UM-SJTU JOINT INSTITUTE
Advanced Lasers and Optics Laboratory
(VE438)

Post Lab Assignment

LAB 1
Polarization, Total Internal Reflection and Single Slit Diffraction

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1 PART A: Polarization

1.1 Measurement data

rotation angle θ	measured voltage mV	rotation angle θ	measured voltage mV
0^o	369.4	90^o	5.3
5^o	368.7	95^o	67.7
10^o	367.6	100^o	150.3
15^o	364.4	105^o	205.4
20^o	359	110^o	239.5
25^o	355.1	115^o	267.0
30^o	354	120^o	286.6
35^o	351.9	125^o	301.9
40^o	342.7	130^o	316.1
45^o	333.0	135^o	327.0
50^o	318.4	140^o	334.0
55^o	304.2	145^o	347.4
60^o	286.5	150^o	352.0
65^o	264.4	155^o	357.5
70^o	239.6	160^o	362.2
75^o	196	165^o	365
80^o	147.4	170^o	366.1
85^o	66.8	175^o	369.0

Table 1: Measurement Data for the 1/4-Wave Plate(rotation angle 0^o)

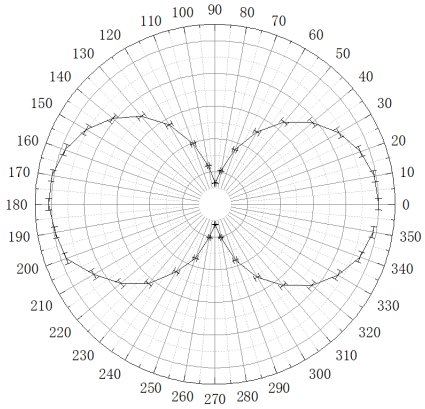

Figure 1: Relation between light amplitude and the rotation angle when rotation angle is 0^o

Rotation angle of 1/4-wave plate 20^o					
Maximum Electric Current $I_0 = 0.5518 \pm 0.0001[\mu A]$					
θ	$I[\mu A] \pm 0.0001[\mu A]$	$\sqrt{I/I_0}$	θ	$I[\mu A] \pm 0.0001[\mu A]$	$\sqrt{I/I_0}$
0^o	0.4874	0.940	180^o	0.5037	0.956
10^o	0.5331	0.983	190^o	0.5393	0.989
20^o	0.5517	1.000	200^o	0.5518	1.000
30^o	0.5405	0.990	210^o	0.5307	0.981
40^o	0.5014	0.953	220^o	0.4780	0.931
50^o	0.4352	0.888	230^o	0.4083	0.860
60^o	0.3498	0.796	240^o	0.3268	0.770
70^o	0.2649	0.693	250^o	0.2455	0.667
80^o	0.1791	0.570	260^o	0.1671	0.550
90^o	0.1133	0.453	270^o	0.0997	0.425
100^o	0.0674	0.350	280^o	0.0611	0.333
110^o	0.0542	0.313	290^o	0.0472	0.293
120^o	0.0715	0.360	300^o	0.0629	0.338
130^o	0.1202	0.467	310^o	0.1051	0.436
140^o	0.1909	0.588	320^o	0.1710	0.557
150^o	0.2757	0.707	330^o	0.2503	0.674
160^o	0.3657	0.814	340^o	0.3355	0.780
170^o	0.4423	0.876	350^o	0.4173	0.870

Table 2: Measurement Data for the 1/4-Wave Plate(rotation angle 20^o)

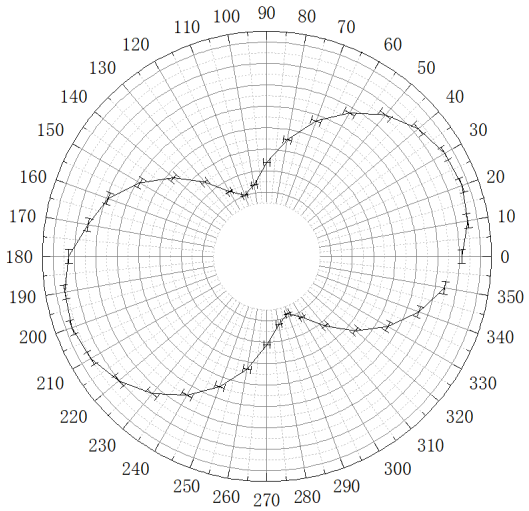

Figure 2: Relation between light amplitude and the rotation angle when rotation angle is 20^o

Rotation angle of 1/4-wave plate 45^o					
Maximum Electric Current $I_0 = 0.3476 \pm 0.0001[\mu A]$					
θ	$I[\mu A] \pm 0.0001[\mu A]$	$\sqrt{I/I_0}$	θ	$I[\mu A] \pm 0.0001[\mu A]$	$\sqrt{I/I_0}$
0^o	0.2903	0.9139	180^o	0.3002	0.9293
10^o	0.3049	0.9366	190^o	0.3092	0.9432
20^o	0.3196	0.9589	200^o	0.3157	0.9530
30^o	0.3321	0.9775	210^o	0.3211	0.9611
40^o	0.3412	0.9908	220^o	0.3237	0.9650
50^o	0.3461	0.9978	230^o	0.3227	0.9635
60^o	0.3476	1.0000	240^o	0.3185	0.9572
70^o	0.3458	0.9974	250^o	0.3110	0.9444
80^o	0.3368	0.9874	260^o	0.3000	0.9290
90^o	0.3260	0.9684	270^o	0.2890	0.9118
100^o	0.3137	0.9500	280^o	0.2776	0.8937
110^o	0.3020	0.9321	290^o	0.2674	0.8771
120^o	0.2929	0.9180	300^o	0.2595	0.8640
130^o	0.2856	0.9064	310^o	0.2527	0.8526
140^o	0.2811	0.8993	320^o	0.2545	0.8557
150^o	0.2828	0.9020	330^o	0.2580	0.8615
160^o	0.2866	0.9080	340^o	0.2642	0.8718
170^o	0.2926	0.9175	350^o	0.2767	0.8922

Table 3: Measurement Data for the 1/4-Wave Plate(rotation angle 45^o)

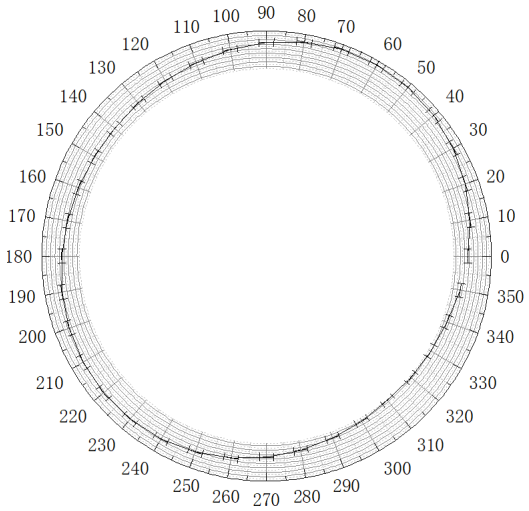

Figure 3: Relation between light amplitude and the rotation angle when rotation angle is 45^o

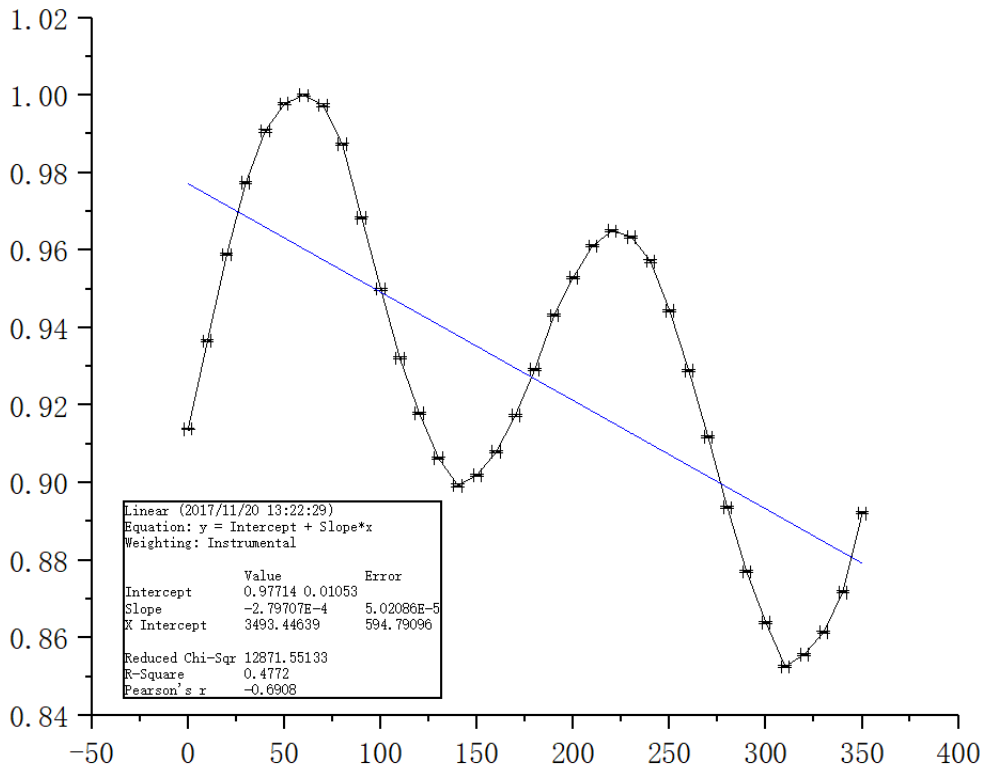

Figure 4: Linear fit plot of light amplitude vs. rotation angle when rotation angle is 45^o

From the linear fit, $\sqrt{\frac{I}{I_0}} = ((-2.80 \cdot 10^{-4} \pm 5.02 \cdot 10^{-5})\theta + (0.977 \pm 0.011))$

Rotation angle of 1/4-wave plate 70^o	
$\theta[^o] \pm [2]^o$	157
$I[\mu A] \pm 0.0001[\mu A]$	0.5241

Table 4: Measurement Data for the 1/4-Wave Plate(rotation angle 70^o)

2 Uncertainty Analysis

2.1 Uncertainty of Data for Demonstration of Malus' Law

For the first pair of data when $\theta = 0^o$

$$u_I = \sqrt{(\frac{\partial(I/I_0)}{\partial I}u)^2 + (\frac{\partial(I/I_0)}{\partial I_0}u)^2} = \sqrt{(\frac{u}{I_0})^2 + (\frac{uI^2}{I_0})} = \sqrt{(\frac{10^{-4}}{0.88})^2 + (\frac{10^{-4} \cdot 0.8763}{0.88^2})^2} = 1.604 \cdot 10^{-4}[\mu A]$$

$$u_r = \frac{u}{I/I_0} \cdot 100\% = \frac{1.604 \cdot 10^{-4}}{0.8763/0.88} \cdot 100\% = 0.02\%$$

Notice that all the angle were measured in the unit of degree, so I must change the unit into rad first

$$u_\theta = \frac{2\pi}{180} = \frac{\pi}{90}$$

$$u_\theta = \sqrt{(\frac{\partial \cos^2\theta}{\partial\theta} \cdot 2)^2} = \sqrt{(\frac{\pi \sin\theta\cos\theta}{45})^2} = 0 = 0$$

$$u_r = \frac{u}{u_\theta} \cdot 100\% = \frac{u_\theta}{\cos^2\theta} \cdot 100\% = \frac{0}{1} = 0\%$$

$\cos^2\theta$	u	$u_r[\%]$	I/I_o	u	$u_r[\%]$
1.000	0.0000	0.00	0.9958	$1.604 \cdot 10^{-4}$	0.02
0.992	0.0061	0.61	0.9971	$1.605 \cdot 10^{-4}$	0.02
0.970	0.0119	1.23	0.9885	$1.598 \cdot 10^{-4}$	0.02
0.933	0.0175	1.87	0.9673	$1.581 \cdot 10^{-4}$	0.02
0.883	0.0224	2.54	0.9185	$1.543 \cdot 10^{-4}$	0.02
0.821	0.0267	3.25	0.8703	$1.506 \cdot 10^{-4}$	0.02
0.750	0.0302	4.02	0.8034	$1.458 \cdot 10^{-4}$	0.02
0.671	0.0328	4.89	0.7352	$1.410 \cdot 10^{-4}$	0.02
0.587	0.0344	5.86	0.6424	$1.351 \cdot 10^{-4}$	0.02
0.500	0.0349	6.98	0.5603	$1.303 \cdot 10^{-4}$	0.02
0.413	0.0344	8.32	0.4567	$1.249 \cdot 10^{-4}$	0.03
0.329	0.0328	9.97	0.3798	$1.216 \cdot 10^{-4}$	0.03
0.250	0.0302	12.1	0.2886	$1.183 \cdot 10^{-4}$	0.04
0.179	0.0267	15.0	0.2143	$1.162 \cdot 10^{-4}$	0.05
0.117	0.0224	19.2	0.1396	$1.147 \cdot 10^{-4}$	0.08
0.067	0.0175	26.0	0.0898	$1.141 \cdot 10^{-4}$	0.13
0.030	0.0119	39.6	0.0474	$1.138 \cdot 10^{-4}$	0.24
0.008	0.0061	79.8	0.0257	$1.137 \cdot 10^{-4}$	0.44
0.000	0.0000	00.0	0.0084	$1.136 \cdot 10^{-4}$	1.35

Table 5: Uncertainty of $\cos^2\theta$ and I/I_0

2.2 Uncertainty for Linearly Polarized Light and the Half-wave Plate

Since I only need to recorded the data of θ and $\triangle$, their uncertainty are both 2^o

2.3 Uncertainty for Circularly and Elliptically Polarized Light and the 1/4-wave Plate

The uncertainty of θ is 2^o, for the first pair of data when the rotation angle is 0^o

$$u = \sqrt{(\frac{\partial\sqrt{I/I_0}}{\partial I}\cdot u_I)^2 + \frac{\partial\sqrt{I/I_0}}{\partial I_0}\cdot u_I)^2} = \sqrt{(\frac{0.5\sqrt{I_0/I}}{I_0}u_I)^2 + ((\frac{0.5I\sqrt{I_0/I}}{I_0^2}u_I)^2}$$

$$u = 0.00005\sqrt{\frac{0.5857}{0.5857}}\sqrt{\frac{1}{0.5857^2} + \frac{1}{0.5857^4}} = 1.689\cdot 10^{-4}$$

$$u_r = \frac{u}{\sqrt{I/I_0}}\cdot 100\% = 0.02\%$$

Rotation angle of 1/4-wave plate 0^o							
Maximum Electric Current $I_0 = 0.5857 \pm 0.0001[\mu A]$							
θ	$\sqrt{I/I_0}$	u	$u_r[\%]$	θ	$\sqrt{I/I_0}$	u	$u_r[\%]$
0^o	1.000	$1.689\cdot 10^{-4}$	0.02	180^o	1.017	$1.660\cdot 10^{-4}$	0.02
10^o	0.988	$1.709\cdot 10^{-4}$	0.02	190^o	0.992	$1.702\cdot 10^{-4}$	0.02
20^o	0.944	$1.789\cdot 10^{-4}$	0.02	200^o	0.968	$1.745\cdot 10^{-4}$	0.02
30^o	0.874	$1.931\cdot 10^{-4}$	0.02	210^o	0.857	$1.971\cdot 10^{-4}$	0.02
40^o	0.777	$2.175\cdot 10^{-4}$	0.03	220^o	0.752	$2.246\cdot 10^{-4}$	0.03
50^o	0.657	$2.571\cdot 10^{-4}$	0.04	230^o	0.633	$2.666\cdot 10^{-4}$	0.04
60^o	0.511	$3.305\cdot 10^{-4}$	0.06	240^o	0.477	$3.544\cdot 10^{-4}$	0.07
70^o	0.356	$4.749\cdot 10^{-4}$	0.13	250^o	0.351	$4.807\cdot 10^{-4}$	0.14
80^o	0.209	$8.079\cdot 10^{-4}$	0.39	260^o	0.207	$8.143\cdot 10^{-4}$	0.39
90^o	0.131	$12.90\cdot 10^{-4}$	0.99	270^o	0.124	$13.60\cdot 10^{-4}$	1.10
100^o	0.241	$7.010\cdot 10^{-4}$	0.29	280^o	0.207	$8.143\cdot 10^{-4}$	0.39
110^o	0.394	$4.285\cdot 10^{-4}$	0.11	290^o	0.124	$4.646\cdot 10^{-4}$	0.13
120^o	0.561	$3.010\cdot 10^{-4}$	0.05	300^o	0.515	$3.278\cdot 10^{-4}$	0.06
130^o	0.703	$2.404\cdot 10^{-4}$	0.03	310^o	0.647	$2.609\cdot 10^{-4}$	0.04
140^o	0.817	$2.068\cdot 10^{-4}$	0.03	320^o	0.772	$2.188\cdot 10^{-4}$	0.03
150^o	0.912	$1.853\cdot 10^{-4}$	0.02	330^o	0.867	$1.949\cdot 10^{-4}$	0.02
160^o	0.975	$1.732\cdot 10^{-4}$	0.02	340^o	0.943	$1.791\cdot 10^{-4}$	0.02
170^o	1.012	$1.669\cdot 10^{-4}$	0.02	350^o	0.980	$1.723\cdot 10^{-4}$	0.02

Table 6: Uncertainty for the 1/4-Wave Plate(rotation angle 0^o)

Rotation angle of 1/4-wave plate 20^o							
Maximum Electric Current $I_0 = 0.5518 \pm 0.0001[\mu A]$							
θ	$\sqrt{I/I_0}$	u	$u_r[\%]$	θ	$\sqrt{I/I_0}$	u	$u_r[\%]$
0^o	0.940	$1.996 \cdot 10^{-4}$	0.02	180^o	0.956	$1.963 \cdot 10^{-4}$	0.02
10^o	0.983	$1.909 \cdot 10^{-4}$	0.02	190^o	0.989	$1.897 \cdot 10^{-4}$	0.02
20^o	1.000	$1.876 \cdot 10^{-4}$	0.02	200^o	1.000	$1.876 \cdot 10^{-4}$	0.02
30^o	0.990	$1.895 \cdot 10^{-4}$	0.02	210^o	0.981	$1.913 \cdot 10^{-4}$	0.02
40^o	0.953	$1.968 \cdot 10^{-4}$	0.02	220^o	0.931	$2.015 \cdot 10^{-4}$	0.02
50^o	0.888	$2.112 \cdot 10^{-4}$	0.02	230^o	0.860	$2.181 \cdot 10^{-4}$	0.02
60^o	0.796	$2.356 \cdot 10^{-4}$	0.03	240^o	0.770	$2.437 \cdot 10^{-4}$	0.02
70^o	0.693	$2.707 \cdot 10^{-4}$	0.04	250^o	0.667	$2.812 \cdot 10^{-4}$	0.03
80^o	0.570	$3.293 \cdot 10^{-4}$	0.06	260^o	0.550	$3.409 \cdot 10^{-4}$	0.03
90^o	0.453	$4.140 \cdot 10^{-4}$	0.09	270^o	0.425	$4.413 \cdot 10^{-4}$	0.04
100^o	0.350	$5.367 \cdot 10^{-4}$	0.15	280^o	0.333	$5.637 \cdot 10^{-4}$	0.06
110^o	0.313	$5.985 \cdot 10^{-4}$	0.19	290^o	0.293	$6.414 \cdot 10^{-4}$	0.10
120^o	0.360	$5.211 \cdot 10^{-4}$	0.14	300^o	0.338	$5.556 \cdot 10^{-4}$	0.16
130^o	0.467	$4.019 \cdot 10^{-4}$	0.09	310^o	0.436	$4.298 \cdot 10^{-4}$	0.10
140^o	0.588	$3.189 \cdot 10^{-4}$	0.05	320^o	0.557	$3.370 \cdot 10^{-4}$	0.06
150^o	0.707	$2.654 \cdot 10^{-4}$	0.04	330^o	0.674	$2.785 \cdot 10^{-4}$	0.04
160^o	0.814	$2.304 \cdot 10^{-4}$	0.03	340^o	0.780	$2.406 \cdot 10^{-4}$	0.03
170^o	0.876	$2.142 \cdot 10^{-4}$	0.02	350^o	0.870	$2.157 \cdot 10^{-4}$	0.02

Table 7: Uncertainty for the 1/4-Wave Plate(rotation angle 20^o)

Rotation angle of 1/4-wave plate 45^o							
Maximum Electric Current $I_0 = 0.3476 \pm 0.0001[\mu A]$							
θ	$\sqrt{I/I_0}$	u	$u_r[\%]$	θ	$\sqrt{I/I_0}$	u	$u_r[\%]$
0^o	0.9139	$4.794 \cdot 10^{-4}$	0.03	180^o	0.9293	$4.714 \cdot 10^{-4}$	0.03
10^o	0.9366	$4.678 \cdot 10^{-4}$	0.03	190^o	0.9432	$4.645 \cdot 10^{-4}$	0.03
20^o	0.9589	$4.569 \cdot 10^{-4}$	0.03	200^o	0.9530	$4.597 \cdot 10^{-4}$	0.03
30^o	0.9775	$4.482 \cdot 10^{-4}$	0.03	210^o	0.9611	$4.558 \cdot 10^{-4}$	0.03
40^o	0.9908	$4.422 \cdot 10^{-4}$	0.03	220^o	0.9650	$4.540 \cdot 10^{-4}$	0.03
50^o	0.9978	$4.390 \cdot 10^{-4}$	0.03	230^o	0.9635	$4.547 \cdot 10^{-4}$	0.03
60^o	1.0000	$4.381 \cdot 10^{-4}$	0.03	240^o	0.9572	$4.577 \cdot 10^{-4}$	0.03
70^o	0.9974	$4.392 \cdot 10^{-4}$	0.03	250^o	0.9444	$4.639 \cdot 10^{-4}$	0.03
80^o	0.9874	$4.451 \cdot 10^{-4}$	0.03	260^o	0.9290	$4.716 \cdot 10^{-4}$	0.03
90^o	0.9684	$4.524 \cdot 10^{-4}$	0.03	270^o	0.9118	$4.805 \cdot 10^{-4}$	0.03
100^o	0.9500	$4.612 \cdot 10^{-4}$	0.03	280^o	0.8937	$4.902 \cdot 10^{-4}$	0.03
110^o	0.9321	$4.700 \cdot 10^{-4}$	0.03	290^o	0.8771	$4.995 \cdot 10^{-4}$	0.03
120^o	0.9180	$4.773 \cdot 10^{-4}$	0.03	300^o	0.8640	$5.070 \cdot 10^{-4}$	0.03
130^o	0.9064	$4.833 \cdot 10^{-4}$	0.03	310^o	0.8526	$5.138 \cdot 10^{-4}$	0.03
140^o	0.8993	$4.872 \cdot 10^{-4}$	0.03	320^o	0.8557	$5.120 \cdot 10^{-4}$	0.03
150^o	0.9020	$4.857 \cdot 10^{-4}$	0.03	330^o	0.8615	$5.085 \cdot 10^{-4}$	0.03
160^o	0.9080	$4.825 \cdot 10^{-4}$	0.03	340^o	0.8718	$5.025 \cdot 10^{-4}$	0.03
170^o	0.9175	$4.775 \cdot 10^{-4}$	0.03	350^o	0.8922	$4.910 \cdot 10^{-4}$	0.03

Table 8: Uncertainty for the 1/4-Wave Plate(rotation angle 45^o)

3 Conclusion

In this experiment, I studied the polarization of light and verified the Malus' law. I also learned how to use full-wave plate, half-wave plate and quarter-wave plate to produce different kinds of polarization of light such as elliptically and circular polarized light.

3.1 Demonstration of Malus' Law

In this part, I observe the change in light intensity indirectly by measuring the electric current. I fist measured the max current I_0 and then recorded different current I according to different angle θ for which the analyer has changed. I plot the relation of I/I_0 vs. $\cos^2\theta$, and through linear fit, I got.

$$I/I_0 = (1.0130 \pm 0.0123)\cos^2\theta + 0.0272 \pm 0.0065$$

The slope very close to the theoretical value 1 according to the Malus' Law. The intercept and the uncertainty are both very tiny, which proves that a full-wave plate won't change the light's polarization state.

3.2 Linearly Polarized Light and the Half-Wave Plate

In this part, I insert the half-wave plate and find the angle $\triangle\theta$ for which the analyzer rotated to find the extinction after I rotated the plate for θ. Then I plot $\triangle\theta$ vs. θ and I find it's a line which seems to pass through the origin. Here are answers for questions in procedures part.

(a)

$$I = I_0 \cos^2\theta \cos^2(\frac{\pi}{2} - \theta) = 0$$

$$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

So the light extinction can be found 4 times.

(b)

$$I = I_0 \cos^2\theta \cos^2(\theta_1) = 0$$

If $\theta_1 = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ then it's always light extinction, otherwise there will be two extinctions when $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

(c) For a half-wave plate, $\triangle = \frac{(2k+1)\lambda}{2}$, where $k = 0, 1, 2\cdots$
$\delta = \frac{2\pi}{\lambda}\triangle = (2k+1)\pi = \pi \Rightarrow \frac{E_x^2}{A_O^2} + \frac{E_y^2}{A_e^2} + 2\frac{E_x E_y}{A_o A_e}\cos\delta = 0 \Rightarrow E_y = -\frac{A_e}{A_o}E_x$ which cause a linear polarization

3.3 Circularly and Elliptically Polarization Light and the Quarter-Wave Plate

In this part, I first rotated the quarter-wave plate for $0^o, 20^o, 45^o$, and 75^o and recorded the electric current whenever I rotate the analyzer for 10^o. For the data measured when the plate was rotated for $0^o, 20^o$ and 45^o, I plot the relation between $\sqrt{I/I_0}$ vs. θ in polar coordinates.

When the quarter-wave plate was not rotated, the light will be elliptically polarized and it looks like an eight in the plot. For the figure plotted from data measured when the rotate angle is 20^o, the light is also elliptically polarized and the plot is very similar to last one, it's seems the figure has been rotated for 20^o counter-clockwise from last plot.

When the quarter-wave plate was rotated for 75^o, the plot is like a circle, which means the light is circularly polarized but the circle is not perfect enough, I think it's because the error caused by the apparatus and the light from other devices in the lab such as mobile phone or other lasers. I think forbid students to use mobile phones in the experiment can reduce such errors. The linear fit get:

$$\sqrt{\frac{I}{I_0}} = ((-2.80 \cdot 10^{-4} \pm 5.02 \cdot 10^{-5x})\theta + (0.977 \pm 0.011))$$

The plot seems incline badly, but the slope and uncertainty are actually very small, which means the line is horizontal and I won't change much when θ is changed.

When the plate was rotated for 75^o I only measure the max current $I = 0.5241 \pm 0.0001[\mu A]$ with the angle rotated $\theta = 157^o \pm 2^o$. For data measured when the plate was rotated for 20^o, when $\theta = 200^o$, I got the max current $I = 0.5518 \pm 0.0001[\mu A]$, and angle difference $200^o - 157^o = 43^o$ is similar to the $75^o - 20^o = 45^o$. I think the angle for which the quarter-wave plate is rotated is equal to the angle difference between the angles at which the max current is generated.

4 Data Sheet

Data sheet is attach to the report

5 Reference

Krzyzosiak,M. Lab Manual of Exercise 4.
Qin Tian, Zeng Ming, Zhao Xijian, Krzyzosiak,M. Handbook-Uncertainty Analysis.