VP390 Problem Set 8

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1 Problem 1

$\langle v^2 \rangle = \int_0^\infty v^2 f(v)\mathrm{d}v = \int_0^\infty 4\pi v^4 (\frac{m}{2\pi k_B T})^{3/2} \exp(-\frac{mv^2}{2k_B T})\mathrm{d}v = \int_0^\infty v^2 f(E)\mathrm{d}E = \int_0^\infty \frac{2E}{m} f(E)\mathrm{d}E$

$$\mathbf{Pr}(|0\rangle^{\bigotimes n-1}) = |\frac{1}{2^n}\sum_{x=0}^{2^{n-1}}(-1)^{f(x)}|^2$$

$\langle v^2 \rangle = \frac{2}{m}\langle E \rangle = \frac{3}{m} k_B T$

2 Problem 2

(a) We can use the Maxwell distribution for the energy:
$n(E_i) = 2^{(i+1)(i+2)/2}\frac{1}{Z}\exp(-\frac{E_i}{k_B T})$
$\frac{n(E_3)}{n(E_1)} = \frac{1024}{3}\exp(-12.1/0.026) \approx 0$
$\frac{n(E_2)}{n(E_1)} = \frac{64}{3}\exp(-10.2/0.026) \approx 0$

(b) $Z = \sum_i e^{-E_i/k_B T} \approx e^{-E_1/k_B T} + e^{-E_2/k_B T} + e^{-E_3/k_B T}$
Assume $k_B T = \alpha$, then $Z = \frac{1}{e^{13.6/\alpha} + e^{3.4/\alpha} + e^{1.5/\alpha}}$
$f_B(E_2) = \frac{e^{3.4/\alpha}}{e^{13.6/\alpha} + e^{3.4/\alpha} + e^{1.5/\alpha}} = 0.01 \to \alpha = 2.22 \to T \approx 25784K$

(c) $f(E_3) = \frac{e^{1.5/\alpha}}{e^{13.6/\alpha} + e^{3.4/\alpha} + e^{1.5/\alpha}} \approx 4 \times 10^{-3}$

3 Problem 3

(a) $f(u)\mathrm{d}u = C\exp(-\frac{Au^2}{k_B T})\mathrm{d}u$
$\int_{-\infty}^{\infty} f(v)\mathrm{d}v = \int_{-\infty}^{\infty} C\exp(-\frac{Au^2}{k_B T})\mathrm{d}u = C\sqrt{\frac{\pi k_B T}{A}} = 1$
$C = \sqrt{\frac{A}{\pi k_B T}}$

(b) $\langle E \rangle = \langle Au^2 \rangle = \int_{-\infty}^{\infty} Au^2 \sqrt{\frac{A}{\pi k_B T}}\exp(-\frac{Au^2}{k_B T})\mathrm{d}u$
$2A\sqrt{\frac{A}{\pi k_B T}}\int_0^\infty u^2\exp(-\frac{Au^2}{k_B T})\mathrm{d}u = 2A\sqrt{\frac{A}{\pi k_B T}}\frac{1}{4}\sqrt{\pi}(\frac{A}{k_B T})^{-\frac{3}{2}} = \frac{1}{2}k_B T$

4 Problem 4

When under same conditions, compared to classic particles, fermions will be affected by an extra pressure due to the Pauli repulsion. Therefore, fermions gases will have a higher pressure.

5 Problem 5

We can only use the Boltzmann distribution when $\frac{h}{\sqrt{3mk_BT}}(\frac{N}{V})^{1/3} << 1$
$PV = NTR \rightarrow \frac{N}{V} = \frac{P}{TR} \rightarrow \frac{6.63\times10^{-34}}{\sqrt{3\times2.02\times1.66\times10^{-27}\times1.38\times10^{-23}T}}(\frac{101325}{8.314T})^{1/3} = 1$
$4.09 \times 10^{-8}T^{-5/6} = 1 \rightarrow T = 1.36 \times 10^{-9}K$

6 Problem 6

(a) For copper, $\langle E\rangle = \frac{3}{5}E_F = \frac{3}{5} \times 7.06 = 4.236\text{eV}$

(b) For lithium, $\langle E\rangle = \frac{3}{5}E_F = \frac{3}{5} \times 4.77 = 2.862\text{eV}$

7 Problem 7

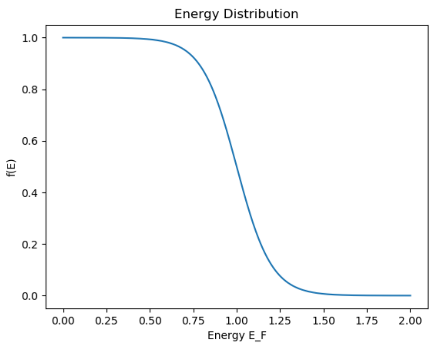

Figure 1: $f_{FD}(E)$ vs. E when $T = 0.1T_F$

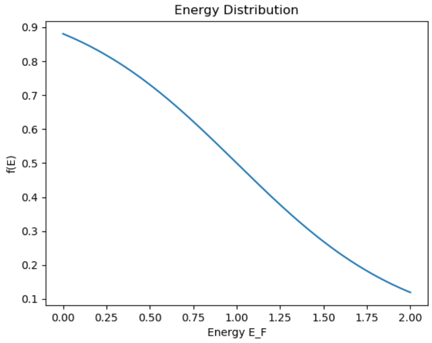

Figure 2: $f_{FD}(E)$ vs. E when $T = 0.5T_F$

8 Problem 8

The fermi energy for Silver is 5.49eV and the corresponding fermi temperature is $6.38 \times 10^4 K$
$k_B T = 8.617 \times 10^{-5} \times 300 = 0.02585\text{eV} << 5.49$
$\int_{E_F}^{\infty} f_{FD}(E)\mathrm{d}E = \int_{5.49}^{\infty} \frac{1}{e^{(E-5.49)/0.02585}+1}\mathrm{d}E = 0.02585\ln(2) = 0.0179$
$\int_{0}^{\infty} f_{FD}(E)\mathrm{d}E = \int_{0}^{\infty} \frac{1}{e^{(E-5.49)/0.02585}+1}\mathrm{d}E = 0.02585\ln(e^{(5.49/0.02585)}+1) = 5.49$
The percentage is $\frac{0.0179}{5.49} \approx 0.003 = 0.3\%$

9 Problem 9

(a) For 2D, $N = \frac{1}{4}\pi R^2 = \frac{\pi}{4}(\frac{E}{E_0})$
$g(E) = \frac{\mathrm{d}N}{\mathrm{d}E} = \frac{\pi}{4E_0} = \frac{mL^2}{2\hbar^2\pi}$
Since the particle is electron $g_e(E) = 2g(E) = \frac{mL^2}{\hbar^2\pi}$

(b) For 1D, $N = \frac{1}{2}R = \frac{1}{2}\sqrt{\frac{E}{E_0}}$
$g(E) = \frac{\mathrm{d}N}{\mathrm{d}E} = \frac{1}{4}\sqrt{\frac{2mL^2}{\hbar^2\pi^2 E}}$
Since the particle is electron $g_e(E) = 2g(E) = \frac{1}{2}\sqrt{\frac{2mL^2}{\hbar^2\pi^2 E}}$

10 Problem 10

$\frac{N}{L} = (8.61 \times 10^28)^{1/3} \approx 4.42 \times 10^9/\text{m}$
$E_F = \frac{h^2}{32m}(\frac{N}{L})^2 = \frac{(6.63\times10^{-34})^2}{32\times9.11\times10^{-31}}(4.42 \times 10^9)^2 \approx 1.84\text{eV}$