Problem Set 9

Due: 31 July 2020, 12.30 p.m.

Problem 1. Use the Maxwell's distribution of molecular speeds to calculate $\langle v^2 \rangle$ for molecules of a gas.

Hint. To find the Gaussian integral that shows up in this problem, differentiate both sides of $\int_0^\infty e^{-\alpha u^2}\, du = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$ twice with respect to the parameter α.

(*2 points*)

Problem 2. Consider a sample containing hydrogen atoms at 300 K. What distribution function can you use to describe it? (a) Compute the number of atoms in the first ($n = 2$) and second ($n = 3$) excited states compared to those in the ground state ($n = 1$). Include the effects of degeneracy in your calculations. (b) At what temperature would 1 percent of the atoms be in the $n = 2$ state? (c) At the temperature found in (b), what fraction of the atoms will be in the $n = 3$ state?

(*3 × 1 point*)

Problem 3. This problem is related to the equipartition theorem. Consider a system in which the energy of a particle is given by $E = Au^2$, where A is a constant and u is any coordinate or momentum that can vary from $-\infty$ to $+\infty$. (a) Write the probability of the particle having u in the range $(u, u + du)$ and calculate the normalization constant C in terms of A. (b) Calculate the average energy $\langle E \rangle = \langle Au^2 \rangle$ and show that $\langle E \rangle = \frac{1}{2} k_B T$.

Hint. Use the same method as in Problem 1 to evaluate the integral in (b).

(*2 + 2 points*)

Problem 4. Given two containers at the same temperature, one filled with a gas of classical molecules, one with a fermion gas, which will have higher pressure? Support your answer (argue without doing any calculations).

(*2 points*)

Problem 5. A container at 300 K contains H_2 gas at a pressure of one atmosphere. At this temperature H_2 obeys the Boltzmann distribution (see Problem 2). To what temperature must the H_2 gas be cooled before quantum effects become important and the use of the Boltzmann distribution is no longer appropriate?

(*Hint*: Equate the de Broglie wavelength at the average energy to the average spacing between molecules, using the ideal gas law $pV = nRT$ to compute the density.)

(*3 points*)

Problem 6. Find the average energy of the electrons at $T = 0$ K in (a) copper ($E_F = 7.06$ eV) and (b) lithium ($E_F = 4.77$ eV).

(*1 + 1 point*)

Problem 7. Use a computer to plot $f_{\mathrm{FD}}(E)$ versus E for (a) $T = 0.1T_{\mathrm{F}}$ and (b) T $= 0.5T_{\mathrm{F}}$, where $T_{\mathrm{F}} = E_{\mathrm{F}}/k_{\mathrm{B}}$.

Assume $\mu(T) = E_F$ in both cases.

(1 + 1 points)

Problem 8. Show that for $T = 300$ K, only about 0.1 percent of the free electrons in metallic silver have an energy greater than E_{F}.

Hint. Argue that (1) most of the excited electrons are within about $2k_{\mathrm{B}}T$ above the Fermi energy; (2) for $k_{\mathrm{B}}T \ll E_{\mathrm{F}}$ the number of excited electrons $N_{\mathrm{ex}} \approx g(E_{\mathrm{F}})k_{\mathrm{B}}T$.

(3 points)

Problem 9. Find the density of states for the free electron gas in (a) 2D, (b) 1D.

(3/2 + 3/2 points)

Problem 10. Calculate the Fermi energy for magnesium in a long, very thin wire (1D).

(2 points)