VP390 Problem Set 2

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1 Problem 1

(a) $m(u^2) = \frac{m}{\sqrt{1-u^2/c^2}} = \sqrt{3}m$

Since $m_{ij} = m(u^2)[\delta_{ij} + \frac{1}{c^2-u^2}u_iu_j] = \sqrt{3}m(\delta_{ij} + \frac{3}{c^2}u_iu_j)$

Then $mij = \sqrt{3}m \begin{pmatrix} 1+1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1+1 \end{pmatrix} = \sqrt{3}m \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

(b) $F = ma = \sqrt{3}m \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \sqrt{3}m \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

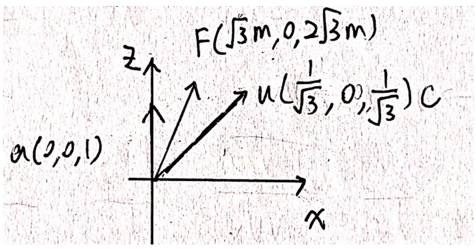

Figure 1: The y component is all 0, so the plot only show the x-z plane

2 Problem 2

$E^2 = p^2c^2 + E_0^2$ where $E_0 = mc^2$
Since $p = p_1, E = E_1 + E_2$
Then $m^2 = m_1^2 + m_2^2$
So the new particle's rest mass m is $\sqrt{m_1^2 + m_2^2}$
$E = K_1 + m_1c^2 + m_2c^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}}$
Then the new particle's velocity is $c\sqrt{1 - \frac{m_1^2c^4 + m_2^2c^4}{(K_1+m_1c^2+m_2c^2)^2}}$

3 Problem 3

If it's possible, then the total energy for the electron after collision is :

$E_{electron} = E_{photon} + E_0$ where $E_0 = mc^2$
For the momentum $p = \frac{E_{photon}}{c}$
Since $E^2 = p^2c^2 + E_0^2$
Then $E_{electron} = \sqrt{E_{photon}^2 + E_0^2} \neq E_{photon} + E_0$
So it's impossible.

4 Problem 4

$L_z = n^2\hbar^2$
$L_{\text{ph}} = n'^2\hbar^2 - n^2\hbar$

5 Problem 5

$R(\lambda) = \frac{1}{4}cu(\lambda) = \frac{2\pi}{\lambda^5}\frac{hc^2}{e^{hc/\lambda k_{\text{B}}T}-1}$
Let $R(\lambda)' = 0$, we can get $\frac{2c^3h^2\pi e^{ch/k_{\text{B}}T\lambda}}{(-1+e^{ch/k_{\text{B}}T\lambda})^2k_{\text{B}}T\lambda^7} - \frac{10c^2h\pi}{(-1+e^{ch/k_{\text{B}}T\lambda})\lambda^6} = 0$
Ignore the root where $\lambda = 0$ we can get $\frac{che^{ch/k_{\text{B}}T\lambda}}{(-1+e^{ch/k_{\text{B}}T\lambda})\lambda k_{\text{B}}T} = 5$
Let $x = \frac{ch}{k_{\text{B}}T\lambda}$ we get $\frac{xe^x}{e^x-1} = 5$
The solution $x \approx 4.96$, so $\lambda_m T = \frac{xk_{\text{B}}}{ch} \approx 2.9 \times 10^{-3}(m \cdot K)$

6 Problem 6

(a)

Since $\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta)$
Then $\frac{c}{\nu_2} = \frac{h}{mc}(1 - \cos\theta) + \frac{c}{\nu}$
The energy of scattered photon is $h\nu_2 = \frac{hc}{\frac{h}{mc}(1-\cos\theta)+\frac{c}{\nu}} = \frac{hm\nu c^2}{h\nu+mc^2-h\nu\cos\theta}$

For the electron, the total energy will be $E_e = h\nu + mc^2 - h\nu_2$, and $K_c = E_e - mc^2 = h\nu - h\nu_2$
$K_c = E_e - mc^2 = h\nu(1 - \frac{mc^2}{h\nu+mc^2-h\nu\cos\theta})$
When $\cos\theta = -1$, we can get the max $K_c = \frac{h\nu}{1+\frac{mc^2}{2h\nu}}$

(b)

$K_c = \frac{2E^2}{2E+mc^2}$, then $2E^2 - 2EK_c - mc^2K_c = 0$
$E = \frac{2K_c \pm \sqrt{4K_c^2+8mc^2K_c}}{4}$, keep the positive solution we can get
$E = 1.133 \times 10^{-13}J$
$\lambda = \frac{c}{\nu} = \frac{hc}{E} = 1.75 \times 10^{-12}m$

7 Problem 7

$\Psi_{12} = |\Psi_1 + \Psi_2|^2 = |\Psi_1|^2 + |\Psi_2|^2 + 2\text{Re}(\Psi_1\Psi_2^*)$
$\Psi_1 = \frac{1}{\sqrt{2}}e^{-\frac{y^2}{2}}\cos(\omega t - ay) + \frac{1}{\sqrt{2}}e^{-\frac{y^2}{2}}\sin(\omega t - ay)i$

$\Psi_1 = \frac{1}{\sqrt{2}} e^{-\frac{y^2}{2}} \cos(\omega t - ay - by) + \frac{1}{\sqrt{2}} e^{-\frac{y^2}{2}} \sin(\omega t - ay - by) i$
$|\Psi_1|^2 = |\Psi_2|^2 = \frac{e^{-y^2}}{2}$
$\mathrm{Re}(\Psi_1 \Psi_2^*) = \frac{e^{-y^2}}{2} \cos(\omega t - ay) \cos(\omega t - ay - by) + \frac{e^{-y^2}}{2} \sin(\omega t - ay) \sin(\omega t - ay - by)$
$\mathrm{Re}(\Psi_1 \Psi_2^*) = \frac{e^{-y^2}}{2} \cos by$
$\Psi_{12} = e^{-y^2}(1 + \cos by)$