Review Questions
(week of 1 June 2020)

1. What does the de Broglie hypothesis say?
2. Write down the de Broglie relations between (a) the momentum and the wave number, (b) the energy and the angular frequency.
3. The length of the de Broglie wave of an electron in the first Bohr orbit of a hydrogen atom is much .... than that of a cricket ball moving at 10 m/s.
4. Write down the general form of the classical wave equation (1D). Give two specific examples from classical physics.
5. What is the superposition principle? From which property of the classical wave equation does it follow?
6. ξ_1 and ξ_2 are solutions to the same classical wave equation. Is $\xi_1 + 2\xi_2$ also a solution? What about $(\xi_1 + 2\xi_2)^2$?
7. What is a (classical) wave packet?
8. What is the group velocity? Is it equal to the phase velocity?
9. Given the angular frequency and the wave number, how can we find the phase velocity? Group velocity?
10. What do we mean when we say that a wave is dispersive?
11. What is the classical uncertainty relation? Illustrate with a sketch.
12. How do we represent the state of a particle (a system) in quantum mechanics?
13. What is the interpretation of the wave function?
14. In quantum mechanics, do we have the concept of the trajectory in the same sense as we do in classical mechanics?
15. Interpret the expressions

$$|\Psi(x,t)|^2\,\mathrm{d}x$$

$$\int_0^a\int_0^a\int_0^a |\Psi(x,y,z,t)|^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$\int_0^R\int_0^\pi\int_0^{2\pi} |\Psi(r,\vartheta,\varphi,t)|^2\,r^2\mathrm{d}\varphi\,\mathrm{d}\vartheta\,\mathrm{d}r$$

$$\int_{-\infty}^{\infty} \Psi^*(x,t)x\Psi(x,t)\mathrm{d}x$$

$$\int_{-\infty}^{\infty} \Psi^*(x,t)x^2\Psi(x,t)\mathrm{d}x$$

16. What do we mean by saying that a wave function is normalized?
17. Is the average value always equal to the most probable value? Explain (a sketch may be helpful).
18. When is a function said to be square-integrable?

19. Let $\Psi(x,t)$ describe a quantum particle in a certain state. How do we calculate the average position of the particle? The variance of the position? (Give formulas.)

20. What is a stationary state? Give an example of a wave function representing a stationary state.

21. Does the average position of a particle in a stationary state depend on time? What about the variance of its position?

22. Does $\Psi(x,t) = \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}E_1 t}\psi_1(x) + e^{-\frac{i}{\hbar}E_2 t}\psi_2(x)\right)$ represent a stationary state? Explain or check by a direct calculation.

23. f, g, h are square-integrable functions, α is a complex number, and i denotes the imaginary unit ($i^2 = -1$)
    $\langle f, g - ih\rangle = \ldots\ldots\ldots\ldots$
    $\langle f, g\rangle^* = \ldots\ldots\ldots\ldots$
    $\langle \alpha f, g\rangle = \ldots\ldots\ldots\ldots,$ $\qquad$ $\langle f, \alpha g\rangle = \ldots\ldots\ldots\ldots$
    $\langle 3if + g, 2h\rangle = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
    $\langle f, f\rangle = 0$ if and only if $\ldots\ldots\ldots\ldots$

24. Using the inner product symbol, write down the formula for the average value of the position of a particle in the state represented by the wave function Ψ.

25. When are two functions said to be orthogonal? (See Problem Set 3 for a definition.)

26. ψ_1 and ψ_2 are orthogonal and normalized. Are $\psi_1 + \psi_2$ and $\psi_1 - \psi_2$ orthogonal? normalized?

27. How do we define the $L^2(\mathbb{R})$ space? Is it a linear (vector) space?

28. ** How, using the inner product, can we define a norm in $L^2(\mathbb{R})$? A metric?

    Note. Star-marked items are optional (and will not appear in quizzes). The more stars, the more optional.

29. *** What is a unitary space?

30. *** What is a Hilbert space? Is $L^2(\mathbb{R})$ a Hilbert space?

31. **** What is the difference between the Banach and the Hilbert spaces?

32. Write down the (time-dependent) Schrödinger equation.

33. What does linearity of the Schrödinger equation imply (from the point of view of physics)?

34. What is the superposition principle? (in quantum mechanics)

35. How do we represent physical (i.e. measurable) quantities in quantum mechanics?

36. How do we calculate the average value of a physical quantity in quantum mechanics?

37. $\Delta_x \cdot \Delta_{p_x} \geq \frac{\hbar}{2}$ is known as the $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ uncertainty principle.
    What is its meaning?

38. Operator $\hat{A}$ is called Hermitian if and only if $\langle f, \hat{A}g\rangle = \ldots\ldots\ldots\ldots.$

39. $\hat{x}^\dagger = \ldots\ldots\ldots\ldots,$ $\qquad$ $\hat{p}_x^\dagger = \ldots\ldots\ldots\ldots$

40. What is the form of the position operator? momentum operator? kinetic energy?

41. Show that $\hat{p}_x$ is Hermitian [exercise, not a quiz question].

42. Use the fact that $\hat{p}_x$ is Hermitian, and properties of the inner product to show that $\hat{p}_x^2$ is Hermitian.

43. What is the Hamiltonian of a quantum system?

44. The Hamiltonian is the operator representing $\ldots\ldots\ldots\ldots\ldots$ of a quantum particle.

45. A particle with moves in the harmonic potential field $V(x) = \frac{1}{2}Ax^2$, where $A > 0$ is a constant. Write down the Hamiltonian of this particle.

46. The average value of a physical quantity A, represented by the operator $\hat{A}$, in state Ψ is calculated as

$$\langle A \rangle_\Psi = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

47. What is the form of the position operator? momentum operator? kinetic energy of a particle with mass m? total energy of that particle moving in a potential field $V(x)$?

48. The operator representing the total energy of a particle or a quantum system is called ....

49. A particle moves in the harmonic potential field $V(x) = \frac{1}{2}Ax^2$, where $A > 0$ is a constant. Write down the Hamiltonian of this particle.

50. *True or false?* If the Hamiltonian does not depend explicitly of time, there exist solutions to the (time-dependent) Schrödinger equation that describe stationary states.

51. Write down a generic equation for eigenvalues and eigenfunctions of an operator. Identify which symbols represent the operator, the eigenvalue and the eigenfunction.

52. Write down the Schödinger equation, both time-dependent and stationary. Identify all symbols.

53. *True of false?* If the Hamiltonian does not depend explicitly of time, there exist solutions to the (time-dependent) Schrödinger equation that describe stationary states.

54. If ψ_E is a solution of the stationary Schrödinger equation, then the corresponding solution of the full (time-dependent) Schrödinger equation is $\Psi_E = \ldots\ldots\ldots \psi_E$.