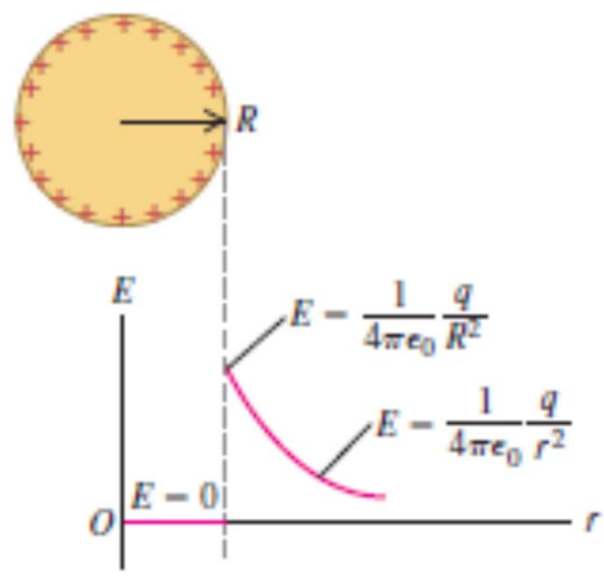
Chapter 22

Gauss's Law

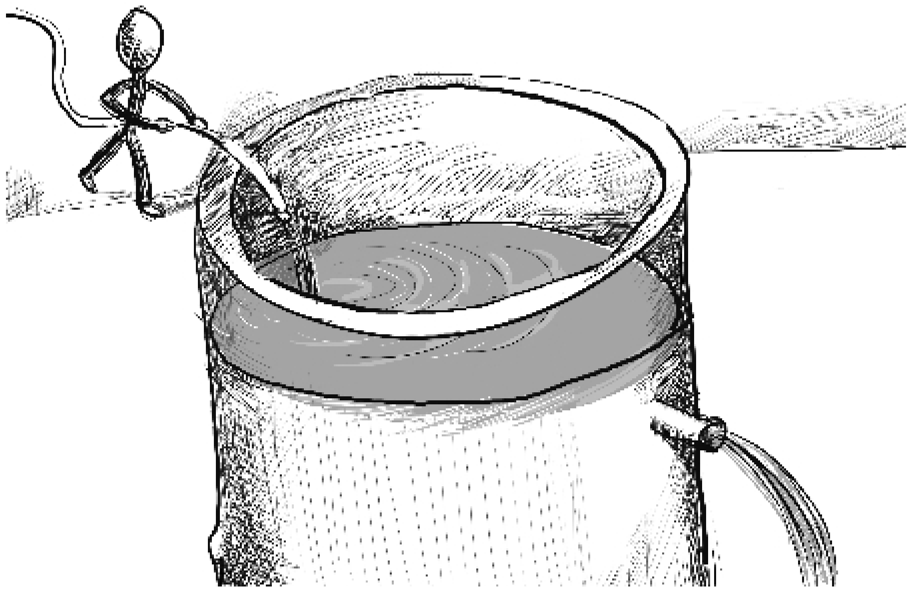


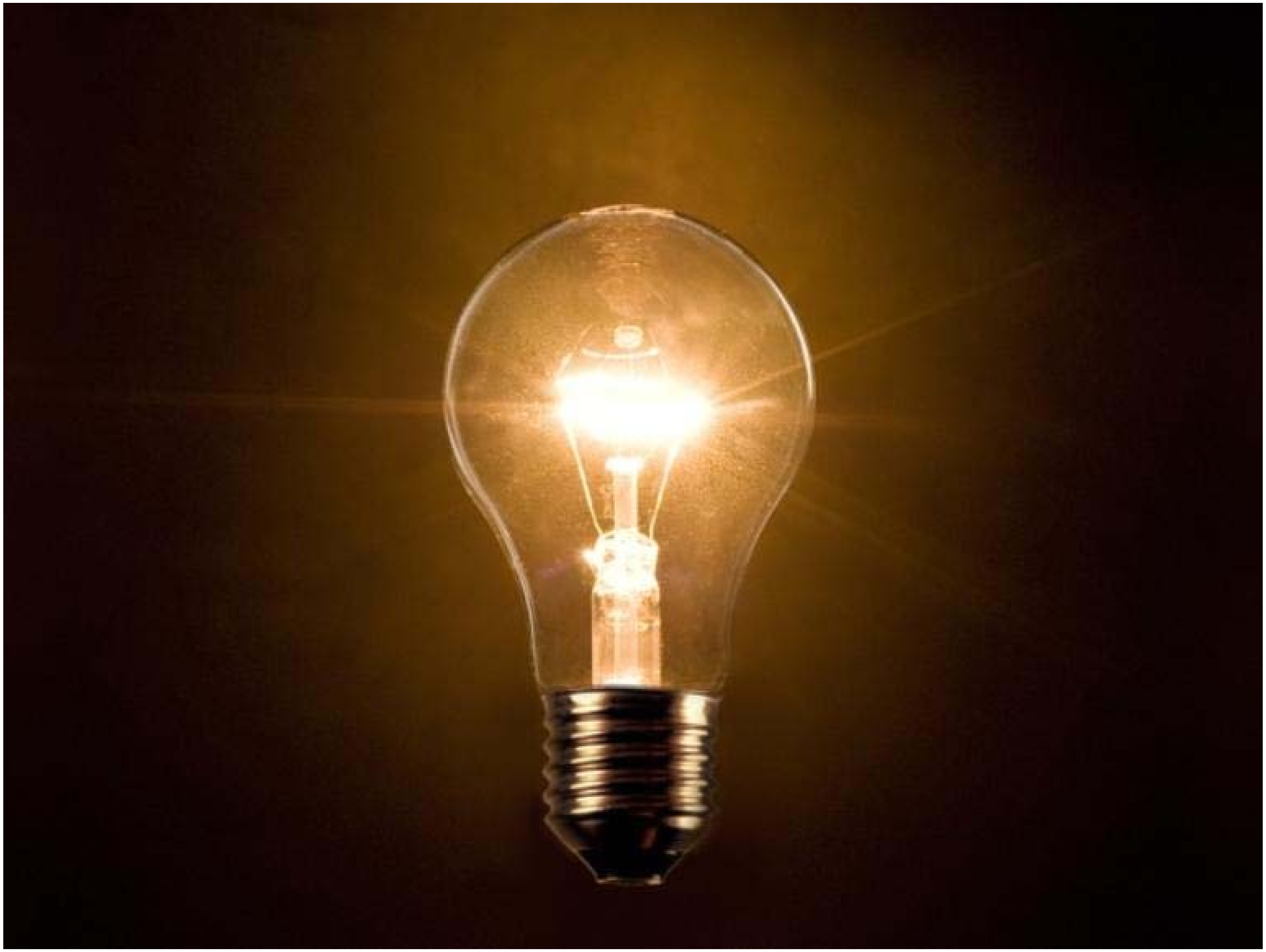
Goals for Chapter 22

- To use the electric field at a surface to determine the charge within the surface
- To learn the meaning of electric flux and how to calculate it
- To learn the relationship between the electric flux through a surface and the charge within the surface
- To use Gauss's law to calculate electric fields
- To learn where the charge on a conductor is located

Introduction

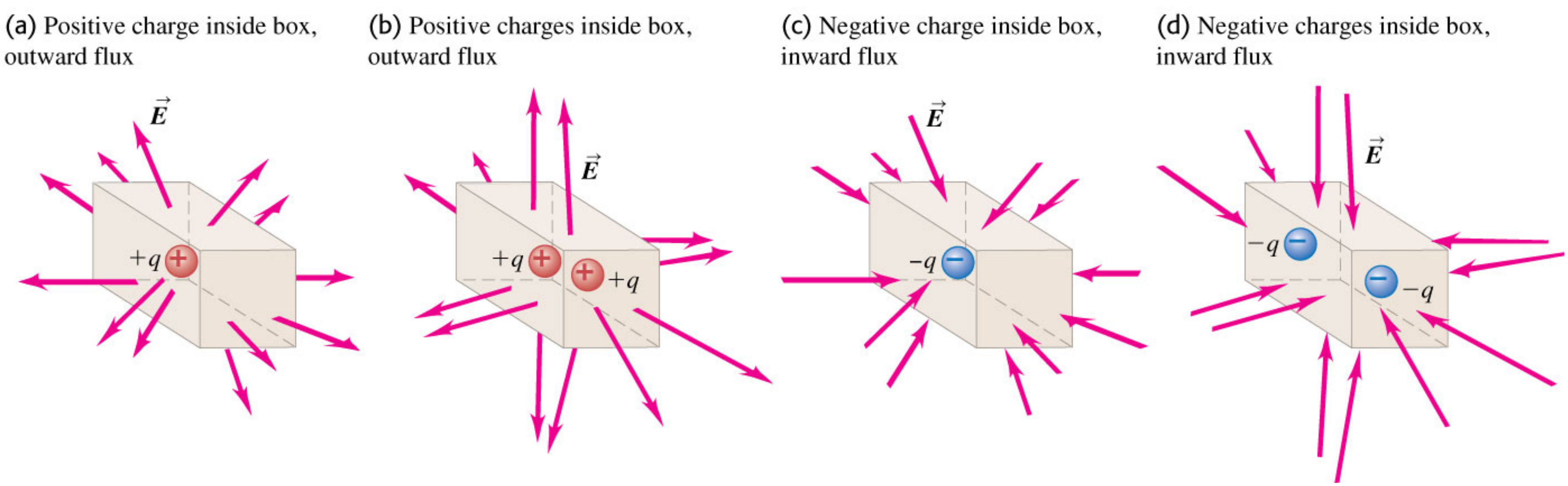
- Would the girl's hair stand on end if she were inside the metal sphere?
- Symmetry properties play an important role in physics.
- Gauss's law will allow us to do electric-field calculations using symmetry principles.





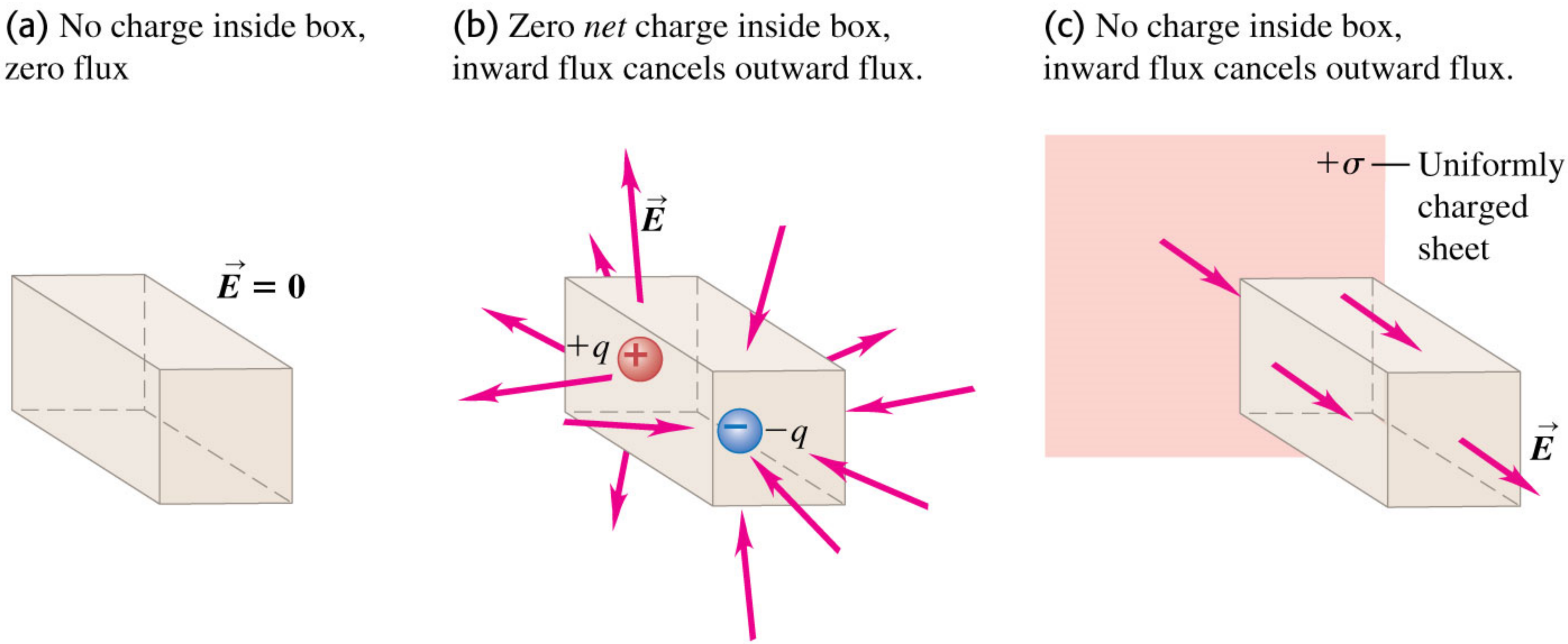
Charge and electric flux

- Positive charge within the box produces outward *electric flux* through the surface of the box, and negative charge produces inward flux. (See Figure 22.2 below.)



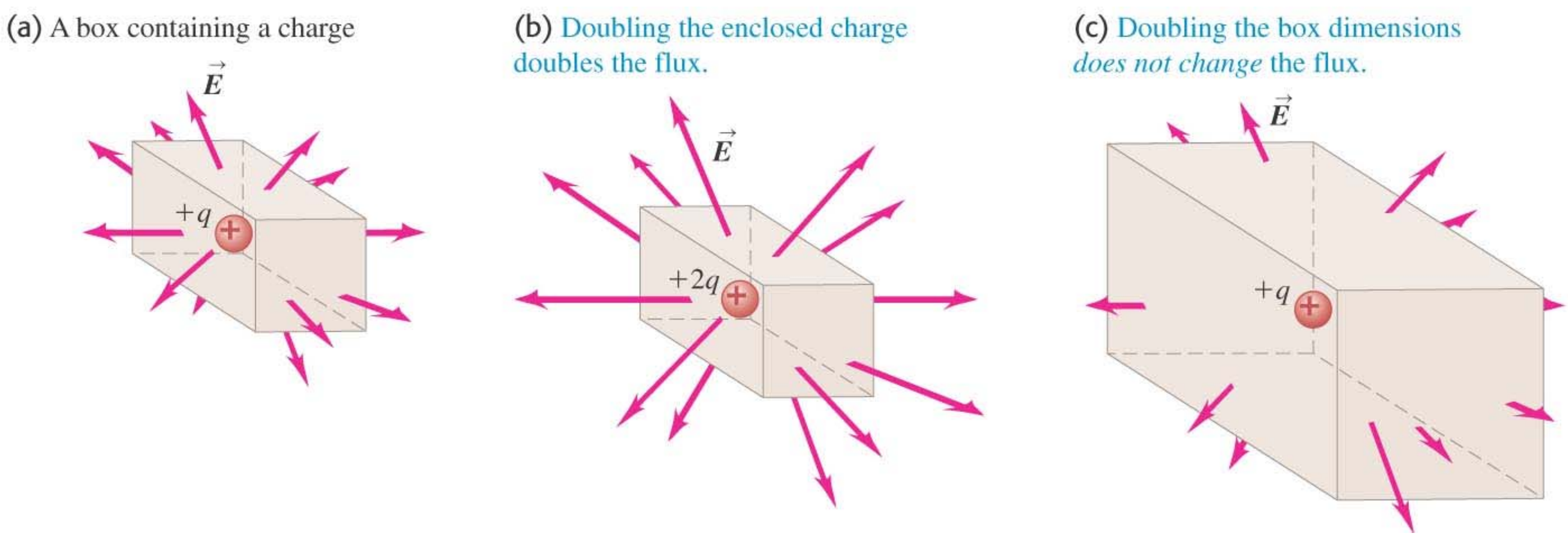
Zero net charge inside a box

- Figure 22.3 below shows three cases in which there is zero *net* charge inside a box and no net electric flux through the surface of the box.

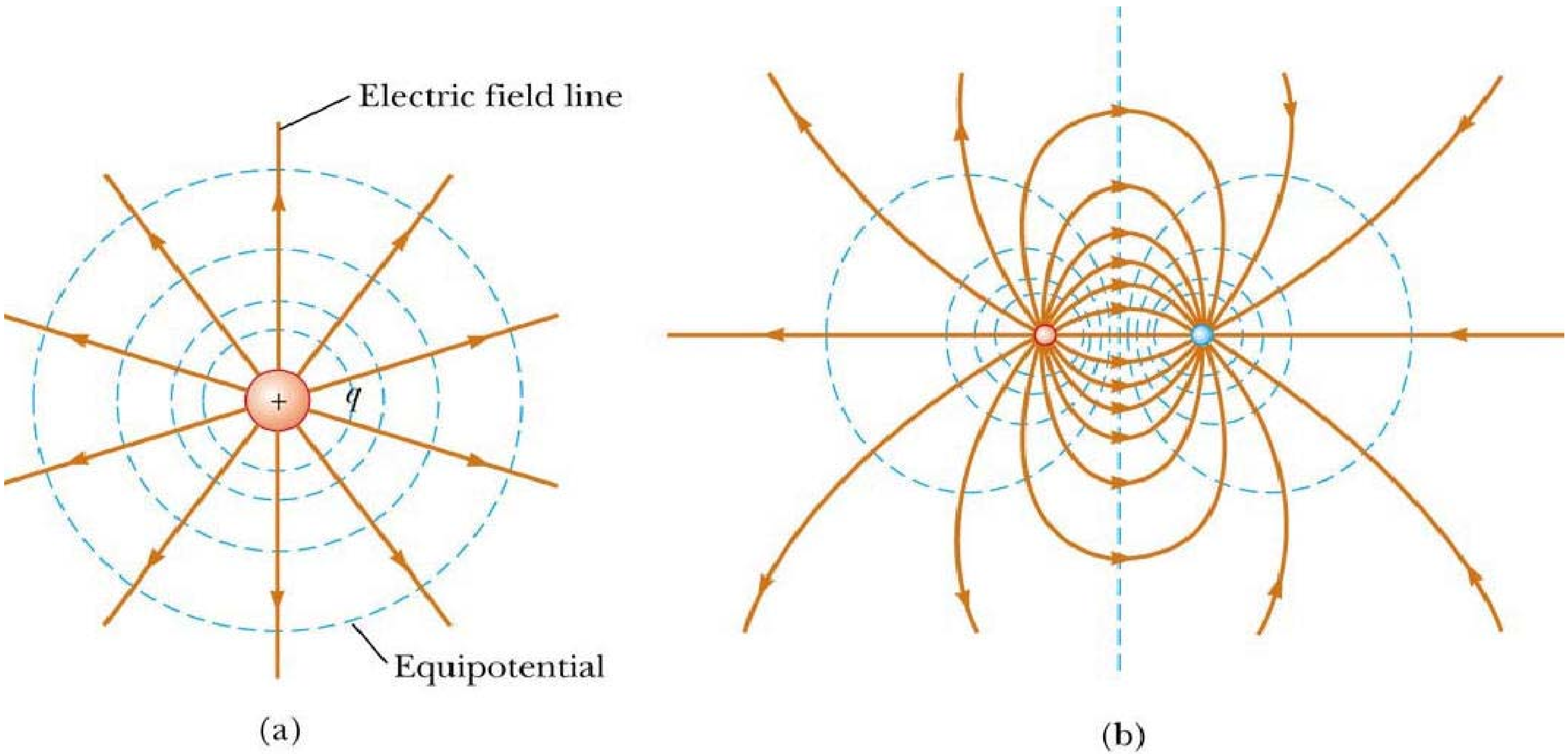


What affects the flux through a box?

- As Figure 22.4 below shows, doubling the charge within the box doubles the flux, but doubling the size of the box does not change the flux.
- Follow the discussion of charge and flux in the text.



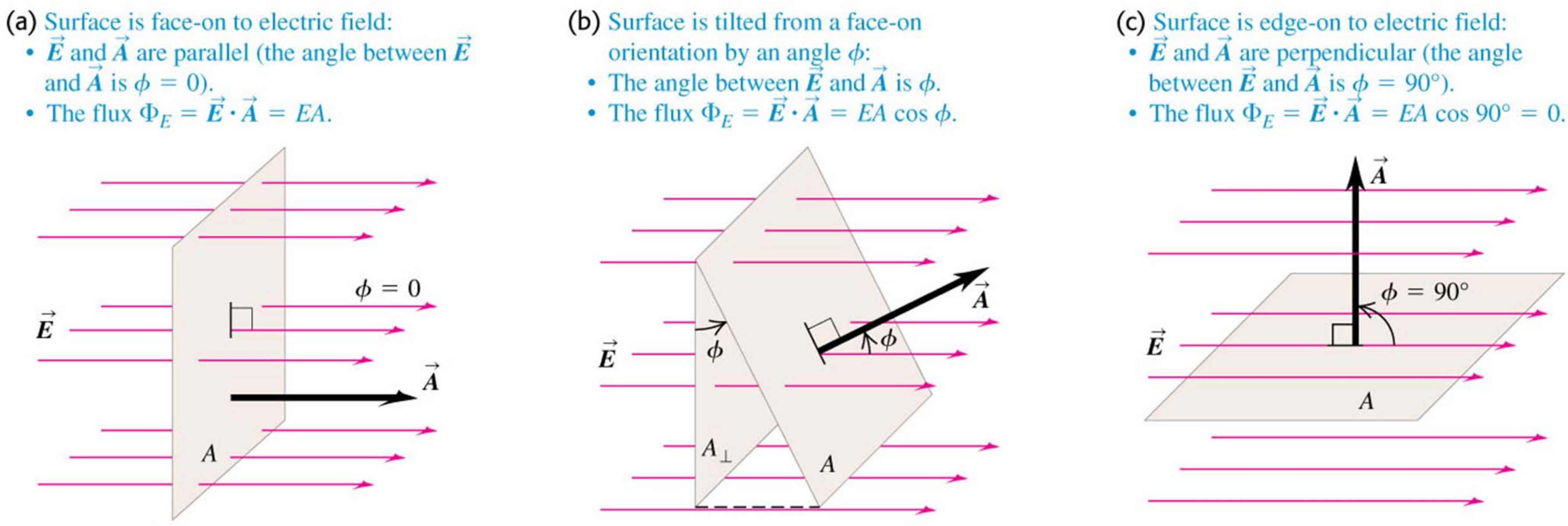
What affects the flux through a box?



Calculating electric flux

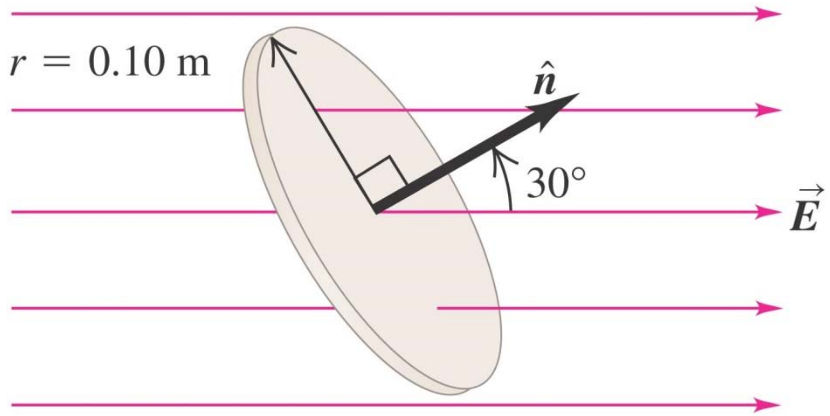
- Follow the discussion in the text of flux calculation for uniform and nonuniform fields, using Figure 22.6 below.

$$\Phi_E = \vec{E} \cdot \vec{A} \qquad \Phi_E = EA\cos\phi$$



Electric flux through a disk

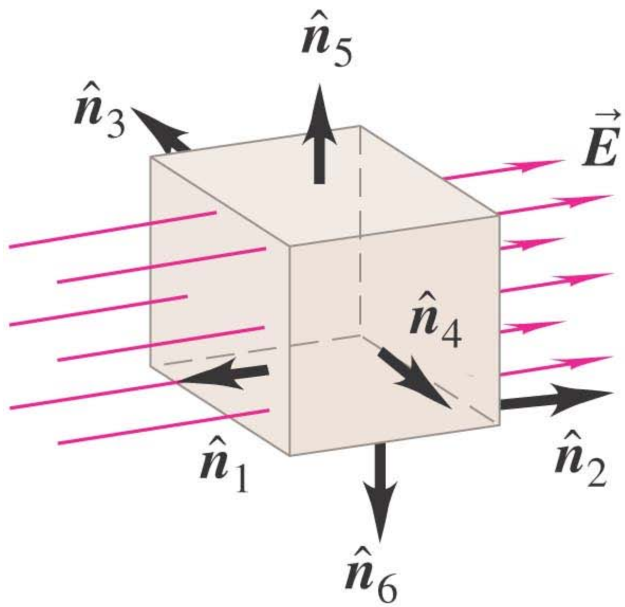
- Follow Example 22.1 to evaluate flux through a disk. Use Figure 22.7 below.



$$\Phi_E = \int E\cos\phi\, dA = \int E_\perp\, dA = \int \vec{E}\cdot d\vec{A} \quad \text{(general definition of electric flux)} \qquad (22.5)$$

Electric flux through a cube

(a)



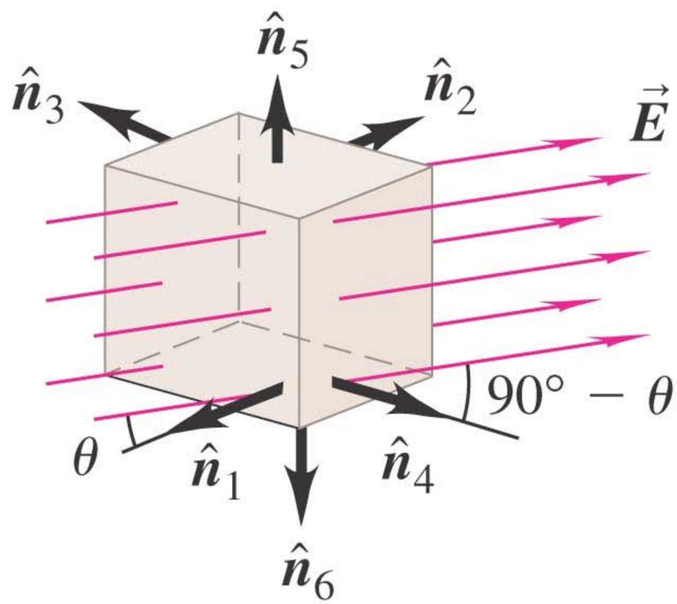
$$\Phi_{E1} = \vec{E} \cdot \hat{n}_1 A = EL^2 \cos 180^\circ = -EL^2$$

$$\Phi_{E2} = \vec{E} \cdot \hat{n}_2 A = EL^2 \cos 0^\circ = +EL^2$$

$$\Phi_{E3} = \Phi_{E4} = \Phi_{E5} = \Phi_{E6} = EL^2 \cos 90^\circ = 0$$

$$\begin{aligned}\Phi_E &= \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6} \\ &= -EL^2 + EL^2 + 0 + 0 + 0 + 0 = 0\end{aligned}$$

(b)



$$\Phi_{E1} = \vec{E} \cdot \hat{n}_1 A = EL^2 \cos(180^\circ - \theta) = -EL^2 \cos \theta$$

$$\Phi_{E2} = \vec{E} \cdot \hat{n}_2 A = +EL^2 \cos \theta$$

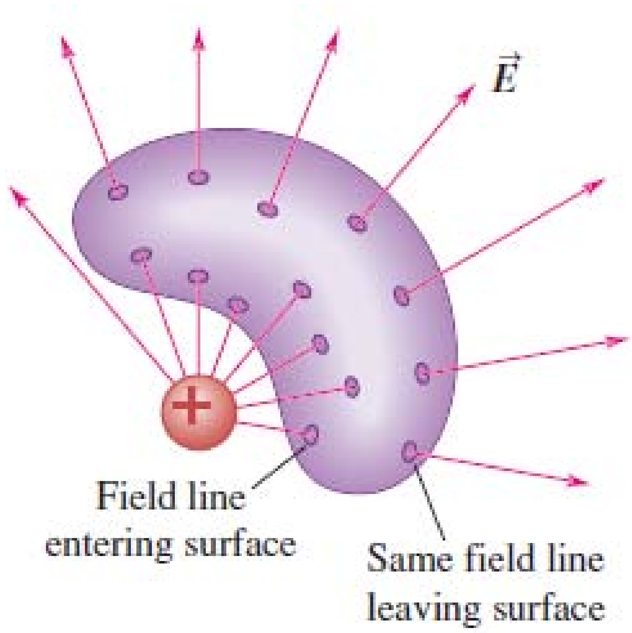
$$\Phi_{E3} = \vec{E} \cdot \hat{n}_3 A = EL^2 \cos(90^\circ + \theta) = -EL^2 \sin \theta$$

$$\Phi_{E4} = \vec{E} \cdot \hat{n}_4 A = EL^2 \cos(90^\circ - \theta) = +EL^2 \sin \theta$$

$$\Phi_{E5} = \Phi_{E6} = EL^2 \cos 90^\circ = 0$$

e total flux $\Phi_E = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6}$

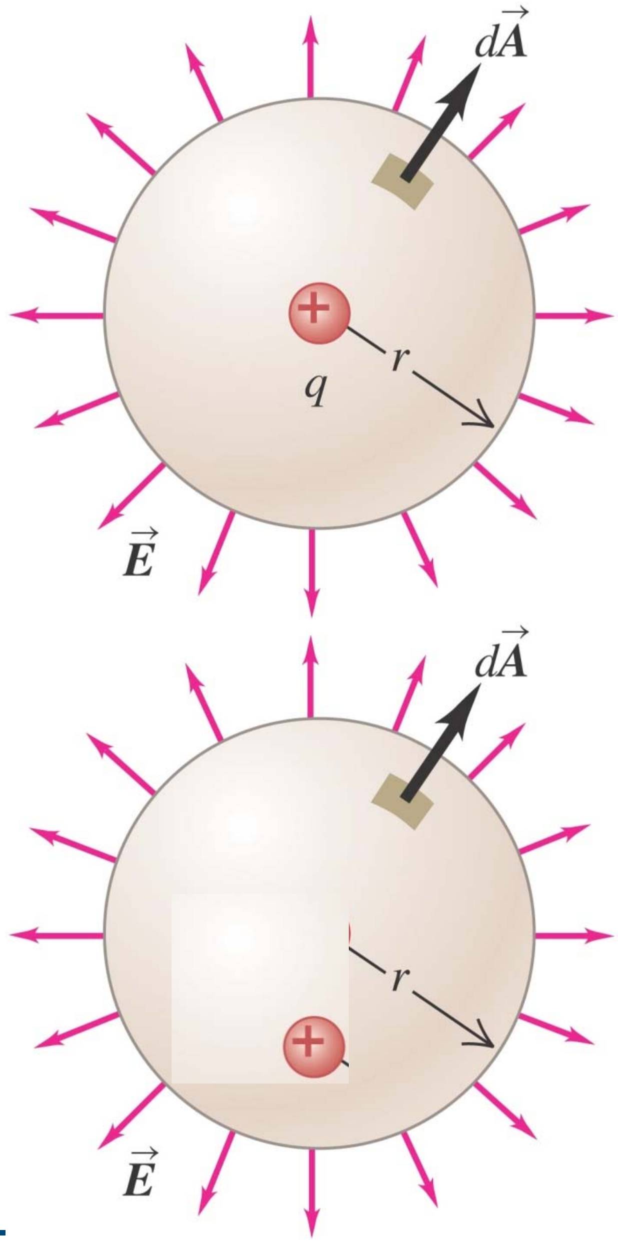
Electric flux through a cube



$$\Phi_{E1} = \vec{E}\cdot\hat{n}_1 A = EL^2\cos(180^\circ - \theta) = -EL^2\cos\theta$$
$$\Phi_{E2} = \vec{E}\cdot\hat{n}_2 A = +EL^2\cos\theta$$
$$\Phi_{E3} = \vec{E}\cdot\hat{n}_3 A = EL^2\cos(90^\circ + \theta) = -EL^2\sin\theta$$
$$\Phi_{E4} = \vec{E}\cdot\hat{n}_4 A = EL^2\cos(90^\circ - \theta) = +EL^2\sin\theta$$
$$\Phi_{E5} = \Phi_{E6} = EL^2\cos 90^\circ = 0$$

e total flux $\Phi_E = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6}$

Electric flux through a sphere



$$\Phi_E = EA = \frac{q}{4\pi\epsilon_0 r^2}4\pi r^2 = \frac{q}{\epsilon_0}$$

$$= \frac{3.0 \times 10^{-6}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = 3.4 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C}$$

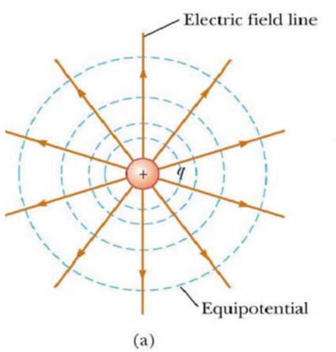
?

Gauss's Law

- Gauss's law is an alternative to Coulomb's law and is completely equivalent to it.
- Carl Friedrich Gauss, shown below, formulated this law.

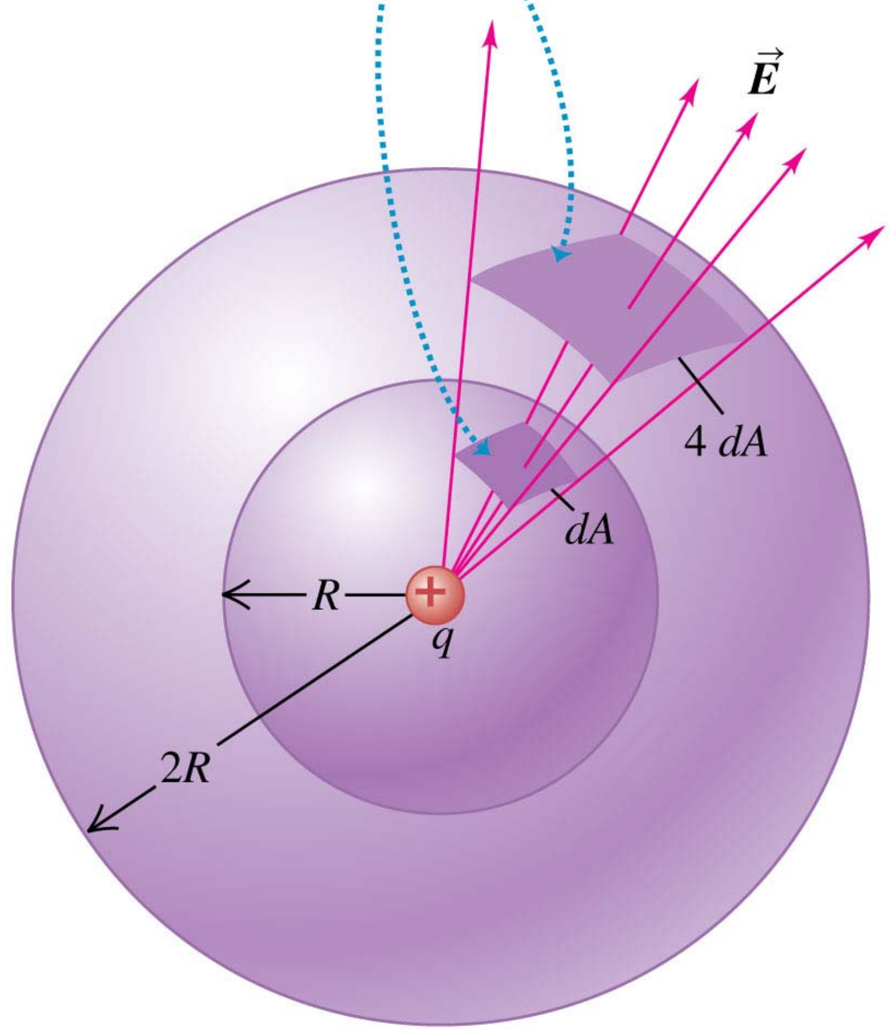
Point charge centered in a spherical surface

- The flux through the sphere is independent of the size of the sphere and depends only on the charge inside. Figure 22.11 at the right shows why this is so.



(a)

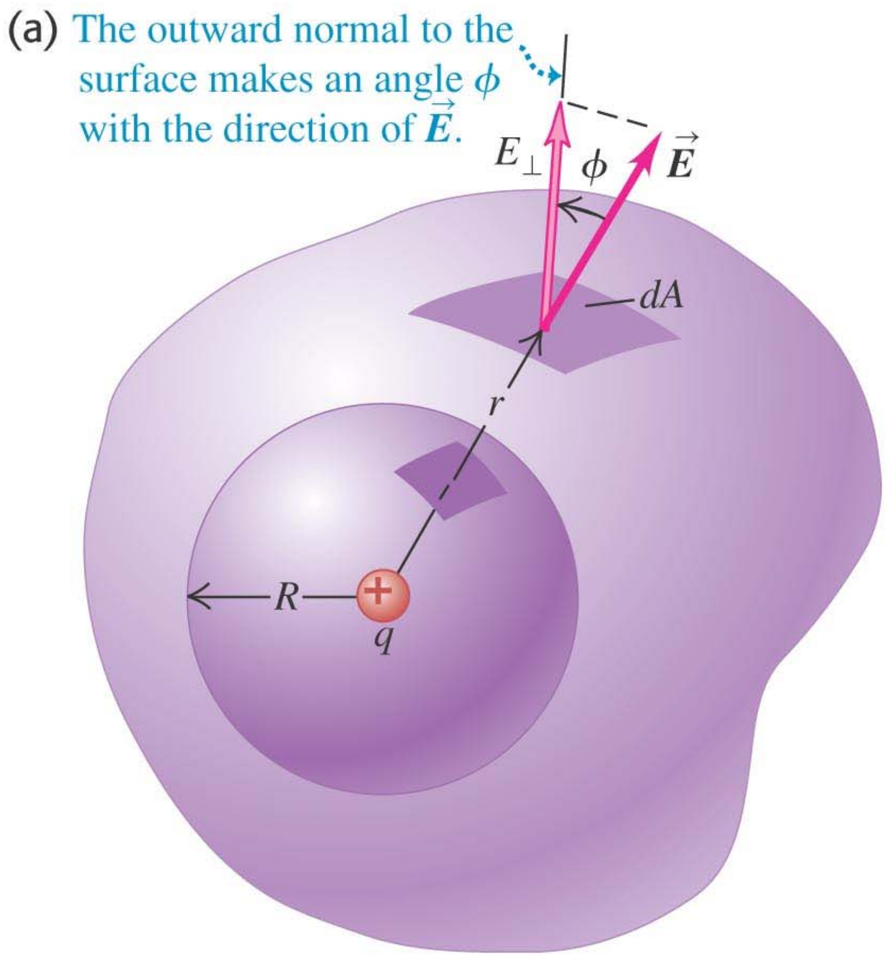
$$\Phi_E = EA = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

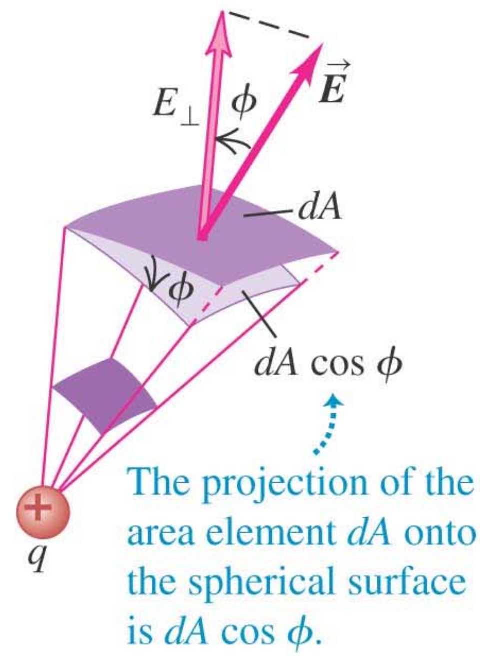


Point charge inside a nonspherical surface

- As before, the flux is independent of the surface and depends only on the charge inside. (See Figure 22.12 below.)

$$\Phi_E = \int E \cos\phi \, dA$$





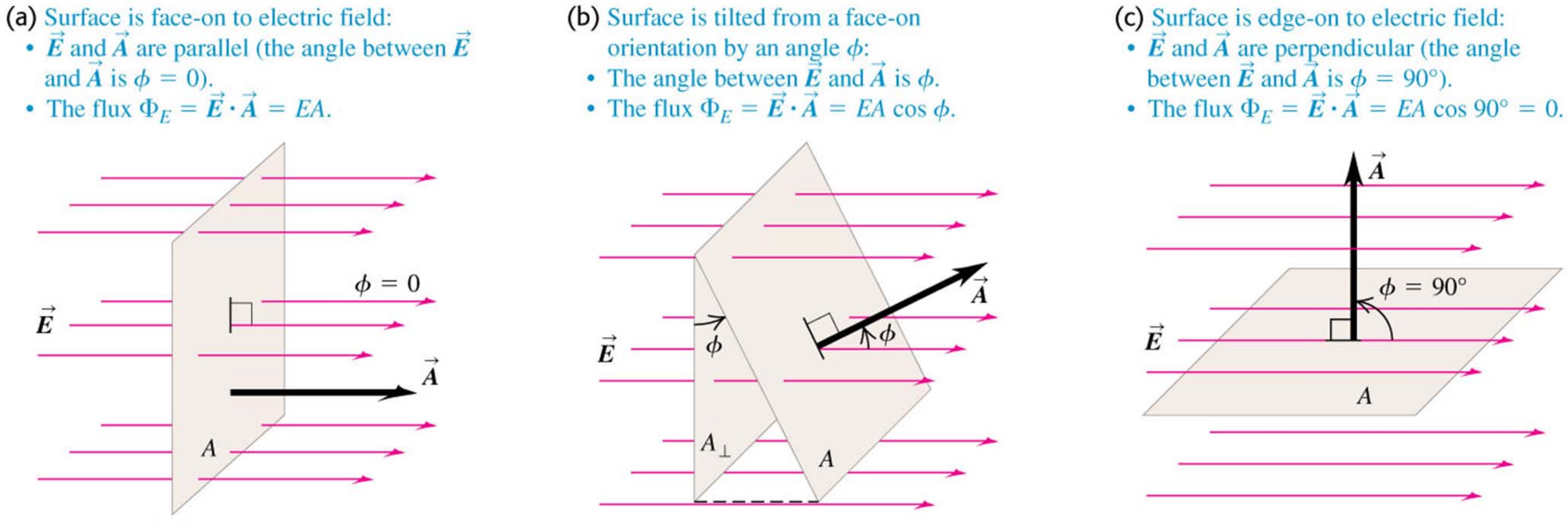
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

$$\Phi_E = EA = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}(4\pi R^2)$$

Calculating electric flux

- Follow the discussion in the text of flux calculation for uniform and nonuniform fields, using Figure 22.6 below.

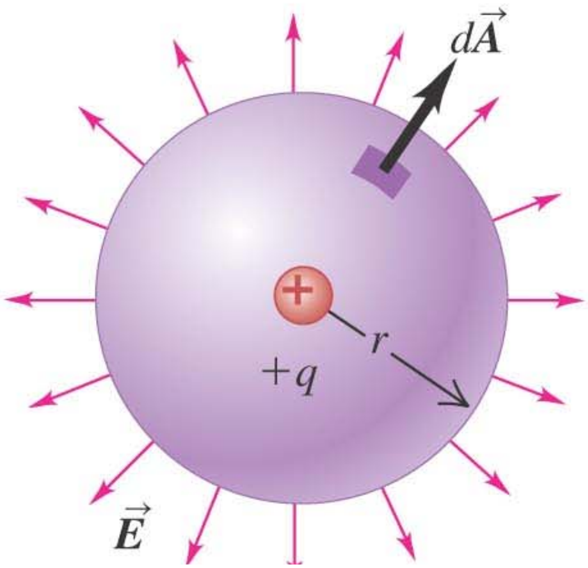
$$\Phi_E = \vec{E} \cdot \vec{A} \qquad \Phi_E = EA\cos\phi$$



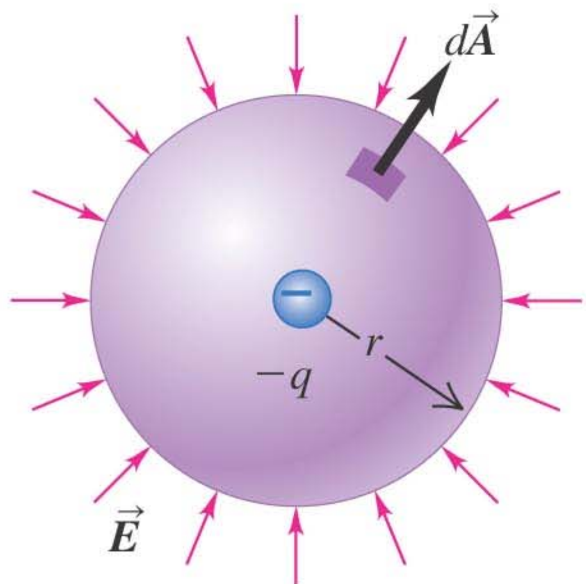
Positive and negative flux

- Figure 22.14 below shows that flux is positive if the enclosed charge is positive, and negative if the charge is negative.

(a) Gaussian surface around positive charge: positive (outward) flux



(b) Gaussian surface around negative charge: negative (inward) flux



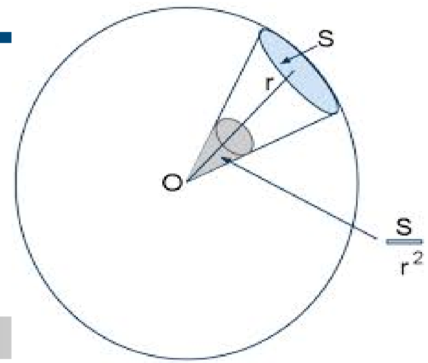
$$\Phi_E = \oint E_\perp \, dA = \oint \left(\frac{q}{4\pi\epsilon_0 r^2}\right) dA = \frac{q}{4\pi\epsilon_0 r^2} \oint dA = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Phi_E = \oint E_\perp \, dA = \oint \left(\frac{-q}{4\pi\epsilon_0 r^2}\right) dA = \frac{-q}{4\pi\epsilon_0 r^2} \oint dA = \frac{-q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{-q}{\epsilon_0}$$

Gauss’s Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)} \qquad (22.8)$$

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0.



Gauss's law

- Consider charge in a generic surface S
- Surround charge with spherical surface S_1 concentric to charge
- Consider cone of solid angle $d\Omega$ from charge to surface S through the little sphere
- Electric flux through little sphere:

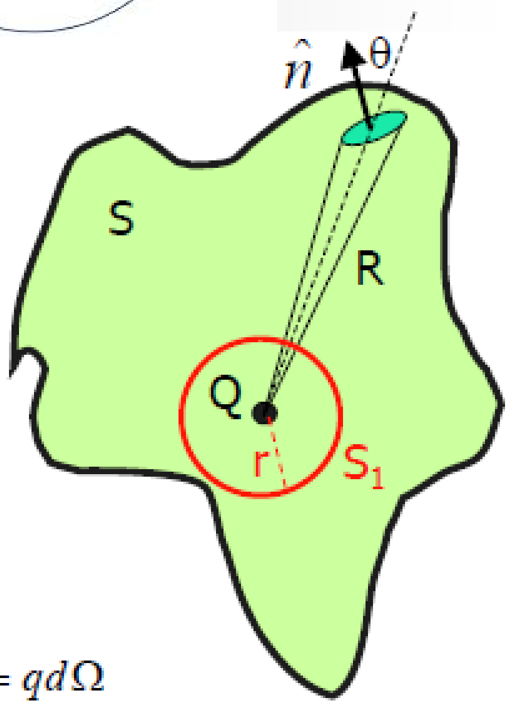
$$d\Phi_{S1} = \vec{E}\bullet d\vec{A} = (\frac{q}{r^2}\hat{r})(r^2 d\Omega\hat{r}) = q d\Omega$$

- Electric flux through surface S:

$$d\Phi_S = \vec{E}\bullet d\vec{A} = (\frac{q}{R^2}\hat{r})\bullet(\frac{R^2 d\Omega}{\cos\theta}\hat{n}) = q d\Omega \frac{\hat{r}\bullet\hat{n}}{\cos\theta} = q d\Omega$$

- $d\Phi_S = d\Phi_{S1} \rightarrow \Phi_S = \Phi_{S1} = 4\pi Q$

$$\boxed{\Phi = \oint_S \vec{E}\bullet d\vec{A} = 4\pi Q_{encl} \text{ is valid for ANY shape S.}}$$



$$\boxed{\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}},}$$

the divergence theorem:

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau.$$

$$Q_{\text{enc}} = \int_{\mathcal{V}} \rho\, d\tau.$$

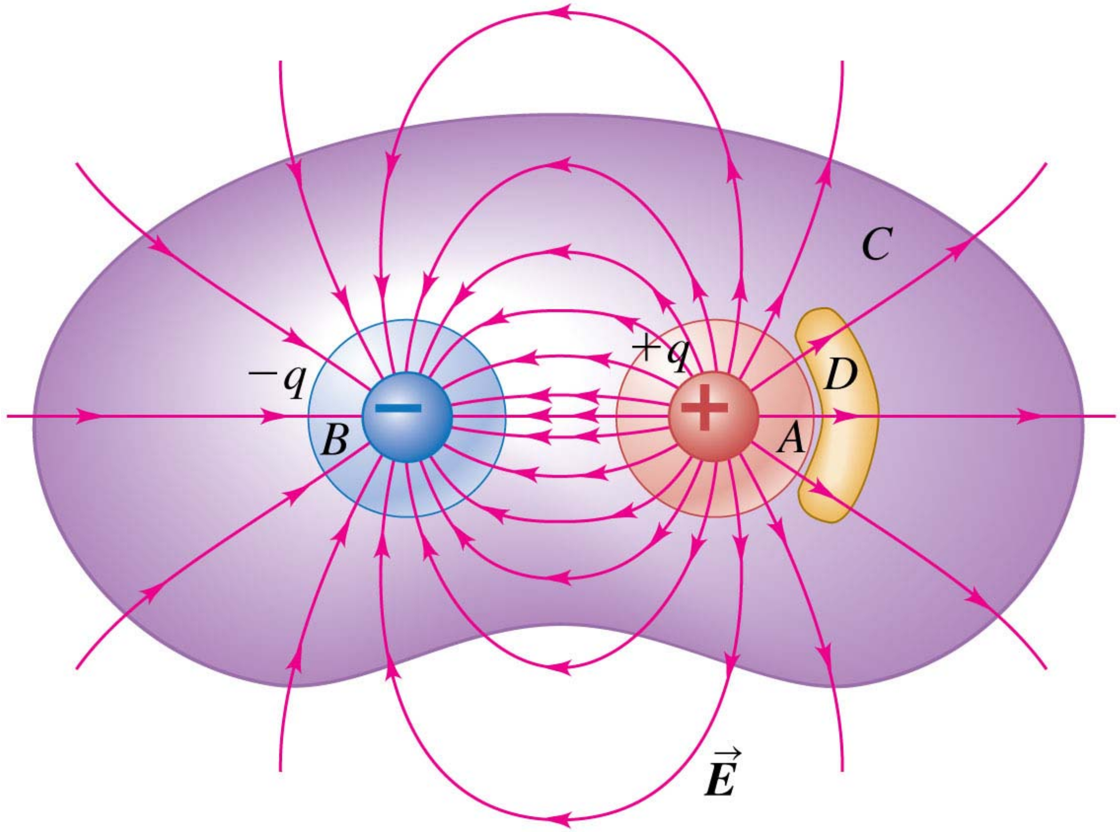
$$\int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau = \int_{\mathcal{V}} \left(\frac{\rho}{\epsilon_0}\right) d\tau.$$

y volume, the integrands must be equa

$$\boxed{\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho.}$$

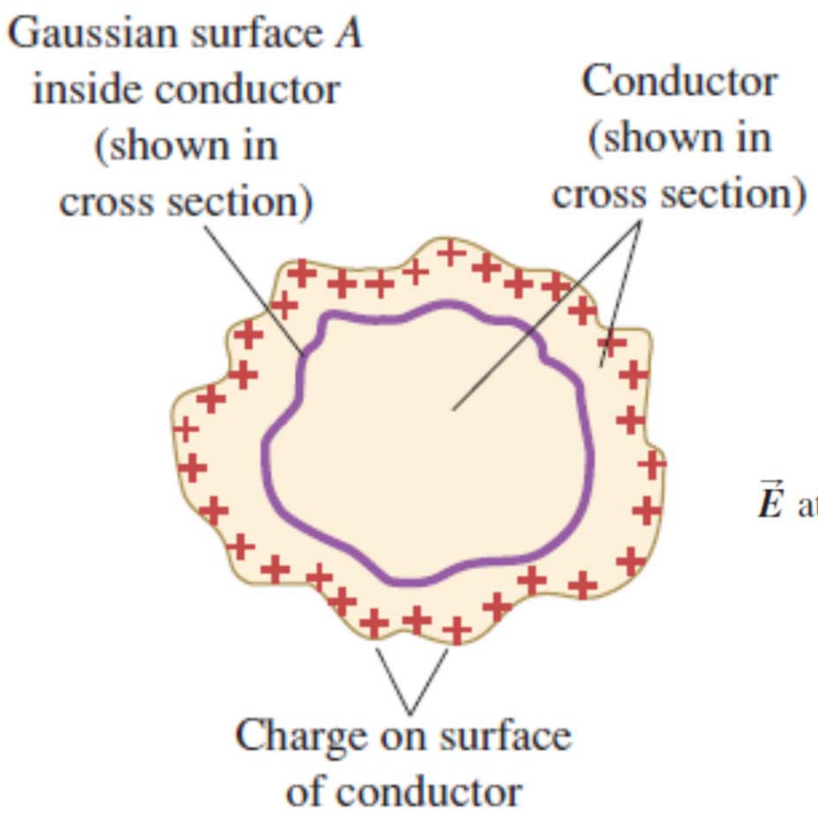
General form of Gauss's law

- Follow the text discussion of the general form of Gauss's law.
- Follow Conceptual Example 22.4 using Figure 22.15 below.



Applications of Gauss's law: conductor - VIP

- Under **electrostatic conditions**, any excess charge on a conductor resides entirely on its *surface*. (See Figure 22.17 below left.)

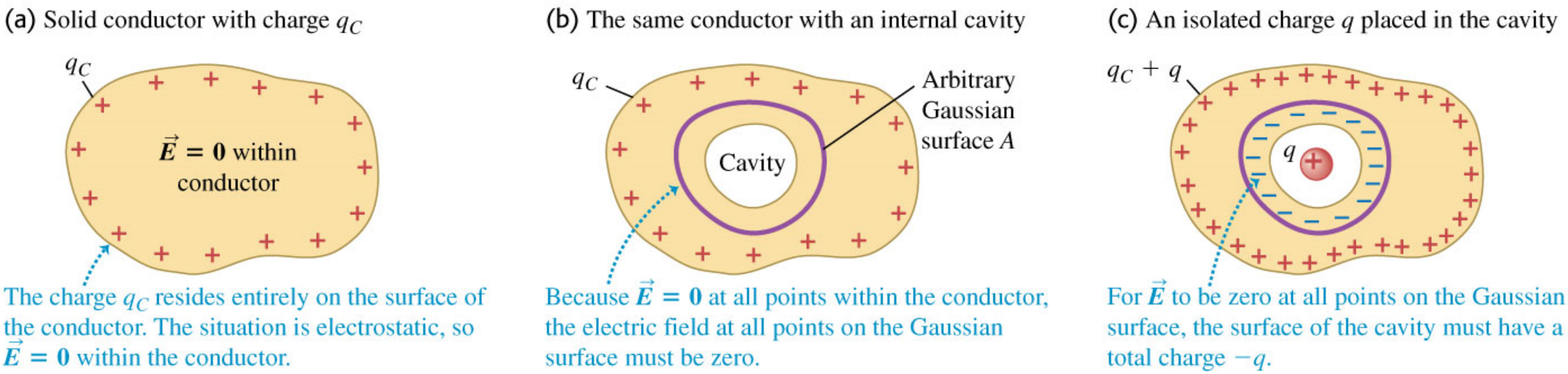


$\vec{E}$ at every point in the interior of a conducting material is zero.

shrinking the surface like a collapsing balloon

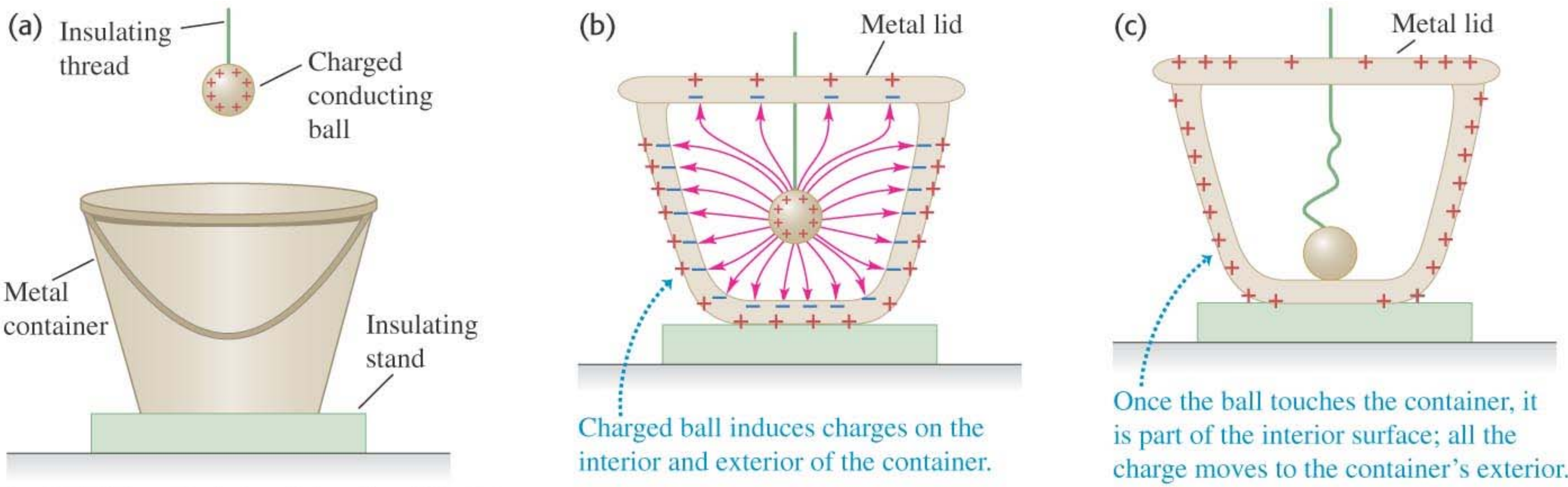
Charges on conductors

- In the text, follow the discussion on charges in a conductor having a cavity within it. Figure 22.23 below will help the explanation.

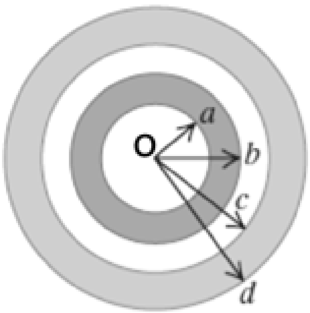


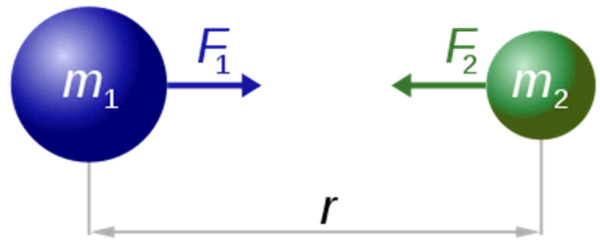
Testing Gauss's law experimentally

- Follow the discussion of *Faraday's icepail experiment*, which confirmed the validity of Gauss's law. Use Figure 22.25 below.



3. (25%) A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d. the inner shell has total charge +q, and the outer shell has charge -q (a) Find the total charge on each surfaces (b) Calculate the electric field EVERYWHERE (magnitude and direction) in terms of and the distance *r* from the common center O of the two shells. (c) Find the capacitance between the inner shell and the outer shell.





4.8.3 Gauss's Law for Gravity $F_1 = F_2 = G\dfrac{m_1 \times m_2}{r^2}$

What is the gravitational field inside a spherical shell of radius a and mass m?

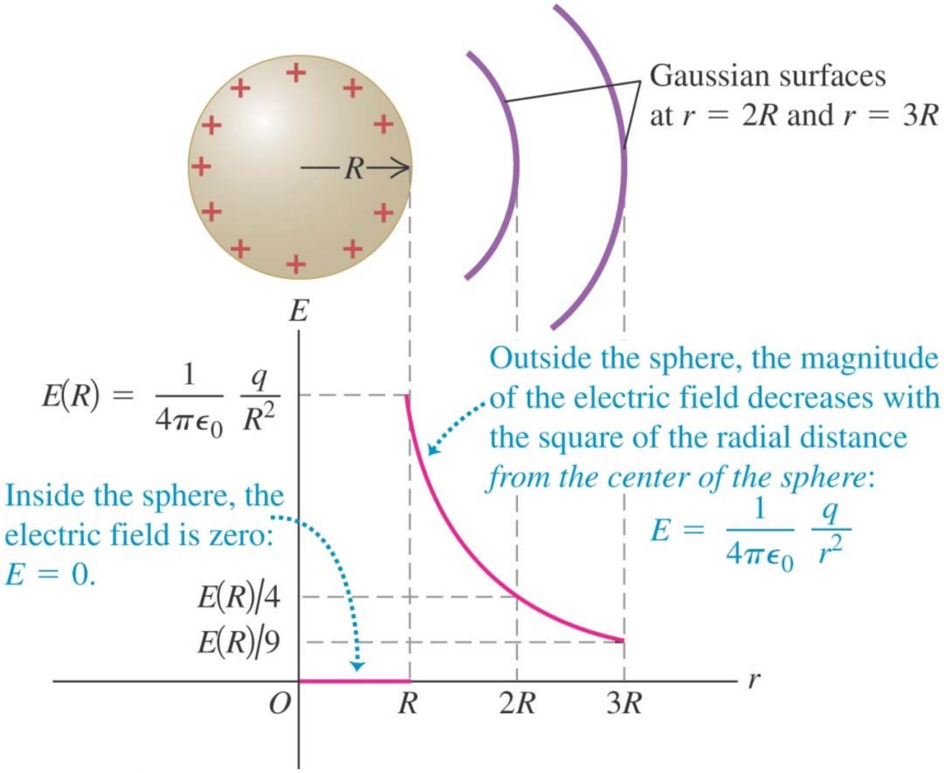
Solution:

Since the gravitational force is also an inverse square law, there is an equivalent Gauss's law for gravitation:

$$\Phi_g = -4\pi G m_{\text{enc}} \tag{4.8.2}$$

The only changes are that we calculate gravitational flux, the constant $1/\varepsilon_0 \to -4\pi G$, and $q_{\text{enc}} \to m_{\text{enc}}$. For $r \le a$, the mass enclosed in a Gaussian surface is zero because the mass is all on the shell. Therefore the gravitational flux on the Gaussian surface is zero. This means that the gravitational field inside the shell is zero!

Applications of Gauss's law: conductor



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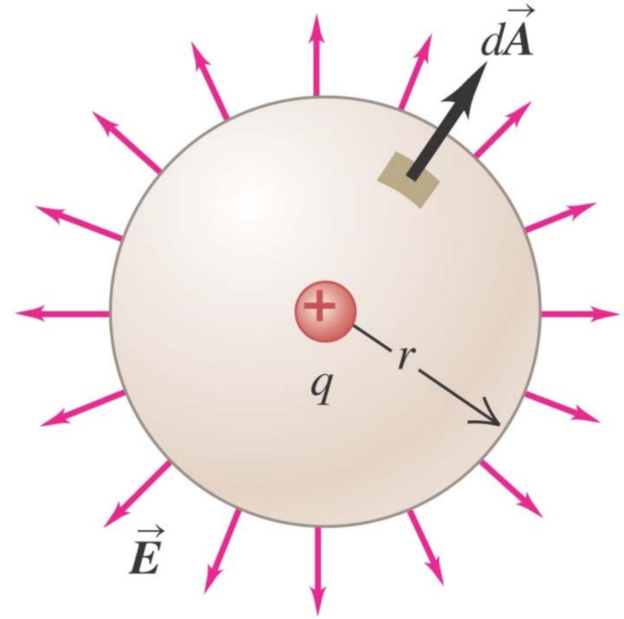
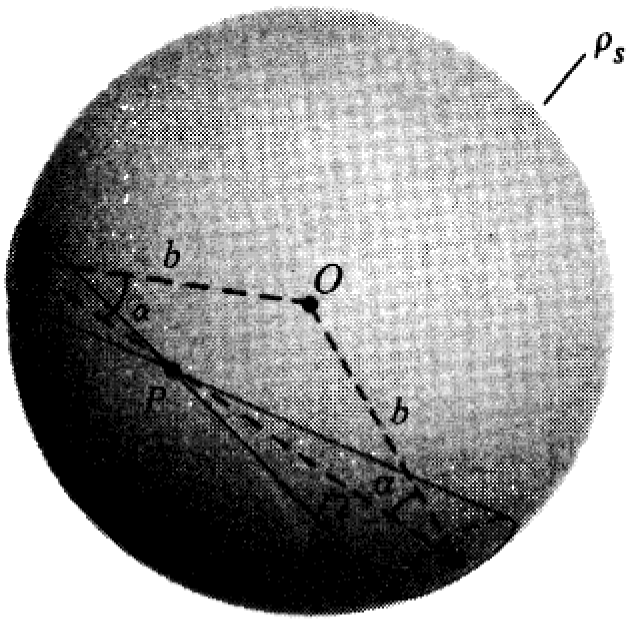


$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)}$$



:es ds_1 and ds_2 is, from Eq. (3–12),

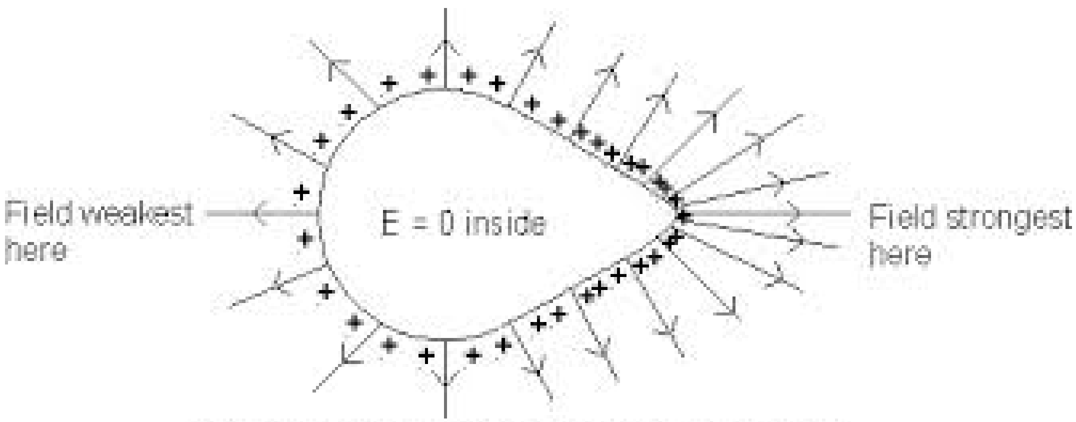
$$dE = \frac{\rho_s}{4\pi\epsilon_0}\left(\frac{ds_1}{r_1^2} - \frac{ds_2}{r_2^2}\right).$$

ıngle $d\Omega$ equals

$$d\Omega = \frac{ds_1}{r_1^2}\cos\alpha = \frac{ds_2}{r_2^2}\cos\alpha.$$

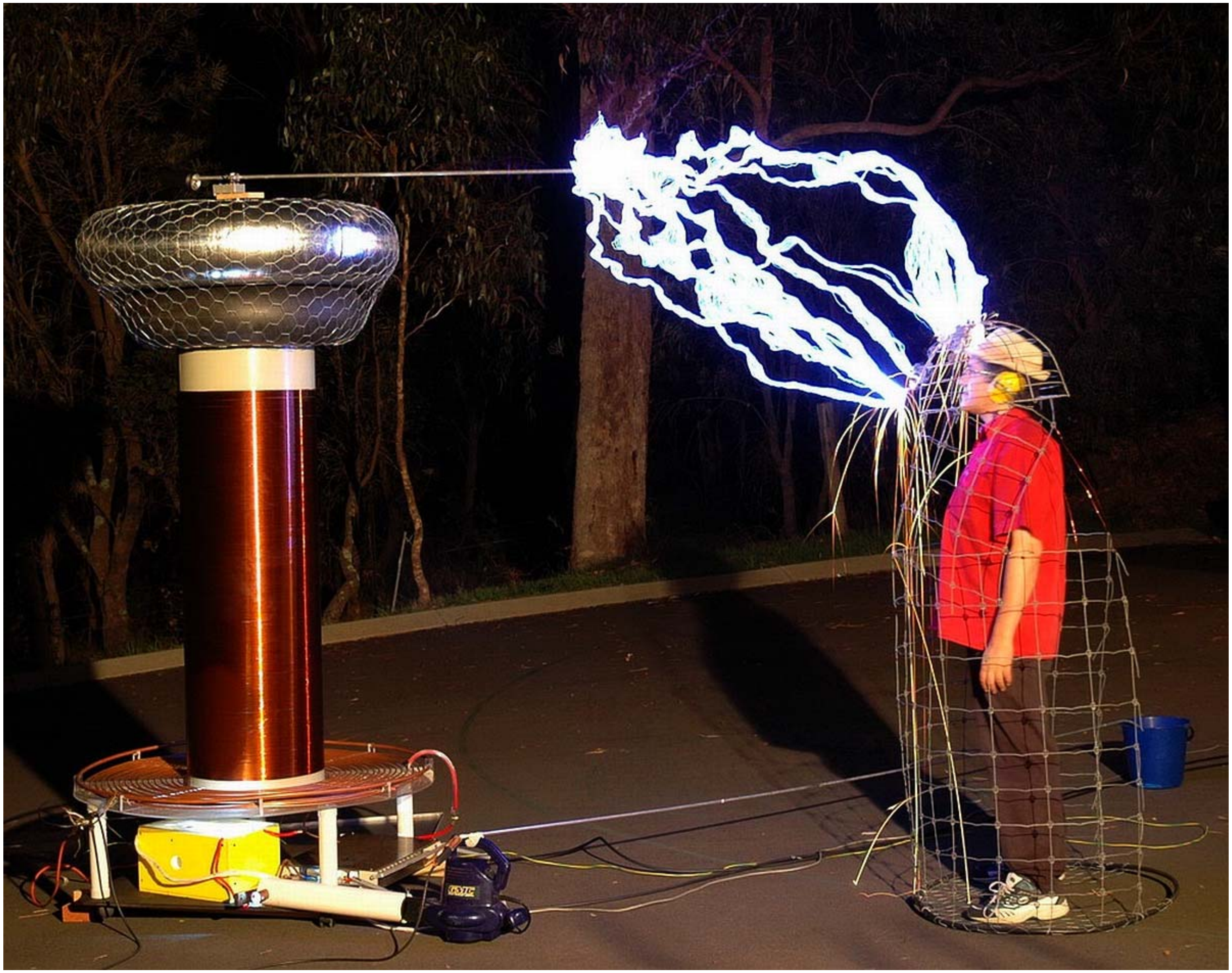
: expressions of dE and $d\Omega$, we find that

$$dE = \frac{\rho_S}{4\pi\epsilon_0}\left(\frac{d\Omega}{\cos\alpha} - \frac{d\Omega}{\cos\alpha}\right) = 0.$$



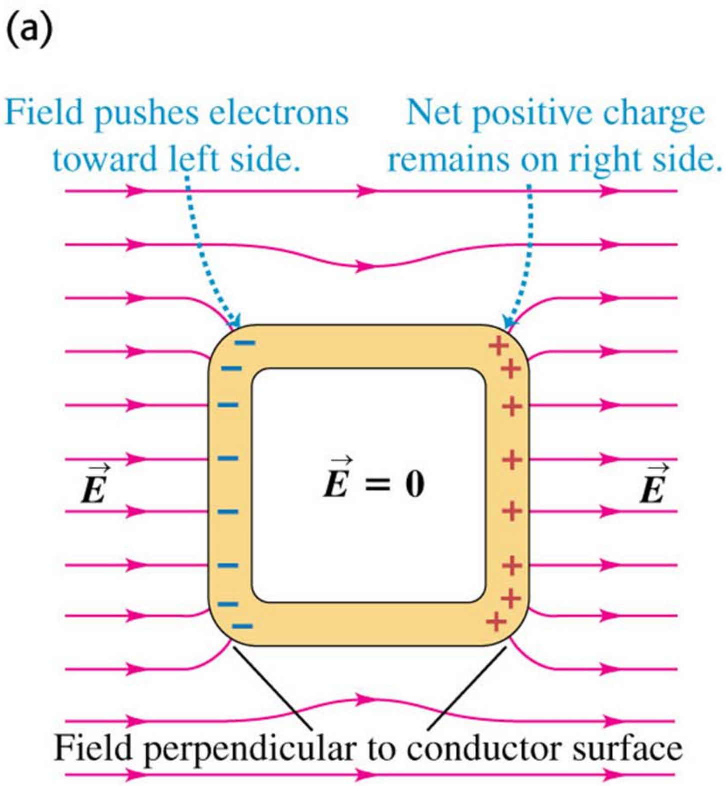
(d) Electric field & charge distribution around a pear-shaped conductor

Faraday's Cage



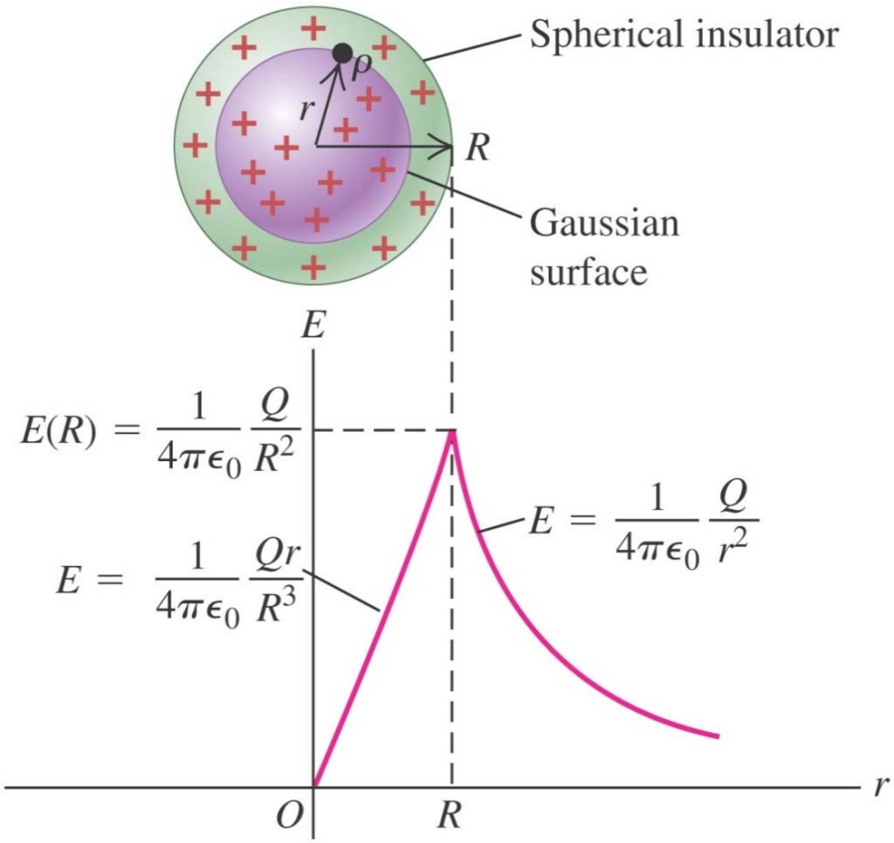
Electrostatic shielding

- A conducting box (a *Faraday cage*) in an electric field shields the interior from the field. (See Figure 22.27 below.)



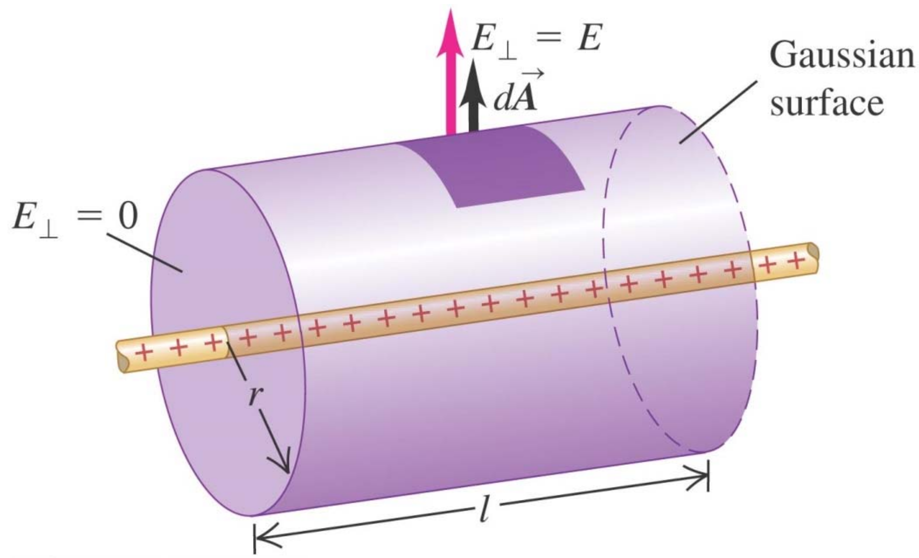
A uniformly charged sphere

- Follow Example 22.9 using Figure 22.22 below, for the field both inside and outside a sphere uniformly filled with charge.
- Follow Example 22.10 for the charge on a hollow sphere.



Field of a line charge

- Follow Example 22.6 using Figure 22.19 below, for a uniform line charge.



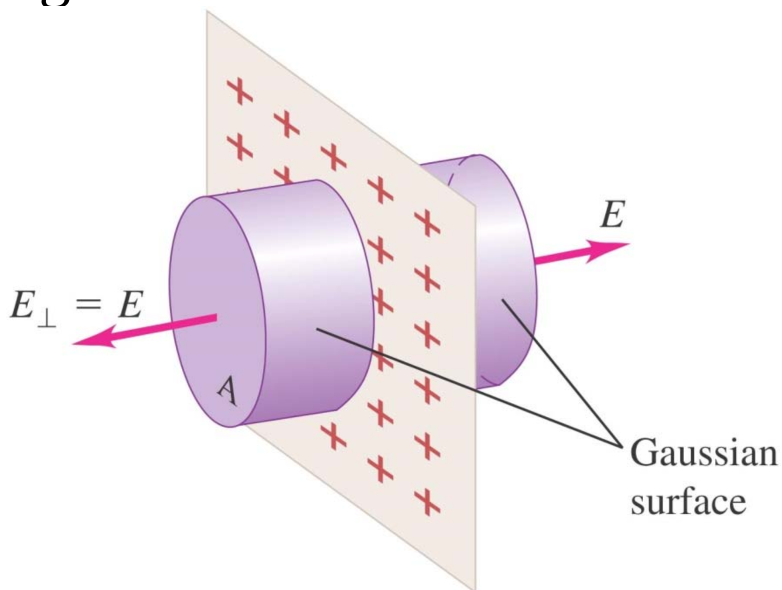
$Q_{encl} = \lambda l$, and so from Gauss's law, Eq. (22.8),

$$\Phi_E = 2\pi r l E = \frac{\lambda l}{\epsilon_0} \quad \text{and}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} \qquad \text{(field of an infinite line of charge)}$$

Field of a sheet of charge

- Follow Example 22.7 using Figure 22.20 below, for an infinite plane sheet of charge.



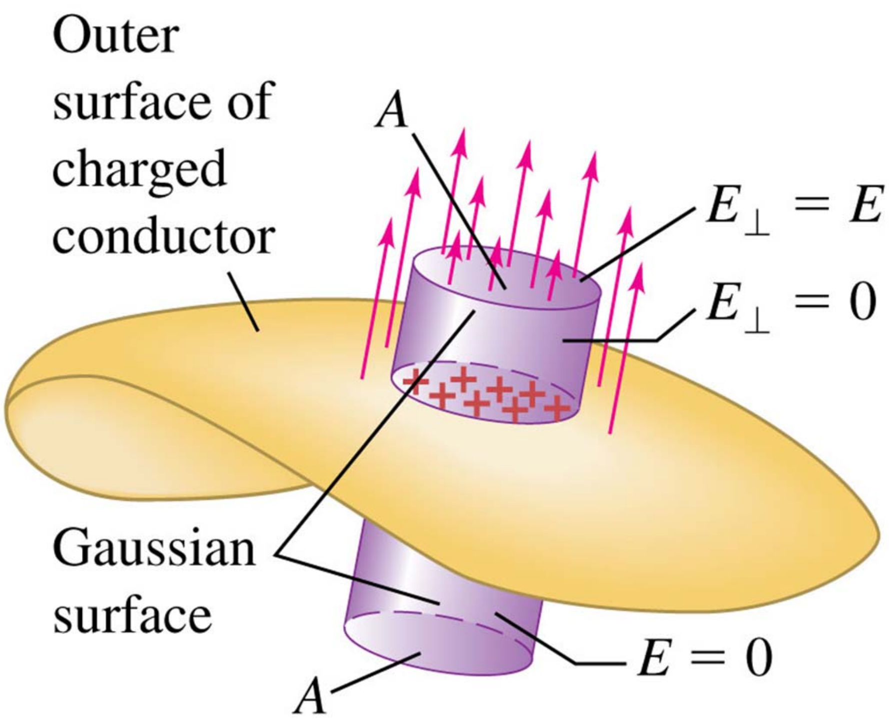
enclosed charge is $Q_{encl} = \sigma A$, and so from Gauss's law,

$$2EA = \frac{\sigma A}{\epsilon_0} \quad \text{and}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad \text{(field of an infinite sheet of charge)}$$

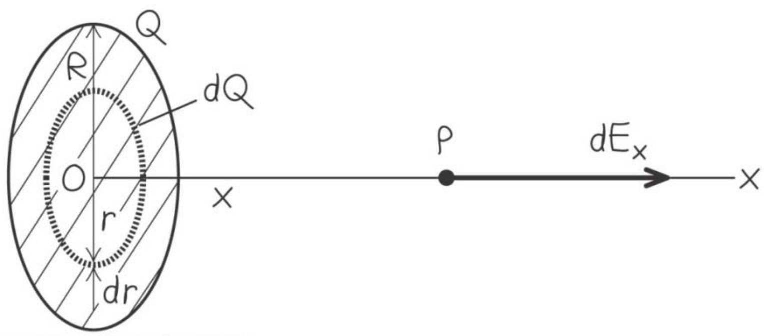
Field at the surface of a conductor

- The electric field at the surface of any conductor is always perpendicular to the surface and has magnitude σ/ε_0.
- Follow the discussion in the text, using Figure 22.28 at the right.
- Follow Conceptual Example 22.12.
- Follow Example 22.13, which deals with the electric field of the earth.



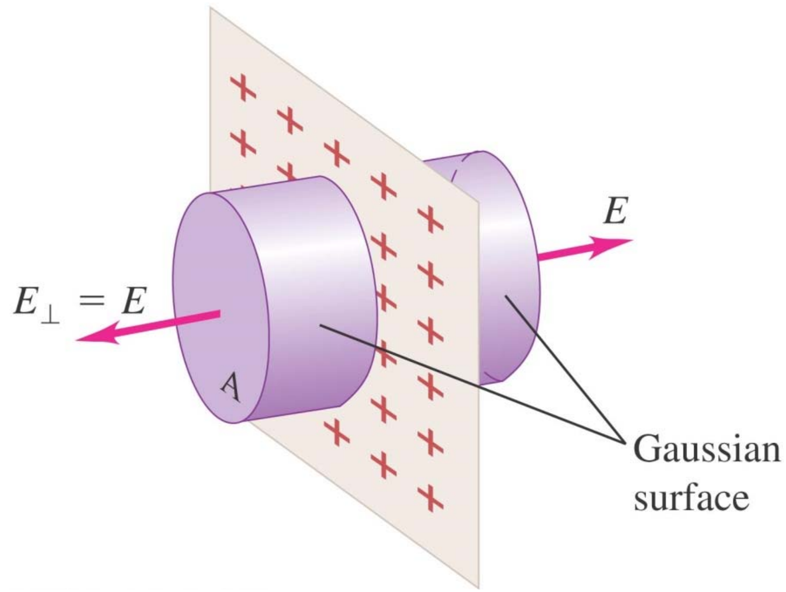
Field of a sheet of charge

- Follow Example 22.7 using Figure 22.20 below, for an infinite plane sheet of charge.



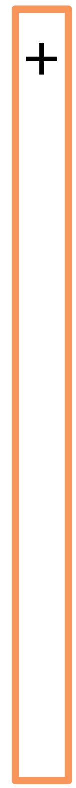
$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}}+\frac{1}{x}\right]$$
$$= \frac{\sigma}{2\epsilon_0}\left[1-\frac{1}{\sqrt{(R^2/x^2)+1}}\right] \qquad (21.11)$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad (21.12)$$



Field between one conducting plates

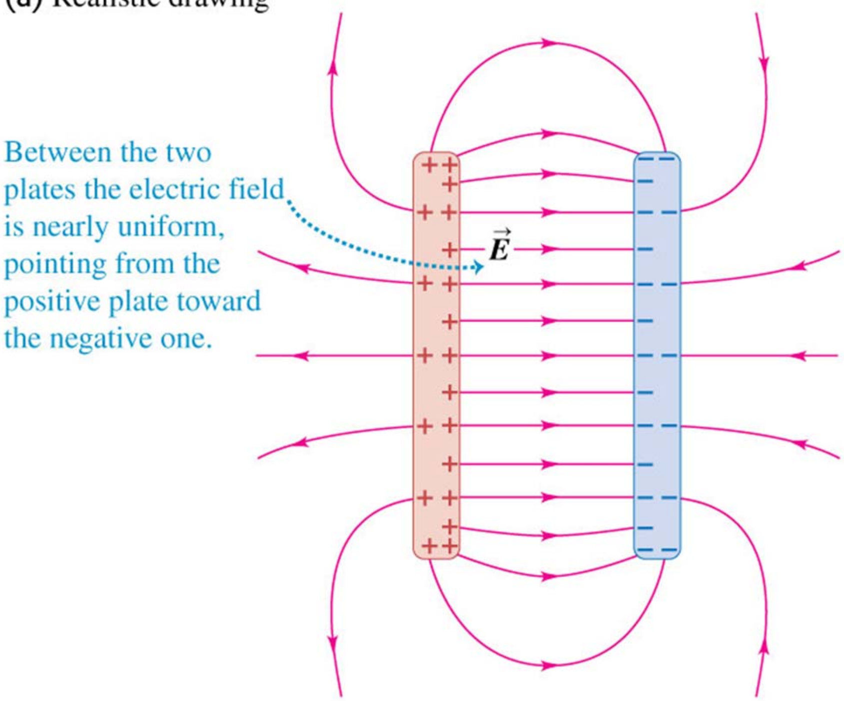
- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.

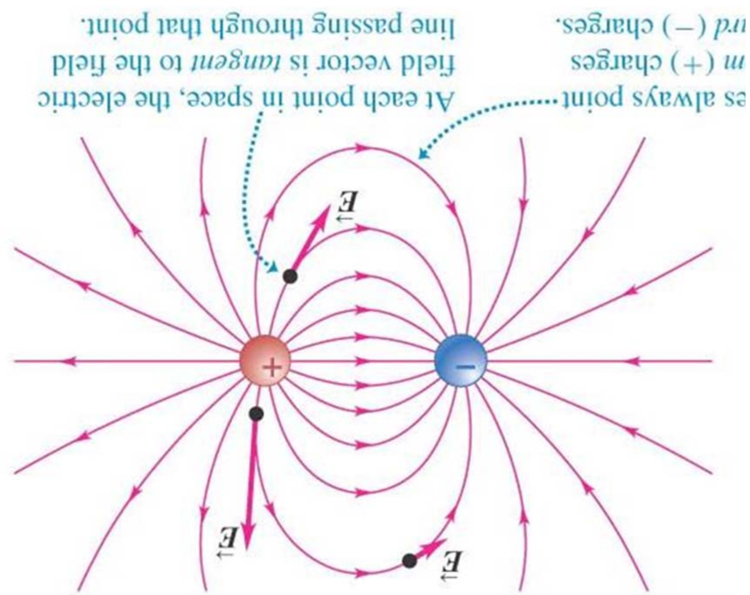


Field between two parallel conducting plates

- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.

(a) Realistic drawing

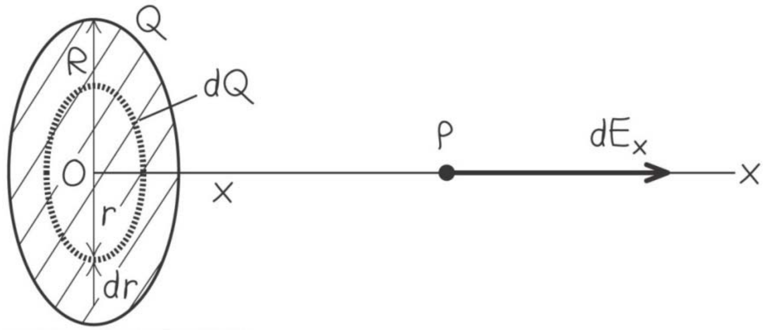




(b) Two equal and opposite charges (a dipole)

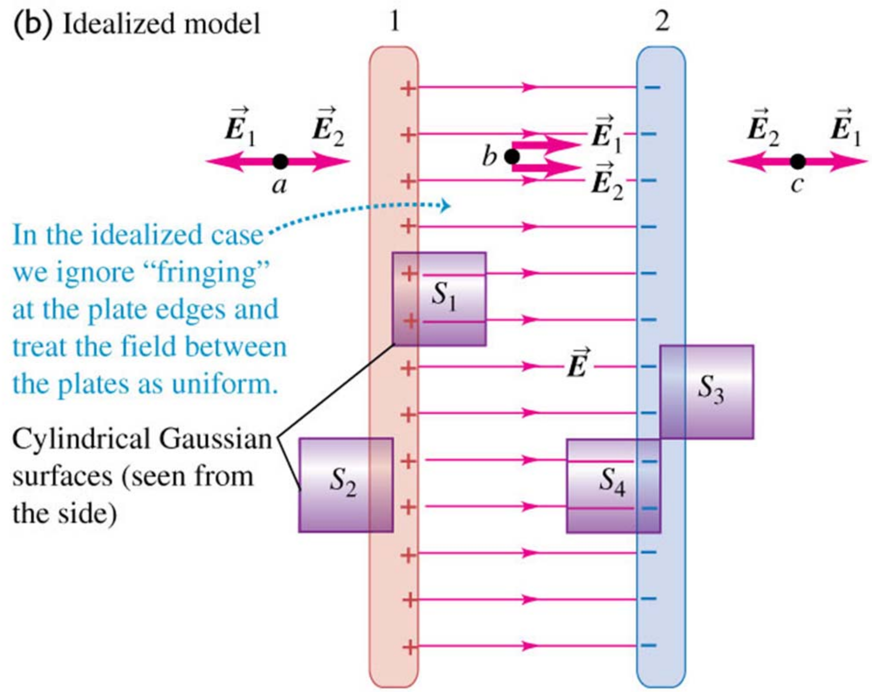
Field between two parallel conducting plates

- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.



$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right]$$
$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right] \qquad (21.11)$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad (21.12)$$

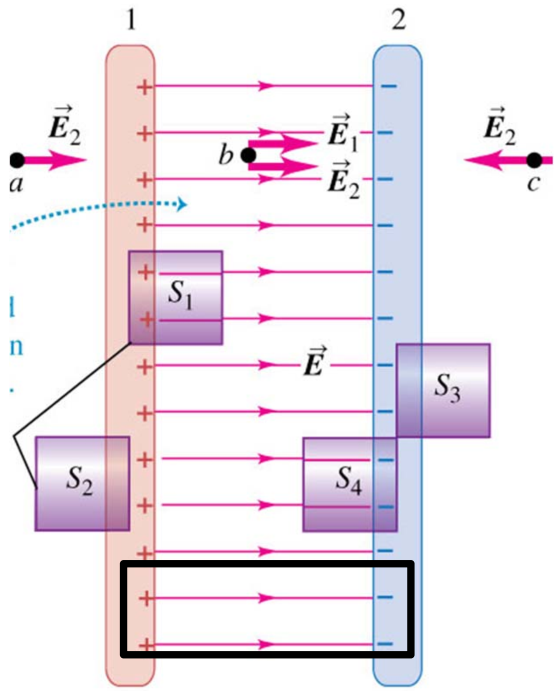


$$EA = \sigma A/\epsilon_0$$

$$E = \frac{\sigma}{\epsilon_0} \text{ (field between oppositely charged conducting plates)}$$

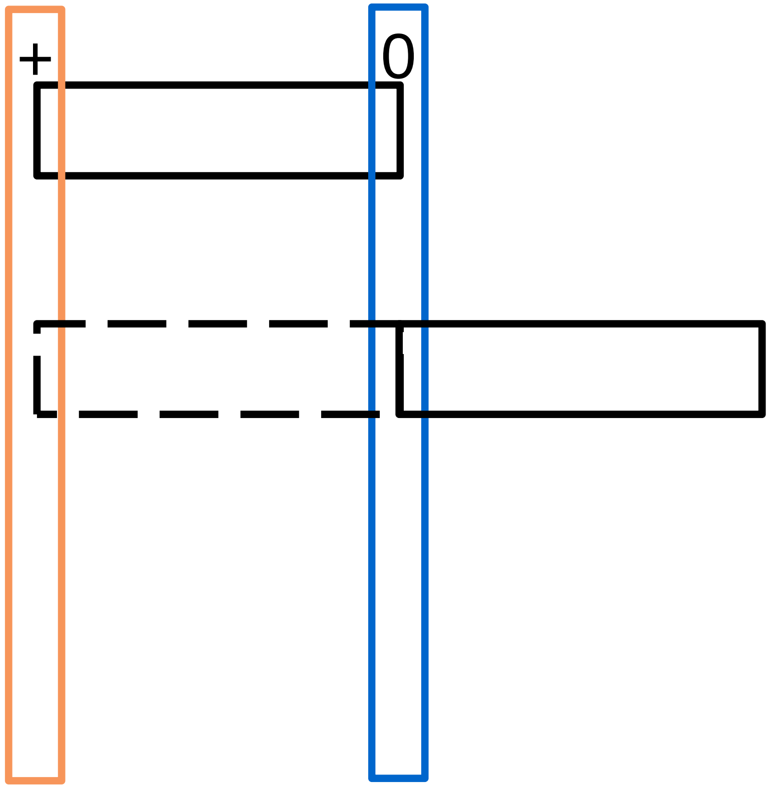
Field between two parallel conducting plates

- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.

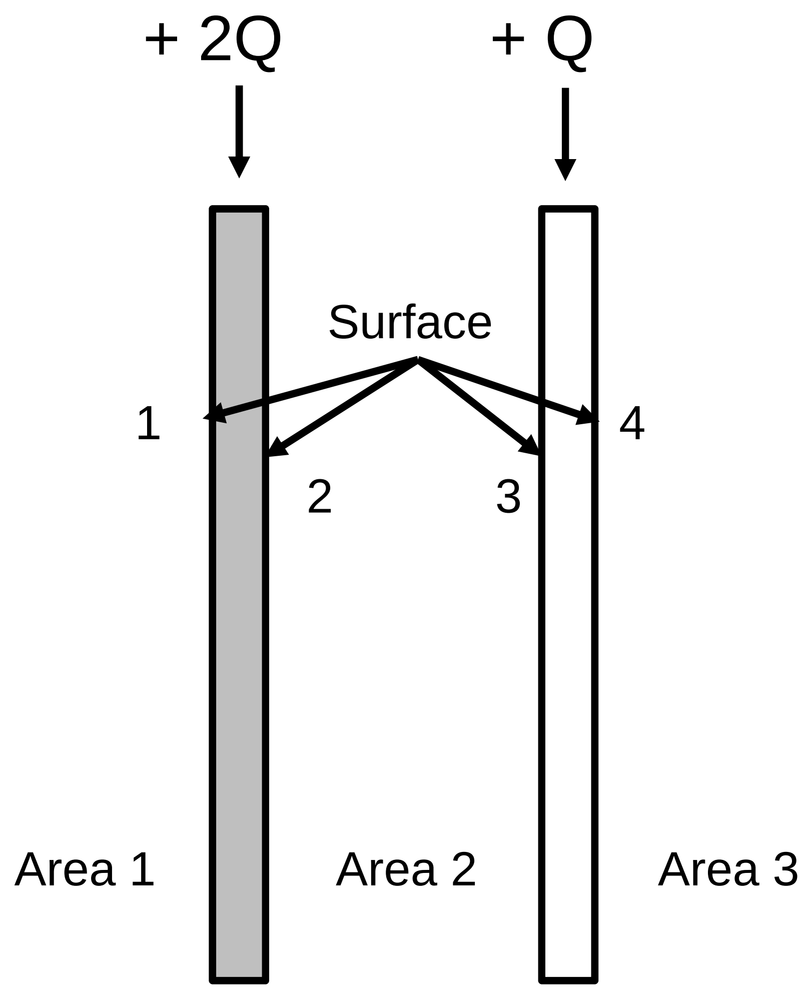


Field between two parallel conducting plates

- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.



Field between two parallel conducting plates



3. (25%) A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d. the inner shell has total charge +q, and the outer shell has charge -q (a) Find the total charge on each surfaces (b) Calculate the electric field EVERYWHERE (magnitude and direction) in terms of and the distance r from the common center O of the two shells. (c) Find the capacitance between the inner shell and the outer shell.