From Static to dynamics

Static → From Steady states to time dependent effects → dynamics

$$\frac{\partial}{\partial t} = 0 \qquad\qquad \frac{\partial}{\partial t} \neq 0$$

Faraday's discovery and the **birth of electrical engineering** **(EC)**

Orsted 1820: Discovery of the close connection between electricity and magnetism

- Current in a wire **creates** a magnetic field (compass deviation)
- Wire carrying current in a magnetic field **manifests** action - reaction

Involves steady current

and Faraday 1821:

- **Changing** magnetic field **creates** a changing electric field
- **Changing** electric field **creates** a changing magnetic field

Involves time varying current

FORCE and / or **TORQUE**

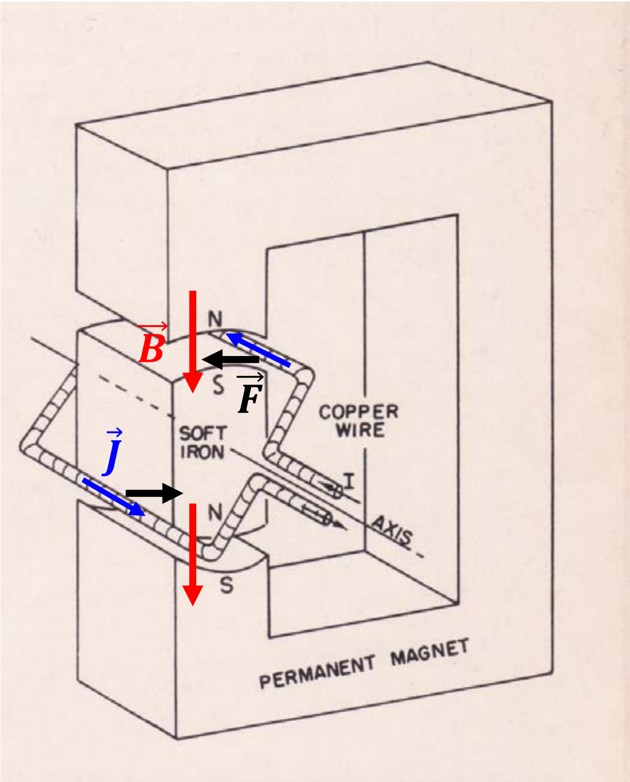
Can be used to produce **WORK**

Birth of electrical engineering

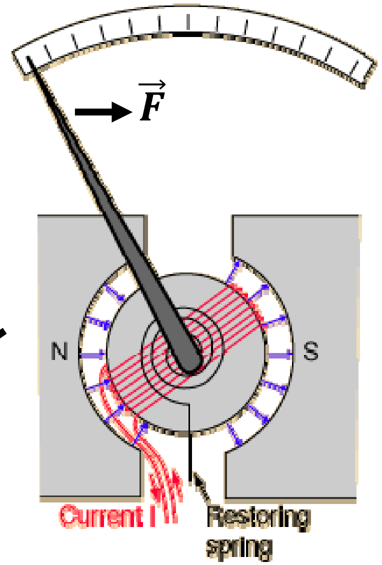
Magnetic field produces mechanical force

Application of Lorentz force

Make a motor (current reversed each half-turn)



Torque in action



Makes a sensitive instrument
for measuring current

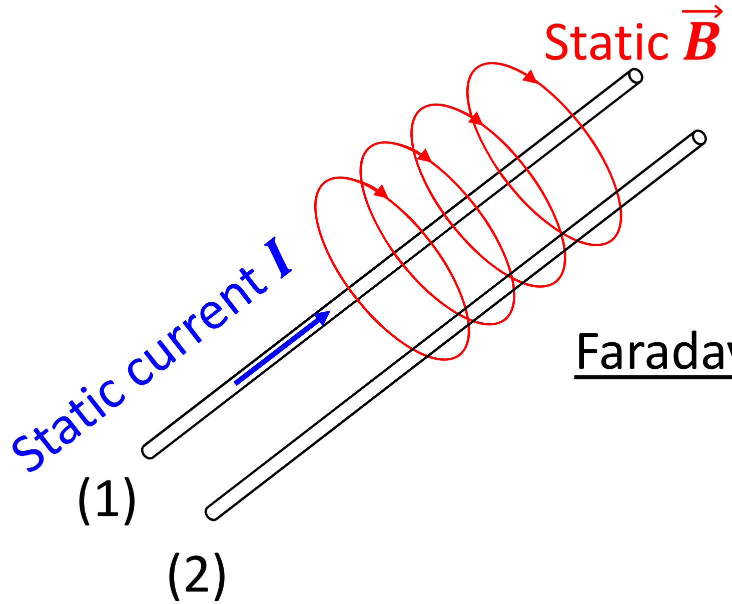
Electromagnetic Motor

Galvanometer

From Feynman lecture (Volume II)

<u>Orsted 1820</u>: Current in a wire creates magnetic field

<u>Faraday 1821</u>: Magnetic field **MAY** create an electric field ⇨ Generating thus a current in wire (2)



Nothing appears in wire (2) If current in (1) is steady and wires immobile

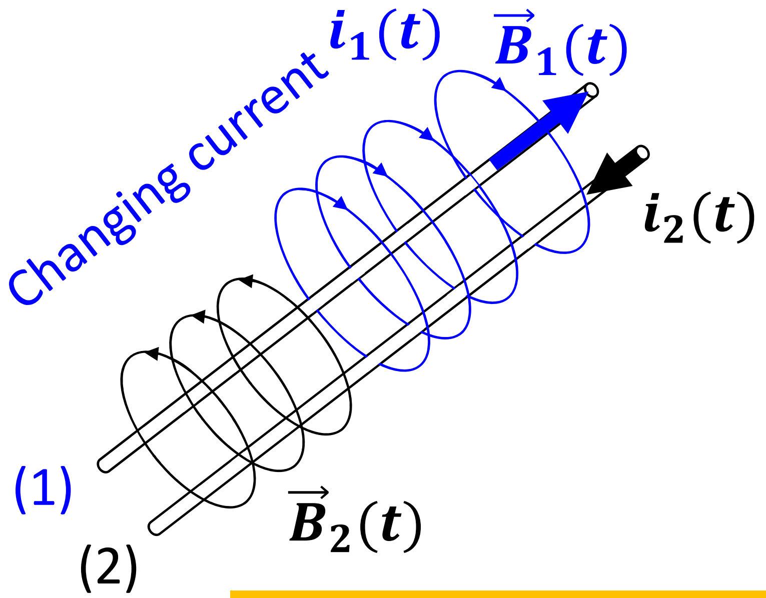
<u>Faraday's discovery</u> ⇨ From Static to **dynamics**

Requires that something changes with **TIME**

Current induction

No Current without $\vec{E}$

A varying current in wire (1) → A varying current $\boldsymbol{i_2(t)}$ appears in wire (2)



Induction of **electric field** in wire (2) driving electrons creating thus a **current**

EMF (ElectroMotive Force)

$\boldsymbol{i_1(t)}$ → $\boldsymbol{i_2(t)}$

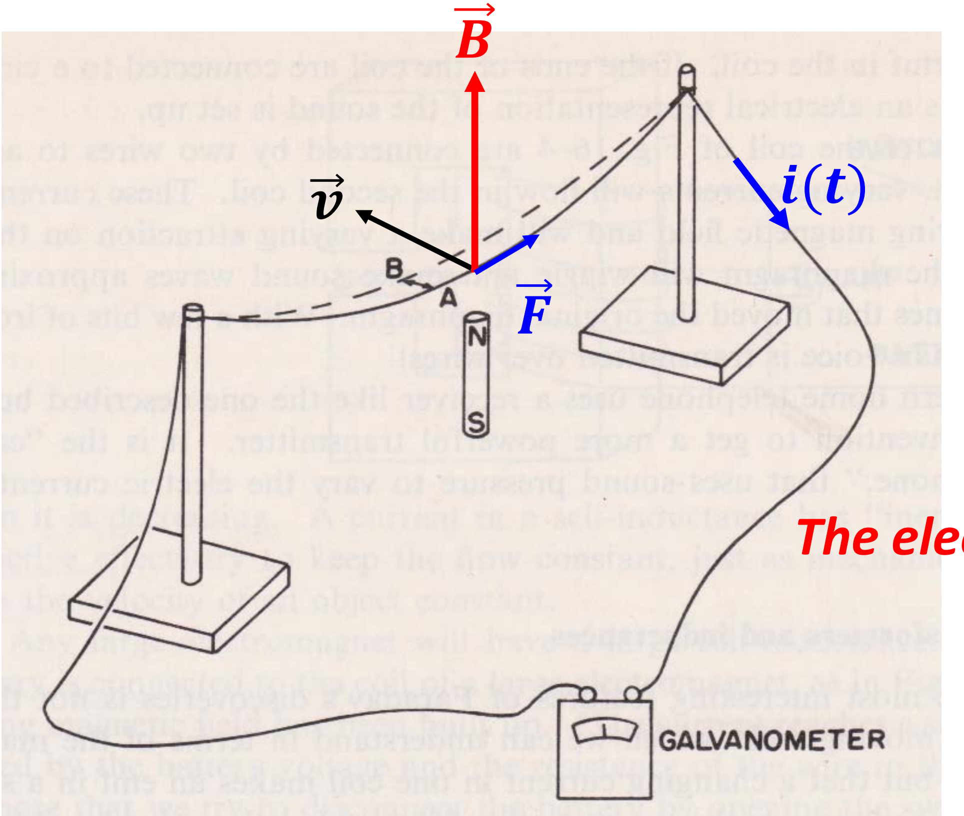
$\vec{J}_1 = -\vec{J}_2$ → Opposing magnetic field

Essential feature discovered by Faraday: **Time changing processes**

Is something missing in this figure? Yes: A second magnetic field is generated by wire (2)

Current induction: Lorentz force was not known to Faraday

$\vec{F} = q\vec{v} \times \vec{B}$



$\vec{v}$ = **mechanical velocity**

Current induced by **Lorentz force**

Concept of motional emf

Why the galvanometer shows a current while it is far from the Lorentz force ?

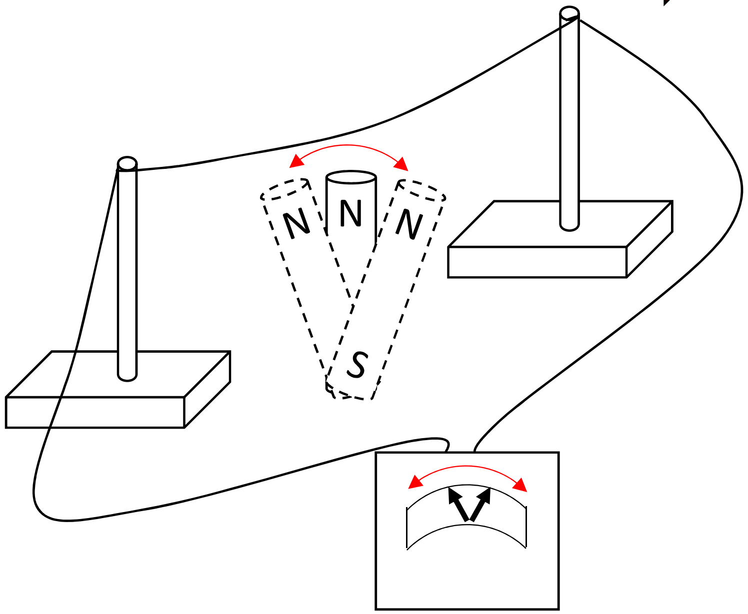
The electrons move by electric repulsion over long distances

First telegraph discovered by Gauss and Weber

From Feynman lecture (Volume II)

Another way of Current induction discovered by Faraday

No Lorentz force → **By shaking the magnet** → **Relativistic effect**



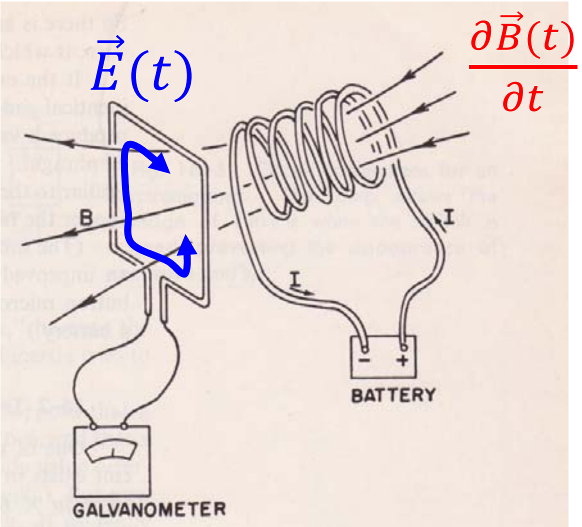
- No current induction if magnet is static
- Current induced by change in magnetic field $\frac{\partial B}{\partial t}$

Concept of **non**motional emf

Again why the galvanometer shows a current while it is far from the Lorentz force ?

There is a net integrated push around the complete circuit. This net push MUST come from an electric field which then leads to the repulsive push over long distance

Faraday's discovery: **emf**



From Feynman lecture (Volume II)

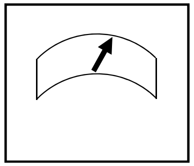
Current induced by change in magnetic field $\boxed{\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}}$

The current noticed in the loop is due to motion of electrons

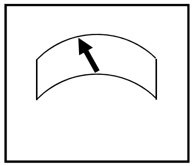
- A force has pushed the electrons ⇨
- An electric field has been generated by $\boxed{\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}}$

$$\boxed{\vec{\boldsymbol{E}}(\boldsymbol{t}) \propto \frac{\partial \vec{\boldsymbol{B}}(\boldsymbol{t})}{\partial \boldsymbol{t}}}$$

1) We switch **ON** the battery



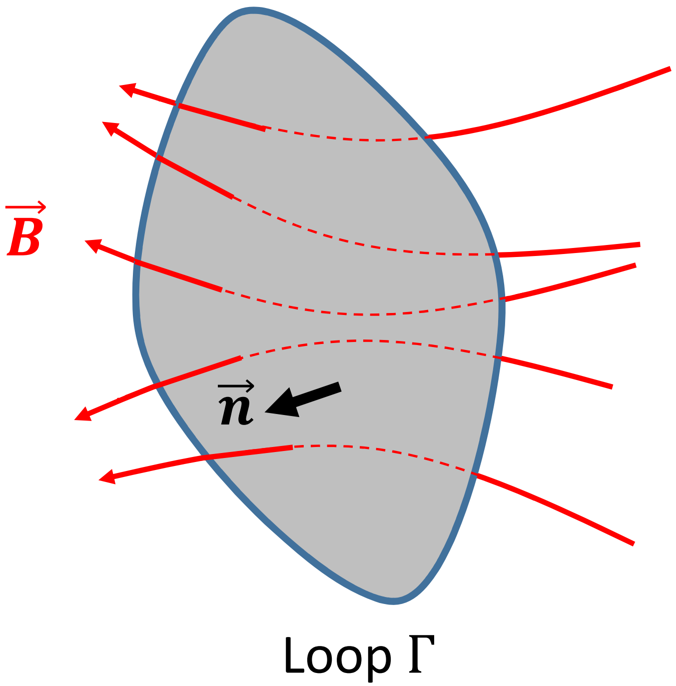
2) We switch **OFF** the battery



emf can be generated in a wire by:

- Moving mechanically the wire near a magnet or vice versa
- Changing current in a nearby wire

Faraday's rule: Is it the changing magnetic field or changing flux that matters?



$$\frac{\partial \vec{B}(t)}{\partial t} \longleftrightarrow \frac{\partial \Phi}{\partial t}$$

Area kept unchanged

$$\Phi = \int \vec{B}.d\vec{A} = \int \boldsymbol{B}_{\perp} dA$$

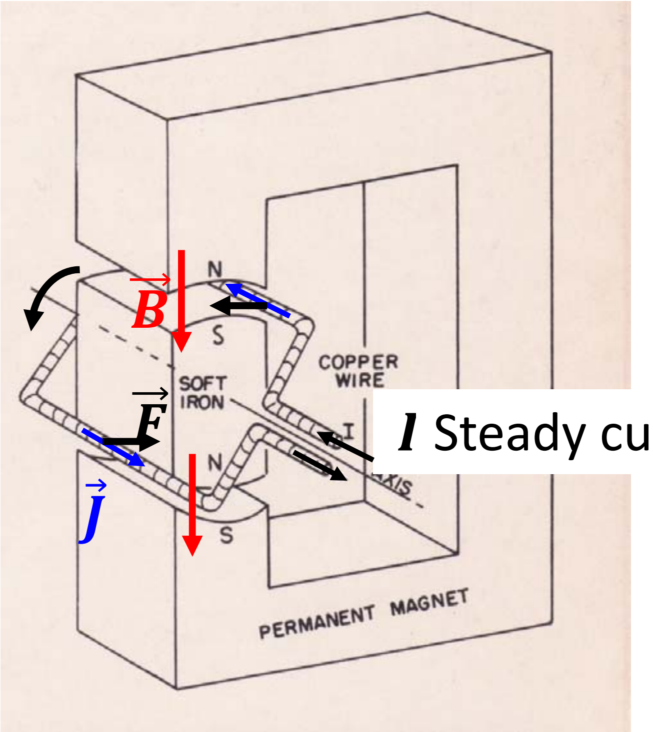
Normal component of $\vec{B}$ integrated over the whole area of the loop

emf: ONLY and **ONLY IF** the flux is changing with time

- Changing $\vec{B}$
- Changing $\vec{A}$
- Changing both $\vec{A}$ and $\vec{B}$

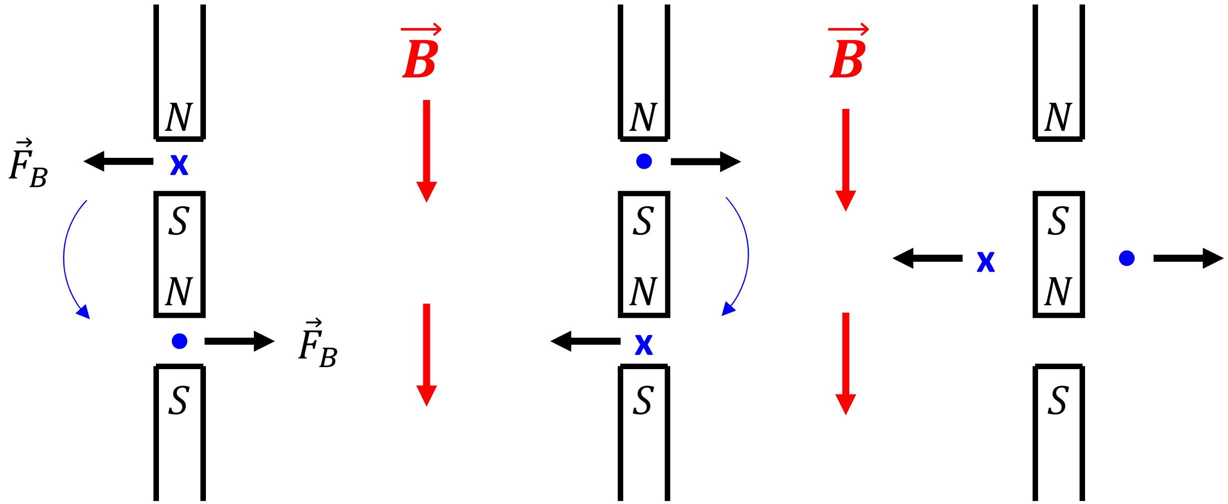
Motor

The loop rotates due to **Lorentz force**



I Steady current

From Feynman lecture (Volume II)



Torque ⇨ Forward

Torque ⇨ backward

No torque ⇨ stop

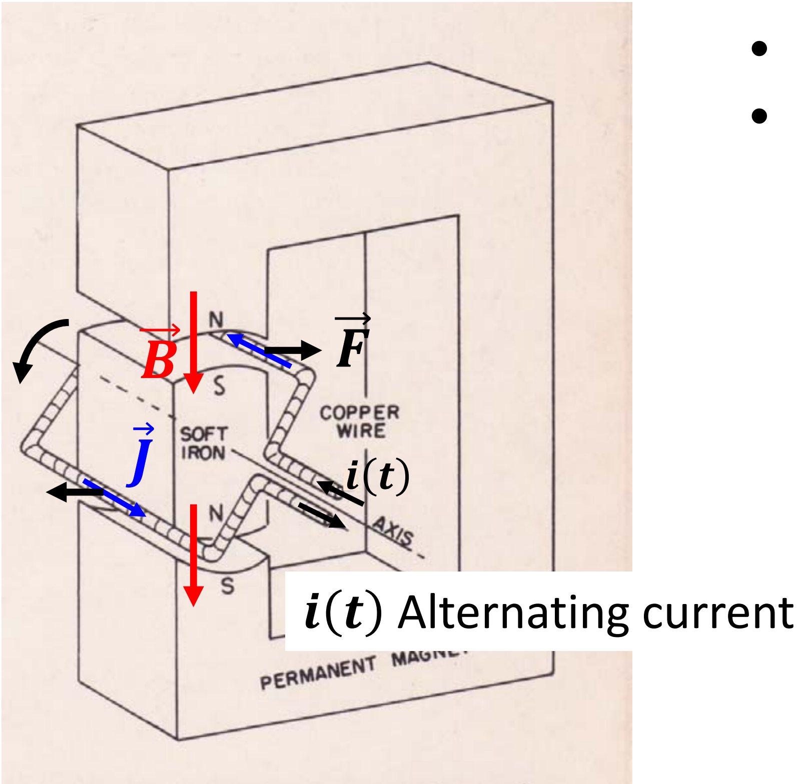
Forces acting along the same line

Current **MUST** be reversed each half-turn <u>otherwise the loop stops rotating</u>

From steady to variable current $I \rightarrow i(t)$

Alternating current generator

The loop rotates **mechanically** → Induction of an emf: **E**lectro**M**otive **F**orce



From Feynman lecture (Volume II)

- Area of the loop is fixed
- The normal component of loop's area $\vec{n}$ is changing

↓

Changing flux $\Phi(t)$

↓

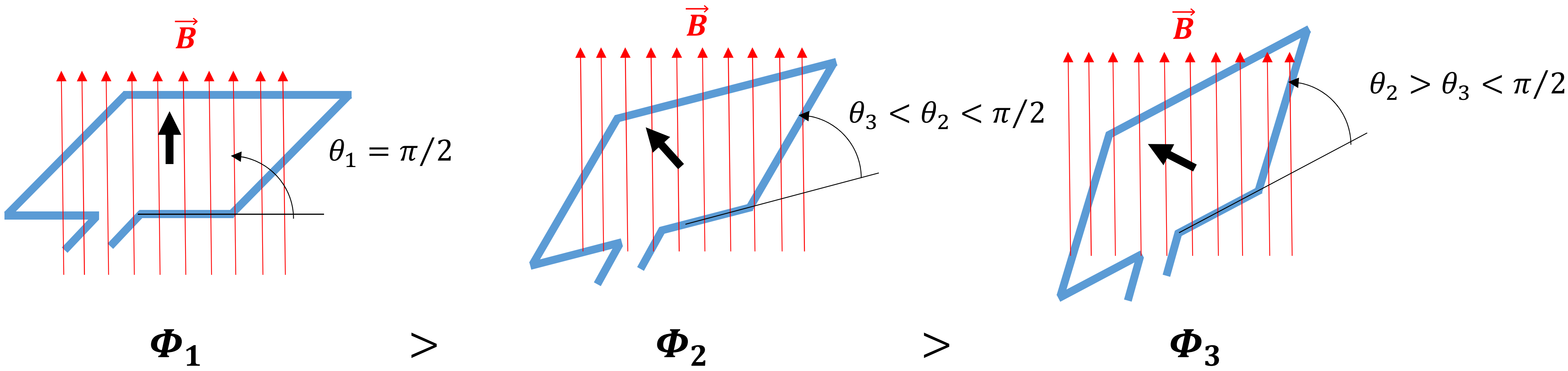
Electric field generated in the wire

Circulation of $\vec{E}(t)$ along the loop

↓

Generation of an alternating current $i(t)$ with frequency = frequency to the mechanical rotation

Emf = rate of change of magnetic flux through the loop



What matters in the flux is the **normal component of the field** along the unit vector of the area

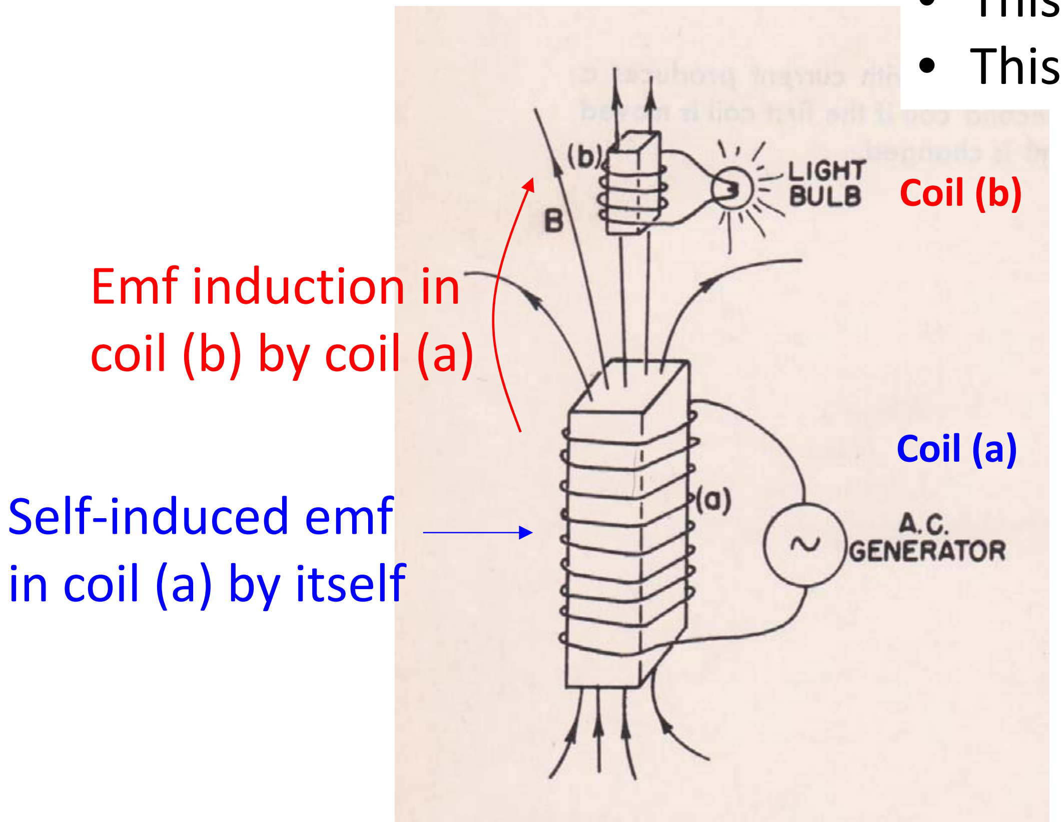
From $\boldsymbol{\Phi_1}$ to $\boldsymbol{\Phi_3}$

$$\frac{\partial \Phi}{\partial t} \rightarrow E(t) \rightarrow emf \rightarrow i(t)$$

Lenz's rule

- The induced emf creates a current
- This current generates a magnetic field
- This magnetic field tends to oppose the original one

The polarity of the induced emf is **ALWAYS** such as to oppose the change of the flux of the original magnetic field



From Feynman lecture (Volume II)

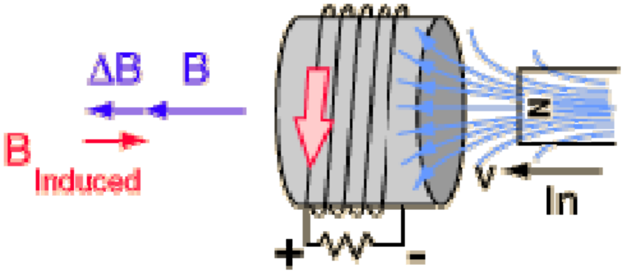
$$emf = -N\frac{d\Phi}{dt}$$

N = # of turns

Φ = Flux of B through area A

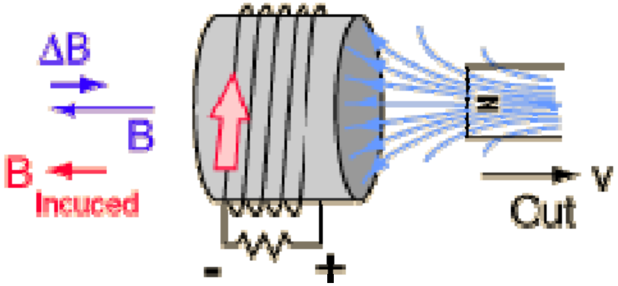
Lenz's law

Lenz's law



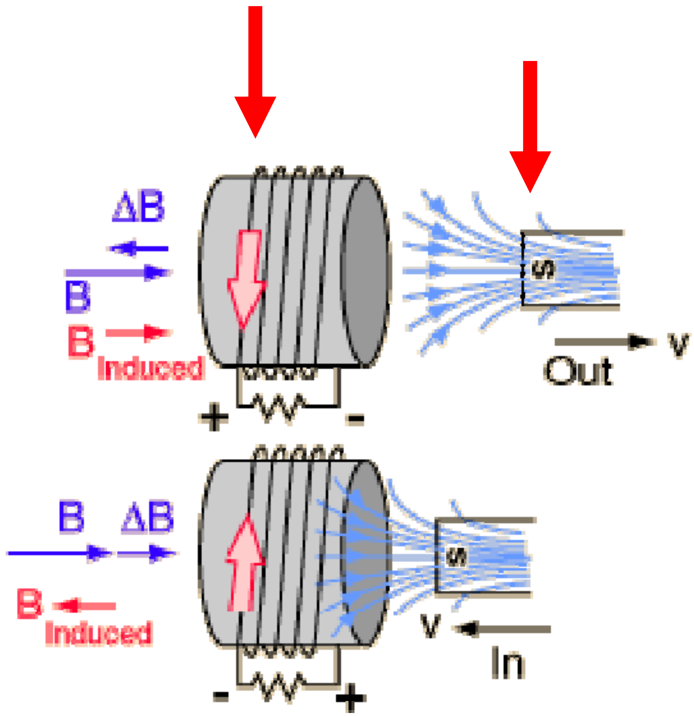
⇨ Flux increases ⇨ B increases by ΔB

The current generated by the emf goes in a direction such as to induce a magnetic field in the opposite direction



⇨ When the flux is decreased the emf is reversed

Caution! Here the arrows are reversed as it is the south pole of the magnet that is moving in and out



Towards time dependent Maxwell's equations

$$\vec{\nabla}.\vec{E}(t) = \frac{\rho(t)}{\varepsilon_0}$$

Gauss' law

$$\vec{\nabla}.\vec{B}(t) = 0$$

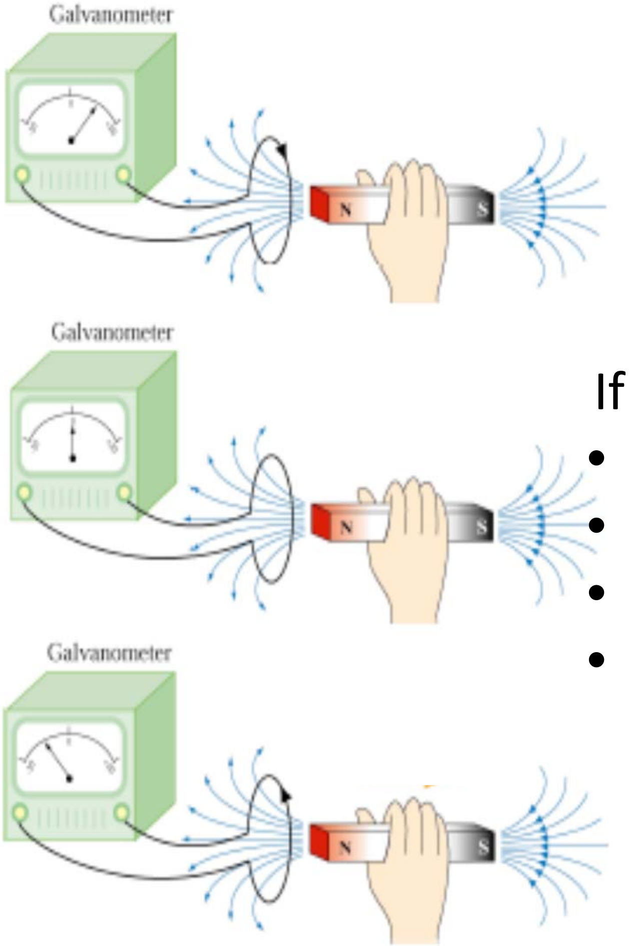
No magnetic charges

$$\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

Faraday's law ($\vec{E}$ is no longer conservative)

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law completed by Maxwell



Faraday's law of induction

Relative motion induces current in the loop

There **MUST** be an induced electric field Along the loop

$$\oint \vec{E}_{nc}.d\vec{l} \neq 0$$

$$\rightarrow \boldsymbol{i(t)}$$

If no motion of the loop and magnet

- **NO** change of the flux
- **NO** electric force
- **NO** motion of the free charges
- **NO** current in the loop

Electrostatic ⇨ Conservative field

$$\oint \vec{E}_c.d\vec{l} = 0$$

Faraday's law: electromotive force

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Motional emf

$$\mathcal{E} = \oint \underbrace{(\vec{v} \times \vec{B})}_{\vec{F}/q} . d\vec{l} = -\frac{d\Phi_B}{dt}$$

Non motional emf
Faraday's law

$$\mathcal{E} = \oint \vec{E}_{nc} . d\vec{l} = -\frac{d\Phi_B}{dt}$$

Non-electrostatic field

$$\oint \vec{E}_{nc} . d\vec{l} = -\frac{d\Phi_B}{dt}$$

Integral form

What is the link between these two relationships?

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Differential form

Faraday's law

$$\oint \vec{E}_{nc}.d\vec{l} = -\frac{d\Phi_B}{dt} \quad + \quad \oint \vec{E}_c.d\vec{l} = 0 \quad \Rightarrow \quad \oint \underbrace{(\vec{E}_c + \vec{E}_{nc})}_{\vec{E}}.d\vec{l} = -\frac{d\Phi_B}{dt}$$

$\vec{E}$ (superposition principle)

Gauss's theorem

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \vec{B}.d\vec{A} = -\iint \frac{d\vec{B}}{dt}.d\vec{A}$$

Stoke's theorem

$$\oint \vec{E}.d\vec{l} = \iint (\vec{\nabla} \times \vec{E}).d\vec{A}$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

True always

$$if\ \frac{\partial \vec{B}}{\partial t} = \vec{0} \Rightarrow \oint(\vec{E}_c + \underbrace{\vec{E}_{nc}}_{=0}).d\vec{l} = 0$$

$$\vec{\nabla} \times \vec{E} = \vec{0}$$ Electrostatic

The Heart of Faraday's law of induction

$$\Phi_B$$

$$\Downarrow$$

$$\frac{d\Phi_B}{dt}$$

Area

$$\frac{dA}{dt}$$

Motional emf

Magnetic field

$$\frac{dB}{dt}$$

Non motional emf (if we can change B without moving a magnet nor changing the area)

$$\Downarrow$$

Faraday's law

$$\frac{d\Phi_B}{dt} = \frac{dBA}{dt}$$

B constant

Part of the circuit is moving ⇨ $\vec{v} \times \vec{B} \neq \vec{0}$

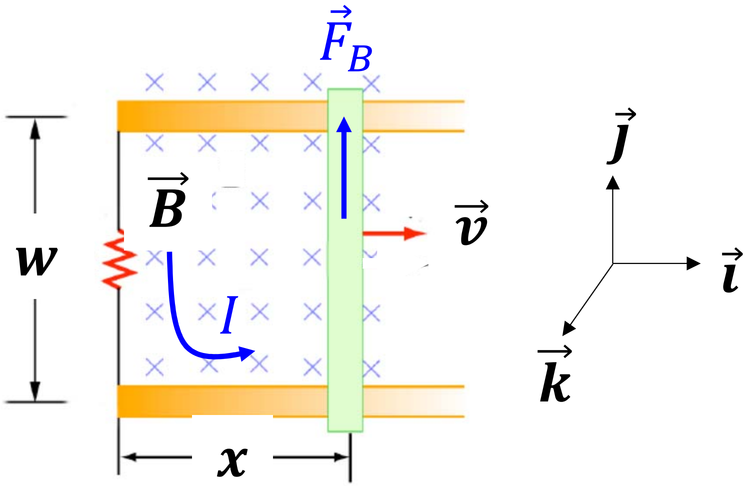
A changes ⇨ $\frac{dA}{dt} = w\frac{dx}{dt}$

A constant

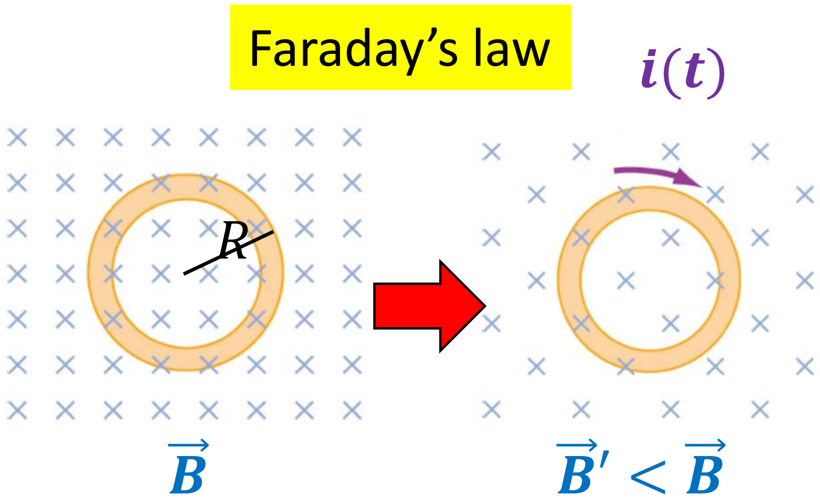
Circuit is immobile ⇨ $\vec{v} \times \vec{B} = \vec{0}$

B changes ⇨ $\frac{dB}{dt} \to E(t) \to \vec{F} = q\vec{E}$

In both cases electrons move due to Coulomb force

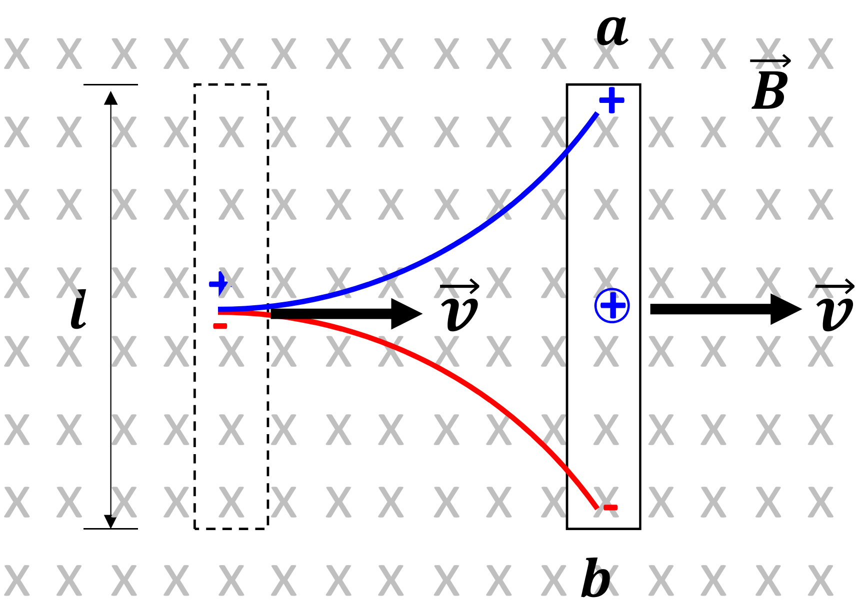


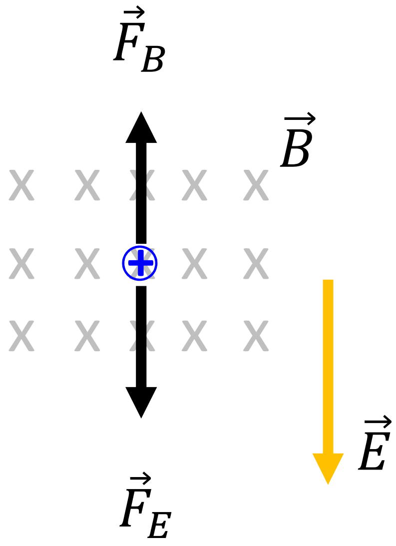
Area $A = wx$

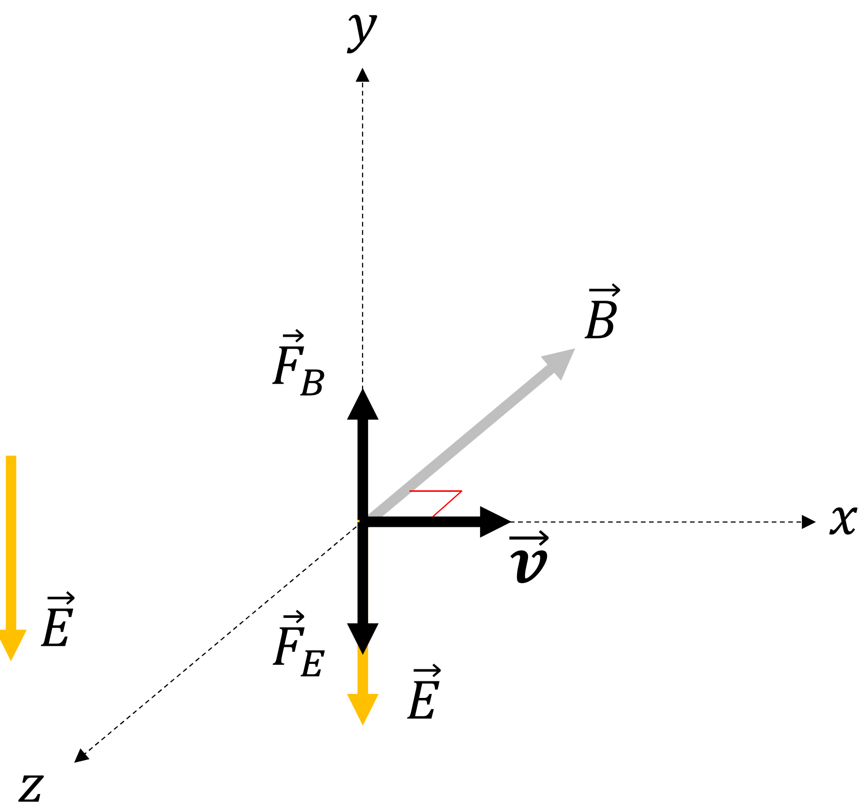


Area $A = 2\pi R$

Motional emf







An electric field is building up inside the conducting bar !

Equilibrium !

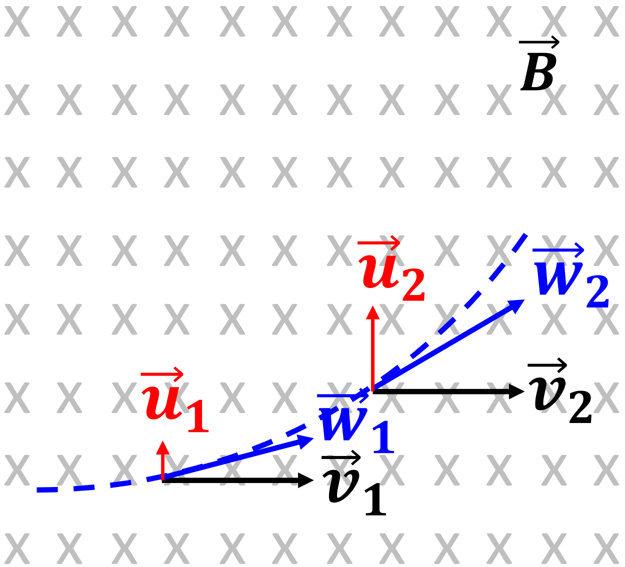
$$\vec{F}_B + \vec{F}_E = \vec{0}$$

Equilibrium $\Rightarrow$ $qvB = qE$ $\Rightarrow$ $E = vB$ $\Rightarrow$ $V_{ab} = V_a - V_b = El$

$$\boxed{V_{ab} = Blv}$$

Motion $\Rightarrow$ **Motional** emf = Potential difference $\Rightarrow$ $\boldsymbol{\mathcal{E} = V_{ab} = Blv}$

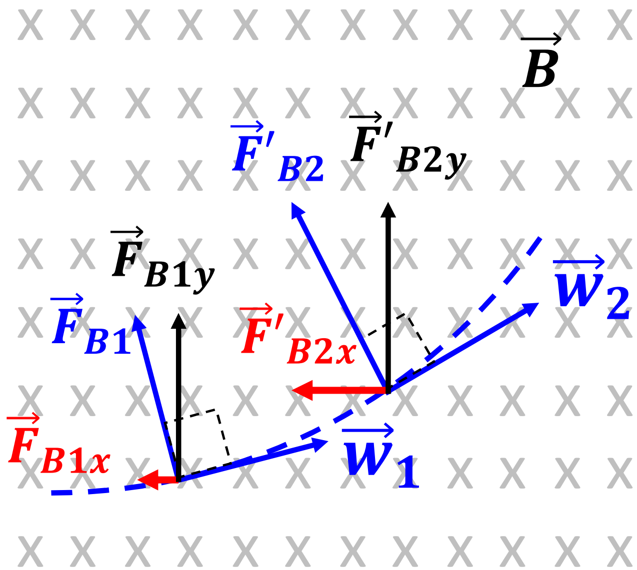
For **Motional** emf both $\boldsymbol{B}$ and $\boldsymbol{v}$ are required to get a potential difference



$$\vec{F}_B \xrightarrow{\perp} \vec{w}$$

$$\vec{F}_{By} \xrightarrow{\perp} \vec{v}$$

$$\vec{F}_{Bx} \xrightarrow{\perp} \vec{u}$$



None of these forces do work on the charge

BUT a force $\vec{F}_{Bx}$ opposing the motion of the <u>bar</u> is building-up

Work **MUST** be done against this force to keep the bar moving at constant velocity $\vec{v}$

Opposite force $\vec{F}_{Bx} = -quB\vec{\imath}$

Work done per unit charge against this force must be $$\frac{\Delta \mathrm{W}}{q} = -\int_{x}^{x+\Delta x} -uB\vec{\imath}.dx\vec{\imath}$$

$$\frac{\Delta \mathrm{W}}{q} = -\Delta V = V_{ab} = -\int_{x}^{x+\Delta x} -uBdx = uB\Delta x$$

$$\Delta x = v\Delta t$$

$$\Delta l = u\Delta t$$

$$\Rightarrow \Delta x = \frac{v\Delta \mathrm{l}}{u} \Rightarrow$$

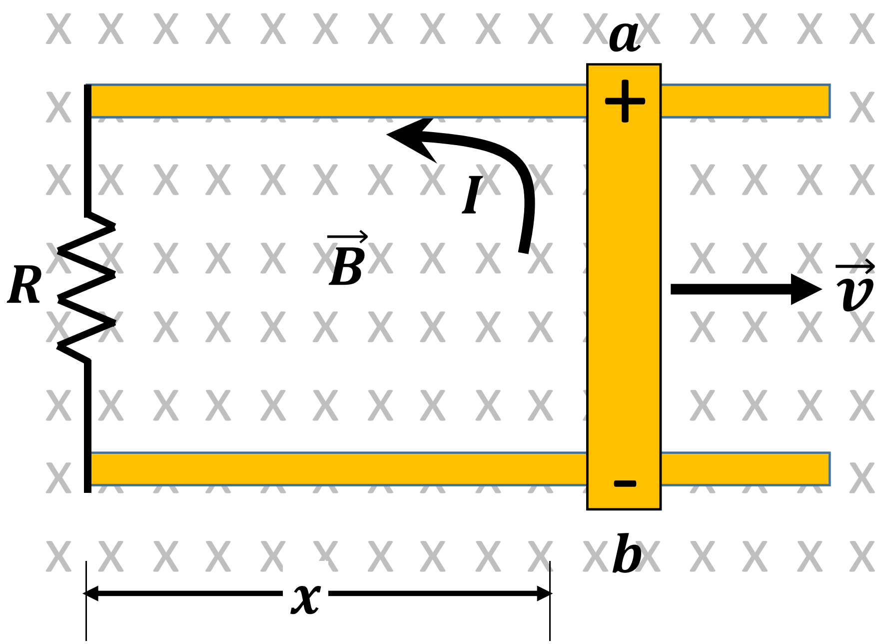
$$\boxed{\frac{\Delta \mathrm{W}}{q} = -\Delta V = V_{ab} = \mathcal{E} = \boldsymbol{Blv}}$$

Motional emf found in slide # 22

This work is useless because the circuit is **not complete !**

The circuit is complete and the bar moves at constant velocity

The charges will **no longer accumulate** at the ends of the bar and a current is generated Counterclockwise: <u>The motion of the charges in the circuit is due to coulomb repulsion</u>

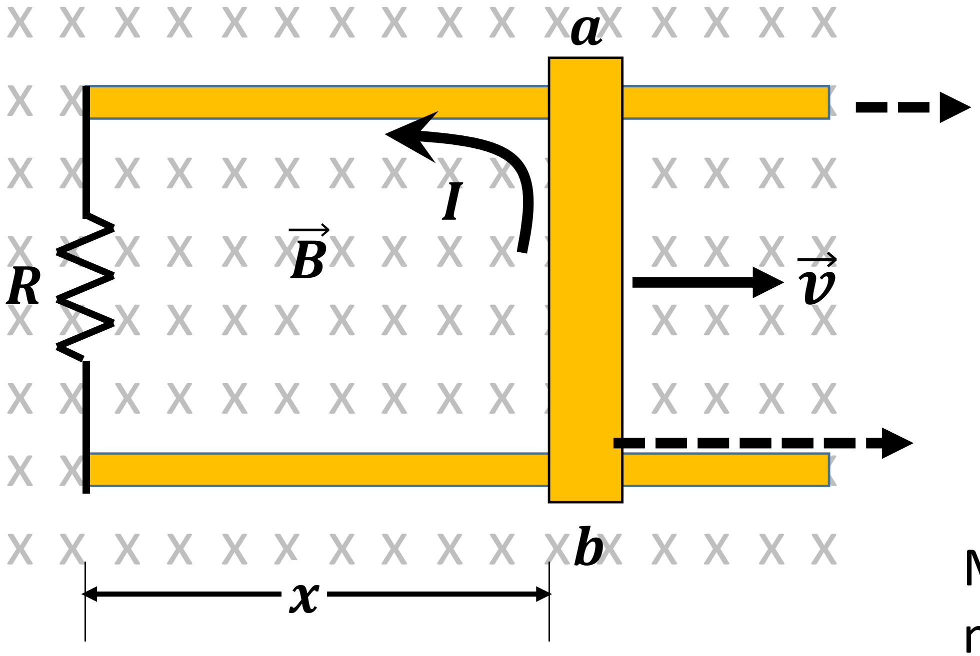


Question: Why putting a resistance R in the circuit ?

Answer: To reduce the current to a minimum in order to avoid generating an important magnetic field which may then disturb the external one

Remember: In electrostatic the test charge must be weak enough to avoid perturbing the source

The circuit is complete and the bar moves at constant velocity



Stationary U-shaped conductor

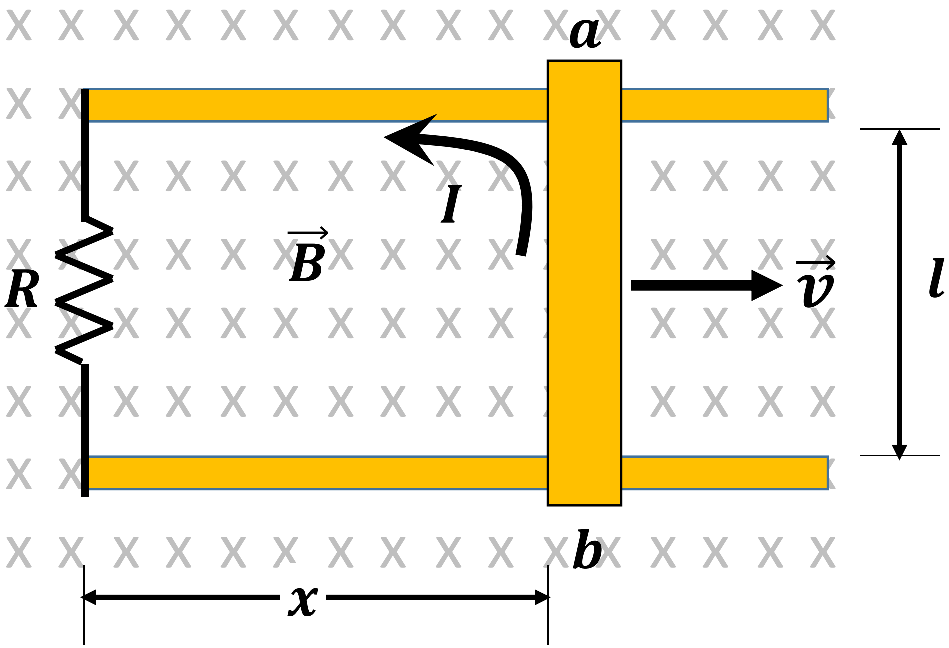
Moving conductor rod

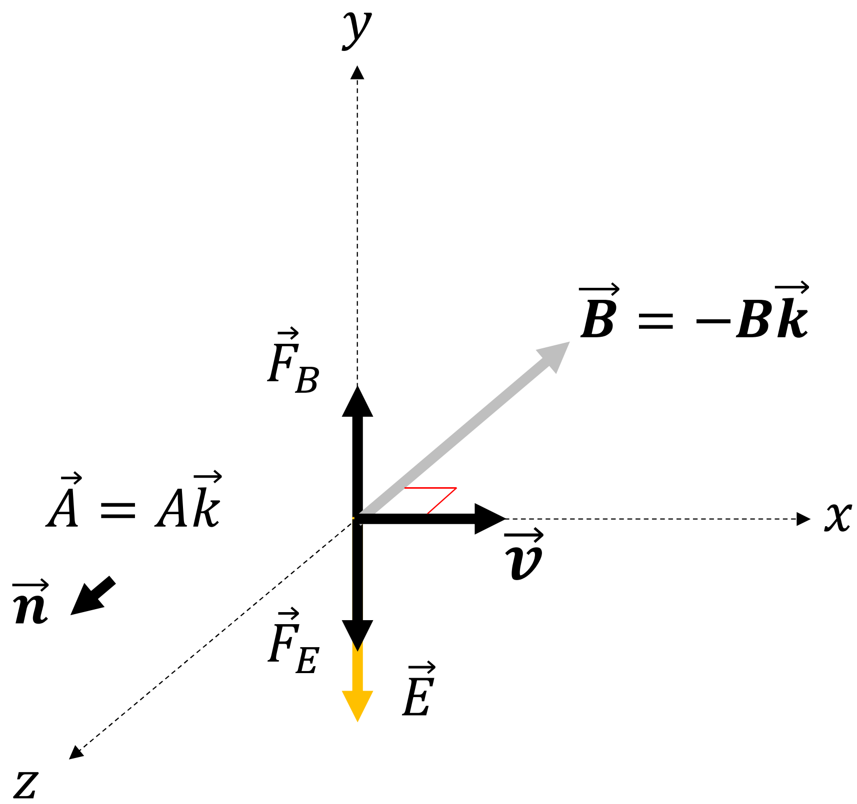
Magnetic force drives charges to the ends of the moving rod from which they redistribute in the stationary U-shaped part

Moving rod = source of electromotive force

Faraday's law: electromotive force

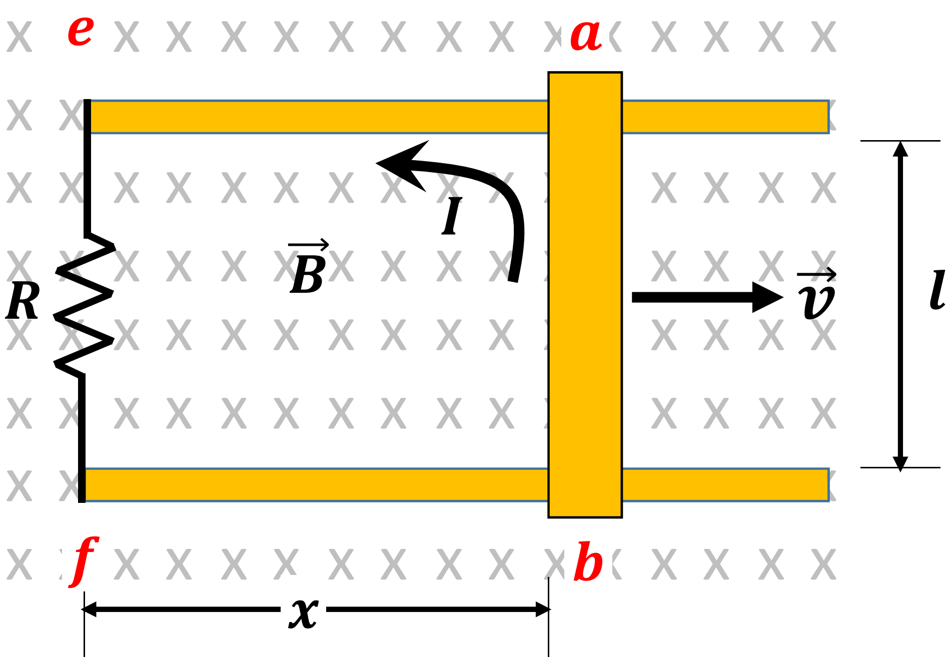
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$





The magnetic flux changes because the area of the circuit is changing

$$\Phi_B = \vec{B}.\vec{A} = -Blx \Rightarrow \frac{d\Phi_B}{dt} = \frac{d}{dt}(-Blx) = -Bl\frac{dx}{dt} \Rightarrow \boxed{\frac{d\Phi_B}{dt} = -Blv}$$



$$\frac{d\Phi_B}{dt} < 0$$

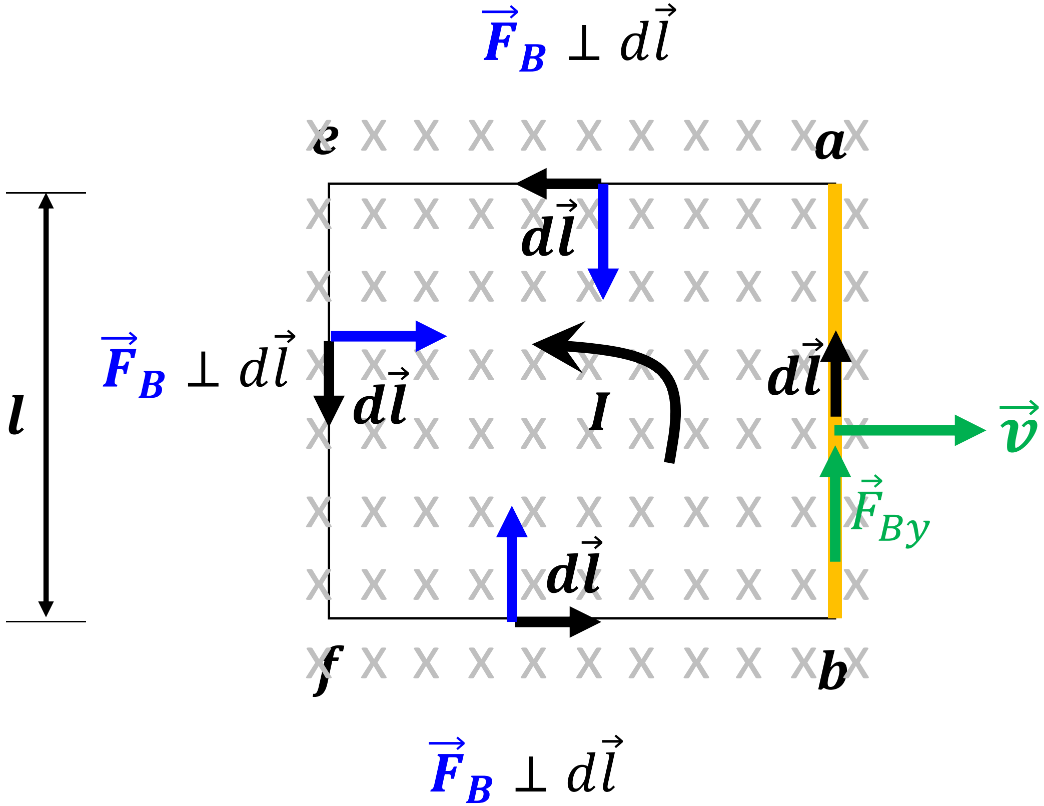
The closed path for the charges is $aefb$

From slides # 17

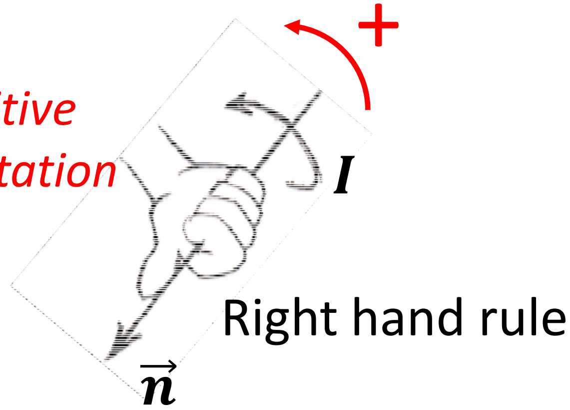
$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

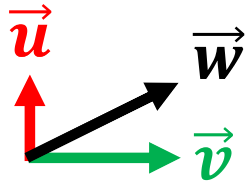
The magnetic field generated by the induced current opposes the original

The electromotive force



Conventional positive trigonometric orientation





$$\vec{F}_{By} = q\vec{v} \times \vec{B} = qv\vec{\imath} \times B\left(-\vec{k}\right)$$
$$= qvB\vec{\jmath}$$

$\vec{F}_B \parallel d\vec{l}$ along the moving rod

Slide #17

$$\mathcal{E} = \oint \frac{\vec{F}_{By}}{q} . d\vec{l} = \oint vB\vec{\jmath}.dy\vec{\jmath}$$

The only contribution is along the moving bar

$$\mathcal{E} = Blv$$

Final exam 2017

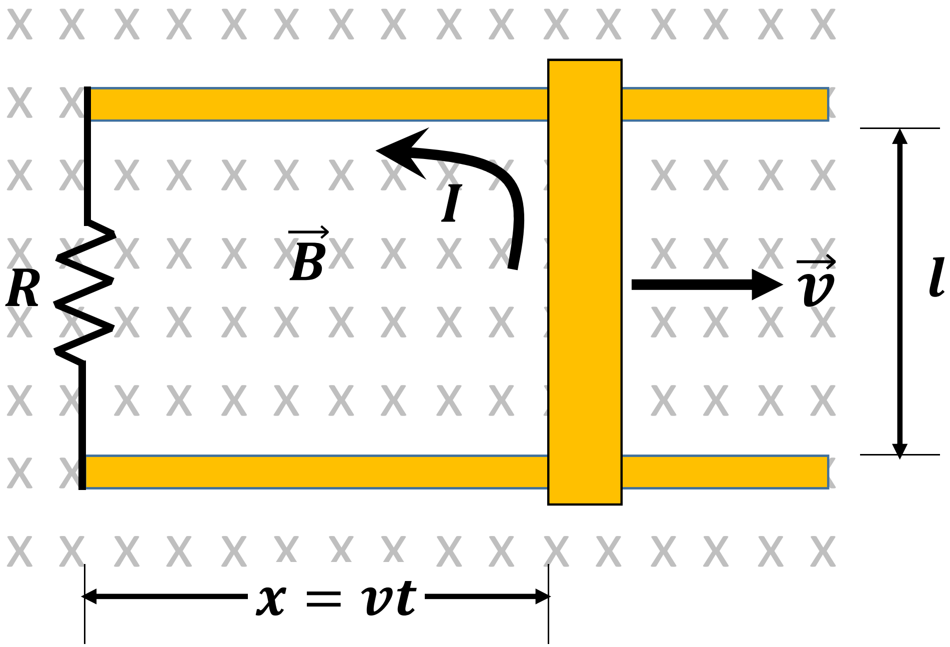
A metal bar of mass m slides without friction on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field B, pointing into the page, fills the entire region.

1) If the bar moves to the right at speed v, what is the current in the resistor and in what direction does it flow ?

2) What is the magnetic force on the bar and what is it direction?

3) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t?

1) If the bar moves to the right at speed v, what is the current in the resistor and in what direction does it flow ?



1) The flux through the loop increases. The area of the loop is lvt, so that the flux and its time derivative through the loop is,

$$\Phi_B = -lvtB \rightarrow \frac{d\Phi_B}{dt} = -Blv$$

By Faradays law, this produces and emf in the loop, whose magnitude is

$$emf = -\frac{d\Phi_B}{dt} = +Blv = IR$$

What does the sign mean?

Circulation according to trigonometric orientation

From which we can extract the current $I = +\frac{Blv}{R}$

2) What is the magnetic force on the bar and what is it direction?

Laplace $F = Idl \times B$

Direction of the force?

To the left otherwise we have discovered the perpetual motion or creation of energy from nothing !

The bar must slow down to ultimately stop

Force directed to the left

$$F = IlB = -\frac{B^2 l^2 v}{R}$$

3) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t?

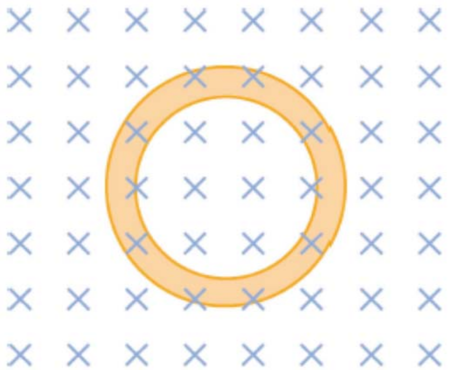
The second Newton's law

$$m\frac{dv}{dt} = -\frac{B^2l^2v}{R}.$$

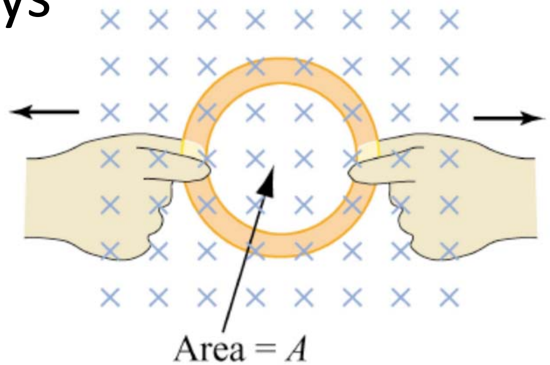
$$v(t) = v_0 e^{-\frac{B^2l^2t}{Rm}}$$

Stationary closed conducting path (coil)

The flux can change following two ways

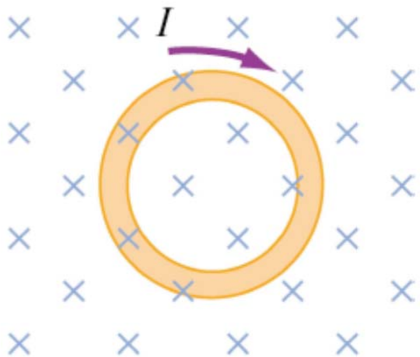


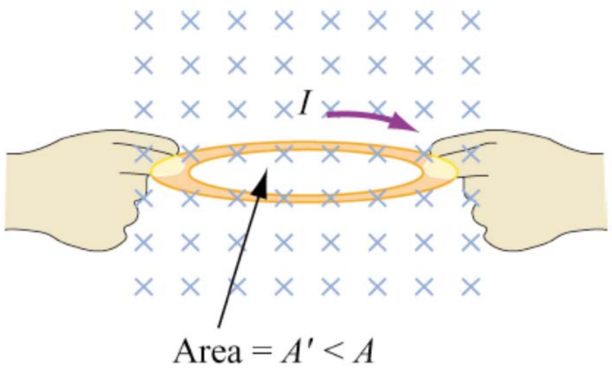
$$\boldsymbol{\Phi_B} = \overrightarrow{\boldsymbol{B}}.\overrightarrow{\boldsymbol{A}}$$



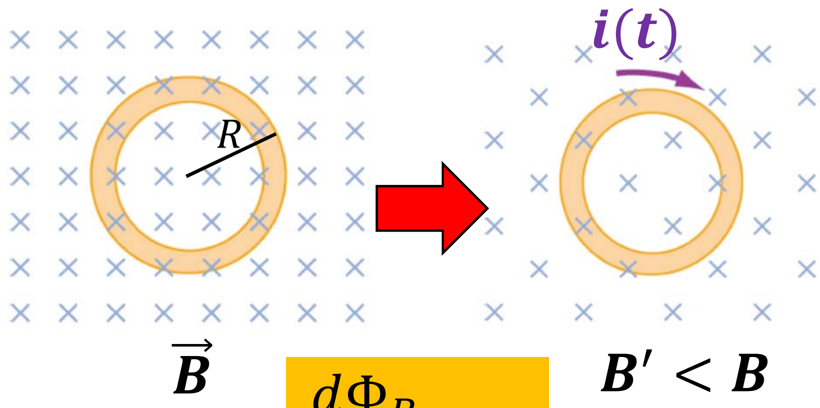
$\vec{B}$ changes

$\vec{A}$ changes

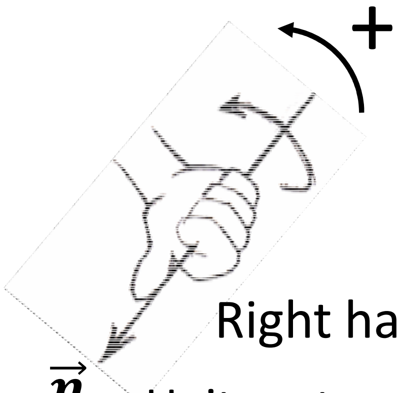




Genius conjecture of Faraday

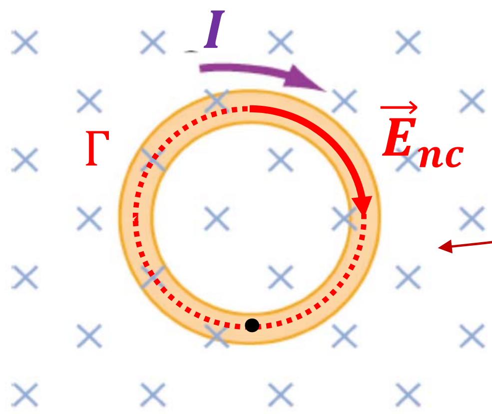


$$\frac{d\Phi_B}{dt} < 0$$



Right hand rule

$\vec{n}$ Unit vector of area



The circuit is immobile ⇨ $\vec{v} = \vec{0}$ ⇨ $\vec{F}_B = \vec{0}$

$$\boxed{\vec{F} = q(\vec{E} + \underbrace{\vec{v} \times \vec{B}}_{=\vec{0}})}$$

$$\frac{d\Phi_B}{dt} \longrightarrow \mathcal{E}(\text{emf}) = \oint_{\Gamma} \vec{E}_{nc}.\, d\vec{l}$$

Slide #17

Faraday's Conjecture

$$\vec{E} \perp \vec{B}$$

$\vec{E}_{nc}$ circulates tangentially to the loop

Electromotive force for both motional and non motional case

$$\varepsilon = \oint (\vec{v} \times \vec{B}).\, d\vec{l}$$

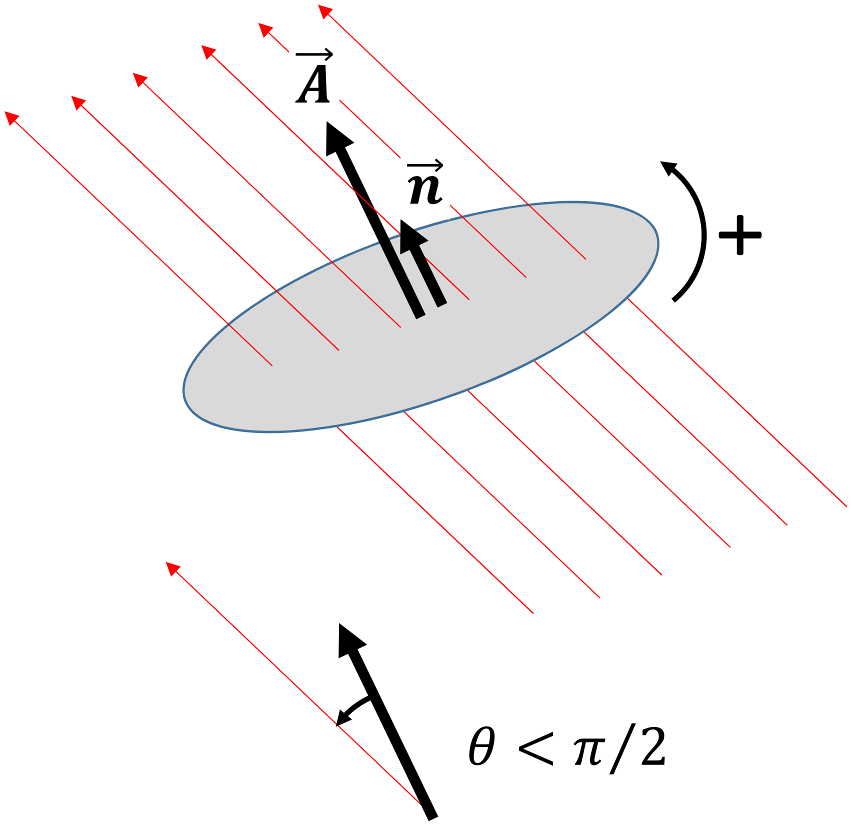
For motional electromotive force

$$\varepsilon = -\frac{d\Phi_B}{dt} = \oint_{\Gamma} \vec{E}_{nc}.\, d\vec{l}$$

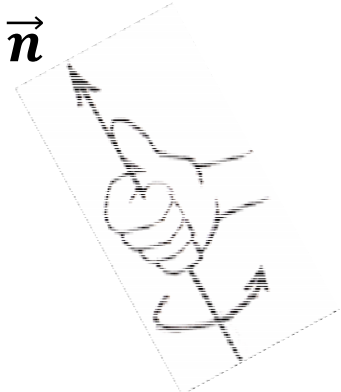
For non-motional electromotive force: Faraday’s law

Lenz’s law

Lenz's law and sign convention for emf

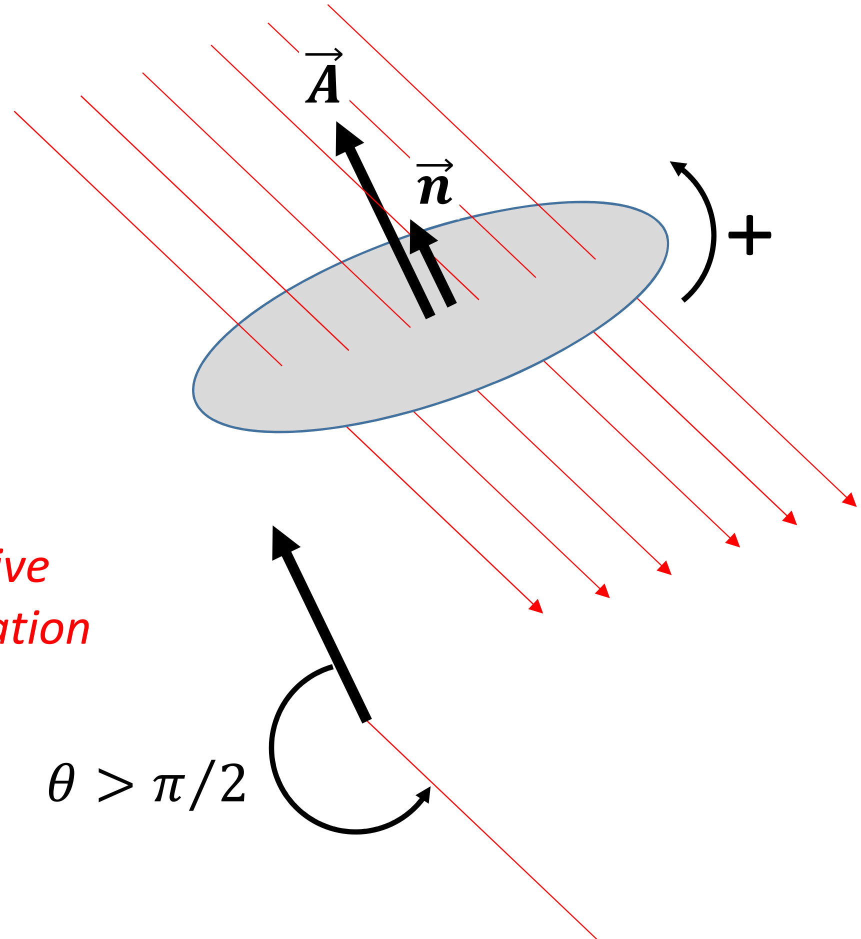


$\Phi_B = \vec{B}.\vec{A} > 0$



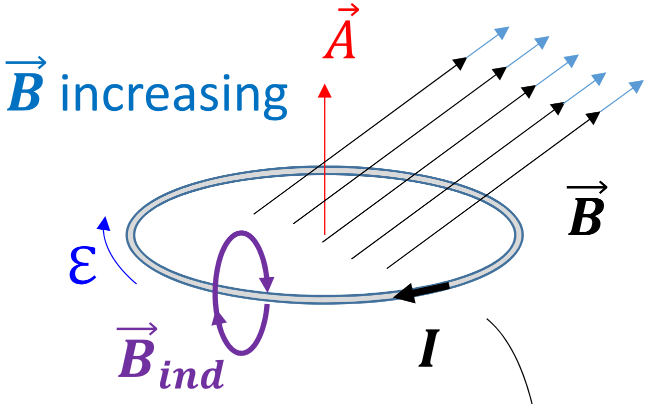
$\vec{A} \equiv thumb$

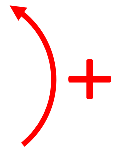
Conventional positive trigonometric orientation



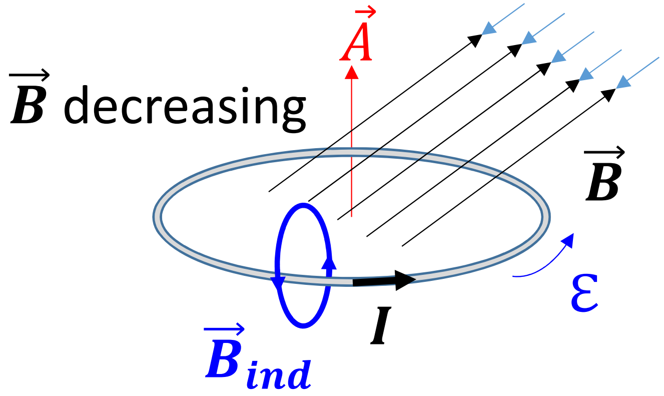
$\Phi_B = \vec{B}.\vec{A} < 0$

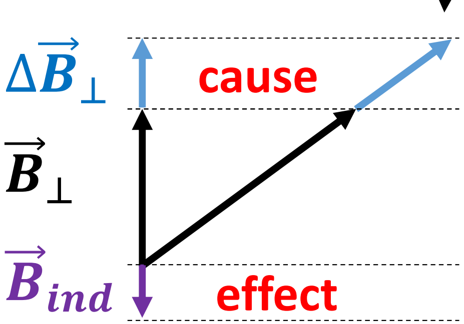
$$\boldsymbol{\cos\theta > 0}$$
$$\boldsymbol{\Phi_B > 0}$$



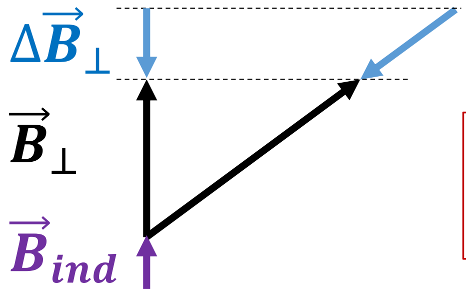


Trigonometric orientation





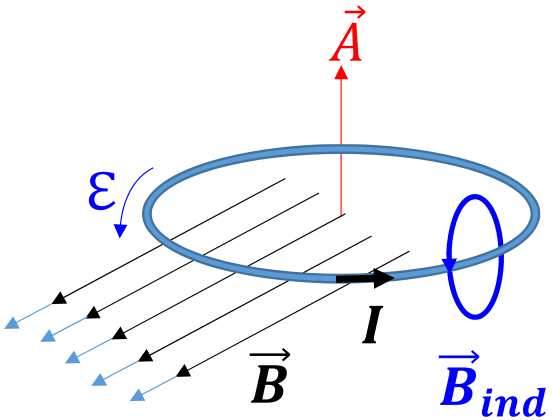
$$\boxed{\frac{d\Phi_B}{dt} > 0}$$

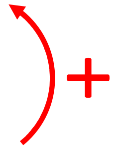


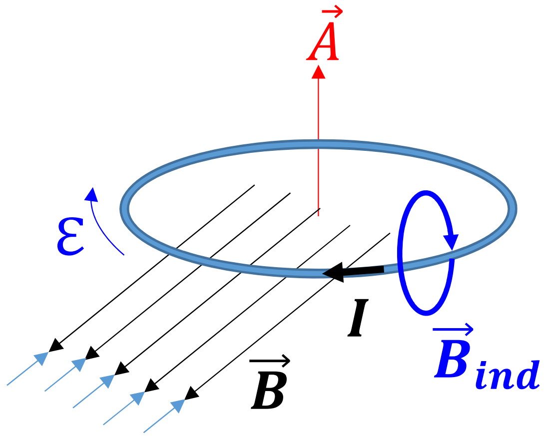
$$\boxed{\frac{d\Phi_B}{dt} < 0}$$

Induced emf ⇨ induces current ⇨ induces $\vec{B}_{ind}$ ⇨ opposing the **cause of the effect**

$$\cos\theta < 0$$
$$\Phi_B < 0$$







$\vec{B}$ increasing

$\vec{B}$ decreasing

$$\boxed{\frac{d\Phi_B}{dt} < 0}$$

$$\boxed{\frac{d\Phi_B}{dt} > 0}$$

$\vec{B}_{\perp}$ parallel $\vec{A}$

$\boldsymbol{\Phi_B = \vec{B}.\vec{A} > 0}$

$\theta < \pi/2$

$\boldsymbol{\frac{d\Phi_B}{dt} > 0}$ $\boldsymbol{\frac{d\Phi_B}{dt} < 0}$

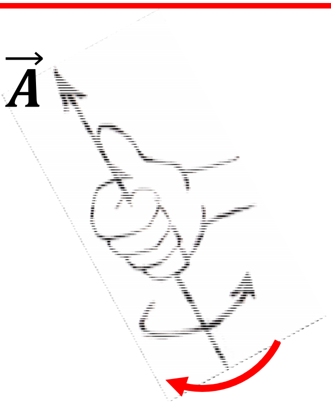
$\vec{B}_{\perp}$ anti-parallel $\vec{A}$

$\boldsymbol{\Phi_B = \vec{B}.\vec{A} < 0}$

$\theta > \pi/2$

$\boldsymbol{\frac{d\Phi_B}{dt} > 0}$ $\boldsymbol{\frac{d\Phi_B}{dt} < 0}$

$\boldsymbol{\vec{n}, \vec{A}}$

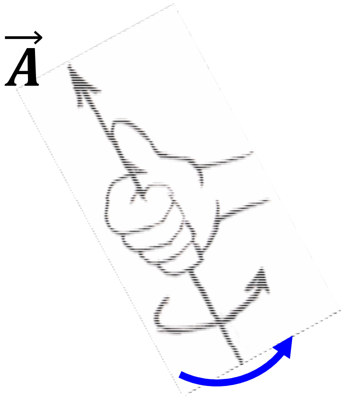


$\boldsymbol{\vec{A} \equiv thumb}$

$\boldsymbol{\mathcal{E} < 0}$

$\boldsymbol{\mathcal{E} > 0}$

$\boldsymbol{\vec{n}, \vec{A}}$



$\boldsymbol{\vec{A} \equiv thumb}$

Summarizing Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{E}_{nc}.\, d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \underbrace{\left(\vec{E}_c + \vec{E}_{nc}\right)}_{\vec{E}}.\, d\vec{l} = -\frac{d\Phi_B}{dt}$$

Stoke's theorem + Gauss's theorem

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Faraday

Varying **magnetic field** gives rise to an induced **electric field**

Maxwell

Remarkable symmetry of nature

Varying **electric field** field gives rise to an induced **magnetic field**

Varying **electric field** field gives rise to an induced **magnetic field**

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J}$$

Ampere's law

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \boxed{\frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}}$$

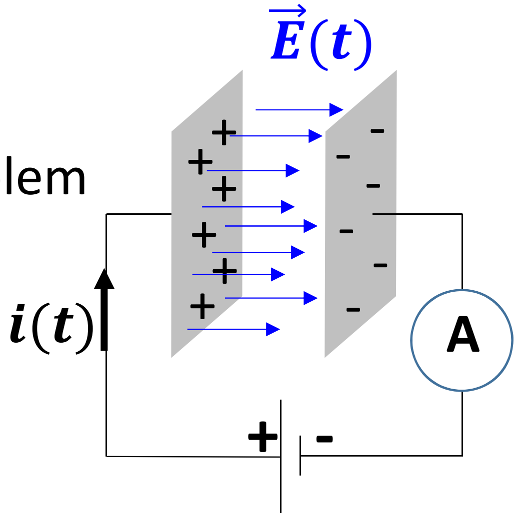
Maxwell's law

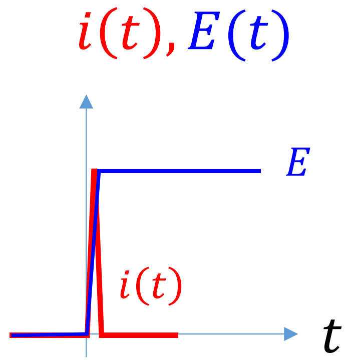
The term that brings symmetry

Displacement current

Displacement current or how Maxwell saved Ampere's law

The capacitor problem

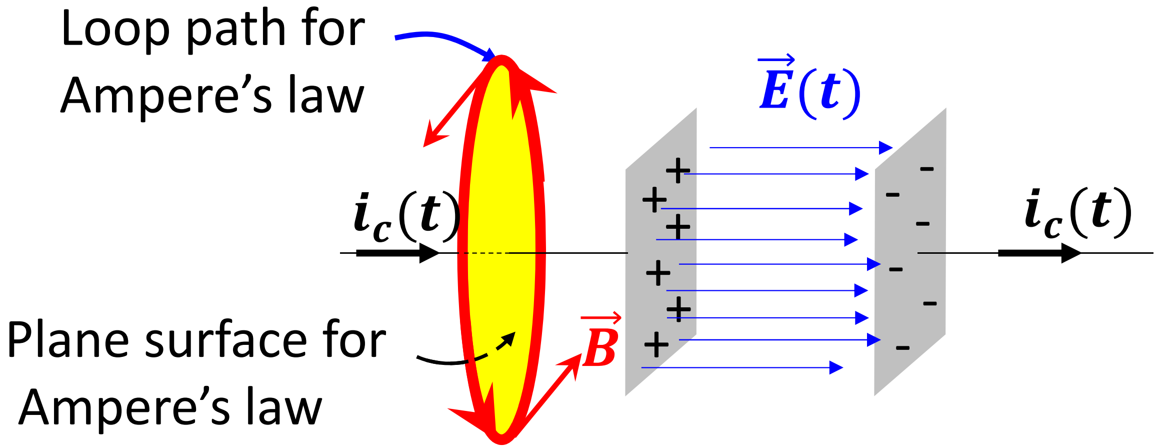




This is not a closed loop **BUT** a current $i(t)$ flows in the circuit

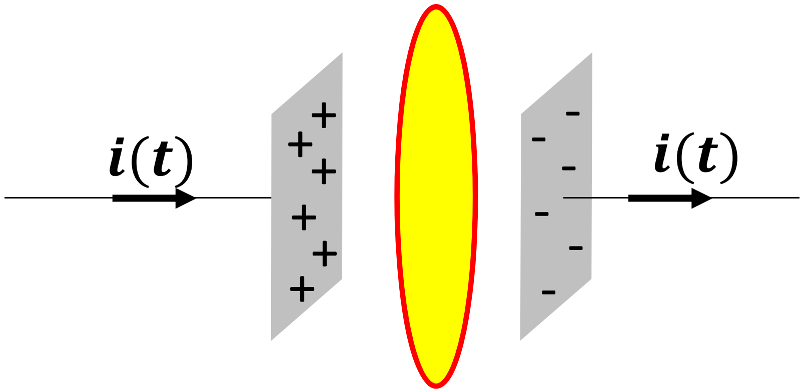
Clearly no electron is jumping from one plate to the other !

Displacement current or how Maxwell saved Ampere's paradoxical law



Disk crossed by the wire

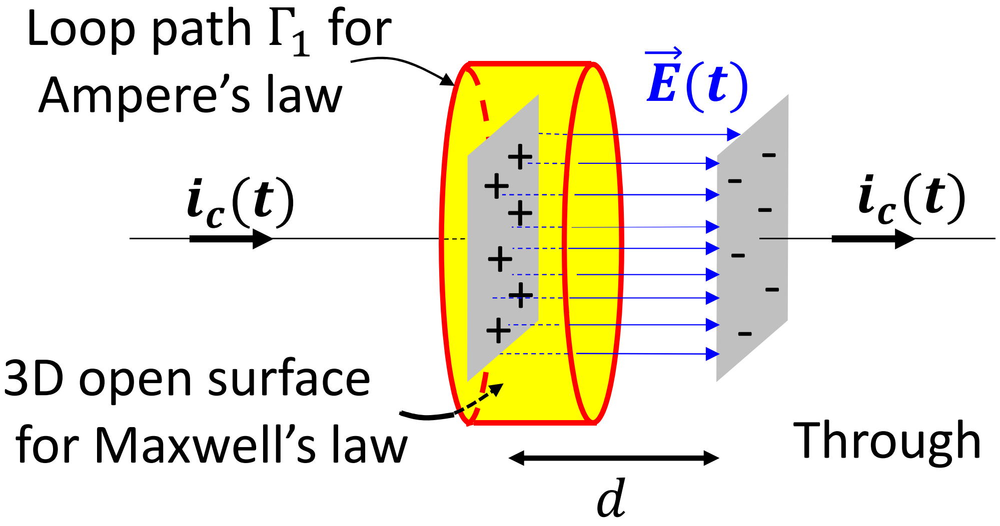
$$\oint \vec{B}(t).d\vec{l} = \mu_0 I_{encl} = \mu_0 \boldsymbol{i_c(t)_{encl}}$$



Disk **NOT** crossed by the wire

No electron moving through the surface thus **NO** magnetic field ?

$$\oint \vec{B}(t).d\vec{l} = 0$$



Cylinder open on the left hand side = Maxwell's surface.

<u>The wire does not cross Maxwell's surface</u>

Through Maxwell's surface a flux $\boldsymbol{\Phi_E(t)}$ is building-up

$$Q(t) = CV(t) = \frac{\varepsilon_0 A}{\cancel{d}} E(t)\cancel{d} = \varepsilon_0 AE(t) = \varepsilon_0 \boldsymbol{\Phi_E(t)}$$

$$\boldsymbol{i(t)} = \frac{\boldsymbol{dQ(t)}}{\boldsymbol{dt}} = \underbrace{\varepsilon_0 \frac{\boldsymbol{d\Phi_E(t)}}{\boldsymbol{dt}}}_{i_D(t)}$$

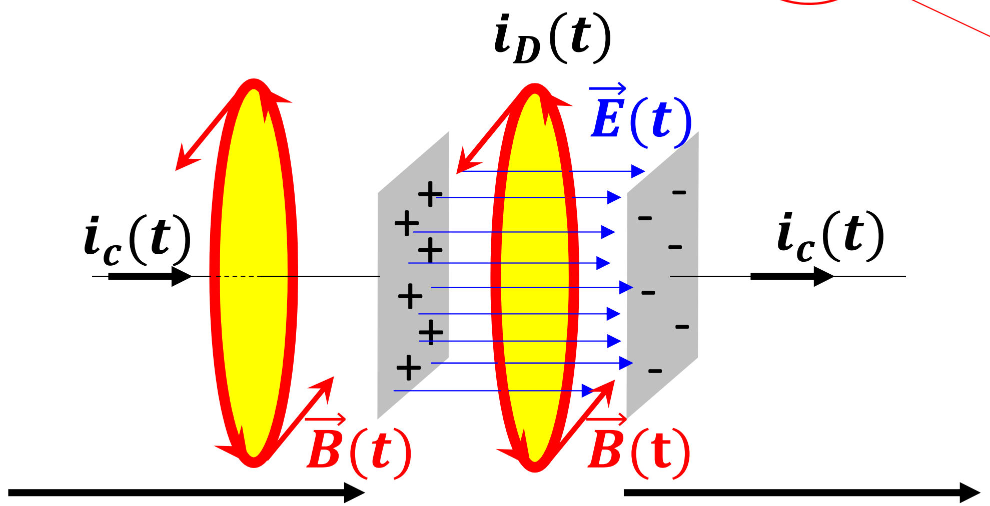
Maxwell's genius idea $\Rightarrow$ $i_D(t) = \varepsilon_0 \dfrac{d\Phi_E(t)}{dt}$

Final step: Fourth Maxwell's equation

$$\oint \vec{B}(t).d\vec{l} = \mu_0 I_{encl} = \mu_0[i_c(t) + i_D(t)]_{encl}$$

$$\mu_0 i_D(t) = \mu_0\varepsilon_0 \frac{d\Phi_E(t)}{dt}$$

c^2



Continuity

$i_c(t) \neq 0$	$i_c(t) = 0$	$i_c(t) \neq 0$
$i_D(t) = 0$	$i_D(t) \neq 0$	$i_D(t) = 0$
$\vec{E}(t) = 0$	$\vec{E}(t) \neq 0$	$\vec{E}(t) = 0$

What is the major outcome of Maxwell's contribution?

Time-varying field of either kind induces a field of the other kind in neighboring region

Waves

Towards the prediction of electromagnetic **disturbances** consisting of $\vec{E}$ and $\vec{B}$ and propagating in free space without requiring any medium

How did Maxwell’s correct Ampere?

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}$$

Ampere’s law …

… completed by Maxwell

$$\oint \vec{B}(t).d\vec{l} = \mu_0 I_{encl} = \mu_0[i_c(t) + i_D(t)]_{encl} \Rightarrow \oint \vec{B}(t).d\vec{l} - \mu_0 i_D(t)_{encl} = \mu_0 i_c(t)_{encl}$$

$$i_D(t) = \varepsilon_0 \frac{d\Phi_E(t)}{dt} \Rightarrow \oint \vec{B}(t).d\vec{l} - \mu_0\varepsilon_0 \frac{d\Phi_E(t)}{dt} = \mu_0 i_c(t)_{encl}$$

Stoke's theorem + Gauss's theorem

$$\oint \vec{B}.d\vec{l} = \iint (\vec{\nabla} \times \vec{B}).d\vec{A}$$

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{d}{dt} \iint \vec{E}.d\vec{A} = \mu_0\varepsilon_0 \iint \frac{d\vec{E}}{dt}.d\vec{A}$$

$$\boxed{\vec{\nabla} \times \vec{B} - \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}}$$

This is all about the story with Maxwell's equations

$$\vec{\nabla}.\vec{E}(t) = \frac{\rho(t)}{\varepsilon_0}$$

$$\vec{\nabla}.\vec{B}(t) = 0$$

No magnetic charges or monopoles

Gauss' law

$$\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

Faraday's law

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law completed by Maxwell

In electric field, the stored energy is:

$$U = \frac{\varepsilon_0}{2} \int E^2 dV \qquad \boxed{\frac{\varepsilon_0}{2} E^2}$$

All space

In a magnetic field, the stored energy stored:

$$U = \frac{1}{2\mu_0} \int B^2 dV \qquad \boxed{\frac{1}{2\mu_0} B^2}$$

All space