VP390 HW1

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1. According to time dilation, let lab be S and rocket be S', then

$$t=\frac{t'}{\sqrt{1-v^2/c^2}}=\frac{10}{\sqrt{1-0.8^2}}=16.67\,(s)$$

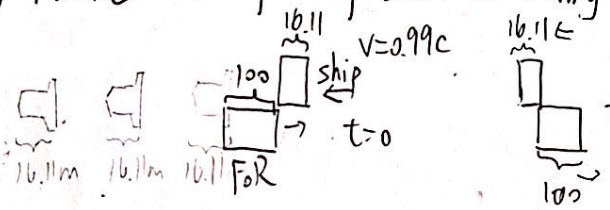
2. (a) Let earth be S and the spaceship be S', then

$$L=L_0\sqrt{1-\frac{V^2}{c^2}}=100\sqrt{1-0.85^2}=52.68(m)$$

(b) $$u_x'=\frac{u_x-V}{1-Vu_x/c^2}=\frac{0.65c+0.85c}{1+0.85^2}=0.99c$$

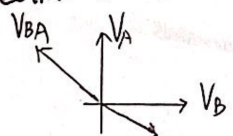
(c) $$L=L_0\sqrt{1-\frac{u_x'^2}{c^2}}=100\sqrt{1-0.99^2}=16.11\,(m)$$ (d) $$t=\frac{52.68}{0.85c}=2.07\times10^{-7}(s)$$

(e) Assume the spaceship seen is moving from left to right



$$t=\frac{116.11}{0.99c}=3.9\times10^{-7}(s).$$

3. (a) Let B be S', let the direction of V_B be X-axis positive direction, and V_A's direction be y-axis



Assume $V_{BA}=V_{BAx}+V_{BAy}$, where V_{BAx} is along X-axis, V_{BAy} is along y-axis

Then $V_{BAx}=\frac{0-V_B}{1-0}=-V_B$ $\quad V_{BAy}=\frac{V_A}{1-0}\sqrt{1-\frac{V_B^2}{c^2}}=V_A\sqrt{1-\frac{V_B^2}{c^2}}$

The angle with X-axis is $\pi-\arctan\frac{V_A\sqrt{1-V_B^2/c^2}}{V_B}$, $|V_{BA}|=\sqrt{V_A^2+V_B^2-\frac{V_A^2V_B^2}{c^2}}$

(b) Similarly to last question, $V_{ABy}=-V_A$, $V_{ABx}=V_B\sqrt{1-\frac{V_A^2}{c^2}}$

So $|V_{AB}|=|V_{BA}|=\sqrt{V_A^2+V_B^2-\frac{V_A^2V_B^2}{c^2}}$, angle is $-\arctan\frac{V_A}{V_B\sqrt{1-V_A^2/c^2}}$

(c) Since V_B and V_A have different direction, so their velocity both change differently for both observers. As a result, the angle is also different

4. (a) Assume it takes t seconds for the star to travel from A to B

then $\Delta t = t - \frac{Vt\cos\theta}{c}$ $\Delta h = Vt\sin\theta$

$$V_A = \frac{\Delta h}{\Delta t} = \frac{cV\sin\theta}{c - V\cos\theta}$$

(b) Let $V_A' = 0$, we can get $\theta = \pm\arccos\theta$

(c) Let $\theta = \arccos\theta$, then $V_A = \frac{cV\sqrt{1-\frac{V^2}{c^2}}}{c - V\cdot\frac{V}{c}} = \frac{cV}{\sqrt{c^2-V^2}}$

Since $\frac{V}{\sqrt{c^2-V^2}}$ can be larger than 1, V_A can be larger than c

For example, when $V > \frac{\sqrt{2}}{2}c$. $V_A > C$

5. (a) $a_x' = \frac{du_x'}{dt'} = \frac{\frac{du_x'}{du_x}\cdot\frac{du_x}{dt}}{\frac{dt'}{dt}}$ $\frac{du_x}{dt} = a_x$ $\frac{du_x'}{du_x} = \frac{c^2(c^2-v^2)}{(c^2-uv)^2}$ $\frac{dt'}{dt} = \frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{uv}{c^2}} \Rightarrow a_x' = \frac{a_x}{(1-\frac{vu_x}{c^2})^3}(1-\frac{v^2}{c^2}$

when $\frac{v}{c} \to 0$, $a_x' \to a_x$

(b) $a_y' = \frac{du_y'}{dt'} = \frac{\frac{du_y'}{du_y}\cdot\frac{du_y}{dt} + \frac{du_y'}{du_x}\frac{du_x}{dt}}{\frac{dt'}{dt}}$ $\frac{du_y}{dt} = a_y$ $\frac{du_y'}{du_y} = \frac{\sqrt{1-v^2/c^2}}{1-vu_x/c^2}$ $\frac{du_y'}{du_x} = \frac{cv\sqrt{c^2-v^2}\,u_y}{(c^2-vu_x)^2}$

Then $a_y' = \frac{a_y(1-vu_x/c^2) + a_x v u_y/c^2}{(1-vu_x/c^2)^3}(1-v^2/c^2)$ similarly $a_z' = \frac{a_z(1-vu_x/c^2) + a_x v u_z/c^2}{(1-vu_x/c^2)^3}(1-v^2/c^2)$

when $\frac{v}{c} \to 0$, $a_y' \to a_y$ $a_z' \to a_z$, so it will result in Galilean result when $v \ll c$

6. The distance between two events is 1.3×10^5 m

The time interval between them is ~~32832000s~~ 1.2×10^9 s

When we want them happen at the same place, $x_2' - x_1' = 0$

So $\frac{x_1 - vt_1}{\sqrt{1-v^2/c^2}} = \frac{x_2 - vt_2}{\sqrt{1-v^2/c^2}} \Rightarrow v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{1.3\times10^5}{1.2\times10^9}$ ~~= 0.04 (m/s)~~ 1.1×10^{-4} (m/s)

As a result, for all FoR moving along the line ~~across~~ across where the two events happen at ~~0.4 m/s~~ 1.1×10^{-4} (m/s), they'll happen at the same place.

(b) Similarly, $t_B' = t_A'$ so that $t_A - \frac{v x_A}{c^2} = t_B - \frac{v x_B}{c^2}$

$$v = \frac{c^2 (t_A - t_B)}{x_A - x_B} = \frac{(3\times10^8)^2 \times 3283200}{1.3\times10^5} = 2.3\times10^{18} > c$$

As a result, there doesn't exist such FoR

7.

Δs^2	A	B	C	D
A	0	-24	20	36
B	-24	0	24	0
C	20	24	0	-16
D	36	0	-16	0

For AC, AD, BC, they're time-like, so there may be a causal relation between.

For AB, CD, they're space-like, so they must have no casual relation.

For BD, it's $\Delta s^2 = 0$, so it's light-like, and they happen at same time without causal relation