JOINT INSTITUTE
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Diode Circuit

VE311 Electronic Circuits (Summer 2019)

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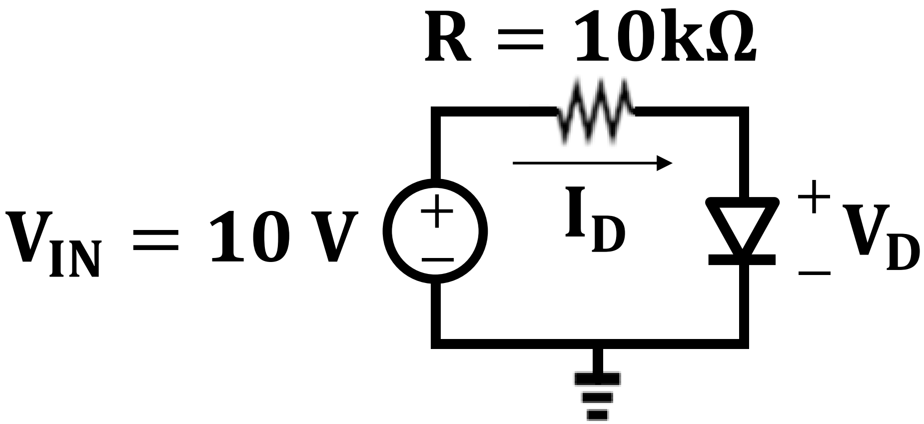
Last Updated on **May 27, 2019**

Solving Techniques



$$V_{IN} = I_D R + V_D$$

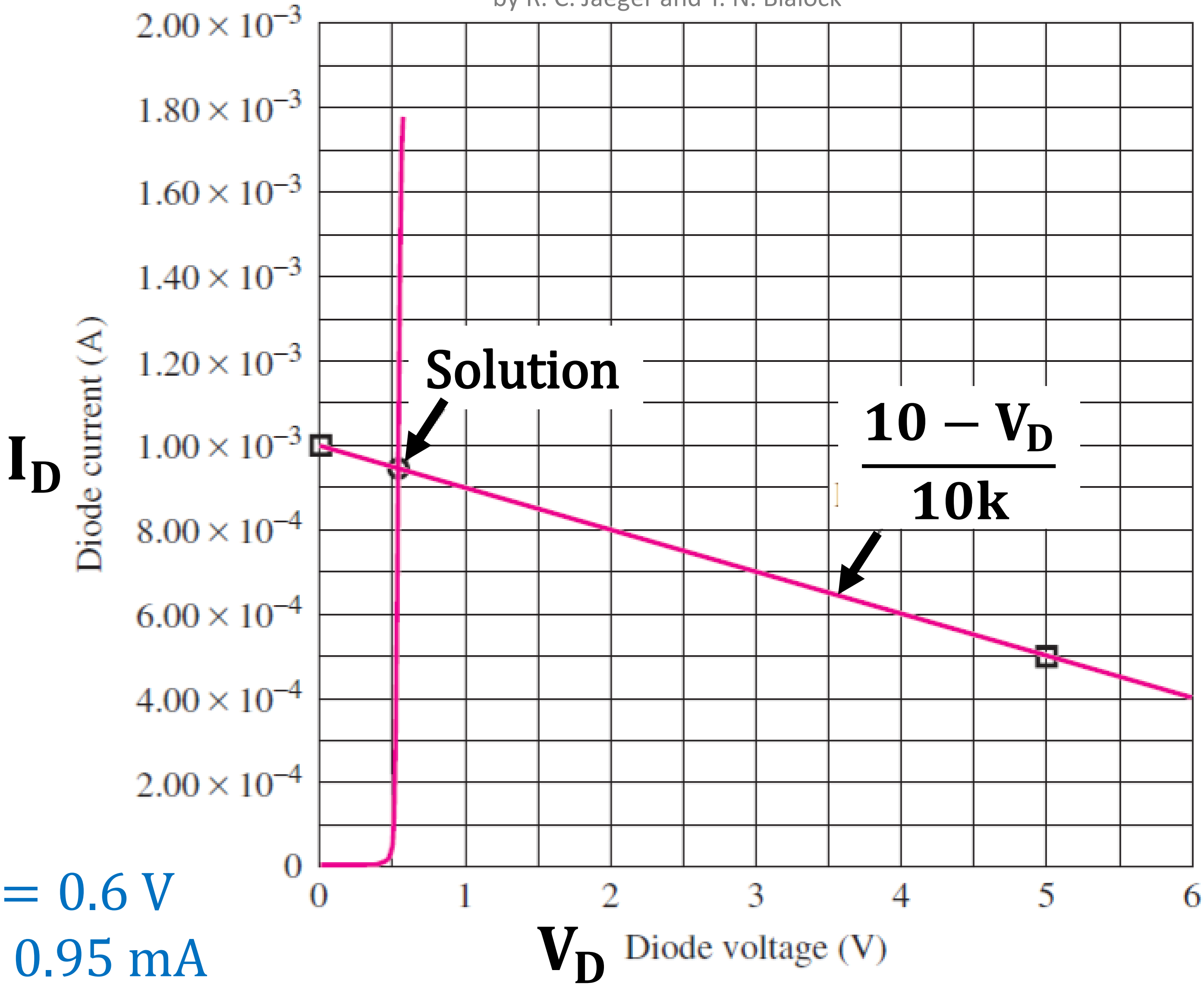
$$I_D = ?$$

$$V_D = ?$$

1. **Graphical analysis**
2. **Mathematical analysis**
3. **Simplified analysis**
   (ideal diode)
4. **Simplified analysis**
   (constant voltage drop)

Graphical Analysis

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock



$V_D = 0.6$ V

$I_D = 0.95$ mA

Mathematical Analysis

$$V_{IN} = I_D R + V_D$$

$$V_{IN} = \left[I_S\left(e^{\frac{qV_D}{kT}} - 1\right)\right] R + V_D$$

$$10 = \left[10^{-13}\left(e^{\frac{V_D}{0.0258}} - 1\right)\right] 10^4 + V_D$$

Above is a transcendental equation which does not have a close-form **analytical solution**. So, through trial and error, we seek a **numerical solution**.

function xd = diode(vd)

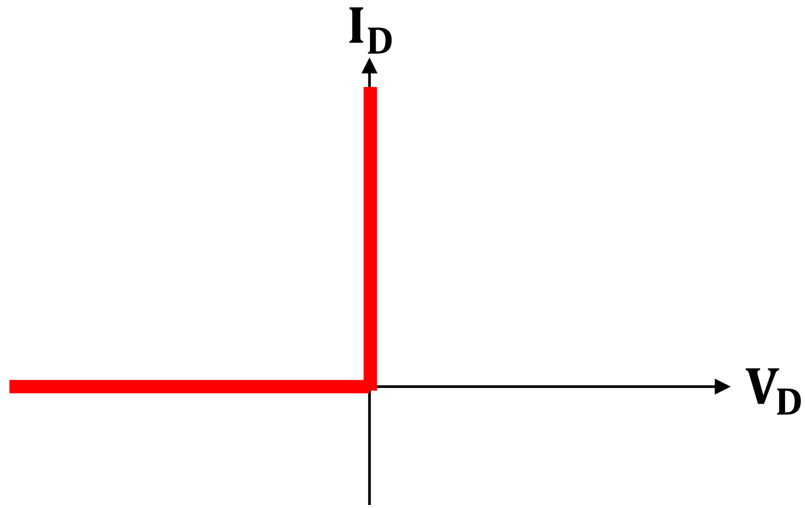
$$xd = 10 - (10^{-9}) * \left(\exp\left(\frac{vd}{0.0258}\right) - 1\right) - vd$$

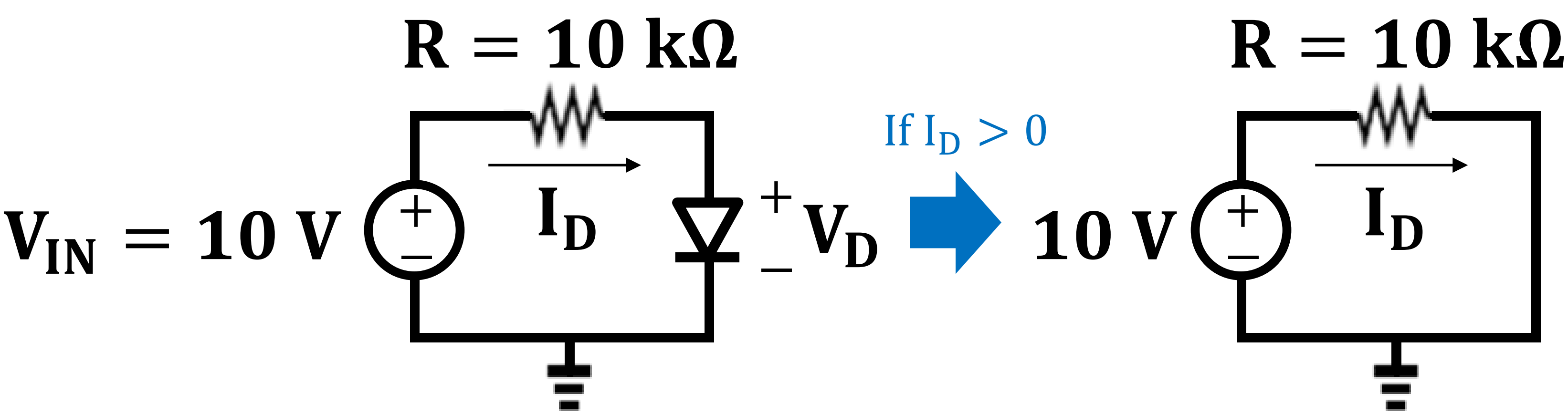
Use MATLAB to plot xd as a function of vd, and find out a vd that makes xd closest to zero.

$$V_D = 0.5742\ V$$

$$I_D = 0.944\ mA$$

Simplified Analysis (Ideal Diode)

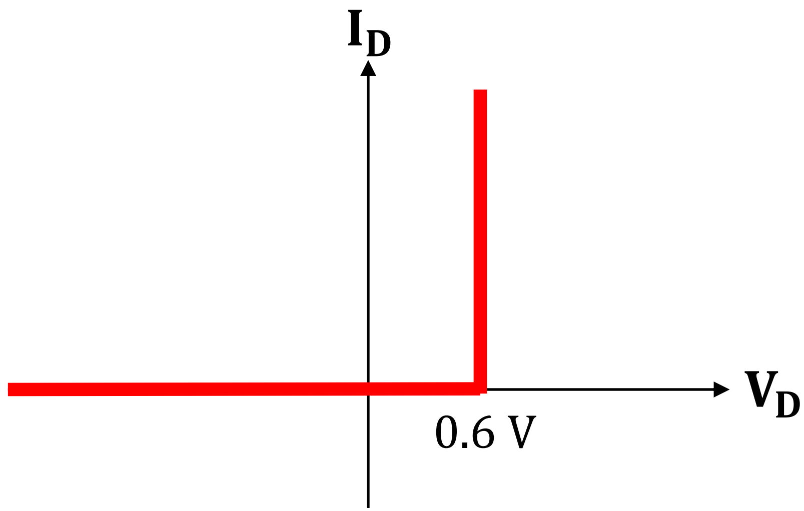


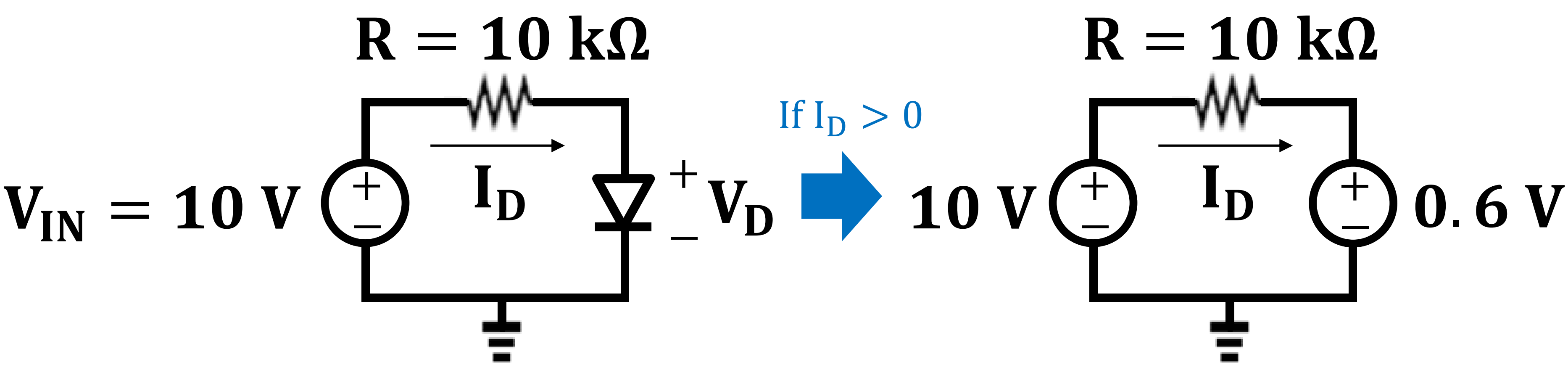


$$V_D = 0\ \mathrm{V}$$

$$I_D = \frac{(10 - 0)\ \mathrm{V}}{10\ \mathrm{k\Omega}} = 1\ \mathrm{mA}$$

Simplified Analysis (Constant Voltage Drop)





$$V_D = 0.6\ V$$

$$I_D = \frac{(10 - 0.6)\ V}{10\ k\Omega} = 0.94\ mA$$

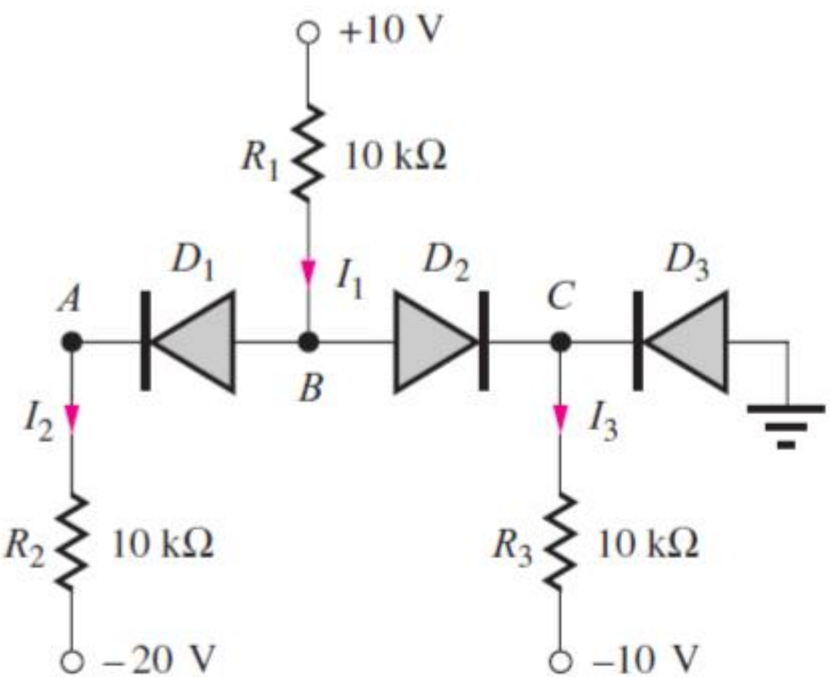
V_D a power supply?
What happens if $V_{IN} = 0.5\ V$?

Comparison

	V_D	I_D
Graphical Analysis	0.6 V	0.95 mA
Mathematical Analysis	0.5742 V	0.944 mA
Ideal Diode Model	0 V	1 mA
Constant Voltage Drop Model	0.6 V	0.94 mA

Example

Use constant voltage drop model ($V_{on} = 0.6$ V) to calculate V_D and I_D of each diode.



- Assume no current flowing through D_3

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} + \frac{V_B - 0.6 + 10}{10k}$$

$$V_B = -6.27 \text{ V}$$

$$V_C = -6.87 \text{ V} \quad \Rightarrow \quad D_3 \text{ in forward bias}$$

Assumption NOT valid

- Assume no current flowing through D_2

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k}$$

$$V_B = -4.7 \text{ V}$$

$$V_C = -0.6 \text{ V} \quad \Rightarrow \quad D_2 \text{ indeed in reverse bias}$$

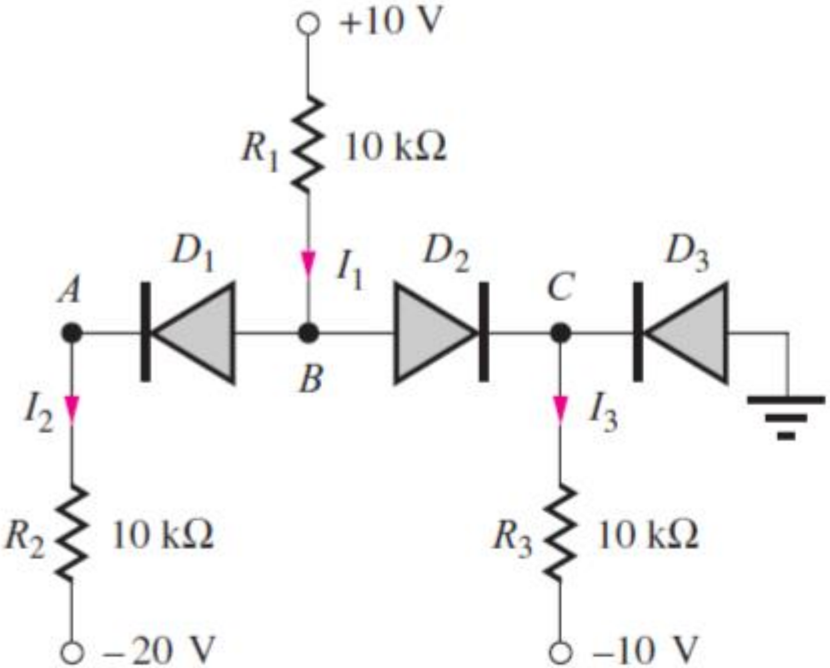
Assumption valid

$V_{D1} = 0.6$ V $V_{D2} = -4.1$ V $V_{D3} = 0.6$ V

$I_{D1} = 1.47$ mA $I_{D2} = 0$ mA $I_{D3} = 0.94$ mA

Example

Use ideal diode model ($V_{on} = 0$ V) to calculate V_D and I_D of each diode.



- Assume D_3 in reverse bias

$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k} + \frac{V_B + 10}{10k}$$

$V_B = -6.67$ V

$V_C = -6.67\text{ V} \Rightarrow D_3$ in forward bias

Assumption NOT valid

- Assume D_2 in reverse bias

$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k}$$

$V_B = -5$ V

$V_C = 0\text{ V} \Rightarrow D_2$ indeed in reverse bias

Assumption valid

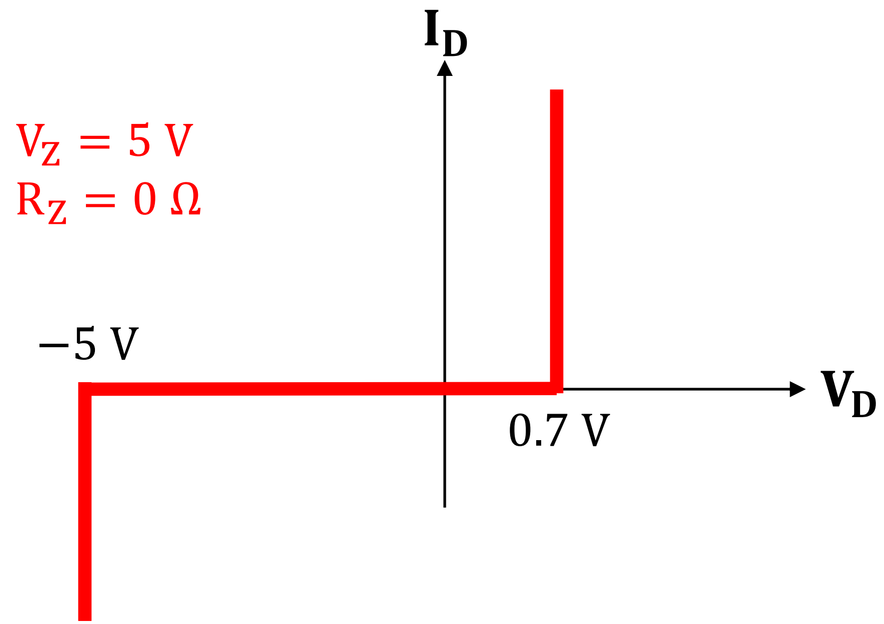
$V_{D1} = 0$ V
$I_{D1} = 1.5$ mA

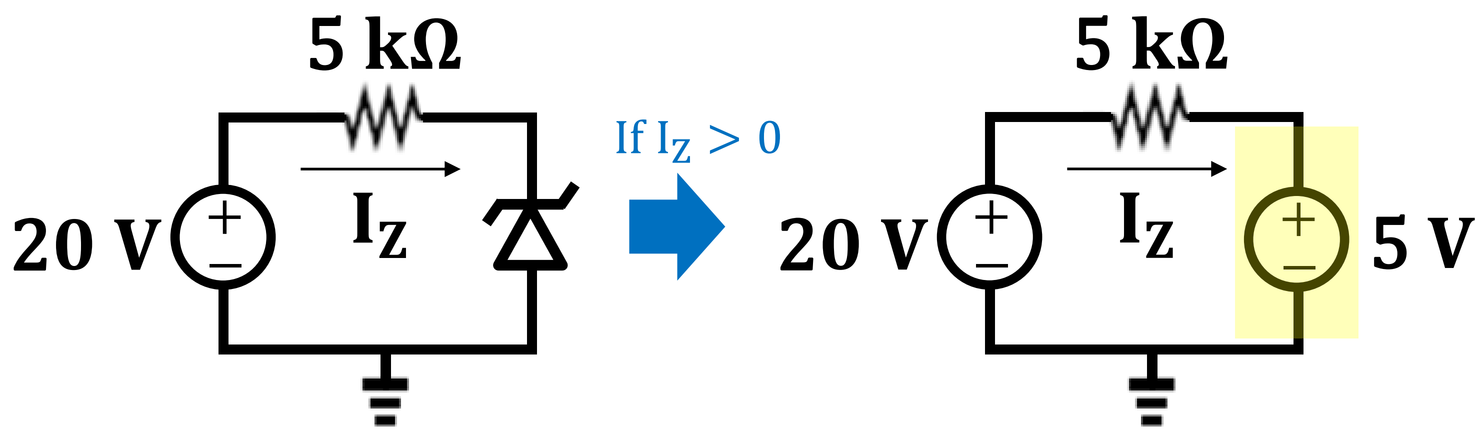
$V_{D2} = -5$ V
$I_{D2} = 0$ mA

$V_{D3} = 0$ V
$I_{D3} = 1$ mA

Zener Diode Circuit

Simplified Analysis for Zener Diode ($R_Z = 0$)

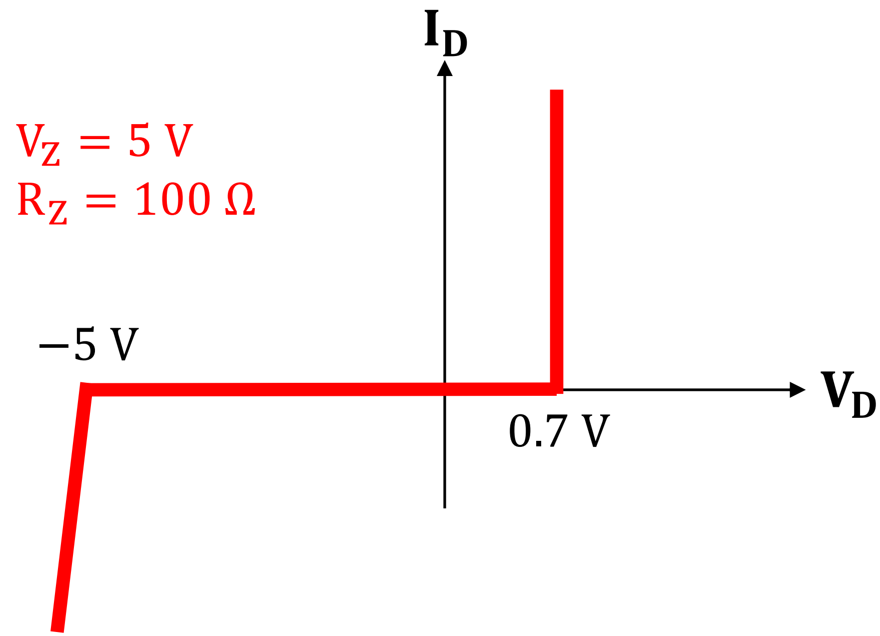


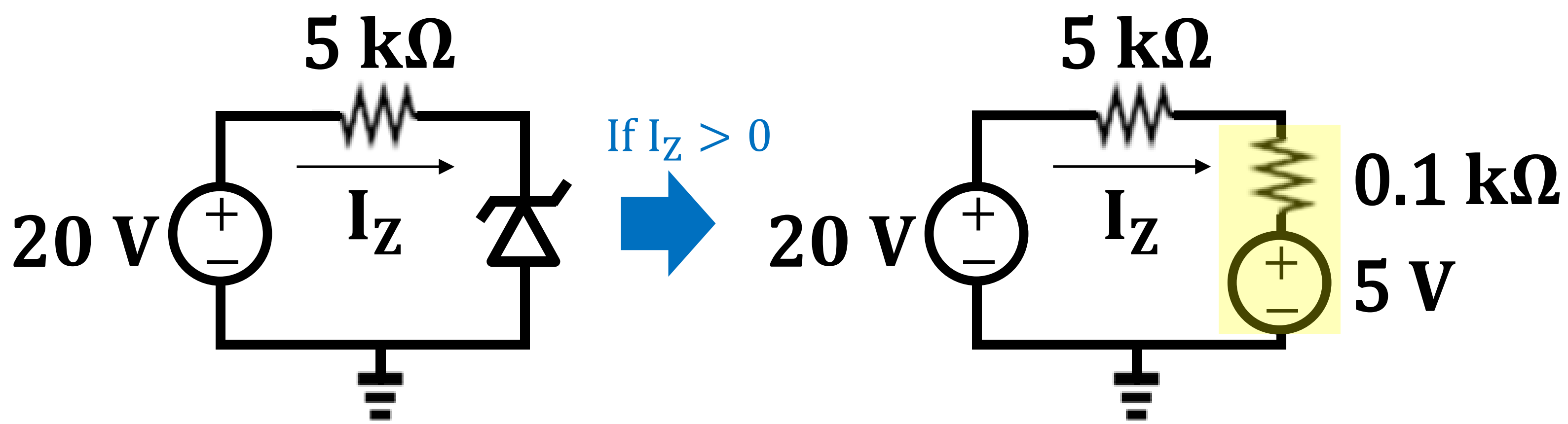


$$I_Z = \frac{20 - 5}{5000} = 3 \times 10^{-3}\ (\mathrm{A})$$

What happens if $V_{IN} = 1$ V?

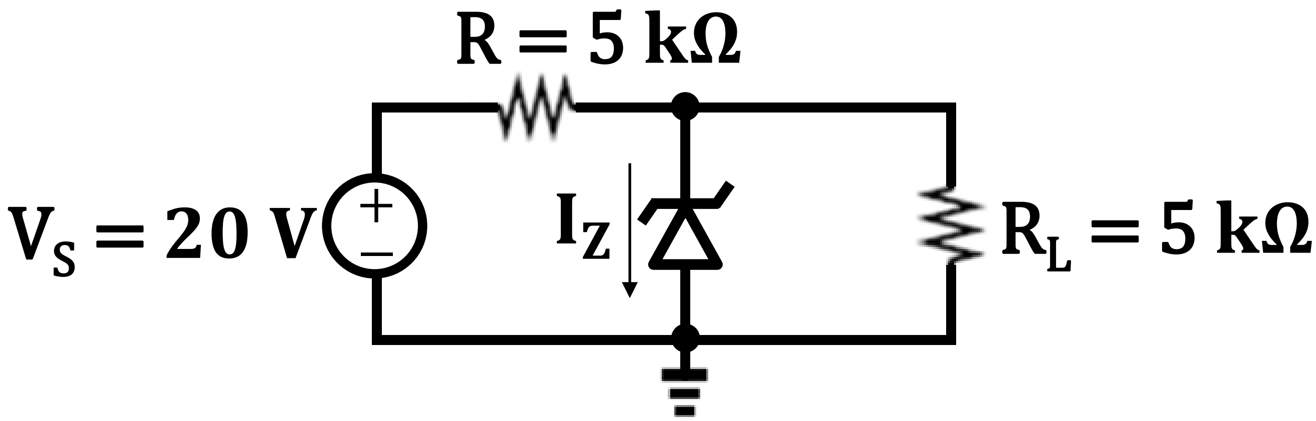
Simplified Analysis for Zener Diode ($R_Z \neq 0$)



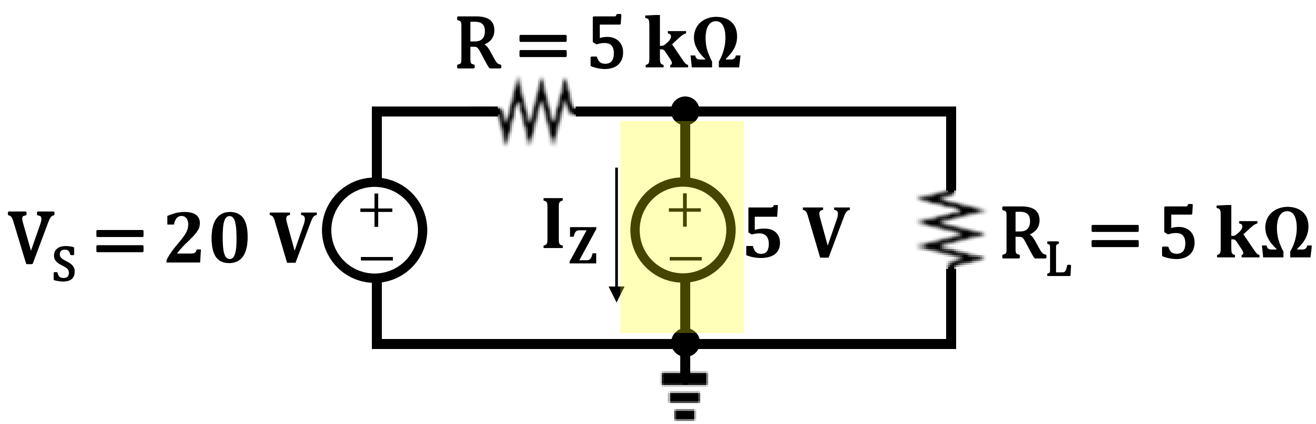


$$I_Z = \frac{20 - 5}{5100} = 2.94 \times 10^{-3}\ (\mathrm{A})$$

Voltage Regulator Using Zener Diode ($R_Z = 0$)



If $I_Z > 0$

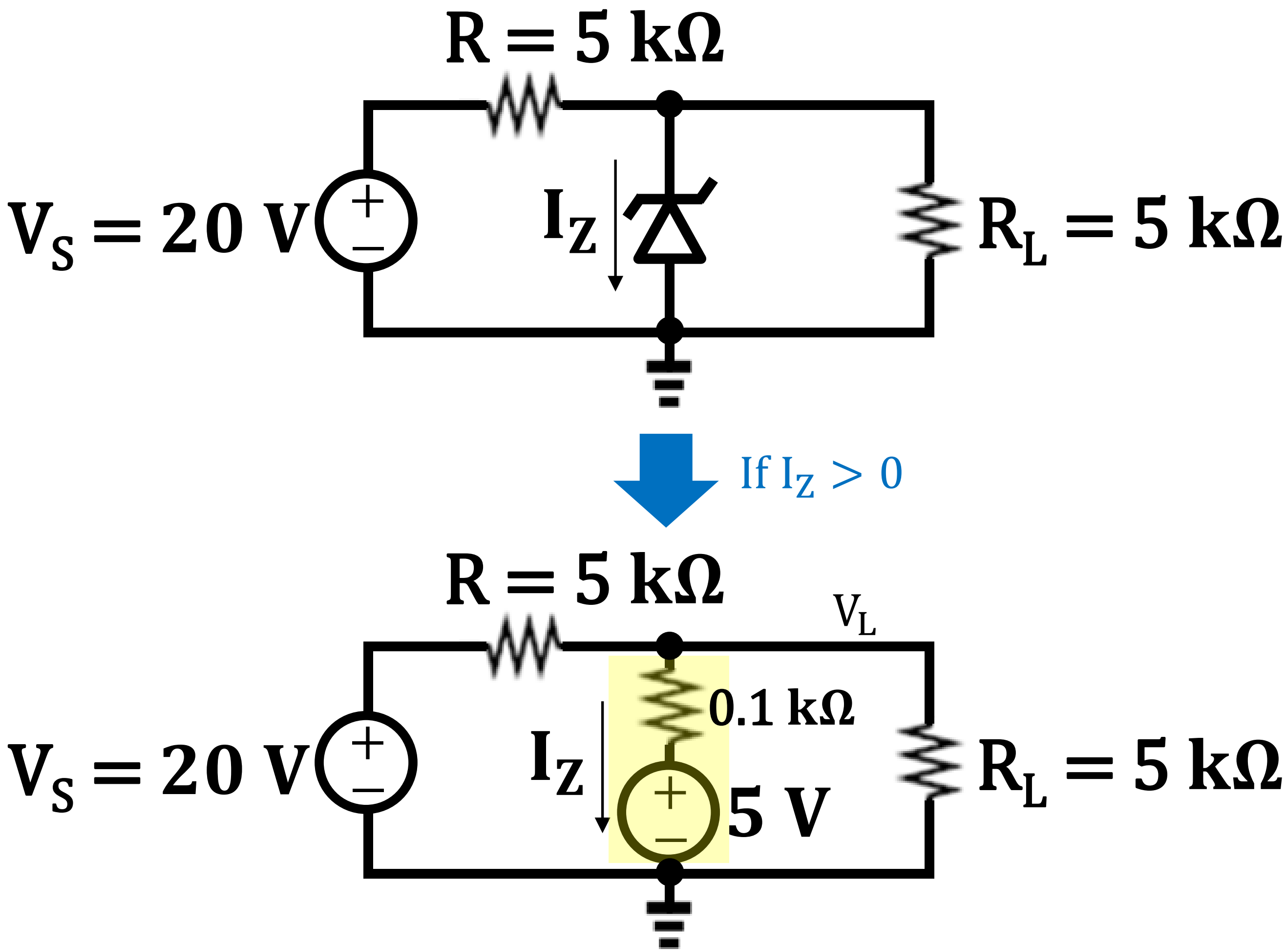


$$I_Z = \frac{20-5}{5k} - \frac{5}{5k} = 2\text{ mA} > 0$$

As long as the zener diode operates in reverse breakdown region ($I_Z > 0$), a constant voltage (5 V) appears across R_L.

What is the smallest R_L for reverse breakdown to happen? Answer: $R_{L,min} = 1.67\,k\Omega$

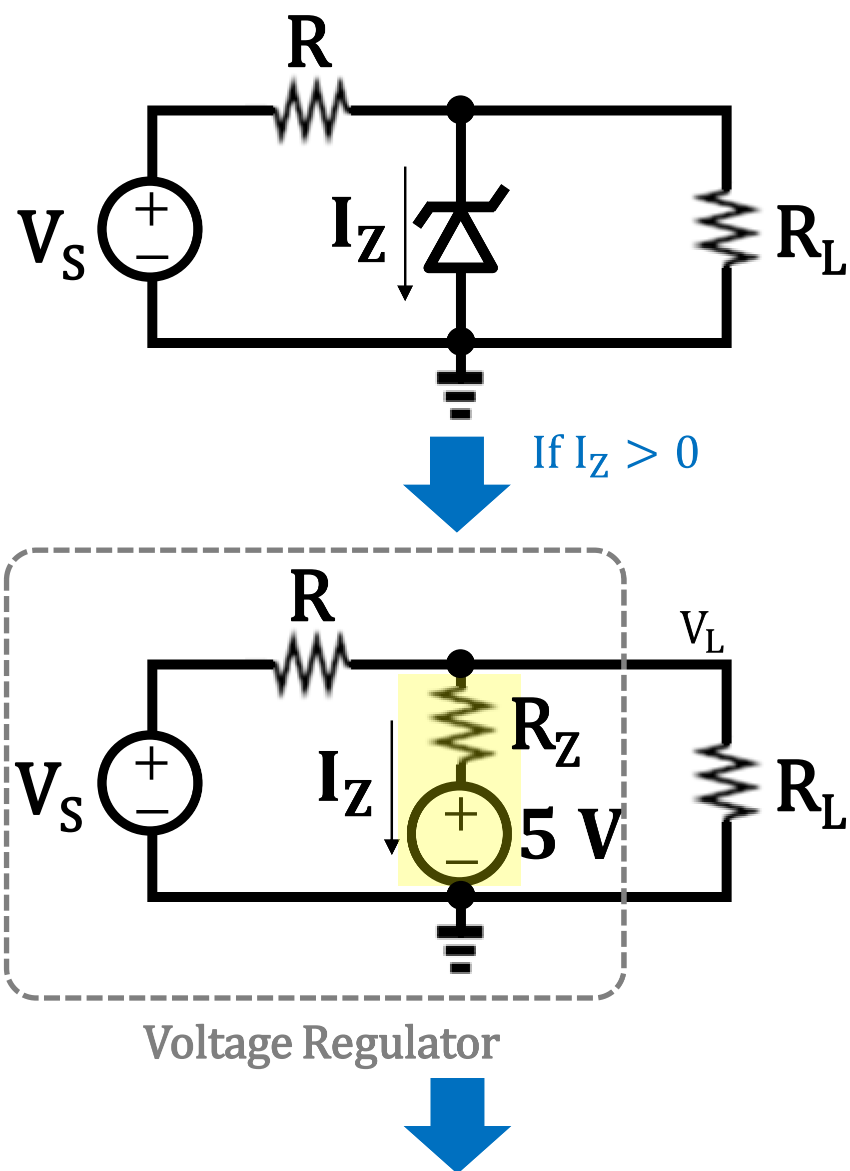
Voltage Regulator Using Zener Diode ($R_Z \neq 0$)



$$\frac{20 - V_L}{5k} = \frac{V_L - 5}{0.1k} + \frac{V_L}{5k} \qquad V_L = 5.1923\ \mathrm{V}$$

$$I_Z = \frac{5.19 - 5}{0.1k} = 1.9\ \mathrm{mA} \ > 0$$

Line Regulation and Load Regulation



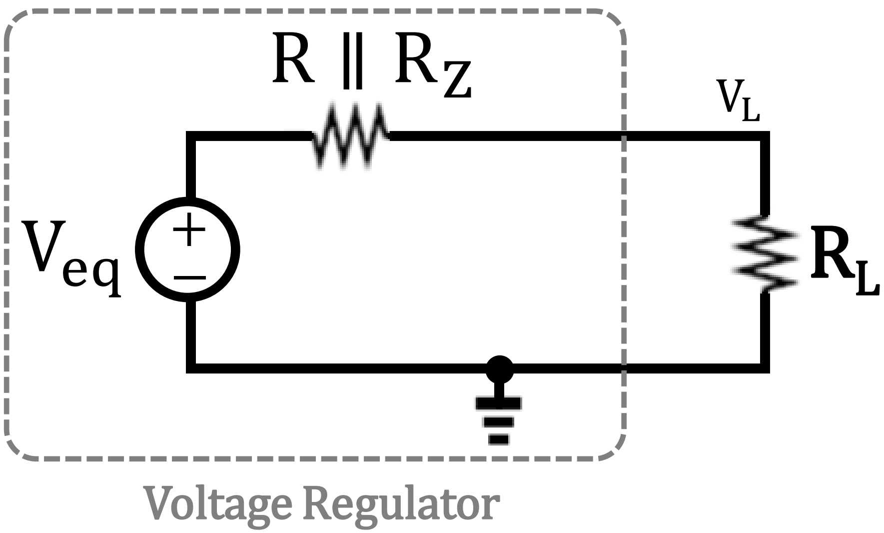
- **Line Regulation:** how sensitive the output voltage (V_L) is to input voltage (V_S) changes, when $R_L = \infty$.

$$\text{Line Regulation} = \frac{dV_L}{dV_S} = \frac{R_Z}{R + R_Z}$$

- **Load Regulation:** output impedance of the voltage regulator.

$$\text{Load Regulation} = \frac{dV_L}{dI_L} = R \parallel R_Z$$

Thevinen Equivalent Circuit of Voltage Regulator



$$V_{eq} = 5 + \frac{V_S - 5}{R + R_Z} R_Z$$

$$V_L = V_{eq} \frac{R_L}{(R \parallel R_Z) + R_L}$$

Numerical Test: $V_S = 20\ \mathrm{V},\ R = 5\ \mathrm{k\Omega},\ R_Z = 0.1\ \mathrm{k\Omega},\ V_Z = 5\mathrm{V},\ R_L = 5\ \mathrm{k\Omega}$

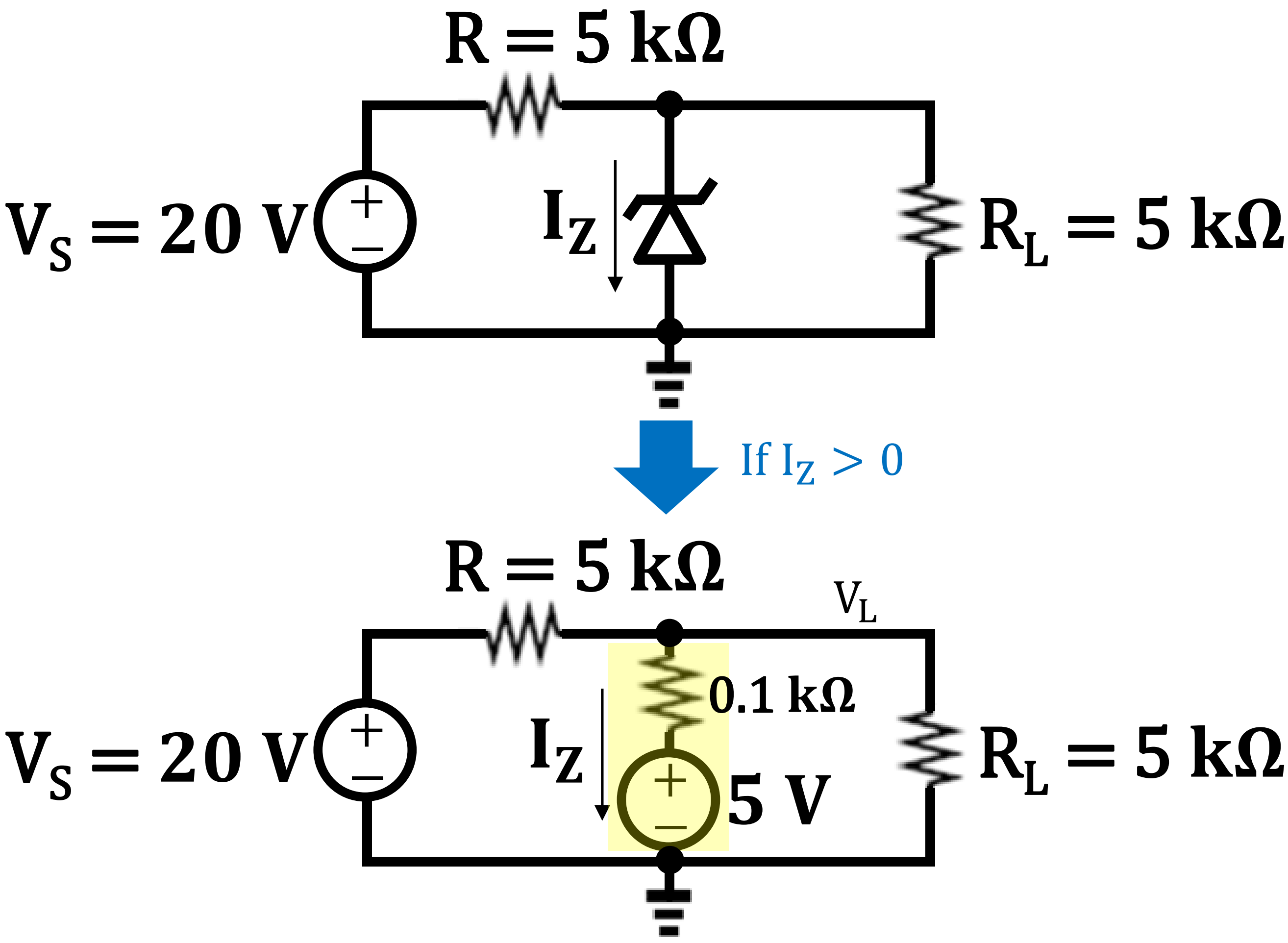
$$V_{eq} = 5 + \frac{20 - 5}{5\mathrm{k} + 0.1\mathrm{k}} 0.1\mathrm{k} = 5.2941\ (\mathrm{V})$$

$$V_L = 5.2941 \times \frac{5k}{\frac{5k \times 0.1k}{5k + 0.1k} + 5k} = 5.1923\ (V)$$

The result exactly the same as on page 14.

Example

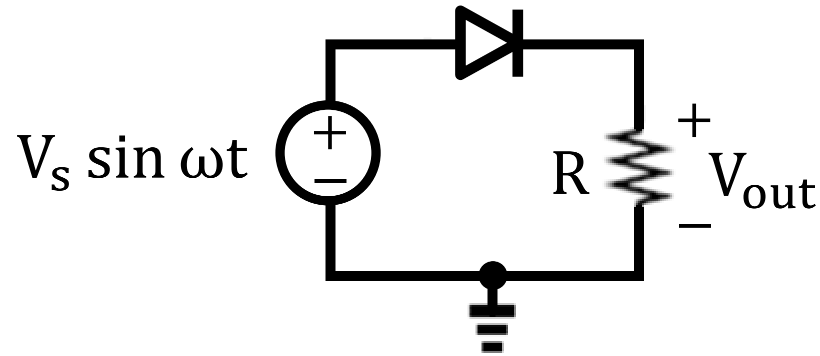
What are the line and load regulations for the circuit below?



$$\text{Line Regulation} = \frac{0.1\text{k}}{5\text{k} + 0.1\text{k}} = 19.6\ \text{mV/V} \quad \text{Load Regulation} = 5\text{k} \parallel 0.1\text{k} = 98\ \Omega$$

Half-Wave Rectifier

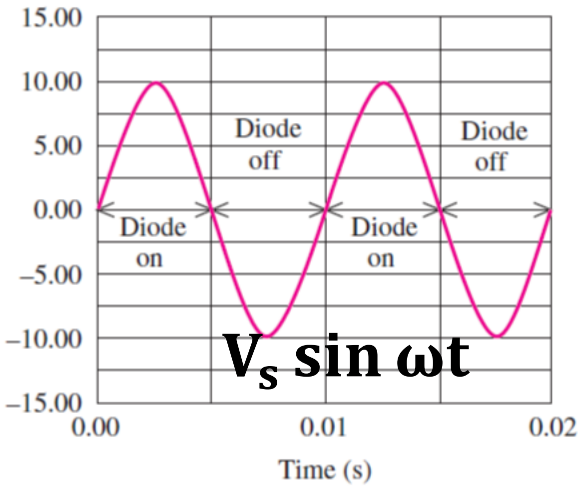
Half-Wave Rectifier with Resistive Load



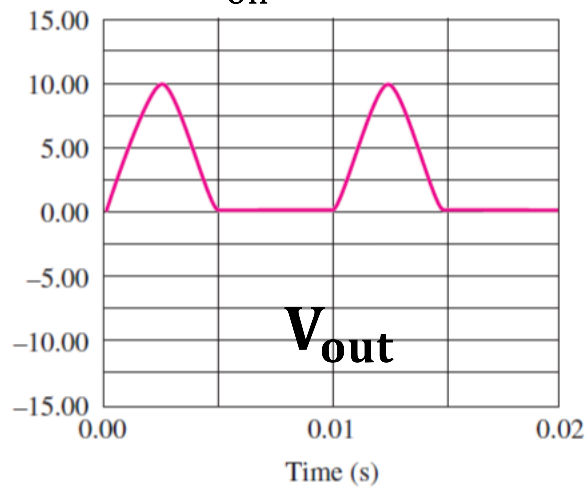
$V_s = 10\ \mathrm{V}$

$$\omega = 2\pi f = 2\pi \frac{1}{0.01} = 200\pi\ (\mathrm{rad/sec})$$

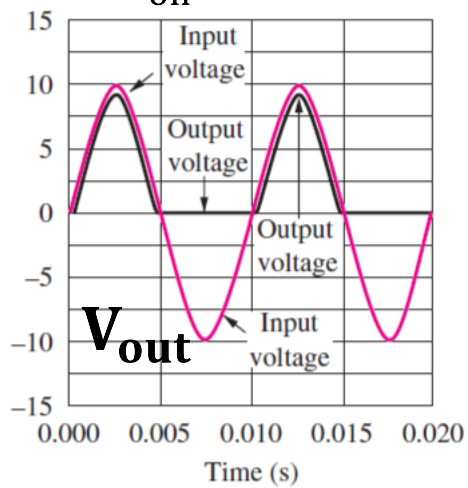
V_{out} is not DC.



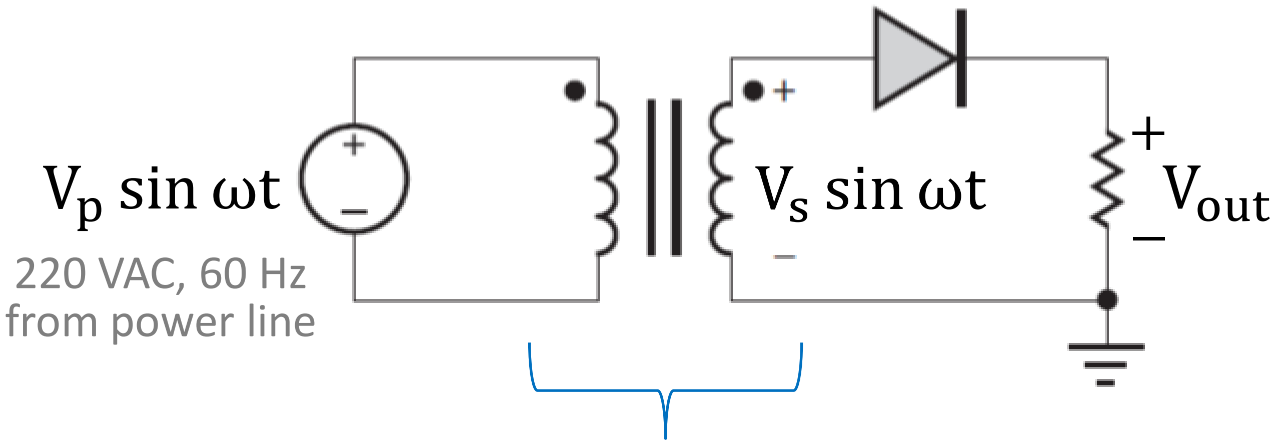
$V_{on} = 0\ \mathrm{V}$



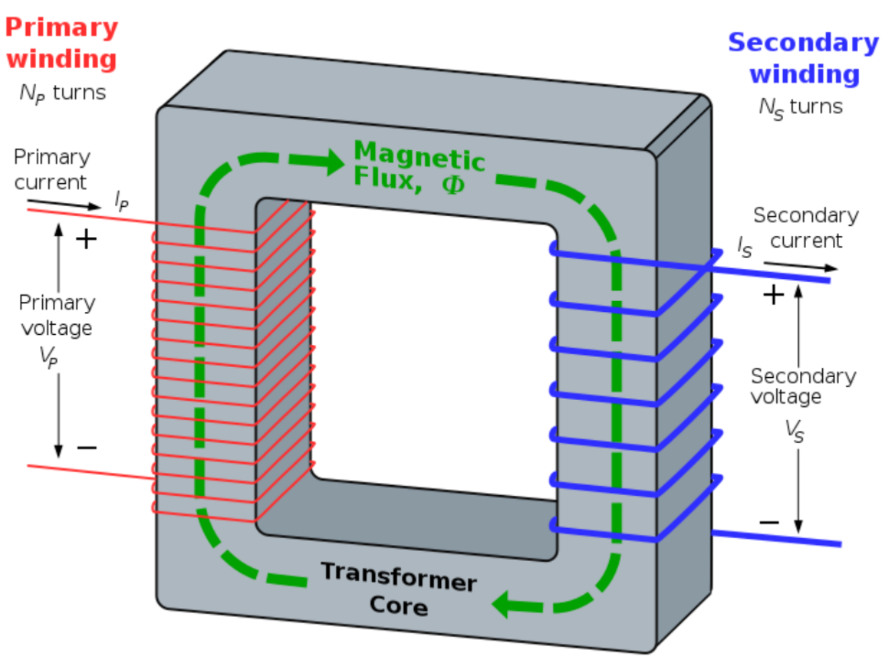
$V_{on} = 0.7\ \mathrm{V}$



Transformer



The output of an ideal transformer can be represented as an ideal voltage source.



Source: Daniels, A (1976). Introduction to Electrical Machines. Macmillan Publishers.

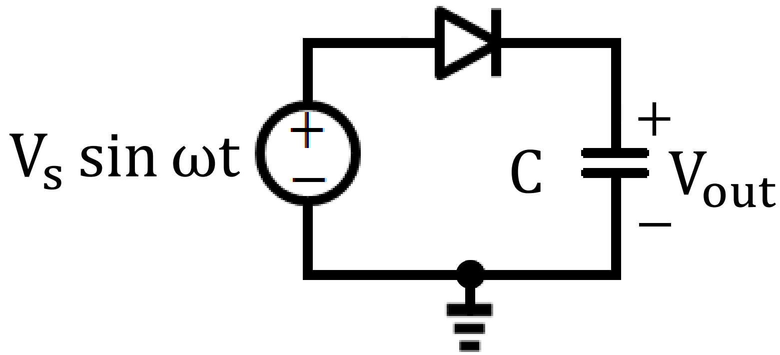
Working Principle:
By Faraday's law of induction

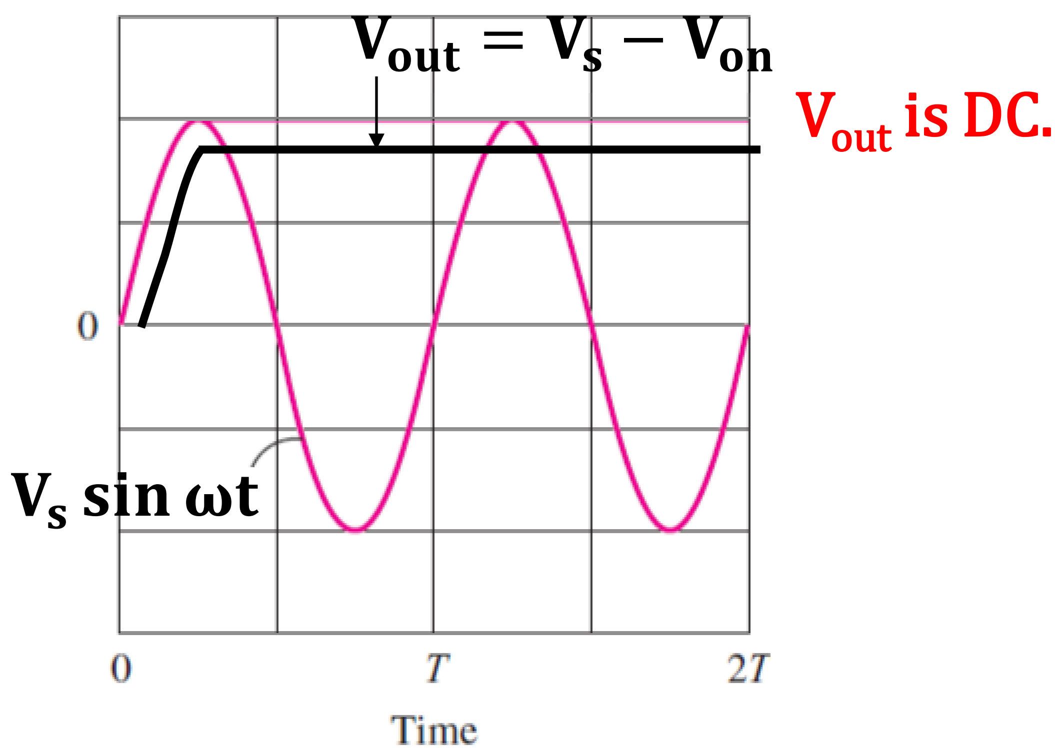
$$V_s = -N_s \frac{d\emptyset}{dt}$$

$$V_p = -N_p \frac{d\emptyset}{dt}$$

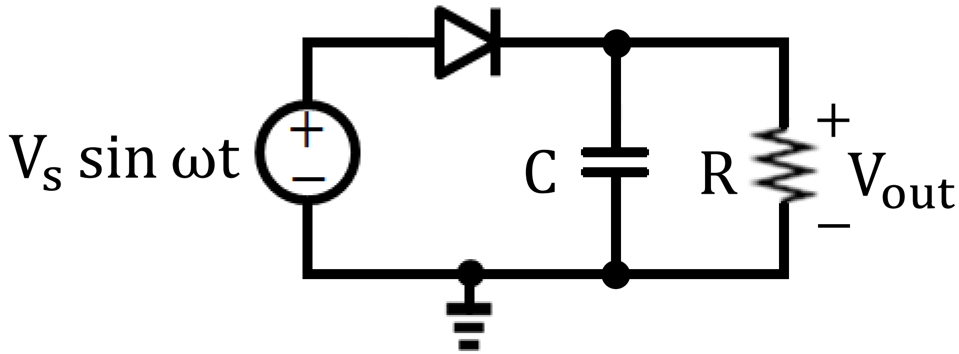
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

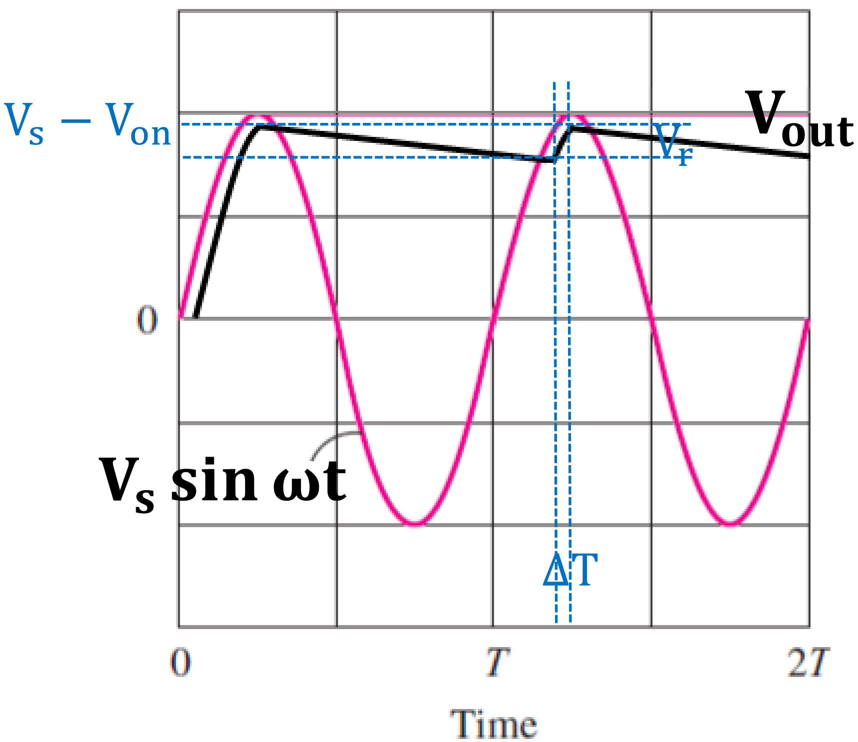
Half-Wave Rectifier with Capacitive Load





Half-Wave Rectifier with RC Load (I)





$$V_{dc} = \boxed{\mathbf{V_s - V_{on}}}$$

$$I_{dc} = \frac{V_{dc}}{R}$$

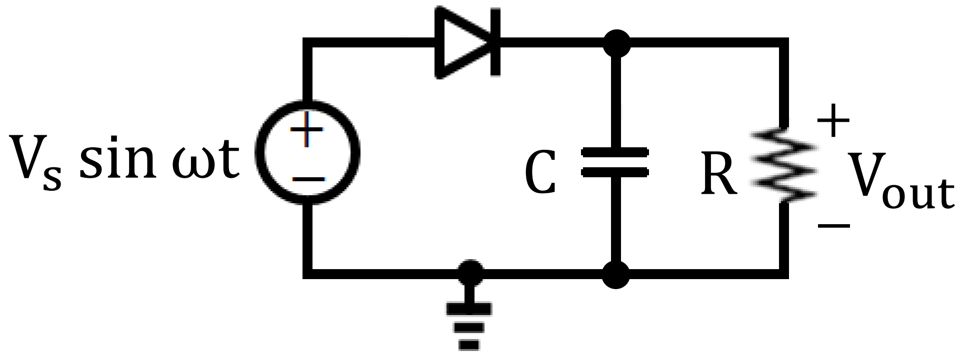
ripple voltage

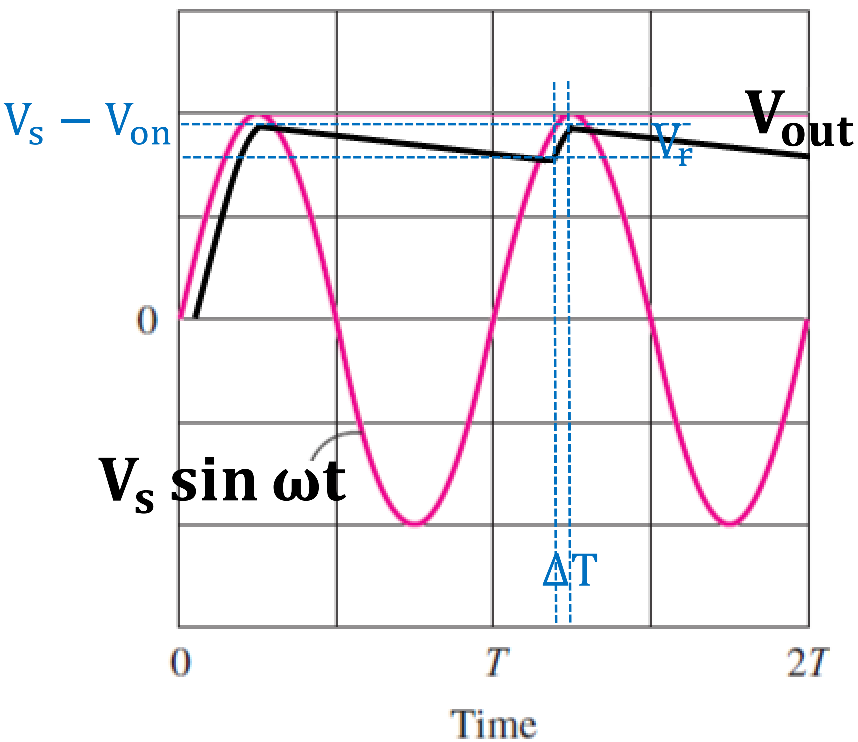
$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T-\Delta T}{RC}}\right)$$

$$\cong (V_s - V_{on})\left(\frac{T - \Delta T}{RC}\right) \quad if\,(T - \Delta T) \ll RC$$

$$\cong \boxed{\mathbf{(V_s - V_{on})\left(\frac{T}{RC}\right)}} \quad if\,\Delta T \ll T$$

Half-Wave Rectifier with RC Load (II)





conduction angle and interval

$$V_s \sin\left[\omega\left(\frac{5T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r$$

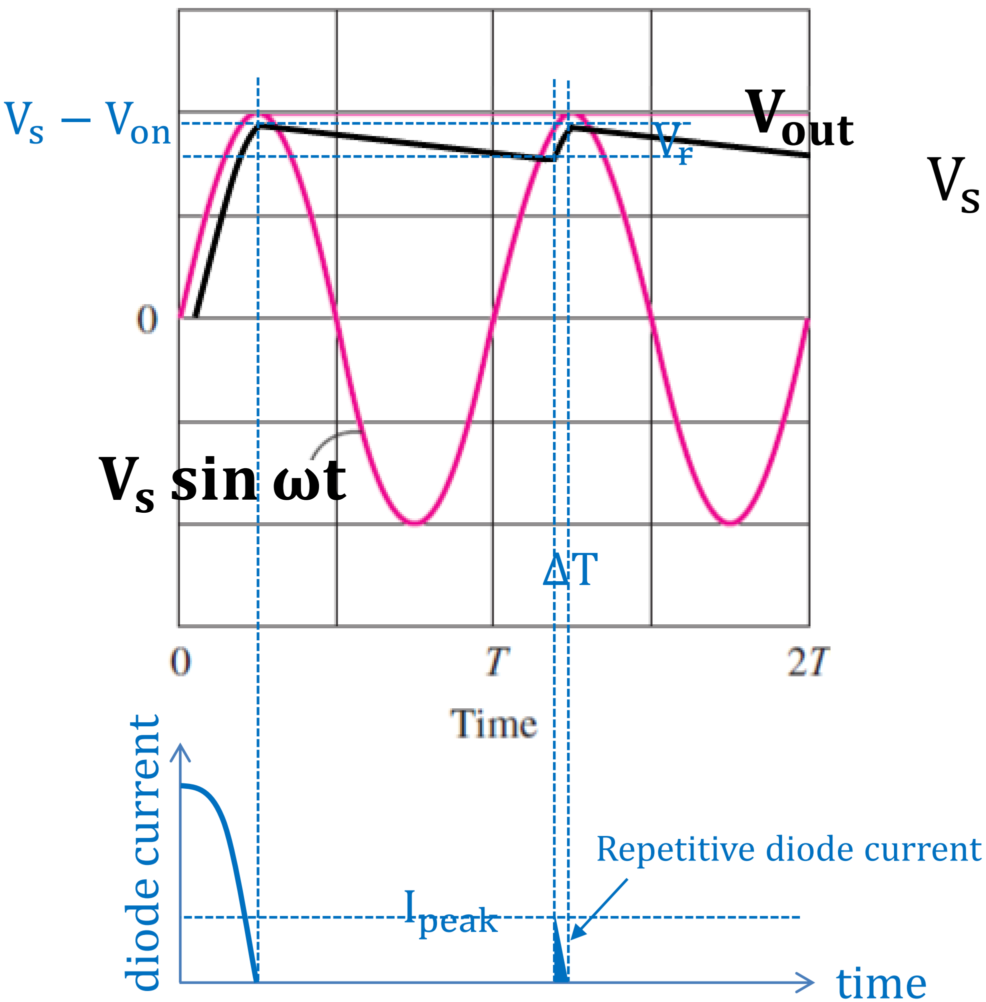
$$V_s \sin\left(\frac{5\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r$$

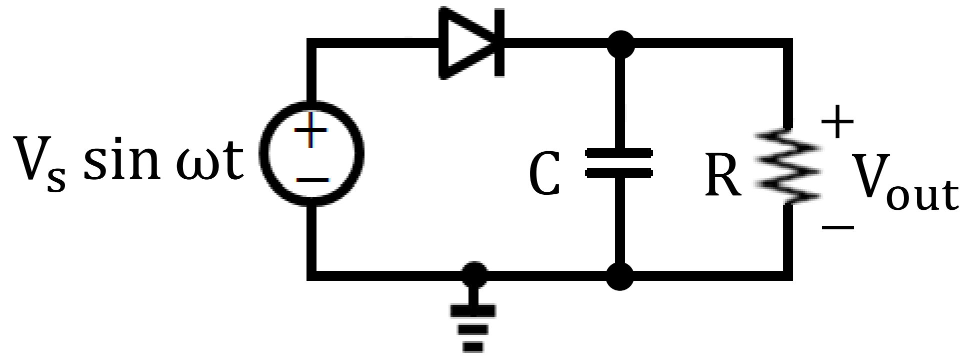
$$V_s \cos\theta_c = V_s - V_r$$

$$\cos\theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{{\theta_c}^2}{2} \quad \textit{if } \theta_c \textit{ very small}$$

$$\boxed{\boldsymbol{\theta_c = \sqrt{\frac{2V_r}{V_s}}}} \qquad \boxed{\boldsymbol{\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}}}}$$

Half-Wave Rectifier with RC Load (III)





The charge filled on C during ΔT is discharged during $T - \Delta T$.

$$Q \cong \frac{I_{peak} \Delta T}{2} = I_{dc}(T - \Delta T) \cong I_{dc} T$$

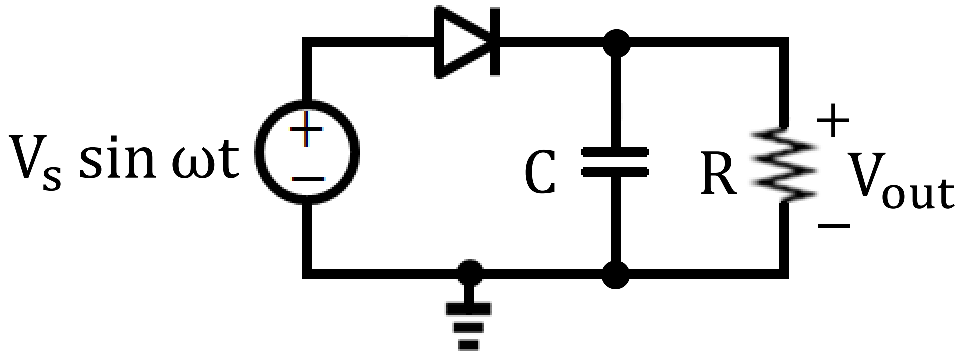
$$\boxed{\mathbf{I_{peak} = \frac{2 I_{dc} T}{\Delta T}}}$$

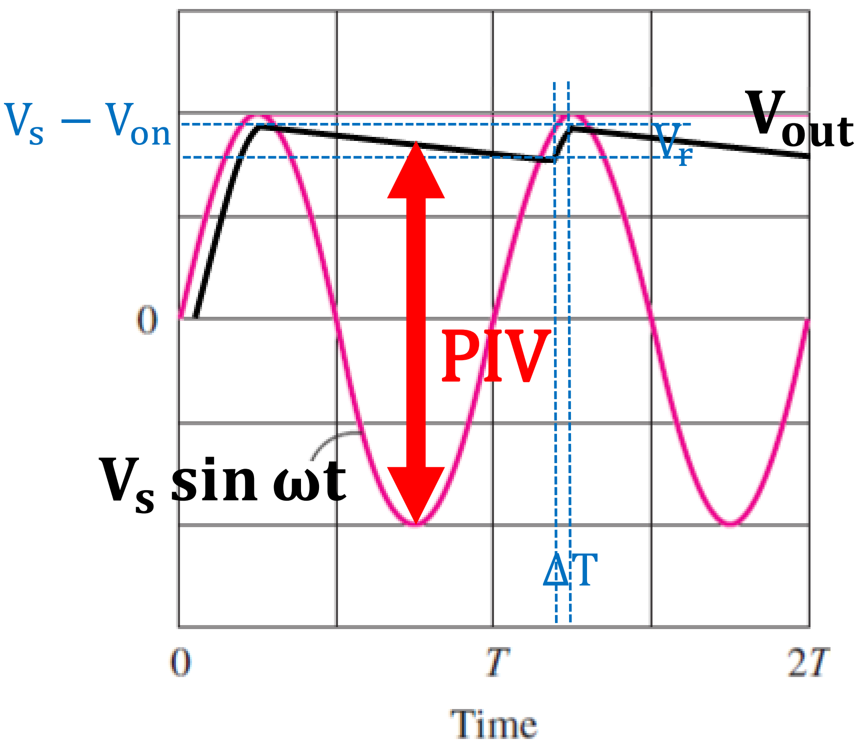
During charging period (ΔT), almost all diode current goes to C.

$$\left|\frac{1}{SC}\right| = \frac{1}{2\pi \frac{1}{T} C} = \frac{T}{2\pi C} \ll R \quad \text{if } RC \gg T$$

$$\boxed{\mathbf{I_{surge} = C \frac{dV_s \sin \omega t}{dt} = \omega C V_s} \quad \text{at } t = 0}$$

Half-Wave Rectifier with RC Load (IV)





Peak-inverse-voltage (PIV) $\cong 2V_s - V_{on}$

If too large, the diode breaks down.

Example

Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, conduction angle and diode peak current for a half-wave rectifier driven from a transformer having a secondary voltage of 12.6 V_{rms} (60 Hz) with R = 15 Ω and C = 25,000 μF. Assume $V_{on} = 1$ V.

$$V_{dc} = 12.6\sqrt{2} - 1 = 16.8\ (\mathrm{V})$$

$$I_{dc} = \frac{16.8}{15} = 1.12\ (\mathrm{A})$$

$$V_r \cong V_{dc}\frac{T}{RC} = 16.8\frac{\frac{1}{60}}{15 \times 25000 \times 10^{-6}} = 0.747\ (\mathrm{V})$$

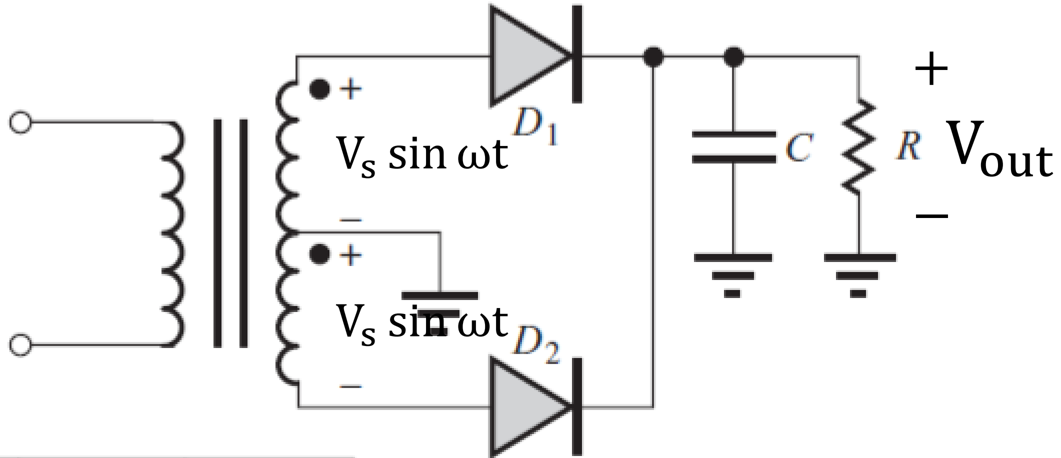
$$\theta_c \cong \sqrt{\frac{2V_r}{V_s}} = \sqrt{\frac{2 \times 0.747}{12.6 \times \sqrt{2}}} = 0.29\ (\mathrm{rad})\ \text{or}\ 16.6^\circ$$

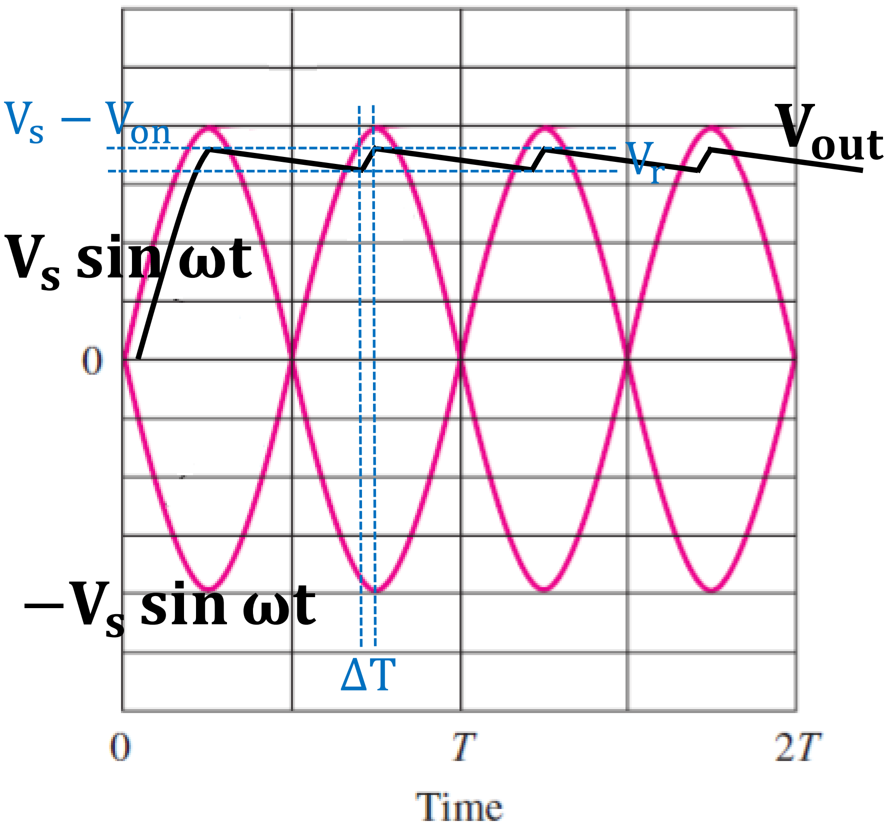
$$\Delta T \cong \frac{\theta_c}{\omega} = \frac{0.29}{2\pi \times 60} = 7.69 \times 10^{-4}\ (\mathrm{sec})$$

$$I_{peak} = \frac{2 \times 1.12 \times \frac{1}{60}}{7.69 \times 10^{-4}} = 48.6\ (\mathrm{A})$$

- Make sure all assumptions are valid.
- Since R is small (15 Ω), C needs to be large (25,000 μF) to maintain a low V_r.
- The diode must be able to handle these repetitively high peak currents.

Full-Wave Rectifier (I)





$$V_{dc} = \boxed{\mathbf{V_s - V_{on}}}$$

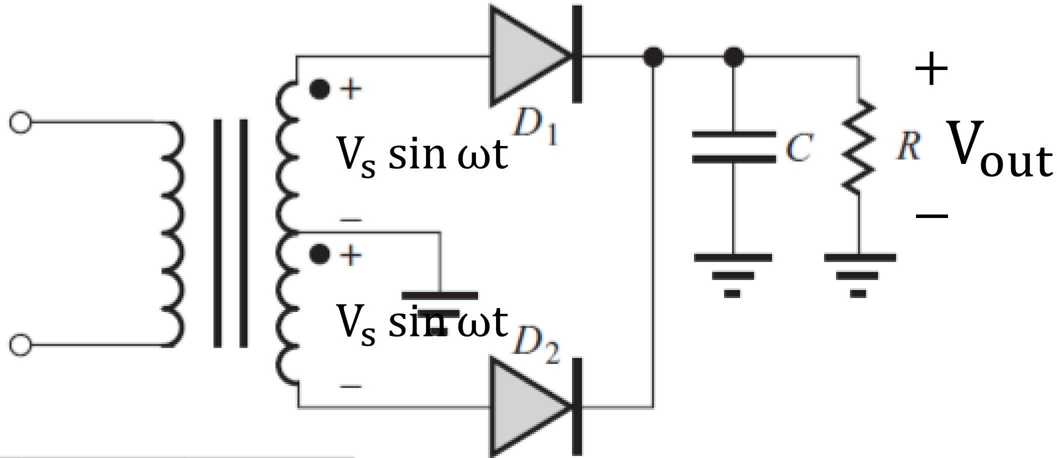
$$I_{dc} = \frac{V_{dc}}{R}$$

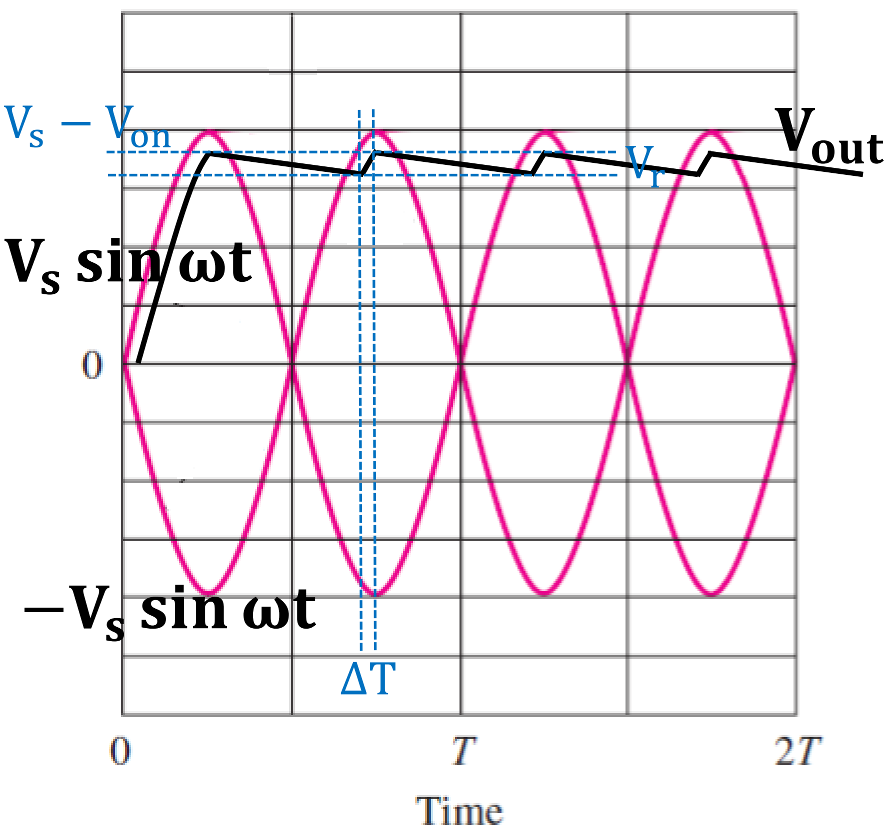
$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T/2 - \Delta T}{RC}}\right)$$

$$\cong (V_s - V_{on})\left(\frac{T/2 - \Delta T}{RC}\right) \quad if \left(\frac{T}{2} - \Delta T\right) \ll RC$$

$$\cong \boxed{\mathbf{(V_s - V_{on})\left(\frac{T}{2RC}\right)}} \quad if\, \Delta T \ll \frac{T}{2}$$

Full-Wave Rectifier (II)





$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r$$

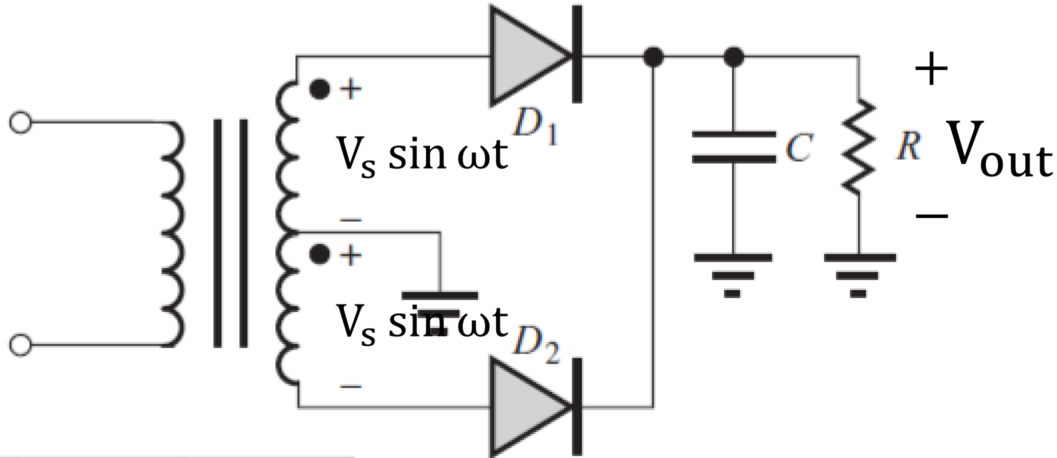
$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r$$

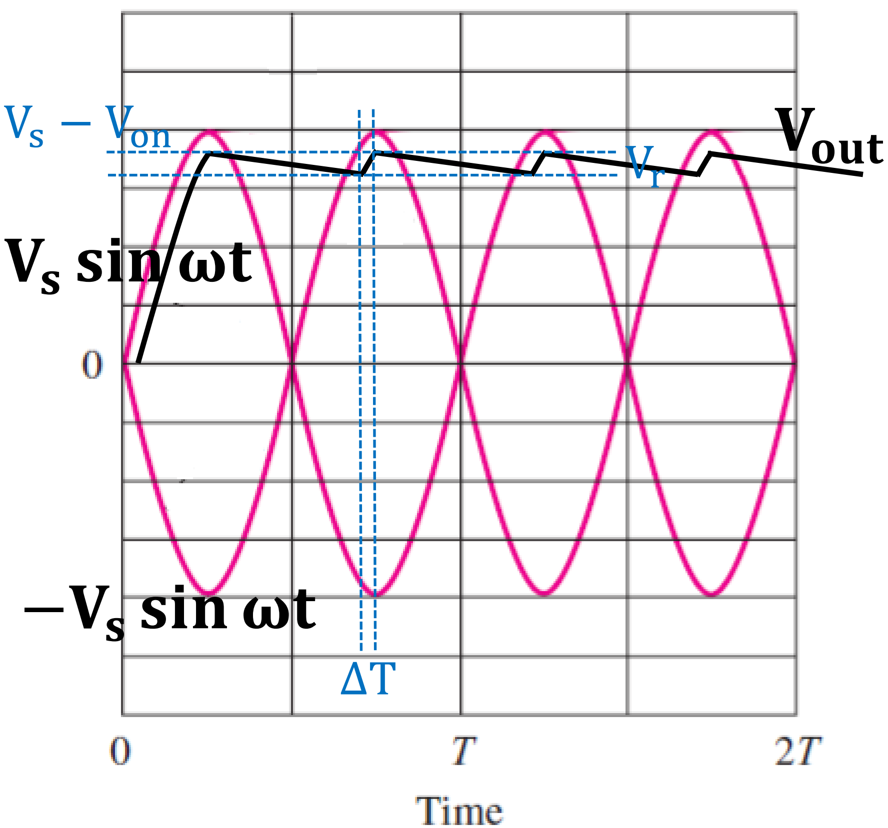
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c{}^2}{2} \quad \textit{if } \theta_c \textit{ very small}$$

$$\boxed{\boldsymbol{\theta_c = \sqrt{\frac{2V_r}{V_s}}}} \qquad \boxed{\boldsymbol{\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}}}}$$

Full-Wave Rectifier (III)





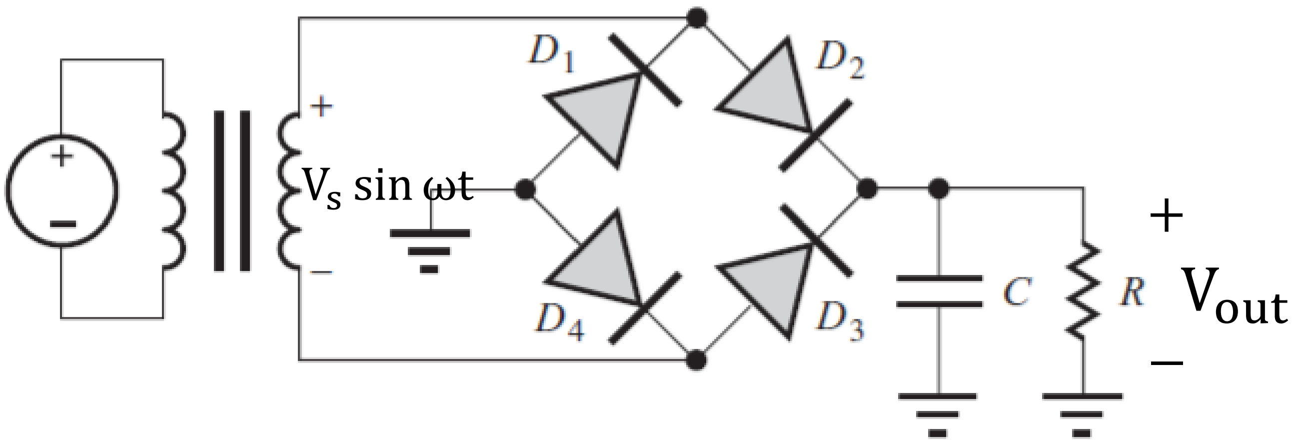
$$Q \cong \frac{I_{peak}\Delta T}{2} = I_{dc}\left(\frac{T}{2} - \Delta T\right) \cong I_{dc}\frac{T}{2}$$

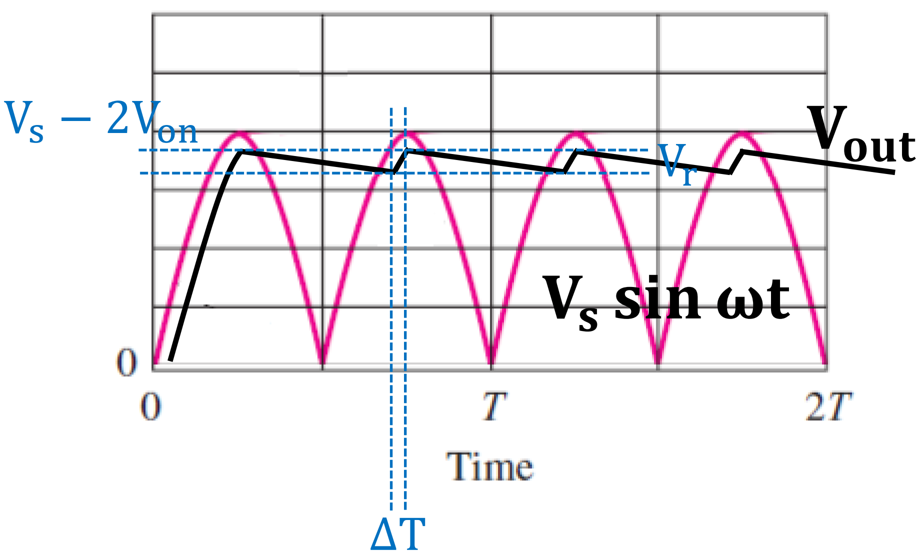
$$\boxed{\mathbf{I_{peak} = \frac{I_{dc}T}{\Delta T}}}$$

$$\boxed{\mathbf{I_{surge} = \omega C V_s}}$$

$$\boxed{\mathbf{PIV = 2V_s - V_{on}}}$$

Full-Wave Bridge Rectifier (I)





$$V_{dc} = \boxed{\mathbf{V_s - 2V_{on}}}$$

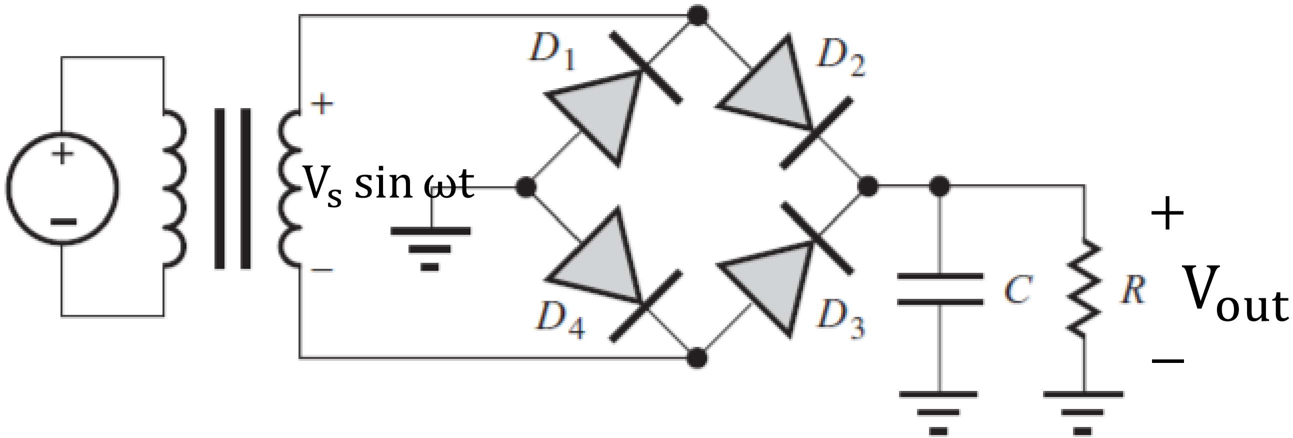
$$I_{dc} = \frac{V_{dc}}{R}$$

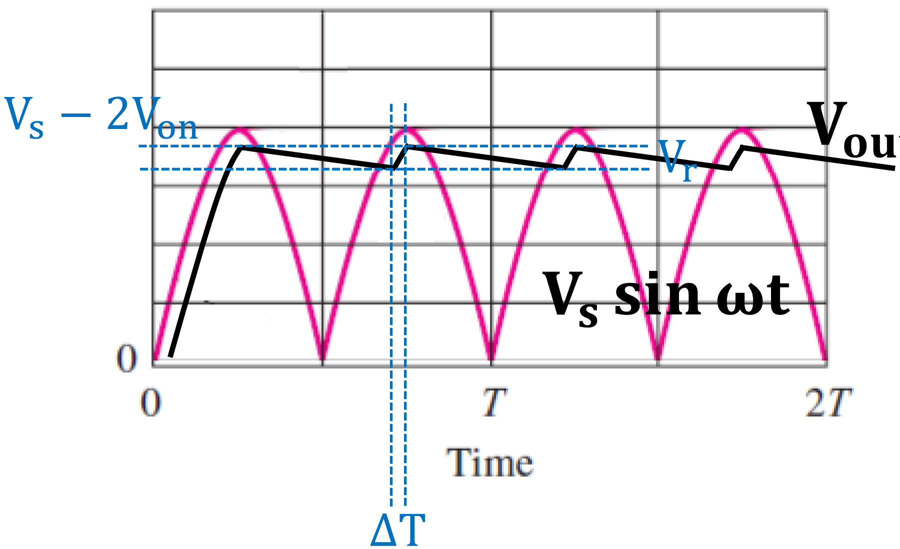
$$V_r = (V_s - 2V_{on})\left(1 - e^{-\frac{T/2 - \Delta T}{RC}}\right)$$

$$\cong (V_s - 2V_{on})\left(\frac{T/2 - \Delta T}{RC}\right) \quad if \left(\frac{T}{2} - \Delta T\right) \ll RC$$

$$\cong \boxed{\mathbf{(V_s - 2V_{on})\left(\frac{T}{2RC}\right)}} \quad if\ \Delta T \ll \frac{T}{2}$$

Full-Wave Bridge Rectifier (II)





$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - 2V_{on} = (V_s - 2V_{on}) - V_r$$

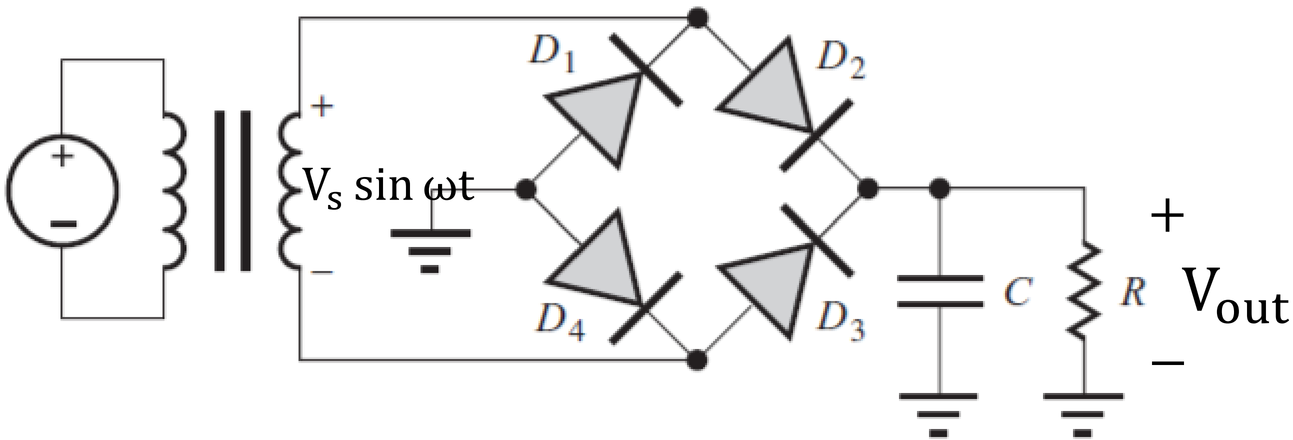
$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - 2V_{on} = (V_s - 2V_{on}) - V_r$$

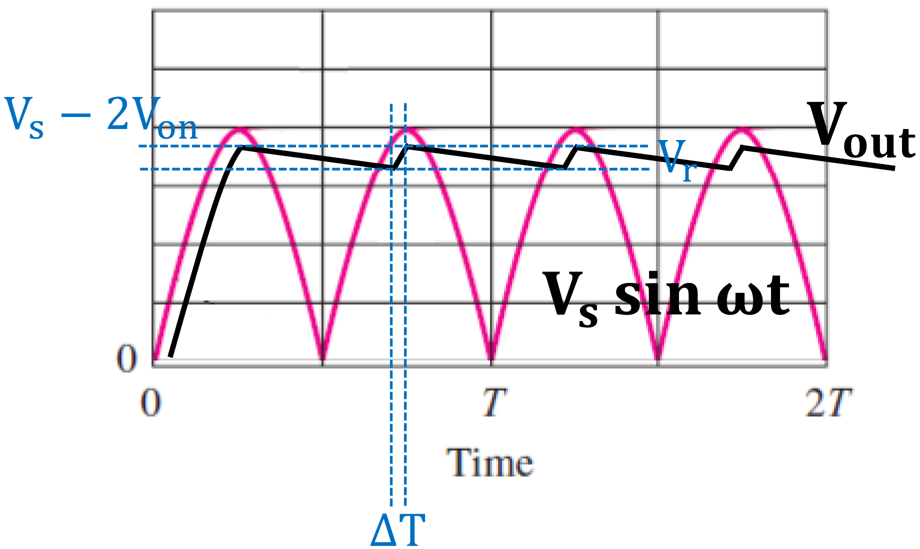
$$V_s \cos\theta_c = V_s - V_r$$

$$\cos\theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{{\theta_c}^2}{2} \quad \textit{if } \theta_c \textit{ very small}$$

$$\boxed{\boldsymbol{\theta_c = \sqrt{\frac{2V_r}{V_s}}}} \qquad \boxed{\boldsymbol{\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}}}}$$

Full-Wave Bridge Rectifier (III)





$$Q \cong \frac{I_{peak}\Delta T}{2} = I_{dc}\left(\frac{T}{2} - \Delta T\right) \cong I_{dc}\frac{T}{2}$$

$$\boxed{\mathbf{I_{peak} = \frac{I_{dc}T}{\Delta T}}}$$

$$\boxed{\mathbf{I_{surge} = \omega C V_s}}$$

$$\boxed{\mathbf{PIV = V_s - V_{on}}}$$

Example

Design a full-wave bridge rectifier to provide a dc output voltage 15 V with no more than 1 percent ripple at a load current of 2A. ($V_{on} = 1$ V, $T = 1/60$ sec)

$$V_{dc} = 15\ (V)$$
$$V_r < 0.15\ (V)$$
$$I_{dc} = 2\ (A)$$
$$\text{Load resistance} = 15/2 = 7.5\ (\Omega)$$

The required transformer voltage $V_s = 15 + 2 = \mathbf{17\ (V)\ or\ \frac{17}{\sqrt{2}}\ (V_{rms})}$

$$V_r \cong (V_s - 2V_{on})\left(\frac{T}{2RC}\right) = 15\left(\frac{1}{2 \times 60 \times 7.5 \times C}\right) = 0.15 \Rightarrow \mathbf{C = 0.111\ (F)}$$

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} = \frac{1}{2\pi \times 60}\sqrt{\frac{2 \times 0.15}{17}} = \mathbf{0.352 \times 10^{-3}\ (sec)}$$

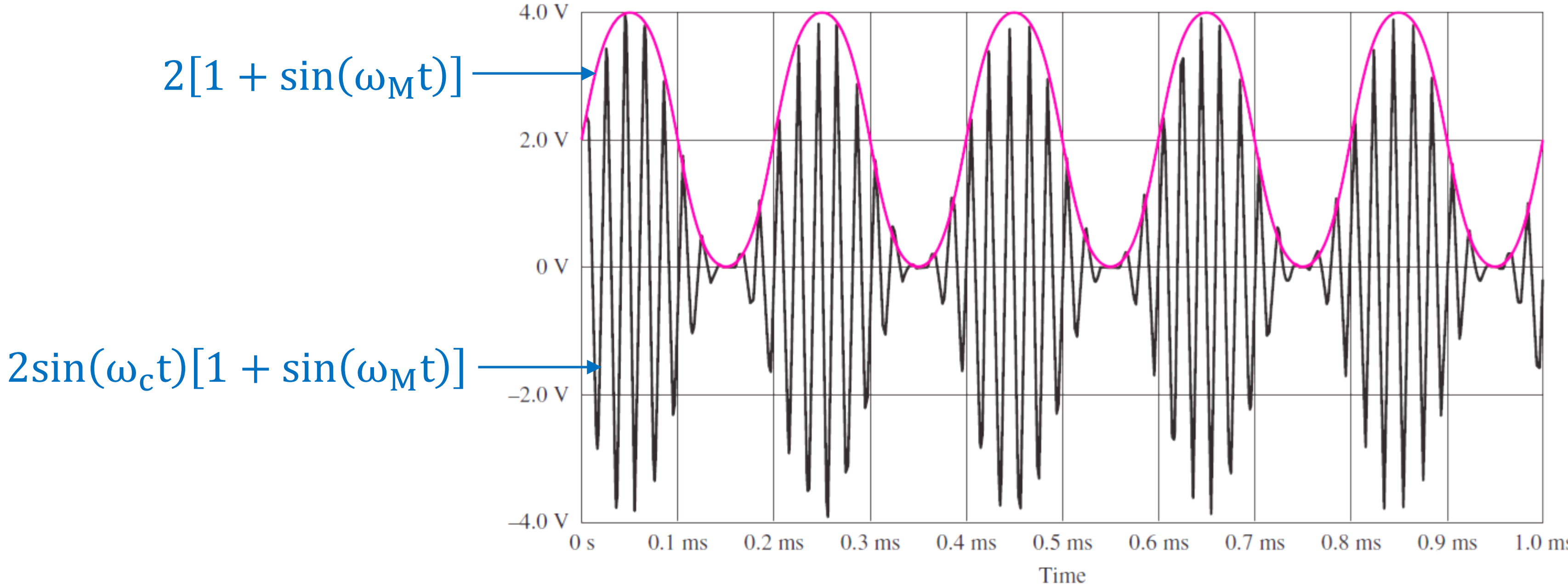
$$I_{peak} = \frac{I_{dc}T}{\Delta T} = \frac{2 \times \frac{1}{60}}{0.352 \times 10^{-3}} = \mathbf{94.7\ (A)}$$

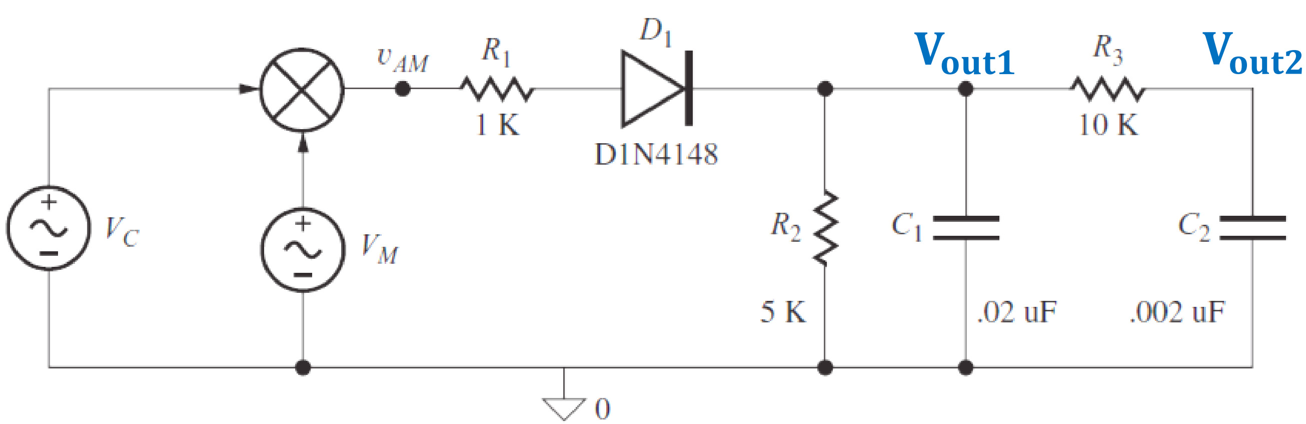
$$I_{surge} = \omega C V_s = 2\pi \times 60 \times 0.111 \times 17 = \mathbf{711\ (A)}$$

Make sure the diodes can handle these large currents

Example

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.





Example

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.