Problem Set 10

Due: 27 July 2016, 12.30 p.m.

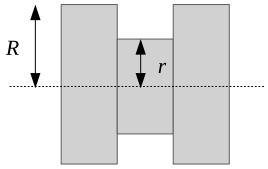
Problem 1. A uniform solid ball and a ring start rolling without slipping from the top of an an inclined plane with slope angle α. The initial velocity of the ball is zero. What should the initial linear velocity of the ring be, so that both objects travel the same distance in the same time t?

(*4 points*)

Problem 2. A yo-yo is made of two identical cylinders of radius R attached at their centers to a cylindrical axle with radius $r < R$. All three cylinders have the same height and are made of the same material (with constant density of mass). A string is attached to and wrapped around the axle. Holding the free end of the string firm, we release the yo-yo.

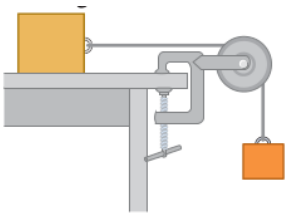
Find the acceleration of the center of mass of the yo-yo. Assume that the string does not slip while unwinding.

(*3 points*)



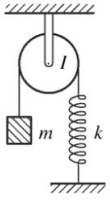
Problem 3. A block with mass m_1 resting on a horizontal, frictionless surface is attached to another block with mass m_2 by a thin, light wire that passes over a pulley. The pulley has the shape of a uniform disk with mass M and radius R. After the system is released, find (a) the tension in the wire on both sides of the pulley, (b) the acceleration of the box, and (c) the horizontal and vertical components of the force that the axle exerts on the pulley.

(*3/2 + 3/2 + 1 points*)

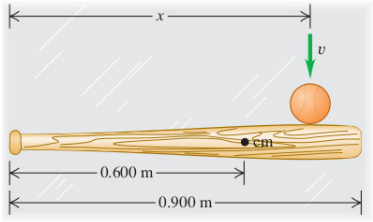


Problem 4. A block with mass m is attached to one end of a light rope that runs over a pulley with the moment of inertia I and radius R. The other end of the rope is attached to a spring with spring constant k. Find the period of oscillations of this system.

(*4 points*)



Problem 5. A baseball bat rests on a frictionless, horizontal surface. The bat has a length of $l = 0.9$ m, a mass of $m = 0.8$ kg, and its center of mass is $d = 0.6$ m from the handle end of the bat (see figure below). The moment of inertia of the bat about the axis (perpendicular to the surface) through the center of mass is $I_{\text{cm}} = 0.0530\ \text{kg}\cdot\text{m}^2$. The bat is struck by a baseball traveling perpendicular to the bat. The impact applies an impulse of magnitude $J = \int_{t_1}^{t_2} F\,dt$ at a point a distance x from the handle end of the bat. What must x be so that the handle end of the bat remains at rest as the bat begins to move?



The point on the bat that you will locate is called the *center of percussion*. Hitting a pitched ball at the center of percussion of the bat minimizes the "sting" the batter experiences on the hands.

Hint. Consider the motion of the center of mass and the rotation about the center of mass. Find x so that these two motions combine to give $v = 0$ for the end of the bat just after the collision.

(*6 points*)