Ve215 Electric Circuits

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Chapter 12

Three-Phase Circuits

12.1 Introduction

So far, we have dealt with *single-phase* circuits. A single-phase ac power system consists of a generator connected through a pair of wires (a transmission line) to a load. Figure 12.1(a) depicts a single-phase two-wire system.

Figure 12.1(a) Single-phase two-wire system.

Circuits or systems in which the sources operate at the same frequency but different phases are known as *polyphase*. Figure 12.3 shows a three-phase four-wire system.

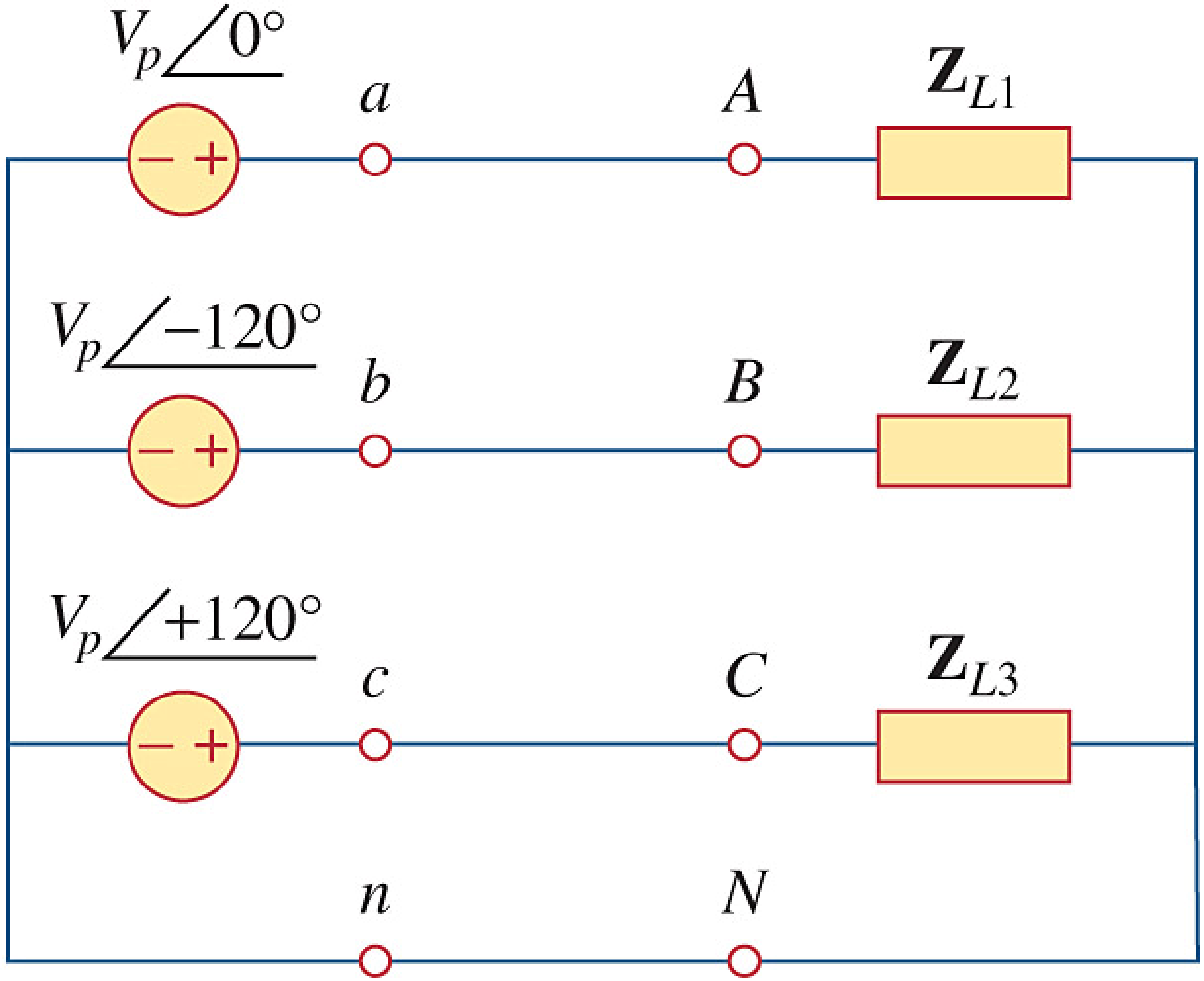

Figure 12.3 Three-phase four-wire system.

As distinct from a single-phase system, a three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°. In this chapter, we study three-phase systems.

12.2 Balanced Three-Phase Voltages

Three-phase voltages are often produced with a three-phase generator (or alternator) whose cross-sectional view is shown in Fig. 12.4. The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*).

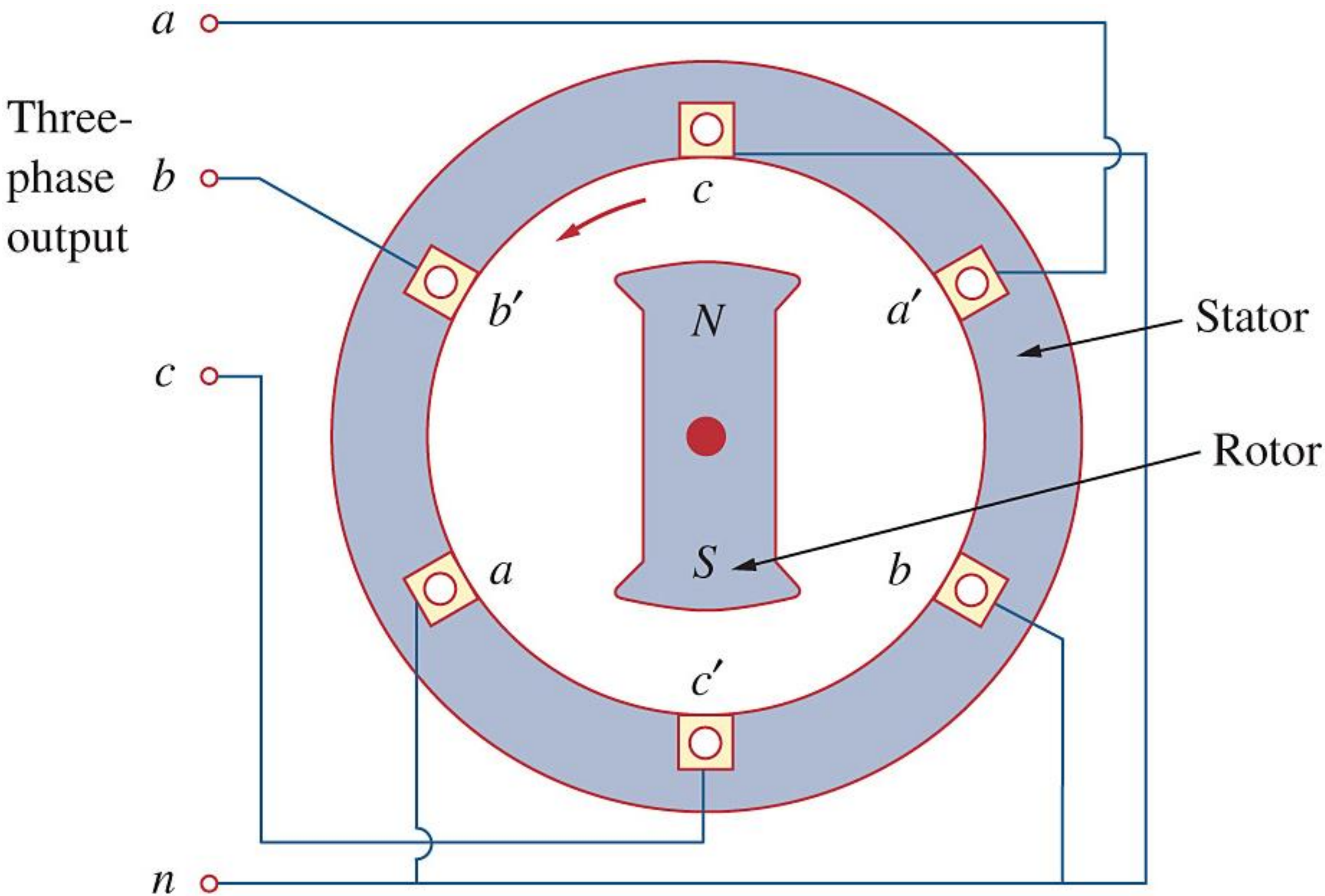

Figure 12.4 A three-phase generator.

As the rotator rotates, the flux through each coil varies sinusoidally with time, inducing a sinusoidal voltage. Because the coils are placed 120° apart, the induced voltages are out of phase by 120° (Fig. 12.5).

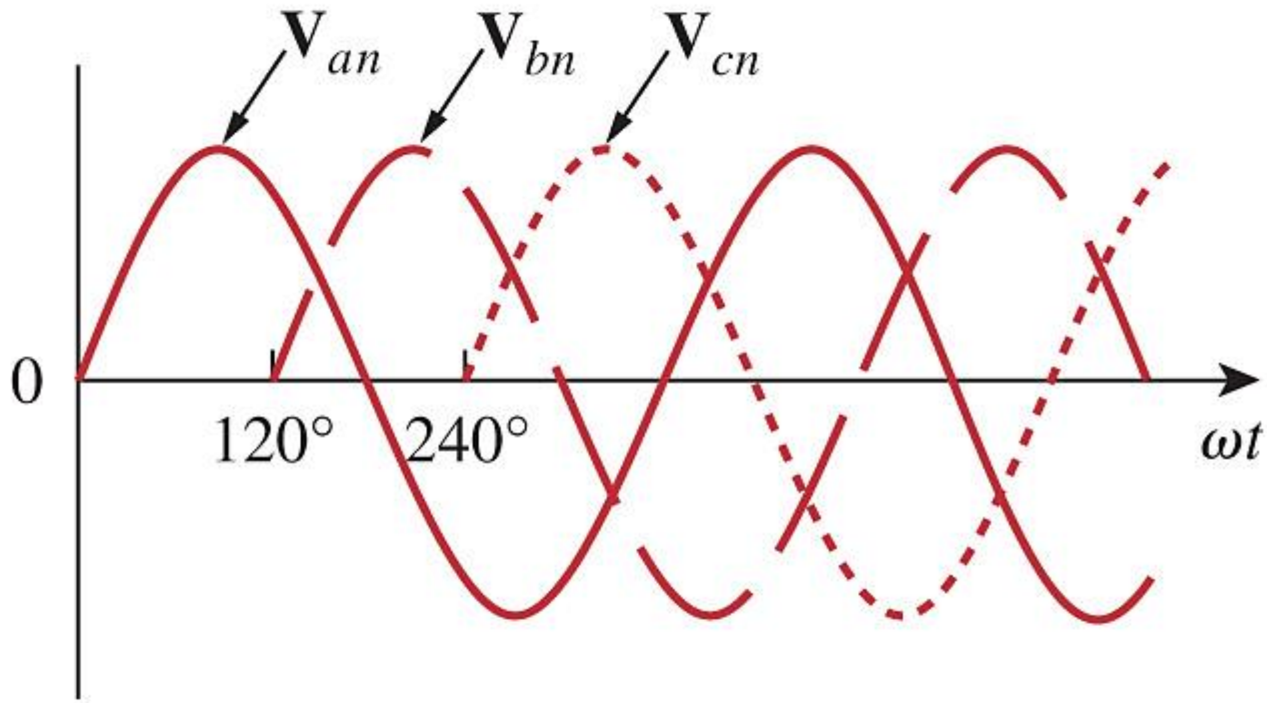

Figure 12.5 The generated voltages are 120° apart from each other.

A typical three-phase system consists of three voltage sources connected to loads by three or four wires. A three-phase system is equivalent to three single-phase systems. The voltage sources can be either wye- or delta-connected, as shown in Fig. 12.6.

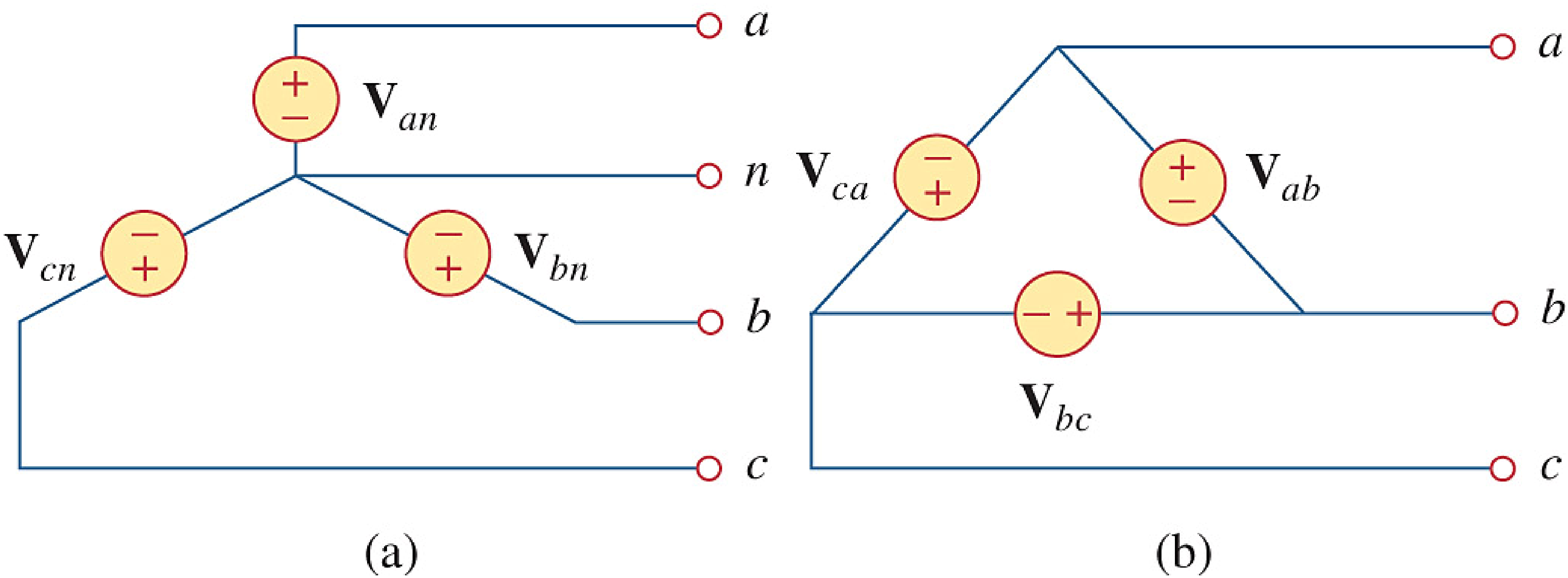

Figure 12.6 Three-phase voltage sources: (a) wye-connected, (b) delta-connected.

Let us consider the wye-connected voltages in Fig. 12.6(a). The voltages $\dot{V}_{an}$, $\dot{V}_{bn}$, and $\dot{V}_{cn}$ are respectively between lines $a, b,$ and c, and the neutral line n. These voltages are called *phase voltages*.

If the voltages have the same amplitude and frequency and are out of phase with each other by 120°, the voltages are said to be *balanced*. This implies that (Fig. 12.7)

$$\dot{V}_{an} + \dot{V}_{bn} + \dot{V}_{cn} = 0$$

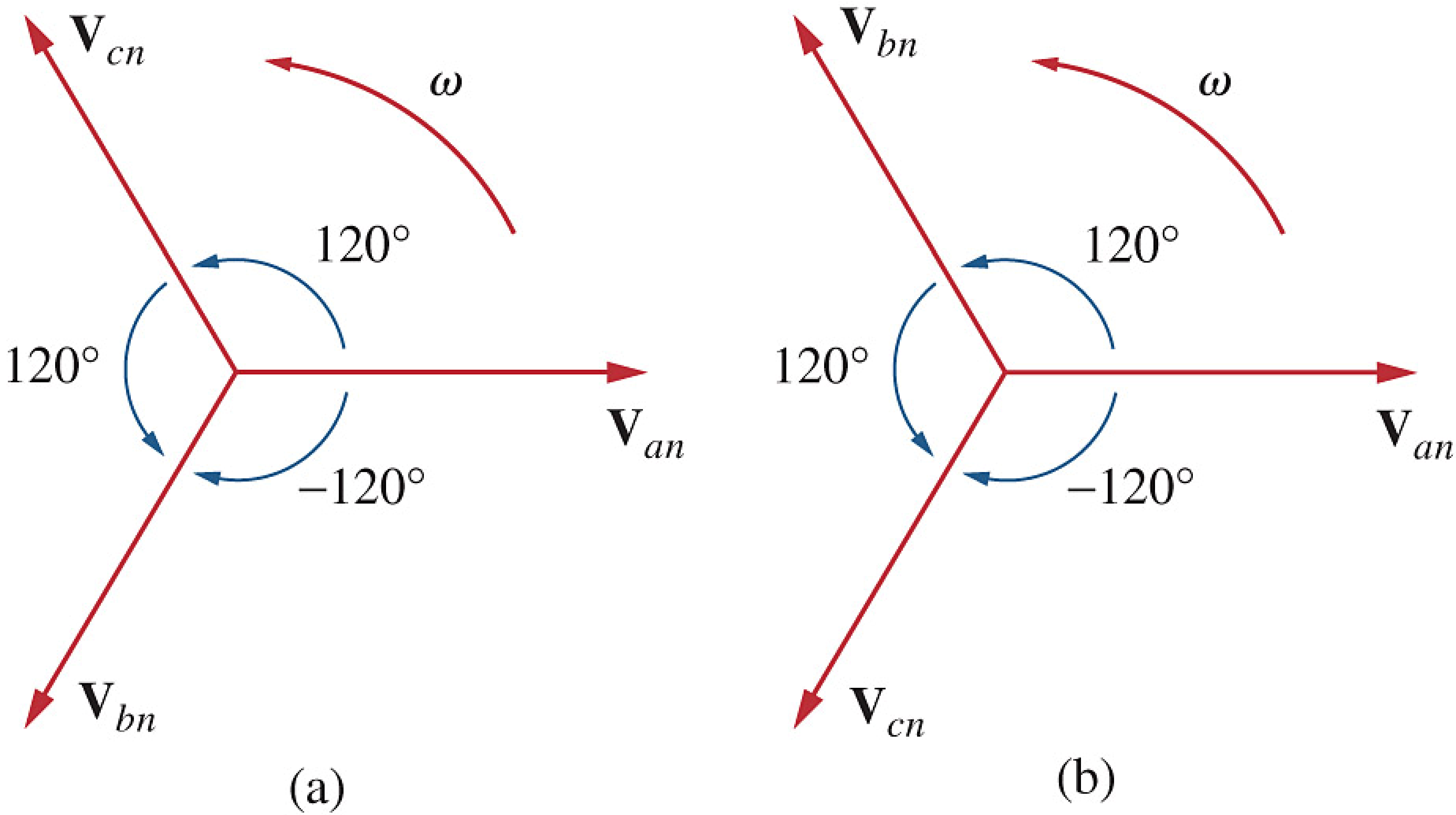

Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

In Fig. 12.7(a),

$$\dot{V}_{an} = V_p \angle 0°$$

$$\dot{V}_{bn} = V_p \angle -120°$$

$$\dot{V}_{cn} = V_p \angle -240° = V_p \angle +120°$$

where V_p is the effective or rms value of the phase voltages. This is known as the *abc sequence* or *positive sequence*.

In Fig. 12.7(b),

$$\dot{V}_{an} = V_p \angle 0°$$

$$\dot{V}_{cn} = V_p \angle -120°$$

$$\dot{V}_{bn} = V_p \angle -240° = V_p \angle +120°$$

This is called the *acb sequence* or *negative sequence.*

Like the source connections, a three-phase load can be either wye-connected or delta-connected, as shown in Fig. 12.8. The neutral line in Fig. 12.8(a) may or may not be there, depending on whether the system is four- or three-wire.

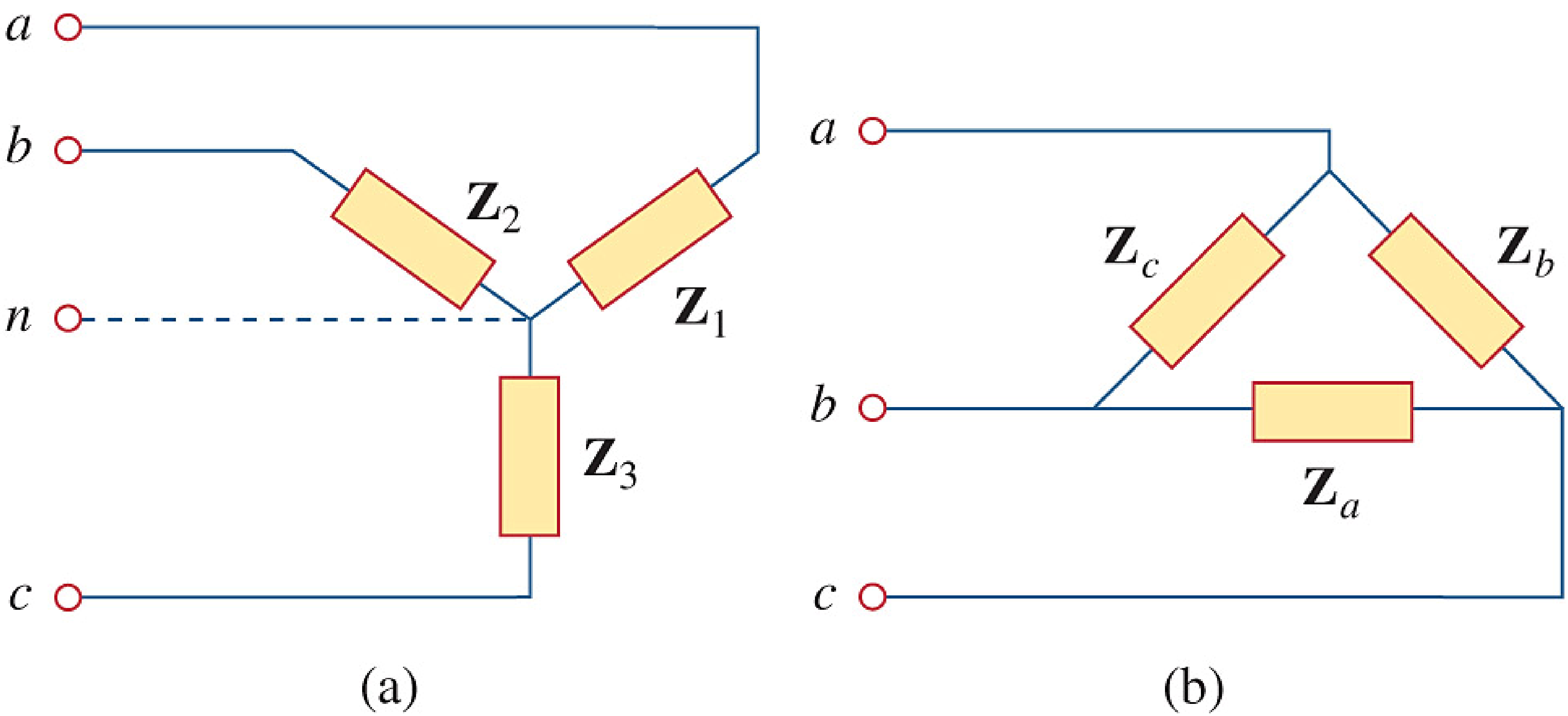

Figure 12.8 Two possible three-phase load configurations: (a) a wye-connected load, (b) a delta-connected load.

A wye- or delta-connected load is said to be balanced if the phase impedances are equal in magnitude and in phase.

For a balanced wye-connected load,

$$Z_1 = Z_2 = Z_3 = Z_Y$$

For a balanced delta-connected load,

$$Z_A = Z_B = Z_C = Z_\Delta$$

A wye-connected load can be transformed into a delta-connected load, or vice versa.

$$Z_{\Delta} = 3Z_{Y} \text{ or } Z_{Y} = \frac{1}{3} Z_{\Delta}$$

Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

1. Y-Y connection (i.e., Y-connected source with a Y-connected load).
2. Y-Δ connection.
3. Δ-Δ connection.
4. Δ-Y connection.

12.3 Balanced Wye-Wye Connection

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load. Figure 12.9 shows a balanced four-wire Y-Y system, which can be simplified to that shown in Fig. 12.10.

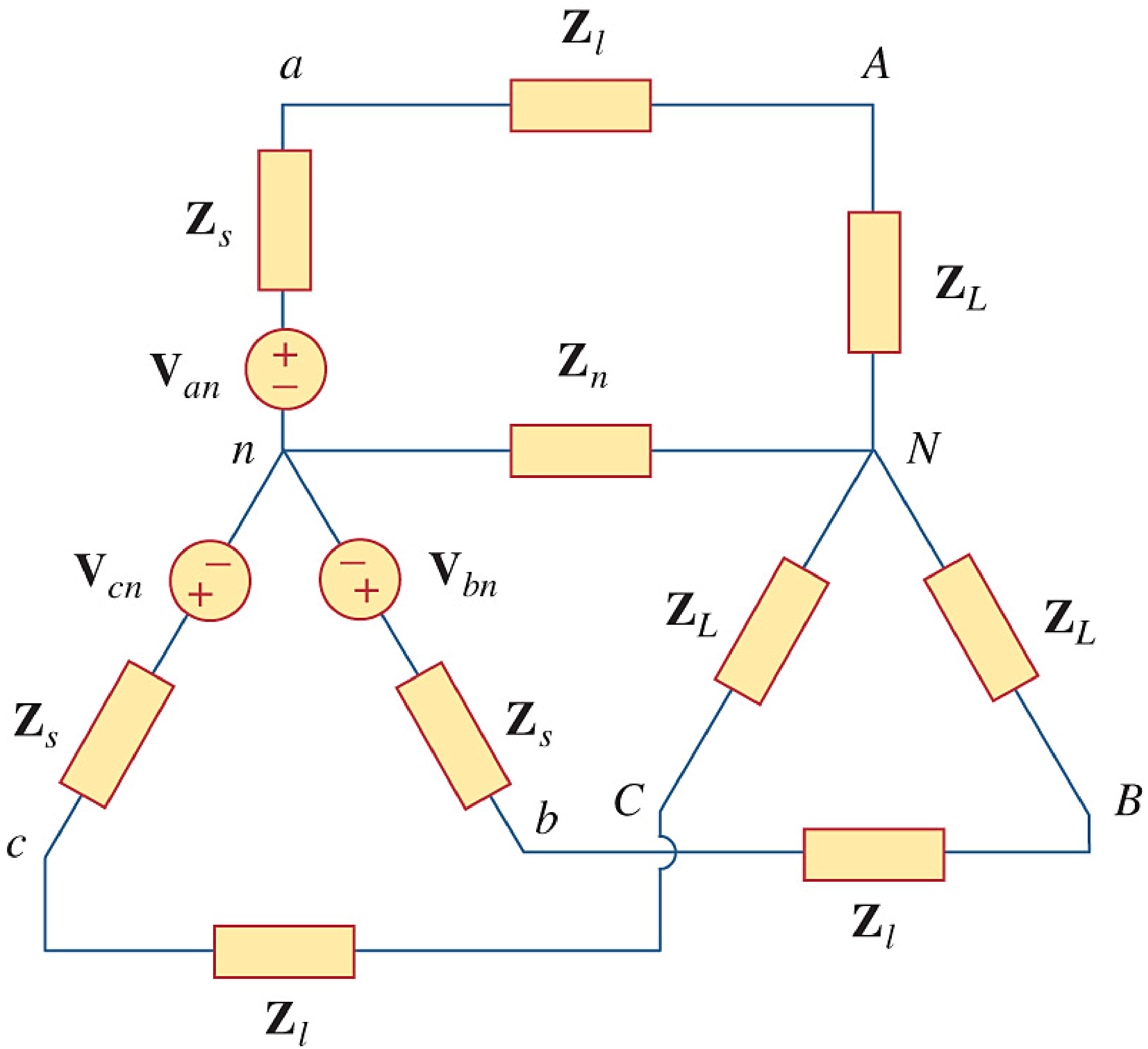

Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

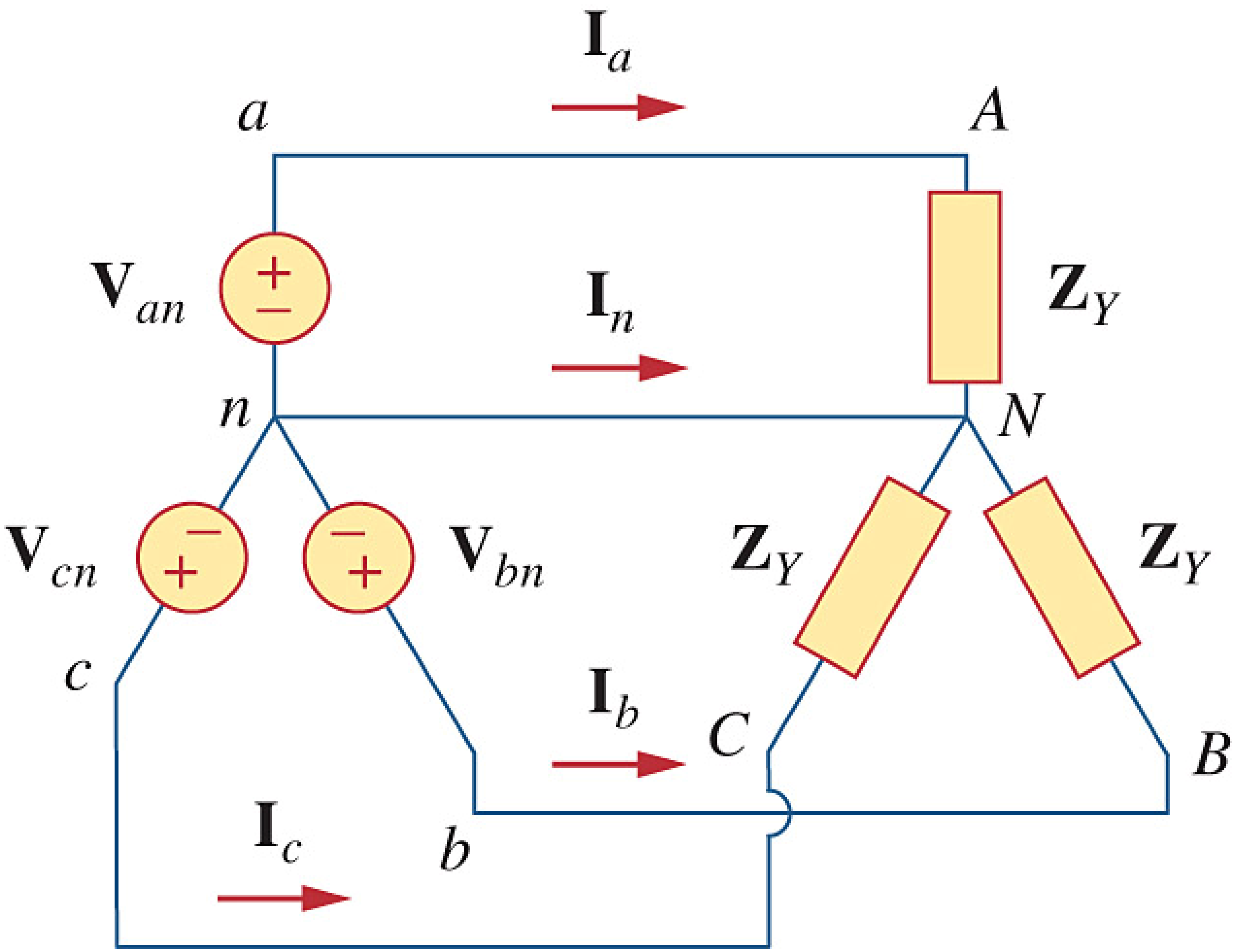

Figure 12.10 Balanced Y-Y connection.

Proof :

$$\dot{V}_{an} = \dot{I}_a (Z_s + Z_l + Z_L) - \dot{I}_n Z_n$$

$$\dot{V}_{bn} = \dot{I}_b (Z_s + Z_l + Z_L) - \dot{I}_n Z_n$$

$$\dot{V}_{cn} = \dot{I}_c (Z_s + Z_l + Z_L) - \dot{I}_n Z_n$$

$$\dot{V}_{an} + \dot{V}_{bn} + \dot{V}_{cn}$$

$$= (\dot{I}_a + \dot{I}_b + \dot{I}_c)(Z_s + Z_l + Z_L) - 3\dot{I}_n Z_n$$

but $\dot{I}_a + \dot{I}_b + \dot{I}_c + \dot{I}_n = 0$, i.e., $\dot{I}_a + \dot{I}_b + \dot{I}_c = -\dot{I}_n$

$$\dot{V}_{an} + \dot{V}_{bn} + \dot{V}_{cn} = -\dot{I}_n(Z_s + Z_l + Z_L) - 3\dot{I}_n Z_n$$

$$= -\dot{I}_n(Z_s + Z_l + Z_L + 3Z_n) = 0$$

$$\dot{I}_n = 0$$

$$\dot{V}_{nN} = \dot{I}_n Z_n = 0$$

so the neutral line can be replaced with

an open circuit or a short circuit.

Consider Fig. 12.10. Assuming the positive sequence, the phase voltages (or line-to-neutral volatges) are

$$\dot{V}_{an} = V_p \angle 0°$$

$$\dot{V}_{bn} = V_p \angle -120°$$

$$\dot{V}_{cn} = V_p \angle -240°$$

The line voltages (or line-to-line volatges) are

$$\dot{V}_{ab} = \dot{V}_{an} - \dot{V}_{bn} = \sqrt{3}V_p\angle 30°$$

$$\dot{V}_{bc} = \dot{V}_{bn} - \dot{V}_{cn} = \sqrt{3}V_p\angle -90° = \dot{V}_{ab}\angle -120°$$

$$\dot{V}_{ca} = \dot{V}_{cn} - \dot{V}_{an} = \sqrt{3}V_p\angle -210° = \dot{V}_{ab}\angle -240°$$

$$\dot{V}_{ab} + \dot{V}_{bc} + \dot{V}_{ca} = 0$$

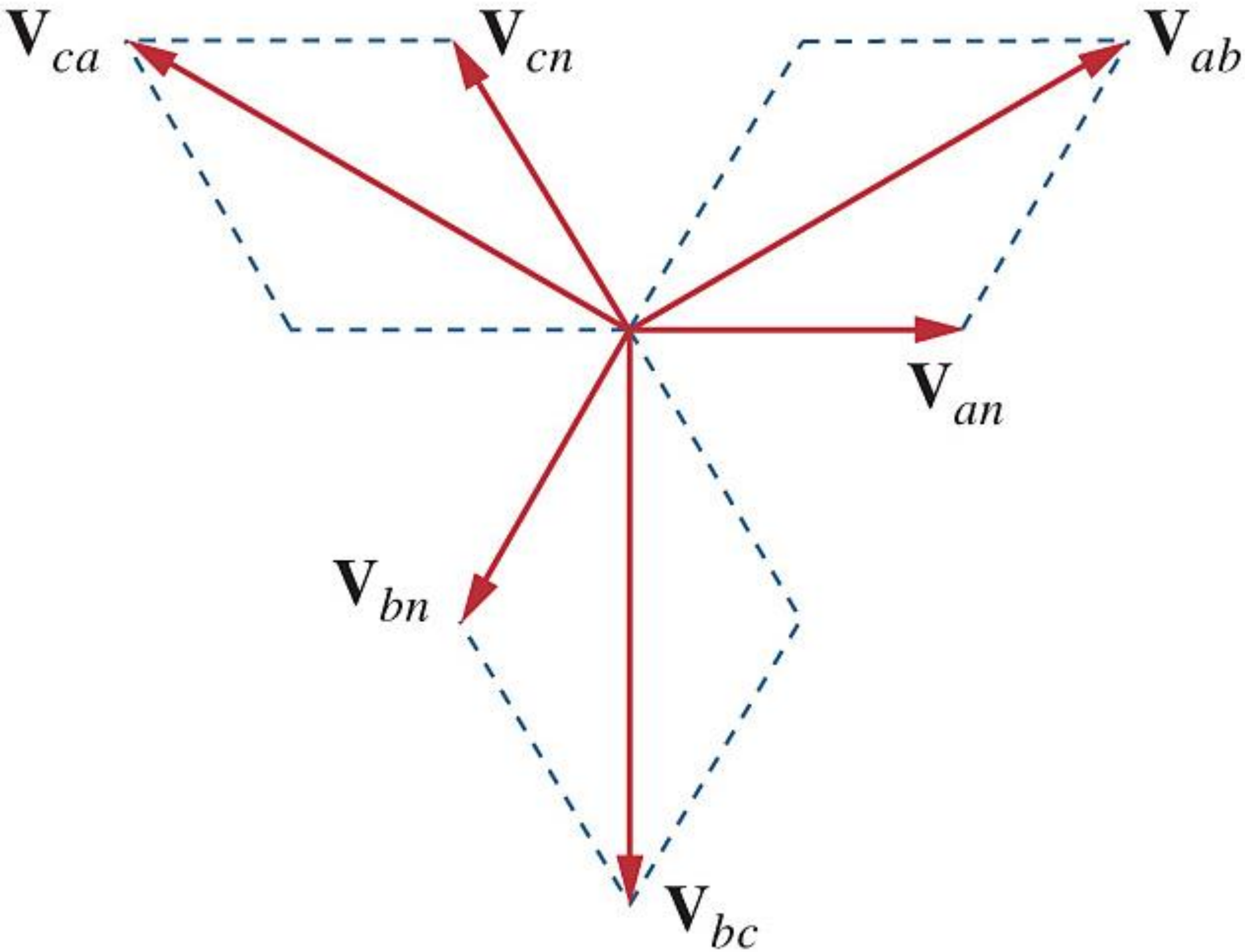

Figure 12.11(b) Phasor diagram illustrating the relationship between line voltages and phase voltages.

The magnitude of the line voltages is $\sqrt{3}$ times the magnitude of the phase volatges.

$$V_L = \sqrt{3} V_p$$

where

$$V_p = \left|\dot{V}_{an}\right| = \left|\dot{V}_{bn}\right| = \left|\dot{V}_{cn}\right|$$

$$V_L = \left|\dot{V}_{ab}\right| = \left|\dot{V}_{bc}\right| = \left|\dot{V}_{ca}\right|$$

The line voltages lead their corresponding phase volatges by 30°.

$$\dot{V}_{ab} = \dot{V}_{an}\angle 30°$$

$$\dot{V}_{bc} = \dot{V}_{bn}\angle 30°$$

$$\dot{V}_{ca} = \dot{V}_{cn}\angle 30°$$

The line currents are

$$\dot{I}_a = \frac{\dot{V}_{an}}{Z_Y}$$

$$\dot{I}_b = \frac{\dot{V}_{bn}}{Z_Y} = \dot{I}_a \angle -120°$$

$$\dot{I}_c = \frac{\dot{V}_{cn}}{Z_Y} = \dot{I}_a \angle -240°$$

Since $\dot{I}_n = 0$, the neutral line can be removed without affecting the system. In fact, in long distance power transmission, conductors in multiples of three are used with the earth itself as the neutral ground. Power systems designed in this way are well ground at all critical points to ensure safety.

While the line current is the current in each line, the phase current is the current in each phase of the source or load. In the Y-Y system, the line current is the same as the phase current.

An alternative way of analyzing a balanced Y-Y system is to do so on a per phase basis. We look at one phase, say phase *a*, and analyze the single-phase equivalent circuit in Fig. 12.12.

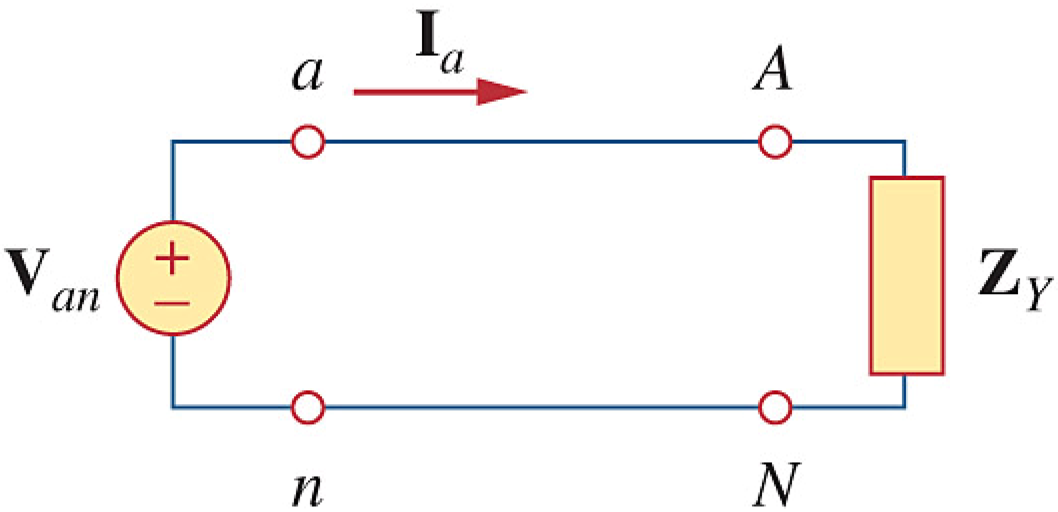

Figure 12.12 A single-phase equivalent circuit.

Practice Problem 12.2 A Y-connected balanced three-phase generator with an impedance of $0.4+j0.3\ \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24+j19\ \Omega$ per phase.The line joining the generator and the load has an impedance of $0.6+j0.7\ \Omega$ per phase. Assuming a positive sequence for the source voltages and that $\dot{V}_{an}=120\angle 30°$ V, find the line voltages and the line currents.

Solution :

$$Z_Y = Z_s + Z_l + Z_L = (0.4 + j0.3) + (24 + j19)$$

$$+(0.6 + j0.7) = 25 + j20\ (\Omega)$$

$$\dot{I}_a = \frac{\dot{V}_{an}}{Z_Y} = \frac{120\angle 30^\circ}{25 + j20} \approx \frac{120\angle 30^\circ}{32.0156\angle 38.66^\circ}$$

$$\approx 3.7482\angle -8.66^\circ\ (\text{A})$$

$$\dot{I}_b = \dot{I}_a \angle -120^\circ = 3.7482\angle -128.66^\circ\ (\text{A})$$

$$\dot{I}_c = \dot{I}_a \angle -240^\circ = 3.7482\angle -248.66^\circ\ (\text{A})$$

The "ideal" line voltages are

$$\dot{V}_{ab} = \dot{V}_{an}\sqrt{3}\angle 30^\circ = 120\angle 30^\circ \times \sqrt{3}\angle 30^\circ$$

$$\approx 207.8461\angle 60^\circ \text{ (V)}$$

$$\dot{V}_{bc} = \dot{V}_{ab}\angle -120^\circ = 207.8461\angle -60^\circ \text{ (V)}$$

$$\dot{V}_{ca} = \dot{V}_{ab}\angle -240^\circ = 207.8461\angle -180^\circ \text{ (V)}$$

The line voltages seen from the generator terminals are

$$\dot{V}_{a'b'} = \dot{I}_a(Z_l + Z_L) - \dot{I}_b(Z_l + Z_L)$$

$$= \dot{I}_a\sqrt{3}\angle 30^\circ(Z_l + Z_L)$$

$$= \frac{\dot{V}_{an}}{Z_Y}\sqrt{3}\angle 30^\circ(Z_l + Z_L) = \dot{V}_{ab}\left(\frac{Z_l + Z_L}{Z_Y}\right)$$

$$= 207.8461\angle 60^\circ \times \left(\frac{24.6 + j19.7}{25 + j20} \right)$$

$$\approx 207.8461\angle 60^\circ \times \frac{31.5159\angle 38.69^\circ}{32.0156\angle 38.66^\circ}$$

$$\approx 204.6020\angle 60.03^\circ \text{ (V)}$$

$$\dot{V}_{b'c'} = 204.6020\angle -59.97^\circ \text{ (V)}$$

$$\dot{V}_{c'a'} = 204.6020\angle -179.97^\circ \text{ (V)}$$

The line voltages seen from the load terminals are

$$\dot{V}_{AB} = \dot{I}_a Z_L - \dot{I}_b Z_L = \dot{V}_{ab} \frac{Z_L}{Z_Y}$$

$$= 207.8461\angle 60^\circ \times \left(\frac{24+19}{25+j20} \right)$$

$$\approx 207.8461\angle 60^\circ \times \frac{30.6105\angle 38.37^\circ}{32.0156\angle 38.66^\circ}$$

$$\approx 198.7242\angle 59.71^\circ \text{ (V)}$$

$$\dot{V}_{BC} \approx 198.7242\angle -60.29^\circ \text{ (V)}$$

$$\dot{V}_{CA} \approx 198.7242\angle -180.29^\circ \text{ (V)}$$

The line voltage drops are

$$\dot{I}_a Z_l = 3.7482\angle -8.66^\circ \times (0.6 + j0.7)$$

$$\approx 3.4558\angle 40.74^\circ \text{ (V)}$$

$$\dot{I}_b Z_l = 3.4558\angle -79.26^\circ \text{ (V)}$$

$$\dot{I}_b Z_l = 3.4558\angle -199.26^\circ \text{ (V)}$$

12.4 Balanced Wye-Delta Connection

A Balanced Y-Δ system consists of a balanced Y-connected source feeding a balanced Δ-connected load, as shown in Fig. 12.14.

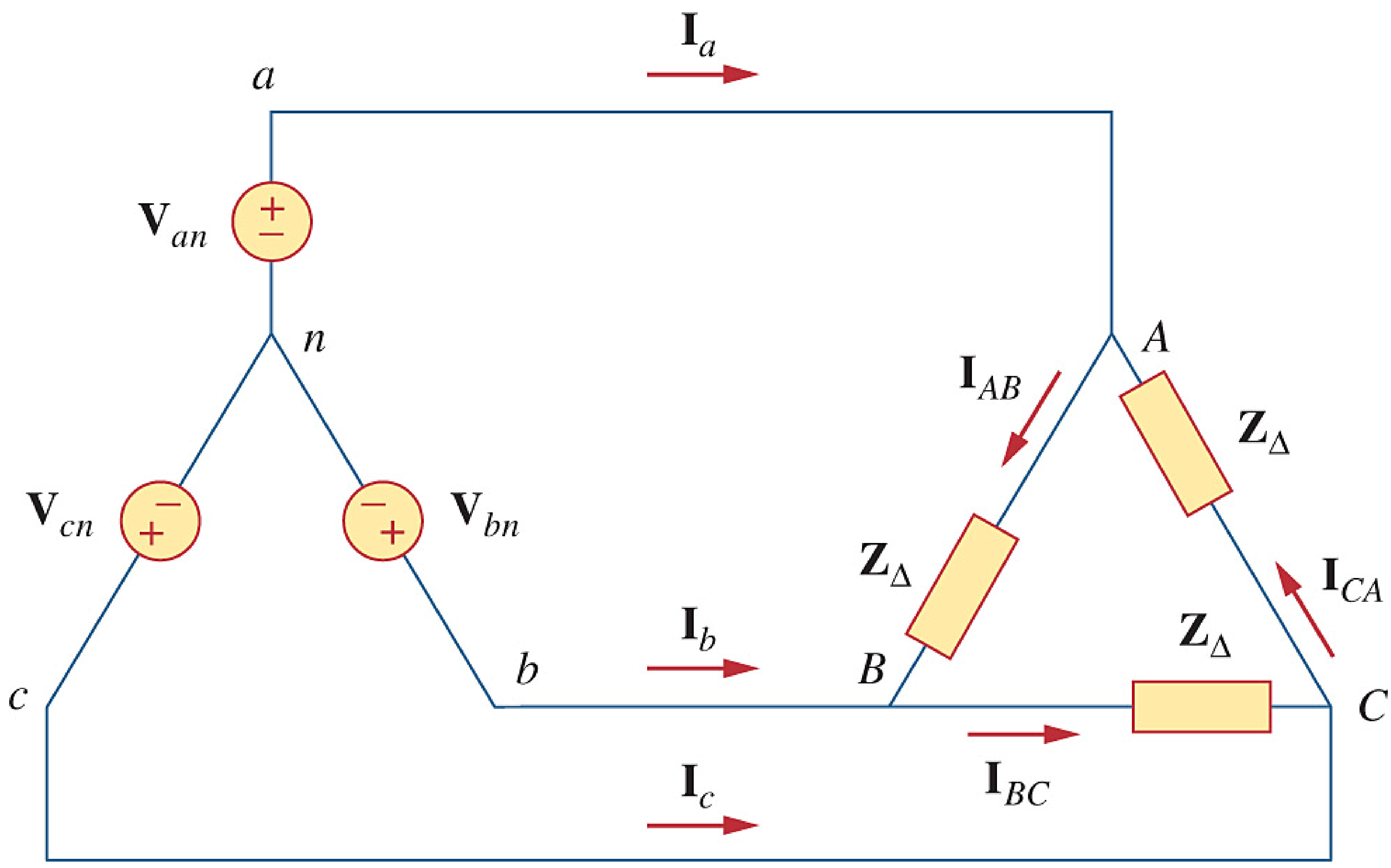

Figure 12.14 Balanced wye-delta connection.

Assume that

$$\dot{V}_{an} = V_p \angle 0^\circ$$

$$\dot{V}_{bn} = V_p \angle -120^\circ$$

$$\dot{V}_{cn} = V_p \angle -240^\circ$$

The line voltages are

$$\dot{V}_{ab} = \sqrt{3} V_p \angle 30^\circ = \dot{V}_{an} \sqrt{3} \angle 30^\circ = \dot{V}_{AB}$$

$$\dot{V}_{bc} = \dot{V}_{ab} \angle -120^\circ = \dot{V}_{BC}$$

$$\dot{V}_{ca} = \dot{V}_{ab} \angle -240^\circ = \dot{V}_{CA}$$

The phase currents are

$$\dot{I}_{AB} = \frac{\dot{V}_{AB}}{Z_{\Delta}}$$

$$\dot{I}_{BC} = \frac{\dot{V}_{BC}}{Z_{\Delta}} = \dot{I}_{AB} \angle -120°$$

$$\dot{I}_{CA} = \frac{\dot{V}_{CA}}{Z_{\Delta}} = \dot{I}_{AB} \angle -240°$$

The line currents are

$$\dot{I}_a = \dot{I}_{AB} - \dot{I}_{CA} = \dot{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\dot{I}_b = \dot{I}_{BC} - \dot{I}_{AB} = \dot{I}_{BC}\sqrt{3}\angle -30^\circ = \dot{I}_a\angle -120^\circ$$

$$\dot{I}_c = \dot{I}_{CA} - \dot{I}_{BC} = \dot{I}_{CA}\sqrt{3}\angle -30^\circ = \dot{I}_a\angle -240^\circ$$

showing that

$$I_L = \sqrt{3}I_p$$

where

$$I_L = \left|\dot{I}_a\right| = \left|\dot{I}_b\right| = \left|\dot{I}_c\right|,\ I_p = \left|\dot{I}_{AB}\right| = \left|\dot{I}_{BC}\right| = \left|\dot{I}_{CA}\right|$$

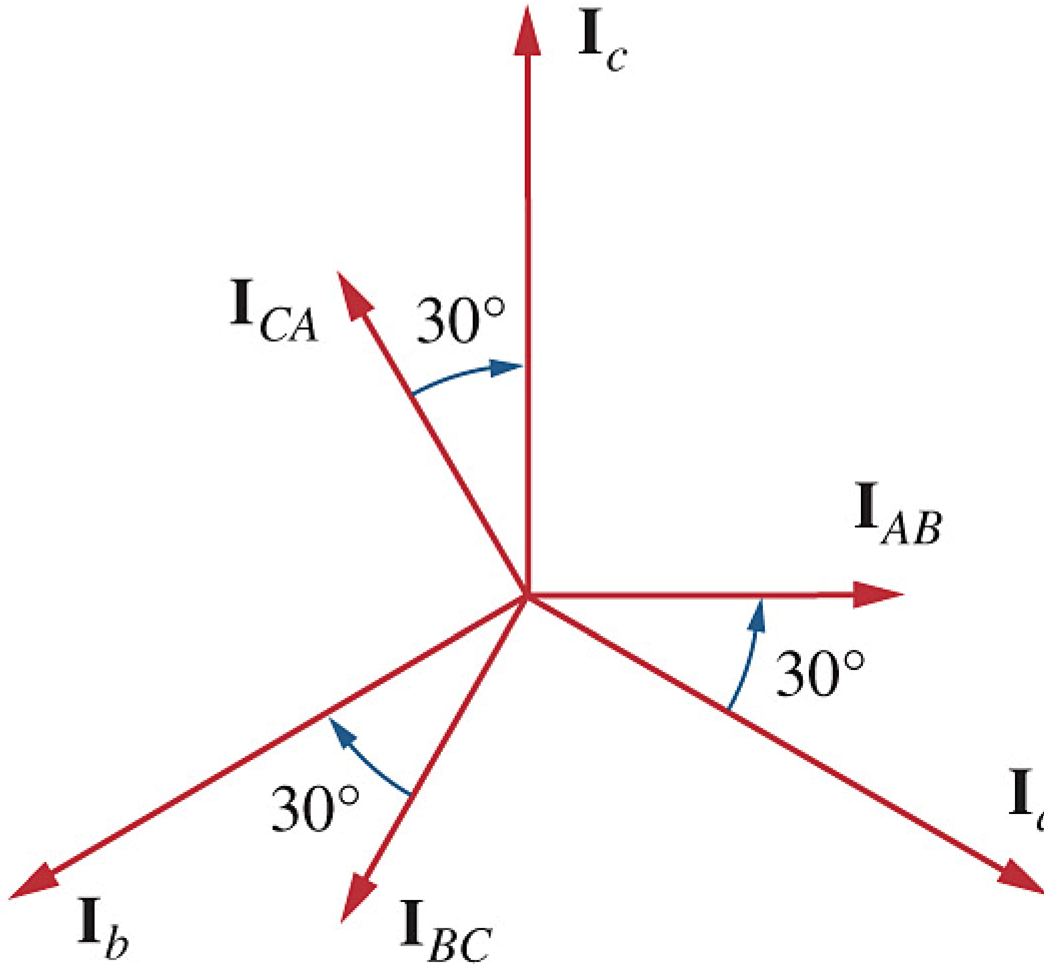

Figure 12.15 Phasor diagram illustrating the relationship between phase and line currents.

Practice Problem 12.3 One line voltage of a balanced Y-connected source is $\dot{V}_{AB} = 240\angle -20°$ V. If the source is connected to a Δ-connected load of $20\angle 40°\ \Omega$, find the phase and line currents. Assume the *abc* sequence.

Solution :

$$\dot{I}_{AB} = \frac{\dot{V}_{AB}}{Z_\Delta} = \frac{240\angle -20°}{20\angle 40°} = 12\angle -60°\ \text{(A)}$$

$$\dot{I}_{BC} = \dot{I}_{AB}\angle -120° = 12\angle -180° \text{ (A)}$$

$$\dot{I}_{CA} = \dot{I}_{AB}\angle +120° = 12\angle 60° \text{ (A)}$$

$$\dot{I}_{a} = \dot{I}_{AB}\sqrt{3}\angle -30° \approx 20.7846\angle -90° \text{ (A)}$$

$$\dot{I}_{b} = \dot{I}_{a}\angle +240° = 20.7846\angle 150° \text{ (A)}$$

$$\dot{I}_{c} = \dot{I}_{a}\angle +120° = 20.7846\angle 30° \text{ (A)}$$

12.5 Balanced Delta-Delta Connection

A balanced Δ-Δ system is one in which both the balanced source and balanced load are Δ-connected, as shown in Fig. 12.17.

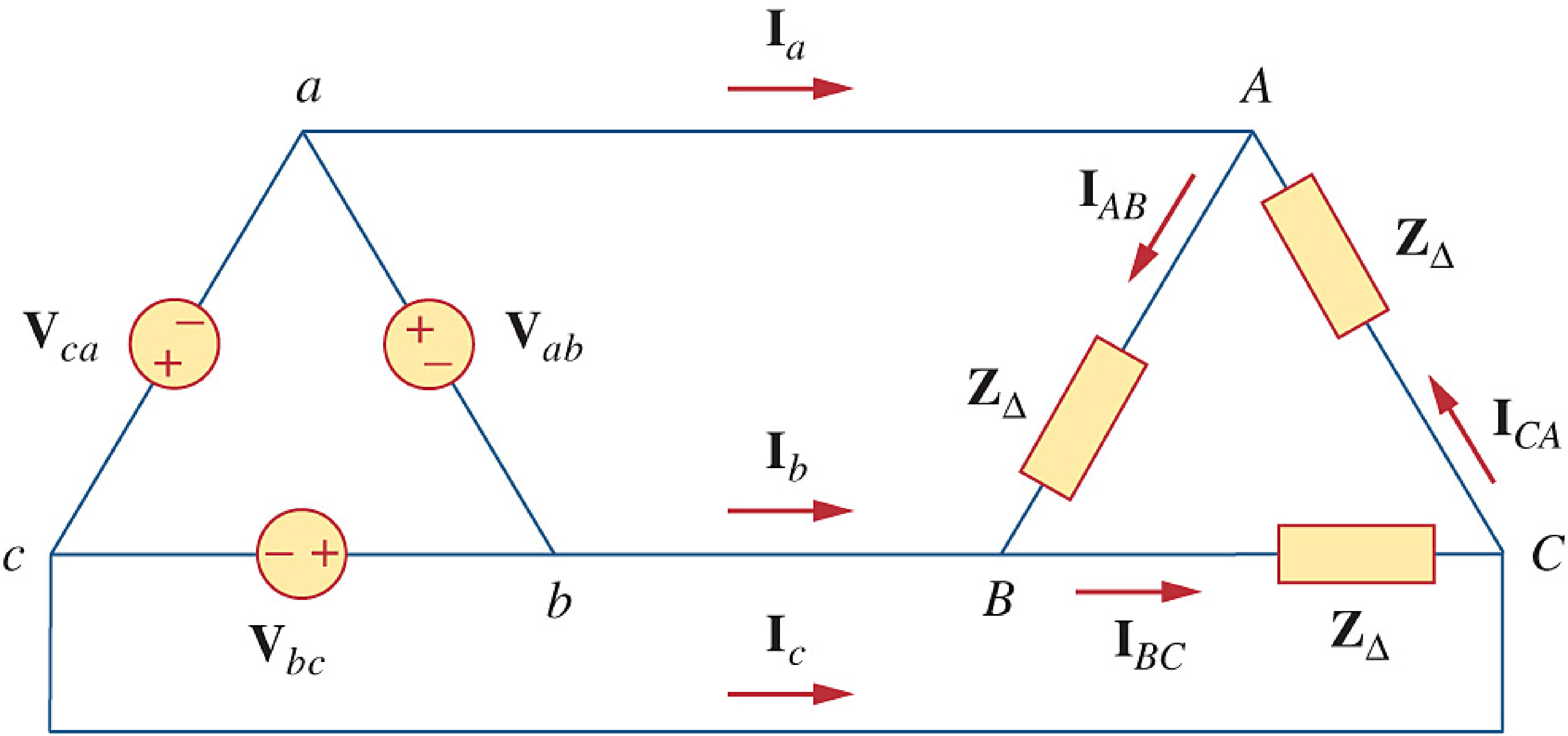

Figure 12.17 A balanced delta-delta connection.

Assume that

$$\dot{V}_{ab} = V_p \angle 0°$$

$$\dot{V}_{bc} = V_p \angle -120°$$

$$\dot{V}_{ca} = V_p \angle +120°$$

The line voltages are

$$\dot{V}_{ab} = \dot{V}_{AB}, \dot{V}_{bc} = \dot{V}_{BC}, \dot{V}_{ca} = \dot{V}_{CA}$$

The phase currents are

$$\dot{I}_{AB} = \frac{\dot{V}_{AB}}{Z_{\Delta}}$$

$$\dot{I}_{BC} = \frac{\dot{V}_{BC}}{Z_{\Delta}} = \dot{I}_{AB}\angle -120°$$

$$\dot{I}_{CA} = \frac{\dot{V}_{CA}}{Z_{\Delta}} = \dot{I}_{AB}\angle +120°$$

The line currents are

$$\dot{I}_a = \dot{I}_{AB} - \dot{I}_{CA} = \dot{I}_{AB}\sqrt{3}\angle -30°$$

$$\dot{I}_b = \dot{I}_{BC} - \dot{I}_{AB} = \dot{I}_{BC}\sqrt{3}\angle -30° = \dot{I}_a\angle -120°$$

$$\dot{I}_c = \dot{I}_{CA} - \dot{I}_{BC} = \dot{I}_{CA}\sqrt{3}\angle -30° = \dot{I}_a\angle -240°$$

showing that

$$I_L = \sqrt{3} I_p$$

where

$$I_L = \left|\dot{I}_a\right| = \left|\dot{I}_b\right| = \left|\dot{I}_c\right|, \ I_p = \left|\dot{I}_{AB}\right| = \left|\dot{I}_{BC}\right| = \left|\dot{I}_{CA}\right|$$

12.6 Balanced Delta-Wye Connection

A balanced Δ-Y system consists of a balanced Δ-connected source feeding a balanced Y-connected load, as shown in Fig. 12.18.

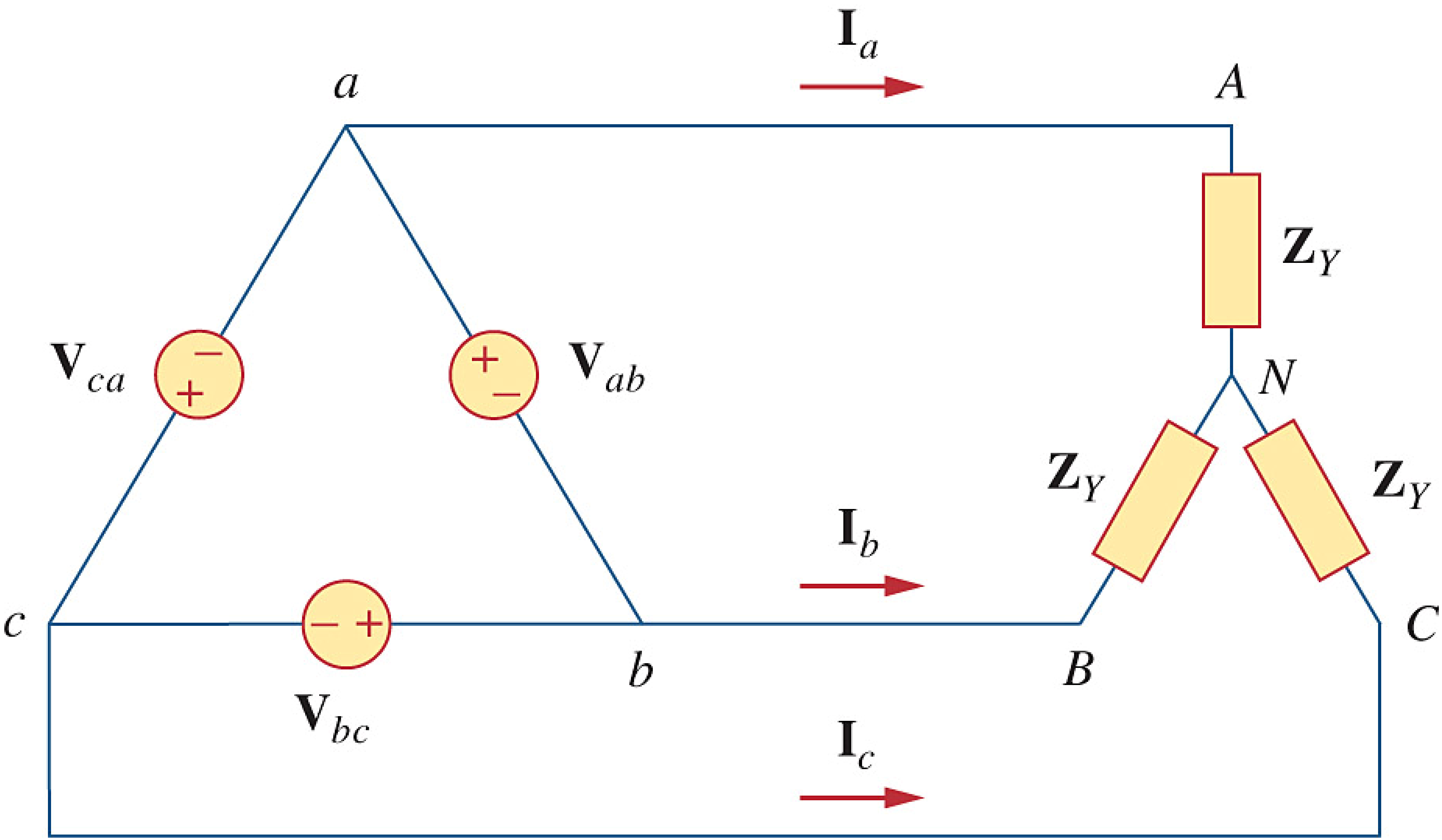

Figure 12.18 A balanced delta-wye connection.

Assume that

$$\dot{V}_{ab} = V_p \angle 0°$$

$$\dot{V}_{bc} = V_p \angle -120°$$

$$\dot{V}_{ca} = V_p \angle +120°$$

Replace the Y-connected load with its equivalent Δ-connected load and recall that for a Δ-connected load,

$$\dot{I}_a = \dot{I}_{AB} \sqrt{3} \angle -30°$$

$$\dot{I}_a = \dot{I}_{AB}\sqrt{3}\angle -30^\circ = \frac{\dot{V}_{AB}}{Z_\Delta}\sqrt{3}\angle -30^\circ$$

$$= \frac{\dot{V}_{ab}}{3Z_Y}\sqrt{3}\angle -30^\circ = \frac{\dot{V}_{ab}}{\sqrt{3}Z_Y}\angle -30^\circ$$

$$\dot{I}_b = \frac{\dot{V}_{bc}}{\sqrt{3}Z_Y}\angle -30^\circ = \dot{I}_a\angle -120^\circ$$

$$\dot{I}_c = \frac{\dot{V}_{ca}}{\sqrt{3}Z_Y}\angle -30^\circ = \dot{I}_a\angle +120^\circ$$

12.7 Power in a Balanced System

We begin by examining the instantaneous power absorbed by the load. For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t$$

$$v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°)$$

If $Z_Y = Z\angle\theta$, the phase currents are

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta)$$

$$i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120°)$$

$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120°)$$

The total instantaneous power is

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c$$

$$
\begin{aligned}
&= 2V_p I_p[\cos\omega t\cos(\omega t-\theta) \\
&+\cos(\omega t-120^\circ)\cos(\omega t-\theta-120^\circ) \\
&+\cos(\omega t+120^\circ)\cos(\omega t-\theta+120^\circ)] \\
&= 2V_p I_p\{\frac{\cos\theta+\cos(2\omega t-\theta)}{2} \\
&+\frac{\cos\theta+\cos(2\omega t-\theta-240^\circ)}{2} \\
&+\frac{\cos\theta+\cos(2\omega t-\theta+240^\circ)}{2}] \\
&= 3V_p I_p\cos\theta
\end{aligned}
$$

Thus the total instantaneous power in a balanced three-phase system is constant – it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ-connected. This is one important reason for using a three-phase system to generate and distribute power.

Since the total instantaneous power is independent of time, the average power per phase P_p for the Y- or Δ-connected load is $p/3$, or

$$P_p = V_p I_p \cos\theta$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin\theta$$

The apparent power per phase is

$$\left|S_p\right| = V_p I_p$$

The complex power per phase is

$$S_p = P_p + jQ_p = \dot{V}_p \dot{I}_p^*$$

The total average power is

$$P = P_a + P_b + P_c = 3P_p$$

$$= 3V_p I_p \cos\theta$$

For a Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}V_p$, whereas for a Δ-connected load, $I_L = \sqrt{3}I_p$ but $V_L = V_p$. Thus,

$$P = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$$

The total reactive power is

$$Q = 3Q_p = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta$$

The total complex power is

$$S = 3S_p = 3\dot{V}_p \dot{I}_p^* = 3I_p^2 Z_p = \frac{3V_p^2}{Z_p^*}$$

where Z_p is the load impedance per phase.

$$S = P + jQ = 3V_p I_p \cos\theta + j3V_p I_p \sin\theta$$

$$= \sqrt{3} V_L I_L \cos\theta + j\sqrt{3} V_L I_L \cos\theta$$

Remember that V_p, I_p, V_L, and I_L are all rms values and that θ is the angle of the load impedance or the angle between the phase voltage and the phase current.

Consider Fig. 12.21. For the single-phase two-wire system, $P_L = V_L I_L$, the power loss in the two wires is

$$P_{loss} = 2 I_L^2 R = 2 \left(\frac{P_L}{V_L} \right)^2 R$$

For the three-phase three-wire system, $P_L = \sqrt{3} V_L I'_L$, the power loss in the three wires is

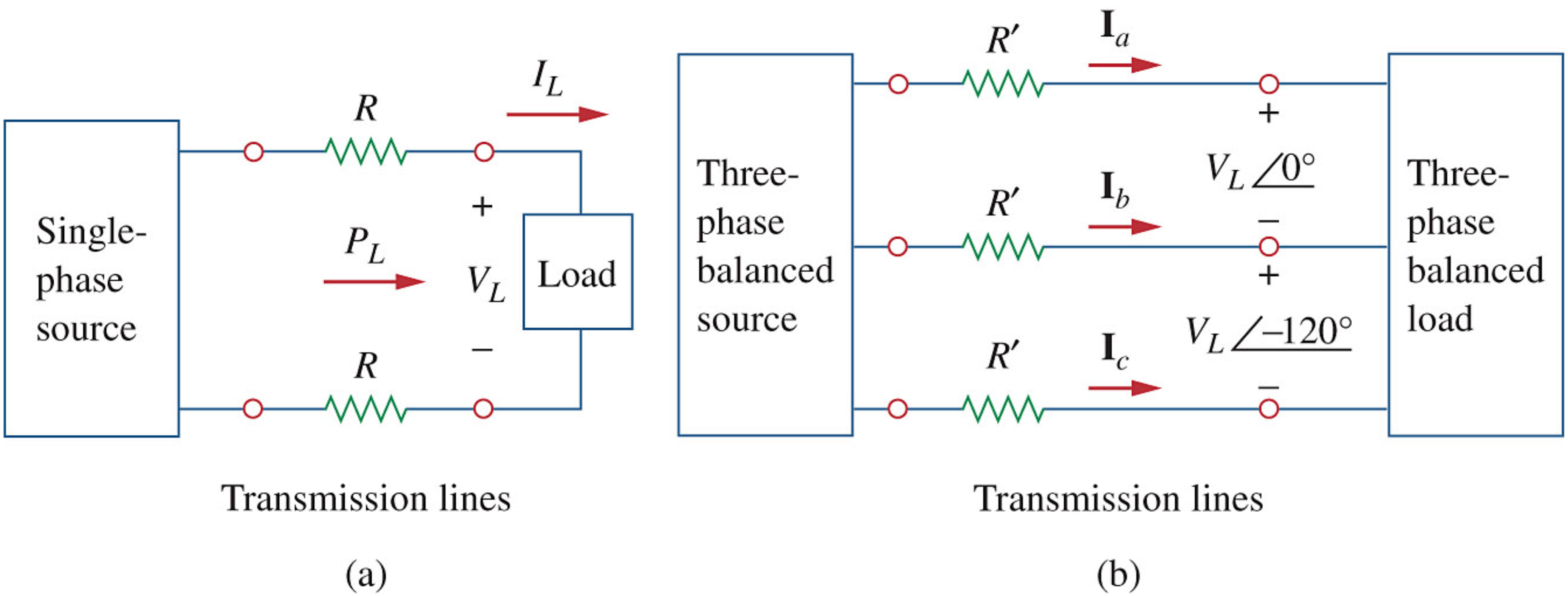

Figure 12.21 Comparing the power loss in (a) single-phase system, and (b) a three-phase system.

$$P'_{loss} = 3I_L'^2 R' = 3\left(\frac{P_L}{\sqrt{3}V_L}\right)^2 R' = \left(\frac{P_L}{V_L}\right)^2 R'$$

The equations show that for the same total power delivered P_L and the same line voltage V_L,

$$\frac{P_{loss}}{P'_{loss}} = \frac{2R}{R'} = \frac{2\rho l / (\pi r^2)}{\rho l / (\pi r'^2)} = \frac{2r'^2}{r^2}$$

If the same power loss is tolerated in both systems, then $r^2 = 2r'^2$, so

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 l)}{3(\pi r'^2 l)}$$

$$= \frac{2r^2}{3r'^2} = \frac{4}{3}$$

This is the second major advantage of three-phase systems for power distribution.

Example 12.8 Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line current, and (c) the kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

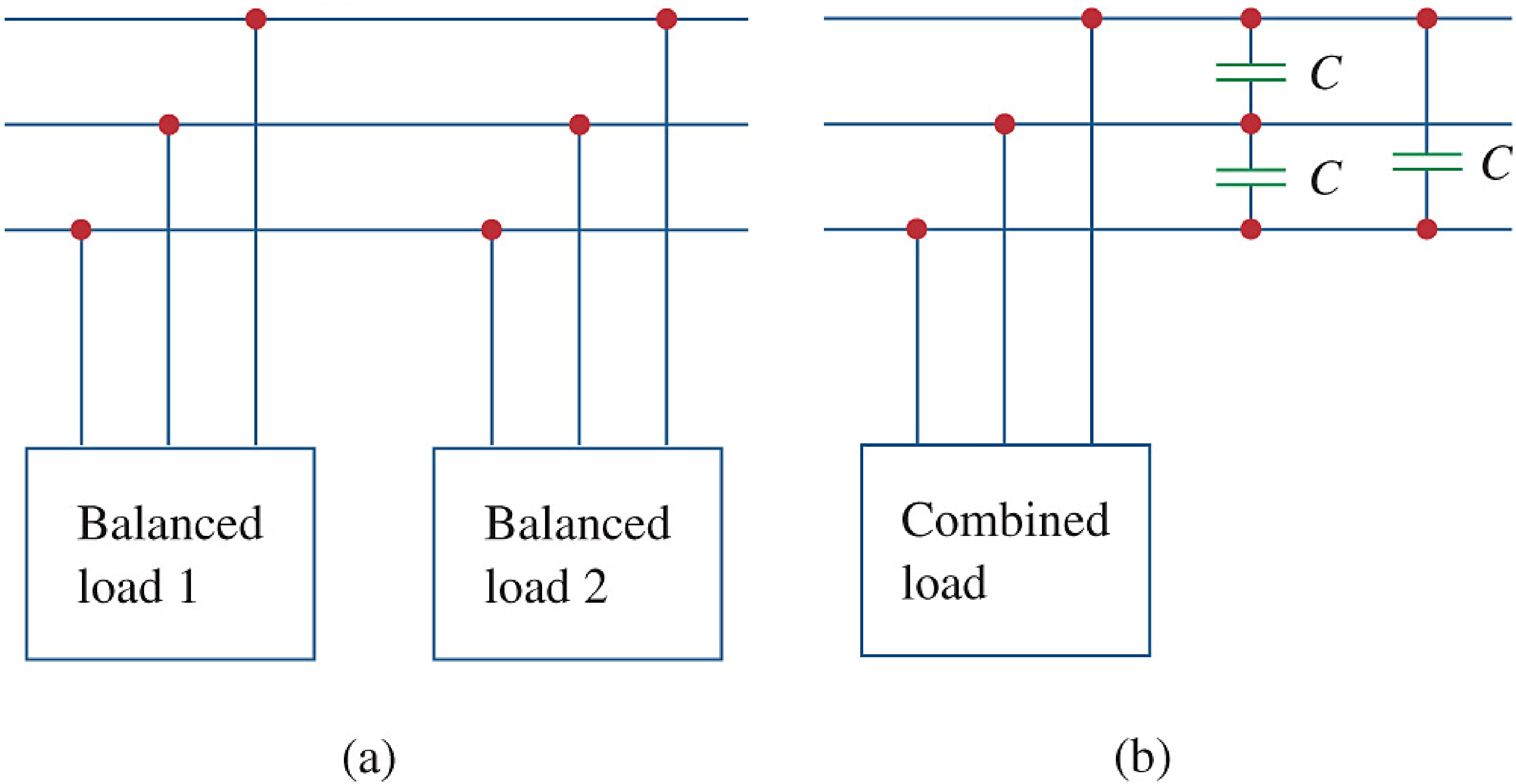

Figure 12.22 (a) The original balanced loads, (b) the combined load with improved power factor.

Solution :

$$\text{(a)}\ \cos\theta_1 = 0.6 \Rightarrow \sin\theta_1 = 0.8$$

$$Q_1 = P_1 \tan\theta_1 = 30 \times \frac{0.8}{0.6} = 40\ \text{(kVAR)}$$

$$S_1 = P_1 + jQ_1 = 30 + j40\ \text{(kVA)}$$

$$\cos\theta_2 = 0.8 \Rightarrow \sin\theta_2 = 0.6$$

$$P_2 = Q_2 / \tan\theta_2 = 45/(0.6/0.8) = 60\ \text{(kW)}$$

$$S_2 = P_2 + jQ_2 = 60 + j45\ \text{(kVA)}$$

$$S = S_1 + S_2 = (30 + j40) + (60 + j45)$$

$$= 90 + j85 \text{ (kVA)}$$

$$P = 90 \text{ kW}, Q = 85 \text{ kVA}$$

(b) $\theta = \tan^{-1}\left(\frac{85}{90}\right) \approx 43.36°$

$$P = \sqrt{3} V_L I_L \cos\theta \Rightarrow I_L = \frac{P}{\sqrt{3} V_L \cos\theta}$$

$$I_L = \frac{90 \times 10^3}{\sqrt{3} \times 240 \times 10^3 \times \cos 43.36°} \approx 0.2978 \text{ (A)}$$

$$S = S_1 + S_2 = (30 + j40) + (60 + j45)$$

$$= 90 + j85 \text{ (kVA)}$$

$$P = 90 \text{ kW}, \ Q = 85 \text{ kVA}$$

(b) $\theta = \tan^{-1}\left(\dfrac{85}{90}\right) \approx 43.36°$

$$P = \sqrt{3} V_L I_L \cos\theta \Rightarrow I_L = \frac{P}{\sqrt{3} V_L \cos\theta}$$

$$I_L = \frac{90 \times 10^3}{\sqrt{3} \times 240 \times 10^3 \times \cos 43.36°} \approx 0.2978 \text{ (A)}$$

$$\text{(c) } \theta_{old} = \tan^{-1}\left(\frac{85}{90}\right) \approx 43.36°$$

$$\theta_{new} = \cos^{-1} 0.9 \approx 25.84°$$

$$\left|Q_C\right| = P(\tan\theta_{old} - \tan\theta_{new})$$

$$= 90 \times (\tan 43.36° - \tan 25.84°)$$

$$\approx 41.40 \text{ (kVAR)}$$

This reactive power is for the three capacitors. For each capacitor,

$$\left|Q_C'\right| = \left|Q_C\right| / 3 \approx 13.80 \text{ (kVAR)}$$

$$C = \frac{\left|Q_C'\right|}{\omega V_L^2} = \frac{13.80 \times 10^3}{2\pi \times 60 \times (240 \times 10^3)^2}$$

$$\approx 6.3551 \times 10^{-10} \text{ (F)}$$

$$\approx 635.51 \text{ pF}$$

12.8 Unbalanced Three-Phase Systems

An unbalanced system is caused by two possible situations: (1) the source voltages are not equal in magnitude and/or differ in phase by angles that are unequal, or (2) load impedances are unequal.

Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis. Figure 12.23 shows an unbalanced system that consists of balanced source voltages and an unbalanced Y-connected load.

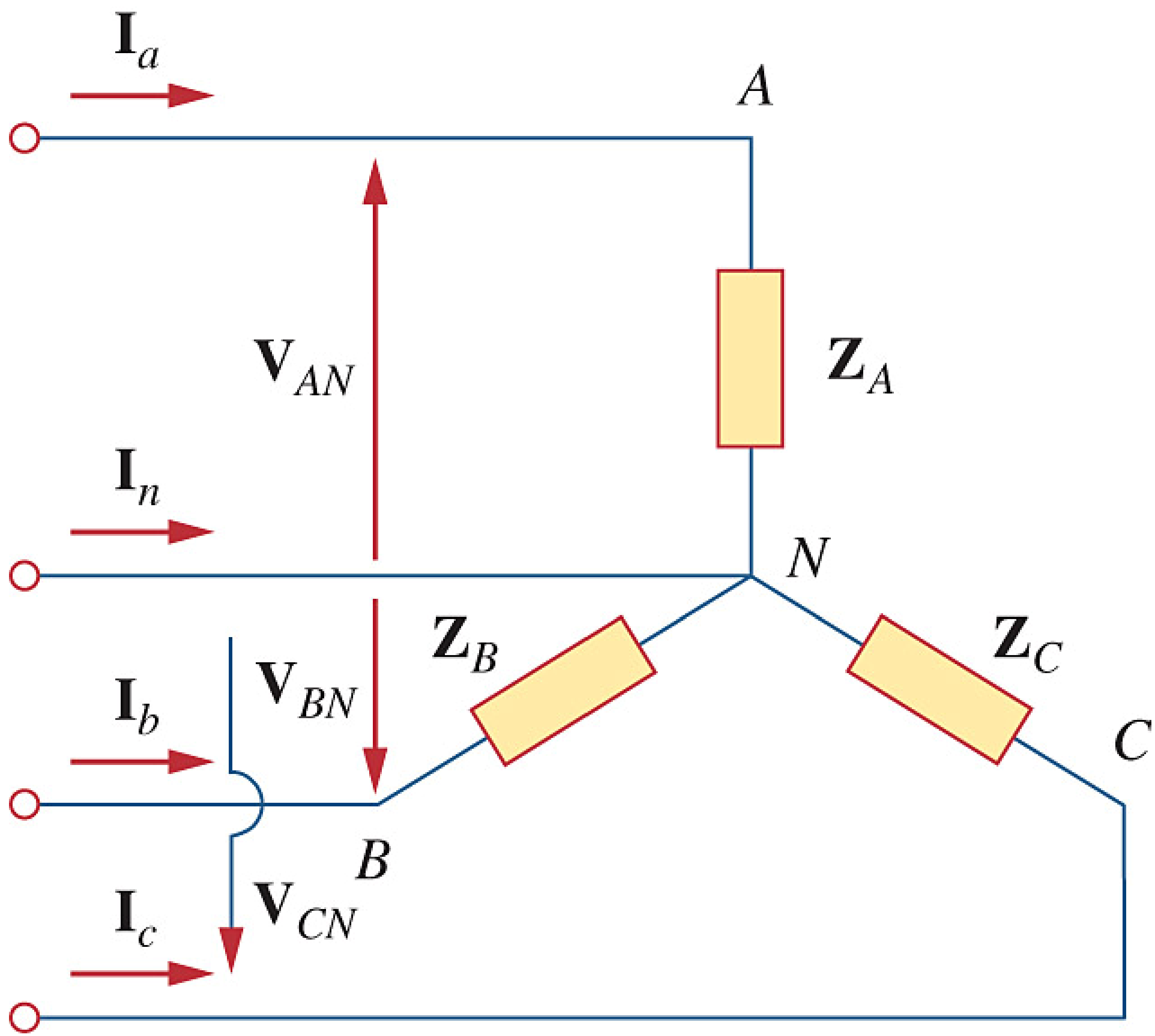

Figure 12.23 Unbalanced three-phase Y-connected load.

Example 12.9 The unbalanced Y-load of Fig. 12.23 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $Z_A = 15\ \Omega$, $Z_B = 10 + j5\ \Omega$, $Z_C = 6 - j8\ \Omega$.

Solution :

$$\dot{I}_a = \frac{\dot{V}_{AN}}{Z_A} = \frac{100\angle 0°}{15} \approx 6.6667\angle 0°\ (\text{A})$$

$$\dot{I}_b = \frac{\dot{V}_{BN}}{Z_B} = \frac{100\angle +120^\circ}{10 + j5} \approx 8.9443\angle 93.43^\circ \text{ (A)}$$

$$\dot{I}_c = \frac{\dot{V}_{CN}}{Z_C} = \frac{100\angle -120^\circ}{6 - j8} \approx 10\angle -66.87^\circ \text{ (A)}$$

$$\dot{I}_n = -(\dot{I}_a + \dot{I}_b + \dot{I}_c)$$

$$= -(6.6667\angle 0^\circ + 8.9443\angle 93.43^\circ + 10\angle -66.87^\circ)$$

$$\approx -(6.6667 - 0.5351 + j8.9283 + 3.9282 - j9.1962)$$

$$= -10.0598 + j0.2679$$

$$\approx 10.0634\angle 178.47^\circ \text{ (A)}$$

Practice Problem 12.10 Find the line currents in the unbalanced three-phase circuit of Fig. 12.26 and the real power absorbed by the load.

Solution :

$$\dot{I}_{AB} = \frac{\dot{V}_{AB}}{Z_A} = \frac{220\angle 0^\circ}{-j5} = 44\angle 90^\circ \text{ (A)}$$

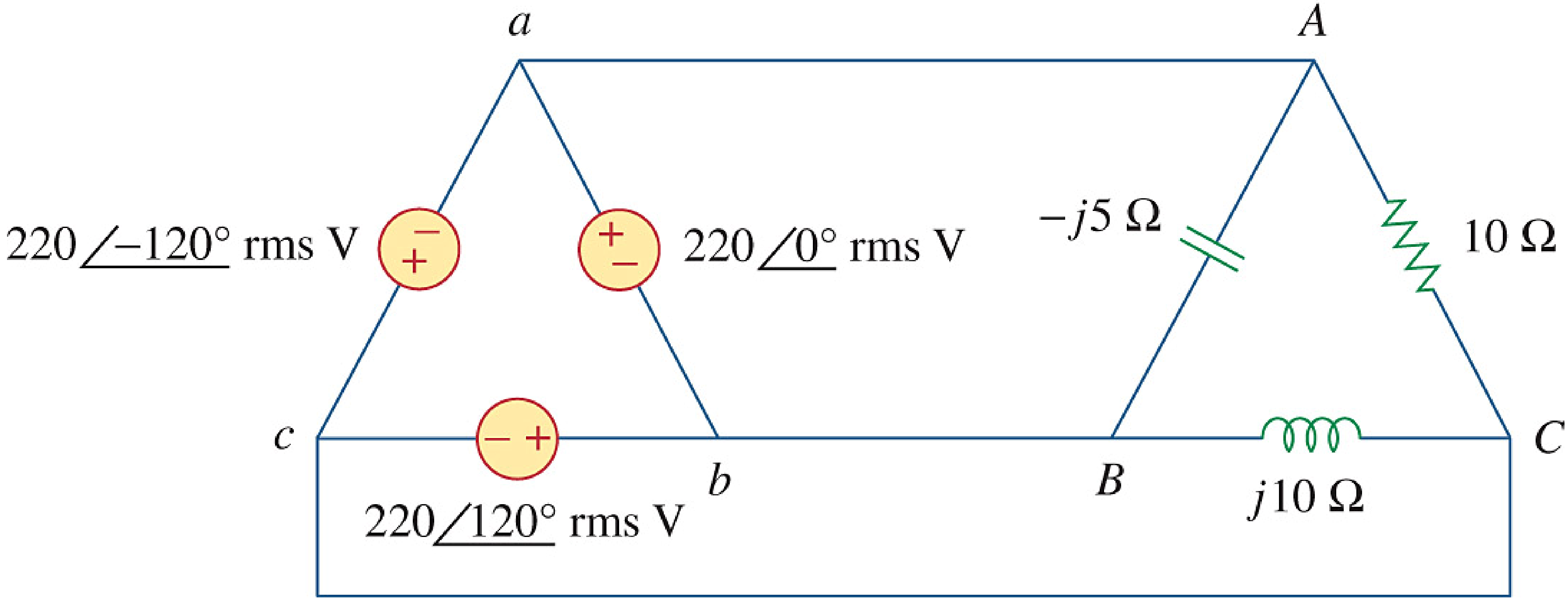

Figure 12.26

$$\dot{I}_{BC} = \frac{\dot{V}_{BC}}{Z_B} = \frac{220\angle 120^\circ}{j10} = 22\angle 30^\circ \text{ (A)}$$

$$\dot{I}_{CA} = \frac{\dot{V}_{CA}}{Z_C} = \frac{220\angle -120^\circ}{10} = 22\angle -120^\circ \text{ (A)}$$

$$\dot{I}_a = \dot{I}_{AB} - \dot{I}_{CA} = 44\angle 90^\circ - 22\angle -120^\circ$$

$$= j44 - (-11 - j19.0526)$$

$$= 11 + j63.05266 \approx 64.0049\angle 80.10^\circ \text{ (A)}$$

$$\dot{I}_b = \dot{I}_{BC} - \dot{I}_{AB} = 22\angle 30° - 44\angle 90°$$

$$= (19.0526 + j11) - j44$$

$$= 19.0526 - j33 \approx 38.1051\angle -60° \text{ (A)}$$

$$\dot{I}_c = \dot{I}_{CA} - \dot{I}_{BC} = 22\angle -120° - 22\angle 30°$$

$$= (-11 - j19.0526) - (19.0526 + j11)$$

$$= -30.0526 - j30.0526 \approx 42.5008\angle -135° \text{ (A)}$$

$$P_L = P_{CA} = I_{CA}^2 R = 22^2 \times 10 = 4{,}840 \text{ (W)}$$

12.10 Applications

Three - Phase Power Measurement The wattmeter is the instrument for measuring the average power.

The traditional analog wattmeter is an electrodynamic instrument. The device consists of two coils. One coil, called the *current coil*, is stationary and carries a current proportional to the load current.

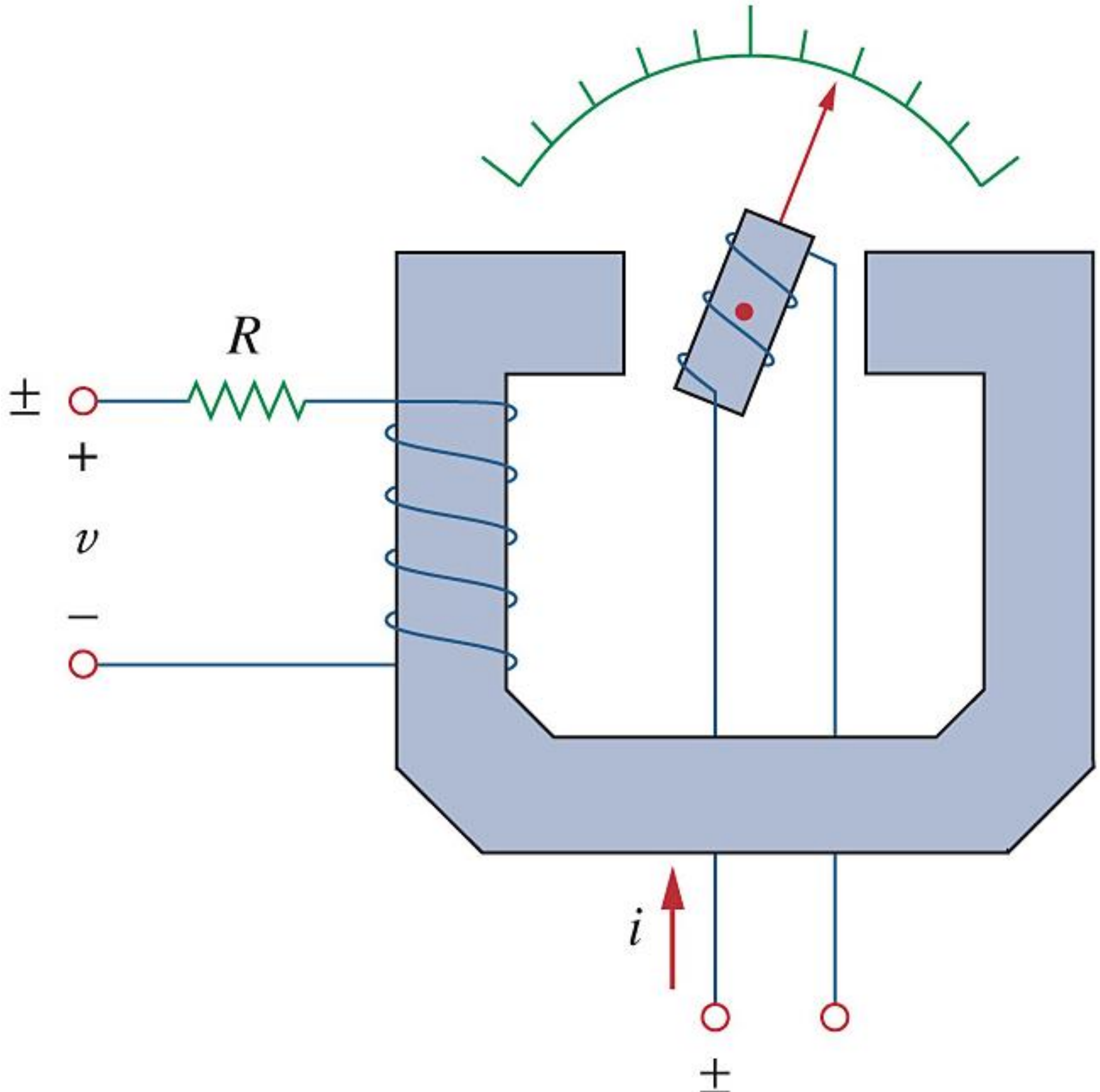

Figure 11.30 A wattmeter.

The second coil, called the *potential coil*, is movable and carries a current proportional to the load voltage. The average deflection of the pointer (or needle) attached to the movable coil is proportional to the average value of the product $v(t)i(t)$. The dircetion in which the pointer deflects depends on the instantaneous polarity of $v(t)$ and the instantaneous direction of $i(t)$. Therefore each coil has one terminal marked $\pm$. The pointer deflects upscale when the $\pm$ terminal of the current coil is toward the source, while the $\pm$ terminal of the potential coil is connected to the same line as the current coil. Reversing both coil connections still results in upscale deflection. However, reversing one coil and not the other results in downscale deflection and no wattmeter reading.

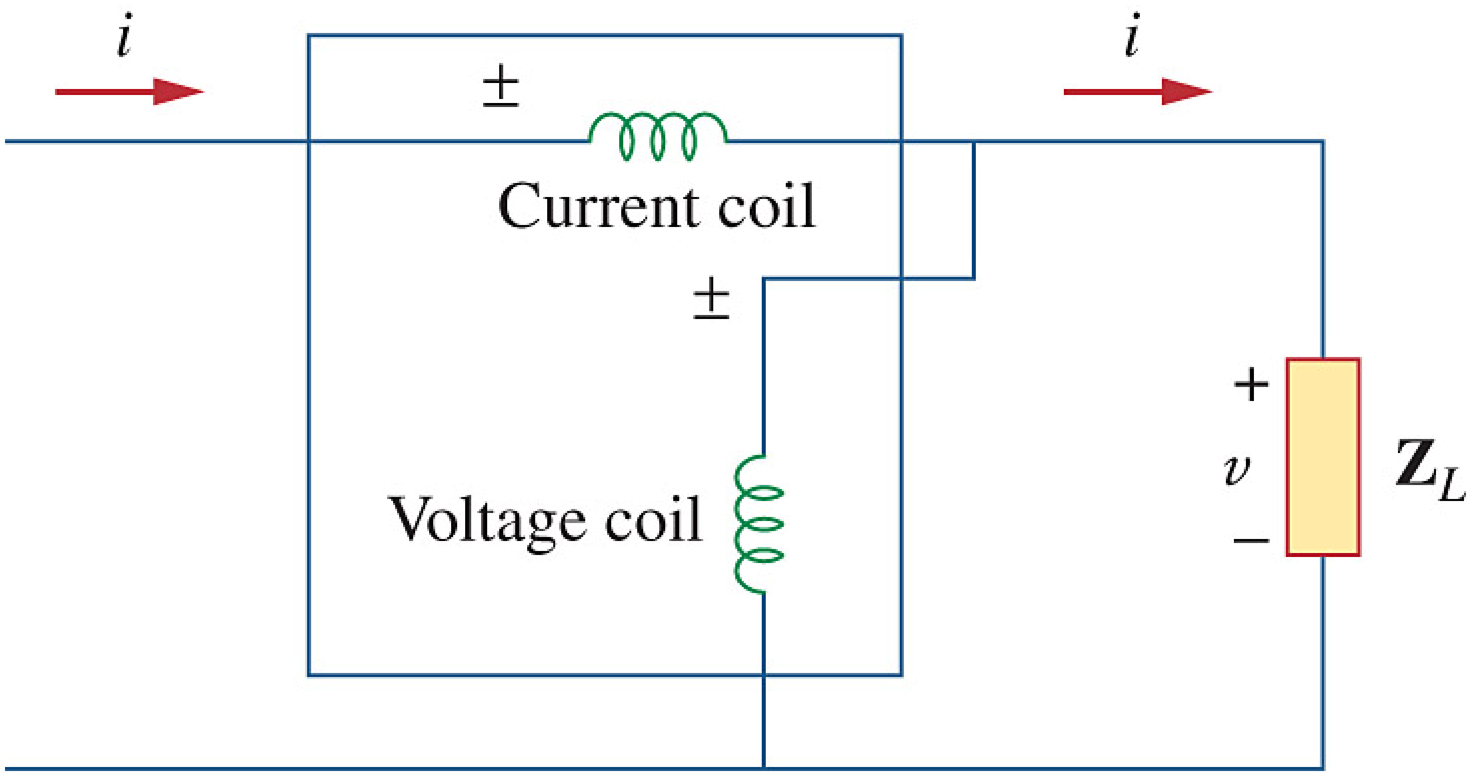

Figure 11.31 The wattmeter connected to the load.

A single wattmeter can also measure the average power in a three-phase system that is balanced, so that $P_1 = P_2 = P_3$; the total power is three times the reading of that one wattmeter.

$$P_T = 3P_1$$

However, two or three wattmeters are necessary to measure power if the system is unbalanced.

The three-wattmeter method, shown in Fig. 12.33, will work regardless of whether the load is balanced or unbalanced, wye- or delta-connected. The total average power is the algebraic sum of the three wattmeter readings,

$$P_T = P_1 + P_2 + P_3$$

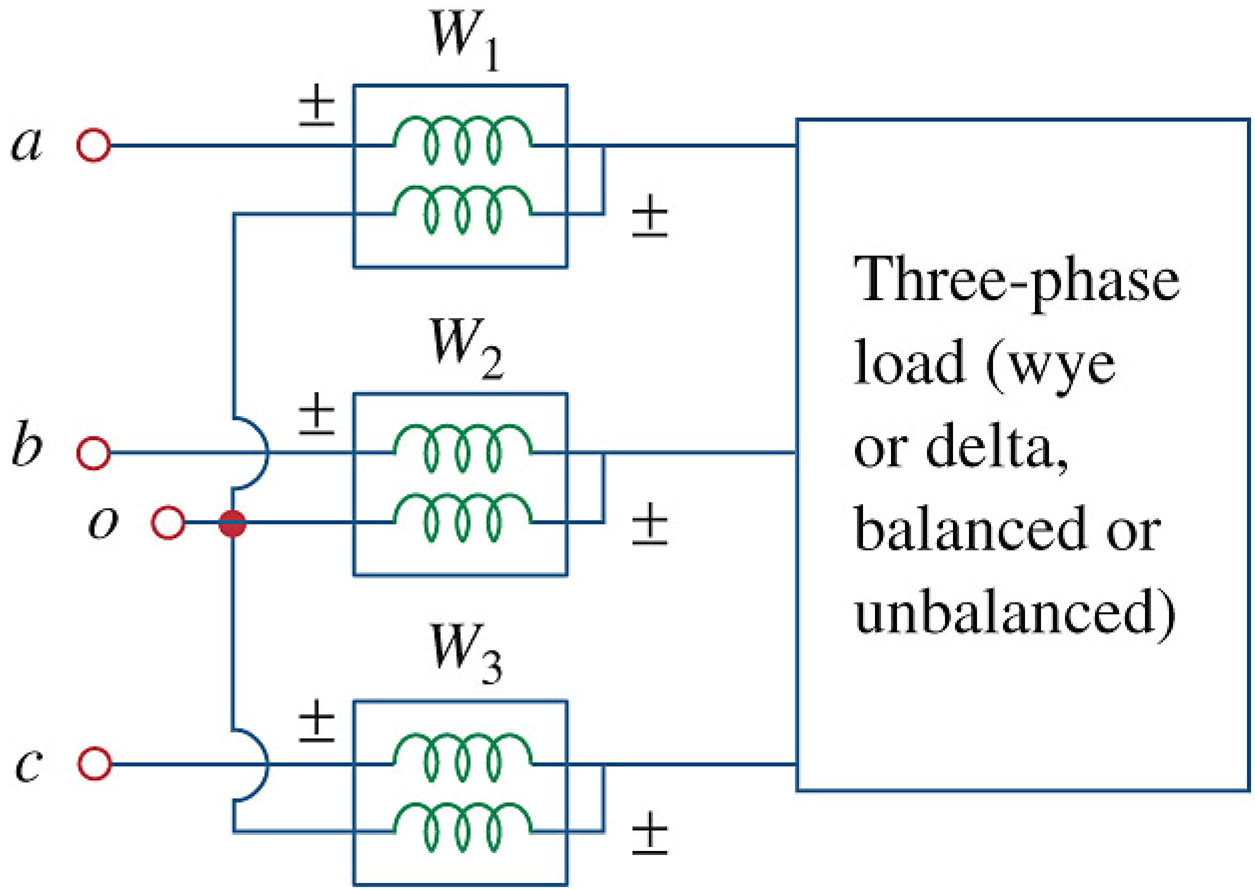

Figure 12.33 Three-wattmeter method for measuring three-phase power.

where P_1, P_2, and P_3 correspond to the readings of wattmeters W_1, W_2, and W_3, respectively. Notice that the reference point o is selected arbitrarily. If the load is wye-connected, point o can be connected to the neutral point n. For a delta-connected load, point o can be connected to any point.

If point o is connected to point b, for example, the potential coil in wattmeter W_2 reads zero and $P_2 = 0,$ indicating that wattmeter W_2 is not necessary. Thus, two wattmeters are sufficient to measure the total power.

The two-wattmeter method is the most commonly used method for three-phase power measurement. The two wattmeters must be properly connected to any two phases, as shown in Fig. 12.34. The total real power is equal to the algebraic sum of the two wattmeter readings,

$$P_T = P_1 + P_2$$

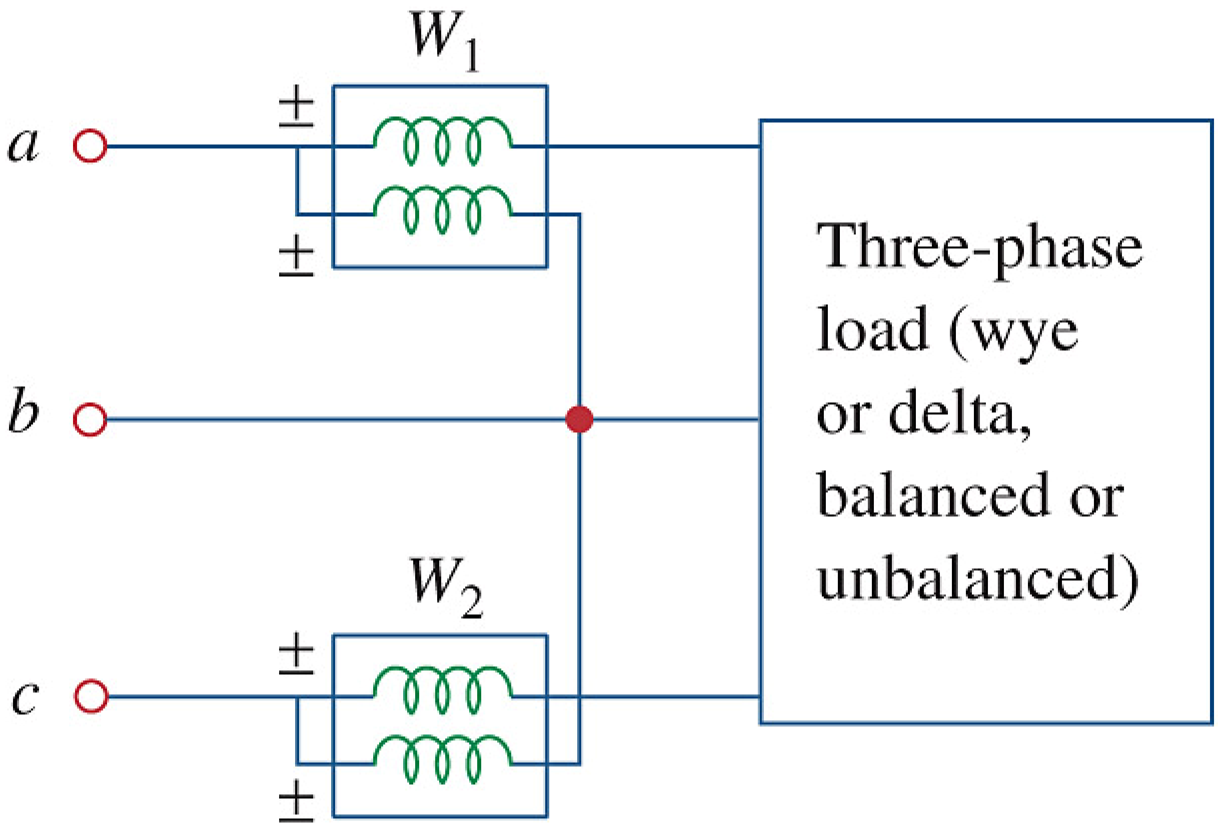

Figure 12.34 Two-wattmeter method for measuring three-phase power.

Proof :

$$P_1 = \mathrm{Re}\left(\dot{V}_{ab}\dot{I}_a^*\right),\ P_2 = \mathrm{Re}\left(\dot{V}_{cb}\dot{I}_c^*\right)$$

$$P_1 + P_2 = \mathrm{Re}\left(\dot{V}_{ab}\dot{I}_a^*\right) + \mathrm{Re}\left(\dot{V}_{cb}\dot{I}_c^*\right)$$

$$= \mathrm{Re}\left(\dot{V}_{ab}\dot{I}_a^* + \dot{V}_{cb}\dot{I}_c^*\right)$$

If the load is wye-connected,

$$\dot{V}_{ab}\dot{I}_a^* + \dot{V}_{cb}\dot{I}_c^* = (\dot{V}_{an} - \dot{V}_{bn})\dot{I}_a^* + (\dot{V}_{cn} - \dot{V}_{bn})\dot{I}_c^*$$

$$= \dot{V}_{an}\dot{I}_a^* - \dot{V}_{bn}(\dot{I}_a^* + \dot{I}_c^*) + \dot{V}_{cn}\dot{I}_c^*$$

$$= \dot{V}_{an}\dot{I}_a^* - \dot{V}_{bn}(\dot{I}_a + \dot{I}_c)^* + \dot{V}_{cn}\dot{I}_c^*$$

If $\dot{I}_a + \dot{I}_b + \dot{I}_c = 0$, then

$$\dot{V}_{ab}\dot{I}_a^* + \dot{V}_{cb}\dot{I}_c^* = \dot{V}_{an}\dot{I}_a^* + \dot{V}_{bn}\dot{I}_b^* + \dot{V}_{cn}\dot{I}_c^*$$

$$P_1 + P_2 = \mathrm{Re}\left(\dot{V}_{an}\dot{I}_a^* + \dot{V}_{bn}\dot{I}_b^* + \dot{V}_{cn}\dot{I}_c^*\right)$$

$$= \mathrm{Re}(\dot{V}_{an}\dot{I}_a^*) + \mathrm{Re}(\dot{V}_{bn}\dot{I}_b^*) + \mathrm{Re}(\dot{V}_{cn}\dot{I}_c^*)$$

$$= P_a + P_b + P_c = P_T$$

The condition $\dot{I}_a + \dot{I}_b + \dot{I}_c = 0$ implies that the two-wattmeter method cannot be used in a four-wire system unless $\dot{I}_n = 0$.

If the load is delta-connected,

$$\dot{V}_{ab}\dot{I}_a^* + \dot{V}_{cb}\dot{I}_c^* = \dot{V}_{AB}(\dot{I}_{AB} - \dot{I}_{CA})^* + \dot{V}_{CB}(\dot{I}_{CA} - \dot{I}_{BC})^*$$

$$= \dot{V}_{AB}(\dot{I}_{AB} - \dot{I}_{CA})^* - \dot{V}_{BC}(\dot{I}_{CA} - \dot{I}_{BC})^*$$

$$= \dot{V}_{AB}\dot{I}_{AB}^* + \dot{V}_{BC}\dot{I}_{BC}^* - (\dot{V}_{AB} + \dot{V}_{BC})\dot{I}_{CA}^*$$

$$= \dot{V}_{AB}\dot{I}_{AB}^* + \dot{V}_{BC}\dot{I}_{BC}^* + \dot{V}_{CA}\dot{I}_{CA}^*$$

$$P_1 + P_2 = \mathrm{Re}\left(\dot{V}_{AB}\dot{I}_{AB}^* + \dot{V}_{BC}\dot{I}_{BC}^* + \dot{V}_{CA}\dot{I}_{CA}^*\right)$$

$$= \mathrm{Re}(\dot{V}_{AB}\dot{I}_{AB}^*) + \mathrm{Re}(\dot{V}_{BC}\dot{I}_{BC}^*) + \mathrm{Re}(\dot{V}_{CA}\dot{I}_{CA}^*)$$

$$= P_{AB} + P_{BC} + P_{CA} = P_T$$

It can be shown that for a balanced three-phase system,

$P_1 = V_L I_L \cos(\theta + 30°)$, $P_2 = V_L I_L \cos(\theta - 30°)$

(If $\theta > 60°$, W_1 reads negative; If $\theta < -60°$, W_2 reads negative.)

$$P_1 + P_2 = \sqrt{3} V_L I_L \cos\theta = P_T$$

$$P_2 - P_1 = V_L I_L \sin\theta = \frac{Q_T}{\sqrt{3}}$$

$$\tan\theta = \frac{Q_T}{P_T} = \sqrt{3}\,\frac{P_2 - P_1}{P_2 + P_1}$$

Practice Problem 12.14 Let the line voltage $V_L = 208$ V and the wattmeter readings of the balanced system in Fig. 12.35 be $P_1 = -560$ W and $P_2 = 800$ W. Determine:

(a) the total average power

(b) the total reactive power

(c) the power factor

(d) the phase impedance

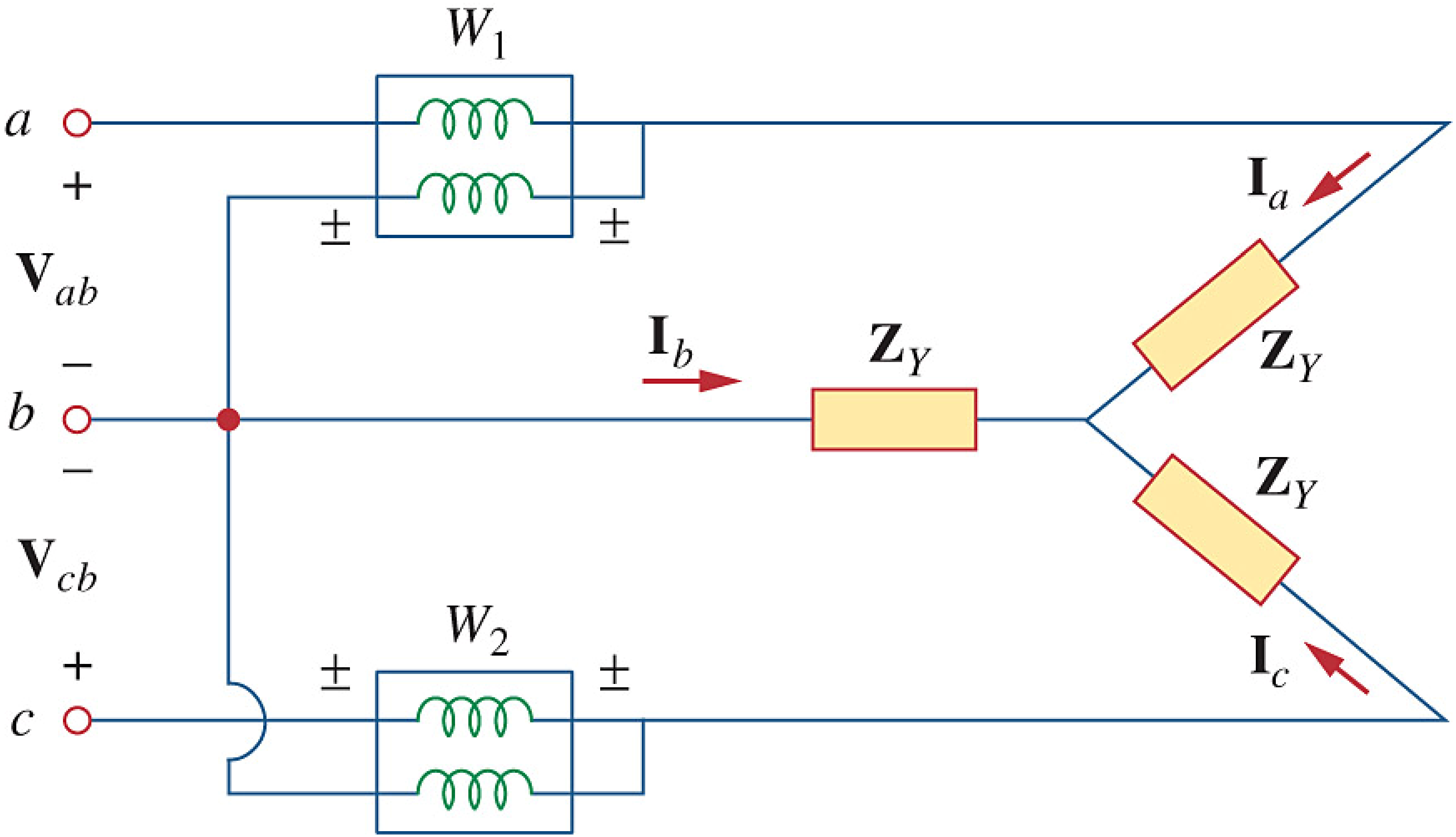

Figure 12.35 Two-wattmeter method applied to a balanced wye load.

Solution :

(a) $P_T = P_1 + P_2 = -560 + 800 = 240 \text{ (W)}$

(b) $Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}[800 - (-560)]$

$\approx 2355.5891 \text{ (VAR)}$

(c) $\theta = \tan^{-1}\left(\dfrac{Q_T}{P_T}\right) = \tan^{-1}\left(\dfrac{2355.5891}{240}\right)$

$\approx 84.18°$

$\text{pf} = \cos\theta = \cos 84.18° \approx 0.1014 \text{ lagging}$

(d) $P_p = P_T / 3 = 240/3 = 80$ (W)

$$P_p = V_p I_p \cos\theta \Rightarrow I_p = \frac{P_p}{V_p \cos\theta}$$

$$\left|Z_Y\right| = \frac{V_p}{I_p} = \frac{V_p}{P_p / (V_p \cos\theta)} = \frac{V_p^2 \cos\theta}{P_p}$$

$$= \frac{V_L^2 \cos\theta}{3P_p} = \frac{208^2 \times 0.1014}{3 \times 80} \approx 18.2790\ (\Omega)$$

$Z_Y = 18.2790\angle 84.18^\circ\ \Omega,$ inductive.