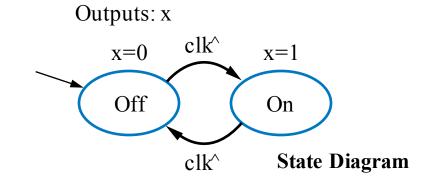
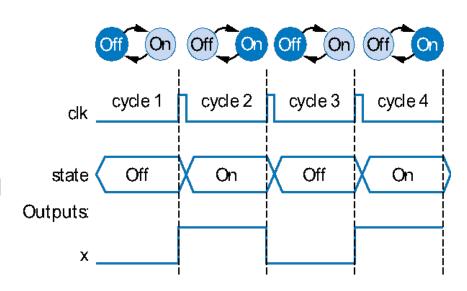
# Topic 10 Finite State Machine

#### **Describing Behavior of Sequential Circuit**

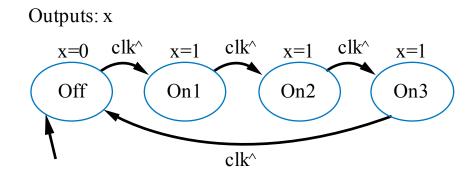
- Finite-State Machine (FSM)
  - A way to describe desired
     behavior of a sequential circuit
  - Consists of a set of states, transitions between states, and maybe inputs and outputs
    - present state: currently happening
    - next state: next to happen
  - Example: Toggle output x every clock cycle
    - Two states: "Off" and "On"
    - Corresponding outputs: x=0 or 1
    - No input
    - Transition from Off to On, or On to Off, on rising clock edge
    - Arrow with no starting state points to initial state

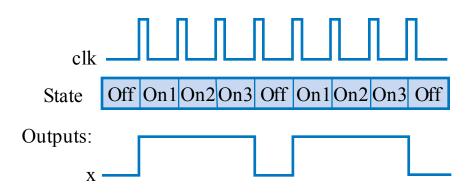




#### **Example: Output Special Pattern**

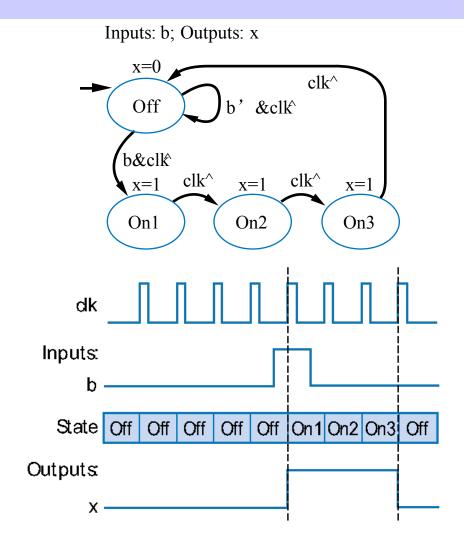
- Want a circuit to output
  - 0, 1, 1, 1, 0, 1, 1, 1, ...
    - One bit at a time
    - Each bit for one clock cycle
- Can be described as FSM
  - Four states
    - Each state corresponds to an output, 0, 1, 1, 1
    - Then repeat
  - Transition on rising clock edge to next state





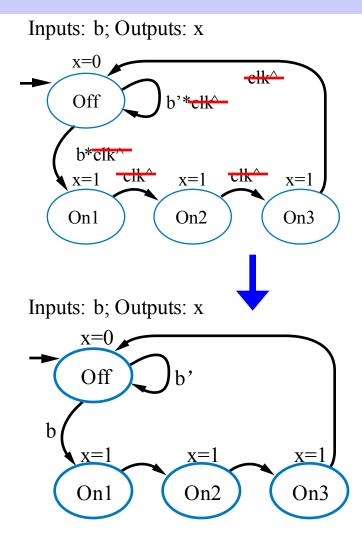
#### **Example: FSM with Input**

- b is a push button, output x to stay on for exactly 3 clock cycles no matter how long b is pushed
- Wait in "Off" state while b is 0 (b')
- When b is 1 (and rising clock edge), transition to On1
  - Sets x=1
  - On next two clock edges, transition to On2, then On3, which also set x=1
- So x=1 for three cycles after button pressed
- Potential issue: if button b stays on, what will happen?



#### State Diagram Simplification: Clock Implicit

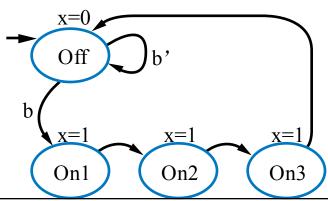
- Synchronous FSMs FSM behaviors synchronized to active edge of clock
  - Asynchronous FSMs -- less common, advanced topic
- Make implicit all state transitions are triggered by rising edge of clock
- Make implicit Unlabeled path means transition is triggered only by clock, inputs don't matter



#### **FSM Definition**

- FSM consists of
  - Set of states
    - Ex: {Off, On1, On2, On3}
  - Set of inputs, set of outputs
    - Ex: Inputs: {b}, Outputs: {x}
  - Initial state
    - Ex: "Off"
  - Set of transitions
    - Describes next states
    - Ex: Has 5 transitions
  - Set of actions (outputs)
    - Sets outputs while in states
    - Ex: x=0, x=1, x=1, and x=1

Inputs: b; Outputs: x



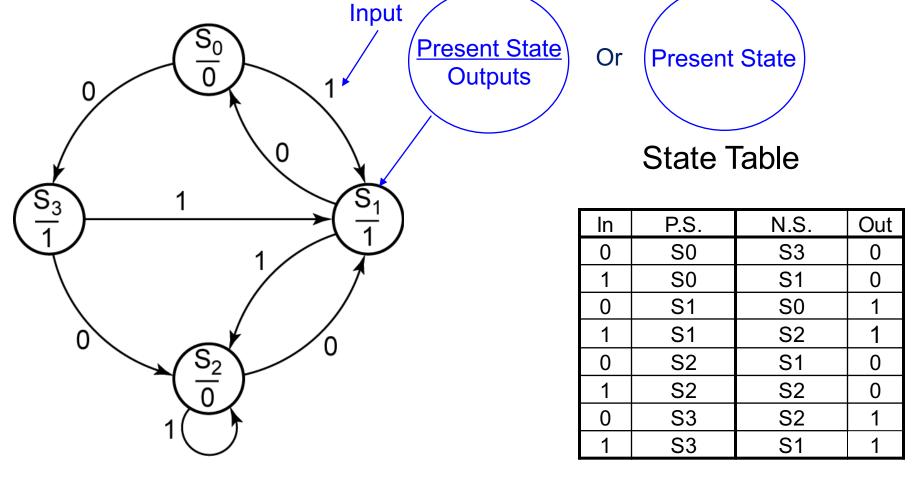
FSM can be represented graphically, known as state diagram

	]	Inputs	;	(	Dutpu	ts
	s 1	s0	b	Х	n1	n0
Off	0	0	0	0	0	0
	0	0	1	0	0	1
On1	0	1	0	1	1	0
	0	1	1	1	1	0
On2	1	0	0	1	1	1
	1	0	1	1	1	1
On3	1	1	0	1	0	0
	1	1	1	1	0	0

FSM can also be represented in tabular form, known as **state table** 

#### **State Diagram and State Table**

State Diagram

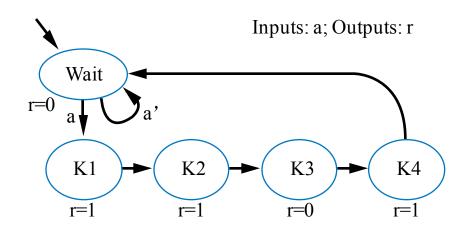


**Outputs** 

#### **Example: Secure Car Key**

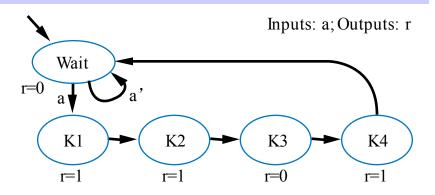
- All new car keys contain a tiny computer chip
  - When car starts, car's computer (under engine hood) requests identifier from key
  - Key transmits identifier
    - If not, computer shuts off car
- FSM
  - Wait until computer requestsID (a=1)
  - Transmit ID (in this case, 1101)

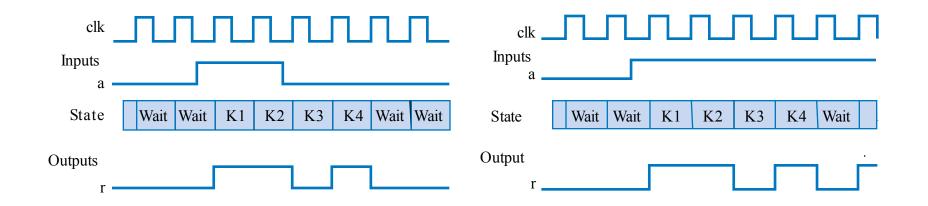




#### FSM Example: Secure Car Key (cont.)

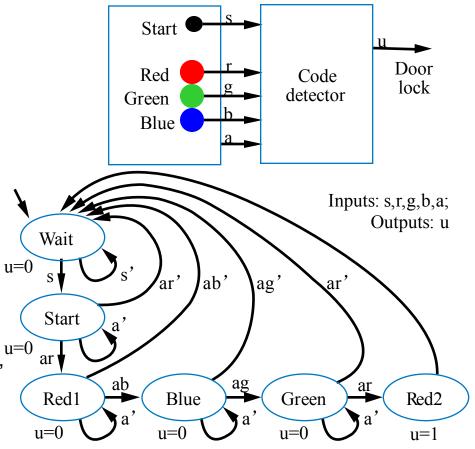
 Timing diagrams show states and output values for different input waveforms





#### **Example: Digital Lock**

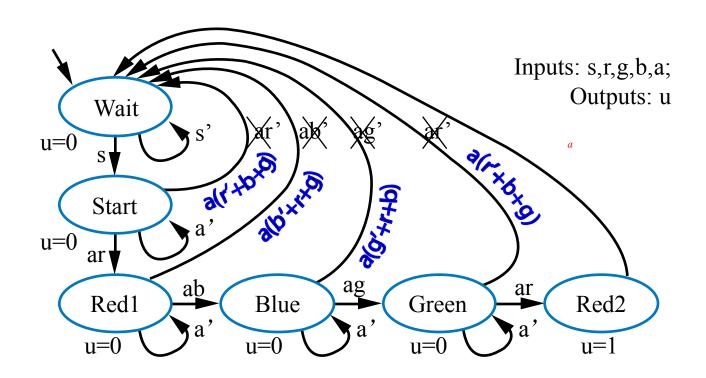
- Unlock door (u=1) only when buttons pressed in sequence:
  - start, then red, blue, green, red
- Input buttons: s, r, g, b
- Input a indicates that some color button pressed
- FSM
  - Wait for start (s=1) in "Wait"
  - Once started, go to "Start", then
    - If see red, go to "Red1"
    - Then, if see blue, go to "Blue"
    - Then, if see green, go to "Green"
    - Then, if see red, go to "Red2", and u=1
    - Wrong button at any step, return to "Wait", without opening door



Q: Can you trick this FSM to open the door, without knowing the code?

A: Yes, hold all buttons simultaneously

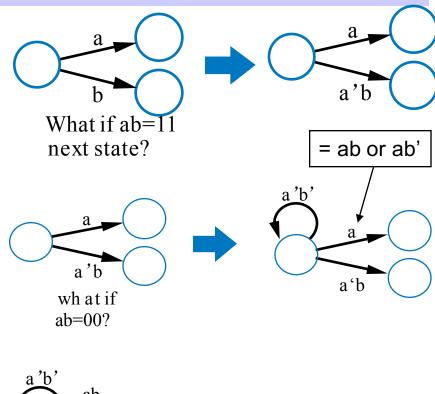
#### **Improve FSM for Code Detector**

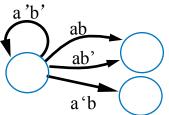


 New transition conditions detect if wrong button pressed, returns to "Wait"

#### **Common State Transition Property**

- Only one condition should be true, among all transitions leaving a state
- One condition must be true
  - For any input combination
- All conditions must be considered when leaving a state





#### Pitfall is Common

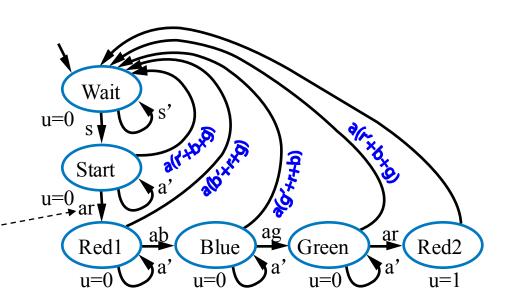
- Recall code detector FSM
  - Do the transitions obey the two required transition properties?

#### NO!

– How would it go wrong?

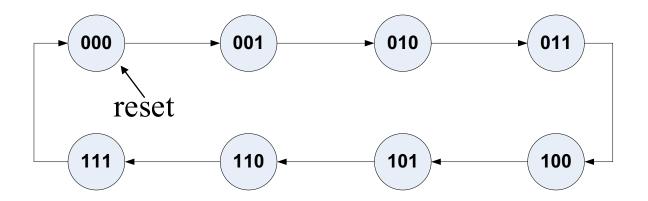
E.g. arbg = 1111 How to solve?

Answer: ar should be arb'g' (likewise for ab, ag, ar)



## Implementation of FSM Example: Synchronous Binary Counter

- FSM that counts binary numbers counter
- An n-bit binary counter can count in binary from 0 up to 2<sup>n</sup>-1 and repeat
- An *n*-bit binary counter consists of *n* flip-flops
- All the flip-flops are synchronized to the same clock synchronous counter
- May be implemented by different type of flip-flops
- Example: a 3-bit binary counter can count through this sequence



## **Synchronous Binary Counter Design**

- An FSM (without external inputs)
  - State Table

Pre	esent S	tate	Next State				
Q2	Q1	Q0	Q2 <sup>+</sup>	Q1+	Q0+		
0	0	0	0	0	1		
0	0	1	0	1	0		
0	1	0	0	1	1		
0	1	1	1	0	0		
1	0	0	1	0	1		
1	0	1	1	1	0		
1	1	0	1	1	1		
1	1	1	0	0	0		

#### Counter Implemented with D Flip-Flop

- Use D flip flops to hold values: Q+ = D upon active edge
- Next state equations

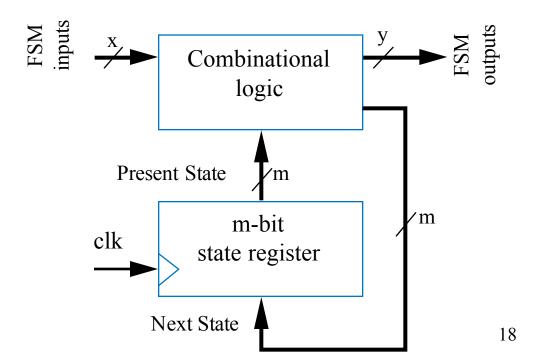
Pres	sent S	tate	Ne	Next State			D flip flop input			
Q2	Q1	Q0	Q2+	Q1+	Q0+	D2	D1	D0		
0	0	0	0	0	1	0	0	1		
0	0	1	0	1	0	0	1	0		
0	1	0	0	1	1	0	1	1		
0	1	1	1	0	0	1	0	0		
1	0	0	1	0	1	1	0	1		
1	0	1	1	1	0	1	1	0		
1	1	0	1	1	1	1	1	1		
1	1	1	0	0	0	0	0	0		

### Synchronous Binary Counter with D Flip-Flop

 3-bit binary counter by D FF Q2 3-bit **Binary** clock Counter Q0 reset reset Q2 Q0 Q1 Combinational part Sequential part Q0 D2 Q2 **D1** Q1 D0 clock-DFF2 DFF1 DFF0 clear Q2 Q10 clear clear 17 reset

#### Standard FSM Architecture

- How to design sequential circuit?
  - Design as FSM
  - Use standard architecture
    - State register -- to store the present state
    - Combinational logic -- to compute outputs, and next state
  - Known as controller



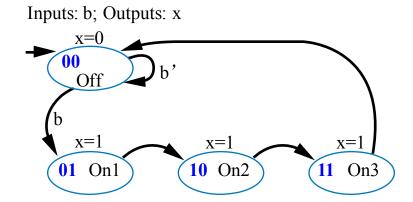
## FSM (Controller) Design

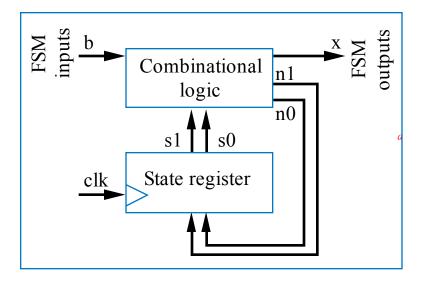
#### Five step FSM design process

	Step	Description
Step 1	Capture the FSM	Create an FSM that describes the desired behavior of the controller.
Step 2	Create the architecture	Create the standard architecture by using a state register of appropriate width, and combinational logic with inputs being the state register bits and the FSM inputs and outputs being the next state bits and the FSM outputs.
Step 3	Encode the states	Assign a unique binary number to each state. Each binary number representing a state is known as an <i>encoding</i> . Any encoding will do as long as each state has a unique encoding.
Step 4	Create the state table	Create a truth table for the combinational logic such that the logic will generate the correct FSM outputs and next state signals. Ordering the inputs with state bits first makes this truth table describe the state behavior, so the table is a state table.
Step 5	Implement the combinational logic	Implement the combinational logic using any method.

#### FSM Design Example: Push Button

- Step 1: Capture the FSM
  - Already done
- Step 2: Create architecture
  - 2-bit state register (for 4 states)
  - Input b, output x
  - Present state signals (s1, s0)
  - Next state signals (n1, n0)
- Step 3: Encode the states
  - Any encoding with unique representation for each state will work

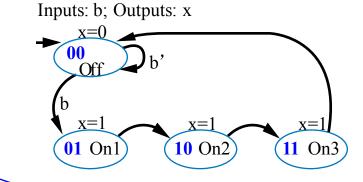


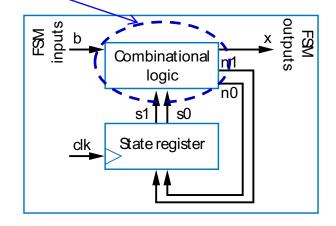


#### FSM Design Example: Push Button (cont.)

Step 4: Create state table

		Inputs	S	Outputs			
		Present state		Х	Ne sta		
Off	0 0	0 0	0 1	0 0	0 0	0	
On1	0 0	1 1	0 1	1 1	1 1	0	
On2	1 1	0 0	0 1	1 1	1 1	1 1	
On3	1 1	1 1	0 1	1 1	0 0	0	
C	ombina In	tional i	Logic	Coml	bination Outpu	nal Logi its	

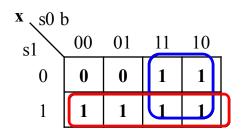




#### FSM Design Example: Push Button (cont.)

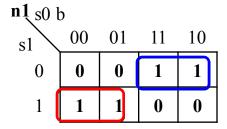
Step 5: Implement combinational logic

$$x = s1 + s0$$

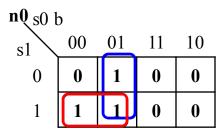


	]	Inputs	<b>S</b>	Outputs			
	s 1	s0	b	Х	n1	n0	
Off	0	0	0	0	0	0	
	0	0	1	0	0	1	
On1	0	1	0	1	1	0	
	0	1	1	1	1	0	
On2	1	0	0	1	1	1	
	1	0	1	1	1	1	
On3	1	1	0	1	0	0	
	1	1	1	1	0	0	

$$n1 = s1's0 + s1s0'$$

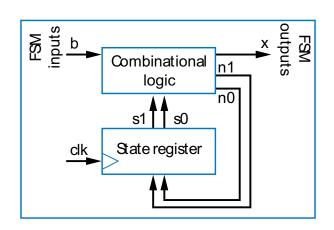


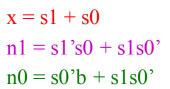
$$n0 = s0$$
'b +  $s1s0$ '

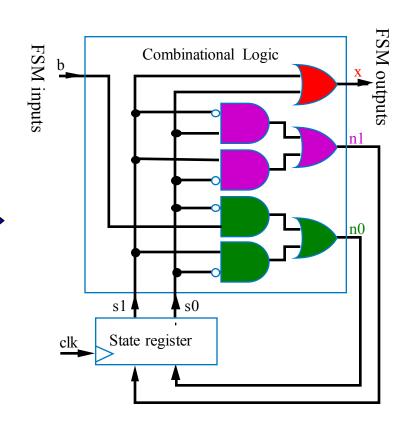


#### FSM Design Example: Push Button (cont.)

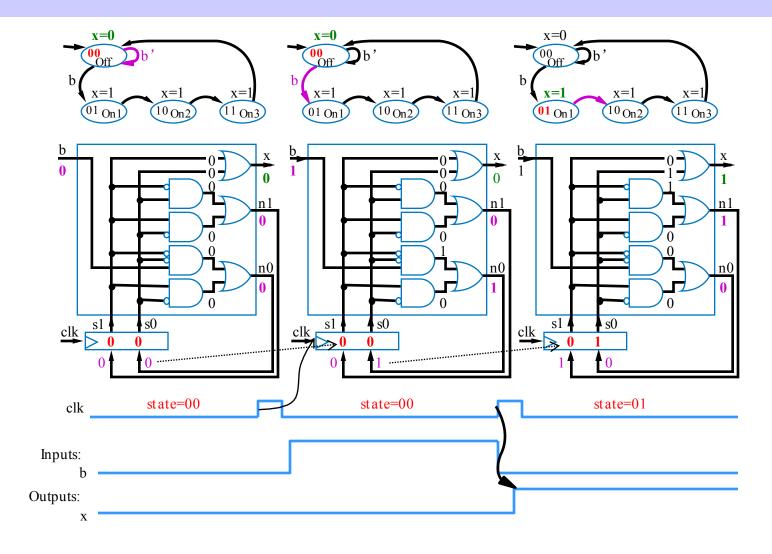
 Step 5: Implement combinational logic (cont)



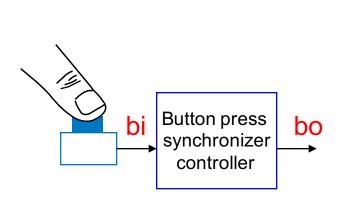


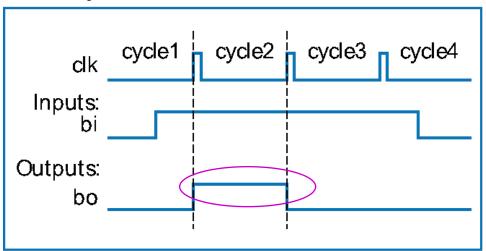


#### **Understanding the Controller's Behavior**



## FSM Design Example: Button Press Synchronizer

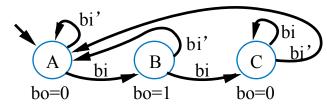




 Want simple sequential circuit that converts button press to single clock cycle duration, regardless of length of time that button actually pressed

## FSM Design Example: **Button Press Synchronizer (cont.)**

FSM inputs: bi; FSM outputs: bo



Step 1: FSM

FSM inputs: bi; FSM outputs: bo

bo=1

Step 3: Encode states

00

bo=0

bi'

bo=0

FSM nputs Combinational logic n0s1State register

Combinational logic

Inputs Outputs s1 s0 bi n1 n0 bo 0 0 0 0 0

0 0 0 1 1 0 1

0 1 0 0 0 0 0

unused

Step 4: State table

may be 'x'

Step 2: Create architecture

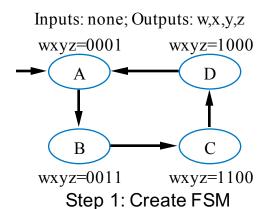
n1 = s1's0bi + s1s0bin0 = s1's0'bibo = s1's0bi' + s1's0bi = s1s0

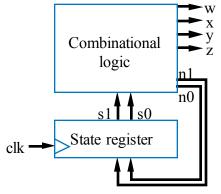
Combinational logic bo bi n0s0State register

Step 5: Create combinational circuit

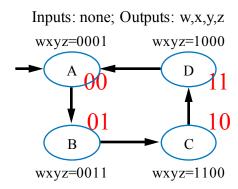
### **FSM Example: Sequence Generator**

- Want generate sequence 0001, 0011, 1100, 1000, (repeat)
  - Each value for one clock cycle





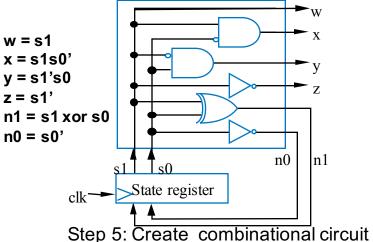
Step 2: Create architecture



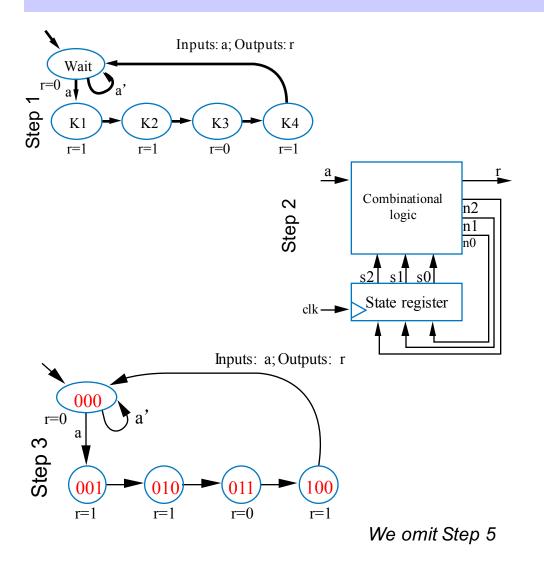
Step 3: Encode states

	Inp	uts	Outputs					
	s 1	s 0	W	X	У	Ζ	n1	n0
$\overline{A}$	0	0	0	0	0	1	0	1
В	0	1	0	0	1	1	1	0
C	1	0	1	1	0	0	1	1
D	1	1	1	0	0	0	0	0

Step 4: Create state table



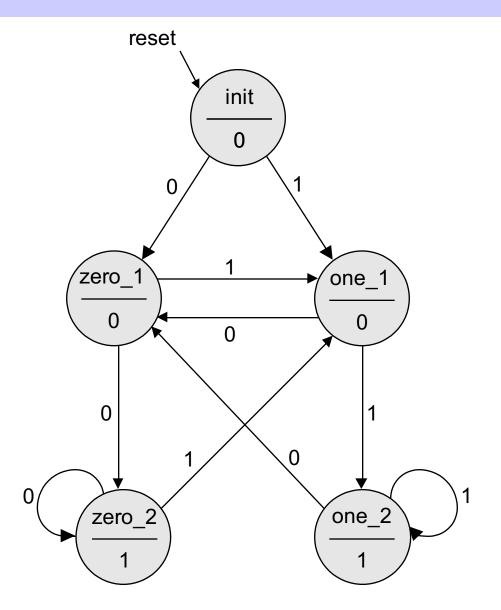
### FSM Example: Secure Car Key



		Inputs				Outputs			
	s 2	s1	s0	а	r	n2	n1	n0	
Wait	0 0	0 0	0 0	0 1	0 0	0 0	0 0	0 1	
K1	0 0	0 0	1 1	0 1	1 1	0 0	1 1	0 0	
K2	0 0	1 1	0 0	0 1	1 1	0 0	1 1	1 1	
K3	0	1 1	1 1	0 1	0	1 1	0	0	
K4	1 1	0	0	0 1	1 1	0	0	0	
Unused	1 1 1 1 1	0 0 1 1 1	1 1 0 0 1 1	0 1 0 1 0	00000	0 0 0 0 0	0 0 0 0 0	0 0 0 0	
Step 4						647		28	

may be 'x'

## **Verilog Modeling of FSM**

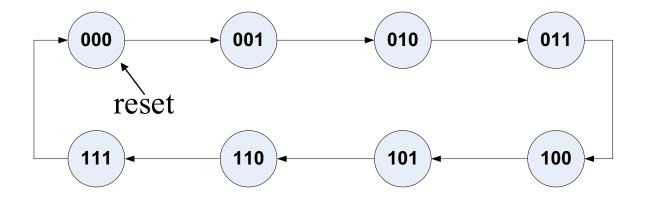


```
Module example FSM (clock, reset, in bit, out bit);
  input clock, reset, in bit;
  output out bit;
  reg [2:0] curr state, next state;
  parameter init = 3'b000;
  parameter zero 1 = 3'b001;
  parameter one 1 = 3'b010;
  parameter zero 2 = 3'b011;
  parameter one 2 = 3'b100;
  always @ (posedge clock or posedge reset) <-----
                                                    State register
    if (reset == 1) curr state <= init;</pre>
    else
                    curr state <= next state;</pre>
                                              Combinational logic
  always @ (curr state or in bit) ← — —
                                              for next state
    case (curr state)
      init: if (in bit == 0) next state <= zero 1; else
            if (in bit == 1) next state <= one 1; else</pre>
                              next state <= init;</pre>
```

```
zero 1: if (in bit == 0) next state <= zero 2; else
               if (in bit == 1) next state <= one 1; else</pre>
                                  next state <= init;</pre>
      zero 2: if (in bit == 0) next state <= zero 2; else</pre>
               if (in bit == 1) next state <= one 1; else</pre>
                                  next state <= init;</pre>
      one 1: if (in bit == 0) next state <= zero 1; else
               if (in bit == 1) next state <= one 2; else</pre>
                                  next state <= init;</pre>
      one 2: if (in bit == 0) next state <= zero 1; else
               if (in bit == 1) next state <= one 2; else</pre>
                                  next state <= init;</pre>
      default:
                                  next state <= init;</pre>
    endcase
  assign out bit = ((curr state==zero 2)||(curr state==one 2))
                     ? 1 : 0;
endmodule
                       Combinational logic for FSM outputs
```

#### **Revisit: Binary Counter as an FSM**

- FSM that counts binary numbers counter
- An *n*-bit binary counter can count in binary from 0 up to 2<sup>*n*</sup>-1 and repeat
- An *n*-bit binary counter consists of *n* flip-flops
- All the flip-flops are synchronized to the same clock synchronous counter
- May be implemented by different type of flip-flops
- Example: a 3-bit binary counter can count through this sequence



Characteristic table and equation of T-FF

Т	Q <sup>+</sup>		
0	Q	No Change	
1	Q'	Complement	

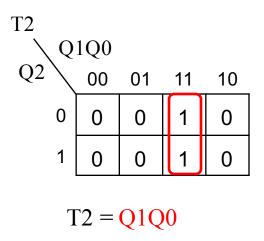
Excitation table of T-FF

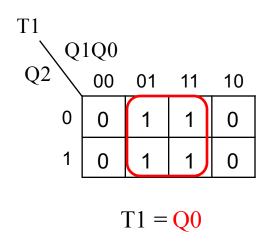
Q	Q <sup>+</sup>		Т	Q	Q <sup>+</sup>		T
0	0	No Change	0	X	Q	No Change	0
0	1	Toggle	1	X	Q'	Toggle	1
1	0	Toggle	1				
1	1	No Change	0				

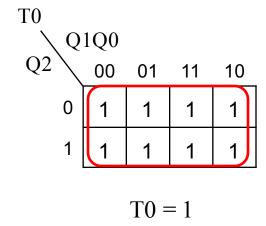
 $\mathbf{Q}^+ = \mathbf{T} \oplus \mathbf{Q}$ 

Pre	esent S	tate	Next State *				T Input	t	
Q2	Q1	Q0	Q2 <sup>+</sup>	Q1+	Q0+	T2	T1	T0	
0	0	0	0	0	1	0	0	1	_
0	0	1	0	11	0	0	1	1	
0	1	0	0	1	1	0	0	1	
0	1	1	1	0	0	1	1	1	
1	0	0	1	0	1	0	0	1	
1	0	1	1	1	0.	0	1	1	
1	1	0	1	1	1	0	0	1	
1	1	1	0	0	0	1	1	1	
		'				•			2

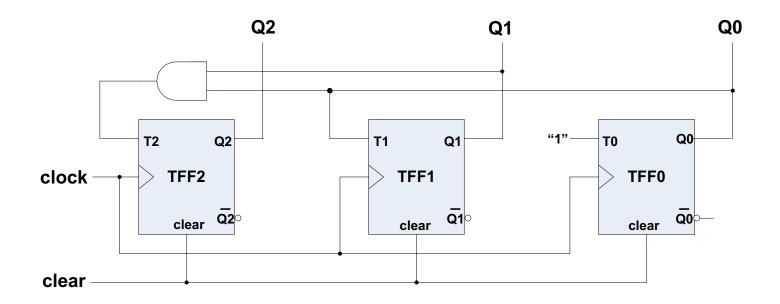
T input equations in terms of Qs can be found from the state table







• 3-bit binary counter by T FF



Characteristic table and equation of JK-FF

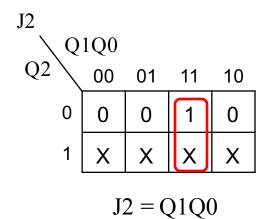
_	J	K	Q <sup>+</sup>	Action	_
	0	0	Q	Hold	$\mathbf{Q}^+ = \mathbf{J}\mathbf{Q'} + \mathbf{K'}\mathbf{Q}$
	0	1	0	Reset	
	1	0		Set	
	1	1	Q'	Toggle	

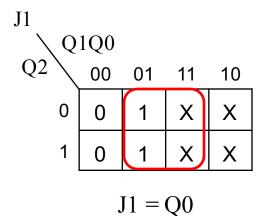
Excitation table of JK-FF

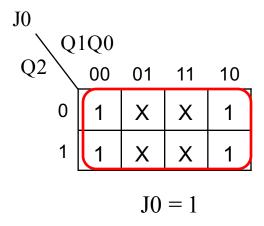
Q	Q <sup>+</sup>	Action	J	K	J	K
0	0	Reset/Hold	0/0	1/0	0	X
0	1	Set/Toggle	1/1	0/1	1	X
1	0	Reset/Toggle	0/1	1/1	X	1
1	1	Set/Hold	1/0	0/0	Χ	0

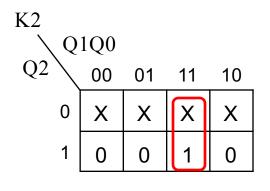
State Table with JK input Intermediate columns **Present State** Next State ✓ JK Inputs Q2 Q1 Q0 Q0 J2 K2 J1 Q2 Q1 **K**1 J0 K0 X 0 0 0 0 0 0 1 X X X 0 0 0 0 X X 0 X 1 X 0 X \_0 X X 0 X X X 0 0 0 X X X 0 X X 0

#### J-K Input equations

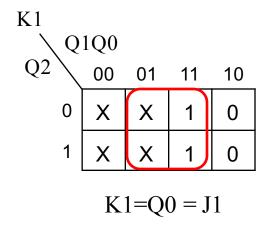


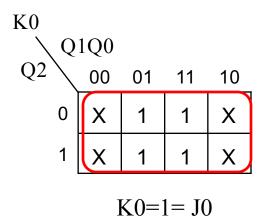






K2 = Q1Q0 = J2





Circuit diagram

