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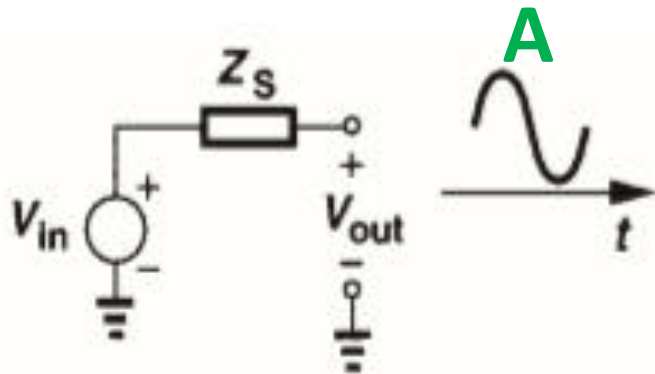
FET Differential Pair

Ve311 Electronic Circuits (Summer 2019)

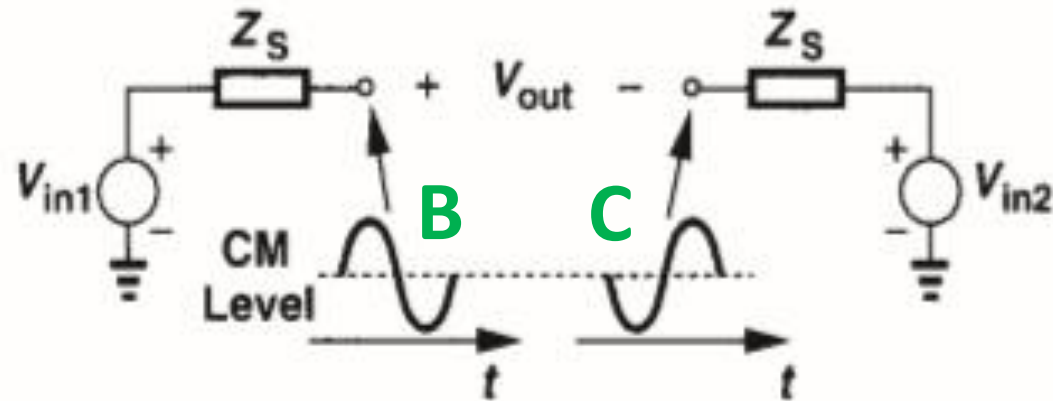
Dr. Chang-Ching Tu

Single-Ended vs Differential Signals

Single-ended



Differential

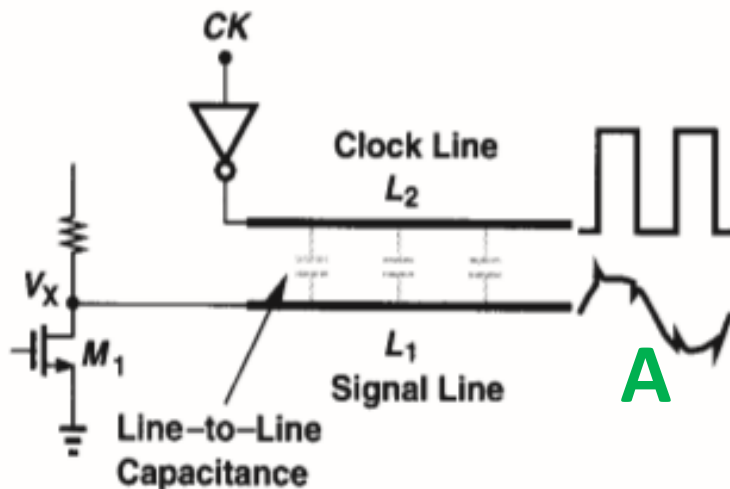


- $B - C = A$ (matters)
- $(B + C) / 2 = \text{common-mode level}$ (doesn't matter)
- Single-ended signal: a voltage signal measured with respect to ground
- Differential signal: a voltage signal measured between two nodes, each having equal amplitude and opposite phase around a common-mode (CM) level

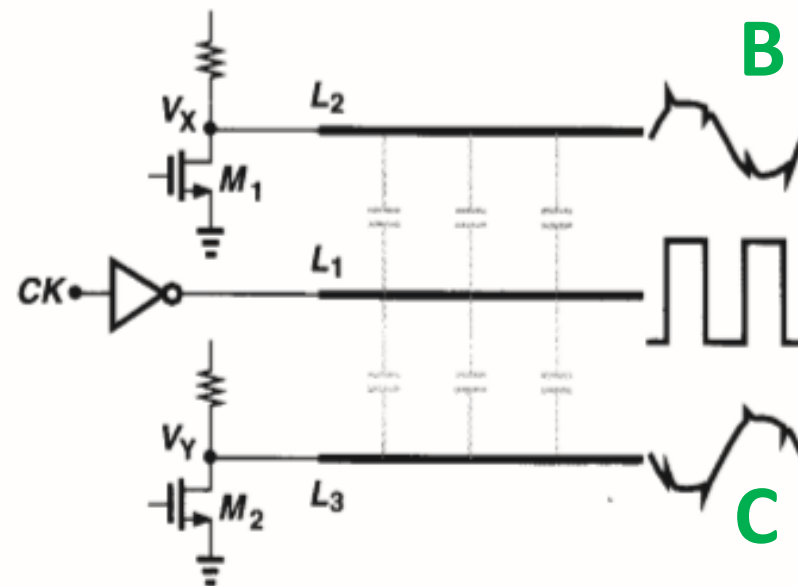
Advantages of Differential Operation

Common-Mode Noise Rejection

Single-ended



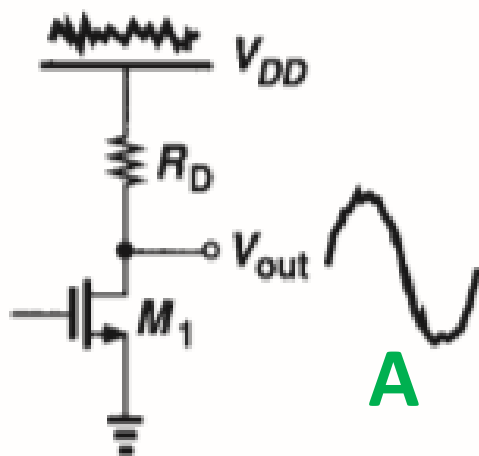
Differential



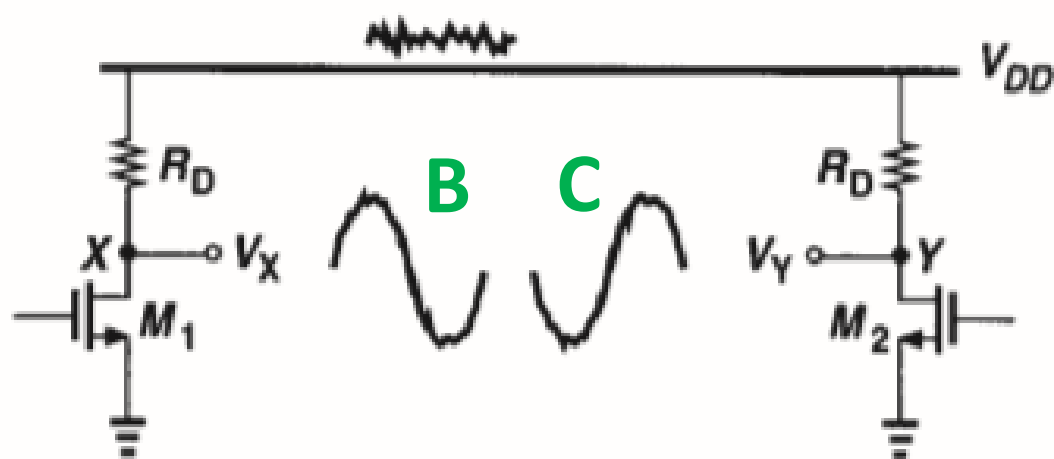
- A corrupted; B corrupted; C corrupted
- $(B + C) / 2 = \text{CM corrupted}$
- $(B - C)$ not corrupted

Common-Mode Noise Rejection

Single-ended



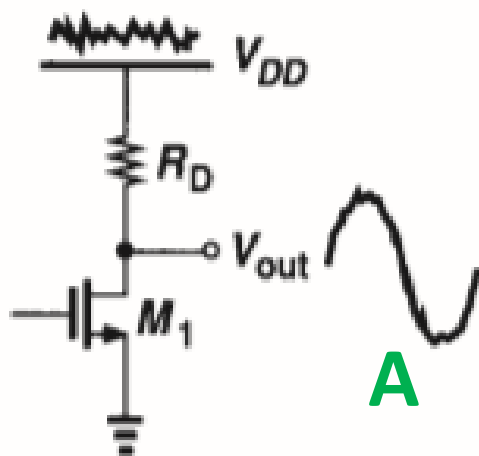
Differential



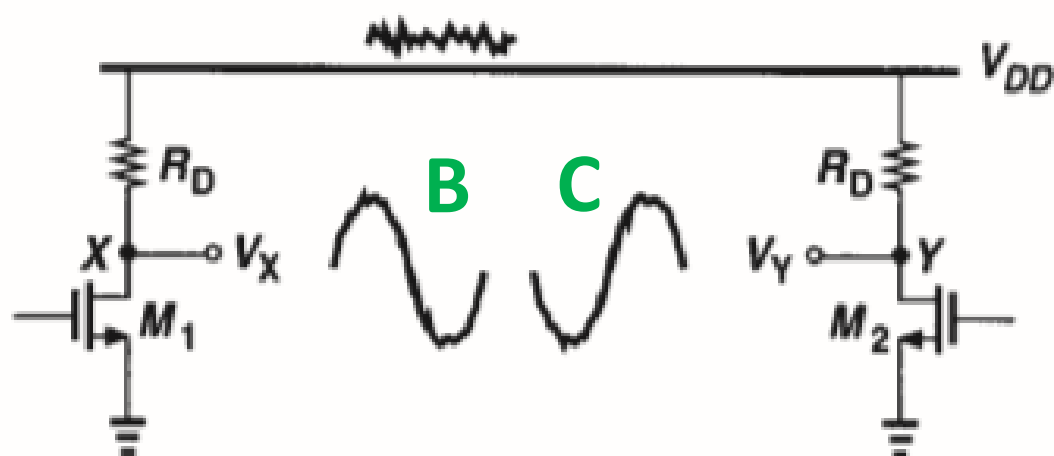
- A corrupted; B corrupted; C corrupted
- $(B + C) / 2 = \text{CM corrupted}$
- $(B - C)$ not corrupted

Increased Output Swing

Single-ended



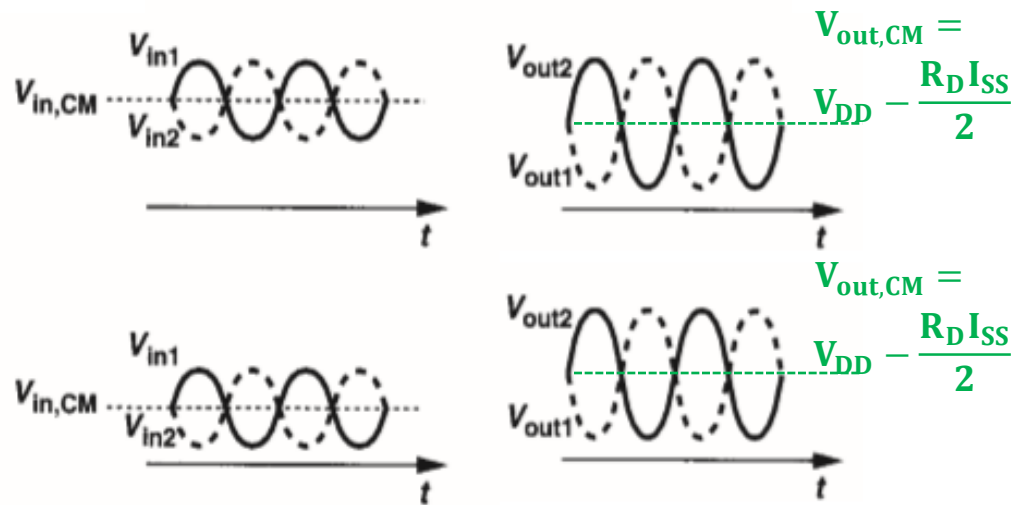
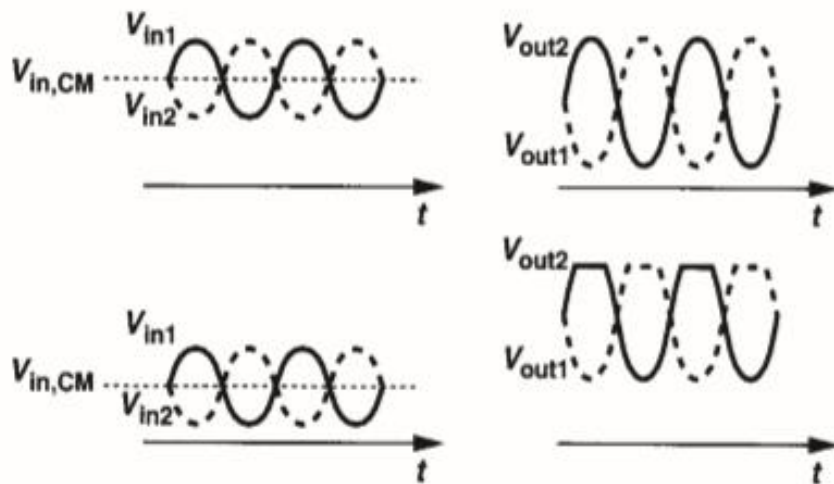
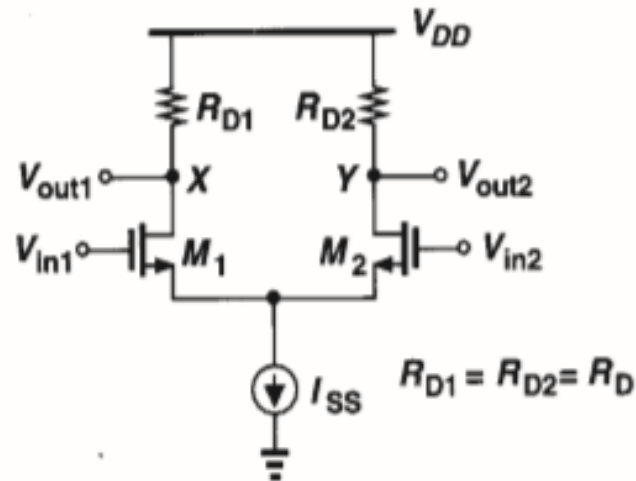
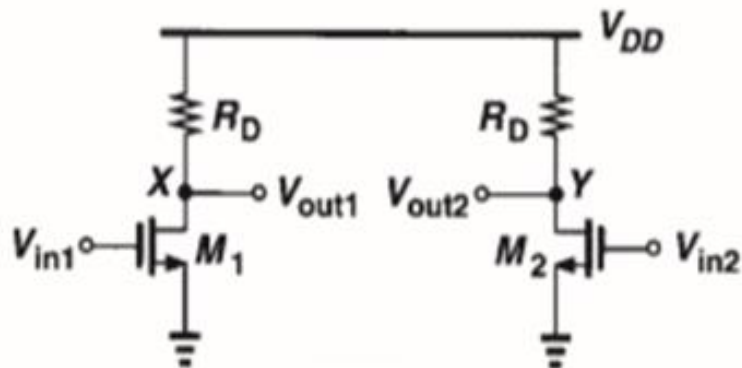
Differential



- $(V_{GS1} - V_{TH1}) \leq A \leq V_{DD}$
- $(V_{GS1,2} - V_{TH1,2}) - V_{DD} \leq (B - C) \leq V_{DD} - (V_{GS1,2} - V_{TH1,2})$

DC and Small-Signal Analysis

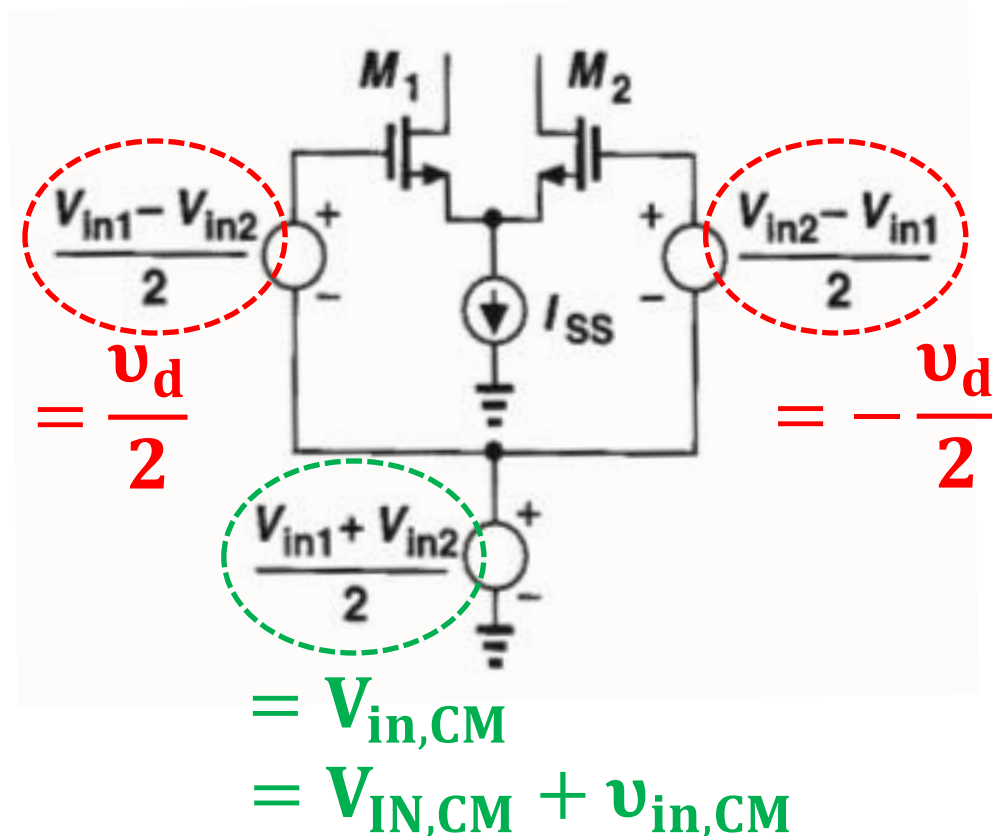
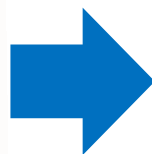
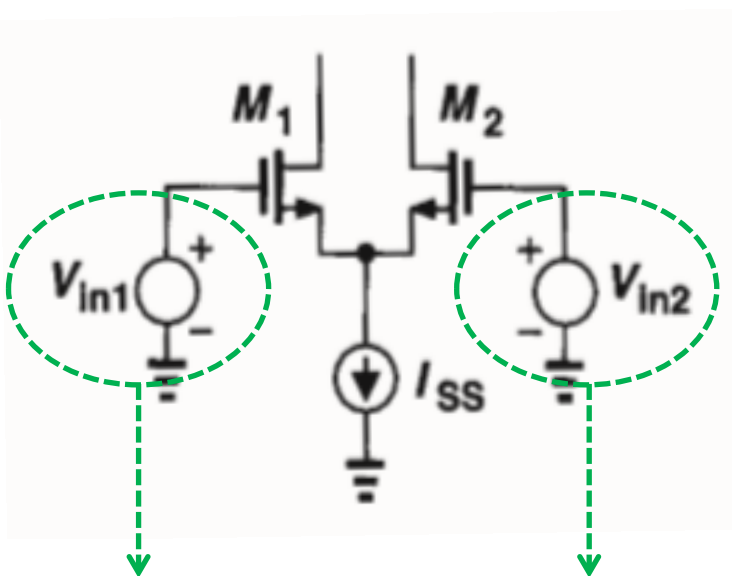
$V_{in,CM}$ and $V_{out,CM}$



- $V_{out,CM}$ dependent on $V_{in,CM}$

- $V_{out,CM}$ independent from $V_{in,CM}$
- Better design

Common-Mode + Differential-Mode



$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d}$$

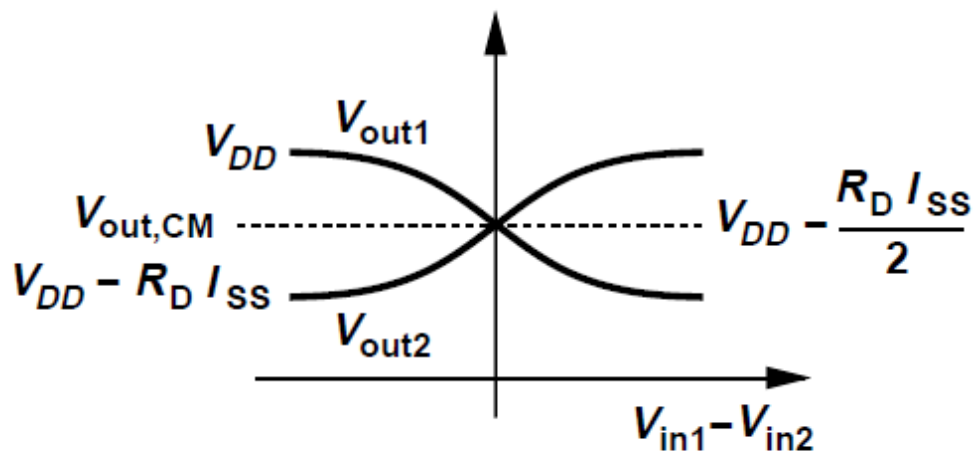
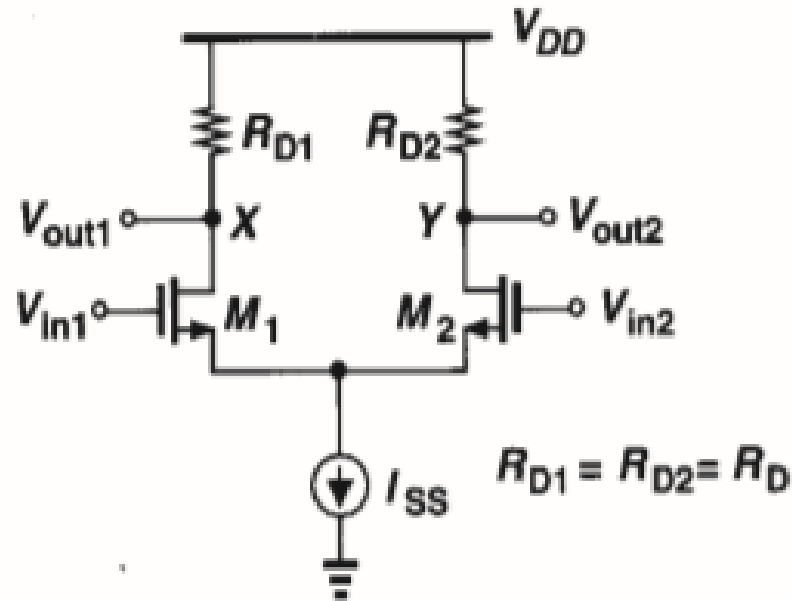
$$A_{CM-DM} = \frac{v_{out1} - v_{out2}}{v_{in,CM}}$$

$$A_{CM} = \frac{v_{out,CM}}{v_{in,CM}}$$

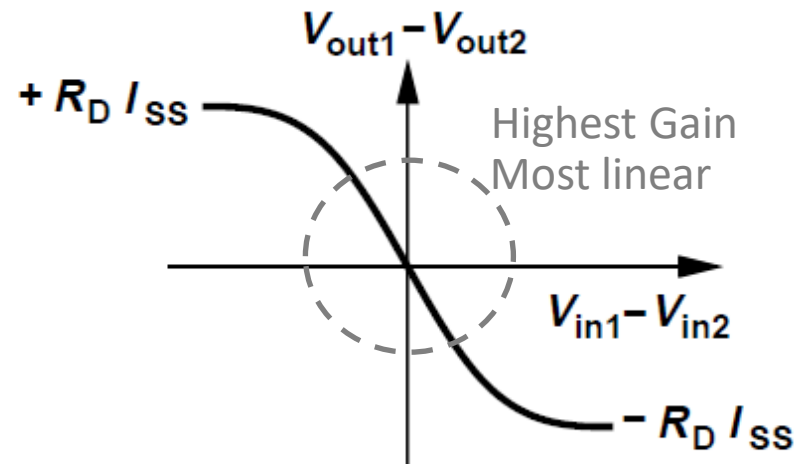
$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

Differential-Mode (Qualitative Analysis)

Qualitative Analysis



(a)

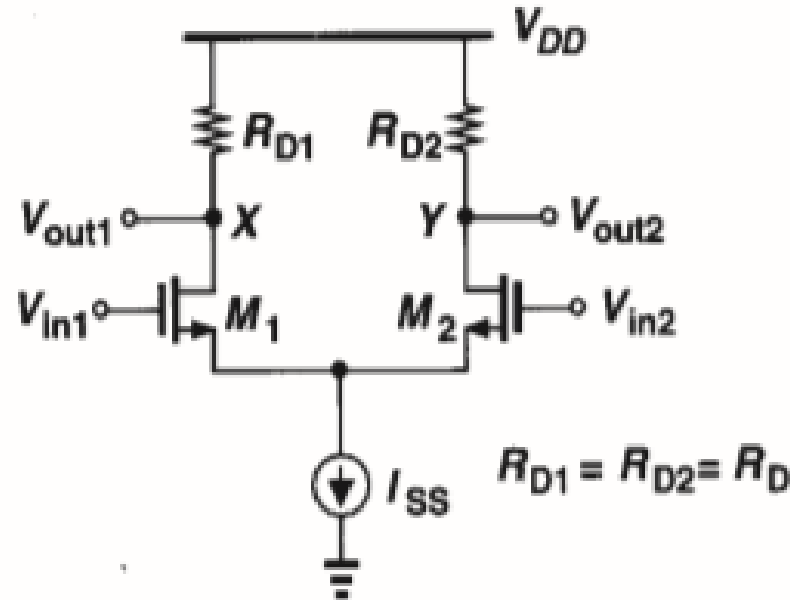


(b)

Differential-Mode (DC Analysis)

DC
Analysis

$\lambda = 0 \quad \gamma = 0$



$$V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} = (V_{GS1} - V_{TH}) - (V_{GS2} - V_{TH}) = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$(V_{in1} - V_{in2})^2 = \frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}} + \frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}} - 2 \frac{\sqrt{4I_{D1}I_{D2}}}{\mu_n C_{ox} \frac{W}{L}} = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1}I_{D2}})$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 = I_{SS} - 2\sqrt{I_{D1}I_{D2}}$$

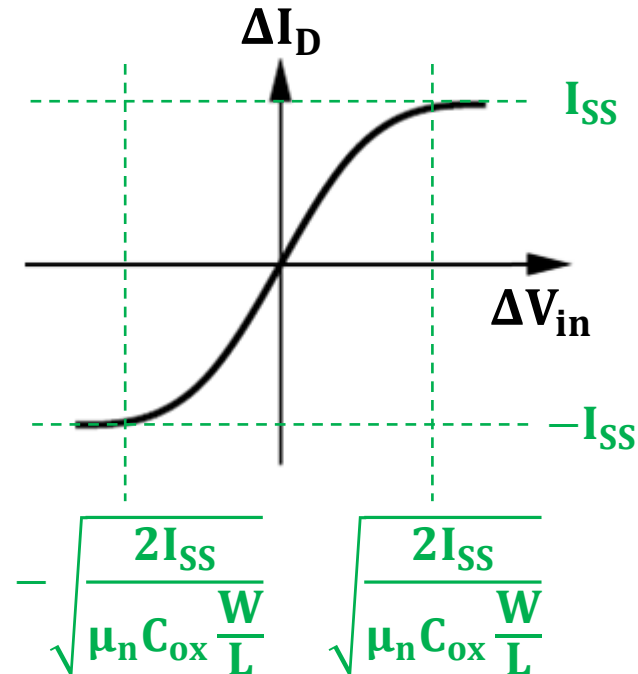
Differential-Mode (DC Analysis)

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1} I_{D2}}$$

$$\frac{1}{4} \left(\mu_n C_{ox} \frac{W}{L} \right)^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 I_{SS} = 4I_{D1} I_{D2}$$

$$\frac{1}{4} \left(\mu_n C_{ox} \frac{W}{L} \right)^2 \underbrace{(V_{in1} - V_{in2})^4}_{= \Delta V_{in}^4} + \cancel{I_{SS}^2} - \mu_n C_{ox} \frac{W}{L} \underbrace{(V_{in1} - V_{in2})^2}_{= \Delta V_{in}^2} I_{SS} = \cancel{I_{SS}^2} - \underbrace{(I_{D1} - I_{D2})^2}_{= \Delta I_D^2}$$

$$\Delta I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Delta V_{in} \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}$$



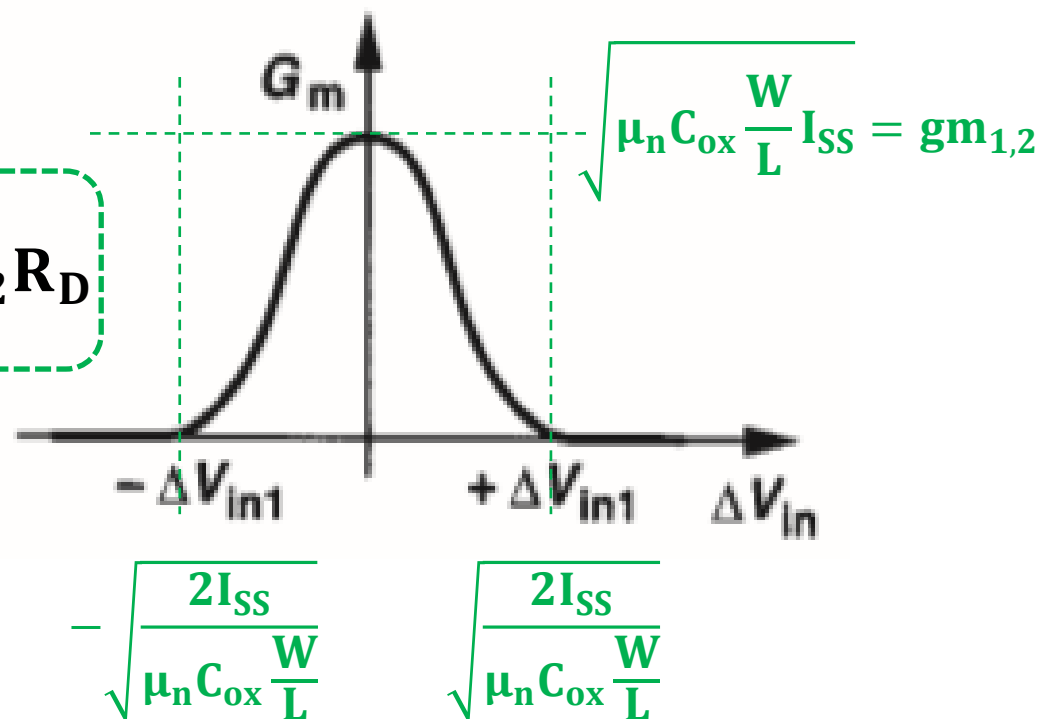
Differential-Mode (DC Analysis)

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}}$$

At $\Delta V_{in} = 0$,

$$A_{DM} = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = -gm_{1,2} R_D$$

- Larger I_{SS} leads to higher G_m and wider input range.
- Smaller W/L leads to lower G_m but wider input range.

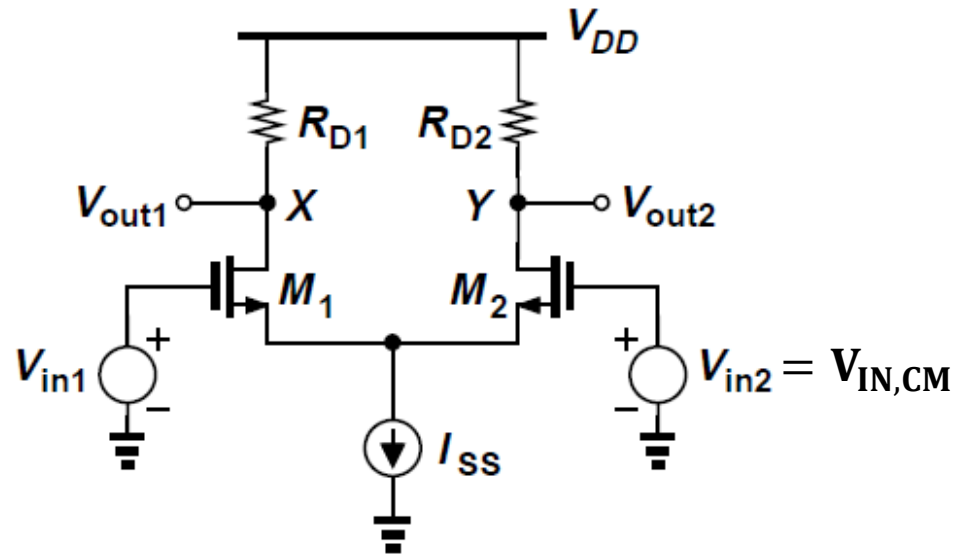


Differential-Mode (Small-Signal, Superposition)¹⁴

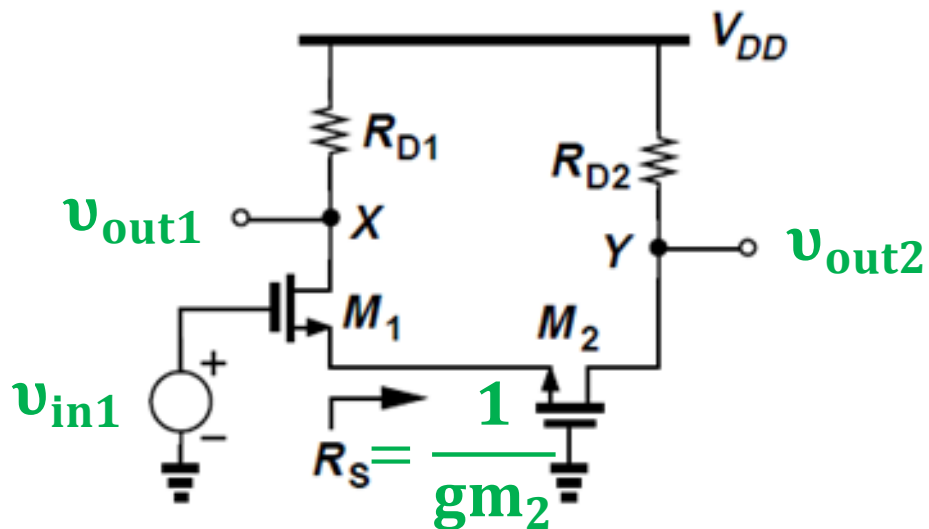
Small-signal
Analysis

$$\lambda = 0 \quad \gamma = 0$$

Complete circuit



Small-signal circuit

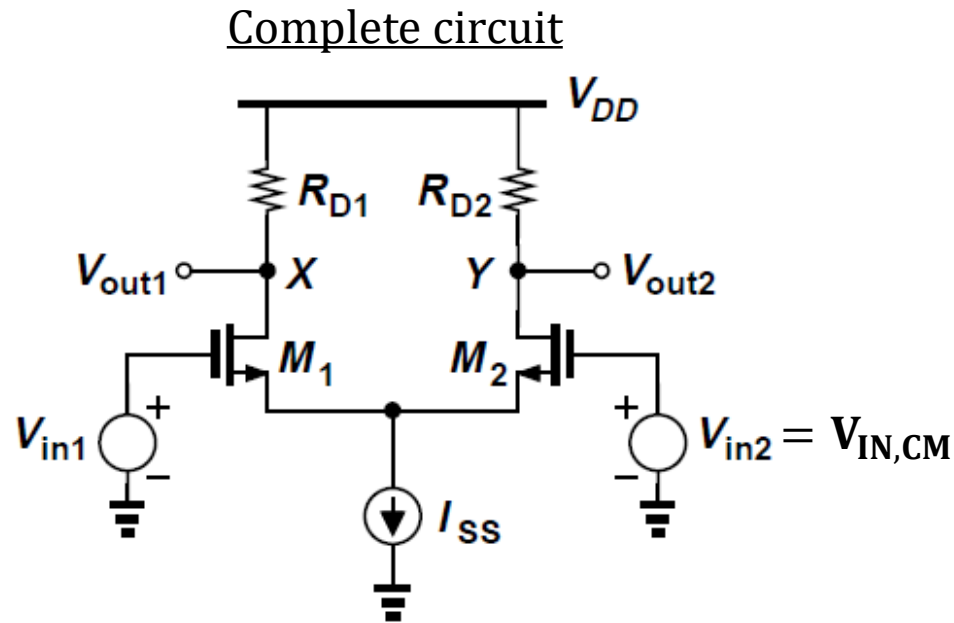


$$v_{out1} = - \frac{R_D}{\frac{1}{gm_1} + \frac{1}{gm_2}} v_{in1}$$

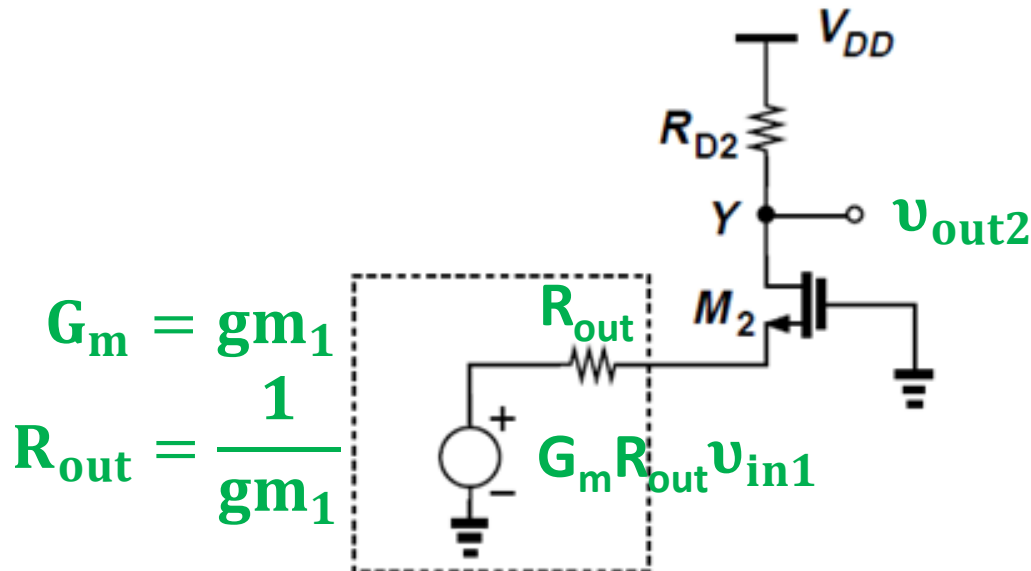
Differential-Mode (Small-Signal, Superposition)¹⁵

Small-signal
Analysis

$$\lambda = 0 \quad \gamma = 0$$



Small-signal circuit



$$v_{out2} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1}$$

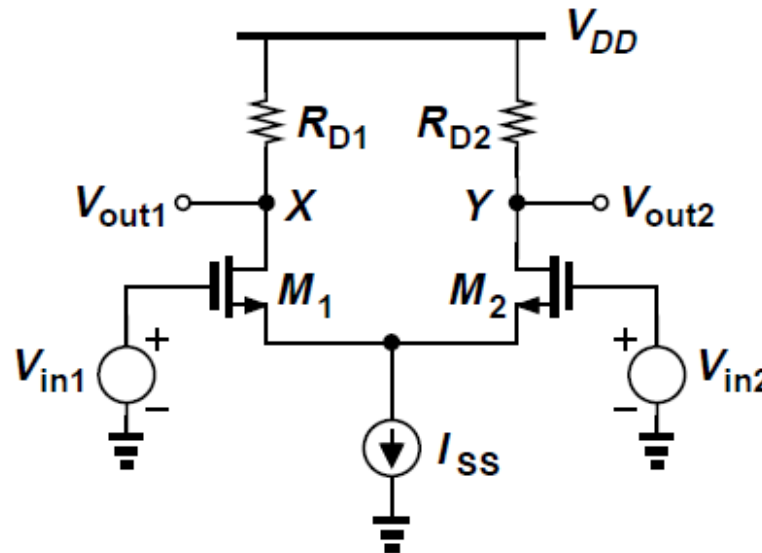
$$G_m = g_{m1}$$

$$R_{out} = \frac{1}{g_{m1}}$$

Differential-Mode (Small-Signal, Superposition)¹⁶

Small-signal
Analysis

$$\lambda = 0 \quad \gamma = 0$$



$$v_{out1} - v_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1} = -g_m R_D v_{in1} \quad (1)$$

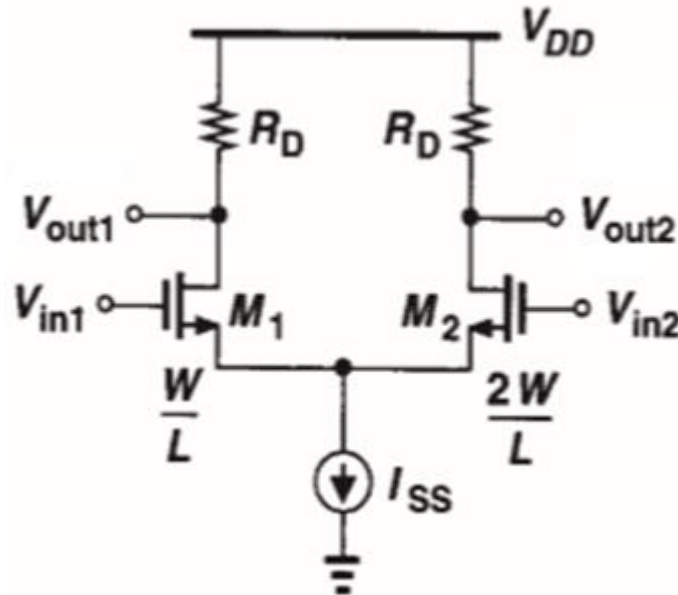
$$v_{out1} - v_{out2} = \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in2} = g_m R_D v_{in2} \quad (2)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -g_m R_D$$

(1) + (2)

Example

Calculate the A_{DM} of the differential pair below if the biasing conditions of M_1 and M_2 are the same.



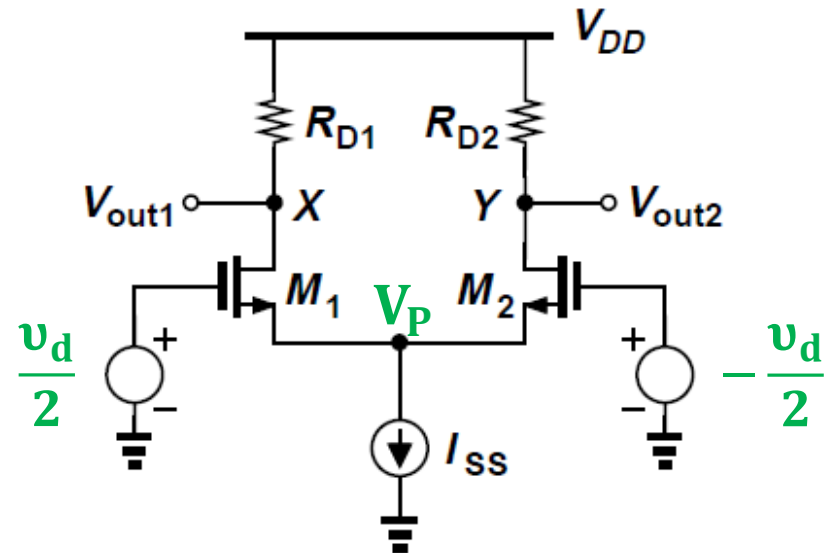
$$\left\{ \begin{aligned} v_{out1} - v_{out2} &= -\frac{2R_D}{\frac{1}{gm_1} + \frac{1}{2gm_1}} v_{in1} = -\frac{4}{3} gm_1 R_D v_{in1} \\ v_{out1} - v_{out2} &= \frac{2R_D}{\frac{1}{gm_1} + \frac{1}{2gm_1}} v_{in2} = \frac{4}{3} gm_1 R_D v_{in2} \end{aligned} \right.$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -\frac{4}{3} gm_1 R_D$$

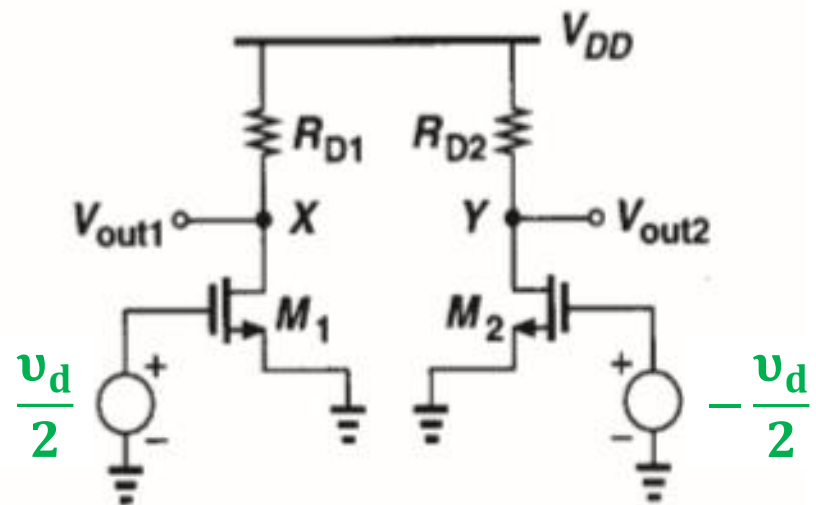
Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$



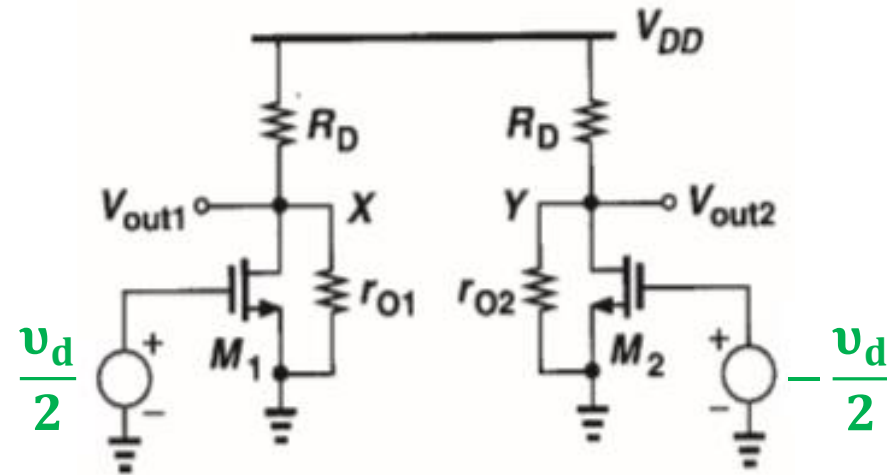
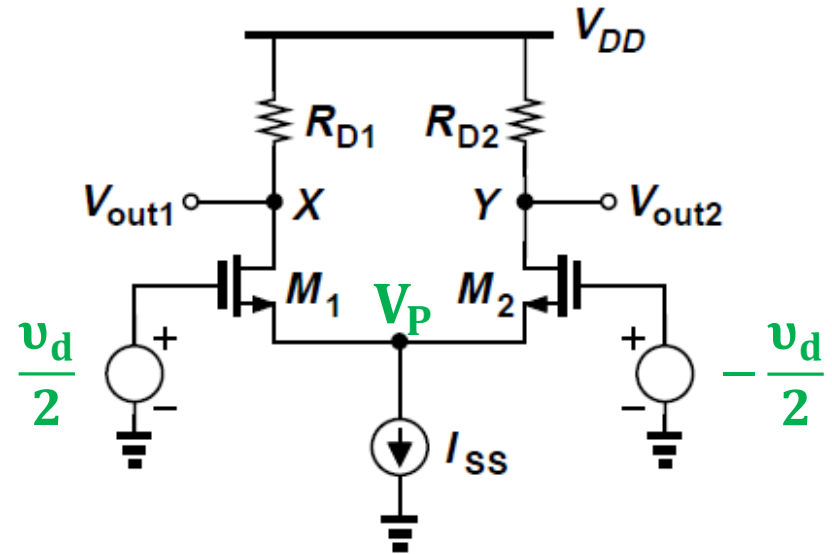
- Assume the circuit is fully symmetric.
- For $i_{d1} + i_{d2} = 0$ and $gm_1 \frac{v_d}{2} + gm_2 \left(-\frac{v_d}{2}\right) = 0$, V_P must be a constant voltage in DC analysis and a virtual ground in small-signal analysis.



Differential-Mode (Small-Signal, Half-circuit)

Small-signal
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$v_{out1} = -gm(R_D \parallel r_o) \frac{v_d}{2}$$

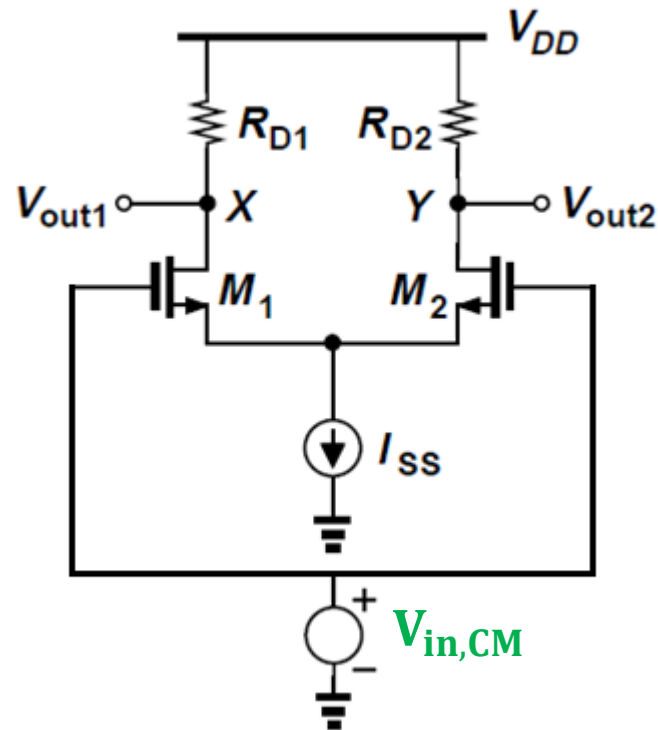
$$v_{out2} = -gm(R_D \parallel r_o) \left(-\frac{v_d}{2}\right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} = -gm(R_D \parallel r_o)$$

Common-Mode Response

Small-signal
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



If the circuit is fully symmetric,

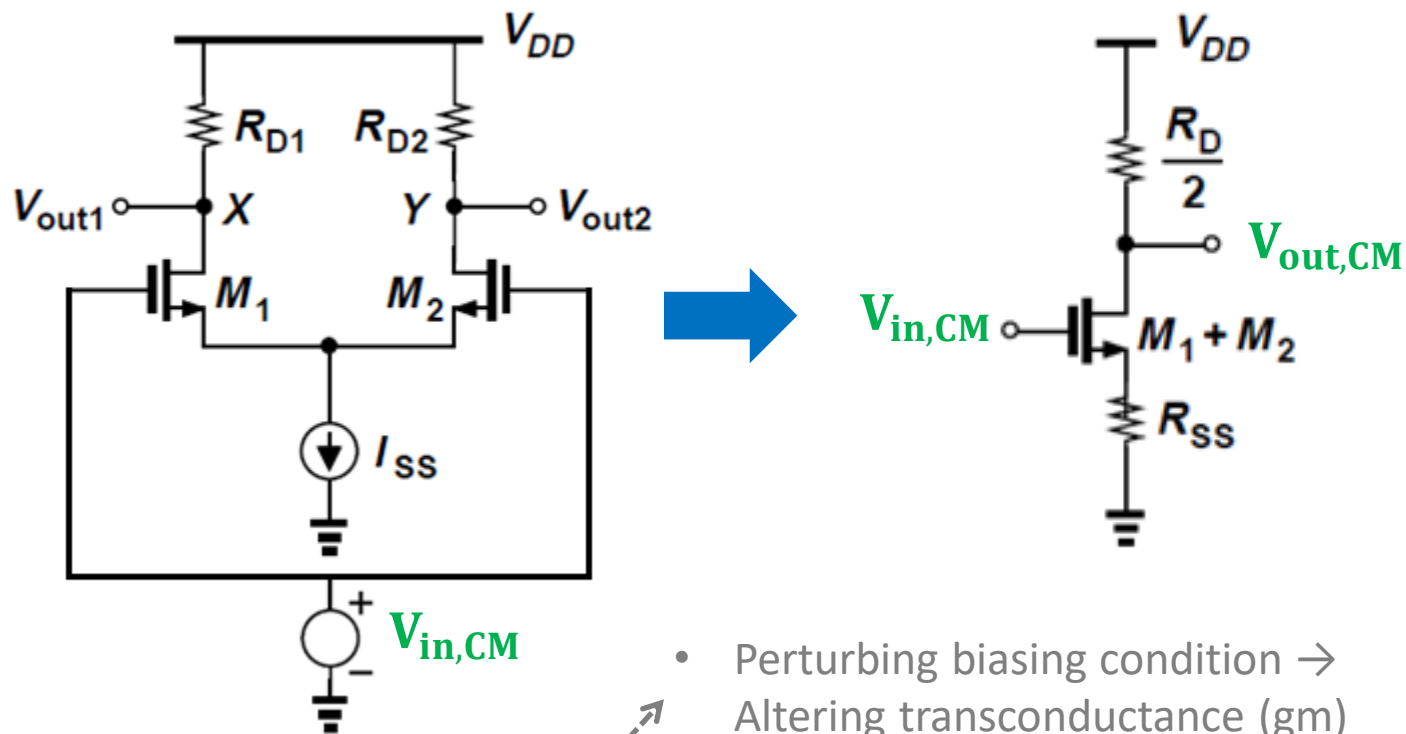
$$A_{\text{CM-DM}} = \frac{v_{\text{out1}} - v_{\text{out2}}}{v_{\text{in,CM}}} = 0$$

$$\text{CMRR} = \left| \frac{A_{\text{DM}}}{A_{\text{CM-DM}}} \right| = \infty$$

Common-Mode Response

Small-signal
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



- Perturbing biasing condition \rightarrow Altering transconductance (g_m)

If the circuit is fully symmetric, $A_{CM} = \frac{v_{out,CM}}{v_{in,CM}}$

$$= \frac{-2g_m \frac{r_o}{2}}{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb}) \frac{r_o}{2} R_{SS} \right]} \cdot \frac{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb}) \frac{r_o}{2} R_{SS} \right] \frac{R_D}{2}}{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb}) \frac{r_o}{2} R_{SS} \right] + \frac{R_D}{2}}$$

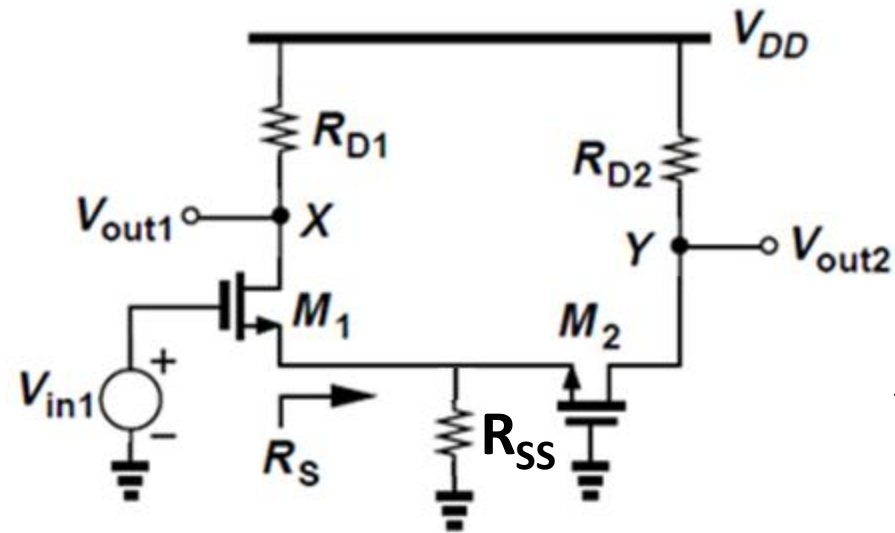
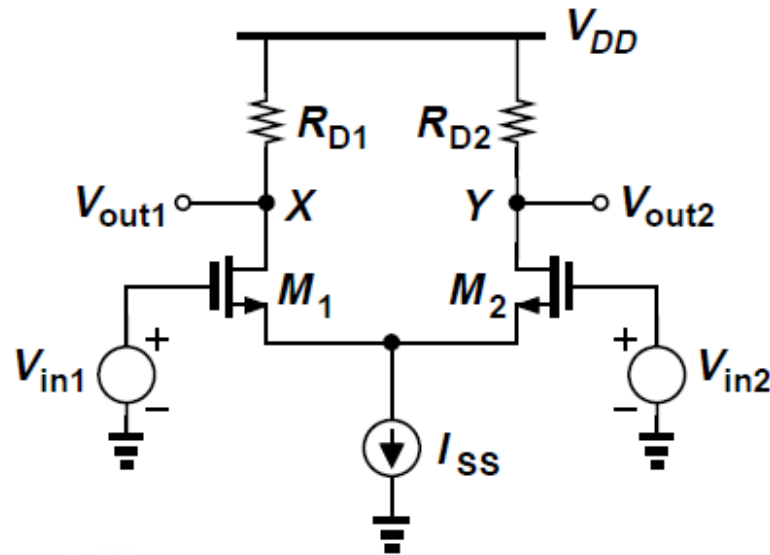
$= 0$ if $R_{SS} = \infty$

A_{DM} for Finite R_{SS}

Differential-Mode (Small-Signal, Superposition)²³

Small-signal
Analysis

$$\lambda = 0 \quad \gamma = 0$$



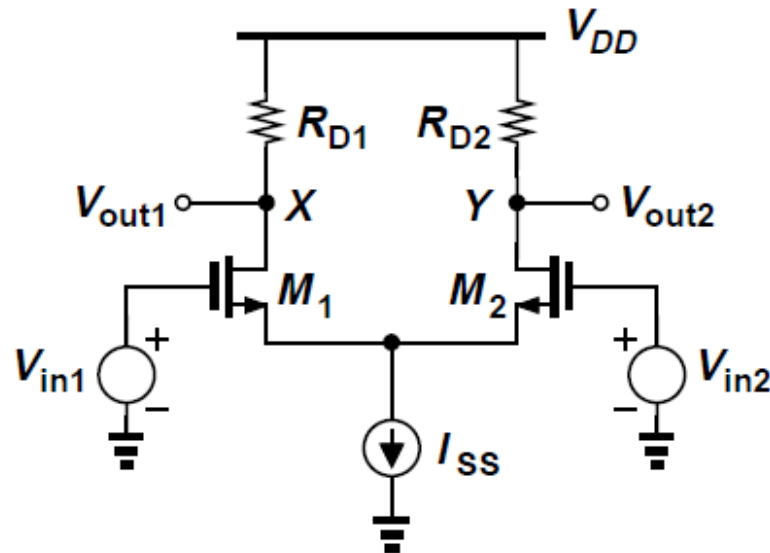
$$R_S = \frac{1}{g_{m2}} \parallel R_{SS}$$

$$v_{out1} = - \frac{R_D}{\frac{1}{g_{m1}} + \left(\frac{1}{g_{m2}} \parallel R_{SS} \right)} v_{in1}$$

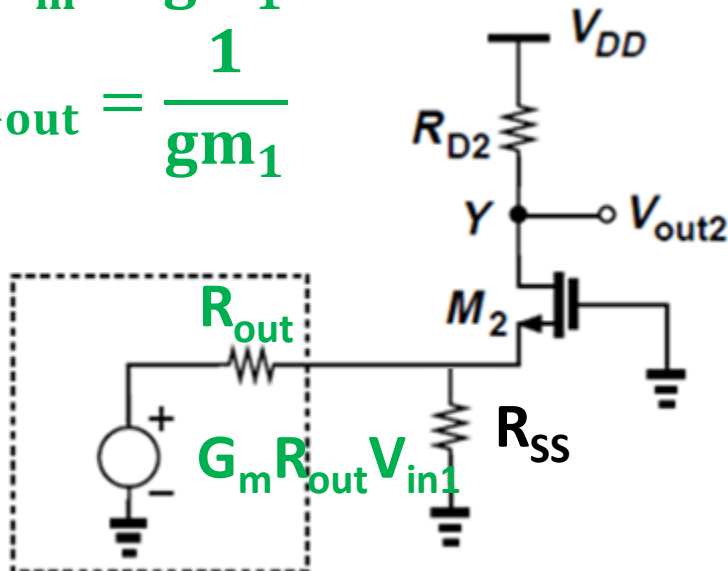
Differential-Mode (Small-Signal, Superposition)²⁴

Small-signal
Analysis

$$\lambda = 0 \quad \gamma = 0$$



$$G_m = g_{m1}$$
$$R_{out} = \frac{1}{g_{m1}}$$

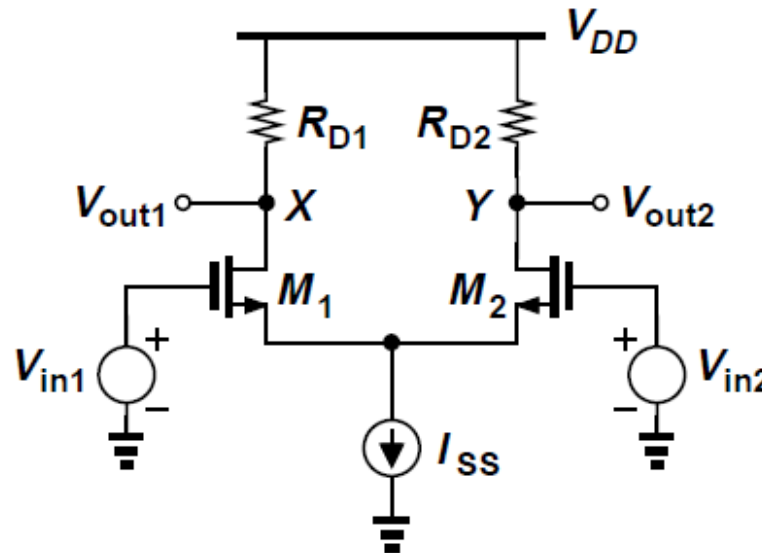


$$v_{out2} = \frac{\frac{R_{SS}}{R_{SS} + \frac{1}{g_{m2}}} R_D}{\frac{1}{g_{m1}} + \left(\frac{1}{g_{m2}} \parallel R_{SS} \right)} v_{in1}$$

Differential-Mode (Small-Signal, Superposition)²⁵

Small-signal
Analysis

$$\lambda = 0 \quad \gamma = 0$$



$$v_{out1} - v_{out2} = -\frac{(gm_1 + 2R_{SS}gm_1gm_2)R_D}{1 + (gm_1 + gm_2)R_{SS}} v_{in1} = -gmR_D v_{in1} \quad (1)$$

$$v_{out1} - v_{out2} = \frac{(gm_2 + 2R_{SS}gm_1gm_2)R_D}{1 + (gm_1 + gm_2)R_{SS}} v_{in2} = gmR_D v_{in2} \quad (2)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -gmR_D$$

(1) + (2)

A_{DM} with MOS Loads

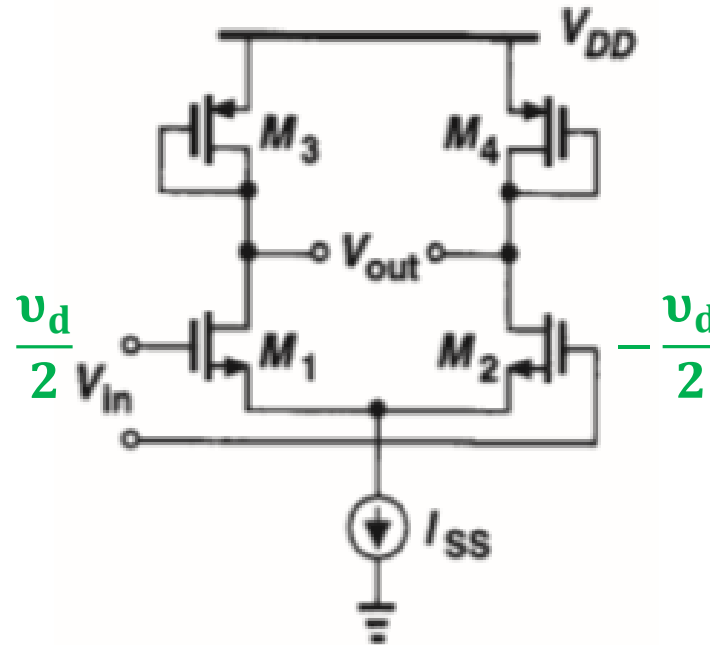
Differential-Mode (Small-Signal, Half-circuit)

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Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$

- Higher A_{DM}
 - Smaller $(W/L)_P$
 - Larger $(V_{SGP} - V_{THP})$
 - Smaller $V_{in,CM}$ headroom



$$v_{out1} = -g_{mN} \left(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right) \frac{v_d}{2}$$

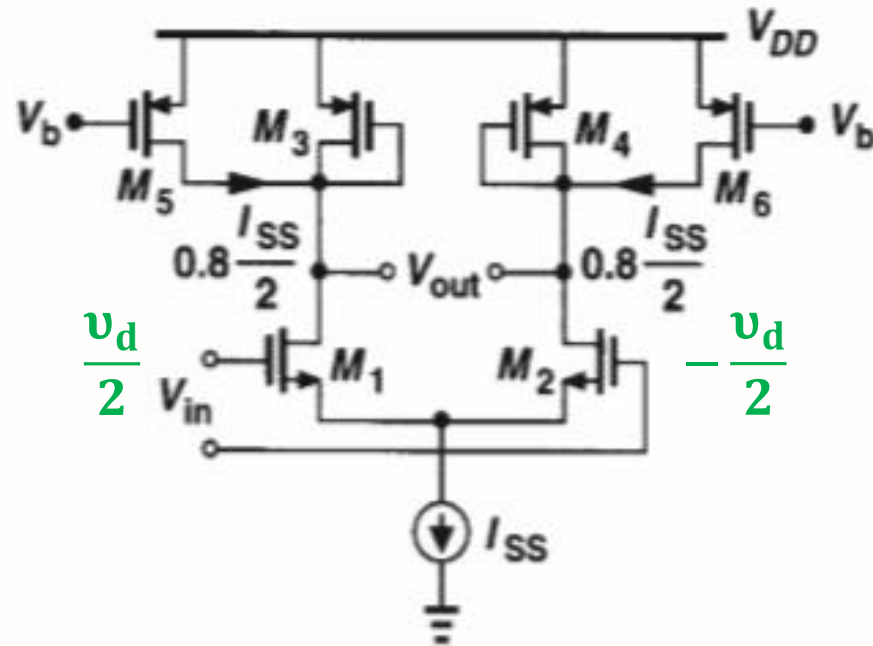
$$v_{out2} = -g_{mN} \left(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right) \left(-\frac{v_d}{2} \right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} = -g_{mN} \left(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right) \approx -\frac{g_{mN}}{g_{mP}} \approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

Differential-Mode (Small-Signal, Half-circuit)

Small-signal
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$v_{out1} = -gm_{1,2} \left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{gm_{3,4}} \parallel r_{o5,6} \right) \frac{v_d}{2}$$

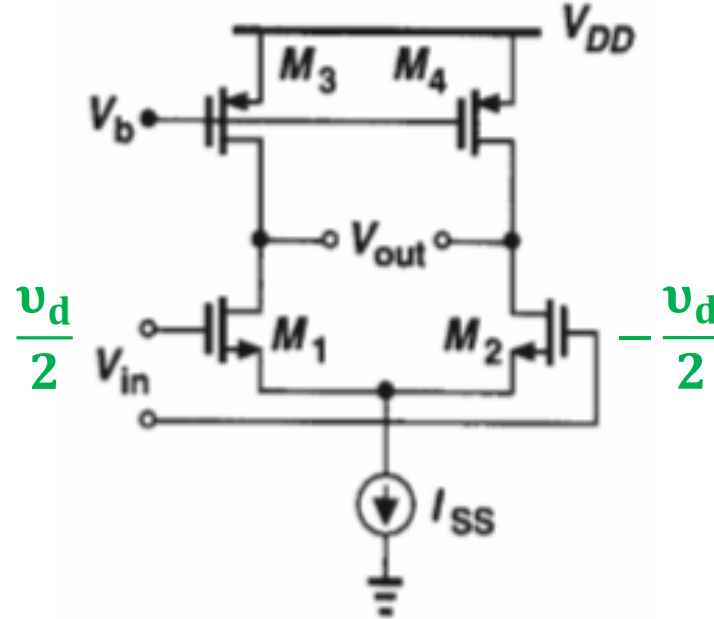
$$v_{out2} = -gm_{1,2} \left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{gm_{3,4}} \parallel r_{o5,6} \right) \left(-\frac{v_d}{2} \right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} \approx -\frac{gm_{1,2}}{gm_{3,4}} \approx -\sqrt{\frac{5\mu_n(W/L)_{1,2}}{\mu_p(W/L)_{3,4}}}$$

Differential-Mode (Small-Signal, Half-circuit)

Small-signal
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$v_{out1} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \frac{v_d}{2}$$

$$v_{out2} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \left(-\frac{v_d}{2}\right)$$

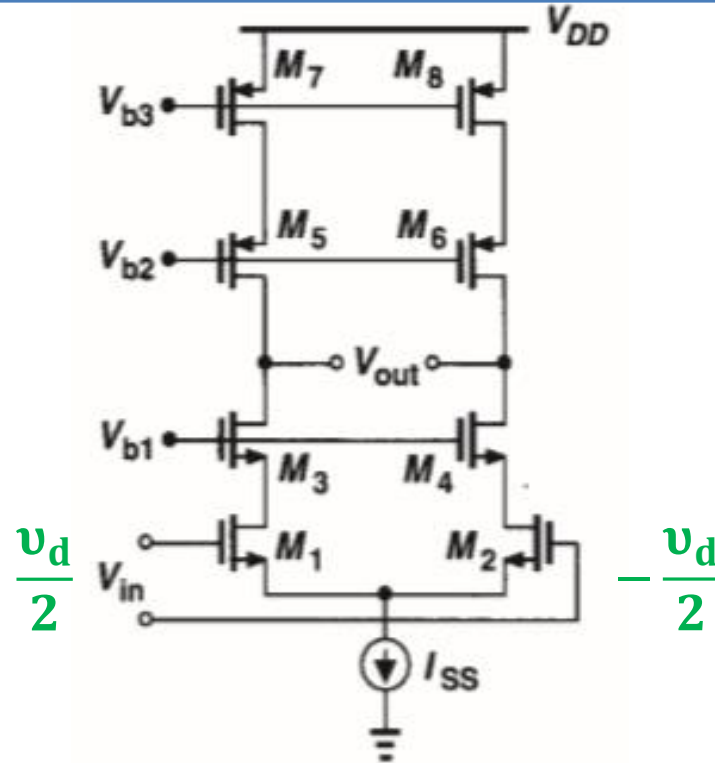
$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4})$$

Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$

- High R_{out}
 \rightarrow High A_{DM}
 \rightarrow Small $V_{in,CM}$ headroom



$$v_{out1} \cong -g_{m1,2} \left\{ [r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel [r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \right\} \frac{v_d}{2}$$

$$v_{out2} \cong -g_{m1,2} \left\{ [r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel [r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \right\} \left(-\frac{v_d}{2} \right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} \cong -g_{m1,2} \left[(g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2} \parallel (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8} \right]$$