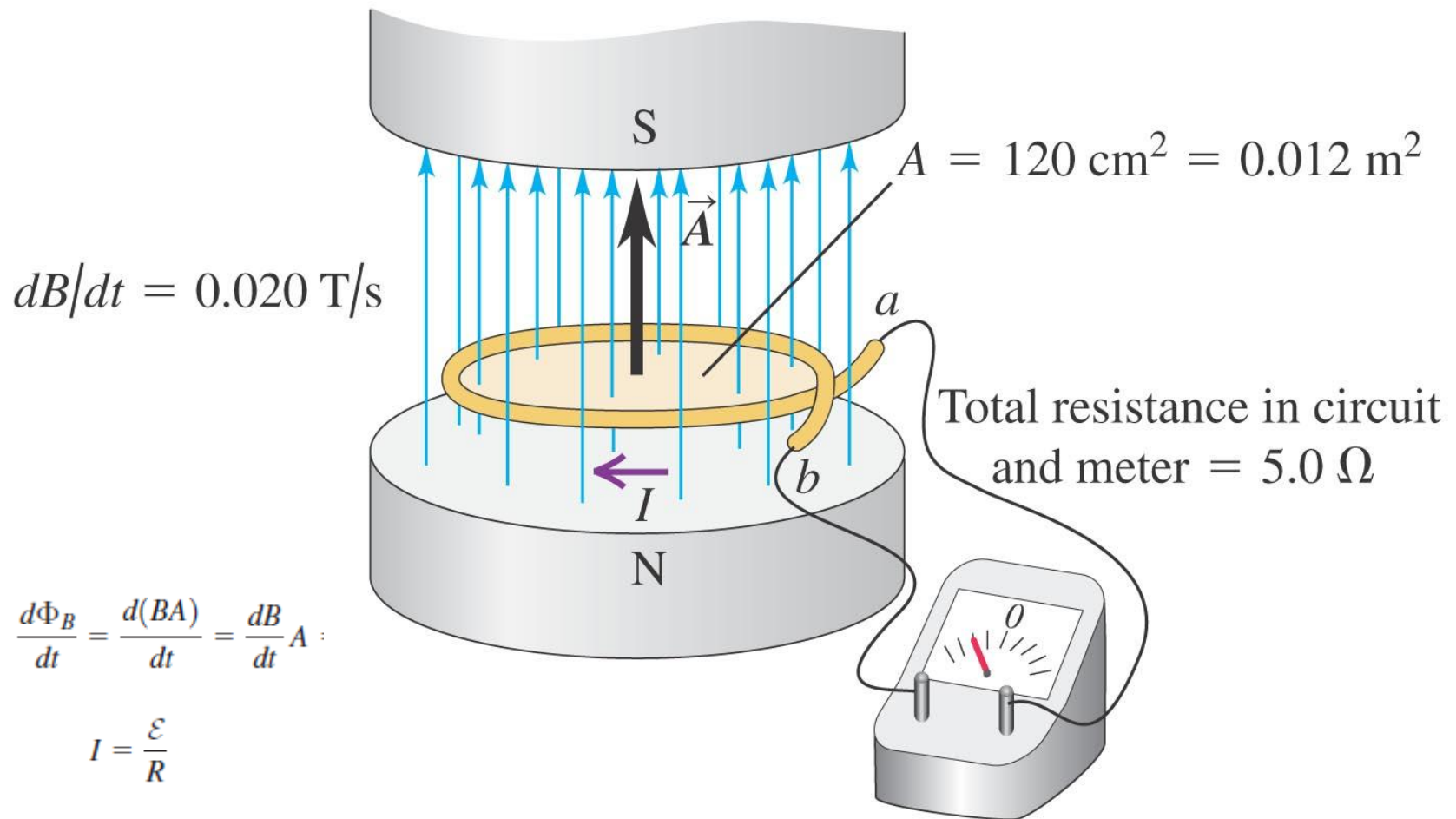

Chapter 29

Electromagnetic Induction

Emf and the current induced in a loop

- Follow Example 29.1 using Figure 29.5 below.



Goals for Chapter 29

- To examine experimental evidence that a changing magnetic field induces an emf
- To learn how Faraday's law relates the induced emf to the change in flux
- To determine the direction of an induced emf
- To calculate the emf induced by a moving conductor
- To learn how a changing magnetic flux generates an electric field
- To study the four fundamental equations that describe electricity and magnetism

Introduction

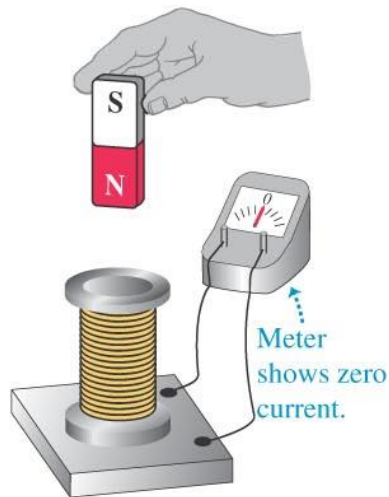
- How is a credit card reader related to magnetism?
- Energy conversion makes use of electromagnetic induction.
- Faraday's law and Lenz's law tell us about induced currents.
- Maxwell's equations describe the behavior of electric and magnetic fields in *any* situation.



Induced current

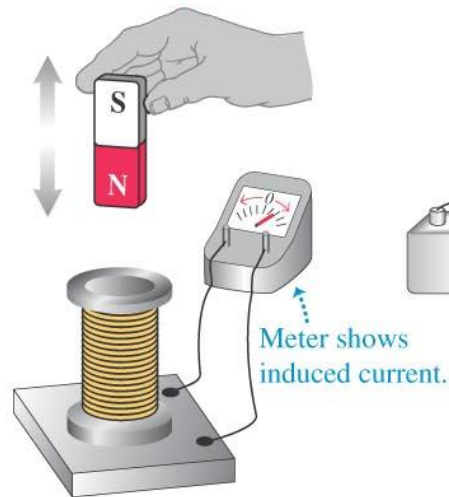
- A changing magnetic flux causes an *induced current*. See Figure 29.1 below.
- The *induced emf* is the corresponding emf causing the current.

(a) A stationary magnet does NOT induce a current in a coil.

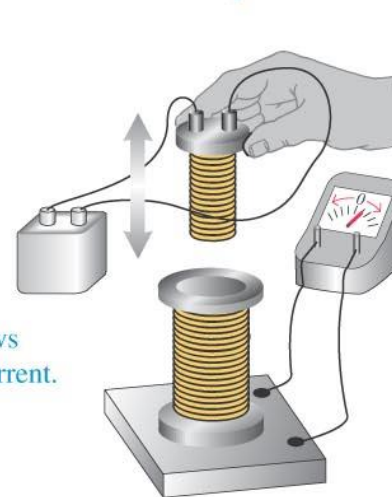


All these actions DO induce a current in the coil. What do they have in common?*

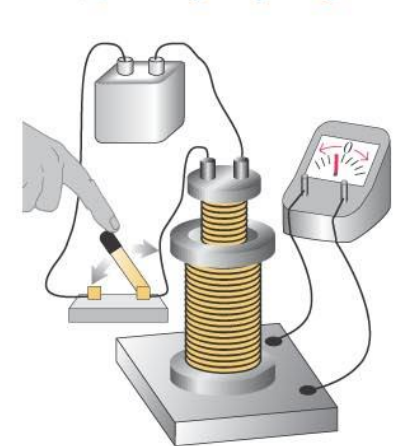
(b) Moving the magnet toward or away from the coil



(c) Moving a second, current-carrying coil toward or away from the coil

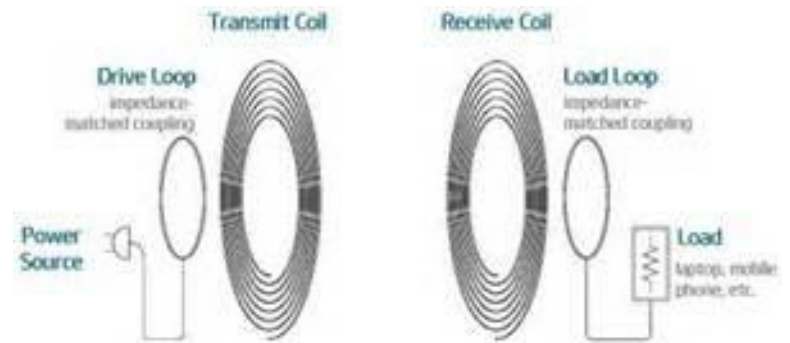


(d) Varying the current in the second coil (by closing or opening a switch)



*They cause the magnetic field through the coil to *change*.

Induced current

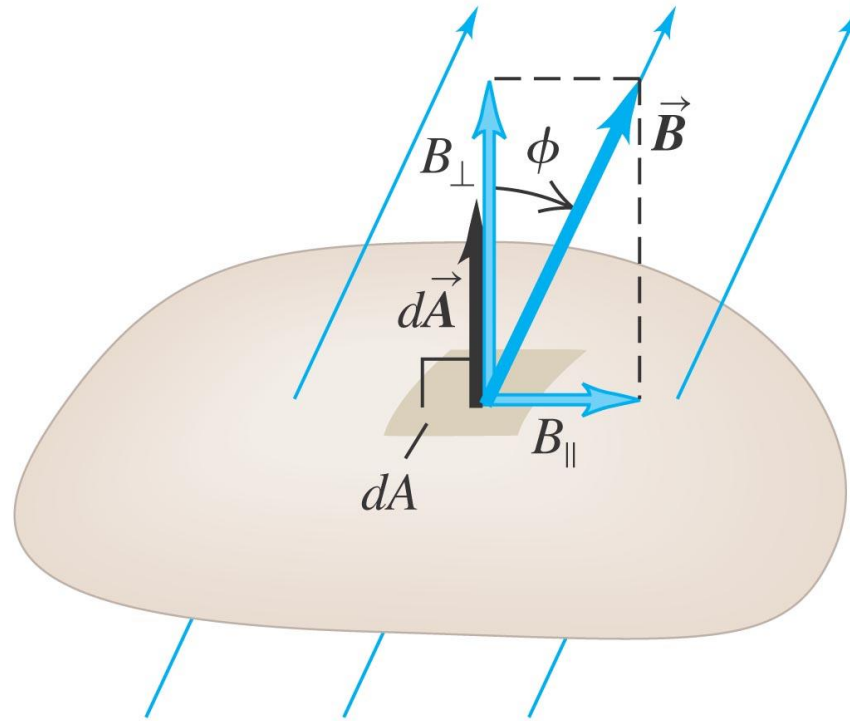


Induced current



Magnetic flux through an area element

- Figure 29.3 below shows how to calculate the magnetic flux through an element of area.



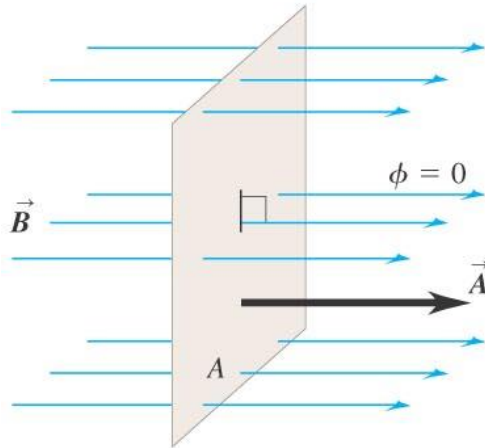
Magnetic flux through element of area $d\vec{A}$:
$$d\Phi_B = \vec{B} \cdot d\vec{A} = B_{\perp} dA = B dA \cos \phi$$

Faraday's law

- The flux depends on the orientation of the surface with respect to the magnetic field. See Figure 29.4 below.

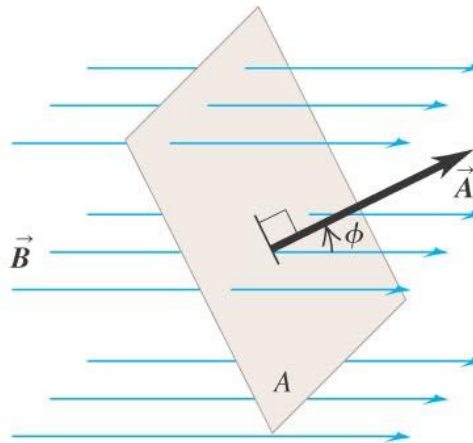
Surface is face-on to magnetic field:

- \vec{B} and \vec{A} are parallel (the angle between \vec{B} and \vec{A} is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.



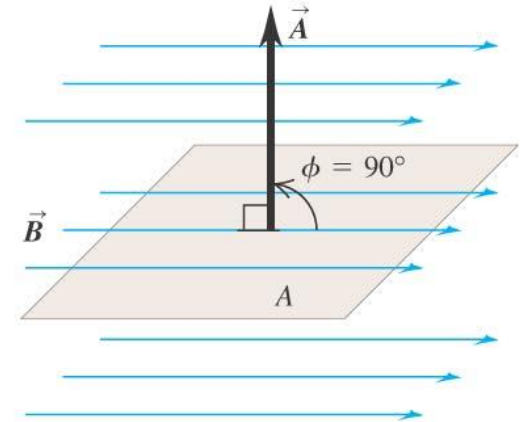
Surface is tilted from a face-on orientation by an angle ϕ :

- The angle between \vec{B} and \vec{A} is ϕ .
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi$.



Surface is edge-on to magnetic field:

- \vec{B} and \vec{A} are perpendicular (the angle between \vec{B} and \vec{A} is $\phi = 90^\circ$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos 90^\circ = 0$.



Faraday's law of induction states:

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

In symbols, Faraday's law is

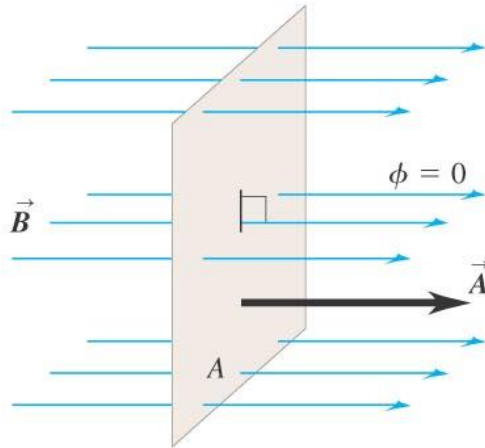
$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law of induction}) \quad (29.3)$$

Faraday's law

- The flux depends on the orientation of the surface with respect to the magnetic field. See Figure 29.4 below.

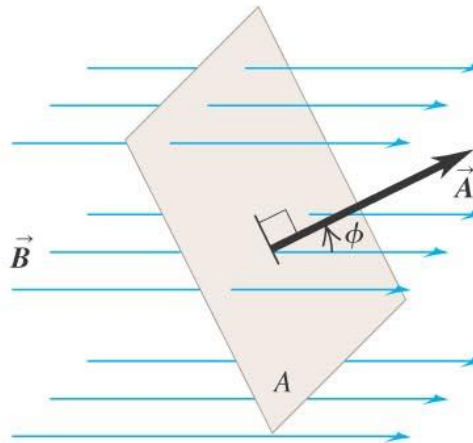
Surface is face-on to magnetic field:

- \vec{B} and \vec{A} are parallel (the angle between \vec{B} and \vec{A} is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.



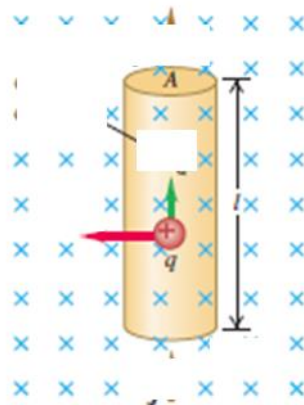
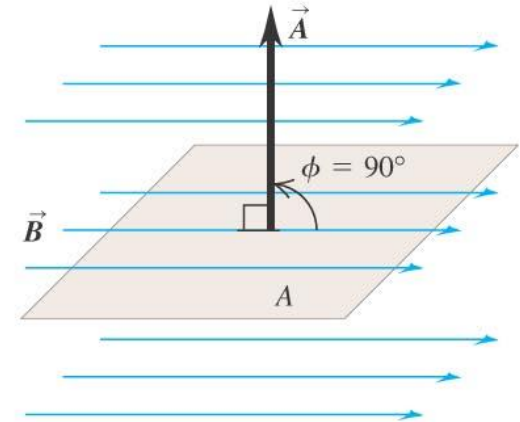
Surface is tilted from a face-on orientation by an angle ϕ :

- The angle between \vec{B} and \vec{A} is ϕ .
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi$.



Surface is edge-on to magnetic field:

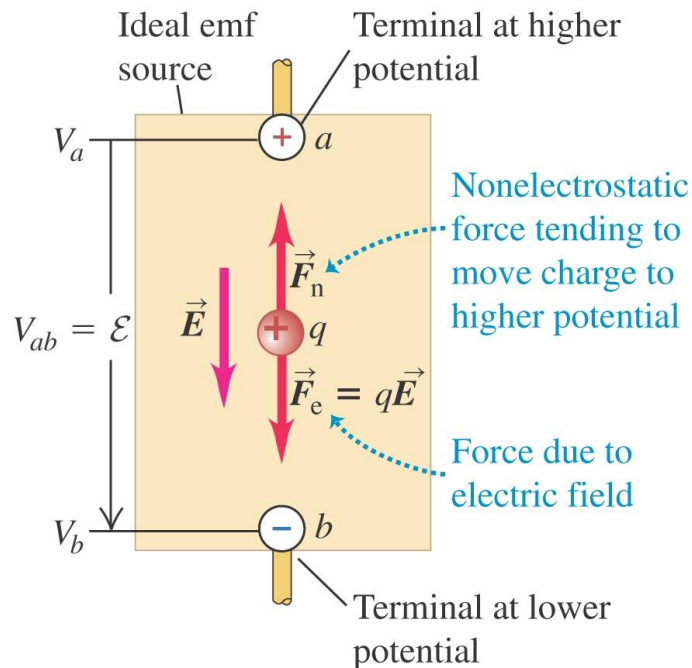
- \vec{B} and \vec{A} are perpendicular (the angle between \vec{B} and \vec{A} is $\phi = 90^\circ$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos 90^\circ = 0$.



$$\vec{F} = q\vec{v} \times \vec{B}$$

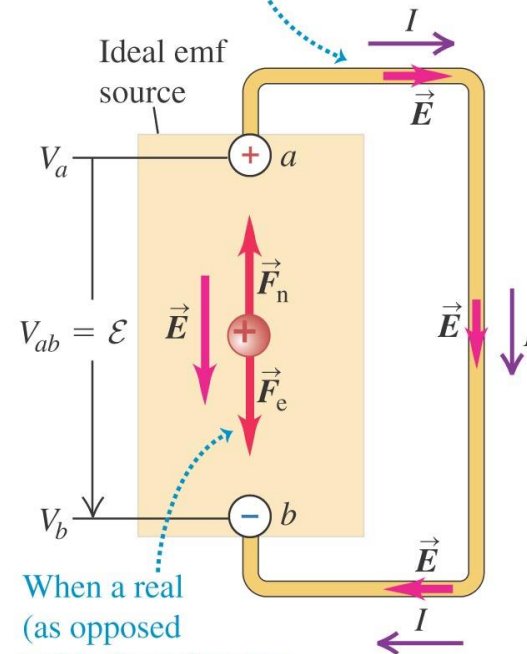
Electromotive force and circuits

- An *electromotive force (emf)* makes current flow. In spite of the name, an emf is *not* a force.
- The figures below show a source of emf in an open circuit (left) and in a complete circuit (right).



When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

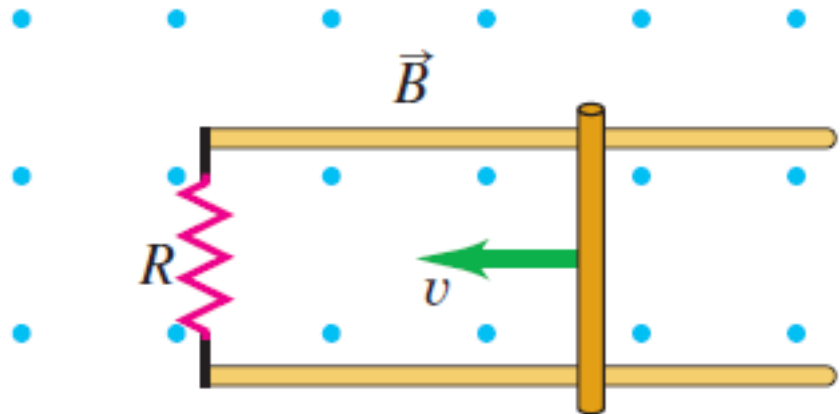
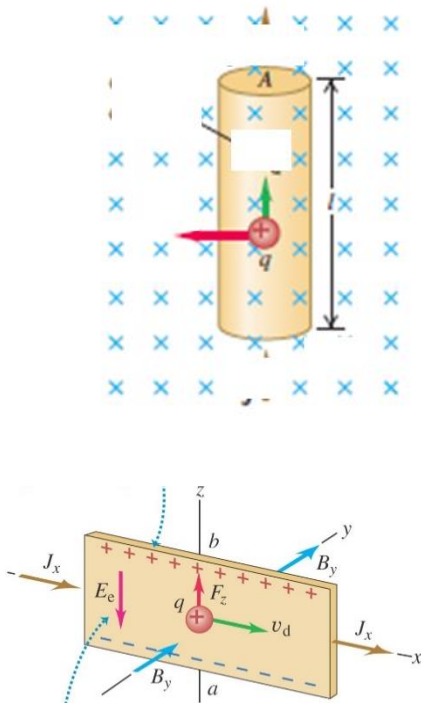
Potential across terminals creates electric field in circuit, causing charges to move.



When a real (as opposed to ideal) emf source is connected to a circuit, V_{ab} and thus F_e fall, so that $F_n > F_e$ and \vec{F}_n does work on the charges.

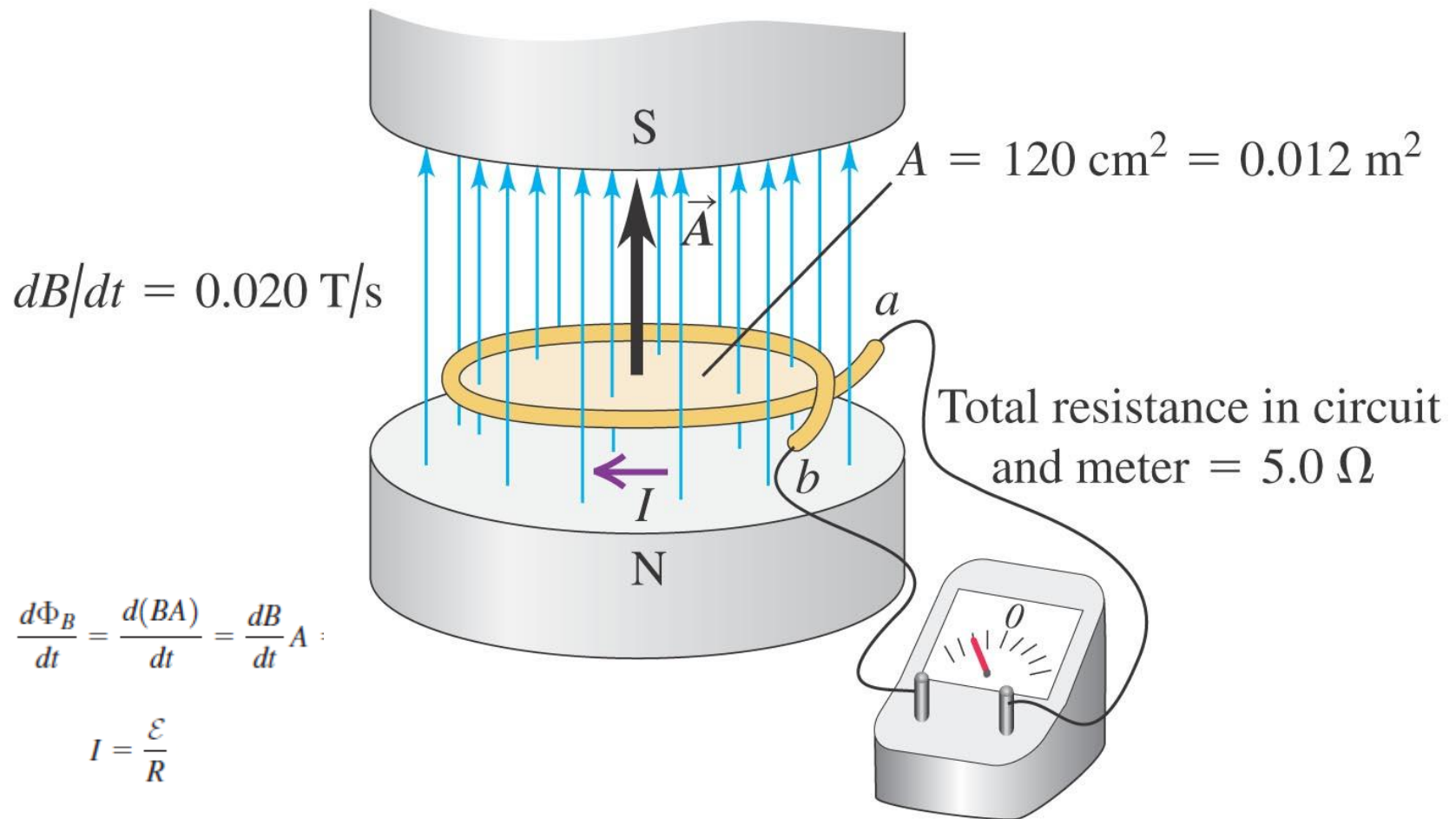
Electromotive force and circuits

- An *electromotive force* (emf) makes current flow. In spite of the name, an emf is *not* a force.
- The figures below show a source of emf in an open circuit (left) and in a complete circuit (right).



Emf and the current induced in a loop

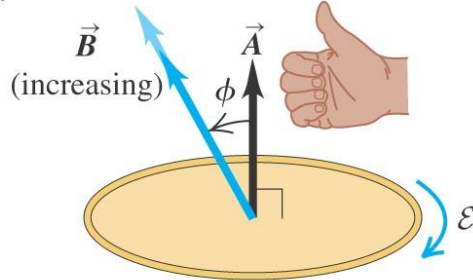
- Follow Example 29.1 using Figure 29.5 below.



Direction of the induced emf

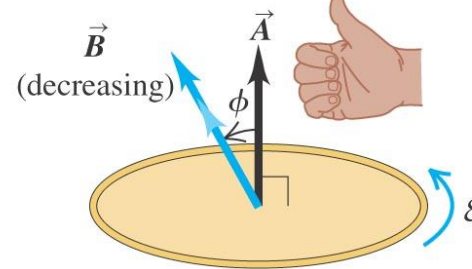
- Follow the text discussion on the direction of the induced emf, using Figure 29.6 below.

(a)



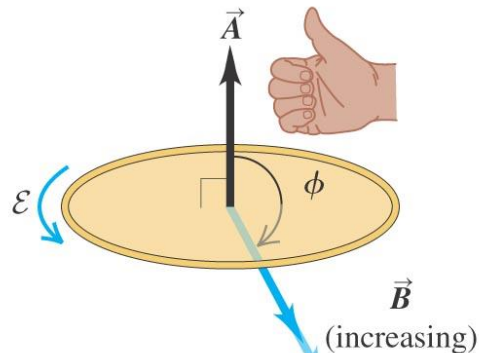
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

(b)



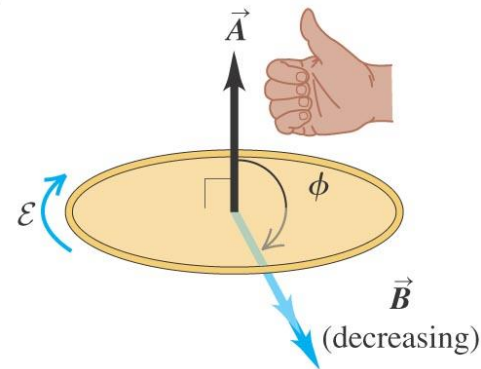
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

(c)



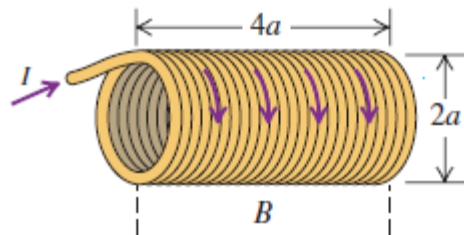
- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

(d)



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

induced emf

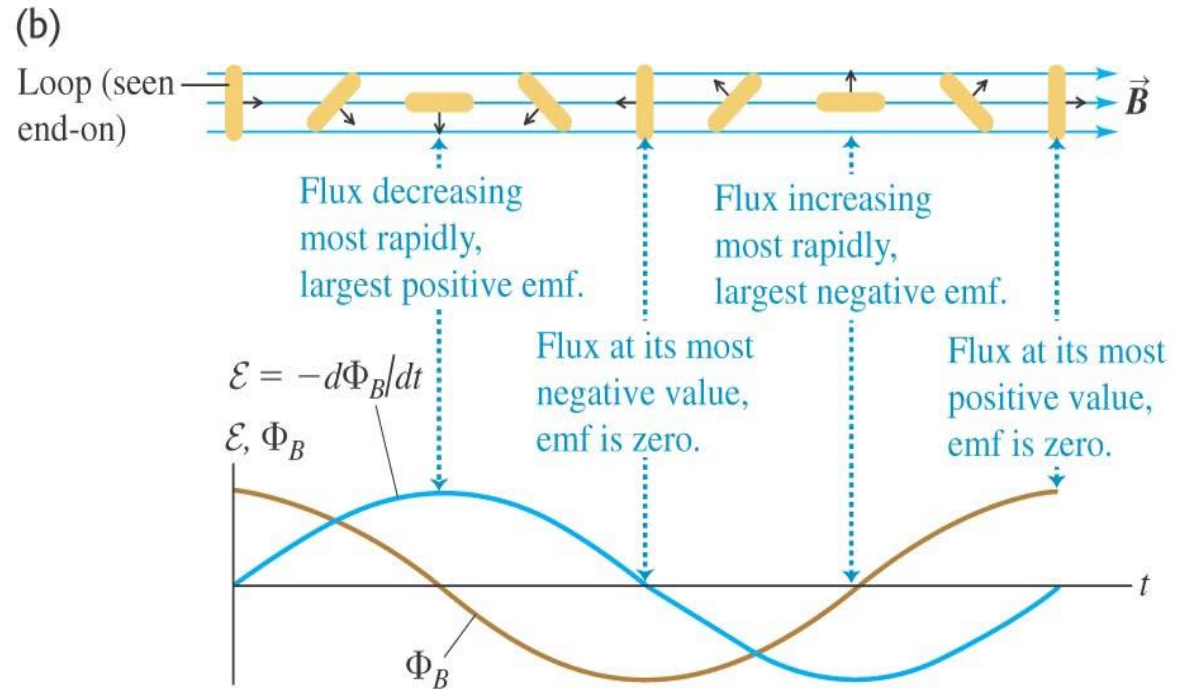
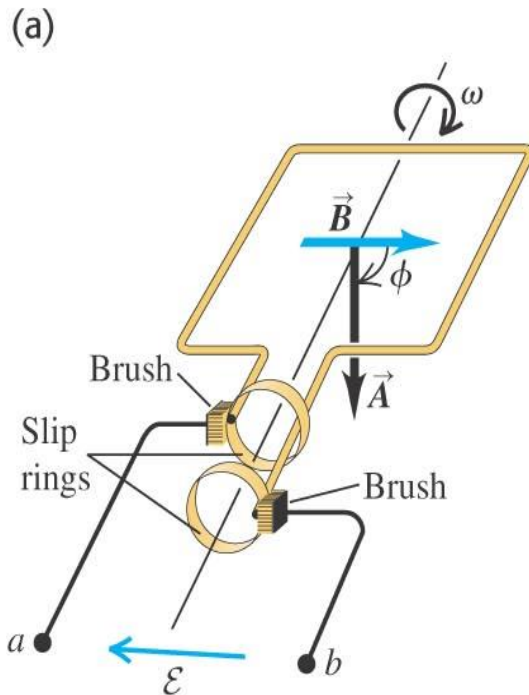
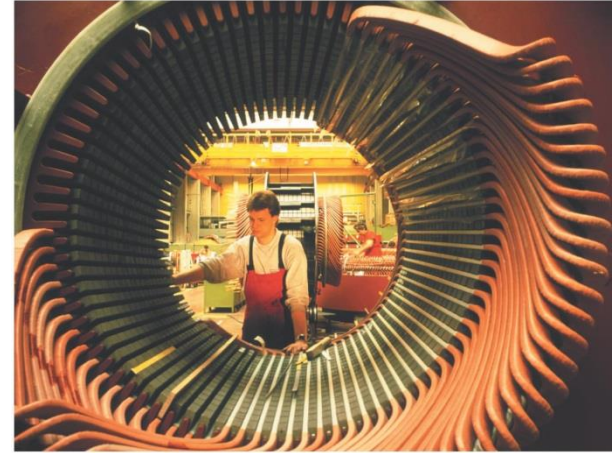


$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

A simple alternator

- Follow Example 29.3 using Figures 29.8 (below) and 29.9 (right).

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA \cos \omega t) = \omega BA \sin \omega t$$

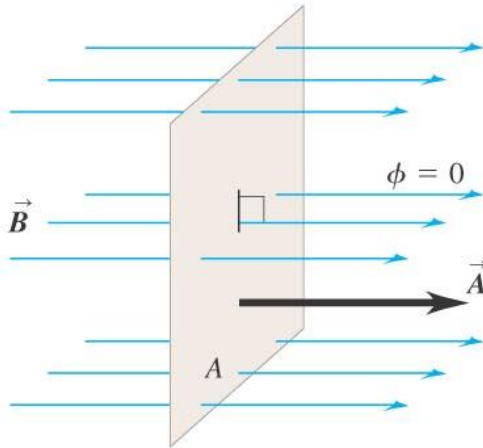


Faraday's law

- The flux depends on the orientation of the surface with respect to the magnetic field. See Figure 29.4 below.

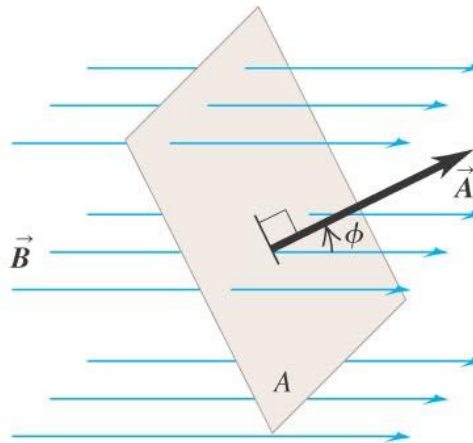
Surface is face-on to magnetic field:

- \vec{B} and \vec{A} are parallel (the angle between \vec{B} and \vec{A} is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.



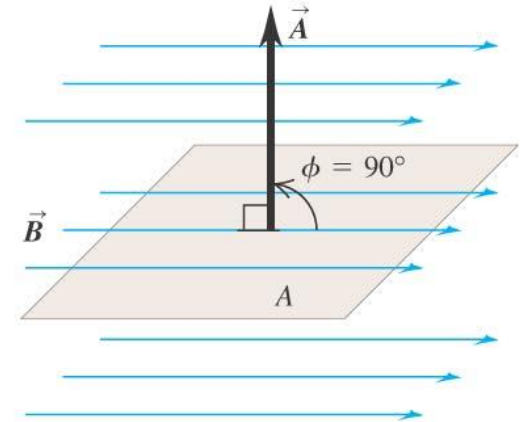
Surface is tilted from a face-on orientation by an angle ϕ :

- The angle between \vec{B} and \vec{A} is ϕ .
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi$.



Surface is edge-on to magnetic field:

- \vec{B} and \vec{A} are perpendicular (the angle between \vec{B} and \vec{A} is $\phi = 90^\circ$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos 90^\circ = 0$.



Faraday's law of induction states:

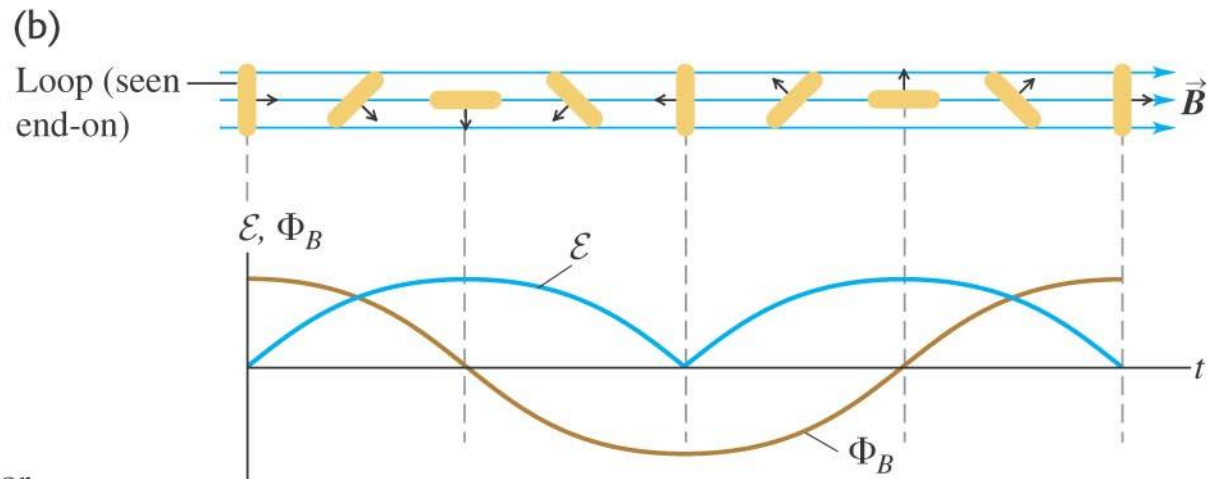
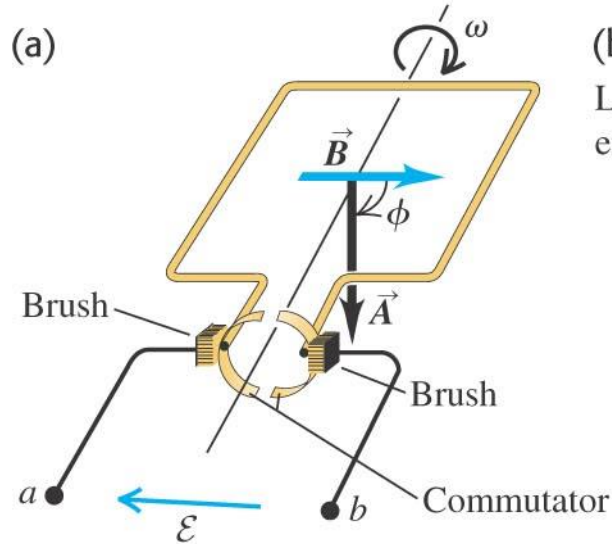
The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

In symbols, Faraday's law is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law of induction}) \quad (29.3)$$

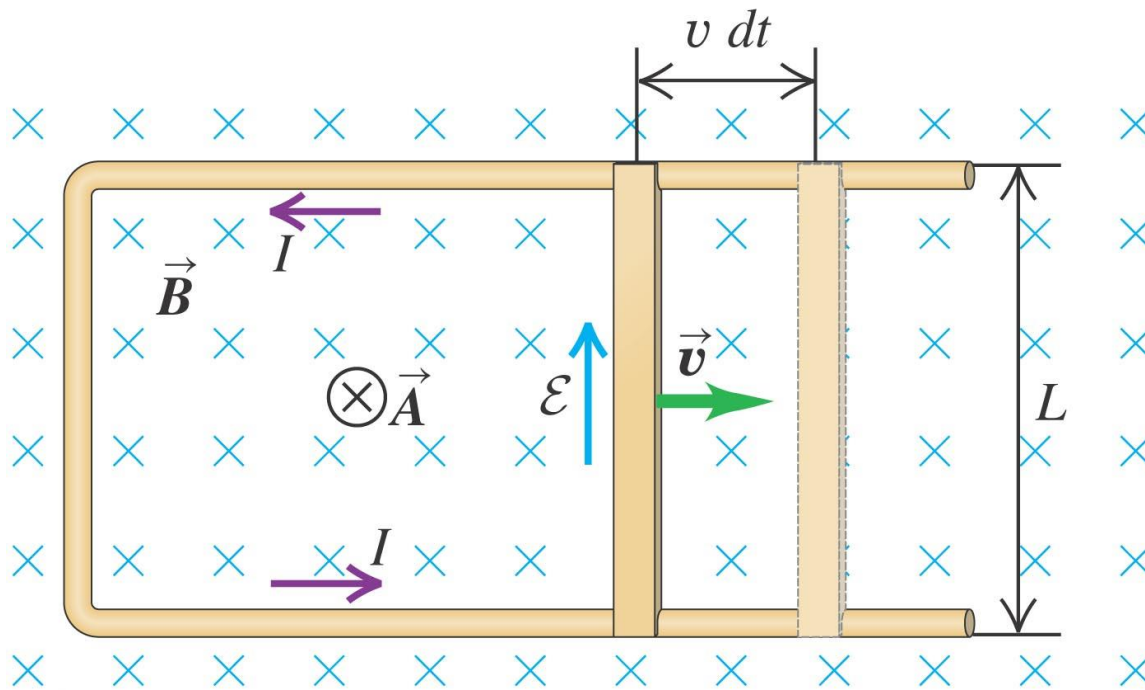
DC generator and back emf in a motor

- Follow Example 29.4 using Figure 29.10 below.



Slidewire generator

- Follow Example 29.5 using Figure 29.11 below.

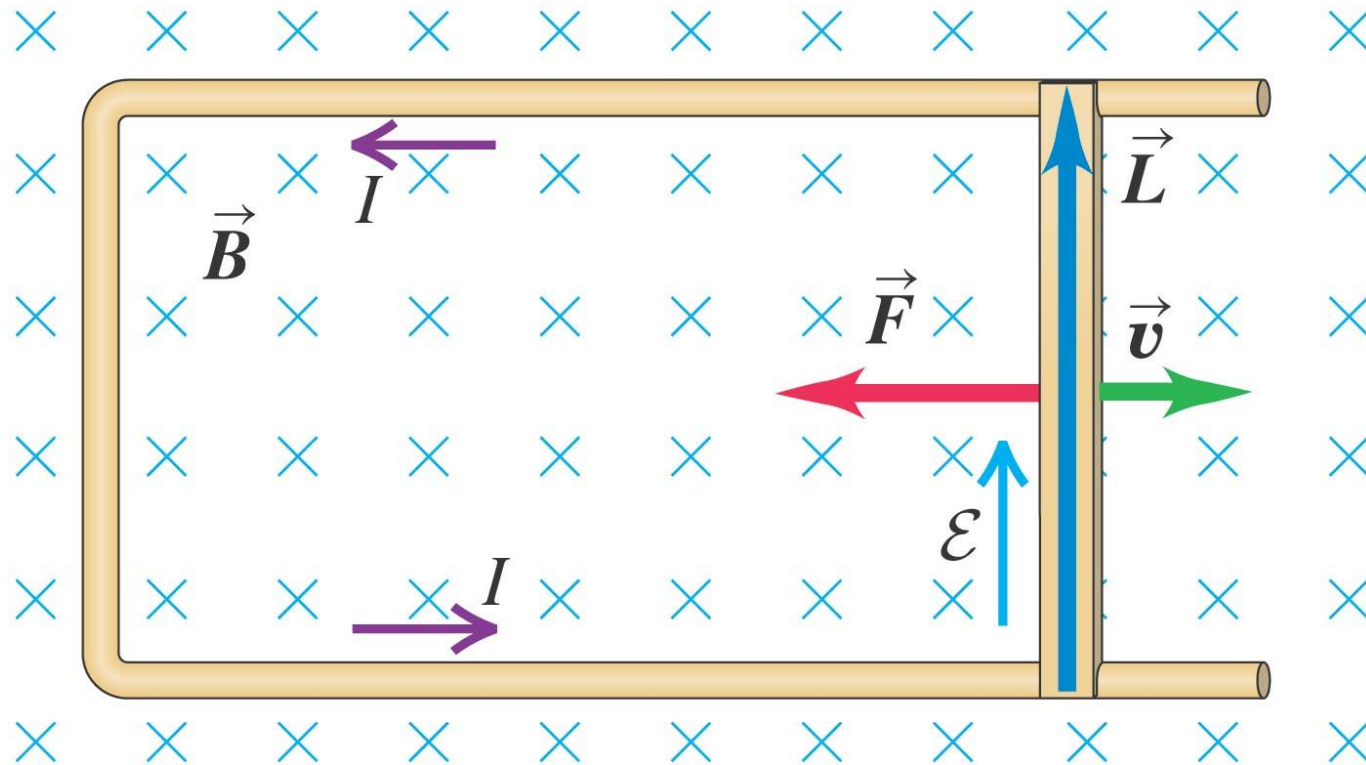


$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt}$$

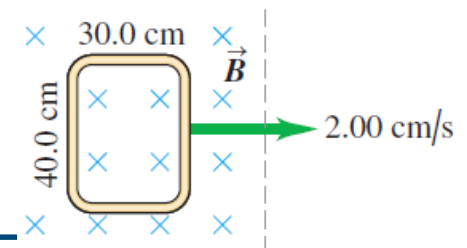
$$\mathcal{E} = -B\frac{Lv dt}{dt} = -BLv$$

Work and power in the slidewire generator

- Follow Example 29.6 using Figure 29.12 below.



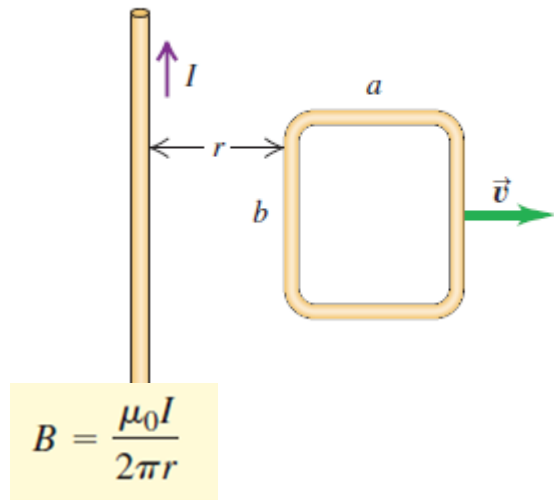
$$d\vec{F} = I d\vec{l} \times \vec{B}$$



Work and power in the slidewire generator

- Follow Example 29.6 using Figure 29.12 below.

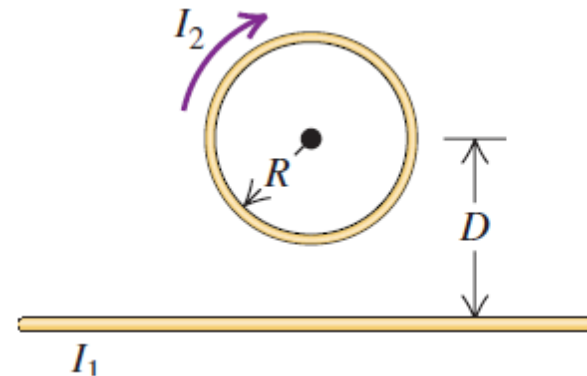
Figure P29.53



$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



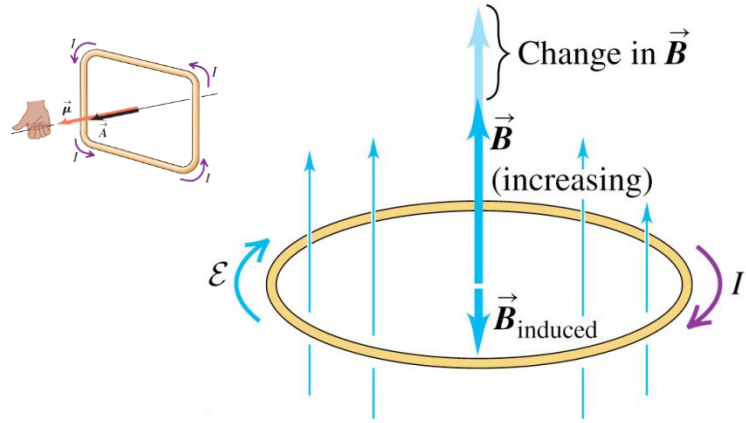
The direction of any magnetic induction effect is such as to oppose the cause of the effect.

Lenz's law

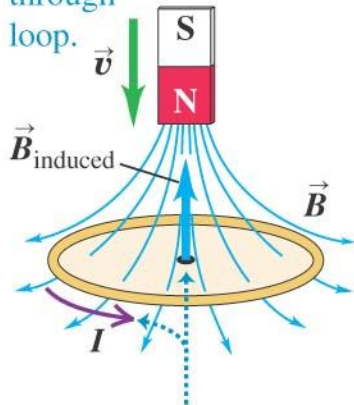
- *Lenz's law*: The direction of any magnetic induction effect is such as to oppose the cause of the effect.

Lenz's law and the direction of induced current

- Follow Example 29.8 using Figures 29.13 (right) and 29.14 (below).

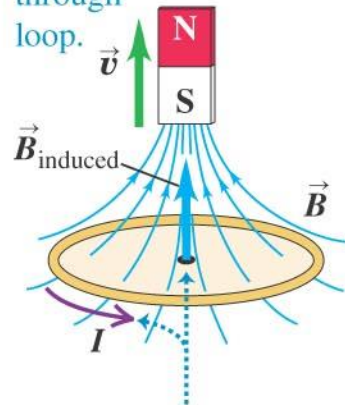


- (a) Motion of magnet causes *increasing downward flux* through loop.

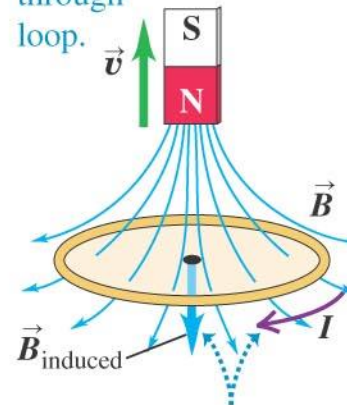


The induced magnetic field is *upward* to oppose the flux change. To produce this induced field, the induced current must be *counterclockwise* as seen from above the loop.

- (b) Motion of magnet causes *decreasing upward flux* through loop.

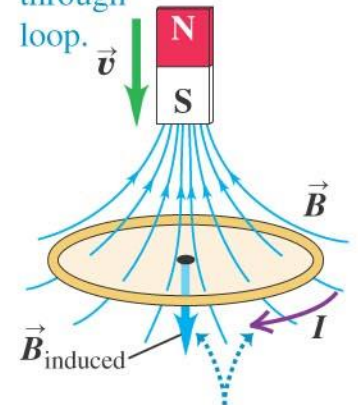


- (c) Motion of magnet causes *decreasing downward flux* through loop.



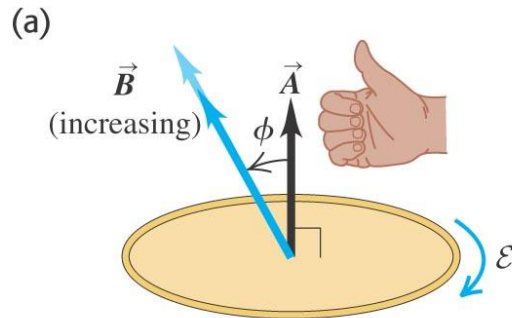
The induced magnetic field is *downward* to oppose the flux change. To produce this induced field, the induced current must be *clockwise* as seen from above the loop.

- (d) Motion of magnet causes *increasing upward flux* through loop.

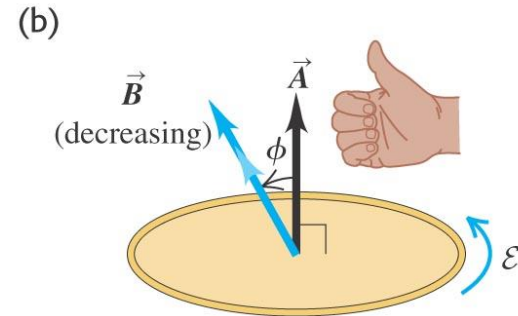


What about F?

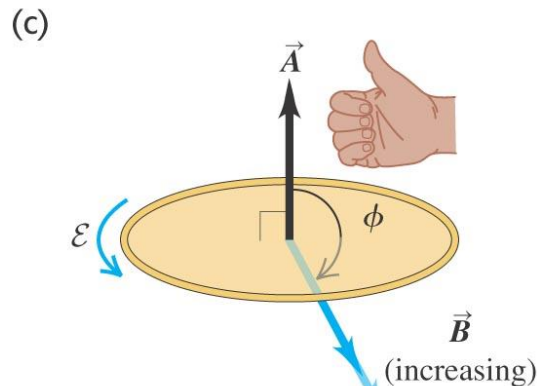
Lenz's law and the direction of induced current



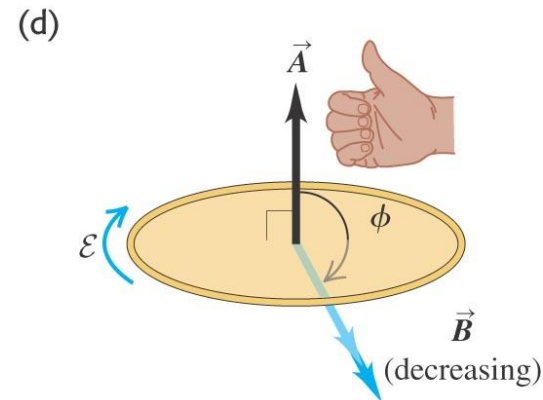
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).



- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

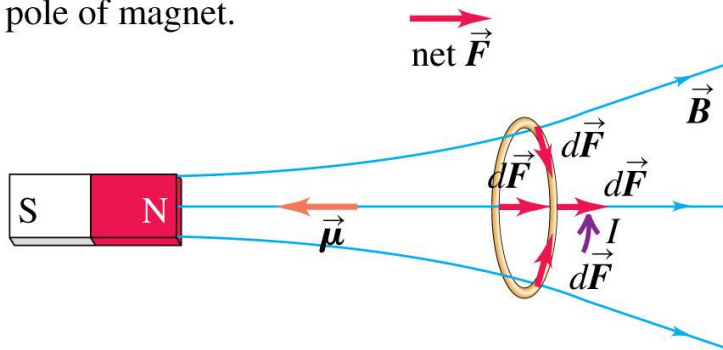


- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

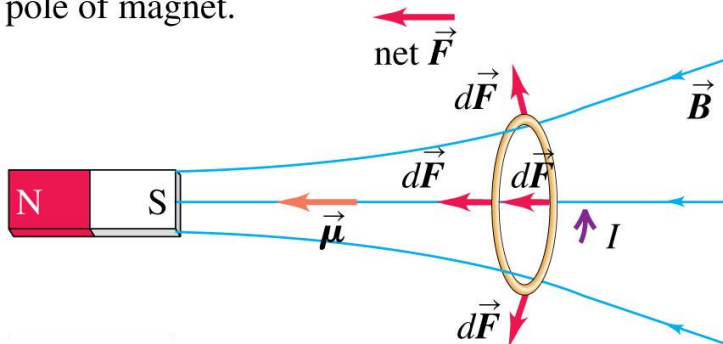
How magnets work

- Follow the discussion in the text of magnetic dipoles and how magnets work. Use Figures 27.36 (below) and 27.37 (right).

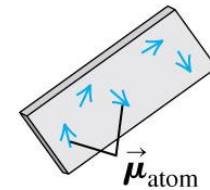
(a) Net force on this coil is away from north pole of magnet.



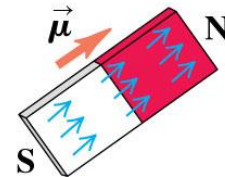
(b) Net force on same coil is toward south pole of magnet.



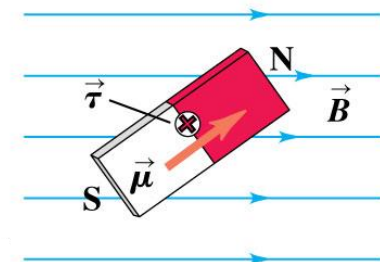
(a) Unmagnetized iron: magnetic moments are oriented randomly.



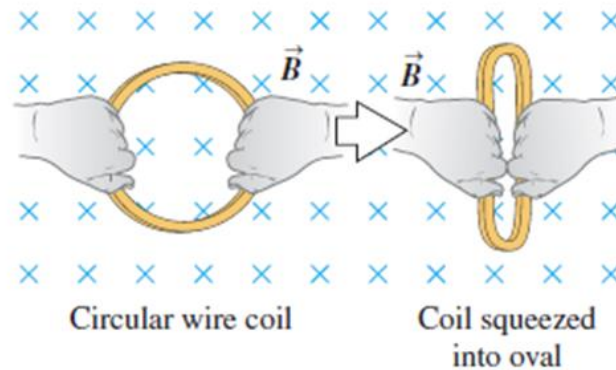
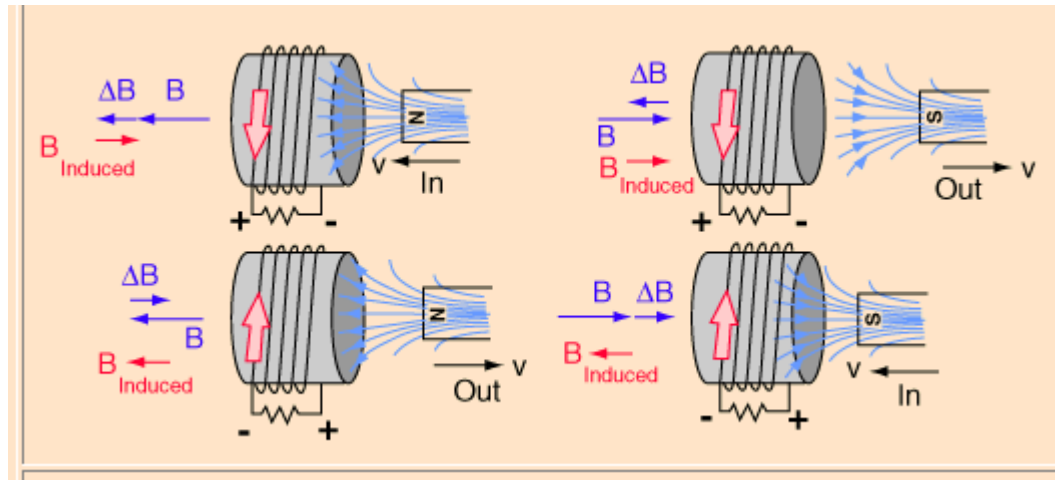
(b) In a bar magnet, the magnetic moments are aligned.



(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the \vec{B} field.



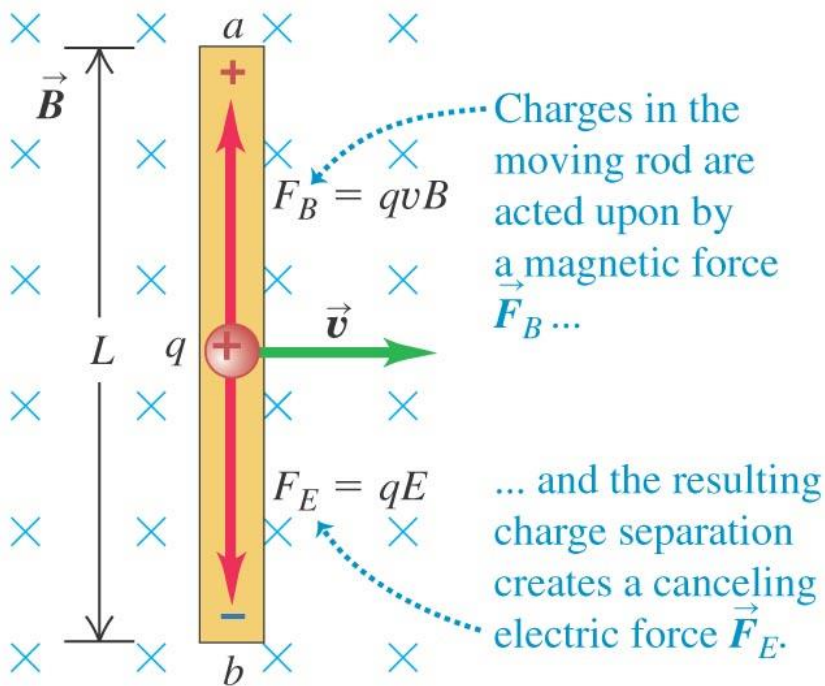
Lenz's law and the direction of induced current



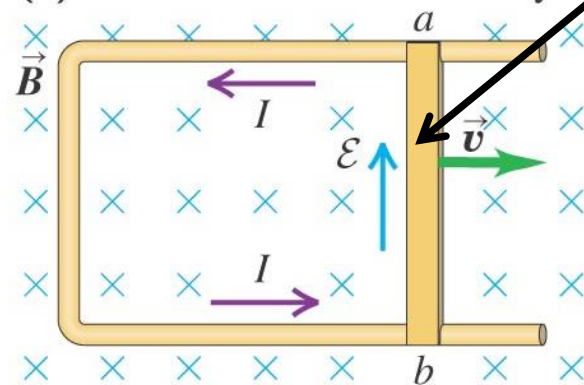
Motional electromotive force

- The *motional electromotive force* across the ends of a rod moving perpendicular to a magnetic field is $\xi = vBL$. Figure 29.15 below shows the direction of the induced current.
- Follow the general form of motional emf in the text.

(a) Isolated moving rod



(b) Rod connected to stationary conductor



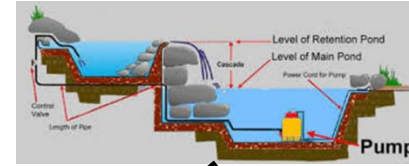
The motional emf \mathcal{E} in the moving rod creates an electric field in the stationary conductor.

$$qE = qvB :$$

$$V_{ab} = EL = vBL$$

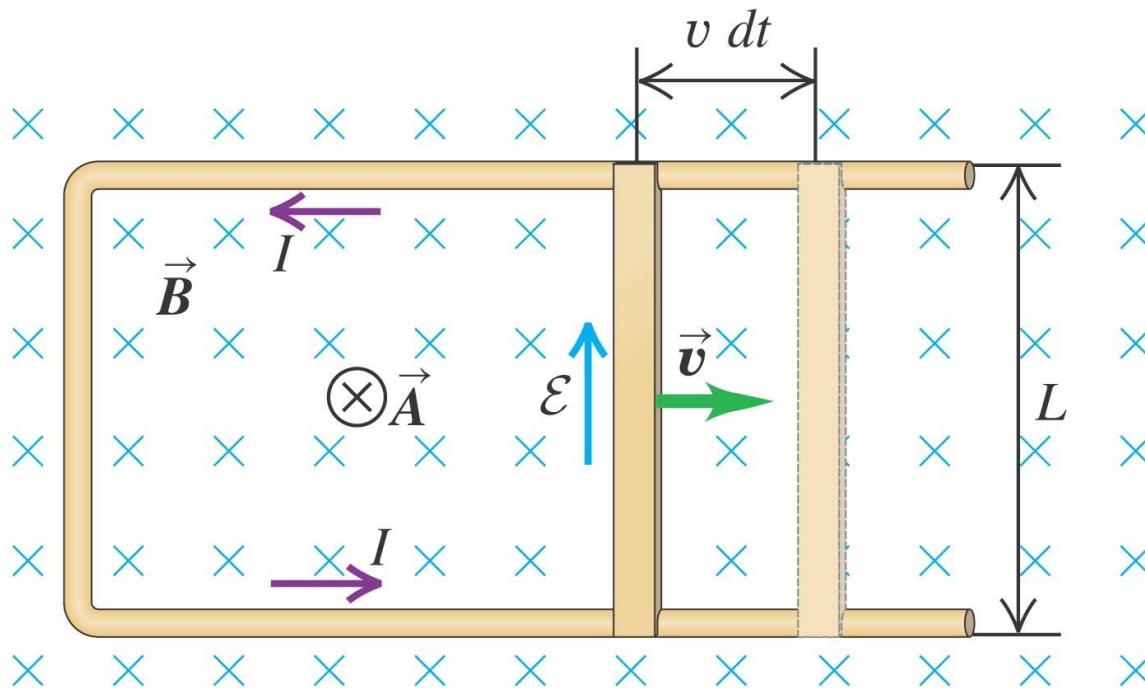
$$\mathcal{E} = vBL$$

(motional emf; length and velocity perpendicular to uniform \vec{B})



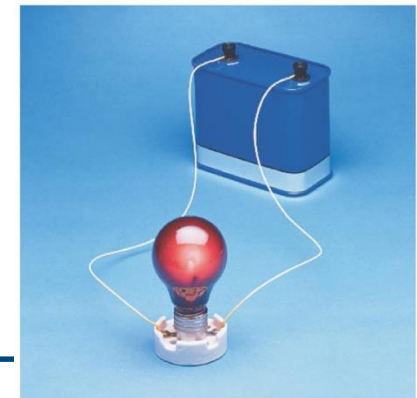
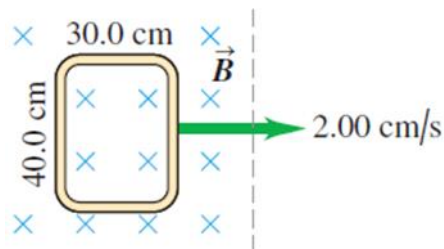
Slidewire generator

- Follow Example 29.5 using Figure 29.11 below.



$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt}$$

$$\mathcal{E} = -B\frac{Lv\,dt}{dt} = -BLv$$



A slidewire generator and a dynamo

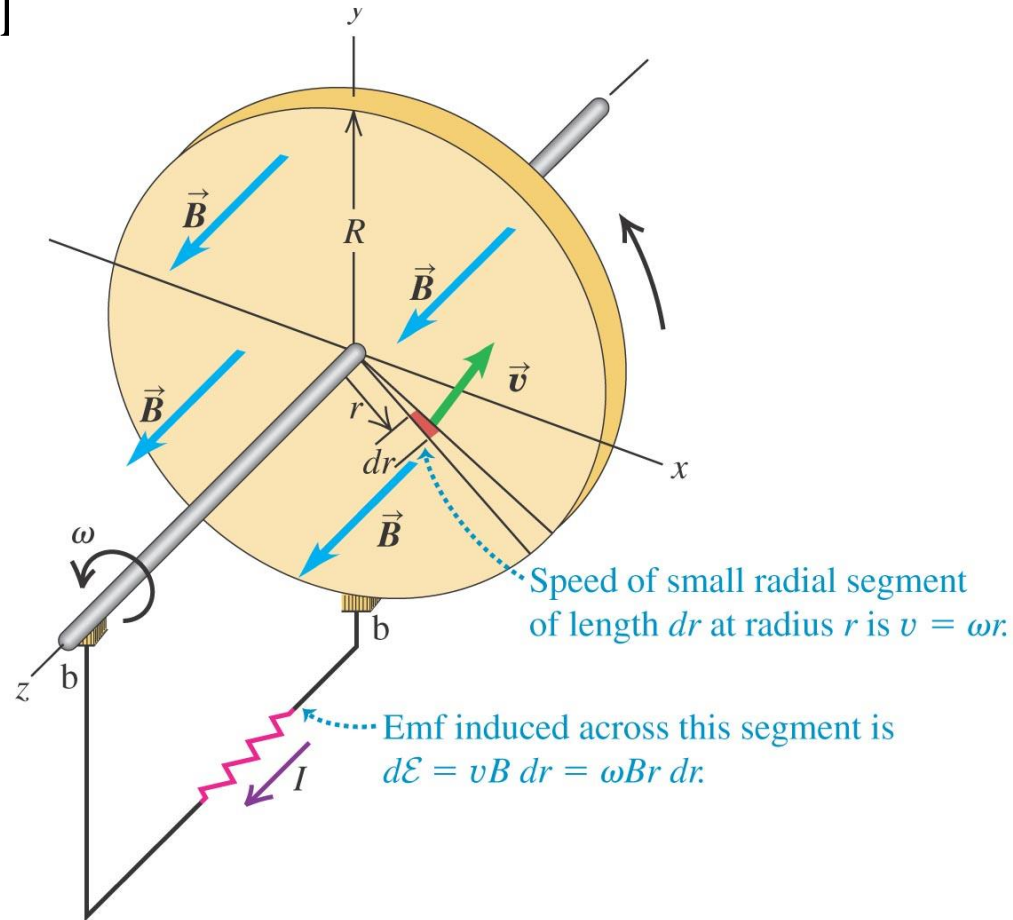
- Follow Example 29.9 for the slidewire generator.
- Follow Example 29.10 for the Faraday disk dynamo.

$$d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

Motional emf: General Form

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

The Faraday disk dynamo

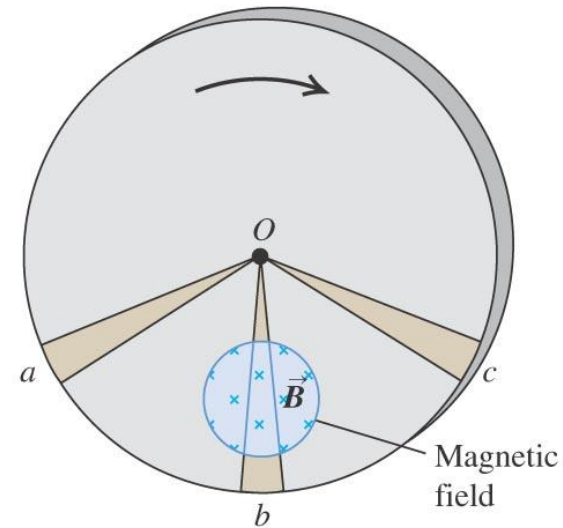


$$\mathcal{E} = \int_0^R \omega Br dr = \frac{1}{2} \omega BR^2$$

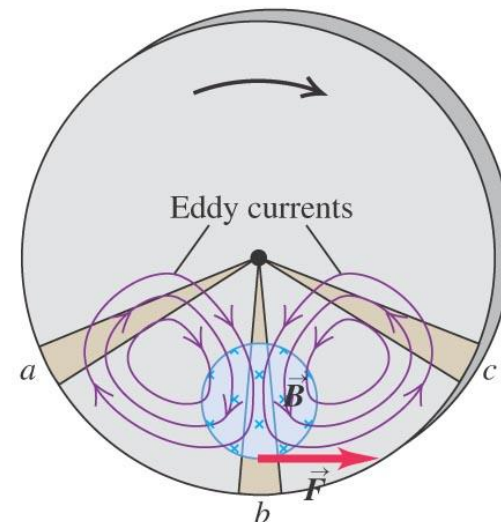
Eddy currents

- Follow the text discussion of *eddy currents*, using Figure 29.19 at the right.

(a) Metal disk rotating through a magnetic field

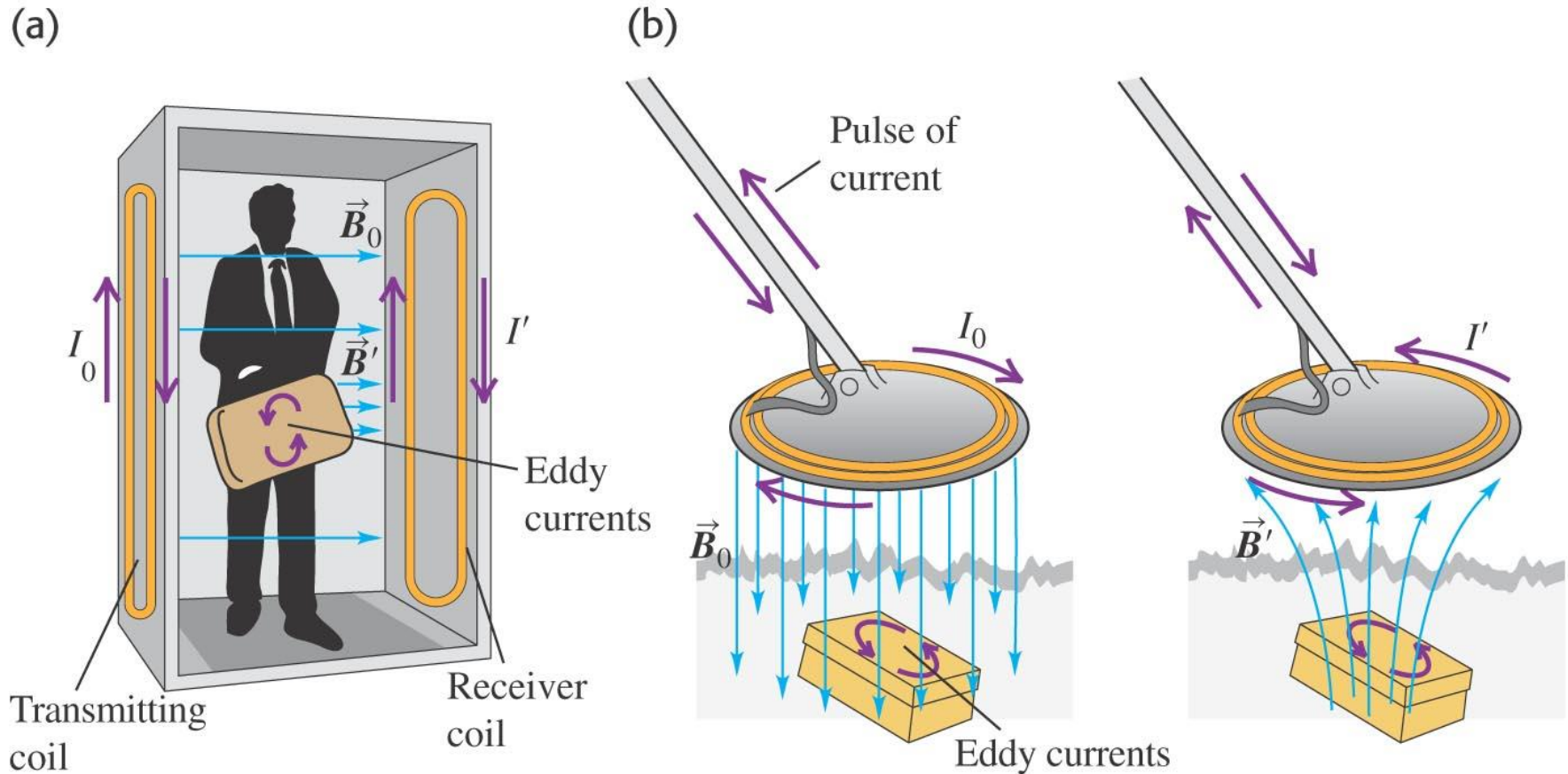


(b) Resulting eddy currents and braking force



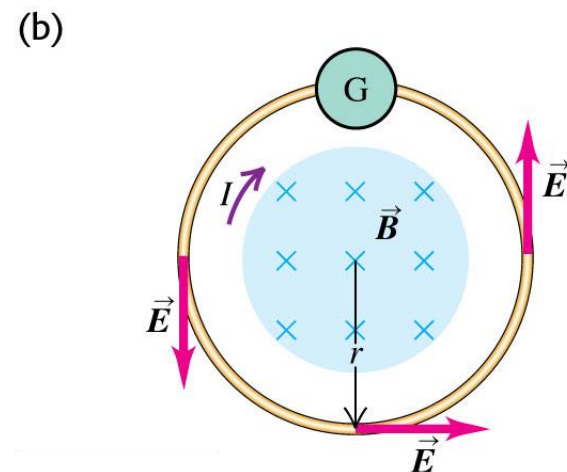
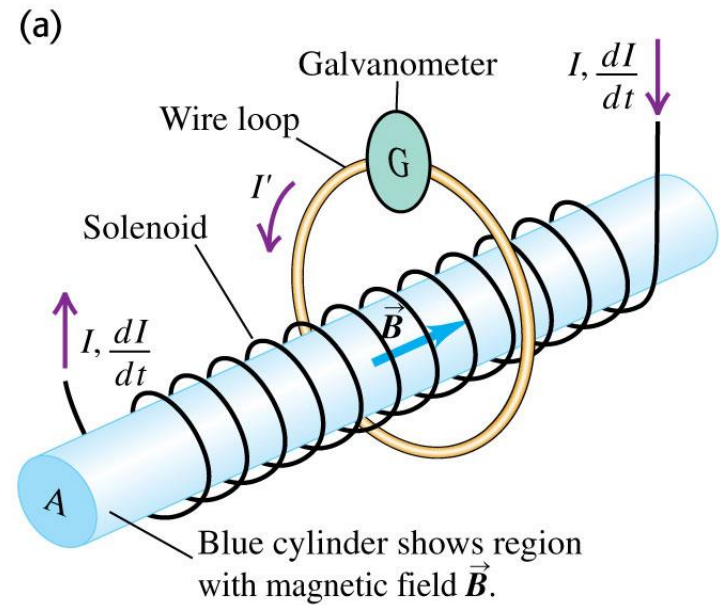
Using eddy currents

- Figure 29.20 below illustrates an airport metal detector and a portable metal detector, both of which use eddy currents in their design.



Induced electric fields

- Changing magnetic flux causes an *induced electric field*.
- See Figure 29.17 at the right to see the induced electric field for a solenoid.
- Follow the text discussion for Faraday's law restated in terms of the induced electric field.
- Follow Example 29.11 using Figure 29.17.



Induced electric fields

$$\Phi_B = BA = \mu_0 n I A$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$$

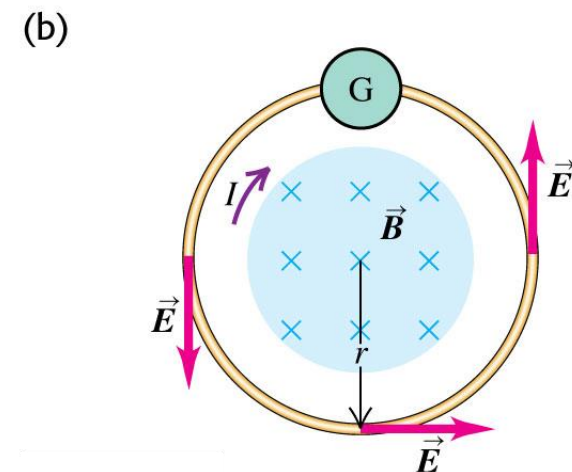
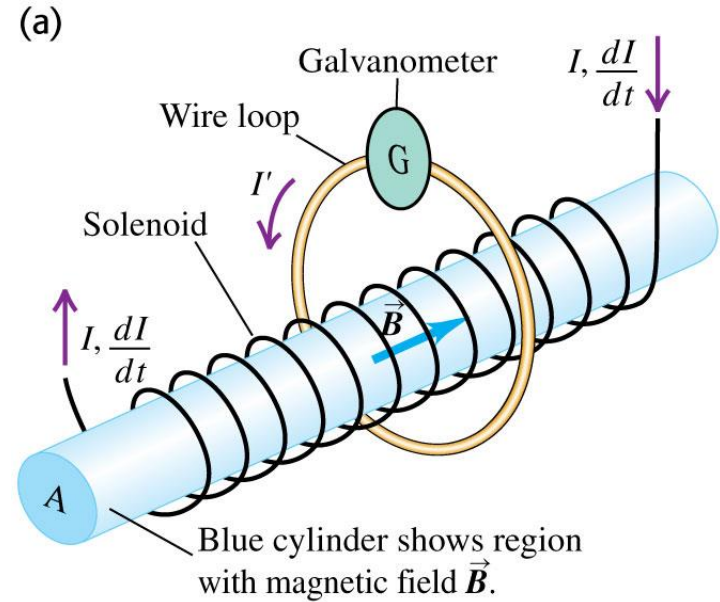
$$V_{ab} = \mathcal{E} \quad (\text{ideal source of emf}) \quad ; I' = \mathcal{E}/R.$$

induced electric field $\oint \vec{E} \cdot d\vec{l} = \mathcal{E}$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad (\text{stationary integration path})$$

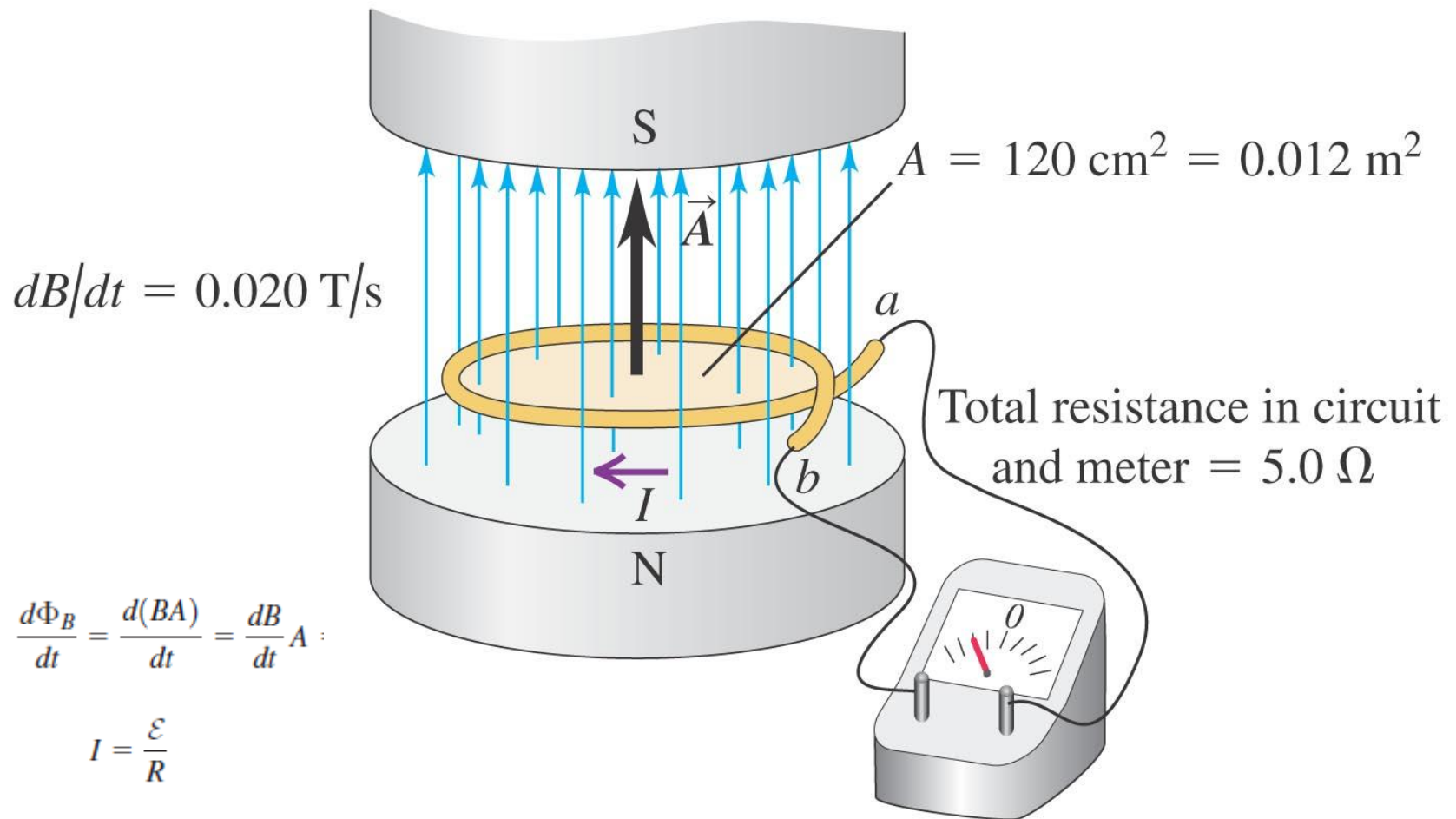
$$\oint \vec{E} \cdot d\vec{l} = 2\pi r E, \text{ and Eq. (29.10) gives}$$

$$E = \frac{1}{2\pi r} \left| \frac{d\Phi_B}{dt} \right|$$



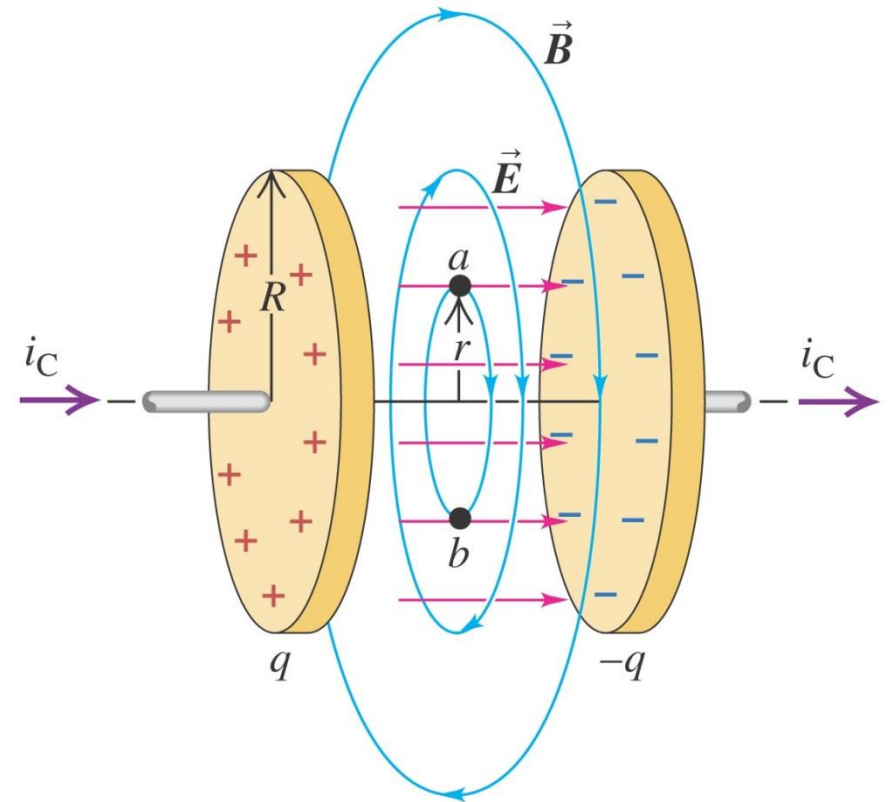
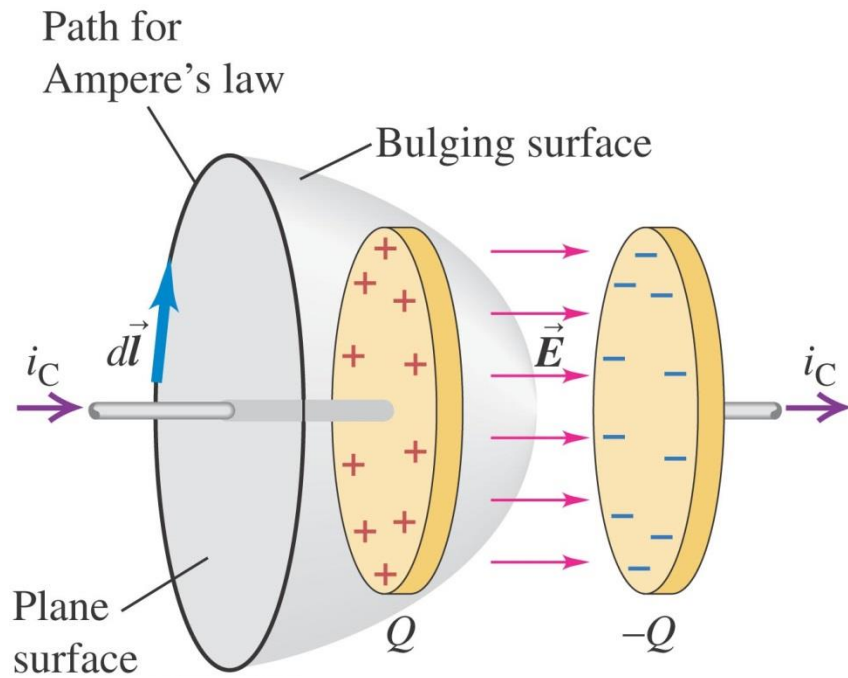
Emf and the current induced in a loop

- Follow Example 29.1 using Figure 29.5 below.



Displacement current

- Follow the text discussion displacement current using Figures 29.21 and 29.22 below.



Displacement current

Substituting these expressions for \bar{C} and v into $q = Cv$, we can express the capacitor charge q as

$$q = Cv = \frac{\epsilon A}{d}(Ed) = \epsilon EA = \epsilon \Phi_E \quad (29.12)$$

where $\Phi_E = EA$ is the electric flux through the surface.

As the capacitor charges, the rate of change of q is the conduction current, $i_C = dq/dt$. Taking the derivative of Eq. (29.12) with respect to time, we get

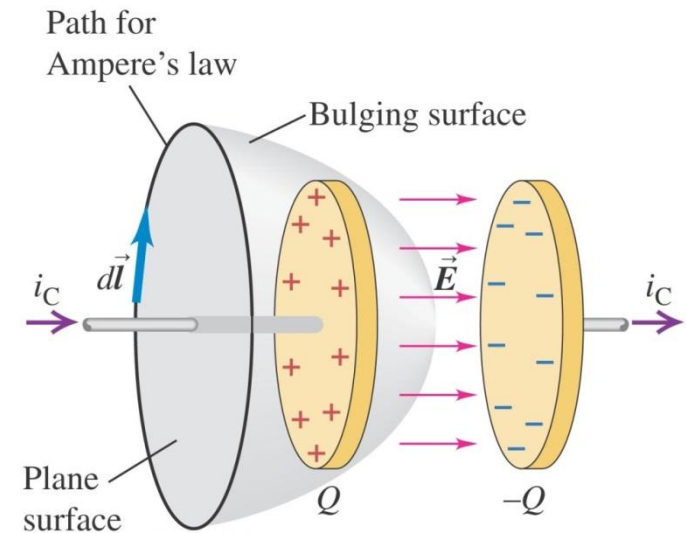
$$i_C = \frac{dq}{dt} = \epsilon \frac{d\Phi_E}{dt} \quad (29.13)$$

Now, stretching our imagination a little, we invent a fictitious **displacement current** i_D in the region between the plates, defined as

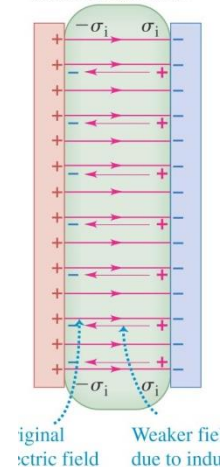
$$i_D = \epsilon \frac{d\Phi_E}{dt} \quad (\text{displacement current}) \quad (29.14)$$

rent i_C , in Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(i_C + i_D)_{\text{encl}} \quad (\text{generalized Ampere's law}) \quad (29.15)$$



(c) Induced charges create electric field

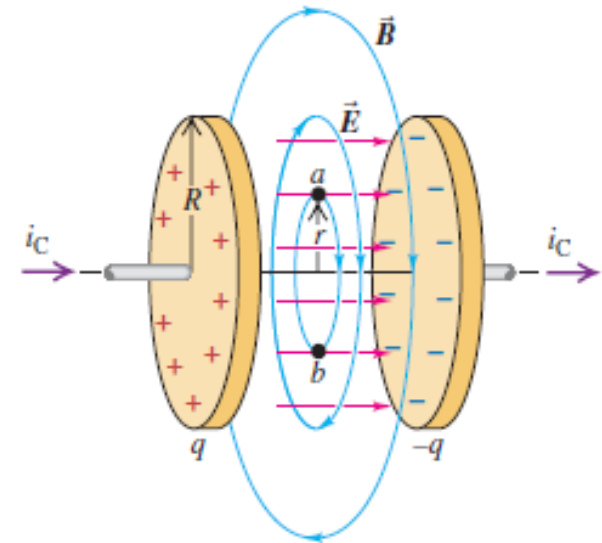


Displacement current

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_C$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_C$$

$$i_D = i_C$$



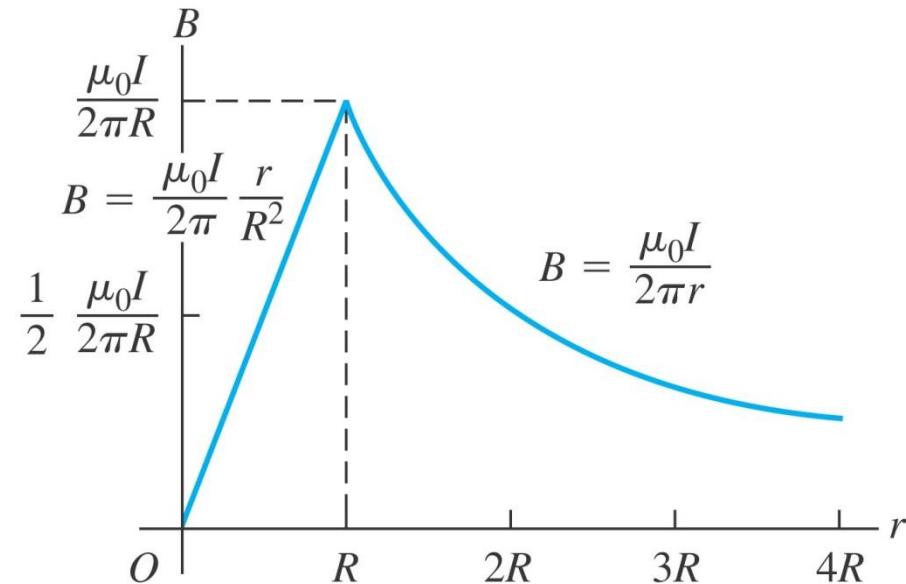
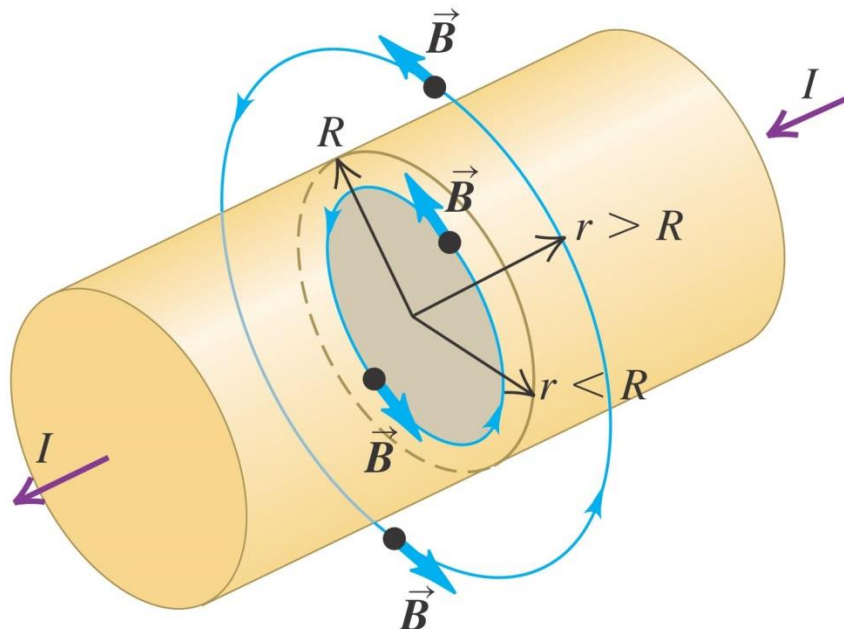
$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

$$\Phi_E = EA$$

$$(i_D / \pi R^2) (\pi r^2).$$

Magnetic fields of long conductors

- Read Problem-Solving Strategy 28.2.
- Follow Example 28.7 for a long straight conductor.
- Follow Example 28.8 for a long cylinder, using Figures 28.20 and 28.21 below.



Maxwell's equations

✓ Gauss's law for the electric field

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad (\text{Gauss's law for } \vec{E})$$

✓ Gauss's law for the magnetic field

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad (\text{Gauss's law for } \vec{B})$$

✓ Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \quad (\text{Ampere's law})$$

✓ Faraday's law.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law})$$

Maxwell's equations

- ✓ Gauss's law for the electric field
- ✓ Gauss's law for the magnetic field
- ✓ Ampere's law
- ✓ Faraday's law.

(i) $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$ (Gauss's law),

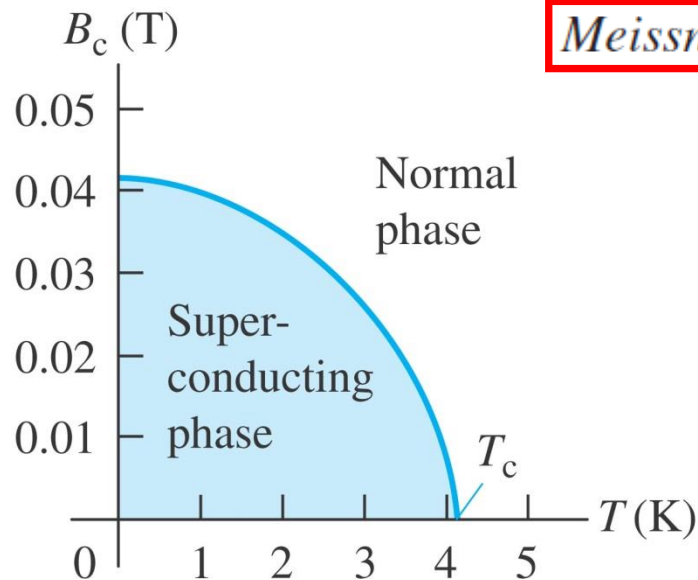
(ii) $\nabla \cdot \mathbf{B} = 0$ (no name),

(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Faraday's law),

(iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (Ampère's law).

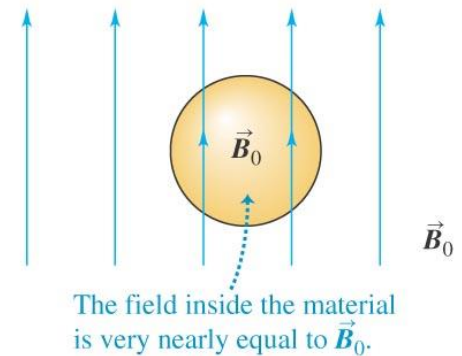
Superconductivity

- When a superconductor is cooled below its *critical temperature*, it loses all electrical resistance.
- Follow the text discussion using Figures 29.23 (below) and 29.24 (right).

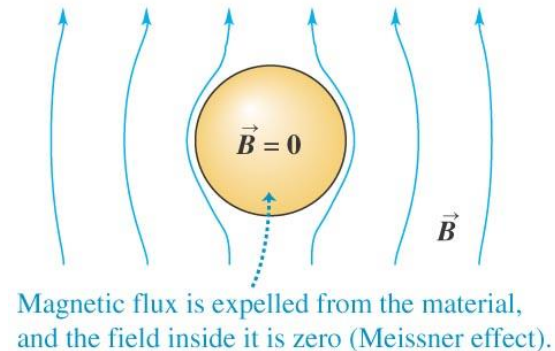


Meissner effect.

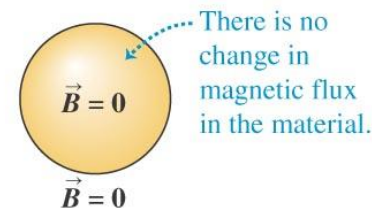
(a) Superconducting material in an external magnetic field \vec{B}_0 at $T > T_c$.



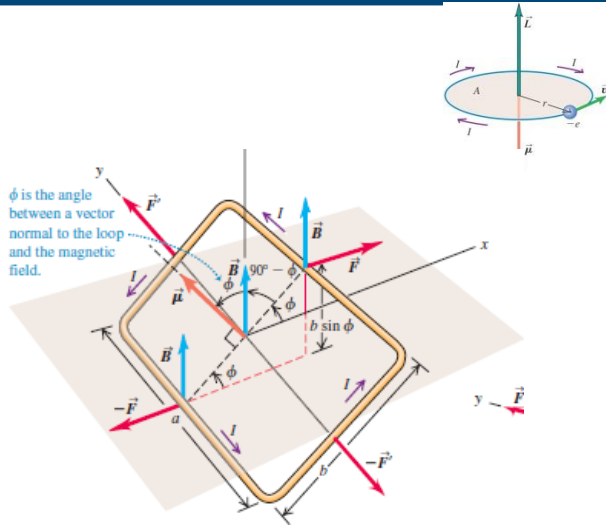
(b) The temperature is lowered to $T < T_c$, so the material becomes superconducting.



(c) When the external field is turned off at $T < T_c$, the field is zero everywhere.



The Bohr magneton and paramagnetism



$$\mu = \frac{e}{2m} \left(\frac{h}{2\pi} \right) = \frac{eh}{4\pi m}$$

the Bohr magneton, denoted by μ_B

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

the magnetization of the material, denoted by \vec{M} :

$$\vec{M} = \frac{\vec{\mu}_{\text{total}}}{V}$$

the total magnetic field \vec{B} in the material is

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$$

as μ and is called the permeability

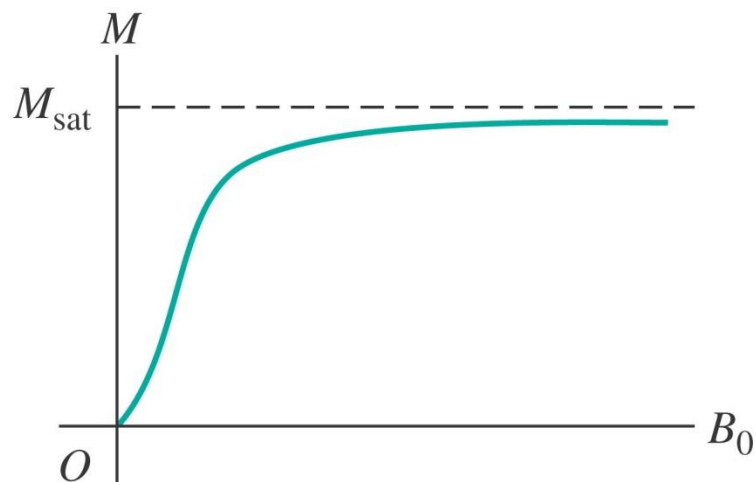
$$\mu = K_m \mu_0 \quad (2)$$

Table 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20^\circ\text{C}$

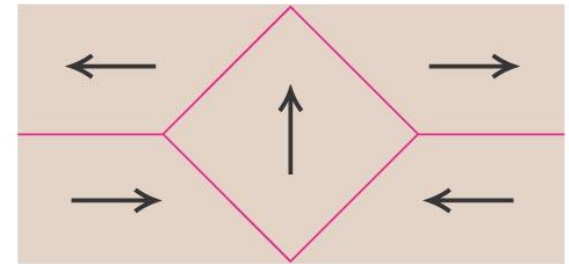
Material	$\chi_m = K_m - 1 (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	-16.6
Mercury	-2.9
Silver	-2.6
Carbon (diamond)	-2.1
Lead	-1.8
Sodium chloride	-1.4
Copper	-1.0

Diamagnetism and ferromagnetism

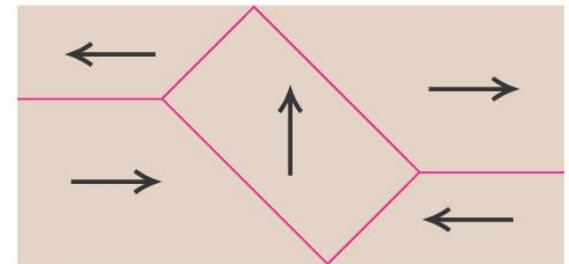
- Figure 28.27 at the right shows how *magnetic domains* react to an applied magnetic field.
- Figure 28.28 below shows a magnetization curve for a ferromagnetic material.



(a) No field



(b) Weak field



(c) Stronger field

