~~VE320~~ PanChongdan
HW1

1. For gold: $E=h\nu$

$$\lambda=\frac{c}{\nu}=\frac{ch}{E}=\frac{3\times10^{8}\times1.054\times10^{-34}\times2\pi}{4.9\times1.6\times10^{-19}}=2.54\times10^{-7}(m)$$

For Cesium: similarly, $\lambda=6.54\times10^{-7}(m)$

2. (a) $p=\frac{h}{\lambda}=\frac{6.626\times10^{-34}}{550\times10^{-9}}=1.2\times10^{-27}\,(kg\cdot m/s)$

$v=\frac{p}{m}=\frac{1.2\times10^{-27}}{9.11\times10^{-31}}=1317\,(m/s)$

$E=\frac{1}{2}pv=\frac{1}{2}\times1.2\times10^{-27}\times1317=7.9\times10^{-25}\,(J)$

(b) $p=\frac{6.626\times10^{-34}}{440\times10^{-9}}=1.505\times10^{-27}\,(kg\cdot m/s)$

$v=\frac{p}{m}=\frac{1.505\times10^{-27}}{9.11\times10^{-31}}=1653\,(m/s)$

$E=\frac{1}{2}pv=\frac{1}{2}\times1.505\times10^{-27}\times1653=1.24\times10^{-25}\,(J)$

(c) For photon, $p=\frac{h}{\lambda}$, so they have the same momentum

3. $E=\frac{3}{2}kT=\frac{3}{2}\times1.38\times10^{-23}\times300=6.21\times10^{-21}\,(J)=3.88\times10^{-2}\,(eV)$

$v=\sqrt{\frac{2E}{m}}=\sqrt{\frac{2\times6.21\times10^{-21}}{9.11\times10^{-31}}}=1.17\times10^{5}\,(m/s)$

$p=mv=9.11\times10^{-31}\times1.17\times10^{5}=1.06\times10^{-25}\,(kg\cdot m/s)$

$\lambda=\frac{h}{p}=\frac{6.626\times10^{-34}}{1.06\times10^{-25}}=6.23\times10^{-9}\,(m)$

4. (a) $p=\frac{h}{\lambda}=\frac{6.626\times10^{-34}}{85\times10^{-10}}=7.8\times10^{-26}\,(kg\cdot m/s)$

$v=\frac{p}{m}=\frac{7.8\times10^{-26}}{9.11\times10^{-31}}=85568.5\,(m/s)$

$E=\frac{1}{2}mv^2=\frac{1}{2}\times9.11\times10^{-31}\times85568.5^2=3.34\times10^{-21}\,(J)=0.021\,(eV)$

$E=\frac{1}{2}mv^2=\frac{1}{2}\times 9.11\times 10^{-31}\times(8\times 10^3)^2=2.92\times 10^{-23}(J)=1.822\times 10^{-4}(eV)$

$p=mv=9.11\times 10^{-31}\times 8\times 10^3=7.3\times 10^{-27}(kg\cdot m/s)$

$\lambda=\frac{h}{p}=\frac{6.626\times 10^{-34}}{7.3\times 10^{-27}}=9.09\times 10^{-8}(m)=9.09\times 10^{2}(\text{Å})$

5. $\Psi(x,t)=A(\cos n\pi x)e^{j\omega t}$

$\Psi(x,t)\times\Psi(x,t)^*=A^2\cos^2 n\pi x.$

$\therefore A^2\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos^2 n\pi x\,dx=1$

$\therefore A^2\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{\cos 2n\pi x+1}{2}dx=1$

$\therefore A^2(\frac{1}{2}+\int_{-n\pi}^{n\pi}\frac{\cos y}{4n\pi}dy)=1$

$\therefore\ A=\pm\sqrt{2}$

6. (a) $E_1=\frac{h^2}{8ma^2}n^2=\frac{(6.63\times 10^{-34})^2}{8\times 9.11\times 10^{-31}\times(10\times 10^{-10})^2}=6.03\times 10^{-20}(J)=0.377(eV)$

$E_2=0.377\times 4=1.508(eV)$

$E_3=0.377\times 9=3.393(eV)$

(b) $\Delta E=E_3-E_2=3.393-1.508=1.885(eV)=3.016\times 10^{-19}(J)$

$\Delta E=h\nu=\frac{hc}{\lambda}\Rightarrow\lambda=\frac{6.63\times 10^{-34}\times 3\times 10^{8}}{3.016\times 10^{-19}}=6.6\times 10^{-7}(m)$

7. For $x\le 0, V(x)=0$, $-\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi(x)}{\partial x^2}=E\psi(x)\Rightarrow\psi_1(x)=A_1e^{jk_1x}+B_1e^{-jk_1x}$, $k_1=\sqrt{\frac{2mE}{\hbar^2}}=8.57\times 10^{9}(m^{-1})$

similarly, for $x>0$, $\psi(x)=C_1e^{jk_2x}+D_1e^{-jk_2x}$, $k_2=\sqrt{\frac{2m(E-V_0)}{\hbar^2}}=j4.29\times 10^{9}(m^{-1})$

since $\psi(x)$ is finite, $D_1=0$, then $\psi_2(x)=C_1e^{-4.29\times 10^{9}x}$

$\psi(x)$ must be continuous at $x=0$, then $A_1+B_1=C_1$

Also $\left.\frac{d\psi_1(x)}{dx}\right|_{x=0}=\left.\frac{d\psi_2(x)}{dx}\right|_{x=0}$, then $jk_1A_1-jk_1B_1=-k_2C_1$

$\Rightarrow\begin{cases}B_1=(\frac{3}{5}-\frac{4}{5}j)A_1\\C_1=(\frac{8}{5}-\frac{4}{5}j)A_1\end{cases}$

$\frac{\psi(5\times 10^{-10})^2}{\psi(0)}=e^{2k_2x}=0.1376$ similarly, for 15Å, it's 2.606×10^{-6}, for 40Å, it's 1.286×10^{-16}

$\therefore$ The answer is 0.1376, 2.606×10^{-6} and 1.286×10^{-16}

$$-\frac{\hbar^2}{2m}\frac{\partial \psi^2(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x)$$

$$\psi(x)=Ae^{-jkx}+Be^{jkx} \qquad k=\frac{\sqrt{2m(E-V)}}{\hbar}$$

For $x>a$ $k=j\infty$, so $A=0$, then $\psi_3(x)=Be^{-\infty x}=0$

For $0\le x\le a$, $k_2=\frac{\sqrt{2mE}}{\hbar}$, $\psi_2(x)=A_2e^{-jk_2x}+B_2e^{jk_2x}=A\cos k_2x+jB\sin k_2x$

For $x<0$, $k_3=\frac{\sqrt{2m(E-V_0)}}{\hbar}$, since $\psi_3(x)$ must be finite, then $B_3=0$

then $\psi_3(x)=A_3e^{kx}$, $k=\frac{\sqrt{2m(V_0-E)}}{\hbar}$

(b) boundary equations:

$\psi_2(a)=0 \Rightarrow j\frac{A}{B}=\tan k_2a$

$\psi_2(a)'=0 \Rightarrow -j\frac{B}{A}=\tan k_2a$ $\Rightarrow A^2=B^2$

$\psi_2(0)=\psi_3(0) \Rightarrow A=A_3$

$\psi_2'(0)=\psi_3'(0) \Rightarrow k_2jB=kA_3$

(c) $A=A_3=\frac{k_2jB}{k}$

$\therefore \tan k_2a=\frac{jA}{B}=-\frac{k_2}{k}=-\sqrt{\frac{E}{V_0-E}}$

$$\tan\frac{a\sqrt{2m(V_0-E)}}{\hbar}=-\sqrt{\frac{E}{V_0-E}}$$

the value for E is not continuous, so it's not quantized