Problem Set 1

Due: 21 May 2020, 12.30 p.m.

Problem 1. Consider a clock at rest at the origin of the laboratory frame. When observed from the reference frame of a rocket moving with respect to the laboratory at $v = 0.8c$, the clock ticks slow. When 10 s have elapsed on the rocket clock, how many have ticked by on the lab clock?

(*2 points*)

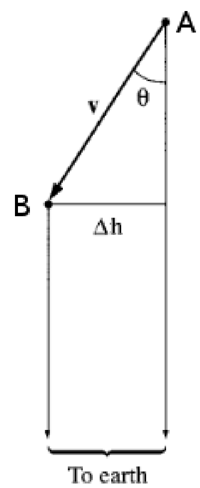
Problem 2. Two spaceships, each 100 m long when measured at rest, travel toward each other with speeds of $0.85c$ relative to Earth. (a) How long is each ship as measured by someone on Earth? (b) How fast is each ship traveling as measured by an observer on the other? (c) How long is one ship when measured by an observer on the other? (d) At time $t = 0$ on Earth, the fronts of the ships are together as they just begin to pass each other. At what time on Earth are their ends together? (e) Sketch accurately scaled diagrams in the frame of reference of one of the ships showing the passing of the other ship.

(*5 × 1 points*)

Problem 3. Two rockets A and B leave a space station with velocity vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ relative to the station frame S, perpendicular to each other. (a) Determine the velocity of A relative to B, $\mathbf{v}_{BA}$. (b) Determine the velocity of B relative to A, $\mathbf{v}_{AB}$. (c) Explain why $\mathbf{v}_{AB}$ and $\mathbf{v}_{BA}$ do not point in opposite directions.

(*2 + 2 + 2 points*)

Problem 4. Every two years, more or less, *The New York Times* publishes an article in which some astronomer claims to have found an object traveling faster than the speed of light. Many of these reports result from a failure to distinguish what is *seen* from what is *observed*—that is, from a failure to account for light travel time. Here is an example: A star is traveling with speed v at an angle θ to the line of sight (see figure above). (a) What is its apparent speed across the sky? (Suppose the light signal from B reaches Earth at a time Δt after the signal from A, and the star has meanwhile advanced a distance Δh across the sky; by apparent speed we mean $\Delta h/\Delta t$.) (b) What angle θ gives the maximum apparent speed? (c) Show that the apparent speed can be much greater than c, even if v itself is less than c.



(*3 + 3/2 + 3/2 points*)

Problem 5. Starting with *the relativistic velocity addition rule*, derive the corresponding transformation for the components of acceleration, *i.e.* express the components of $\mathbf{a}'$ in terms of components of $\mathbf{a}$ (and possibly $\mathbf{u}$).

Do you get the "Galilean" result in the limit $v \ll c$?

Hint. Follow the method we used in class to derive *the relativistic velocity addition rule.*

(3 + 1 points)

Problem 6. Is it possible to find a frame of reference where the birth of Confucius and the Battle of An happened (a) in the same place, (b) at the same instant of time. If so, define such frame of reference. (Assume that Earth is an inertial frame of reference and neglect its curvature.)

(3/2 + 3/2 points)

Problem 7. Four events are marked on the Minkowski diagram in the figure below, with both the x and the ct axes having the same units. Find and classify the space–time intervals for all possible pairs of events (make a table). For each pair, comment on the character of the causal relation between the events implied by the space–time interval.

(6 × 1 points)