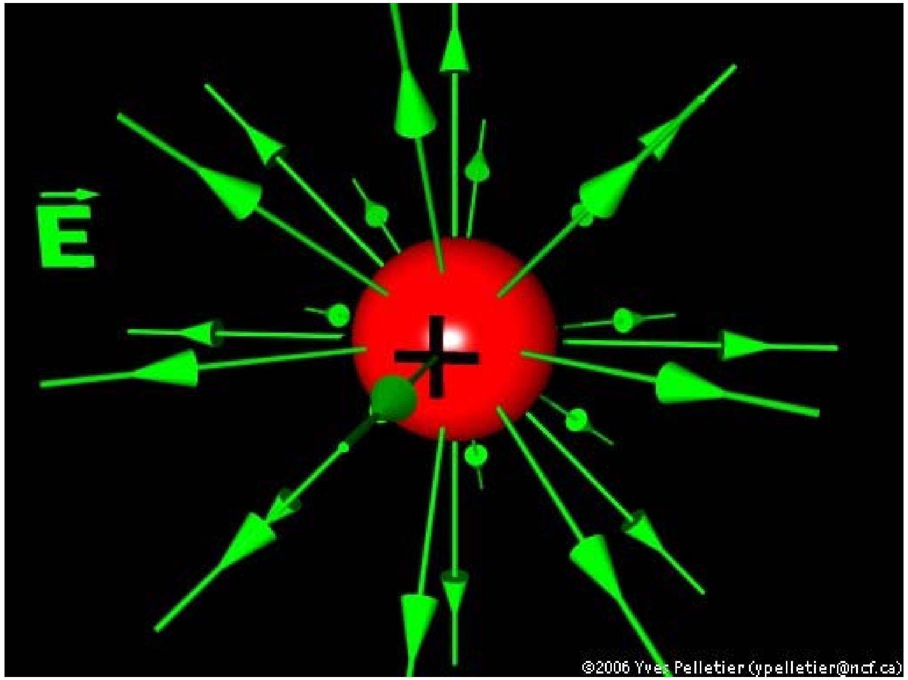
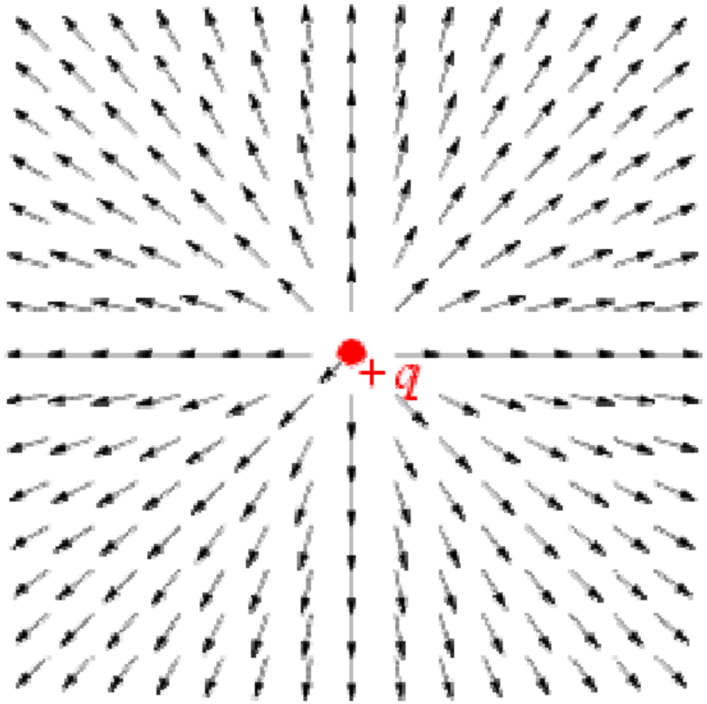
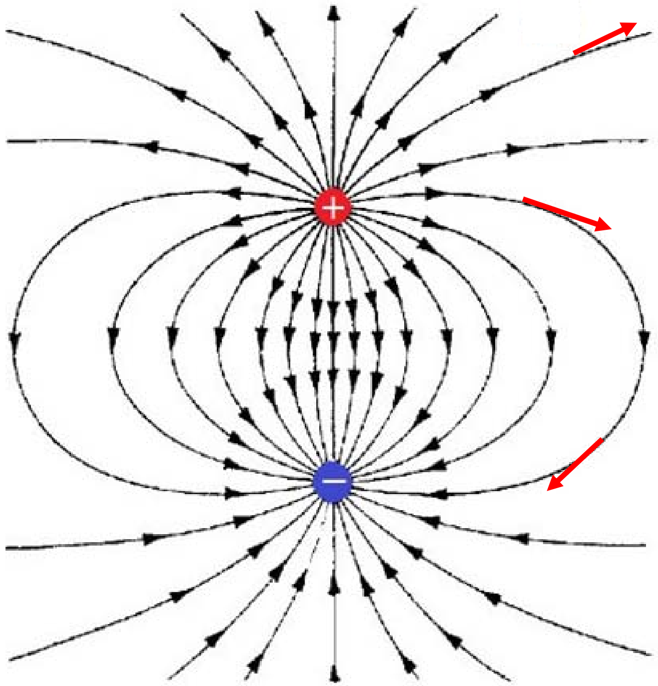
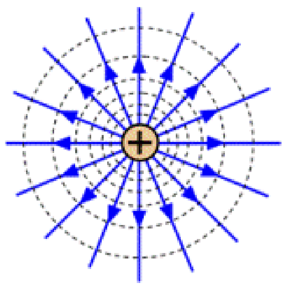
Electric **field** of a single charge

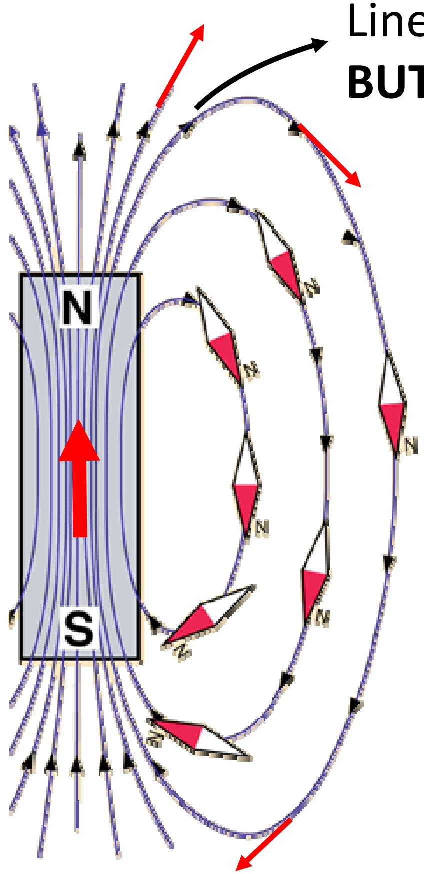




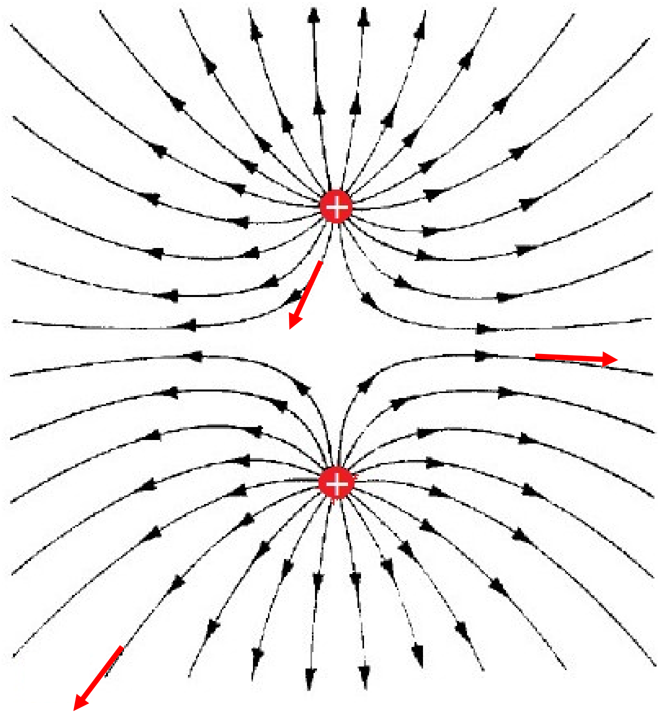
Field lines keep track of the direction of the field







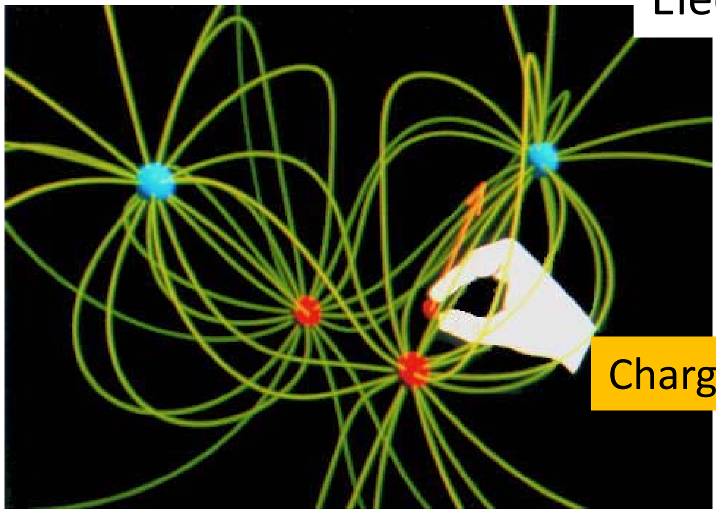
Lines are going from N to S outside the magnet
BUT from S to N inside the magnet !



Field vectors are always tangent to the field lines

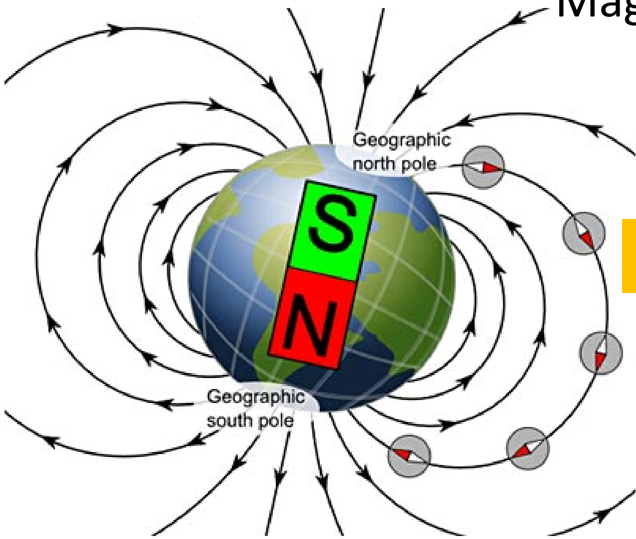
Action at distance:
Electric field of multiple charges and Magnetic field of a dipole magnet

Electric field lines



Charge test

Magnetic field lines



Magnetic dipole test

We can imagine how complicated will be the trajectory of a test charge left in this field !

A non negligible test charge may perturb these field lines !

Major difficulty with field line concept

They do not fit with superposition principle

We add vectors $$\vec{E}_{tot} = \sum_i \vec{E}_i$$

How can we add field lines?

Example of the difficulty with field line concept

Two charges moving in space, both at the same speed and parallel to each other

<u>For a static observer:</u> There is a magnetic field thus magnetic field lines

<u>For an observer moving with the same speed:</u> Do the field lines move with the charges?

Impossible to say as the field disappears. Why?

For the moving observer the charges are at rest ⇨ No magnetic field
BUT for another static observer the field still exists thus the field lines !

<u>Best:</u> Restrict to the abstract idea of field vector and scalar

$$\vec{\boldsymbol{E}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t}),\ \vec{\boldsymbol{B}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t}),\ \boldsymbol{V}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t}),\boldsymbol{T}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t})\ \text{etc...}$$

According to Faraday and Maxwell

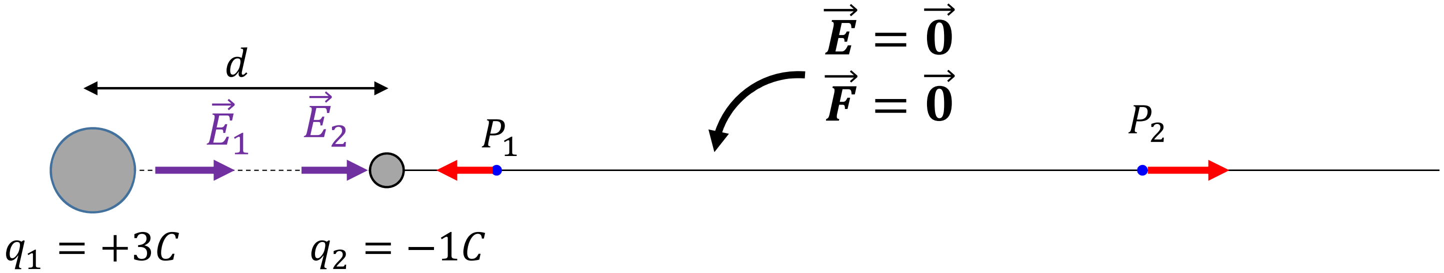
Fields $\Rightarrow$ $\vec{\boldsymbol{E}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t}), \vec{\boldsymbol{B}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t})$ $\Leftrightarrow$ 'sphere of influence'

- Something which propagates in space
- Something which stores energy and momentum

"Substance or fluid" that mediates interactions between bodies

To be taken with great care

System of two charges: Towards dipoles



What does a charge test $+q$ feel at P_1 and P_2 ?

Consequence of Coulomb's law

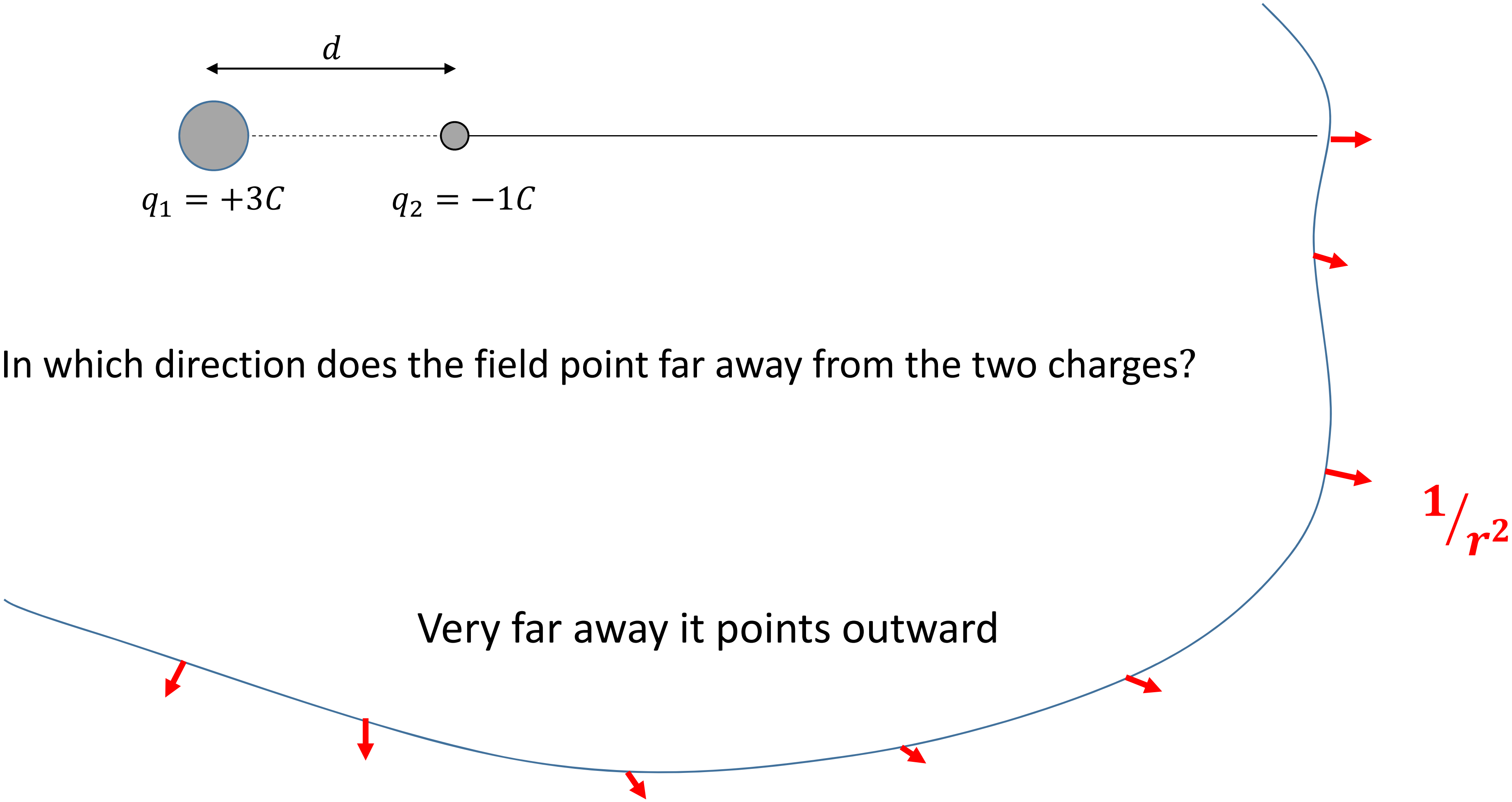
- At P_1 the test charge feels mostly the **attractive** force from q_2 (because of ${}^{1}/_{r^2}$)
- At P_2 ($r >> d$) the test charge feels mostly the **repelling** force from $(q_1 + q_2) = +2C$

Can we guess what happens between P_1 and P_2?

- Somewhere in between P_1 and P_2 the field must be zero thus the force: Why?

Because the fields are in opposite directions at P_1 and P_2

System of two charges: Towards dipoles

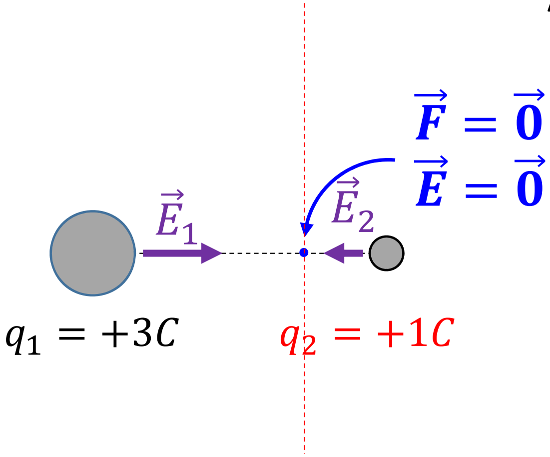


In which direction does the field point far away from the two charges?

Very far away it points outward

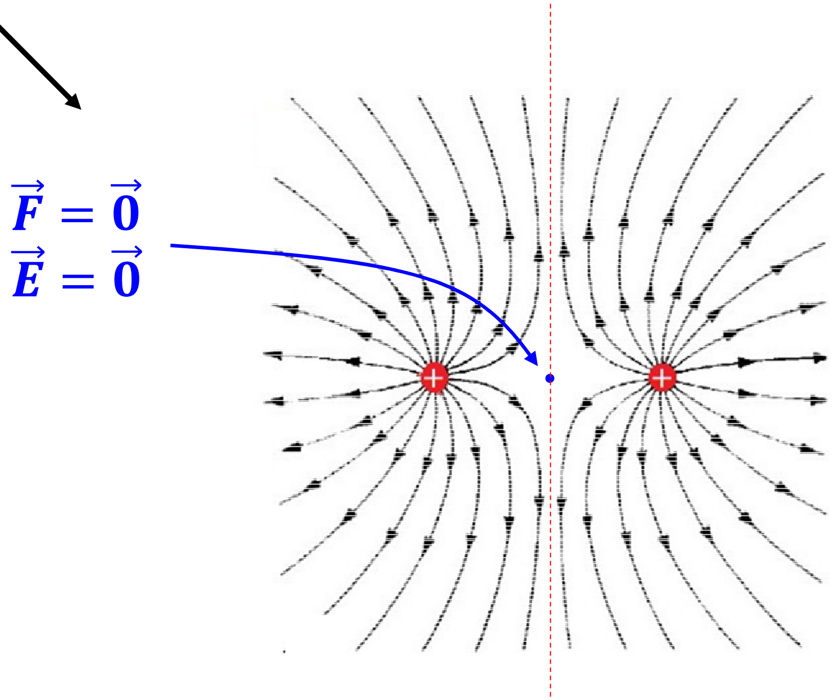
Are these locations **points of equilibrium** ?

Wait for next lecture



A point near the moon in the line joining the moon to the earth the gravitation is zero

Notice that here the charges **repel** each other while the moon and the earth attract each other



Is the field zero all along this line?

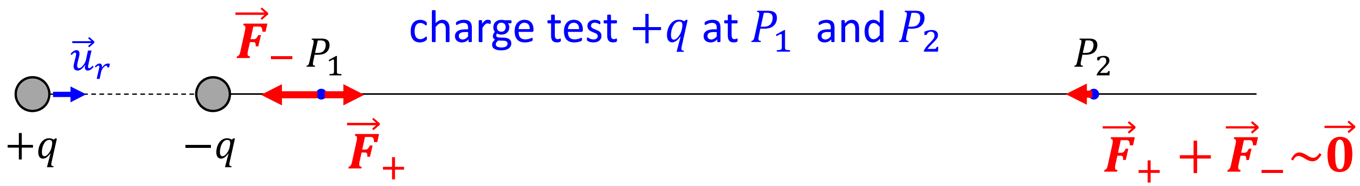
No ! Just at the blue point

Consequence for a dipole

For a single charge q ⇨ $\vec{E} \propto \frac{1}{r^2}\vec{u}_r$

Dipole ≡ equal but opposite charges

What about if we consider a dipole ?



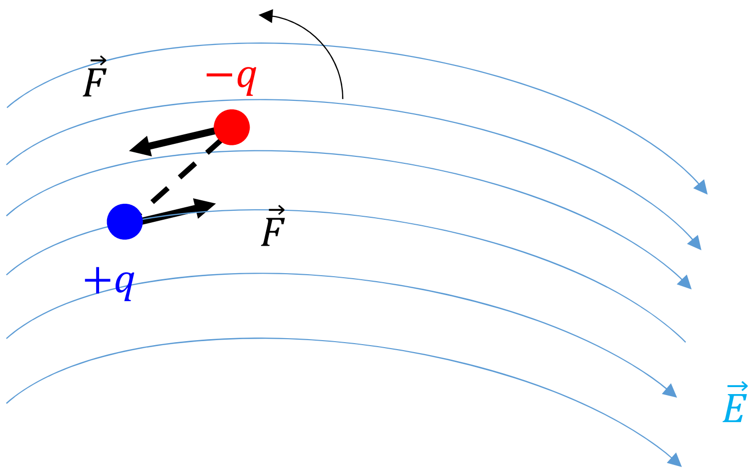
⇨ $\vec{E} \propto \frac{1}{r^n}\vec{u}_r$ $\quad n > 2$ The field strength falls off much more rapidly

Is there a point in space where the field is zero?

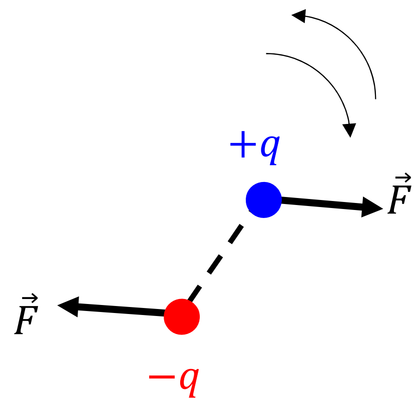
There is **NOT** a single point in space where the field is zero except at infinity

The dipole is a very interesting and useful system

Any symmetric molecule will convert into a dipole once place in an electric field

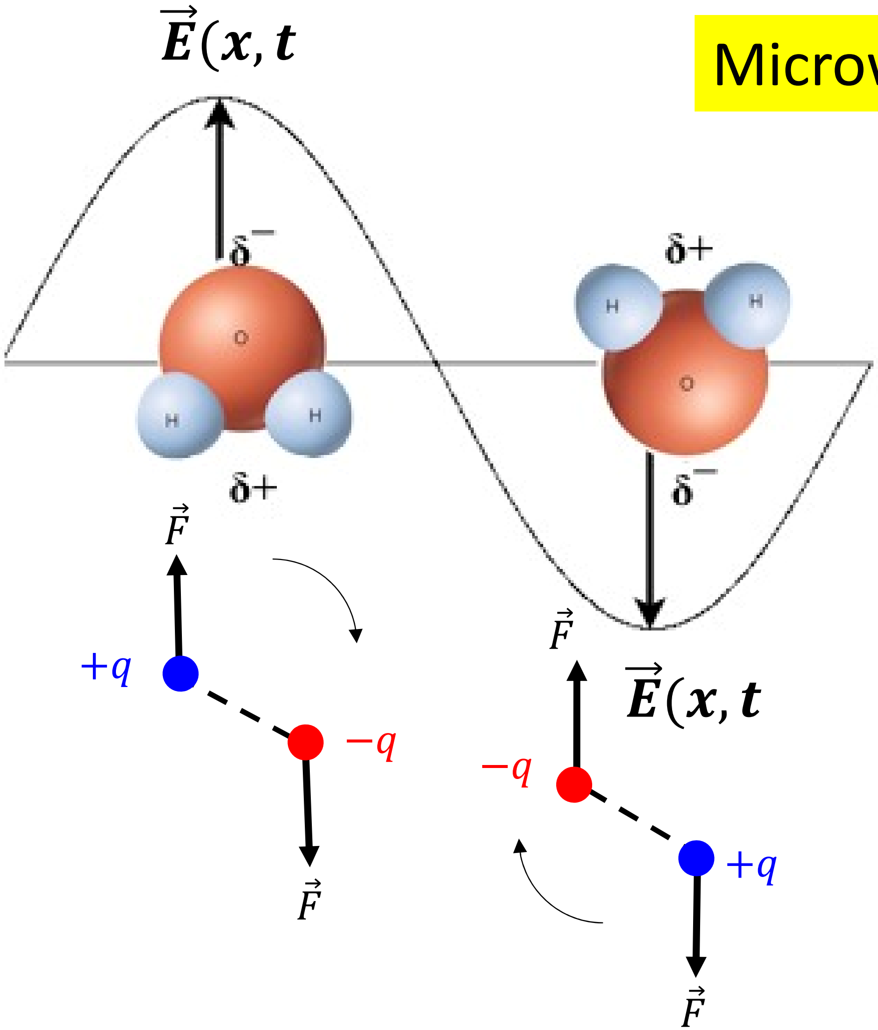


Torque → overshot



Oscillation of the dipole

Can we get something useful from this mechanism?



Microwave oven: 2.45 GHz

Oscillation of the Electric field

Continuous rotation of the molecule

Gain of energy

Inelastic collisions with neighboring molecules

Energy transfer

Newtonian mechanism and Maxwell's electromagnetism at once!

Summary of all forces acting on charges

Derived from experiment

$$\boxed{\vec{F}_E \propto \frac{1}{r^2}}$$

Electric force: Static charges

$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

Magnetic force: charge moving in a magnetic field

$$\boxed{\vec{F}_{Lorentz} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

EM force: Holds when charges are moving in a magnetic field

Equation of motion

If $v \ll c \Rightarrow \vec{F} = \frac{dm\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Newton

$$\vec{F} = \frac{d}{dt}\left[\frac{m\vec{v}}{(1 - v^2/c^2)^{1/2}}\right] = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Einstein

Knowing the electric and magnetic forces ⇨ knowing the motion

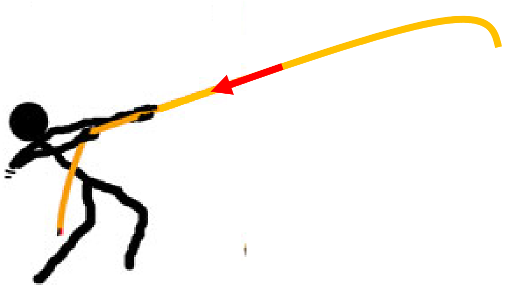
How are these forces produced ? ⇨ VE230

Force acting on an electron

Mechanical (Newtonian) force

$$\vec{F} = m\vec{a}$$

Pulling a conducting wire



This guy is setting mechanically electrons into motion

The force acting on the electron is in the same direction and parallel to $\vec{v}$

What happens if a magnet is nearby?

- Charges (electrons) can be set into motion by moving **mechanically** the conducting wire containing such electrons

- Reversibly by setting **electrically** into motion free electrons inside the conducting wire, the whole wire can be set into motion **if a magnet is nearby**

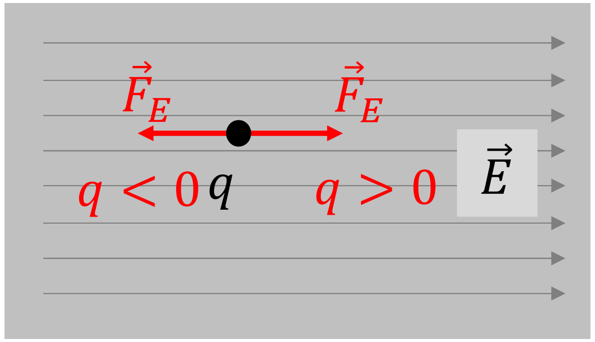
Lorentz force acting on a charge

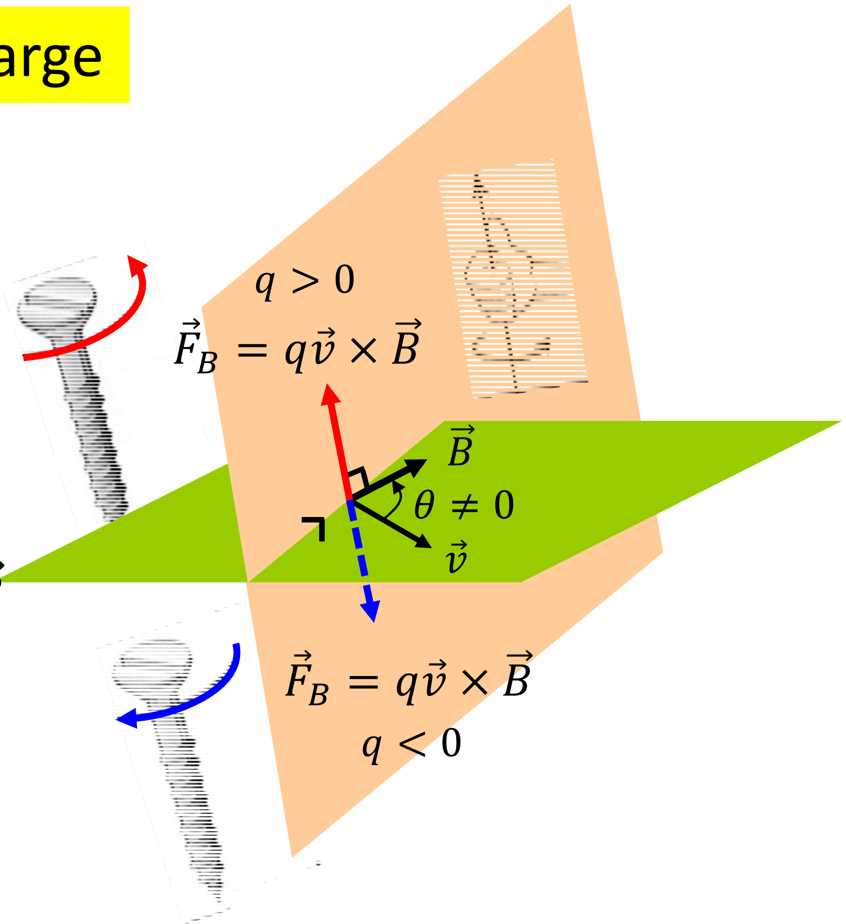
Lorentz force

$$\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$\vec{F} = \vec{F}_E + \vec{F}_B$$

$$\vec{F}_E = q\vec{E} \qquad\qquad \vec{F}_B = q\vec{v} \times \vec{B}$$





The electric force acting on the charge is parallel to $\vec{E}$

$\vec{E}$ acts on a **static** and **moving** charge
$\vec{B}$ acts on a moving charge **ONLY**

The magnetic force acting on the charge is perpendicular to $\vec{v}$ and $\vec{B}$

Concept #9: Work and Energy

As for the gravitational force, **electrostatic** force is **conservative**:
It derives from the potential energy

$$F_E = -\frac{dV}{dx} \Rightarrow dV = -F_E dx \Rightarrow W_{a\to b} = V(b) - V(a) = -\int_a^b F_E dx$$

From Newton

$$F = m\frac{dv}{dt} \Rightarrow Fdx = m\frac{dv}{dt}dx = m\frac{dx}{dt}dv = mvdv \Rightarrow K(b) - K(a) = \int_a^b Fdx$$

$$\frac{1}{2}mv^2$$

Miracle

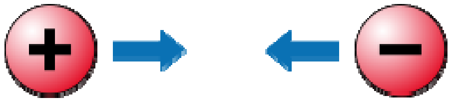
If the only acting force is the conservative one $\Rightarrow$ $F = F_E$

$$K(b) - K(a) = -[V(b) - V(a)] \Rightarrow K(b) + V(b) = K(a) + V(a)$$ Conservation of energy

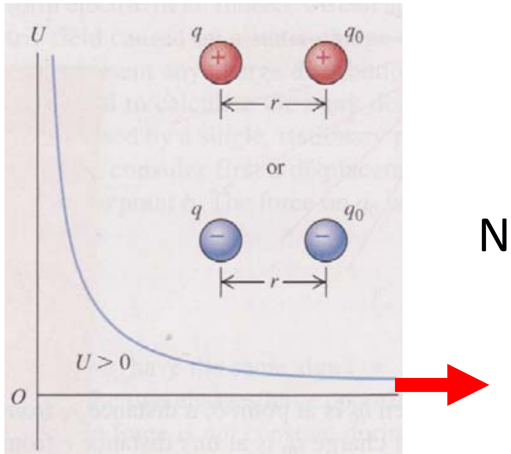
A conservative force ALWAYS acts to "push" the system toward lower energy

Energy is lowered when the charges are at **infinite** distance

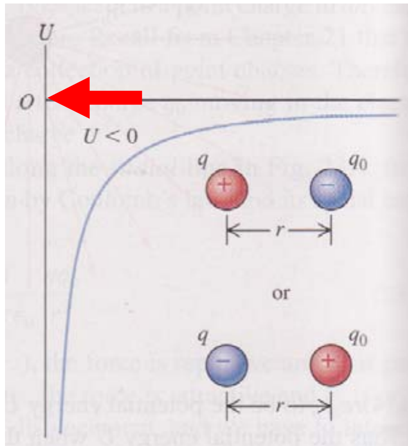
Nature favors situation with minimum energy



Energy is lowered when the charges are as **close** as possible



From University Physics, 11th edition

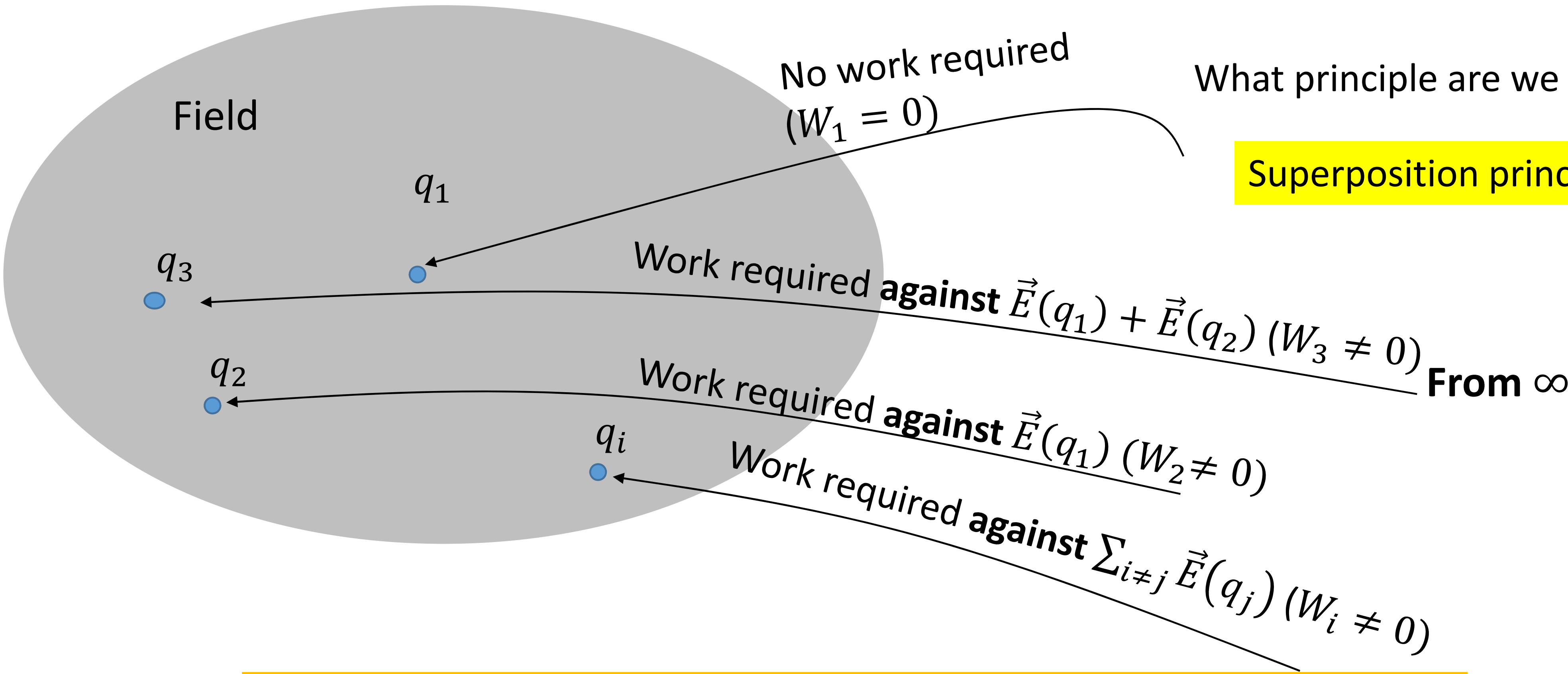


Nucleus predicted unstable !

Classical mechanics

Atom predicted unstable !

From empty space to space with imported charges

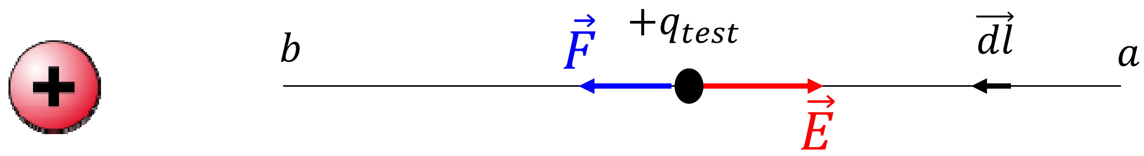


What principle are we applying?

Superposition principle

Why does an external force do a work **AGAINST** the electric field?

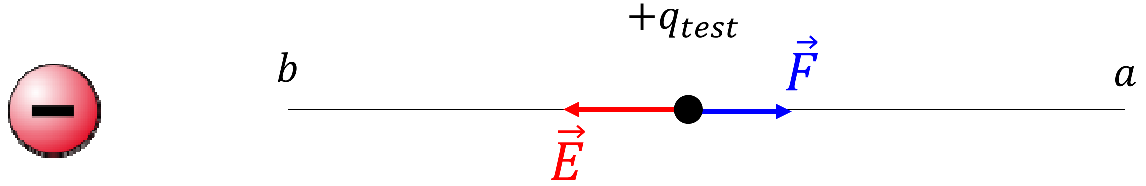
Work required to assemble charges



$$W_{a\to b} = -\int_a^b \vec{F}.d\vec{l} = -\int_a^b q_{test}\vec{E}.d\vec{l}$$

We push harder and harder **Against** the repulsion

$\vec{F}$ is conservative field



$$\frac{W_{a\to b}}{q_{test}} = -\int_a^b \vec{E}.d\vec{l}$$

any path

We retain harder and harder **Against** the attraction

$$\frac{W_{a\to b}}{q_{test}} = V(b) - V(a) = -\int_a^b \vec{E}.d\vec{l}$$

The electric potential V

Charge and motion

- No motion ⇨ **Electrostatic** $\vec{F}_E$
- Steady state motion ⇨ **Electrostatic and Magnetostatic** $q\vec{E} + \vec{F}_B$
- Acceleration ⇨ **electromagnetic wave** $\dfrac{\partial}{\partial t}$

<u>**Concepts of flux**</u> and <u>**circulation**</u> of the vector fields: two fundamental properties

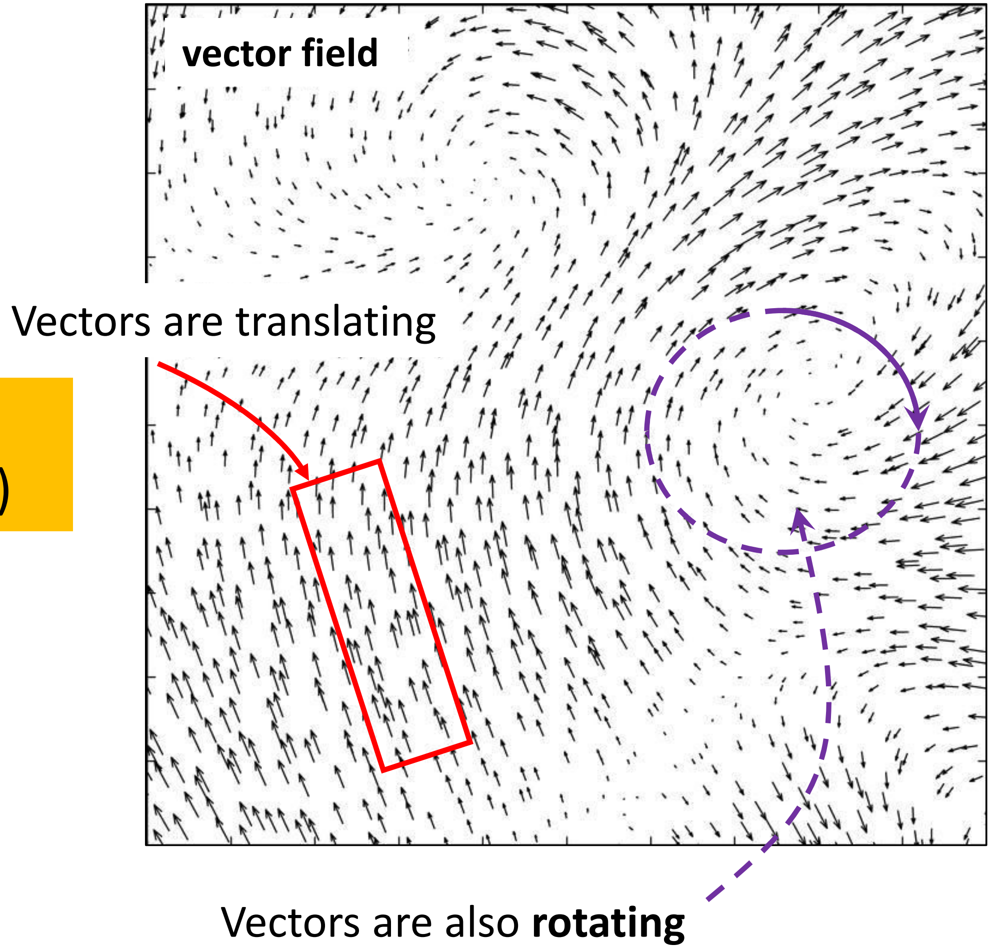
Vector field may undergo

- Translation (vibration is a kind of translation)
- Rotation (circulation along a curved path)

1) Flux through a surface (closed **OR** not)
2) Circulation along a curved path (closed **OR** not)

- Flux
- Circulation

→ Describe the laws of electromagnetism



Concept #10: Flux of the vector fields

Vectors **flowing** in and out of the closed surface

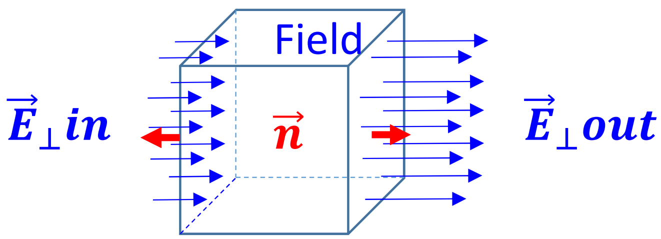
Flux = net outward flow

Flux = (average normal component) **.** (surface area)

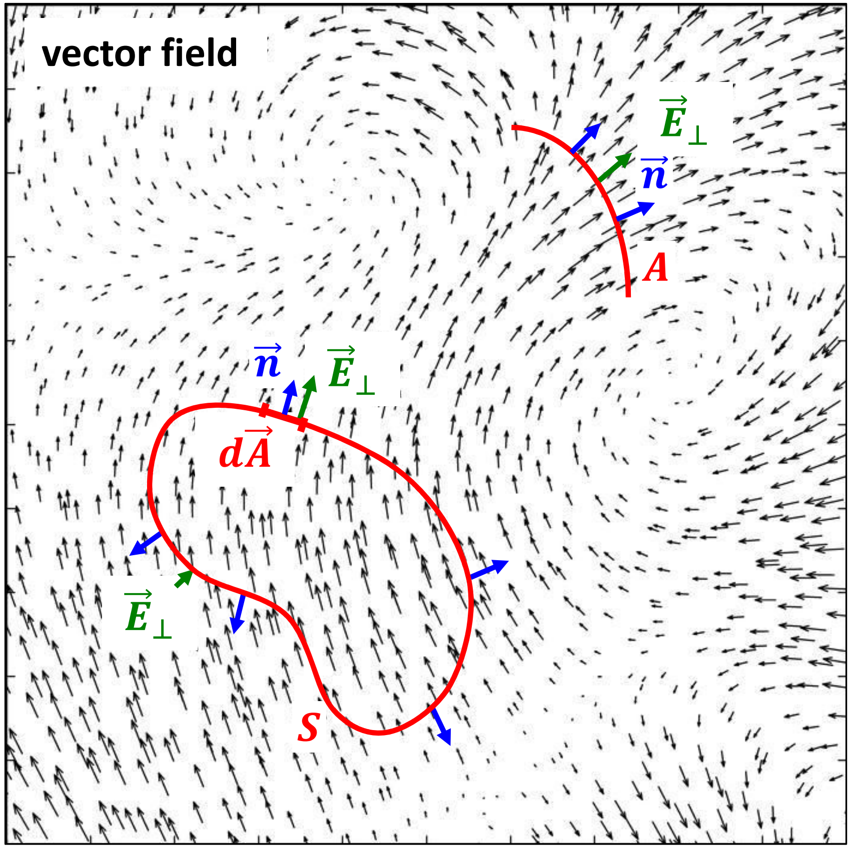
$$\boxed{\int \vec{E}_{\perp} . d\vec{A}}$$

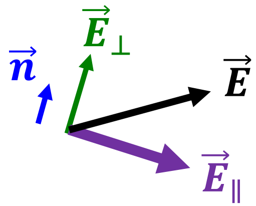
Why normal component only?

Intensity matters !

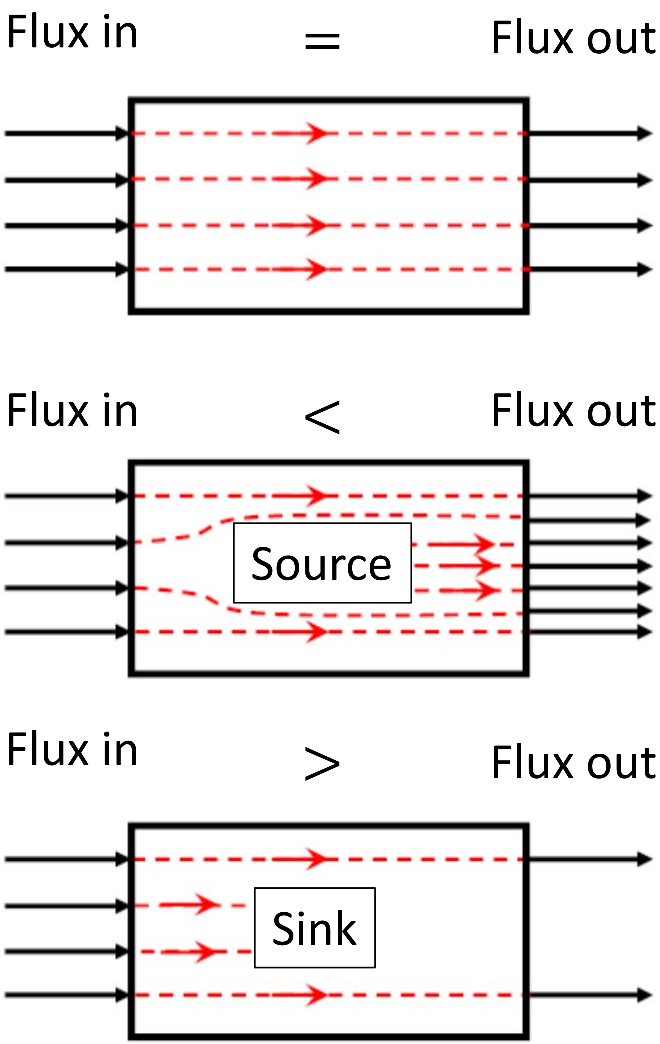


$E_{\perp} in < E_{\perp} out \quad \Rightarrow \quad Flux > 0$

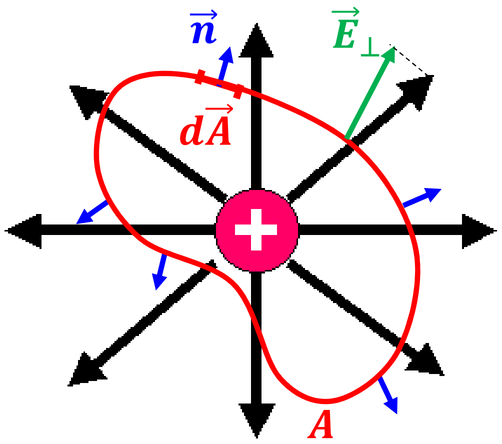




$\vec{E}_{\parallel} . \vec{n} dA = 0$



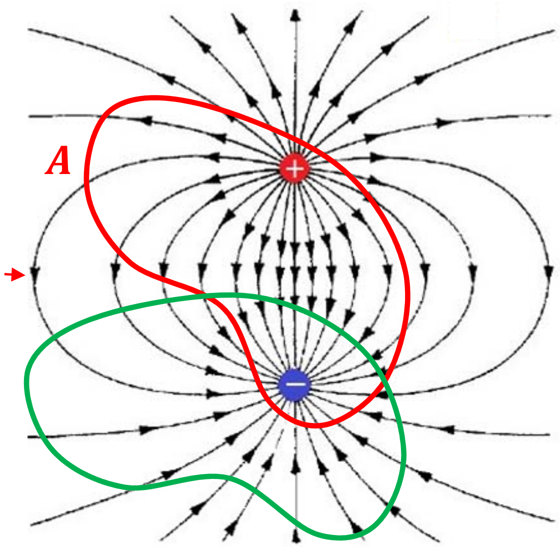
The net flux through a closed surface indicates whether there is a source or sink within the enclosed volume



Source

Flux > 0 ⇔ DIVERGENCE

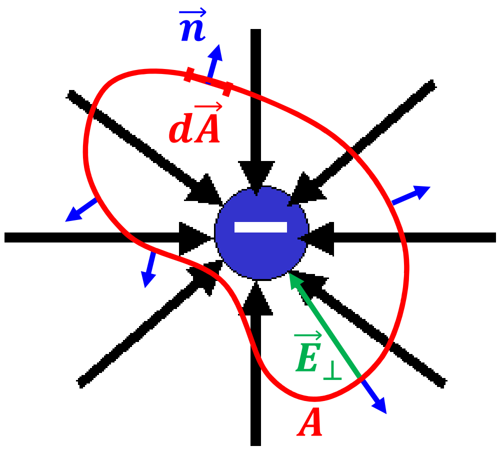
Closed 3D surfaces



Flux = 0

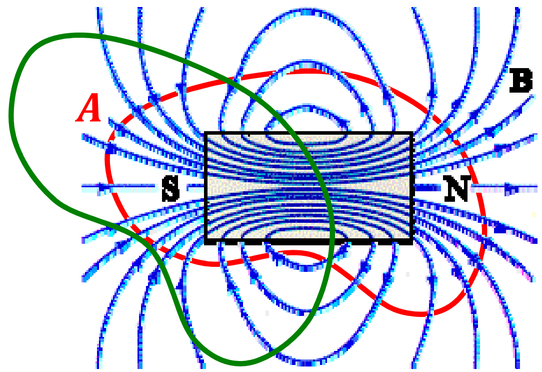
Flux < 0

Electric monopoles exist



Sink

Flux < 0 ⇔ CONVERGENCE
= -DIVERGENCE



Flux = 0

Flux = 0

Inside the magnet lines go from S to N

Magnetic monopoles DO NOT exist

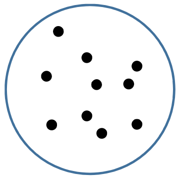
The first Maxwell's equation (in vacuum) ⬄ Gauss law

1) The flux of $\vec{E}$ through any closed surface = $\dfrac{\boldsymbol{net}\ charge\ inside}{\varepsilon_0}$

Electric monopoles exist

To make the dimension coherent

*Question #1



Why we may have charges inside but **NET flux = 0 ?**

Answer to *Question #1

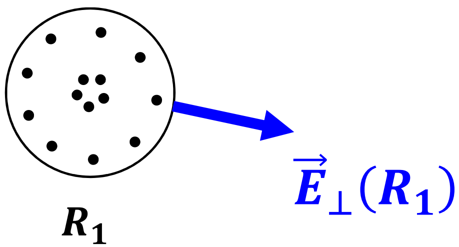
$$\sum_i q_i(>0) = \sum_j q_j(<0)$$

If **Net** charge > 0 ⇨ Divergence ⇨ Source

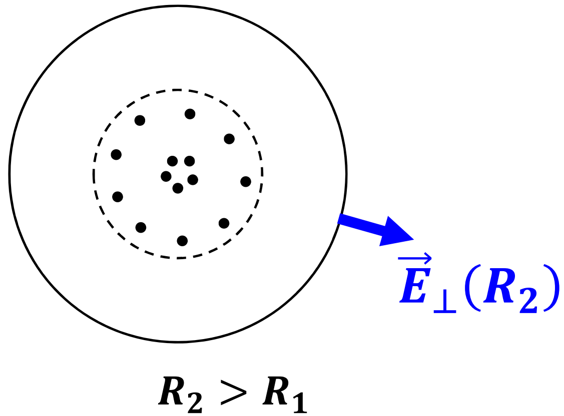
If **Net** charge < 0 ⇨ convergence ⇨ Sink

How powerful and beautiful is the idea of flux?

Net charge inside > 0



Electric field at the surface



Flux $= E_\perp \times 4\pi R^2 = Cte$ ...is conserved through closed surface... ⇨ If the surface increases, the field must decrease

Charge $+q$ and the idea that the field of a point charge is <u>spherically symmetric</u> ⇨ Coulomb's law

Coulomb's law derives from Gauss's law $E_\perp \times 4\pi R^2 = \dfrac{q}{\varepsilon_0}$ ⇨ $E = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{R^2}$

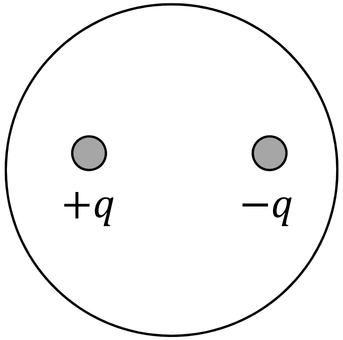
*Question #2

Would Gauss's law exist if the Coulomb law were $\propto 1/r^n$ with $n \neq 2$?

Answer to *Question #2

NO! wait for lecture on Gauss's law

Gauss's law applied to a dipole



Flux = (average normal component) **.** (surface area)

$$\int \vec{\boldsymbol{E}}_{\perp}.\,\boldsymbol{d}\vec{\boldsymbol{A}} = \mathbf{0}$$

What can we say about the Electric field outside the surface?

Not much ! Gauss's law is not of great help in this case and in non <u>symmetric systems</u> in general

Gauss's law + **symmetry** → Helpful in solving problems

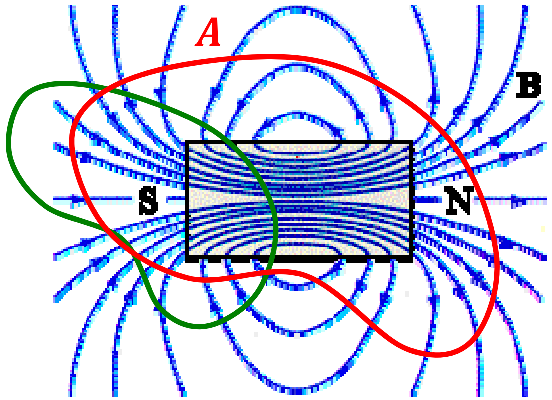
Gauss's law + **asymmetry** → Useless

… Unless some imagination comes about…

The second Maxwell's equation (in vacuum) ⇔ Gauss law

2) The flux of $\vec{B}$ through any closed surface = 0

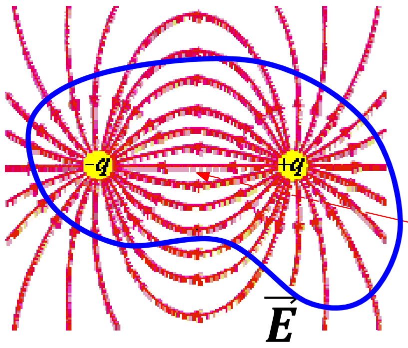
Magnetic monopoles **DO NOT** exist
Field lines close on themselves



Flux = 0

Flux = 0

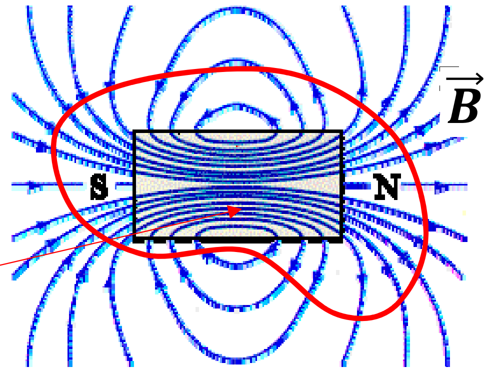
<u>Perfect symmetry</u> of these two laws if we compare an electric dipole with a magnet



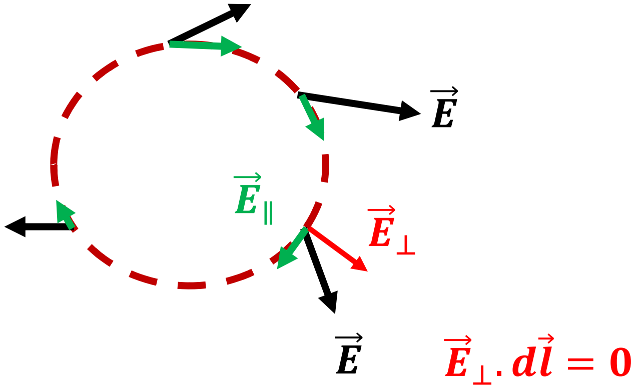
Flux of $\vec{E}$ and $\vec{B}$ = 0

!

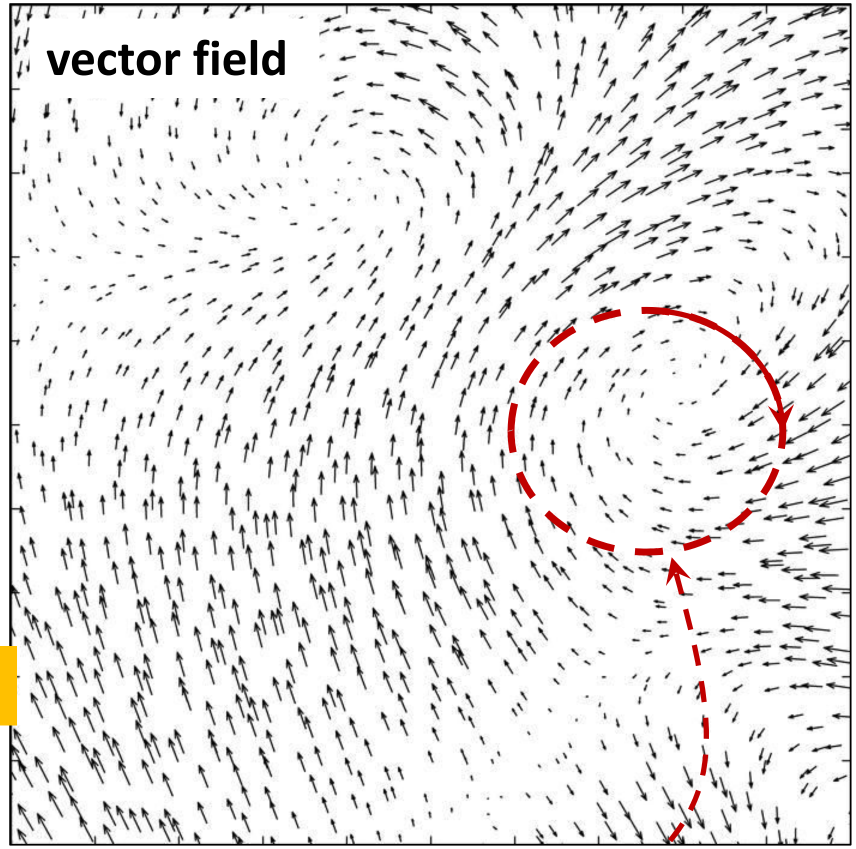
Between the poles direction changes



Concept #11: Circulation of the vector fields



Circulation = <u>net rotational motion</u>



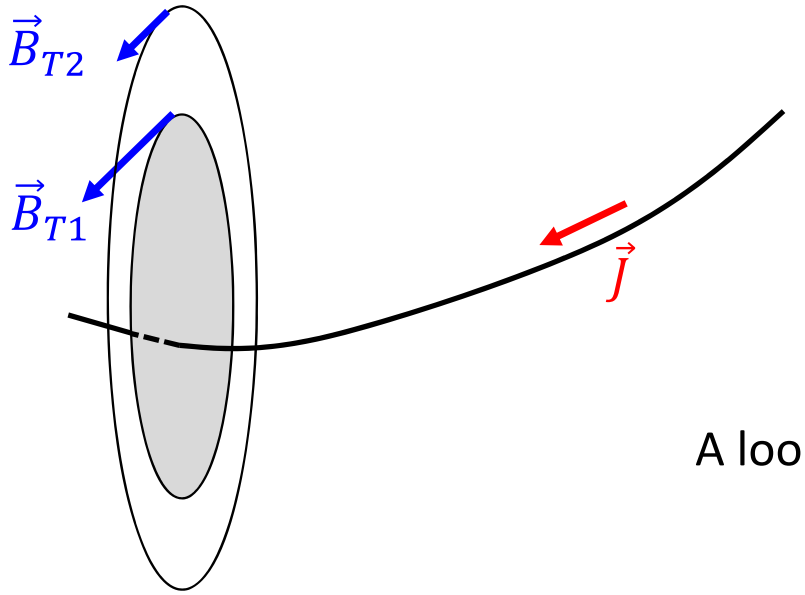
Circulation = (average tangential component) **.** (distance around)

$$\boxed{\int \vec{E}_{\parallel}.d\vec{l}}$$

Why tangential component only?
Intensity matters !

Vectors are **rotating**

How powerful is the idea of circulation?



If $\vec{J}$ is fixed, the circulation must be constant

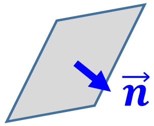
A loop with radius R_2 is larger than a loop with radius R_1

Circulation = (average tangential component) **.** (distance around)

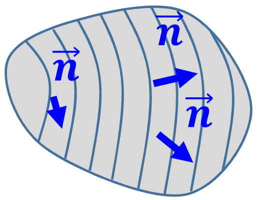
$\vec{B}_{T2} < \vec{B}_{T1}$

Connection of the two concepts: Flux and circulation

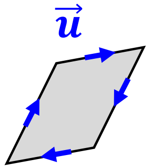
Flux → Open surface



Flux → Closed surface



Circulation → Closed path (loop))



This open surface is bounded by a closed loop

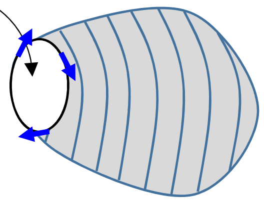
A relation must exist

Flux | Open surface

Circulation | Closed path (loop)

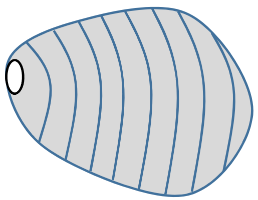
Circulation

opening

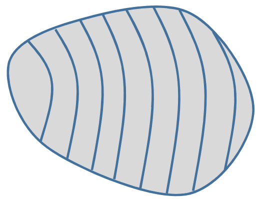


Loop

Closed loop shrinks



What happens to the circulation?



Closed surface

Laws of electromagnetism

$\vec{E}(x, y, z, t)$

Flux through closed surface

Circulation along closed path (loop)

$\vec{B}(x, y, z, t)$

Flux through closed surface

Circulation along closed path (loop)

open surface bounded by a loop

Circulation	$\longleftrightarrow$	Flux
=		=
Line integral		Surface integral

Stokes Theorem

Laws of electromagnetism

Circulation ⟷ Flux

= =

Line integral ⟷ Surface integral

Stokes Theorem

↑

From Stokes to Gauss

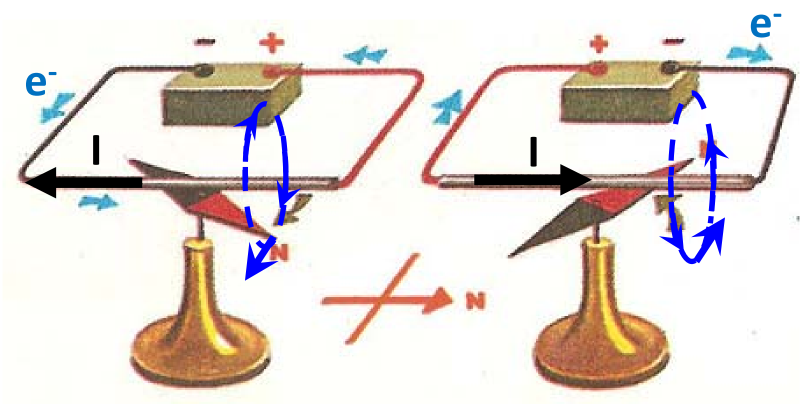
↓

Gauss's theorem

Closed Surface integral ⟷ Volume integral

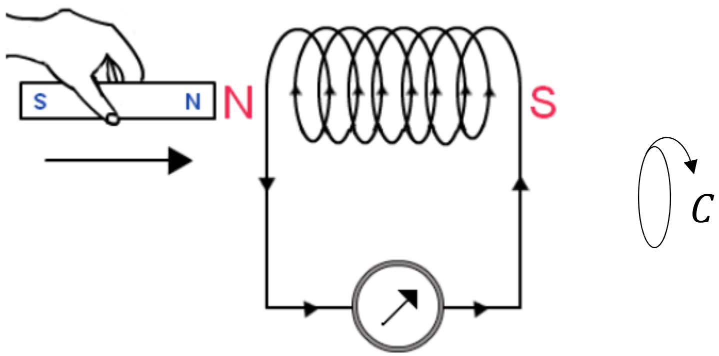
Field induction: Œrsted vs Faraday

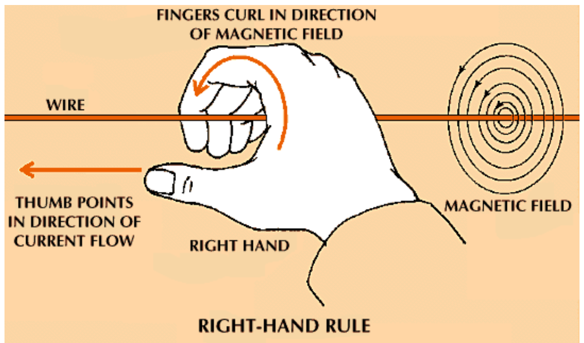
Œrsted's experiment (1820)



Steady or **varying** Current flow generates $\overrightarrow{B}$ ⇨ Lorentz force

Faraday's experiment (1821)

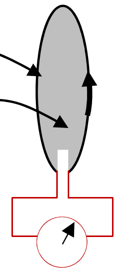




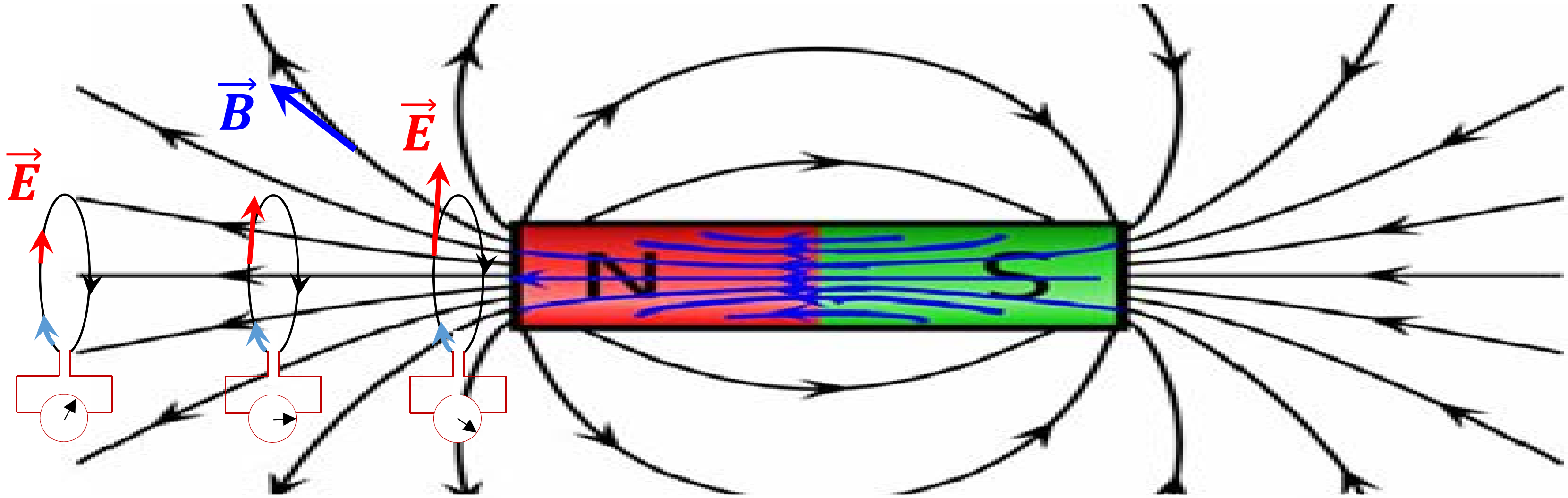
Only Varying $\overrightarrow{B}$ generates $\overrightarrow{E}$ ⇨ Lorentz force responsible for the measured current

The third Maxwell's equation: Faraday's experiment

loop C
Surface S



Circulation of $\vec{E}$ around loop C is responsible for the measured current

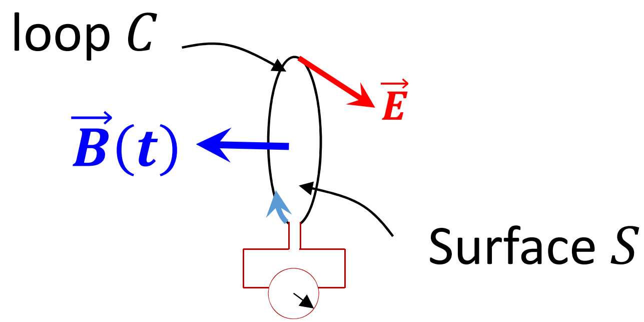


Configurations 1 2 3

Flux 1 < Flux 2 < Flux 3

A relation must exist between the **circulation of $\vec{E}$ around C** and the **flux of $\vec{B}$ through surface S**

The third Maxwell's equation



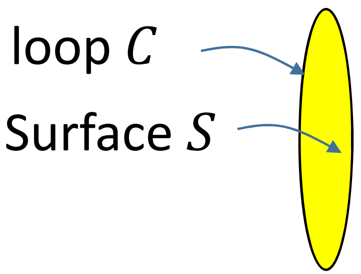
3) Circulation of $\vec{E}$ around C (bordering a surface S) $= \frac{d}{dt}$ (flux of $\vec{B}$ through S)

Sign and circulation directions will be claryfied later

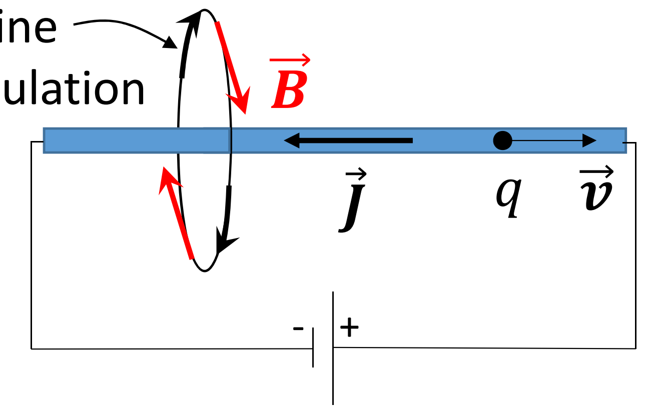
We can foresee Stokes theorem behind this statement

The fourth Maxwell's equation: Œrsted's experiment, Biot & Savart's and Ampere's law

Œrsted's experiment: Biot & Savart's and Ampere's law



- Magnetic field line
- Direction of circulation

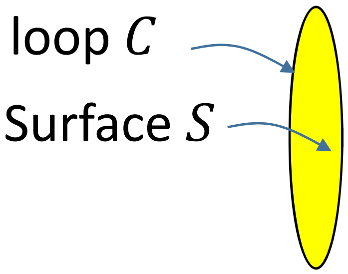


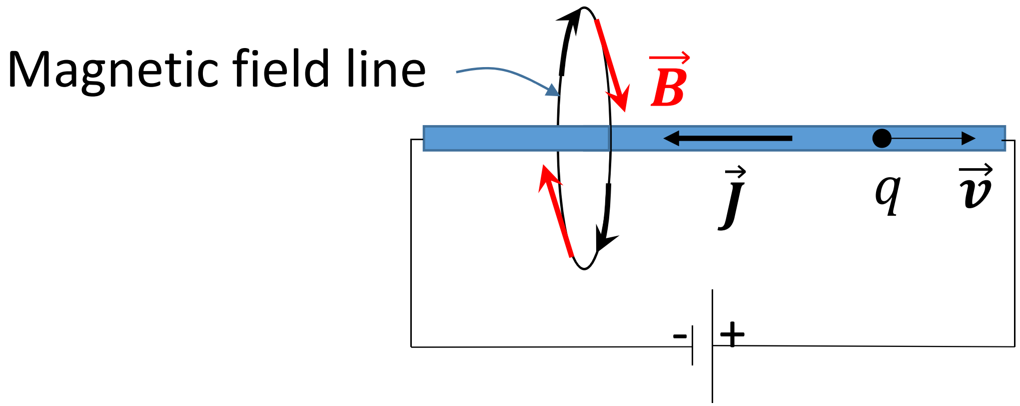
A relation must exist between the **current flow** $\vec{J}$ through **surface S** and the **circulation of** $\vec{B}$ **around** C

Biot & Savart's and Ampere's law

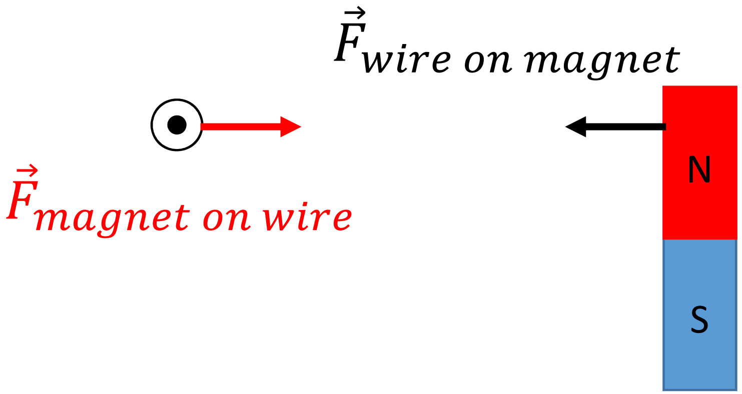
$$4)\ c^2[\text{Circulation of } \vec{B} \text{ around C (bordering a surface } S)] = \frac{flux\ of\ electric\ current\ through\ S}{\varepsilon_0}$$

Œrsted's experiment

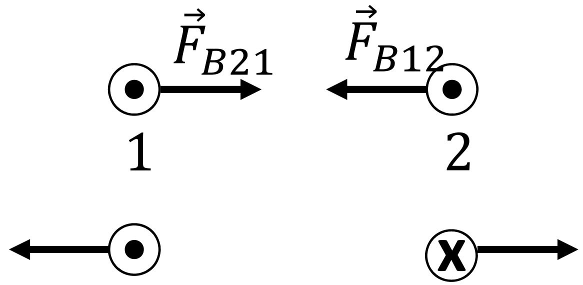


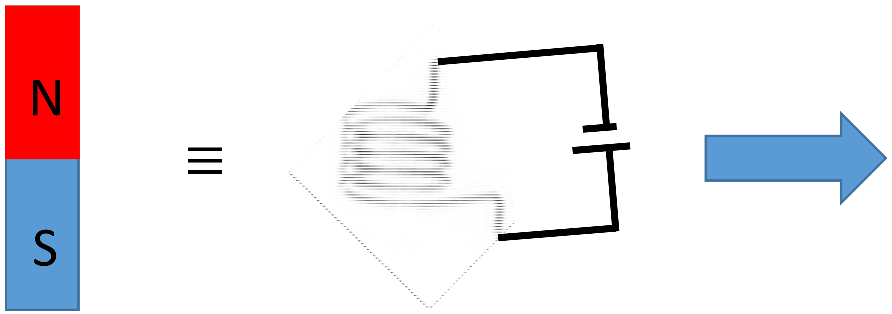


Action - reaction $\quad \vec{F} = q\vec{v} \times \vec{B}$

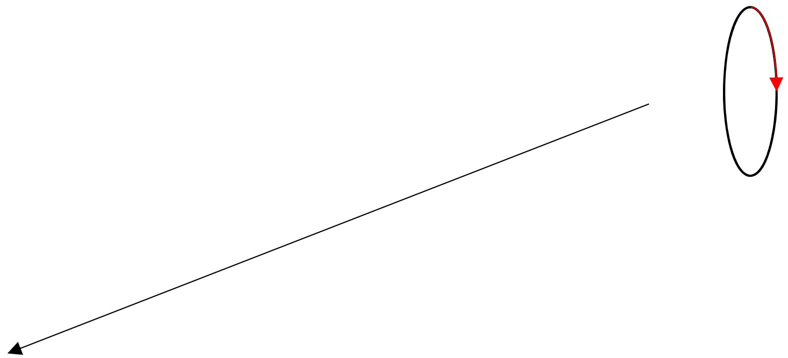


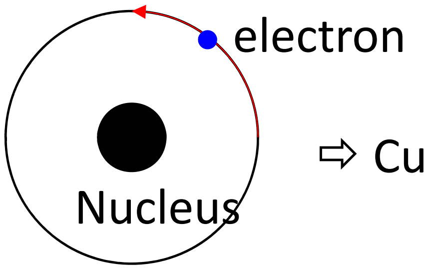
We may replace the magnet by another current carrying wire



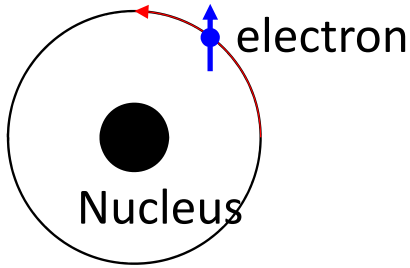


Magnet is made of little permanent current loops





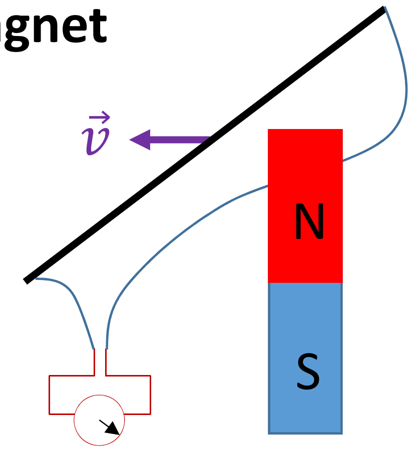
⇨ Current ⇨ $\vec{B}$



Orbital motion

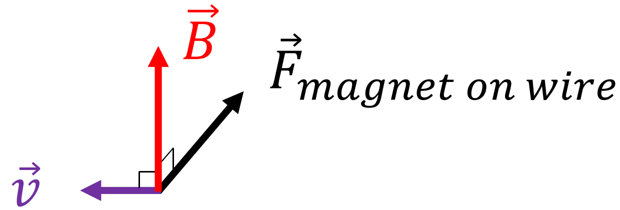
Spin effect

Pulling a conducting wire nearby a magnet



Mechanical motion of electron in the presence of a magnet

Lorentz force



***Question #3 Is that all what happens? Electrons set into motion in the wire and circuit ?

Answer to*question #3

Principle of action-reaction requires that the magnet also moves. In which direction?

Close look at the fourth Ampere's law: Genius idea of Maxwell

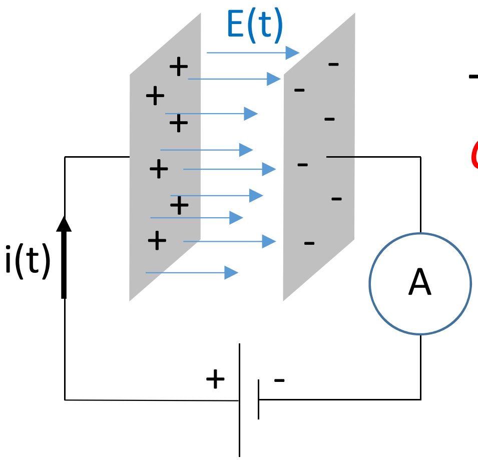
4) $\boldsymbol{c}^2$[Circulation of $\overrightarrow{\boldsymbol{B}}$ around C (bordering a surface S)] = $\dfrac{flux\ of\ electric\ current\ through\ S}{\varepsilon_0}$

Ampere and Biot & Savart's law based on Œrsted's experiment

Something is missing in this relation

Maxwell’s discovery of the displacement current

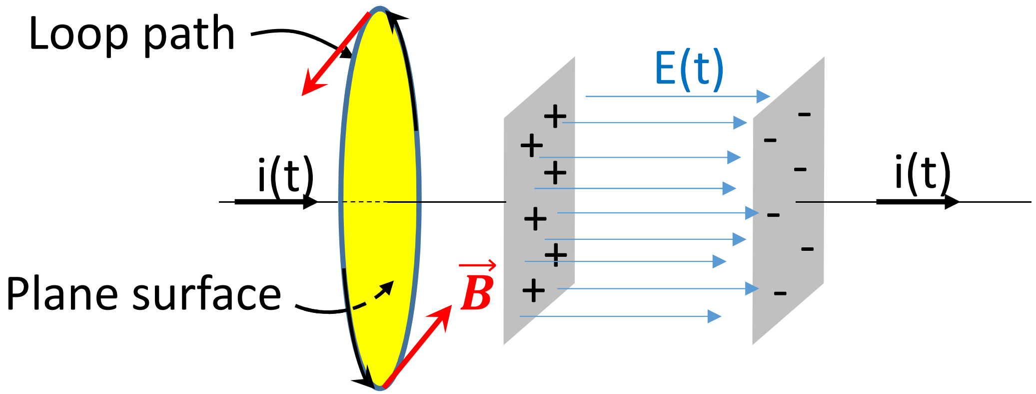
The capacitor problem



This is not a closed circuit **BUT** a current i(t) flows

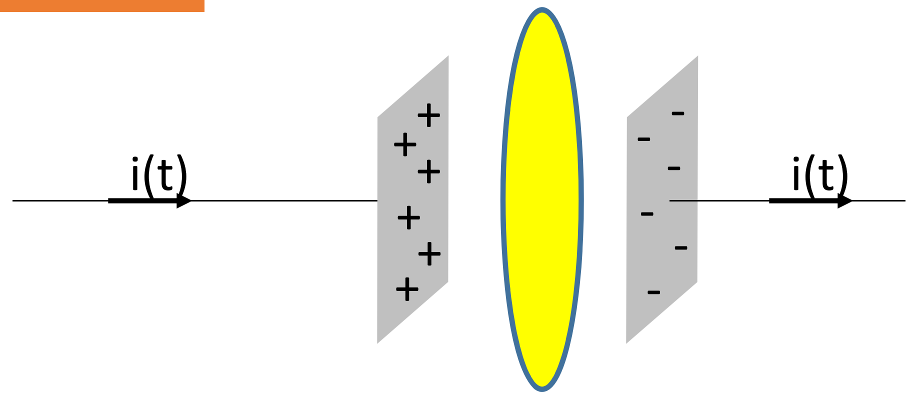
Clearly no electron is jumping from one plate to the other !

Ampere’s law

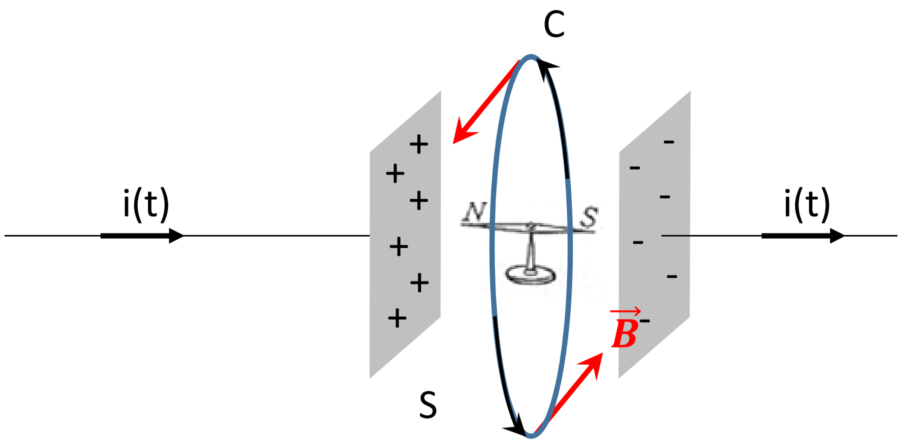


The wire crosses the disk

The wire does **NOT** cross the disk

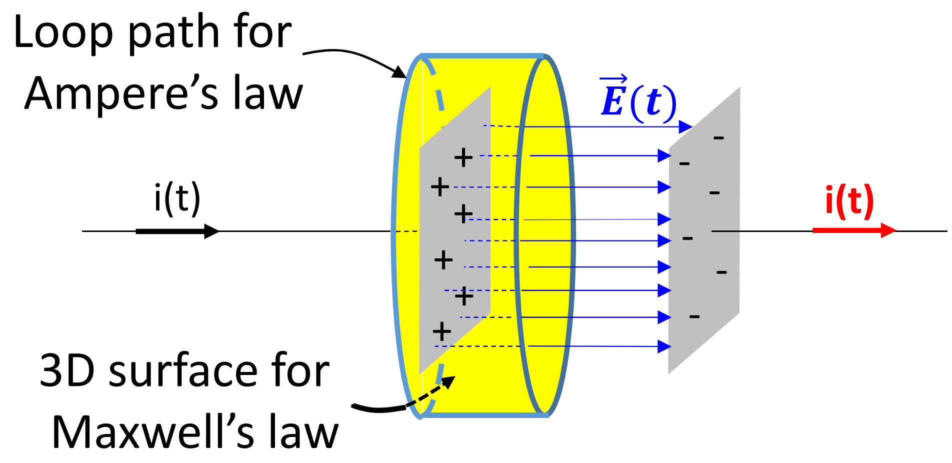


No electron moves through the surface thus **NO** magnetic field ?



compass ROTATES

There is a magnetic field
Where does it come from?



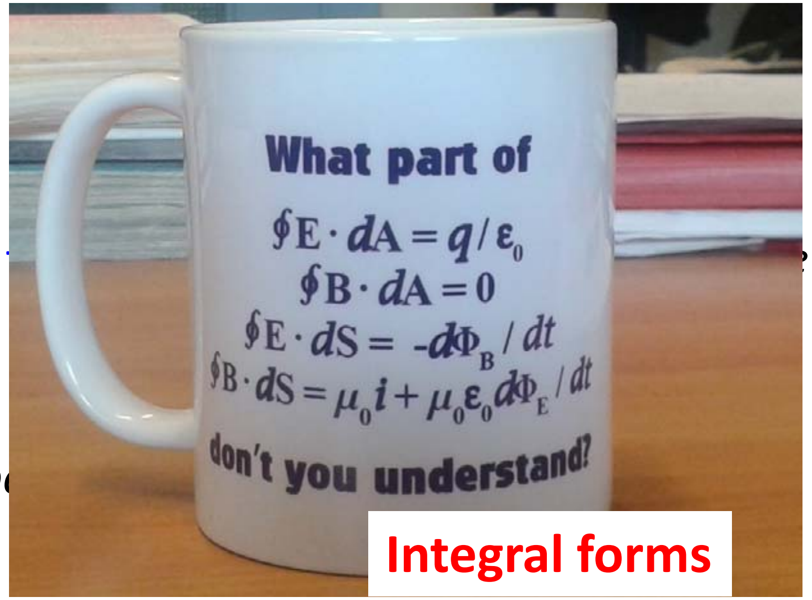
Cylinder open on the left hand side = Maxwell's surface

<u>The wire does not cross it</u>

4) c^2[Circulation of $\vec{B}$ around C (bordering a surface S)] = $\dfrac{flux\ of\ electric\ current\ through\ S}{\varepsilon_0}$ + $\underbrace{\dfrac{d}{dt}\text{(flux of }\vec{E}\text{ through S)}}_{\text{Maxwell's discovery}}$

Summary of the <u>complete</u> set of electrodynamics laws

...Differential form



1) The flux of $\vec{E}$ through any **closed** surface = $\dfrac{net\ charge\ inside}{\varepsilon_0}$

Poisson

2) The flux of $\vec{B}$ through any **closed** surface = 0

Gauss

No magnetic monop

3) Circulation of $\vec{\boldsymbol{E}}$ around a curve (bordering a surface S) = $\dfrac{d}{dt}$ (flux of $\vec{\boldsymbol{B}}$ through S)

Faraday

4) $\boldsymbol{c^2}$[Circulation of $\vec{\boldsymbol{B}}$ around C (bordering a surface S)] = $\underbrace{\dfrac{flux\ of\ electric\ current\ through\ S}{\varepsilon_0}}_{\text{Œrsted, Biot \& Savart Ampere}} + \underbrace{\dfrac{d}{dt}\ (\text{flux of } \vec{\boldsymbol{E}} \text{ through } S)}_{\text{Maxwell}}$

1) The flux of $\vec{E}$ through any closed surface = $\dfrac{net\ charge\ inside}{\mu_0}$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

2) Circulation of $\vec{\boldsymbol{E}}$ around C (bordering a surface S) = $\dfrac{d}{dt}$ (flux of $\vec{\boldsymbol{B}}$ through S)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Stokes Theorem

Gauss's theorem

3) $\boldsymbol{c^2}$[Circulation of $\vec{\boldsymbol{B}}$ around C (bordering a surface S)] = $\dfrac{d}{dt}$ (flux of $\vec{\boldsymbol{E}}$ through S) + $\dfrac{flux\ of\ electric\ current\ through\ S}{\varepsilon_0}$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{\boldsymbol{j}}$$

4) The flux of $\vec{B}$ through any closed surface = 0

$$\vec{\nabla}.\vec{B} = 0$$

Maxwell's discovery that **speed of light** emerges from **static** phenomena !

$$\boxed{c^2 = \frac{1}{\varepsilon_0 \mu_0}}$$

Comments on the so-called universal constants

G: gravitational constant
ε_0: Electric permittivity
μ_0: Magnetic permeability

All appear as proportionality constants

The gravitational force does not depend on the intervening medium

Hence *G* is a universal constant

ε and μ depend on the medium

- Electric permittivity ε describes how an electric field affects and is affected by a medium. It is determined by the ability of a material to polarize in response to an applied field, and thereby to cancel, partially, the field inside the material

- Similarly, magnetic permeability μ is the ability of a substance to acquire magnetization in magnetic fields. It is a measure of the extent to which magnetic field can penetrate matter

They have different values for different media.

strictly speaking ε and μ are not universal constants

$\varepsilon\mu$ gives the speed v of electromagnetic radiation in the medium through $\varepsilon\mu = 1/v^2$

$$\boxed{\text{Index of refraction } n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}}$$

Different media = different index of refraction

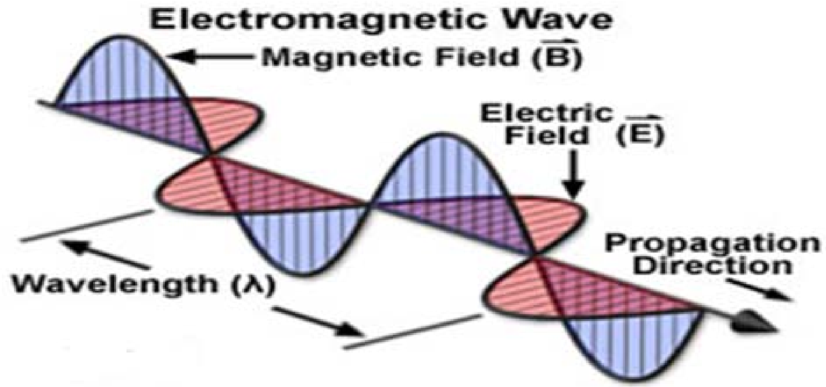
Electromagnetic wave production and radiation

Setting $\vec{J}(t)$ in a circuit

Sets electrons into accelerating motion

$\vec{E}(t)$ $\vec{B}(t)$

Far from the source



Emission of EM waves

$\vec{E}(t)$ and $\vec{B}(t)$ are self sustaining each other

Some interesting questions

***Question #4

Why c^2 appears in the fourth Maxwell's equation?

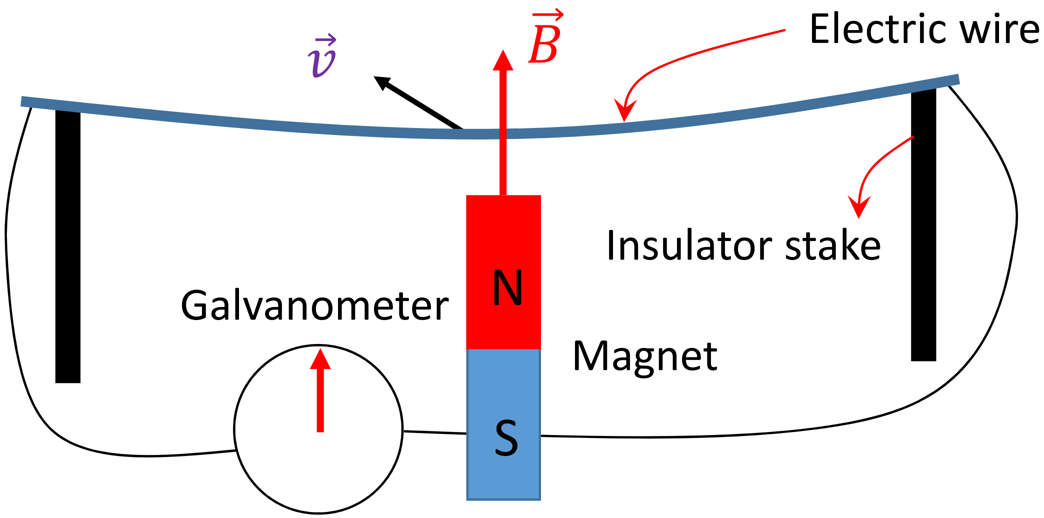
4) $\boldsymbol{c^2}$[Circulation of $\vec{\boldsymbol{B}}$ around C (bordering a surface S)] = $\frac{flux\ of\ electric\ current\ through\ S}{\varepsilon_0} + \frac{d}{dt}$ (flux of $\vec{\boldsymbol{E}}$ through S)

Answer to *Question #4

Because B is a relativistic effect in electricity: Wait for last lecture

Illustration: The electrons in the electric wire are at rest. A magnet nearby has no effect on immobile electrons

- Could we initiate current in the wire ?
- If yes how ?



Move the wire sideway mechanically

Electrons in the wire move: Lorentz force

Current induced in the wire

If the wire is immobile and we move the magnet

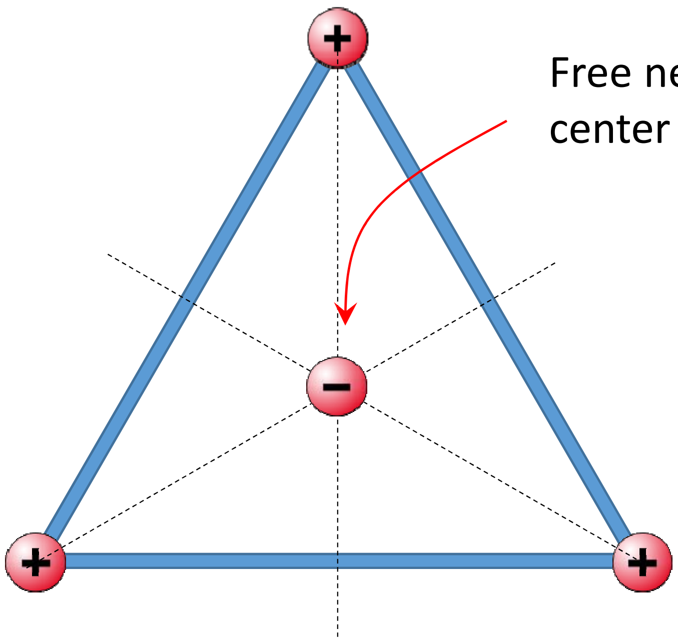
Relativity principle requires that we observe the same effect

Moving B creates a changing E ⇨ Electric force ⇨ Current in the wire

REMARQUABLE CONSEQUENCE
of relativity before its discovery

***Question #5

Three positive charges rigidly bound together



Free negative charge put at the center of the **equilateral** triangle

Is the negative charge

- In a stationary state ?
- In equilibrium ?

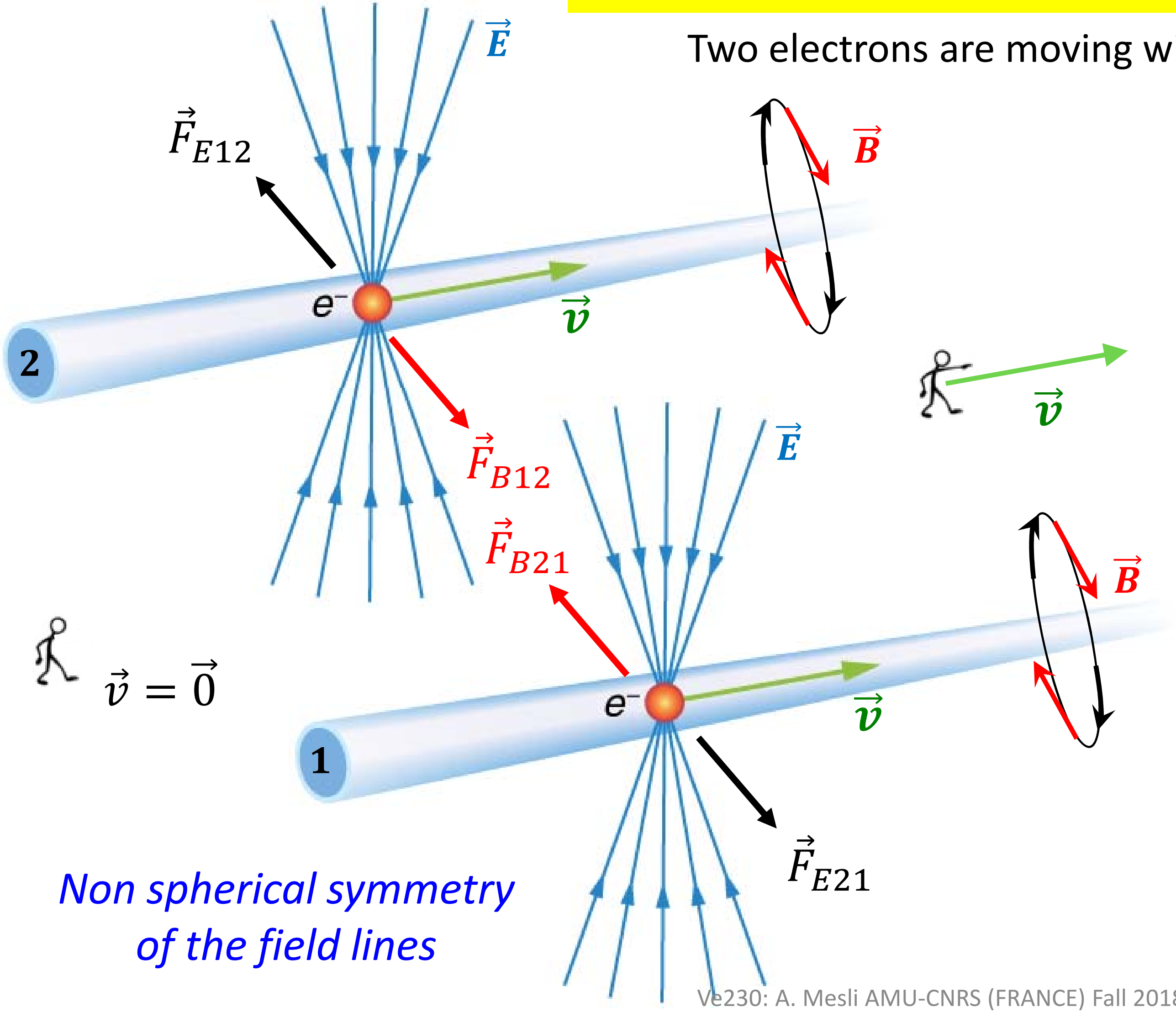
Answer to*question #5

In a stationary state **NOT** in equilibrium

Gauss law will demonstrate this

Are the electric and magnetic fields virtual

Two electrons are moving with the same speed $\vec{v}$ and along parallel paths



Non spherical symmetry of the field lines

For a stationary observer should the electrons

- Repel as predicted from the $\vec{E}$ field or
- Attract as predicted from the $\vec{B}$ field ?

For the same observer riding with the same speed as the two electrons?

Does he still observe:

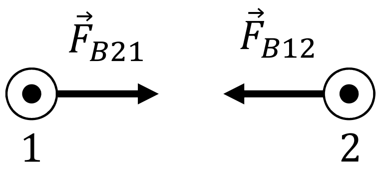
- The electric field ?
- The magnetic field ?

Special relativity:

Whether the observer is immobile or moving with the charges,
All physical laws remain unchanged in any inertial frame

***Question #6

Two wires carrying the same current in the same direction (out of the page)



According to Lorentz force they attract each other via the magnetic field force
BUT <u>what about the Electric forces du to charges inside the wires? Do they play any role ?</u>

Answer to **question #6

- The net charges everywhere in the two wires $Q = 0$ ⇨ The net electrical forces between the wires are zero

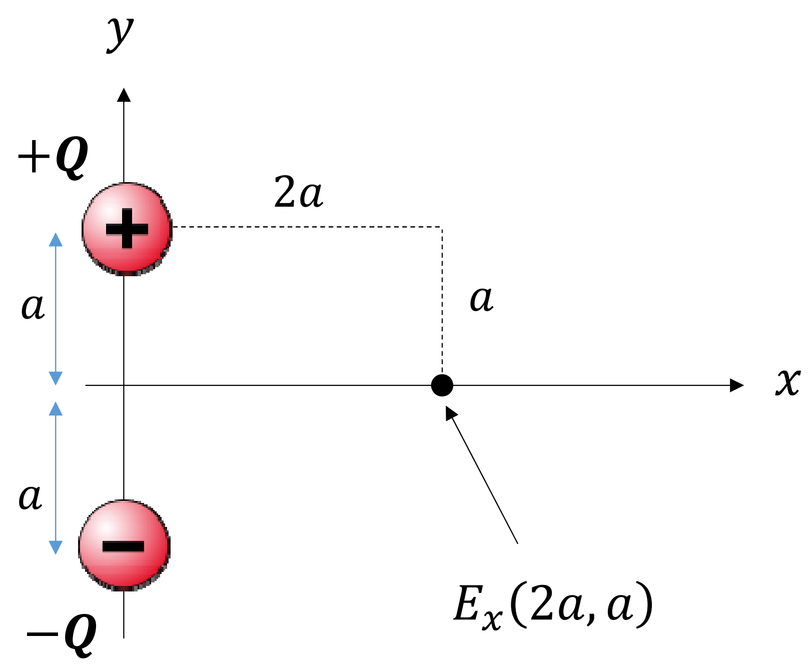
- The relativistic magnetic forces and fields will be of the same sort as in the case of two beams of charges of a single sign. This is true even in the frame of reference of what we think as the moving charges, that is, the electrons

- In the frame of reference moving at the drift velocity of these current-carrying electrons, it is the protons or positively charged ions that are moving in the opposite direction

- Consequently in any frame of reference for current-carrying wires in parallel, the net electrical force will be zero, and there will be a net attractive magnetic force.

Principle of superposition

Consider the following dipole



Which of the following statements about $E_x(2a,a)$ is true?

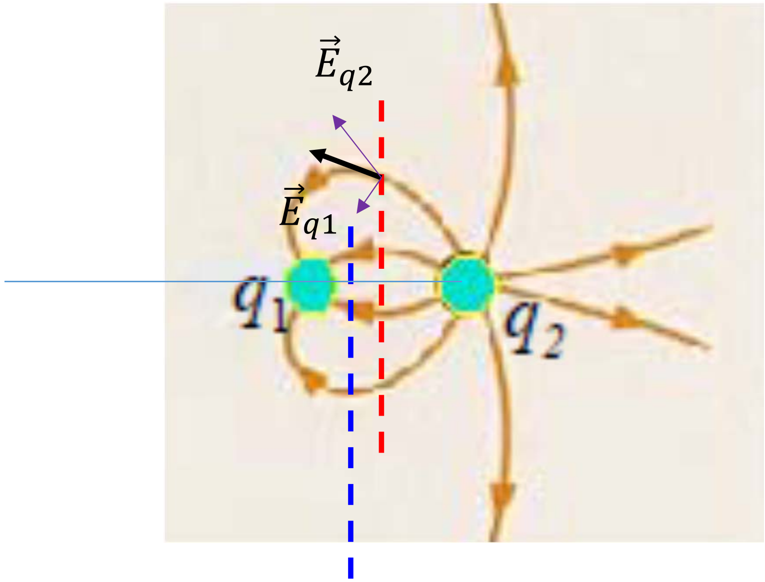
a) $E_x(2a,a) < 0$

b) $\boxed{E_x(2a,a) = 0}$

c) $E_x(2a,a) > 0$

BUT $E_y(2a,a) < 0$

There is **NOT** a single point in space where the field is zero



Question #7

While examining the electric field lines produced by the two charges in the figure, say which statement is true

* 1) q_1 and q_2 have the same sign

*** 2) q_1 and q_2 have the opposite signs and $|q_1| > |q_2|$

3) q_1 and q_2 have the opposite signs and $|q_1| < |q_2|$

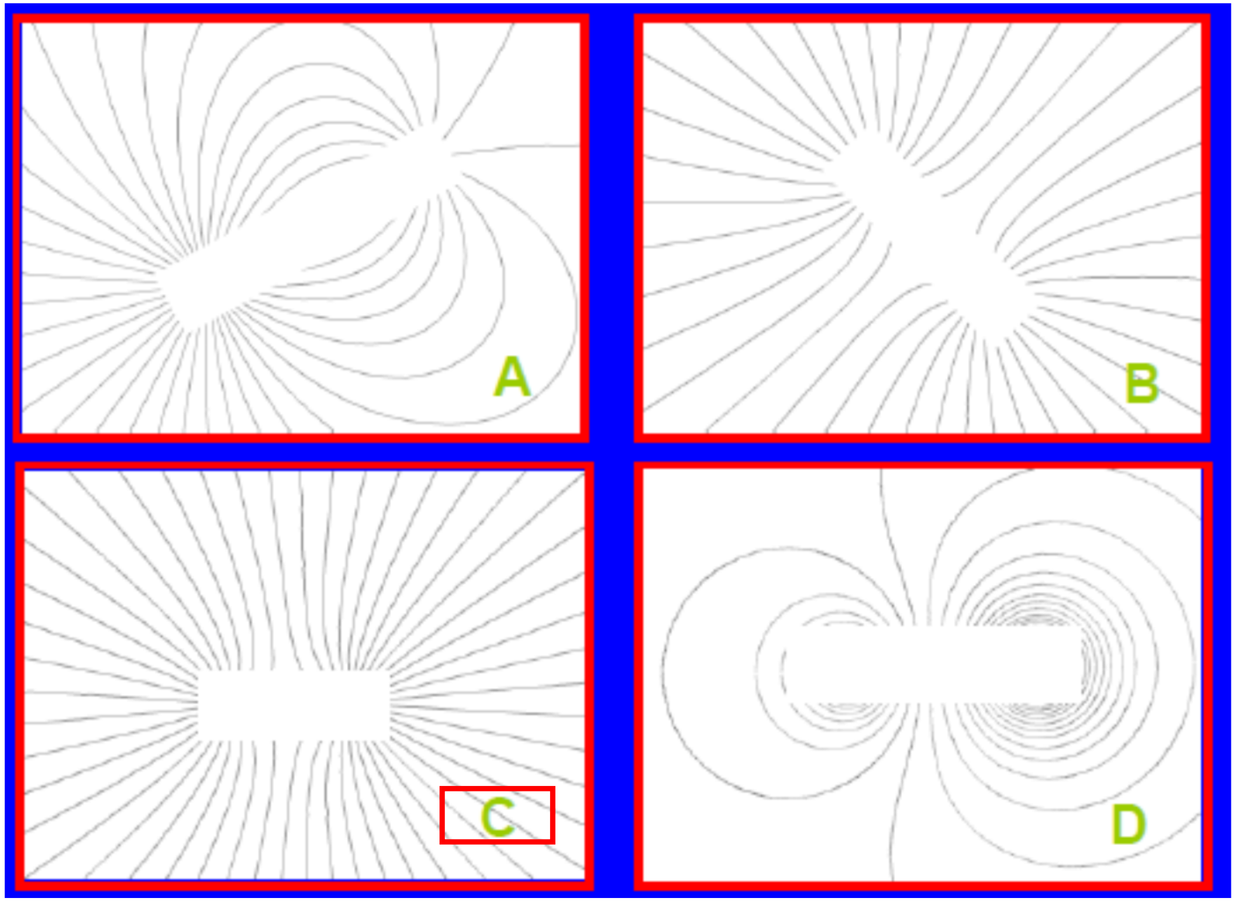
Answers to Question #7

1) **No !** Filed lines go from q_2 to q_1 ➡ $q_2 > 0$ and $q_1 < 0$

2) Along a line of symmetry between the two charges the E-field still has a positive $y -$ component. If the charges were equal, the y-component would be zero therefore $|q_2| \neq |q_1|$

3) The inflection point (the point where the $y -$ component cancels) is close to q_1 ➡ $|q_1| < |q_2|$

*Question #8



Answers to *Question #8

Which of the following field pictures best represent the electric field from the two charges that have the same sign **BUT** different magnitude ?

Electrostatics I: Summary of basic considerations

- Electromagnetism involves particles that have a property called : **Charge**

Charge is as fundamental as **Mass**

- **Mass** is linearly accelerated by gravitational force
- **Charge** is linearly accelerated by electric force

Work is done by → Gravitational force / Electric force

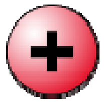
*Question #9

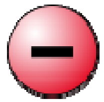
Does magnetic force do work?

Answer to*question #9

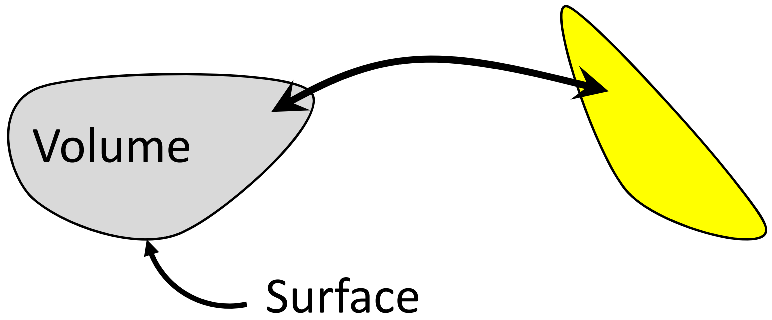
NO because it is always perpendicular to the velocity of the moving charge

- Electric charge: two types

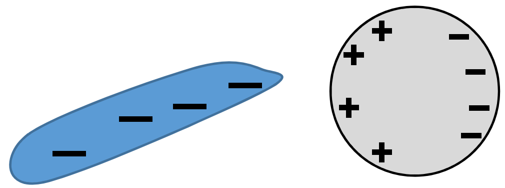




- **Transfer** of charges by contact or friction:
  Charging – discharging process



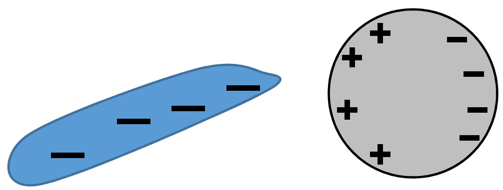
- **Induction** at distance of charges in a body (neutral or initially charged)
- Affects the distribution of charges **NOTHING** else



- Some bodies allow charges to move in the volume and the surface: **Conductors**

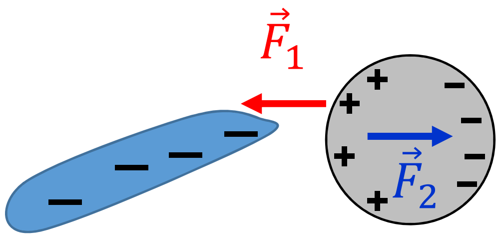
- Some bodies **DO NOT** allow charges to move in the volume and the surface: **Dielectrics**

**Question #10



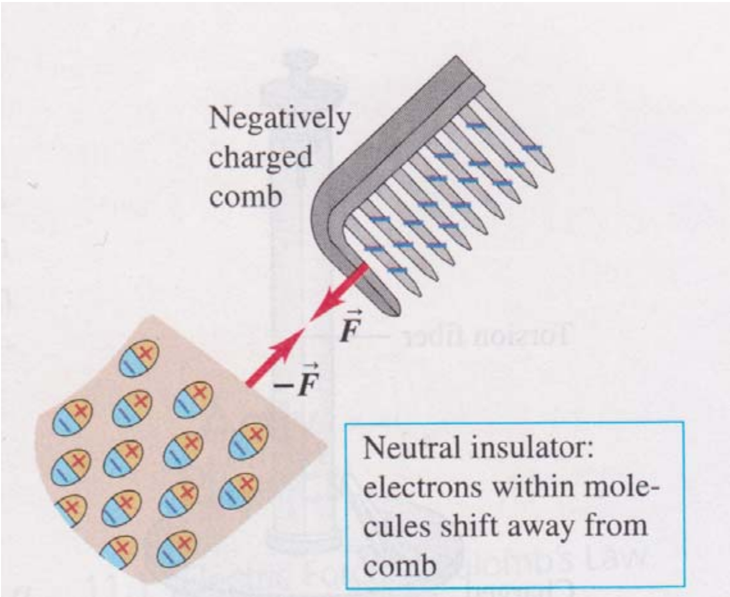
Why can't we accumulate more positive charges by **induction** on the left of the sphere?

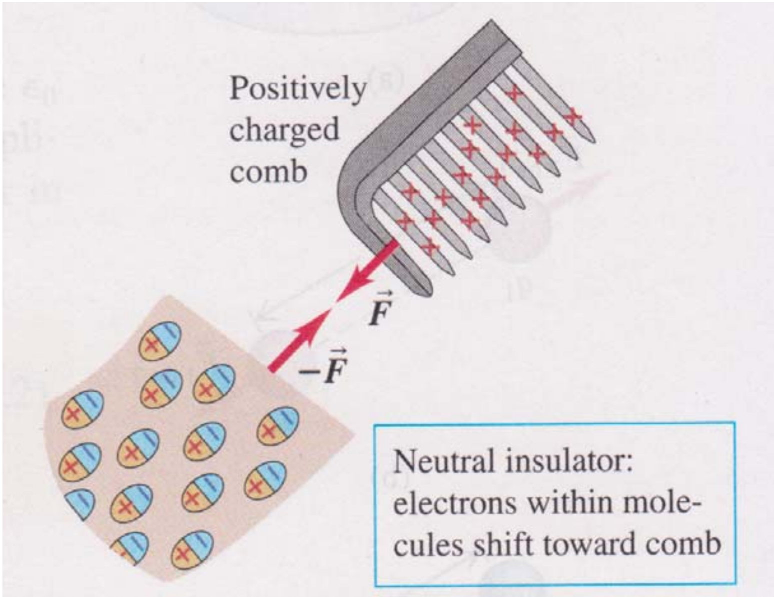
Answer to*Question #10



Stationary state is reached when $\sum_i \vec{F}_i = \vec{0}$

- Dielectrics interact via induced dipoles: **Polarization**





From University Physics, 11th edition

A charged object of either sign (+ or -) attracts **ALWAYS** an uncharged dielectric

Summary for Electrostatic

- **Charge (+ and -)**
- **Electric force**
- **Coulomb's law ⇨ Gauss's law**
- **Electric field $\vec{E}(r)$**
- **Field lines**
- **Electric potential $V(r)$**
- **Conservative field**
- **Electric potential energy**
- **Potential lines**

From an empty space …

…to the presence of objects of different nature:

Impact of the external field

- **Induction**
- **Conductors**

- **Polarization**
- **Dielectrics**

Boundary considerations

Static

If $\frac{\partial}{\partial t} = 0$ (no time dependence) in all four Maxwell's equations

Electrostatics

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

Magnetostatics

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{J}}$$

Two completely different domains

Electrostatics

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$ $\Leftrightarrow$ Electrostatic field is conservative

$$\vec{E} = -\vec{\nabla}\mathrm{V}$$

$$\vec{\nabla}.\vec{E} = \vec{\nabla}.\left(-\vec{\nabla}\mathrm{V}\right) = -\nabla^2 V = \frac{\rho}{\varepsilon_0}$$

$\Rightarrow$ Poisson equation $\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0}$

Next Lectures

- **Electrostatics I: Basic concepts**
- Position of the problem: Coordinate systems
- Scalar versus Vector fields: Operators
- Electrostatics II: Gauss law
- Dipoles
- Conductors
- Dielectrics
- Electrostatic energy
- Magnetostatic
- The magnetic field and its applications
- The vector potential
- Induced currents
- Law of induction
- Maxwell equations
- Wave polarization and propagation
- Relation to special theory of relativity