

UM-SJTU JOINT INSTITUTE  
Intro to Circuits  
(VE215)

LABORATORY REPORT

EXERCISE 5  
Filter Lab

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# 1 Introduction and Theoretical Background

## 1.1 Objectives

1. Learn about four types of filters Low-Pass, High-Pass, Band-Pass, and Band-reject.
2. Learn about transfer functions.
3. Predict the theoretical result and make comparison with lab data.

## 1.2 Theoretical Background

### 1.2.1 Filters

Filters are everywhere in our lives. The circuits built to operate on signals usually apply filters. For example, telephone lines pass the sounds at frequencies between about 100Hz and 3kHz and practically blocks all other frequencies.

### 1.2.2 Transfer Function

Mathematically, the transfer function is used to analyze what the circuit did to the signal:

$$TransferFunction = \frac{OutputSignal}{InputSignal}$$

The function can also be expressed as

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$

The magnitude of the transfer function is called voltage gain, often measured as the ratio of the peak-to-peak (ppk) voltages:

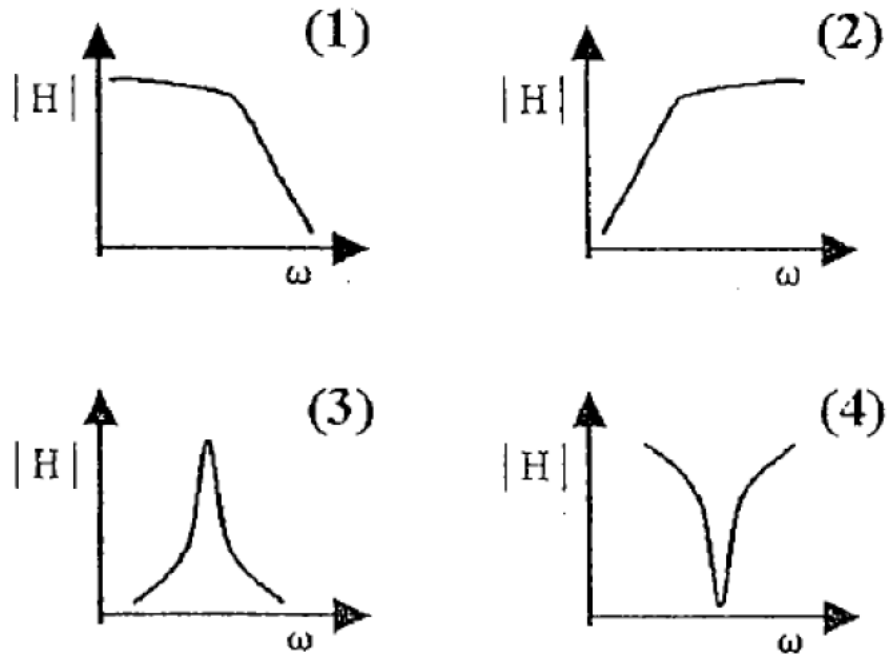
$$|H(\omega)| = \left| \frac{V_{out}(\omega)}{V_{in}(\omega)} \right| = \frac{V_{Outppk}(\omega)}{V_{Inppk}(\omega)}$$

It is convenient to express and plot the magnitude of the transfer function on the logarithmic scale using decibels:

$$|H(\omega)|_{dB} = 20 \cdot \log_{10} \frac{V_{Outppk}(\omega)}{V_{Inppk}(\omega)}$$

Since both ppk voltages are always positive, the transfer function magnitude is positive and thus can always be converted to decibels. The use of decibels allows us to review data over a broad range.

## Types of filters



In the figure above are the four main families of filters:

(1): Low-Pass; (2): High-Pass; (3): Band-Pass; (4): Band-reject

## Summary of the characteristics of ideal filters.

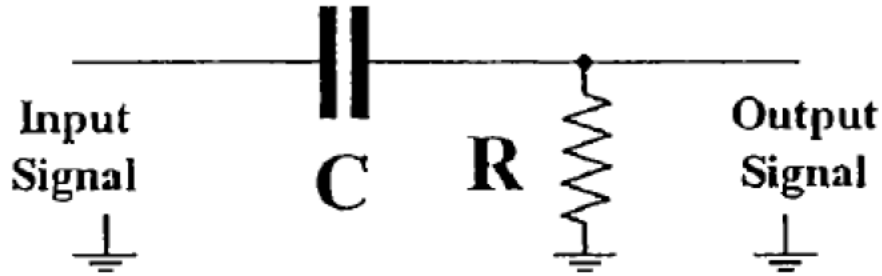
Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

$\omega_c$  is the cutoff frequency for lowpass and highpass filters;  $\omega_0$  is the center frequency for bandpass and bandstop filters.

Filter circuits, which you are going to build in this lab, contain resistors, capacitors, and inductors. They are all passive filters.

### 1.2.3 High-Pass Filter

The high-pass filter we are going to build uses a capacitor and a resistor.



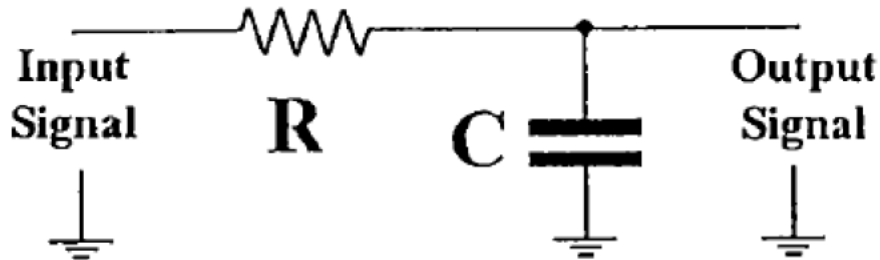
For the high-pass filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

Note that  $H(0)=0, H(\infty)=1$ . Hence, it would only let high frequency pass.

### 1.2.4 Low-Pass Filter

The low-pass filter we are going to build uses a capacitor and a resistor.



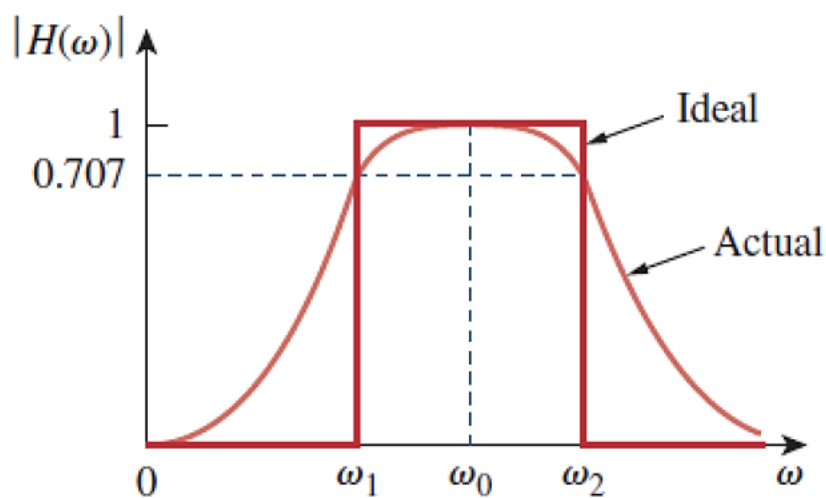
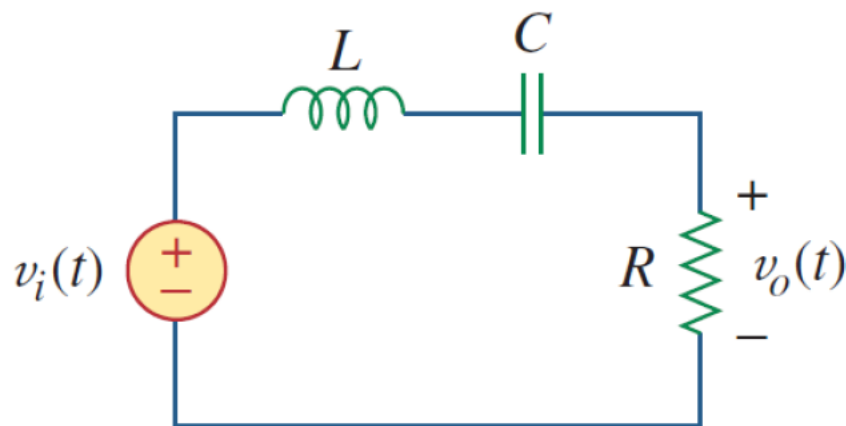
For the low-pass filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

Note that  $H(0)=1, H(\infty)=0$ . Hence, it would only let low frequency pass.

### 1.2.5 Band-Pass Filter

The band-pass filter we are going to build uses a capacitor, an inductor and a resistor.



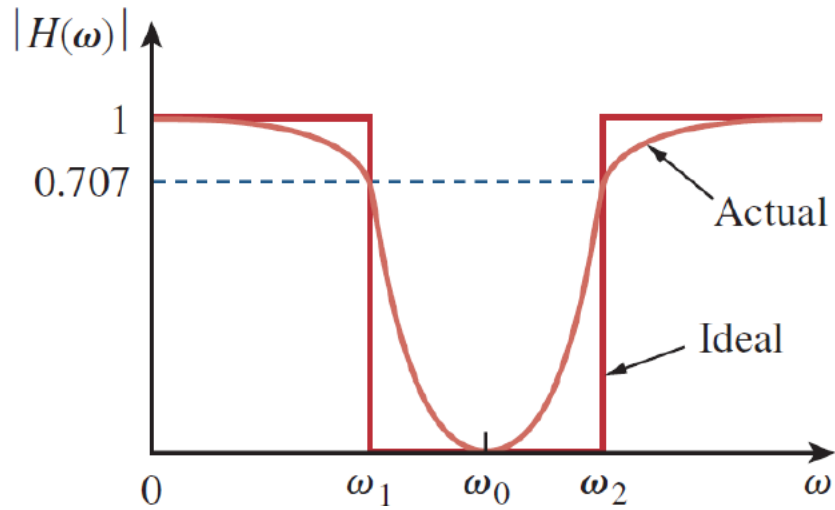
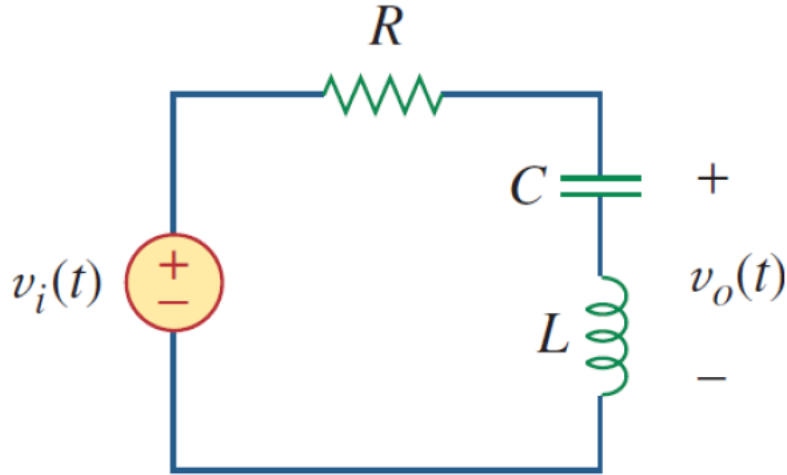
For the band-pass filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

Note that  $H(0)=0$ ,  $H(\infty)=0$ . The band-pass filter passes a band of frequencies centered on the center frequency  $\omega_0$ , which is given by  $\omega_0 = \frac{1}{\sqrt{LC}}$ .

### 1.2.6 Band-Stop Filter

The band-stop filter we are going to build uses a capacitor, an inductor and a resistor.



For the band-reject filter:

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

Note that  $H(0)=0$ ,  $H(\infty)=0$ . The band-stop filter rejects a band of frequencies centered on the center frequency  $\omega_0$ , which is given by  $\omega_0 = \frac{1}{\sqrt{LC}}$ .

## 2 Procedures

1. I filled the expected data column according to the pre-lab assignments.
2. I constructed the circuit for each type of filter. Resistor:  $R = 982\Omega$ ; Capacitor:  $C = 0.1\mu F$ ; Inductor:  $L = 1mH$ .

3. I set the Input Signal in the function generator to be Sine Wave with amplitude of 5  $V_{ppk}$  and changed the frequency accordingly.
4. I used the oscilloscope to detect the amplitudes of the Input and Output signals and recorded them respectively in the first two column in the tables.
5. For the band-reject filter, when the frequency approached the critical frequency at which the Transfer Function Magnitude reached its minimum, the Output Signal Amplitude was changing rapidly.
6. I calculated with the experimental data for the Transfer function magnitude and Transfer function magnitude, in dB columns.

## 3 Results

### 3.1 Low-pass Filter

Frequency	Input signal amplitude $V_{ppk}$	Output signal amplitude, $V_{ppk}$	Transfer function magnitude	<b>Expected</b> transfer function magnitude	Transfer function magnitude in dB	<b>Expected</b> transfer function magnitude, in dB
1MHz	4.78	0.0197	0.0041	0.0016	-47.7443	-55.8085
100kHz	4.82	0.114	0.0237	0.0162	-35.8097	-35.8070
50kHz	4.82	0.213	0.0442	0.0324	-29.7891	-29.7898
10kHz	4.86	0.92	0.1893	0.1600	-15.9176	-15.9184
5kHz	4.86	1.65	0.3395	0.3083	-10.2205	-10.2191
1kHz	5.03	4.14	0.8231	0.8510	-1.4014	-1.4010
500Hz	5.07	4.78	0.9428	0.9556	-0.3948	-0.3948

### 3.2 High-pass Filter

Frequency	Input signal amplitude $V_{ppk}$	Output signal amplitude, $V_{ppk}$	Transfer function magnitude	<b>Expected</b> transfer function magnitude	Transfer function magnitude in dB	<b>Expected</b> transfer function magnitude, in dB
1MHz	4.82	4.78	0.9917	0.9999	-0.0724	-0.0011
100kHz	4.82	4.78	0.9917	0.9999	-0.0011	-0.0724
50kHz	4.86	4.78	0.9835	0.9995	-0.1445	-0.0046
10kHz	4.86	4.68	0.9630	0.9871	-0.3275	-0.1126
5kHz	4.86	4.46	0.9177	0.9513	-0.4337	-0.4339
1kHz	4.94	2.57	0.5202	0.5251	-5.6766	-5.5952
500Hz	5.03	1.45	0.2882	0.2948	-10.8061	-10.6096
100Hz	5.11	0.346	0.0677	0.0616	-23.3882	-24.2107

### 3.3 Hand-pass Filter

Frequency	Input signal amplitude $V_{ppk}$	Output signal amplitude, $V_{ppk}$	Transfer function magnitude	<b>Expected</b> transfer function magnitude	Transfer function magnitude in dB	<b>Expected</b> transfer function magnitude, in dB
1MHz	5.11	0.3	0.0587	0.1545	-24.6272	-16.2240
500kHz	5.07	1.35	0.2663	0.2986	-11.4926	-10.4796
100kHz	5.07	4.22	0.8323	0.8485	-1.5944	-1.4267
1850kHz	5.07	4.62	0.9112	0.9611	-0.8077	-0.3449
10kHz	5.11	4.70	0.9198	0.9952	-0.0416	-0.7261
1kHz	5.11	2.59	0.5266	0.5266	-5.9032	-5.5703
500Hz	5.11	1.55	0.2921	0.2921	-10.3626	-10.6018



### 3.4 Band-reject Filter

Frequency	Input signal amplitude $V_{ppk}$	Output signal amplitude, $V_{ppk}$	Transfer function magnitude	<b>Expected</b> transfer function magnitude	Transfer function magnitude in dB	<b>Expected</b> transfer function magnitude, in dB
1MHz	5.11	4.90	0.9589	0.9880	-0.3645	-0.1049
500kHz	5.11	4.94	0.9544	0.9544	-0.2942	-0.4056
500kHz	5.11	4.78	0.9354	0.8863	-0.5801	-1.0481
200kHz	5.07	4.14	0.8166	0.7860	-1.7598	-2.0911
100kHz	5.07	2.59	0.5108	0.5292	-5.8350	-5.5282
50kHz	5.07	1.25	0.2465	0.2763	-12.1637	-11.1721
10kHz	5.07	0.59	0.1164	0.0976	-18.6809	-20.2092
5kHz	5.03	1.43	0.2843	0.2804	-10.9245	-11.0435
1kHz	5.03	4.14	0.8231	0.8501	-1.6909	-1.4105
500Hz	5.03	4.74	0.9423	0.9555	-0.5162	-0.3956

## 4 Conclusion

Through this lab work, I have a general idea about four kinds of filters, which are Low-pass, High pass, Band-pass and Band-reject and their transfer function in  $V_{ppk}$  and dB. Before the in-lab work, I first calculate the expected transfer function magnitude in  $V_{ppk}$  and dB according to the transfer function of every filter. Then I set a sine wave whose amplitude is  $5V_{ppk}$  at different frequencies and record the input signal and output signal amplitude to calculate the transfer function magnitude. Notice  $\omega = 2\pi \cdot f$ ,  $R = 982\Omega$ ,  $C = 0.1\mu F$  and  $L = 1mH$

For the low-pass filter,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

the transfer function magnitude is becoming bigger as the frequency increases, and my results is similar to the expected magnitude.

For the high-pass amplitude,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

the transfer function magnitude is becoming smaller as the frequency increases, and my results is similar to the expected magnitude.

For the band-pass filter,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

the transfer function magnitude will first become bigger and then scale down as the frequency increases, and my results is similar to the expected magnitude except when the frequency is  $1MHz$ . In my lab work, I've measured the signal at this frequency many times

For the band-reject filter,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

the transfer function magnitude will first become smaller and then grow bigger as the frequency increases, and my results is similar to the expected magnitude.

## 5 Reference

- *VE215FA2017 Filter LabManual*
- *Circuits Make Sense*, Alexander Ganago, Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor.
- Charles K. Alexander, Matthew N.O. Sadiku. *Fundamentals of Electirc Circuits*. New York: McGraw-Hill, 2013. Print.