Review Questions
(week of 8 June 2020)

1. What physical system may the infinite square potential well model?
2. What are the conditions the wave function of a particle moving in an infinite potential well should satisfy at the points where the two impenetrable walls are placed?
3. What do we mean by saying that the energy spectrum of a quantum system is discrete?
4. *True of false?* Energy levels in the infinite potential well are equally-spaced.
5. How does the energy of a particle in the infinite potential well depend on the width of the well? mass of the particle? quantum number n?
6. In an infinite potential well, the energy levels depend on the quantum number n as .... [Answer: n^2]
7. Which state of a quantum system is called the *ground state*?
8. Sketch the energy level ladder for the infinite potential well model. Indicate which energy corresponds to the ground state. Do the same for the second excited state.
9. *True of false?* The ground state in the infinite potential well of width 10 nm has lower energy than the ground state in the well of width 20 nm.
10. *True of false?* For particles in an infinite potential well, the ground state energy increases with increasing mass of the particle.
11. Sketch the wave function of the ground state and the first excited state in the infinite potential well with the walls at $x = 0$ and $x = L$. Is the probability of finding a particle in the intervals $(L/4, 3L/4)$ and $(0, L/2)$ equal? (answer the question without doing any calculations) How is it in classical mechanics?
12. How many nodes (zeros) in the interval $0 < x < L$ does the wave function of a particle moving in an infinite potential well have if $n = 7$?
13. The eigenfunction corresponding to the energy level E_n $(n = 1, 2, 3, \ldots)$ in the infinite potential well has .... nodes.
14. Explain the idea of the Bohr's correspondence principle using a particle in a box (infinite potential well) as an example.
15. Eigenvalues of Hermitian operators are .... and eigenfunctions corresponding to different eigenvalues are ....
16. Why are Hermitian operators so useful in quantum mechanics? [just list their properties]
17. Name an important property of the set of all eigenfunctions of a Hermitian operator. [Hint: see the next question.]
18. *True or false?* Any function from the space $\mathcal{L}^2([0, a])$ (i.e. the space of square-integrable functions on the interval $[0, a]$) can be represented as a linear combination of $\psi_n(x) = \sqrt{\frac{2}{a}}\sin(n\pi x/a)$, (that is wavefunctions that are solutions to the stationary Schrödinger equation for the infinite potential well with walls placed at $x = 0$ and $x = a$).

19. Initially, a quantum particle is in the state represented by the wave function $\Psi(x,0) = \frac{1}{\sqrt{7}}\psi_1(x) + \sqrt{\frac{6}{7}}\psi_{100}(x)$, with ψ_1 and ψ_{100} being eigenfunctions of $\hat{H}$ corresponding to the eigenvalues $E_1 = A$ and $E_{100} = 2A$, where A is a real constant.

    Write down $\Psi(x, t > 0)$. Find $|\Psi(x,t)|^2$. Does Ψ represent a stationary state?

20. Suppose that, initially, a particle is described by a wave function Φ that is a superposition of two eigenfunctions of $\hat{H}$. Is it possible that Φ is also an eigenfunction $\hat{H}$? If so, when?

21. A particle is in the ground state in an infinite potential well with walls at $x = 0$ and $x = L$. Suddenly, at $t = 0$, the right wall is moved to $x = 2L$. (a) Right after the well has expanded (*i.e.*, at $t = 0^+$), is the particle in a stationary state? (b) Is it possible that a measurement of the particle's energy performed at $t = 0^+$ returns the value corresponding to the ground-state energy of the narrow well? (*Hint*: See homework for the answer.)

22. Suppose that we have 50 copies of a quantum particle, each in the state described by $\psi = \frac{1}{\sqrt{5}}\psi_4 + \sqrt{\frac{2}{5}}\psi_9 + \sqrt{\frac{2}{5}}\psi_{16}$, where ψ_n is the solution of the stationary Schrödinger equation corresponding to energy $E_n = \epsilon\sqrt{n}$, where ϵ is a positive constant and $n = 1, 2, 3 \ldots$.

    Is this ψ an eigenfunction of $\hat{H}$? If we perform a measurement of energy on each of the particles in the state represented by ψ, what are the probabilities that we get the values: 2ϵ, 3ϵ, 4ϵ, 5ϵ? What is the average energy of a particle in this state?

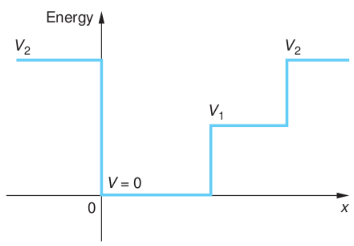
23. Does the average energy of a particle in a stationary state depend on time?

24. What are classical turning points?

25. What is a classically forbidden region?

26. What is an important feature of the graph of the wave function in a classically accessible region? [*Answer: oscillatory behavior.*]

27. Sketch the wave function corresponding to a particle with energy $V_1 < E < V_2$ in the finite potential well shown below.

    Your sketch should properly reflect features of the wave function, such as the amplitude and wavelength of the oscillatory part of the wave function.

28. Is it possible for a quantum particle penetrate into a classically inaccessible region? Give an example.

29. Does the fact that a particle penetrates into a classically forbidden region imply that the kinetic energy of that particle becomes negative? Explain briefly.

30. Why cannot we have a term $\sim e^{-\kappa x}$ in the wave function of a particle in the region $x < -a$ of the finite potential well $V(x) = -V_0$ for $|x| \leq a$ and zero otherwise.

31. Can we set parameters of a rectangular potential well, so that there is no bound state? Exactly one bound state?

32. On the graph $V = V(x)$ for the finite rectangular potential well mark the values of the total energy that correspond to bound states. The same for unbound states.

33. What is the symmetry of the ground-state wave function in a rectangular potential well? First excited state?

    Sketch their graphs.