Problem Set 6

Due: 6 July 2020, 12.30 p.m.

Problem 1. Use the expression for $|\Psi(x,t)|^2$ given in class for the Gaussian wave packet and plot three curves $|\Psi(x,t)|^2$ for four different instants t (including $t=0$). Comment on the shape of the curve in terms of localization of the particle.

Use a computer and attach the graphs (list the values of the packet's parameters that you have used to plot the curves).

(*2 points*)

Problem 2. (a) An electron beam is sent through a potential barrier $2 \cdot 10^{-10}$ m wide. The transmission coefficient exhibits a third maximum at $E = 100$ eV. What is the height of the barrier?

(b) An electron beam is incident on a barrier of height 10 eV. At $E = 10$ eV, the transmission coefficient $T = 3.37 \times 10^{-3}$ What is the width of the barrier?

(*2 + 2 points*)

Problem 3. Show that $T + R = 1$ for all one-dimensional barrier problems.

(*4 points*)

Problem 4. Show that for scattering on a rectangular potential well (that is $V(x) = -V_0$ for $|x| \le a$ and $V(x) = 0$ otherwise), the inverse of the transmission coefficient

$$\frac{1}{T} = 1 + \frac{1}{4}\frac{V_0^2}{E(E+V_0)}\sin^2(2k_2 a),$$

where $k_2 = \sqrt{2m(E+V_0)/\hbar^2}$ and $E > 0$.

Find the energies of incident particles for which transmission is perfect.

(*8 points*)

Problem 5. The scattering cross-section for scattering of electrons by a rare gas of krypton atoms exhibits a low-energy minimum at $E \approx 0.9$ eV. Assuming that the diameter of the atomic well seen by electrons is 1 Bohr radius, calculate its depth.

(*2 points*)

Problem 6. Check the following properties of the commutator

(a) $[\hat{A}, \hat{B}] = -[\hat{B}, \hat{A}]$

(b) $[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$,

(c) $[\hat{A} + \hat{B}, \hat{C}] = [\hat{A}, \hat{C}] + [\hat{B}, \hat{C}]$

(d) $[\hat{A}, \gamma] = 0$,

(e) $[\hat{A} - \alpha, \hat{B} - \beta] = [\hat{A}, \hat{B}]$

(f) $[\hat{A}, f(\hat{A})] = 0$,

where $\hat{A}, \hat{B}, \hat{C}$ are operators, α, β, γ are numbers (complex, in general), and f is an analytic function.

(*5 × 1/2 + 3/2 points*)

Problem 7. (a) Find $[\hat{H}, \hat{p}]$ for a free particle (i.e. $V(x) \equiv 0$ for all x) and comment on consequences of the result.

(b) Check that any eigenfunction of the 1D momentum operator is also an eigenfunctions of the Hamiltonian of a free particle. Is the opposite true?

(c) Find $[\hat{H}, \hat{p}]$ for a particle in a potential field $V = V(x)$ and comment on consequences of the result.

(3/2 + 3/2 + 2 points)

Problem 8. For a particle in a 1D potential field show that we have the following uncertainty principle for the total energy and position: $\Delta_E \Delta_x \geq \frac{\hbar}{2m} \langle p_x \rangle$.

Hints: (1) What is $[\hat{H}, \hat{x}]]$ equal to? (2) The general (Schrödinger–Robertson) uncertainty principle.

(2 points)

Problem 9. Consider a system of two linear polarizers A and B placed one after another: A is oriented so that its polarization axis is vertical and the polarization axis of B is horizontal. Light polarized in the vertical direction is incident on A.

(a) Will there be any photons detected behind B?

(b) How will the answer change if we place a third linear polarizer C between A and B, and the polarization axis of C forms an angle θ with the polarization axis of A. Discuss the cases (i) $\theta = \pi/4$, (ii) $0 < \theta < \pi/2$.

How would you explain your answers in the language of quantum mechanics and quantum measurement?

(1 + 3 points)