UM-SJTU JOINT INSTITUTE
PHYSICS LABTORATORY
(VE216)

LABTORATORY REPORT

LAB 1
LTI SYSTEMS

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1 Objectives

- Become familiar with the laboratory equipment: power supply, signal generator, digital oscilloscope, computer data acquisition system (Scope Connect).
- Review basic concepts of linear time-invariant systems.
- Illustrate several possible ways to determine the impulse response of a physical system from measured data.
- Use linearity, time-invariance and impulse response to compute the output of an LTI system when the input is a step, a pulse, or a more complicated signal.
- Measure the output response of the series RC circuit for a variety of inputs, including a step, a combination of a step and a ramp, and a sinusoid
- Compare results to those computed as part of pre-lab assignment

2 Theoretical Background

An RC circuit is used so that the computations are easy and physically meaningful. The same procedures can be applied to much more complicated systems.

2.1 RC Circuit

The RC circuit shown below is an example of a simple LTI system. Of course, there are many other LTI systems that do not involve circuits at all.

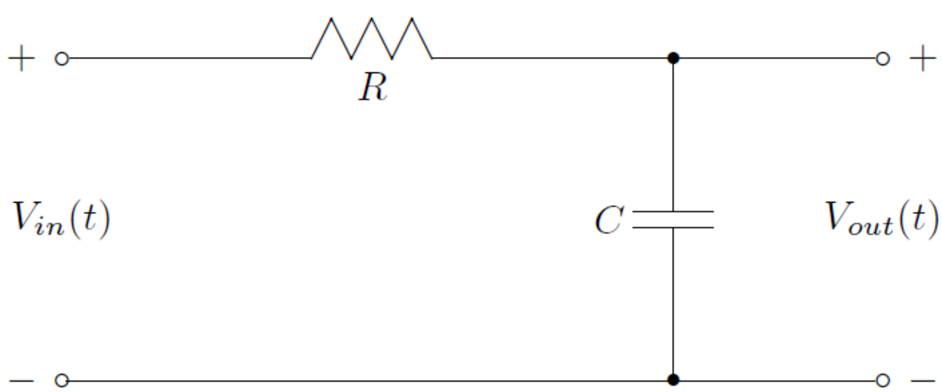

Figure 1: RC Circuit

We will take the system input to be the voltage $V_{in}(t)$, while the system output is the voltage, $V_{out}(t)$, dropped across the capacitor. Notice that these voltages, in general, will be functions of time,t.

2.2 When is a Linear Circuit a Linear System?

Using Kirchoffs current and voltage laws, one can easily derive a differential equation model of the RC-circuit in Figure 1, namely

$$RC\frac{\mathbf{d}V_{out}(t)}{\mathbf{d}t} + V_{out}(t) = V_{in}(t) \tag{1}$$

Appealing to basic knowledge of ODEs from a sophomore level math course, the total solution is seen to be

$$V_{out}(t) = V_0 e^{-t/RC} + \int_0^t \frac{1}{RC} e^{-(t-\tau)/RC} V_{in}(\tau)d\tau, \tau > 0 \tag{2}$$

where the initial condition at time zero is $V_{out}(0) = V_0$. It is very easy to verify that $V_{out}(t)$ is a linear function $V_{in}(t)$ if, and only if, $V_{out}(0) = V_0 = 0$, that is, the initial voltage on the capacitor has to be zero. This point is emphasized because you will have to assure this in the laboratory by either waiting for the charge to decay on the capacitor or by shorting the capacitor with a wire.

2.3 Impulse Response

The impulse response, $h(t)$, of an LTI system is, by definition, the output response when the input of the system is a delta function, $\delta(t)$. Of course, the delta function is a mathematical idealization. In practice, $h(t)$ can be well approximated by the response of the system when the input is a pulse of very short duration (compared with the response time of the system) and unit area, such as $p_{\triangle} = \frac{1}{\triangle}((u(t) - u(u - \triangle)))$ for $\triangle > 0$ sufficiently small.

Note that in order to keep the area of the pulse equal to unity, the amplitude has to increase as the pulse duration gets shorter. Often, this is a problem in a practical system as a large voltage pulse may fry an amplifier, for example. One way to get around this is to use linearity and realize that if the input is scaled by "b", then the output will be scaled by "b" as well. Consequently, if the measured response is divided by "b", an approximation of the impulse response is obtained.

2.4 Step Response

For any LTI system the output can be expressed as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau \tag{3}$$

where $x(t)$ denotes the input and $*$ denotes the convolution operation. The output resulting when the input is a unit step function, $x(t) = u(t)$, is called the unit step response. Simple manipulation leads to

$$y_{step}(t) = u(t)*h(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} h(\tau)u(t-\tau)d\tau + \int_{t}^{\infty} h(\tau)u(t-\tau)d\tau \tag{4}$$

which, because the unit step function is equal to 1 for $t-\tau > 0$ and equal to 0 for $t-\tau < 0$ step response simplifies to

$$y_{step}(t) = \int_{-\infty}^{t} h(\tau) d\tau \tag{5}$$

By taking the derivative of $y_{step}(t)$ with respect to t, we obtain, by the fundamental theorem of calculus,

$$\frac{\mathbf{d}y_{step}(t)}{\mathbf{d}t} = \frac{\mathbf{d}}{\mathbf{d}t} \int_{-\infty}^{t} h(\tau) d\tau = h(t) \tag{6}$$

Thus the impulse response can be computed from the unit step response by calculating the derivative of the step response with respect to time. This is a useful observation because it is sometimes easier to apply a step input to a physical system than it is to apply (an approximation of) an impulse.

We now have two ways of determining the impulse response from data. A third way will be hinted at a little later.

3 Experimental Procedure

3.1 Step

- Function generator: Utility → Output Setup → Load → High Z
- Oscillator: Trigger Menu → $\begin{cases} \text{1.Trigger Mode} \rightarrow \text{Basic} \\ \text{2.Edge Trigger (Rising Edge)} \\ \text{3.Trigger Settings} \rightarrow \text{DC Coupling} \end{cases}$
- RC Circuit: $R = 1K\Omega$, and $C = 1\mu F$

3.2 Part 1: Step Response

- Function generator:
  Square Wave   Vpp: 1V   Frequency: 100Hz
- Oscillator: CH1: 200mV/div   CH2: 200mV/div   Time:2ms

3.3 Part 2: Pulse Response

- Function generator: Pulse frequency: 100Hz
  1. Width: 1ms   A:100mV
  2. Width: 0.5ms   A:200mV

3.4 Part 3: Ramp Response

- Function generator:
  Ramp   Vpp:100mV   frequency:100Hz

3.5 Part 4: Sine Response

- Function generator: 10 Vpp
- Get the result of V_{out}/V_{in}, time shift and phase shift when the frequency is at 50, 500, 5000 respectively
- Compare the results with ideal case

4 Experimental Results

4.1 Part 1:Step Response

Figure 2: Step Response

4.2 Part 2:Pulse Response

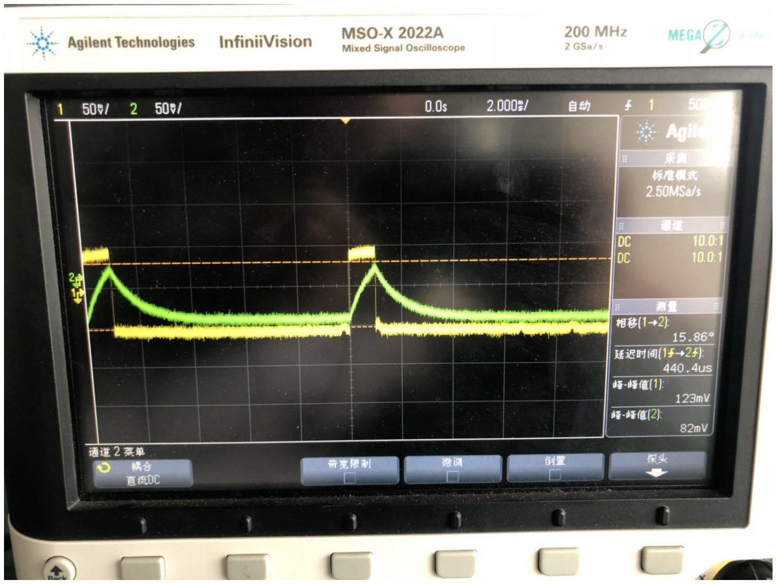

Figure 3: Pulse Response 1

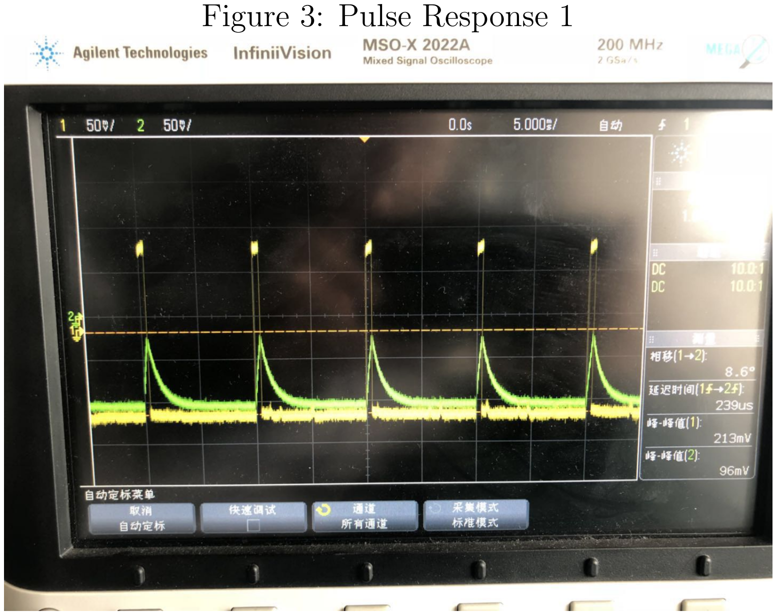

Figure 4: Pulse Response 2

4.3 Part 3:Ramp Response

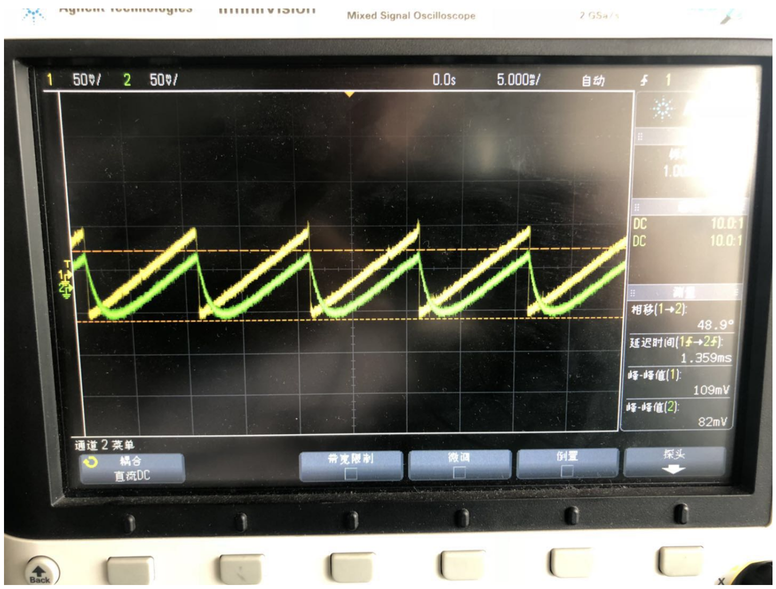

Figure 5: Ramp Response

4.4 Part 4:Sine Response

Frequency (Hz)	Vout/Vin	Time Shift	Phase Shift
50	0.9626	1.02[ms]	-18.8^o
500	0.3689	410[μs]	-73^o
5k	0.0359	45[μs]	-82^o

5 Error Analysis and Discussion

5.1 Error Analysis

5.1.1 Part 1:Step Response

In our first part of the experiment, the square wave function can be formed by infinite step function, so we compare our figure with the step response, whose figure looks like this

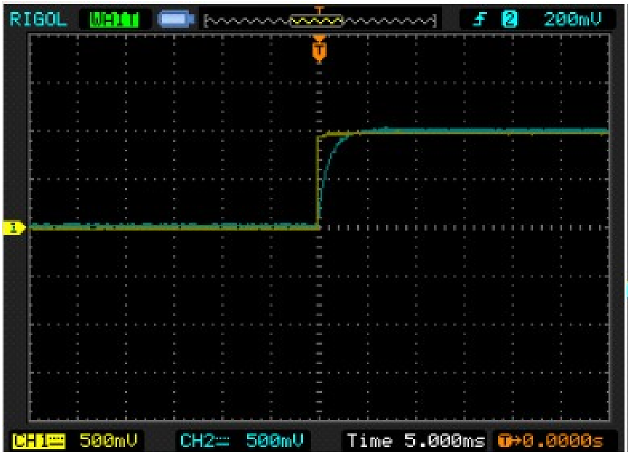

Figure 6: Step Response

As we can see, this figures is quite similar to the one in experimental result, which means we got the right results in our first part.

5.1.2 Part 2:Pulse Response

In our second part, we dealt with the pulse train function, and compared our figure with the figure of pulse response

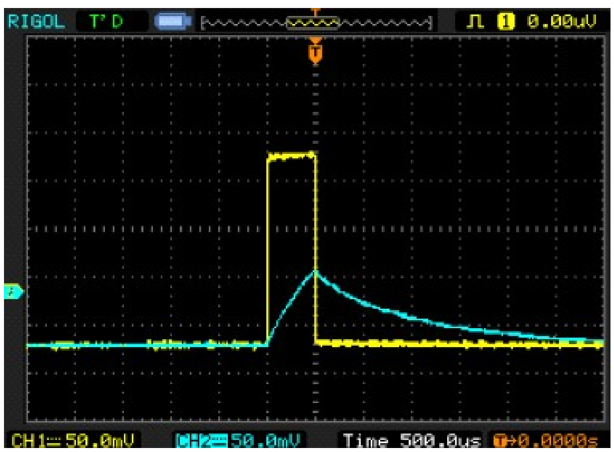

Figure 7: Pulse Response

Also, its quite similar to the figure we got in experimental results, which means we got the right result in second part.

5.1.3 Part 3:Ramp Response

In the third part, compare our figure to the ramp response

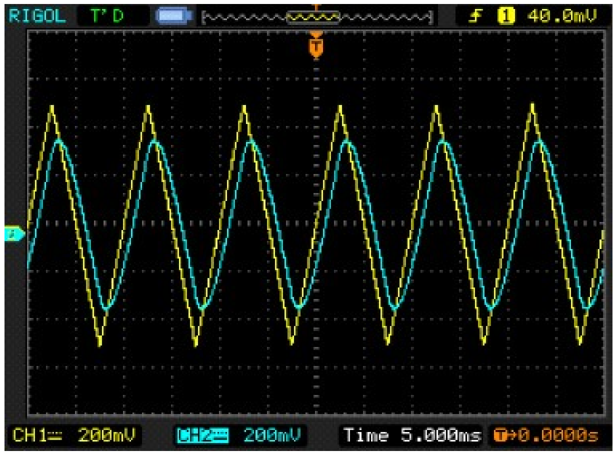

Figure 8: Ramp Response

Obviously, our figure is quite same to the standard ramp response, so we succeed in getting the true results.

5.1.4 Sine Response

In part 4 we focused on the sine response, and figured out the values of V_{out}/V_{in}, Time Shift and Phase Shift at different Frequency(50Hz, 500Hz and 5kHz)

Frequency (Hz)	Vout/Vin	Time Shift	Phase Shift
50	0.9626	1.02[ms]	-18.8^o
500	0.3689	410[μs]	-73^o
5k	0.0359	45[μs]	-82^o

Table 1: Experimental Results

Frequency (Hz)	Vout/Vin	Time Shift	Phase Shift
50	0.9540	0.9689[ms]	-17.4^o
500	0.3033	402[μs]	-72.3^o
5k	0.0318	49[μs]	-88.2^o

Table 2: Theoretical Results Calculated in Prelab Exercise

Frequency (Hz)	Vout/Vin	Time Shift	Phase Shift
50	0.0086	0.0511[ms]	1.4^o
500	0.0656	8[μs]	0.7^o
5k	0.0041	4[μs]	6.2^o

Table 3: Absolute Error

Frequency (Hz)	Vout/Vin	Time Shift	Phase Shift
50	0.90%	5.27%	8.05%
500	21.62%	1.99%	0.97%
5k	1.29%	8.16%	7.03%

Table 4: Relative Error

As shown on the tables above, compare to the theoretical results we got before, they are quite close to each other, the relative error is below 8%.

5.2 Discussion of Error

Though we did a good job and got the similar results comparing to the theoretical results, but there still are 4 errors in Part 4, we think there are 2 reasons.

1. First, is the unavoidable error, which is caused by the instruments we used. In the experiment, we were supposed to use the resistor with 1000Ω, but actually, when we measured the resistor, the actual value is 985Ω, and since in the ideal results, we assumed that R is constant and equals to 1000Ω, so obviously this will cause the errors, and its unavoidable since we cant find a ideal resistor. Also, the connecting clamp might also have resistance, however, we ignore them in our ideal results.

2. Second, is the avoidable error, which is caused by ourselves. Though we called it the avoidable error, but actually its hard for us to avoid it, but we can try to decrease it. In the experiment, when we were reading the data from screen, actually the data is shaking, it was jumping up and down in a small range. And we just randomly chose one value in this range. This would cause some errors. To reduce the influence of this kind of errors, maybe next time we should choose the value which is exactly at the middle of the range, and repeat this experiment three or four times, choose the average value as the final answer.

6 Conclusion

6.1 Prelab

By doing the prelab assignments, we take a review of the basic concepts of linear timeinvariant systems, like using the convolution to calculate the output signal. And also, we reviewed some knowledge about RC circuit we learned last year in Ve 215. And we got the ideal output when the input is a step, a pulse, or a more complicated siganl.

6.2 During Lab

We used and became familar with the laboratory equipments, like the power supply, signal generator and digital oscilloscope. By measuring the data and getting the figures by ourselves, we had a deeper understanding of the concepts in LTI system and RC circuit.

6.3 After Lab

we comparing the ideal results with the figures and data we got from the lab, and find they are quite similar to each other, which confirmed the concepts we learned before. Also, after we analyzing the errors, we got to know how to improve our steps next time. All in all, this lab is quite successful, we got the results we want, we reviewed the previous knowledge and learned new knowledge at the same time.