(b) Find the parameters of the second-order system with the open-loop transfer function

$$G_L(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

that will give the same values for M_r and ω_r as the third-order system. Compare the values of BW of the two systems.

M_r, ω_r, and BW of third-order and second-order systems

9-24. Repeat Problem 9-23 for the transfer function given in Problem 9-22(f). (Although the approach used here may render results that are better or at least as good as those obtained by the method outlined in Section 7-10, the match here is restricted only to a prototype second-order system as the approximating low-order system.)

9-25. Sketch or plot the Bode diagrams of the forward-path transfer functions given in Problem 9-2. Find the gain margin, gain-crossover frequency, phase margin, and the phase-crossover frequency for each system.

Gain and phase margins from Bode plots

9-26. The forward-path transfer functions of unity-feedback control systems are given in the following. Plot the Bode diagram of $G(j\omega)/K$ and do the following: **(1)** Find the value of K so that the gain margin of the system is 20 dB. **(2)** Find the value of K so that the phase margin of the system is 45°.

(a) $G(s) = \dfrac{K}{s(1 + 0.1s)(1 + 0.5s)}$ $\quad$ **(b)** $G(s) = \dfrac{K(s + 1)}{s(1 + 0.1s)(1 + 0.2s)(1 + 0.5s)}$

(c) $G(s) = \dfrac{K}{(s + 3)^3}$ $\quad$ **(d)** $G(s) = \dfrac{K}{(s + 3)^4}$

(e) $G(s) = \dfrac{Ke^{-s}}{s(1 + 0.1s + 0.01s^2)}$ $\quad$ **(f)** $G(s) = \dfrac{K(1 + 0.5s)}{s(s^2 + s + 1)}$

9-27. The forward-path transfer functions of unity-feedback control systems are given in the following equations. Plot $G(j\omega)/K$ in the gain-phase coordinates of the Nichols chart, and do the following: **(1)** Find the value of K so that the gain margin of the system is 10 dB. **(2)** Find the value of K so that the phase margin of the system is 45°. **(3)** Find the value of K so that $M_r = 1.2$.

(a) $G(s) = \dfrac{10K}{s(1 + 0.1s)(1 + 0.5s)}$ $\quad$ **(b)** $G(s) = \dfrac{5K(s + 1)}{s(1 + 0.1s)(1 + 0.2s)(1 + 0.5s)}$

(c) $G(s) = \dfrac{10K}{s(1 + 0.1s + 0.01s^2)}$ $\quad$ **(d)** $G(s) = \dfrac{10Ke^{-s}}{s(1 + 0.1s + 0.01s^2)}$

9-28. The Bode diagram of the forward-path transfer function of a unity-feedback control system is obtained experimentally, and is shown in Fig. 9P-28 when the forward gain K is set at its nominal value.

(a) Find the gain and phase margins of the system from the diagram as best you can read them. Find the gain and phase-crossover frequencies.

(b) Repeat part (a) if the gain is doubled from its nominal value.

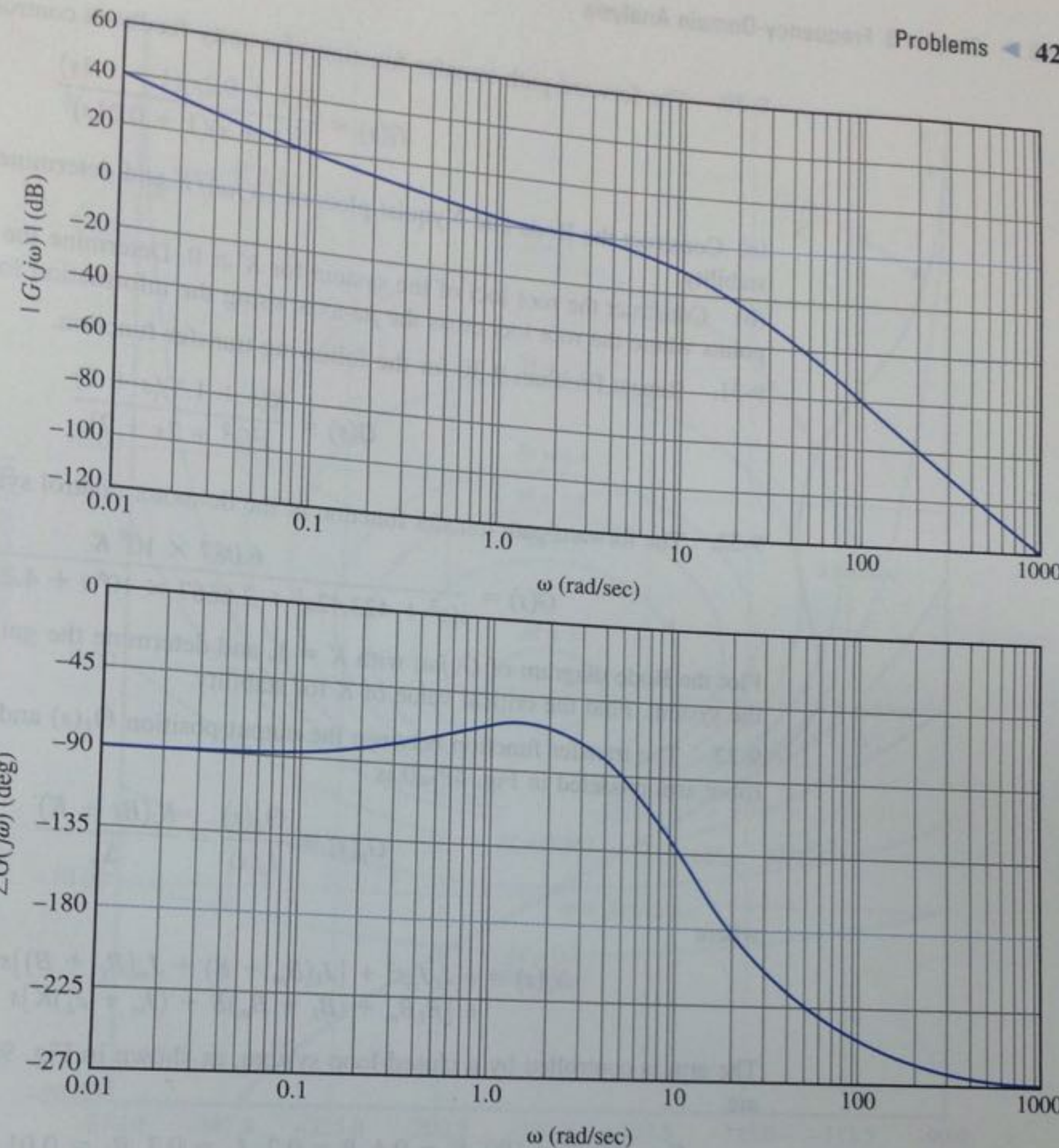
(c) Repeat part (a) if the gain is 10 times its nominal value.

(d) Find out how much the gain must be changed from its nominal value if the gain margin is to be 40 dB.

(e) Find out how much the loop gain must be changed from its nominal value if the phase margin is to be 45°.

(f) Find the steady-state error of the system if the reference input to the system is a unit-step function.

(g) The forward path now has a pure time delay of T_d sec, so that the forward-path transfer function is multiplied by $e^{-T_d s}$. Find the gain margin and the phase margin for $T_d = 0.1$ sec. The gain is set at nominal.



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(h) With the gain set at nominal, find the maximum time delay T_d the system can tolerate without going into instability.

ions of Bode

9-29. Repeat Problem 9-28 using Fig. 9P-28 for the following parts.

(a) Find the gain and phase margins if the gain is four times its nominal value. Find the gain- and phase-crossover frequencies.

(b) Find out how much the gain must be changed from its nominal value if the gain margin is to be 20 dB.

(c) Find the marginal value of the forward-path gain for system stability.

(d) Find out how much the gain must be changed from its nominal value if the phase margin is to be 60°.

(e) Find the steady-state error of the system if the reference input is a unit-step function and the gain is twice its nominal value.

(f) Find the steady-state error of the system if the reference input is a unit-step function and the gain is 20 times its nominal value.

(g) The system now has a pure time delay so that the forward-path transfer function is multiplied $e^{-T_d s}$. Find the gain and phase margins when $T_d = 0.1$ sec. The gain is set at its nominal value.

(h) With the gain set at 10 times its nominal, find the maximum time delay T_d the system can tolerate without going into instability.

9-30. The forward-path transfer function of a unity-feedback control system is

$$G(s) = \frac{K(1 + 0.2s)(1 + 0.1s)}{s^2(1 + s)(1 + 0.01s)^2}$$

(a) Construct the Bode and Nyquist plots of $G(j\omega)/K$ and determine the range of K for system stability.

(b) Construct the root loci of the system for $K \geq 0$. Determine the values of K and ω at the points where the root loci cross the $j\omega$-axis, using the information found from the Bode plot.

9-31. Repeat Problem 9-30 for the following transfer function.

$$G(s) = \frac{K(s + 1.5)(s + 2)}{s^2(s^2 + 2s + 2)}$$

9-32. The forward-path transfer function of the dc-motor control system described in Fig. 3P-41 is

$$G(s) = \frac{6.087 \times 10^8 K}{s(s^3 + 423.42s^2 + 2.6667 \times 10^6 s + 4.2342 \times 10^8)}$$

Plot the Bode diagram of $G(j\omega)$ with $K = 1$, and determine the gain margin and phase margin of the system. Find the critical value of K for stability.

9-33. The transfer function between the output position $\Theta_L(s)$ and the motor current $I_a(s)$ of the robot arm modeled in Fig. 4P-20 is

$$G_p(s) = \frac{\Theta_L(s)}{I_a(s)} = \frac{K_i(Bs + K)}{\Delta_o}$$

where

$$\begin{aligned}\Delta_o(s) = s\{&J_LJ_ms^3 + [J_L(B_m + B) + J_m(B_L + B)]s^2 \\ &+ [B_LB_m + (B_L + B_m)B + (J_m + J_L)K]s + K(B_L + B_m)\}\end{aligned}$$

The arm is controlled by a closed-loop system, as shown in Fig. 9P-33. The system parameters are

$$K_a = 65,\ K = 100,\ K_i = 0.4,\ B = 0.2,\ J_m = 0.2,\ B_L = 0.01,\ J_L = 0.6, \text{ and } B_m = 0.25.$$

(a) Derive the forward-path transfer function $G(s) = \Theta_L(s)/E(s)$.

(b) Draw the Bode diagram of $G(j\omega)$. Find the gain and phase margins of the system.

(c) Draw $|M(j\omega)|$ versus ω, where $M(s)$ is the closed-loop transfer function. Find M_r, ω_r, and BW.

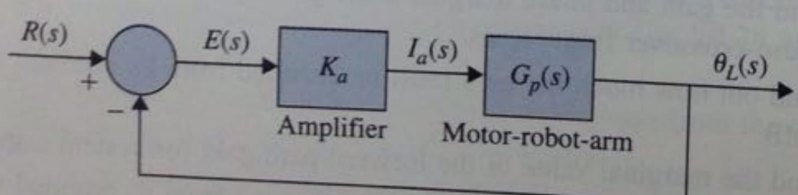

Figure 9P-33

9-34. The gain-phase plot of the forward-path transfer function of $G(j\omega)/K$ of a unity-feedback control system is shown in Fig. 9P-34. Find the following performance characteristics of the system.

(a) Gain-crossover frequency (rad/sec) when $K = 1$.

(b) Phase-crossover frequency (rad/sec) when $K = 1$.

where K and a are real constants. Find the values of K and a.

The design strategy is to place the closed-loop poles at $-10 + j10$ and $-10 - j10$, and then adjust the values of K and a to satisfy the steady-state requirement. The value of a is large so that it will not affect the transient response appreciably. Find the maximum overshoot of the designed system.

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10-2. Repeat Problem 10-1 if the ramp-error constant is to be 9. What is the maximum value of K_v that can be realized? Comment on the difficulties that may arise in attempting to realize a very large K_v.

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10-3. A control system with a PD controller is shown in Fig. 10P-3.

(a) Find the values of K_P and K_D so that the ramp-error constant K_v is 1000 and the damping ratio is 0.5.

(b) Find the values of K_P and K_D so that the ramp-error constant K_v is 1000 and the damping ratio is 0.707.

(c) Find the values of K_P and K_D so that the ramp-error constant K_v is 1000 and the damping ratio is 1.0.

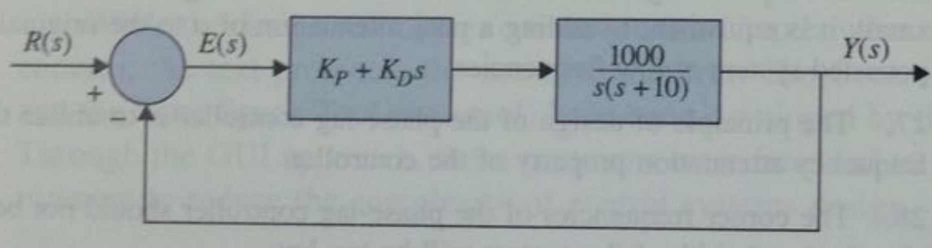

Figure 10P-3

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10-4. For the control system shown in Fig. 10P-3, set the value of K_P so that the ramp-error constant is 1000.

(a) Vary the value of K_D from 0.2 to 1.0 in increments of 0.2 and determine the values of phase margin, gain margin, M_r, and BW of the system. Find the value of K_D so that the phase margin is maximum.

(b) Vary the value of K_D from 0.2 to 1.0 in increments of 0.2 and find the value of K_D so that the maximum overshoot is minimum.

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10-5. Consider the second-order model of the aircraft attitude control system shown in Fig. 7-25. The transfer function of the process is

$$G_p(s) = \frac{4500K}{s(s + 361.2)}$$

(a) Design a series PD controller with the transfer function $G_c(s) = K_D + K_P s$ so that the following performance specifications are satisfied:

Steady-state error due to a unit-ramp input ≤ 0.001
Maximum overshoot ≤ 5 percent
Rise time $t_r \leq 0.005$ sec
Settling time $t_s \leq 0.005$ sec

(b) Repeat part (a) for all the specifications listed, and, in addition, the bandwidth of the system must be less than 850 rad/sec.