Limits of Classical Physics

Structure of Matter

- Rutherford's Nuclear Atom Model
- Bohr Model of the Hydrogen Atom
- Frank-Hertz Experiment

Dual Nature of Light

- Blackbody Radiation and Ultraviolet Catastrophe
- Photoelectric Effect
- Compton Effect

Particles or waves?

Young Double-slit Experiment

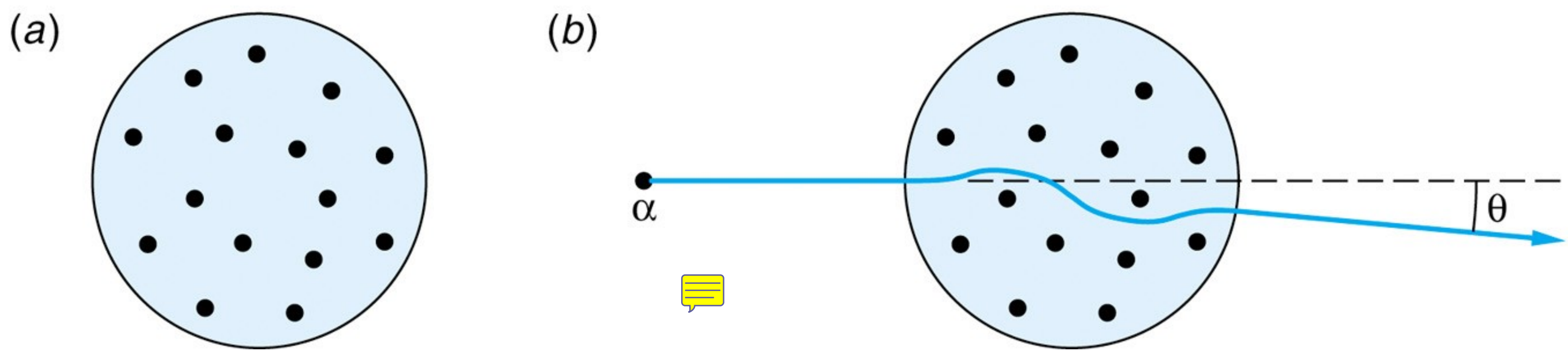
STRUCTURE of MATTER

Rutherford's Nuclear Model of Atom

Model of Atom. First Attempts

Known facts: atom's diameter ~10^{-10}m = Å; mass of the electron is only small fraction of the total mass of the atom

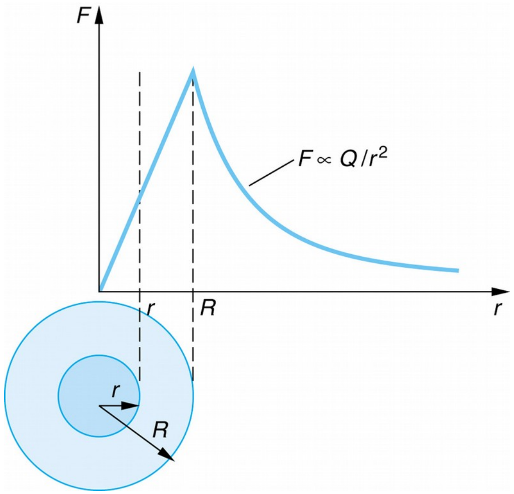
Thomson's Model: Positive charge uniformly distributed over the atom's volume.



Problem: Contrary to experimental evidence, this model does not predict any significant scattering of alpha particles.

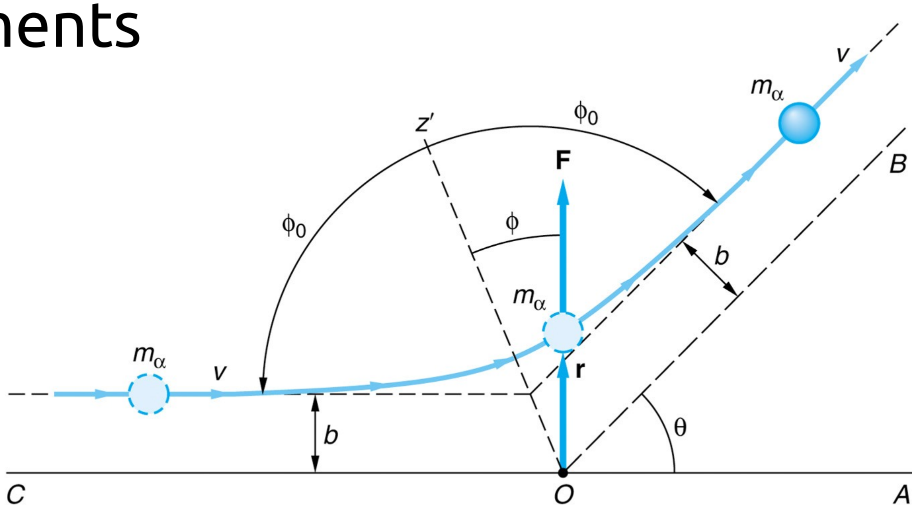
Illustrations: Tipler's textbook, except where noted.

Rutherford's Nuclear Model of Atom

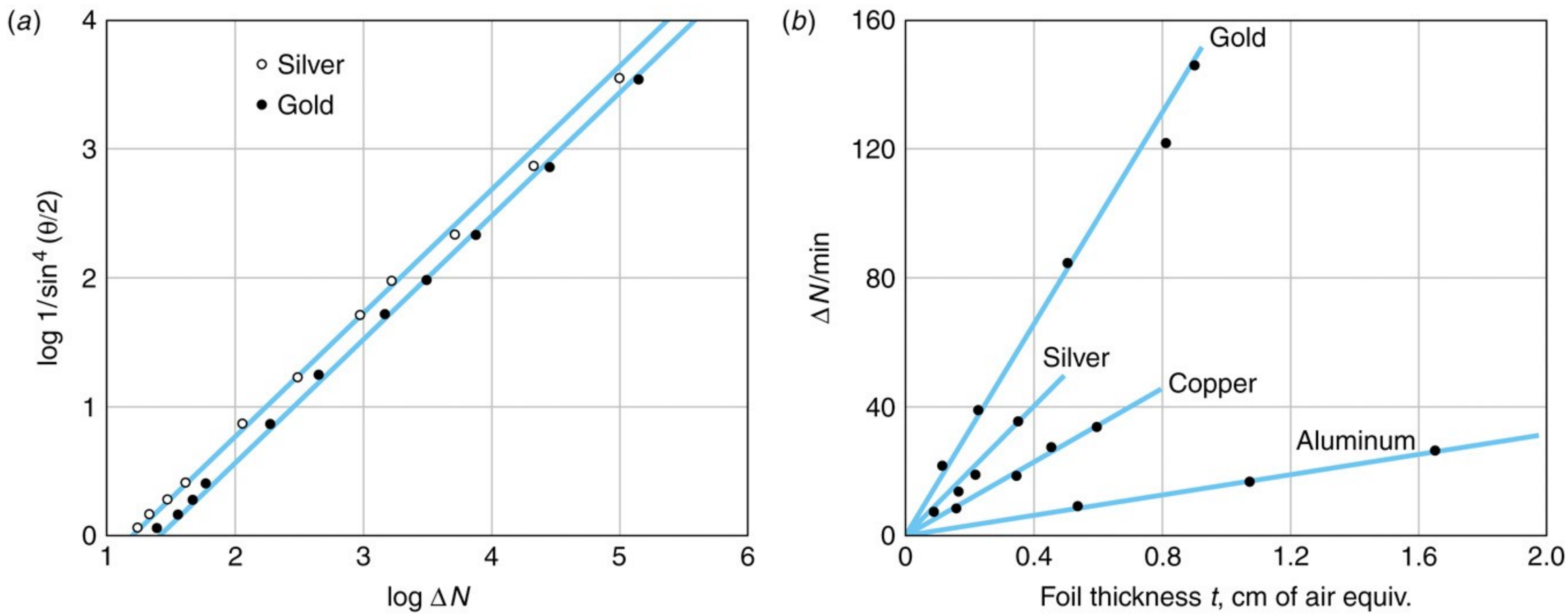


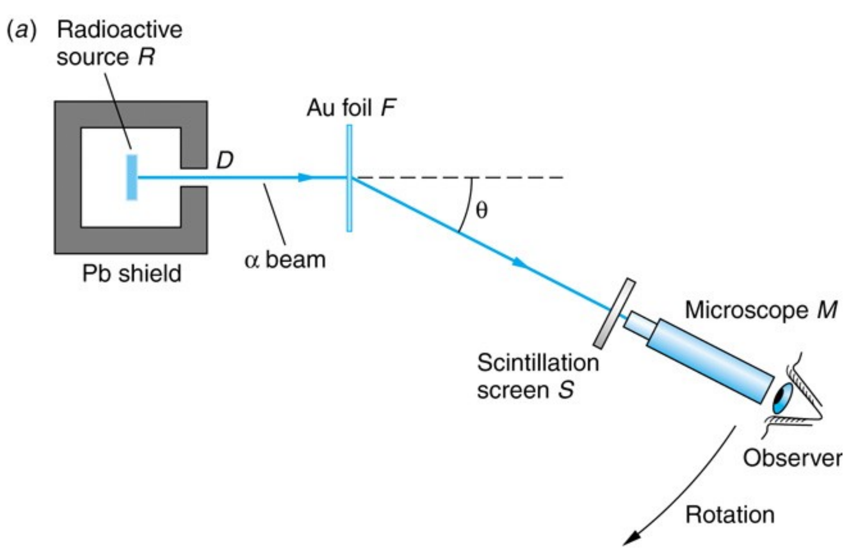
Idea: positive charge concentrated in a massive spherical nucleus with diameter much less than the size of atom

Results: consistent with the data from scattering experiments



Rutherford's Nuclear Model of Atom





Au foil thickness

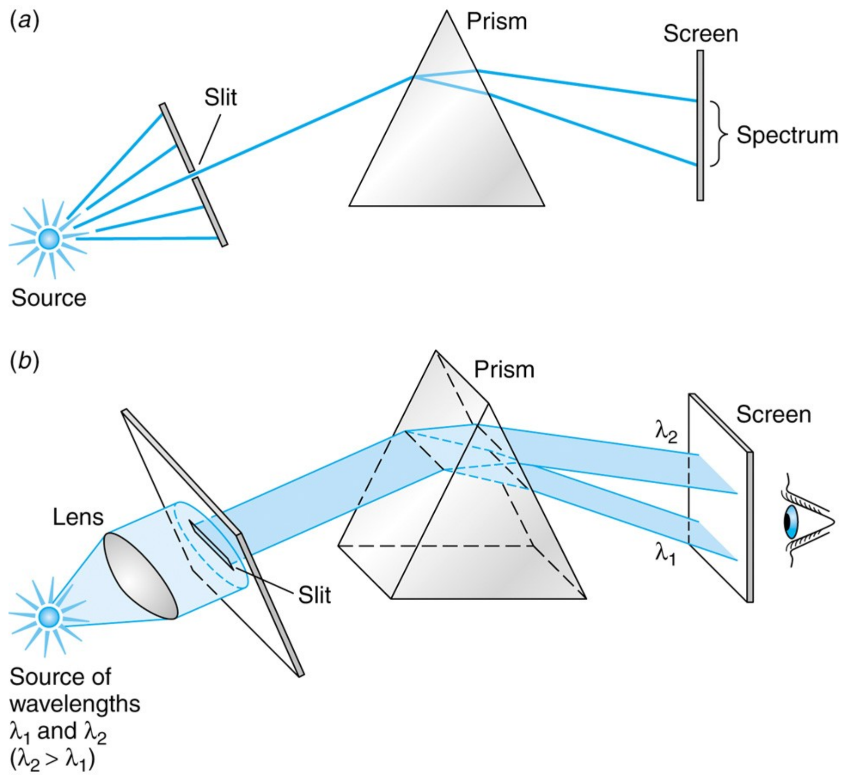
$$\Delta N \propto \frac{\text{thickness}}{\sin^4 \frac{\theta}{2}}$$

number of alpha particles scattered

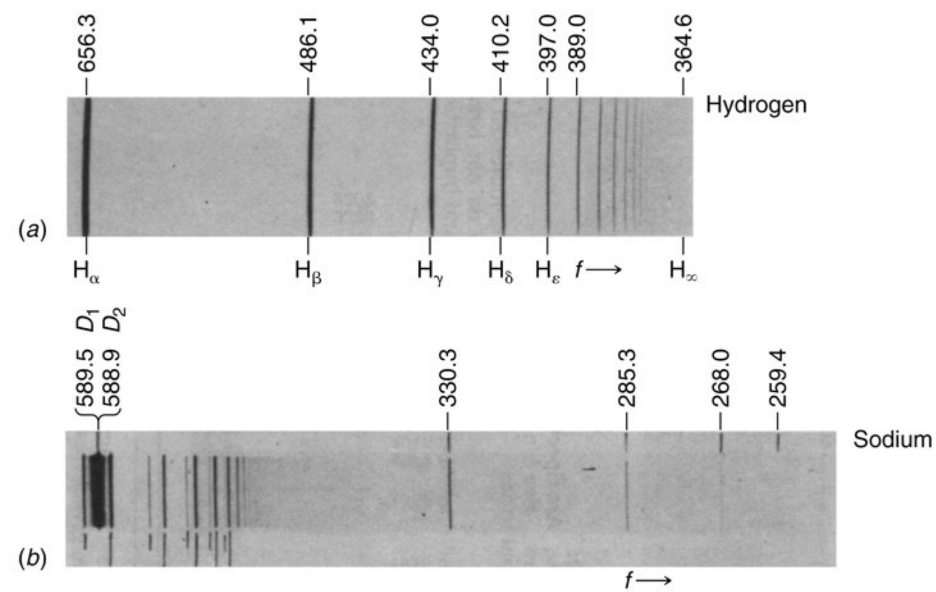
Bohr Model of the Hydrogen Atom

Introduction. Spectroscopy and Atomic Spectra

Spectroscopic Measurements – Idea



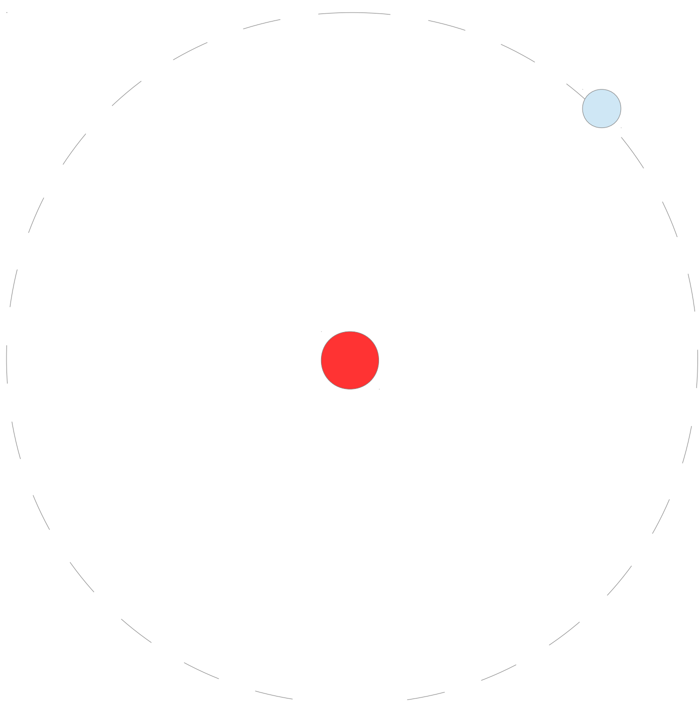
… and results (atomic spectra)



Question: How to explain the discrete spectra?

Bohr Model (I)

The electron in the hydrogen atom (or hydrogen-like ions) moves in a circular orbit around the nucleus as a result of Coulomb attraction between the electron and the positively charged nucleus.



Coulomb force as the centripetal force (magnitude)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

Total energy (non-relativistic)

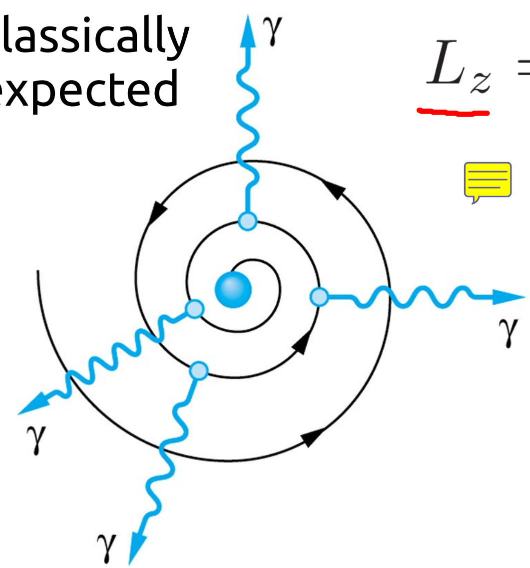
$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = \\ = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

Bohr Model (II)

The electron can only move in an orbit, for which the orbital angular momentum about the nucleus L_z is a multiple of $\hbar = h/2\pi$.

Electron moving in such an orbit does not radiate. Thus, the total energy of the atom remains constant. The atom is in a so-called "stationary state".

classically expected



$$L_z = mvr = n\hbar \Longrightarrow (mvr)^2 = n^2\hbar^2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

combine both

$$r_n = \frac{4\pi\hbar^2\varepsilon_0}{mZe^2}n^2 = \frac{n^2}{Z}a_0$$

radius of the *n*-th Bohr orbit

$$a_0 = \frac{4\pi\hbar^2\varepsilon_0}{me^2} = 0.529\dot{A}$$

Bohr radius

total energy of electron in the *n*-th orbit
[*n* – quantum number]

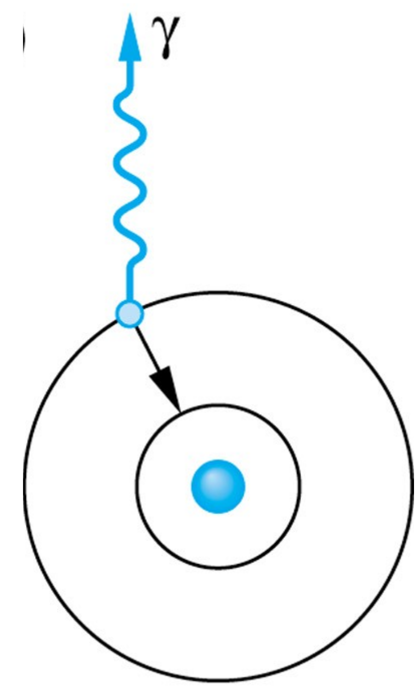
$$E_n = -\frac{mZ^2e^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -E_1\frac{Z^2}{n^2}$$

$$E_1 = 13.6\ \text{eV}$$

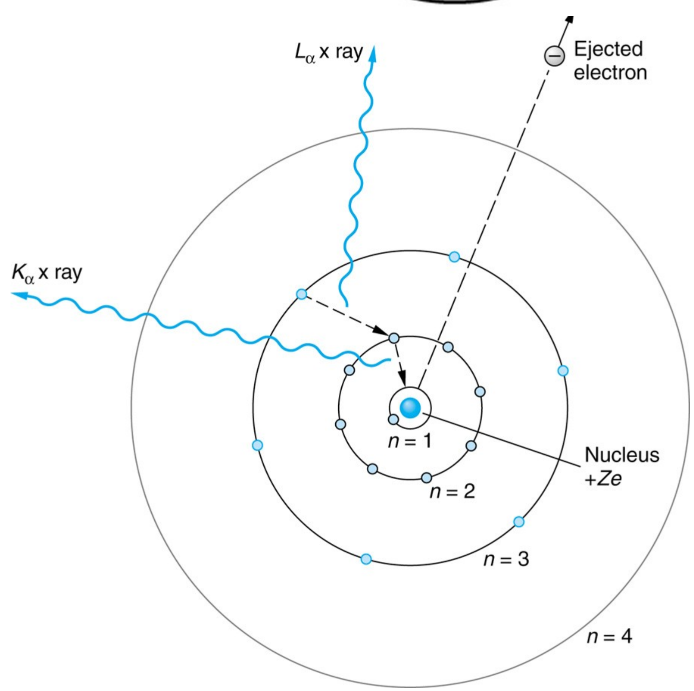
discrete (quantized) energy levels

Bohr Model (III). Electromagnetic Radiation

The atom radiates only when the electron undergoes a transition from one stationary state to another. The inverse of the wavelength of the emitted electromagnetic radiation is



$$\lambda^{-1} = \frac{E_f - E_i}{hc} = \frac{E_1 Z^2}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$



The Correspondence Principle

In the limit of large quantum numbers, quantum results must correspond with classical ones.

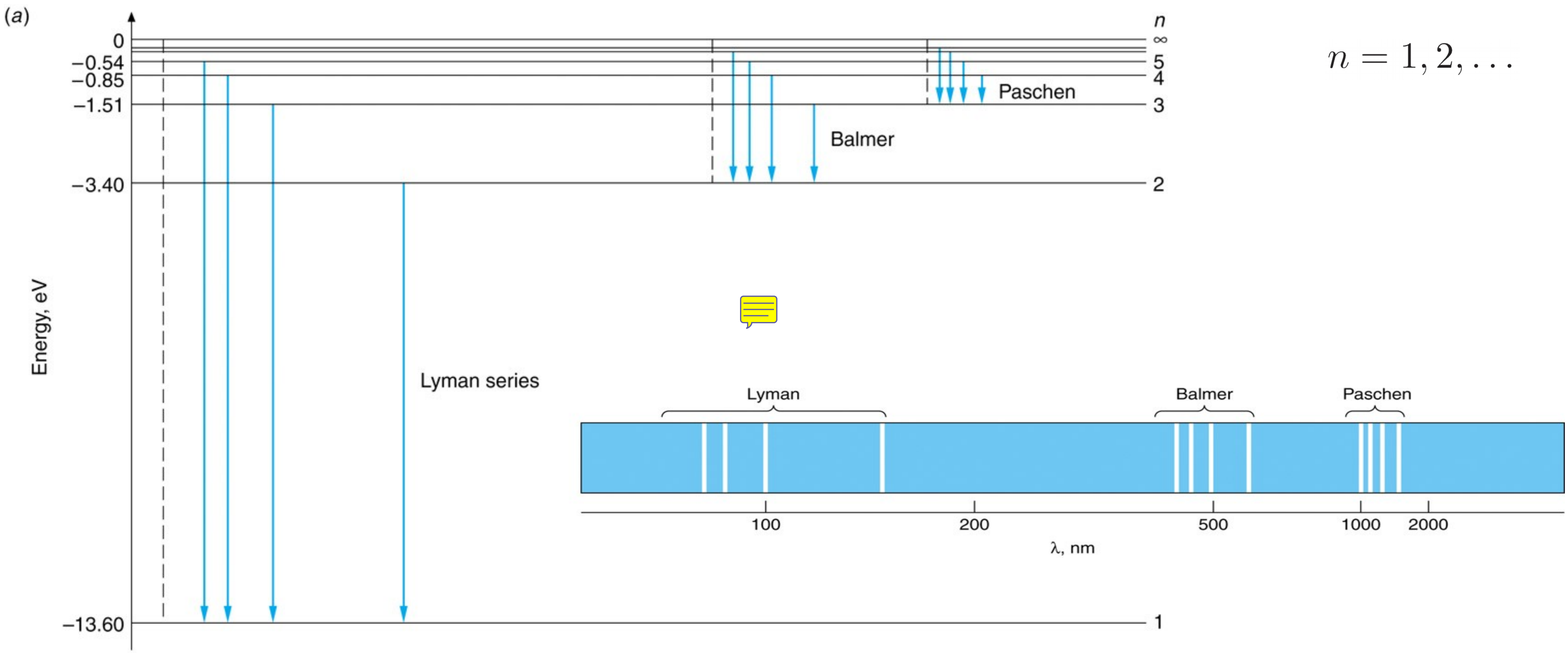
Bohr Model. Discussion

Hydrogen atom (*Z=1; n=1, 2,...*)

$$r_n = n^2 a_0$$

$$E_n = -\frac{E_1}{n^2}$$

$$n = 1, 2, \ldots$$



$$\lambda^{-1} = \frac{E_1}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Bohr Model. Concluding Remarks

Successful in explanation of the structure of the hydrogen's atomic spectrum.

Can be refined to include some relativistic effects by allowing elliptical orbits (explanation of the fine structure of spectra).

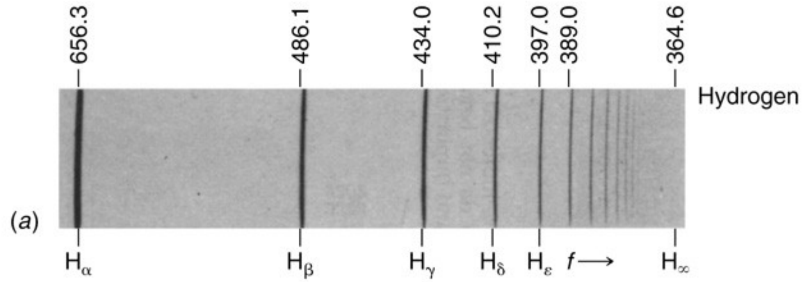
$$E_n = -\frac{\alpha^2 mc^2}{2}\frac{1}{n^2}, \qquad a_0 = \frac{\hbar}{\alpha mc}$$

energy levels

Bohr radius

$$\frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137} = \alpha$$

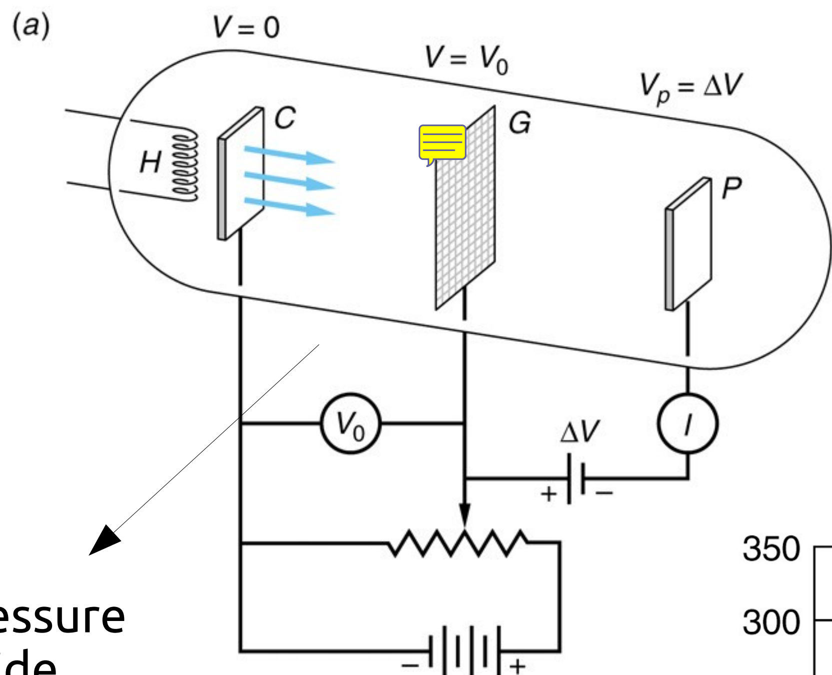
fine-structure constant



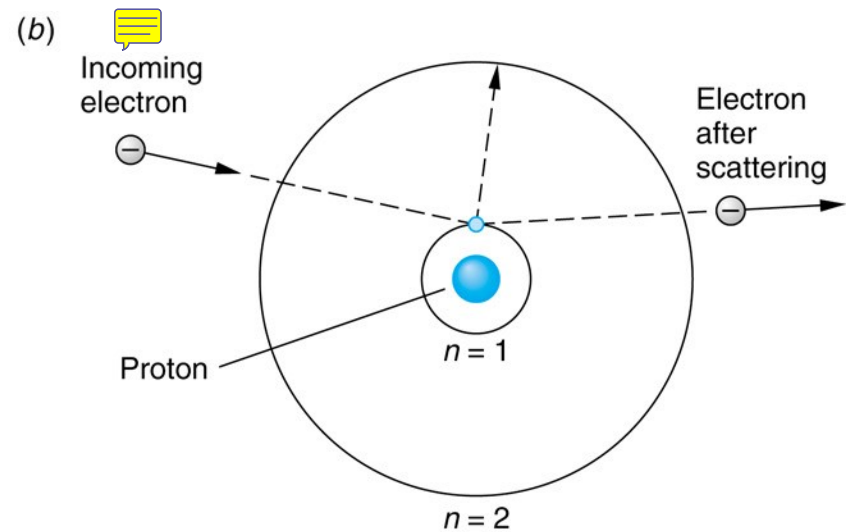
Frank-Hertz Experiment

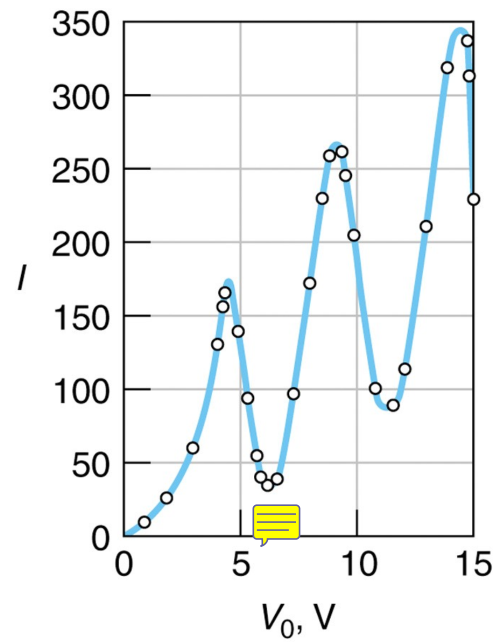
Experimental Setup, Idea, and Result

Provides experimental evidence for discrete structure of atomic energy levels.



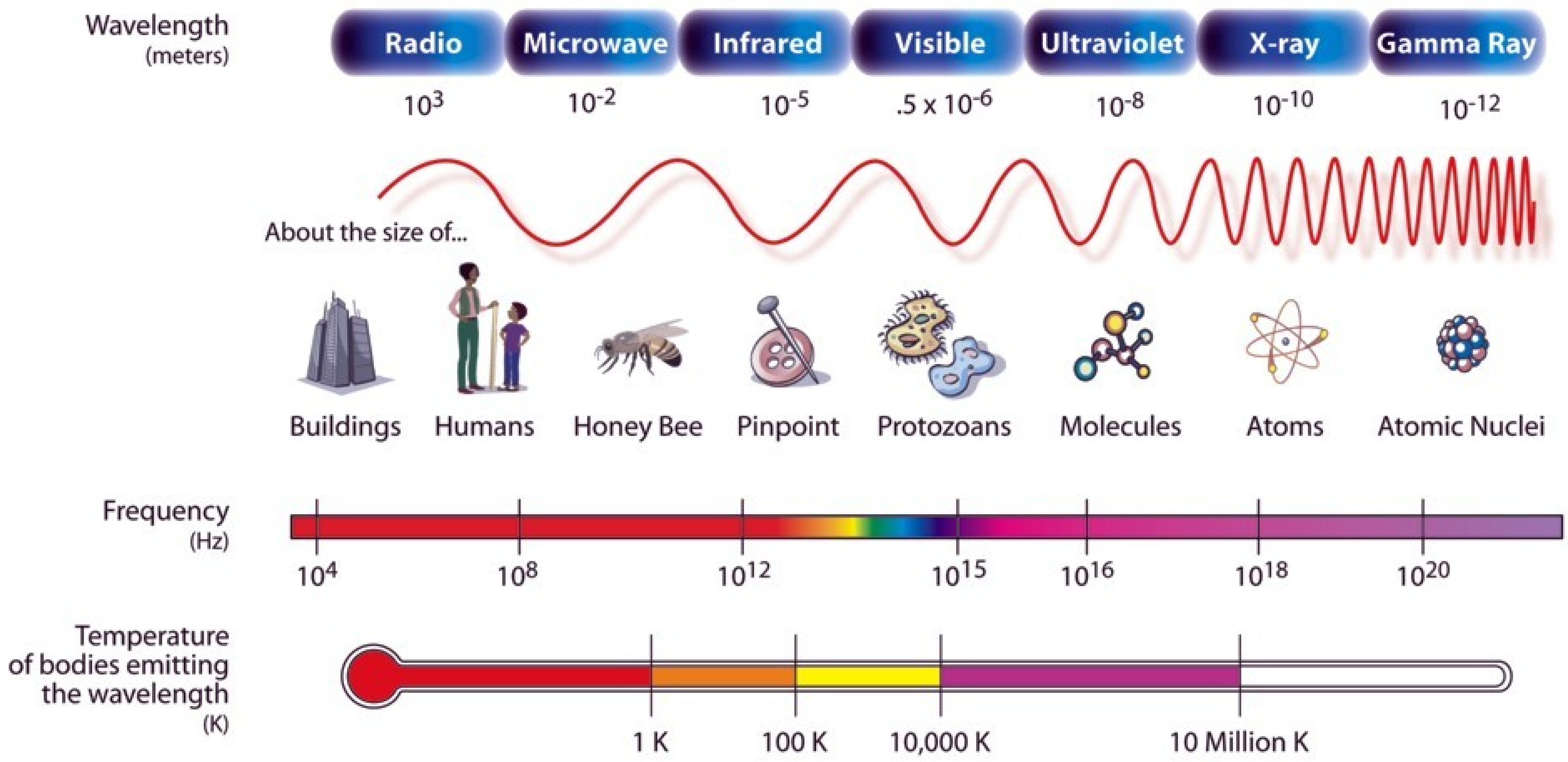
low-pressure
gas inside





NATURE
of
ELECTROMAGNETIC RADIATION

Electromagnetic Spectrum



http://mynasadata.larc.nasa.gov/

Blackbody Radiation

Thermal Radiation

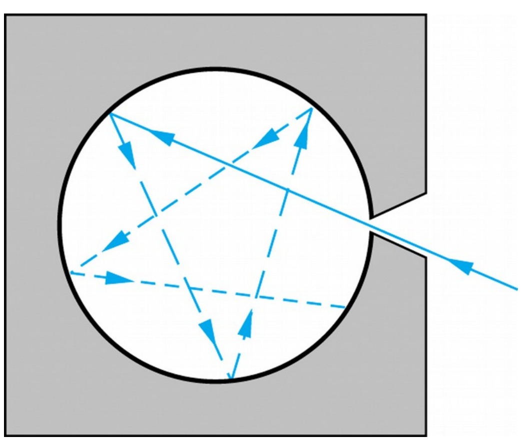
e-m radiation absorption →
→ kinetic energy of atoms increases →
→ *T* increases → **e-m radiation emission**
→ kinetic energy of atoms decreases →
→ *T* decreases

ENERGY
rate of emission = rate of absorption

thermal equilibrium

http://physics.uoregon.edu/~soper/Light/blackbody.html

Ideal Blackbody Model – cavity in thermal equilibrium



- *completely absorbs incident radiation*
- *rate of emission = rate of absorption* [thermal equilibrium]

Ideal Blackbody Radiation. Experimental Features

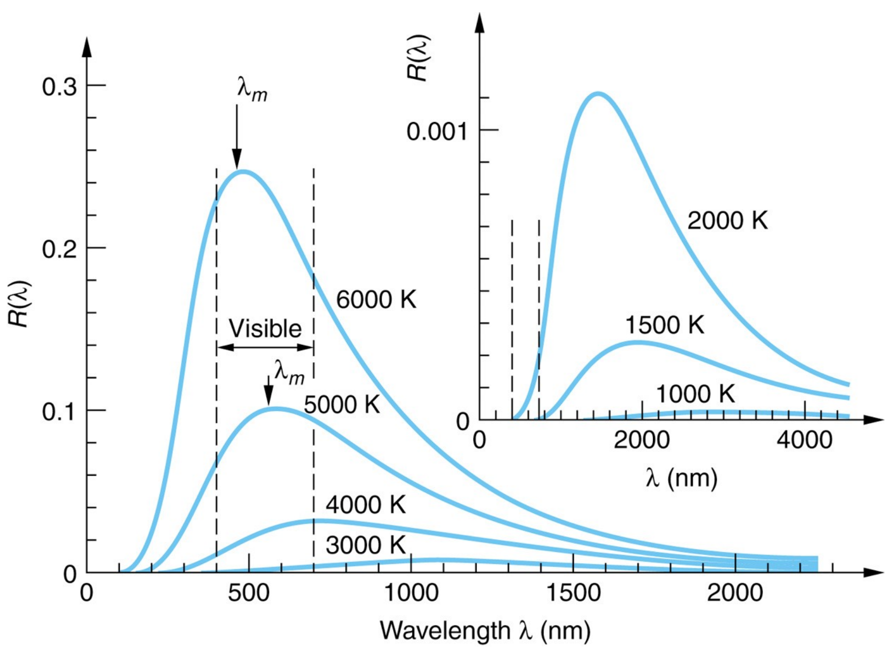
total power radiated

energy radiated per unit time (power) from unit area

$$\frac{P}{A} = \sigma T^4 \qquad \sigma = \mathrm{const} = 5.67 \times 10^{-8}\ [\mathrm{W/m^2K^4}]$$

Stefan-Boltzmann law
(first found empirically)

spectral distribution and Wien displacement law



$$\lambda_{\mathrm{m}} T = \mathrm{const}$$

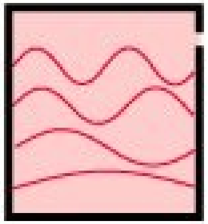
Wien displacement law

$$\frac{P}{A} = \int_0^\infty R(\lambda)\, d\lambda$$

$R(\lambda)\, d\lambda =$ power emitted from unit area; wavelength $(\lambda, \lambda + d\lambda)$

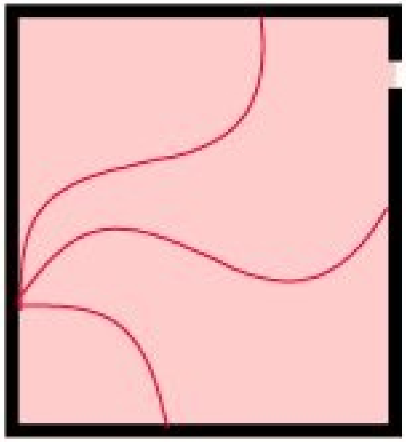
Theoretical Description

The radiated energy can be considered to be produced by standing waves (resonant modes) of the cavity that is radiating.

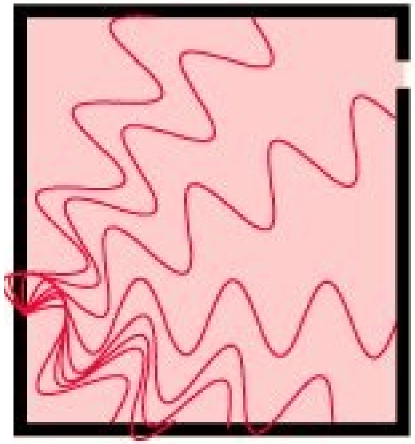


Number of modes per unit frequency per unit volume $\rho(\nu) = \dfrac{8\pi\nu^2}{c^3}$

or per unit wavelength per unit volume $\rho(\lambda) = \dfrac{8\pi}{\lambda^4}$



For higher frequencies more modes can fit into the cavity.



Images: http://hyperphysics.phy-astr.gsu.edu

Theoretical Description

Number of modes per unit frequency per unit volume $\rho(\nu) = \dfrac{8\pi\nu^2}{c^3}$

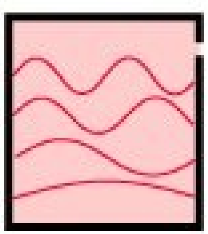
or per unit wavelength per unit volume $\rho(\lambda) = \dfrac{8\pi}{\lambda^4}$

spectral distribution

$$R(\lambda) = \frac{1}{4}\, c u(\lambda)$$

number of modes of e-m field oscillations per unit volume x average energy per mode (spectral energy density)

Radiation modes in a hot cavity provide a test of quantum theory



	#Modes per unit frequency per unit volume	Average energy per mode
CLASSICAL	$\dfrac{8\pi\nu^2}{c^3}$	kT
QUANTUM	$\dfrac{8\pi\nu^2}{c^3}$	$\dfrac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

Image: http://hyperphysics.phy-astr.gsu.edu

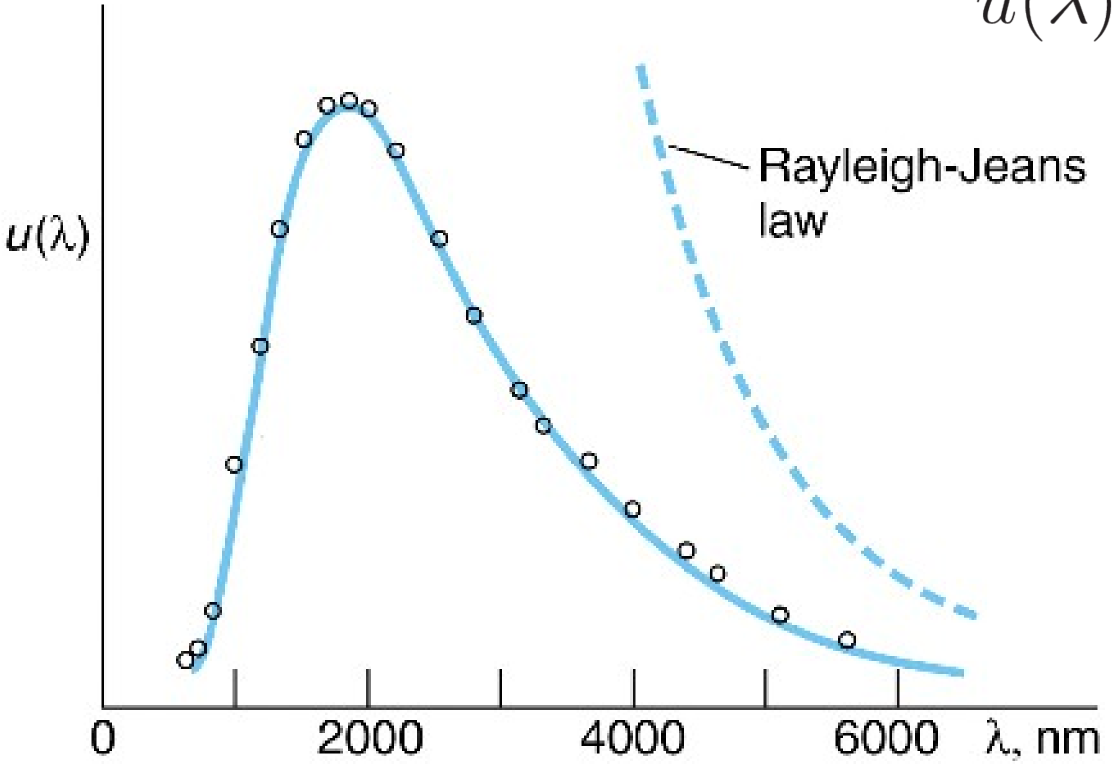
Theoretical Explanation (classical)

$$u(\lambda) = \frac{8\pi}{\lambda^4} k_{\mathrm{B}} T$$

Rayleigh-Jeans formula

PROBLEM

for short wavelengths (high frequencies)



$$u(\lambda) \to \infty \implies \int_0^\infty u(\lambda)\, d\lambda = \infty$$

ultraviolet catastrophe

Theoretical Explanation (quantum)

Planck's postulate

$$E_n = nh\nu, \qquad n = 0, 1, 2, \ldots$$

Energy of electromagnetic radiation is *quantized*.

$$f_n = \frac{e^{-E_n/k_B T}}{\sum_{n=0}^{\infty} e^{-E_n/k_B T}} = \frac{e^{-nh\nu/k_B T}}{\sum_{n=0}^{\infty} e^{-nh\nu/k_B T}}$$

probability of finding the system with energy E_n (Boltzmann distribution)

$$\langle E \rangle = \sum_{n=0}^{\infty} E_n f_n = \frac{h\nu}{e^{h\nu/k_B T} - 1} = \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$$

average energy per mode

$$u(\lambda) = \frac{8\pi}{\lambda^4} \langle E \rangle = \frac{8\pi}{\lambda^5} \frac{hc}{e^{hc/\lambda k_B T} - 1}$$

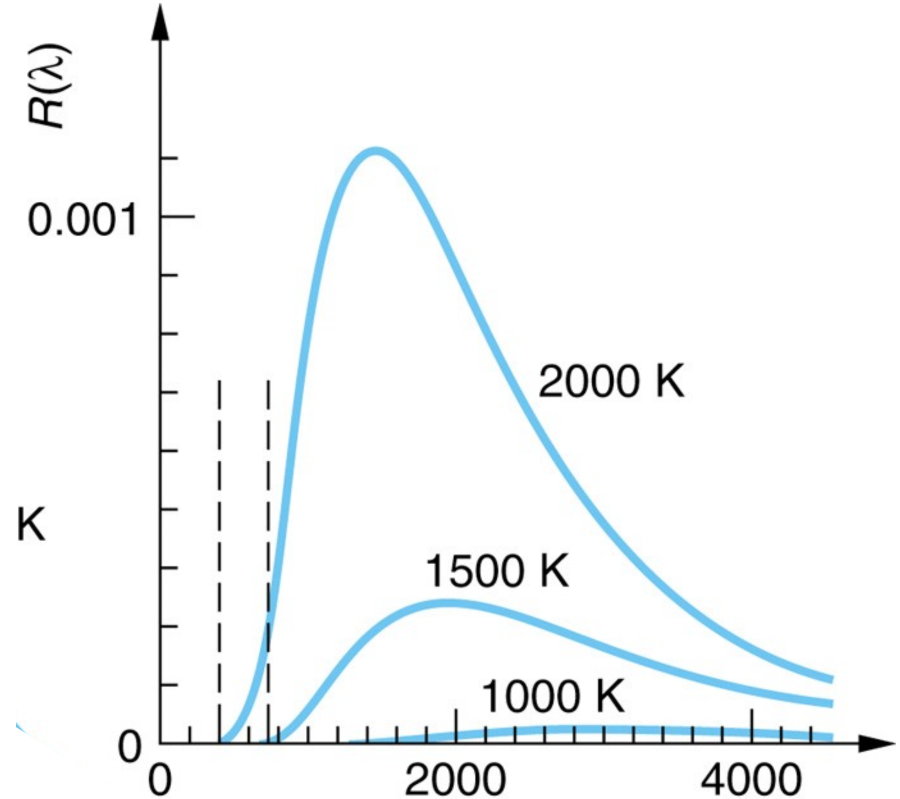
Planck's law

Planck's Law. Discussion

limit cases

short-wave limit $$\frac{hc}{\lambda k_B T} \gg 1$$

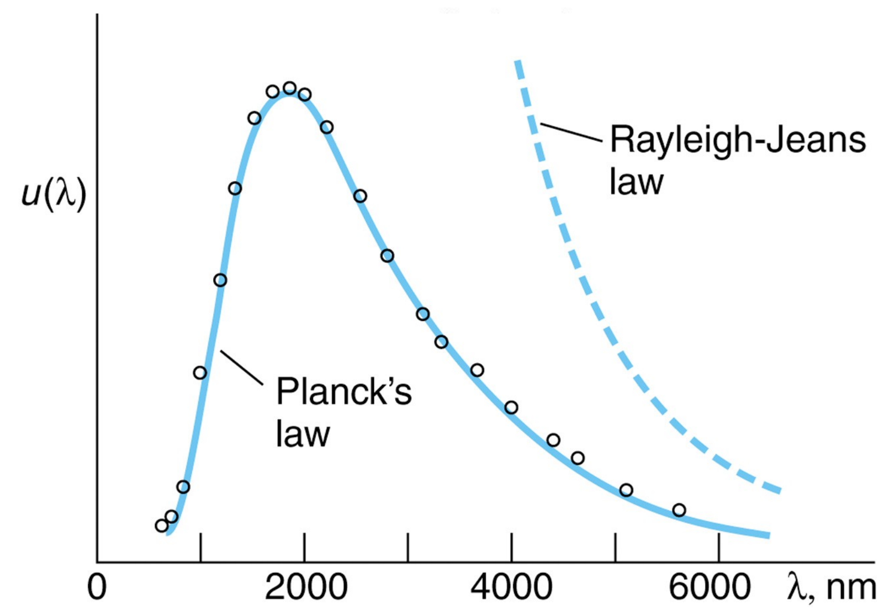
$$u(\lambda) = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda k_B T}} \to 0, \text{ as } \lambda \to 0$$



long-wave limit

$$\frac{hc}{\lambda k_B T} \ll 1 \qquad e^{\frac{hc}{\lambda k_B T}} \approx 1 + \frac{hc}{\lambda k_B T}$$

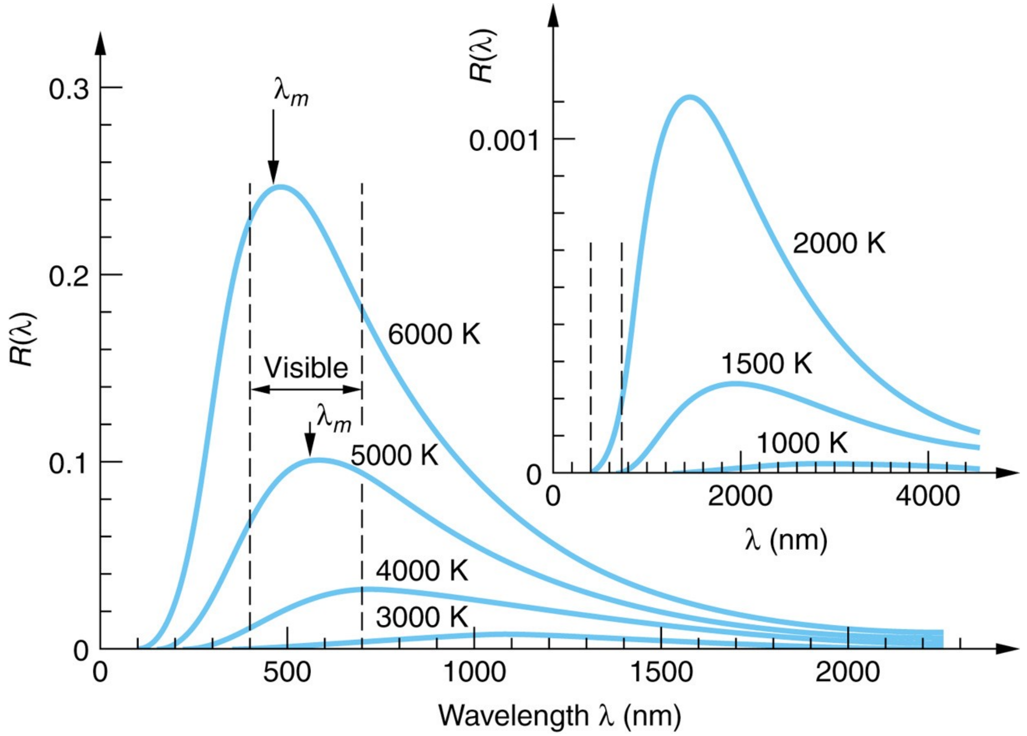
$$u(\lambda) = \frac{8\pi}{\lambda^4} k_B T$$ **Rayleigh-Jeans**



Both Stefan-Boltzmann and Wien laws follow
[see recitation class and assignment]

Blackbody Radiation. Summary

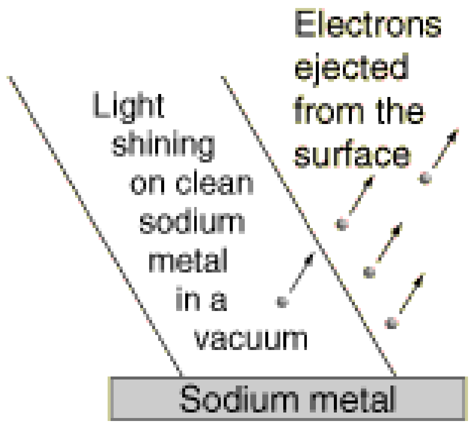
- key assumption: energy of e-m radiation is quantized
- results consistent with experimental data
- consistent with the classical result of Rayleigh-Jeans in the low-frequency (long-wave) limit
- no UV catastrophe in the high-frequency (short-wave) limit

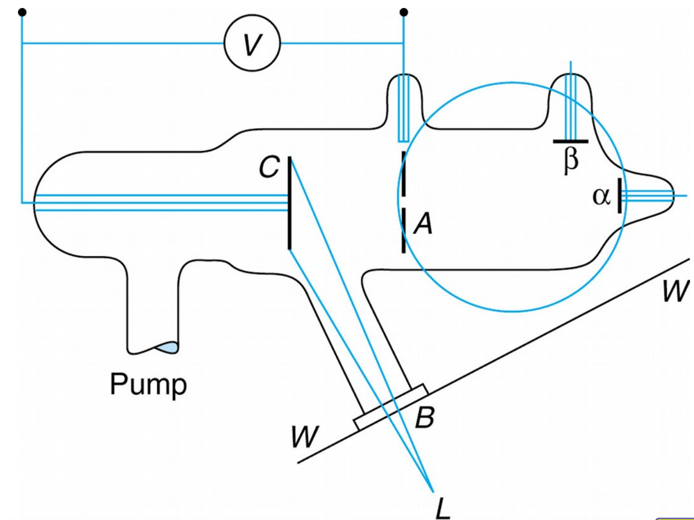


Photoelectric Effect

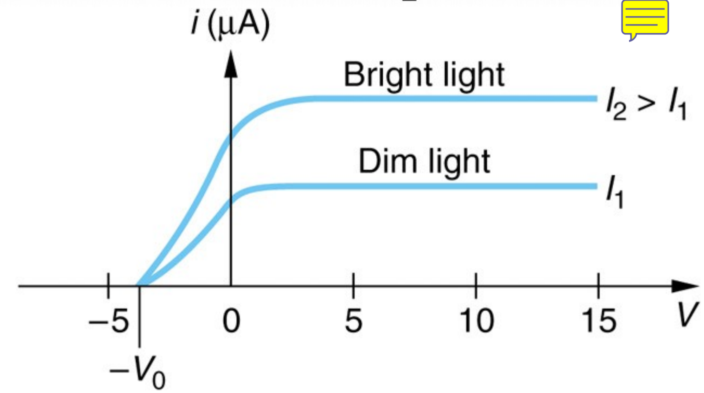
Photoelectric Effect. Basic Features

Idea and experimental setup



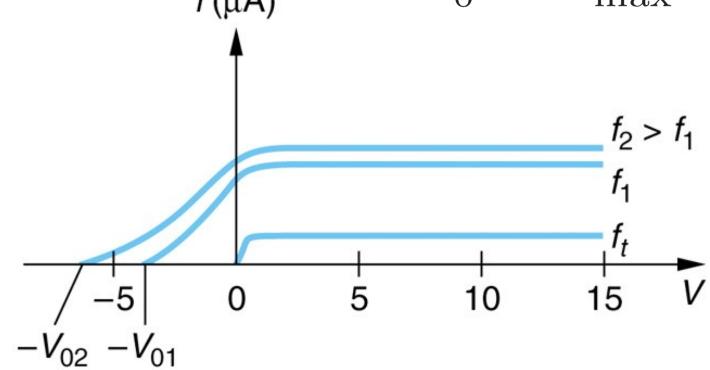


(a)



$$eV_0 = K_{\max}$$

(b)



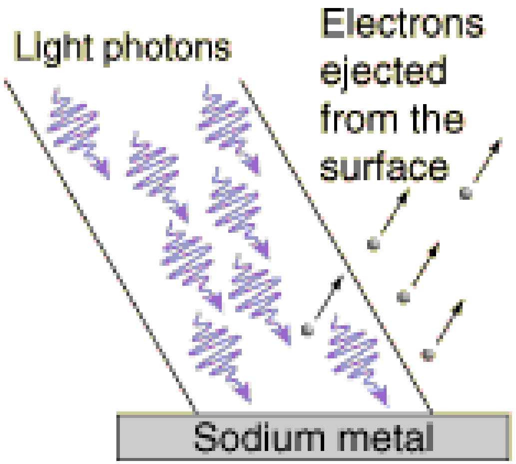
observations

- maximum current proportional to intensity of incident light
- no threshold intensity, effect present even for small intensities
- stopping potential independent of intensity, but different for different frequencies

[latter two features point to the role of frequency]

Photoelectric Effect. Einstein's Explanation

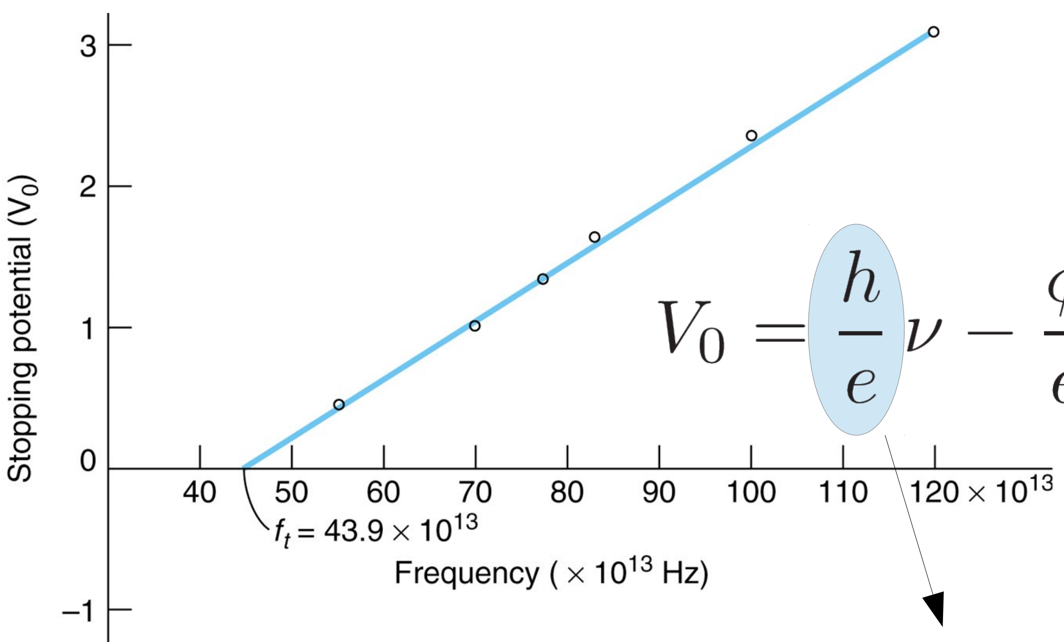
$$E = h\nu$$



$$K_{\max} = \frac{1}{2}mu^2$$

$$\boxed{h\nu = \phi + K_{\max}}$$

ϕ energy needed to remove a single electron from inside of the metal to the surface (work function)



$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

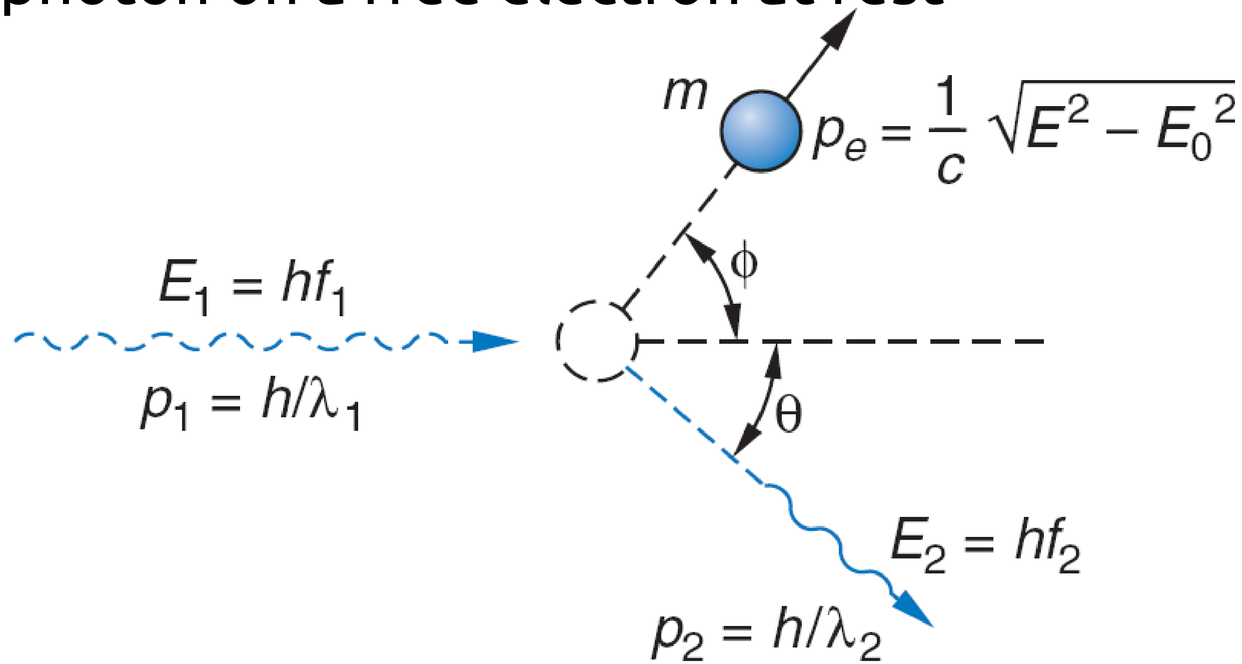
h/*e* ratio can be found

Element	Work function (eV)
Na	2.28
Cs	1.95
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

Compton Effect

Compton Scattering

scattering of a photon on a free electron at rest



conservation of energy
and momentum (relativistic)

$$E_1 + E_0 = E + E_2$$

$$\mathbf{p}_1 = \mathbf{p}_e + \mathbf{p}_2$$

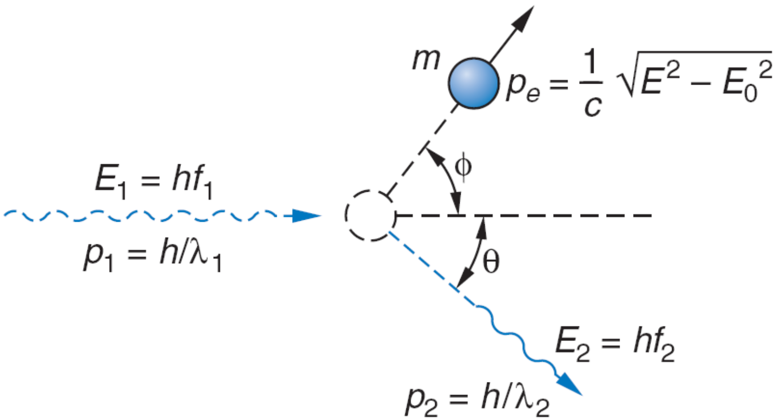
$$\longrightarrow$$

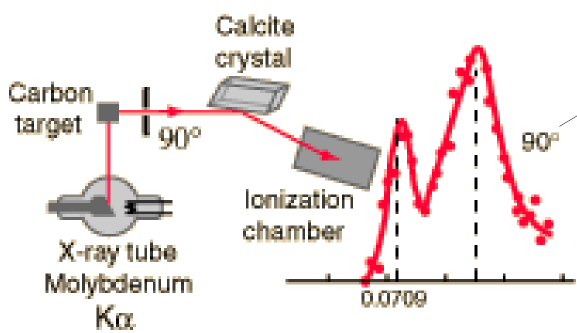
$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta)}$$

rest energy
of the electron

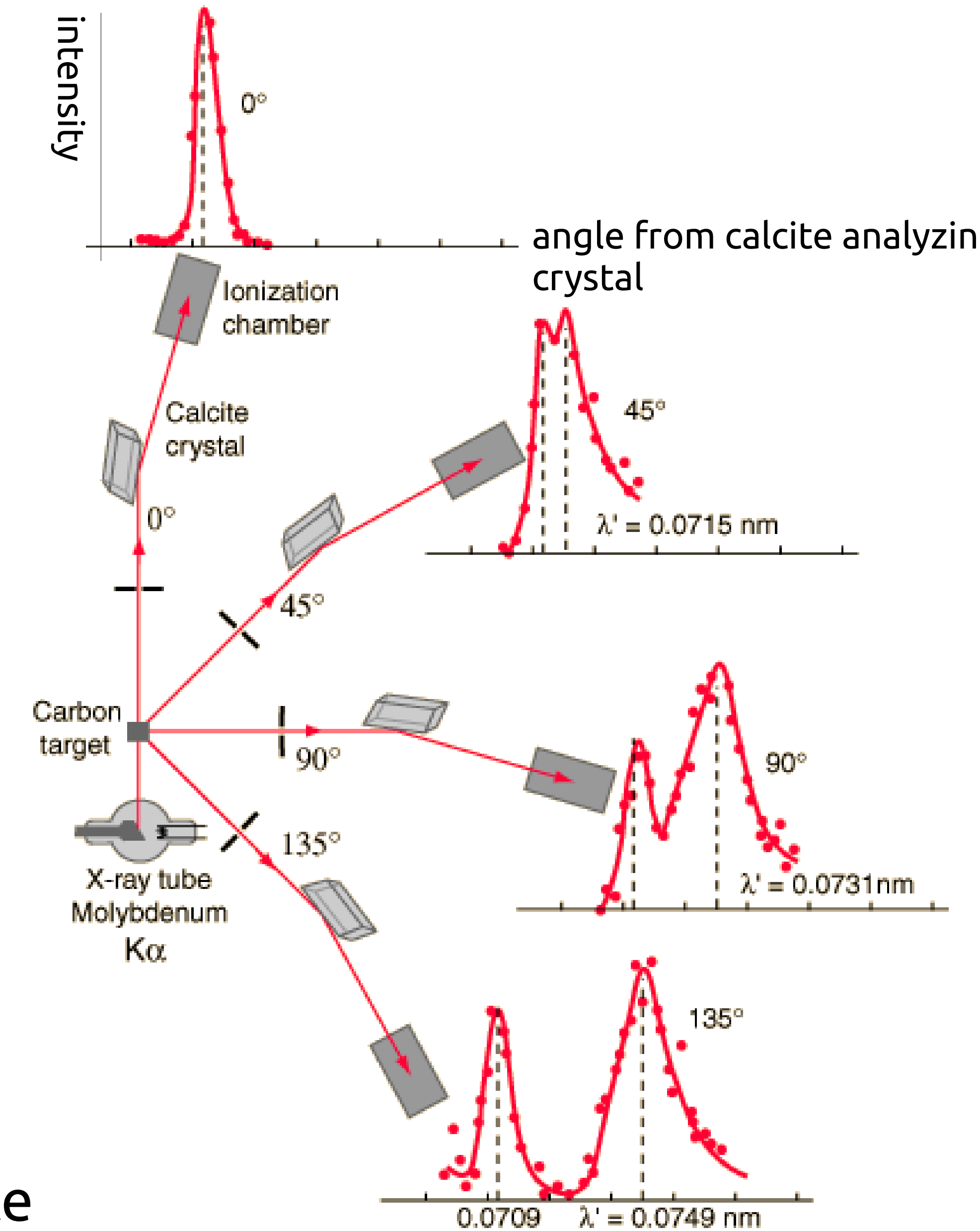
Compton wavelength $\lambda_C = \frac{h}{mc} = 2.43 \times 10^{-12}$ m

Compton Scattering. Experiment





scattered by graphite at this angle



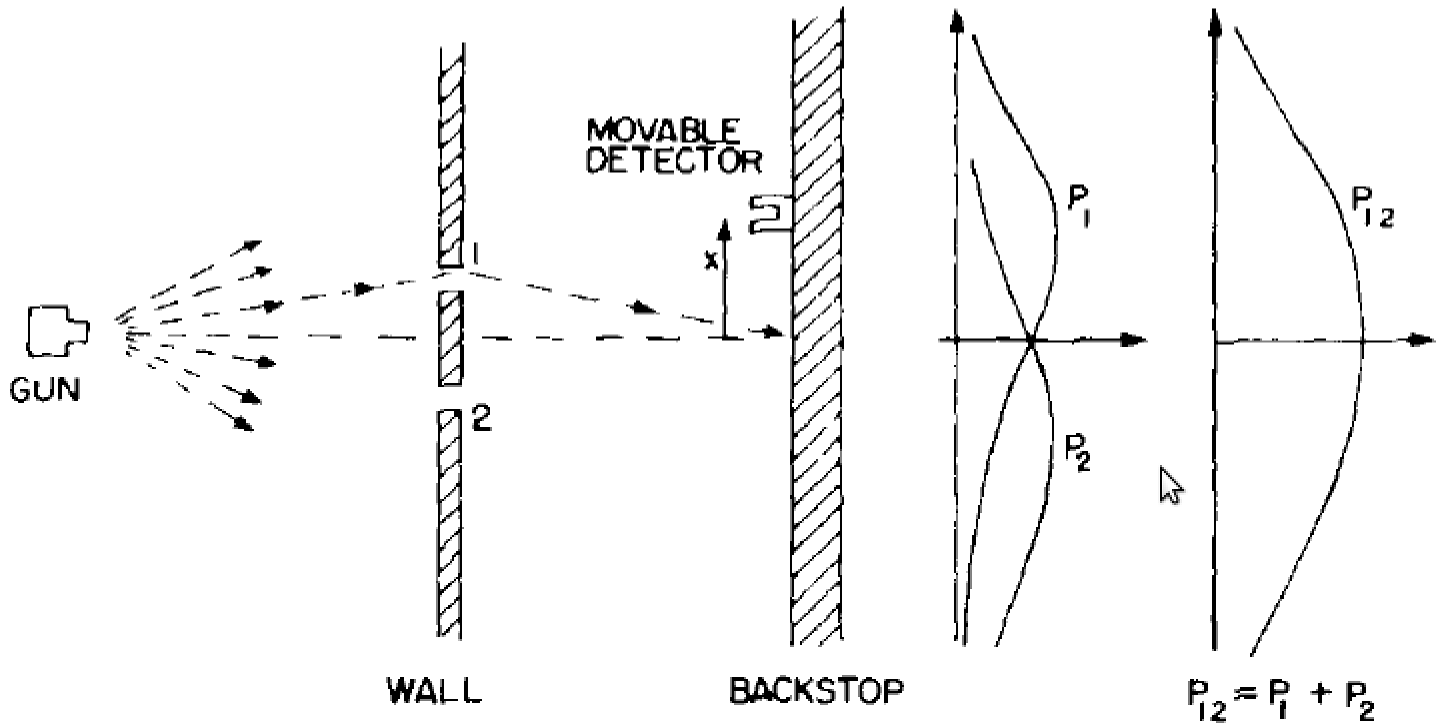
Conclusion

e-m radiation can be treated
as a collection of particles with definite
momentum moving at *c* (zero rest-mass)

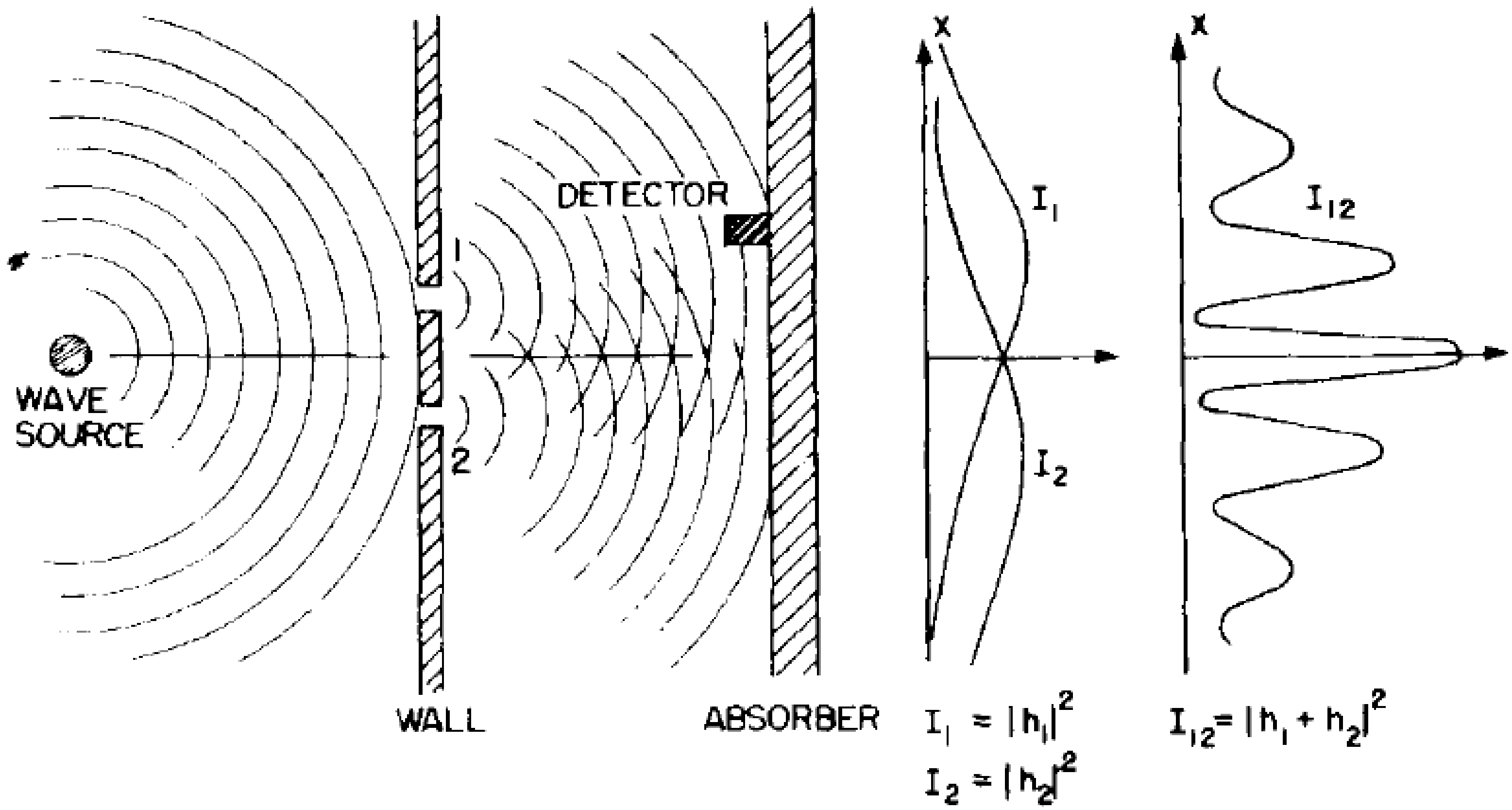
PARTICLES or WAVES?

Young (Double-Slit) Experiment

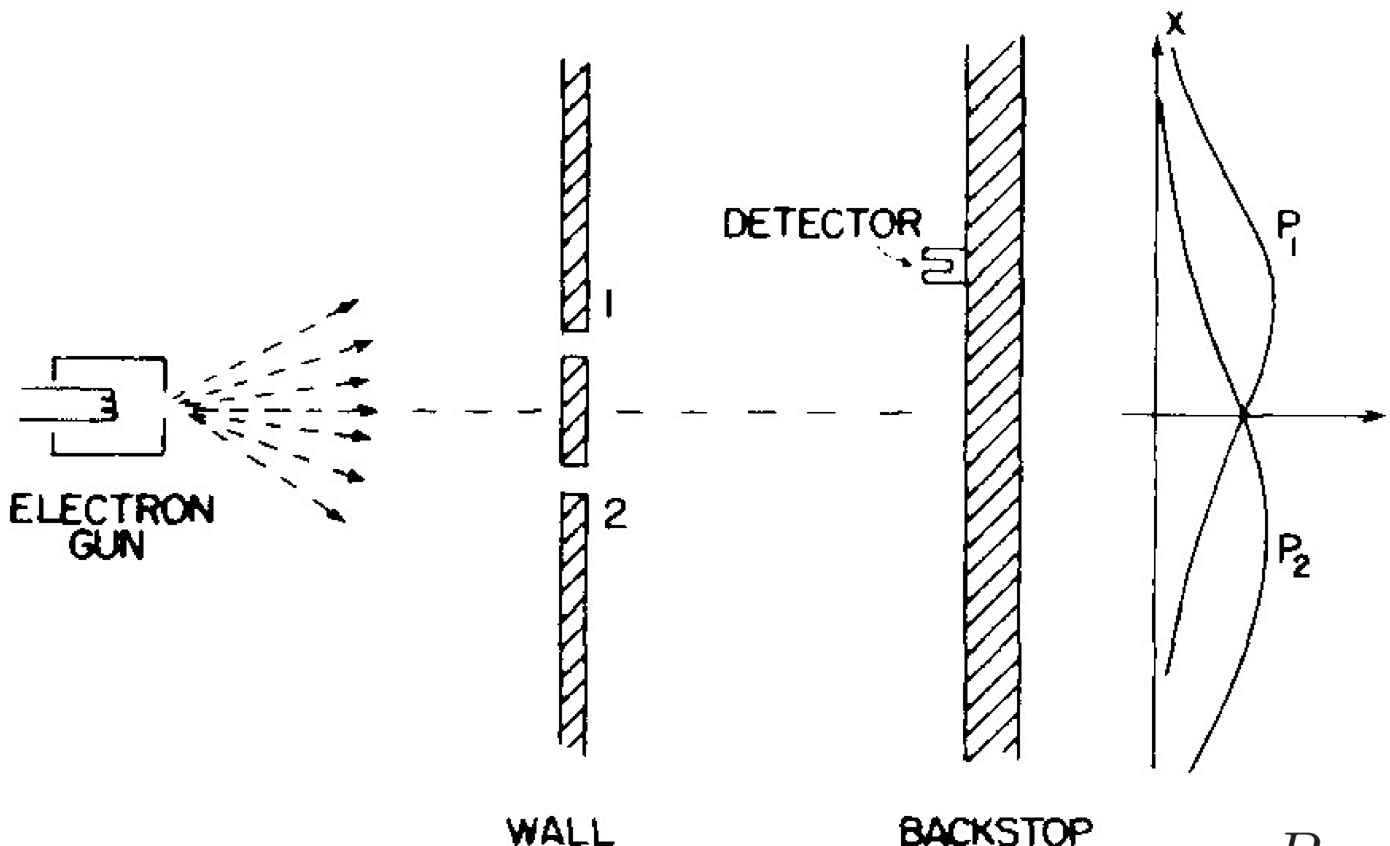
An Experiment with Bullets...



...and Water Waves



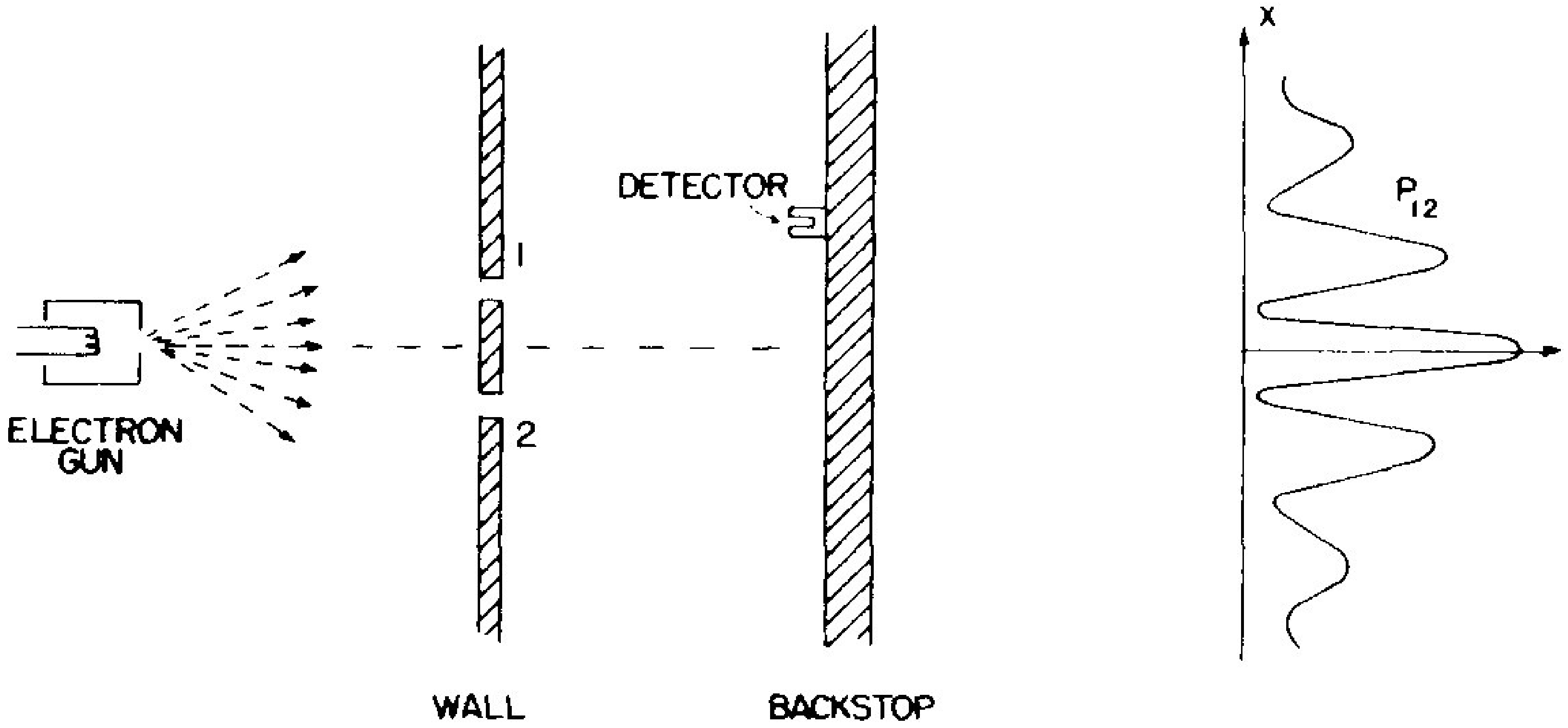
Electrons. Only One Slit Open



$$P_1 = |\Psi_1|^2$$

$$P_2 = |\Psi_2|^2$$

Electrons. Both Slits Open



$$P_{12} = |\Psi_1 + \Psi_2|^2 = (\Psi_1^* + \Psi_2^*)(\Psi_1 + \Psi_2) =$$

$$= |\Psi_1|^2 + |\Psi_2|^2 + \Psi_2^*\Psi_1 + \Psi_1^*\Psi_2 = |P_1|^2 + |P_2|^2 + 2\,\mathrm{Re}(\Psi_1\Psi_2^*)$$

only slit 1 open

only slit 2 open

interference
term

Electrons. What waves do interfere here?

de Broglie hypothesis (1924)

All matter exhibits wave-like properties with

$$\lambda = \frac{h}{p}$$ (wavelength)

$$\nu = \frac{E}{h}$$ (frequency)

de Broglie waves (or matter waves)

Or, equivalently, in terms of the wave-number $k = 2\pi/\lambda$ and angular frequency $\omega = 2\pi\nu$

$$k = \frac{p}{\hbar}, \qquad \omega = \frac{E}{\hbar}$$

$$\hbar = \frac{h}{2\pi}$$

(Dirac's constant "*h-bar*")

Example. The length of de Broglie wave of a free non-relativistic particle $$\lambda = \frac{h}{mv}$$

(a) electron in the first Bohr orbit: $\lambda = 3.32 \times 10^{-10}$ m (~ distances the electron travels)

(b) ball (10 g) moving at 10 m/s: $\lambda = 6.62 \times 10^{-35}$ m (is it measurable?)

Classical Waves (review). Wave eqn, plane waves

Classical wave equation

$$\frac{\partial^2 \tilde{\xi}}{\partial x^2} = \frac{1}{v_{\mathrm{ph}}^2} \frac{\partial^2 \tilde{\xi}}{\partial t^2}$$

$\tilde{\xi}$ →
- displacement (mechanical waves)
- electric field/magnetic field (electromagnetic waves – light)
- ...

$\xi = \alpha \tilde{\xi}_1 + \beta \tilde{\xi}_2$

Comments

(1) linear equation: the superposition principle holds

(2) describes a wave propagating with phase speed v_{ph}

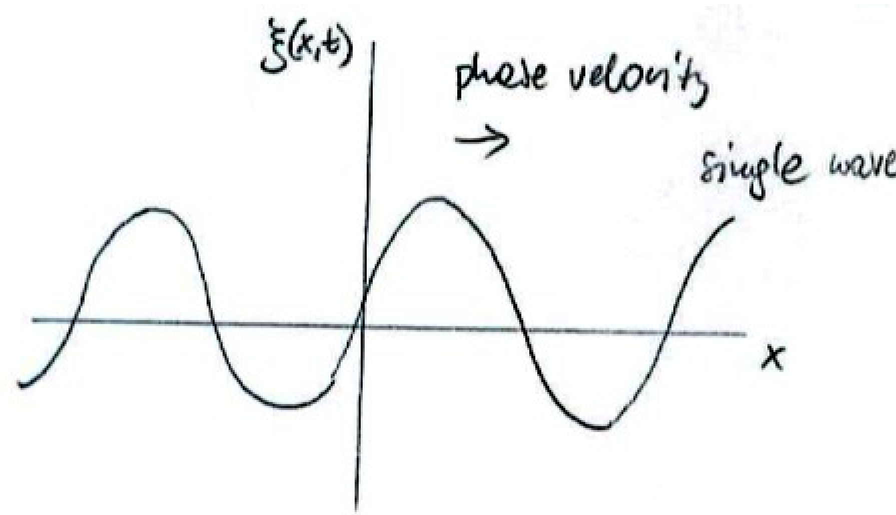
(3) linearity allows to operate on complex quantities;
(the real part represents a physical quantity)

Plane waves (harmonic/sinusoidal waves)

$$\tilde{\xi}(x,t) = \xi_0 e^{i(kx - \omega t)} = \xi_0 e^{ik\left(x - \frac{\omega}{k} t\right)}$$

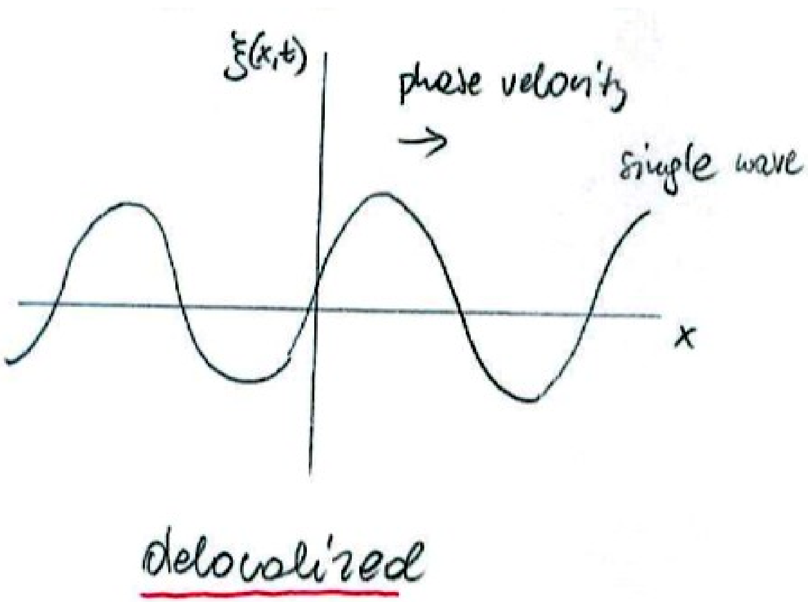
$$\xi(x,t) = \mathrm{Re}\,\tilde{\xi}(x,t) = \xi_0 \cos\left[k\left(x - \frac{\omega}{k} t\right)\right]$$

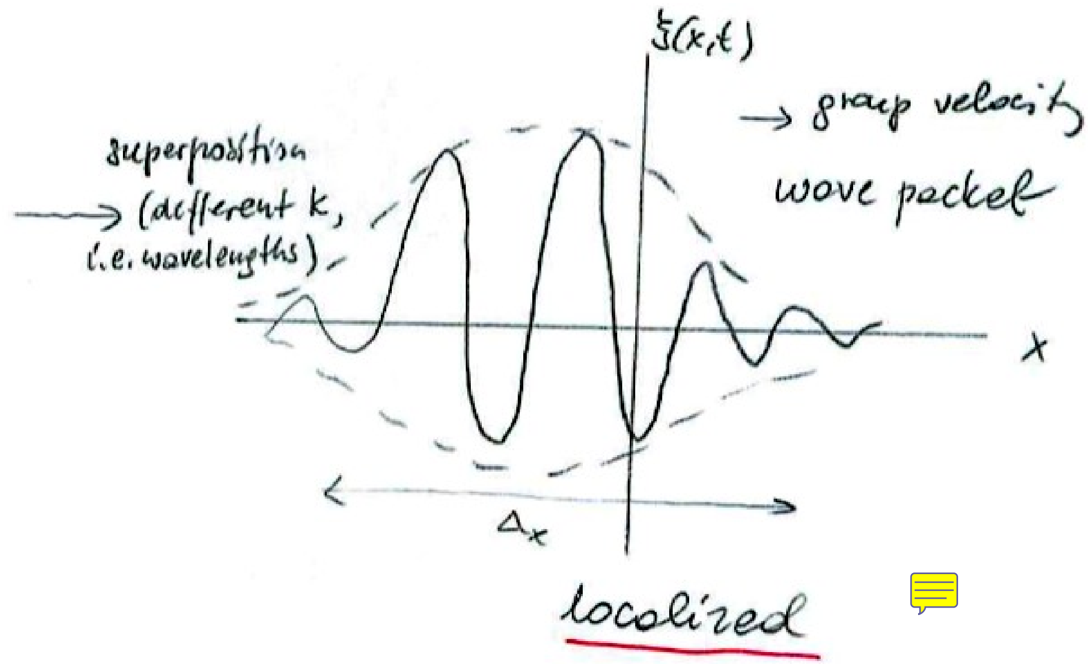
Phase velocity $v_{\mathrm{ph}} = \dfrac{\omega}{k}$



Waves (short review). Wave packets

What happens if we add multiple plane waves with different lengths (wave numbers)?





phase velocity

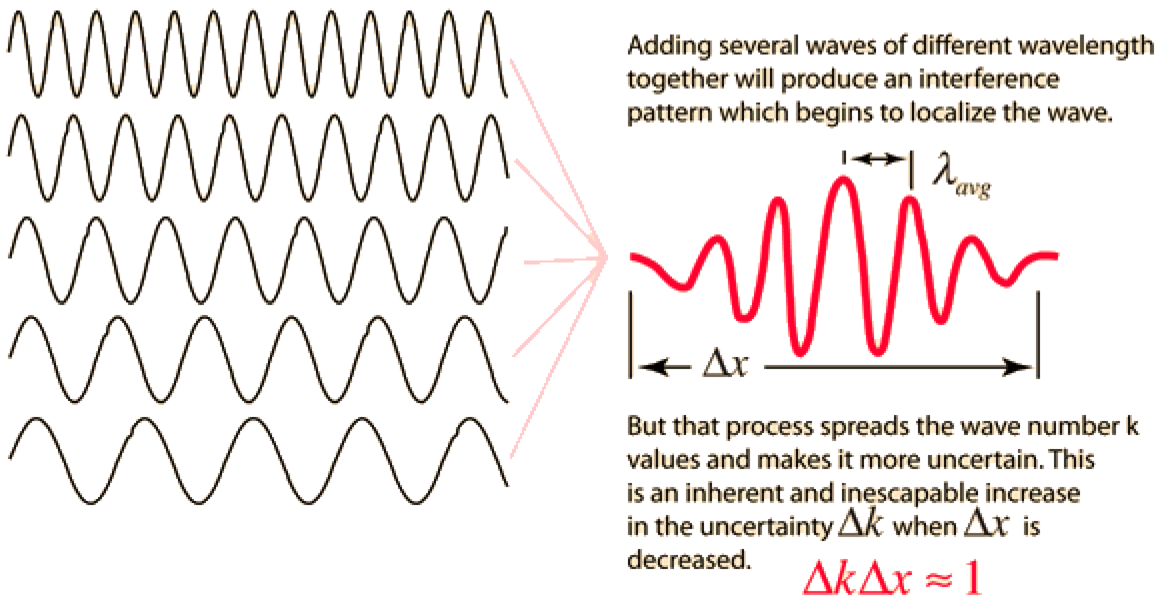
$$v_{\mathrm{ph}} = \frac{\omega}{k}$$

group velocity

$$v_{\mathrm{g}} = \frac{d\omega}{dk} = v_{\mathrm{ph}} + k\frac{dv_{\mathrm{ph}}}{dk}$$

describes dispersion
(waves with different lengths
travel with different phase speeds)

Waves (short review). Wave packets

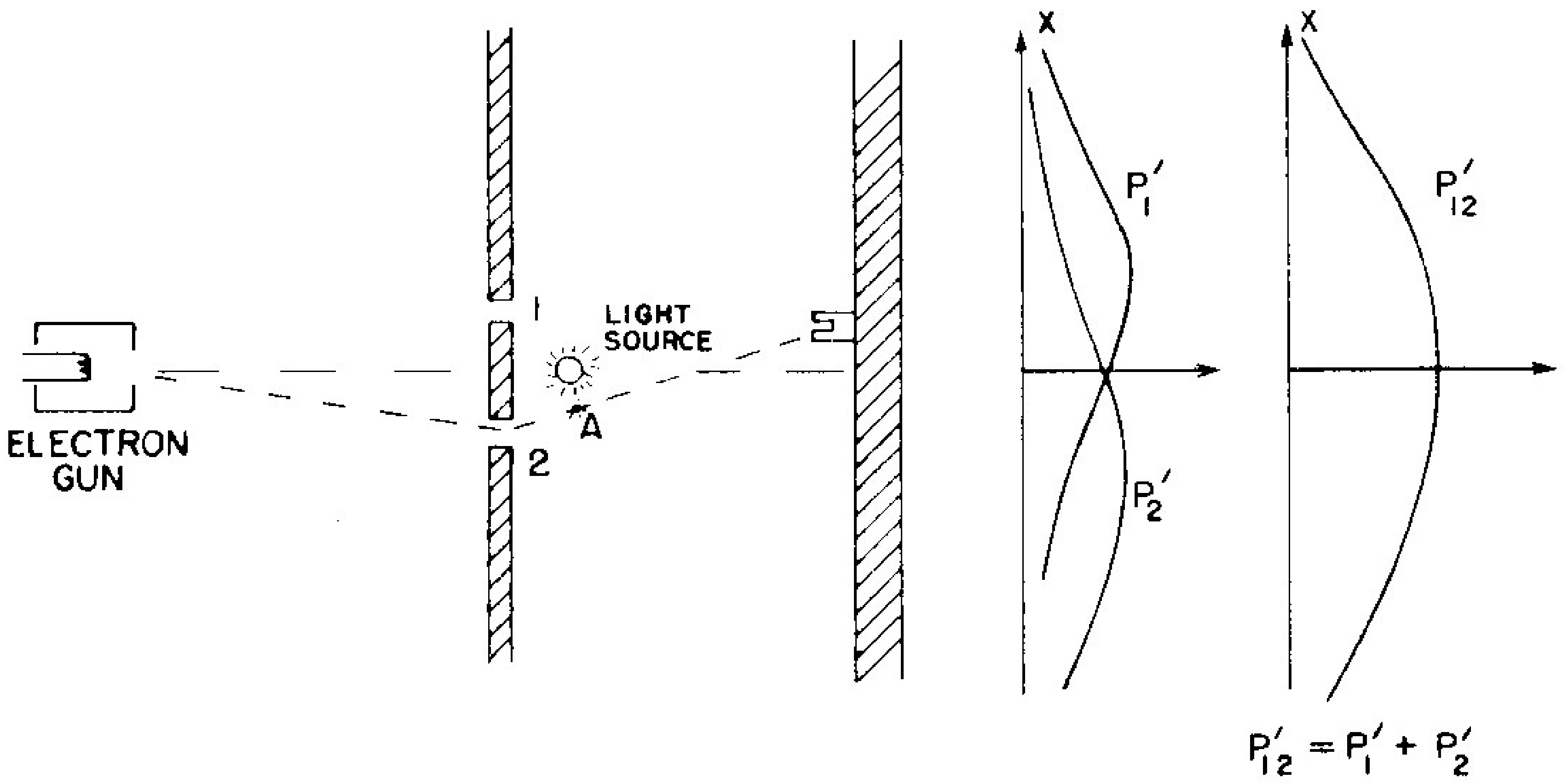


Classical uncertainty principle

better localization in space requires adding waves with a larger spread in wave numbers (and vice versa)

$$\Delta_x \cdot \Delta_k \sim 1$$

That Experiment Again. Watching Electrons...



If we are able to detect which of the slits the electrons have passed through, the interference pattern disappears.

Measurement in QM is “destructive”.