And when the charges start moving…

… A <u>magnetic field</u> builds up and adds to the <u>electric field</u>

Superposition principle

Magnetostatic

At constant velocity

We are still considering static phenomena although the charges are moving !!

Concepts

- Electric field
- Permittivity ε_0
- Electric force ON a stationary charge
- Electric field OF a stationary charge
- Coulomb's law
- Gauss's law
- Scalar potential
- Electric dipole

- Magnetic field
- Permeability μ_0
- Magnetic force ON a moving charge
- Magnetic field OF moving charge
- Biot & Savart's law
- Ampere's law
- Vector potential
- Magnetic "dipole"

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = \vec{0}$$

Gauss's law: Flux through a closed surface

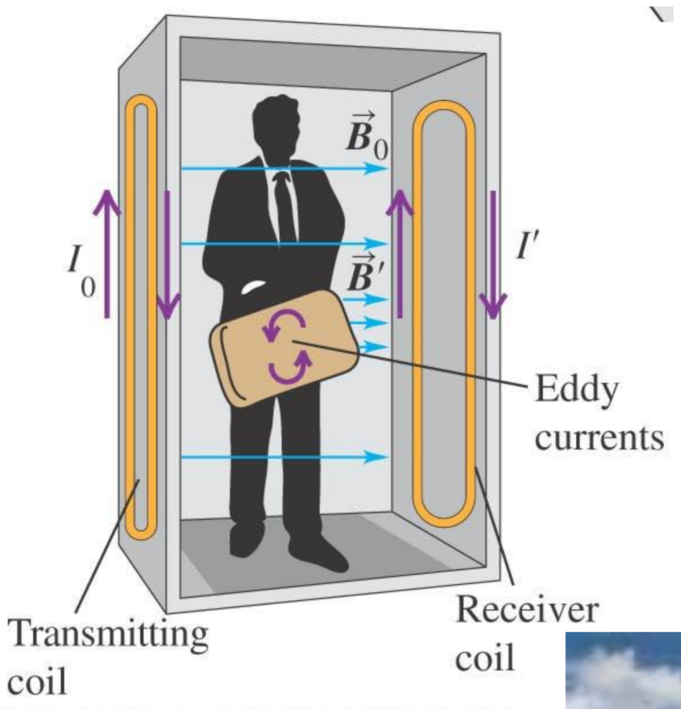
Stokes' theorem: Flux through a surface defined by a closed path

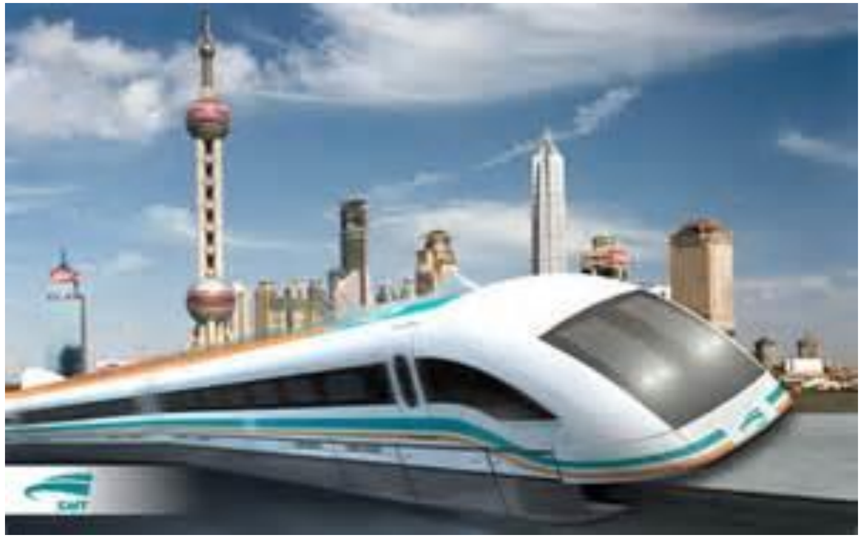
Closed surface $\Leftrightarrow$ path $= 0$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Can we gain any practical use of the magnetic field?





Magnetostatic versus Electrostatic: A perfect correspondence

Electric force

The presence of the electric field is taken as an experimental fact

Coulomb law

How a charge **(moving or not)** creates an electric field

Gauss law

Exploits symmetry in relating $\vec{E}$ to source (charge distribution)

Magnetic force

The presence of the magnetic field is taken as an experimental fact

Biot & Savart law

How a **moving** charge creates a magnetic field

Ampere's law

Exploits symmetry in relating $\vec{B}$ to source (current)

Two types of forces apply to a charge: Lorentz force

- One due to electric field $\vec{E}$:

  The *force* $\vec{F}_E(\vec{r})$depends **ONLY** on where the charge is (whether it is moving or not)

- One due to magnetic field $\vec{B}$:

  The force$\vec{F}_B(\vec{r}, \vec{v})$depends on where the charge is but also on **HOW FAST** it moves with respect to the magnetic field

Superposition principle: In case both fields exist and the test charge is moving

$$\vec{F} = \vec{F}_E(\vec{r}) + \vec{F}_B(\vec{r}, \vec{v})$$

$$\vec{F}(\vec{r}, \vec{v}) = q\left[\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r})\right]$$

$$\vec{F}(\vec{r},\vec{v}) = q\left[\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r})\right]$$

$$\vec{F}(\vec{r},\vec{v}) \;//\; \vec{r} \;//\; \vec{E}(\vec{r})$$

The electric force does work

$$\vec{F}(\vec{r},\vec{v}) \perp \vec{v} \perp \vec{B}(\vec{r})$$

The magnetic force **NEVER** does work

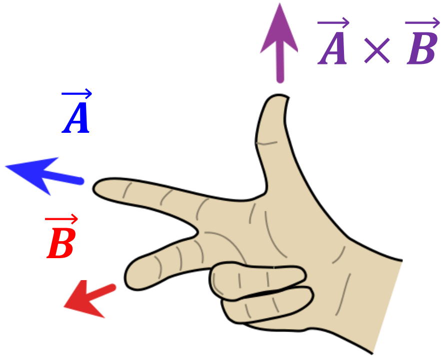
If a charge moves in $\vec{E}$ and $\vec{B}$, the <u>principle of superposition</u> applies

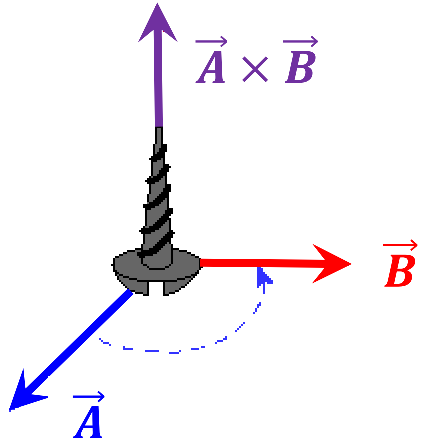
and the charge undergoes the Lorentz force

Main tool to help understanding magnetism

Wire carrying current

Lorentz force on moving charge





Right hand rule

Screwing and unscrewing rule

$$\vec{F}(\vec{r},\vec{v}) = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

$\vec{B} = \vec{0}$

$$\vec{F}(\vec{r}) = q\vec{E}$$

$\vec{B} \neq \vec{0}$

$\vec{v} = \vec{0}$

$$\vec{F}(\vec{r}) = q\vec{E}$$

$\vec{v} \neq \vec{0}$

$\vec{v} \,//\, \vec{B}$

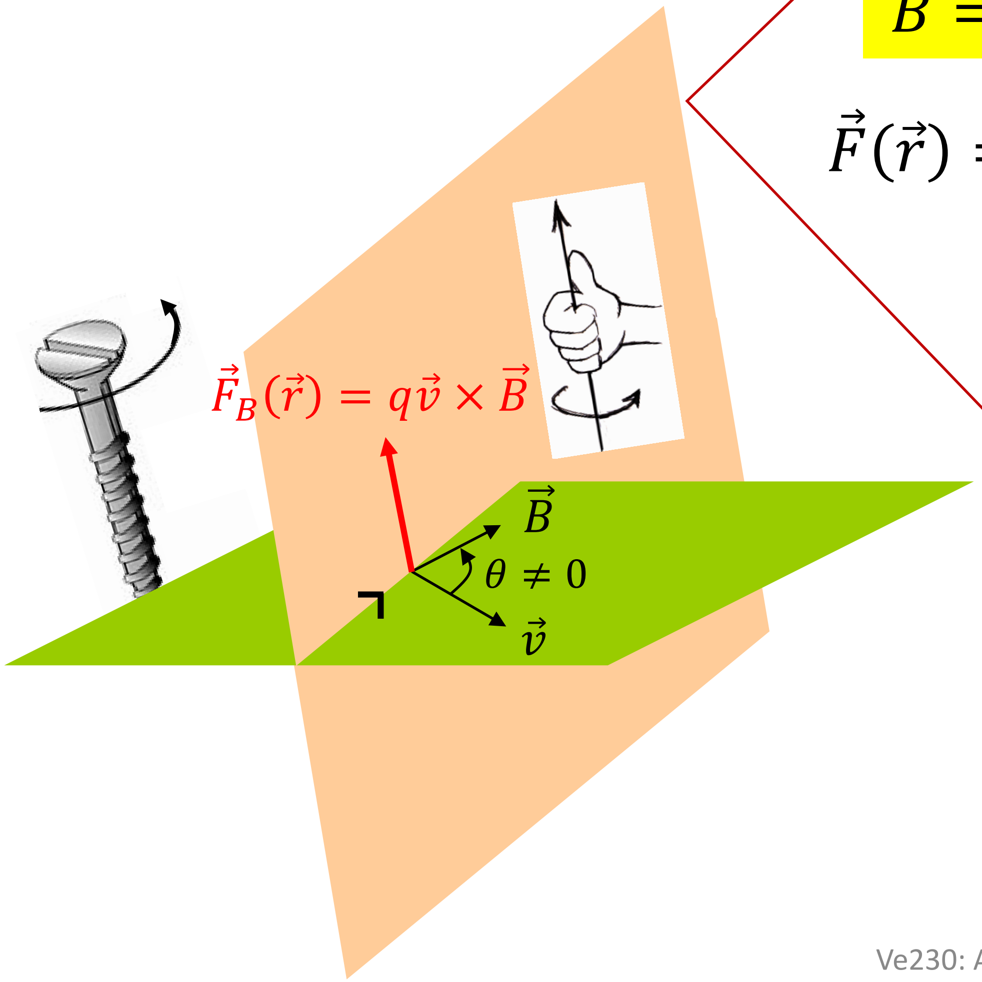
$$\vec{F}(\vec{r}) = q\vec{E}$$

$\vec{B}$

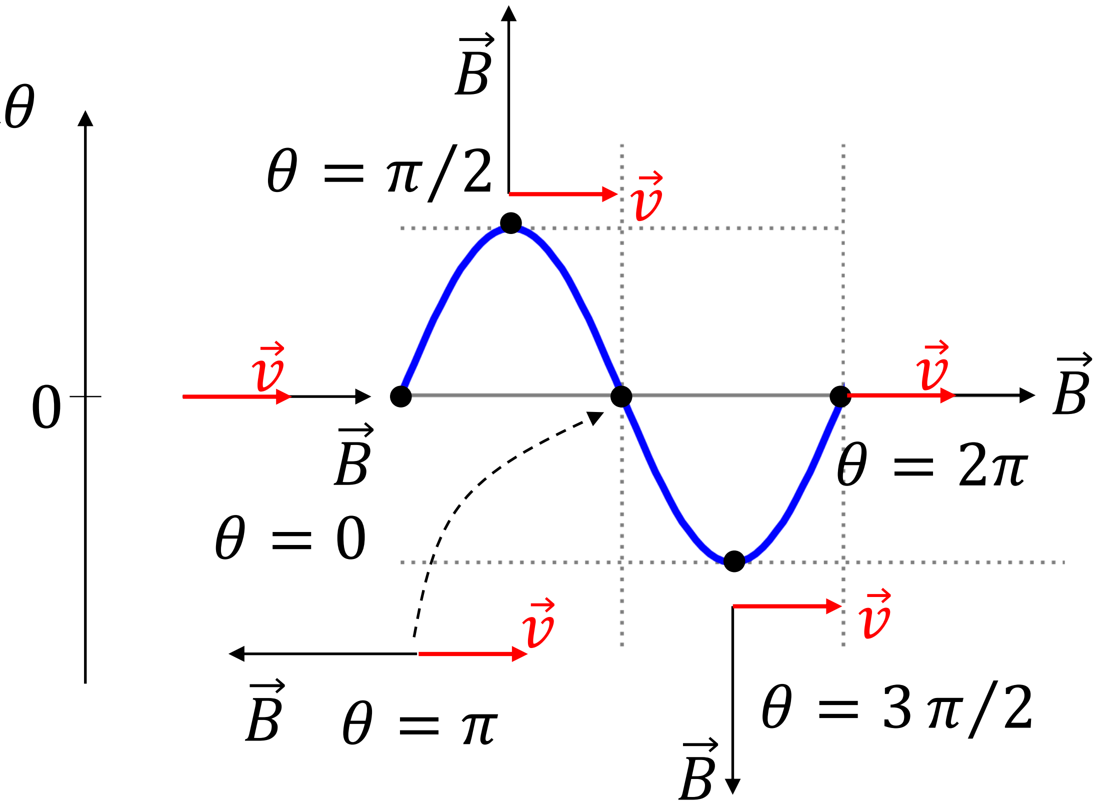
$\theta \neq n\pi$

$\vec{v}$

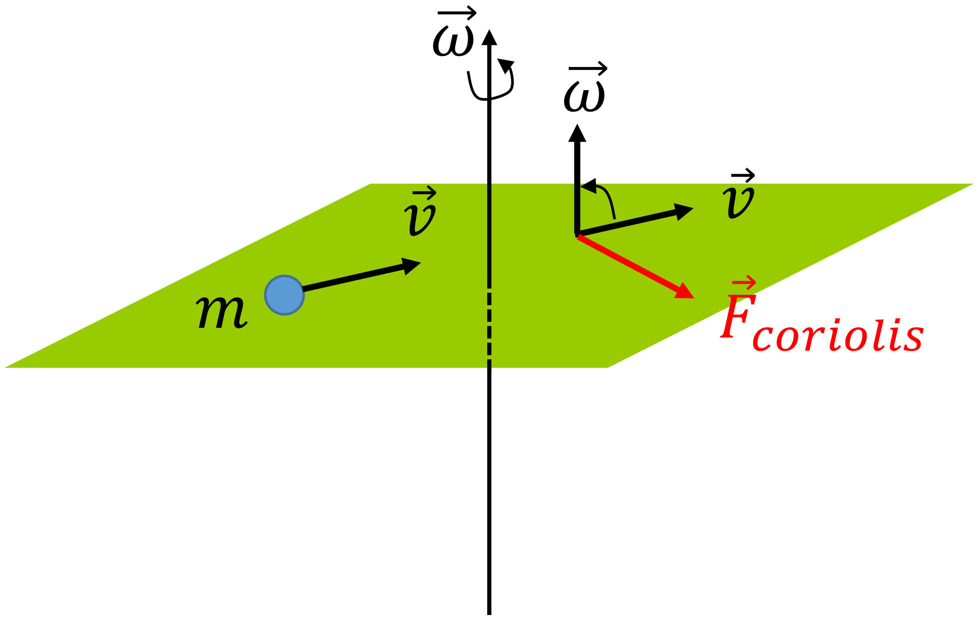
$$\vec{F}(\vec{r},\vec{v}) = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$



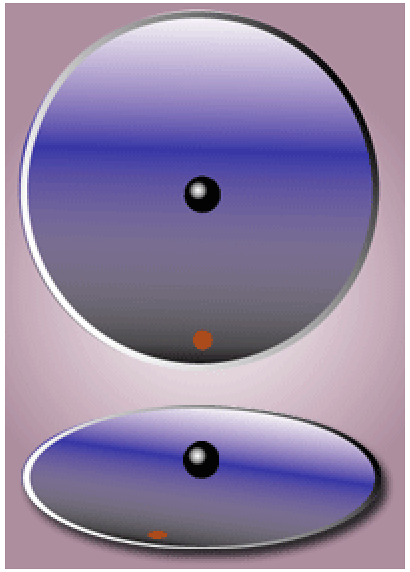
$$F_B = qvB\sin\theta$$



Mechanical equivalence of Lorentz force



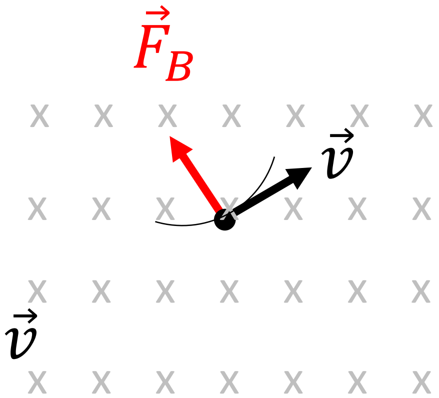
$$\vec{F}_C = 2m\vec{v} \times \vec{\omega}$$



External observer

Observer sitting on the red point

Motion path of a charge in a magnetic field



$\vec{B} \perp \vec{v}$

The magnetic force affects the direction of $\vec{v}$ only **NOT** the speed?

$$dW = \vec{F}_B . d\vec{l} = 0 \qquad \vec{F}_{//} = \vec{0}$$

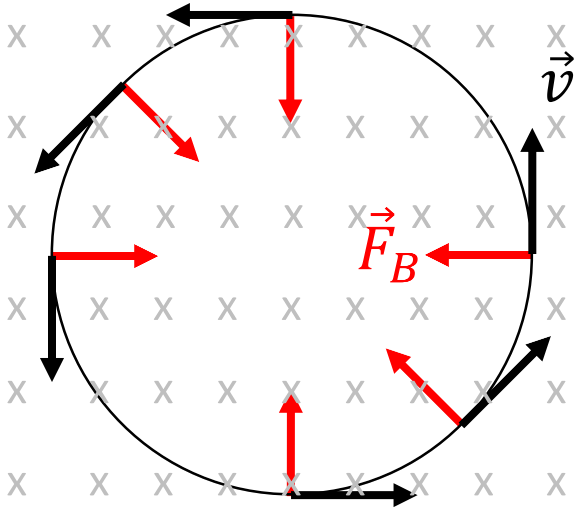
The magnetic force does not do any work on the moving charge

Work - Energy theorem $\Delta K = \Delta W = \vec{F}_B . \Delta\vec{l} = \vec{F}_B .(\vec{v}\Delta t) = 0$

NO work ⇨ **NO** change in kinetic energy ⇨ **NO** change of speed

Consequences:

- Only direction changes
- Particle moves in a circle



$$\vec{F}(\vec{r},\vec{v}) = q\left[\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r})\right]$$

Electric force directed <u>parallel</u> to the vector position

Magnetic force <u>perpendicular</u> to the plane defined by the two vectors $\vec{v}$ and $\vec{B}$

$$[E] = \frac{N}{C} = \frac{V}{m} = \frac{kg\,m\,s^{-2}}{C}$$

$$[B] = \frac{Ns}{Cm} = \frac{Vs}{m^2} = \frac{Weber}{m^2} \rightarrow Flux/unit\ area$$

$$[B] = \frac{F}{qv} = \frac{kg}{As^2} = Gauss = 10^{-4}\ Tesla$$

$(6-7)10^5$ G – Inside an atom and in a medical magnetic resonance imaging machine

50 G – A typical refrigerator magnet

0.25– 0.60 G – The Earth’s magnetic field at its surface

$10^{-9} - 10^{-8}$ G – The magnetic field of the human brain

Question:

- But what exactly is the Magnetic field?

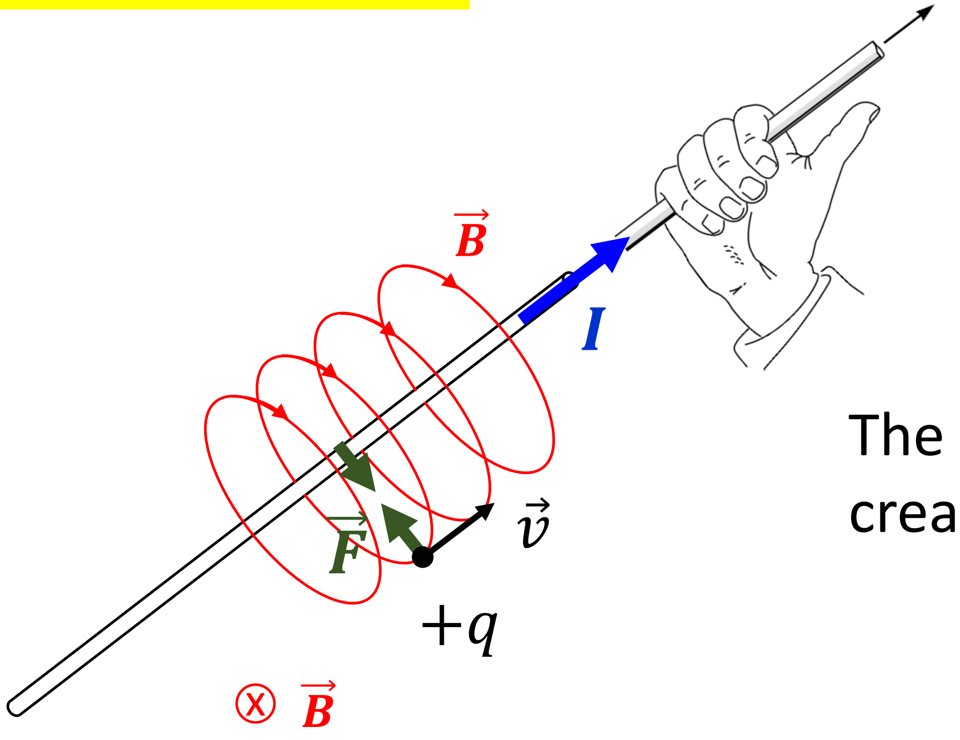
- Why should there exist some field that acts **ONLY** on **moving** charges?

↓

Answer: Special relativity

Postulate: *Physics must be consistent in every "frame of reference"*

Laboratory frame



The magnetic field generated by the current carrying wire creates a Lorentz **attractive force** on the moving charge

$$F = qvB$$

Charge frame: charge is not moving

There can be no magnetic force!

Physics must be consistent in both frames of reference

↓

There must be some attractive force in the charge frame

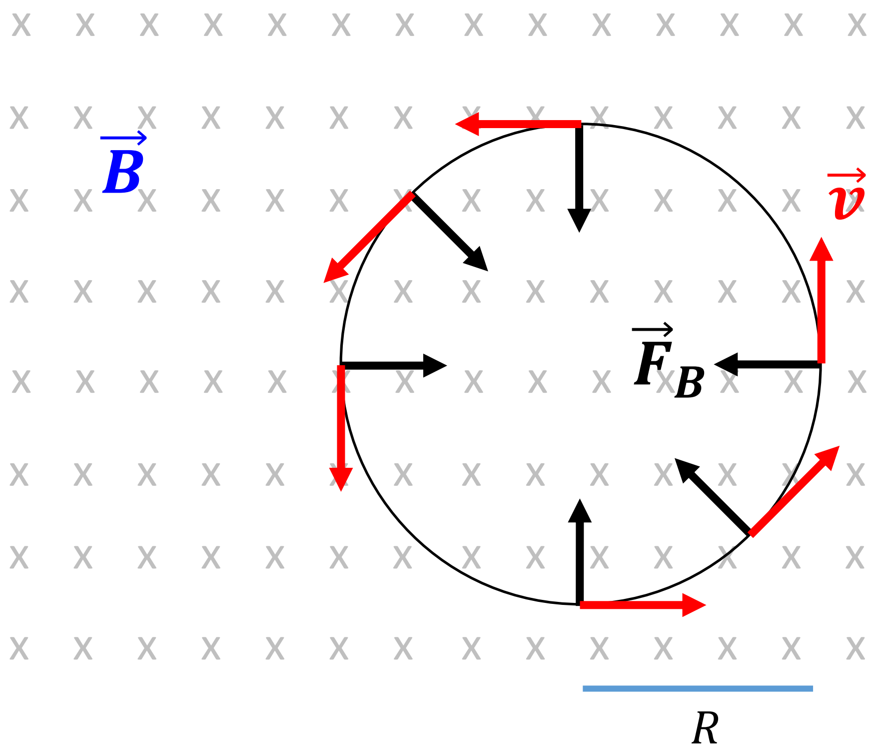
↓

What is this force ???

Special relativity ⇒ Electric $E = vB$ $\boldsymbol{F = qE}$ ⇒ $\boldsymbol{F = qvB}$

Application of the Lorentz force

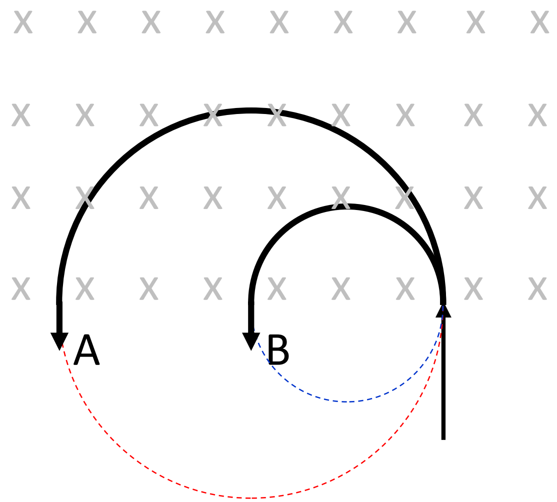
- Mass spectrometer
- Thomson experiment
- Velocity selector



Mass spectrometer

From mechanics

Centripetal force = $\dfrac{mv^2}{R} = qvB$



$$\boxed{R = \frac{mv}{qB}}$$

Two particles A and B with:

- $q_A = q_B$
- $m_A \neq m_B$

enter the magnetic field with the same speed.

What is the relation between mass and curvature?

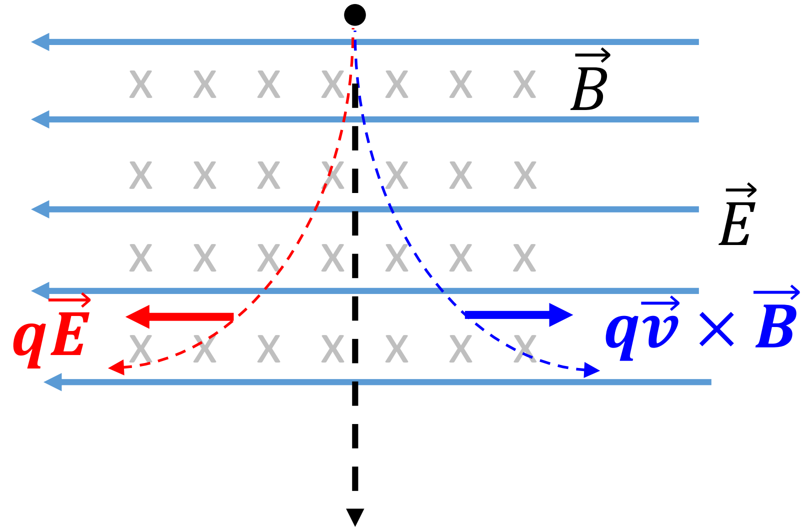
Bigger mass ⇨ larger inertia

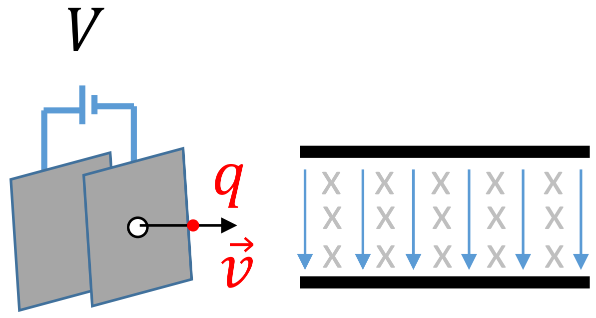
⇨ less acceleration ⇨ Thus larger radius

Question: *Given a charge q moving with a velocity $\vec{v}$, it crosses a magnetic field that we want to measure. How can we proceed ?*

Could be a bonus question !

Thomson experiment
Discovery of the electron





$$\frac{1}{2}mv^2 = eV$$

Adjust the electric field

$$qE = qvB$$

$$v = \sqrt{\frac{2eV}{m}} = \frac{E}{B}$$

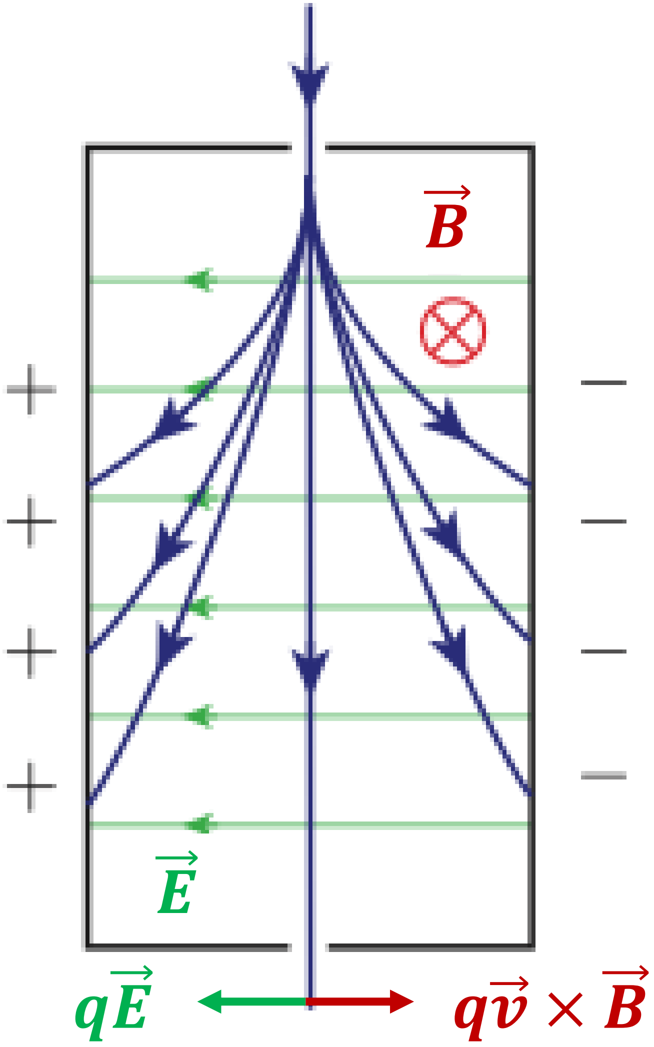
$$\boxed{\frac{e}{m} = \frac{E^2}{2VB^2}}$$

$$\boxed{B = \frac{E}{v}}$$

Does not depend on q !

Velocity selector

Same particles getting in with different velocities



$$v = \frac{E}{B}$$

- By adjusting E and B appropriately, we can select particles with a particular speed to accelerate for other purposes.

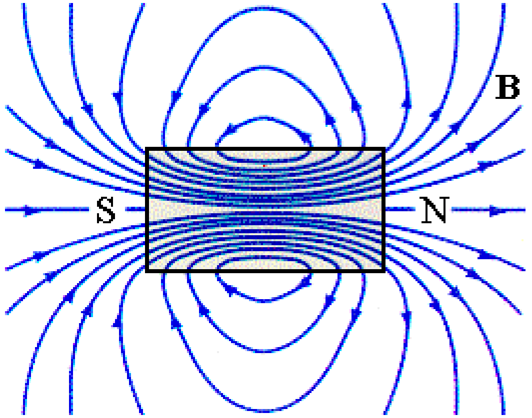
- If all particles have the same mass, then the setup can be used as a source of mono-kinetics particles used for accelerators

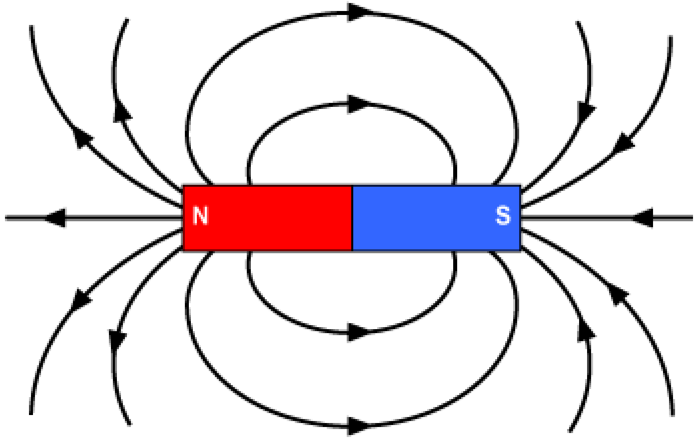
Origin of the magnetic field

- Magnet
- Current carrying wires

Magnet: Magnetic field lines

Which one of these representations is correct ?





- Outside the magnet lines go from N to S
- Inside the magnet lines go from S to N

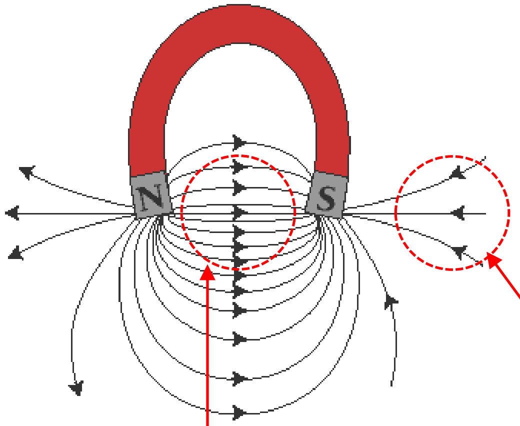
This is not correct

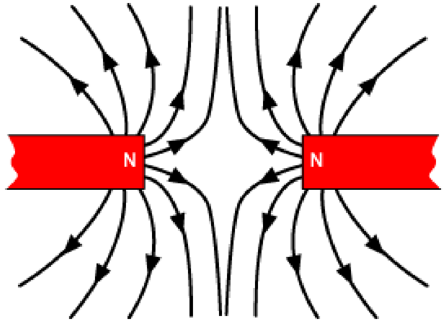
Every magnetic field line forms a loop
There is no start and no end to the field lines

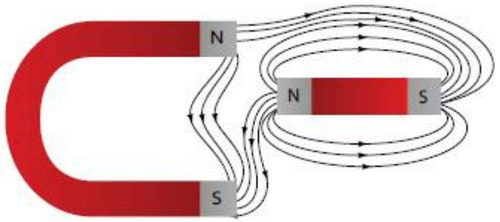
Source and sink for magnetic field lines cannot be separated

As for electrostatic, the magnetic field vectors are tangent to the field lines

As at every point is space the field is unique => Lines **NEVER** cross

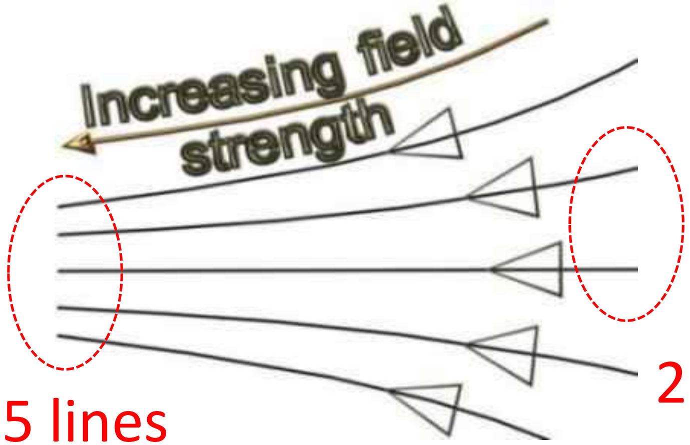






Less field lines
Weak Field

More field lines
Strong Field



5 lines

2 lines

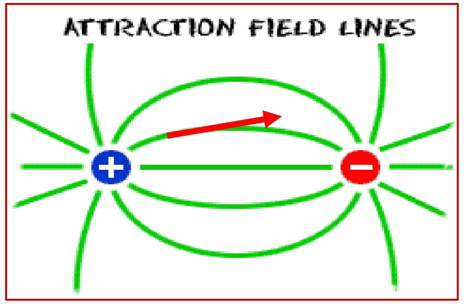
Field lines

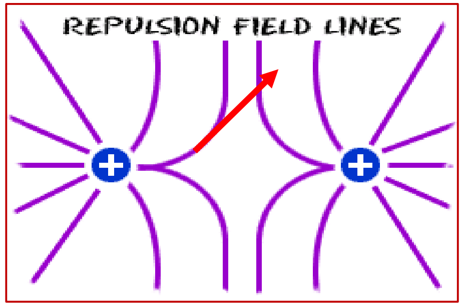
Electric monopole exists
$\boldsymbol{E}$ lines start at **+'s** and end at **–'s**

Magnetic monopole does not exist
$\boldsymbol{B}$ lines form loops

Comparison makes sense for dipoles…**BUT**…

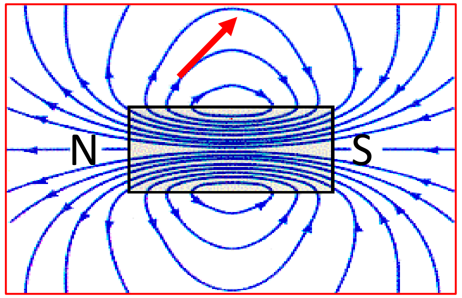
Lines are concave everywhere

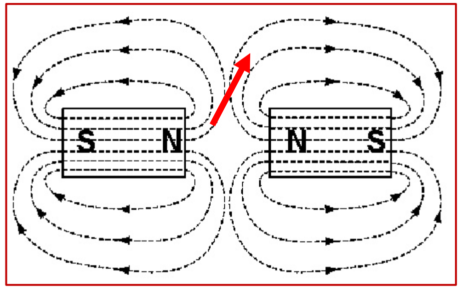




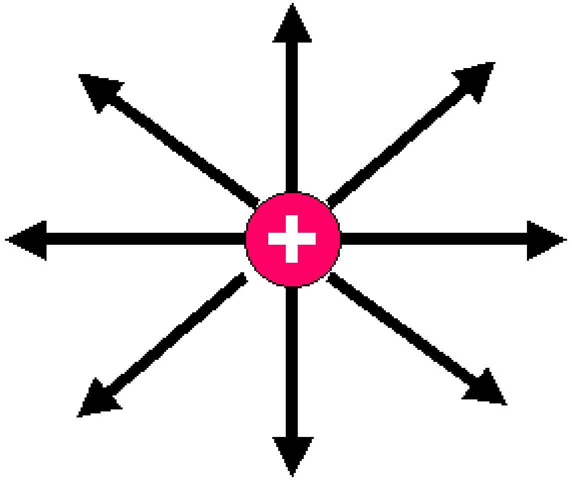
Vector field ($\boldsymbol{E}$ or $\boldsymbol{B}$) is always Tangent to the field line

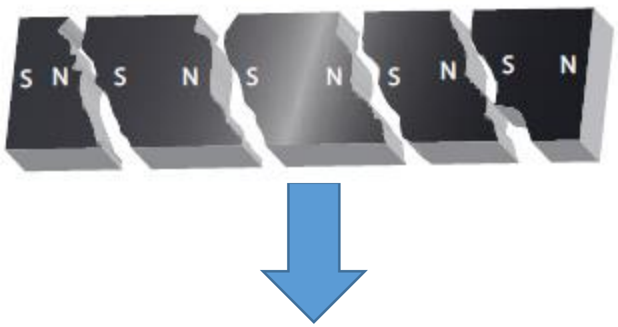
Lines are concave outside magnet and convex inside





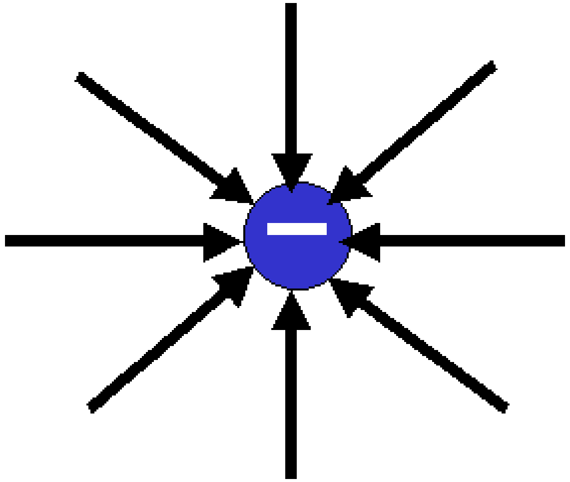
Magnetic flux and Gauss law for magnetism

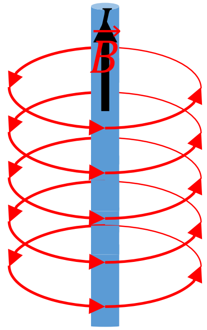




Magnetic monopole
Does not exist

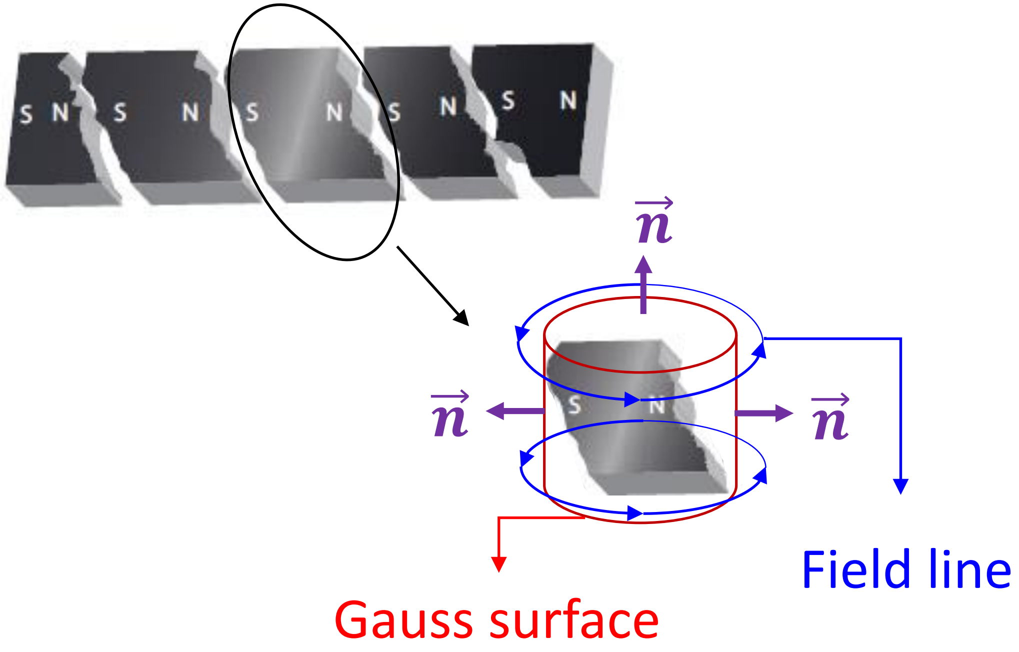
- Charges radiate outward (inward) for $+q$ $(-q)$
- $\vec{E}$ lines have a start $(+q)$ and end $(-q)$





- $\vec{B}$ lines encircles the current
  - $\vec{B}$ lines never end

Magnetic flux and Gauss law for magnetism



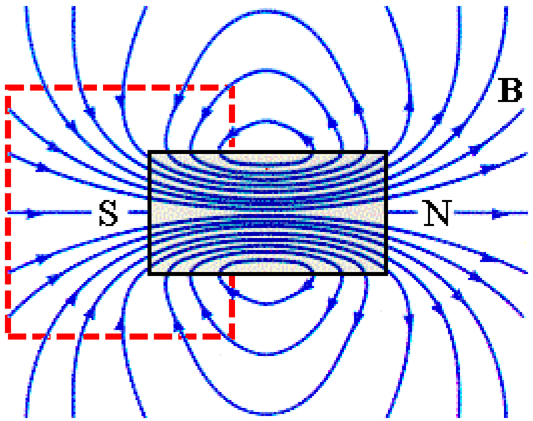
$$\boxed{\emptyset_B = \oint \vec{B}.d\vec{A} = 0}$$

Or Gauss theorem

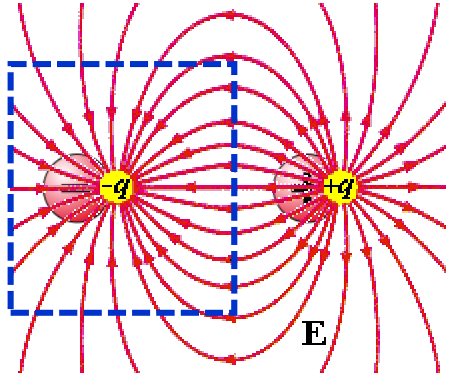
$$\boxed{\vec{\nabla}.\vec{B} = 0}$$

It does not mean that $\vec{B} = \vec{0}$ inside the Gaussian surface

Divergence of the field



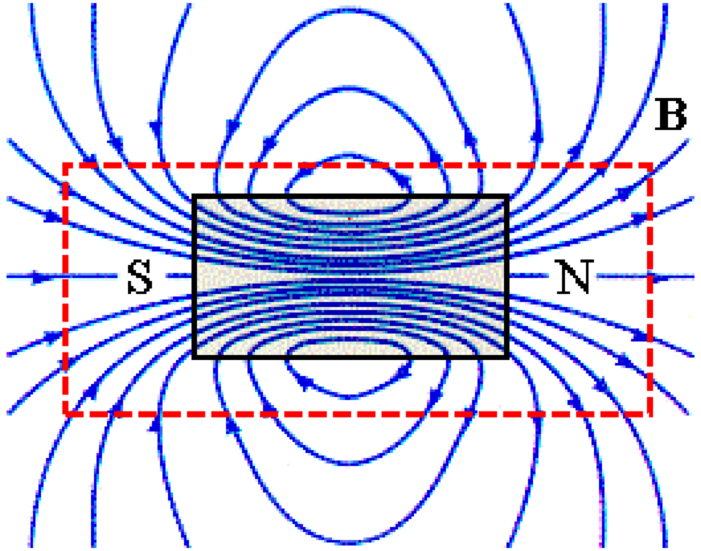
Two of Maxwell's equations

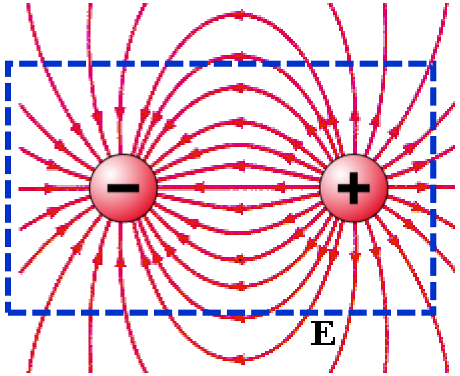


B never diverges: Always loops around the pole

E diverges: Extends to or from infinity to the pole

$\vec{\nabla}.\vec{B} = 0$ ← Gauss Laws → $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$

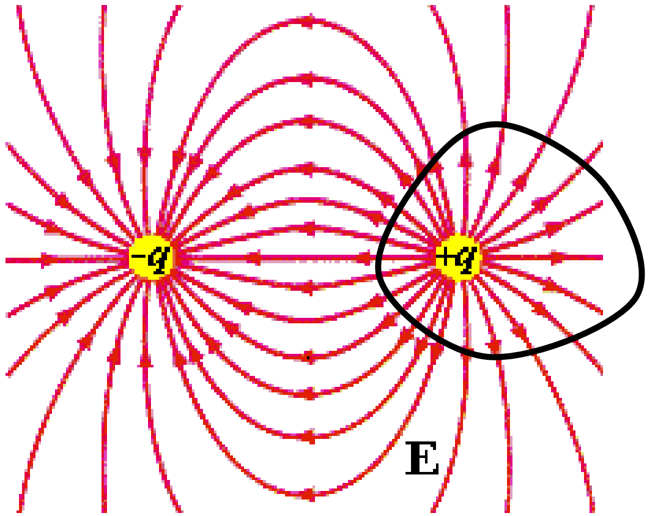




$\vec{\nabla}.\vec{E} = 0$

$\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$

- $\vec{E}$ diverges out of or toward the charges
- $\vec{E}$ is the response to the charges located somewhere
- ρ is the source of $\vec{E}$ like a source of fluid
- ε_0 is the ability of vacuum to adapt the field lines in response to the charge source



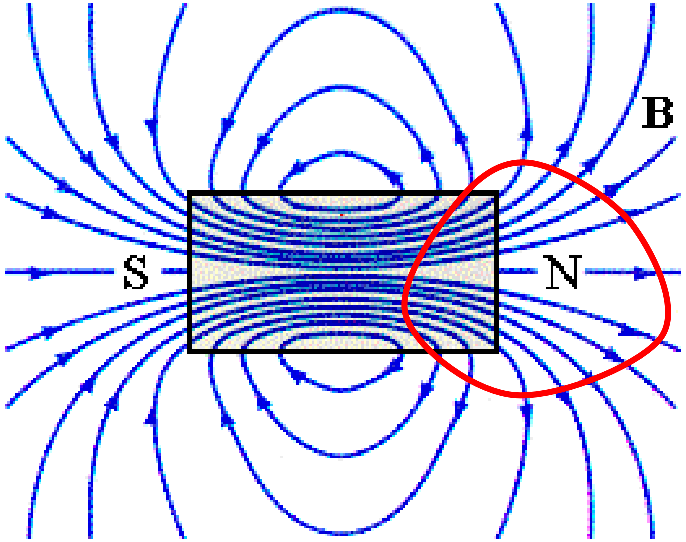
Inside this closed path the lines are all going out of the charge $+q$ and they are all going toward $-q$

With a magnet things are fundamentally different.

$\vec{\nabla}.\vec{B} = 0$

$\vec{B}$ has no isolated source

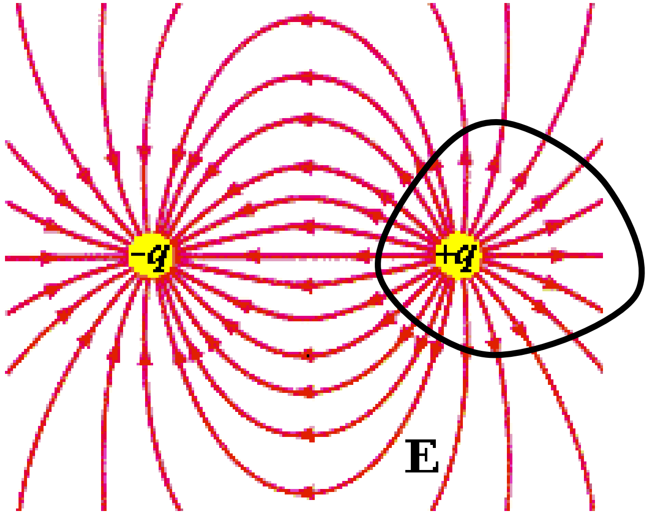
$\vec{B}$ lines have no start and no end



The area inside the closed path is divided into two regions:

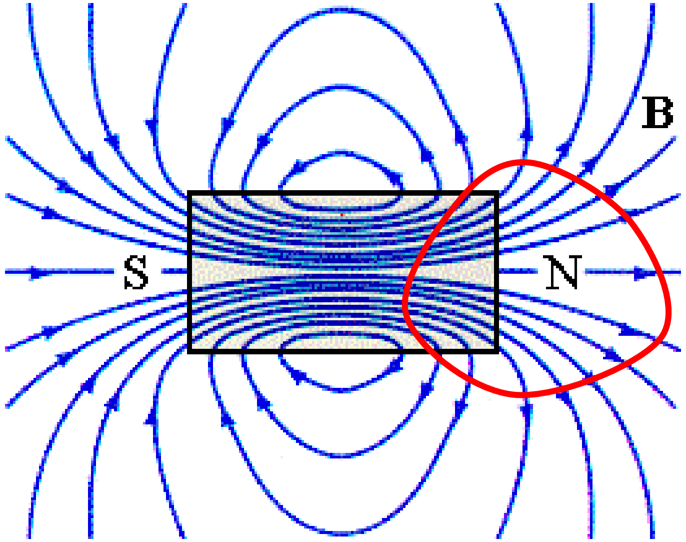
- Region I outside the magnet: The lines are all going out of the North pole
- Region II inside the magnet: The lines are back to the North pole coming from the South pole

The density of field lines ⇔ Strength of field



Source and sink are independent

Huge difference



Source and sink are **NOT** independent

Current as a source of magnetic field

Biot & Savart's law

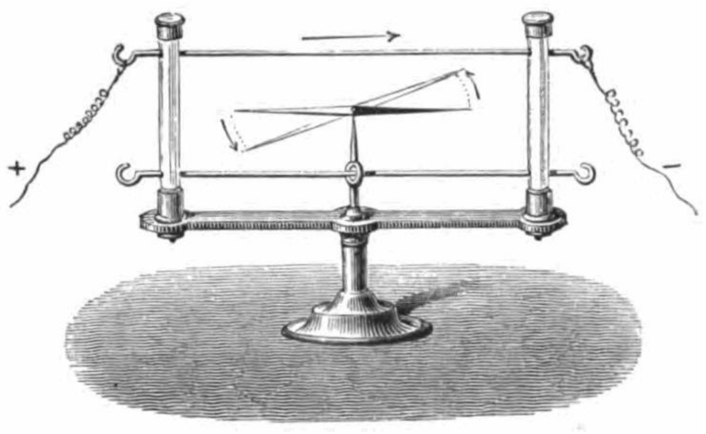
What was known at the time of Biot & Savart?

Coulomb's law (1785)

$$\vec{E} = \frac{1}{4\pi\boldsymbol{\varepsilon_0}}\frac{q}{r^2}\vec{e}_r$$

- $\vec{E}$ is along the unit vector $\vec{\boldsymbol{e}}_{\boldsymbol{r}}//\vec{\boldsymbol{r}}$
- $\vec{E}$ is proportional to the charge $\boldsymbol{q}$
- $\vec{E}$ is inversely proportional to the charge $\boldsymbol{r^2}$
- $\vec{E}$ is inversely proportional to the permittivity $\boldsymbol{\varepsilon_0}$

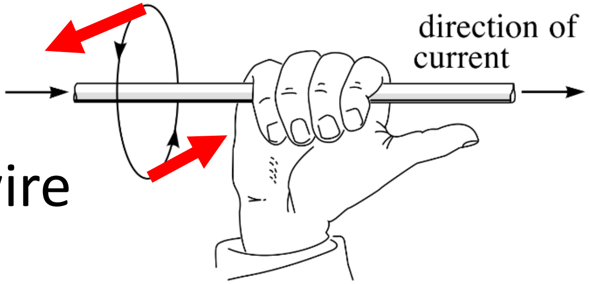
1820 Oersted experiment: experimental facts on the magnetic field



Charges set into motion generate a magnetic field

Magnetic force or field

Compass above the wire



Compass below the wire

Inverting the direction of motion of the charges inverts the direction of the magnetic field

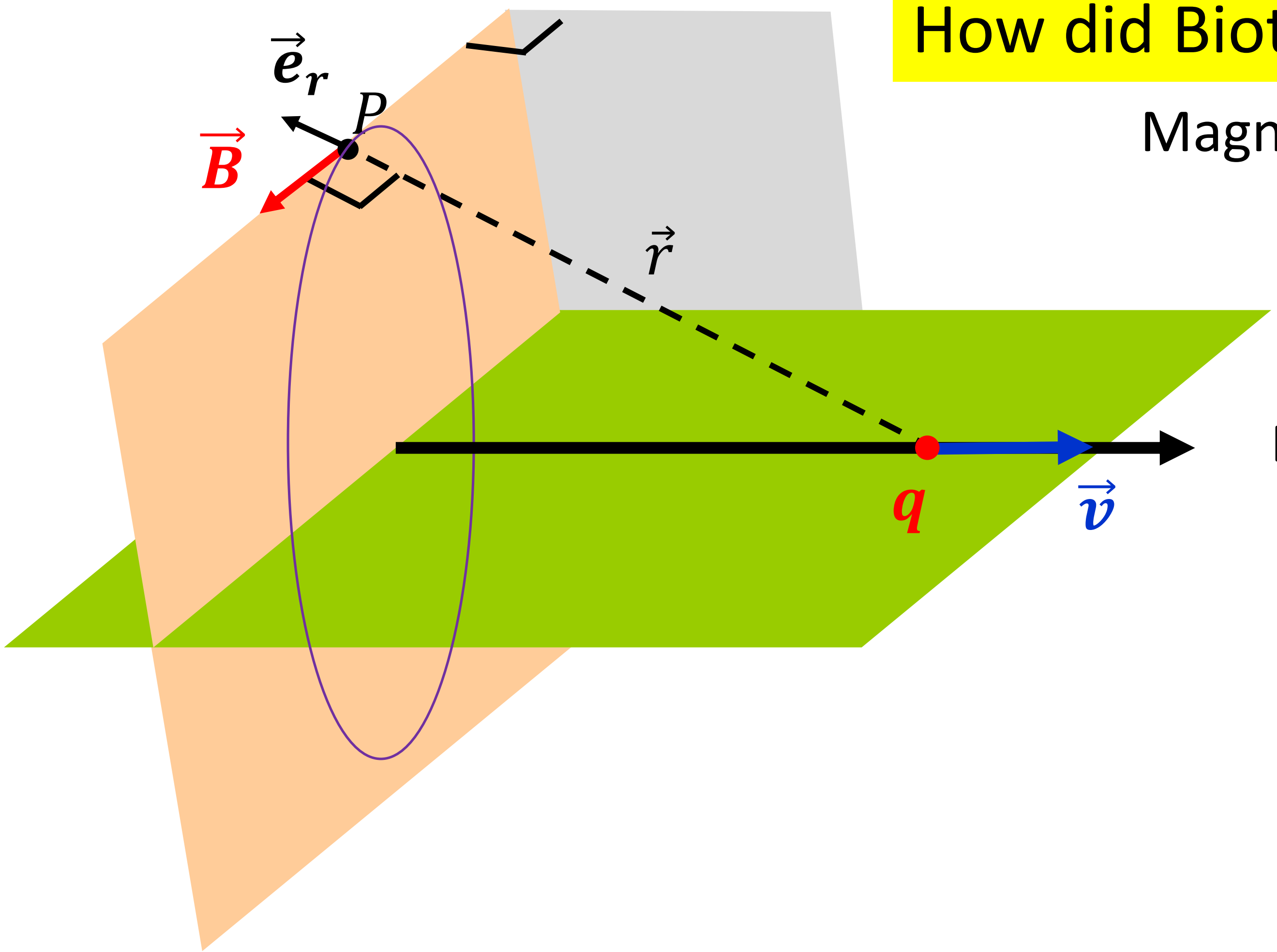
1820 Biot & Savart law: linking charge motion to magnetic field

What are the ingredients determining the law resulting from Orsted's observation?

- $\vec{B}$ is proportional to the velocity $\vec{\boldsymbol{v}}$
- $\vec{B}$ is perpendicular to the plane defined by $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{e}}_{\boldsymbol{r}}$
  (The magnetic field is thus proportional to $\vec{\boldsymbol{v}} \times \vec{\boldsymbol{e}}_{\boldsymbol{r}}$)
- $\vec{B}$ is proportional to the moving charge $\boldsymbol{q}$
- $\vec{B}$ is inversely proportional to the distance to the test point $\boldsymbol{r}^{\mathbf{2}}$
- $\vec{B}$ is proportional to the permeability μ_0 (constant)

How did Biot & Savart to establish their law?

Magnetic field of a moving charge



Direction of motion of the charge

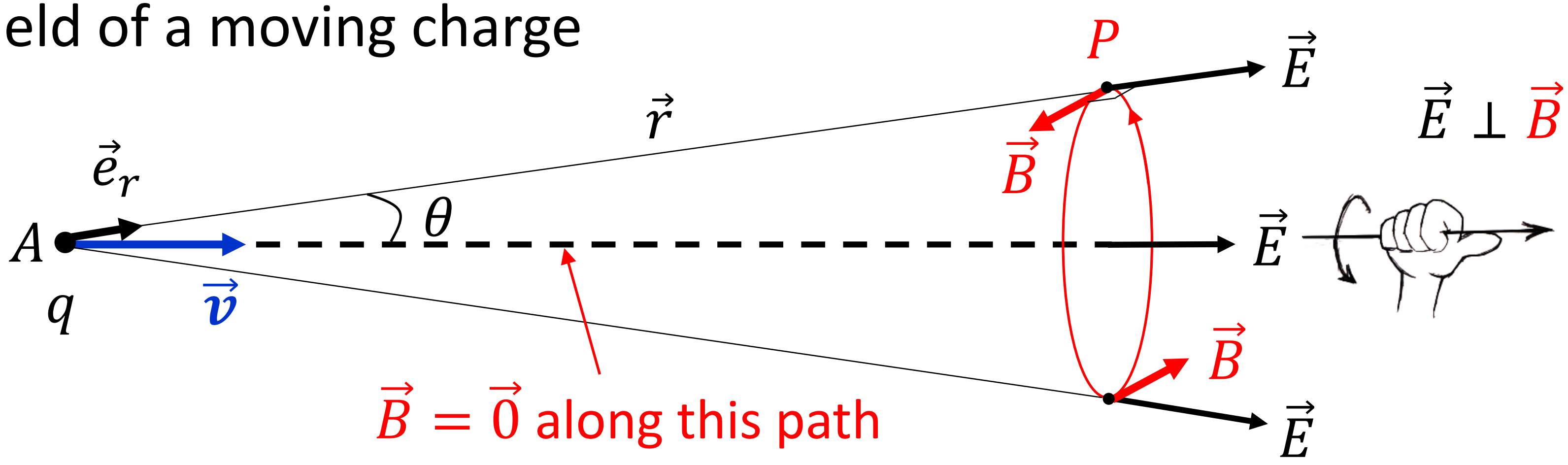
$\vec{B}$ is perpendicular to the plan $(\vec{v}, \vec{e}_r)$

$\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{e}_r$

$\vec{B}$ must result from a cross product of $\vec{v}$ and $\vec{e}_r$

Origin of the magnetic field: some more imagination

Magnetic field of a moving charge



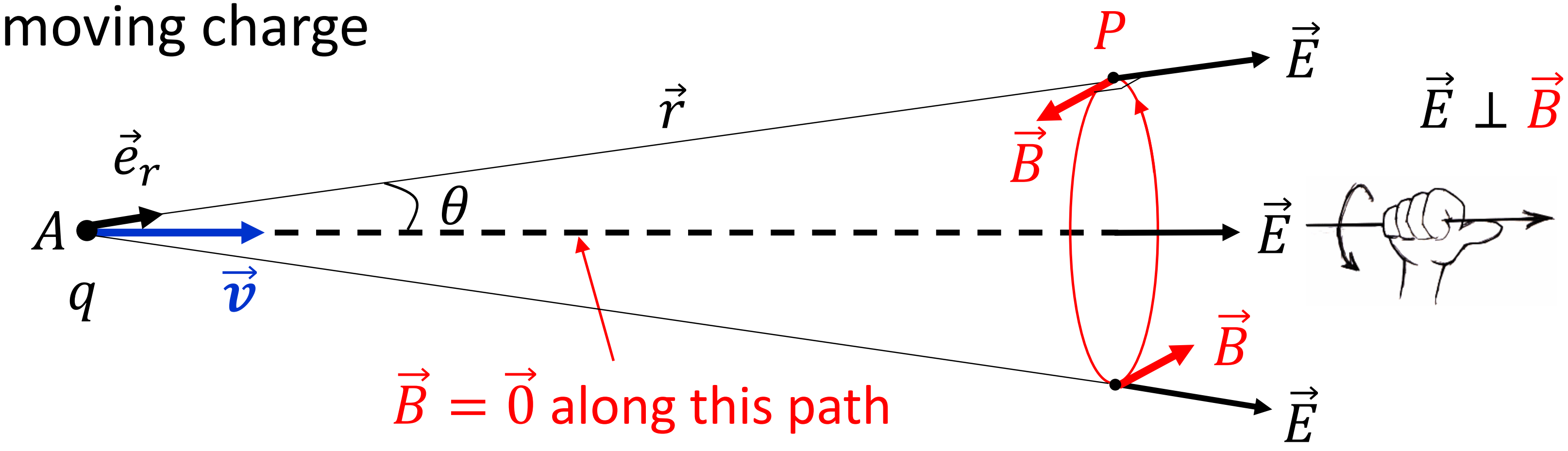
- $\vec{B}$ must decrease with increasing r

Why not following $1/r^2$ as in coulomb law !

- $\vec{B}$ must result from a cross product of $\vec{v}$ and $\vec{e}_r$
- As for the electric field, there must be a permeability of $\vec{B}$ in vacuum: $\boldsymbol{\mu_0}$

Origin of the magnetic field

Magnetic field of a moving charge



Coulomb's law

Biot & Savart's law

Electrostatic

$$\vec{E} = \frac{1}{4\pi \boldsymbol{\varepsilon_0}} \frac{q}{r^2} \boxed{\vec{e}_r}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} F/m$$

$\vec{E}$ is along the unit vector $\vec{e}_r$

Magnetostatic

$$\vec{B} = \frac{\boldsymbol{\mu_0}}{4\pi} \frac{q}{r^2} \boxed{\vec{v} \times \vec{e}_r}$$

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

$\vec{B}$ is perpendicular to the plan $(\vec{v}, \vec{e}_r)$

Origin of the magnetic field

Electric and Magnetic fields produced by a steady moving charge

Electrostatic

$$\vec{E} = \frac{1}{4\pi\boldsymbol{\varepsilon_0}}\frac{q}{r^2}\vec{e}_r$$

Magnetostatic

$$\vec{B} = \frac{\boldsymbol{\mu_0}}{4\pi}\frac{q}{r^2}\vec{v}\times\vec{e}_r$$

Velocity of the source

$$\frac{E}{B} = \frac{1}{\varepsilon_0\mu_0 v}$$

Maxwell

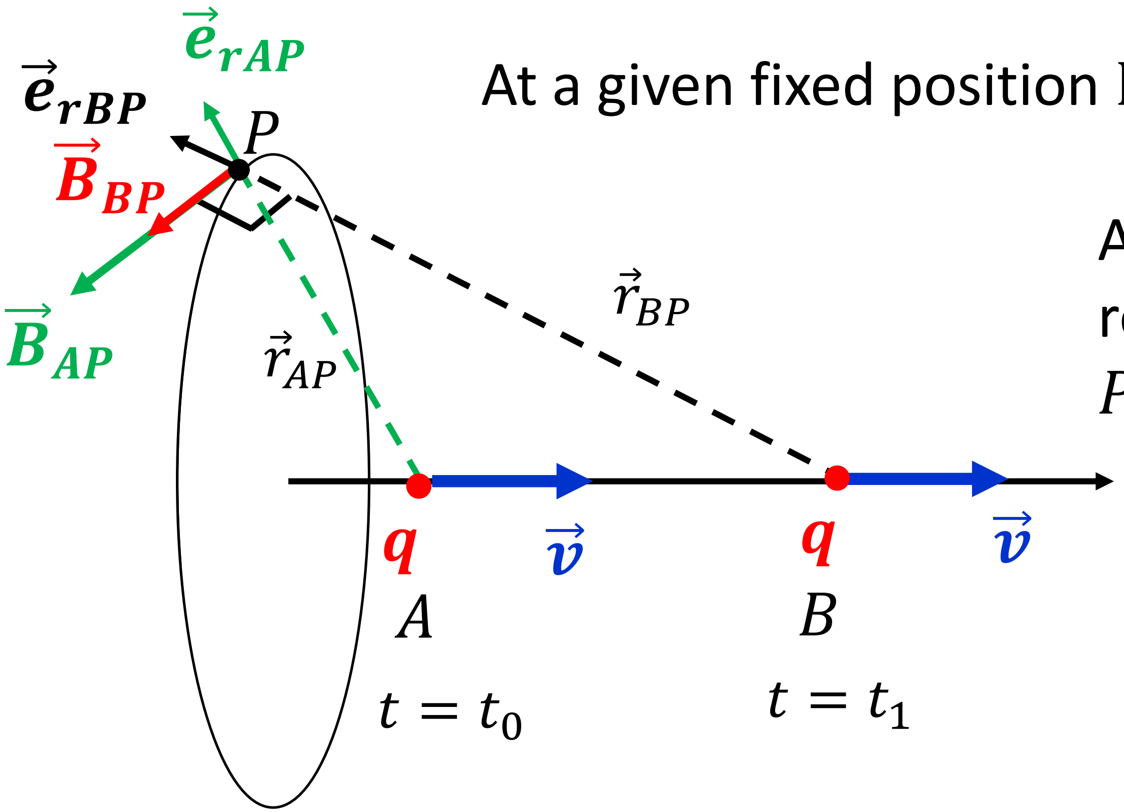
$$\frac{1}{c^2} = \varepsilon_0\mu_0$$

$$\boldsymbol{\frac{E}{B} = \frac{c^2}{v}}$$

If the source of $\vec{E}$ and $\vec{B}$ is EM waves

$$\boldsymbol{\frac{E}{B} = c}$$

Conceptual difficulty with a single moving charge

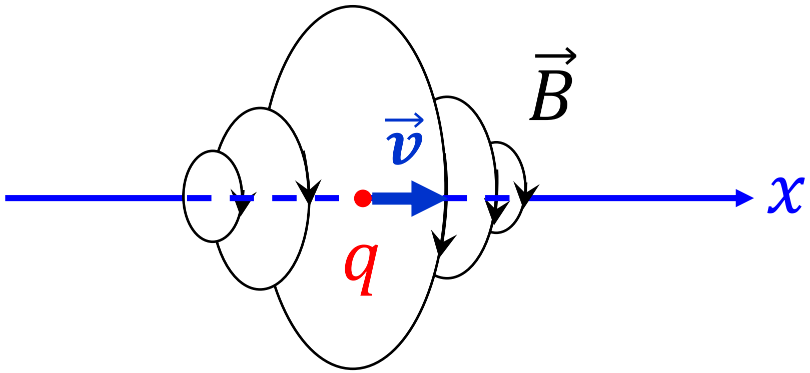


At a given fixed position $\mathrm{P}[\vec{r}(t)]$ → The field is ***non-stationary***

As soon as the particle starts moving from A at t_0 reaching B at later time t_1, The field at position P has changed intensity as $\vec{r}_{AP}(t_0) \neq \vec{r}_{BP}(t_1)$

$$\vec{B}[\vec{r}(t)] = \frac{\mu_0}{4\pi} \frac{q}{\vec{r}(t)^2} \vec{v} \times \vec{e}_r$$

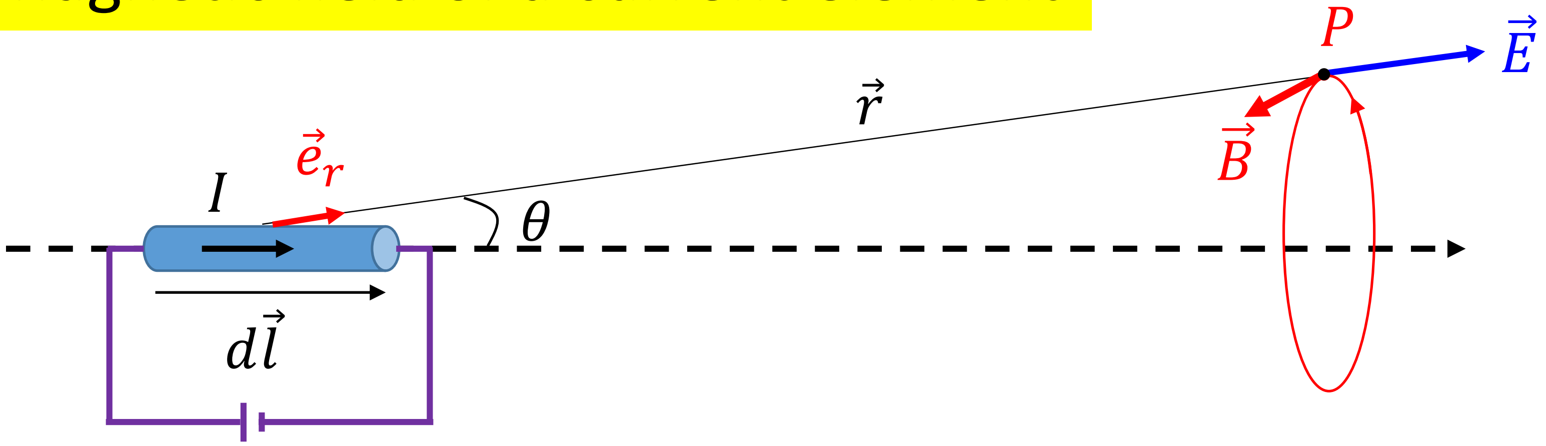
$\boxed{\vec{B}(t) \text{ at } P \text{ is time dependent}}$



The process is no longer magneto<u>static</u>

Magnetic field of a current element

Biot and Savart law: 1820



$$dQ = \rho dV = nqAdl$$

$\vec{v}$ = drift velocity

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{dQ}{r^2}\vec{v}\times\vec{e}_r \Rightarrow d\vec{B} = \frac{1}{4\pi\mu_0}\frac{nqAdl}{r^2}\vec{v}\times\vec{e}_r \Rightarrow d\vec{B} = \frac{1}{4\pi\mu_0}\frac{\overbrace{nqAv}^{I}}{r^2}d\vec{l}\times\vec{e}_r$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}d\vec{l}\times\vec{e}_r}$$

For a complete circuit

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I}{r^2}d\vec{l}\times\vec{e}_r}$$

Flux of magnetic field and Gauss's theorem

Demonstration of $\vec{\nabla}.\vec{B} = 0$ and $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ based on Biot & Savart law

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I}{r^2}\,d\vec{l} \times \vec{e}_r$$ With $$\vec{e}_r \to \frac{\vec{\mathrm{r}} - \vec{\mathrm{r}}'}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|} \qquad \frac{1}{r^2} \to \frac{1}{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')^2} \qquad d\vec{l} \to \mathrm{d}\vec{l'}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\boxed{\mathrm{I}}}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}\,\mathrm{d}\vec{l'} \times (\vec{\mathrm{r}} - \vec{\mathrm{r}}') = \frac{\mu_0 \mathrm{I}}{4\pi}\int \mathrm{d}\vec{l'} \times \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}$$

$\vec{\nabla}.\vec{B}(\vec{r})$?

Steady current $\to \frac{dI}{dt} = 0$

$$\vec{\nabla}.\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\vec{\nabla}.\left(\int \mathrm{d}\vec{l'} \times \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}\right) = \frac{\mu_0 I}{4\pi}\int \vec{\nabla}.\left[d\vec{\mathrm{l}'} \times \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}\right]$$

$\vec{\nabla}$ acts on $\vec{r}$ only

$$\vec{\nabla}.\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\vec{\nabla}.\left(\int d\vec{l'} \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right) = \frac{\mu_0 I}{4\pi}\int \vec{\nabla}.\left[d\vec{l'} \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right]$$

$$\vec{U} = d\vec{l'} \qquad \vec{V} = \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{\nabla}.(\vec{U}\times\vec{V}) = \vec{V}.(\vec{\nabla}\times\vec{U}) - \vec{U}.(\vec{\nabla}\times\vec{V})$$

$$\vec{\nabla}.\left(d\vec{l'} \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right) = \cancel{\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}.\vec{\nabla}\times d\vec{l'}} - \boxed{d\vec{l'}.\left(\vec{\nabla}\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right)}$$

This acts on r'

This acts on r

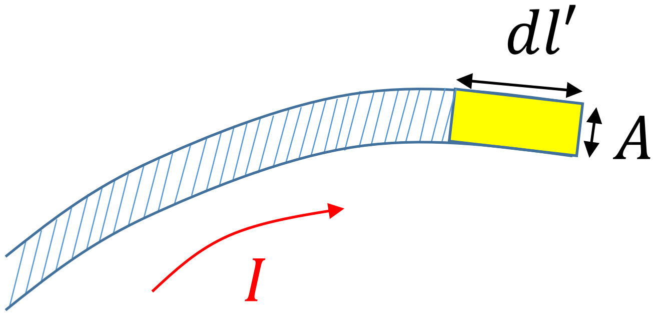
$$\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = -\vec{\nabla}\frac{1}{\vec{r}-\vec{r}'}$$

$$\cancel{-d\vec{l'}.\left(\vec{\nabla}\times\vec{\nabla}\frac{1}{\vec{r}-\vec{r}'}\right)}$$

$$\boxed{\vec{\nabla}.\vec{B} = 0}$$

There are **NO** independent sources or sinks for magnetic fields

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$



$$I\vec{dl'} = \vec{J}(\vec{r}')A.dl'$$
$$= \vec{J}(\vec{r}')d\mathcal{V}'$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\vec{\nabla} \times \left(\int \mathrm{d}\vec{l'} \times \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}\right)$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \underbrace{\vec{\nabla} \times \left(\vec{J}(\vec{r}') \times \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}\right)} d\mathcal{V}'$$

$$\vec{U} = \vec{J}(\vec{r}')$$

$$\vec{V} = \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{|\vec{\mathrm{r}} - \vec{\mathrm{r}}'|^3}$$

$$\vec{\nabla} \times (\vec{U} \times \vec{V}) = (\vec{V}.\vec{\nabla})\vec{U} - (\vec{U}.\vec{\nabla})\vec{V} + \vec{U}(\vec{\nabla}.\vec{V}) - \vec{V}(\vec{\nabla}.\vec{U})$$

$$\vec{V}.\vec{\nabla} = V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y} + V_z\frac{\partial}{\partial z}$$

$$\vec{\nabla} \times (\vec{U} \times \vec{V}) = (\vec{V}.\vec{\nabla})\vec{U} - (\vec{U}.\vec{\nabla})\vec{V} + \vec{U}(\vec{\nabla}.\vec{V}) - \vec{V}(\vec{\nabla}.\vec{U})$$

$$\rightarrow \cancel{\left(\underbrace{\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}_{\text{Acting on } r}.\vec{\nabla}\right)\vec{J}(\vec{r}')} \uparrow = \vec{0}$$

$$\rightarrow \vec{V}\left(\cancel{\vec{\nabla}.\vec{J}(\vec{r}')}\right) = \vec{0}$$

$\uparrow$

Acting on r

$$-\left(\vec{J}(\vec{r}').\vec{\nabla}\right)\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{U}(\vec{\nabla}.\vec{V}) \rightarrow \vec{J}(\vec{r}')\vec{\nabla}.\frac{(\vec{r}-\vec{r}')}{(\vec{r}-\vec{r}')^3} = 4\pi\vec{J}(\vec{r}')\delta(\vec{r}-\vec{r}')$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int -(\vec{J}(\vec{r}').\vec{\nabla}) \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d\mathcal{V}' + \mu_0 \int \vec{J}(\vec{r}')\delta(\vec{r} - \vec{r}') d\mathcal{V}'$$

$$= \boxed{-\frac{\mu_0}{4\pi} \int \cancel{\underbrace{(\vec{J}(\vec{r}').\vec{\nabla}) \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}}_{-(\vec{J}(\vec{r}').\vec{\nabla}') \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}}} d\mathcal{V}'} + \mu_0 \vec{J}(\vec{r})$$

For steady current

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t} = 0$$

$$\boxed{\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})}$$

Ampere's law