UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP141)

LABTORATORY REPORT

EXERCISE 1
Measurements of the Moment of Inertia

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1 Introduction and Theoretical Background

1.1 Objectives

The main objective of this exercise is to get familiar with the constant-torque method for measuring the moment of inertia of a rigid body. Student will study the dependence of the moment of inertia on mass distribution and on the choice of the rotation axis. The parallel axis (Steiner's) will be verified in this exercise as well. Student's measurement technique skills will also be developed by learning how to measure time using a photo-gate electronic timer.

1.2 Basic Concepts of moment of inertia

A rigid body's moment of inertia about an axis is a quantitative characteristics that denes the body's resistance (inertia) to a change of angular velocity in rotation about that axis. The moment of inertia of a rigid body rotating about a fixed axis is determined not only by the mass of the body, but also by its distribution. The moment of inertia of a rigid body about a certain rotation axis can be calculated analytically as long as the body has regular shape or uniformly distributed mass. Otherwise, the calculation may be difficult. Experimental methods turn out to be more useful in such cases.

For rigid body which has an irregular shape or non-uniformly distributed mass, its moment of inertia can be expressed as following equation

$$I = m_1 r_1^2 + m_1 r_2^2 + \cdots + \sum_i m_i r_i^2 \tag{1}$$

I:Moment of Inertia of a body for a given rotation axis
m_i:Masses of the particles making up the body
r_i:Perpendicular distances of the particles from rotation axis

1.3 Second Law of Dynamics for Rotational Motion

According to the second law of dynamics for rotational motion about a fixed axis

$$\tau_z = I\beta_z \tag{2}$$

relates the component of the torque τ_z about the axis of rotation with the moment of inertia about this axis, and the angular acceleration component β_c. Therefore, the moment of inertia I can be found once the torque and the resulting angular acceleration are measured.

The moment of inertia is an additive quantity, i.e. if the moment of inertia of a rigid body (A) about an axis is I_A and the moment of inertia of another rigid body (B) about the same axis is I_B, then the moment of inertia of the combined rigid body AB composed of A and B, about the same axis of rotation, is

$$I_{AB} = I_A + I_B \tag{3}$$

1.4 Parallel Axis Theorem

If the moment of inertia of a rigid body with mass m about an axis through the body's center of mass is I_0, then for any axis parallel to that axis, the moment of inertia is

$$I = I_0 + md^2 \tag{4}$$

where d is the distance between the axes. This result is known as the parallel axis theorem or Steiner's theorem.

2 Measurement Setup and Measurement Method

2.1 Measurement of the Moment of Inertia

The measurement setup is shown in Figure 1. It consists of a turntable with an integrated photo-gate system used for time measurements.

In order to understand the idea of the measurement method, suppose that the empty turntable is initially rotating and its moment of inertia with respect to the rotation axis is I_1. Since the bearings of the turntable are not frictionless, there will be a non-zero frictional torque M_μ causing the turntable to decelerate with angular acceleration β_1, so that the second law of dynamics for rotational motion of the empty turntable reads (furthermore we will skip the subscript z)

$$M_\mu = -I_1\beta_1 \tag{5}$$

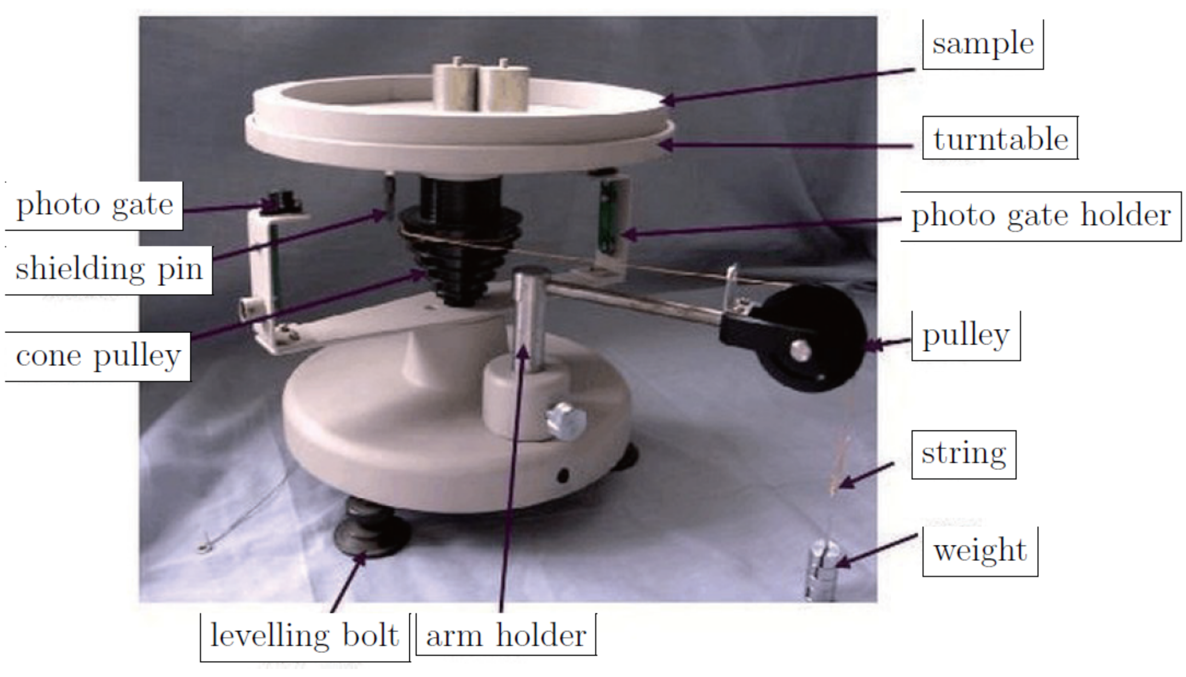

Figure 1: Measurement Setup.

Below the turntable, there is a conical pulley of radius R, with a light and inextensible string wound on it. The axis of the cone coincides with the axis of rotation. Attached to the other end of the string passing through a disk pulley, there is a weight with mass m, free to move downwards after it is released. If the mass moves downwards with constant acceleration a, the tension in the string T is constant and $T = m(g - a)$. This constant tension force, on the other end of the string, provides constant torque on the cone pulley that in turn makes the turntable rotate the with constant angular acceleration. If the turntable rotates with angular acceleration β_2, then $a = R\beta_2$ (we assume that the string does not slip on the pulleys). The torque the string exerts on the pulley (and hence the turntable) is then $TR = m(g - R\beta_2)R$. Consequently, taking into account the frictional torque, the net torque on the turntable is $TR - M_\mu$, and the equation of motion for the turntable reads

$$m(g - R\beta_2)R - M_\mu = I_1\beta_2 \tag{6}$$

Eliminating M_μ from Eqs. (3) and (4), we find

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1} \tag{7}$$

Similarly, if a rigid body with an unknown moment of inertia is placed on the turntable, we may find

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3} \tag{8}$$

where β_3 is the magnitude of angular deceleration of the turntable with the body, and β_3 is its angular acceleration, when the mass m is released and moves downwards.

Using the fact that the moment of inertia is an additive quantity, the moment of inertia of the rigid object placed on the turntable, with respect to the axis of rotation, may be found as the difference

$$I_3 = I_2 - I_1 \tag{9}$$

2.2 Measurement of Angular Acceleration

At the edge of the turntable two shielding pins are fixed. As the turntable rotates, the pins generate a series of signals in the photo-gate with the phase interval of π. An integrated counter-type electronic timer is used to measure the consecutive number k and the time t of the photo-gate signal.

If (k,t) is a set of the measurement data, the corresponding angular position is

$$\theta = k\pi = \omega_0 t + \frac{1}{2}\beta t^2 \tag{10}$$

where ω_0 is the initial angular speed. Now, by performing a quadratic fit to the measurement data, we can find the angular acceleration.

3 Apparatus

As shown in the measurement setup part,we have many apparatus in our exercise. I classify them into two parts.

3.1 Tools

The first part is tools, including the electronic timer, vernier calliper, and electronic balance. However, the tools have their limit of precision, which may cause systematic errors in my results.

Figure 2: Calliper.

	Calliper	Electronic balance	Timer
Resolution	0.002cm	0.1g	0.0001s
Relative uncertainty	/		0.004%

Table 1: Precision of the measurement instruments

3.2 Materials

The second part is materials that we use and measure to help us indirectly examine the moment of inertia. The materials include one Disk, two hoops of different radiant, two similar cylinders, one cone pulley, and one weight.

In this experiment, I need to measure the moment of inertia of the empty turntable, the hoop, the disk and the cylinder, as well as verify the parallel-axis theorem by placing two cylinders o the axis while keeping their center of mass on the axis in order to keep the rotation steady. The distances between the holes and the center of the turntable are 45, 60, 75, 90, 105 mm, respectively.

4 Measurement Procedure

1. Measure the mass of the weight, the hoop, the disk, and the cylinder, as well as the radius of the cone pulley and the cylinder (follow the instructor's requirements). Calculate the moment of inertia of the hoop and the disk analytically. Use a reliable source to find the local value of the acceleration due to gravity in Shanghai.

2. Turn the electronic timer on and switch it to mode 1-2 (single gate, multiple pulses).

3. Place the instrument close to the edge of the desk and stretch the disk pulley arm outside, so that the weight can move downwards unobstructed.

4. Level the turntable with the bubble level.

5. Make the turntable rotating and press the start button on the timer. After at least 8 signals are recorded, stop the turntable and record the data in your data sheet.

6. Attach the weight to one end of the string. Place the string on the disk pulley, thread through the hole in the arm, and wind the string around the 3rd ring of the cone pulley. Adjust the arm holder so that the string goes through the center of the hole.

7. Release the weight and start the timer. Stop the turntable when the weight hits the floor. Write down the recorded data.

8. The angular acceleration can be found by plotting $\theta = k\pi$ against t and performing a quadratic fit using data processing software. (The magnitude of the angular acceleration is equal to the coefficient next to t^2 multiplied by two. The uncertainty of the angular acceleration can be read directly from the fitting result.) The moment of inertia of the empty turntable is found by using the formulae in Section 3 and the data from step 5 and 7. Repeat steps $5 \sim 7$ with a rigid object placed on the turntable. Eq. (7) is used to find the moment of inertia of the rigid object.

   In the experiment, each student should measure the moment of inertia of the empty turntable, the hoop, the disk and the cylinder, as well as verify the parallel-axis theorem by placing two cylinders of the axis while keeping their center of mass on the axis in order to keep the rotation steady.

   The distances between the holes and the center of the turntable are ca. 45, 60, 75, 90, 105 mm, respectively. The timer's resolution is 0:0001 s, and the error is 0:004

5 Calculation and Results

5.1 Relative Parameters of Materials

In the exercise, I use a calliper to measure the diameter of the disk, the inner diameter and external diameter of two hoops, the two cylinder's diameter, the cone pulley's diameter and distance between the holes that I choose.

	1	2	3	4
Disk $\varnothing$ [cm] $\pm$ 0.002[cm]	23.988	24.000	24.002	24.010
Hoop $\varnothing_1$[cm] $\pm$ 0.002[cm]	21.380	21.388	21.224	21.404
Hoop $\varnothing_2$[cm] $\pm$ 0.002[cm]	23.596	23.620	23.640	23.542
Cylinder A $\varnothing$ [cm] $\pm$ 0.002[cm]	3.000	2.996	2.992	2.984
Cylinder B $\varnothing$ [cm] $\pm$ 0.002[cm]	2.996	2.986	2.988	2.986
Cone pulley $\varnothing$ [cm] $\pm$ 0.002[cm]	5.014	5.020	5.010	5.014
Hole ① d [cm] $\pm$ 0.002[cm]	4.002	5.014	/	
Hole ② d [cm] $\pm$ 0.002[cm]	3.992	5.024	/	
Hole ③ d [cm] $\pm$ 0.002[cm]	5.482	6.504	/	
Hole ④ d [cm] $\pm$ 0.002[cm]	5.488	6.482	/	

Table 2: Calliper measurements.

The hole numbers is consistent with my choice in table 4. The distance of the hole to the rotation center is measured as: the inner distance and the outer distance. Other parameters are measured 4 times.

In addition, I also use the electronic balance to measure the mass of disk, hoop, two cylinders and the weight

Disk [g] $\pm$ 0.1[g]	490.2	Hoop [g] $\pm$ 0.1[g]	429.4
Cylinder A [g] $\pm$ 0.1[g]	166.0	Cylinder B [g] $\pm$ 0.1[g]	165.9
Weight [g] $\pm$ 0.1[g]	54.5		

Table 3: Mass measurements

5.2 Theoretical Moment of inertia of Disk, Hoops and Cylinders

Since I've measure the relative parameter of the materials, they will be used to calculate the theoretical value of moment of inertia

5.2.1 Theoretical Moment of Inertia of the Disk

$$R_{disk} = \frac{1}{2}\bar{\varnothing}_{disk} = 12.000 \pm 0.008[cm]$$

$$I_{disk} = \frac{1}{2}M_{disk}R_{disk}^2 = 3.529 \times 10^{-3} \pm 4.47 \times 10^{-6}[kg \cdot m^2]$$

5.2.2 Theoretical Moment of Inertia of Hoops

$$R_{hoop1} = \frac{1}{2}\bar{\varnothing}_1 = 10.675 \pm 0.066[cm]$$

$$R_{hoop2} = \frac{1}{2}\bar{\varnothing}_2 = 11.800 \pm 0.034[cm]$$

The moment of inertia of the hoop is estimated by:

$$I_{hoop} = \frac{1}{2}M_{hoop}(R^2_{hoop1} + R^2_{hoop2}) = 5.435 \times 10^{-3} \pm 3.66 \times 10^{-5}[kg \cdot m^2]$$

5.2.3 Theoretical Moment of Inertia after cylinders are added

The cylinders' radius are:

$$R_A = \frac{1}{2}\bar{\varnothing}_A = 1.497 \pm 0.006[cm]$$

$$R_B = \frac{1}{2}\bar{\varnothing}_B = 1.495 \pm 0.004[cm]$$

If they're rotating around the their center, their moment of inertia are:

$$I_A = \frac{1}{2}m_A R_A^2 = 1.860 \times 10^{-5} \pm 9.797 \times 10^{-8}[kg \cdot m^2]$$

$$I_B = \frac{1}{2}m_B R_B^2 = 1.853 \times 10^{-5} \pm 6.907 \times 10^{-8}[kg \cdot m^2]$$

The distance from the four holes to the center are:

$$d_1 = 4.508 \pm 0.0014[cm]$$
$$d_2 = 4.508 \pm 0.0014[cm]$$
$$d_3 = 5.993 \pm 0.0014[cm]$$
$$d_4 = 5.985 \pm 0.0014[cm]$$

Then according to Parallel Axis Theorem
If cylinder A and cylinder B are placed in hole ① and hole ② respectively,

$$I_{A1B2} = I_A + m_A d_1^2 + I_B + m_B d_2^2 = 7.116 \times 10^{-4} \pm 1.6 \times 10^{-5}[kg \cdot m^2]$$

If cylinder A and cylinder B are placed in hole ③ and hole ④ respectively,

$$I_{A3B4} = I_A + m_A d_3^2 + I_B + m_B d_4^2 = 1.228 \times 10^{-3} \pm 5.06 \times 10^{-7}[kg \cdot m^2]$$

5.3 Measurement of the Moment of Inertia

I use constant-torque method to find the moment of inertia in this exercise. To calculate the moment of inertia, I've already measured following physical quantities:

The acceleration due to gravity: $g = 9.7494 \pm 0.001[m/s^2]$
The mass of the weight: $m = 54.5 \pm 0.1[g]$
The radius of the cone pulley: $R_{cone} = \frac{1}{2}\varnothing_c\bar{o}ne = 2.508 \pm 0.035[cm]$

	Deceleration					Acceleration				
Empty turntable	k	1	2	3	4	k	1	2	3	4
	t [s]	1.1452	2.3156	3.5037	4.7182	t [s]	0.8993	1.5529	2.0941	2.5797
	k	5	6	7	8	k	5	6	7	8
	t [s]	5.9521	7.2158	8.5009	9.8189	t [s]	3.0134	3.4372	3.8926	4.4138
	Deceleration					Acceleration				
With disk	k	1	2	3	4	k	1	2	3	4
	t [s]	1.0060	2.0264	3.0550	4.0988	t [s]	2.0963	3.3393	4.2880	5.0304
	k	5	6	7	8	k	5	6	7	8
	t [s]	5.1515	6.2199	7.2978	8.3924	t [s]	5.6946	6.3570	7.0251	7.6953
	Deceleration					Acceleration				
With hoop	k	1	2	3	4	k	1	2	3	4
	t [s]	0.9790	1.9636	2.9603	3.9624	t [s]	1.0913	1.9277	2.6298	3.2728
	k	5	6	7	8	k	5	6	7	8
	t [s]	4.9770	5.9975	7.0308	8.0703	t [s]	3.9143	4.5606	5.2084	5.8611
	Deceleration					Acceleration				
Cylinder A in hole ①, B in ②	k	1	2	3	4	k	1	2	3	4
	t [s]	0.6053	1.2131	1.8268	2.4432	t [s]	1.4368	2.2513	2.8846	3.4224
	k	5	6	7	8	k	5	6	7	8
	t [s]	3.0656	3.6907	4.3220	4.9561	t [s]	3.8963	4.3277	4.7248	5.1044
	Deceleration					Acceleration				
Cylinder A in hole ③, B in ④	k	1	2	3	4	k	1	2	3	4
	t [s]	0.6683	1.3434	2.0218	2.7073	t [s]	0.5142	2.4129	3.2828	3.9550
	k	5	6	7	8	k	5	6	7	8
	t [s]	3.3963	4.0925	4.7923	5.4996	t [s]	4.5208	5.0210	5.4726	5.8974

Table 4: Time measurements

The chosen hole ① and hole ① is a symmetric pair, and hole ③and hole ④ is another symmetric pair. With these data, I can use equation(10) to draw $k \times \pi$ vs t images. Then I can get the angular acceleration through image fitting to calculate the moment of inertia.

5.3.1 Moment of Inertia of the Empty Turntable

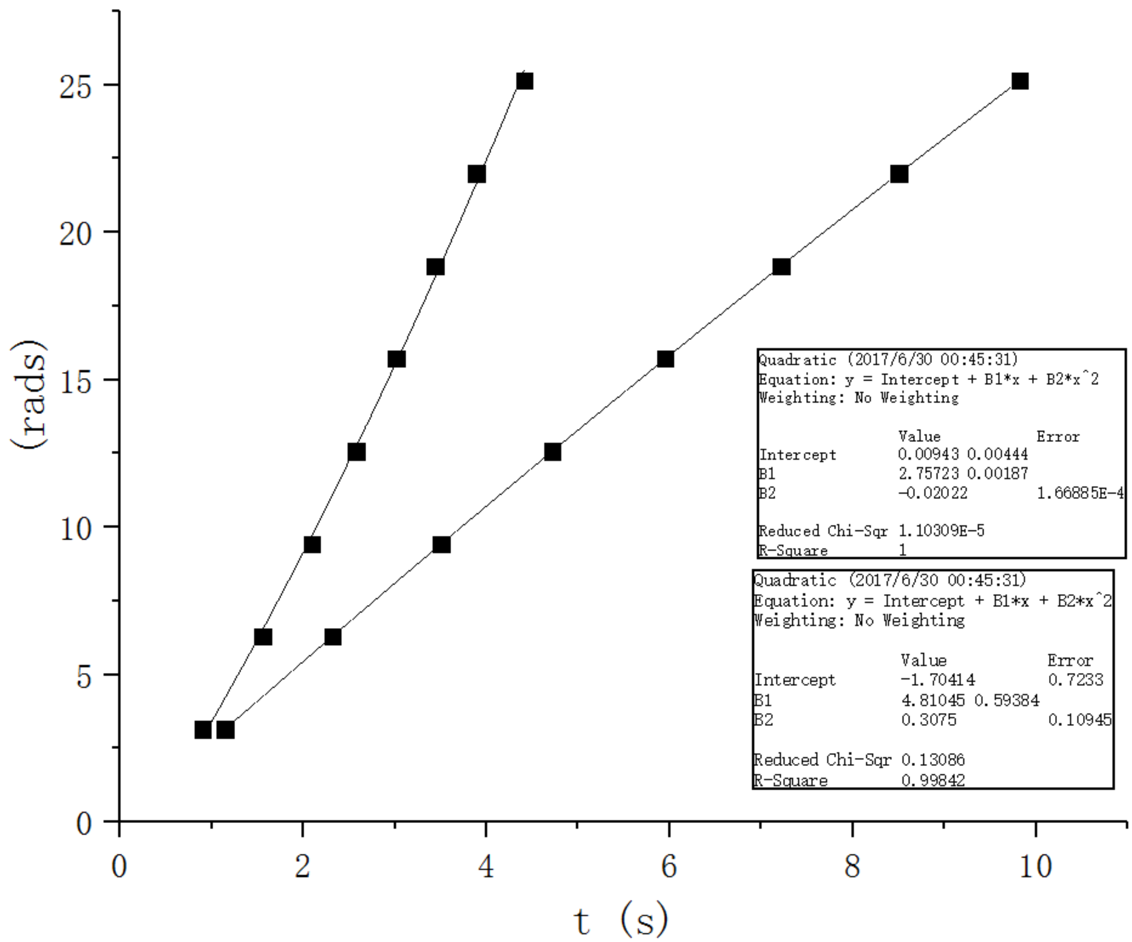

Figure 3: Measurement data with error bars and a parabola fit for the relation between angle of rotation and time. The value for the obtained fit is two values of B2 in the tables.

From the fitting:

$$\beta_1 = 2 \times B2 = -0.0403 \pm 5.3684 \times 10^{-4} [rad/s^2]$$

$$\beta_2 = 2 \times B2 = 0.0615 \pm 0.10945 [rad/s^2]$$

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1} = 0.0203 \pm 3.4 \times 10^{-4} [kg \cdot m^2]$$

5.3.2 Moment of Inertia of the Disk

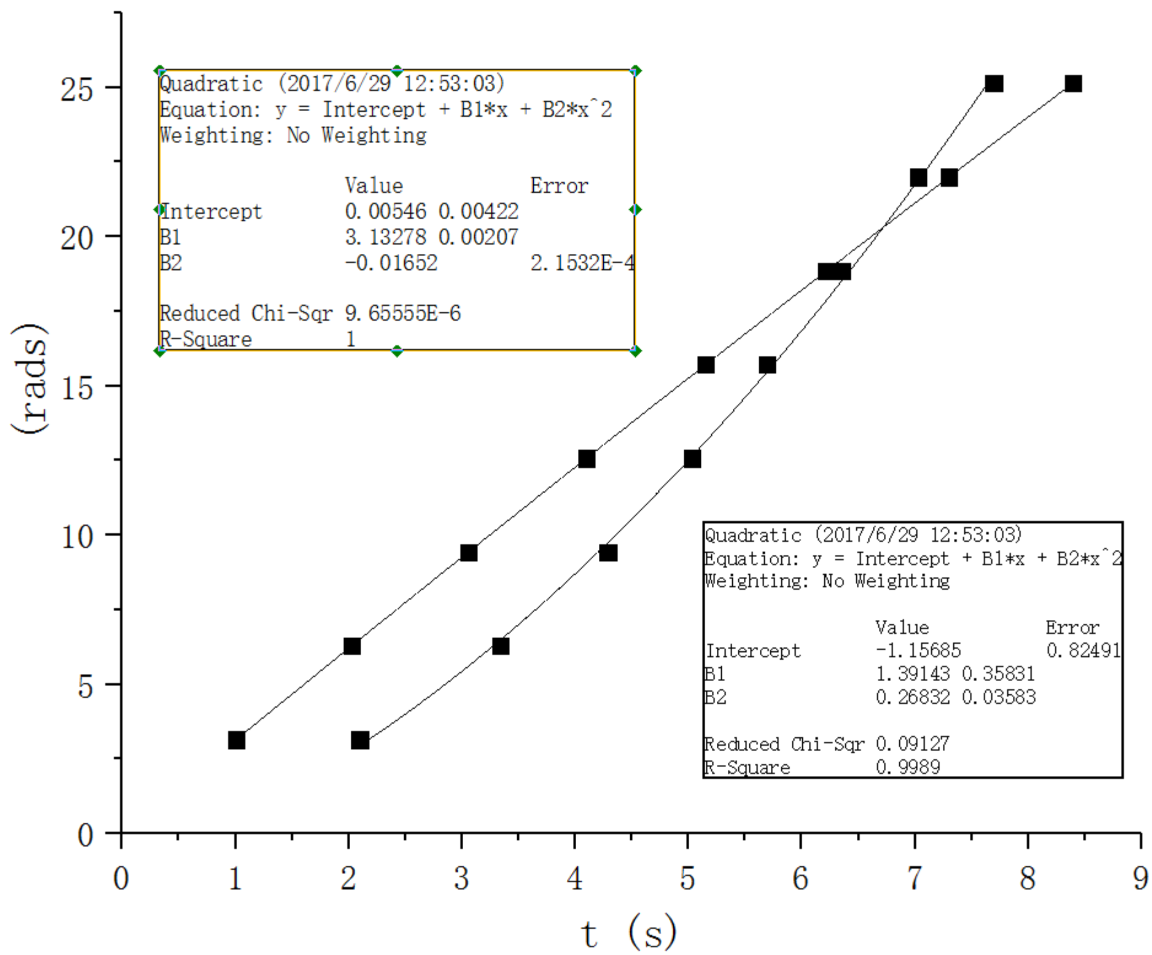

Figure 4: Measurement data with error bars and a parabola fit for the relation between angle of rotation and time. The value for the obtained fit is two values of B2 in the tables.

From the fitting:

$$\beta_3 = 2 \times B2 = -0.0330 \pm 4.3064 \times 10^{-4}[rad/s^2]$$

$$\beta_4 = 2 \times B2 = 0.5366 \pm 0.0717[rad/s^2]$$

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3} = 0.0234 \pm 3 \times 10^{-4}[kg \cdot m^2]$$

$$I_{disk} = I_2 - I_1 = 3.1 \times 10^{-3} \pm 4.534 \times 10^{-4}[kg \cdot m^2]$$

5.3.3 Moment of Inertia of the Hoop

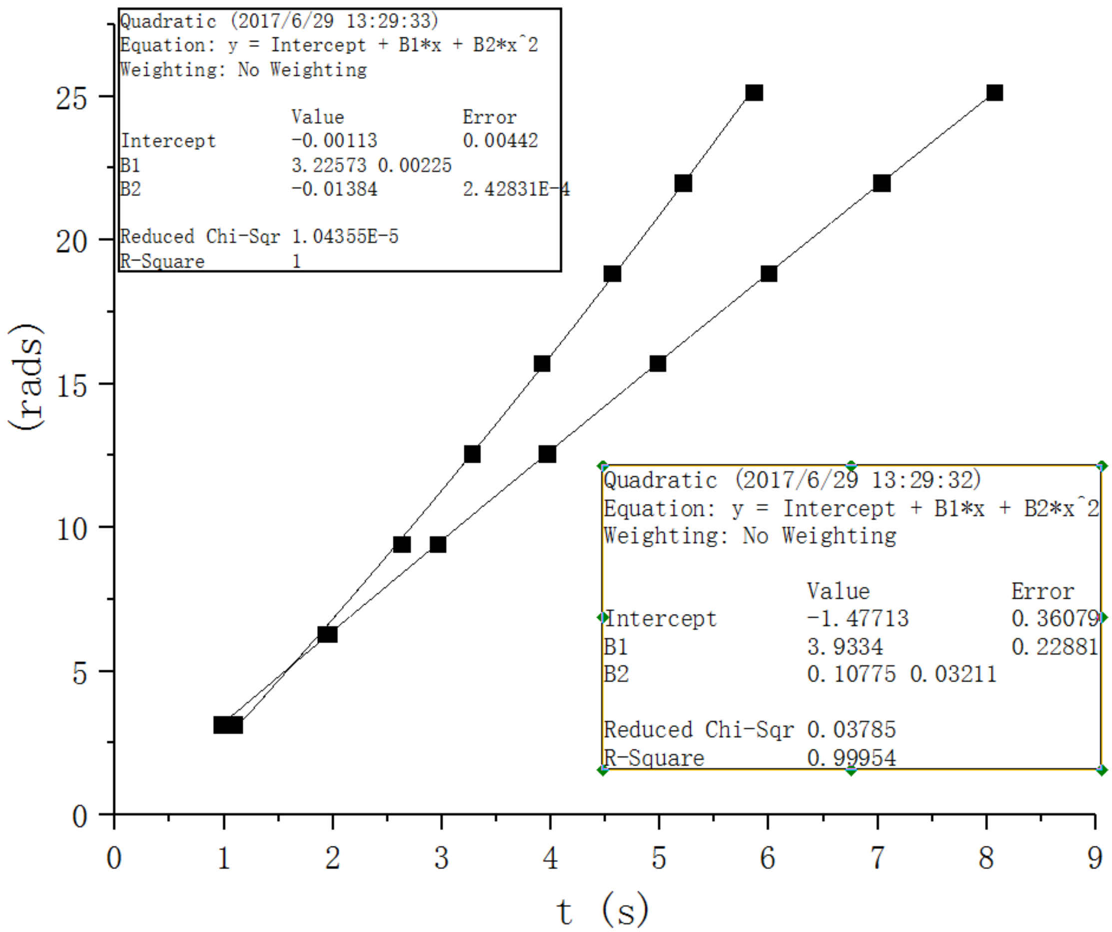

Figure 5: Measurement data with error bars and a parabola fit for the relation between angle of rotation and time. The value for the obtained fit is two values of B2 in the tables.

From the fitting:

$$\beta_3 = 2 \times B2 = -0.0277 \pm 4.8566 \times 10^{-4}[rad/s^2]$$

$$\beta_4 = 2 \times B2 = 0.2155 \pm 0.0642[rad/s^2]$$

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3} = 2.38 \times 10^{-2} \pm 1.45 \times 10^{-2}[kg \cdot m^2]$$

$$I_{hoop} = I_2 - I_1 = 3.45 \times 10^{-3} \pm 1.25 \times 10^{-3}[kg \cdot m^2]$$

5.3.4 Moment of Inertia of with Cylinder A in Hole① and Cylinder B in Hole②

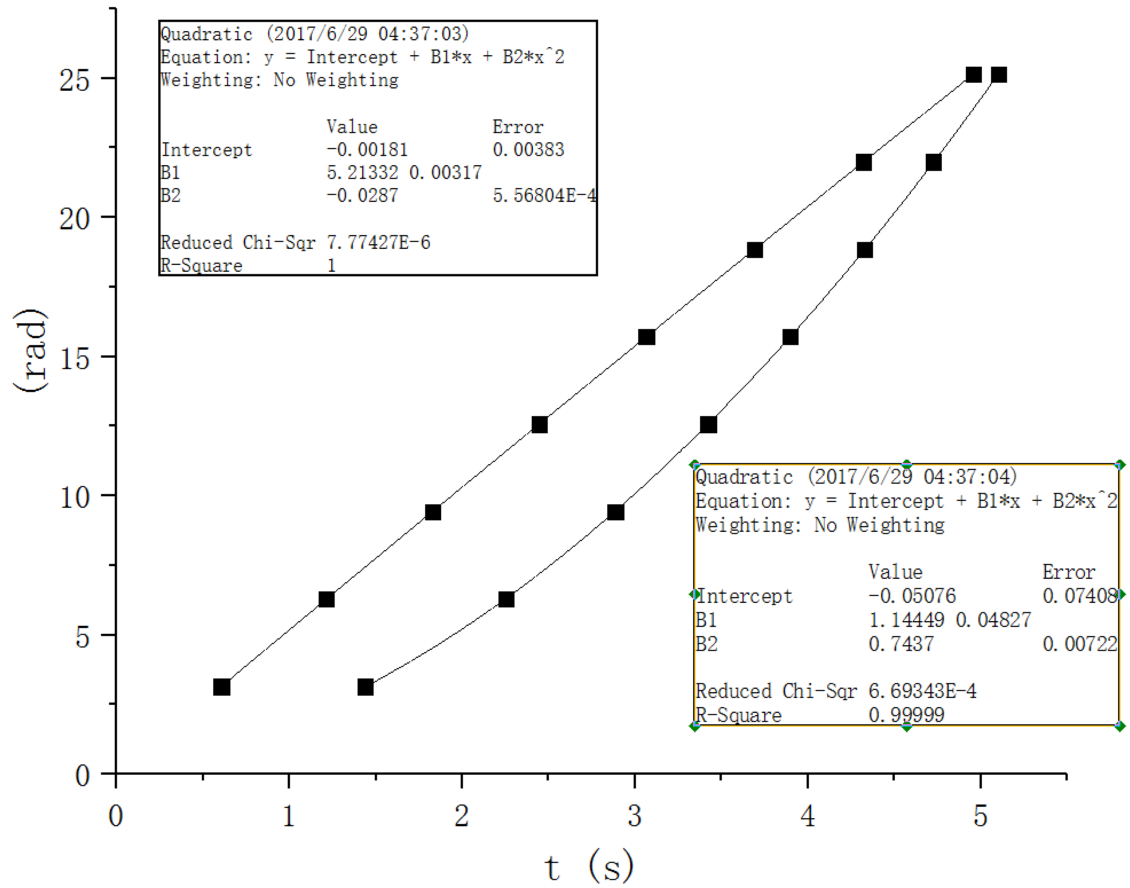

Figure 6: Measurement data with error bars and a parabola fit for the relation between angle of rotation and time. The value for the obtained fit is two values of B2 in the tables.

From the fitting:

$$\beta_3 = 2 \times B2 = -0.0574 \pm 1.1136 \times 10^{-3}[rad/s^2]$$

$$\beta_4 = 2 \times B2 = 1.4874 \pm 0.0144[rad/s^2]$$

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3} = 1.913 \times 10^{-2} \pm 1.15 \times 10^{-4}[kg \cdot m^2]$$

$$I_{A1B2} = I_2 - I_1 = -1.17 \times 10^{-3} \pm 3.589 \times 10^{-4}[kg \cdot m^2]$$

5.3.5 Moment of Inertia of with Cylinder A in Hole③ and Cylinder B in Hole④

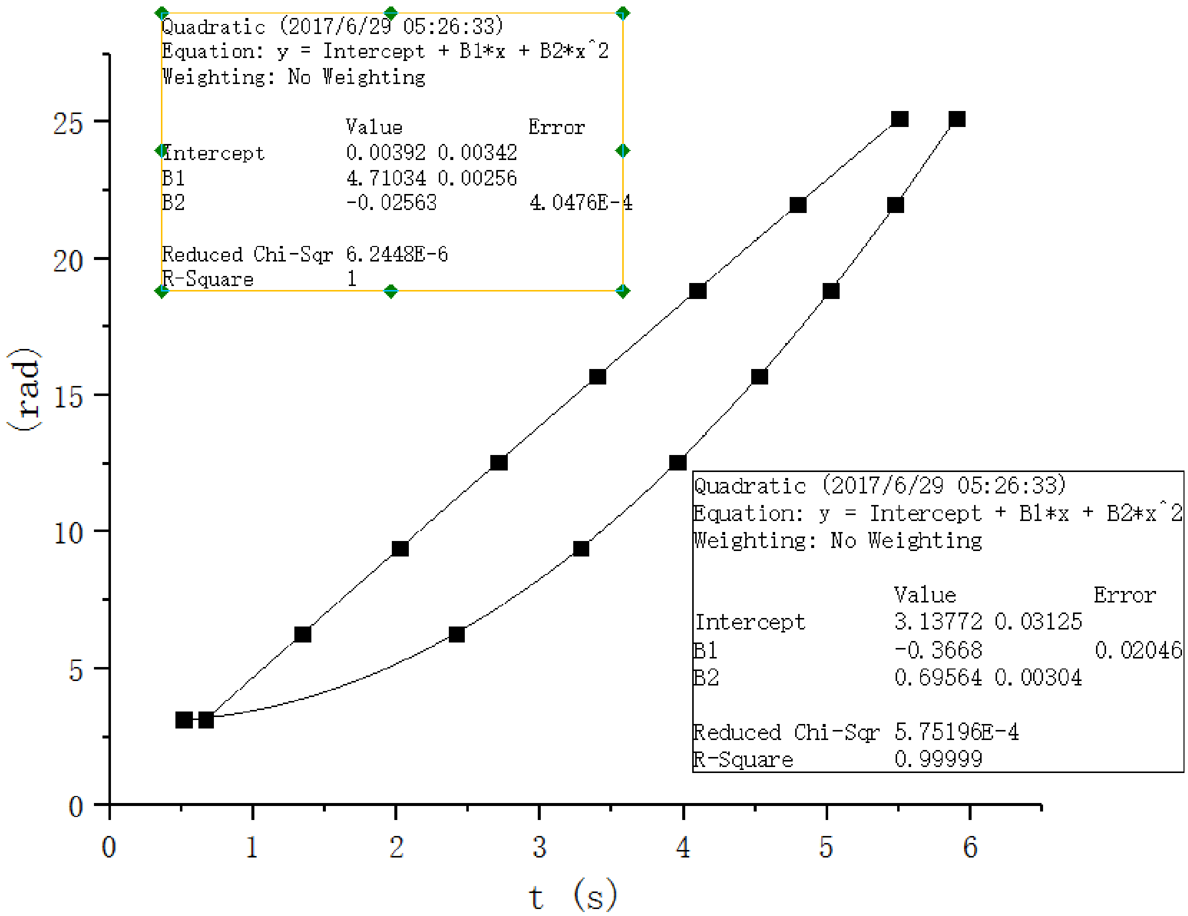

Figure 7: Measurement data with error bars and a parabola fit for the relation between angle of rotation and time. The value for the obtained fit is two values of B2 in the tables.

From the fitting:

$$\beta_3 = 2 \times B2 = -0.0513 \pm 8.0952 \times 10^{-4}[rad/s^2]$$

$$\beta_4 = 2 \times B2 = 1.3913 \pm 0.0061[rad/s^2]$$

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3} = 1.919 \times 10^{-2} \pm 1.05 \times 10^{-4}[kg \cdot m^2]$$

$$I_{A3B4} = I_2 - I_1 = -1.11 \times 10^{-3} \pm 3.558 \times 10^{-4}[kg \cdot m^2]$$

6 Measurement Uncertainty Analysis

6.1 Range Measurement Uncertainty

6.1.1 Multiple measurements yield to type A uncertainty

For multiple measurements' uncertainty:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$S_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2}$$

$$\triangle_A = \frac{S_X t_{0.95}}{\sqrt{n}}$$

The value of $t_0.95$ and $\frac{t_{0.95}}{\sqrt{n}}$ are giving in Table 4

n	3	4	5	6	7	8	9	10	15	20	≥ 100
$t_0.95$	4.30	3.18	2.78	2.57	2.45	2.36	2.31	2.26	2.14	2.09	≤ 1.97
$\frac{t_0.95}{\sqrt{n}}$	2.48	1.59	1.204	1.05	0.926	0.834	0.770	0.715	0.553	0.467	≤ 0.139

Table 5: The values of $t_{0.95}$ and $\frac{t_{0.95}}{\sqrt{n}}$

The total uncertainty $u = \sqrt{(\triangle_A)^2 + (\triangle_B)^2}$ and relative uncertainty $u_r = \frac{u}{X} \times 100\%$. For example, we can calculate the uncertainty of disk's diameter as the following process:

$$\bar{X} = \frac{1}{4}\sum_{i=1}^{4} x_i = 24.000[cm]$$

$$S_X = \sqrt{\frac{1}{3}\sum_{i=1}^{4}(x_i - 24.000)^2} = 0.009$$

$$\triangle_A = 1.59 S_X = 0.014$$

$$\triangle_B = 0.002[cm]$$

$$u = \sqrt{0.014^2 + 0.002^2} = 0.015[cm]$$

$$u_r = \frac{0.015}{24.000} \times 100\% = 0.06\%$$

6.1.2 Single measurement yields to type B uncertainty

Since the calliper's resolution is 0.002[cm], $\triangle_B = \triangle_\varnothing = 0.002[cm]$ When I measured the distance from holes to the center of the disk, I use the average value of the inner distance and external distance.

$$d = \frac{d_1 + d_2}{2} \tag{11}$$

Since it's indirect measurement,its uncertainty is estimated:

$$u_d = \sqrt{(\frac{\partial d}{\partial d_1})^2 {u_1}^2 + (\frac{\partial d}{\partial d_2})^2 {u_2}^2} = \sqrt{(\frac{1}{2})^2 (0.002[cm])^2 + (\frac{1}{2})^2 (0.002[cm])^2} = 0.0014[cm]$$

6.1.3 The Final Uncertainty of Calliper Measurement

Object	Average	±Uncertainty [cm]	Relative uncertainty
Disk ∅ [cm]	24.000	0.015	0.06%
Hoop $\varnothing_1$ [cm]	21.349	0.133	0.63%
Hoop $\varnothing_2$ [cm]	23.600	0.067	0.29%
Cylinder A ∅ [cm]	2.993	0.011	0.37%
Cylinder B ∅ [cm]	2.989	0.008	0.26%
Cone pulley ∅ [cm]	5.015	0.007	0.14%
Hole ① d [cm]	4.508	0.0014	0.03%
Hole ② d [cm]	4.508	0.0014	0.03%
Hole ③ d [cm]	5.993	0.0014	0.02%
Hole ④ d [cm]	5.985	0.0014	0.02%

Table 6: Uncertainty of calliper measurements

6.2 Mass Measurements Uncertainty

A single measurement for mass yields only type B uncertainty $\triangle_B = \triangle_\varnothing = 0.1[g]$. $u = \triangle_B = 0.1[g]$.

Object	Mass	±Uncertainty [g]	Relative uncertainty
Disk [g]	490.2	0.1	0.02%
Hoop [g]	429.4	0.1	0.02%
Cylinder A [g]	166.0	0.1	0.06%
Cylinder B [g]	165.9	0.1	0.06%
Weight [g]	54.5	0.1	0.18%

Table 7: Uncertainty of electronic balance measurements

6.3 Time Measurements Uncertainty

A single measurement for time yields only type B uncertainty $\triangle_B = \triangle_{\varnothing} = 0.0001[s]$. $u = \triangle_B = 0.0001[s]$ and $u_r = 0.004\%$

6.4 Uncertainty of Theoretical Moment of Inertia

The theoretical moment of inertia is calculated through other parameters measured so I should use the formula for combined uncertainty to estimate it. First, we should know its resolution and relative uncertainty.

Object	$u[kg \cdot m^2]$	$u_r[\%]$
With disk	4.47×10^{-6}	0.13
With Hoop	3.66×10^{-5}	0.67
Cylinder A in hole ① and Cylinder B in hole ②	1.6×10^{-5}	2.2
Cylinder A in hole ③ and Cylinder B in hole ④	5.06×10^{-7}	0.04

Table 8: Uncertainty of theoretical moment of inertia

6.4.1 Calculation of uncertainty of theoretical I_{disk}

$$u = \sqrt{(\frac{\partial I}{\partial m} \cdot u_m)^2 + (\frac{\partial I}{\partial R} \cdot u_R)^2} = \sqrt{(\frac{R^2}{2} \cdot u_m)^2 + (MR \cdot u_R)^2} = 4.47 \times 10^{-6}[kg \cdot m^2]$$

$$u_r = \frac{u}{I_{disk}} \times 100\% = 0.13\%$$

6.4.2 Calculation of uncertainty of theoretical I_{hoop}

$$u = \sqrt{(\frac{\partial I}{\partial m} \cdot u_m)^2 + (\frac{\partial I}{\partial R_1} \cdot u_{R_1})^2 + (\frac{\partial I}{\partial R_2} \cdot u_{R_2})^2}$$

$$= \sqrt{(\frac{(R_1 + R_2)^2}{2} \cdot u_m)^2 + (MR_1 \cdot u_{R_1})^2 + (MR_2 \cdot u_{R_2})^2} = 3.66 \times 10^{-5}[kg \cdot m^2]$$

$$u_r = \frac{u}{I_{hoop}} \times 100\% = 0.67\%$$

6.4.3 Calculation of uncertainty of theoretical I_{A1B2} and I_{A3B4}

When Cylinder A is placed at ① and Cylinder B is placed at ② Then

$$u_{A1B2} = \sqrt{(\frac{\partial I_{A1B2}}{\partial m_A} \cdot u_{m_A})^2 + (\frac{\partial I_{A1B2}}{\partial m_B} \cdot u_{m_B})^2 + \frac{\partial I_{A1B2}}{\partial d_1} \cdot u_{d_1})^2 + \frac{\partial I_{A1B2}}{\partial d_2} \cdot u_{d_2})^2 + \frac{\partial I_{A1B2}}{\partial u_A} \cdot u_A)^2 + (\frac{\partial I_{A1B2}}{\partial u_B}}$$

$$= 1.6 \times 10^{-5} [kg \cdot m^2]$$

When Cylinder A is placed at ③ and Cylinder B is placed at ④ Then

$$u_{A1B2} = \sqrt{(\frac{\partial I_{A1B2}}{\partial m_A} \cdot u_{m_A})^2 + (\frac{\partial I_{A1B2}}{\partial m_B} \cdot u_{m_B})^2 + \frac{\partial I_{A3B4}}{\partial d_3} \cdot u_{d_3})^2 + \frac{\partial I_{A3B4}}{\partial d_4} \cdot u_{d_4})^2 + \frac{\partial I_{A3B4}}{\partial u_A} \cdot u_A)^2 + (\frac{\partial I_{A3B4}}{\partial u_B}}$$

$$= 5.06 \times 10^{-7} [kg \cdot m^2]$$

6.5 Uncertainty of Angular Acceleration and Deceleration

Since we can get the angular acceleration and deceleration through image fitting, the software "origin" can also give us their uncertainty.

Uncertainty of β $[rad/s^2]$		
	Deceleration	Acceleration
Empty table	1.6689×10^{-4}	0.1095
With disk	2.1532×10^{-4}	0.0358
With hoop	2.4283×10^{-4}	0.0321
Cylinder A in hole ① and Cylinder B in hole ②	5.568×10^{-4}	0.0072
Cylinder A in hole ③ and Cylinder B in hole ④	4.0476×10^{-4}	0.003

Table 9: Uncertainty of β

6.6 Uncertainty of Measured Moment of Inertia

Uncertainty of I		
	$u[kg \cdot m^2]$	$u_r[\%]$
Empty table	3.4×10^{-4}	1.67
With disk	4.534×10^{-4}	14.6
With hoop	1.25×10^{-2}	36.23
Cylinder A in hole ① and Cylinder B in hole ②	3.589×10^{-4}	30.67
Cylinder A in hole ③ and Cylinder B in hole ④	3.558×10^{-4}	32.05

Table 10: Uncertainty of measured moment of inertia

To calculate the uncertainty of I_0

$$u_{I_0} = \sqrt{(\frac{\partial I_0}{\partial m}u_m)^2 + (\frac{\partial I_0}{\partial R}u_R)^2 + (\frac{\partial I_0}{\partial \beta_1}u_{\beta_1})^2 + (\frac{\partial I_0}{\partial \beta_2}u_{\beta_2})^2 + (\frac{\partial I_0}{\partial g}u_g)^2}$$

$$= \sqrt{(\frac{(g - R\beta_2)R}{\beta_2 - \beta_1}u_m)^2 + (\frac{(g - 2R\beta_2)m}{\beta_2 - \beta_1}u_R)^2 + (\frac{(g - R\beta_2)mR}{(\beta_2 - \beta_1)^2}u_{\beta_1})^2 + (\frac{(g - R\beta_1)mR}{(\beta_2 - \beta_1)^2}u_{\beta_2})^2 + (\frac{-mR}{\beta_2 - \beta_1}u_g)^2}$$

$$= 3.4 \times 10^{-4}$$

$$u_{rI_0} = \frac{u_{I_0}}{I_0} \times 100\% 1.67\%$$

Similarly, the resolution and relative uncertainty of I_{disk}, I_{hoop}, I_{A1B2}, and I_{A3B4} can be calculated as u_{I_0} and u_{rI_0}

$$u_{Idisk} = 4.534 \times 10^{-4}[kg \cdot m^2]$$

$$u_{r_{disk}} = 14.6\%$$

$$u_{Ihoop} = 1.25 \times 10^{-2}[kg \cdot m^2]$$

$$u_{r_{hoop}} = 36.23\%$$

$$u_{IA1B2} = 3.589 \times 10^{-4}[kg \cdot m^2]$$

$$u_{r_{A1B2}} = 30.67\%$$

$$u_{IA3B4} = 3.558 \times 10^{-4}[kg \cdot m^2]$$

$$u_{r_{A3B4}} = 32.05\%$$

7 Conclusion and Discussion

7.1 Conclusion

In this experiment, I use constant-torque method to measure the moment of inertia of a rigid body. Parallel axis theorem can also be verified by comparing my measured moment of inertia to my theoretically calculated data.

Through this exercise, I have a rough idea about the concept of rigid body, its moment of inertia, rotational motion, and parallel axis theorem. In addition, I learn how to use the calliper, electronic balance, and timer to measure relative physics quantity with high accuracy after this experiment.

Moreover, uncertainty analysis is very important in the exercise. It's the first time I analyse the uncertainty of physical quantities. In the exercise, lots of data are recorded, which leads to many calculation of uncertainty. When I use the data measured, I must first calculate the random errors and systematic errors then I can get the total measurement uncertainty.

7.2 Discussion

7.2.1 Error Analysis

Object	Measured I $kg \cdot m^2$	Theoretical I $kg \cdot m^2$	Relative error
With disk	$3.1 \times 10^{-3} \pm 4.534 \times 10^{-4}$	$3.529 \times 10^{-3} \pm 4.47 \times 10^{-6}$	12%
With hoop	$3.45 \times 10^{-3} \pm 1.25 \times 10^{-2}$	$5.435 \times 10^{-3} \pm 3.66 \times 10^{-5}$	36%
Cylinder A in hole ① and Cylinder B in hole ②	$-1.17 \times 10^{-3} \pm 3.589 \times 10^{-4}$	$7.116 \times 10^{-4} \pm 1.6 \times 10^{-5}$	160.8%
Cylinder A in hole ③ and Cylinder B in hole ④	$-1.11 \times 10^{-3} \pm 3.558 \times 10^{-4}$	$1.228 \times 10^{-3} \pm 5.06 \times 10^{-7}$	190%

Table 11: Relative error of moment of inertia

From the table of moment of inertia, I can observe the relative error is quite bigger than excepted, especially when I put the cylinders on the holes. As a result, the error analysis is extremely important.

Since the errors of disk and hoop's moment of inertia are not so big, I guess my calculation of I_0 is not. The main problem lies in the calculation of $I_A 1B2$ and $I_A 3B4$, because it should never be negative value. I think external force might be exerted on equipments during my measurement and cause the errors.

What's more, I found my timer is very unstable in my experiment because sometimes it'll miss the data or record the time twice. It caused me a lot of trouble. For sake of stability, I did the first three measurement again but I can't repeat the last two due to the limited time, so it may lead big errors in my data.

7.3 Suggestions and Improvement

1. Give the table an appropriate initial angular velocity. If it's too fast or too slow, it will lead to huge uncertainty and lack accuracy.
2. Never exert an external force on the materials because it will change the angular acceleration or deceleration measured.
3. Better timer is preferred.
4. I should improve my efficiency and save time in my experiment.

8 Data Sheet

Data sheet is attach to the report

9 Reference

Young, H.D., Freedman R.A. University Physics. Chapter 9,10.
Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Lab Manual of Exercise 1.
Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Handbook-Uncertainty Analysis.