JOINT INSTITUTE
交大密西根学院

MODERN PHYSICS (Summer 2020)

Problem Set 2

Due: 2 June 2020, 12.30 p.m.

Problem 1. A particle with rest mass m moves in an inertial frame of reference S. At an instant of time its velocity in S is found to be $\mathbf{u} = \left(\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}\right) c$.

(a) Find the 3×3 matrix representing the relativistic mass tensor of this particle at that instant of time.

(b) If, at that same instant of time, the acceleration $\mathbf{a}$ of the particle is along the direction $(0, 0, 1)$ in S, what is the direction of the force $\mathbf{F}$ this acceleration is due to? Draw a figure indicating the directions of all three vectors $\mathbf{u}, \mathbf{a}$, and $\mathbf{F}$ in this problem and comment on the result.

(*2 + 2 points*)

Problem 2. A particle with rest mass m_1 and kinetic energy K_1 collides with a resting particle of rest mass m_2. After the collision the two particles move together as a single particle. Find the rest mass of the new particle and its velocity.

(*3 points*)

Problem 3. A photon collides with a stationary electron of rest mass m. Is it possible for the photon to impart all its energy to the electron? Explain.

(*3 points*)

Problem 4. The angular momentum of an isolated system is constant (when referred to any origin). Within the Bohr model derive an expression for the angular momentum carried L_{ph} away by a photon emitted in the transition of a hydrogen atom from the stationary state with n to another stationary state with n'.

(*1 point*)

Problem 5. Derive the Wien displacement law $\lambda_m T = \text{const}$. Calculate the value of the constant.

Hint. Use a computer to solve the resulting transcendental equation.

(*3 points*)

Problem 6. The maximum kinetic energy K_C given to the electron in a Compton scattering event plays a role in the measurement of gamma-ray spectra using scintillation detectors. The maximum is referred to as the *Compton edge*.

(a) Show that

$$K_C = \frac{h\nu}{1 + \frac{mc^2}{2h\nu}},$$

where ν is the incident phonon's frequency, m the rest mass of the electron, c speed of light, and h is the Planck's constant.

(b) Suppose that the Compton edge in a particular experiment is found to be 520 keV. What were the wavelength and energy of the incident gamma rays?

(*3 + 1 points*)

Problem 7. In a double-slit experiment, a detector traces across a screen placed at the y axis. If one slit is closed, the amplitude

$$\Psi_1(y,t) = \frac{1}{\sqrt{2}} e^{-\frac{y^2}{2}} e^{i(\omega t - ay)}$$

is measured, and if the other slit is closed, the amplitude

$$\Psi_2(y,t) = \frac{1}{\sqrt{2}} e^{-\frac{y^2}{2}} e^{i(\omega t - ay - by)}$$

is measured. What is the intensity pattern along the y axis if both slits are open?

(*3 points*)