Homework 1

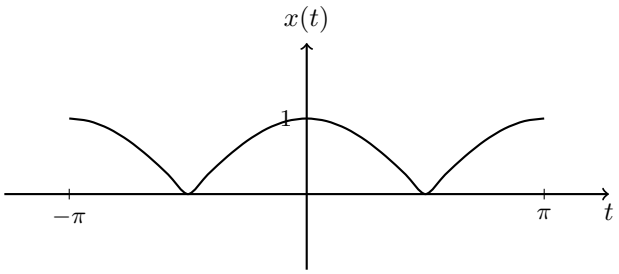
HW Notes:

- Problems where the number of points are followed by an exclamation are basic skill problems and will be graded without partial credit.
- Box your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit.

  For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

Problems:

1. Consider the cosinusoidal signal illustrated below.



   (a) [5!] Find the mathematical representation for this signal.

   (b) [5!] Carefully sketch and find a mathematical expression for the output signal of an integrator system, i.e., $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, where $x(t)$ is under the interval $[-\pi, \pi]$.

2. [15!] Calculate the average value, power and energy of signal $x(t) = \begin{cases} e^{-t} & t > 0, \\ 0 & \text{otherwise.} \end{cases}$ Is it an energy signal, power signal, or neither?

3. Suppose $x_1(t)$ and $x_2(t)$ are two periodic signals with fundamental periods $T_1 > 0$ and $T_2 > 0$ respectively.

   (a) [5!] Show that if T_1/T_2 is rational, then $x(t) = x_1(t) + x_2(t)$ is periodic.

   (b) [5!] Show that if T_1/T_2 is rational, then $x(t) = x_1(t)x_2(t)$ is periodic and the least common multiple of T_1 and T_2 is a period of $x(t)$.

   (c) [10!] Determine whether the following signals are periodic. If so, find a period. If not, specify the reason.

$$x_1(t) = \sin(\pi t/3)\cos(\pi t/4) + \sin(\pi t/5)\sin(\pi t/2)$$

$$x_2(t) = \sin\left(\sqrt{3}\pi t/3\right) + \sin(\pi t/5)$$

4. [15!] Indicate whether the following systems are Memoryless, Time Invariant, Linear, Causal, Stable. Justify your answers.

   (a) $y(t) = x(t-2) + x(2-t)$

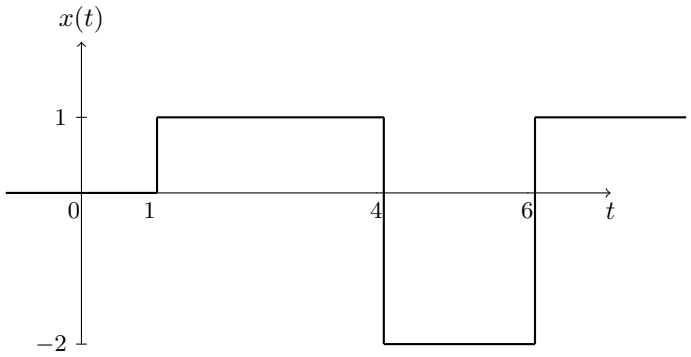
(b) $y(t) = x(t/3)$

(c) $y(t) = \cos(x(t))$

(d) $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$

(e) $y(t) = \frac{d}{dt}x(t)$

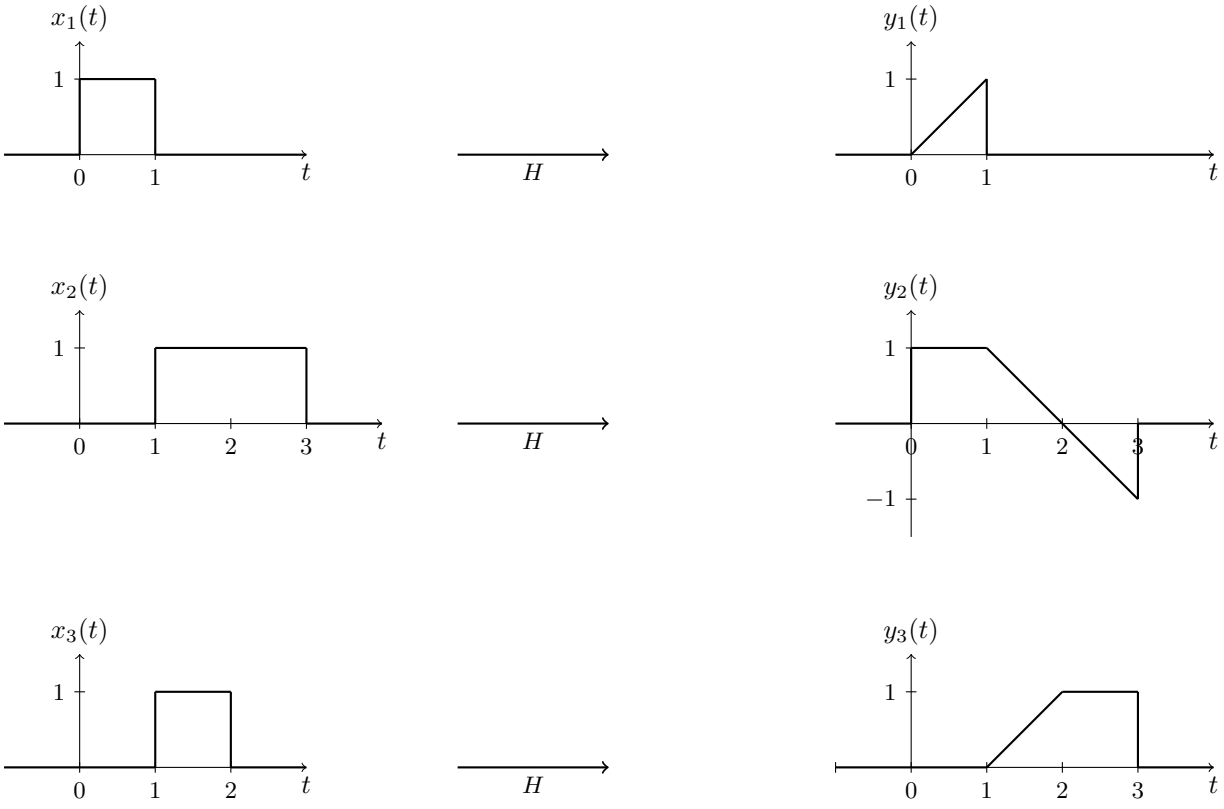
5. Consider the signal illustrated below.

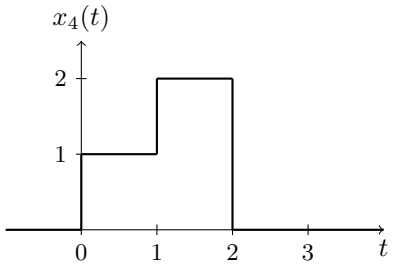


(a) [5!] Express the signal $x(t)$ using a sum of step functions.

(b) [5!] Find the derivative of the signal and carefully sketch it.

6. A linear system H has following input-output pairs. Answer the following question, and justify your answers.





(a) [5!] Could this system be causal?

(b) [5!] Could this system be time invariant?

(c) [5!] Could this system be memoryless?

(d) [5!] What is the output for the input $x_4(t)$, sketch it.

7. Consider the signal $s(t) = \left(\frac{t-1}{2}\right)^2 rect(\frac{t-1}{2})$

(a) [3!] Make s sketch of $s(t)$.

(b) [7!] Evaluate $\int_{-\infty}^{\infty} s(t)x(t)dt$, where $x(t) = \delta(t - \frac{1}{2}) + \delta(t-2) - \delta(3t-4)$