Homework 4

Skills and Concepts:

- FT pairs, FT properties, PFE, Parsevals relation (Energy density spectrum), convolution property and LTI systems, application of FT to diffeq systems,

HW Notes:

- Problems where the number of points are followed by an exclamation point are basic skill problems and will be graded without partial credit.
- $\boxed{\text{Box}}$ your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit.

  For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

Problems:

1. [10!] **Please finish the course evaluation on Canvas.**
2. [10!] Use the table of FT pairs and FT properties, find the FT of each of the following signals (*DO NOT USE INTEGRATION*):
   (a) [2!] $x(t) = 2\text{rect}\left(\frac{t-2}{4}\right)$
   (b) [2!] $x(t) = e^{-3t}\text{rect}\left(\frac{t-2}{4}\right)$
   (c) [3!] $x(t) = t\text{rect}\left(\frac{t-2}{4}\right)$
   (d) [3!] $x(t) = \cos(4\pi t)\text{rect}\left(\frac{t-2}{4}\right)$
3. [10!] Compute the FT of each of the following signals:
   (a) [3!] $[e^{-\alpha t}\cos\omega_0 t]u(t), \alpha > 0$
   (b) [3!] $e^{-3|t|}\sin 2t$
   (c) [4!] $t^2 e^{-(t/2)^2}$
4. [7!] Find a mathematical expression of the inverse FT of $F(\omega) = \text{sinc}^3(\omega/2)$.
   Hint: the inverse FT formula would probably be a hard way to do it.
5. [8!] Consider a real signal $f(t)$ and let

$$f(t) \stackrel{\mathcal{F}}{\longleftrightarrow} F(\omega), \quad F(\omega) = \text{real}\{F(\omega)\} + j\ \text{imag}\{F(\omega)\}$$

   and

$$f(t) = f_e(t) + f_o(t)$$

   where $f_e(t)$ and $f_o(t)$ are the even and odd component of $f(t)$ respectively. Show that

$$f_e(t) \stackrel{\mathcal{F}}{\longleftrightarrow} \text{real}\{F(\omega)\} \qquad\qquad f_o(t) \stackrel{\mathcal{F}}{\longleftrightarrow} j\ \text{imag}\{F(\omega)\}$$

6. [10!] Find the FT of the following signal: $x(t) = \sum_{n=-\infty}^{\infty} 2\delta(t-6n) - \delta(t-6n-2) - \delta(t-6n+2)$. sketch the magnitude of the spectrum.

7. [10!] Determine the continuous-time signal corresponding to the following transform.
   (a) [4!] $X(j\omega) = cos(4\omega + \pi/3)$
   (b) [6!] $X(j\omega)$ as given by magnitude and phase plots in Figure 1.

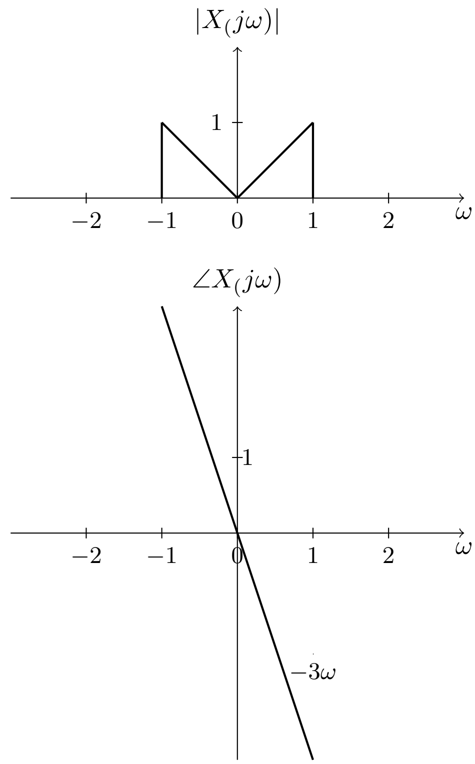
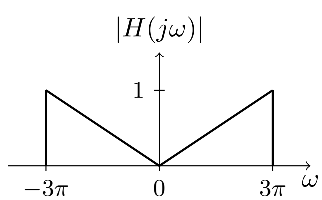


Figure 1. magnitude and phase plots

8. [15!] Shown in the Figure 2(a) is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals x(t) below, determine the filter output signal $y(t)$.
   (a) $x(t) = \cos(2\pi t + \theta)$
   (b) $x(t) = \cos(4\pi t + \theta)$
   (c) $x(t)$ is a half-wave rectified sine wave of period 1, as sketched in Figure 2(b). The math expression of $x(t)$ is

$$x(t) = \begin{cases} \sin(2\pi t) & , m \le t \le m + \frac{1}{2} \\ 0 & , (m + \frac{1}{2}) \le t \le m \end{cases}.$$



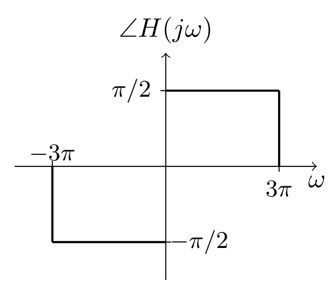


Figure 2(a): Frequency response $H(j\omega)$.

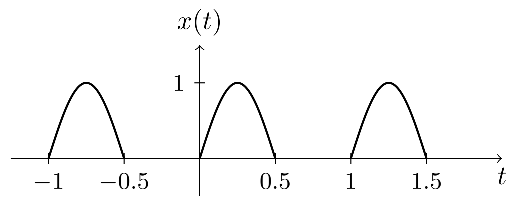


Figure 2(b): Sketch of $x(t)$ in problem 7(c).

9. [10!] Find the energy of the signal $x(t) = t\text{sinc}^2(t)$ by Fourier methods.

10. [10!] A LTI system has the following frequency response:

$$H(j\omega) = \frac{-\omega^2 + j\omega + 1}{(-\omega^2 + 6j\omega + 25)(j\omega + 2)}.$$

(a) [5!] Find the impulse response of the LTI system.

(b) [5!] Find the differential equation corresponding to the LTI system.
Hint: write $H(\omega) = Y(\omega)/X(\omega)$ and cross multiply.