



Review Questions

(week of 15 June 2020)

- 1. Why is the harmonic oscillator model important in physics?
- 2. The harmonic approximation works well if the amplitude of oscillations about a stable equilibrium position is
- 3. What plays the role of a "spring constant" in the harmonic approximation?
- 4. Name a phenomenon in solid state (or molecular) physics where the quantum harmonic oscillator model may be useful.
- 5. What are the main features of solutions to the stationary Schrödinger for the quantum harmonic oscillator.
- 6. True of false? Energy levels in the harmonic potential well are equally-spaced.
- 7. What are zero-point oscillations?
- 8. On the graph V = V(x) mark the energy levels (enough to mark the first 3 levels) of a quantum oscillator. Sketch the corresponding wave functions.
- 9. What does it mean that the ground state of a quantum harmonic oscillator saturates the Heisenberg uncertainty principle? Do excited states saturate it, too?
- 10. The energy levels of the harmonic oscillator are $E_n = \ldots,$ where $n = \ldots, \ldots$
- 11. The wave functions of a particle in the harmonic well are of the form $\sim e^{-\xi^2}$, where $H_n(\xi)$ are polynomials.
- 12. ψ_1 and ψ_2 are solutions to the Schrödinger equation for a harmonic oscillator, corresponding to the energy levels E_1 and E_2 , respectively. Are ψ_1 and ψ_2 orthogonal? Explain why (without any calculations!).
- 13. Explain how oscillations of the N atom between two geometrical arrangements in a NH₃ molecule can be explained using the double-well model.
- 14. If a double well is formed of two identical rectangular finite-depth wells close to each other, and initially a particle is located in the left well, it will eventually "tunnel" to the right one. The tunnelling time increases/decreases with the separation of the wells.

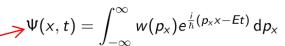
Hint. If the wells are far away, the two lowest energy levels are very close to each other, so $|E_2 - E_1|$ is very small. Does it then take a long or a short time for $e^{-\frac{i}{\hbar}(E_2 - E_1)t}$ to change significantly?

Or you may play around with this applet to get the right answer:

https://phet.colorado.edu/en/simulation/covalent-bonds

- 15. In the double well described above, is the state representing a particle localized in the left well an eigenstate of the Hamiltonian? Is it a stationary state?
- 16. True or false? the eigenvalues of the momentum operator $\hat{p}_x = -i\hbar\partial/\partial x$ are all real numbers.
- 17. True or false? The eigenfunction of the momentum operator $\hat{p}_x = -i\hbar\partial/\partial x$ corresponding to the eigenvalue p is $\psi_p(x) = Ce^{\frac{i}{\hbar}px}$.
- 18. How are eigenfunctions of the momentum operator normalized?
- 19. What is a classical wave packet?

$$\langle \phi_{p_{\mathsf{x}}}, \phi_{p_{\mathsf{x}'}} \rangle = \frac{1}{2\pi\hbar} \int\limits_{-\infty}^{\infty} \mathrm{e}^{\frac{i}{\hbar}(p_{\mathsf{x}}' - p_{\mathsf{x}})\mathsf{x}} \, \mathsf{d}\mathsf{x} = \delta(p_{\mathsf{x}}' - p_{\mathsf{x}}).$$



- 20. How do we construct a classical wave packet?
- 21. What is the phase velocity/group velocity?



- 22. The wave function of a free particle with momentum p_x (and energy $E = p_x^2/2m$) travelling in the positive x axis direction is $\Psi(x,t) = \dots$
- 23. Is the wave function describing a free particle with definite momentum square-integrable?
- 24. For a particle with definite momentum, we have $\Delta_{p_x} = 0$. Using the Heisenberg uncertainty principle, argue what should be the uncertainty of this particle's position. Explain how the form of the free-particle wave function (for example for a particle moving in the positive x axis direction), is compatible with the Heisenberg uncertainty principle.
- 25. Why is the function $\Psi(x,t) = Ce^{\frac{i}{\hbar}(p_x x Et)}$, where $E = p_x^2/2m$ not suitable for representing a localized free quantum particle?
- 26. Underline the correct answer: A particle described by $\Psi(x,t) = Ce^{\frac{i}{\hbar}(p_x x Et)}$ has definite/indefinite position and definite/indefinite momentum.
- 27. How do we construct a wave packet describing a quantum particle?
- 28. True or false: A quantum particle described by a wave packet has both definite position and definite momentum.
- 29. What does it mean that there is a trade-off between definiteness of position and definiteness of momentum of a particle described by a wave packet. Name the principle that implies this trade-off.
- 30. Suppose a free quantum particle moving in the positive x axis direction is described by a Gaussian packet $\Psi(x,t)$. Sketch $|\Psi(x,t)|^2$ at two instants of time t_1 and t_2 where $t_1 < t_2$.
- 31. What do we mean when we say that a Gaussian packet spreads with time?
- 32. Why don't we see the spreading effect for wavepackets describing macroscopic objects?

