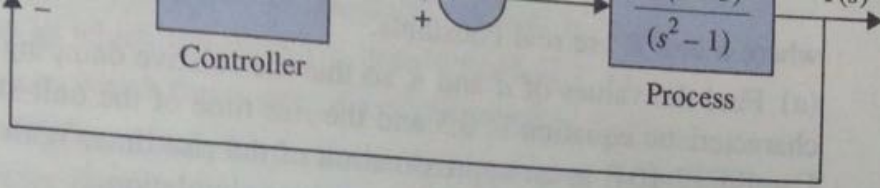

7-12. The unit-step response of a linear control system is shown in Fig. 7P-12. Find the transfer function of a second-order prototype system to model the system.

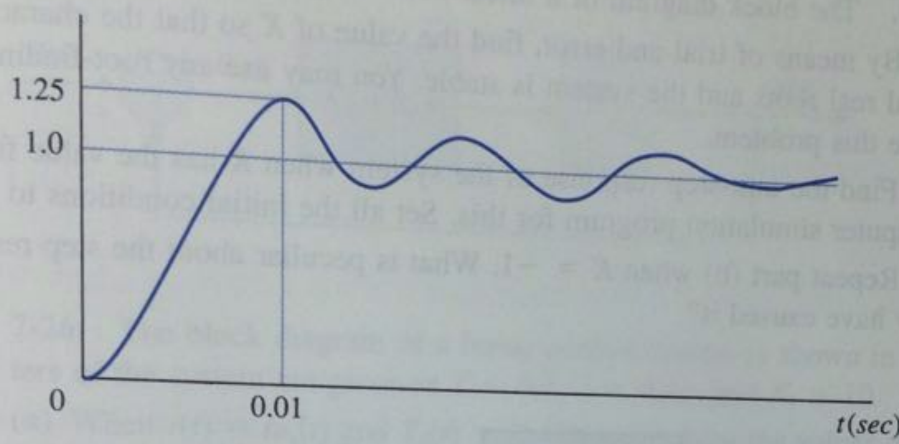

Figure 7P-12

7-13. For the control system shown in Fig. 7P-7, find the values of K and K_t so that the maximum overshoot of the output is approximately 4.3 percent and the rise time t_r is approximately 0.2 sec. Use Eq. (7-104) for the rise-time relationship. Simulate the system with any time-response simulation program to check the accuracy of your solutions.

7-14. Repeat Problem 7-13 with a maximum overshoot of 10 percent and a rise time of 0.1 sec.

7-15. Repeat Problem 7-13 with a maximum overshoot of 20 percent and a rise time of 0.05 sec.

7-16. For the control system shown in Fig. 7P-7, find the values of K and K_t so that the maximum overshoot of the output is approximately 4.3 percent and the delay time t_d is approximately 0.1 sec. Use Eq. (7-102) for the delay-time relationship. Simulate the system with a computer program to check the accuracy of your solutions.

7-17. Repeat Problem 7-16 with a maximum overshoot of 10 percent and a delay time of 0.05 sec.

7-18. Repeat Problem 7-16 with a maximum overshoot of 20 percent and a delay time of 0.01 sec.

7-19. For the control system shown in Fig. 7P-7, find the values of K and K_t so that the damping ratio of the system is 0.6 and the settling time of the unit-step response is 0.1 sec. Use Eq. (7-108) for the settling time relationship. Simulate the system with a computer program to check on the accuracy of your results.

7-20. **(a)** Repeat Problem 7-19 with a maximum overshoot of 10 percent and a settling time of 0.05 sec.

(b) Repeat Problem 7-19 with a maximum overshoot of 20 percent and a settling time of 0.01 sec.

7-21. Repeat Problem 7-19 with a damping ratio of 0.707 and a settling time of 0.1 sec. Use Eq. (7-109) for the settling time relationship.

7-22. The forward-path transfer function of a control system with unity feedback is

$$G(s) = \frac{K}{s(s+a)(s+30)}$$

where a and K are real constants.

(a) Find the values of a and K so that the relative damping ratio of the complex roots of the characteristic equation is 0.5 and the rise time of the unit-step response is approximately 1 sec. Use Eq. (7-104) as an approximation of the rise time. With the values of a and K found, determine the actual rise time using computer simulation.

(b) With the values of a and K found in part (a), find the steady-state errors of the system when the reference input is (i) a unit-step function, and (ii) a unit-ramp function.

7-23. The block diagram of a linear control system is shown in Fig. 7P-23.

(a) By means of trial and error, find the value of K so that the characteristic equation has two equal real roots and the system is stable. You may use any root-finding computer program to solve this problem.

(b) Find the unit-step response of the system when K has the value found in part (a). Use any computer simulation program for this. Set all the initial conditions to zero.

(c) Repeat part (b) when $K = -1$. What is peculiar about the step response for small t, and what may have caused it?

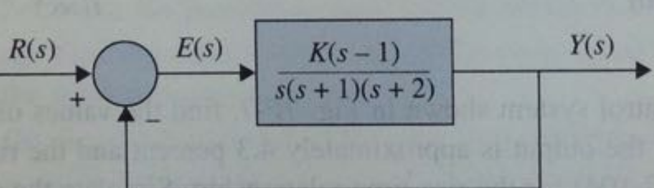

Figure 7P-23

7-24. A controlled process is represented by the following dynamic equations:

$$\frac{dx_1(t)}{dt} = -x_1(t) + 5x_2(t)$$

$$\frac{dx_2(t)}{dt} = -6x_1(t) + u(t)$$

$$y(t) = x_1(t)$$

The control is obtained through state feedback with

$$u(t) = -k_1x_1(t) - k_2x_2(t) + r(t)$$

where k_1 and k_2 are real constants, and $r(t)$ is the reference input.

(a) Find the locus in the k_1-versus-k_2 plane (k_1 = vertical axis) on which the overall system has a natural undamped frequency of 10 rad/sec.

(b) Find the locus in the k_1-versus-k_2 plane on which the overall system has a damping ratio of 0.707.

(c) Find the values of k_1 and k_2 such that $\zeta = 0.707$ and $\omega_n = 10$ rad/sec.

(d) Let the error signal be defined as $e(t) = r(t) - y(t)$. Find the steady-state error when $r(t) = u_s(t)$ and k_1 and k_2 are at the values found in part (c).