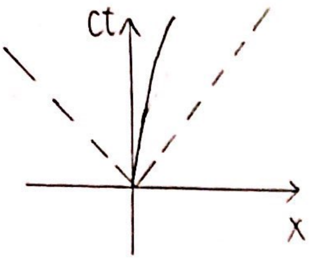
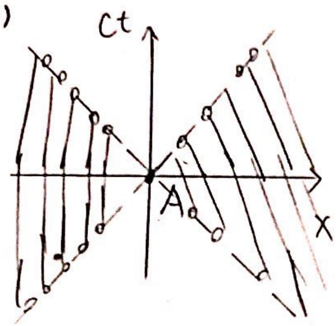
PART II

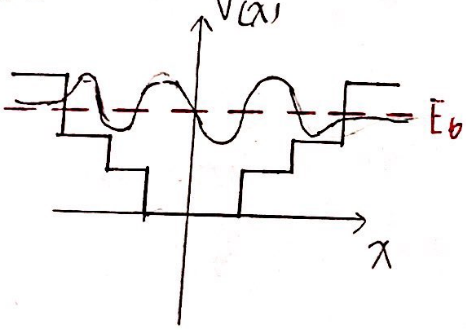
Q1. (a)



(b)



Q2



Q3

(a) $E_1 = \varepsilon \quad E_2 = 8\varepsilon \quad E_8 = 512\varepsilon$

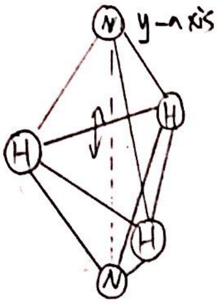
$$\psi(x,t) = \sqrt{\frac{2}{5}}e^{-\frac{i}{\hbar}\varepsilon t}\psi_1 + \sqrt{\frac{2}{5}}e^{-\frac{i}{\hbar}8\varepsilon t}\psi_2 + \sqrt{\frac{1}{5}}e^{-\frac{i}{\hbar}512\varepsilon t}\psi_8$$

(b) $Pr(E = 8\varepsilon = E_2) = C_2^2 = 0.4$

Q4 Heisenberg uncertainty principle claims that for any particle, $\Delta x \Delta p \geq \frac{\hbar}{2}$ so there is a trade-off between the particle's position and momentum.

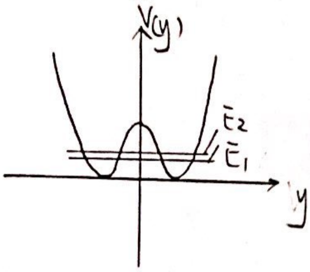
The more accurate we know about the particle's position, the less we know about its momentum, and vice versa.

Q5.



According to the stucture of NH_3, we know the arrangement of H is equilateral triangle and symmetric, the N atom also oscillates up and down symmetric

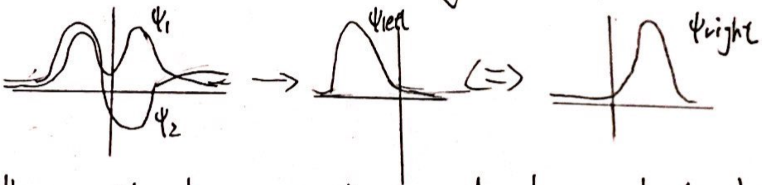
In the y-axis, which is N's direction, the potential is:



Since the N-atom can move between left and right half even when $E<V$ in QM! ...is

In a double-well

For $\psi_{Left} = \frac{1}{\sqrt{2}}(\psi_1+\psi_2)$ $\psi_{right} = \frac{1}{\sqrt{2}}(\psi_1-\psi_2)$ where ψ_1 is even ψ_2 is odd



ψ_{left} will change to ψ_{right} and change back between two configurations

Hence it can be modeled as 1D oscillation potential well

Q6

(a) $Pr([0, \frac{L}{2}]) = Pr([\frac{L}{2}, 0]) < Pr([\frac{L}{4}, \frac{3}{4}L])$

(b) $P = \frac{30}{L^5}\hbar \int_0^L x(x-L)\frac{\partial x(x-L)}{\partial x}\,dx = 0$

Q7 The lowest levels are $E_0 = \frac{3}{2}\hbar\omega$ and $E_1 = \frac{7}{2}\hbar\omega$

because since the left part of $V(x)$ is ∞, the $\psi(x)=0$ for $x \le 0$

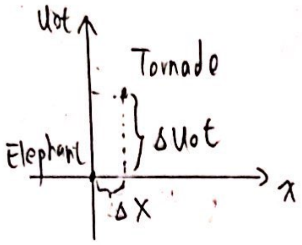
and the energy level should be odd function.

Hence it should be $E_1 = \frac{3}{2}\hbar\omega$, $E_3 = \frac{7}{2}\hbar\omega$ in original oscillator model.

1\. a) We just replace all $c = 3\times10^8$ m/s with $u_0 = 250$ m/s, because they're the max velocity for all objects

b) Yes because simultaneous is relative since Thomas is moving, he can see Manuel and David's clock with different time

(c)



$\Delta u_0 t = 250\times5\times60 = 75000\ (m)$

$\Delta X = 60000\ m$

Then $\Delta s^2 = (\Delta u_0 t)^2 - (\Delta x)^2 > 0$, so the two instance are time like

Manuel is reasonable

d) Yes, if they're all not moving

Q2. a) $U_0 q = \frac{1}{2}mc^2$

$$u_0 = \frac{mc^2}{2q}$$

b) Before acceleration, the particle has energy $E = mc^2$

After, $E = mc^2 + qV_0$, so $K = qV_0 = \frac{mc^2}{\sqrt{1-u^2/c^2}} - mc^2$

then $u = c\sqrt{1 - \frac{(mc^2)^2}{(qV_0 + mc^2)^2}}$

Q3 For the stationary equation: $\hat{H}\psi(x) = E\psi(x)$

$$\begin{cases} \text{For } x \le -a,\ \psi_1(x) = A_1 e^{K_1 x} \text{ where } K_1^2 = -\frac{2m(E-V_1)}{\hbar^2} \\ \text{For } x \ge a,\ \psi_3(x) = A_3 e^{K_3 x} \text{ where } K_3^2 = -\frac{2m(E-V_2)}{\hbar^2} \\ \text{For } x < |a|,\ \psi_2(x) = C_1\cos K_2 x + C_2 \sin K_2 x \text{ where } K_2^2 = \frac{2m}{\hbar^2}(E+V_0) \\ \psi_1(-a) = \psi_2(-a) \\ \psi_1'(-a) = \psi_2'(-a) \\ \psi_3(a) = \psi_2(a) \\ \psi_3'(a) = \psi_2'(a) \end{cases}$$

$$\int_{-\infty}^{\infty}|\psi(x)|^2 dx = 1$$

Q4:

(a) $\phi = \begin{cases} \frac{1}{a} & |x| < \frac{a}{2} \\ 0 & |x| > \frac{a}{2} \end{cases}$

$\psi = \sum C_n \psi_n$ where $C_n = \langle \psi_n , \phi \rangle$

(b) i $p = -i\hbar \frac{d}{dx} (\frac{1}{a}) = 0$

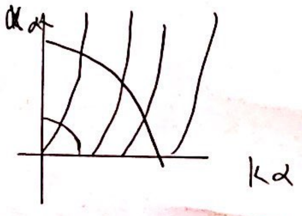
ii

Q5 (a) $\Delta p \Delta x = \sqrt{\langle p^2 \rangle_\psi - \langle p \rangle_\psi^2} \sqrt{\langle x^2 \rangle_\psi - \langle x \rangle_\psi^2}$

$\langle p^2 \rangle_\psi = \langle -i\hbar \frac{d[\langle p \rangle_\psi + i \varepsilon (x - \langle x \rangle_\psi)] \psi x}{dx} , \psi x \rangle$

Then $\Delta p \Delta x \geq \frac{\hbar}{2}$

(b)



Q6 According to the diagram

since $(ka)^2 + (ka)^2 = \frac{2ma^2}{\hbar} V_0$

we can see the nodes of different states have different radius.

since $E = \frac{k^2 \hbar^2}{2m} - V_0$. plug it back, we can get different E_n

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