

# VE 320 Summer 2019

## Introduction to Semiconductor Devices

Instructor: Rui Yang (杨睿)

Office: JI Building 434

[rui.yang@sjtu.edu.cn](mailto:rui.yang@sjtu.edu.cn)

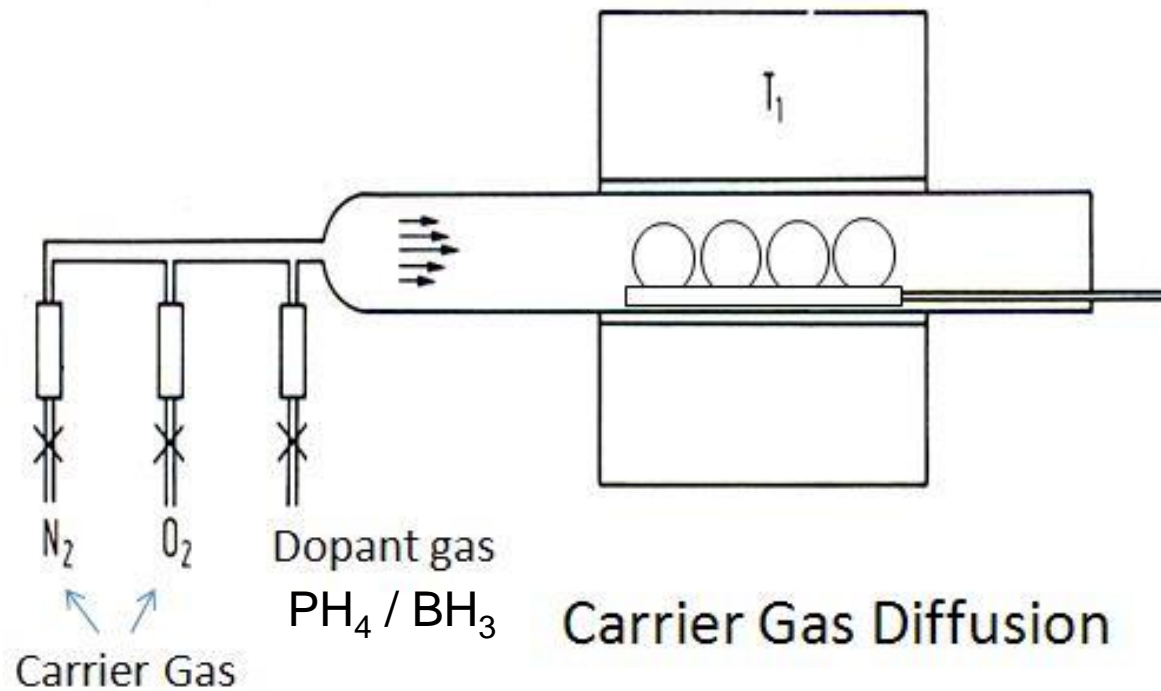


# Lecture 7

## pn Junction (Chapter 7)

# PN junction formation

- Doping by Thermal Diffusion



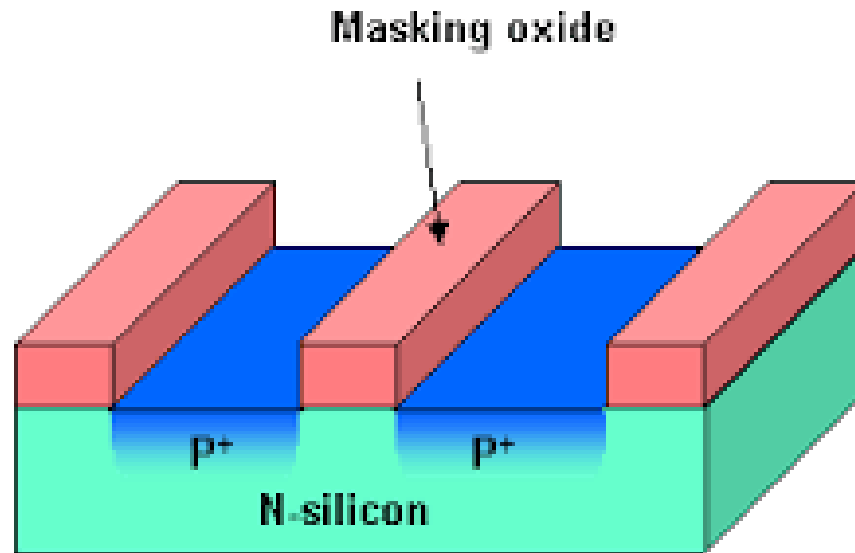
# PN junction formation

- Doping by Thermal Diffusion



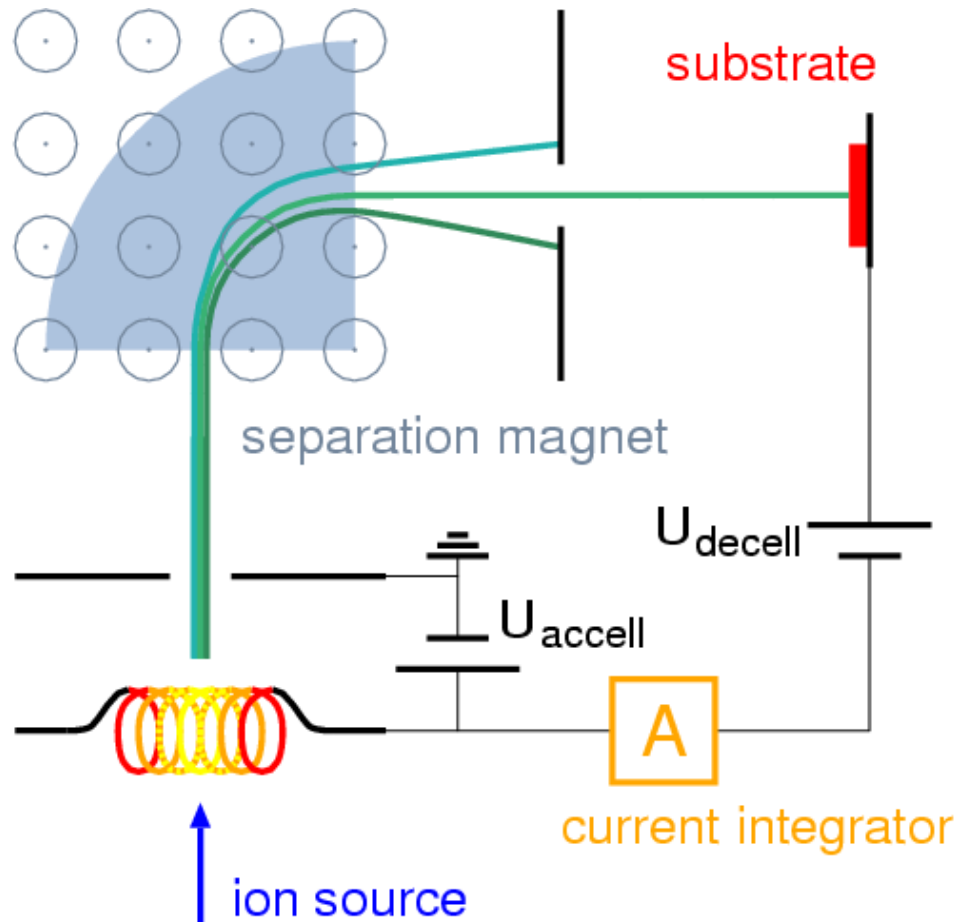
# PN junction formation

- Doping by Thermal Diffusion



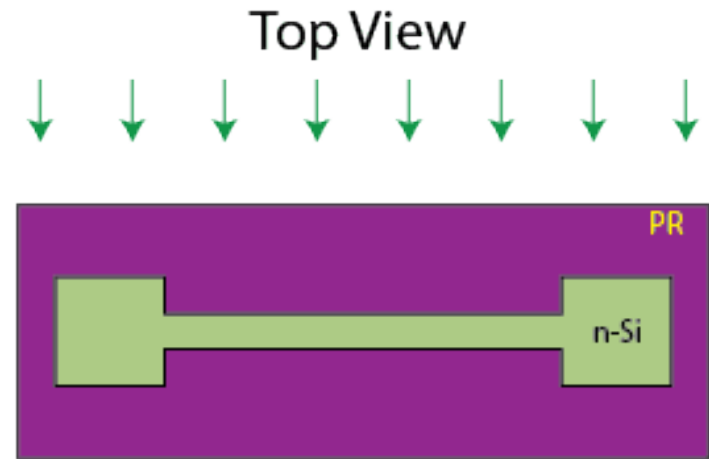
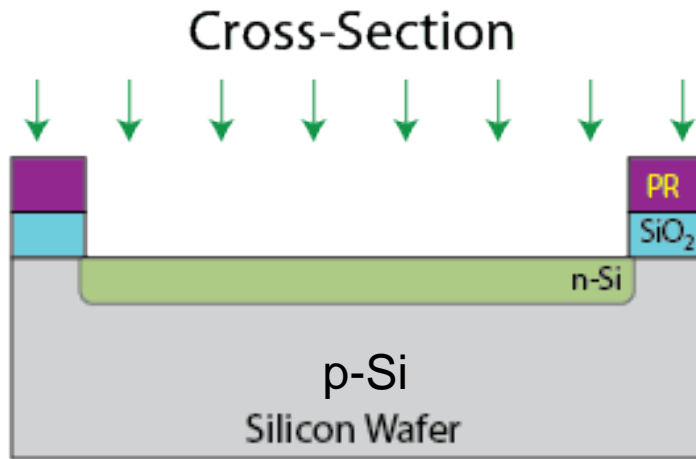
# PN junction formation

- Doping by Ion Implantation



# PN junction formation

- Doping by Ion Implantation

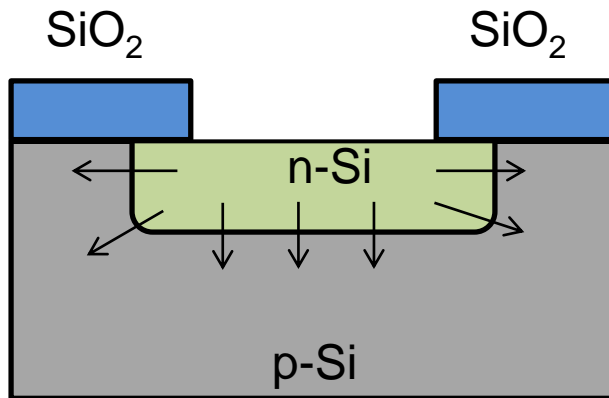


Followed by rapid thermal annealing:

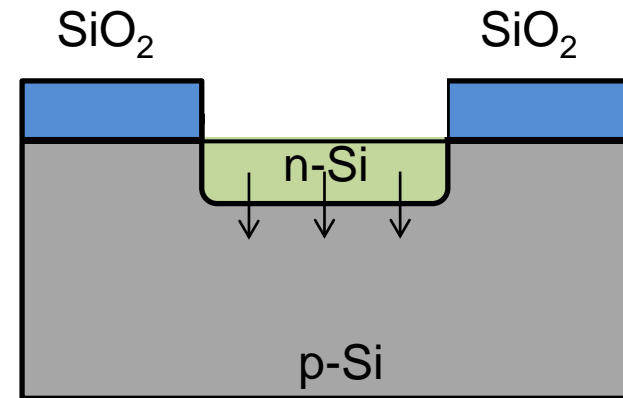
- chemically activate dopants
- not cause physical diffusion

# PN junction formation

- Doping by Thermal Diffusion

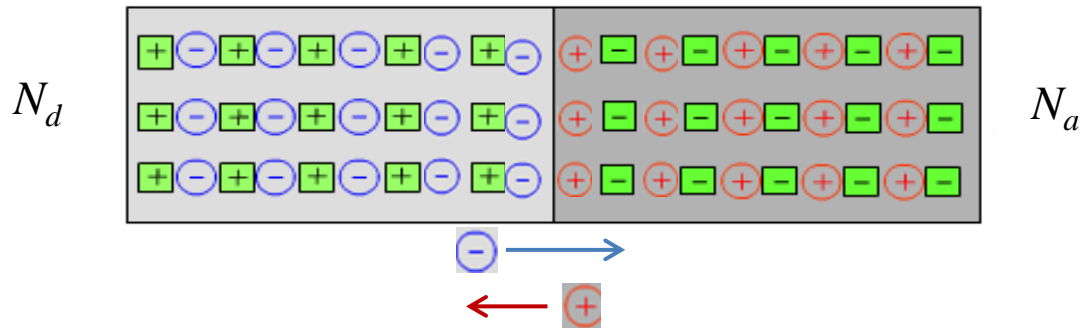


- Doping by Ion Implantation

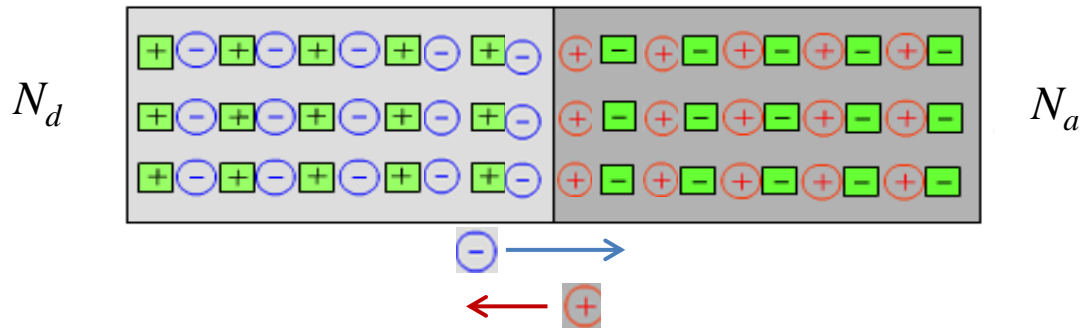




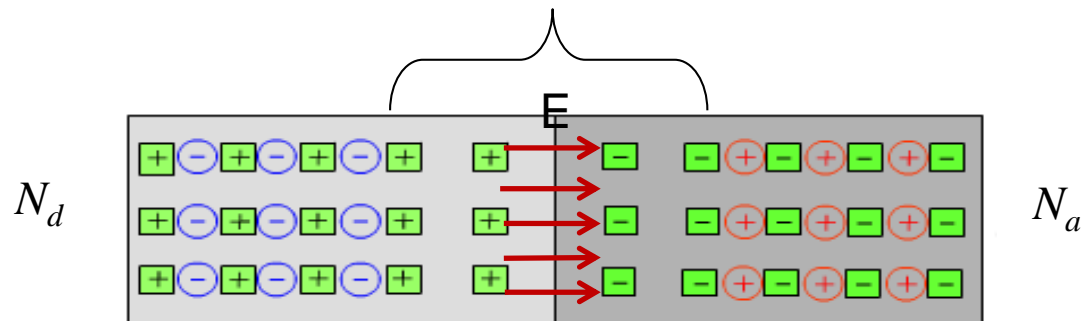
# PN junction electrostatics



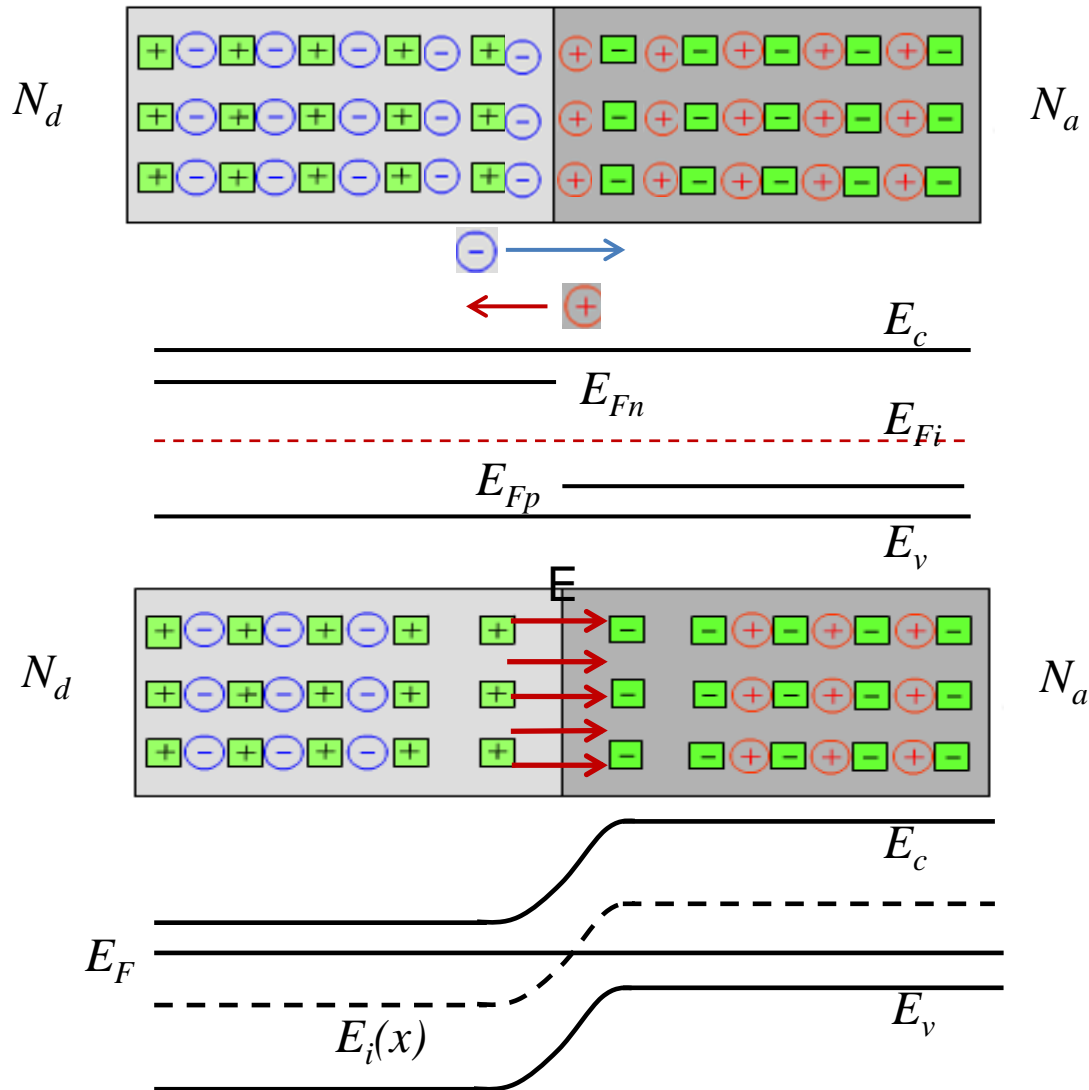
# PN junction electrostatics



space charge region / depletion region



# PN junction electrostatics



# PN junction electrostatics

- Built-in Potential

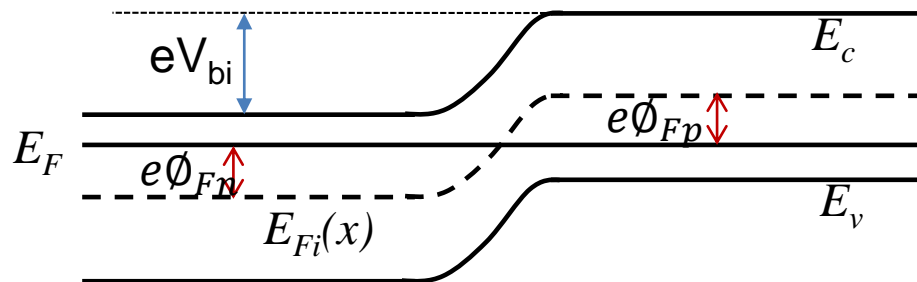
Assume complete ionization

$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) = n_i \exp\left(\frac{e\phi_{Fn}}{kT}\right) \longrightarrow \phi_{Fn} = \frac{-kT}{e} \ln\left(\frac{N_d}{n_i}\right)$$

$$p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) = n_i \exp\left(\frac{e\phi_{Fp}}{kT}\right) \longrightarrow \phi_{Fp} = +\frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right)$$

$$\Rightarrow V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| = kT \ln\left(\frac{N_a}{n_i}\right) + kT \ln\left(\frac{N_d}{n_i}\right) = kT \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

Example:  $N_a = 10^{17} \text{ cm}^{-3}, N_d = 10^{17} \text{ cm}^{-3}, \Rightarrow V_{bi} = 0.026 * \ln\left(\frac{10^{17} 10^{17}}{10^{20}}\right) = 0.84 \text{ eV}$

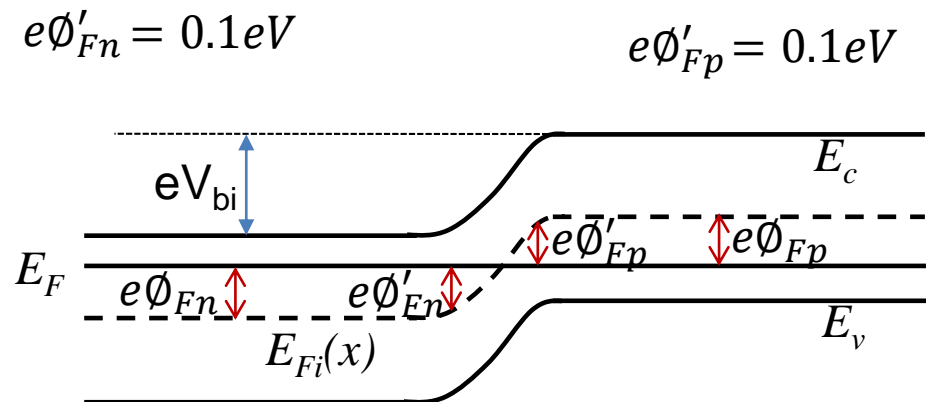


# PN junction electrostatics

- Charge carrier distribution

$$n = n_i \exp\left(\frac{E_F - E_{Fi}(x)}{kT}\right) = N_d \exp\left(\frac{0.1\text{eV}}{0.026\text{eV}}\right) \approx \frac{N_d}{50}$$

$$p = n_i \exp\left(\frac{E_{Fi}(x) - E_F}{kT}\right) = N_a \exp\left(\frac{0.1\text{eV}}{0.026\text{eV}}\right) \approx \frac{N_a}{50}$$



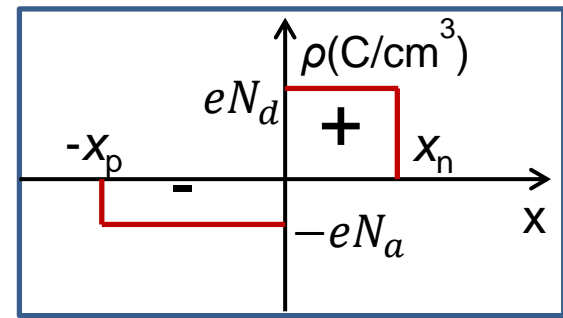
# PN junction electrostatics

- Potential function: Poisson's equation

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon} = -\frac{dE(x)}{dx}$$

$$= -\frac{e}{\varepsilon} [N_d(x) - N_a(x) + p(x) - n(x)]$$

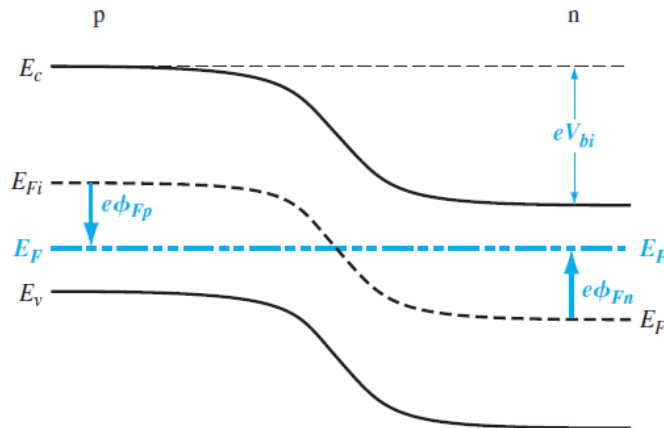
$\rho(x)$ : volume charge density



Depletion region: no mobile charge  $n=p=0$

$$\rho(x) = -eN_a \quad -x_p < x < 0$$

$$\rho(x) = eN_d \quad 0 < x < x_n$$

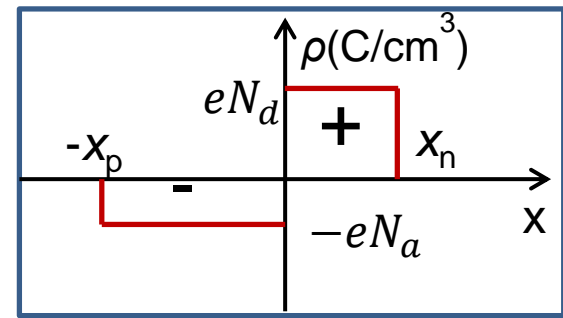


# PN junction electrostatics

- Potential function: Poisson's equation

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{dE(x)}{dx}$$

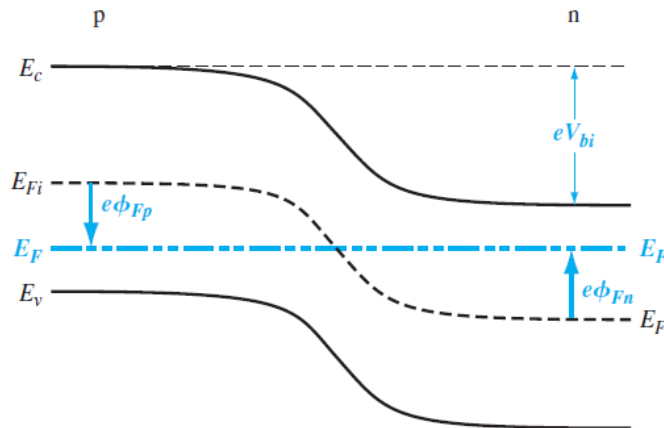
$$\rho(x) = -eN_a \quad -x_p < x < 0$$



Electric field in the p region (in the depletion region): integration

$$E = \int \frac{\rho(x)}{\epsilon_s} dx = - \int \frac{eN_a}{\epsilon_s} dx = \frac{-eN_a}{\epsilon_s} x + C_1$$

$$E = 0 \text{ at } x = -x_p \quad \text{So} \quad E = \frac{-eN_a}{\epsilon_s} (x + x_p) \quad -x_p \leq x \leq 0$$

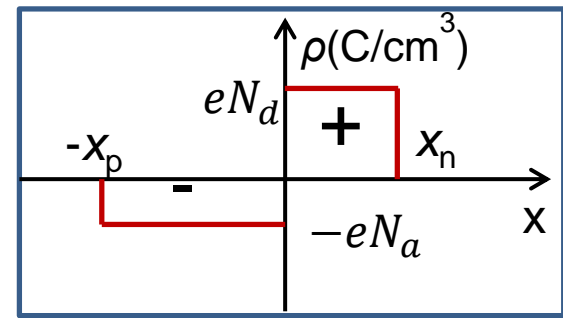


# PN junction electrostatics

- Potential function: Poisson's equation

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{dE(x)}{dx}$$

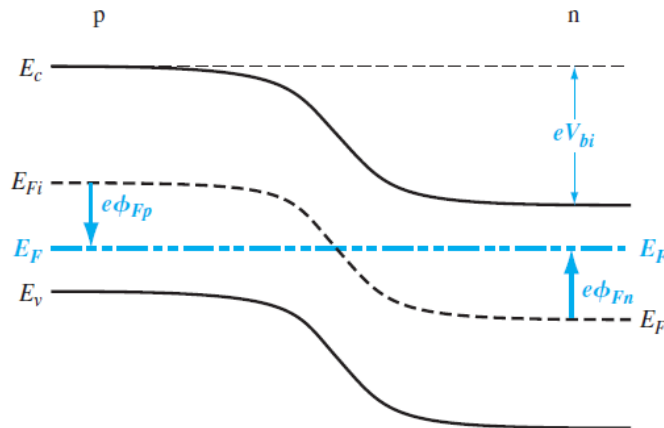
$$\rho(x) = -eN_a \quad -x_p < x < 0$$



Electric field in the n region: integration

$$E = \int \frac{(eN_d)}{\epsilon_s} dx = \frac{eN_d}{\epsilon_s} x + C_2$$

$$E = 0 \text{ at } x = x_n \quad \text{So} \quad E = \frac{-eN_d}{\epsilon_s} (x_n - x) \quad 0 \leq x \leq x_n$$



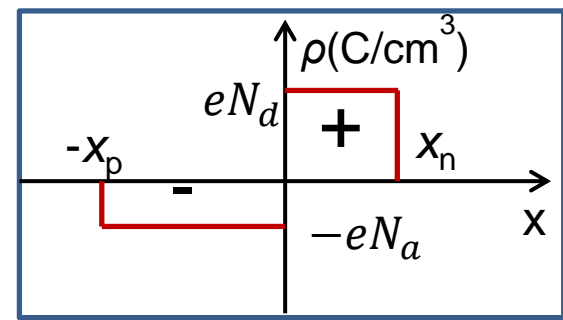


# PN junction electrostatics

- Potential function: Poisson's equation

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{dE(x)}{dx}$$

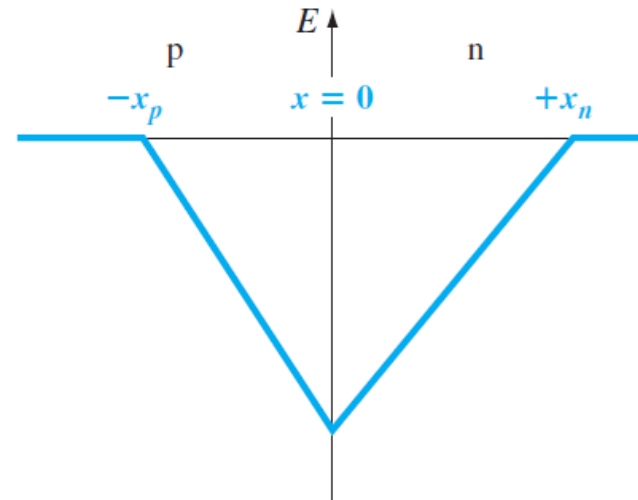
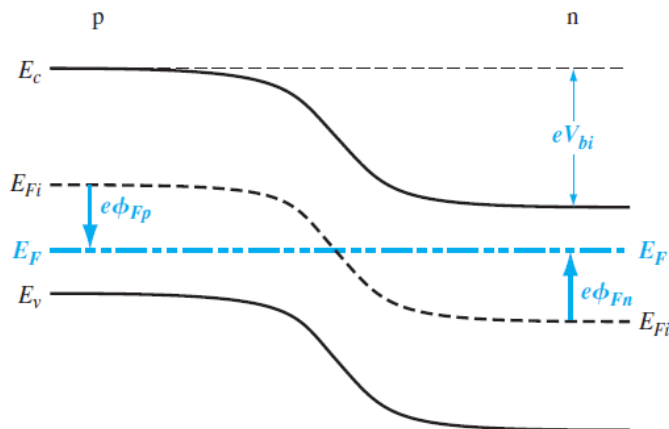
$$\rho(x) = -eN_a \quad -x_p < x < 0$$



Electric field at  $x=0$ : continuous

$$N_a x_p = N_d x_n$$

the number of negative charges per unit area in the p region is equal to the number of positive charges per unit area in the n region



# PN junction electrostatics

Potential: integrate electric field

In the p region (in the depletion region)

$$\phi(x) = - \int E(x) dx = \int \frac{eN_a}{\epsilon_s} (x + x_p) dx$$

$$\phi(x) = \frac{eN_a}{\epsilon_s} \left( \frac{x^2}{2} + x_p \cdot x \right) + C'_1$$

Only potential difference is important, so set potential=0 at  $x=x_p$

$$C'_1 = \frac{eN_a}{2\epsilon_s} x_p^2$$

$$\phi(x) = \frac{eN_a}{2\epsilon_s} (x + x_p)^2 \quad (-x_p \leq x \leq 0)$$

In the n region

$$\phi(x) = \int \frac{eN_d}{\epsilon_s} (x_n - x) dx = \frac{eN_d}{\epsilon_s} \left( x_n \cdot x - \frac{x^2}{2} \right) + C'_2$$

Potential is continuous at  $x=0$

$$C'_2 = \frac{eN_a}{2\epsilon_s} x_p^2$$

$$\phi(x) = \frac{eN_d}{\epsilon_s} \left( x_n \cdot x - \frac{x^2}{2} \right) + \frac{eN_a}{2\epsilon_s} x_p^2 \quad (0 \leq x \leq x_n)$$

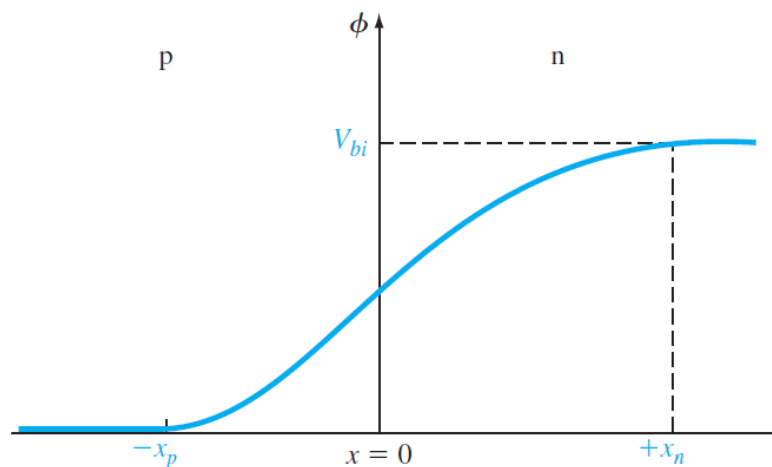
# PN junction electrostatics

Potential: quadratic function of distance

$$V_{bi} = |\phi(x = x_n)| = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$$

Potential energy

$$E = -e\phi$$



# PN junction electrostatics

Preciously we have  $N_a x_p = N_d x_n \longrightarrow x_p = \frac{N_d x_n}{N_a}$

From  $V_{bi} = |\phi(x = x_n)| = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$

We have  $x_n = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$

Space charge width, or the width of the depletion region, extending into the n-type region for the case of zero applied voltage

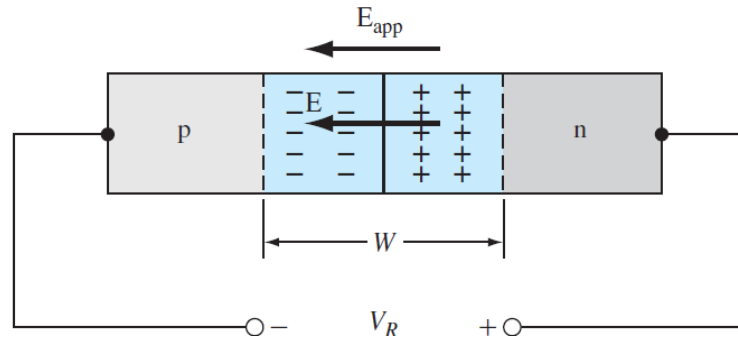
$$x_p = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_d}{N_a} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2} \quad \text{In the p-type region}$$

Total depletion or space charge width

$$\begin{aligned} W &= x_n + x_p \\ &= \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} \end{aligned}$$

# PN junction electrostatics

- Reverse applied bias: Fermi energy no longer constant

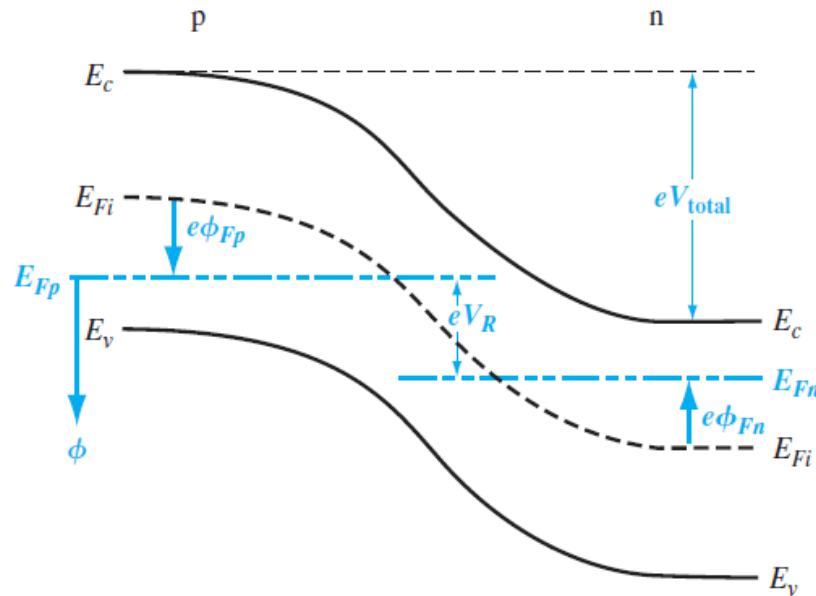


$$\begin{aligned} V_{\text{total}} &= |\phi_{Fn}| + |\phi_{Fp}| + V_R \\ &= V_{bi} + V_R \end{aligned}$$

No longer thermal equilibrium

# PN junction electrostatics

- Reverse applied bias: Fermi energy no longer constant



Space charge region: electric field increase  
impurity concentration not changed  $\longrightarrow$  Space charge width  $W$  increases

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

# PN junction electrostatics

- Reversed applied bias: Fermi energy no longer constant

Electric field:

$$E = \frac{-eN_a}{\epsilon_s}(x + x_p) \quad -x_p \leq x \leq 0$$

$$E = \frac{-eN_d}{\epsilon_s}(x_n - x) \quad 0 \leq x \leq x_n$$

Maximum electric field:  $x=0$ , metallurgical junction

$$E_{\max} = \frac{-eN_d x_n}{\epsilon_s} = \frac{-eN_a x_p}{\epsilon_s}$$

$$E_{\max} = - \left\{ \frac{2e(V_{bi} + V_R)}{\epsilon_s} \left( \frac{N_a N_d}{N_a + N_d} \right) \right\}^{1/2}$$

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

$$E_{\max} = \frac{-2(V_{bi} + V_R)}{W}$$

# PN junction electrostatics

- Junction capacitance

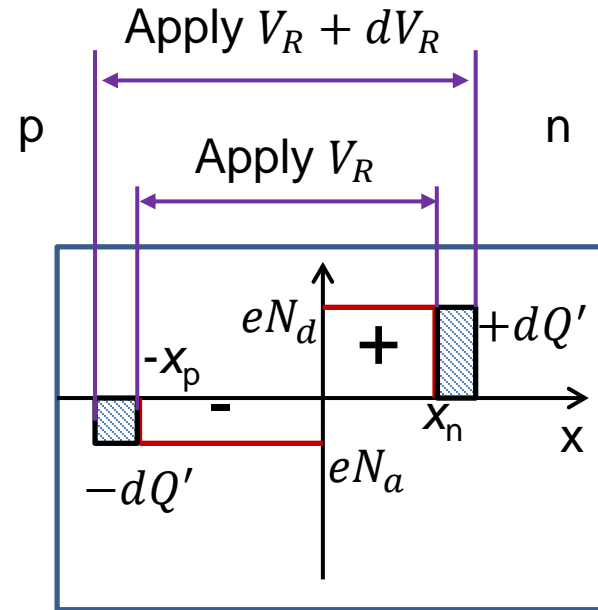
$$C' = \frac{dQ'}{dV_R}$$

$$dQ' = eN_d dx_n = eN_a dx_p$$

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}$$

$$C' = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$$



Depletion layer  
capacitance



# pn Junction with reverse bias

- One-sided junction:

If  $N_a \gg N_d$ , the junction is a p<sup>+</sup>n junction

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{eN_d} \right\}^{1/2}$$

$$x_p \ll x_n$$

$$W \approx x_n$$

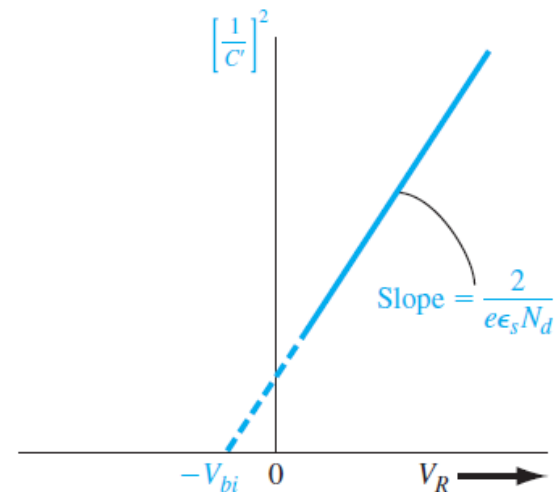
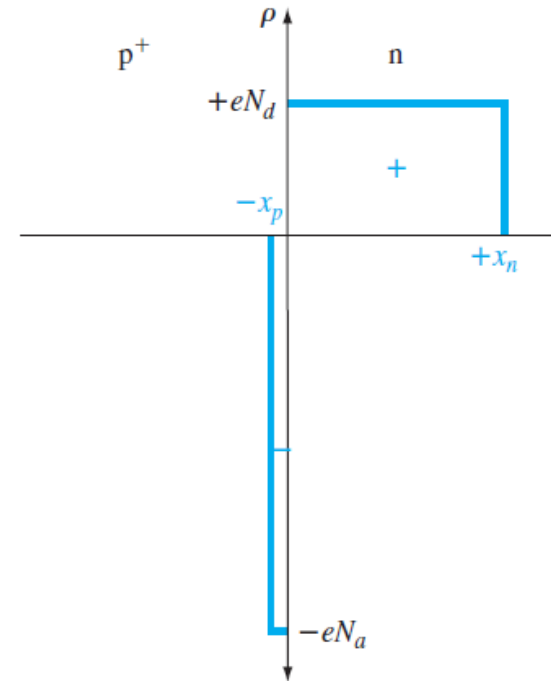
Almost the entire space charge layer extends into the low-doped region of the junction.

$$C' \approx \left\{ \frac{e\epsilon_s N_d}{2(V_{bi} + V_R)} \right\}^{1/2}$$

$$\left( \frac{1}{C'} \right)^2 = \frac{2(V_{bi} + V_R)}{e\epsilon_s N_d}$$

Assume: uniform doping in both semiconductor regions, the abrupt junction approximation, and a planar junction.

$V_{bi}$  and  $N_d$  can be determined

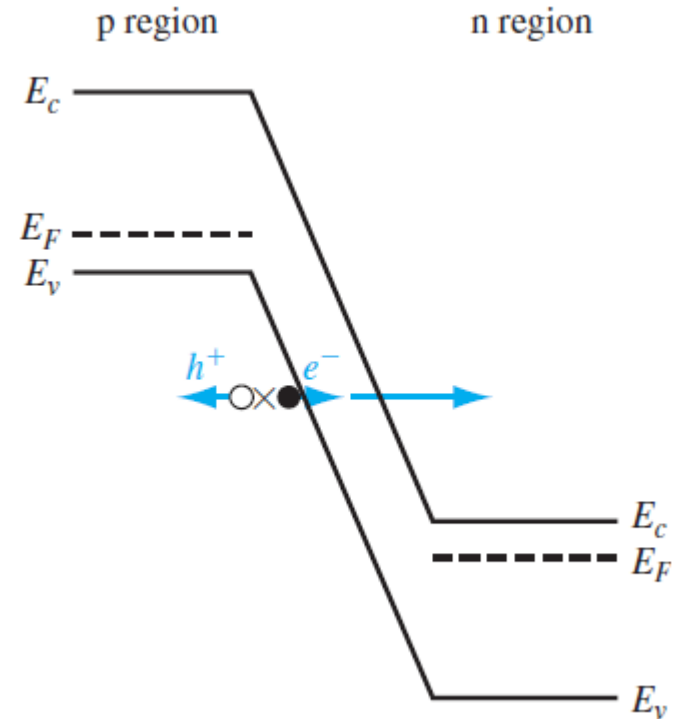


# pn Junction breakdown

- Reverse bias: breakdown at large voltage  
Zener effect and the **avalanche effect**

Zener breakdown: tunneling mechanism.

- In highly doped pn junctions
- Conduction and valence bands on opposite sides of the junction are sufficiently close during reverse bias
- Electrons may “tunnel” directly from the valence band on the p side into the conduction band on the n side.

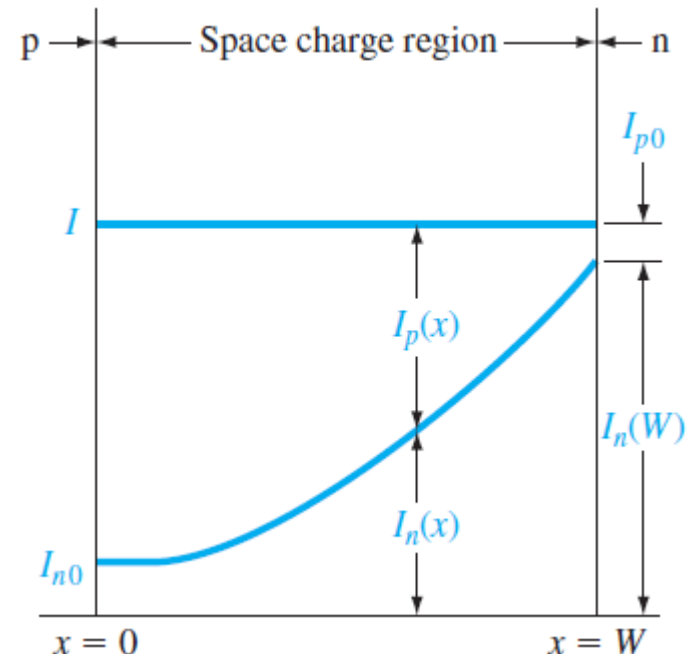
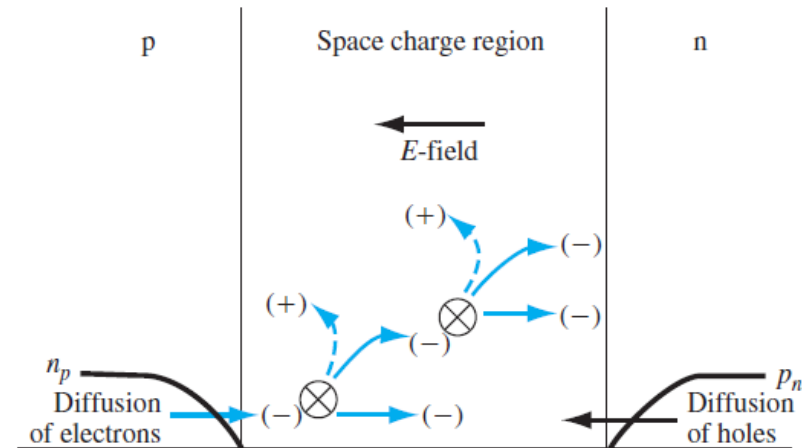


# pn Junction breakdown

- Reverse bias: breakdown at large voltage  
Zener effect and the **avalanche effect**

Avalanche breakdown: collision, dominant mechanism

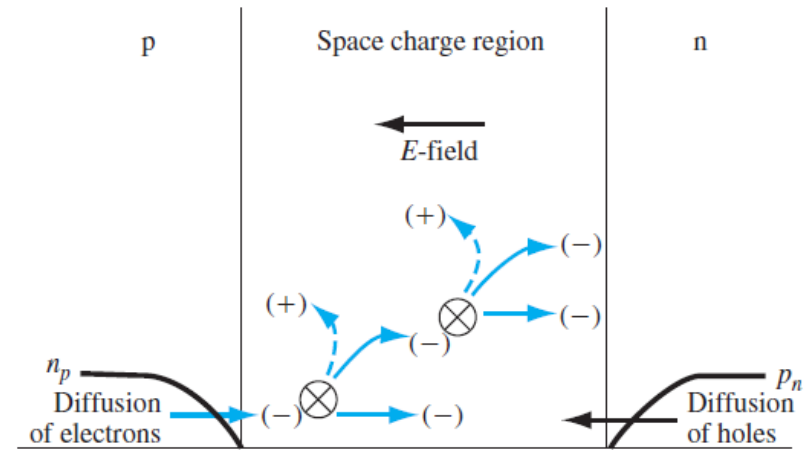
- Electrons and/or holes, moving across the space charge region, acquire sufficient energy from the electric field to create New electron–hole pairs by colliding with atomic electrons within the depletion region
- Electrons and holes move in opposite directions due to the electric field and thereby create a reverse-biased current
- Avalanche: the newly generated electrons and/or holes may acquire sufficient energy to ionize other atoms-> more and more



# pn Junction breakdown

- Reverse bias: breakdown at large voltage  
Zener effect and the **avalanche effect**

Avalanche breakdown: collision, dominant mechanism



# pn Junction breakdown

Avalanche breakdown:

Reverse-biased electron current  $I_{n0}$  enters the depletion region at  $x = 0$ ,

At  $x=W$ ,

$$I_n(W) = M_n I_{n0}$$

where  $M_n$  is a multiplication factor

Hole current: large at  $x=0$

Total current: constant through the pn junction

Incremental electron current  $dI_n(x) = I_n(x)\alpha_n dx + I_p(x)\alpha_p dx$

$\alpha_n$  and  $\alpha_p$  are the ionization rates, or the number of electron-hole pairs generated per unit length

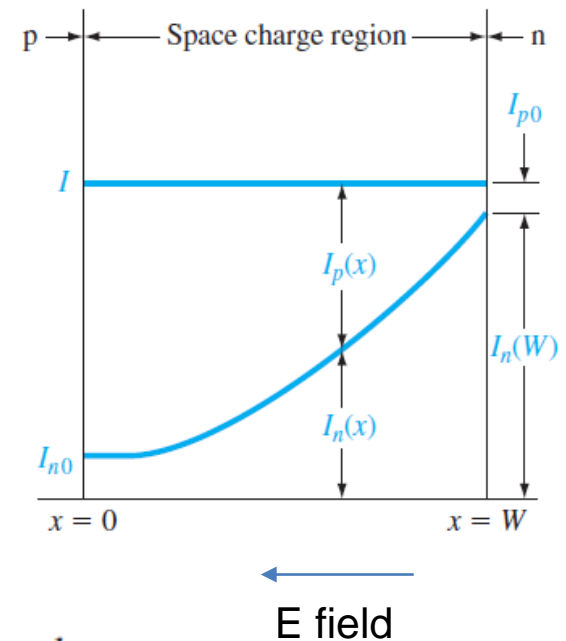
Total current is a constant  $I = I_n(x) + I_p(x)$

$$\frac{dI_n(x)}{dx} + (\alpha_p - \alpha_n)I_n(x) = \alpha_p I$$

Assume electron and hole ionization rates are equal  $\alpha_n = \alpha_p \equiv \alpha$

Integrate:

$$I_n(W) - I_n(0) = I \int_0^W \alpha dx$$



# pn Junction breakdown

Avalanche breakdown:

$$\frac{M_n I_{n0} - I_n(0)}{I} = \int_0^W \alpha dx$$

Since  $M_n I_{n0} \approx I$  and since  $I_n(0) = I_{n0}$

$$1 - \frac{1}{M_n} = \int_0^W \alpha dx$$

Avalanche breakdown:  $M_n \rightarrow \infty$

$$\int_0^W \alpha dx = 1$$

For one-sided p<sup>+</sup>n junction:

$$E_{\max} = \frac{e N_d x_n}{\epsilon_s}$$

Depletion width:

$$x_n \approx \left\{ \frac{2\epsilon_s V_R}{e} \cdot \frac{1}{N_d} \right\}^{1/2}$$

Breakdown voltage:

$$V_B = \frac{\epsilon_s E_{\text{crit}}^2}{2eN_B}$$

