Review Questions
(week of 15 June 2020)

1. Why is the harmonic oscillator model important in physics?
2. The harmonic approximation works well if the amplitude of oscillations about a stable equilibrium position is ....
3. What plays the role of a "spring constant" in the harmonic approximation?
4. Name a phenomenon in solid state (or molecular) physics where the quantum harmonic oscillator model may be useful.
5. What are the main features of solutions to the stationary Schrödinger for the quantum harmonic oscillator.
6. *True of false?* Energy levels in the harmonic potential well are equally-spaced.
7. What are zero-point oscillations?
8. On the graph $V = V(x)$ mark the energy levels (enough to mark the first 3 levels) of a quantum oscillator. Sketch the corresponding wave functions.
9. What does it mean that the ground state of a quantum harmonic oscillator saturates the Heisenberg uncertainty principle? Do excited states saturate it, too?
10. The energy levels of the harmonic oscillator are $E_n = \ldots\ldots\ldots$, where $n = \ldots\ldots\ldots, \ldots\ldots\ldots, \ldots\ldots\ldots$
11. The wave functions of a particle in the harmonic well are of the form $\sim e^{-\xi^2}\ldots\ldots\ldots$, where $H_n(\xi)$ are .... polynomials.
12. ψ_1 and ψ_2 are solutions to the Schrödinger equation for a harmonic oscillator, corresponding to the energy levels E_1 and E_2, respectively. Are ψ_1 and ψ_2 orthogonal? Explain why (without any calculations!).
13. Explain how oscillations of the N atom between two geometrical arrangements in a NH_3 molecule can be explained using the double-well model.
14. If a double well is formed of two identical rectangular finite-depth wells close to each other, and initially a particle is located in the left well, it will eventually "tunnel" to the right one. The tunnelling time increases/decreases with the separation of the wells.

    Hint. If the wells are far away, the two lowest energy levels are very close to each other, so $|E_2 - E_1|$ is very small. Does it then take a long or a short time for $e^{-\frac{i}{\hbar}(E_2-E_1)t}$ to change significantly?

    Or you may play around with this applet to get the right answer:

    `https://phet.colorado.edu/en/simulation/covalent-bonds`
15. In the double well described above, is the state representing a particle localized in the left well an eigenstate of the Hamiltonian? Is it a stationary state?
16. *True or false?* the eigenvalues of the momentum operator $\hat{p}_x = -i\hbar\partial/\partial x$ are all real numbers.
17. *True or false?* The eigenfunction of the momentum operator $\hat{p}_x = -i\hbar\partial/\partial x$ corresponding to the eigenvalue p is $\psi_p(x) = Ce^{\frac{i}{\hbar}px}$.
18. How are eigenfunctions of the momentum operator normalized?
19. What is a classical wave packet?

$$\langle \phi_{p_x}, \phi_{p_{x'}} \rangle = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} e^{\frac{i}{\hbar}(p'_x - p_x)x}\, \mathrm{d}x = \delta(p'_x - p_x).$$

$$\Psi(x,t) = \int_{-\infty}^{\infty} w(p_x) e^{\frac{i}{\hbar}(p_x x - Et)}\, \mathrm{d}p_x$$

20. How do we construct a classical wave packet?

21. What is the phase velocity/group velocity?

22. The wave function of a free particle with momentum p_x (and energy $E = p_x^2/2m$) travelling in the positive x axis direction is $\Psi(x,t) = \ldots\ldots\ldots\ldots\ldots\ldots$

23. Is the wave function describing a free particle with definite momentum square-integrable?

24. For a particle with definite momentum, we have $\Delta_{p_x} = 0$. Using the Heisenberg uncertainty principle, argue what should be the uncertainty of this particle's position. Explain how the form of the free-particle wave function (for example for a particle moving in the positive x axis direction), is compatible with the Heisenberg uncertainty principle.

25. Why is the function $\Psi(x,t) = Ce^{\frac{i}{\hbar}(p_x x - Et)}$, where $E = p_x^2/2m$ not suitable for representing a localized free quantum particle?

26. Underline the correct answer: A particle described by $\Psi(x,t) = Ce^{\frac{i}{\hbar}(p_x x - Et)}$ has *definite/indefinite* position and *definite/indefinite* momentum.

27. How do we construct a wave packet describing a quantum particle?

28. *True or false*: A quantum particle described by a wave packet has both definite position and definite momentum.

29. What does it mean that there is a trade-off between definiteness of position and definiteness of momentum of a particle described by a wave packet. Name the principle that implies this trade-off.

30. Suppose a free quantum particle moving in the positive x axis direction is described by a Gaussian packet $\Psi(x,t)$. Sketch $|\Psi(x,t)|^2$ at two instants of time t_1 and t_2 where $t_1 < t_2$.

31. What do we mean when we say that a Gaussian packet spreads with time?

32. Why don't we see the spreading effect for wavepackets describing macroscopic objects?