UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP241)

LABORATORY REPORT

EXERCISE 5
RC, RL, and RLC Circuits

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Date: March 27, 2018

1 Objectives

1. Understand the physics of alternating-current circuits, in particular the processes of charging and discharging of capacitors.
2. Understand the phenomenon of electromagnetic induction in inductive elements, and other dynamic processes in RC, RL, and RLC series circuits.
3. Study the methods for measuring the amplitude-frequency and the phase-frequency characteristics of RC, RL, and RLC series circuits.
4. Find the resonance frequency of a RLC circuit as well as the quality factor of the circuit from the amplitude-frequency curve.

2 Theoretical Background

Resistors, capacitors, and inductors are basic elements of electric circuits. Depending on a particular arrangement of these elements, RC, RL, RLC alternating-current (AC) circuits may display various features, including transient, steady state, and resonant behavior.

2.1 Transient Processes in RC, RL, RLC Series Circuits

2.1.1 RC Series Circuits

In a RC circuit, the process of charging or discharging of the capacitor is an example of a transient process.

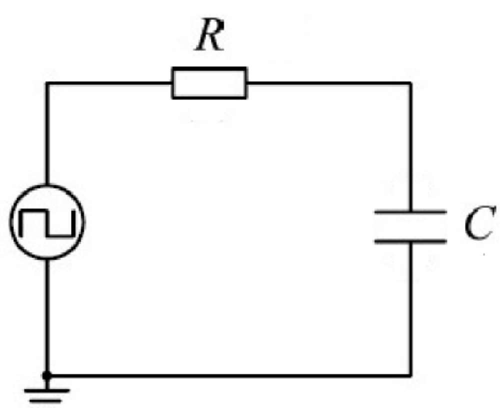

Figure 1: RC series circuit

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Figure 1 shows a RC series circuit in which a square-wave signal is used as the source signal. In the first half of the cycle, the square-wave voltage is $U_(t) = \varepsilon$ and it charges the capacitor. In the second half-cycle, the square-wave voltage is 0, and the capacitor discharge through the resistor. The loop equation for the charging process is

$$RC\frac{\mathrm{d}U_C}{\mathrm{d}t}+U_c=\varepsilon \tag{1}$$

With the initial condition $U_C(t=0)=0$, the solution of Eq.(1) can be found as

$$U_C=\varepsilon(1-e^{-\frac{t}{RC}})$$

$$U_R=iR=\varepsilon e^{-\frac{t}{RC}}$$

Hence the voltage across the capacitor U_C increases exponentially with time t, whereas the voltage on the resistor U_R decreases exponentially with time. The curves $U(t)$, $U_C(t)$, and $U_R(t)$ are shown in Figure 2

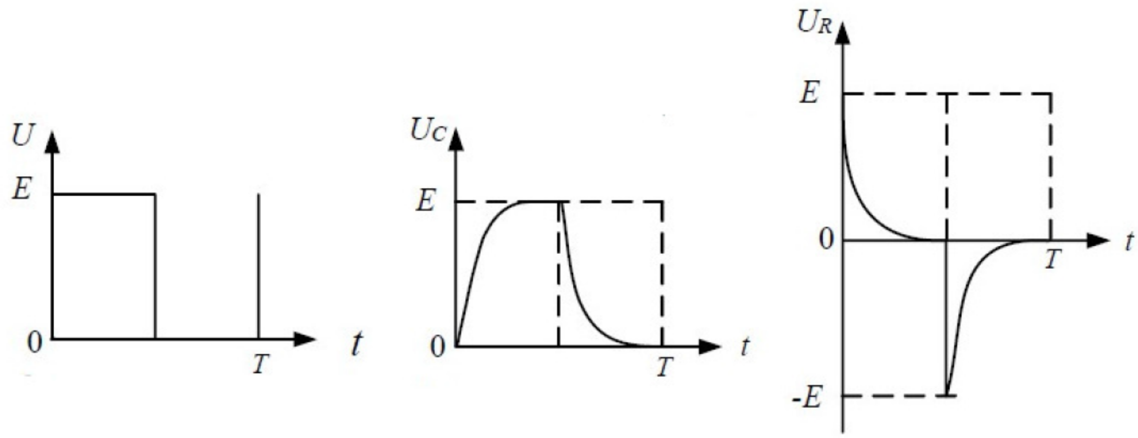

Figure 2: Charging and discharging curves for a RC series circuit.

For the discharging process, the loop rule gives

$$RC\frac{\mathrm{d}U_c}{\mathrm{d}t}+U_C=0 \tag{2}$$

The solution of Eq.(2), with the initial condition $U_C=(t=0)=\varepsilon$, is

$$U_C=\varepsilon e^{-\frac{t}{RC}}$$

and consequently,

$$U_R=iR=-\varepsilon e^{-\frac{t}{RC}}$$

where the magnitudes of both U_C and U_R decrease exponentially with time. Since $RC=\tau$ has the units of time, it is called the time constant of the circuit, and characterizes the dynamics of the transient process. There is another characteristics related to the time constant, easier to measure in experiments, which is called the half-life period $T_{\frac{1}{2}}$. The half-life period is the time needed for U_C to decrease to a half of the initial value (or increase to a half of the terminal value), and may be also used to characterize the dynamics of the transient process. Both quantities, in the process with exponential dynamics discussed above, are related by the equation

$$T_{\frac{1}{2}} = \tau \ln 2 \approx 0.693\tau$$

2.2 *RL* Series Circuit

A similar analysis can be carried out for a *RL* series circuit. In this case,

$$\tau - \frac{L}{R}, T_{\frac{1}{2}} = \frac{L}{R} \ln 2$$

2.3 *RLC* Series Circuit

First, let us discuss the situation when a power source is suddenly plugged into a *RLC* circuit. Then the voltage across the capacitor satisfies the differential equation

$$LC\frac{\mathrm{d}^2 U_C}{\mathrm{d}t^2} + RC\frac{\mathrm{d}U_C}{\mathrm{d}t} + U_C = \varepsilon \tag{3}$$

following again from the loop rule. Dividing both sides of the equation by LC and introducing the symbols

$$\beta = \frac{R}{2L}, \omega_0 = \frac{1}{\sqrt{LC}} \tag{4}$$

Eq.(3) can be written as

$$\frac{\mathrm{d}^2 U_C}{\mathrm{d}t^2} + 2\beta\frac{\mathrm{d}U_C}{\mathrm{d}t} + \omega_0^2 U_C = \omega_0^2 \varepsilon \tag{5}$$

Note that Eq. (5) is an inhomogeneous differential equation and it is mathematically equivalent to the equation of motion of a damped harmonic oscillator with a constant driving force. Therefore, the complementary homogeneous equation is fully analogous to the equation of motion of a damped harmonic oscillator, with β being the damping coefficient, and ω_0 the natural angular frequency. Moreover, after a specific solution to the inhomogeneous equation is found, a unique solution to the initial value problem consisting of Eq.(5) and the initial conditions

$$U_C = (t = 0) = 0, \frac{\mathrm{d}U_C}{\mathrm{d}t}(t = 0) = 0 \tag{6}$$

can be found.

Exactly as for mechanical oscillations, depending on the relation between β and ω_0, there are three regimes, as implied by the solution of the complementary homogeneous equation:

1. If $\beta^2 - \omega_0^2 < 0$ (weak damping), the system is in the underdamped regime and the solution to the initial value problem is of the form

$$U_C = \varepsilon - \varepsilon e^{-\beta t}(\cos\omega t + \frac{\beta}{\omega}\sin t)$$

where $\omega = \sqrt{\omega_0^2 - \beta^2}$

2. If $\beta^2 - \omega_0^2 < 0$ (strong damping), the system is in the overdamped regime and the solution to the initial value problem is of the form

$$U_C = \varepsilon - \frac{\varepsilon}{2\gamma}e^{-\beta t}[(\beta+\gamma)e^{\gamma t} - (\beta-\gamma)e^{-\gamma t}]$$

where $\omega = \sqrt{\beta^2 - \omega_0^2}$

3. f $\beta^2 - \omega_0^2 = 0$ the system is said to be critically damped, and

$$U_C = \varepsilon - \varepsilon(1+\beta t)e^{-\beta t} \tag{7}$$

When the circuit reaches a steady state, the power source is suddenly removed ($\varepsilon = 0$). The differential equation for the discharging process is similar to that of the charging process, and there are also three regimes of the process.

The above discussion is valid for an ideal circuit and a step-signal source with zero internal resistance. In the experiment, the ideal system is replaced by a square-wave source with a small internal resistance. The period of the square-signal must be much greater than the time constant of the circuit. Note that, according to the above equations, the voltage across the capacitor U_C will finally reach ε regardless of the regime (Figure 5, charging phase).

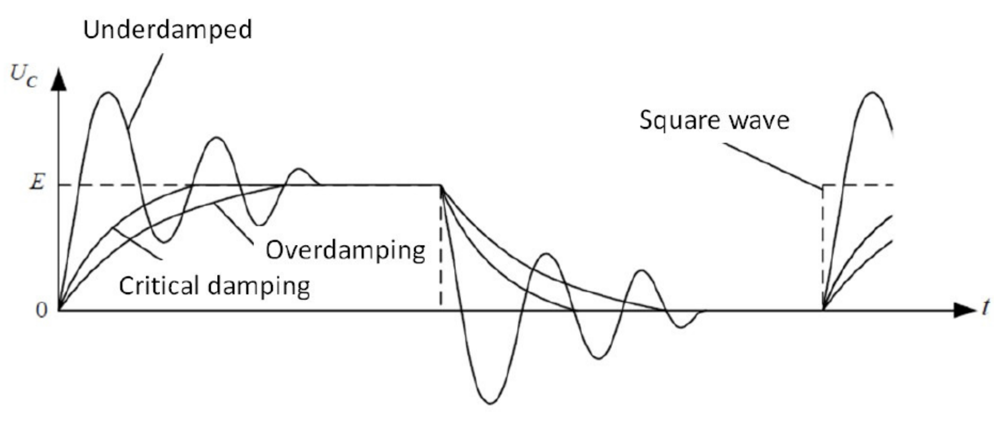

Figure 3: Three different regimes of transient processes in a RLC series circuit.

2.4 RC, RL Steady State Circuits

When a sinusoidal alternating input voltage is provided to a RC (or RL) series circuit, the amplitude and the phase of the voltage across the capacitor and the resistor will change with the frequency of the input voltage. Then the amplitude vs. frequency relation and the phase vs. frequency relation can be obtained by measuring the voltage across the elements in the circuit for different input signal frequencies

$$\varphi = \tan^{-1}(\frac{U_L}{U_R}) = \tan^{-1}(\frac{\omega L}{R}), \varphi = (-\frac{U_C}{U_R}) = \tan^{-1}(-\frac{1}{\omega RC})$$

2.5 RLC Resonant Circuit

2.5.1 RLC Series Circuit

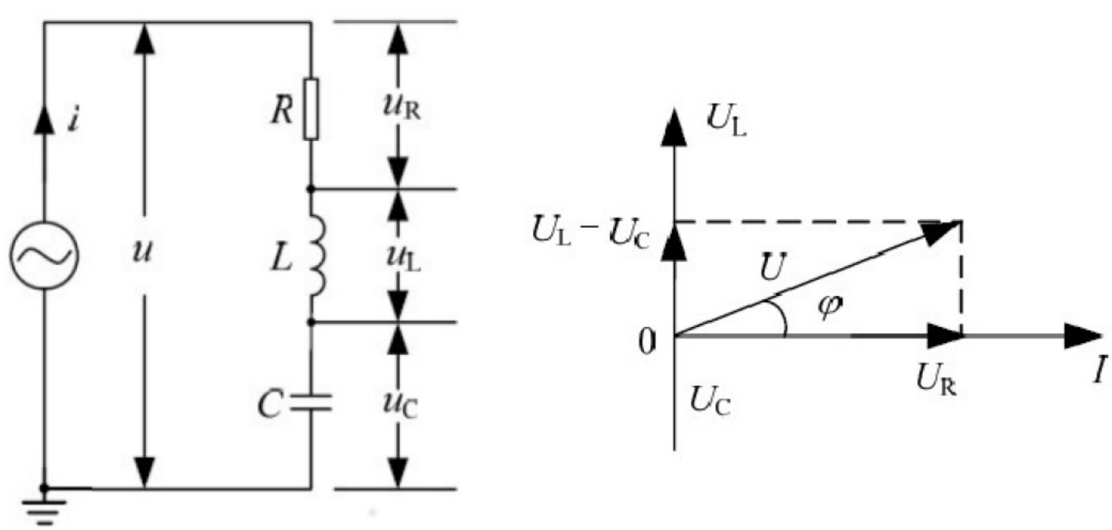

Figure 4: RLC series circuit.

A generic RLC series circuit is shown in Figure 4. The impedance and the phase difference in the RLC circuit can be calculated by using the phasers technique. Representing the current I by a vector along the horizontal axis, the phase differences between the current and the voltages across the resistor, coil, and capacitor are

$$\varphi_R = 0, \varphi_L = \frac{\pi}{2}, \varphi = -\frac{\pi}{2}$$

respectively. The corresponding voltage amplitudes across the elements are

$$U_R = IZ = IZ, U_Z = IZ_L = I\omega L, U_C = IZ_C = \frac{I}{\omega C}$$

Hence the voltage amplitude are

$$U = \sqrt{U_R^2 + (U_L - U_C)^2} \quad \text{or} \quad U = I\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \tag{8}$$

and the total impedance

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \tag{9}$$

with the phase difference between the current and the voltage in the circuit

$$\varphi = \tan^{-1}(\frac{U_L - U_C}{U_R}) = \tan^{-1}(\frac{\omega L - \frac{1}{\omega C}}{R})$$

2.6 Resonance

If the frequency of the input signal provided by the source satisfies the condition

$$\omega_0 L = \frac{1}{\omega_0 C}, \quad \text{or, equivalently,} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

the total impedance will reach a minimum, $Z_0 = R$. Note that the resistance R in a real circuit includes the internal resistance and all kinds of alternating-current power losses, so its actual value will be greater than the theoretical one. When the current reaches its maximum, $I_m = \frac{U}{R}$, the circuit is said to be at resonance. The frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

at which the resonance phenomenon occurs, is called the resonance frequency.

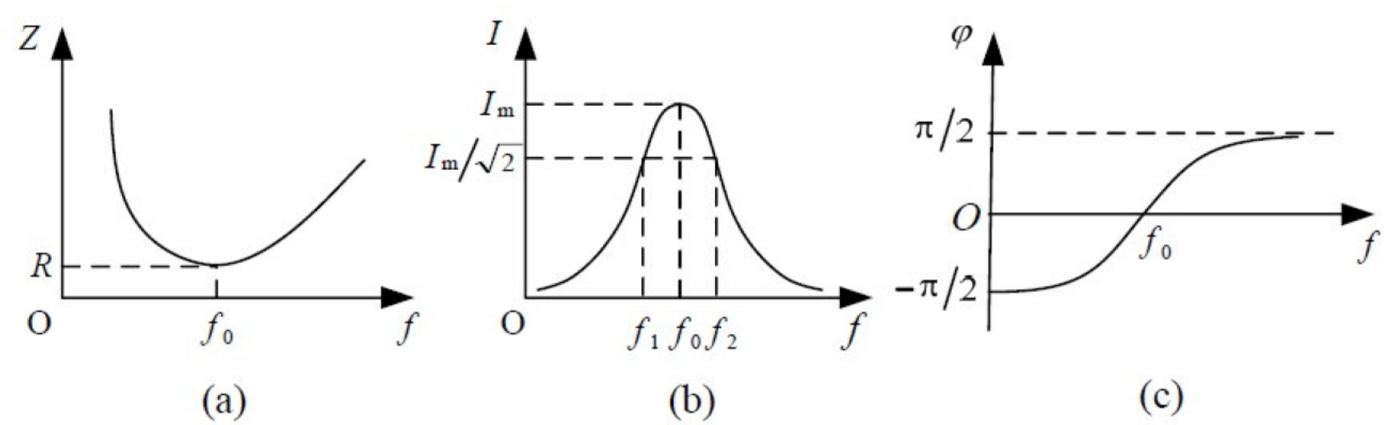

Figure 5: The impedance, the current and the phase difference as functions of the frequency for a *RLC* series circuit (generic sketches).

The total impedance Z, the current I, and the phase difference $\varphi = \varphi_u - \varphi_i$ all depend on the frequency, with generic shapes of the three curves shown in Figure 5. According to Eqs. (8) and (9), when the frequency is low ($f < f_0, \frac{1}{\omega C} > \omega L$), then $\varphi < 0$. In this situation the total voltage lags behind the current and the circuit is said to be capacitive. When the circuit is resonant ($f < f_0, \frac{1}{\omega C} = \omega L$), then $\varphi = 0$ and the voltages across the capacitor and the inductor should be equal. The circuit is said to be resistive.

2.7 Quality Factor in Resonant Circuits

Since $I_m = \frac{U}{R}$, the voltages across the resistor, the inductor, and the capacitor are

$$U_R = I_m R = U$$

$$U_L = I_M Z_L = \frac{U}{R}\omega L$$

$$U_C = I_M Z_C = \frac{U}{R}\frac{1}{\omega_0 C} = U_L$$

respectively. For a circuit driven at the resonance frequency, the ratio of U_L or U_C to U is called the quality factor Q of a resonant circuit

$$Q = \frac{U_L}{U} = \frac{\omega_0 L}{R} \quad \text{or} \quad Q = \frac{U_C}{U} = \frac{1}{\omega_0 RC}$$

When the total voltage is fixed, the greater Q is, the greater U_L and U_C are. The value of Q can be used to quantify the efficiency of resonant circuits.

The quality factor can be also found as

$$Q = \frac{f_0}{f_2 - f_1}$$

where f_1 and f_2 are two frequencies such that $I(f_1) = I(f_2) = \frac{I_m}{\sqrt{2}}$

3 Apparatus

The measurement setup consists of the following main elements: a signal generator, an oscilloscope, a digital multimeter, a wiring board, a fixed resistor $100\Omega(2W)$, a variable resistor $2k\Omega(2W)$, two capacitors $0.47\mu F$ and $0.1\mu F$, as well as two inductors ($10mH$ and $33mH$).

4 Measurement Procedure

4.1 RC, RL Series Circuit

1. I chose a capacitor and an inductor to assemble a circuit with the fixed-resistance 100Ω resistor, adjusted the output frequency of the square-wave signal provided by

the signal generator, and observed the change of the waveform when the time constant is smaller or greater than the period of the square wave. Then I chose the frequency that allows the capacitor to fully charge/discharge and use the PRINT function of the oscilloscope to store the waveforms.

2. I adjusted display parameters of the oscilloscope and measured $T_{\frac{1}{2}}$ for the studied circuits. In the report, I calculate the time constant and compare it with the theoretical value.

4.2 *RLC* Series Circuit

1. I chose a capacitor and an inductor to assemble a *RLC* series circuit with the variable resistor and observed the waveform of the capacitor voltage in the underdamped, critically damped, and overdamped regimes. I used the PRINT function of the oscilloscope to store the waveforms.

2. I adjusted the variable resistor to the critically damped regime. According to the definition of the half-life period $T_{\frac{1}{2}}$, we have $\beta T_{\frac{1}{2}} = 1.68$. By finding the value of $T_{\frac{1}{2}}$, the time constant can be found as $\tau = \frac{1}{\beta} = T_{\frac{1}{2}}/1,68$. In the report, my result with the theoretical value.

4.3 *RLC* Resonant Circuit

I applied a sinusoidal input voltage U_i to the *RLC* series circuit, changed the frequency, then observed the change of the voltage U_R for a fixed resistor R, as well as the phase difference between U_R and U_i. I measured how UR changes with U_i and calculated the phase difference according to Figure 4. The graphs $\frac{I}{I_m}$ vs. $\frac{f}{f_0}$ and φ vs. $\frac{f}{f_0}$ are plotted in the report. I also estimated the resonance frequency and calculate the quality factor Q.

5 Results and Calculations

5.1 *RC* Series Circuit

R	$98.55 \pm 0.01[\Omega]$
f	$4000 \pm 0.001[\mathrm{Hz}]$
ε	$4.00 \pm 0.001[\mathrm{V}]$
C	$100.52 \pm 0.01[\mathrm{nF}]$
$T_{1/2}$	$7.000 \pm 0.001[\mu s]$

Table 1: $T_{1/2}$ measurement data for a *RC* series circuit

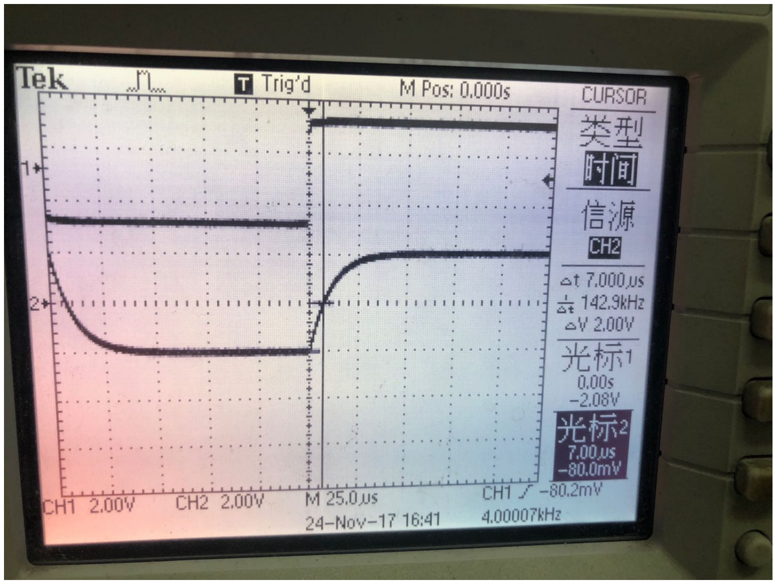

Figure 6: Figure of RC series circuit's waveform

The theoretical value of time constant:

$$\tau_{th} = RC = 98.55 \cdot 100.52 = 9.906 \pm 1.41 \cdot 10^{-3}[\mu s]$$

The experimental value of time constant:

$$\tau_{ex} = \frac{T_{1/2}}{\ln 2} = 10.10 \pm 1.44 \cdot 10^{-3}[\mu s]$$

5.2 RL Series Circuit

R	$98.55 \pm 0.01[\Omega]$
f	$500 \pm 0.001[\text{Hz}]$
ε	$4.08 \pm 0.001[\text{V}]$
L	$0.01 \pm 0[[\text{H}]$
$T_{1/2}$	$64.00 \pm 0.001[\mu s]$

Table 2: $T_{1/2}$ measurement data for a RL series circuit

The theoretical value of time constant:

$$\tau_{th} = \frac{L}{R} = \frac{0.01}{98.55} = 101.5 \pm 0.01[\mu s]$$

The experimental value of time constant:

$$\tau_{ex} = \frac{T_{1/2}}{\ln 2} = 92.33[\mu s]$$

5.3 *RLC* Series Circuit

L	$0.01 \pm 0[[\mathrm{H}]$
C	$100.52 \pm 0.01[\mathrm{nF}$
ε	$4.48 \pm 0.001[\mathrm{V}]$
f	$200 \pm 0.001[\mathrm{Hz}]$
βt	1.68
$T_{1/2}$	$48.00 \pm 0.001[\mu s]$

Table 3: $T_{1/2}$ measurement data for a critically damped RLC series circuit

The experimental value of time constant:

$$\tau_{ex} = \frac{T_{1/2}}{\ln 2} = 69.25 \pm 1.44 \cdot 10^{-3}[\mu s]$$

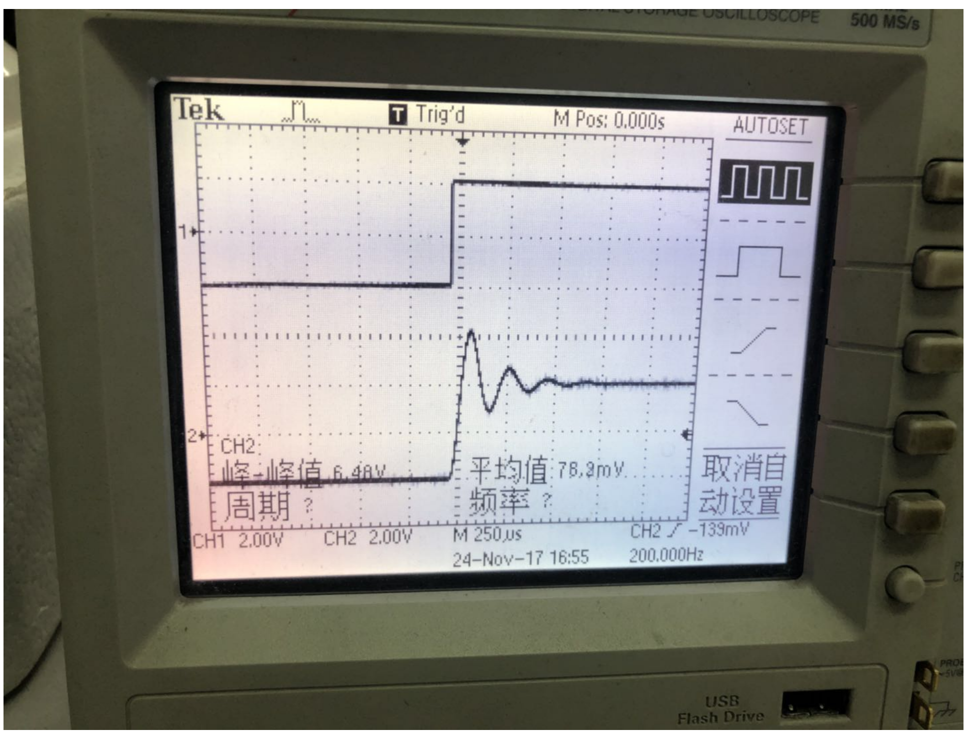

Figure 7: Figure for underdamped regime waveform.

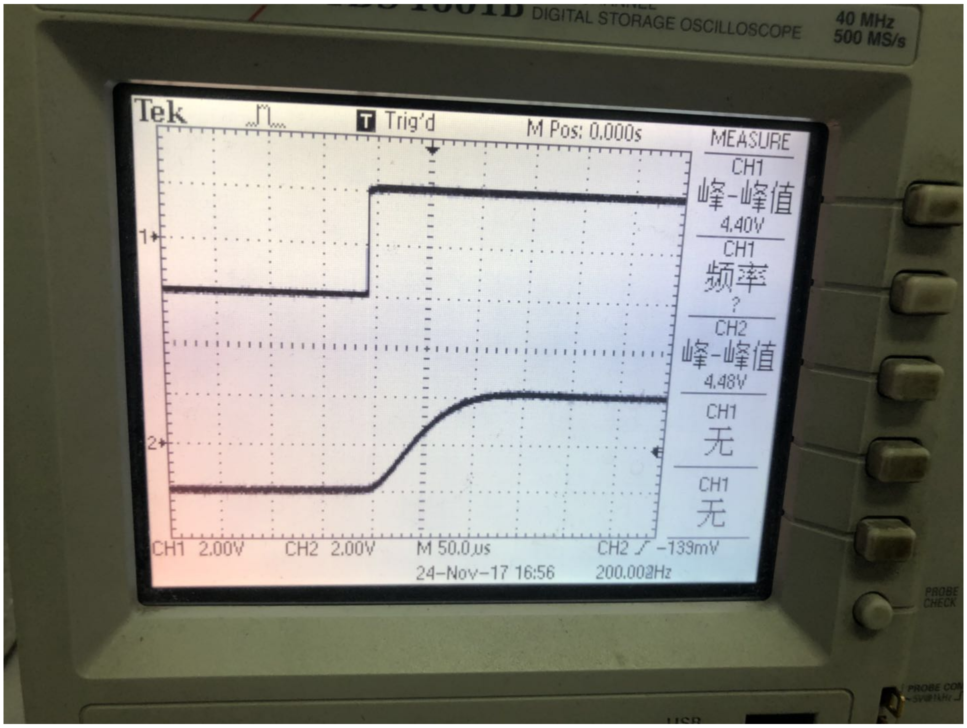

Figure 8: Figure for critically damped regime waveform.

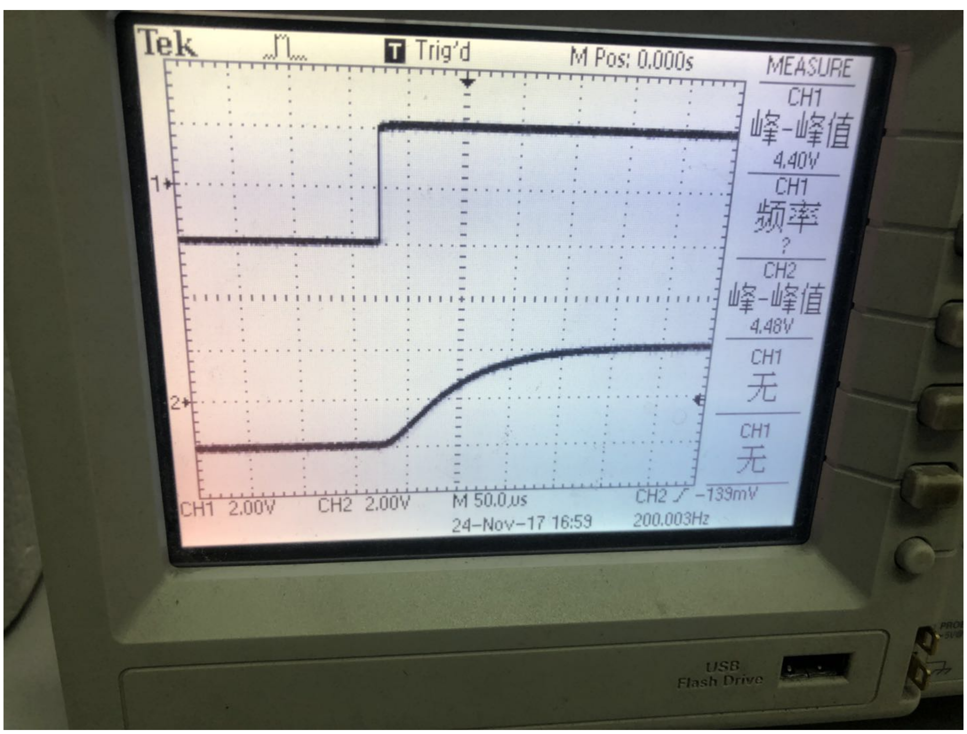

Figure 9: Figure for overdamped regime waveform.

5.4 RLC Resonant Circuit

R	$98.55 \pm 0.01[\Omega]$	
L	$0.01 \pm 0[[\mathrm{H}]$	
C	$100.52 \pm 0.01[\mathrm{nF}$	
f_0	$5000 \pm 0.001[\mathrm{Hz}]$	
ε	$4.24 \pm 0.001[\mathrm{V}]$	
	$U_R[V] \pm 0.001[V]$	$f[\mathrm{Hz}] \pm 0.001[\mathrm{Hz}]]$
1	0.56	1000
2	0.64	1500
3	0.80	2000
4	1.04	2500
5	1.36	3000
6	1.76	3500
7	2.48	4000
8	2.88	4250
9	3.44	4500
10	3.84	4750
11	3.92	5000
12	3.76	5250
13	3.44	5500
14	3.12	5750
15	2.72	6000
16	2.16	6500
17	1.84	7000
18	1.60	7500
19	1.36	8000
20	1.28	8500
21	1.20	9000

Table 4: Measurement data for the U_R vs. f dependence for a RLC resonant circuit

$$I_m = \frac{U_m}{R} = \frac{3.92}{98.55} = 39.8 \pm 0.398[mA]$$

$$I_1 = \frac{U_1}{R} = \frac{0.56}{98.55} = 5.68 \pm 5.77 \cdot 10^{-2}[mA]$$

$$\frac{I_1}{I_m} = \frac{5.68}{39.8} = 0.143$$

$$\frac{f_1}{f_0} = \frac{1000}{5000} = 0.2$$

	$U_R[V] \pm 0.001[V]$	f[Hz] $\pm$ 0.001[Hz]]	I/I_m	f/f_0
1	0.56	1000	0.143	0.20
2	0.64	1500	0.163	0.30
3	0.80	2000	0.204	0.40
4	1.04	2500	0.265	0.50
5	1.36	3000	0.347	0.60
6	1.76	3500	0.449	0.70
7	2.48	4000	0.633	0.80
8	2.88	4250	0.735	0.85
9	3.44	4500	0.877	0.90
10	3.84	4750	0.980	0.95
11	3.92	5000	1.000	1.00
12	3.76	5250	0.959	1.05
13	3.44	5500	0.877	1.10
14	3.12	5750	0.796	1.15
15	2.72	6000	0.694	1.20
16	2.16	6500	0.551	1.30
17	1.84	7000	0.469	1.40
18	1.60	7500	0.408	1.50
19	1.36	8000	0.347	1.60
20	1.28	8500	0.327	1.70
21	1.20	9000	0.306	1.80

Table 5: Calculated data for the U_R vs. f for a RLC resonant circuit

5.4.1 Calculation of Q

So the f_1 and f_2 according to the table is 4250 and 6000$\pm$ respectively. So the experimental value for quality factor is:

$$Q_{ex} = \frac{f_0}{f_2 - f_1} = \frac{5000}{6000 - 4250} = 2.857 \pm 2.38 \cdot 10^{-6}$$

The theoretical value of quality factor is

$$Q_{th} = \frac{\sqrt{LC}}{RC} = \frac{\sqrt{0.01 \cdot 100.52}}{98.55 \cdot 100.52} = 3.2 \pm 1.03 \cdot 10^{-8}$$

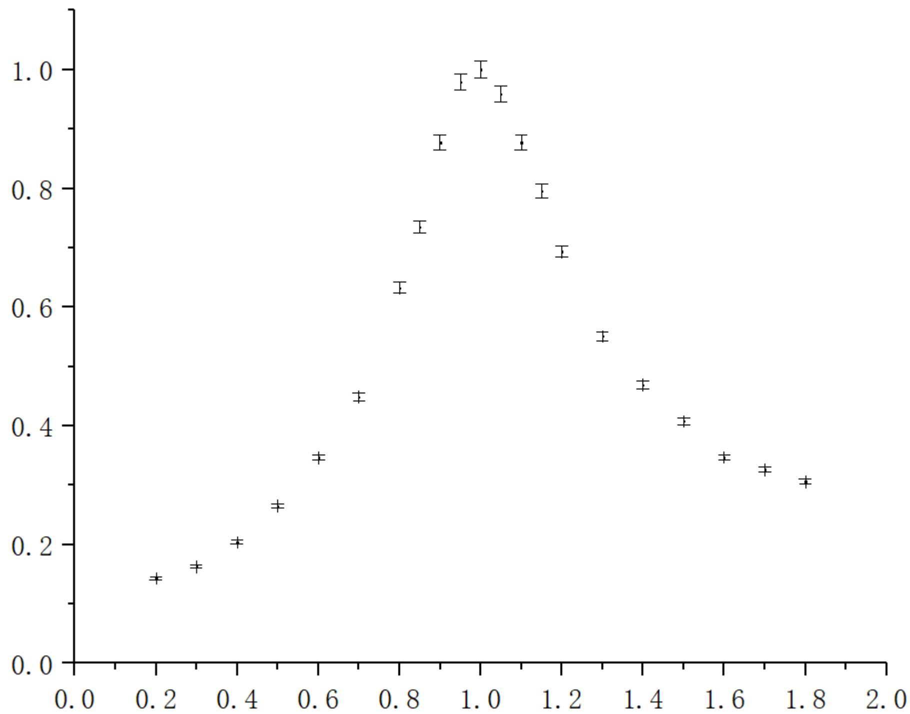

Figure 10: Plot for I/I_m vs. f/f_0

5.5 Calculation of ϕ

For the theoretical value of ϕ,

$$\phi_{th} = \arctan \frac{2\pi f L - \frac{1}{2\pi f C}}{R} = \arctan \frac{2\pi \cdot 1000 \cdot 0.01 - \frac{1}{2\pi \cdot 1000 \cdot 100.52 \cdot 10^{-9}}}{98.55} = -1.506[\text{rad}]$$

	$U_R[V] \pm 0.001[V]$	f[Hz] $\pm$ 0.001[Hz]]	ϕ_{th}	f/f_0
1	0.56	1000	-1.506	0.20
2	0.64	1500	-1.469	0.30
3	0.80	2000	-1.424	0.40
4	1.04	2500	-1.367	0.50
5	1.36	3000	-1.288	0.60
6	1.76	3500	-1.170	0.70
7	2.48	4000	-0.973	0.80
8	2.88	4250	-0.820	0.85
9	3.44	4500	-0.613	0.90
10	3.84	4750	-0.342	0.95
11	3.92	5000	-0.027	1.00
12	3.76	5250	0.277	1.05
13	3.44	5500	0.528	1.10
14	3.12	5750	0.716	1.15
15	2.72	6000	0.853	1.20
16	2.16	6500	1.031	1.30
17	1.84	7000	1.138	1.40
18	1.60	7500	1.208	1.50
19	1.36	8000	1.258	1.60
20	1.28	8500	1.294	1.70
21	1.20	9000	1.323	1.80

Table 6: Calculated data for the ϕ_{th} vs. f for a RLC resonant circuit

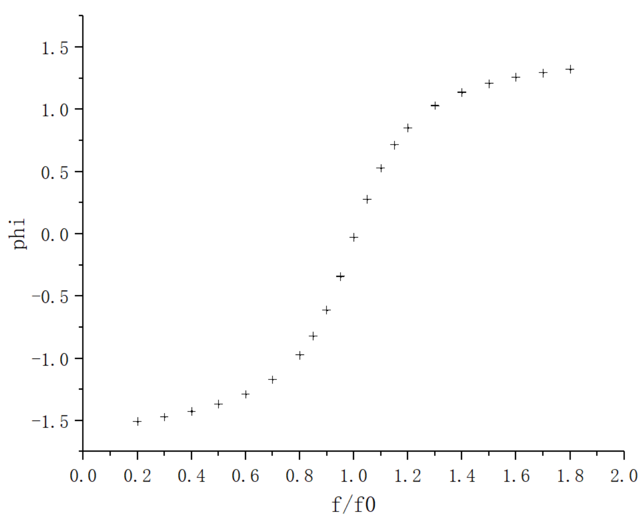

Figure 11: Plot for ϕ_{th} vs. f/f_0

For the experimental value of ϕ,

$$\phi_{ex} = \arccos \frac{U_R}{U_m} = -\arccos \frac{0.56}{3.92} = -1.427[\text{rad}]$$

	$U_R[V] \pm 0.001[V]$	$f[\text{Hz}] \pm 0.001[\text{Hz}]]$	ϕ_{ex}	f/f_0
1	0.56	1000	-1.427	0.20
2	0.64	1500	-1.407	0.30
3	0.80	2000	-1.365	0.40
4	1.04	2500	-1.302	0.50
5	1.36	3000	-1.217	0.60
6	1.76	3500	-1.105	0.70
7	2.48	4000	-0.885	0.80
8	2.88	4250	-0.746	0.85
9	3.44	4500	-0.500	0.90
10	3.84	4750	-0.203	0.95
11	3.92	5000	0.000	1.00
12	3.76	5250	0.287	1.05
13	3.44	5500	0.500	1.10
14	3.12	5750	0.650	1.15
15	2.72	6000	0.804	1.20
16	2.16	6500	0.987	1.30
17	1.84	7000	1.082	1.40
18	1.60	7500	1.150	1.50
19	1.36	8000	1.217	1.60
20	1.28	8500	1.238	1.70
21	1.20	9000	1.260	1.80

Table 7: Calculated data for the ϕ_{ex} vs. f for a RLC resonant circuit

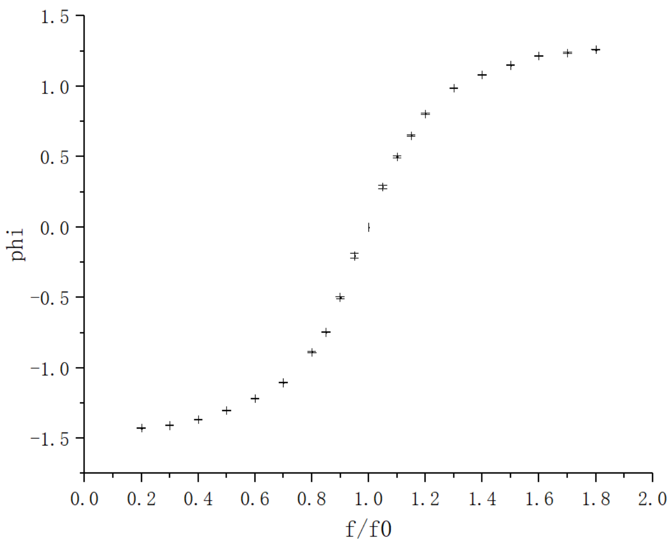

Figure 12: Plot for ϕ_{ex} vs. f/f_0

6 Uncertainty Analysis

6.1 RC Series Circuit

For uncertainty of τ_{th}

$$u = \sqrt{(R \cdot u_C)^2 + (c \cdot u_R)^2} = \sqrt{(98.55 \cdot 0.01)^2 + (100.52 \cdot 0.01)} = 1.41 \cdot 10^{-3}[\mu s]$$

The relative uncertainty:

$$u_r = \frac{u}{\tau_{th}} \cdot 100\% = \frac{1.41 \cdot 10^{-3}}{9.906} \cdot 100\% = 0.01\%$$

For uncertainty of τ_{ex}

$$u = u_T \cdot \frac{1}{\ln 2} = \frac{0.001}{\ln 2} = 1.44 \cdot 10^{-3}[\mu s]$$

The relative uncertainty:

$$u_r = \frac{u}{\tau_{th}} \cdot 100\% = \frac{1.44 \cdot 10^{-3}}{10.10} \cdot 100\% = 0.01\%$$

6.2 RL Series Circuit

For uncertainty of τ_{th}

$$u = \frac{u_R L}{R^2} = \frac{0.01 \cdot 0.01}{98.55^2} = 0.01[\mu s]$$

The relative uncertainty:

$$u_r = \frac{u}{\tau_{th}} \cdot 100\% = \frac{0.01}{101.5} \cdot 100\% = 0.01\%$$

For uncertainty of τ_{ex}

$$u = u_T \cdot \frac{1}{\ln 2} = \frac{0.001}{\ln 2} = 1.44 \cdot 10^{-3} [\mu s]$$

The relative uncertainty:

$$u_r = \frac{u}{\tau_{th}} \cdot 100\% = \frac{1.44 \cdot 10^{-3}}{92.33} \cdot 100\% = 0.001\%$$

6.3 RLC Series Circuit

For uncertainty of τ_{ex}

$$u = u_T \cdot \frac{1}{\ln 2} = \frac{0.001}{\ln 2} = 1.44 \cdot 10^{-3} [\mu s]$$

The relative uncertainty:

$$u_r = \frac{u}{\tau_{th}} \cdot 100\% = \frac{1.44 \cdot 10^{-3}}{69.25} \cdot 100\% = 0.002\%$$

For uncertainty of I_m

$$u = \sqrt{(\frac{u_U}{R})^2 + (\frac{U u_R}{R^2})^2} = \sqrt{(\frac{0.001}{98.55})^2 + (\frac{0.56 \cdot 0.01}{98.55^2})^2} = 5.77 \cdot 10^{-2} [mA]$$

For uncertainty of $\frac{I}{I_m}$

$$u = \sqrt{(\frac{u_I}{I_m})^2 + (\frac{u_m I}{I_m^2})^2} = \sqrt{(\frac{5.77 \cdot 10^{-2}}{39.8})^2 + (\frac{0.398 \cdot 5.68}{39.8^2})^2} = 2 \cdot 10^{-3}$$

The relative uncertainty

$$u_r = \frac{u}{I/I_m} \cdot 100\% = \frac{0.002}{0.1429} \cdot 100\% = 1.42\%$$

For uncertainty of $\frac{f}{f_0}$

$$u = \sqrt{(\frac{u_f}{f_0})^2 + (\frac{f u_f}{f_0^2})^2} = \sqrt{(\frac{0.001}{5000})^2 + (\frac{1000 \cdot 0.001}{5000^2})^2} = 2 \cdot 10^{-7}$$

The relative is approaching 0.

	I/I_m	Uncertainty	Relative Uncertainty [%]	f/f_0	Uncertainty
1	0.143	0.002	1.42	0.20	$2.04 \cdot 10^{-7}$
2	0.163	0.002	1.42	0.30	$2.09 \cdot 10^{-7}$
3	0.204	0.003	1.42	0.40	$2.15 \cdot 10^{-7}$
4	0.265	0.004	1.42	0.50	$2.24 \cdot 10^{-7}$
5	0.347	0.005	1.42	0.60	$2.33 \cdot 10^{-7}$
6	0.449	0.006	1.42	0.70	$2.44 \cdot 10^{-7}$
7	0.633	0.009	1.41	0.80	$2.56 \cdot 10^{-7}$
8	0.735	0.010	1.41	0.85	$2.62 \cdot 10^{-7}$
9	0.877	0.012	1.41	0.90	$2.69 \cdot 10^{-7}$
10	0.980	0.014	1.41	0.95	$2.75 \cdot 10^{-7}$
11	1.000	0.014	1.41	1.00	$2.83 \cdot 10^{-7}$
12	0.959	0.014	1.41	1.05	$2.90 \cdot 10^{-7}$
13	0.877	0.012	1.41	1.10	$2.97 \cdot 10^{-7}$
14	0.796	0.011	1.41	1.15	$3.04 \cdot 10^{-7}$
15	0.694	0.010	1.41	1.20	$3.12 \cdot 10^{-7}$
16	0.551	0.008	1.41	1.30	$3.28 \cdot 10^{-7}$
17	0.469	0.007	1.42	1.40	$3.44 \cdot 10^{-7}$
18	0.408	0.006	1.42	1.50	$3.61 \cdot 10^{-7}$
19	0.347	0.005	1.42	1.60	$3.77 \cdot 10^{-7}$
20	0.327	0.005	1.42	1.70	$3.94 \cdot 10^{-7}$
21	0.306	0.004	1.42	1.80	$4.12 \cdot 10^{-7}$

Table 8: Uncertainty data for the U_R vs. f for a RLC resonant circuit

6.3.1 Uncertainty of Q

Uncertainty of Q_{ex}

$$u = u_f\sqrt{(\frac{1}{f_2 - f_1})^2 + 2(\frac{f_0}{f_2 - f_1})^2}$$

$$u = 0.001\sqrt{(\frac{1}{6000 - 4250})^2 + 2 \cdot (\frac{5000}{(6000 - 4250)^2})^2} = 2.38 \cdot 10^{-6}$$

The relative uncertainty is close to 0[%]
Uncertainty if Q_{th}

$$u = \sqrt{(\sqrt{\frac{L}{C}}\frac{u_R}{R^2})^2 + (\frac{C^{-3/2}u_C\sqrt{L}}{2R})^2} = 1.03 \cdot 10^{-8}$$

The relative uncertainty is also very close to 0[%]

6.4 Uncertainty of ϕ

Uncertainty of ϕ_{th}

$$u_\phi = \frac{1}{1+(\frac{2\pi fL-\frac{1}{2\pi fC}}{R})^2}\sqrt{(\frac{u_C}{2\pi RfC^2})^2+[(2\pi fL-\frac{1}{2\pi fC}\frac{u_R}{R^2})]^2+[(\frac{2\pi L}{R}+\frac{1}{2\pi CRf^2})\cdot u_f]^2}$$

u_ϕ

	ϕ_{ex}	Uncertainty	Relative Uncertainty [%]	f/f_0	Uncertainty
1	-1.506	$1.832\cdot10^{-5}$	0.001	0.20	$2.04\cdot10^{-7}$
2	-1.468	$3.078\cdot10^{-5}$	0.002	0.30	$2.09\cdot10^{-7}$
3	-1.424	$4.853\cdot10^{-5}$	0.003	0.40	$2.15\cdot10^{-7}$
4	-1.367	$7.628\cdot10^{-5}$	0.01	0.50	$2.24\cdot10^{-7}$
5	-1.288	$1.241\cdot10^{-4}$	0.01	0.60	$2.33\cdot10^{-7}$
6	-1.170	$2.157\cdot10^{-4}$	0.02	0.70	$2.44\cdot10^{-7}$
7	-0.972	$4.092\cdot10^{-4}$	0.04	0.80	$2.56\cdot10^{-7}$
8	-0.820	$5.775\cdot10^{-4}$	0.07	0.85	$2.62\cdot10^{-7}$
9	-0.613	$8.026\cdot10^{-4}$	0.13	0.90	$2.69\cdot10^{-7}$
10	-0.341	$1.03\cdot10^{-3}$	0.30	0.95	$2.75\cdot10^{-7}$
11	-0.027	$1.13\cdot10^{-3}$	4.18	1.00	$2.83\cdot10^{-7}$
12	0.278	$1.02\cdot10^{-3}$	0.37	1.05	$2.90\cdot10^{-7}$
13	0.528	$8.078\cdot10^{-4}$	0.15	1.10	$2.97\cdot10^{-7}$
14	0.716	$6.056\cdot10^{-4}$	0.08	1.15	$3.04\cdot10^{-7}$
15	0.853	$4.531\cdot10^{-4}$	0.05	1.20	$3.12\cdot10^{-7}$
16	1.031	$2.700\cdot10^{-4}$	0.03	1.30	$3.28\cdot10^{-7}$
17	1.138	$1.770\cdot10^{-4}$	0.02	1.40	$3.44\cdot10^{-7}$
18	1.208	$1.254\cdot10^{-4}$	0.01	1.50	$3.61\cdot10^{-7}$
19	1.258	$9.422\cdot10^{-5}$	0.01	1.60	$3.77\cdot10^{-7}$
20	1.294	$7.400\cdot10^{-5}$	0.01	1.70	$3.94\cdot10^{-7}$
21	1.323	$6.014\cdot10^{-5}$	0.004	1.80	$4.12\cdot10^{-7}$

Table 9: Uncertainty data for the ϕ_{th} vs. f for a RLC resonant circuit

Uncertainty of ϕ_{ex}

$$u_\phi = \sqrt{(\frac{u_R}{U_m\sqrt{1-(\frac{U_R}{U_m})^2}})^2+(\frac{u_mU_R}{U_m^2\sqrt{1-(\frac{U_R}{U_m})^2}})^2} = \frac{1}{\sqrt{U_m^2-U_R^2}}\sqrt{(u_R)^2+(\frac{u_mU_R}{U_m})^2}$$

$$u = \frac{1}{\sqrt{3.92^2-0.56^2}}\sqrt{0.01^2+(\frac{0.01\cdot0.56}{3.92})^2} = 2.6\cdot10^{-3}[\text{rad}]$$

$$u_\phi = \frac{u}{\phi_{ex}} \cdot 100\% = \frac{2.6 \cdot 10^{-3}}{0.143} \cdot 100\% = 0.18[\%]$$

	ϕ_{th}	Uncertainty	Relative Uncertainty [%]	f/f_0	Uncertainty
1	-1.427	0.0026	0.18	0.20	$2.04 \cdot 10^{-7}$
2	-1.406	0.0026	0.19	0.30	$2.09 \cdot 10^{-7}$
3	-1.365	0.0027	0.19	0.40	$2.15 \cdot 10^{-7}$
4	-1.302	0.0027	0.21	0.50	$2.24 \cdot 10^{-7}$
5	-1.217	0.0029	0.24	0.60	$2.33 \cdot 10^{-7}$
6	-1.105	0.0031	0.28	0.70	$2.44 \cdot 10^{-7}$
7	-0.886	0.0039	0.44	0.80	$2.56 \cdot 10^{-7}$
8	-0.746	0.0047	0.63	0.85	$2.62 \cdot 10^{-7}$
9	-0.500	0.0071	1.42	0.90	$2.69 \cdot 10^{-7}$
10	-0.203	0.0178	8.76	0.95	$2.75 \cdot 10^{-7}$
11	0.000	0.0000	0.00	1.00	$2.83 \cdot 10^{-7}$
12	0.287	0.0012	4.36	1.05	$2.90 \cdot 10^{-7}$
13	0.500	0.0071	1.42	1.10	$2.97 \cdot 10^{-7}$
14	0.650	0.0054	0.83	1.15	$3.04 \cdot 10^{-7}$
15	0.804	0.0043	0.54	1.20	$3.12 \cdot 10^{-7}$
16	0.987	0.0035	0.35	1.30	$3.28 \cdot 10^{-7}$
17	1.082	0.0032	0.29	1.40	$3.44 \cdot 10^{-7}$
18	1.150	0.0030	0.26	1.50	$3.61 \cdot 10^{-7}$
19	1.217	0.0028	0.24	1.60	$3.77 \cdot 10^{-7}$
20	1.238	0.0028	0.23	1.70	$3.94 \cdot 10^{-7}$
21	1.260	0.0028	0.22	1.80	$4.12 \cdot 10^{-7}$

Table 10: Uncertainty data for the ϕ_{ex} vs. f for a RLC resonant circuit

7 Conclusion and Discussion

In this experiment, I examined the RC, RL, RLC circuits and get familiar with the usage of oscilloscope. For the RC circuit, I studied the process of charging and discharging process of capacitors. I took a photo of the waveform from the oscilloscope and got two time constant theoretically and experimentally:

$$\tau_{th} = 9.906 \pm 1.41 \cdot 10^{-3}[\mu s], \tau_{ex} = 10.10 \pm 1.44 \cdot 10^{-3}[\mu s]$$

The experimental value is a bit bigger than the theoretical one, and their uncertainty is very small. For the RL circuit, I paid much attention to the phenomenon of electromagnetic induction in inductive elements and also got two time constant theoretically and experimentally:

$$\tau_{th} = 101.5 \pm 0.01[\mu s], tau_{ex} = 92.33[\mu s]$$

In contrast to RC circuit, the experimental value of time constant is smaller than theoretical one. They also have very small uncertainty.

Then I studied RLC circuit, which includes more dynamics process. First, I changed the frequency and took three photos when the circuit is underdamped, critically damped, and overdamped respectively. Second, I recorded the frequency and the corresponding voltage cross the resistor so I can calculate the time constant and quality factor.

$$\tau = 69.25 \pm 1.44 \cdot 10^{-3}[\mu s]$$

$$Q_{ex} = 2.857 \pm 2.38 \cdot 10^{-6}, Q_{th} = 3.2 \pm 1.03 \cdot 10^{-8}$$

The experiment value is smaller than the theoretical value and they both have small uncertainty.

Third, I plot the figure I/I_m vs. f/f_0 and I found I/I_m will first become bigger as f approaching f_0 and will become smaller if f continues increasing. In addition, the plot is roughly symmetric about $f = f_0$. However, it's a little different from the theoretical plot, I think it's because the dots are not enough. If I measured more value, they'll be more similar. Also, my dots are not uniformly distributed, which may be a reason.

In the end, I calculated ϕ experimentally and theoretically and plot the figure ϕ vs. f/f_0 in with two kinds of ϕ. The two figures are very similar to the theoretical figure. ϕ will grow bigger as f increases. For the further improvement, I think if we're asked to measure more times, we'll get more accurate plots, but it'll also cost more time.

8 Data Sheet

Data sheet is attach to the report

9 Reference

Krzyzosiak,M. Lab Manual of Exercise 5.

Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Handbook-Uncertainty Analysis.