VP390 RECITATION CLASS

TWO

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Relativistic momentum

Newton's second law is not valid in relativistic region.

- The acceleration should be transformed in Lorentz transformation.
- If Newton's second law is still valid, then everything can exceed the speed of light after a enough long time of acceleration.

Modification:

$$\overline{F} = \frac{d\overline{p}}{dt}$$

Redefine the momentum and find its derivative with respect to time.

Rest mass

The momentum of a particle moving with velocity $\boldsymbol{u}$ is given by:

$$\bar{p} = \frac{m(0)\bar{u}}{\sqrt{1-u^2/c^2}}$$

where $m(0)$ is known as the **rest mass** of the particle.

- what will happen if $u/c \rightarrow 0$?
- if $u=c$ and p is still finite, then what condition must be satisfied? And is there any real case?

Equation of motion

Plug the relativistic momentum into the following equation:

$$\overline{F} = \frac{d\overline{p}}{dt}$$

The final result is given by:

$$F_i = m_{ij}\dot{u}_j = m(u^2)[\delta_{ij} + \frac{1}{c^2 - u^2}u_i u_j]\dot{u}_j$$

where mij is known as the **mass tensor**.

Generally, is the force still parallel to the acceleration? And in what case it will?

Relativistic energy

From the work-kinetic energy theorem:

$$\delta W = \bar{F} \cdot d\bar{r} = dK$$

Then the rate of work done should be equal to the rate of kinetic energy change:

$$\frac{\delta W}{dt} = \bar{F} \cdot \frac{d\bar{r}}{dt} = m_{ij}\dot{u}_j u_i = \frac{dK}{dt}$$

Plug in the mass tensor and do the derivative:

$$\frac{dK}{dt} = \frac{d}{dt}[m(u^2)c^2] \Rightarrow K = \frac{mc^2}{\sqrt{1-u^2/c^2}} + \text{const.}$$

How to obtain the constant?

Relativistic energy

$$K = \frac{m(0)c^2}{\sqrt{1-u^2/c^2}} - m(0)c^2 = [m(u) - m(0)]c^2$$

$$K + m(0)c^2 = \frac{m(0)c^2}{\sqrt{1-u^2/c^2}} = E$$

where E is the total energy. And what is E when the particle is at rest? i.e. $K=0$.

$$\boxed{E = mc^2}$$

This formula indicates that mass and energy can be converted to each other.

Energy & Momentum

Defne the four–momentum as:

$$p^{\mu} = (\frac{E}{c}, \bar{p})$$

For massive particles ($m>0$):

$$\bar{p} = \frac{m\bar{u}}{\sqrt{1-u^2/c^2}}, \quad E = \frac{mc^2}{\sqrt{1-u^2/c^2}}, \quad p^{\mu} = mu^{\mu}$$

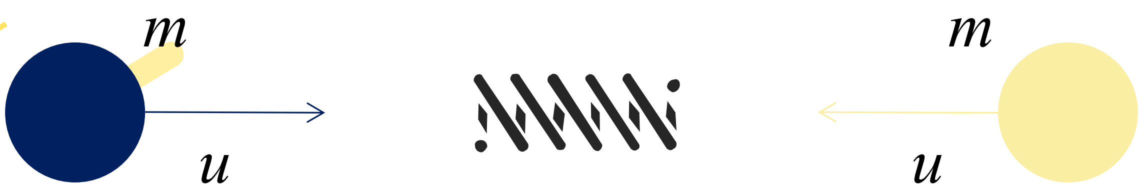
$$p^{\mu}p_{\mu} = (\frac{E}{c})^2 - p^2 = \frac{(mc)^2 - m^2u^2}{1-u^2/c^2} = m^2c^2$$

$$E^2 = p^2c^2 + m^2c^4 \quad \text{or} \quad E = \sqrt{p^2c^2 + m^2c^4}$$

What about particles which has a speed closed or equal to the speed of light?

Exercise 1

Two identical balls with mass m come towards each other at speed u in the laboratory FoR. There is a spring in the middle between them, and can lock both balls immediately when they collide with the spring. Please find the rest mass of the system after the collision.



Exercise 2

An elementary particle of mass M completely absorbs a photon, after which its mass is $1.01M$.

(a) What was the energy of the incoming photon?

(b) Why is that energy greater than $0.01Mc^2$?

Exercise 3

Two identical particles collide with each other relativistically. Before the collision, one particle has energy E_{10} and another particle is at rest. After the collision, both particles have the same energy E and the same scattering angle θ. Please find:

(a). The relativistic momentum of each particle after collision;

(b). The scattering angle θ.

Thank you very much!!!