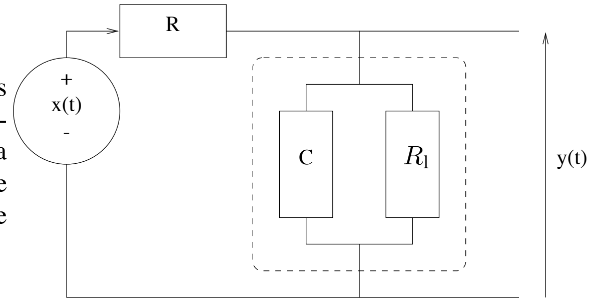
6. (15 points)



The analysis of the RC circuit that we discussed repeatedly in class was somewhat idealized because it assumed an ideal capacitor. Real capacitors slowly leak charge. We can model this leakage by incorporating a large resistance R_l in parallel with the ideal capacitor, as shown in the figure to the right. (The ideal capacitor and the resistance R_l within the dashed box together model a realistic capacitor.)

Determine the impulse response of the system described above.
Hint: check your result as $R_l \to \infty$.
Let $Z(\omega)$ be the impedance of the realistic capacitor: $Z(\omega)^{-1} = j\omega C + 1/R_l$. By complex-impedance voltage divider: $H(\omega) = Y(\omega)/H(\omega) = Z(\omega)/(Z(\omega) + R) = 1/(1 + RZ(\omega)^{-1}) = 1/[1 + R(j\omega C + 1/R_l)] = \boxed{\dfrac{1}{(1 + R/R_l) + j\omega RC}}$.

From FT table, $\boxed{h(t) = (1/RC)e^{-t/\tau}u(t) \text{ where } \tau = \frac{RC}{1+R/R_l} \text{ is the time constant.}}$ The leakage $\boxed{\text{shortens the time constant}}$ of the RC circuit by a factor $1 + R/R_l$. As R_l increases, the time constant approaches its ideal value $\tau = RC$.

[20 correct. lots of mixing ω and t in expressions.] *(HW 10.1)*

end