▶ REFERENCES

Complex Variables, Laplace Transforms

1. F. B. Hildebrand, *Methods of Applied Mathematics*, 2nd Ed., Prentice Hall, Englewood Cliffs, NJ, 1965.
2. B. C. Kuo, *Linear Networks and Systems*, McGraw-Hill Book Company, New York, 1967.
3. C. R. Wylie, Jr., *Advanced Engineering Mathematics*, 2nd Ed., McGraw-Hill Book Company, New York, 1960.

Partial-Fraction Expansion

4. C. Pottle, "On the Partial Fraction Expansion of a Rational Function with Multiple Poles by Digital Computer," *IEEE Trans. Circuit Theory*, Vol. CT-11, 161–162, Mar. 1964.
5. B. O. Watkins, "A Partial Fraction Algorithm," *IEEE Trans. Automatic Control*, Vo. AC-16, 489–491, Oct. 1971.

▶ PROBLEMS

MATLAB

2-1. Find the poles and zeros of the following functions (including the ones at infinity, if any). Mark the finite poles with × and the finite zeros with o in the s-plane.

(a) $G(s) = \dfrac{10(s+2)}{s^2(s+1)(s+10)}$ **(b)** $G(s) = \dfrac{10s(s+1)}{(s+2)(s^2+3s+2)}$

(c) $G(s) = \dfrac{10(s+2)}{s(s^2+2s+2)}$ **(d)** $G(s) = \dfrac{e^{-2s}}{10s(s+1)(s+2)}$

• Laplace transforms

2-2. Find the Laplace transforms of the following functions. Use the theorems on Laplace transforms if applicable.

(a) $g(t) = 5te^{-5t}u_s(t)$ **(b)** $g(t) = (t\sin 2t + e^{-2t})u_s(t)$

(c) $g(t) = 2e^{-2t}\sin 2t\, u_s(t)$ **(d)** $g(t) = \sin 2t \cos 2t\, u_s(t)$

(e) $g(t) = \sum_{k=0}^{\infty} e^{-5kT}\delta(t-kT)$ where $\delta(t)$ = unit-impulse function

• Laplace transforms

2-3. Find the Laplace transforms of the functions shown in Fig. 2P-3. First, write a complete expression for $g(t)$, and then take the Laplace transform. Let $g_T(t)$ be the description of the function over the basic period and then delay $g_T(t)$ appropriately to get $g(t)$. Take the Laplace transform of $g(t)$ to get

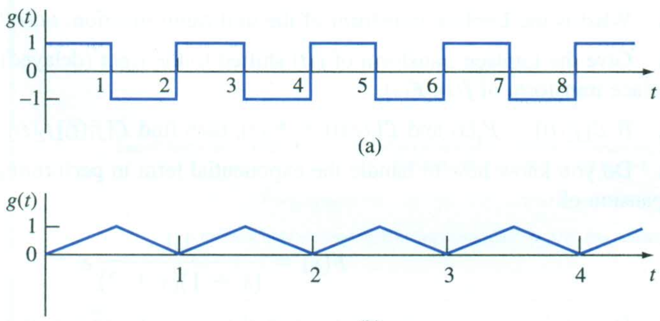

Figure 2P-3

• Laplace transforms

2-4. Find the Laplace transform of the following function.

$$g(t) = \begin{cases} t+1 & 0 \le t < 1 \\ 0 & 1 \le t < 2 \\ 2-t & 2 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$

• Laplace transforms

2-5. Solve the following differential equations by using the Laplace transform.

(a) $\dfrac{d^2 f(t)}{dt^2} + 5\dfrac{df(t)}{dt} + 4f(t) = e^{-2t}u_s(t)$ Assume zero initial conditions.

(b) $\dfrac{dx_1(t)}{dt} = x_2(t)$

$\dfrac{dx_2(t)}{dt} = -2x_1(t) - 3x_2(t) + u_s(t) \qquad x_1(0) = 1, x_2(0) = 0$

MATLAB

2-6. Find the inverse Laplace transforms of the following functions. Perform partial-fraction expansion on $G(s)$ first, then use the Laplace transform table. Use TFtool or any computer program that is available for the partial-fraction expansion.

(a) $G(s) = \dfrac{1}{s(s+2)(s+3)}$ (b) $G(s) = \dfrac{10}{(s+1)^2(s+3)}$

(c) $G(s) = \dfrac{100(s+2)}{s(s^2+4)(s+1)}e^{-s}$ (d) $G(s) = \dfrac{2(s+1)}{s(s^2+s+2)}$

(e) $G(s) = \dfrac{1}{(s+1)^3}$ (f) $G(s) = \dfrac{2(s^2+s+1)}{s(s+1.5)(s^2+5s+5)}$

2-7. Express the following set of first-order differential equations in vector-matrix form:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\frac{dx_1(t)}{dt} = -x_1(t) + 2x_2(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + 3x_3(t) = u_1(t)$$

$$\frac{dx_3(t)}{dt} = -x_1(t) - 3x_2(t) - x_3(t) + u_2(t)$$

• Transfer function

2-8. The following differential equations represent linear time-invariant systems, where $r(t)$ denotes the input, and $y(t)$ the output. Find the transfer function $Y(s)/R(s)$ for each of the systems.

(a) $\dfrac{d^3y(t)}{dt^3} + 2\dfrac{d^2y(t)}{dt^2} + 5\dfrac{dy(t)}{dt} + 6y(t) = 3\dfrac{dr(t)}{dt} + r(t)$

(b) $\dfrac{d^4y(t)}{dt^4} + 10\dfrac{d^2y(t)}{dt^2} + \dfrac{dy(t)}{dt} + 5y(t) = 5r(t)$

(c) $\dfrac{d^3y(t)}{dt^3} + 10\dfrac{d^2y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} + y(t) + 2\displaystyle\int_0^t y(\tau)d\tau = \dfrac{dr(t)}{dt} + 2r(t)$

(d) $2\dfrac{d^2y(t)}{dt^2} + \dfrac{dy(t)}{dt} + 5y(t) = r(t) + 2r(t-1)$

MATLAB

Additional Computer Problems

The following problems are to be solved by the **TFtool**.

2-9. Perform partial-fraction expansion of the following functions.

(a) $G(s) = \dfrac{10(s+1)}{s^2(s+4)(s+6)}$ (b) $G(s) = \dfrac{(s+1)}{s(s+2)(s^2+2s+2)}$

(c) $G(s) = \dfrac{5(s+2)}{s^2(s+1)(s+5)}$ (d) $G(s) = \dfrac{5e^{-2s}}{(s+1)(s^2+s+1)}$

(e) $G(s) = \dfrac{100(s^2+s+3)}{s(s^2+5s+3)}$ (f) $G(s) = \dfrac{1}{s(s^2+1)(s+0.5)^2}$

2-10. Find the inverse Laplace transforms of the functions in Problem 2-9.