Problem Set 5

Due: 26 June 2017, 12.30 p.m.

Problem 1. Recall that the amplitude of steady-state forced oscillations with a sinusoidal driving force $F_{\mathrm{dr}} = F_0 \cos \omega_{\mathrm{dr}} t$ in the presence of linear drag is given by the formula

$$A(\omega_{\mathrm{dr}}) = \frac{F_0}{m\sqrt{\left(\omega_0^2 - \omega_{\mathrm{dr}}^2\right)^2 + \left(\frac{b\omega_{\mathrm{dr}}}{m}\right)^2}}$$

(a) Check that the driving frequency for which the amplitude is maximum, *i.e.* the resonance frequency, is $\omega_{\mathrm{res}} = \sqrt{\omega_0^2 - b^2/2m^2}$.

(b) Suppose that the damping is small and the system is being driven at an angular frequency close to the natural angular frequency ω_0. Show that the amplitude can be then found as

$$A(\omega_{\mathrm{dr}}) \approx \frac{F_0}{2m\omega_0\sqrt{\left(\omega_0 - \omega_{\mathrm{dr}}\right)^2 + \frac{b^2}{4m^2}}}.$$

Plot both the exact and approximate curve $A(\omega_{\mathrm{dr}})$ on the same graph for a few sets of parameters (list the values). Use a computer, attach the graphs to your solution.

(2 + 3 points)

Problem 2. Consider a harmonic oscillator with linear drag, driven by an external force $F_0 \cos \omega_{\mathrm{dr}} t$.

(a) Suppose $b/m = 4\ \mathrm{s}^{-1}$. At what frequency should the force drive the system, to make the phase of steady-state oscillations lag by $\pi/4$ behind the driving force?

(b) For two different driving angular frequencies Ω_1 and Ω_2 the amplitudes of steady-state oscillations are equal. Find the natural frequency of this oscillator.

(3 + 2 points)

Problem 3. Solve Problem 2 (b) and (c) from Problem Set 3 in the non-inertial frame of reference of the moving car.

Please make sure that you have clearly distinguished real forces (due to interactions) and forces of inertia (kinematic corrections due to the fact that the frame of reference is non-inertial).

(1 + 2 points)

Problem 4. A small object with mass m is placed on the inner surface of a container with vertical cross section in the shape of the parabola $y = \frac{1}{2}\alpha x^2$, where $\alpha > 0$. The static coefficient of friction between the object and the pot's surface is μ_s. Find all points such that if we place the object there, it remains at rest.

Consider the following two cases

(a) The container is at rest.

(b) The container rotates with constant angular velocity ω about its axis of symmetry (the object gets in contact with the surface with no relative velocity).

In part (b) clearly identify the frame of reference you are solving the problem in.

(*2 + 4 points*)

Problem 5. Suppose that we have two observers: one at a pole and one on the equator. Assume that rotational motion of the earth was stopped over the time of 5 minutes, with constant angular acceleration (what is the direction of the angular acceleration vector?). Describe an effect that one observer would notice whereas the other one would not.

(*3 points*)

Problem 6. Two hunters were hunting at latitude of 49° on the northern hemisphere. One was shooting at a wolf to the west of him, the other one at a wolf to the south of him. Fortunately, none of them had succeeded and they were both blaming the Coriolis force. Could they use this excuse? Assume the average speed of the bullet $v_0 = 300$ m/s and the time of flight 1 s.

(*3 points*)