UM-SJTU JOINT INSTITUTE
Intro to Circuits
(VE215)

LABORATORY REPORT

EXERCISE 3
Transient Lab

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1 Introduction and Theoretical Background

1.1 Objectives

1. Apply the theory on the step responses in first and second order circuits to series RC and RLC circuits.

2. Build a series RC circuit, observe its responses to input square wave signal of varied frequency, and explain them based on the theor

   (a) Relate the observed capacitor voltage and resistor voltage as functions of time to calculations.

   (b) Explain the changes of both output waveforms in response to the increase of the frequency of the input square wave signal.

   (c) Explain the amplitudes of the capacitor voltage and the resistor voltage related to the amplitude of the input square wave

3. Build a series RLC circuit, observe the three types of its responses to input square wave signal, and relate them to the theory. For the under-damped/ over-damped/ critical damped response, compare the resistance in the circuit measured in the lab with the critical resistance calculated in the pre-lab.

4. Build the simplest second-order circuit, an LC tank, and observe oscillations.

1.2 Theoretical Background

1.2.1 First-order Circuits

Theoretically, the transient responses in electric circuits are described by differential equations. The circuits, whose responses obey the first-order differential equation

$$\frac{\mathsf{d}x(t)}{\mathsf{d}t} + \frac{1}{\tau} \cdot x(t) = f(t)$$

are called $\mathbf{first-ordercircuits}$. Their responses are always monotonic and appear in the form of exponential form

$$x(t) = K_1 \cdot e^{-\frac{t}{\tau}} + K_2$$

A first-order circuit includes the effective resistance R and one energy-storage element, an inductor L, or a capacitor C.

In an RC circuit, the time constant is

$$\tau = RC$$

In an LC circuit, the time constant is

$$\tau = \frac{L}{R}$$

The **falltime** of a signal is defined as the interval between the moment when the signal reaches 90% and the moment when the signal reaches 10% level. Note that the 10% level is reached between 2τ and 3τ. Approximately, you can assume the fall time $\approx 2.2\ \tau$. After $t = 5\tau$, the exponent practically equals zero.

1.2.2 Second-order Circuits

Many circuits involve two energy-storing elements, both an inductor L and a capacitor C. Such circuits require a second-order differential equation description.

$$\frac{\mathsf{d}^2 x(t)}{\mathsf{d}t^2} + 2 \cdot \alpha \cdot \frac{\mathsf{d}x(t)}{\mathsf{d}t} + \omega_0^2 \cdot x(t) = f(t)$$

thus they are called $\mathbf{second-ordercircuits}$.

We will consider only second-order circuits with one inductor and one capacitor. The differential equation includes two parameters: the damping factor α and the undamped frequency ω_0 which are determined by the circuit and its components.

For example, in the series RLC circuit, which you will build and study in this lab,

$$\alpha = \frac{R}{2 \cdot L}, \omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

while in the parallel RLC circuit

$$\alpha = \frac{1}{2 \cdot R \cdot C}, \omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

Depending on the two parameters α and ω_0, second-order circuits can exhibit three types of response.

1.2.3 The Underdamped Response

If $\alpha < \omega_0$

$$x(t) = e^{-\alpha t}(K_1 cos(\omega t) + K_2 sin(\omega t))$$

where $\omega = \sqrt{\omega_0^2 - \alpha^2}$ The underdamped circuit response involves decaying oscillations, which may last for many periods or for less than one period, depending on the damping ratio $\xi = \frac{\alpha}{\omega_0}$, which for the series RLC circuit $\xi = \frac{R}{2L}\sqrt{LC} = \frac{R}{2} \cdot \sqrt{\frac{C}{L}}$. Varying the values of R, L, C, affects the damping ration ξ.

1.2.4 The Critically Damped Response

if $\alpha = \omega_0$

$$x(t) = e^{-\alpha t}(K_1 + K_2 t)$$

and the circuit has the critically damped response.

The critically damped response does not involve oscillations.

For the series RLC circuits,$\alpha = \omega_0$ corresponds to $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ or $R = R_{critical} = 2\sqrt{\frac{L}{C}}$.

If $L = 1mHandC = 10nF$, then $R_{critical} \approx 632\Omega$.

1.2.5 The Overdamped Response

If $\alpha > \omega_0$

$$x(t) = K_1 \cdot e^{s_1 t} + K_2 \cdot e^{s_2 t}$$

where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ and $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

In the series RLC circuits, the overdamped solution is obtained if the resistance is larger than the critical resistance, such that $R > R_{critical} = 2 \cdot \sqrt{\frac{L}{C}}$

Notice that the larger resistance corresponds to the longer delay, and even the faster decay has a much longer fall time than the critically damped response.

One of the most interesting features of series RLC circuits is that increasing the resistance above the critical value results in much longer fall time, or longer delays of responses in digital circuits. Among all monotonic responses, the critically damped is the fastest.

2 Procedure

2.1 First-order Circuit

1. (a) I turned on the function generator, set a square wave at 1 V_{ppk} and 100 Hz, and applied it to the circuit as the input signal.
   (b) I monitored the input signal in Channel 1 and the output in Channel 2 of the oscilloscope .
2. For the For the fastest circuit response, set R_P=0, and set the oscilloscope:
   (a) Vertical scale for the input signal 200mV/div
   (b) Vertical scale for the output signal 200mV/div
   (c) Horizontal scale 5ms/div
3. For the slowest circuit response, set $=$ 10 k, and set the oscilloscope:
   (a) Vertical scale for the input signal 200mV/div
   (b) Vertical scale for the output signal 50mV/div
   (c) Horizontal scale 5ms/div

2.2 Second-order Circuit

1. I set a square wave at 1V_{ppk} and 10 kHz as the input signal on the function generator.
2. I varied R_P to generate three kinds of plot on the oscilloscope.
3. I observed and saved the graph from the oscilloscope.
4. I record the fall time and rise time, the time interval between the neighbouring peaks, $\triangle t$, and the resistance of the potentiometer, R_P.

3 Results

3.1 First-order Circuit

R_1	C
$1k\Omega$	$0.1\mu F$

Table 1: Resistance and capacitance

The setting of the potentiometer	Fastest circuit response	Slowest circuit response
V_{ppk} of the input square wave[V]	1.09	1.11
V_{ppk} of the output square wave[V]	1.09	1.05
Period of the input square wave, T[ms]	10.000	10.000
Rise time of the output waveform,[ms]	0.219	2.344
Fall time of the output waveform,[[ms]	0.21	2.218

Table 2: Table for first-order circuit

3.2 Second-order Circuit

R_2	100Ω
R_P	$10k\Omega$
L	$1mH$
C	$820pF$

Table 3: Resistance, inductance, and capacitance

	Resistance, R_P	Rise time $[ms]$	Fall time $[ms]$	time interval $\triangle t$
Under-damped	639	1.236×10^{-3}	1.208]$times10^{-3}$	99.99×10^{-3}
	1000	1.416×10^{-3}	1.416×10^{-3}	100.0×10^{-3}
critically damped	2109	2.704×10^{-3}	2.772×10^{-3}	
Over-damped	3113	4.668×10^{-3}	4.628×10^{-3}	
	4113	6.208×10^{-3}	6.296×10^{-3}	

Table 4: Table for second-order circuit

3.3 Graphs from waveform

Figure 1: Under-damped waveform when $R_P = 639\Omega$

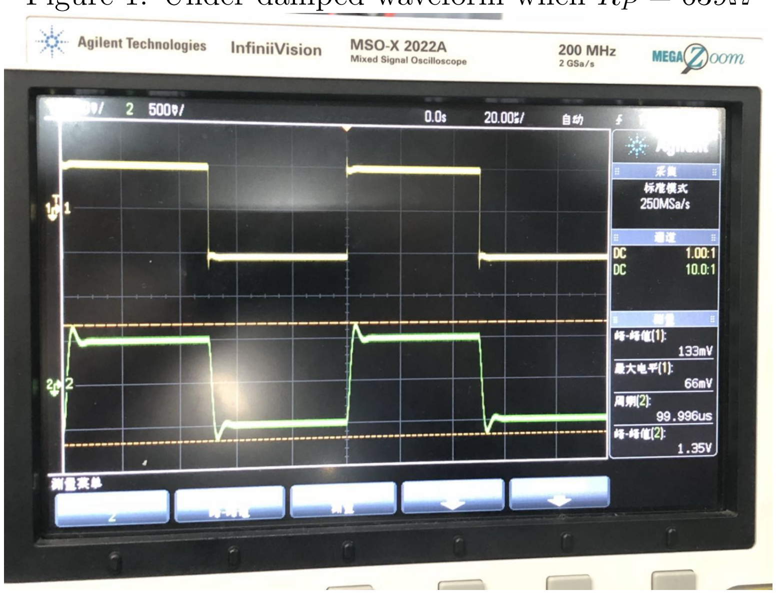

Figure 2: Under-damped waveform when $R_P = 100\Omega$

Figure 3: Critically damped waveform when $R_P = 2109\Omega$

Figure 4: Over-damped waveform when $R_P = 3113\Omega$

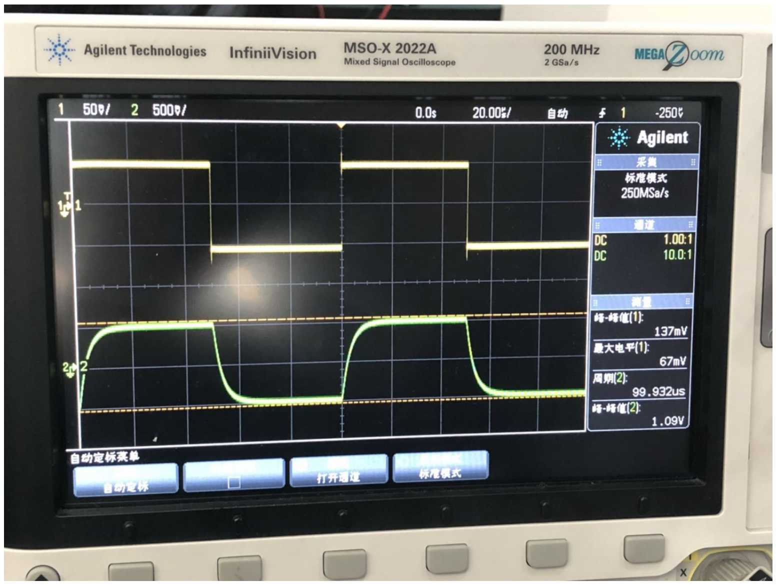

Figure 5: Over-damped waveform when $R_P = 4113\Omega$

4 Conclusion

In this lab work, I built series RC and RLC circuits, and studied the step responses on these corresponding first-order and second-order circuits.

For the first-order circuit, I measured the rise time and fall time when the circuit has fastest response and slowest response respectively. With the increase of R_P the rise time and fall time is also increasing.

When $R_P = 0[\Omega]$, which means the circuit has the fastest response, the V_{ppk} of input square wave and output square wave is very close, which is $1.09[V]$. The period of the input square wave $T = 10[ms]$, which is equal to the theoretical value $\frac{1}{100 \cdot 1000} = 0.01[s]$. According to the pre-lab assignment, I calculated $\tau = 1 \cdot 10^{-4}[s]$, so the theoretical value of rise time and fall time is $2.2\tau = 2.2 \cdot 10^{-4}[s]$, which is very similar to my measured value: $0.219[ms]$ and $0.210[ms]$.

When $R_P = 10[k\Omega]$, which means the circuit has the fastest response, the V_{ppk} of input square wave and output square wave is not as close as fastest response, which is $1.05[V]$ and $1.11[V]$. The period of the input square wave $T = 10[ms]$, which is equal to the theoretical value $\frac{1}{100 \cdot 1000} = 0.01[s]$. According to the pre-lab assignment, I calculated $\tau = 1.1 \cdot 10^{-3}[s]$, so the theoretical value of rise time and fall time is $2.2\tau = 2.42 \cdot 10^{-3}[s]$, which is very similar to my measured value: $2.344[ms]$ and $2.218[ms]$.

For the second-order circuit, I also measure the rise time and fall time during under-damped, critically damped, and over-damped conditions. Similar to first-order circuit, the rise time and fall time also becomes bigger as R_P increasing.

In pre-lab assignment, I'm asked to use $PSpice$ to build the first-order and second-order circuit to stimulate the waveform, and the results are similar to what I get from the screen.

5 Reference

- *VE215FA2017 Transient LabManual*

- *Circuits Make Sense*, Alexander Ganago, Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor.