PHYSICS LABORATORY
VP141

EXERCISE 1

MEASUREMENTS OF THE MOMENT OF INERTIA

This lab manual is based on the materials provided by the Department of Physics, Shanghai Jiaotong University.

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1 Pre-lab Reading

Chapters 9 and 10 (Young and Freedman); mini-tutorial uploaded on Sakai

2 Objectives

In this exercise you will get familiar with the constant-torque method for measuring the moment of inertia of a rigid body. You will study the dependence of the moment of inertia on mass distribution and on the choice of the rotation axis. The parallel axis (Steiner's) theorem will be verified in this exercise as well. You will also develop your measurement technique skills, by learning how to measure time using a photo-gate electronic timer.

3 Theoretical Background

Moment of inertia of a rigid body about an axis is a quantitative characteristics that defines the body's resistance (inertia) to a change of angular velocity in rotation about that axis. This characteristics of the rigid body rotating about a fixed axis is determined not only by the mass of the body, but also by its distribution. The moment of inertia of a rigid body about a certain rotation axis can be calculated analytically. However, if the body has irregular shape or non-uniformly distributed mass, the calculation may be difficult. Experimental methods turn out to be more useful in such cases.

3.1 Second Law of Dynamics for Rotational Motion

According to the second law of dynamics for rotational motion about a fixed axis

$$\tau_z = I\beta_z, \tag{1}$$

relates the component of the torque τ_z about the axis of rotation with the moment of inertia about this axis, and the angular acceleration component β_c. Therefore, the moment of inertia I can be found once the torque and the resulting angular acceleration are measured.

The moment of inertia is an additive quantity, *i.e.* if the moment of inertia of a rigid body (A) about an axis is I_A and the moment of inertia of another rigid body (B) about the same axis is I_B, then the moment of inertia of the combined rigid body AB composed of A and B, about the same axis of rotation, is

$$I_{AB} = I_A + I_B.$$

3.2 Parallel Axis Theorem

If the moment of inertia of a rigid body with mass m about an axis *through the body's center of mass* is I_0, then for any axis *parallel* to that axis, the moment of inertia is

$$I = I_0 + md^2, \tag{2}$$

where d is the distance between the axes. This result is known as the parallel axis theorem or Steiner's theorem.

3.3 Measurement Setup and Measurement Method

The measurement setup is shown in Figure 1. It consists of a turntable with an integrated photo-gate system used for time measurements.

In order to understand the idea of the measurement method, suppose that the empty turntable is initially rotating and its moment of inertia with respect to the rotation axis is I_1. Since the bearings of the turntable are not frictionless, there will be a non-zero frictional torque M_μ causing the turntable to decelerate with angular acceleration β_1, so that the second law of dynamics for rotational motion of the empty turntable reads (furthermore we will skip the subscript z)

$$M_\mu = -I_1\beta_1. \tag{3}$$

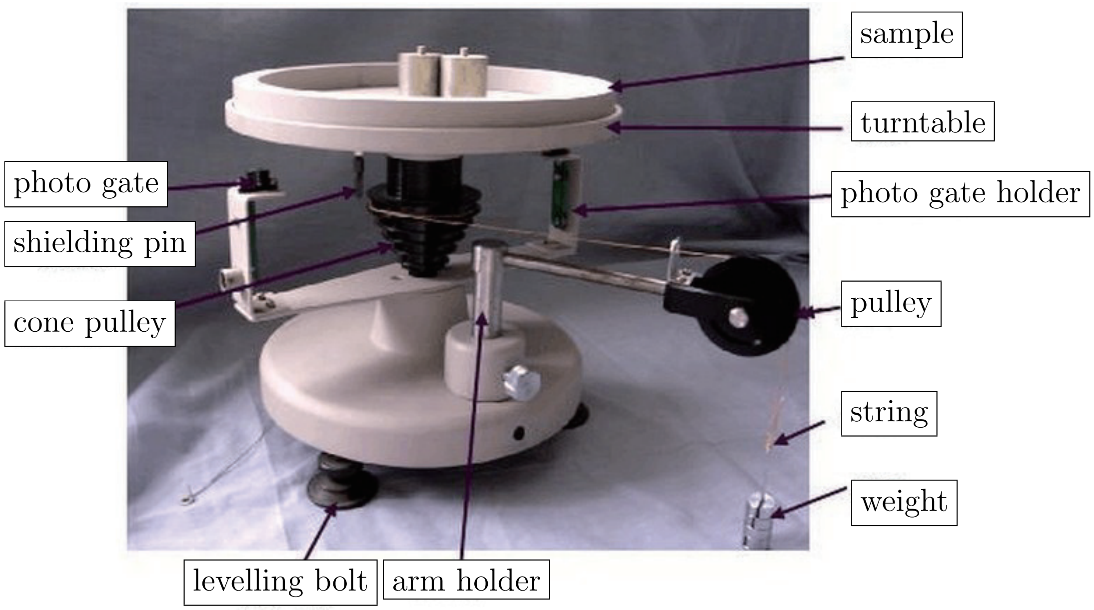

Figure 1. Measurement setup.

Below the turntable, there is a conical pulley of radius R, with a light and inextensible string wound on it. The axis of the cone coincides with the axis of rotation. Attached to the other end of the string passing through a disk pulley, there is a weight with mass m, free to move downwards after it is released. If the mass moves downwards with constant acceleration a, the tension in the string T is constant and $T = m(g - a)$. This constant tension force, on the other end of the string, provides constant torque on the cone pulley that in turn makes the turntable rotate the with constant angular acceleration. If the turntable rotates with angular acceleration β_2, then $a = R\beta_2$ (we assume that the string does not slip on the pulleys). The torque the string exerts on the pulley (and hence the turntable) is then $TR = m(g - R\beta_2)R$. Consequently, taking into account the frictional torque, the net torque on the turntable is $TR - M_\mu$, and the equation of motion for the turntable reads

$$m(g - R\beta_2)R - M_\mu = I_1\beta_2. \tag{4}$$

Eliminating M_μ from Eqs. (3) and (4), we find

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1}. \tag{5}$$

Similarly, if a rigid body with an unknown moment of inertia is placed on the turntable, we may find

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3}, \tag{6}$$

where β_3 is the magnitude of angular deceleration of the turntable with the body, and β_3 is its angular acceleration, when the mass m is released and moves downwards.

Using the fact that the moment of inertia is an additive quantity, the moment of inertia of the rigid object placed on the turntable, with respect to the axis of rotation, may be found as the difference

$$I_3 = I_2 - I_1. \tag{7}$$

3.4 Measurement of Angular Acceleration

At the edge of the turntable two shielding pins are fixed. As the turntable rotates, the pins generate a series of signals in the photo-gate with the phase interval of π. An integrated counter-type electronic timer is used to measure the consecutive number k and the time t of the photo-gate signal.

If (k,t) is a set of the measurement data, the corresponding angular position is

$$\theta = k\pi = \omega_0 t + \frac{1}{2}\beta t^2$$

where ω_0 is the initial angular speed. Now, by performing a quadratic fit to the measurement data, we can find the angular acceleration.

4 Measurement Procedure

1. Measure the mass of the weight, the hoop, the disk, and the cylinder, as well as the radius of the cone pulley and the cylinder (follow the instructor's requirements). Calculate the moment of inertia of the hoop and the disk analytically.

   Use a reliable source to find the local value of the acceleration due to gravity in Shanghai.

2. Turn the electronic timer on and switch it to mode 1-2 (single gate, multiple pulses).

3. Place the instrument close to the edge of the desk and stretch the disk pulley arm outside, so that the weight can move downwards unobstructed.

4. Level the turntable with the bubble level.

5. Make the turntable rotating and press the start button on the timer. After at least 8 signals are recorded, stop the turntable and record the data in your data sheet.

6. Attach the weight to one end of the string. Place the string on the disk pulley, thread through the hole in the arm, and wind the string around the 3rd ring of the cone pulley. Adjust the arm holder so that the string goes through the center of the hole.

7. Release the weight and start the timer. Stop the turntable when the weight hits the floor. Write down the recorded data.

8. The angular acceleration can be found by plotting $\theta = k\pi$ against t and performing a quadratic fit using data processing software. (The magnitude of the angular acceleration is equal to the coefficient next to t^2 multiplied by two. The uncertainty of the angular acceleration can be read directly from the fitting result.)

   The moment of inertia of the empty turntable is found by using the formulae in Section 3 and the data from step 5 and 7. Repeat steps 5 $\sim$ 7 with a rigid object placed on the turntable. Eq. (7) is used to find the moment of inertia of the rigid object.

In the experiment, each student should measure the moment of inertia of the empty turntable, the hoop, the disk and the cylinder, as well as verify the parallel-axis theorem by placing two cylinders off the axis while keeping their center of mass on the axis in order to keep the rotation steady.

The distances between the holes and the center of the turntable are ca. 45, 60, 75, 90, 105 mm, respectively. The timer's resolution is 0.0001 s, and the error is 0.004%.

5 Caution

- Stop the turntable as soon as the weight hits the ground in order to avoid the string on the cone pulley getting stuck.

- Press the timer start button after releasing the weight to ensure that the weight moves with constant acceleration during the measurement.

- Use the 3rd (the middle one) ring of the cone pulley so that sets of at least 8 data in one measurement can be recorded.

- The string must not overlap while being wound on the cone pulley.

- In each measurement the measurement uncertainty should be written down along with the recorded value. The uncertainty analysis should be included in the report.

6 Preview Questions

- Explain what a rigid body is.
- State the parallel axis theorem.
- When the torque of a force with respect to an axis is zero?
- Why should the turntable be levelled before measurements start?
- The method of calculating the angular acceleration from the experimental data that is used in this manual is called the successive differences method. Can you design another method to obtain the angular acceleration from the data? What are the advantages and the disadvantages of your method compared to the successive differences method?
- In order to verify Steiner's theorem, we use at least two cylinders placed at some distance from the center of the turntable, and keep their center of mass at the rotation axis. Why not use one cylinder only, or not keep their center of mass on the axis?
- Estimate the measurement range of this setup. What setup should be used if the moment of inertia of a sample is beyond this range?
- Discuss other uncertainty factors that are not included in this manual and comment on their contribution to the uncertainty of the final result.