VP390 RECITATION CLASS

ONE

Zhang Boyu 张搏宇

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Inertial FoR

What is an inertial FoR?

$$\overline{F} = 0$$

$$\Downarrow$$

Newton's first law:
There exists an inertial FoR

$$\overline{u} = \text{const},\ \overline{a} = 0$$

A FoR moves with respect to an inertial FoR at **constant velocity** is also an inertial FoR.

Non-inertial FoR

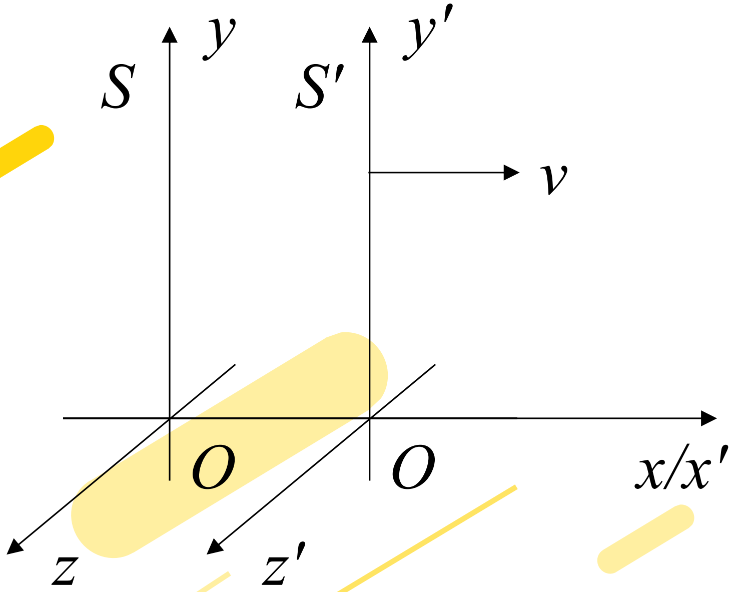
What is an non-inertial FoR?

A FoR moves with an inertial FoR at **varying velocity** is a non-inertial FoR. In a non-inertial FoR, Newton's 2nd law should be modified as:

$$m\bar{a} = \bar{F} + \text{fictitious forces}$$

The fictitious forces are kinematic corrections (with the unit of the force, i.e. [N]). All fictitious forces are **not real forces**.

Galilean transformation



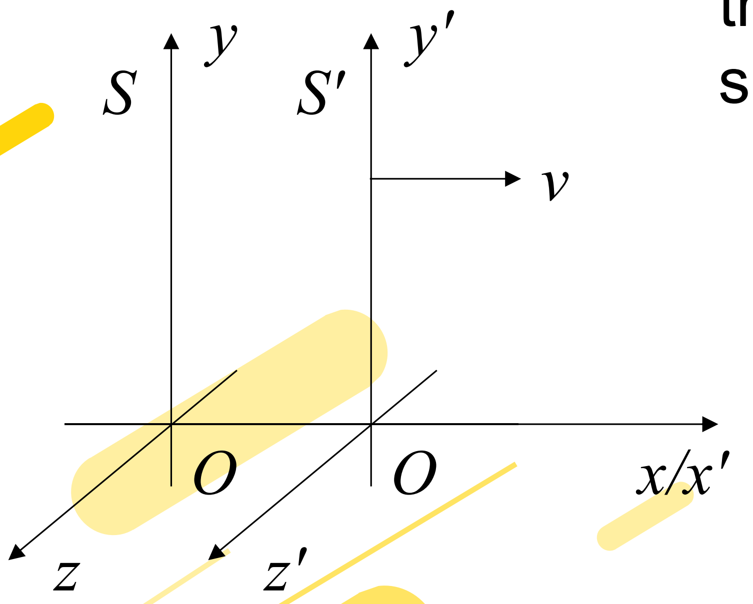
$$\boxed{\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}}$$

or

$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$

In Galilean transformation, time t is not a variable, but an **invariant** parameter.

Galilean transformation



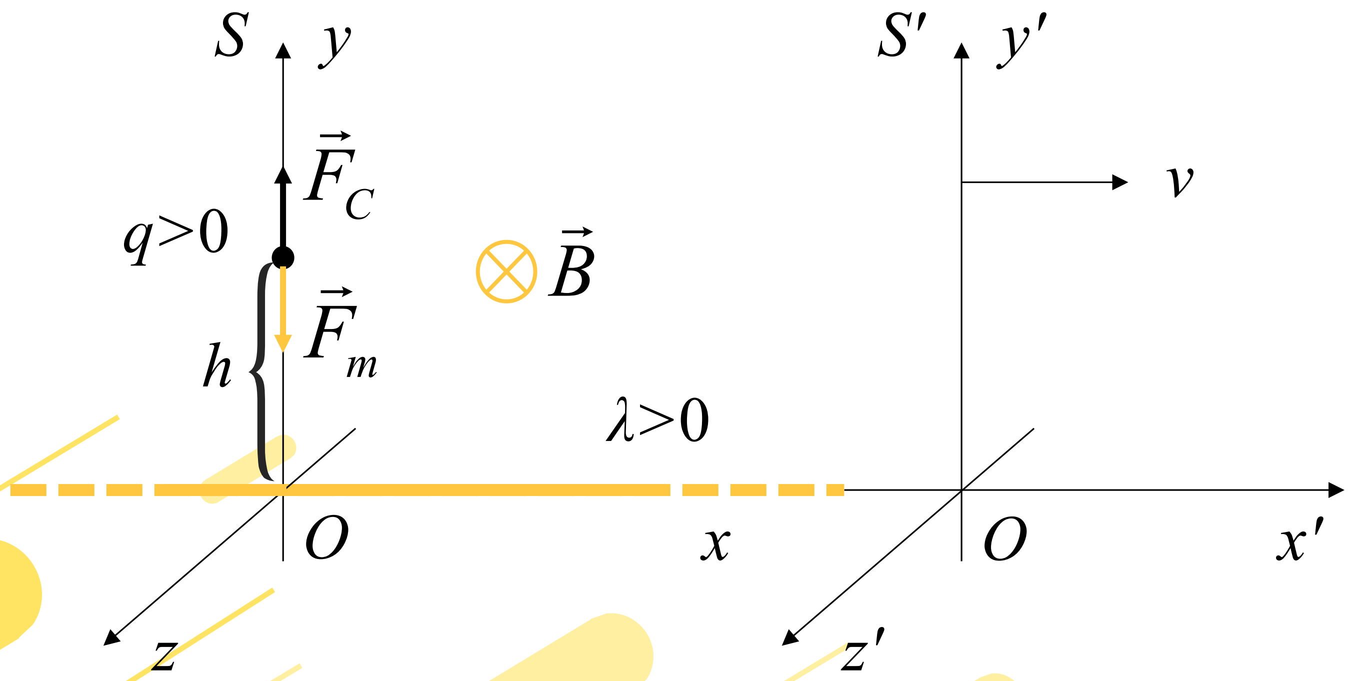
Consequently, Since here it is implicitly assumed that $t = t'$, $d/dt=d/dt'$, take the derivative on both side of the equations in Galilean transformation:

$$\boxed{\begin{aligned} u_x' &= u_x - v \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned}}$$

The equations above form the **non-relativistic velocity addition rule**.

Is Newtonian mechanics universal?

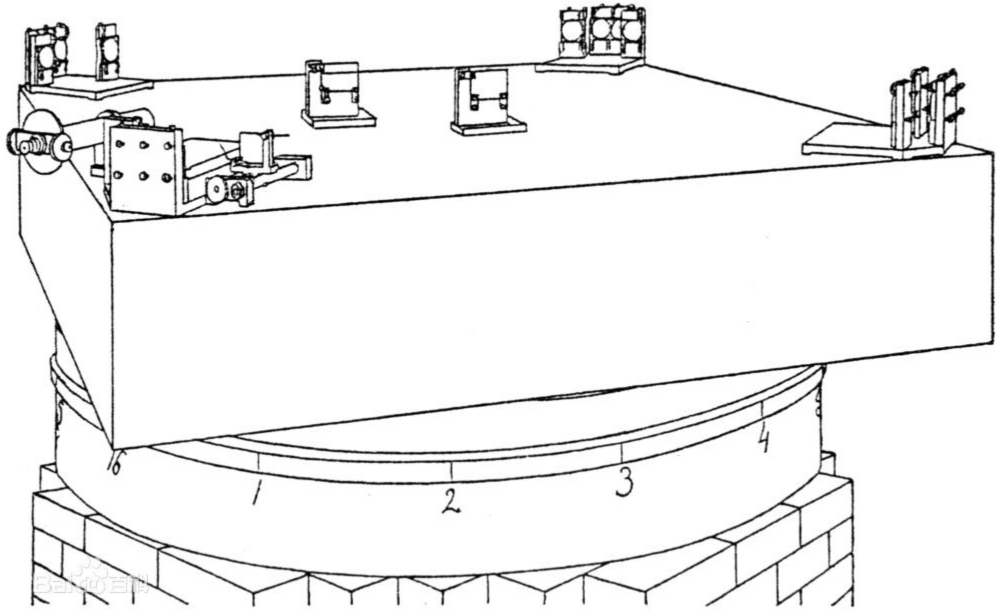
Think about the following case, does Newton's 2nd law still hold?



Are the forces exerted on the point charge still the same in both FoRs? **NO!!!**

Is Newtonian mechanics universal?

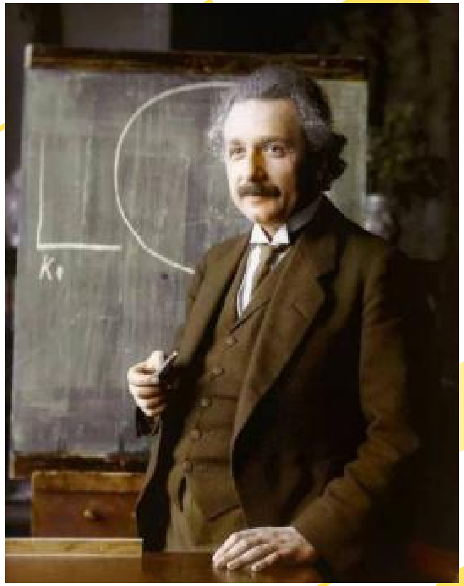
Think about what does Michelson-Morley experiment imply?



Postulates of STR

Albert Einstein raised the following postulates in 1905:

1. The laws of physics are the same in all inertial FoRs.
2. The light in vacuum propagates with the same speed c in all inertial FoRs.



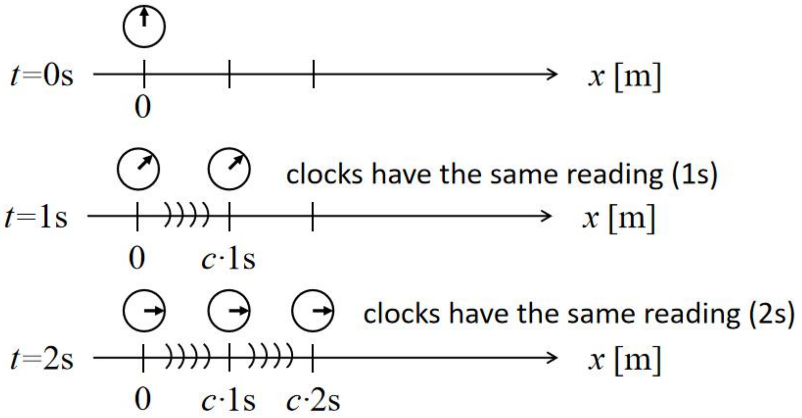
- The speed of light in vacuum $c = 1/\sqrt{\varepsilon_0 \mu_0} \approx 3 \times 10^8$ m/s (can be derived from classical electrodynamics).
- Extends the conclusion drawn (with respect to Newton's laws) from the Galilean transformation to include all laws of physics.
- The speed of light is invariant for all inertial observers in a given medium. It has the same value, regardless of the motion of the observer or the light source.
- Therefore, it is impossible to detect any “absolute motion”.

Vocabularies of STR

EVENT occurs at { some point of space / some instant of time } in a given FoR.

An **OBSERVER** describes an event with respect to his FoR using his "meter sticks" (at rest in his FoR) and his "clocks".

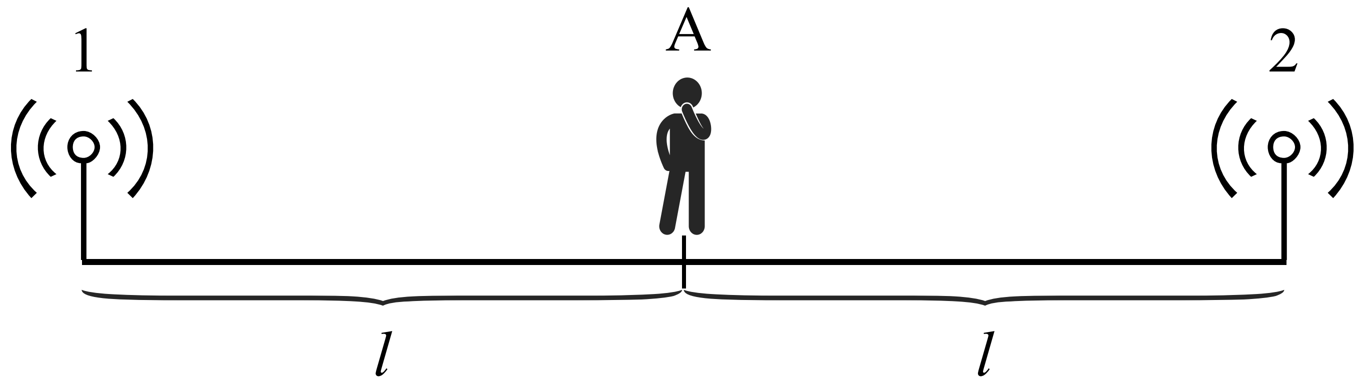
CLOCK SYNCHRONIZATION



A flash fired at $t = 0$ and $x = y = z = 0$ (at origin).

Vocabularies of STR

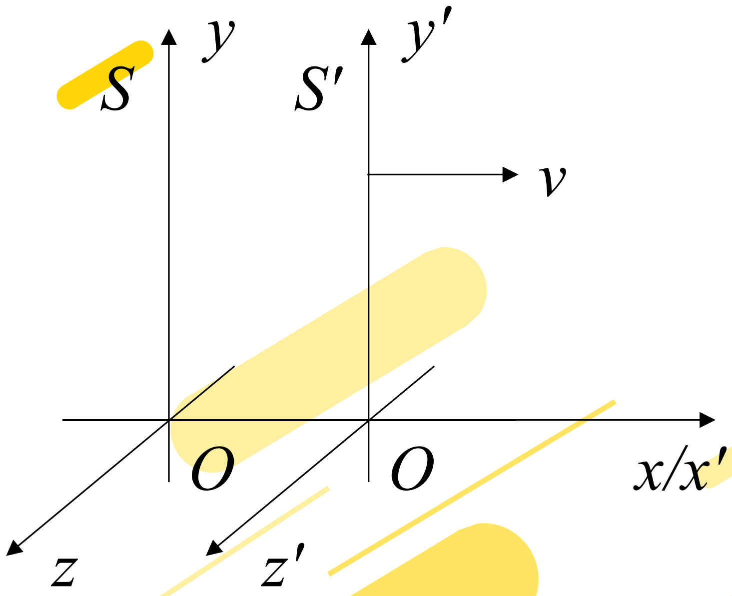
SIMULTANEITY



Firing flash 1 and 2 are simultaneous events for observer A, if the light signals from 1 and 2 arrive at the same instant of time (measured using A's own clocks).

Are clock synchronization and simultaneity absolute? **NO!!!**

Lorentz transformation



$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

inverse transformation

Lorentz transformation

In Lorentz transformation, spatial and temporal variables get ”entangled”. Time is no longer a parameter, but a **variable** that varies in different FoRs.

But is Galilean transformation totally wrong? think about what will happen to Lorentz transformation when $v/c \to 0$?

$$\boxed{\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}}$$

which is exactly the Galilean transformation. Therefore, the Galilean transformation is a special case of the Lorentz transformation, valid for small v.

Temporal effects

How to use Lorentz transformation to prove that simultaneity is relative?

From Lorentz transformation:

$$t_B - t_A = \frac{t_B' - t_A'}{\sqrt{1 - v^2 / c^2}} + \frac{v(x_B' - x_A')}{c^2\sqrt{1 - v^2 / c^2}}$$

Obviously t_B-t_A=0 does not imply that t_B'-t_A'=0. Furthermore, if x_B'-x_A'=0, then:

$$t_B - t_A = \frac{t_B' - t_A'}{\sqrt{1 - v^2 / c^2}}$$

And it is known as the **time dilation**.

Spatial effects

From Lorentz transformation:

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}} - \frac{v(t_2 - t_1)}{\sqrt{1 - v^2/c^2}} = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

And it is konwn as the **length contraction** or **Lorentz-Fitzgerald contraction**.

But think about why we need t_2-t_1=0 here? i.e. Why we need both ends' position to be measured **simultaneously** in S?

Relativistic velocity addition rule

From Lorentz transformation, the new rule of velocity addition is given by:

$$u_x' = \frac{u_x - v}{1 - vu_x / c^2}$$

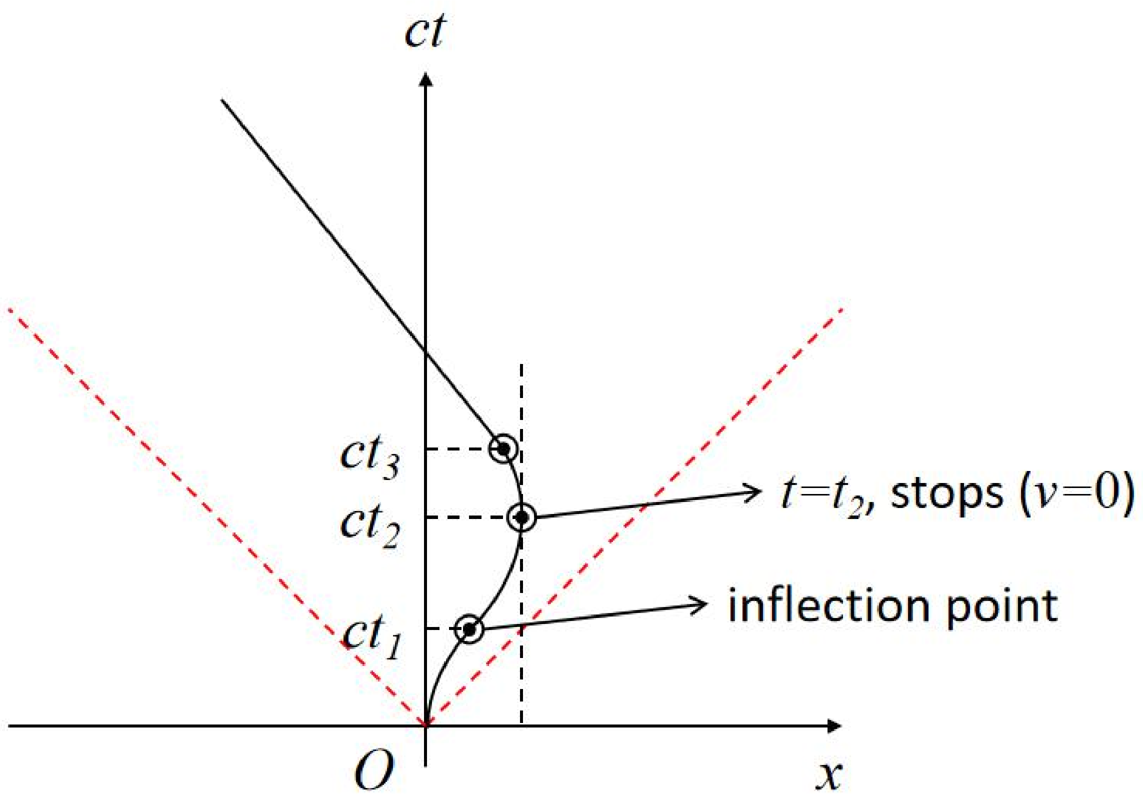
$$u_y' = \frac{u_y}{1 - vu_x / c^2}\sqrt{1 - v^2 / c^2}$$

$$u_z' = \frac{u_z}{1 - vu_x / c^2}\sqrt{1 - v^2 / c^2}$$

Be very careful that velocity also transform in y and z direction!

Minkowski diagrams

The trajectory of a particle plotted in the 4-dimensional spacetime is called the **worldline**.



- $0 \leq t < t_1$, accelerates in +x direction;
- $t_1 \leq t < t_2$, slows down;
- $t_2 \leq t < t_3$, accelerates in -x direction;
- $t > t_3$, moves with constant speed.

Four-position vector

contravariant

$$X^{\mu} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

covariant

$$X_{\mu} = (ct, x, y, z)$$

metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The following hold for four-position vector:

$$g_{\mu\nu} X^{\nu} = \sum_{\nu=0}^{3} g_{\mu\nu} X^{\nu} = (ct, -x, -y, -z) = X_{\mu}$$

$$g_{\mu\nu} X^{\nu} X^{\mu} = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} X^{\nu} X^{\mu} = \sum_{\mu=0}^{3} X_{\mu} X^{\mu} = X_{\mu} X^{\mu}$$

Space-time interval

The spacetime interval (between events A and B) is defined as:

$$
\begin{aligned}
\Delta s^2 &= \Delta X_\mu \Delta X^\mu \\
&= c^2(t_B - t_A)^2 - [(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2] \\
&= c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2
\end{aligned}
$$

An important fact about the space-time interval is that it is an invariant of the Lorentz transformation, Δs^2 = inv. That is, it has the same value in all inertial FoRs.

Space-time interval

There are three different categories of space-time interval:

- $\Delta s^2 > 0$

The spacetime interval is said to be **time-like**, what means that the events are separated temporally. There may be a possible causal relation between the events.

- $\Delta s^2 < 0$

In this case the spacetime interval is called **space-like**, what means that the events are separated spatially. There cannot be a possible causal relation between the events.

- $\Delta s^2 = 0$

The spacetime interval is light-like. There may be a possible causal relation between the events. But only when the events influence others at speed of light.

Exercise 1

A pair of twins were born at the same time on earth, but to tell them apart, we still call it younger and older brother. When they grow up to 20 years old, the older one decides to go on a space trip towards a planet far away from earth then come back at $0.8c$ while the younger one decides to stay on earth. In the younger brother's perspective, his older brother is in a moving FoR, so his time should flow slower and consequently when the older brother comes back, he will be younger than the younger brother. However, in the older brother's perspective, it is his younger brother moving in the opposite direction, so he expect that his younger brother will be younger than himself. But obviously, when the older brother comes back, there is only one person being correct, so who is the one holding the correct view?

Exercise 2

A train is about to enter a tunnel. The length of the tunnel is L_1 and the length of the train is L_2 and we know that when measured in the same FoR, $L_2 > L_1$. The speed of the train is $(1-L_1^2/L_2^2)^{0.5}c$, so for a man standing next to the tunnel, the length of the train will be contracted to L_1 which means that at some moment he can see no part of the train outside the tunnel. However, for a man sitting in the train, the tunnel comes towards the train at $(1-L_1^2/L_2^2)^{0.5}c$ so the length of the tunnel will contract and be even shorter than the train, so it is impossible that the train is “contained” in the tunnel. What happens here?

Exercise 3

Suppose that you are travelling on a train and arrive at the station of a city where the speed limit is very small so that even when you are on the train, the relativistic effect is obvious. When the train is still running, you hear a gunshot and someone gets killed on the platform. Meanwhile, you notice a man who was just reading newspaper stands up and runs towards the crime site. The murderer left the gun on the ground and the man just picks it up. At that time, a policeman arrives at the scene and arrests him. Now your train stops so you tell the policeman what you see and try to prove the man innocent. But the policeman tells you: “well, since you are just on a train which is a moving FoR, what you see cannot prove anything.” How can you argue with the policeman?

Thank you very much!!!

And happy May 20th!