VP390 RECITATION CLASS

THREE

Zhang Boyu 张搏宇

2020. 6. 3

Structure of matter

Known facts: atom's diameter ~10^{-10}m; mass of the electron is only small fraction of the total mass of the atom.

Thompson's model

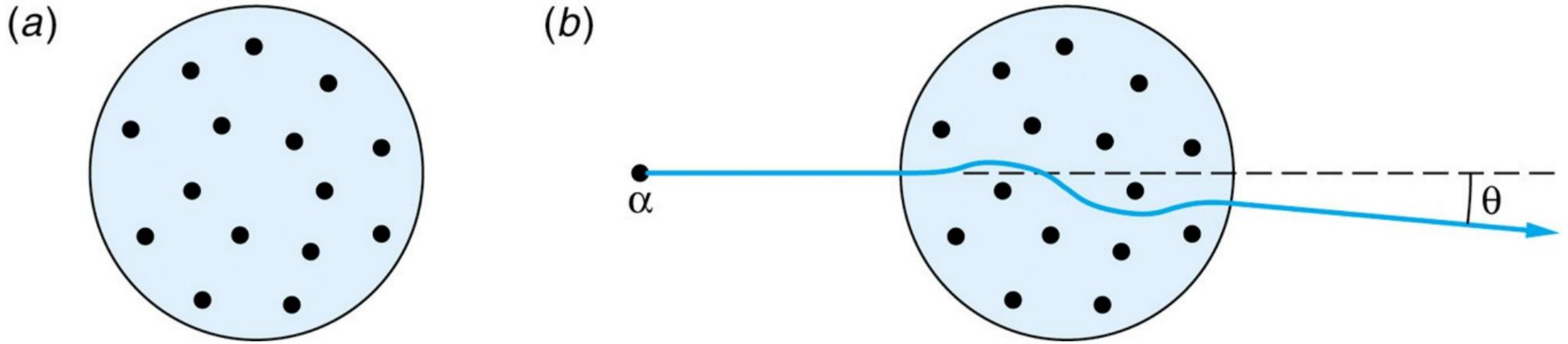
Rutherford's model

Bohr's model

What kind of experiments and results lead to the revolution of the models?

Thompson's model

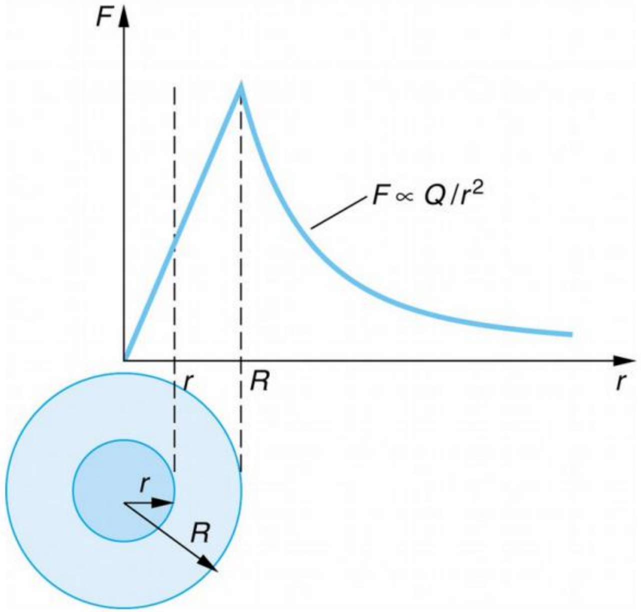
Postulate: positive charge uniformly distributed over the atom's volume.

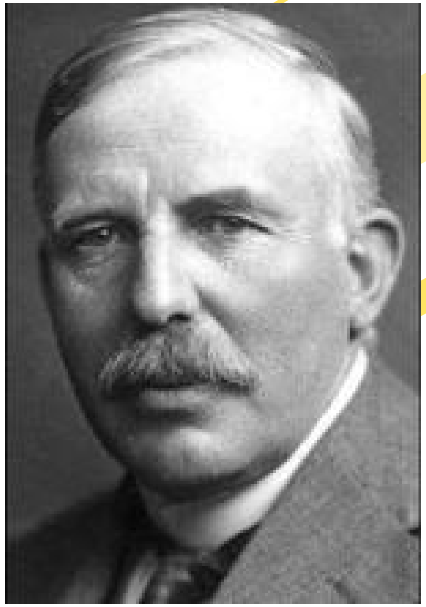


This model does not predict any significant scattering of alpha particles.
But actually it is discovered in the experiment.

Rutherford's model

Postulate: positive charge concentrated in a massive spherical nucleus with diameter much less than the size of atom.

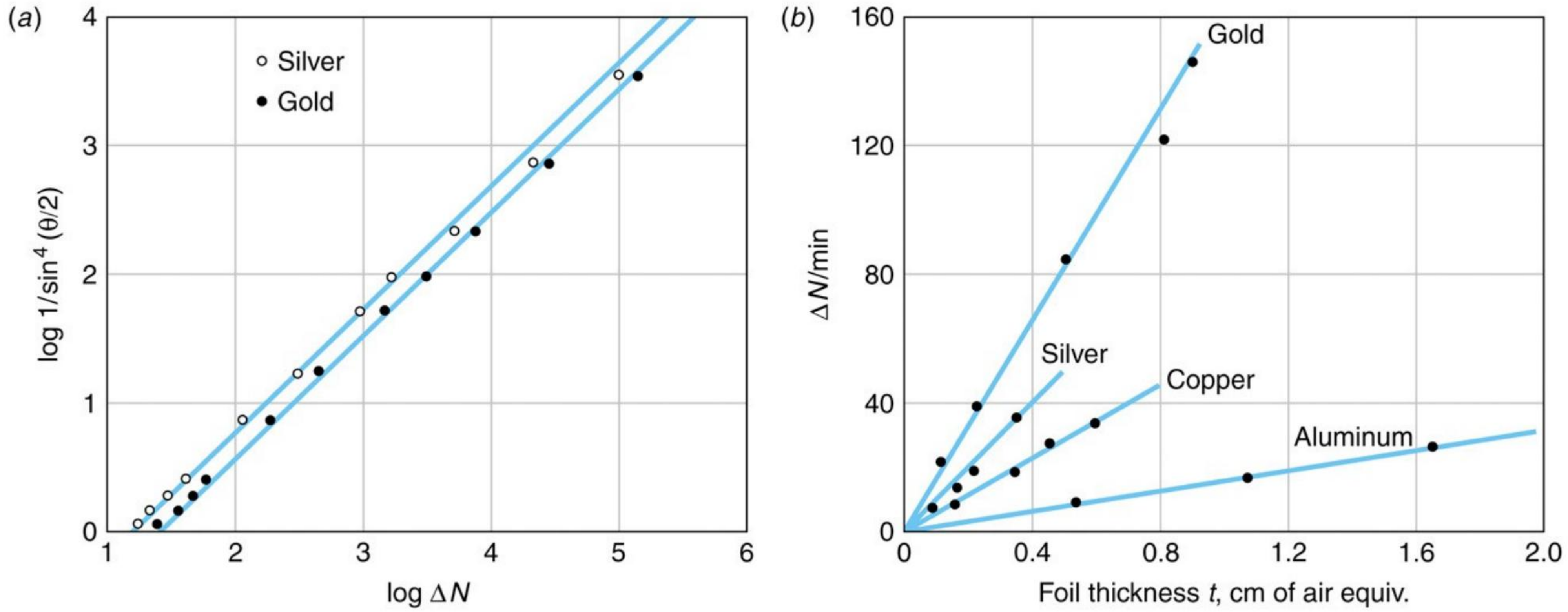




Ernest Rutherford
(1871-1937)

It explains well the phenomenon of scattering which Tompson's model cannot.

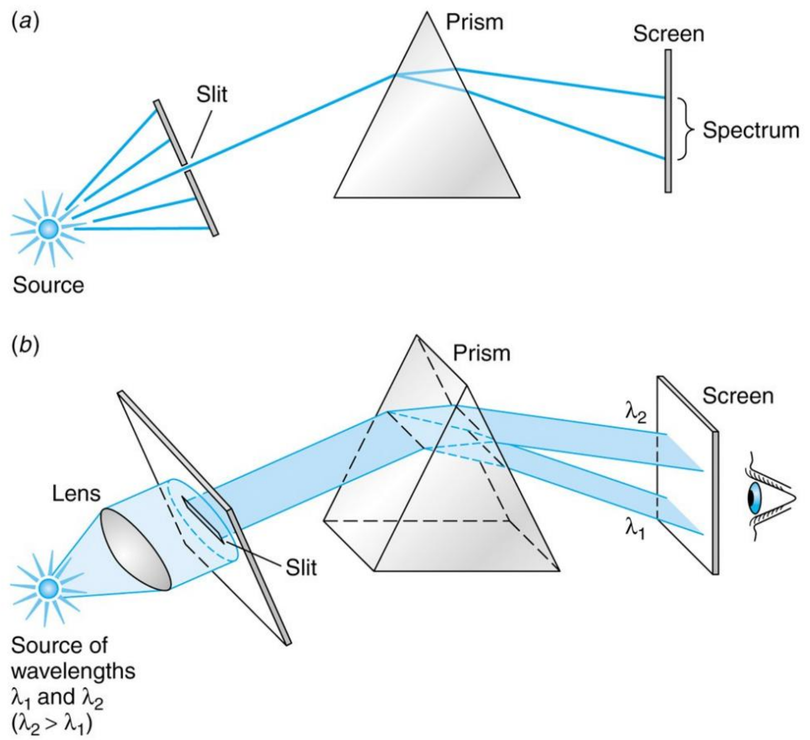
Rutherford's model

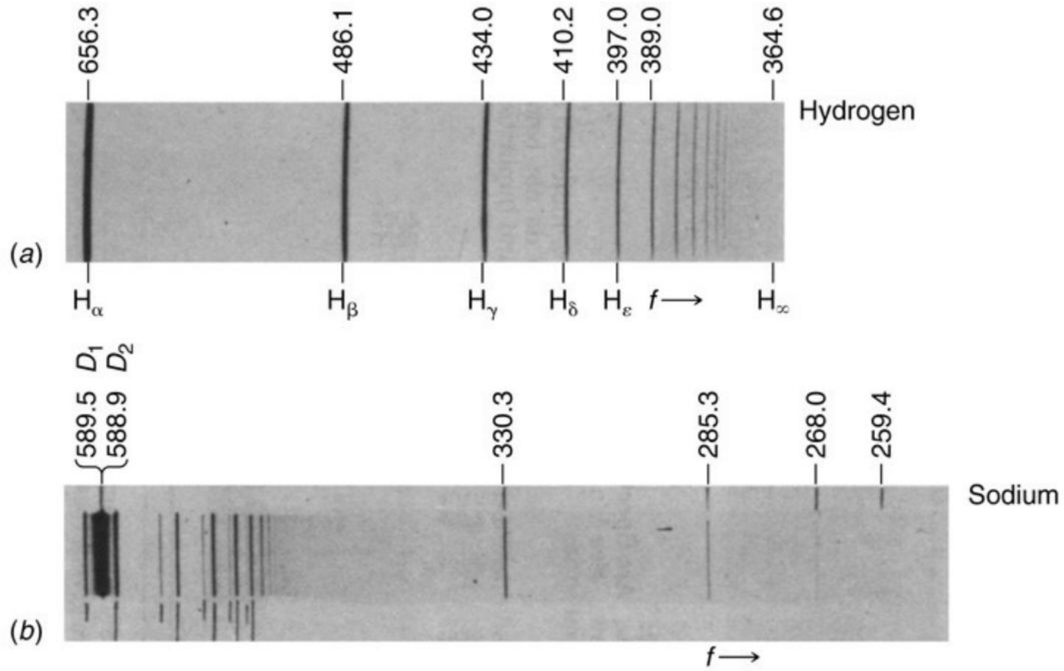


What do the results tell us? How can they be interpretated?

Rutherford's model

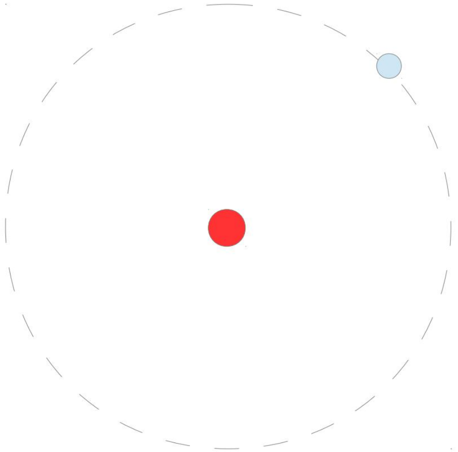
But there is also something that Rutherord's model cannot explain.





How to explain that the atom spectra is discrete?

Bohr model



$$F = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

Niels Bohr
(1885-1962)

$$\begin{cases} L_z = mvr = n\hbar \Rightarrow (mvr)^2 = n^2\hbar^2 \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r^2} = \dfrac{mv^2}{r} \end{cases} \Longrightarrow \boxed{r_n = \frac{4\pi\hbar^2\varepsilon_0}{mZe^2}n^2 = \frac{n^2}{Z}a_0}$$

Bohr model

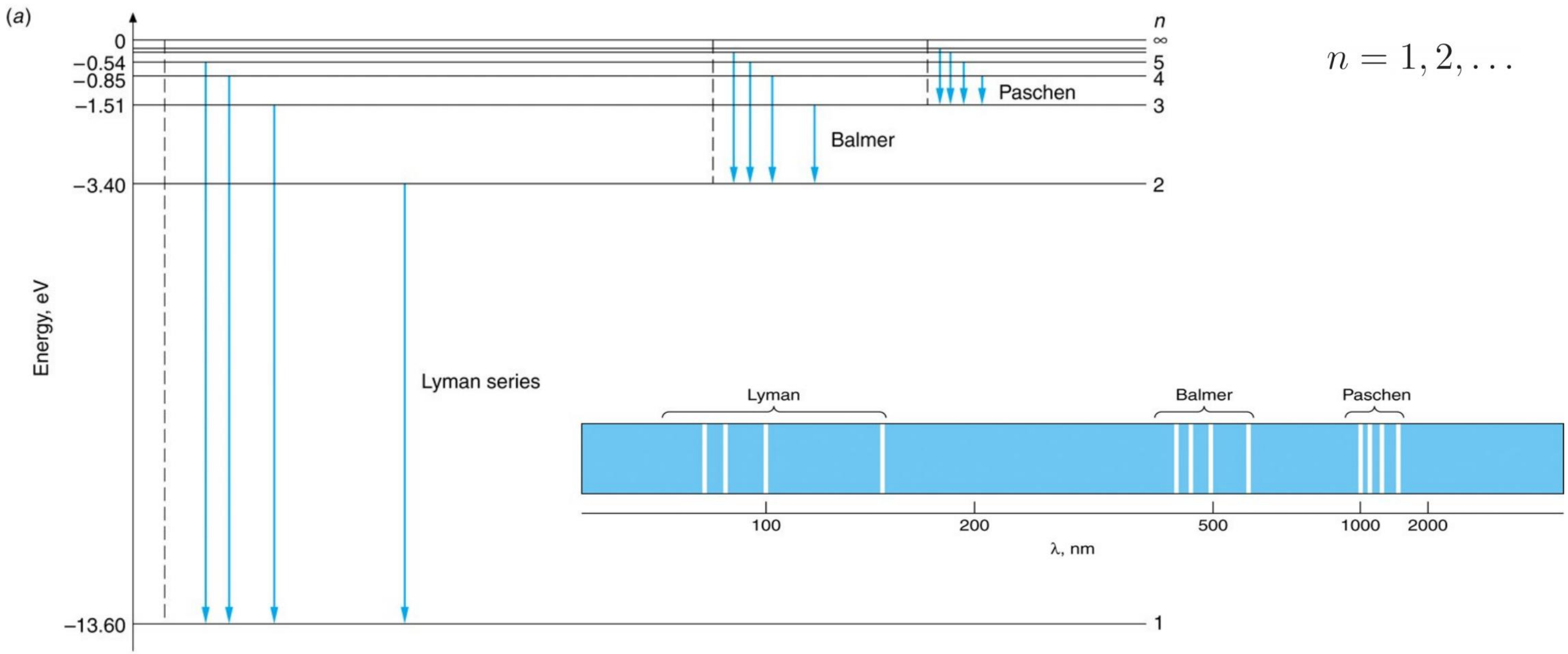
Total energy of electron in the n-th orbit (n is the quantum number):

$$E_n = -\frac{mZ^2e^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = -E_1\frac{Z^2}{n^2}$$

The atom radiates only when the electron undergoes a transition from one stationary state to another. The inverse of the wavelength of the emitted electromagnetic radiation is:

$$\lambda^{-1} = \frac{E_f - E_i}{hc} = \frac{E_1Z^2}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

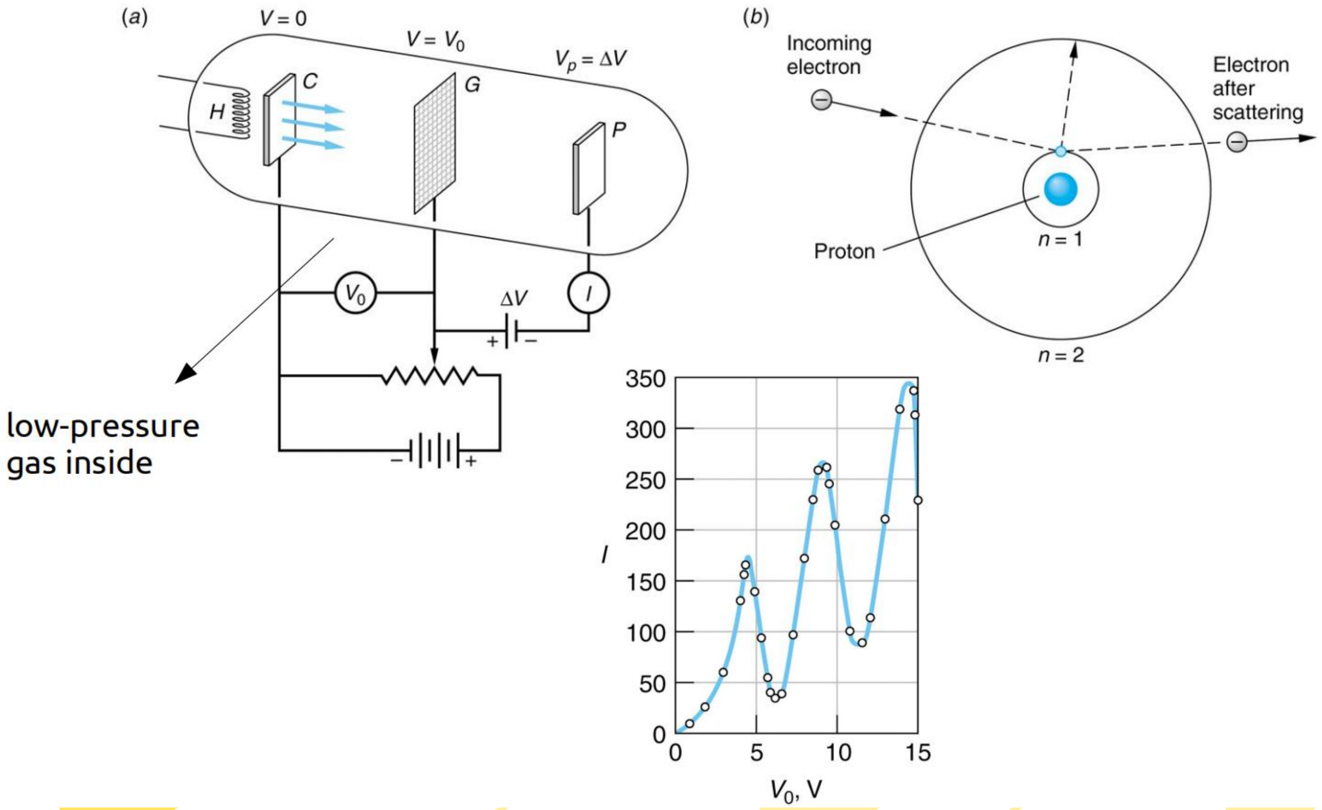
Bohr model



$$n = 1, 2, \ldots$$

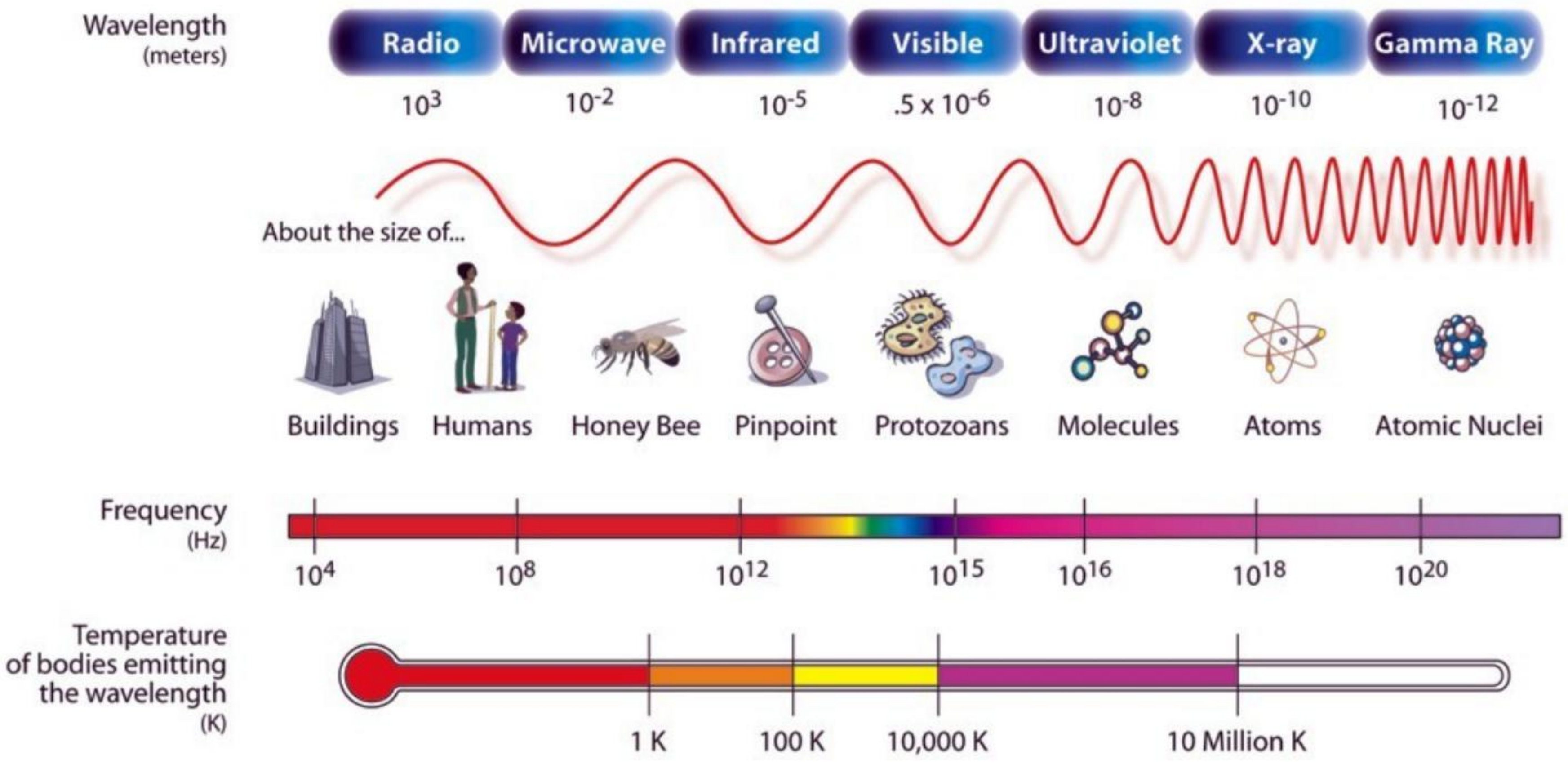
Frank-Hertz experiment

What is the idea of Frank-Hertz experiment and what does the result tell us?



Electrons accelerate in low pressure gas and then overcome potential difference ΔV. An amperemeter is used to detect how many electrons arrive at the detector.

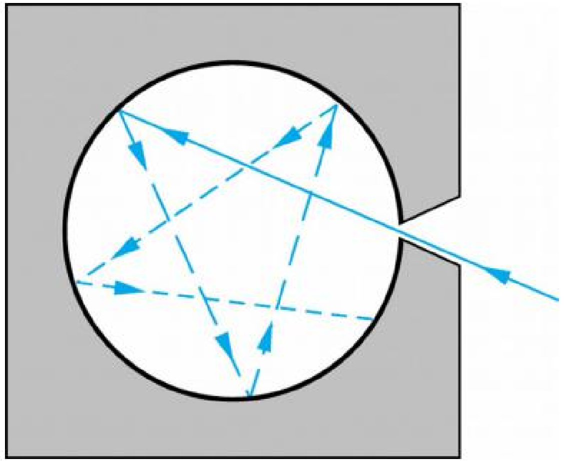
Electromagnetic spectrum



Blackbody model

e-m radiation absorption → kinetic energy of atoms increases → T increases
→ e-m radiation emission→ kinetic energy of atoms decreases → T decreases

thermal equilibrium



Ideal blackbody model: cavity in thermal equilibrium

- completely absorbs incident radiation
- rate of emission = rate of absorption

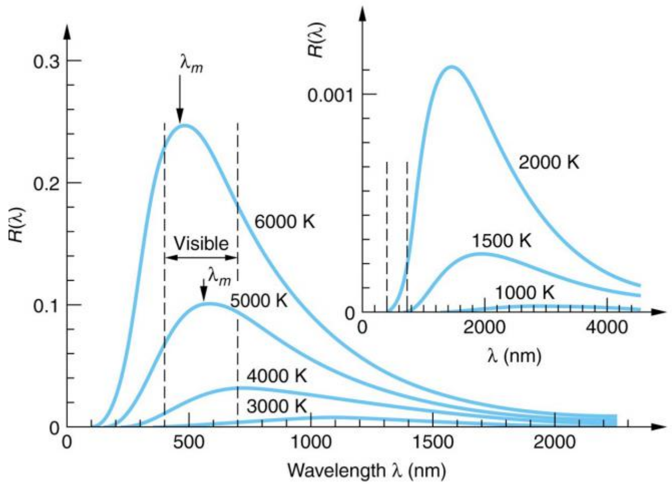
Blackbody radiation

Total power radiated: Stefan-Boltzmann law

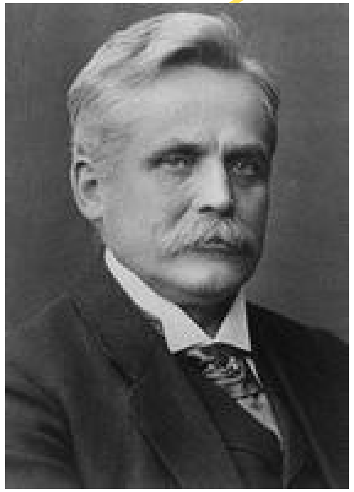
Energy radiated per unit time (power) from unit area:

$$\frac{P}{A} = \sigma T^4 \qquad \sigma = \text{const} = 5.67 \times 10^{-8} [W / m^2 K^4]$$

spectral distribution and Wien displacement law



$$\lambda_m T = \text{const}$$

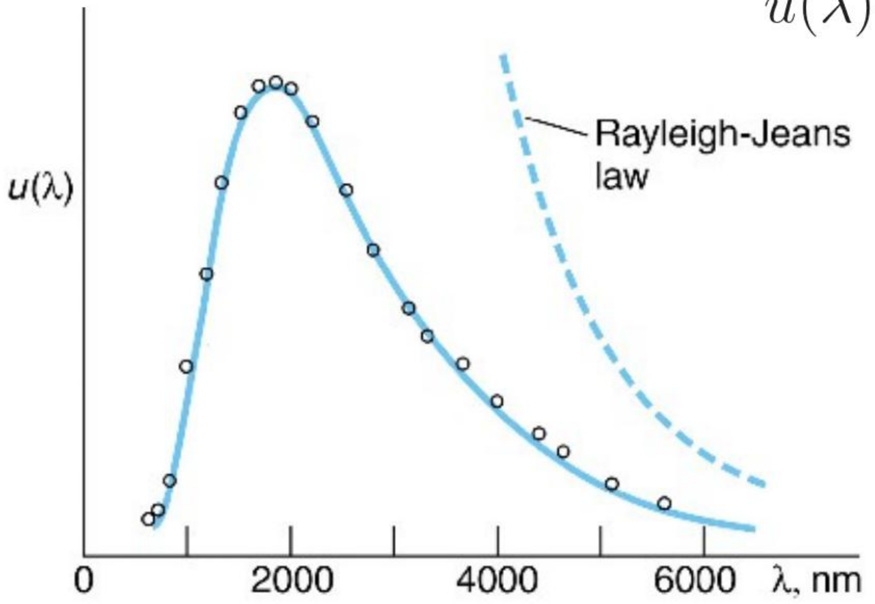


Wilhelm Wien
(1864-1928)

Ultraviolet catastrophe

Rayleigh-Jeans formula: $u(\lambda) = \frac{8\pi}{\lambda^4} k_B T$

$$u(\lambda) \to \infty \Longrightarrow \int_0^{\infty} u(\lambda)\, d\lambda = \infty$$



ultraviolet catastrophe

However, $u(\lambda)$ is the spectral density, so its integral should never be infinity.

Planck's law

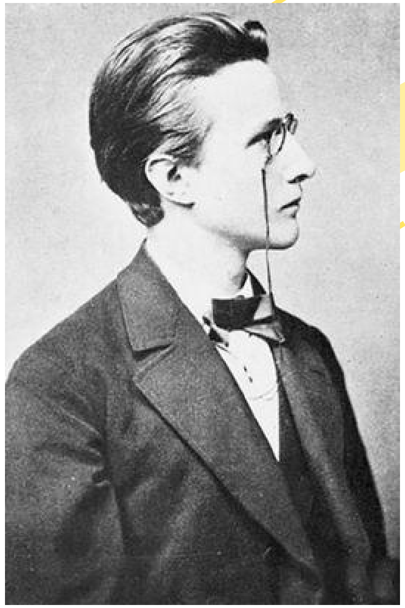
Planck's postulate:

$$E_n = nh\nu, \quad n = 0, 1, 2, \ldots$$

It means that the energy of electromagnetic radiation is **quantized.**

$$f_n = \frac{e^{-E_n/k_BT}}{\sum_{n=0}^{\infty} e^{-E_n/k_BT}} = \frac{e^{-nh\nu/k_BT}}{\sum_{n=0}^{\infty} e^{-nh\nu/k_BT}}$$

$$\langle E \rangle = \sum_{n=0}^{\infty} E_n f_n = \frac{h\nu}{e^{h\nu/k_BT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$$



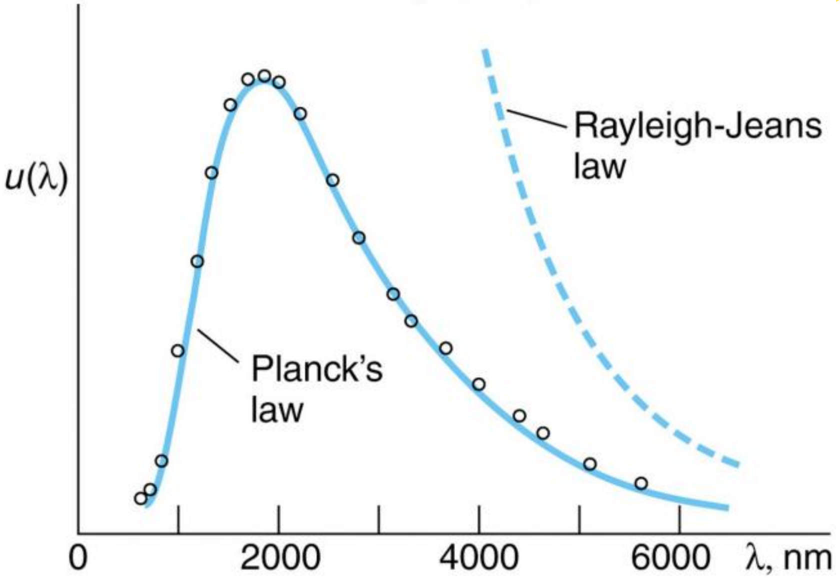
Max Planck
(1858-1947)

Planck's law

Planck's law:

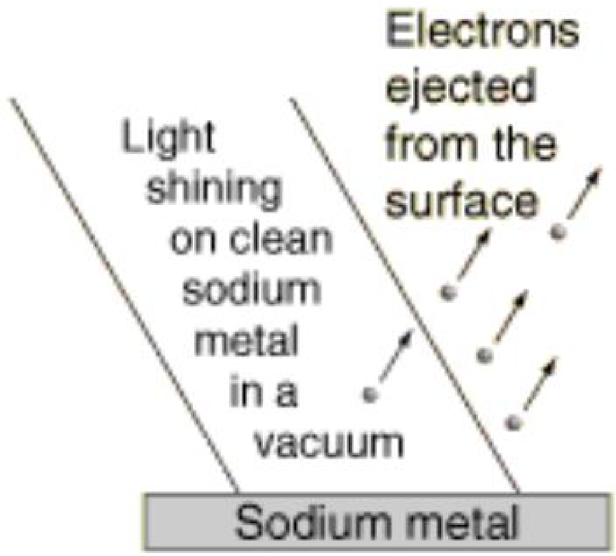
$$\boxed{u(\lambda) = \frac{8\pi}{\lambda^4}\langle E\rangle = \frac{8\pi}{\lambda^5}\frac{hc}{e^{hc/\lambda k_B T}-1}}$$

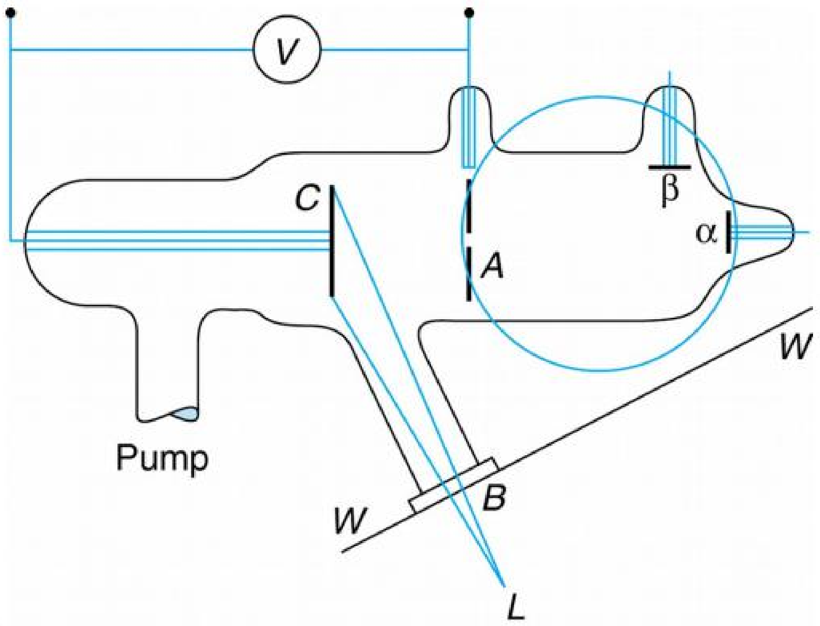
For the limit of long wave length, what will be the approximation of Planck's law?



Photoelectric effect

Idea and experimental setup:

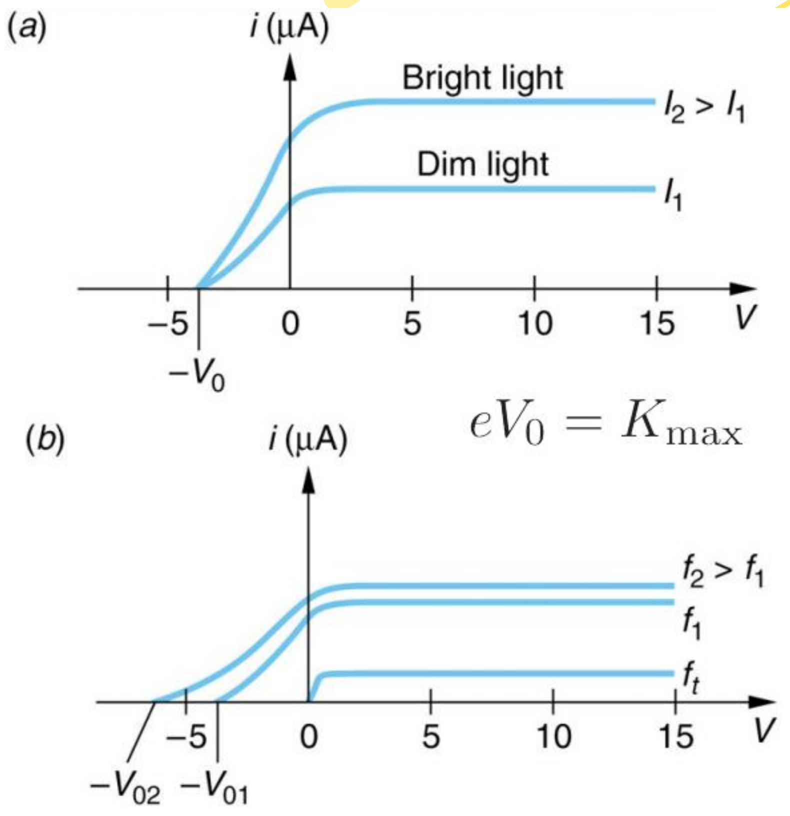




Photoelectric effect

Observations:

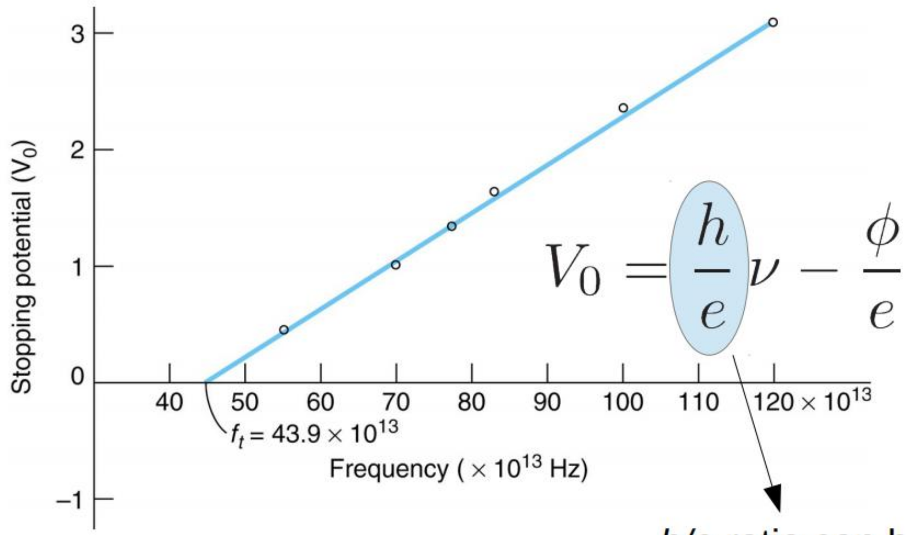
- maximum current proportional to intensity of incident light;
- no threshold intensity, effect present even for small intensities ;
- stopping potential independent of intensity, but different for different frequencies.



$$eV_0 = K_{\max}$$

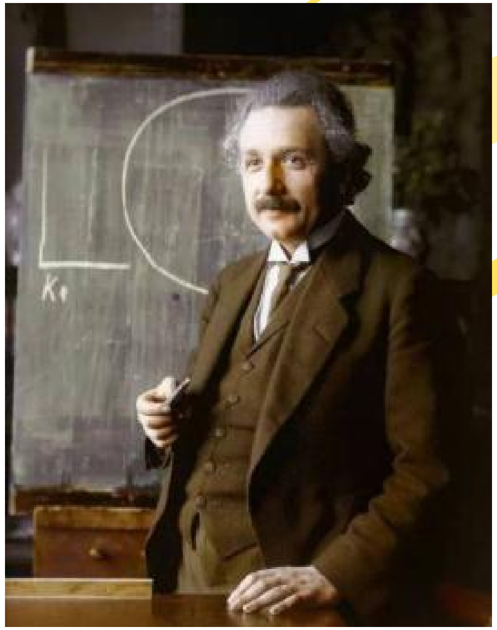
Photoelectric effect

$$\boxed{h\nu = \phi + K_{max}}$$



h/e ratio can be found

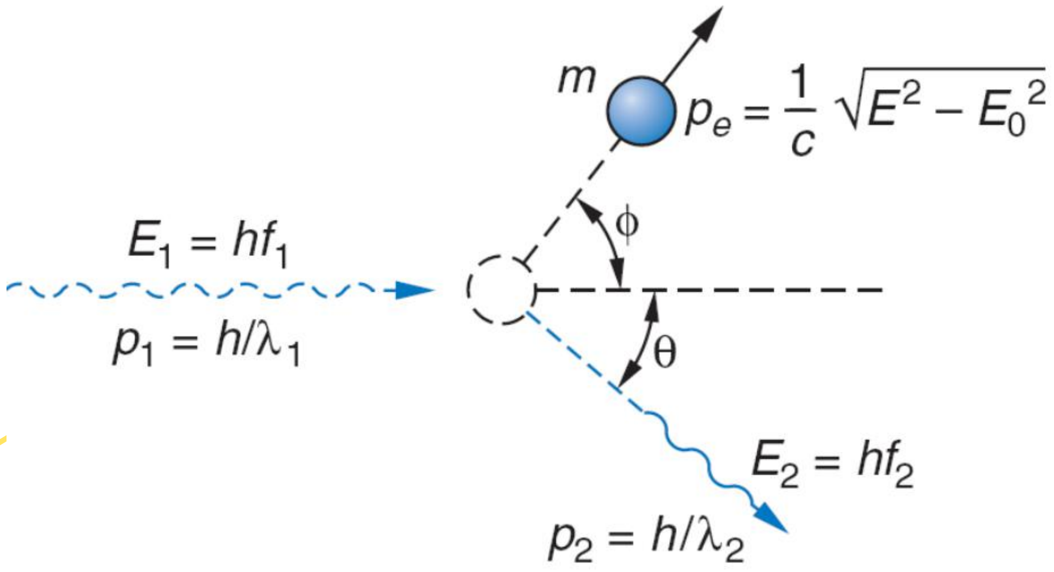
Element	Work function (eV)
Na	2.28
Cs	1.95
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14



Albert Einstein
(1879-1955)

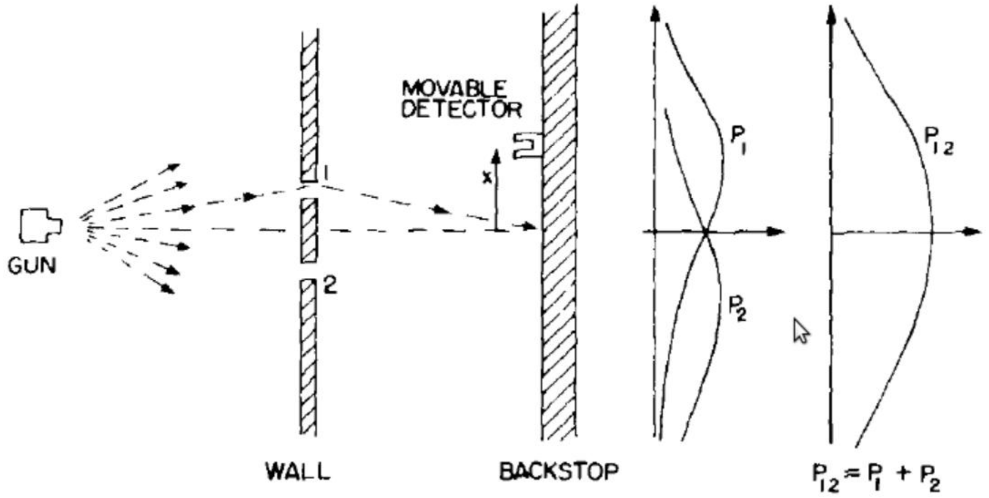
Compton Scattering

scattering of a photon on a free electron at rest

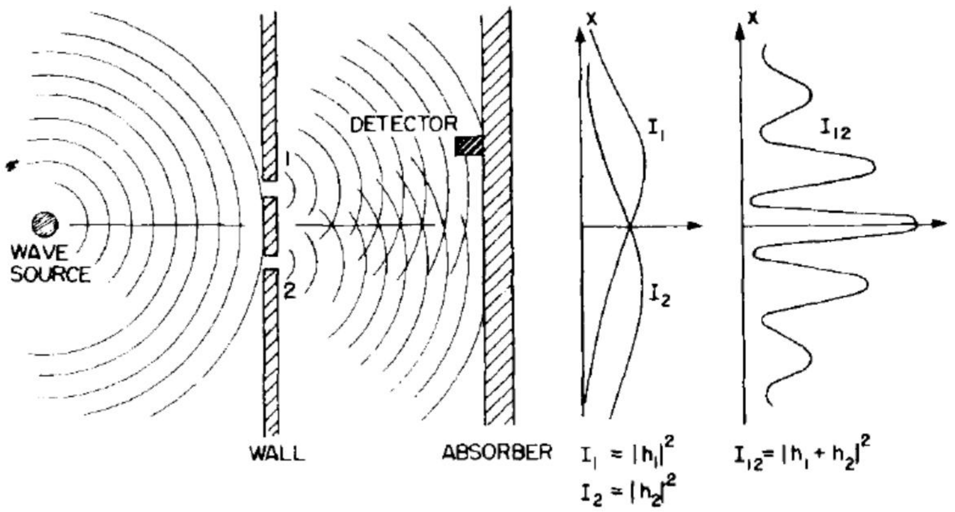


$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta)$$

Particles or waves?



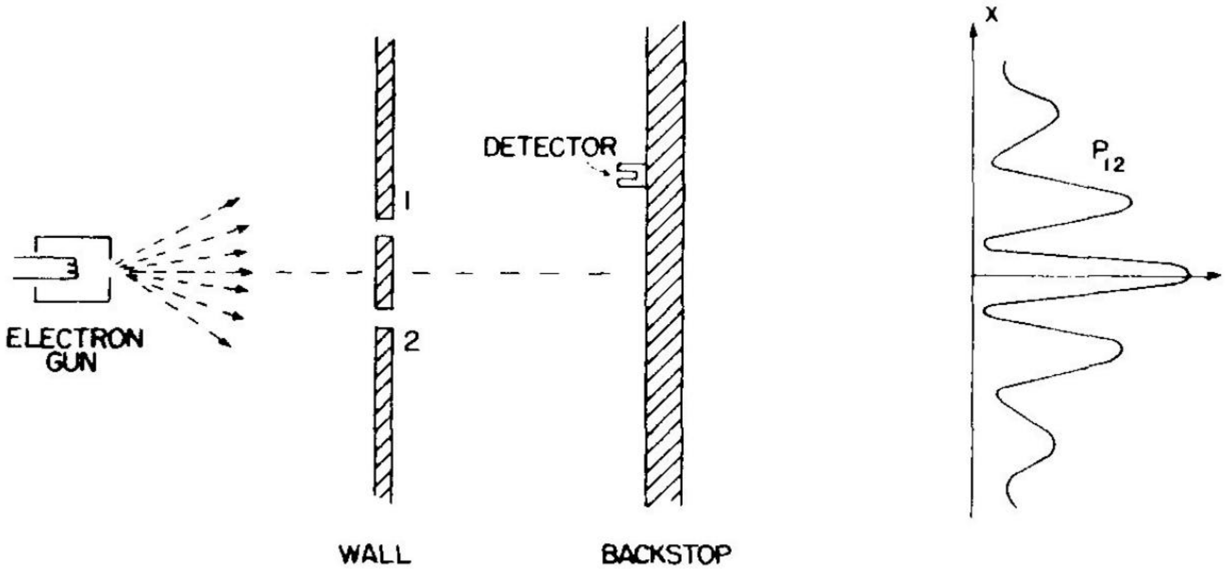
bullet



water

What is the key difference here?

Particles or waves?



$$P_{12} = |\psi_1 + \psi_2|^2 = |P_1|^2 + |P_2|^2 + 2\,\mathrm{Re}(\psi_1 \psi_2^*)$$

What does the result tell us?

Matter waves

de Broglie hypothesis:

All matter exhibits wave-like properties with:

$$\lambda = \frac{h}{p}, \quad \nu = \frac{E}{h}$$

Or, equivalently, in terms of the wave-number and angular frequency:

$$k = \frac{p}{\hbar}, \quad \omega = \frac{E}{\hbar}$$

Louis de Broglie
(1892-1987)

Thank you very much!!!