Review Questions
(week of 6 July 2020)

1. Fill the blanks

   $\hbar^2 \ell(\ell+1)$

   - $\hat{L}^2 Y_{lm}(\vartheta,\varphi) = \hbar^2 \ldots\ldots Y_{lm}(\vartheta,\varphi)$, where $l = \ldots\ldots\ldots\ldots\ldots\ldots$, $m = \ldots$
   - $\hat{L}_z Y_{lm}(\vartheta,\varphi) = \hbar \ldots Y_{lm}(\vartheta,\varphi)$, where $l = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$, $m = .$ $\hat{L}_z \phi = \hbar m \phi$
   - The functions Y_{lm} are called $\ldots\ldots\ldots\ldots\ldots\ldots$

2. Using arguments based on physics rather than mathematics, how can you explain that $|m| \le l$?

3. For $l = 2$ sketch a vector diagram illustrating possible directions of $\mathbf{L}$ in space.

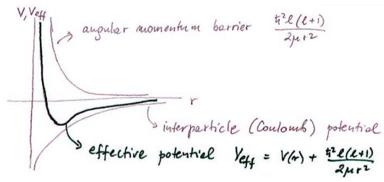
4. What is the rigid rotor model? Give an example of a system that we can describe using this model.

5. What is the origin of the so-called *rotational energy levels* of diatomic molecules? $\hat{L}^2\psi = 2IE\psi$

6. Any two-particle problem with a central potential field $V(r)$ can be always separated into translational motion of $\ldots\ldots\ldots\ldots\ldots\ldots$ and motion of a single particle with mass $\mu = \ldots\ldots\ldots$ about the center with the potential energy $V(r)$.

7. $m_{\text{electron}} \ldots\ldots\ldots m_{\text{proton}}$, hence in the hydrogen atom problem the reduced mass $\mu \approx \ldots\ldots\ldots\ldots$

8. Sketch the effective potential energy of the hydrogen atom.



9. What is the angular momentum barrier?

10. What does it mean that the energy levels of the hydrogen atom are degenerate?

11. What do we mean when we say that the degeneracy of an energy level has been "lifted" (or "removed")?

12. Placing a hydrogen atom in an external magnetic field lifts the degeneracy due to the magnetic quantum number m. This is known as the $\ldots\ldots\ldots\ldots\ldots$ effect [Answer: Zeeman].

    See also: `http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/zeeman.html`

13. What are the so-called selection rules?

14. The selection rules for transitions between the energy levels in the hydrogen atom are $\Delta m = \ldots\ldots$ or $\ldots\ldots$ and $\Delta l = \ldots\ldots$

15. For transitions between states of the hydrogen atom labelled with quantum numbers (n, l, m) mark with "X" these that are forbidden. Indicate the selection rule(s) that is (are) violated.

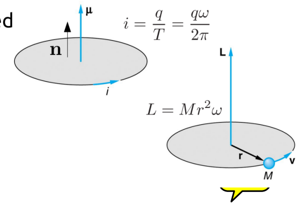
$$(1,0,0) \mapsto (3,\underline{0},0) \qquad (2,1,-1) \mapsto (3,\underline{1},0) \qquad (2,1,-1) \mapsto (3,2,\underline{1}),$$
$$(2,1,-1) \mapsto (3,2,-1) \qquad (3,1,-1) \mapsto (4,\underline{3},-1) \qquad (3,1,-1) \mapsto (4,2,0)$$

16. For the hydrogen atom in the ground state, the most probable distance of the electron from the proton is equal to $\ldots\ldots\ldots\ldots\ldots\ldots$, which is of the order of $10^{\cdots\cdots}$ m.

17. *True or false? For wave functions $\psi_{n,0,m}$ representing states of the Hydrogen atom with $l = 0$, the probability density $|\psi_{n,0,m}|^2$ is spherically symmetric.*

Recall: magnetic moment of a charged classical particle in orbital motion

$$\boldsymbol{\mu} = iA\,\mathbf{n} = \frac{q}{2M}\mathbf{L}$$



18. What is the interpretation of

    - $|\psi|^2 4\pi r^2\,\mathrm{d}r$
    - $\int_0^{a_0} |\psi|^2 4\pi r^2\,\mathrm{d}r$,

    where ψ is the wave function representing the ground state state of the Hydrogen atom and a_0 is the radius of the first Bohr orbit.

19. Using classical electrodynamics explain how is the magnetic dipole moment[1] of a charged particle in orbital motion related to its angular momentum.

20. Why is the magnetic moment of a particle in orbital motion quantized?

21. What happens with a magnetic moment placed in a uniform magnetic field?

22. (mark the correct answer) The potential energy corresponding to magnetic moment–magnetic field interaction is minimum if the magnetic moment is *parallel/anti-parallel/perpendicular* to the uniform external magnetic field.

23. Briefly describe the idea of the Stern–Gerlach experiment. What did it discover?

24. (fill the gaps) The algebra of spin operators is analogous to that of the orbital angular momentum operators and

    (a) $\hat{S}^2\chi = \hbar^2 \ldots\ldots \chi$, where $s = \ldots\ldots\ldots\ldots\ldots\ldots$

    (b) $\hat{S}_z\chi = \hbar \ldots \chi$, where $m_s = \ldots\ldots\ldots\ldots\ldots\ldots$

    $$\hat{S}^2\chi = \hbar^2 s(s+1)\chi, \qquad s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

    $$\hat{S}_z\chi = \hbar m_s\chi, \qquad m_s = -s, s+1, \ldots, s-1, s$$

25. *True or false*? Half-integer values of the spin quantum number s are possible.

26. *True or false*? Half-integer values of the orbital quantum number l are possible.

27. For the electron $s = 1/2$, hence the magnetic moment associated with the spin degree of freedom of the electron has .... (how many?) possible projections onto the z axis.

    2

[1] The magnetic dipole moment is often simply called the *magnetic moment*.