UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP141)

LABORATORY REPORT

EXERCISE 3
SIMPLE HARMONIC MOTION
OSCILLATIONS IN MECHANICAL SYSTEMS

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1 Objectives

In this exercise I'll study simple harmonic motion and learn how to use air track to find the spring constant and effective mass of a spring. I'll also analyse the relationship between the oscillation period and the mass of the oscillator, check whether the oscillation period depends on the the amplitude, and examine the relationship between the maximum speed and the amplitude.

2 Theoretical Background

2.1 Basic Concept of Simple Harmonic Motion

There are various kinds of periodic motion in nature.The simple harmonic motion is the simplest and most fundamental, where the restoring force is proportional to the displacement from the equilibrium position and as a result, the position of a particle depends on time as the sine or cosine function. Discussion of the simple harmonic motion is a basis for studying more complex situations.

2.1.1 Hooke's Law

Within the elastic limit of deformation, the force F_x needed to be applied in order to stretch or compress a spring by the distance x is proportional to that distance,for example:

$$F_x = kx \tag{1}$$

where k is a constant (called the spring constant) characterizing how easy it is to deform the spring. This constant will be found in the present exercise using a measurement device called the *Jolly balance*. The linear relation (1), between the force and the deformation, is known as the *Hooke's Law*. According to Newton's third law of dynamics, the spring exerts a reaction elastic force of the same magnitude but opposite direction. As this force tries to restore the system back to the equilibrium, it is also known as the restoring force.

2.1.2 Equation of Motion of the Simple Harmonic Oscillator

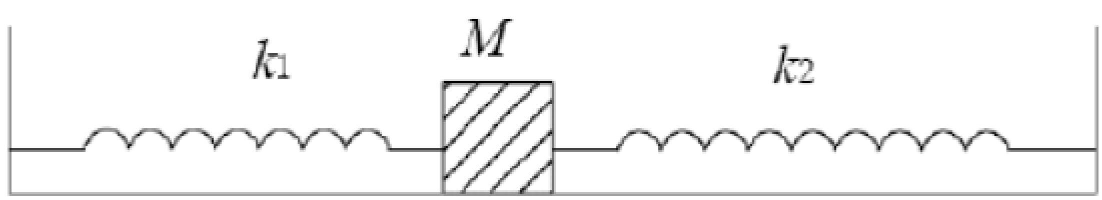

Figure 1: Mass-spring system

As shown in Figure 1, an object with mass M is set on an air track with a spring attached to both of its sides. The purpose of using the air track is to eliminate frictional forces between moving surfaces. The other ends of the springs are fixed to the air track.The spring constants k_1 and k_2 are to be measured with the Jolly balance. The origin ($x = 0$) of the coordinate system is set at the equilibrium position of the mass M.Assuming that the masses of the springs can be ignored, and neglecting damping in the system, the elastic (restoring) forces of the springs are the only forces acting on mass M. According to Newton's second law of dynamics, the equation of motion of mass M is

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + (k_1 + k_2)x = 0 \tag{2}$$

The general solution for equation (2) is

$$x(t) = A\cos(\omega_0 + \varphi_0) \tag{3}$$

where $\omega_0 = \sqrt{(k_1 + k_2)/M}$ is the natural angular frequency of the oscillations, A is their amplitude, and φ_0 is the initial phase. The natural angular frequency is determined by the parameters of the system itself, whereas the initial phase is determined by initial conditions. The natural period of oscillation is

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M}{k_1 + k_2}} \tag{4}$$

In this exercise, the relationship between the oscillation period and the mass of the oscillator will be studied.

2.2 Mass of the Spring

Whenever the mass of the springs cannot be ignored, we take it into account in terms of the so-called effective mass. The effective mass of the oscillator is the sum of the mass of the object and the effective mass of the spring. When we take the effective mass of the spring into account, the angular frequency of the system can be expressed as

$$\omega_0 = \sqrt{\frac{k_1 + k_2}{M + m_0}} \tag{5}$$

where m_0 is the effective mass of the spring, which is 1/3 of the actual mass of the spring.

2.3 Mechanical Energy in Harmonic Motion

The elastic potential energy for a spring-mass system is $U = kx^2/2$ and the kinetic energy of an oscillating mass m is $K = mv^2/2$

At the equilibrium position ($x = 0$), the speed of the mass is maximum $v = v_max$. At this point the total Mechanical Energy is equal to the maximum kinetic energy K_max.

On the other hand, at maximum displacement ($x = A$), the mass is instantaneously at rest ($v = 0$), and the contribution to the total mechanical energy is due to the potential energy only, which is at its maximum $U_m ax$. In the absence of non-conservative forces (such as frictional forces or drag forces), the total mechanical energy is conserved and $K_m ax = U_m ax$, which implies

$$k = \frac{mv_{max}^2}{A^2} \tag{6}$$

3 Apparatus

The measurement equipment consists of the following elements: springs, Jolly balance, air track, electronic timer, electronic balance, and masses.

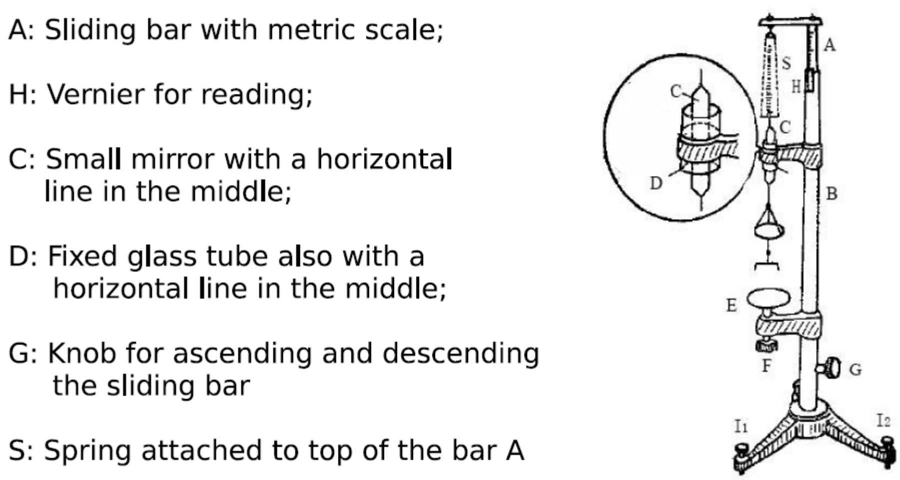

Figure 2: Jolly balance

In order to measure the spring constant using the Jolly balance, we need to place the small mirror C in the tube D and make three lines coincide: the line on the mirror, the line on the glass tube and its reflection in the mirror. First, without adding any weight on the bottom end of the spring, adjust the knob G and make the three lines coincide. Then read the scale L_1.

Second, add mass m to the button of the spring. The spring is stretched and the three lines no longer coincide. Adjust knob G to make them into one line again and read the corresponding number on scale L_2. The spring constant may be then found as

$$k = \frac{mg}{L_2 - L_1} \tag{7}$$

With a series of measurement for different masses m, we may estimate the spring constant by finding a linear fit to the data using the least squares method.

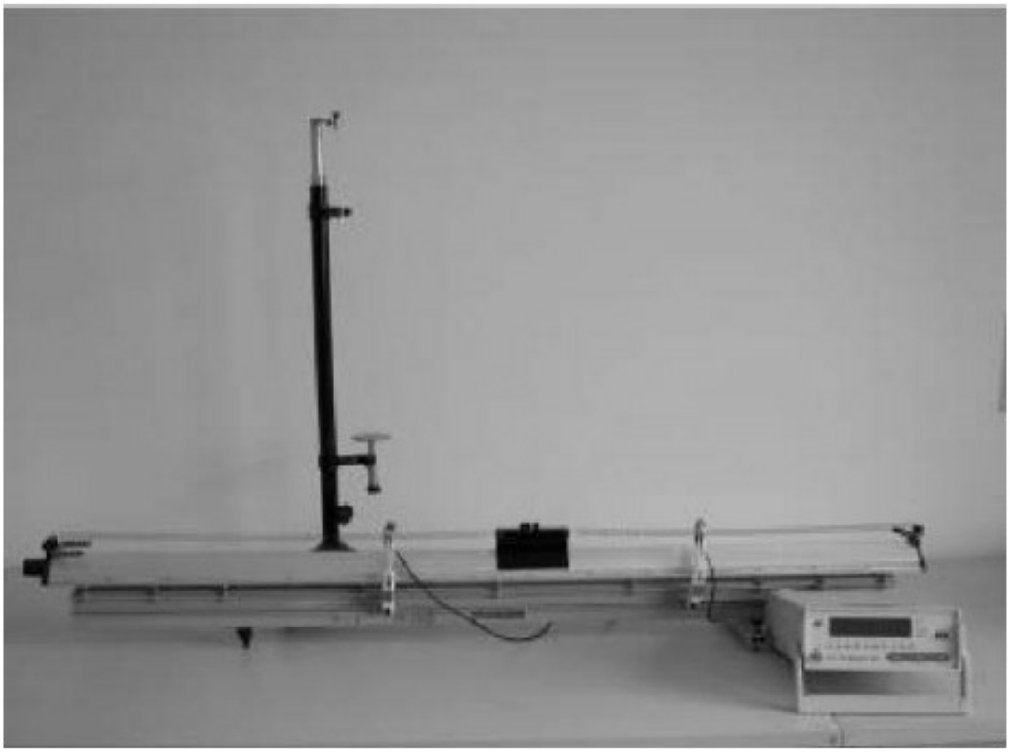

Figure 3: The experiment setup.

A photoelectric measuring system consists of two photoelectric gates and an electronic timer. When a shutter placed on the object passes a gate, it blocks the light emitted from the light source at the top of the gate, and the receiver sends a signal to the electronic timer. Please note that for period measurements we use, we use the I-shape shutter.

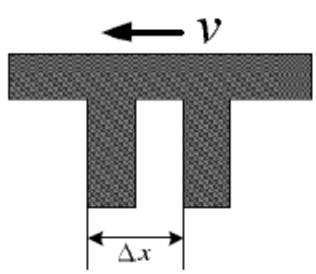

Figure 4: The U-shape shutter.

When measuring the speed of the object, we use a U-shaped shutter (Figure 4), so that the light is blocked twice during a pass. The timer will then record the time interval $\triangle t$ between the two generated signals. After the distance $\triangle x = \frac{1}{2}(x_{in} + x_{out})$ between the two arms of the U-shape shutter is measured, the speed of the object at the point of passing the gate is calculated as $v = \frac{\triangle x}{\triangle t}$.

4 Measurement Procedure

4.1 Spring Constant

1. Adjust the Jolly balance to be vertical: Attach the spring and the mirror as shown in Figure 2. Add a 20 g **preload** and adjust knob I_1 and I_2 to make sure the mirror can move freely through the tube.
   Check whether the Jolly balance is parallel to the spring, and adjust knobs if necessary. You should look at the balance from two orthogonal directions: from one direction, adjust the balance to be parallel to the spring; from the direction orthogonal to the previous one, check if the balance coincides with the spring.

2. Adjust knob G and make the three lines in the tube coincide. Adjust the position of the tube to set the initial position L_0 within $5.0 \sim 10.0$ cm.

3. Record the reading L_0 on the scale, add mass m_1 and record L_1.

4. Keep adding masses and take measurements for six different positions. The order of the masses should be recorded.

5. Estimate the spring constant k_1 using the least squares method.

6. Replace spring 1 with spring 2, repeat the measurements and calculate k_2.

7. Remove the preload and repeat the measurement for springs 1 and 2 connected in series. Calculate k_3 and compare it with the theoretical value.

4.2 Relation Between the Oscillation Period T and the Mass of the Oscillator M

(a) Adjust the air track so that it is horizontal.

   (a) Turn on the air pump and check if any of the holes on the air track are blocked. Call the instructor for help if you find blocked holes.

   (b) Place the cart on the track without any initial velocity. Adjust the track until the object moves slowly back and forth in both directions.
   Adjustment method. The air track has three knobs at the bottom: two on one side and another one on the other side. You can only adjust that single knob.

(b) Horizontal air track

   (a) Attach springs to the sides of the cart, and set up the I-shape shutter. Make sure that the photoelectric gate is at the equilibrium position.

(b) Add weight m_1. Let the cart oscillate about the photoelectric gate. The amplitude of oscillations should be about 5 cm. Release the cart with a caliper or a ruler. Set the timer into the "T" mode. The timer in this mode will automatically record the time of 10 oscillation periods. Record the mass of the cart and the period.

(c) Add weights to the object, repeat Step 2 and take measurements for 5 times.

(d) Analyze the relation between M and T by plotting a graph.

(c) Inclined air track

(a) Inclination of the air track is controlled with plastic plates. Place them under the air track, using three plates at a time.

(b) Repeat the steps in (b) for two different inclinations (for example, with 3 and 6 plates beneath the air track).

(c) Discuss the relation between M and T by plotting a graph.

4.3 Relation Between the Oscillation Period T and the Amplitude A

1. Keep the mass of the cart unchanged and change the amplitude (choose 6 different values). The recommended amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.

2. Apply linear fit to the data and comment on the relation between the oscillation period T and the amplitude A based on the correlation coefficient γ.

4.4 Relation Between the Maximum Speed and the Amplitude

1. Measure the outer distance x_{out} and the inner distance x_{in} of the U-shape shutter by a caliper. Calculate the distance $\triangle x = (x_{out} + x_{in}) = 2$.

2. Change the shutter from I- to U-shape. Set the timer into the "S_1" mode. Let the cart oscillate. Record the second readings of the time interval $\triangle t$ only if the two subsequent readings show the same digits to the left of the decimal point.

3. Change the amplitude (choose 6 different values). The recommended amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.

4. Measure the maximum speed v_max for different values of the amplitude A. Obtain the spring constant from equation (6). Compare this result to that of the first part.

4.5 Mass measurement

1. Adjust the electronic balance every time before you use it. The level bubble should be in the center of the circle.

2. Add weights according to a fixed order. Weigh the cart with the I-shape shutter and with the U-shape shutter. Measure the mass of spring 1 and spring 2.
3. Record the data only after the circular symbol on the scales display disappears.

5 Calculation and Results

5.1 Measurement Results

5.1.1 Spring Constant Measurement Data

spring 1 [mm] ± 0.1[mm]		spring 2 [mm] ± 0.1[mm]		spring 3 [mm] ± 0.1[mm]	
L_0	105.0	L_0	65.5	L_0	50.0
L_1	125.0	L_1	84.6	L_1	91.2
L_2	147.2	L_2	106.6	L_2	134.6
L_3	168.5	L_3	126.3	L_3	178.0
L_4	188.6	L_4	147.3	L_4	217.3
L_5	210.0	L_5	168.5	L_5	260.5
L_6	230.0	L_6	189.1	L_6	301.6

Table 1: Spring constant measurement data

5.1.2 Time Measurements of Ten Periods for T vs. M Relation

Ten periods [ms] ± 0.1[ms]					
horizontal		incline 1		incline 2	
m_1	12893.6	m_1	12897.1	m_1	12897.0
m_2	13059.8	m_2	13058.5	m_2	13057.2
m_3	13213.2	m_3	13215.0	m_3	13215.0
m_4	13373.7	m_4	13371.5	m_4	13371.5
m_5	13529.2	m_5	13528.1	m_5	13526.0
m_6	13682.2	m_6	13680.2	m_6	13677.3

Table 2: Data for the T vs. M relation

5.1.3 Time Measurements of Ten Periods for T vs. A Relation

A [mm] ± 1[mm]		ten periods [ms] ± 0.1[ms]
1	50	12731.3
2	100	12734.1
3	150	12736.2
4	200	12735.3
5	250	12734.1
6	300	12733.7

Table 3: Data for the T vs. A relation

5.1.4 Measurement Data for the v_{max}^2 vs. A^2 Relation

A [mm] ± 1[mm]		$\triangle t$ [ms] ± 0.1[ms]
1	50	40.55
2	100	20.49
3	150	13.84
4	200	10.35
5	250	8.24
6	300	6.89
x_{in} [mm] ± 0.02[mm]		x_{out} [mm] ± 0.02[mm]
4.20		15.44
4.12		15.24
4.14		15.56

Table 4: Data for the v_{max}^2 vs. A^2 relation

5.1.5 Measurement Data for Weight

m[g] ± 0.01[g]	
1	4.66
2	9.47
3	14.12
4	18.88
5	23.67
6	28.42

Table 5: Weight measurement data

5.1.6 Mass Measurement Data

object with I-shape m_{obj}[g] $\pm$ 0.01[g]
178.08
object with U-shape m_{obj}[g] $\pm$ 0.01[g]
187.19
mass of spring 1 m_{spr1}[g] $\pm$ 0.01[g]
11.23
mass of spring 2 m_{spr2}[g] $\pm$ 0.01[g]
10.72

Table 6: Mass measurement data

5.1.7 Other Relative Data

g [m/s]
9.794
mass of preload [g]
20

Table 7: Acceleration due to gravity and mass of preload

5.2 Calculations

5.2.1 Calculation of Equivalent Mass M_0

$$M_0 = m_{obj} + \frac{1}{3}m_{spr1} + \frac{1}{3}m_{spr2}$$

$$M_I = 178.08 + \frac{1}{3} \times 11.23 + \frac{1}{3} \times 10.72 = 185.40[g]$$

$$M_U = 187.19 + \frac{1}{3} \times 11.23 + \frac{1}{3} \times 10.72 = 194.51[g]$$

5.2.2 Calculation of $\triangle x$, v, and mv^2

$$\bar{x}_{in} = \frac{1}{3}(4.20 + 4.12 + 4.14) = 4.15[mm]$$

$$\bar{x}_{out} = \frac{1}{3}(15.44 + 15.24 + 15.56) = 15.41[mm]$$

$$\triangle x = \frac{1}{2}(\bar{x}_{in} + \bar{x}_{out}) = \frac{1}{2}(4.15 + 15.41) = 9.78[mm]$$

$$v = \frac{\triangle x}{\triangle t}$$

$$v_1 = \frac{\triangle x}{\triangle t_1} = \frac{9.78}{40.55} = 0.24[m/s]$$

$$m{v_1}^2 = 194.51 \times (0.24)^2 = 1.1 \times 10^{-2}[kg \cdot m^2/s^2]$$

	$\triangle t$ [ms] ± 0.1[ms]	v[m/s]	$mv^2[kg \cdot m^2/s^2]$
1	40.55	0.24	1.1×10^{-2}
2	20.49	0.48	4.5×10^{-2}
3	13.84	0.71	9.8×10^{-2}
4	10.35	0.94	1.7×10^{-1}
5	8.24	1.19	2.8×10^{-1}
6	6.89	1.42	3.9×10^{-1}

Table 8: Calculated data for v_{max} and mv^2

5.3 Theoretical Value k from data measured by Jolly balance

At the first of the measurement procedure, I used a Jolly balance to measure the spring constant of k_1, k_2, and $k_1 + k_2$ indirectly. The Jolly balance can recorded the length of spring when its restoring force is increasing, then I can get its k through linear fit diagram. The restoring force equals to the total weight connected to the spring

$$F_1 = (m_p + m_1)g = (20 + 4.66) \times 9.794 = 241.52[mN]$$

spring 1 [mm] ± 0.1[mm]		spring 2 [mm] ± 0.1[mm]		spring 3 [mm] ± 0.1[mm]		mg[mN] ± 0.1[mN]	
L_0	105.0	L_0	65.5	L_0	50.0	F_0	195.88
L_1	125.0	L_1	84.6	L_1	91.2	F_1	241.52
L_2	147.2	L_2	106.6	L_2	134.6	F_2	288.63
L_3	168.5	L_3	126.3	L_3	178.0	F_3	334.17
L_4	188.6	L_4	147.3	L_4	217.3	F_4	380.79
L_5	210.0	L_5	168.5	L_5	260.5	F_5	427.70
L_6	230.0	F_6	189.1	L_6	301.6	F_6	474.23

Table 9: Indirect spring constant measurement data

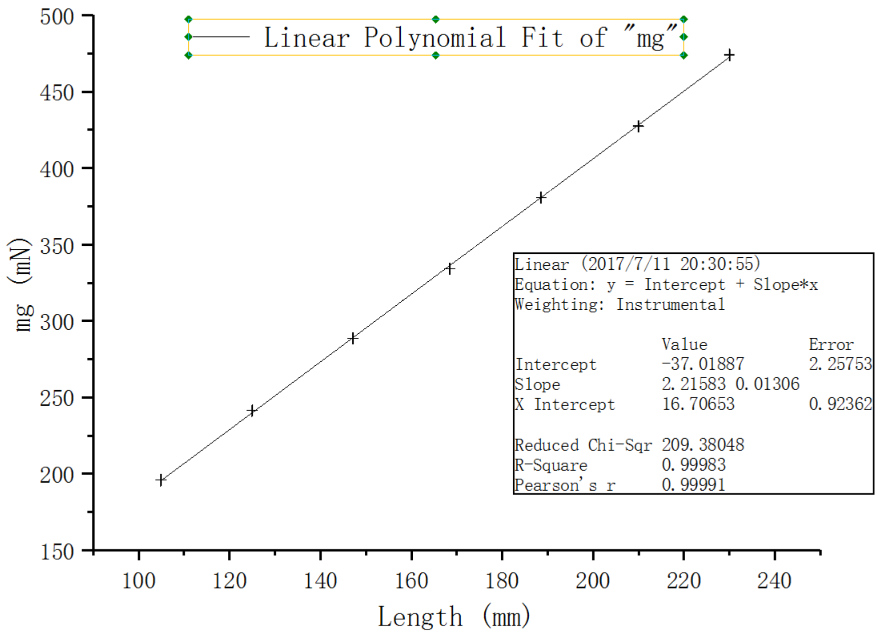

Figure 5: The linear fit diagram for spring 1.

Through the linear fir diagram, I know $k_{spr1} = 2.216 \pm 0.013$[N/m]

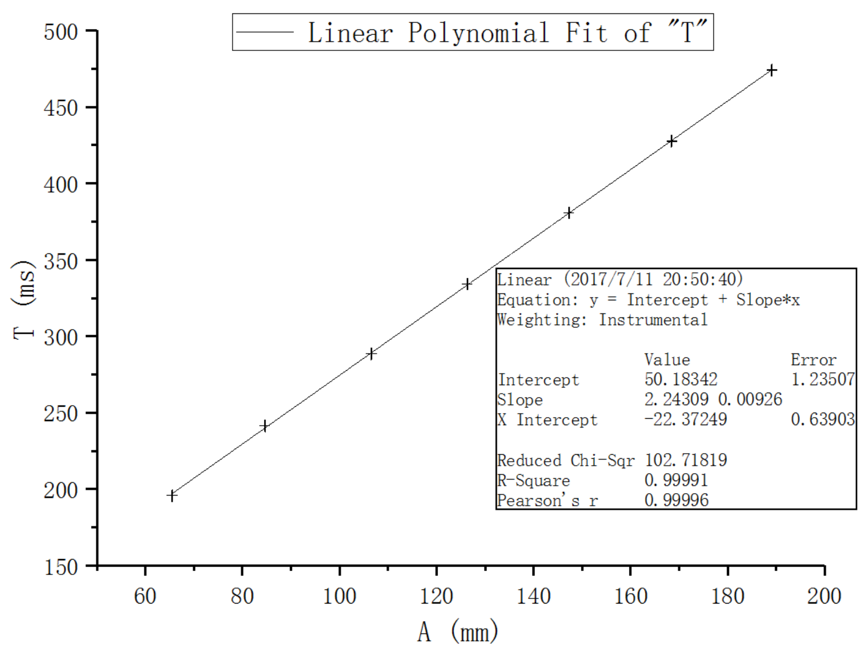

Figure 6: The linear fit diagram for spring 2.

Through the linear fir diagram, I know $k_{spr2} = 2.243 \pm 0.009$[N/m] When I measured the k of the series, I didn't use preload, but the result should be same because k is only relative the change of F, so I can use the data for measurement of k_{spr1} and k_{spr2}.

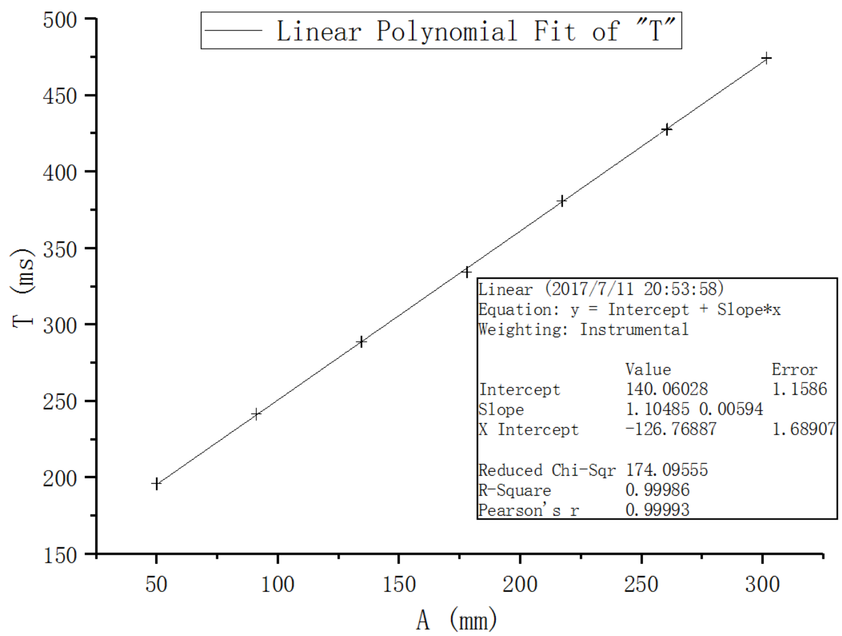

Figure 7: The linear fit diagram for series.

Through the linear fit diagram, I know $k_{sp1+sp2} = 1.105 \pm 0.006$[N/m]

5.4 Diagram for Relation between T and M

Preload was no longer used after the measurement of spring constant, so M is equal to the weight measurement data but I have to add the equivalent mass of object with I-shape.

$$m_1 = 4.66 + 185.40 = 190.06[g] = 0.19[kg]$$

In addition, since T is proportional to $\sqrt{M}$, I use square of one period T^2 as y coordinates to draw the linear fit graph.

Periods [s^2]						M[kg] $\pm 2\times10^{-5}[kg]$
horizontal		incline 1		incline 2		
m_1	1.6624	m_1	1.6634	m_1	1.6633	0.1901
m_2	1.7056	m_2	1.7052	m_2	1.7049	0.1949
m_3	1.7459	m_3	1.7464	m_3	1.7464	0.1995
m_4	1.7886	m_4	1.7880	m_4	1.7880	0.2042
m_5	1.8304	m_5	1.8301	m_5	1.8295	0.2091
m_6	1.8720	m_6	1.8715	m_6	1.8707	0.2138

Table 10: Data for the T vs. M relation

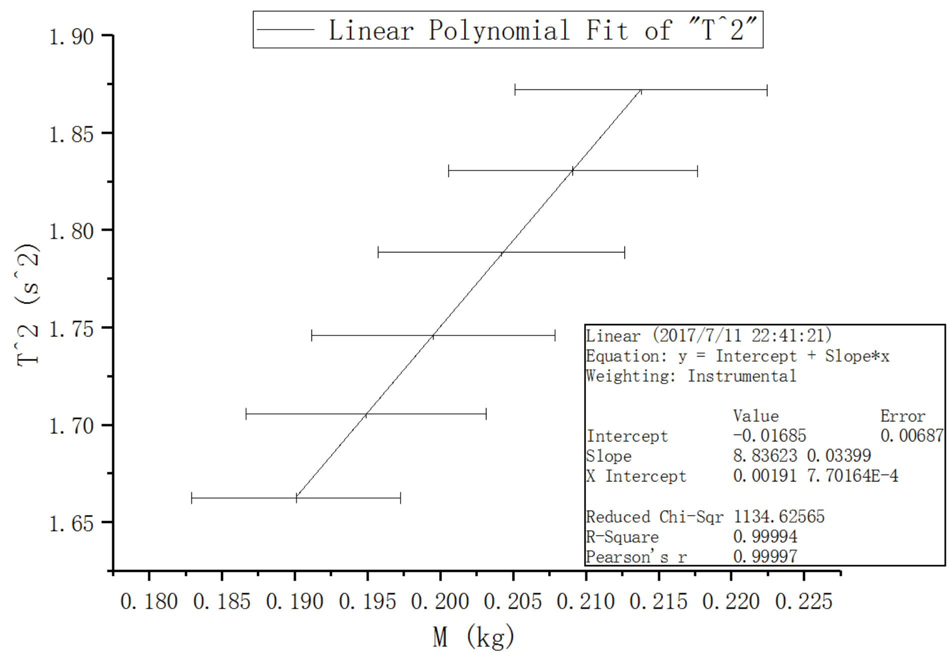

Figure 8: The linear fit diagram for T^2 vs. M when air track is horizontal.

From the diagram, I can see that T^2 is proportional to the value of M when the air track is horizontal. Then I will examine what will happen when the track form an incline with the ground. (The uncertain of error bar is in the uncertainty analysis section)

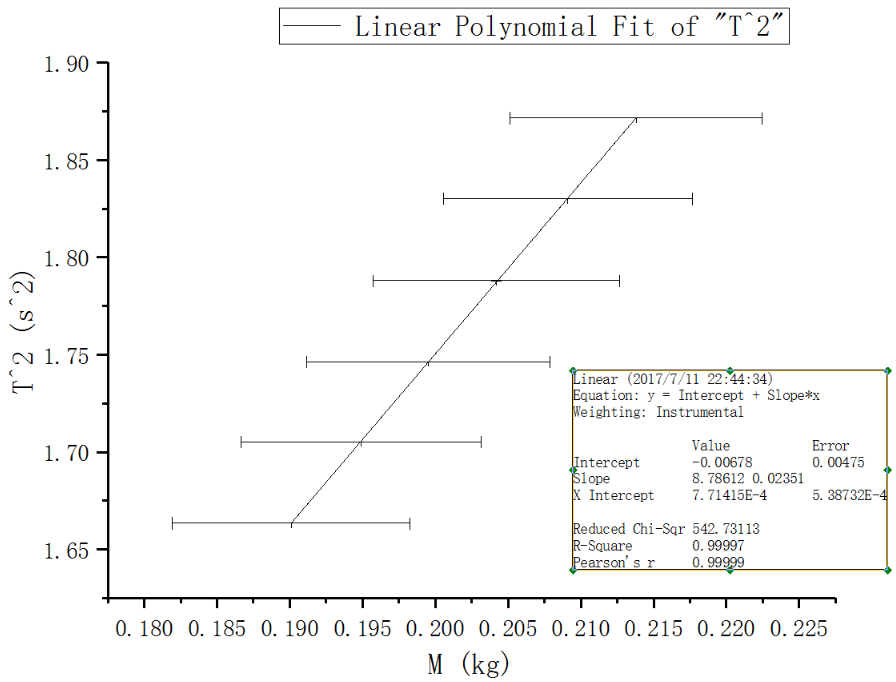

Figure 9: The linear fit diagram for T^2 vs. M when air track forms an incline 1.

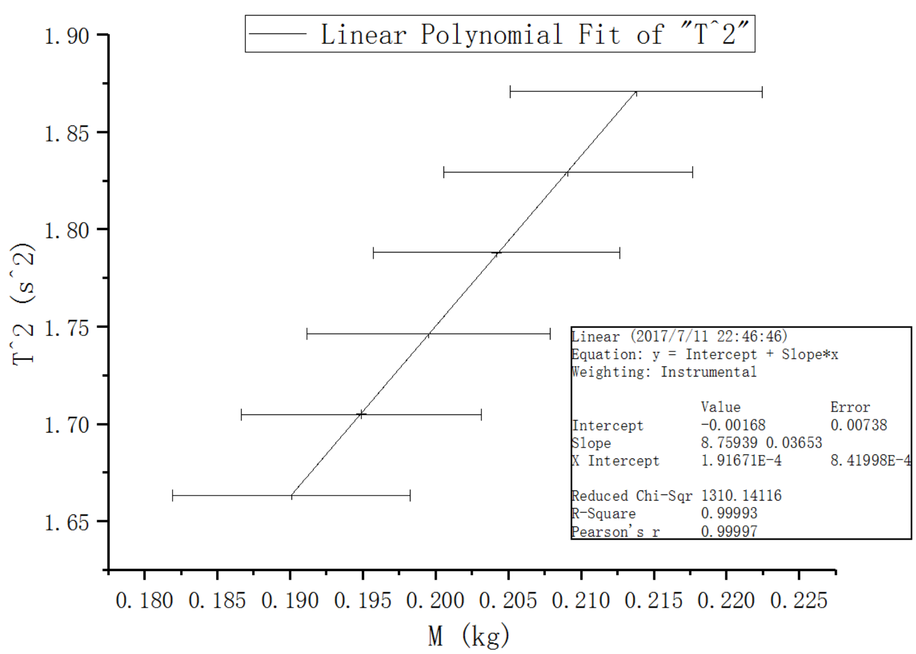

Figure 10: The linear fit diagram for T^2 vs. M when air track forms an incline 2.

From the three diagrams we can see the three slope is $k_1 = 8.836 \pm 0.034$, $k_2 = 8.786 \pm 0.024$, and $k_3 = 8.759 \pm 0.037$, which are very close. As a result, I think the angle between the air track and ground has no effect on the relation between T and M.

5.5 Diagram for Relation between T and A

	A[mm]±1[mm]	ten periods [ms] ± 0.1[ms]
1	50	12731.3
2	100	12734.1
3	150	12736.2
4	200	12735.3
5	250	12734.1
6	300	12733.7

Table 11: Data for T vs. A relation

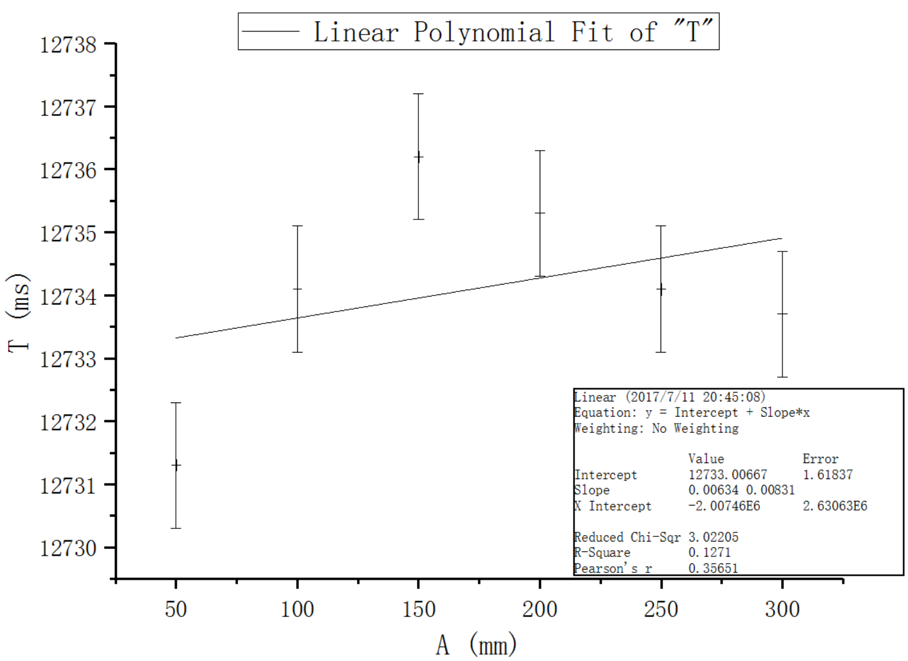

Figure 11: The linear fit diagram for T vs. A relation.

From the diagram I can see that the slope is very small but it still fit all the points. It seems that all the points on the diagram is not well distributed so A has nothing to do with T.

5.6 Diagram for Relation between v_{max}^2 vs. A^2

	A^2[m^2]	$v_{max}^2[m^2/s^2]$
1	0.0025	0.0576
2	0.0100	0.2304
3	0.0225	0.5041
4	0.0400	0.8836
5	0.0625	1.4161
6	0.0900	2.0164

Table 12: Data for v_{max}^2 vs. A^2 relation

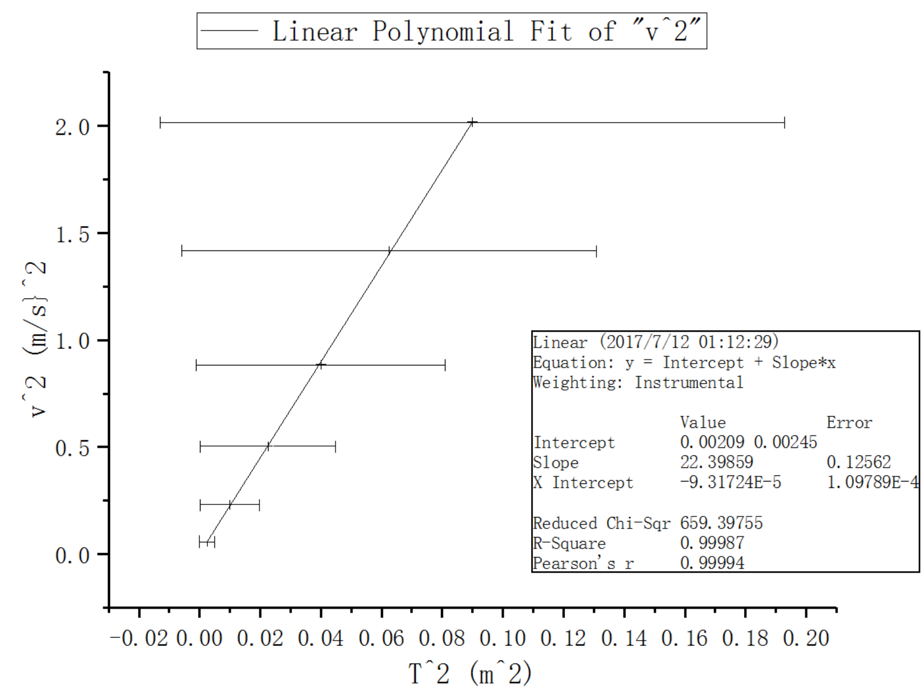

Figure 12: The linear fit diagram for v_{max}^2 vs. A^2.

From the diagram I know that the slope between v_{max}^2 and A^2 is 22.42 ± 0.14 $[1/s^2]$, so V is proportional to A. According to equation (6), $k = M_U \times slope = 0.194 \times 22.42 = 4.36 \pm 0.027[N/m]$

6 Measurement Uncertainty Analysis

6.1 Multiple Measurements Yield to Type A Uncertainty

For multiple measurements' uncertainty:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$S_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2}$$

$$\triangle_A = \frac{S_X t_{0.95}}{\sqrt{n}}$$

The value of $t_{0.95}$ and $\frac{t_{0.95}}{\sqrt{n}}$ are giving in Table 7

n	3	4	5	6	7	8	9	10	15	20	≥ 100
$t_{0.95}$	4.30	3.18	2.78	2.57	2.45	2.36	2.31	2.26	2.14	2.09	≤ 1.97
$\frac{t_{0.95}}{\sqrt{n}}$	2.48	1.59	1.204	1.05	0.926	0.834	0.770	0.715	0.553	0.467	≤ 0.139

Table 13: The values of $t_{0.95}$ and $\frac{t_{0.95}}{\sqrt{n}}$

6.1.1 Type A Uncertainty for x_{in} and x_{out}

$$\bar{x}_{in} = \frac{1}{3}(4.20 + 4.12 + 4.14) = 4.15[mm]$$

$$S_x = \sqrt{\frac{1}{2}\sum_{i=1}^{3}(x_i - \bar{x}_{in})^2} = 0.04$$

$$\triangle_A = \frac{S_X t_{0.95}}{\sqrt{n}} = 0.04 \times \frac{4.3}{\sqrt{3}} = 0.099[mm]$$

$$\bar{x}_{out} = \frac{1}{3}(15.44 + 15.24 + 15.56) = 15.41[mm]$$

$$S_x = \sqrt{\frac{1}{2}\sum_{i=1}^{3}(x_i - \bar{x}_{out})^2} = 0.16$$

$$\triangle_A = \frac{S_X t_{0.95}}{\sqrt{n}} = 0.16 \times \frac{4.3}{\sqrt{3}} = 0.397[mm]$$

6.2 Uncertainty for x_{in}, x_{out}, and $\triangle x$

Since $\triangle_B = 0.02[mm]$ then

$$u_{in} = \sqrt{\triangle_A^2 + \triangle_B^2} = \sqrt{0.02^2 + 0.172^2} = 0.101[mm]$$

$$u_r = \frac{u_{in}}{\bar{x}_{in}} \times 100\% = \frac{0.101}{4.15} \times 100\% = 2.43\%$$

Similarly, $u_{out} = 0.398[mm]$ and $u_r = 2.58\%$

$$\triangle x = \frac{1}{2}(x_{in} + x_{out})$$

$$u_{\triangle x} = \sqrt{(\frac{\partial \triangle x}{\partial x_{in}} \cdot u_{in})^2 + (\frac{\partial \triangle x}{\partial x_{in}} \cdot u_{out})^2} = \sqrt{(\frac{0.101}{2})^2 + (\frac{0.398}{2})^2} = 0.205[mm]$$

$$u_r = \frac{u_{\triangle x}}{\triangle x} \times 100\% = \frac{0.716}{9.78} \times 100\% = 2.10\%$$

	Value [m]	Uncertainty [m]	Relative uncertainty %
x_{in}	4.15×10^{-3}	1.01×10^{-4}	2.43%
x_{out}	1.541×10^{-2}	3.98×10^{-4}	2.58%
$\triangle x$	9.78×10^{-3}	2.05×10^{-4}	2.1%

Table 14: Uncertainty for x_{in}, x_{out}, and $\triangle x$

6.3 Uncertainty for Equivalent Mass

$$u_M = m_{obj} + \frac{1}{3}m_{spr1} + \frac{1}{3}m_{spr}u_m = \sqrt{(\frac{\partial M_0}{\partial m_{obj}} \cdot u_{m(obj)})^2 + (\frac{\partial M_0}{\partial m_{spr1}} \cdot u_{m(spr1)})^2 + (\frac{\partial M_0}{\partial m_{spr2}} \cdot u_{m(spr2)})^2}$$

$$u_I = u_U = \sqrt{0.01^2 + (\frac{1}{3} \times 0.01)^2 + (\frac{1}{3} \times 0.01)^2} = 1 \times 10^{-5}[kg]$$

$$u_{rI} = \frac{u_I}{M_I} \times 100\% = \frac{0.011}{185.4} \times 100\% = 0.01\%$$

$$u_{rU} = \frac{0.011}{194.51} \times 100\% = 0.01\%$$

6.4 Uncertainty for T vs. M Relation

Since I draw a picture of T^2 and M, I must calculate the uncertainty of T^2 and M.

$$T^2 = (\frac{t}{10})^2$$

$$u_{T^2} = \sqrt{(\frac{\partial T^2}{\partial t} \cdot u_t)^2} = \sqrt{\frac{2t}{5} \times 10^{-4}}$$

$$u_1 = \sqrt{0.4 \times 10^{-4} \times 1.28936} = 7.18 \times 10^{-3}[s^2]$$

$$u_r = \frac{u_1}{T^2} \times 100\% = \frac{7.18 \times 10^{-3}}{1.6624} \times 100\% = 0.43\%$$

Horizontal			
	$T^2[s^2]$	Uncertainty $[s^2]$	Relative Uncertainty %
m_1	1.6624	7.18×10^{-3}	0.43%
m_2	1.7056	8.26×10^{-3}	0.48%
m_3	1.7459	8.36×10^{-3}	0.48%
m_4	1.7886	8.46×10^{-3}	0.47%
m_5	1.8304	8.56×10^{-3}	0.47%
m_6	1.8720	8.65×10^{-3}	0.46%

Table 15: Uncertainty for T^2 when air track is horizontal

Incline 1			
	$T^2[s^2]$	Uncertainty $[s^2]$	Relative Uncertainty %
m_1	1.6634	8.16×10^{-3}	0.49%
m_2	1.7052	8.26×10^{-3}	0.48%
m_3	1.7464	8.36×10^{-3}	0.48%
m_4	1.7880	8.46×10^{-3}	0.47%
m_5	1.8301	8.56×10^{-3}	0.47%
m_6	1.8715	8.65×10^{-3}	0.46%

Table 16: Uncertainty for T^2 when air track forms incline 1

Incline 2			
	$T^2[s^2]$	Uncertainty $[s^2]$	Relative Uncertainty %
m_1	1.6633	8.16×10^{-3}	0.49%
m_2	1.7049	8.26×10^{-3}	0.48%
m_3	1.7464	8.36×10^{-3}	0.48%
m_4	1.7880	8.46×10^{-3}	0.47%
m_5	1.8295	8.55×10^{-3}	0.47%
m_6	1.8707	8.65×10^{-3}	0.46%

Table 17: Uncertainty for T^2 when air track forms incline 2

6.5 Uncertainty for A^2 and v^2

$$u_{A^2} = \sqrt{(2A \cdot u_A)^2}$$
$$u_1 = \sqrt{(2A_1)^2} = 100[mm^2] = 1 \times 10^{-4}[m^2]$$
$$u_r = \frac{u_1}{A_1^2} \times 100\% = \frac{100}{2500} \times 100\% = 4\%$$

	A^2 $[m^2]$	Uncertainty $[m^2]$	Relative uncertainty %
1	0.0025	1×10^{-4}	4%
2	0.0100	2×10^{-4}	2%
3	0.0225	3×10^{-4}	1.33%
4	0.0400	4×10^{-4}	1%
5	0.0625	5×10^{-4}	0.8%
6	0.0900	6×10^{-4}	0.67%

Table 18: Uncertainty for A^2

$$u_v = \sqrt{(\frac{\partial v}{\partial \triangle t} \cdot u_{\triangle t})^2 + (\frac{\partial v}{\partial \triangle x} \cdot u_{\triangle x})^2} = \sqrt{(-\frac{\triangle x}{\triangle t^2} \cdot u_{\triangle t})^2 + (\frac{1}{\triangle t} \cdot u_{\triangle x})^2}$$
$$u_{v1} = \sqrt{(-\frac{9.78 \times 10^{-3}}{(4.06 \times 10^{-2})^2} \cdot 10^{-4})^2 + (\frac{1}{4.06 \times 10^{-2}} \cdot 2.05 \times 10^{-4})^2} = 5.09 \times 10^{-3}[m/s]$$
$$u_{r1} = \frac{u_{v1}}{v_1} \times 100\% = \frac{5.09 \times 10^{-2}}{0.24} = 2.12\%$$
$$u_{v^2} = \sqrt{(\frac{\partial v^2}{\partial v} \cdot u_v)^2} = 2v \cdot u_v$$
$$u_{v_1^2} = 2 \times 0.24 \times 5.09 \times 10^{-3} = 2.44 \times 10^{-3}[m/s^2]$$

$$u_{r1} = \frac{u_{v_1^2}}{v_1^2} \times 100\% = \frac{2.44 \times 10^{-3}}{0.0576} \times 100\% = 4.24\%$$

	v $[m/s]$	Uncertainty $[m/s]$	Relative uncertainty %
1	0.24	5.09×10^{-3}	2.12%
2	0.48	1.02×10^{-2}	2.13%
3	0.71	1.57×10^{-2}	2.21%
4	0.94	2.18×10^{-2}	2.32%
5	1.19	2.87×10^{-2}	2.41%
6	1.42	3.62×10^{-2}	2.55%

Table 19: Uncertainty for v

	v^2 $[(m/s)^2]$	Uncertainty $[(m/s)^2]$	Relative uncertainty %
1	0.0576	2.44×10^{-3}	4.24%
2	0.2304	9.79×10^{-3}	4.25%
3	0.5041	2.23×10^{-2}	4.42%
4	0.8836	4.1×10^{-2}	4.64%
5	1.4161	6.83×10^{-2}	4.82%
6	2.0164	1.03×10^{-1}	5.11%

Table 20: Uncertainty for v^2

6.6 Uncertainty for k

$$k = m\frac{v^2}{A^2}$$

$$u_k = \sqrt{(\frac{\partial k}{\partial v^2} \cdot u_{v^2})^2 + (\frac{\partial k}{\partial A^2} \cdot u_{A^2})^2 + (\frac{\partial k}{\partial m} \cdot u_m)^2} = \sqrt{(\frac{m}{A^2} \cdot u_{v^2})^2 + (-\frac{mv^2}{A^4} \cdot u_{A^2})^2 + (\frac{v^2}{A^2} \cdot u_m)^2}$$

$$k_1 = 0.195 \times \frac{0.24^2}{0.05^2} = 4.49[N/m]$$

$$u_{k1} = \sqrt{(\frac{0.195}{0.0025} \times 2.44 \times 10^{-3})^2 + (\frac{0.195 \times 0.0576}{0.0025^2} \times 10^{-4})^2 + (\frac{0.0576}{0.0025} \times 10^{-5})^2} = 0.26[N/m]$$

$$u_{r1} = \frac{u_{k_1}}{k_1} \times 100\% = \frac{0.26}{4.49} \times 100\% = 5.83\%$$

	k[N/m]	Uncertainty[N/m]	Relative uncertainty %
1	4.49	0.26	5.83%
2	4.49	0.19	4.23%
3	4.37	0.19	4.42%
4	4.31	0.2	4.64%
5	4.42	0.21	4.75%
6	4.37	0.22	5.1%

Table 21: Uncertainty for calculated k

6.7 Uncertainty for $\bar{k}$

6.7.1 Multiple Measurement Yield to Type-A Uncertainty

$$\bar{k} = \sum_{i=1}^{6} k_i = 4.41[N/m]$$

$$S_X = 0.072$$

$$\triangle A = S_X \cdot \frac{t_{0.95}}{\sqrt{n}} = 0.072 \times 1.05 = 0.076[N/m]$$

6.7.2 Type-B Uncertainty

$$\triangle B = \sqrt{\sum_{i=1}^{6} (\frac{1}{6} u_k i)^2} = 0.087[N/m]$$

$$u_k = \sqrt{0.076^2 + 0.087^2} = 0.116[N/m]$$

$$k = 4.41 \pm 0.116[N/m]$$

$$u_r = \frac{u_k}{\bar{k}} \times 100\% = 2.62\%$$

6.8 Uncertainty for k through linear fit diagram

Since I draw the diagram of v^2 vs. A^2, the slope equals to $\frac{k}{m}$ and its value is 22.499 ± 0.126.

$$k = m \cdot s = 22.499 \times 0.195 = 4.387[N/m]$$

$$u_k = \sqrt{(\frac{\partial k}{\partial s})^2 + (\frac{\partial k}{\partial s})^2} \times \frac{t_{0.95}}{\sqrt{n-2}} = \sqrt{0.126^2 + 10^{-10}} \times \frac{2.57}{2} = 0.162[N/m]$$

$$u_r = \frac{u_k}{k} \times 100\% = 3.69\%$$

7 Conclusion and Discussion

7.1 Conclusion

In this exercise, Hooke's Law is used to examine simple harmonic motion. In addition, I become familiar with many apparatus which I hadn't met before. I learned how to use Jolly balance to measure the spring constant easily. I also learned the function of air track and air pump which are used to measure the periods and mechanic energy of the spring-mass system.

7.1.1 Measurement of Spring Constant

In the exercise, I used to ways to measure the spring constant. The first one is use a Jolly Balance to measure the increase of the length of the spring when it have a restoring force. Then I can use the Hooke's Law and equation 7 to calculate its spring constant. I use this method to measure two spring and their series.

The second way is to use air track, electronic timer, electronic balance, and calliper to measure v_{max}, m, and A of the system. Then I can draw a A^2 vs. v^2 diagram and use equation 6 to get k.

	Spring Constant [N/m]	Uncertainty [N/m]	Relative uncertainty %
Spring 1	2.216	0.013	0.59%
Spring 2	2.243	0.009	0.40%
Series	1.105	0.006	0.54%
Two springs at both sides	4.410	0.116	2.62%
Linear fit diagram	4.387	0.162	3.69%

Table 22: Data for spring constant

From the table I can draw a conclusion when two springs are connected to each other, their equivalent spring constant is their average value and if they are connected to a object on both sides, their equivalent spring constant is their sum.

7.1.2 Linear Fit Diagram

In the report, I draw eight diagrams. The first three are used to get the spring constant through linear fit. The next three are used to study t vs. M relation. I find that the angle between the air track and horizontal line has nothing to do with the relation between t and M. T^2 is always proportional to M. The seventh diagram is used to examine the relation between T and A. The diagram show that the change of A has no effect on the period of the system. The eight diagram is used to calculate the equivalent spring constant when two springs are connect to both sides of the object according equation 6. The error bar and uncertainty is also included when the diagram is plotted.

7.2 Discussion

7.2.1 Error Analysis

1. The relative uncertainty of equivalent spring constant when two springs are connected to both sides are much bigger than other conditions. It may because I measure the quantity for 6 times and $\triangle A$ yields much of it

2. The 4,5 and 6 diagrams are used to calculating the relation between T vs. M. However, their relation is not linear actually. According to equation 4, it is T^2 that should be proportional to M. So the uncertainty will increase when I do the square calculation of T.

3. In the exercise, every time measured is 10 periods. I think there are two reasons. The first one is the average valued can help use reduce random uncertainty. The second one may be one period is too short for the electronic timer to record.

4. When I recorded the data for v_{max}^2 vs. A^2 relation, I found v_{max} will become smaller and smaller with increase of time due to air drag force. So I think air drag force will also affect other data measured and cause errors.

8 Data Sheet

Data sheet is attach to the report

9 Reference

- Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Lab Manual of Exercise 3.

- Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Handbook-Uncertainty Analysis.