Example

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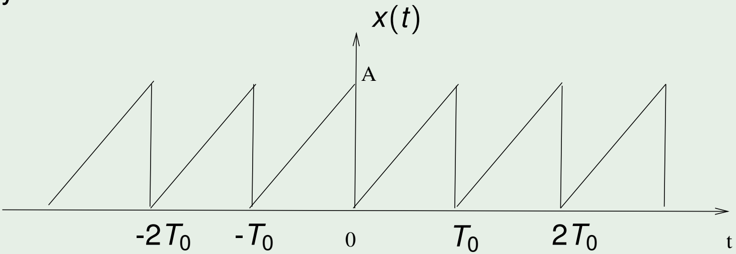
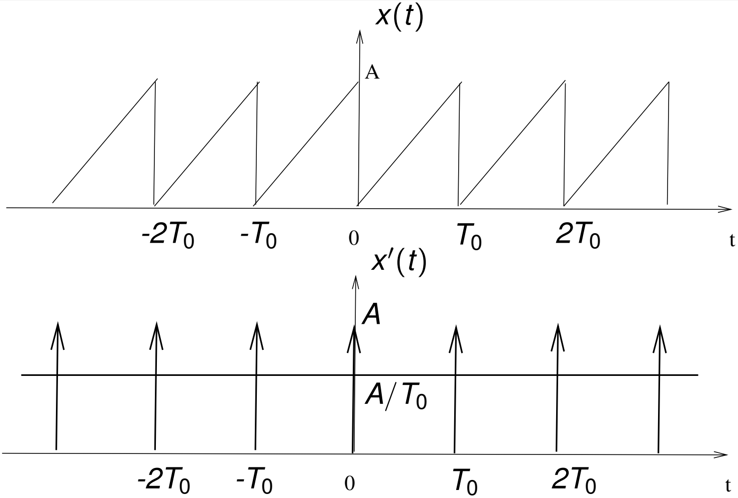
Find the FS of $x(t)$ using the differentiation property. Express your results in a real form.

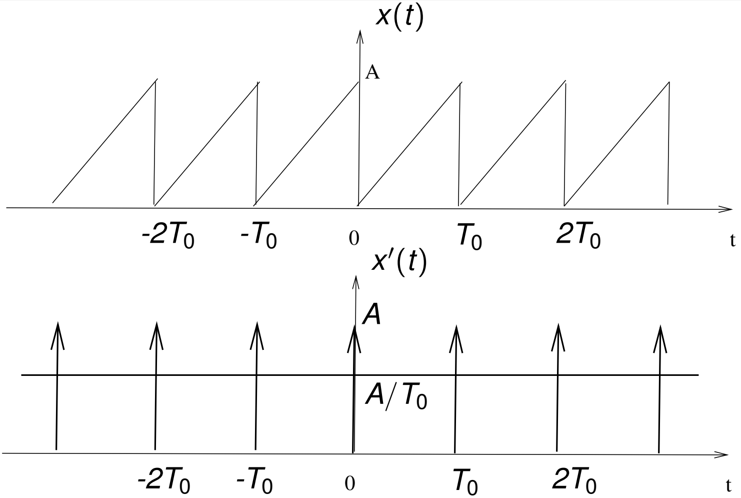


Solution (1)



$$x'(t) = \frac{A}{T_0} + g(t), \quad g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

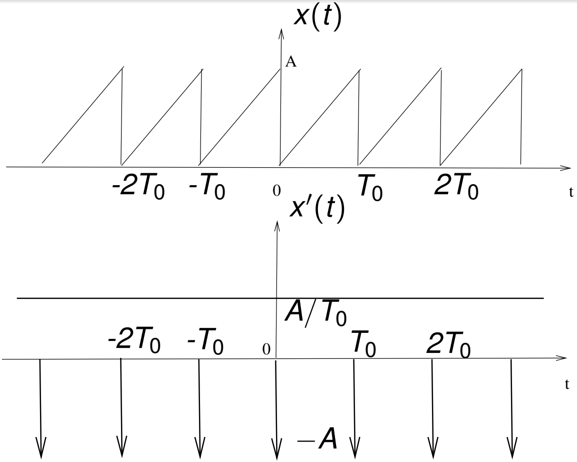
Solution (1)



$$x'(t) = \frac{A}{T_0} + g(t), \quad g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

Wrong!

Solution (2)



$$x'(t) = \frac{A}{T_0} - g(t), \quad g(t) = A\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

Solution (2)

The FS coefficients for $g(t)$ is

$$d'_k = \frac{1}{T_0} \underbrace{\int_{T_0} \delta(t) e^{-jk\omega_0 t}\, dt}_{=1 \text{ sifting property}} = \frac{1}{T_0}, \quad \forall k$$

$$g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t}$$

$$x'(t) = \frac{A}{T_0} - A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

$$d_k = \begin{cases} 0, & k = 0 \\ -A\frac{1}{T_0}, & k \neq 0 \end{cases}$$

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Solution (3)

$$c_k = \begin{cases} \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{T_0}\int_0^{T_0}\frac{A}{T_0}t\,dt = \frac{A}{2}, & k = 0 \\ \frac{d_k}{jk\omega_0} = -\frac{A}{T_0 jk\omega_0} = j\frac{A}{2\pi k}, & k \neq 0 \end{cases}$$

$$x(t) = \frac{A}{2} + \sum_{k=-\infty, k\neq 0}^{\infty} j\frac{A}{2\pi k}e^{jk\omega_0 t}$$

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Solution (4)

$$
\begin{aligned}
x(t) &= \frac{A}{2} + \sum_{k=1}^{\infty} j\left[\frac{A}{2\pi k}e^{jk\omega_0 t} - \frac{A}{2\pi k}e^{-jk\omega_0 t}\right] \\
&= \frac{A}{2} - \frac{A}{\pi}\sum_{k=1}^{\infty} \frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2jk} \\
&= \boxed{\frac{A}{2} - \frac{A}{\pi}\sum_{k=1}^{\infty} \frac{1}{k}\sin(k\omega_0 t)}
\end{aligned}
$$