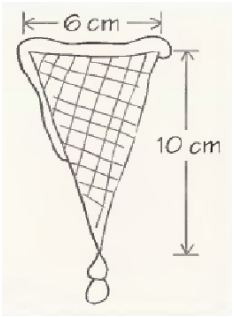
Problem Set 11

Due: 4 August 2017, 12.30 p.m.

Problem 1. An ice-cream cone (shown in the figure below) is filled with melted ice-cream of density $\varrho = 1.2$ g/cm^3. The cone has a diameter $D = 6$ cm at the larger end and is $l = 10$ cm long. Find the pressure at the bottom of the cone.

If a small hole of diameter $d = 1$ mm is opened at the bottom, the ice cream starts to run out. Ignoring the viscosity of the melted ice-cream, find the amount of time it takes the ice cream to run out. Assume that the fluid speed is zero at the top.

(*5 points*)



Problem 2. An open water tank stands on a plane surface. The water surface in the tank is a height h above the plane. A small hole is opened up at a depth y below the surface of the water

(a) Show that the jet of water will hit the plane surface a distance D from the tank, where $D = \sqrt{4y(h-y)}$.

(b) Show that the hole should be placed at a depth $y = h/2$ for the jet to cover a maximum horizontal distance.

(*2 + 2 points*)

Problem 3. A thin, uniform rod has length L and mass M. Calculate the magnitude of the gravitational force the rod exerts on a particle with mass m that is placed at a point along the axis of the rod a distance x from one end. Discuss the limit $x \gg L$?

(*2 points*)

Problem 4. Consider a ring-shaped body and a particle with mass m placed a distance x from the center of the ring, along the line through the center of the ring and perpendicular to its plane (see figure below).

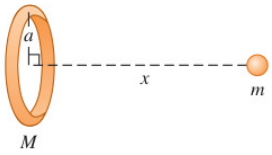
(a) Calculate the gravitational potential energy U of this system. Take the potential energy to be zero when the two objects are far apart.

(b) Show that your answer to part (a) reduces to the expected result when $x \gg a$.

(c) Find the magnitude and direction of the force on the particle, and show that your answer reduces to the expected result when $x \gg a$.

(d) What is the potential energy and the force when $x = 0$?

(*2 + 1/2 + 2 + 1/2 points*)



Problem 5. Suppose you could drill a tunnel through a uniform planet with radius R and mass M, so that it passes through the planet's center (*i.e.* it is drilled along a diameter). Find the equation of motion of an object of mass m that is dropped into the tunnel (without initial velocity) and show that it describes a harmonic oscillator. Find the period of the oscillations and compare it with a period of a satellite orbiting around the planet close to its surface.

Will the answer change if the tunnel is drilled at an angle to the diameter, so that it does not pass through the center? Explain.

(*5 points*)