Problem Set 1

Due: 25 May 2017, 8 a.m.

Problem 1. Estimate the number of piano tuners in Beijing. List all steps of your estimation procedure.

(*3 points*)

Problem 2. The formula for the period T of an object, orbiting close to the surface of a planet, is of the form $T = k \cdot \varrho^A \cdot G^B$, where k is a dimensionless constant, ϱ is the planet's density of mass, and G is the gravitational constant. Find the numbers A and B.

(*3 points*)

Problem 3. *True or false?* If the magnitude of vector $\mathbf{u}$ is less than the magnitude of vector $\mathbf{w}$, then the (Cartesian) x component of $\mathbf{u}$ is less than the x component of $\mathbf{w}$.

Briefly explain your answer.

(*1 point*)

Problem 4. Positions of two particles at an instant of time are given by the vectors $\mathbf{r}_1 = 4\hat{n}_x + 3\hat{n}_y + 8\hat{n}_z$ and $\mathbf{r}_2 = 2\hat{n}_x + 10\hat{n}_y + 5\hat{n}_z$, respectively (in the units of m). Find

(a) the magnitudes of these vectors,

(b) the displacement vector $\mathbf{r}_{12}$ from the position of particle 1 to the position of particle 2 and the unit vector in the direction of the vector $\mathbf{r}_{12}$.

(c) the angles between all pairs of vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_{12}$ (give your answer both in radians and degrees),

(d) the orthogonal projection (vector) of the vector $\mathbf{r}_2$ onto the vector $\mathbf{r}_1$,

(e) the cross product $\mathbf{r}_{12} \times \mathbf{r}_1$.

(*1/2 + 1/2 + 1 + 1 + 1 points*)

Problem 5. A vector $\mathbf{n} = \mathbf{n}(t)$ has unit length for any instant of time t.

(a) Is the magnitude of $\dot{\mathbf{n}}$ equal to one, too? If so, prove it, otherwise find a counterexample.

(b) Show that, for any t, the vectors $\mathbf{n}$ and $\dot{\mathbf{n}}$ are perpendicular to each other.

(*2 + 2 points*)

Problem 6. A particle moves along the x axis with velocity $v_x(t) = v_0 \exp(-t/\alpha)$, where $v_0 = 10$ m/s and $\alpha = 3$ s. At the initial instant $t = 0$ s, the particle is at $x = 0$ m.

(a) Find the acceleration a_x of the particle at any instant of time t.

(b) Find the time after which the particle stops.

(c) What is the distance traveled by the particle from $t = 0$ s until it stops?

(d) Sketch the graphs $a_x(t)$, $v_x(t)$, and $x(t)$.

(e) Find the particle's average velocity between $t = 0$ and $t = 10$ s.

(f) What is its average speed in the same time interval?

(1/2 + 1/2 + 1 + 3/2 + 1/2 + 1 points)

Problem 7. A tourists kayaks from marina A down the river to marina B in 3 hours, and from B to A in 6 hours, paddling always with the same speed with respect to the river. How much time does he need to get from A to B without paddling?

(3 points)

Problem 8. A spare paddle drops from a fisherman's canoe traveling up the river exactly at the moment as it passes under a bridge. After one hour of paddling the fisherman realizes that the paddle is missing He turns around and paddles his canoe back to find the paddle 6 km down the bridge. Find the speed of the river's current.

Assume that the fisherman paddles always with the same speed with respect to the river. Clearly indicate the frame of reference the problem is being solved in.

(4 points)