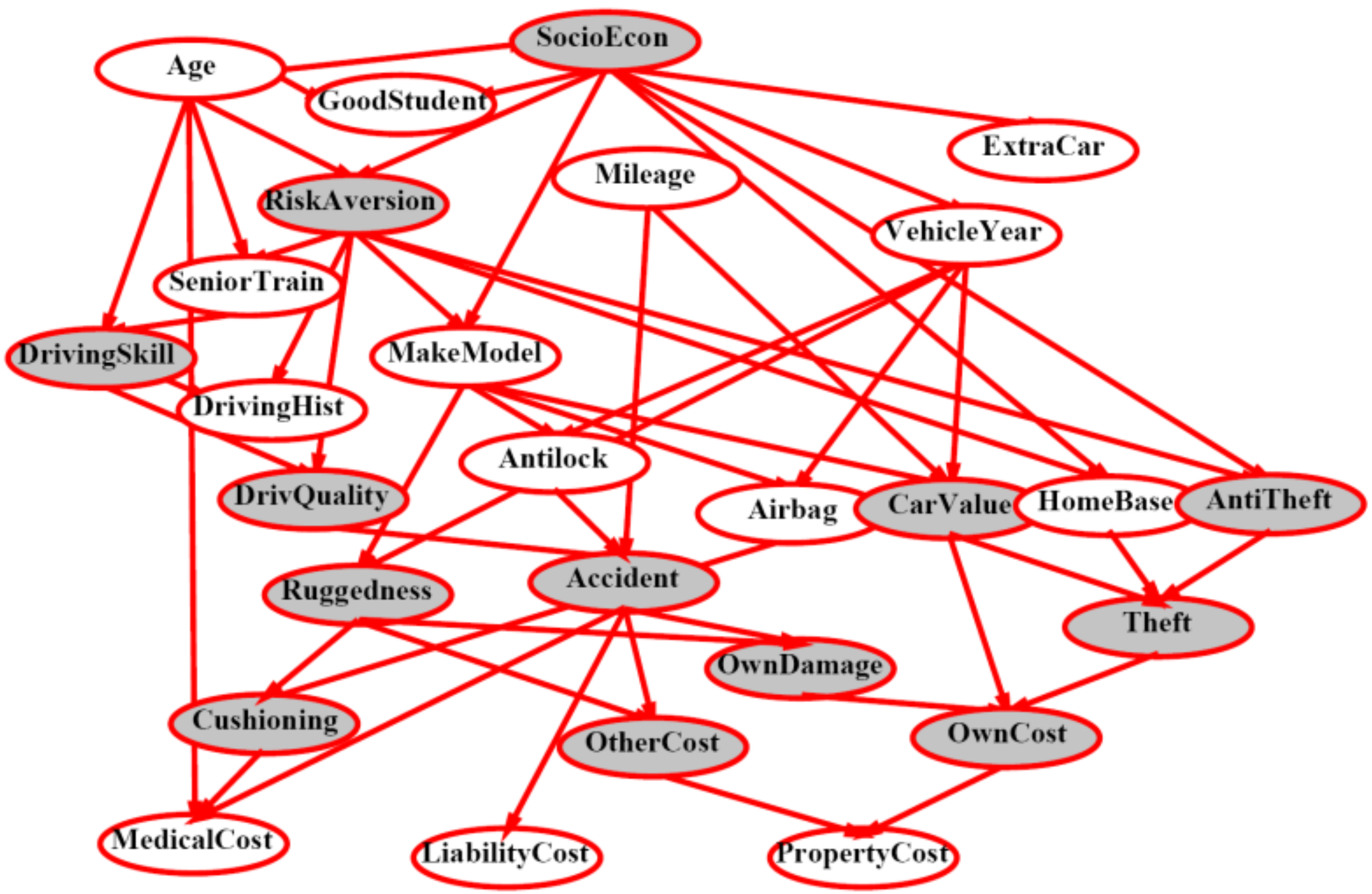
# Bayes' Nets: Big Picture

- Two problems with using full joint distribution tables as our probabilistic models:
  - Unless there are only a few variables, the joint is WAY too big to represent explicitly
  - Hard to learn (estimate) anything empirically about more than a few variables at a time

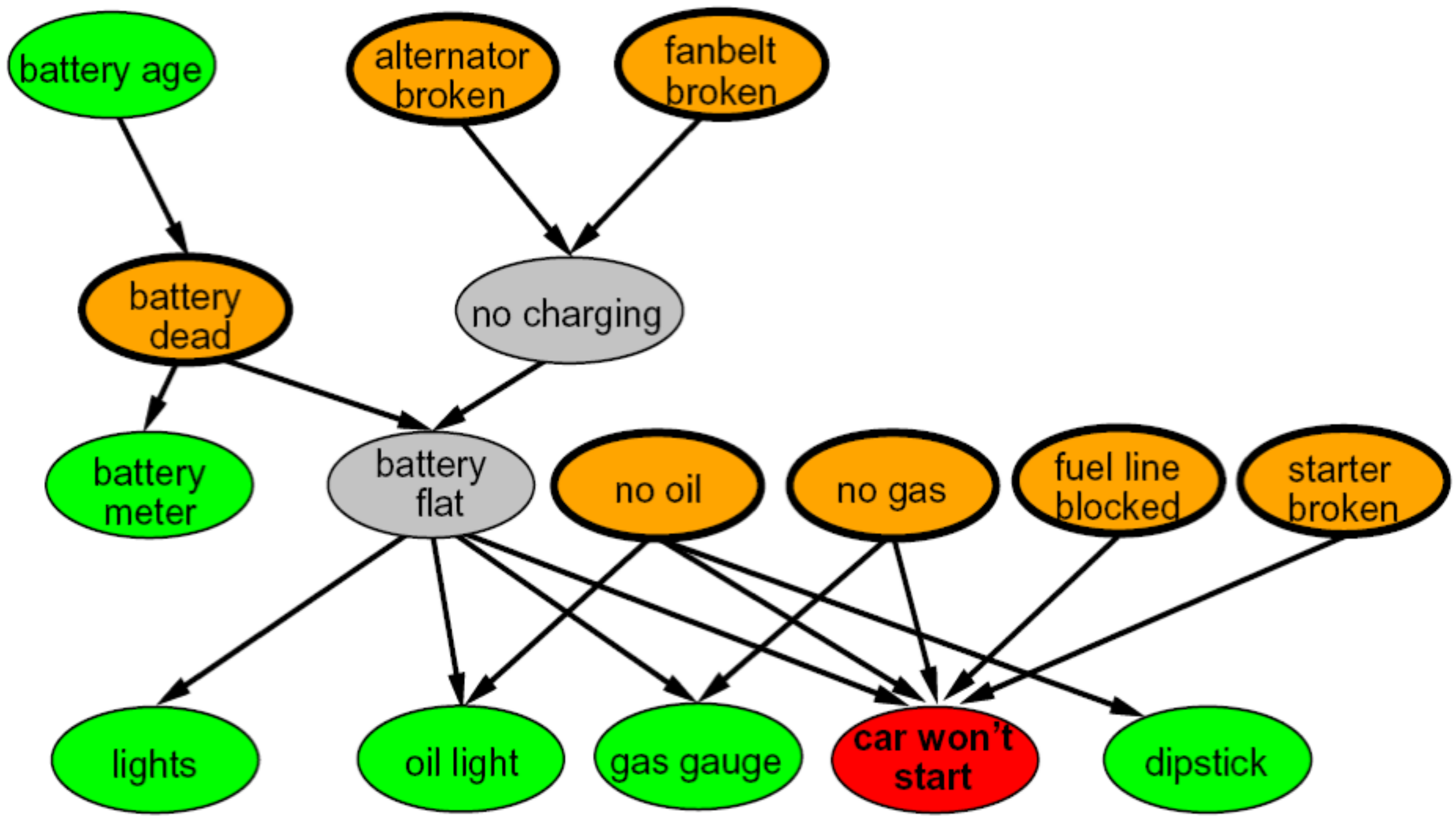
- Bayes' nets: a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
  - More properly called graphical models
  - We describe how variables locally interact
  - Local interactions chain together to give global, indirect interactions



# Example Bayes' Net: Insurance

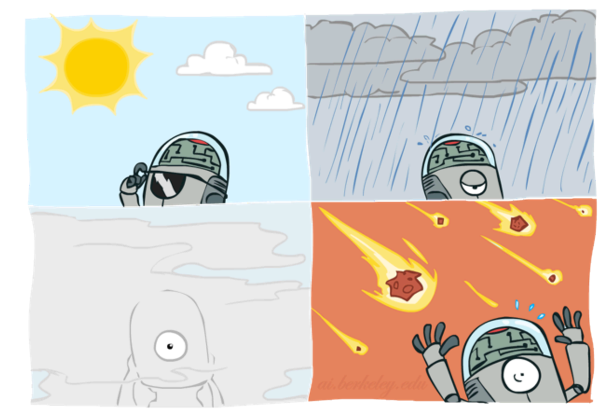


# Example Bayes' Net: Car

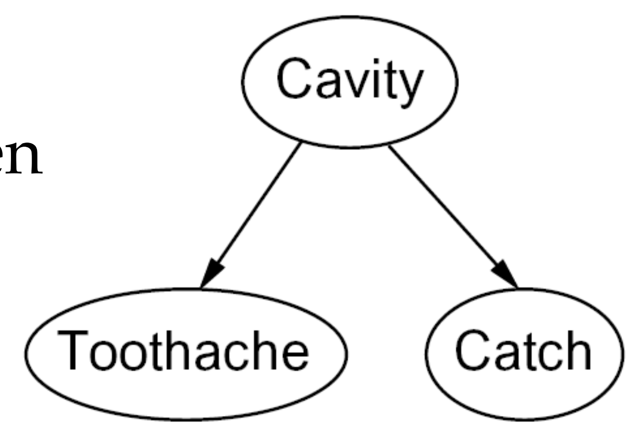


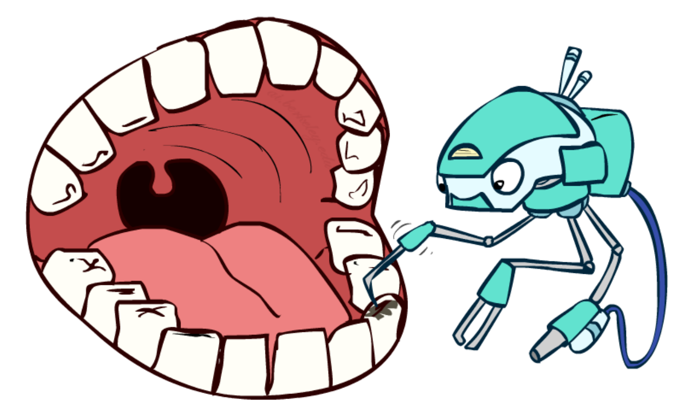
# Bayes' Net Notation

- Nodes: variables (with domains)
  - Can be assigned (observed) or unassigned (unobserved)

- Arcs: interactions
  - Similar to CSP constraints
  - Indicate "direct influence" between variables
  - Formally: encode conditional independence (more later)

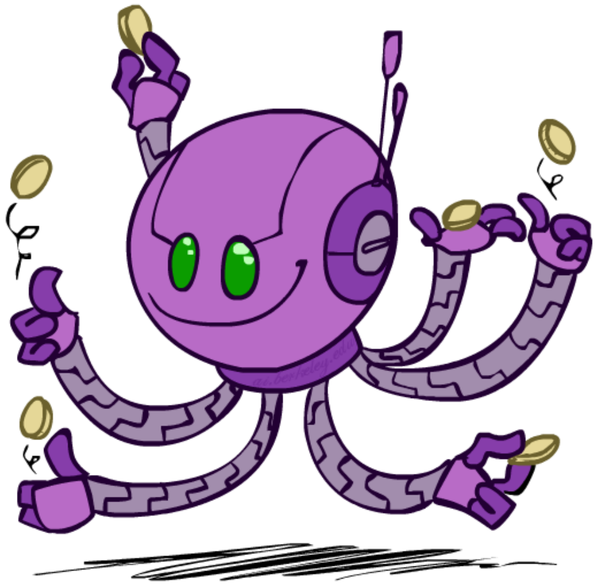


- For now: imagine that arrows mean direct causation (in general, they don't!)

# Example: Coin Flips

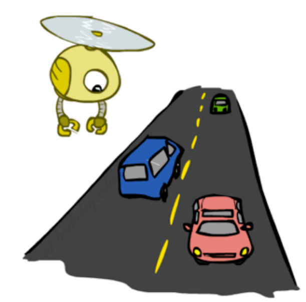
- N independent coin flips

- No interactions between variables: absolute independence

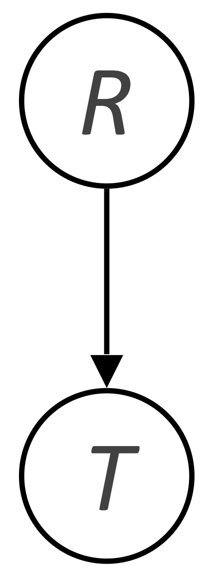
# Example: Traffic

- Variables:
  - R: It rains
  - T: There is traffic



- Model 1: independence

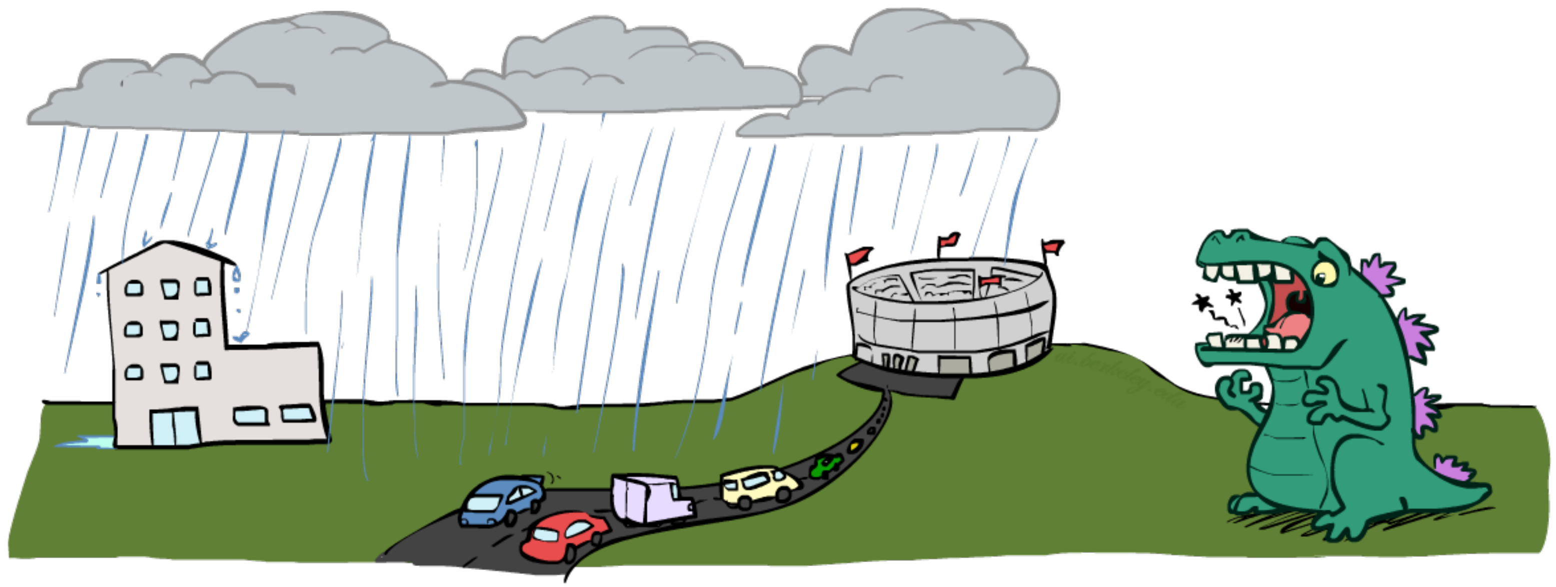

- Model 2: rain causes traffic




- Why is an agent using model 2 better?

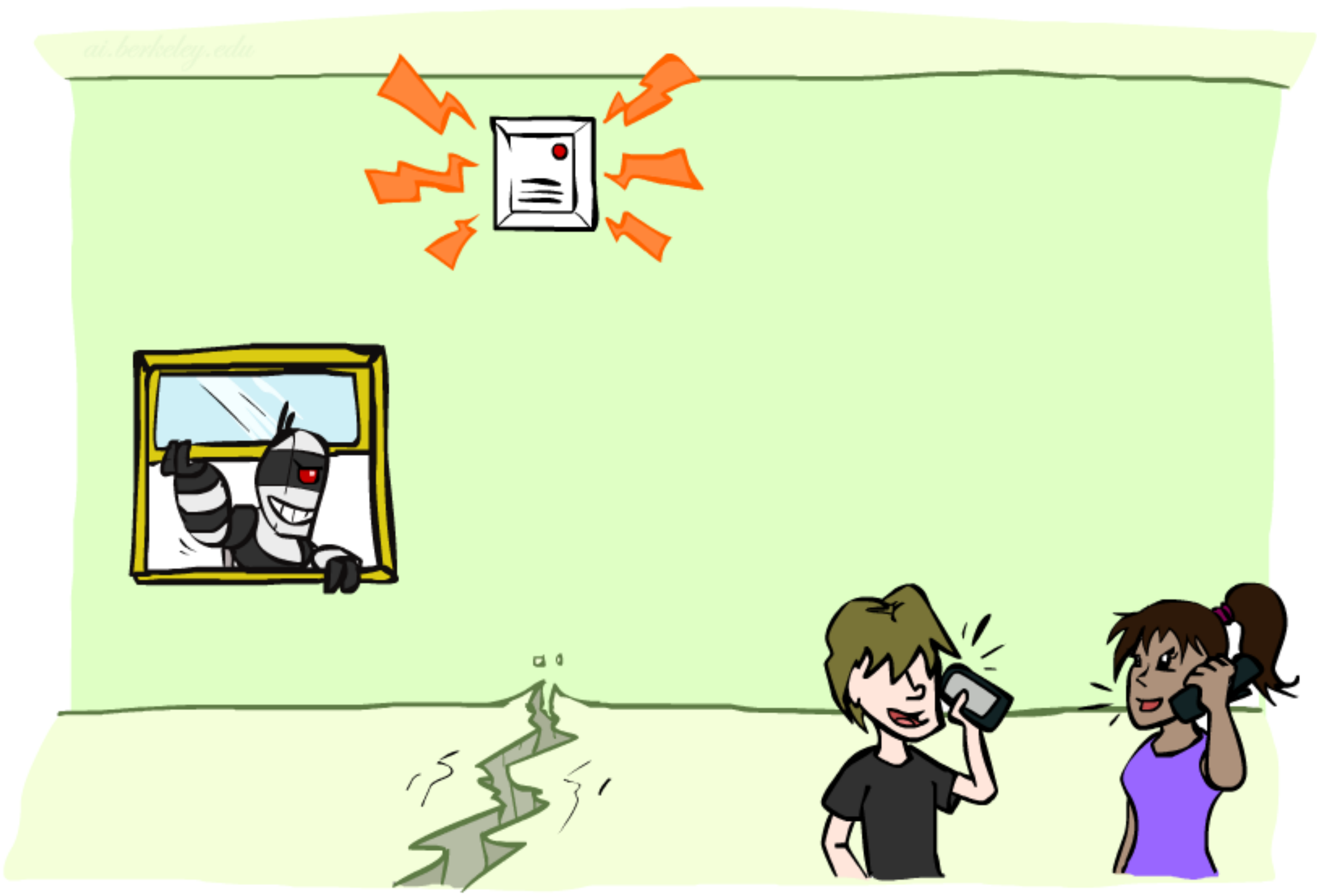
# Example: Traffic II

- Let's build a causal graphical model!
- Variables
  - T: Traffic
  - R: It rains
  - L: Low pressure
  - D: Roof drips
  - B: Ballgame
  - C: Cavity



# Example: Alarm Network

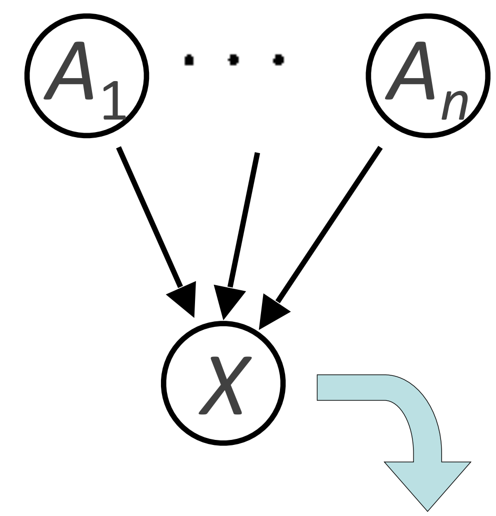
- Variables
  - B: Burglary
  - A: Alarm goes off
  - M: Mary calls
  - J: John calls
  - E: Earthquake!



# Bayes' Net Semantics

# Bayes' Net Semantics

- A set of nodes, one per variable
- A directed, acyclic graph
- A conditional distribution for each node
  - A collection of distributions over X, one for each combination of parents' values




$$P(X|a_1 \ldots a_n) \qquad\qquad P(X|A_1 \ldots A_n)$$

  - CPT: conditional probability table
  - Description of a noisy "causal" process

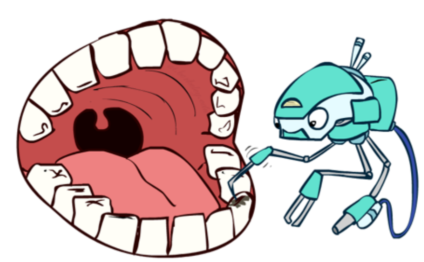
*A Bayes net = Topology (graph) + Local Conditional Probabilities*

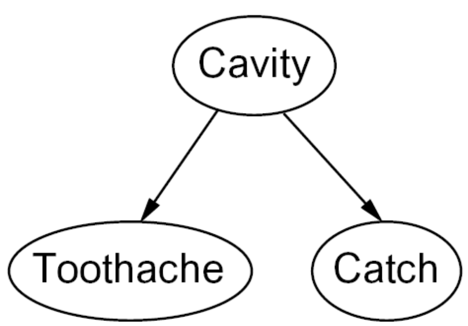
# Probabilities in BNs

- Bayes' nets implicitly encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$

  - Example:






$P(+cavity,\ +catch,\ -toothache)$

# Probabilities in BNs

- Why are we guaranteed that setting

$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | \mathit{parents}(X_i))$$

results in a proper joint distribution?

- Chain rule (valid for all distributions): $P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | x_1 \ldots x_{i-1})$

- Assume conditional independences: $P(x_i | x_1, \ldots x_{i-1}) = P(x_i | \mathit{parents}(X_i))$

→ Consequence: $P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | \mathit{parents}(X_i))$

- Not every BN can represent every joint distribution
  - The topology enforces certain conditional independencies

# Example: Coin Flips

$X_1$ $X_2$ $\cdots$ $X_n$

$P(X_1)$

| | |
|---|---|
| h | 0.5 |
| t | 0.5 |

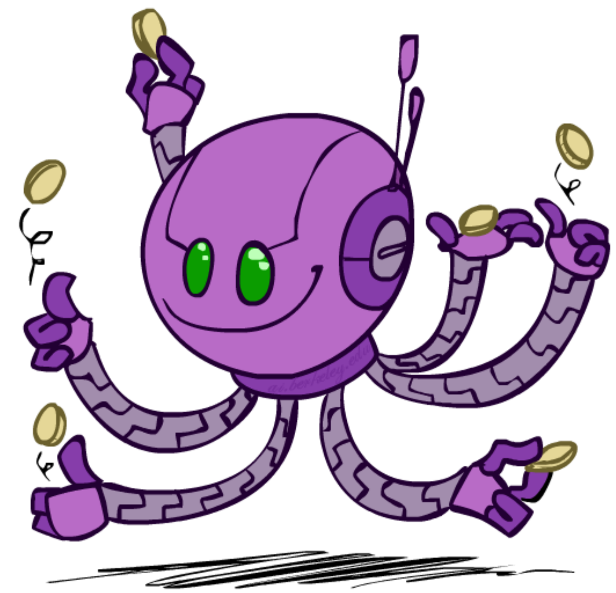
$P(X_2)$

| | |
|---|---|
| h | 0.5 |
| t | 0.5 |

$\cdots$

$P(X_n)$

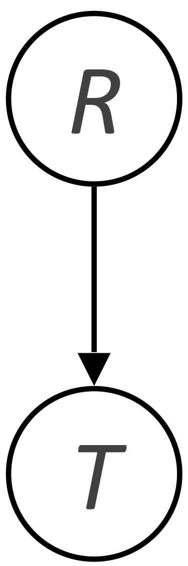
| | |
|---|---|
| h | 0.5 |
| t | 0.5 |



$$P(h, h, t, h) =$$

*Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.*

# Example: Traffic



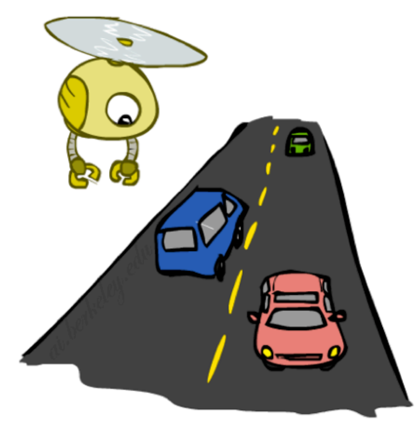

$P(R)$

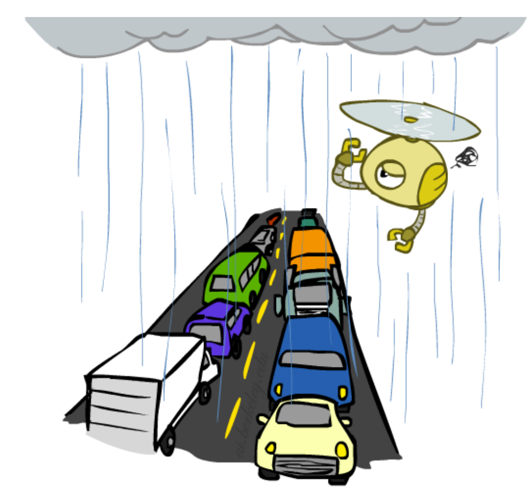
| | |
|---|---|
| +r | 1/4 |
| -r | 3/4 |

$P(T|R)$

| | | |
|---|---|---|
| +r | +t | 3/4 |
| | -t | 1/4 |
| -r | +t | 1/2 |
| | -t | 1/2 |

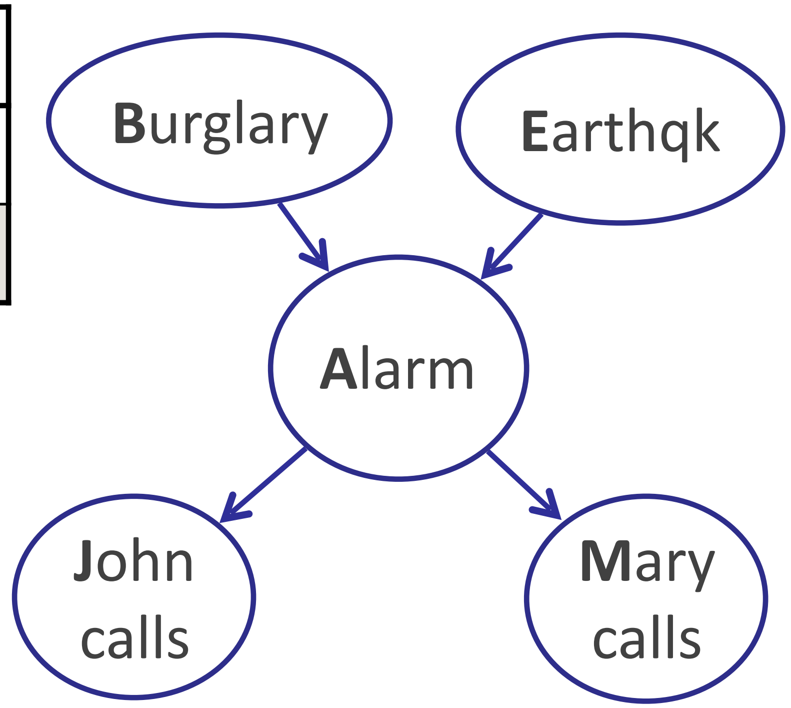
$$P(+r, -t) =$$



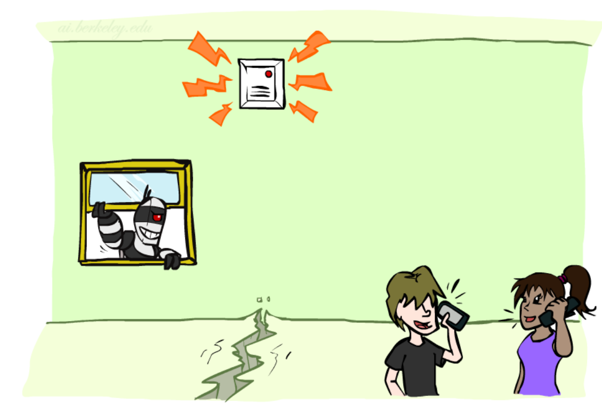


# Example: Alarm Network

| B | P(B) |
|---|---|
| +b | 0.00 |
| -b | 0.99 |




| E | P(E) |
|---|---|
| +e | 0.00 |
| -e | 0.99 |



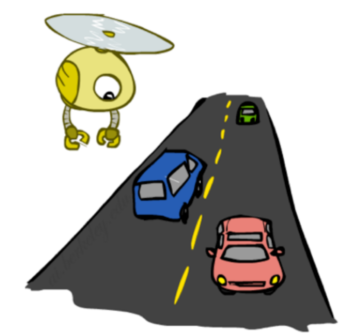
| B | E | A | P(A\|B,E) |
|---|---|---|---|
| +b | +e | +a | 0.95 |
| +b | +e | -a | 0.05 |
| +b | -e | +a | 0.94 |
| +b | -e | -a | 0.06 |
| -b | +e | +a | 0.29 |
| -b | +e | -a | 0.71 |
| -b | -e | +a | 0.001 |
| -b | -e | -a | 0.999 |

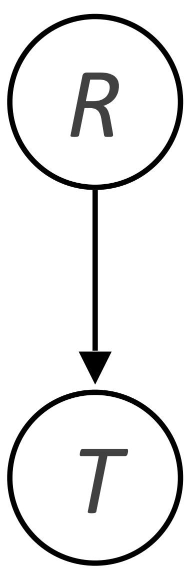
| A | J | P(J\|A) |
|---|---|---|
| +a | +j | 0.9 |
| +a | -j | 0.1 |
| -a | +j | 0.05 |
| -a | -j | 0.95 |

| A | M | P(M\| |
|---|---|---|
| +a | +m | 0.7 |
| +a | -m | 0.3 |
| -a | +m | 0.01 |
| -a | -m | 0.99 |

# Example: Traffic

- Causal direction






$P(R)$

| | |
|---|---|
| +r | 1/4 |
| -r | 3/4 |

$P(T|R)$

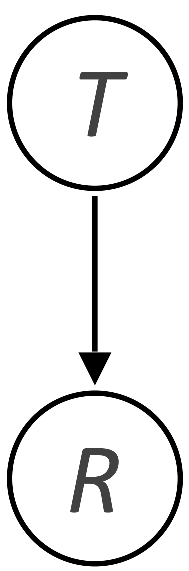
| | | |
|---|---|---|
| +r | +t | 3/4 |
| | -t | 1/4 |
| -r | +t | 1/2 |
| | -t | 1/2 |

$P(T, R)$

| | | |
|---|---|---|
| +r | +t | 3/16 |
| +r | -t | 1/16 |
| -r | +t | 6/16 |
| -r | -t | 6/16 |

# Example: Reverse Traffic

- Reverse causality?




$P(T)$

| | |
|---|---|
| +t | 9/16 |
| -t | 7/16 |

$P(R|T)$

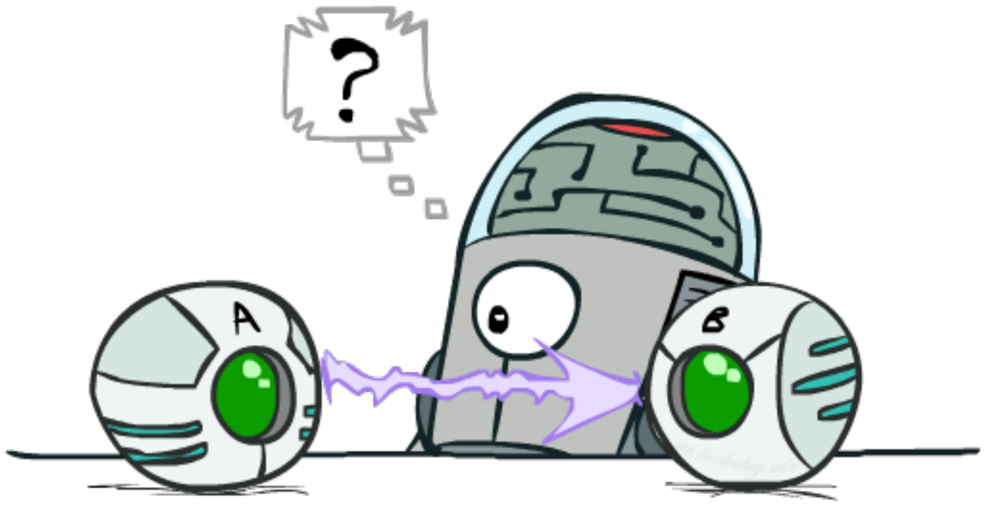
| | | |
|---|---|---|
| +t | +r | 1/3 |
| | -r | 2/3 |
| -t | +r | 1/7 |
| | -r | 6/7 |

$P(T,R)$

| | | |
|---|---|---|
| +r | +t | 3/16 |
| +r | -t | 1/16 |
| -r | +t | 6/16 |
| -r | -t | 6/16 |

# Causality?

- When Bayes' nets reflect the true causal patterns:
  - Often simpler (nodes have fewer parents)
  - Often easier to think about
  - Often easier to elicit from experts




- BNs need not actually be causal
  - Sometimes no causal net exists over the domain (especially if variables are missing)
  - E.g. consider the variables *Traffic* and *Drips*
  - End up with arrows that reflect correlation, not causation

- What do the arrows really mean? $P(x_i|x_1, \ldots x_{i-1}) = P(x_i|\mathit{parents}(X_i))$
  - Topology may happen to encode causal structure
  - Topology really encodes conditional independence

# Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution
  - Key property: conditional independence
- Next: how to answer queries about that distribution
  - Queries about conditional independence and influence
- After that: how to answer numerical queries (inference)
  - Queries about (conditional) probabilities