Homework 6

Skills and Concepts:

- Laplace transform ROC, pole-zero plot, Laplace transform and LTI systems, feedback control, Z-transform ROC

HW Notes:

- Problems where the number of points are followed by an exclamation point are basic skill problems and will be graded without partial credit.
- $\boxed{\text{Box}}$ your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit.

  For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

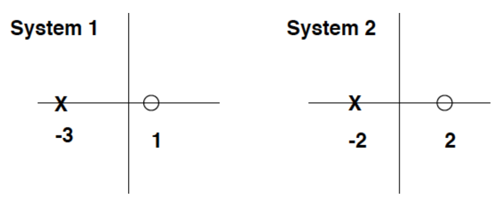
Problems:

1. [5] Is this system stable? Explain. (Note: the system is causal.)

$$2 \cdot 10^6 y(t) + 10^5 \frac{d}{dt} y(t) + 60 \frac{d^2}{dt^2} y(t) + \frac{d^3}{dt^3} y(t) = 8 \cdot 10^6 x(t) - 10^4 \frac{d}{dt} x(t)$$

   Hint: use matlab to compute roots.

2. [10] A unit step signal is applied to a system consisting of two LTI system connected in parallel. The pole-zero plots of each of the system are shown below. Determine the output signal. Assume that each of the system has unit gain at DC.



   Hint: first find the Laplace transform $Y(s)$ of the output signal using the convolution and linearity properties of the Laplace transform, Then take the inverse Laplace transform to get $y(t)$ using PFE. The "unit gain at DC" specifies $H_1(0)$ and $H_2(0)$, which you can use to determine the scaling factor.

3. [20] Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

   (a) Determine the Laplace transform of $x(t)$ and $h(t)$.

   (b) Using the convolution property, determine the Laplace transform $Y(s)$ of the output $y(t)$.

   (c) From the Laplace transform of $y(t)$ as obtained in part(b), determine $y(t)$.

   (d) Verify your result in part (c) by explicitly convolving $x(t)$ and $h(t)$.

4. [10] The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}$$

Determine the response $y(t)$ when the input is

$$e^{-|t|}, \quad -\infty < t < \infty$$

5. [25] In this problem, we consider the construction of various type of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2+4s-6}{s^2+3s+2}$$

To derive the direct-diagram representation of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S, but whose system function is

$$H_1(s) = \frac{1}{s^2+3s+2}$$

With the output of S_1 denoted by $y_1(t)$, the direct-form diagram representation of S_1 is shown in Figure 1. The signals $e(t)$ and $f(t)$ indicates in the figure represent respective inputs into the two integrators.

(a) [5!] Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$, and $d^2y_1(t)/dt^2$.

(b) [1!] How is $dy_1(t)/dt$ related to $f(t)$.

(c) [2!] How is $d^2y_1(t)/dt^2$ related to $e(t)$.

(d) [2!] Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, $y_1(t)$.

(e) [5!] Use the result from the previous part to extend the direct-form block diagram representation of S_1 and create a block diagram representation of S.

(f) [5!] Observing that

$$H(s) = \left(\frac{2(s-1)}{s+2}\right)\left(\frac{s+3}{s+1}\right)$$

draw a block diagram representation for S as a cascade combination of two subsystems.

(g) [5!] Observing that

$$H(S) = 2 + \frac{6}{s+2} - \frac{8}{s+1}$$

draw a block-diagram representation for S as parallel combination of three subsystems.

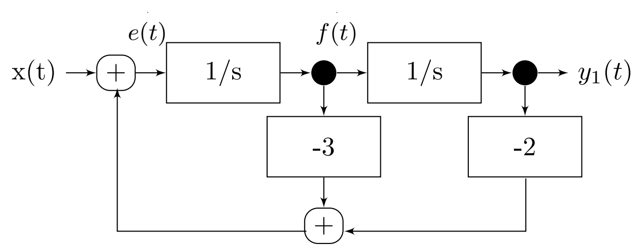


Figure 1

6. [10!] An LTI system has an impulse response $h[n]$ for which the z-transform is

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

(a) [5!] Plot the pole-zero pattern for $H(z)$.

(b) [5!] Using the fact that signals of the form z^n are eigenfunctions of LTI systems, determin the system output for all n if the input $x[n]$ is

$$x[n] = (\frac{3}{4})^n + 3(2)^n$$

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7. [20!] Consider the sequence $x[n] = 2^n u[n]$.

(a) [4!] Is $x[n]$ absolutely summable?

(b) [4!] Does the Fourier transform of $x[n]$ converge?

(c) [4!] For what range of values of r does the Fourier transform of the sequence $r^{-n}x[n]$ converge?

(d) [4!]Determine the z-transform $X(z)$ of $x[n]$, including a specification of the ROC.

(e) [4!]$X(z)$ for $z = 3e^{j\Omega}$ can be thought of as the Fourier transform of a sequence $x_1[n]$, i.e.,

$$2^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z), \qquad x_1[n] \overset{\mathcal{F}}{\longleftrightarrow} X(3e^{j\Omega}) = X_1(e^{j\Omega}).$$

Determine $x_1[n]$.