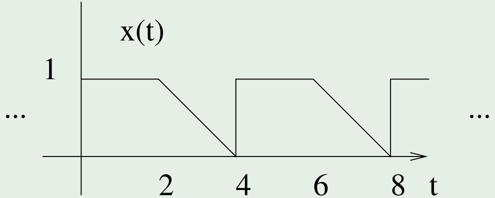
Example

Find the Fourier series representation of the signal $x(t)$ pictured below. Express your results in a real form.



Method 1

$T_0 = 4 \Longrightarrow \omega_0 = \pi/2.$

- $c_0 = (1/4)[2+1] = 3/4.$
- For $k \neq 0$

$$c_k = 1/4 \int_0^2 1 e^{-jk\omega_0 t}\, dt + 1/4 \int_2^4 (2 - t/2) e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{j2\pi k} + \begin{cases} 0, & k \text{ even} \\ \frac{-1}{k^2\pi^2}, & k \text{ odd}, \end{cases}$$

$$\boxed{x(t) = 3/4 + \sum_{k=1}^{\infty} \frac{1}{\pi k} \sin\left(k\frac{\pi}{2}t\right) + \sum_{k=1 \text{ odd}}^{\infty} \frac{-2}{k^2\pi^2} \cos\left(k\frac{\pi}{2}t\right).}$$

Solution (2)

Method 2

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) == \sum_{n=-\infty}^{\infty} \delta(t-4n) - \frac{1}{2}\sum_{n=-\infty}^{\infty} \mathrm{rect}\left(\frac{t-3-4n}{2}\right)$$

The FS coefficients of $y(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t)$ are
$k \neq 0$

$$\begin{aligned}
d_k &= \frac{1}{T_0}\int_{T_0} y(t)e^{-jk\omega_0 t}\,dt = \frac{1}{4}\int_{0}^{4} y(t)e^{-jk\omega_0 t}\,dt \\
&= \frac{1}{4}\int_0^4 \delta(t)e^{-jk\omega_0 t}\,dt - \frac{1}{8}\int_2^4 e^{-jk\omega_0 t}\,dt \\
&= \frac{1}{4} - \frac{1}{4} - \frac{1}{-jk\omega_0}e^{-jk\omega_0 t}\Big|_2^4 \\
&= \frac{1}{4} - \frac{1}{8jk\omega_0}\left(e^{-4jk\omega_0} - e^{-2jk\omega_0}\right)
\end{aligned}$$

Solution (3)

Since $d_k = jk\omega_0 c_k$,

$$
\begin{aligned}
c_k &= \frac{1}{jk\omega_0} d_k = \frac{1}{4jk\omega_0} - \frac{1}{8(jk\omega_0)^2}\left(e^{-4jk\omega_0} - e^{-2jk\omega_0}\right) \\
&= \frac{1}{j2\pi k} + \frac{1}{j2\pi^2 k^2}\left(e^{-j2\pi k} - e^{-j\pi k}\right) \quad (\omega_0 = \pi/2) \\
&= \frac{1}{j2\pi k} + \begin{cases} 0, & k \text{ even} \\ \frac{-1}{k^2\pi^2}, & k \text{ odd}, \end{cases} .
\end{aligned}
$$

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A 2 Hz cosinusoidal signal of 4 volt peak-to-peak amplitude is applied to a system described by the following differential equation

$$3y(t) + 2\frac{\mathrm{d}}{\mathrm{d}t}y(t) = 6x(t) - 4\frac{\mathrm{d}}{\mathrm{d}t}x(t).$$

Determine the output signal. (selected from Exam 2 in Summer 2014)

Solution

$$x(t) = 2\cos\omega_0 t, \qquad \text{with } \omega_0 = 4\pi$$

$$H(\omega) = (6 - 4j\omega)/(3 + 2j\omega)$$

so

$$H(\omega_0) = (6 - j16\pi)/(3 + j8\pi)$$

So

$$|H(\omega_0)| = 2, \qquad \text{and } \angle H(\omega_0) = \angle(6 - j16\pi)/(3 + j8\pi).$$

Thus $\boxed{y(t) = 4\cos\left(4\pi t + \angle\frac{6 - j16\pi}{3 + j8\pi}\right).}$