VG441 Problem Set 1

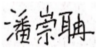
Pan, Chongdan ID:516370910121

June 29, 2020

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

Signature:

1 Problem 1

(a)

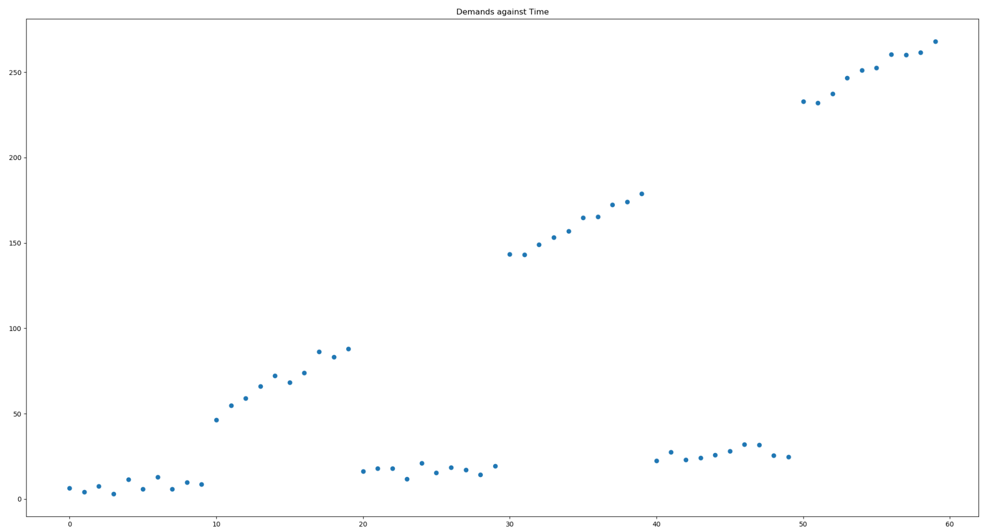


Figure 1: demands against time

(b)

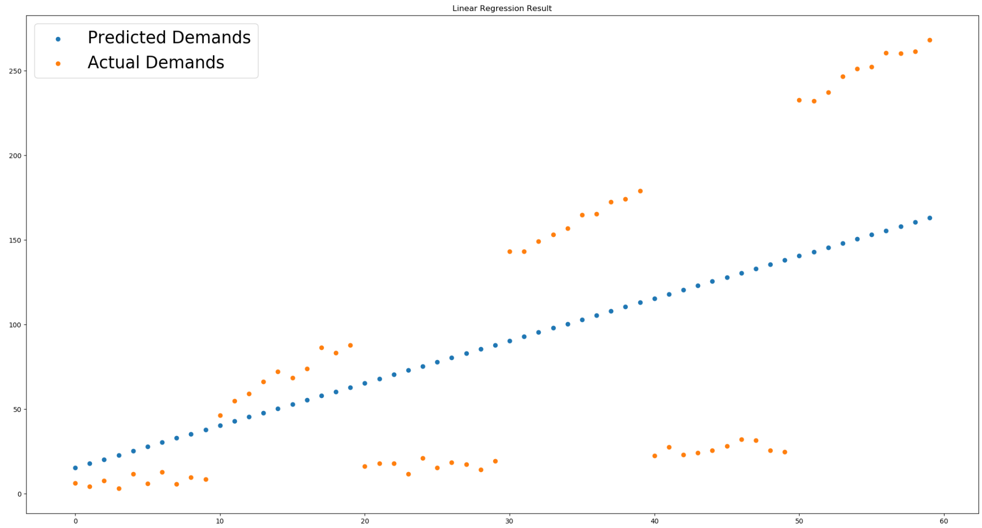


Figure 2: Linear Regression Result

(c)

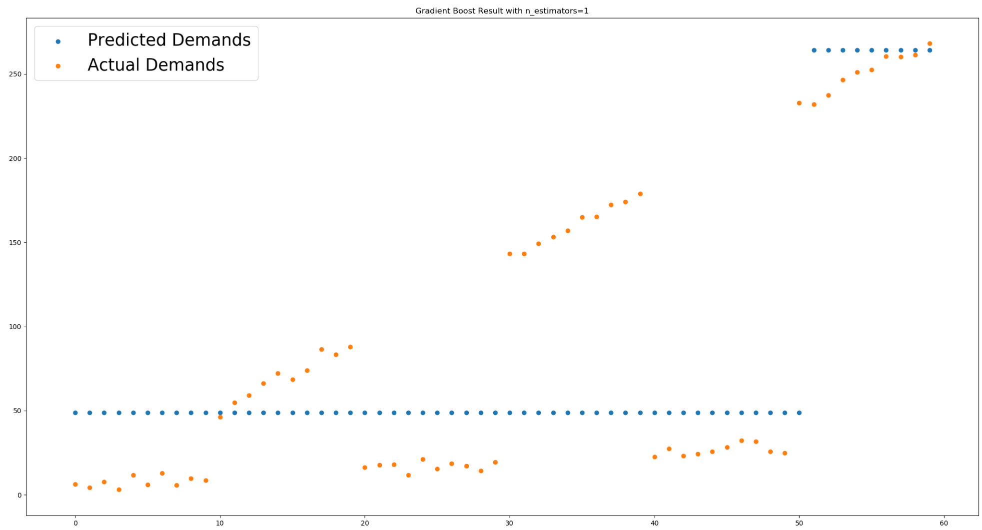


Figure 3: Gradient Boost Result with n_estimators=1

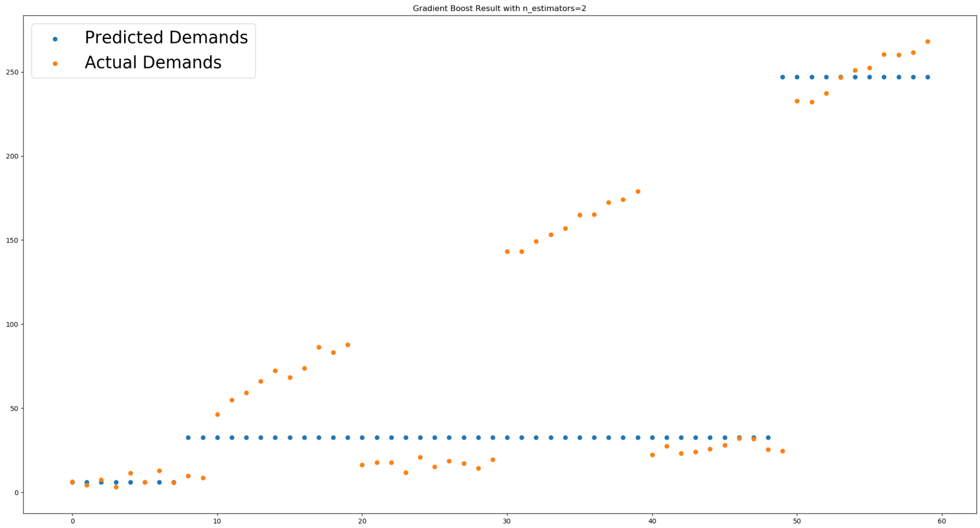


Figure 4: Gradient Boost Result with n_estimators=2

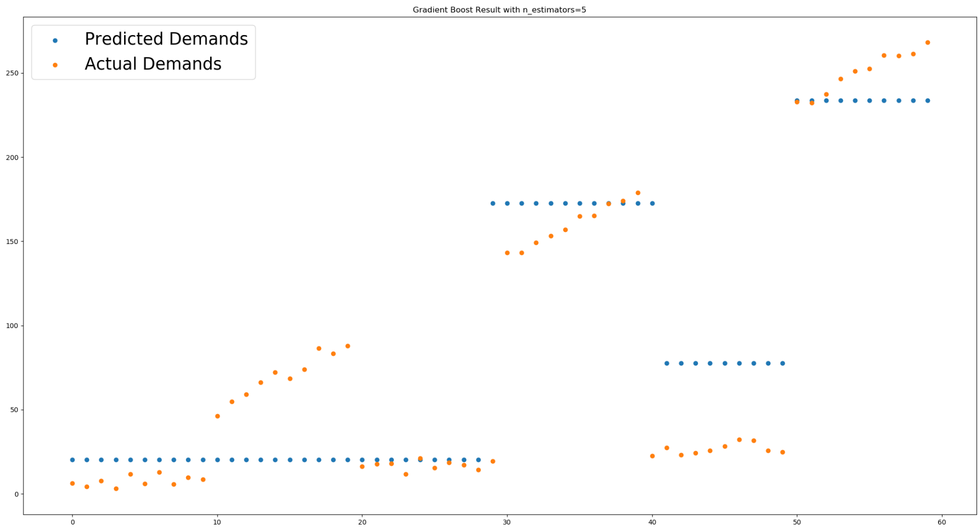


Figure 5: Gradient Boost Result with n_estimators=5

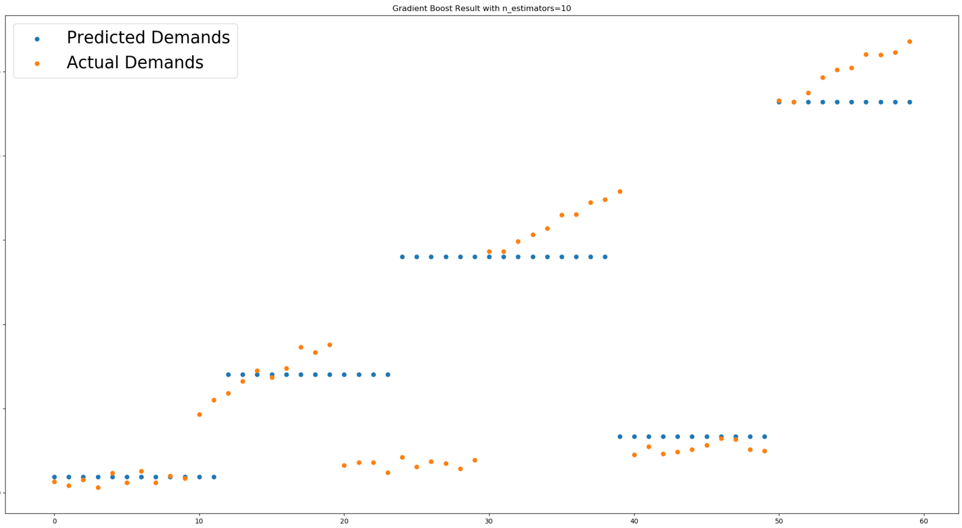


Figure 6: Gradient Boost Result with n_estimators=10

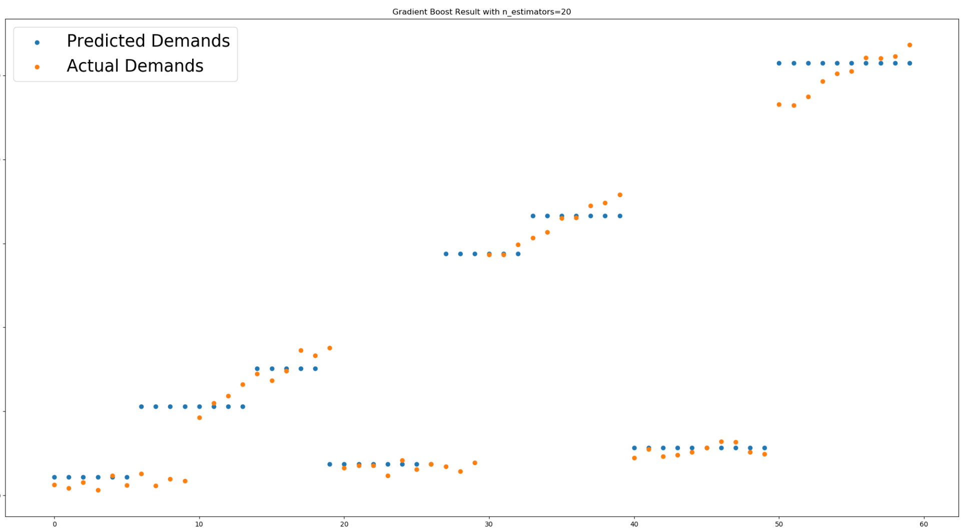


Figure 7: Gradient Boost Result with n_estimators=20

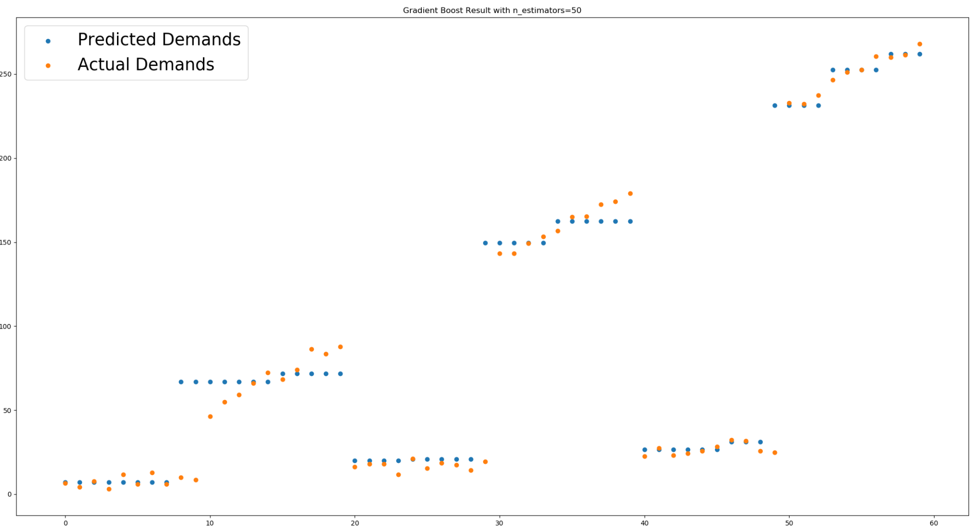


Figure 8: Gradient Boost Result with n_estimators=50

2 Problem 2

(a) $K = \$50$ per order, $h = \$\frac{50}{3}$ per unit per month, $\lambda = 50$ units per month

$$c = \begin{cases} 520 & Q < 12 \\ 510 & 12 \leq Q \leq 64 \\ 495 & 65 \leq Q \leq 128 \\ 485 & Q > 128 \end{cases}$$

Since it's all unit discount model, we calculate the Q^* first.
$Q^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2\times 50\times 50}{\frac{50}{3}}} \approx 17.3$
So the price should be \$510
$g_0(Q_*) = 510 \times 50 + \sqrt{2 \times 50 \times 50 \times \frac{50}{3}} = 25788.68\$$
Now we calculate the cost at breakpoints:
$g_0(1) = 520 \times 50 + \frac{50\times 50}{1} + \frac{50}{3}\frac{1}{2} \approx 28508.33$
$g_1(12) = 510 \times 50 + \frac{50\times 50}{12} + \frac{50}{3}\frac{12}{2} \approx 25808.33\$$
$g_2(65) = 495 \times 50 + \frac{50\times 50}{65} + \frac{50}{3}\frac{65}{2} \approx 25330.13\$$
$g_2(128) = 495 \times 50 + \frac{50\times 50}{128} + \frac{50}{3}\frac{128}{2} \approx 25836.2\$$
$g_3(129) = 485 \times 50 + \frac{50\times 50}{129} + \frac{50}{3}\frac{129}{2} \approx 25334.38\$$
Therefore, the optimal order is not at breakpoints, the optimal quantity is $Q = 65$, which incurs a purchase cost of 495\$ and a total monthly cost of 25330.13\$

(b) If the offer is accepted, the monthly cost will be $50 \times (520 + 50) = 28500\$$, which is larger than our optimal strategy, hence Zeus should not accept the offer.

3 Problem 3

(a) $K = \$1000, h = \$1.2$
$\theta_8 = 0$

$\theta_7 = K + \theta_8 = 1000$

$\theta_6 = K + \min\{\theta_7, hd_7\}$
$= 1000 + \min\{1000, 1.2 \times 290\}$
$= 1000 + \min\{1000, 348\} = 1348$

$\theta_5 = K + \min\{\theta_6, hd_6 + \theta_7, h(d_6 + 2 \times d_7) + \theta_8\}$
$= 1000 + \min\{1348, 1.2 \times 210 + 1000, 1.2 \times (210 + 2 \times 290)\}$
$= 1000 + \min\{1348, 1252, 948\} = 1948$

$\theta_4 = K + \min\{\theta_5, hd_5 + \theta_6, h(d_5 + 2 \times d_6) + \theta_7, h(d_5 + 2 \times d_6 + 3 \times d_7) + \theta_8\}$
$= 1000 + \min\{1948, 1.2 \times 170 + 1348, 1.2 \times (170 + 2 \times 210) + 1000, 1.2 \times (170 + 2 \times 210 + 3 \times 290)\}$
$= \min\{2948, 2552, 2708, 2752\} = 2552$

$\theta_3 = K + \min\{\theta_4, hd_4 + \theta_5, h(d_4 + 2 \times d_5) + \theta_6, h(d_4 + 2 \times d_5 + 3 \times d_6) + \theta_7, h(d_4 + 2 \times d_5 + 3 \times d_6 + 4 \times d_7) + \theta_8\}$
$= 1000 + \min\{2552, 1.2 \times 90 + 1948, 1.2 \times (90 + 2 \times 170) + 1348, 1.2 \times (90 + 2 \times 170 + 3 \times 210) + 1000, 1.2 \times (90 + 2 \times 170 + 3 \times 210 + 4 \times 290)\}$
$= \min\{3552, 3056, 2864, 3272, 3664\} = 2864$

$\theta_2 = K + \min\{\theta_3, hd_3 + \theta_4, h(d_3 + 2 \times d_4) + \theta_5, h(d_3 + 2 \times d_4 + 3 \times d_5) + \theta_6, h(d_3 + 2 \times d_4 + 3 \times d_5 + 4 \times d_6) + \theta_7, h(d_3 + 2 \times d_4 + 3 \times d_5 + 4 \times d_6 + 5 \times d_7) + \theta_8\}$
$= 1000 + \min\{2864, 1.2 \times 105 + 2552, 1.2 \times (105 + 2 \times 90) + 1948, 1.2 \times (105 + 2 \times 90 + 3 \times 170) + 1348, 1.2 \times (105 + 2 \times 90 + 3 \times 170 + 4 \times 210) + 1000, 1.2 \times (105 + 2 \times 90 + 3 \times 170 + 4 \times 210 + 5 \times 290)\}$
$= \min\{3864, 3678, 3290, 3302, 3962, 4702\} = 3290$

$\theta_1 = K + \min\{\theta_2, hd_2 + \theta_3, h(d_2 + 2 \times d_3) + \theta_4, h(d_2 + 2 \times d_3 + 3 \times d_4) + \theta_5, h(d_2 + 2 \times d_3 + 3 \times d_4 + 4 \times d_5) + \theta_6, h(d_2 + 2 \times d_3 + 3 \times d_4 + 4 \times d_5 + 5 \times d_6) + \theta_7, h(d_2 + 2 \times d_3 + 3 \times d_4 + 4 \times d_5 + 5 \times d_6 + 6 \times d_7) + \theta_8\}$
$= 1000 + \min\{3290, 1.2 \times 155 + 2864, 1.2 \times (155 + 2 \times 105 + 2552, 1.2 \times (155 + 2 \times 105 + 3 \times 90) + 1948, 1.2 \times (155 + 2 \times 105 + 3 \times 90 + 4 \times 170) + 1348, 1.2 \times (155 + 2 \times 105 + 3 \times 90 + 4 \times 170 + 5 \times 210) + 1000, 1.2 \times (155 + 2 \times 105 + 3 \times 90 + 4 \times 170 + 5 \times 210 + 6 \times 290)\}$
$= \min\{4290, 4050, 3990, 3710, 3926, 4838, 5926\} = 3710$

Therefore, we should order d_1, d_2, d_3, d_4 at Sunday, and order d_5, d_6, d_7 at Thursday.
The total cost is \$3710

(b) At the beginning, $n = 7, m = 21, s = 1, A[1] = 0, B[1] = \emptyset$

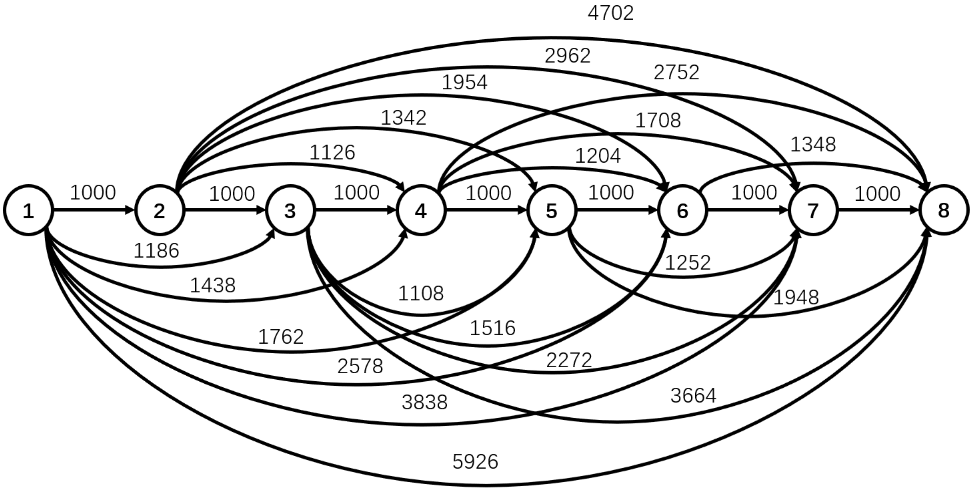


Figure 9: Wagner-Whitin network

$\min A[2] = 1000, B[2] = \{12\}$
$\min A[3] = \min\{2000, 1186\} = 1186, B[3] = \{12, 13\}$
$\min A[4] = \min\{2186, 2126, 1438\} = 1438, B[4] = \{12, 13, 14\}$
$\min A[5] = \min\{2438, 2294, 2342, 1762\} = 1762, B[5] = \{12, 13, 14, 15\}$
$\min A[6] = \min\{2762, 2642, 2702, 2954, 2578\} = 2578, B[6] = \{12, 13, 14, 15, 16\}$
$\min A[7] = \min\{3762, 3014, 3146, 3458, 3962, 3838\} = 3014, B[7] = \{12, 13, 14, 15, 16, 57\}$
$\min A[8] = \min\{4762, 3629, 3710, 4190, 4850, 5702, 5926\} = 3710, B[7] = \{12, 13, 14, 15, 16, 57, 58\}$

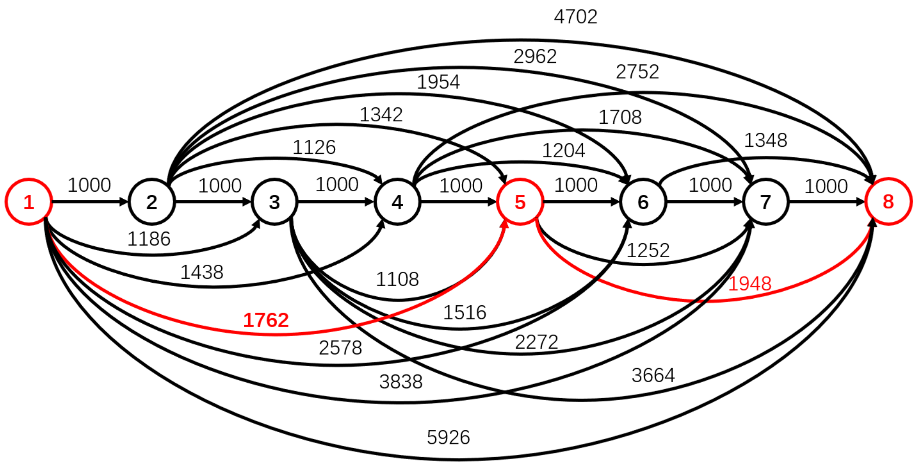


Figure 10: Wagner-Whitin network

Therefore, we should order d_1, d_2, d_3, d_4 at Sunday, and order d_5, d_6, d_7 at Thursday. The result is same to dynamic programming with total cost \$3710

(c) The problem is equal to :

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{7}(1000y_t + 1.2x_t) \\
s.t. \quad & x_t = x_{t-1} + q_t - d_t \quad \forall t = 1, \cdots, 7 \\
& q_t \leq 100000y_t \\
& x_t \geq 0 \\
& q_t \geq 0 \\
& y_t \in \{0, 1\}
\end{aligned}
$$

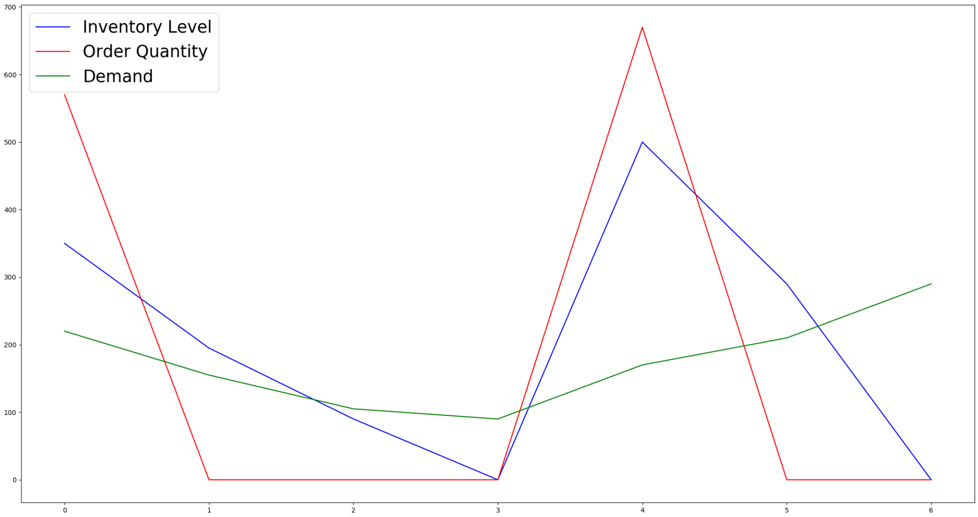
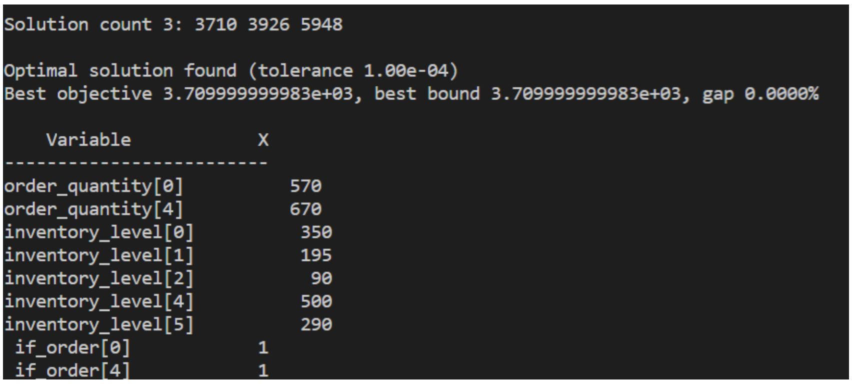


Figure 11: MILP Figure



```
Solution count 3: 3710 3926 5948

Optimal solution found (tolerance 1.00e-04)
Best objective 3.709999999983e+03, best bound 3.709999999983e+03, gap 0.0000%

    Variable            X
-------------------------
order_quantity[0]         570
order_quantity[4]         670
inventory_level[0]         350
inventory_level[1]         195
inventory_level[2]          90
inventory_level[4]         500
inventory_level[5]         290
 if_order[0]            1
 if_order[4]            1
```

Figure 12: MILP Result

The result shows that we should oder d_1, d_2, d_3, d_4 on Sunday, and d_5, d_6, d_7 on Thursday. The optimal cost is \$3710, which is same to last two questions.

4 Problem 4

(a) The duality of the LP problem is:

$$\min \quad 24y_1 + 60y_2$$

s.t.

$$\begin{aligned} 3y_1 + y_2 &\geq 6 \\ 2y_1 + 2y_2 &\leq 14 \\ y_1 + 4y_2 &= 13 \\ y_1 &\geq 0 \\ y_2 &\leq 0 \end{aligned}$$

(b) Assume $X = [x_{11}, \cdots, x_{1j}, x_{21}, \cdots, x_{ij}]^T$, where $(i,j) \in E$
For the two constraints, we can let $MX \leq 1, NX \leq 1$, where $M \in \mathbb{R}^{|I| \times |E|}, N \in \mathbb{R}^{|J| \times |E|}$
We can connect M and N together as: $A = \begin{bmatrix} M \\ N \end{bmatrix}$ where $A \in \mathbb{R}^{(|I|+|J|) \times |E|}$
Then according to duality and $\mathbf{A}^T$, we get $y_i + y_{(|I|+j)} \geq 1 \quad \forall (i,j) \in E$
Hence the duality of the problem is :

$$\min \quad \sum_{k=1}^{|I|+|J|} y_k$$

s.t.

$$\begin{aligned} y_i + y_{(|I|+j)} &\geq 1 \quad \forall (i,j) \in E \\ y_k &\geq 0 \quad \forall k = 1, \cdots, |I| + |J| \end{aligned}$$

Python Code

P1(a)

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
df = pd.DataFrame(pd.read_csv(r"D:\PANDA\Study\VG441\Homework\Midterm 1\demands.csv"))
T=df['time']
D=df['demands']
plt.title('Demands against Time')
plt.scatter(T,D)
plt.show()
```

P1(b)

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn import linear_model
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.model_selection import cross_val_predict
df = pd.DataFrame(pd.read_csv(r"D:\PANDA\Study\VG441\Homework\Midterm 1\demands.csv"))
T=df[['time']]
D=df[['demands']]
X_train,X_test,Y_train,Y_test=train_test_split(T, D, test_size=0.8)
model=linear_model.LinearRegression()
model.fit(X_train, Y_train)
model_score = model.score(X_train,Y_train)
Y_predicted = model.predict(X_test)
print("Mean squared error: %.2f"% mean_squared_error(Y_test, Y_predicted))
print('R2 sq: ',r2_score(Y_test, Y_predicted))
D_predicted = model.predict(T)
plt.title('Linear Regression Result')
plt.scatter(T,D_predicted,label='Predicted Demands')
plt.scatter(T,D,label='Actual Demands')
plt.legend(loc='upper left', fontsize=25)
plt.show()
```

P1(c)

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn import ensemble
```

```
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.model_selection import cross_val_predict
df = pd.DataFrame(pd.read_csv(r"D:\PANDA\Study\VG441\Homework\Midterm 1\demands.csv"))
T=df[['time']]
D=df[['demands']]
n=50
X_train,X_test,Y_train,Y_test=train_test_split(T, D, test_size=0.8)
params = {'n_estimators': n, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'}
model = ensemble.GradientBoostingRegressor(**params)
model.fit(X_train, Y_train)
model_score = model.score(X_train,Y_train)
Y_predicted = model.predict(X_test)
print("Mean squared error: %.2f"% mean_squared_error(Y_test, Y_predicted))
print('R2 sq: ',r2_score(Y_test, Y_predicted))
D_predicted = model.predict(T)
title='Gradient Boost Result with n_estimators='+str(n)
plt.title(title)
plt.scatter(T,D_predicted,label='Predicted Demands')
plt.scatter(T,D,label='Actual Demands')
plt.legend(loc='upper left', fontsize=25)
plt.show()
```

P3(c)

```
import numpy as np
import pandas as pd
from gurobipy import *
import matplotlib.pyplot as plt
d = [220, 155, 105, 90, 170, 210, 290]
T=len(d)
K, h = 1000, 1.2
M = 10e5
#
WW = Model()
q = WW.addVars(T, lb=np.zeros(T), vtype=GRB.CONTINUOUS, name="order_quantity")
x = WW.addVars(T, lb=np.zeros(T), vtype=GRB.CONTINUOUS, name="inventory_level")
y = WW.addVars(T, vtype=GRB.BINARY, name="if_order")
WW.setObjective(quicksum(K*y[t]+h*x[t] for t in range(T)), GRB.MINIMIZE)
c1 = WW.addConstrs(q[t] <= M*y[t] for t in range(T))
c2 = WW.addConstrs(x[t] == x[t-1] + q[t] - d[t] for t in range(1,T))
c3 = WW.addConstr(x[0] == q[0] - d[0])
WW.optimize()
WW.printAttr('X')
t=np.linspace(0,T-1,T)
X=[350,195,90,0,500,290,0]
Q=[570,0,0,0,670,0,0]
```

```
plt.plot(t,X,color='blue',label='Inventory Level')
plt.plot(t,Q,color='red',label='Order Quantity')
plt.plot(t,d,color='green',label='Demand')
plt.legend(loc='upper left', fontsize=25)
plt.show()
```