JOINT INSTITUTE
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MODERN PHYSICS (Summer 2020)

Review Questions
(week of 29 June 2020)

1. The commutator of two operators $\hat{A}$ and $\hat{B}$ is defined as $[\hat{A}, \hat{B}] = \dots\dots\dots\dots\dots\dots$
2. Two operators are said to commute, iff $[\hat{A}, \hat{B}] = \dots\dots\dots\dots\dots\dots$
3. $[\hat{x}, \hat{p_x}] = \dots\dots\dots\dots\dots\dots$ (give the answer only).
4. Two physical quantities are called compatible if the operators representing these quantities $\dots\dots\dots\dots\dots$ (answer: commute).
5. Give an example of a pair of physical quantities that are compatible/are not compatible.
6. What is special about a pair of compatible physical quantities?
7. *True or false?* Two physical quantities that are compatible can be simultaneously measured with arbitrarily small uncertainty.
8. *True of false?* Heisenberg uncertainty principle is a special case of a more general uncertainty principle.
9. State the postulates of quantum mechanics.
10. What is the wave function collapse?
11. The wave function of a particle is of the form $\psi = \sqrt{\frac{2}{3}}\psi_4 + \sqrt{\frac{1}{3}}\psi_5$, where ψ_n is the eigenfunction of the operator $\hat{A}$ representing a physical value A and corresponding to the eigenvalue a_n.
    (a) We measure the physical quantity A on a particle in the state ψ. What are possible outcomes of this measurement and their probabilities?
    (b) If, just after the first measurement, we perform another measurement of A, what are its possible outcomes and their probabilities?
    (c) * If, just after the first measurement is performed, we measure another physical quantity B, is it possible that this second measurement yields a certain value with probability 1? If so, under what conditions?
12. What does it mean that the energy levels of a quantum system are degenerate? Explain using an example.
13. Recall that for the 3D isotropic harmonic oscillator $E_N = \hbar\omega\left(N + \frac{3}{2}\right)$, where $N = n_x + n_y + n_z$. List all sets of quantum numbers corresponding to the energy level $7\hbar\omega/2$.
14. For the 3D isotropic harmonic oscillator, how can the degeneracy of energy levels be lifted?
15. $[\hat{L}_y, \hat{L}_z] = \dots\dots\dots$ (express the answer in terms of the third component of $\hat{L}_x$)
16. $[\hat{L}^2, \hat{L}_z] = \dots\dots\dots$
17. $[\hat{L}^2 = \dots\dots\dots$ (express in terms of the Cartesian components of the orbital angular momentum operator).
18. Discuss consequences of the fact that $[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$.
19. Discuss consequences of the fact that $[\hat{L}^2, \hat{L}_z] = 0$.

$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$
$$\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$
$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$