# Homework 2

**Skills and Concepts:**

- impulse response, convolution for CT LTI systems, properties of convolution, LTI system properties via impulse response, step response, CT systems described by differential equation models

**HW Notes:**

- Problems where the number of points are followed by an exclamation point are basic skill problems and will be graded without partial credit.
- $\boxed{\text{Box}}$ your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit.

  For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

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**Problems:**

1. One of following two statements is correct, and the other is incorrect. The symbol $\boxed{*}$ denotes *convolution*.
   - If $y(t) = h(t) * x(t)$ then $y(t-3) = h(t-3) * x(t-3)$;
   - Or if $y(t) = h(t) * x(t)$ then $y(t-3) = h(t) * x(t-3)$.

   (a) [4!] Give a simple proof of the correct statement.

   (b) [3!] Give a simple counterexample for the incorrect statement.

   (c) [3!] Repeat (a) and (b) for the following two statements. The symbol $\boxed{\cdot}$ denotes *multiplication*.
   - If $y(t) = h(t) \cdot x(t)$ then $y(t-3) = h(t-3) \cdot x(t-3)$;
   - Or if $y(t) = h(t) \cdot x(t)$ then $y(t-3) = h(t) \cdot x(t-3)$.

   Be careful with the notation $h(t) * x(t)$. More precise notation is $(h * x)(t)$, which makes it clear that convolution is an operation on two signals, not a point-wise operation like multiplication.

2. Often we will be convolving two signals that are zero everywhere except over some finite range (called **finite support** signals). Suppose $x_1(t)$ is non-zero over the range $a \le t \le b$ and that $x_2(t)$ is non-zero over the range $c \le t \le d$. Suppose $y(t) = x_1(t) * x_2(t)$.

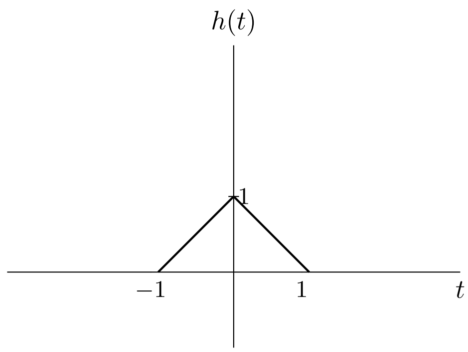
   (a) [5!] Find the range of values of $t$ for which $y(t)$ is possibly non-zero.

   (b) [10!] Compute $rect((t-2)/2) * rect((t+3)/4)$ (express answer with braces and carefully sketch). Check your result with part (a).

3. [10!] Consider a LTI system. Let its impulse response $h(t)$ be the triangular pulse shown below, and $x(t)$ be the **impulse train**

   $$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT),$$

   SKETCH $y(t) = x(t) * h(t)$ for $T = 4, 2, 1.5$ and 1.(No formulae are needed though you still want to label your graphs clearly.)




4. Let $y(t) = (x * h)(t)$. Show the following properties of convolution.

   (a) [5!] $\int_{-\infty}^{\infty} y(t) = \left[\int_{-\infty}^{\infty} x(t)dt\right] \left[\int_{-\infty}^{\infty} h(t)dt\right]$,

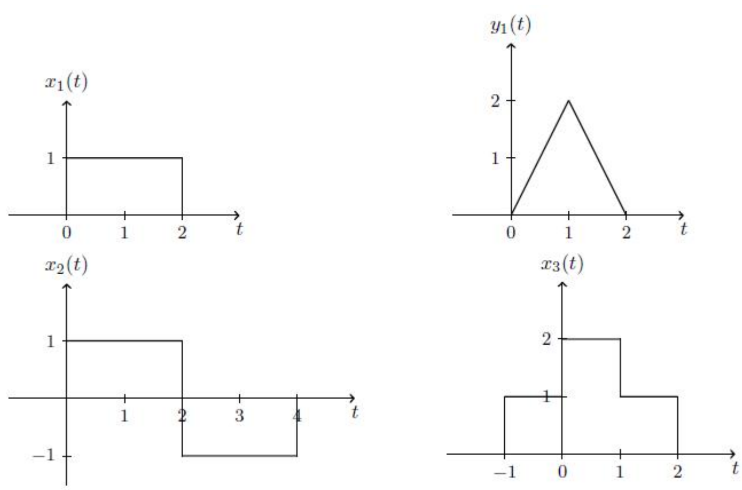
   (b) [5!] $\frac{d}{dt}y(t) = \left[\frac{d}{dt}x(t)\right] * h(t) = x(t) * \left[\frac{d}{dt}h(t)\right]$,

   (c) [5!] Suppose you have a "black box" LTI system and you discover that for a unit-step input signal the response of the system is the function $(3-t)rect(\frac{t-1}{2})$. Determine the impulse response of the system. (*Hint: See (a) and (b).*)

5. [$2 \times 5$!] Consider an LTI system whose response to the signal $x_1(t)$ is the signal $y_1(t)$ which are illustrated below.

   (a) Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted below.

   (b) Determine and sketch carefully the response of the system to the input $x_3(t)$ depicted below.




6. [10!] The triangular pulse is defined as $tri(t) = (1-|t|)rect(t/2)$. Compute $x(t) = tri(t/2) * rect(\frac{t-1}{2})$. Express your answer using braces, and carefully sketch.

7. [$2 \times 5$!] Find the impulse response of the following LTI systems and further determine whether they are causal, stable and static.

   (a) $y(t) = \int_{-\infty}^{t} (t-\tau)e^{-(t-\tau)}x(\tau)d\tau$

   (b) $y(t) = \int_{t-1}^{t+1} e^{-2(t-\tau)}x(\tau)d\tau$

8. [10!] Consider an LTI system $S$ and a signal $x(t) = 2e^{-3t}u(t-1)$. If

$$x(t) \rightarrow y(t)$$

and

$$\frac{dx(t)}{dt} \rightarrow -3y(t) + e^{-2t}u(t)$$

determine the impulse response $h(t)$ of $S$.

9. [10!] Find the expression of response of the CT system described by the linear constant-coefficient differential equation.

$$\frac{d}{dt}y(t) + 10y(t) = 2x(t); y(0) = 1; x(t) = u(t)$$