VE 320 Summer 2019

Introduction to Semiconductor Devices

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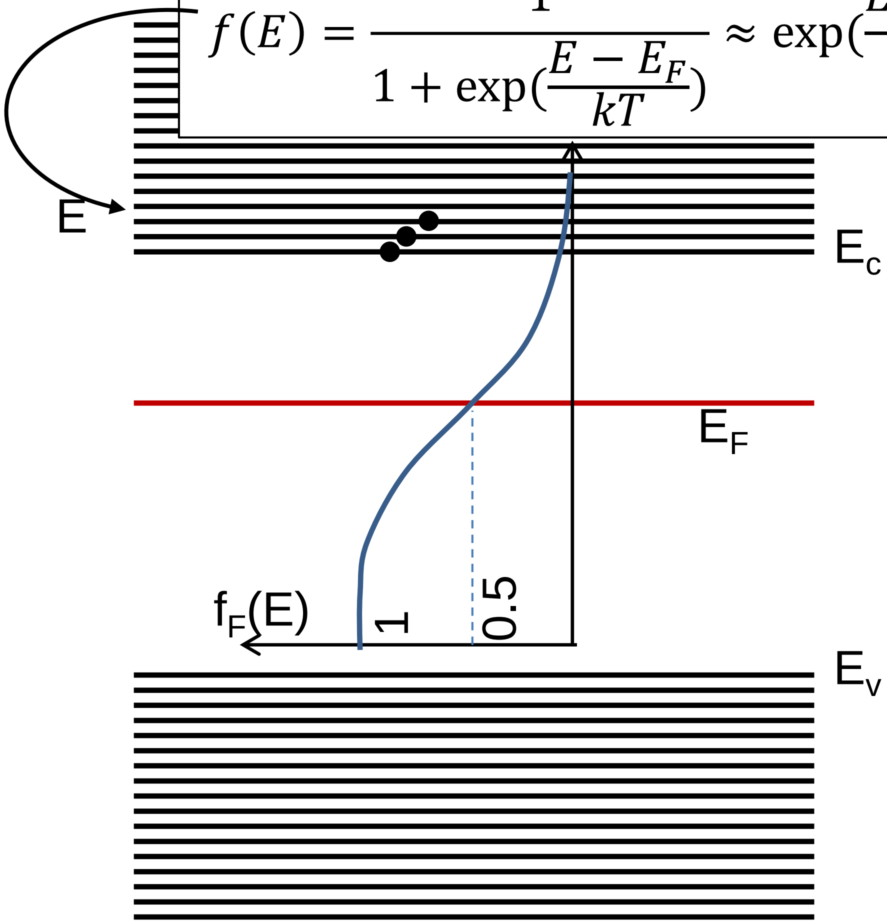
Lecture 4

The semiconductor in equilibrium
(Chapter 4)

Ionization of dopants

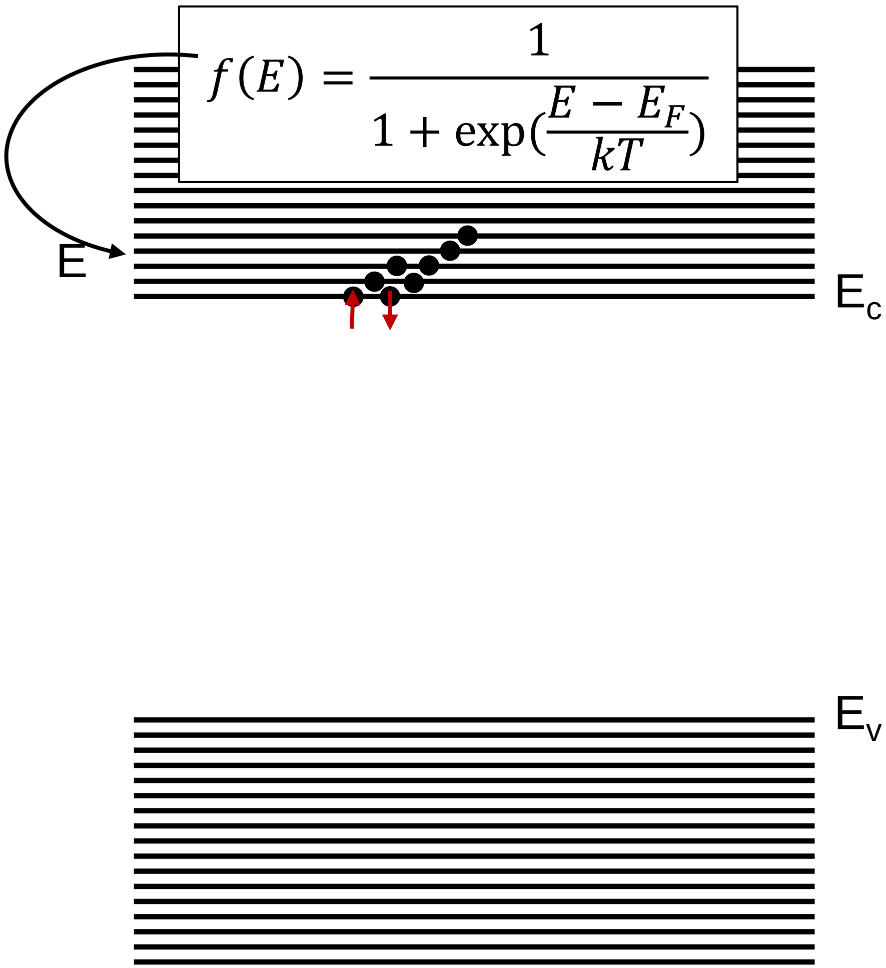
$$f(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})} \approx \exp(\frac{E_F-E}{kT})$$



Non-degenerate Semiconductors:

- Small number of impurity atoms
- Impurities introduce discrete, noninteracting donor energy states in n-type semiconductor
- Boltzmann approximation

Ionization of dopants



Degenerate Semiconductors:

- High impurity concentration
- Donor electrons interact
- Split into a band of energies
- When $n_0 > N_c$, then $E_F > E_c$:

  Degenerate n-type semiconductor

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

- When $p_0 > N_v$, the $E_F < E_v$:

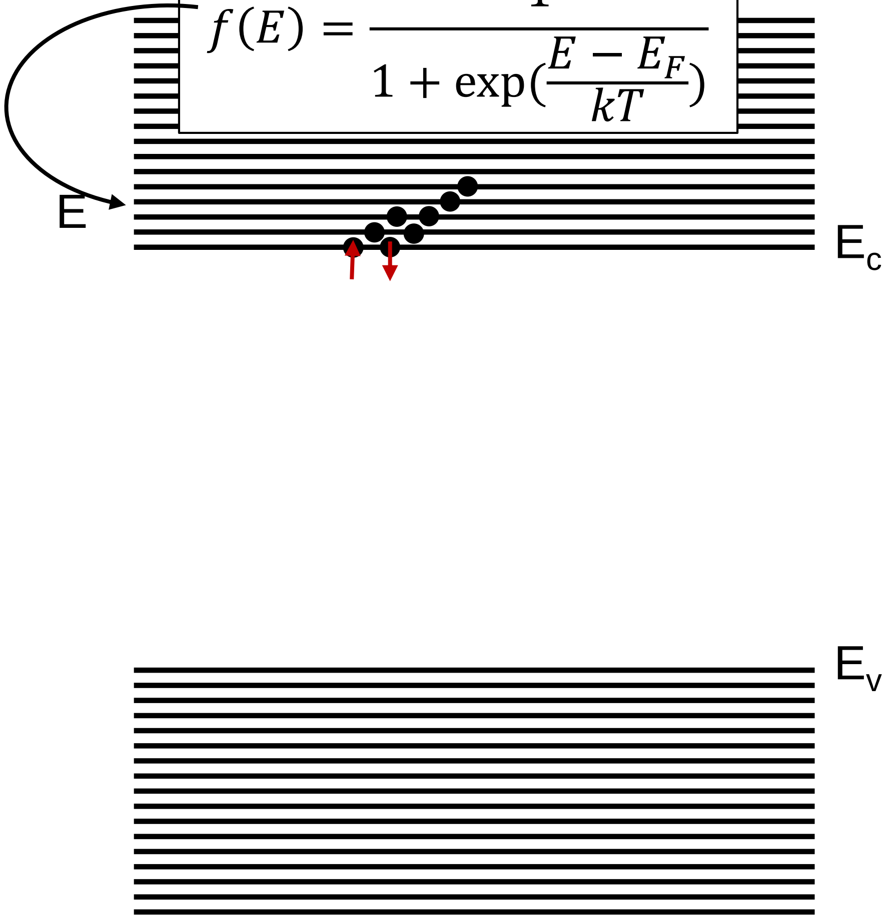
  Degenerate p-type semiconductor

- Fermi distribution, cannot use

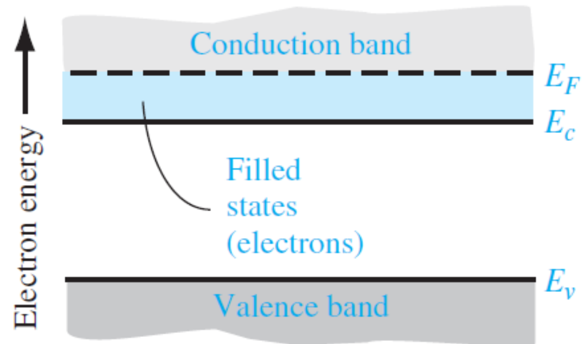
  Boltzmann approximation

Ionization of dopants

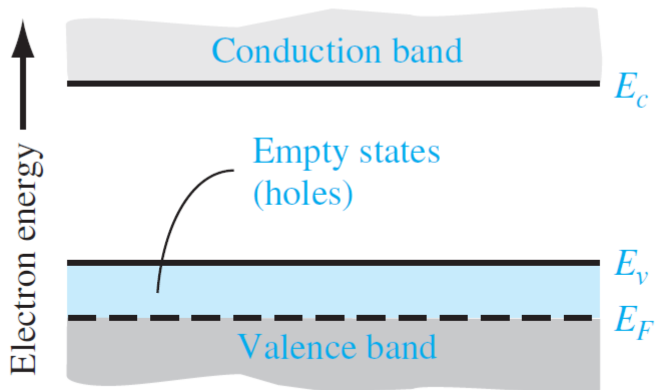
$$f(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$



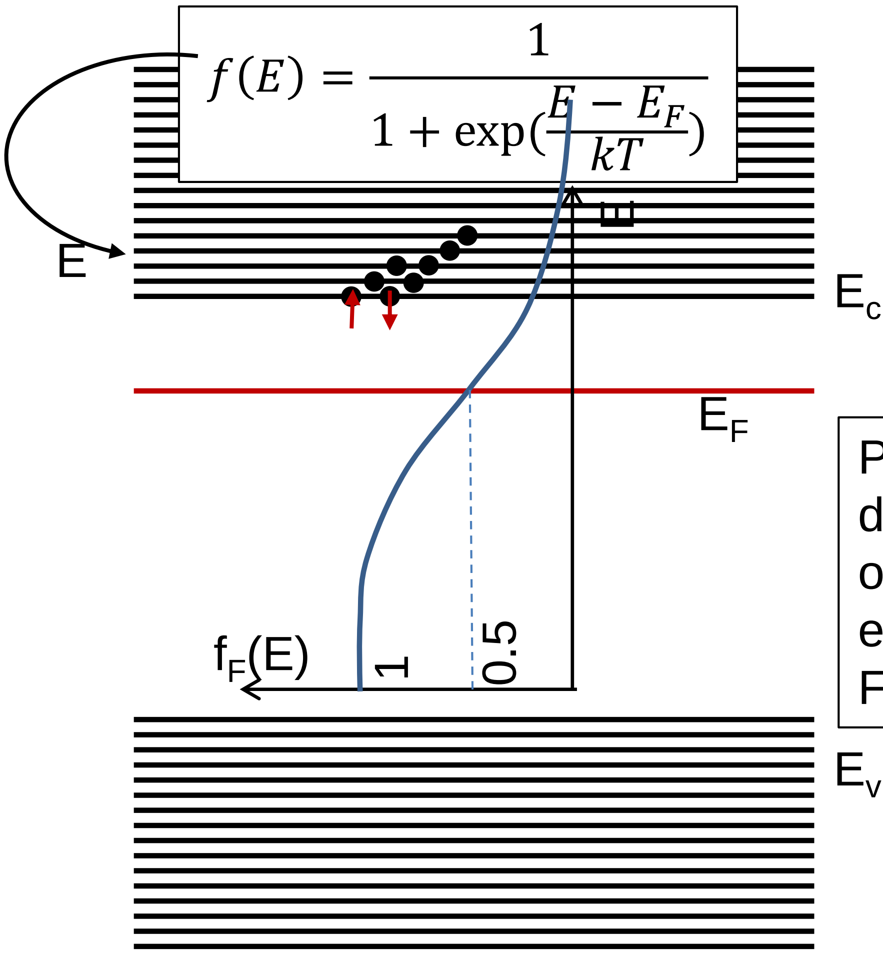
Degenerate n-type semiconductor: the states between E_F and E_c are mostly filled with electrons

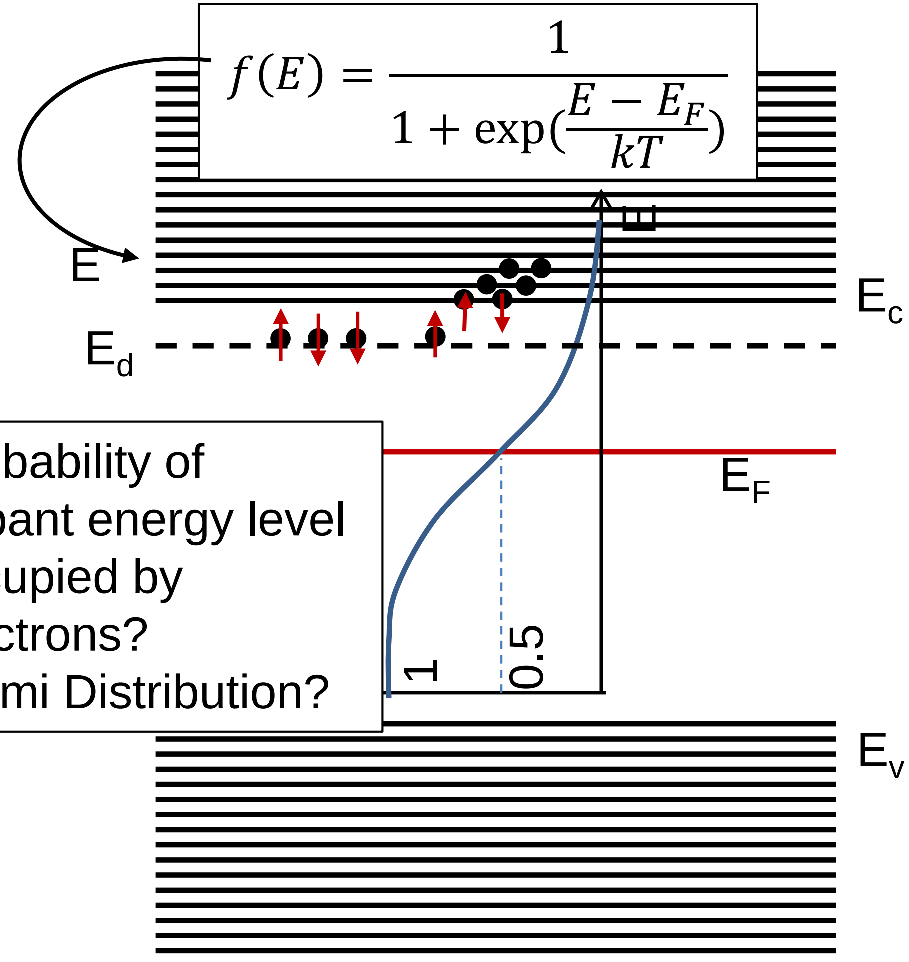


Degenerate p-type semiconductor: states between E_F and E_v are mostly empty



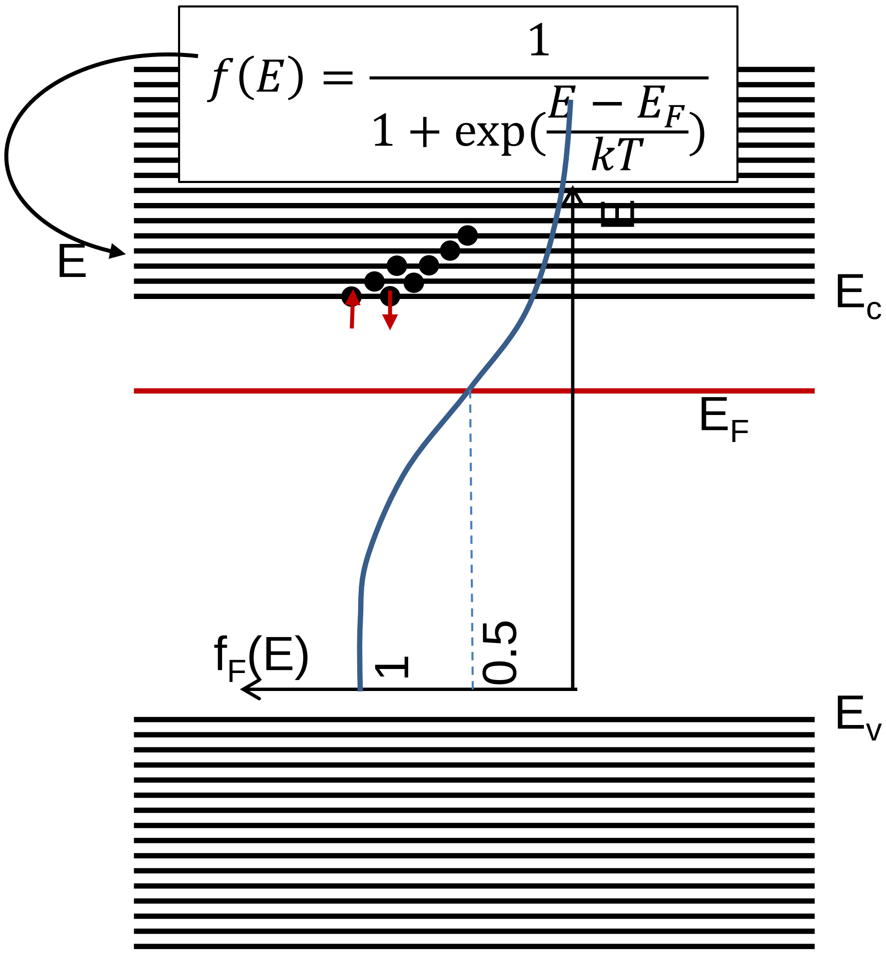
Ionization of dopants



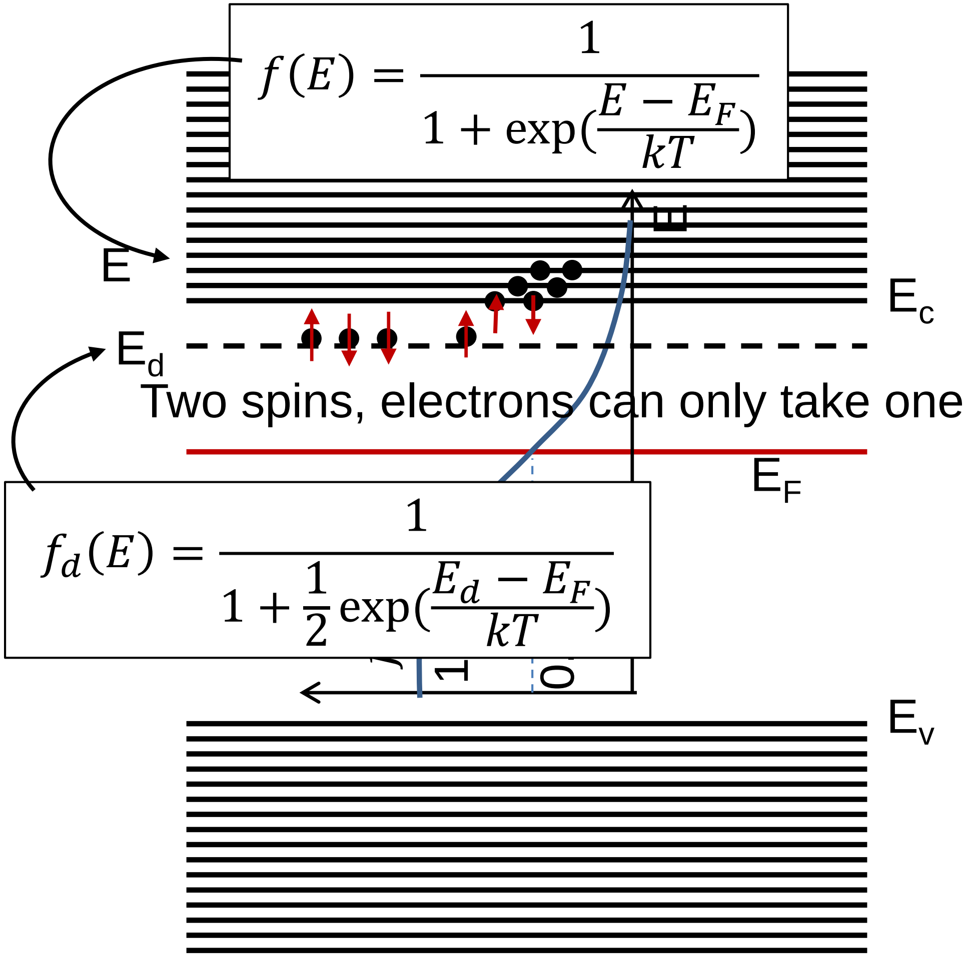


Probability of dopant energy level occupied by electrons?
Fermi Distribution?

Ionization of dopants



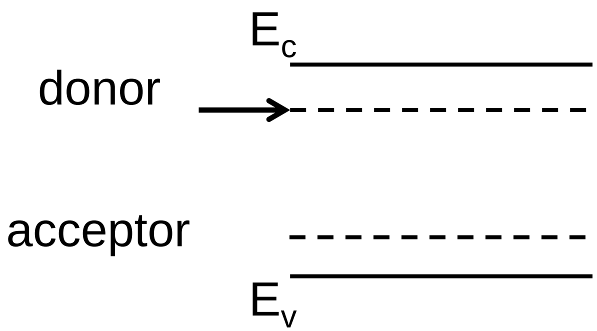
$$f(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$



$$f(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

Ionization of dopants



① Probability of electrons occupying donor energy level

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

The density of electrons occupying the donor level:

$$n_d = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

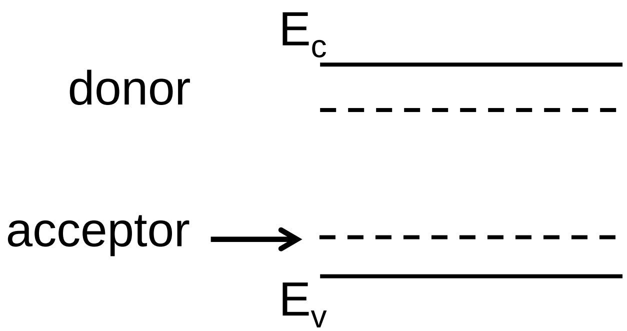
1/2 = 1/*g*,
g is degeneracy factor

E_d: energy of the donor level

$$n_d = N_d - N_d^+$$

N_d^+ is the concentration of ionized donors--Important

Ionization of dopants



① Probability of electrons occupying acceptor energy level

$$f_a(E) = \frac{1}{1 + \frac{1}{2}\exp(\frac{E_F - E_a}{kT})}$$

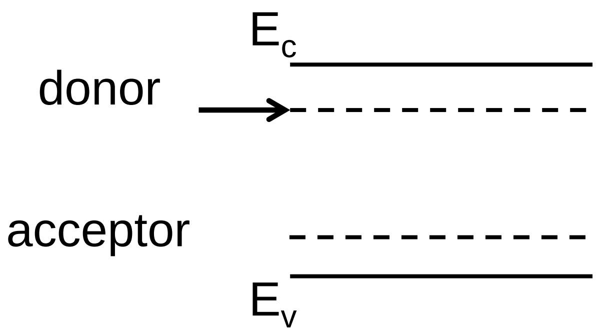
The density of holes occupying the acceptor level:

$$p_a = \frac{N_a}{1 + \frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)} = N_a - N_a^-$$

N_a^- is the concentration of ionized acceptors--Important

The ground state degeneracy factor *g* is normally taken as 4 for the acceptor level in silicon and gallium arsenide because of the detailed band structure

Ionization of dopants



① Probability of electrons occupying dopant energy level

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

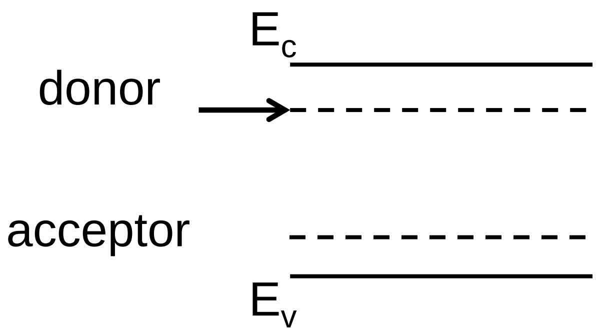
② Ionization rate

$$1 - f_d(E) = \frac{1}{1 + 2\exp(\frac{E_F - E_d}{kT})}$$

$$= \begin{cases} 1/3 & E_d = E_F \\ 1 & E_d \gg E_F + kT \\ 0 & E_d \ll E_F \end{cases}$$

Ionization of dopants

Completer ionization



Ratio of electrons that remains in dopant level to all electrons

$$(E_d - E_F) \gg kT$$

$$n_d \approx \frac{N_d}{\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} = 2N_d \exp\left[\frac{-(E_d - E_F)}{kT}\right]$$

Electrons in the conduction band:

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$\frac{n_d}{n_d + n_0} = \frac{2N_d \exp\left[\frac{-(E_d - E_F)}{kT}\right]}{2N_d \exp\left[\frac{-(E_d - E_F)}{kT}\right] + N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]}$$

$$\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_c}{2N_d}\exp\left[\frac{-(E_c - E_d)}{kT}\right]}$$

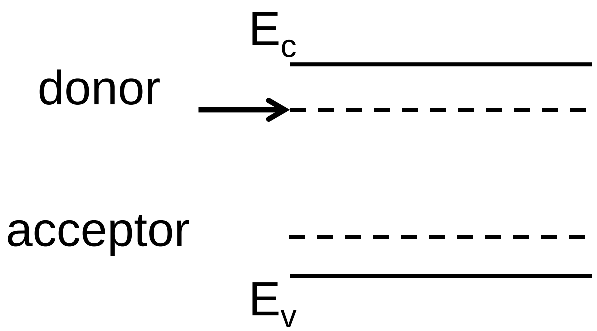
E_c-E_d: ionization energy of the donor electrons

The donor states are essentially completely ionized for a typical doping of 10^{16} cm^{-3}, almost all donor impurity atoms have donated an electron to the conduction band.

Ionization of dopants

E_c-E_d: ionization energy of the donor electrons

Ionization energy: the approximate energy required to elevate the donor electron into the conduction band



Bohr model:
Coulomb force = centripetal force

$$\frac{e^2}{4\pi\epsilon r_n^2} = \frac{m^* v^2}{r_n}$$

Angular momentum is also quantized $\quad m^* r_n v = n\hbar$

Radius of the orbit is quantized $\quad r_n = \frac{n^2\hbar^2 4\pi\epsilon}{m^* e^2}$

Bohr radius $\quad a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_0 e^2} = 0.53\ \text{Å} \qquad \frac{r_n}{a_0} = n^2\epsilon_r\left(\frac{m_0}{m^*}\right)$

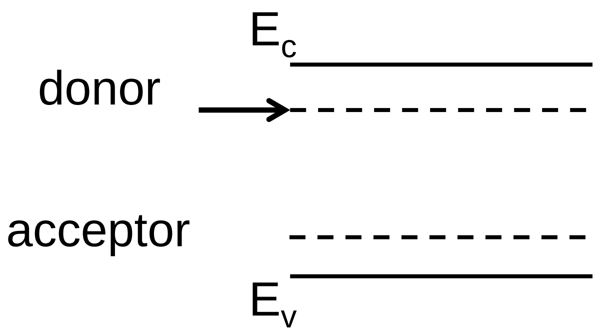
*m** is the conductivity effective mass

Ionization of dopants

E_c-E_d: ionization energy of the donor electrons

Total energy $E = T + V$

T: kinetic energy, V: potential energy



$$T = \frac{1}{2} m^* v^2 \qquad T = \frac{m^* e^4}{2(n\hbar)^2(4\pi\epsilon)^2}$$

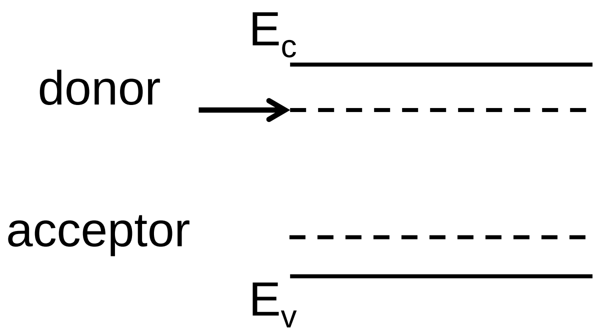
$$V = \frac{-e^2}{4\pi\epsilon r_n} = \frac{-m^* e^4}{(n\hbar)^2(4\pi\epsilon)^2}$$

$$E = T + V = \frac{-m^* e^4}{2(n\hbar)^2(4\pi\epsilon)^2}$$

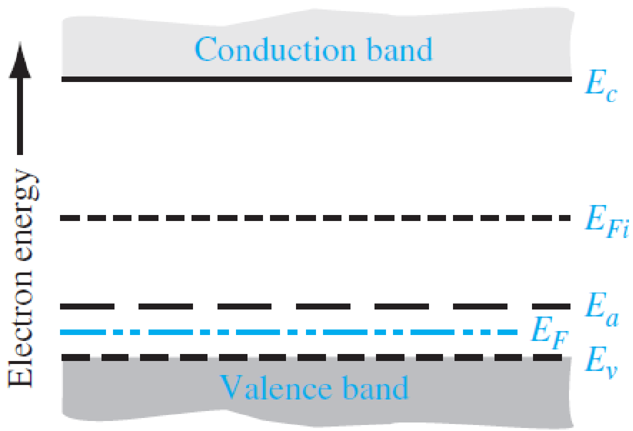
Hydrogen atom in the lowest energy state: E_H = -13.6 eV

Silicon ionization energy: is $E_{Si} = E_H \times m^*/m_0/\epsilon_r^2$ = -25.8 meV << E_g = 1.12eV

Ionization of dopants



p-type: $E_F < E_a$



Freeze-out:

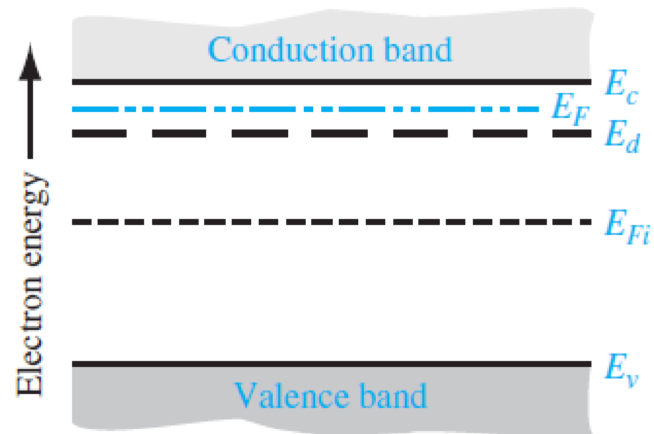
$$n_d = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

T=0K

n-type semiconductor, each donor state must contain an electron, $n_d=N_d$ or $N_d^+=0$

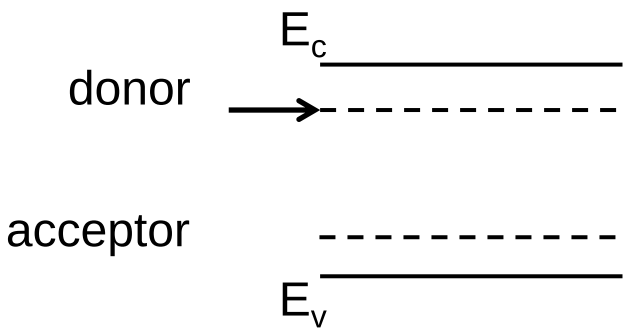
$$\exp\left[(E_d - E_F)/kT\right] = 0$$

$$E_F > E_d$$



No electrons from the donor state are thermally elevated into the conduction band

Ionization of dopants

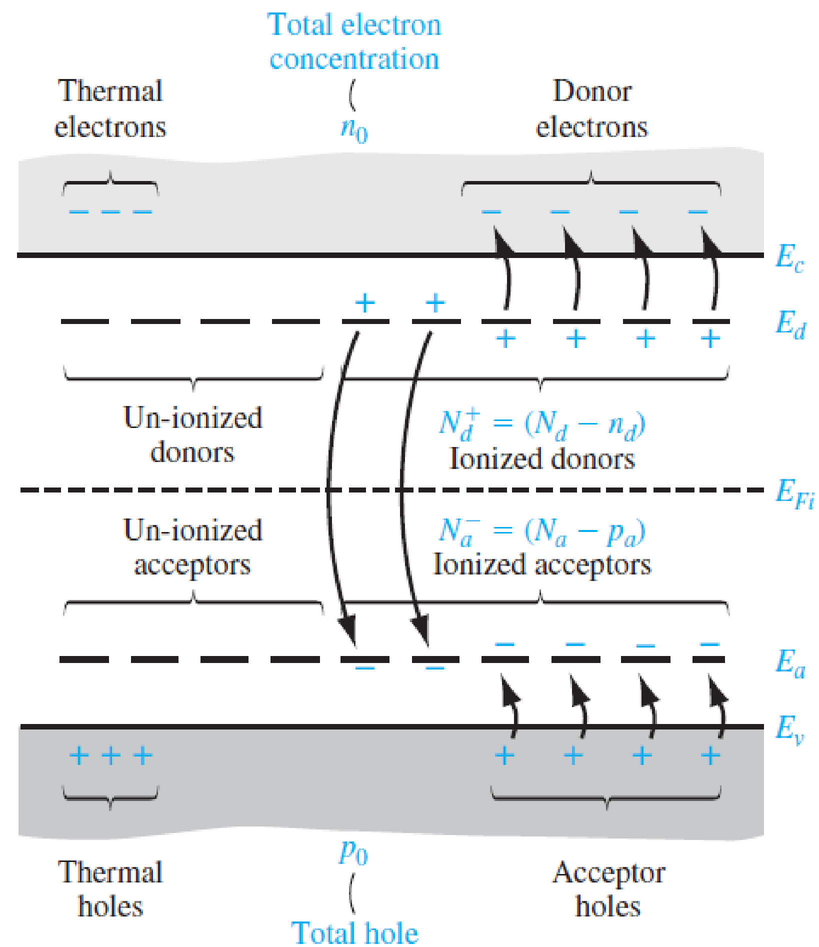


③ Charge neutrality

$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

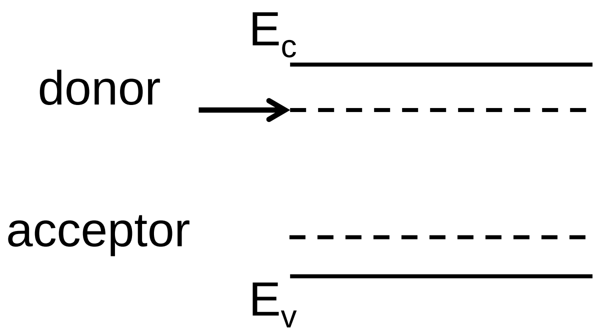
Compensated semiconductors at thermal equilibrium



Compensated semiconductor: contains both donor and acceptor impurity atoms in the same region

n-type compensated semiconductor: $N_d > N_a$
p-type compensated semiconductor: $N_a > N_d$
$N_a = N_d$: completely compensated semiconductor, like intrinsic.

Ionization of dopants



Compensated semiconductors

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$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

Electron concentration in thermal equilibrium

Complete ionization: n_d=0, p_a=0

$$\boxed{n_0 + N_a = N_d + p_0}$$

$$n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$$n_0^2 - (N_d - N_a)n_0 - n_i^2 = 0$$

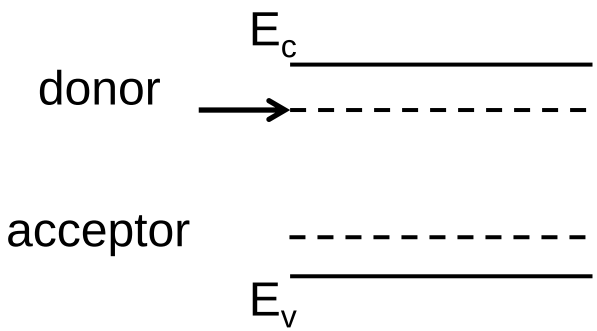
$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Can calculate the electron concentration in an n-type semiconductor, $N_d > N_a$

$$p_0 = \frac{n_i^2}{n_0}$$

N_a can be 0: only n doping

Ionization of dopants



Compensated semiconductors

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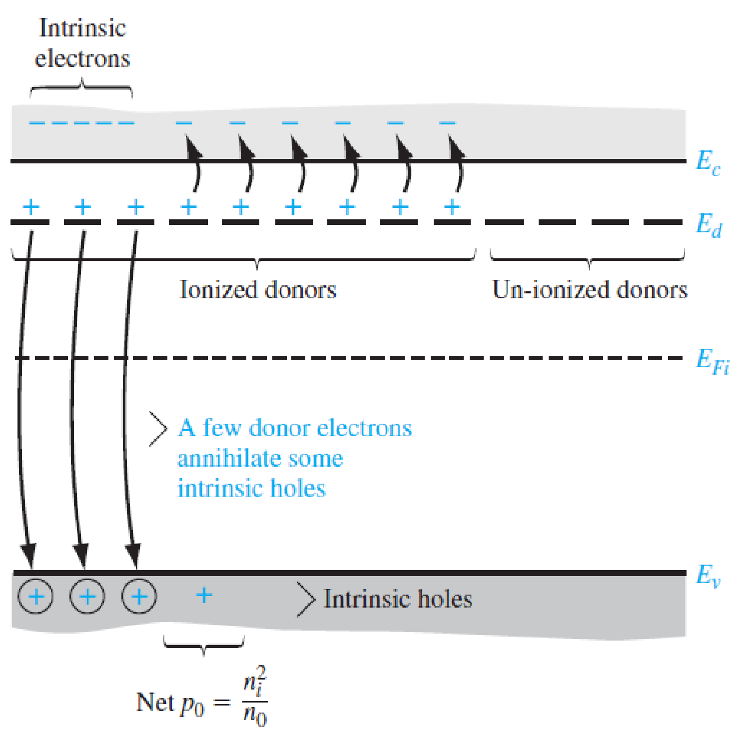
$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

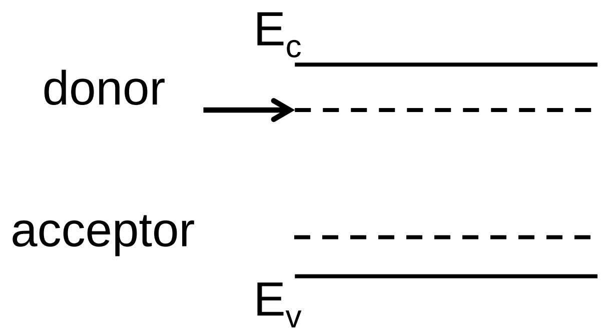
Electron concentration in thermal equilibrium

Complete ionization: n_d=0, p_a=0

$$n_0 > n_i\ ,\ p_0 < n_i$$



Ionization of dopants



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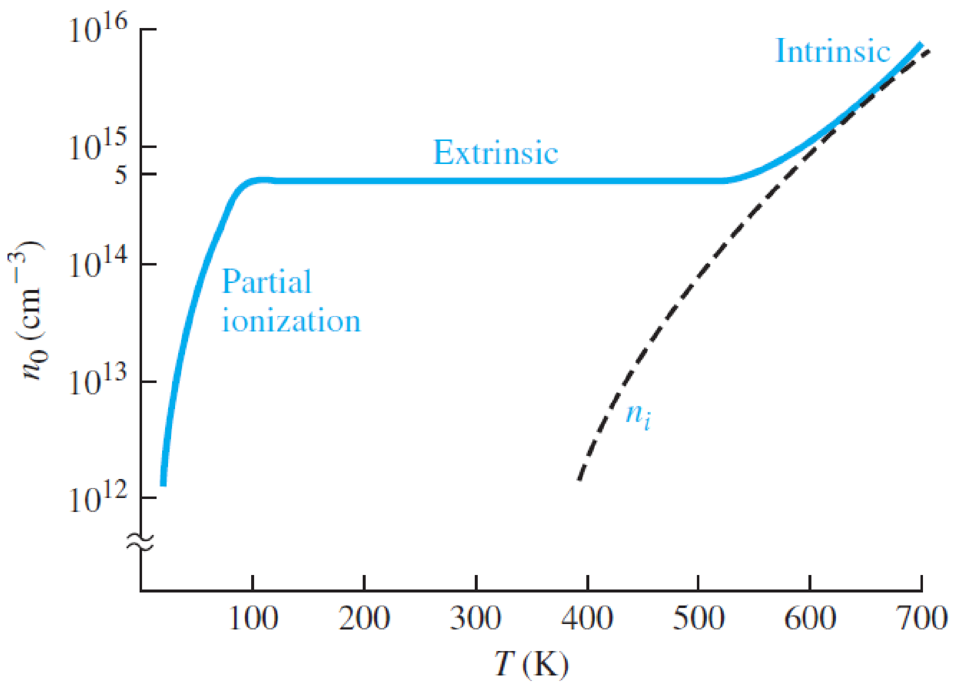
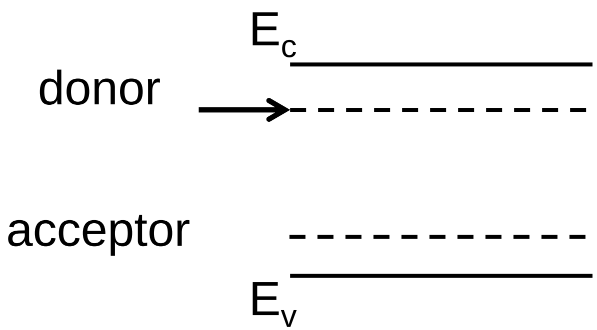
Electron concentration in thermal equilibrium

High temperature: like intrinsic! – Usually bad for semiconductor devices



Ionization of dopants



Compensated semiconductors

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$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

Fermi level position:

Special case: N_a=0, Only n-type doping

Boltzmann approximation

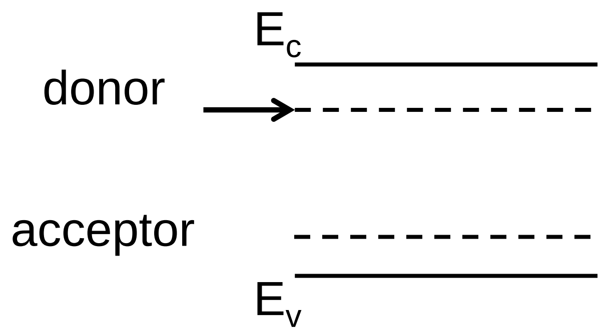
$$\boxed{n_0 = N_d^+ + p_0} \qquad \boxed{n_0 p_0 = n_i^2}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)}$$

$$\boxed{n_i >> N_d^+ \Rightarrow T \text{ very high}}$$

$$\boxed{n_0 = p_0 = n_i = \sqrt{N_c N_v}\exp(-\frac{E_g}{2kT})}$$

Ionization of dopants



Compensated semiconductors

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$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

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Fermi level position:

Special case: N_a=0, Only n-type doping

Boltzmann approximation

$$\boxed{n_0 = N_d^+ + p_0} \quad \boxed{n_0 p_0 = n_i^2}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)}$$

$$\boxed{n_i << N_d^+ \Rightarrow T\ not\ very\ high}$$

$$n_0 \approx N_d^+$$

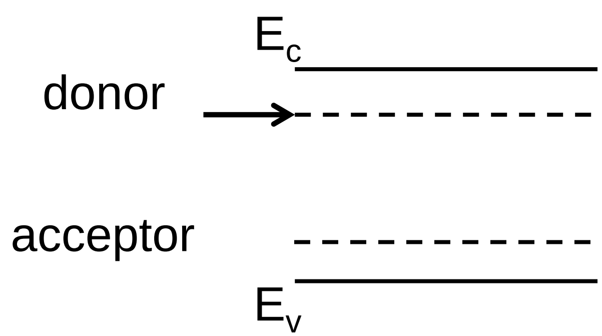
(1) Assuming complete ionization:

$$n_0 \approx N_d$$

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) = kT \ln\left(\frac{N_c}{N_d}\right)$$

N_d↑, E_c-E_F↓, Fermi level closer to the conduction band

Ionization of dopants



Compensated semiconductors

③ Charge neutrality

$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

Fermi level position:

Special case: N_a=0, Only n-type doping

Boltzmann approximation

$$\boxed{n_0 = N_d^+ + p_0} \qquad \boxed{n_0 p_0 = n_i^2}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)}$$

$$\boxed{n_i \ll N_d^+ \Rightarrow T \text{ not very high}}$$

$$n_0 \approx N_d^+$$

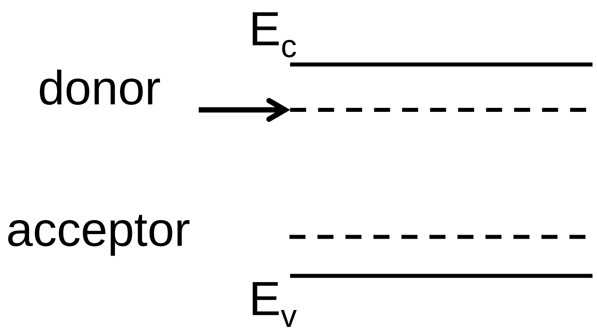
(1) Assuming complete ionization:

$$n_0 \approx N_d$$

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) = kT \ln\left(\frac{N_c}{N_d}\right)$$

Compensated semiconductor:
$N_d \rightarrow N_d - N_a$

Ionization of dopants



Compensated semiconductors

③ Charge neutrality

$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

Fermi level position:

Special case: N_a=0, Only n-type doping

Boltzmann approximation

$$\boxed{n_0 = N_d^+ + p_0} \qquad \boxed{n_0 p_0 = n_i^2}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)}$$

$$\boxed{n_i \ll N_d^+ \Rightarrow T \text{ not very high}}$$

$$n_0 \approx N_d^+$$

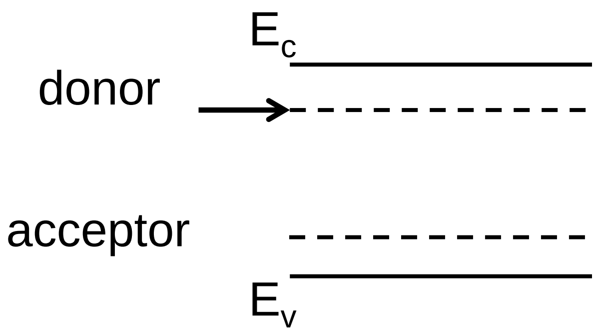
(1) Assuming complete ionization:

$$n_0 \approx N_d$$

Also, $n_0 = n_i \exp\left[(E_F - E_{Fi})/kT\right]$

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)$$

Ionization of dopants



Compensated semiconductors

③ Charge neutrality

$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

Fermi level position:

Special case: N_d=0, Only p-type doping

$$\boxed{n_i << N_a^+ \Rightarrow T\ not\ very\ high}$$

$$p_0 \approx N_a^+$$

(1) Assuming complete ionization:

$$p_0 \approx N_a$$

Boltzmann approximation

$$p_0 = N_v \exp\left[-(E_F - E_v)/kT\right]$$

$$E_F - E_v = kT \ln\left(\frac{N_v}{N_a}\right)$$

$$E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

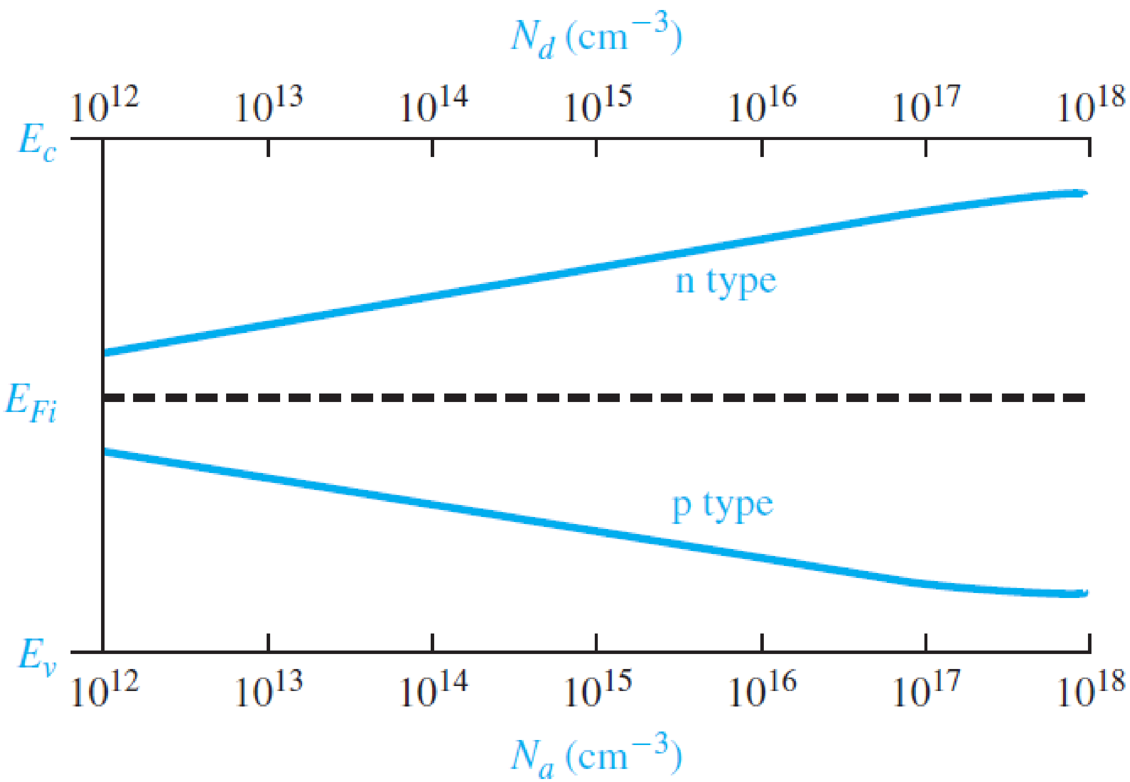
Ionization of dopants

Fermi level position:

$\boxed{T\ not\ very\ high}$

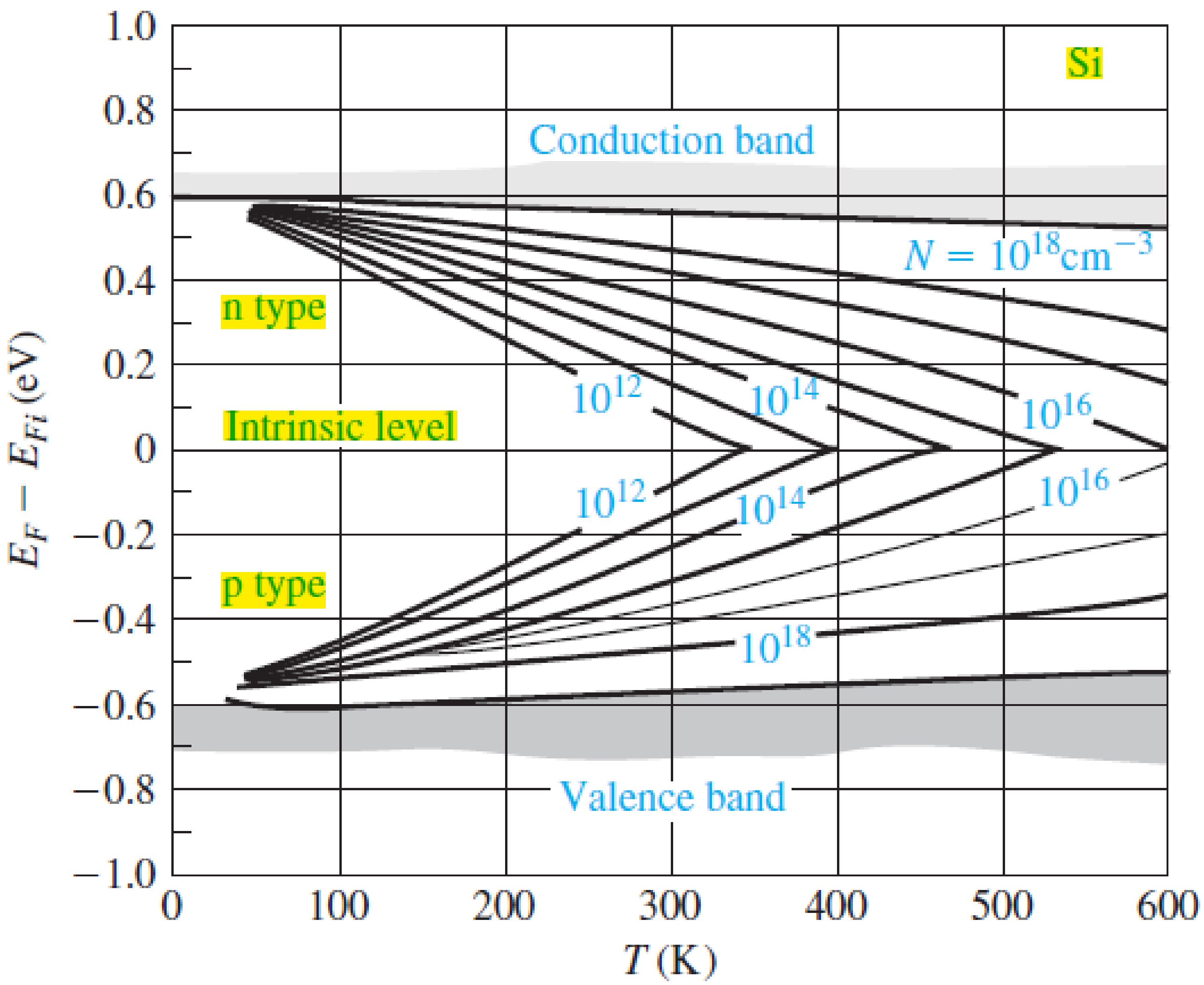
Assuming complete ionization:

Boltzmann approximation

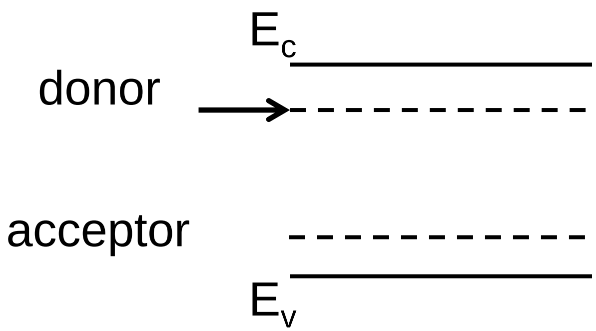


Ionization of dopants

Fermi level position:



Ionization of dopants



Compensated semiconductors

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$$n_0 + N_a^- = N_d^+ + p_0$$

$$n_0 = N_d^+ - N_a^- + p_0$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d: concentration of electrons in the donor states
p_a: concentration of holes in the acceptor states

Fermi level position:

Special case: N_a=0, Only n-type doping

Boltzmann approximation

$$n_0 = N_d^+ + p_0 \qquad n_0 p_0 = n_i^2$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$n_i << N_d^+ \Rightarrow T$ *not very high*

$$n_0 \approx N_d^+$$

(2) Incomplete ionization:

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_d^+ = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}$$

Ionization of dopants

$\boxed{n_i << N_d^+ \Rightarrow T\ not\ very\ high}$

(2) Incomplete ionization:

$$\boxed{n_0 = N_d^+}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}}$$

Ionization of dopants

$\boxed{n_i << N_d^+ \Rightarrow T\ not\ very\ high}$

(2) Incomplete ionization:

$$\boxed{n_0 = N_d^+}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}}$$

$$\Downarrow$$

$$\boxed{\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}}$$

Ionization of dopants

$$\boxed{n_i \ll N_d^+ \Rightarrow T\ not\ very\ high}$$

(2) Incomplete ionization:

$$\boxed{n_0 = N_d^+}$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}}$$

$$\boxed{\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}}$$

$$\frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})} = \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp(\frac{E_F - E_c}{kT})}$$

Ionization of dopants

(2) Incomplete ionization:

$$n_i << N_d^+ \Rightarrow T\ not\ very\ high$$

$$n_0 = N_d^+$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}$$

$$\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}$$

$$\frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})} = \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp(\frac{E_F - E_c}{kT})}$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_d^+ = \frac{N_d}{1 + 2\exp(\frac{E_A}{kT})\frac{n_0}{N_c}}$$

Ionization of dopants

$n_i << N_d^+ \Rightarrow T\ not\ very\ high$

(2) Incomplete ionization:

$n_0 = N_d^+$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})}$$

$$\exp\left(\frac{E_F - E_c}{kT}\right) = \frac{n_0}{N_c}$$

$$\frac{N_d}{1 + 2\exp(\frac{E_F - E_d}{kT})} = \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp(\frac{E_F - E_c}{kT})}$$

$$2\exp\left(\frac{E_A}{kT}\right) n_0^2 + N_c n_0 - N_d N_c = 0$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) = N_d^+ = \frac{N_d}{1 + 2\exp(\frac{E_A}{kT})\frac{n_0}{N_c}}$$

Ionization of dopants

(2) Incomplete ionization:

$$\boxed{2\exp\left(\frac{E_A}{kT}\right)n_0^2 + N_c n_0 - N_d N_c = 0}$$

$$\boxed{n_0 = N_c \times \frac{-1+\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}}$$

$$\Downarrow$$

$$\boxed{n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \Rightarrow \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{-1+\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}}$$

$$\Downarrow$$

$$\boxed{E_F = E_c + kTln(\frac{\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}-1}{4\exp(\frac{E_A}{kT})})}$$

Ionization of dopants

(2) Incomplete ionization, Boltzmann approximation, only n doping:

$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})} = \begin{cases} \sqrt{\frac{N_d N_c}{2}}\exp(-\frac{E_A}{2kT}) & T\ small \\ N_d & T\ big \end{cases}$$

$$E_F = E_c + kTln(\frac{\sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})} - 1}{4\exp(\frac{E_A}{kT})}) = \begin{cases} \frac{E_c + E_d}{2} + \frac{kT}{2} ln\frac{N_d}{2N_c} & T\ small \\ E_c - kTln\frac{N_c}{N_d} & T\ big \end{cases}$$

Ionization of dopants

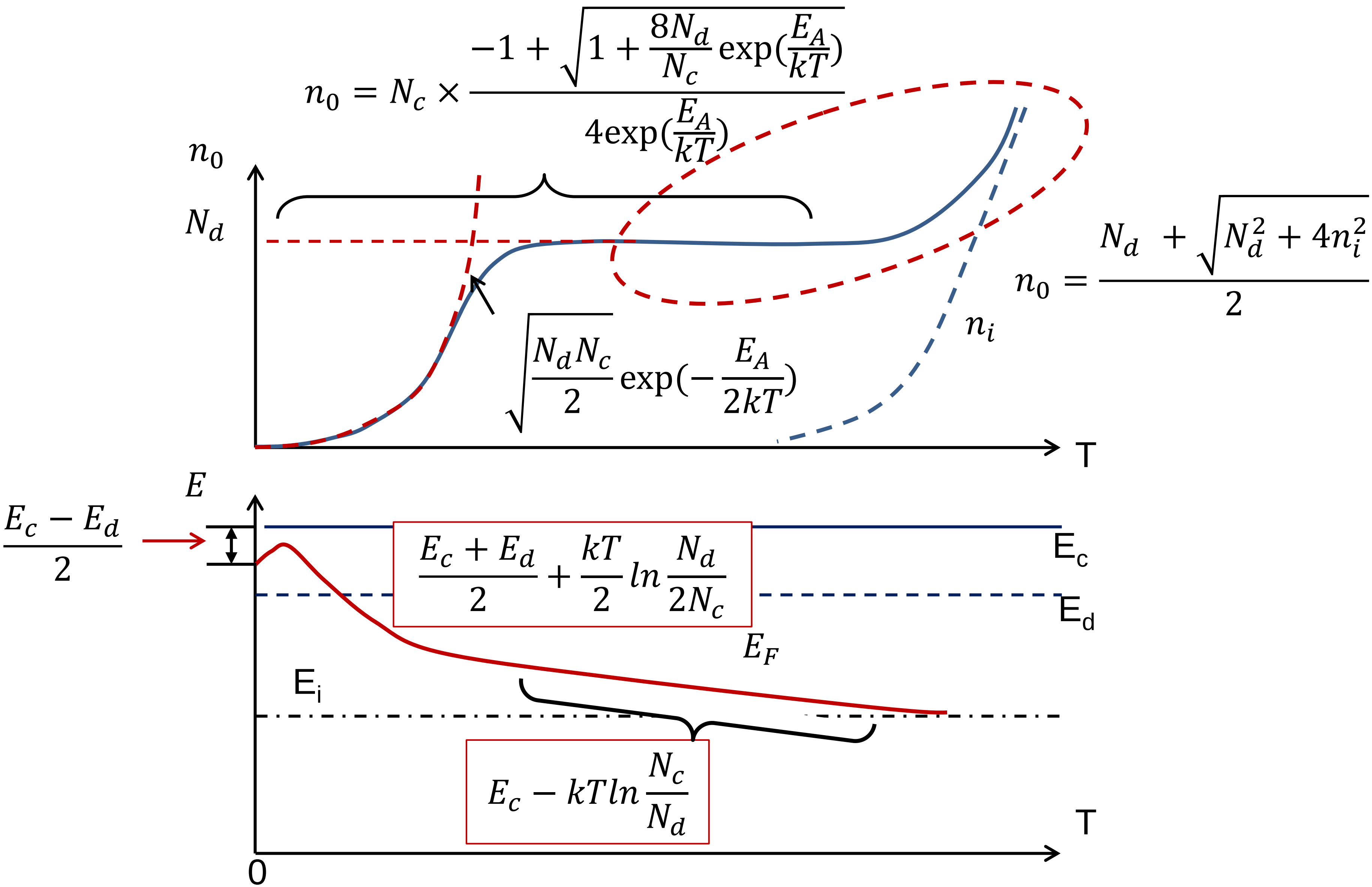
$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})} = \begin{cases} \sqrt{\frac{N_d N_c}{2}}\exp(-\frac{E_A}{2kT}) & T\ small \\ N_d & T\ big \end{cases}$$

$$\boxed{N_c \sim T^{3/2}}$$

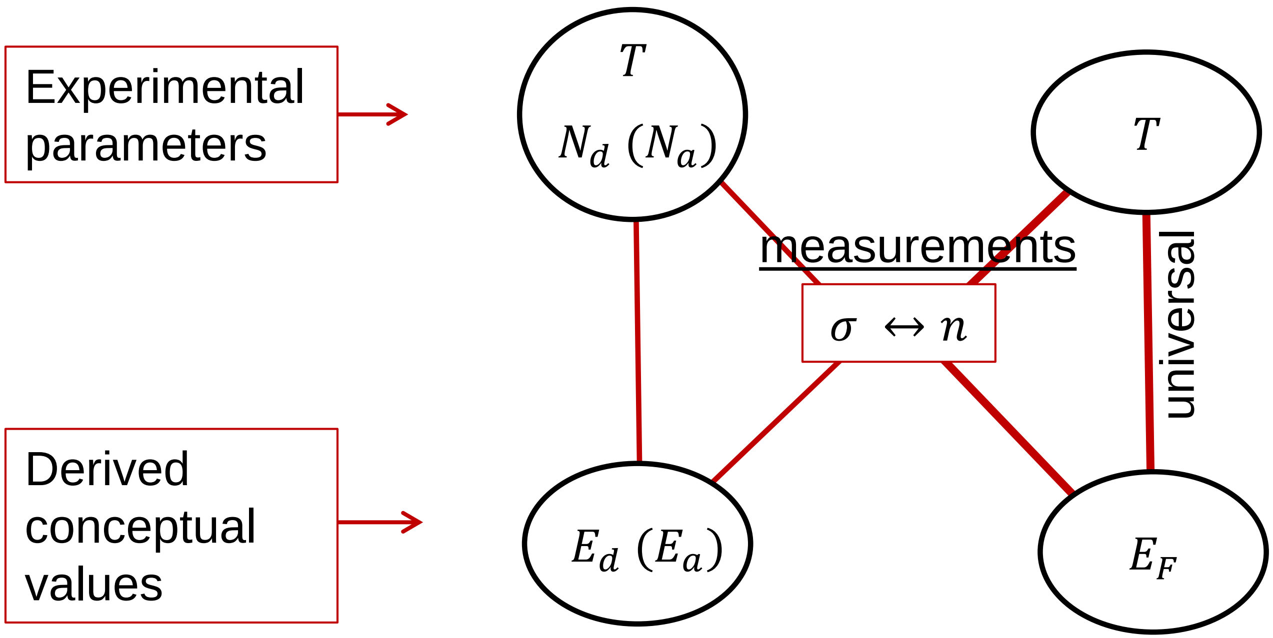
$T \to 0$ 0

$$E_F = E_c + kTln(\frac{\sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})} - 1}{4\exp(\frac{E_A}{kT})}) = \begin{cases} \frac{E_c + E_d}{2} + \frac{kT}{2} ln\frac{N_d}{2N_c} & T\ small \\ E_c - kTln\frac{N_c}{N_d} & T\ big \end{cases}$$

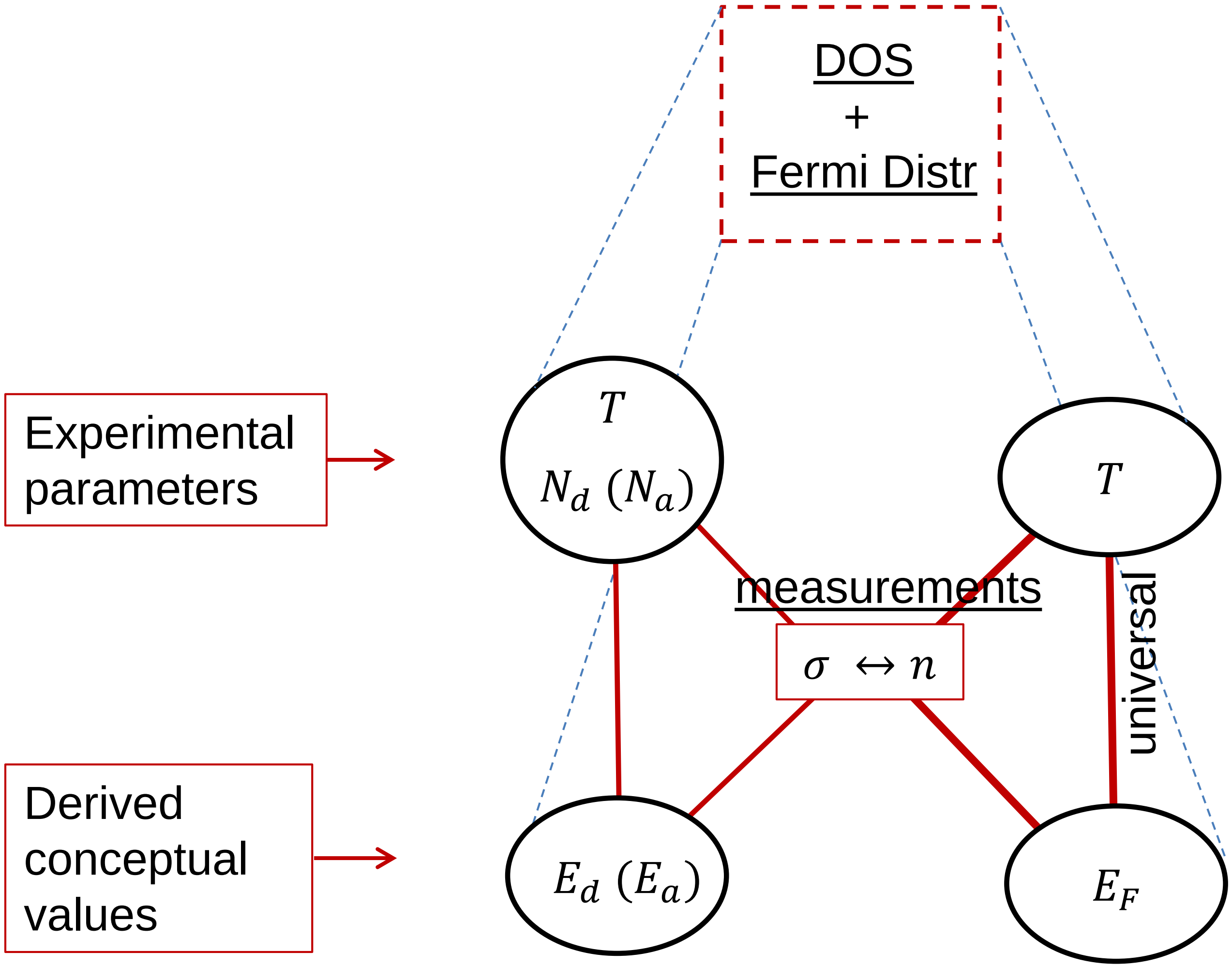
Ionization of dopants



Summary of Lecture 4



Summary of Lecture 4



Summary of Lecture 4

Why is E_F useful?

In thermal equilibrium, the Fermi energy level is a constant throughout a system