UM-SJTU JOINT INSTITUTE
PHYSICS LABTORATORY
(VP241)

LABTORATORY REPORT

EXERCISE 2
The Hall Probe: Characteristics and Applications This lab

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1 Objectives

1. Study the principle of the Hall effect and its applications by using a Hall probe.
2. Verify that Hall voltage is proportional to the magnetic field.
3. Study the sensitivity of an integrated Hall probe by calculating the magnetic field at the center of a solenoid.
4. Measure the distribution of the magnetic field along the axis of the solenoid and compare it with the corresponding theoretical curve.

2 Introduction and Theoretical Background

2.1 Hall Effect

Consider a conducting sheet (made of a metal or a semiconductor) placed in a magnetic field so that the plane of the sheet is perpendicular to the direction of the magnetic field B. When the electric current I passes through the sheet in the direction shown in Figure 1, an electric potential difference between the sides a and b of the sheet is generated. The corresponding electric field is perpendicular to both the direction of the current and the direction of the magnetic field. This effect is known as the Hall effect, and the electric potential difference is called the Hall voltage U_H.

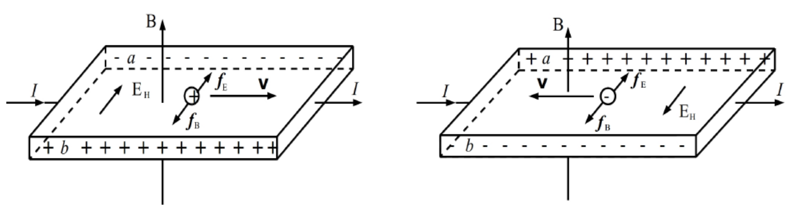


Figure 1: The principle of Hall effect

Microscopically, the Hall effect is caused by the Lorentz force, that is a force acting on charges moving in a magnetic field. The Lorentz force F_B leads to the detection of the moving charges, and their accumulation on one side of the sheet, which in turn increases the magnitude of the transverse electric field E_H (the Hall field). Due to this field, there is an electric force F_E acting upon the charges, and since F_B and F_E act in opposite directions, a balance is eventually reached and U_H stabilizes. The sign of UH depends on the sign of the charge carriers (positive or negative). Therefore the type of charge carriers in semiconductors can be determined by analyzing the sign of U_H.

When the external magnetic field is not too strong, the Hall voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d

$$U_H = R_H \frac{IB}{d} = KIB \tag{1}$$

where R_H is the so-called Hall coefficient and $K = R_H/d = K_H/I$, where K_H is the so-called sensitivity of the Hall element.

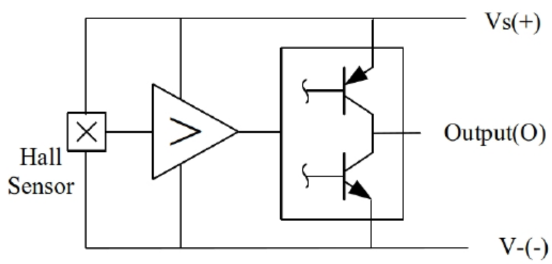
2.2 Integrated Hall Probe

The magnitude of the magnetic field can be found by measuring the Hall voltage with a Hall probe when the sensitivity K_H and the current I are fixed. Since the Hall voltage is usually very small, it should be amplified before the measurement.

Silicon can be used to design both the Hall probe and the integrated circuits, so it is convenient to arrange the Hall probe and the electric circuits into a single device. Such a device is called an integrated Hall probe.

The integrated Hall probe SS495A consists of a Hall sensor, an amplifier, and a voltage compensator (Figure 2). The output voltage U can be read ignoring the residual voltage. The working voltage $U_S = 5V$, and the output voltage U_0 is approximately 2.5 V when the magnetic field is zero. The relation between the output voltage U and the magnitude of the magnetic field is

$$B = \frac{U - U_0}{K_H} \tag{2}$$



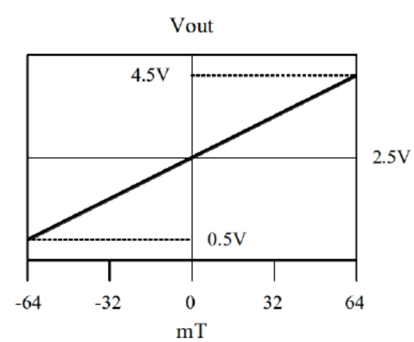


Figure 2: The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

2.3 Magnetic Field Distribution Inside a Solenoid

The magnetic field distribution on the axis of a single layer solenoid can be calculated from the following formula

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L+2x}{2[D^2 + (L+2x)^2]^{\frac{1}{2}}} + \frac{L-2x}{2[D^2 + (L-2x)^2]^{\frac{1}{2}}} 1 \right\} \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_M is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \cdot 10^7 H/m$.

The solenoid used in this exercise has ten layers, and the magnetic field $B(x)$ for each layer can be calculated using Eq. (3). Then the net magnetic on the axis of the solenoid can be found by adding contributions due to all layers. The theoretical value of the magnetic field inside the solenoid with $I_M = 0 : 1A$ is given in Table 1.

x[cm]	B[mT]	X[cm]	B[mT]
±0.0	1.4366	±8.0	1.4057
±1.0	1.4363	±9.0	1.3856
±2.0	1.4356	±10.0	1.3478
±3.0	1.4343	±11.0	1.2685
±4.0	1.4323	±11.5	1.1963
±5.0	1.4292	±12.0	1.0863
±6.0	1.4245	±12.5	0.9261
±7.0	1.4173	±13.0	0.7233

Table 1: Theoretical value of the magnetic field inside the solenoid.

2.4 Study of the Geomagnetic Field with a Hall Probe

The geomagnetic field is the magnetic associated with the Earth. The geomagnetic field lines are shown schematically in Figure 3. The Earth's magnetic field is similar to that of a bar magnet tilted about 11.5^o degrees from the spin axis of the Earth.

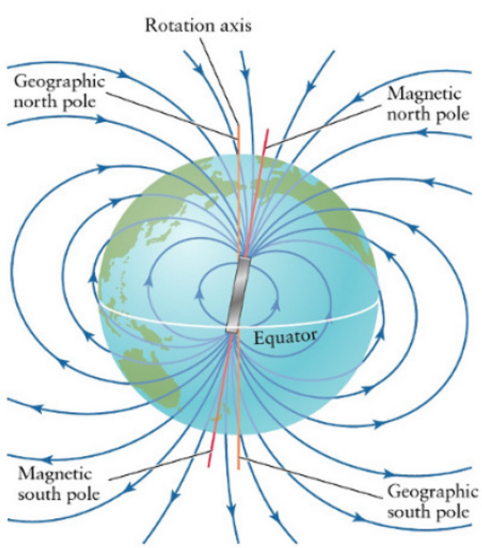


Figure 3: Magnetic field of the Earth.

Figure 4 shows the geomagnetic field distribution of China in 1970. The magnetic inclination is about $44:5^o$ and the magnitude of the magnetic field in Shanghai is about 48000 nT.

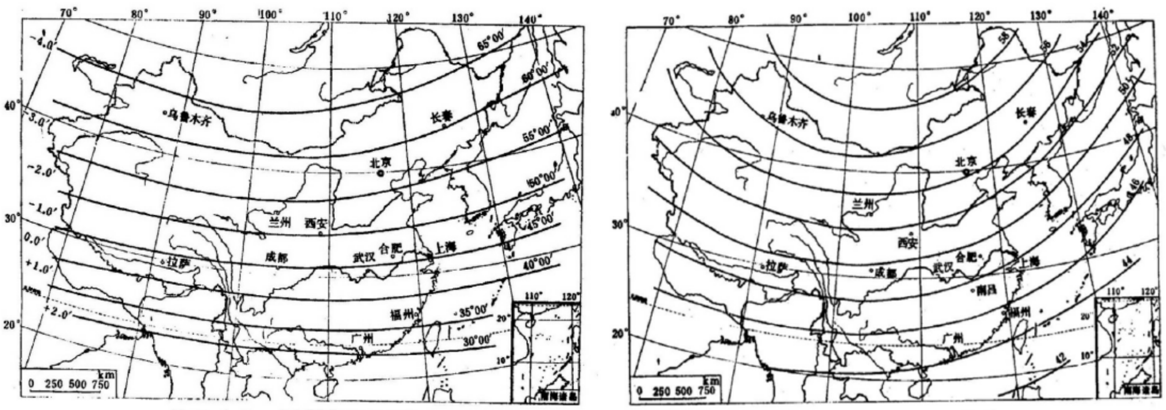


Figure 4: Geomagnetic inclination in China, 1970 (left). The magnitude of the geomagnetic field in China, 1970 (right).

3 Apparatus

The experimental setup shown in Figure 5 consists of an integrated Hall probe SS495A (see Figure 6) with $K_H = 31.25 \pm 1.25 V/T$ or $K_H = 3.125 \pm 0.125 mV/G$, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

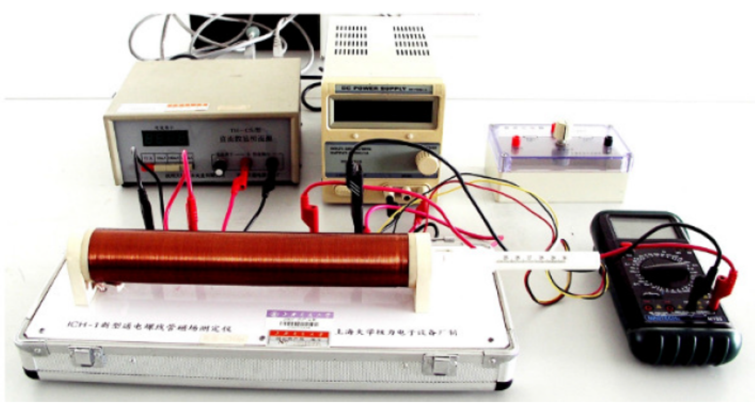


Figure 5: Measurement setup.

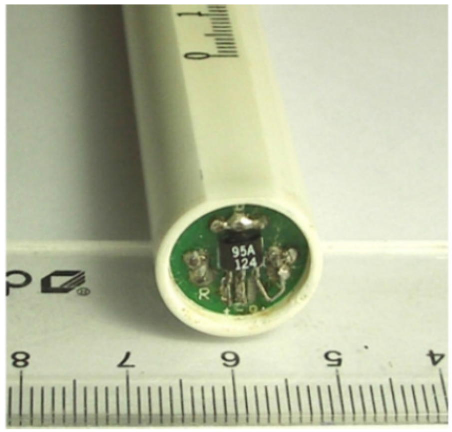


Figure 6: Integrated Hall probe SS495A.

4 Procedure

4.1 Relation between Sensitivity K_H and Working Voltage U_S

1. I placed the integrated Hall probe at the center of the solenoid and set the working voltage at $5V$ to measure the output voltage $U_0(I_M = 0)$ and $U(I_M = 250mA)$. Then I take the theoretical value of $B(x = 0)$ from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).

2. I measured K_H for different values of U_S from 2.8 V to 10 V and calculate $K_H = U_S$ to plot the curve $K_H = U_S$ vs. U_S.

4.2 Relation between Output Voltage U and Magnetic field B

1. I connected the $2.4 \sim 2.6V$ output terminal of the DC voltage divider and the negative port of the voltmeter with $B = 0$, $U_S = 5V$, then adjust the voltage until $U_0 = 0$.

2. I placed the integrated Hall probe at the center of the solenoid and measured the output voltage U for different values of I_M ranging from 0 to 500 mA, with intervals of $50mA$.

3. I explain the relation between $B(x = 0)$ and the Hall voltage U_H in the report by using output voltage U as amplified signal from U_H and the theoretical value of $B(x = 0)$ from Table 1.

4. Then I plot the curve U vs. B to find the sensitivity K_H by a linear fit and compare the value with the theoretical value in given in the Apparatus section.

4.3 Magnetic Field Distribution Inside the Solenoid

1. I measure the magnetic field distribution along the axis of the solenoid for $I_M = 250mA$, record the output voltage U and the corresponding position x. Then find $B = B(x)$. (Use the value of K_H found in the previous part of the experiment).

2. I plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid by using dots for the data measured and a solid line for the theoretical curve. The origin of the plot is at the center of the solenoid.

4.4 Measurement of the Geomagnetic Field

I used the integrated Hall probe to measure the magnitude and the direction of the geomagnetic field.

5 Results & Calculations

5.1 U_0 and U with U_S=5V

5.1.1 Measured Data for U_0 and U with U_S=5V

$U_S \pm 0.05\%[V]$	$U_0(I_M = 0mA) \pm 0.05\% + 6 \cdot 10^{-3}[V]$	$U_0(I_M = 250mA) \pm 0.05\% + 6 \cdot 10^{-3}[V]$
4.99	2.479	2.584

5.1.2 Calculation for K_H

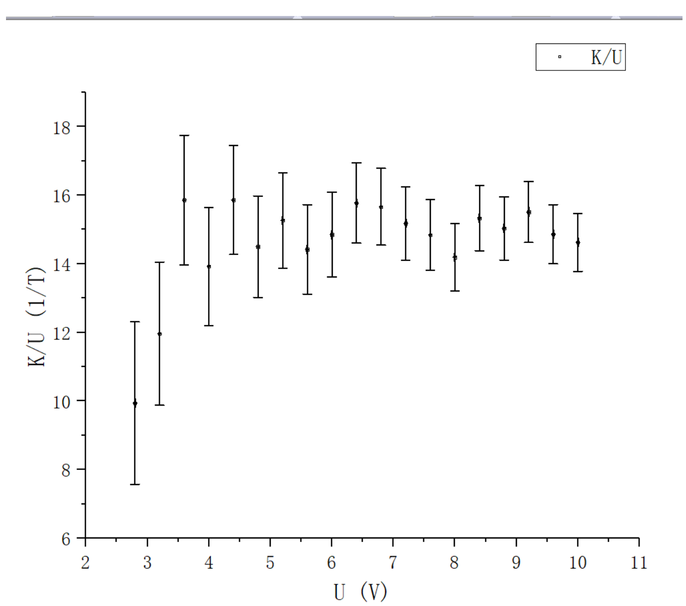
$$K_H = \frac{U - U_0}{B} = \frac{2.584 - 2.479}{1.4366 \cdot 10^{-3}} = 73.90 \pm 7.15[V/T]$$

5.1.3 Relation between Sensitivity K_H and Working Voltage U_S

	$U_S \pm 0.05\%[V]$	$U_0 \pm 0.05\% + 6 \cdot 10^{-3}[V]$	$U \pm 0.05\% + 6 \cdot 10^{-3}[V]$	$K_H[V/T]$	$K_H/U_S[1/T]$
1	2.8	1.452	1.492	27.84	9.94
2	3.2	1.572	1.627	38.28	11.96
3	3.6	1.772	1.854	57.08	15.86
4	4.0	1.982	2.062	55.69	13.92
5	4.4	2.178	2.278	69.61	15.89
6	4.8	2.379	2.479	69.61	14.50
7	5.2	2.580	2.694	79.35	15.26
8	5.6	2.778	2.894	80.75	14.42
9	6.0	2.981	3.109	89.10	14.85
10	6.4	3.175	3.320	100.9	15.77
11	6.8	3.377	3.530	106.5	15.66
12	7.2	3.586	3.743	109.3	15.18
13	7.6	3.781	3.943	112.8	14.84
14	8.0	3.987	4.150	113.5	14.19
15	8.4	4.178	4.363	128.8	15.33
16	8.8	4.384	4.574	132.3	15.03
17	9.2	4.576	4.781	142.7	15.51
18	9.6	4.784	4.989	142.7	14.86
19	10.0	4.979	5.189	146.2	14.62

Table 2: Data of K_H for U_0 and U with different U_S

5.1.4 Plot of Sensitivity K_H/U_S and Working Voltage U_S



5.2 I_M vs. U relation

	$I_M \pm 2\%[mA]$	$U \pm 0.05\% + 6 \cdot 10^{-4}[V]$
1	0	0.0000
2	50	0.0211
3	100	0.0418
4	150	0.0624
5	200	0.0832
6	250	0.1039
7	300	0.1251
8	350	0.1453
9	400	0.1661
10	450	0.1871
11	500	0.2080

5.2.1 Calculation of B

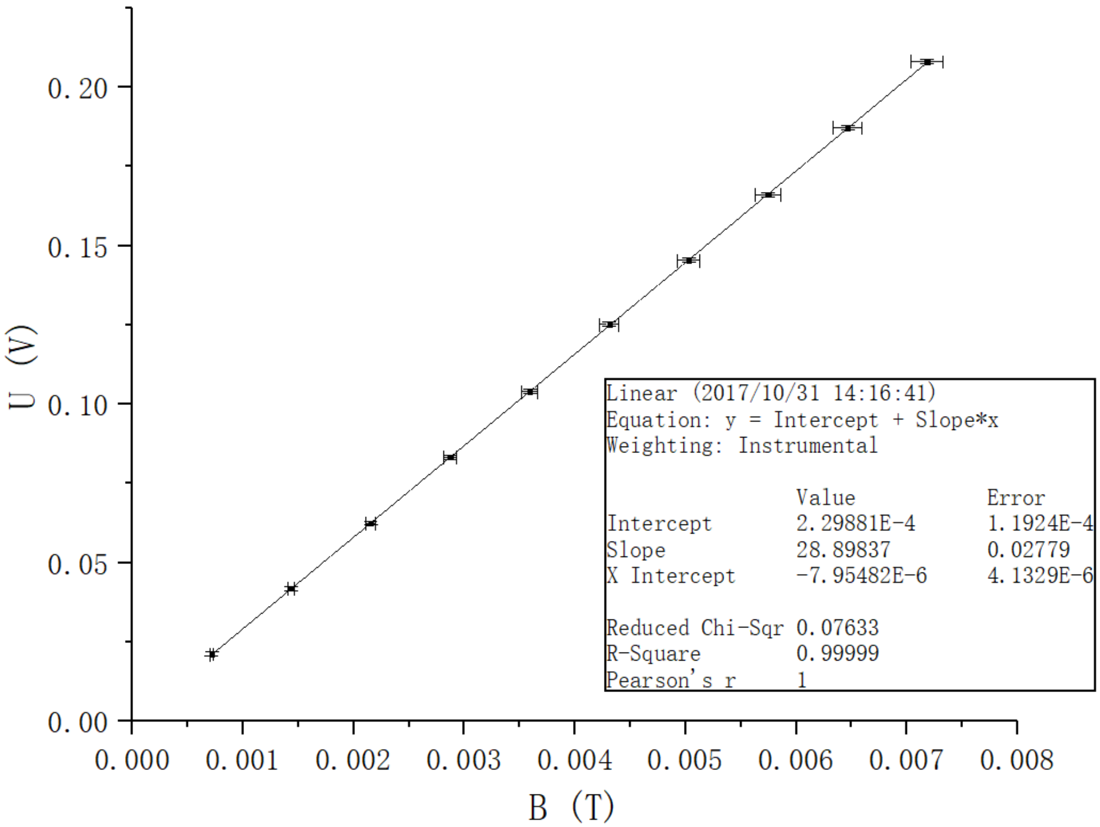
$$B(x) = C(x)I_M$$

$$C(0) = \frac{B(0)}{I_M} = \frac{1.436 \cdot 10^{-3}}{0.1} = 1.4366 \cdot 10^{-2}[T/A]$$

	$B[T]$	$U[V]$
1	0	0.0000
2	$7.184 \cdot 10^{-4}$	0.0211
3	$1.437 \cdot 10^{-3}$	0.0418
4	$2.155 \cdot 10^{-3}$	0.0624
5	$2.873 \cdot 10^{-3}$	0.0832
6	$3.592 \cdot 10^{-3}$	0.1039
7	$4.310 \cdot 10^{-3}$	0.1251
8	$5.028 \cdot 10^{-3}$	0.1453
9	$5.746 \cdot 10^{-3}$	0.1661
10	$6.465 \cdot 10^{-3}$	0.1871
11	$7.183 \cdot 10^{-3}$	0.2080

Table 3: Relation between B and U

5.2.2 Plot for relation between B and U



U_H becomes bigger as B increasing, and they're proportional to each other. According to linear fit, the slope K_H is 28.8983 ± 0.0277, which is a little smaller than the value in apparatus section.

5.3 U vs. x relation

5.3.1 Measured Data for x and U

	$x \pm 0.05[cm]$	$U \pm 0.05\% + 6 \cdot 10^{-4}[V]$		$x \pm 0.05[cm]$	$U \pm 0.05\% + 6 \cdot 10^{-4}[V]$
1	0.0	0.0485	27	15.6	0.1050
2	0.6	0.0679	28	16.2	0.1050
3	1.2	0.0793	29	16.8	0.1048
4	1.8	0.0893	30	17.4	0.1047
5	2.4	0.0944	31	18.0	0.1045
6	3.0	0.0975	32	18.6	0.1045
7	3.6	0.0996	33	19.2	0.1039
8	4.2	0.1019	34	19.8	0.1035
9	4.8	0.1025	35	20.4	0.1031
10	5.4	0.1028	36	21.0	0.1024
11	6.0	0.1034	37	21.6	0.1015
12	6.6	0.1037	38	22.2	0.0999
13	7.2	0.1040	39	22.8	0.0973
14	7.8	0.1043	40	23.4	0.0958
15	8.4	0.1044	41	24.6	0.0897
16	9.0	0.1046	42	25.2	0.0823
17	9.6	0.1047	43	25.8	0.0730
18	10.2	0.1047	44	26.4	0.0548
19	10.8	0.1046	45	27.0	0.0377
20	11.4	0.1046	46	27.6	0.0147
21	12.0	0.1046	47	28.2	0.0088
22	12.6	0.1047	48	28.8	0.0050
23	13.2	0.1048	49		
24	13.8	0.1049	50		
25	14.4	0.1049	51		
26	15.0	0.1050	52		

5.3.2 Calculation for $B(x)$

$$B(x) = \frac{U}{K_H}$$

$$B(0) = \frac{0.0485}{28.898} = 1.678 \cdot 10^{-3}[T]$$

	$x \pm 0.05[cm]$	$B[T]$		$x \pm 0.05[cm]$	$B[T]$
1	0.0	$1.678 \cdot 10^{-3}$	27	15.6	$3.633 \cdot 10^{-3}$
2	0.6	$2.350 \cdot 10^{-3}$	28	16.2	$3.633 \cdot 10^{-3}$
3	1.2	$2.744 \cdot 10^{-3}$	29	16.8	$3.627 \cdot 10^{-3}$
4	1.8	$3.090 \cdot 10^{-3}$	30	17.4	$3.623 \cdot 10^{-3}$
5	2.4	$3.267 \cdot 10^{-3}$	31	18.0	$3.616 \cdot 10^{-3}$
6	3.0	$3.374 \cdot 10^{-3}$	32	18.6	$3.616 \cdot 10^{-3}$
7	3.6	$3.447 \cdot 10^{-3}$	33	19.2	$3.595 \cdot 10^{-3}$
8	4.2	$3.526 \cdot 10^{-3}$	34	19.8	$3.582 \cdot 10^{-3}$
9	4.8	$3.526 \cdot 10^{-3}$	35	20.4	$3.568 \cdot 10^{-3}$
10	5.4	$3.557 \cdot 10^{-3}$	36	21.0	$3.543 \cdot 10^{-3}$
11	6.0	$3.578 \cdot 10^{-3}$	37	21.6	$3.512 \cdot 10^{-3}$
12	6.6	$3.588 \cdot 10^{-3}$	38	22.2	$3.457 \cdot 10^{-3}$
13	7.2	$3.599 \cdot 10^{-3}$	39	22.8	$3.367 \cdot 10^{-3}$
14	7.8	$3.609 \cdot 10^{-3}$	40	23.4	$3.315 \cdot 10^{-3}$
15	8.4	$3.612 \cdot 10^{-3}$	41	24.6	$3.104 \cdot 10^{-3}$
16	9.0	$3.620 \cdot 10^{-3}$	42	25.2	$2.848 \cdot 10^{-3}$
17	9.6	$3.623 \cdot 10^{-3}$	43	25.8	$2.526 \cdot 10^{-3}$
18	10.2	$3.623 \cdot 10^{-3}$	44	26.4	$1.896 \cdot 10^{-3}$
19	10.8	$3.620 \cdot 10^{-3}$	45	27.0	$1.305 \cdot 10^{-3}$
20	11.4	$3.620 \cdot 10^{-3}$	46	27.6	$3.568 \cdot 10^{-4}$
21	12.0	$3.620 \cdot 10^{-3}$	47	28.2	$5.087 \cdot 10^{-4}$
22	12.6	$3.623 \cdot 10^{-3}$	48	28.8	$1.730 \cdot 10^{-4}$
23	13.2	$3.627 \cdot 10^{-3}$	49		
24	13.8	$3.630 \cdot 10^{-3}$	50		
25	14.4	$3.630 \cdot 10^{-3}$	51		
26	15.0	$3.633 \cdot 10^{-3}$	52		

5.3.3 Theoretical Value of B when $I = 250mA$

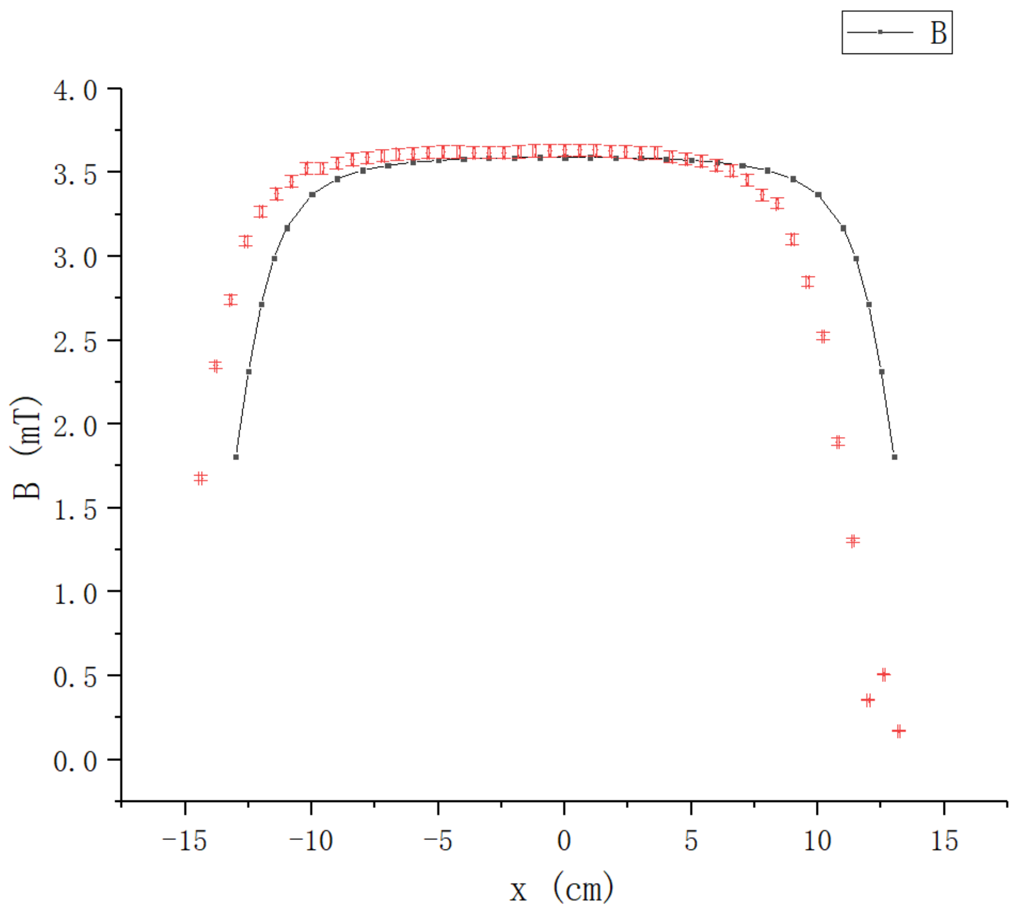
$B' = \frac{B \cdot 0.25}{0.1} = 1.4366 \cdot 2.5 = 3.5915[mT]$

x[cm]	B[mT]	X[cm]	B[mT]
±0.0	3.5915	±8.0	3.5143
±1.0	3.5908	±9.0	3.464
±2.0	3.589	±10.0	3.3695
±3.0	3.5858	±11.0	3.1713
±4.0	3.5808	±11.5	2.9908
±5.0	3.573	±12.0	2.7158
±6.0	3.5613	±12.5	2.3153
±7.0	3.5433	±13.0	1.8083

Table 4: Theoretical value of the magnetic field inside the solenoid.

5.4 Plot of Magnetic field inside the Solenoid

The black line is the theoretical value and red dots are measured value.



6 Uncertainty Analysis

6.1 Uncertainty for Relation between Sensitivity K_H and Working Voltage U_S

when $U_S = 5V$:

$$u = \sqrt{(\frac{\partial K_H}{\partial U_0} \cdot u_0)^2 + (\frac{\partial K_H}{\partial U} \cdot u_u)^2} = 7.15[V/T]$$

$$u_r = \frac{u}{K_H} \cdot 100\% = 9.68\%$$

The uncertainty for $u = \sqrt{(\frac{u_K}{u})^2 + (\frac{K}{u^2} \cdot u_u)^2} = 2.3679[1/T]$

	$U_S \pm 0.05\%[V]$	$K_H[V/T]$	$u_K[V/T]$	$u_r\%$	$u[1/T]$
1	2.8	27.84	6.63	23.81	2.3679
2	3.2	38.28	6.69	17.48	2.0906
3	3.6	57.08	6.80	11.91	1.8889
4	4.0	55.69	6.90	12.39	1.725
5	4.4	69.61	7.00	10.06	1.5909
6	4.8	69.61	7.10	10.20	1.4792
7	5.2	79.35	7.20	9.07	1.3846
8	5.6	80.75	7.30	9.04	1.3036
9	6.0	89.10	7.40	8.31	1.2334
10	6.4	100.9	7.50	7.43	1.1719
11	6.8	106.5	7.60	7.14	1.1177
12	7.2	109.3	7.70	7.04	1.0695
13	7.6	112.8	7.80	6.91	1.0263
14	8.0	113.5	7.90	6.96	0.9875
15	8.4	128.8	8.00	6.21	0.9524
16	8.8	132.3	8.11	6.13	0.9216
17	9.2	142.7	8.21	5.75	0.8924
18	9.6	142.7	8.31	5.82	0.8657
19	10.0	146.2	8.41	5.75	0.841

Table 5: Data of K_H for U_0 and U with different U_S

6.2 Uncertainty for Relation between I_M vs. U

$$B = C(0)I_M$$

$$u_B = C(0) \cdot u_I = 1.4366 \cdot 10^{-2} \cdot 50 \cdot 10^{-3} \cdot 2\% = 1.4366 \cdot 10^{-5}[T]$$

$$u_r = \frac{u_B}{B} \cdot 100\% = 2.00\%$$

$$u_v = 0.0211 \cdot 0.05\% + 6 \cdot 10^{-4} = 6.1055 \times 10^{-4}$$

$$u_r = \frac{6.1055 \times 10^{-4}}{0.0211} \cdot 100\% = 2.89\%$$

	$B[T]$	$u_B[T]$	$u_r[\%]$	$U[V]$	$u_v[V]$	$u_r[\%]$
1	0	0	0	0.0000	0	0
2	$7.184 \cdot 10^{-4}$	$1.4366 \cdot 10^{-5}$	2.00	0.0211	$6.106 \cdot 10^{-4}$	2.89
3	$1.437 \cdot 10^{-3}$	$2.8732 \cdot 10^{-5}$	2.00	0.0418	$6.209 \cdot 10^{-4}$	1.49
4	$2.155 \cdot 10^{-3}$	$4.3098 \cdot 10^{-5}$	2.00	0.0624	$6.312 \cdot 10^{-4}$	1.01
5	$2.873 \cdot 10^{-3}$	$5.7464 \cdot 10^{-5}$	2.00	0.0832	$6.416 \cdot 10^{-4}$	0.77
6	$3.592 \cdot 10^{-3}$	$7.183 \cdot 10^{-5}$	2.00	0.1039	$6.520 \cdot 10^{-4}$	0.63
7	$4.310 \cdot 10^{-3}$	$8.6196 \cdot 10^{-5}$	2.00	0.1251	$6.626 \cdot 10^{-4}$	0.53
8	$5.028 \cdot 10^{-3}$	$1.0056 \cdot 10^{-4}$	2.00	0.1453	$6.727 \cdot 10^{-4}$	0.46
9	$5.746 \cdot 10^{-3}$	$1.1493 \cdot 10^{-4}$	2.00	0.1661	$6.831 \cdot 10^{-4}$	0.41
10	$6.465 \cdot 10^{-3}$	$1.2929 \cdot 10^{-4}$	2.00	0.1871	$6.936 \cdot 10^{-4}$	0.37
11	$7.183 \cdot 10^{-3}$	$1.4366 \cdot 10^{-4}$	2.00	0.2080	$7.04 \cdot 10^{-4}$	0.34

Table 6: Uncertainty for relation between B and U

6.3 Uncertainty for Relation between B and x

$$u_B = \sqrt{(\frac{u_u}{K_H})^2 + (\frac{U \cdot u_K}{K_H^2})^2} = 1.609 \cdot 10^{-6}[T]$$

$$u_r = \frac{u_B}{B} \cdot 100\% = 0.1\%$$

7 conclusion

In this exercise, I studied Hall effect by using an Integrated Hall Probe, which can show the magnitude of the magnetic field indirectly through the Hall voltage.

First, I measure the voltage when $U_S = 5V$ and get the sensitivity of the Hall element $K_H = 73.90 \pm 7.15[V/T]$. It's relative uncertainty is 9.68% is quite big, I think it's because B is so small that the uncertainty of voltage is amplified.

Second, I calculated K_H at different U_S and plot the figure. At first, K_H/U_S increase greatly as U_S becomes bigger, and then it doesn't change very much. The uncertainty for it is also very big, I think it's the same reason in last paragraph.

Third, I measured B indirectly by measuring I_M when $x = 13cm$. I get another K_H,which is the slope of linear fit plot of U vs. B and $K_H = 28.28983 \pm 0.0277[V/T]$, which is a bit smaller than the given value of $31.25 \pm 1.25[V/T]$ from the apparatus section. The relative uncertainty is very small, I think it's due to some measurement error.

At last, I used the K_H which I have got to calculate B at different x in the solenoid by measuring the voltage. I measured more than 40 values to make my plot more accurate, and it's similar to the theoretical line. However, in my plot, my measured dots is on the left side of the theoretical line, I think it's because I didn't know the center of the solenoid when I measuring the voltage.

Due to the time limit, I didn't measure the Geomagnetic field through Hall probe but I still learnt a lot from the lab manual about the Geomagnetic field.

7.1 Suggestions and Improvement

1. I should first find out where the center of solenoid is before measuring the voltage
2. The accuracy of the measurement setup is so big that there is a high uncertainty in first part. I think it should be improved.

8 Data Sheet

Data sheet is attach to the report

9 Reference

Young, H.D., Freedman R.A. University Physics. Chapter 28,31.
Krzyzosiak,M. Lab Manual of Exercise 2.
Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Handbook-Uncertainty Analysis.