VP390 RECITATION CLASS

FOUR

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Wave function

In quantum mechanics the state of a particle is represented by a **wave-function**. The wave function is in general complex–valued. For the wave function in the position representation in 1D and in 3D, respectively:

$$\psi(x,t) \qquad\qquad \psi(\bar{r},t)$$

How to interpretate the wave function?

The square of the magnitude of the wave function represents the **probability** of finding the particle at some position in 1D or 3D at some instant of time.

Max Born
(1882-1970)

Wave function

Functions satisfying the following condition are said to be **normalized**:

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 \, dx = 1 \qquad \iiint_{R^3} |\psi(\bar{r},t)|^2 \, d^3 r = 1$$

Functions satisfying the following condition are said to be **square-integrable**:

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 \, dx < \infty \qquad \iiint_{R^3} |\psi(\bar{r},t)|^2 \, d^3 r < \infty$$

Wave function

average position of the particle:

$$\langle x\rangle_\psi = \int_{-\infty}^{\infty} x|\psi(x,t)|^2 dx = \int_{-\infty}^{\infty} \psi^*(x,t)\,x\,\psi(x,t)dx$$

variance of the position of the particle:

$$(\Delta_x^{(\psi)})^2 = \left\langle (x-\langle x\rangle_\psi)^2 \right\rangle_\psi = \left\langle x^2 - 2x\langle x\rangle_\psi + \langle x\rangle_\psi^2 \right\rangle_\psi$$

$$= \langle x^2\rangle_\psi - 2\langle x\rangle_\psi \langle x\rangle_\psi + \langle x\rangle_\psi^2 = \langle x^2\rangle_\psi - \langle x\rangle_\psi^2$$

standard deviation of the position of the particle:

$$\Delta_x^{(\psi)} = \sqrt{\langle x^2\rangle_\psi - \langle x\rangle_\psi^2}$$

Stationary state

If the modulus square does not depend on time, then the state represented by Ψ is called a **stationary state**:

$$\left|\psi(x,t)\right|^2 = \phi(x)$$

For a stationary state, the **avearge** and the **standard deviation** of the particle is **time-independent**.

Example: $\psi(x,t) = e^{-\frac{i}{h}Et} f(x)$

Inner product

In the linear space formed by the square-integrable functions, the inner product can be introduced as:

$$\langle \psi, \phi \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\phi(x,t)dx$$

For such inner products, they have the following properties:

$$\langle \psi, \phi_1 + \phi_2 \rangle = \langle \psi, \phi_1 \rangle + \langle \psi, \phi_2 \rangle$$

$$\langle \psi, \alpha\phi \rangle = \alpha \langle \psi, \phi \rangle$$

$$\langle \psi, \phi \rangle = \langle \phi, \psi \rangle^*$$

$$\langle \psi, \psi \rangle \geq 0; \quad \langle \psi, \psi \rangle = 0 \Leftrightarrow \psi \equiv 0$$

Hermitian operators

In quantum mechanics, physical quantities are represented by **Hermitian operators**.

What are operators?

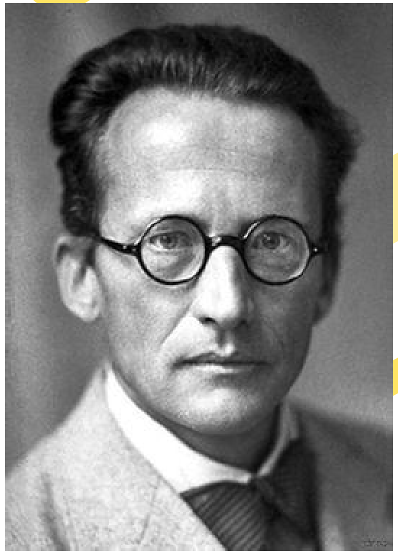
What kind of operators are Hermitian?

Are the following operators Hermitian?

$$\hat{x},\quad \hat{p},\quad \hat{K},\quad \hat{V},\quad \hat{H}$$

Schrödinger equation

$$
-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)+V(x)\psi(x,t)=i\hbar\frac{\partial}{\partial t}\psi(x,t)
$$



Erwin Schrödinger
(1887-1961)

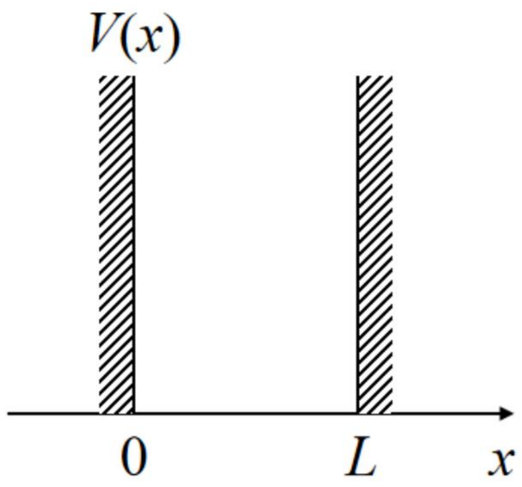
Schrödinger equation is a linear 2nd order differential equation. What does its linearity imply?

stationary Schrödinger equation:

$$
-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x)+V(x)\varphi(x)=E\varphi(x)
$$

How is stationary Schrödinger equation derived from the time-dependent one?

Infinite Potential Well



Potential distribution:

$$V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$$

Stationary Schrödinger equation:

$$\begin{cases} -\dfrac{\hbar^2}{2m}\dfrac{d^2\varphi(x)}{dx^2} = E\varphi(x) \\ \varphi(0) = \varphi(L) = 0 \end{cases}$$

What kind of problem is it and how to solve it?

Exercise 1

An electron moving in a thin metal wire is a reasonable approximation of a particle in a one-dimensional infinite well. The potential inside the wire is constant on average but rises sharply at each end. Suppose the electron is in a wire $1.0\ \mathrm{cm}$ long.

(a). Compute the ground-state energy for the electron.

(b). If the electron's energy is equal to the average kinetic energy of the molecules in a gas at T=$300\ \mathrm{K}$, about $0.03\ \mathrm{eV}$, what is the electron's quantum number n?

Exercise 2

Suppose that the electron in Exercise 1 could be "seen" while in its ground state.
(a) What would be the probability of finding it somewhere in the region $0<x<L/4$?
(b) What would be the probability of finding it in a very narrow region $\Delta x<0.01L$ wide centered at $x = 5L/8$?

Thank you very much!!!