



**Problem Set 7**

Due: 5 July 2017, 12.30 p.m.

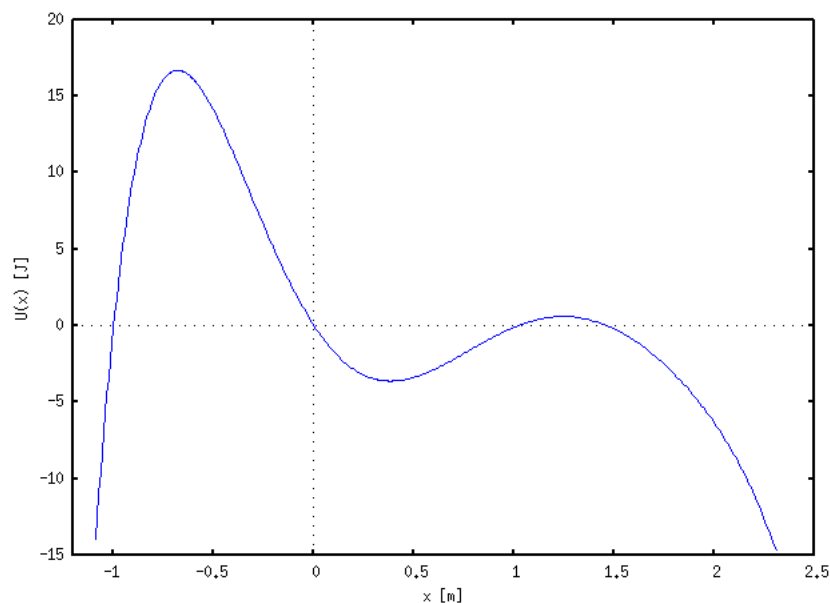
**Problem 1.** Given the potential energy  $U(x, y) = xy^2 + yx^2$ , (a) find the corresponding force field and visualize it (use a computer); (b) on the same graph sketch a few equipotential lines, i.e. lines defined by equation  $U(x, y) = U_0$ , for different values of  $U_0$ ; (c) comment on the graph; (d) calculate work done by this force on a particle moving from (0,0) to (1,1) along a straight line; (e) same as (d) but along the parabola  $y = x^2$ .

*Note.* In (d) and (e) use the simplest possible method.

(3/2 + 1 + 1 + 1 + 1 points)

**Problem 2.** The figure below shows the graph of the potential energy  $U = U(x)$  for a particle moving along the  $x$  axis. Identify equilibrium positions and tell which of them are stable/unstable.

(3/2 points)



**Problem 3.** The Lennard–Jones potential energy  $U = U_0 \left[ \left( \frac{R_0}{r} \right)^{12} - 2 \left( \frac{R_0}{r} \right)^6 \right]$ , also called the L–J potential or 6–12 potential, is often used to approximate the potential energy associated with interaction between a pair of neutral atoms or molecules separated by distance  $r$ .

- (a) Find the corresponding force (it is the force exerted by one atom/molecule on the other). Sketch the graphs of both the potential energy and the force as functions of  $r$ . Which term in the force is responsible for attraction and which for repulsion?
- (b) What is the interpretation of the parameters  $U_0$  and  $R_0$  (both are positive)?
- (c) Introducing a new variable  $x = r - R_0$ , find an approximate expression for the force in the regime  $|x/R_0| \ll 1$ . Interpret your result in terms of oscillations. Find their period.  
*Hint.* The binomial theorem.
- (d) What is oscillating here?

*(3/2 + 1/2 + 3 + 1 points)*

**Problem 4.** What is the period of *small* oscillations about the stable equilibrium position in a potential field with potential energy  $U(x) = U_0 \tan^2 \alpha x$ , where  $U_0$  and  $\alpha$  are constants (what are their units)? The mass of the oscillating particle is  $m$ .

*(2 points)*