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## FET Single State Amplifier

Ve311 Electronic Circuits (Summer 2019)

Dr. Chang-Ching Tu

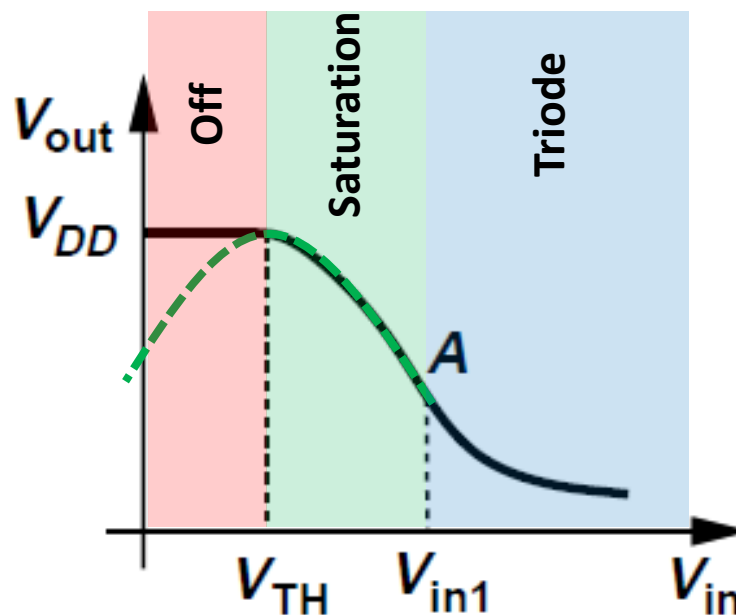
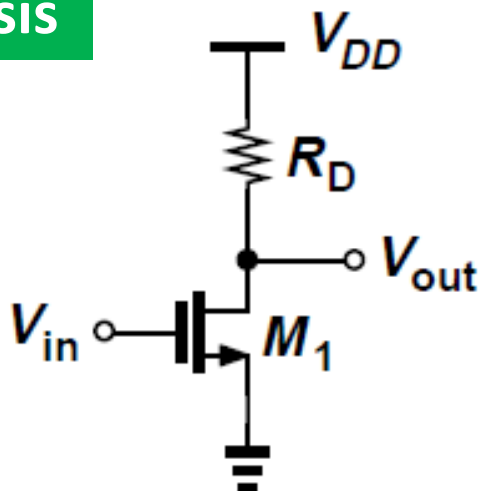
# Common-Source with Resistive Load

# Common-Source with Resistive Load

## DC Analysis

$$\lambda = 0$$

$$\gamma = 0$$



- $V_{in} < V_{TH} \rightarrow M_1$  Off

$$V_{out} = V_{DD}$$

- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$  in Saturation

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

- $V_{in} > V_{in1} \rightarrow M_1$  in Triode

$$V_{out} = V_{DD} - R_D \mu_n C_{ox} \frac{W}{L} [(V_{in} - V_{TH})V_{out} - \frac{1}{2} V_{out}^2]$$

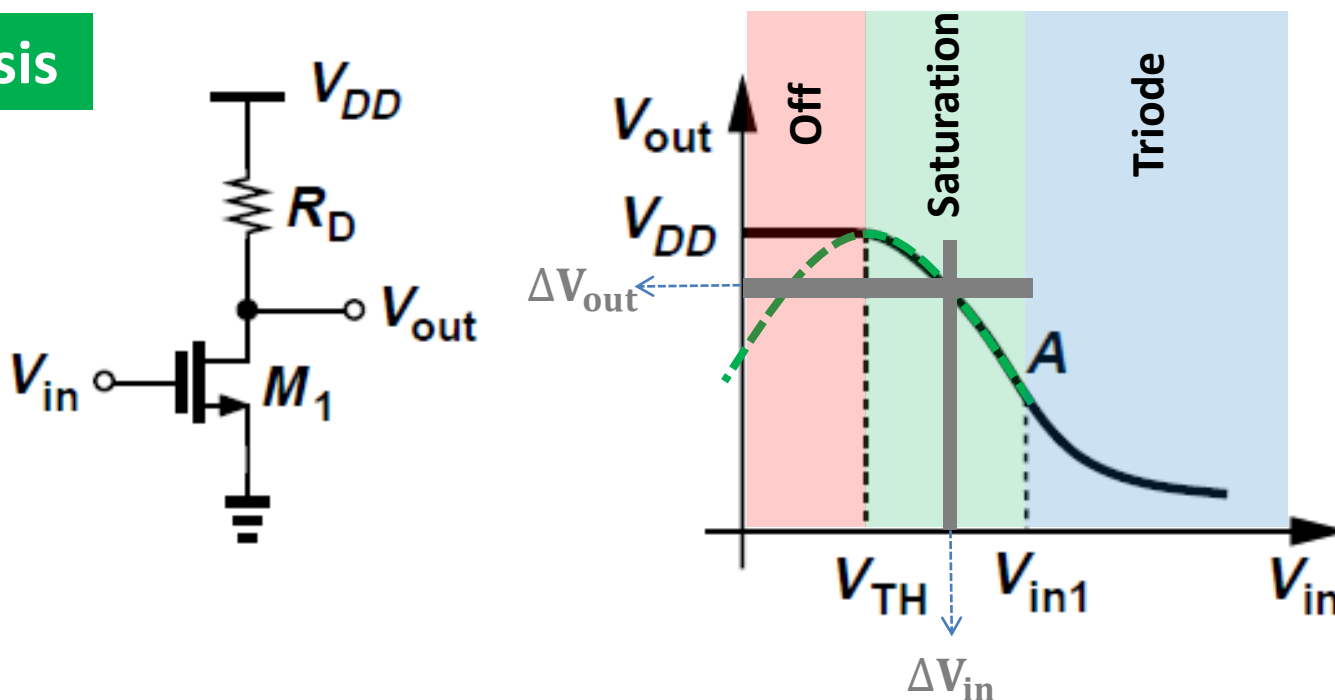
$$\begin{aligned} V_{out} &= V_{in1} - V_{TH} \\ &= V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2 \end{aligned}$$

# Common-Source with Resistive Load

## DC Analysis

$$\lambda = 0$$

$$\gamma = 0$$



$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$$

$$= -g_m \cdot R_D$$

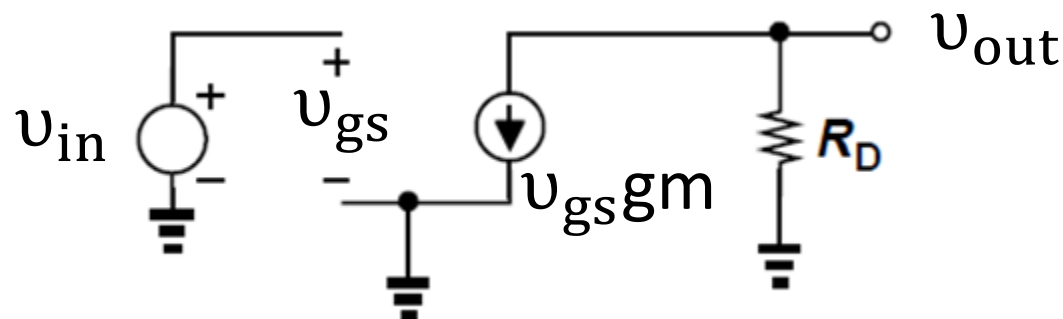
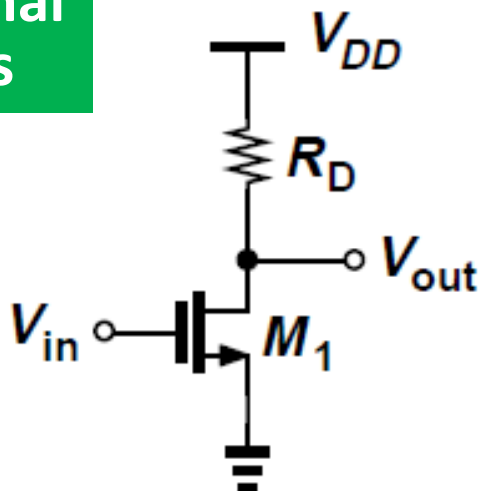
- $V_{gs}$  increases by  $\Delta V_{in} \rightarrow I_d$  increases by  $\Delta V_{in} \cdot g_m \rightarrow V_{out}$  decreases by  $\Delta V_{in} \cdot (g_m \cdot R_D)$

# Common-Source with Resistive Load

Small-signal  
Analysis

$$\lambda = 0$$

$$\gamma = 0$$



$$A_v = \frac{v_{out}}{v_{in}} = -g_m \cdot R_D$$

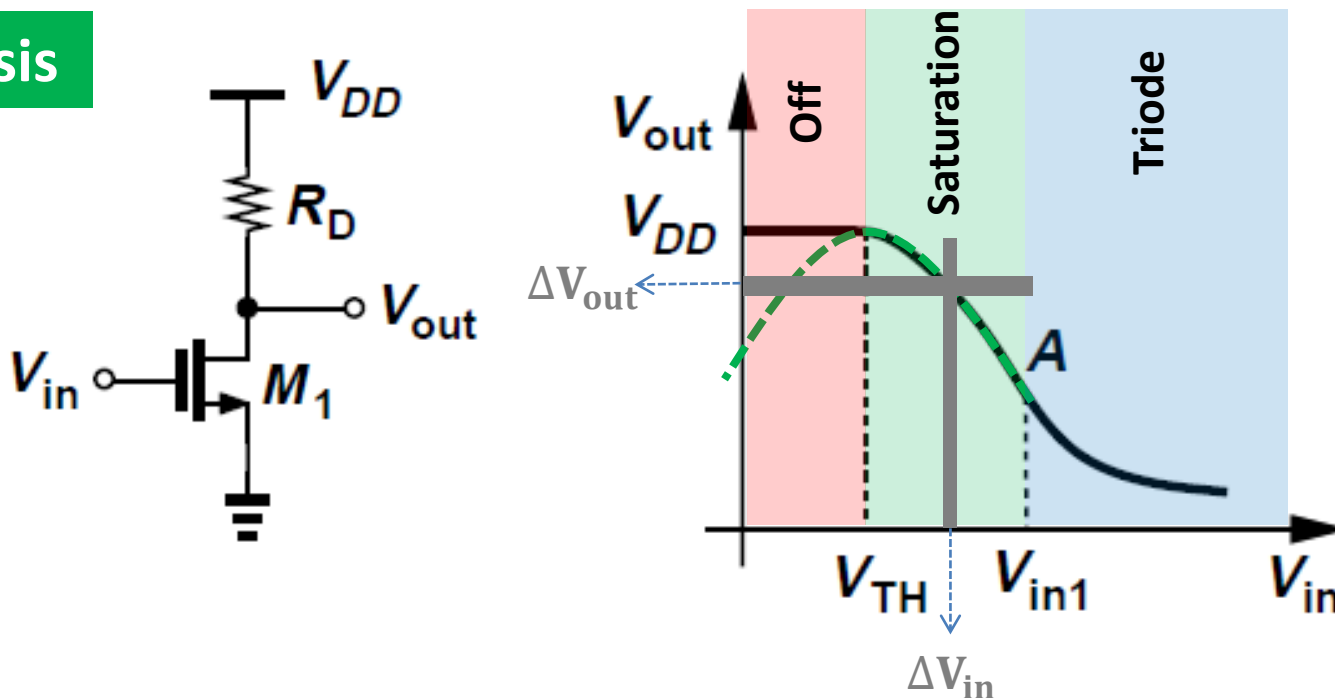
- Small-signal analysis leads to the same result as DC analysis.

# Common-Source with Resistive Load

## DC Analysis

$\lambda \neq 0$

$\gamma \neq 0$



$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{\partial \left[ V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{out}) \right]}{\partial V_{in}}$$

$$= -R_D \underbrace{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH}) (1 + \lambda V_{out})}_{= g_m} - R_D \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \lambda}_{\approx I_D} \underbrace{\frac{\partial V_{out}}{\partial V_{in}}}_{= A_v}$$

$$A_v = \frac{-g_m R_D}{1 + R_D \underbrace{I_D \lambda}_{= 1/r_o}} = -g_m \frac{1}{\frac{1}{R_D} + \frac{1}{r_o}} = \boxed{-g_m (R_D \parallel r_o)}$$

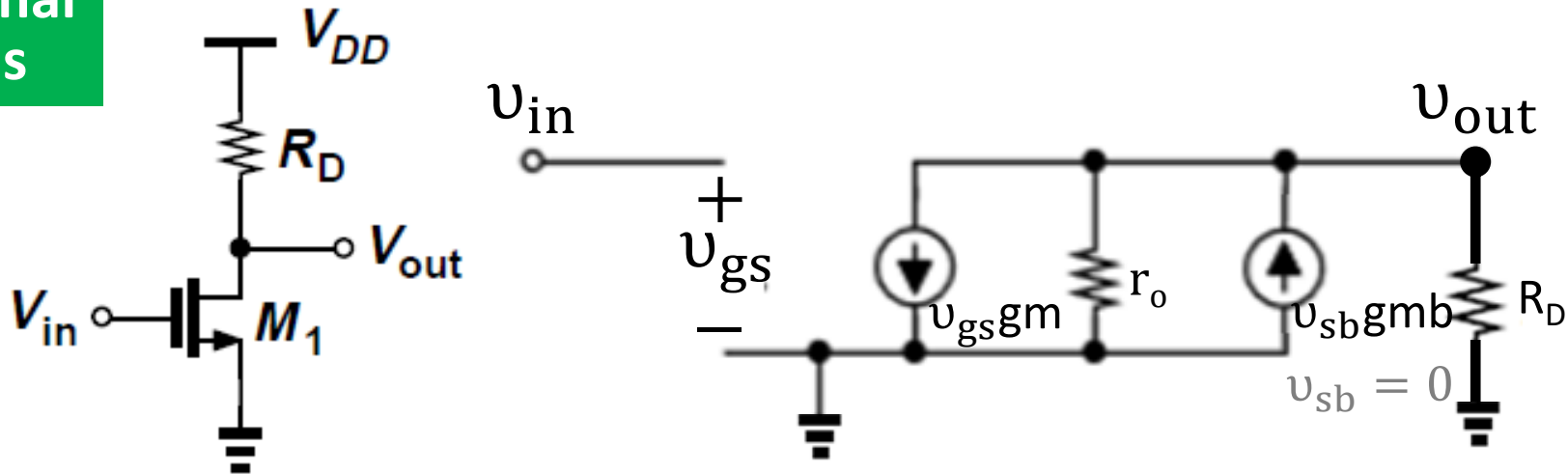
# Common-Source with Resistive Load

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Small-signal  
Analysis

$\lambda \neq 0$

$\gamma \neq 0$

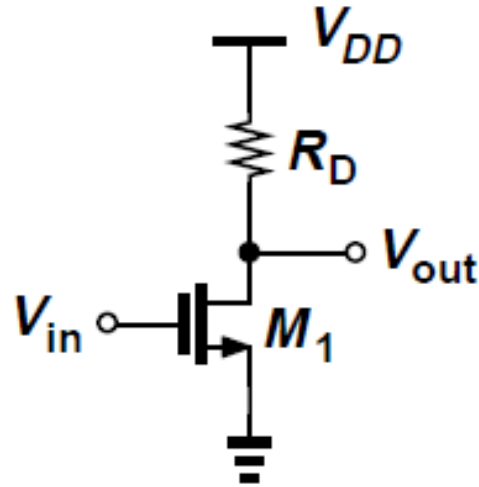


$$A_v = \frac{v_{out}}{v_{in}} = -g_m \cdot (R_D \parallel r_o)$$

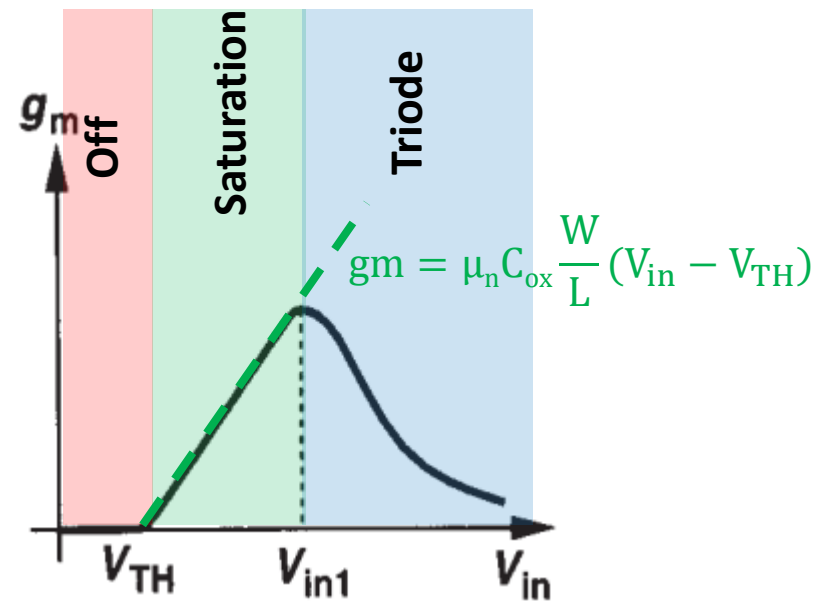
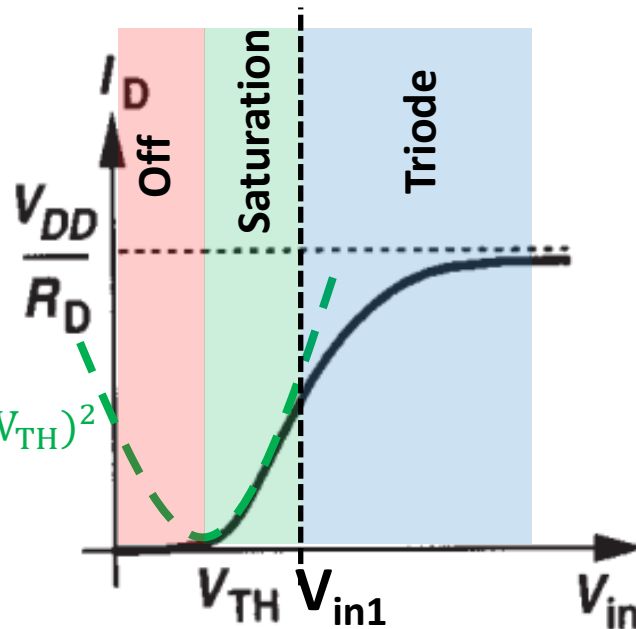
- Small-signal analysis leads to the same result as DC analysis.
- $g_m$  is a function of  $V_{GS}$  and  $V_{DS}$ , while  $r_o$  is a function of  $I_D$ . → **Nonlinearity**

# Example

Sketch the drain current and transconductance of  $M_1$  as a function of input voltage. Assume  $\lambda = \gamma = 0$ .



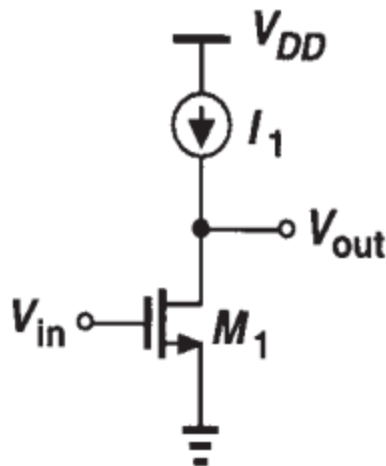
Solution:





# Example

Assuming  $M_1$  in saturation, calculate its small-signal gain.



Solution:

- Small-signal Analysis:

$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1} r_{o1}$$

- DC Analysis:

$$I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{DS})$$

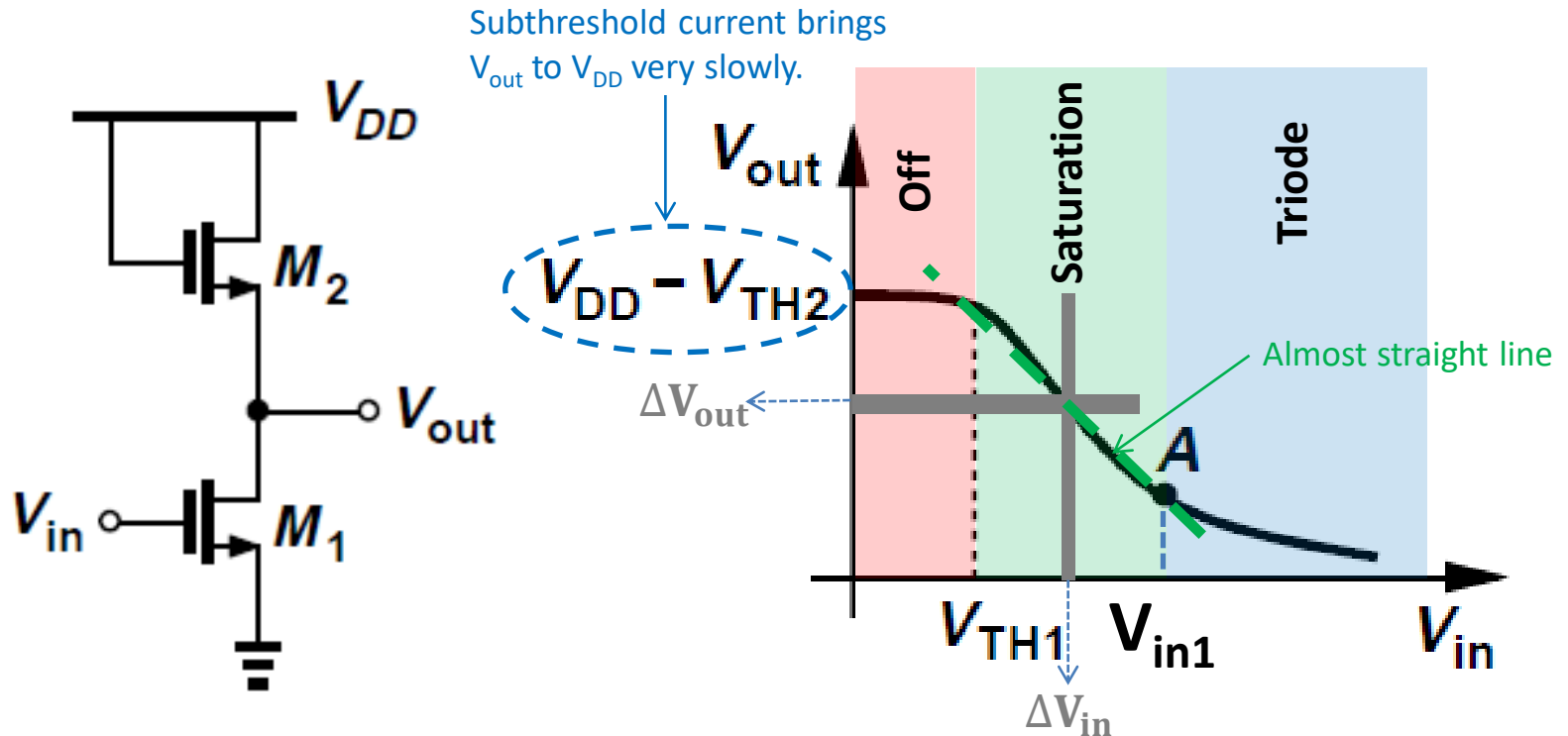
# Common-Source with Diode-Connected Load

# Common-Source with Diode-Connected Load

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## DC Analysis

$$\lambda = 0 \quad \gamma \neq 0$$



$$V_{out} = V_{in1} - V_{TH1}$$

$$\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{in1} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 [V_{DD} - (V_{in1} - V_{TH1}) - V_{TH2}]^2$$

- $V_{gs}$  increases by  $\Delta V_{in} \rightarrow I_d$  increases by  $\Delta V_{in} \cdot g_m \rightarrow V_{out}$  decreases

## DC Analysis

$$\lambda = 0 \quad \gamma \neq 0$$

- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$  in Saturation

$$\frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right)_2 [V_{DD} - V_{out} - V_{TH2}]^2$$

$$\sqrt{\left( \frac{W}{L} \right)_1} (V_{in} - V_{TH1}) = \sqrt{\left( \frac{W}{L} \right)_2} (V_{DD} - V_{out} - V_{TH2})$$

$$\sqrt{\left( \frac{W}{L} \right)_1} = \sqrt{\left( \frac{W}{L} \right)_2} \left( -\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{in}} \right)$$

$$\sqrt{\left( \frac{W}{L} \right)_1} = \sqrt{\left( \frac{W}{L} \right)_2} \left( -\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{out}} \frac{\partial V_{out}}{\partial V_{in}} \right)$$

$$= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1 + \eta}$$

- $\eta$  is a function of  $V_{SB}$ .
- $A_v$  is almost linear for  $M_1$  in saturation.

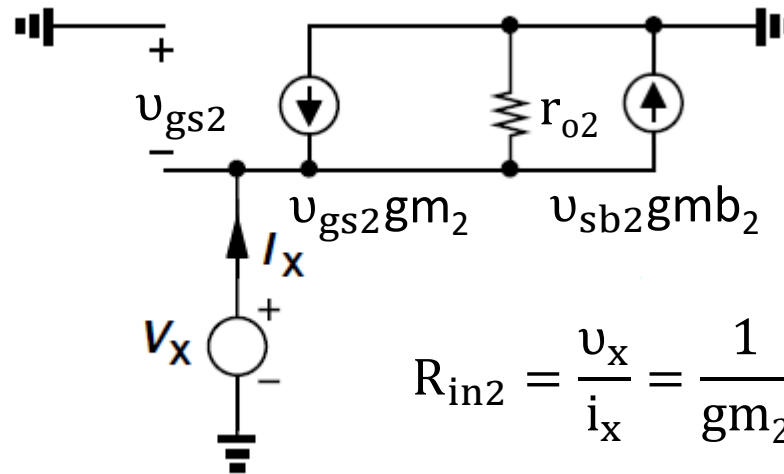
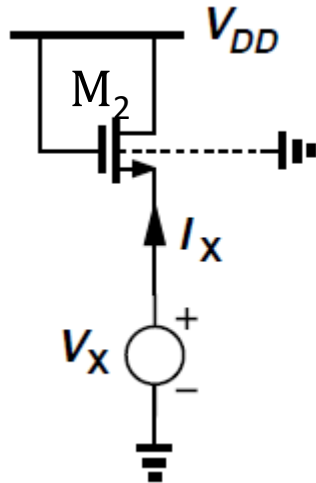


# Common-Source with Diode-Connected Load

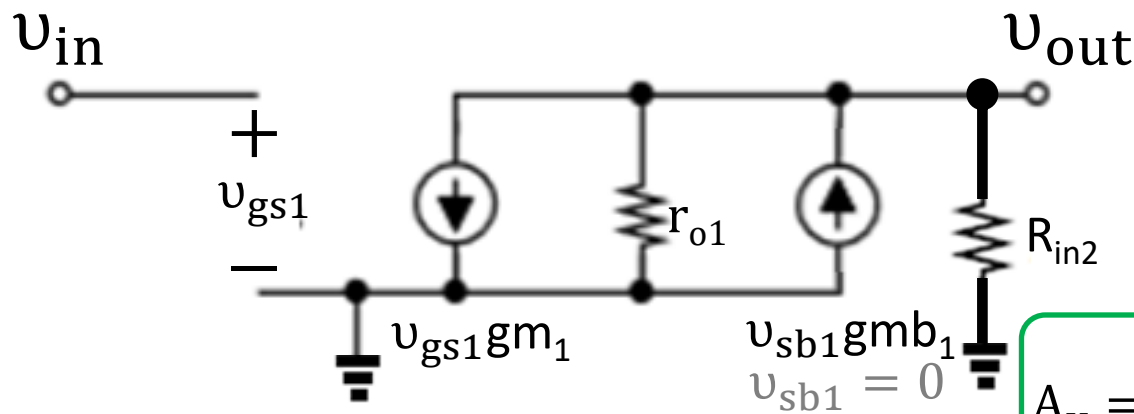
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Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$R_{in2} = \frac{v_x}{i_x} = \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}$$



$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1} \left( \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2} \parallel r_{o1} \right)$$

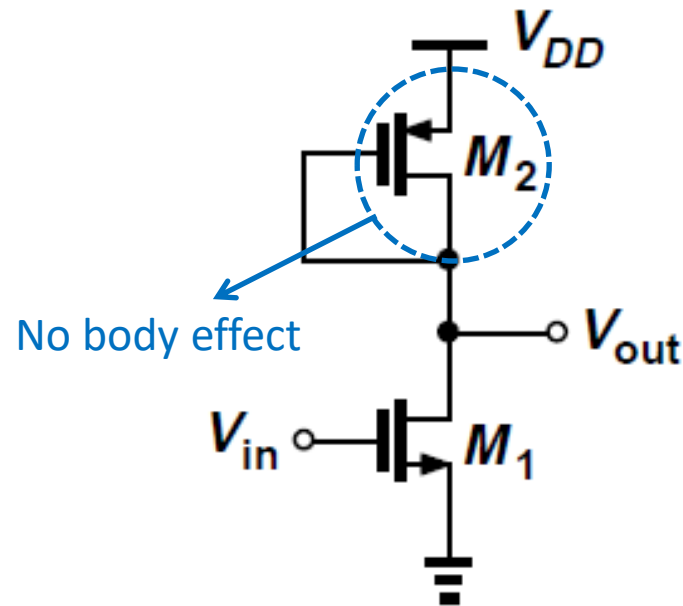
$r_o \gg 1/g_m$

# Common-Source with Diode-Connected Load

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Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



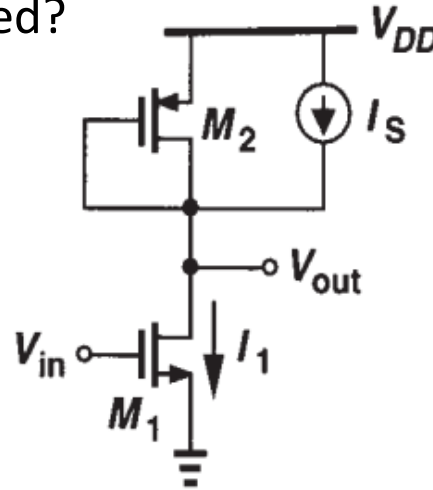
$$\begin{aligned} A_v &= \frac{v_{out}}{v_{in}} \\ &= -g_{m1} \left( \frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{o1} \right) \\ &\approx -\frac{g_{m1}}{g_{m2}} \\ &= -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}} \\ &= -\frac{V_{SG2} - V_{TH2}}{V_{GS1} - V_{TH1}} \end{aligned}$$

$r_o \gg 1/g_m$

- For  $A_v = 10$ ,  $(W/L)_1 \gg (W/L)_2 \rightarrow$  **Disproportionally large transistor**
- For  $A_v = 10$ ,  $(V_{SG2} - V_{TH2}) = 10 \times (V_{GS1} - V_{TH1}) \rightarrow$  **Limited output swing**

# Example

$M_1$  in saturation and  $I_S = 0.75 \times I_1$ . How do the disadvantages of CS stage with diode-connected load get improved?



Solution:

- Small-signal Analysis ( $\lambda = 0$ ):

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{gm_1}{gm_2} = -\frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_2 I_{D2}}} = -\frac{\sqrt{4\mu_n \left(\frac{W}{L}\right)_1}}{\sqrt{\mu_p \left(\frac{W}{L}\right)_2}} = -\frac{4(V_{SG2} - V_{TH2})}{(V_{GS1} - V_{TH1})}$$



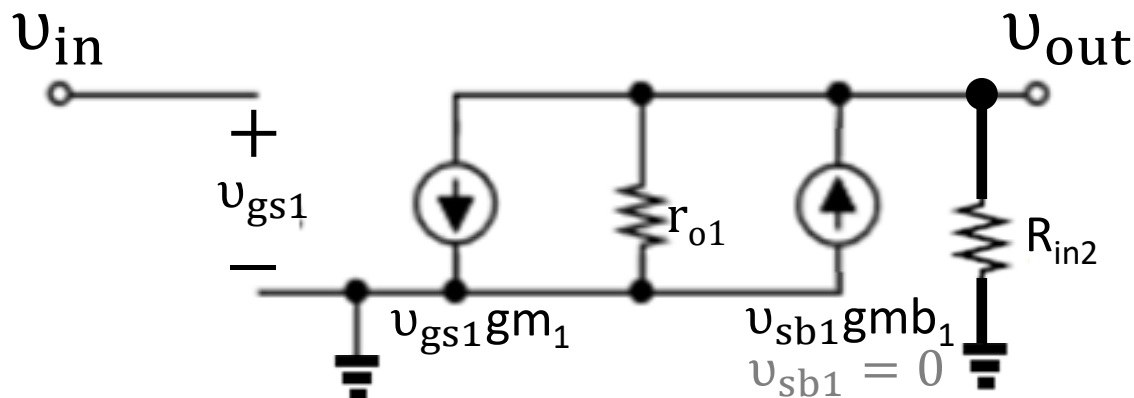
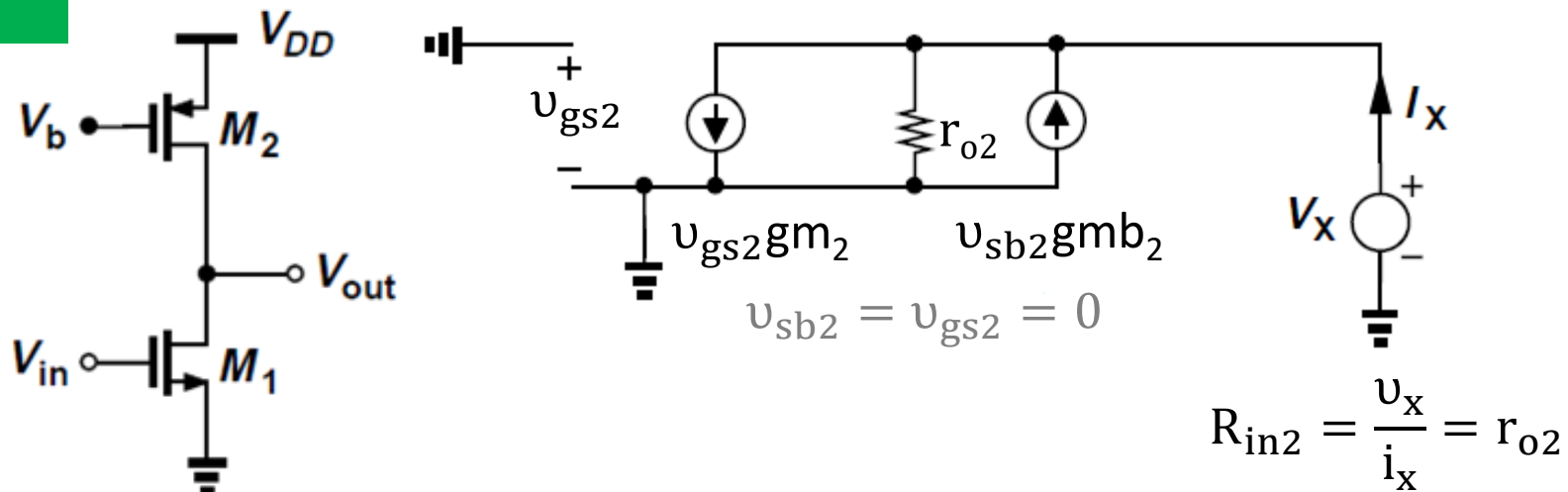
# Common-Source with Current-Source Load

# Common-Source with Current-Source Load

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Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o2} \parallel r_{o1})$$

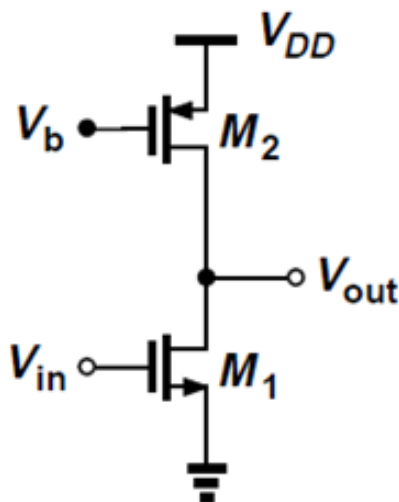
- To achieve high  $A_v$ , the output swing is severely limited in the CS stages with resistive load and diode-connected load.
- Here  $V_{out, max} = V_{DD} - (V_{SG2} - V_{TH2})$ , which can be quite close to  $V_{DD}$ .

# Common-Source with Current-Source Load

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## Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$



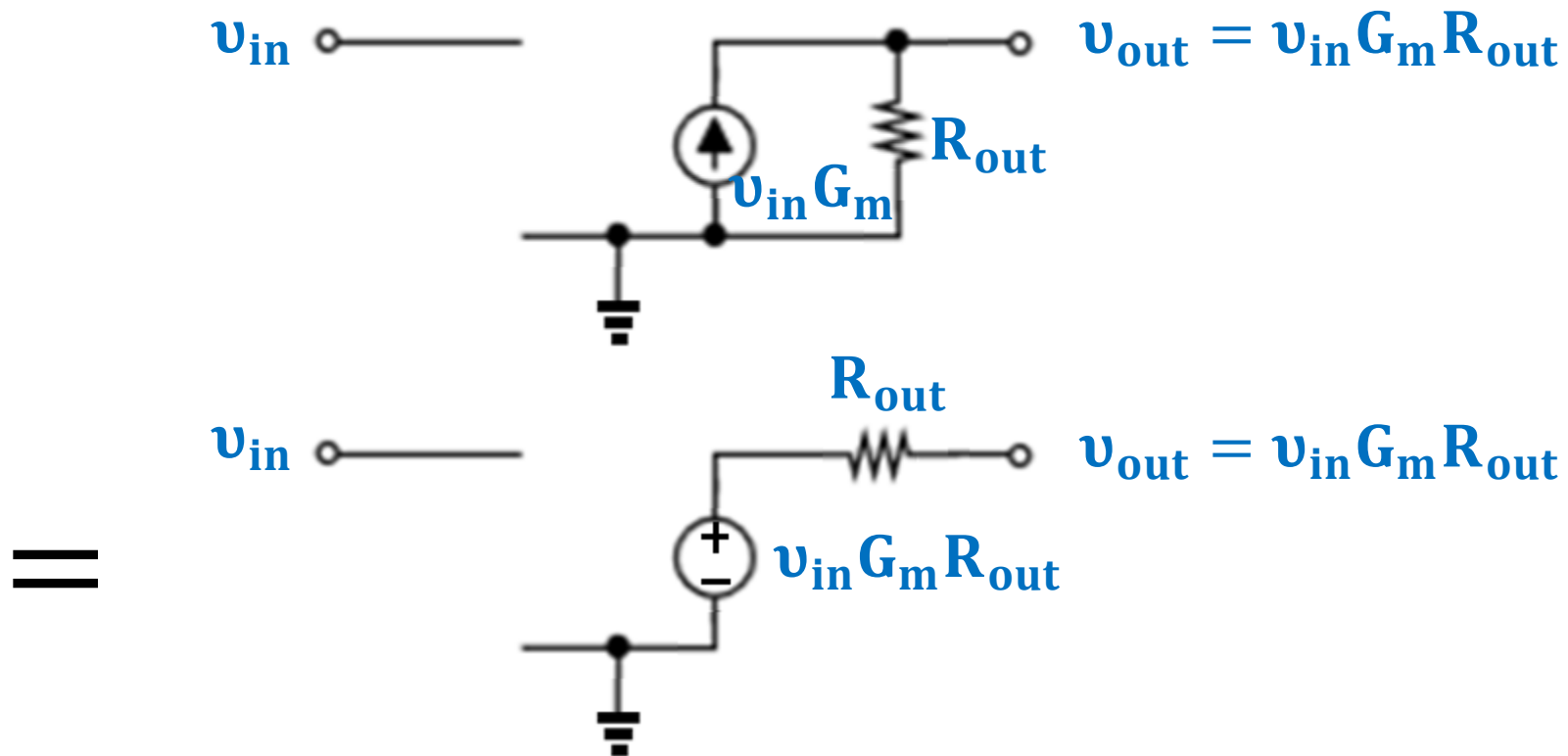
$$A_v = \frac{v_{out}}{v_{in}} \\ = -gm_1(r_{o2} \parallel r_{o1})$$

$$r_o \approx \frac{1}{\lambda I_D} \quad \lambda \propto \frac{1}{L}$$

- $V_{out, max} = V_{DD} - (V_{SG2} - V_{TH2})$
- $V_{out, min} = (V_{GS1} - V_{TH1})$
- For high  $gm_1$  and small  $(V_{GS1} - V_{TH1})$ ,  $W$  of  $M_2$  needs to be large.
- For high  $r_{o1}$  and  $r_{o2}$ ,  $L$  of  $M_1$  and  $M_2$  need to be large and  $L$  of  $M_1$  and  $M_2$  needs to be increased proportionally. The cost is the **large parasitic drain junction capacitance** at the output.

# Common-Source with Source Degradation

# Amplifier Equivalent Circuit

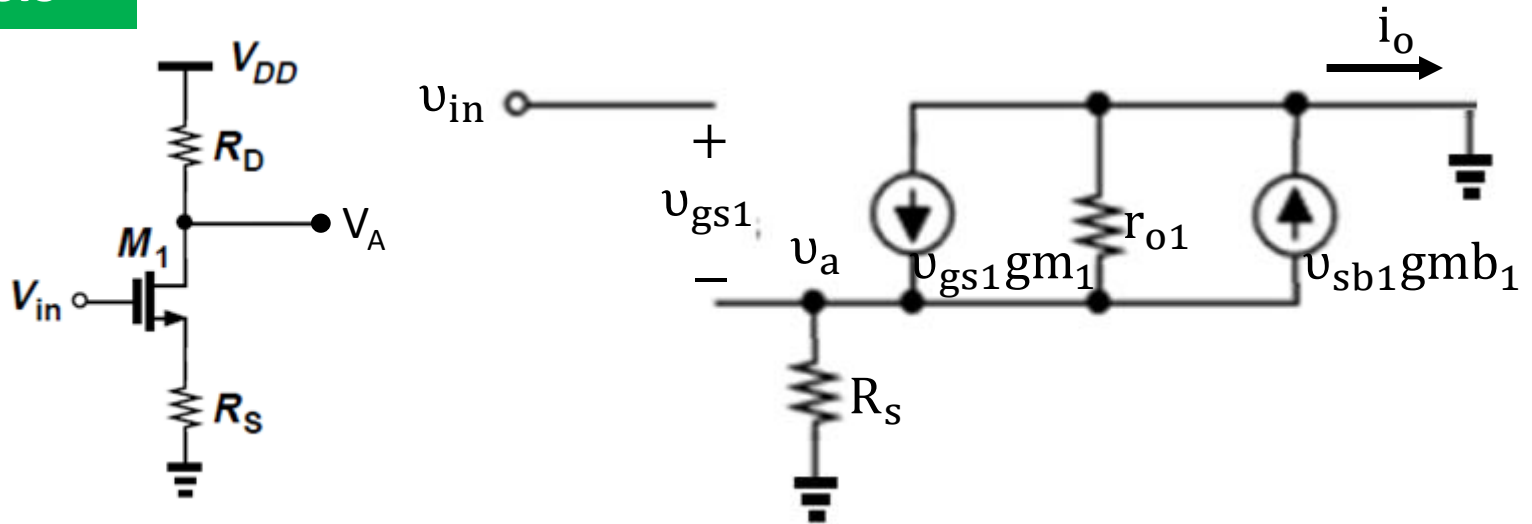


- How to calculate  $G_m$ ?  $v_{out}$  shorted to ground.  $G_m = i_{out}/v_{in}$
- How to calculate  $R_{out}$ ?  $v_{in}$  shorted to ground and  $v_{out}$  connected to  $v_{test}$ .  
 $R_{out} = v_{test}/i_{test}$

# Common-Source with Source Degradation

Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



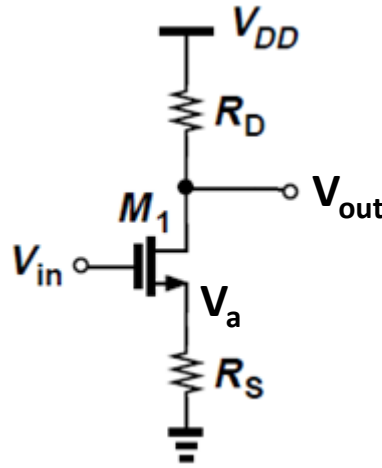
$$\begin{cases} i_o = \frac{-v_a}{R_S} \\ (v_{in} - v_a)g_{m1} + i_o = \frac{v_a}{r_{o1}} + v_a g_{mb1} \end{cases}$$

$$G_m = \frac{i_o}{v_{in}} = \frac{-g_{m1}r_{o1}}{R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S} \approx -\frac{1}{R_S}$$

If  $(g_{m1} + g_{mb1})r_{o1}R_S \gg r_{o1}$  and  $R_S$

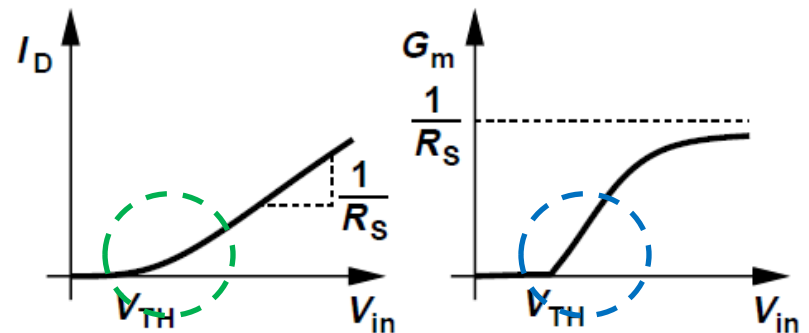
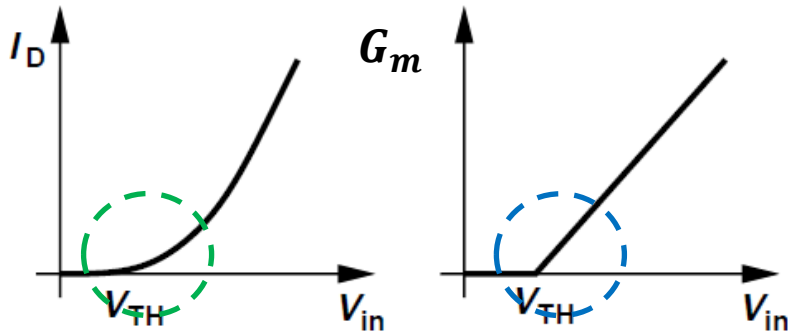
# Common-Source with Source Degradation

## DC Analysis



$R_S = 0$

$R_S \neq 0$

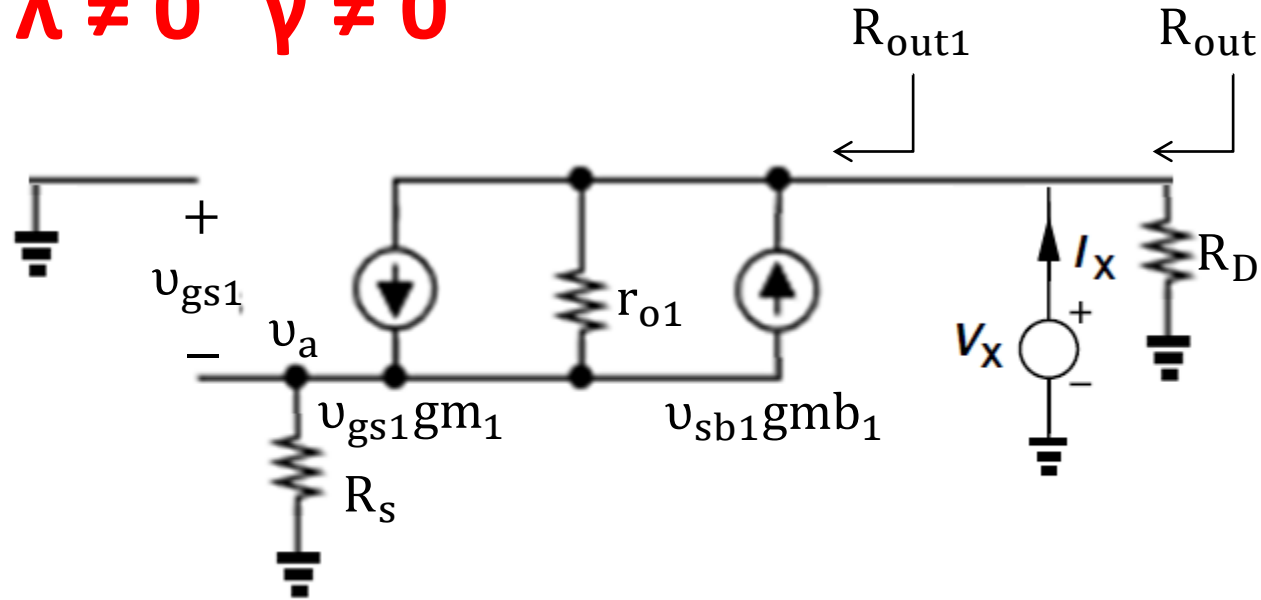


- At low  $V_{in}$  (gm small), turn-on behavior of  $R_S \neq 0$  is similar to that of  $R_S = 0$ .
- At large  $V_{in}$  (gm large), the effect of  $R_S$ , i.e. degradation, becomes more significant.
- $V_{in} = 0 \text{ V} \rightarrow M_1$  off, no current flowing  $\rightarrow V_a = 0 \text{ V}$  and  $V_{out} = V_{DD}$

# Common-Source with Source Degradation

Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$\begin{cases} i_x = \frac{v_a}{R_S} \\ v_a g_{m1} + v_a g_{mb1} + \frac{v_a - v_x}{r_o} + i_x = 0 \end{cases}$$

$$R_{out} = R_{out1} \parallel R_D = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D \approx R_D$$

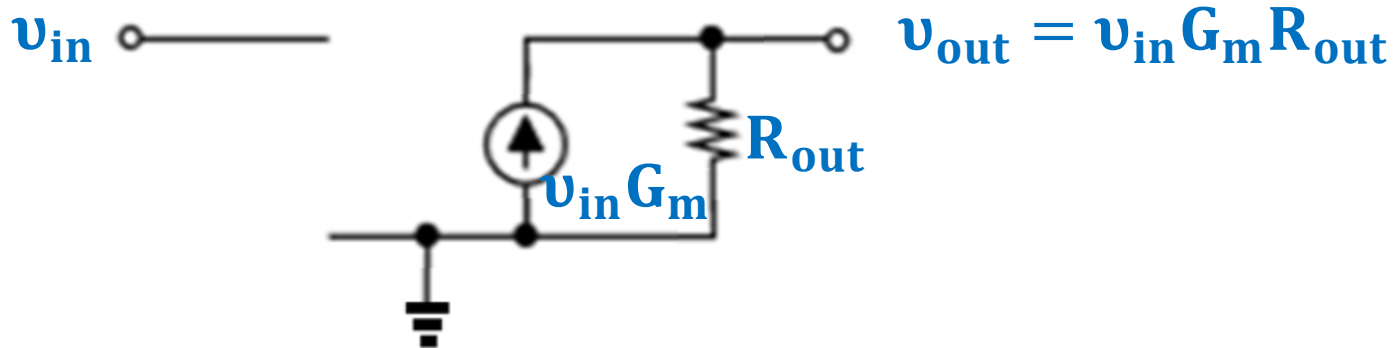
If  $(g_{m1} + g_{mb1})r_{o1}R_S \gg R_D$



# Common-Source with Source Degradation

Small-signal  
Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$



$$A_v = \frac{v_{out}}{v_{in}} = G_m R_{out}$$

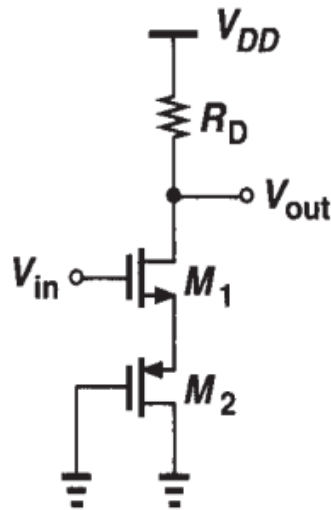
$$= \frac{-g_{m1} r_{o1}}{R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S} \cdot \frac{[R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S] R_D}{[R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S] + R_D}$$

$$\approx -\frac{R_D}{R_S}$$

If  $(g_{m1} + g_{mb1}) r_{o1}$ , the intrinsic gain, is large.

# Example

Assuming  $\lambda = \gamma = 0$ , calculate the small signal voltage gain of the circuit below.



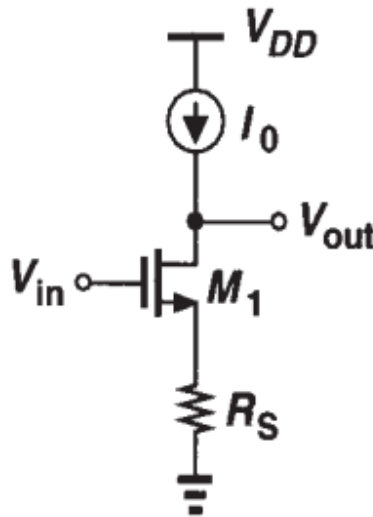
Solution:

$$G_m = -\frac{1}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} \quad R_{out} = R_D$$

$$A_v = G_m R_{out}$$

# Example

Calculate the small signal voltage gain of the circuit below.



Solution:

$$G_m = \frac{-g_{m1}r_{o1}}{r_{o1} + R_S + (g_{m1} + g_{mb1})r_{o1}R_S}$$

$$R_{out} = r_{o1} + R_S + (g_{m1} + g_{mb1})r_{o1}R_S$$

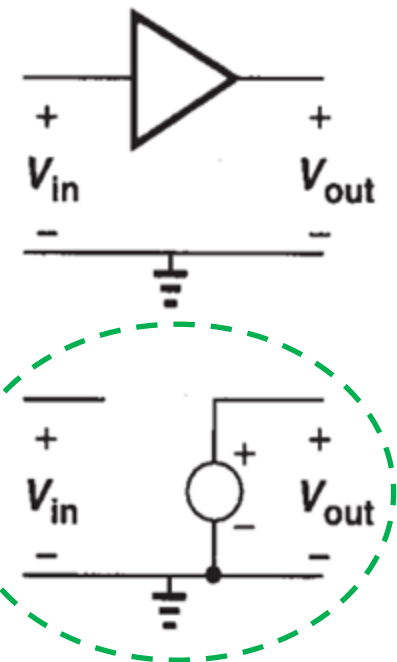
$$A_V = G_m R_{out} = -g_{m1}r_{o1}$$

- $I_0$  is ideal current source  $\rightarrow$  Voltage across  $R_S$  is constant  $\rightarrow M_1$  source shorted to ground
- $R_D$  replaced by current source  $\rightarrow$  Nonlinearity issue arises again

# Source Follower

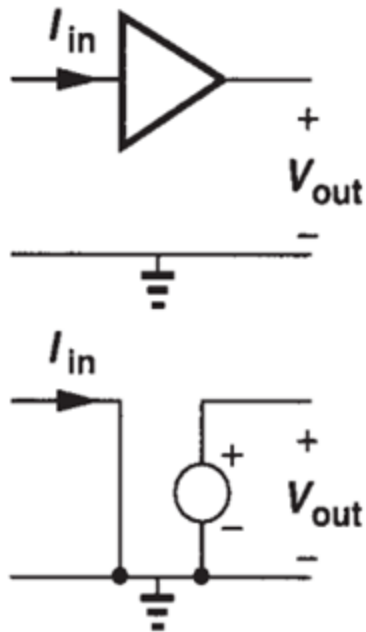
# Ideal Amplifier

Voltage Amp.

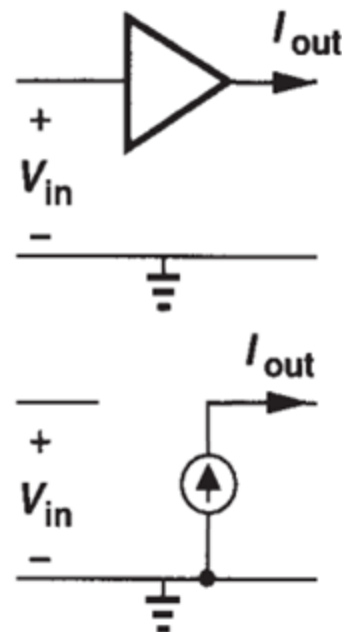


CS + Source Follower

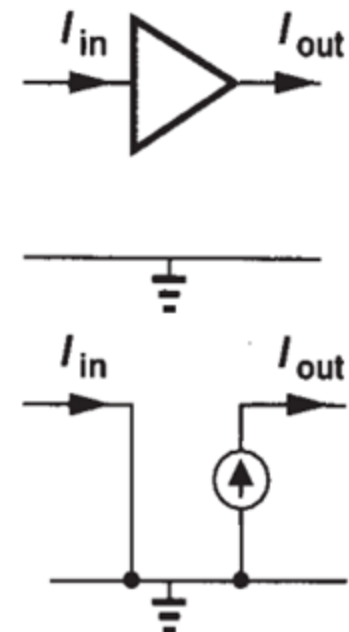
Transimpedance Amp.



Transconductance Amp.



Current Amp.



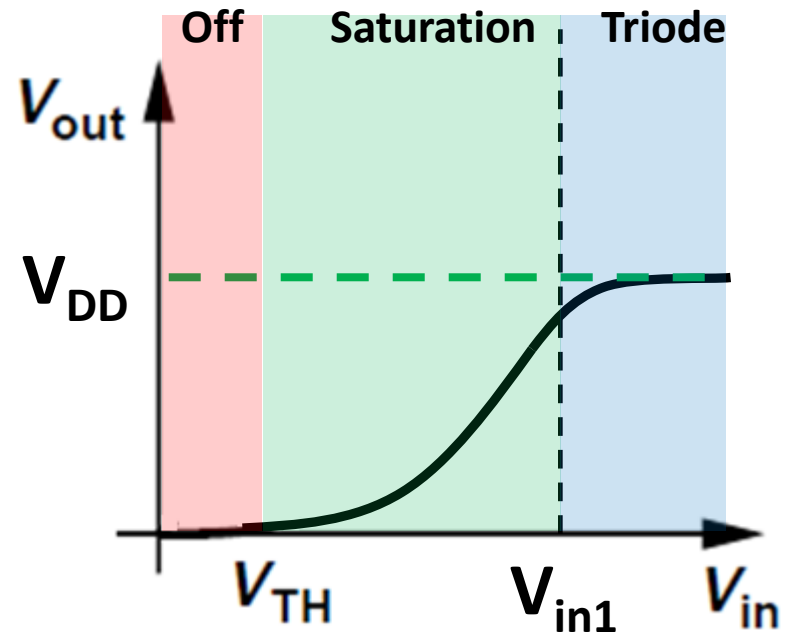
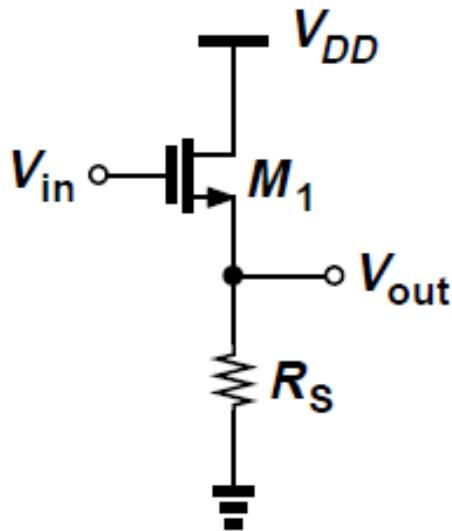
- For driving a low impedance load, source follower, as a buffer, provides **no gain** but **large input impedance** and **low output impedance**.

# Source Follower

## DC Analysis

$$\lambda = 0$$

$$\gamma \neq 0$$



- $V_{in} < V_{TH} \rightarrow M_1$  Off

$$V_{out} = 0$$

- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$  in Saturation

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

- $V_{in} > V_{in1} \rightarrow M_1$  in Triode

$$R_S \mu_n C_{ox} \frac{W}{L} \left[ (V_{in} - V_{out} - V_{TH})(V_{DD} - V_{out}) - \frac{1}{2} (V_{DD} - V_{out})^2 \right] = V_{out}$$

$$\begin{aligned} V_{DD} - V_{out} &= V_{in1} - V_{out} - V_{TH} \\ \rightarrow V_{in1} &= V_{DD} + V_{TH} \end{aligned}$$

# Source Follower

## DC Analysis

$$\lambda = 0$$

$$\gamma \neq 0$$

- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$  in Saturation

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_{in} - V_{out} - V_{TH}) \left( 1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}} \right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$R_S \underbrace{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})}_{= gm} \left( 1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\frac{\partial V_{TH}}{\partial V_{out}} \frac{\partial V_{out}}{\partial V_{in}}}_{= \eta} \right) = \frac{\partial V_{out}}{\partial V_{in}} \gamma$$

$$= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

$$A_v = \frac{gmR_S}{1 + gmR_S(1 + \eta)} = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1 + \eta}$$

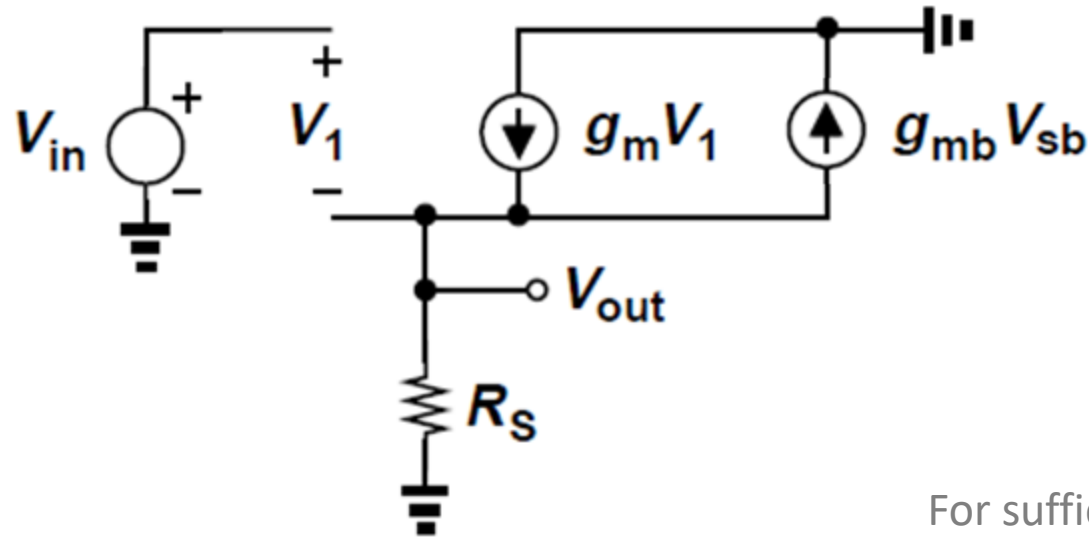
$$\text{If } (gm + gmb)R_S \gg 1$$

# Source Follower

## Small-signal Analysis

$$\lambda = 0$$

$$\gamma \neq 0$$



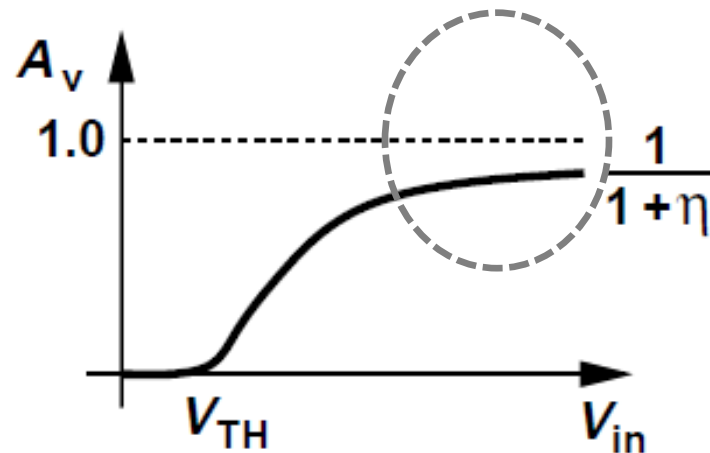
$$G_m = g_m$$

$$R_{out} = R_S \parallel \left( \frac{1}{g_m + g_{mb}} \right)$$

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S} \approx \frac{1}{1 + \eta}$$

$$\text{If } (g_m + g_{mb}) R_S \gg 1$$

For sufficiently large  $V_{in}$ ,  $I_D$  and thus  $g_m$ .



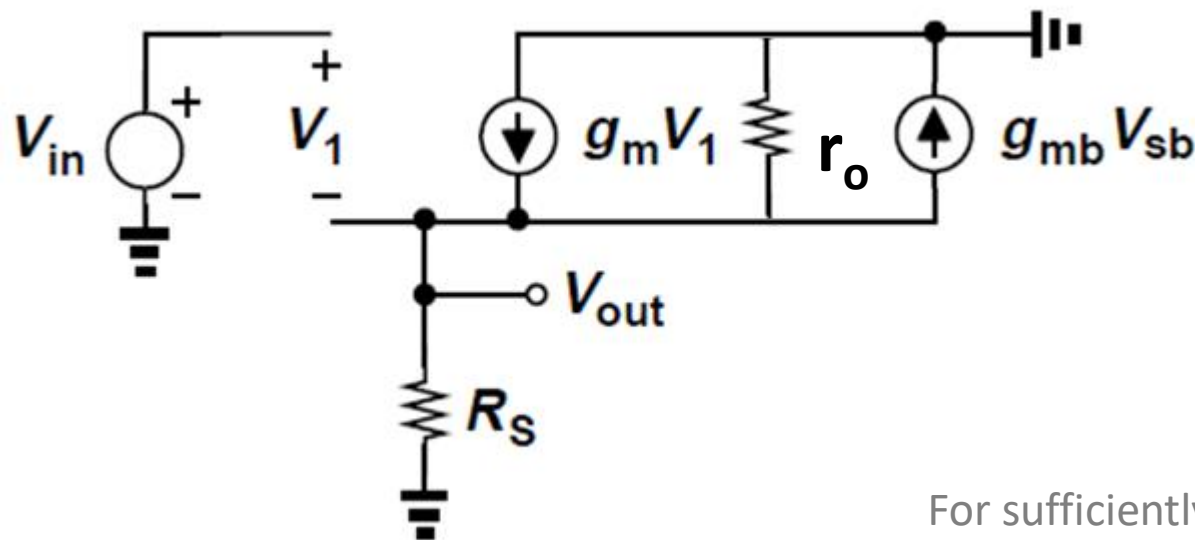


# Source Follower

## Small-signal Analysis

$$\lambda \neq 0$$

$$\gamma \neq 0$$

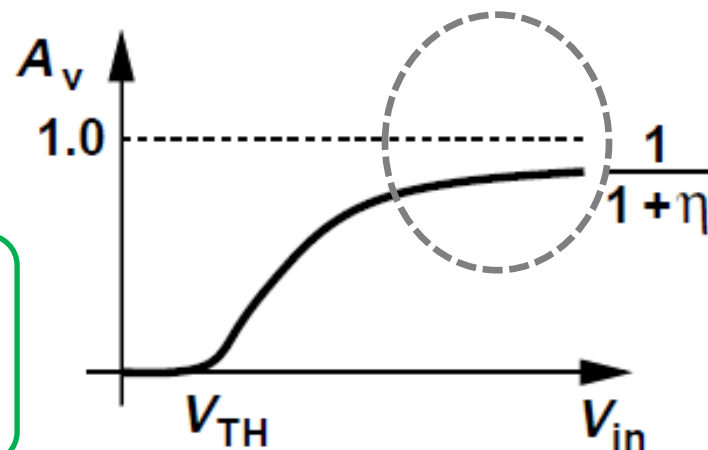


$$G_m = g_m$$

$$R_{out} = r_o \parallel R_S \parallel \left( \frac{1}{g_m + g_{mb}} \right)$$

$$A_v = \frac{g_m r_o R_S}{r_o + R_S + (g_m + g_{mb}) r_o R_S} \approx \frac{1}{1 + \eta}$$

For sufficiently large  $V_{in}$ ,  $I_D$  and thus  $g_m$ .



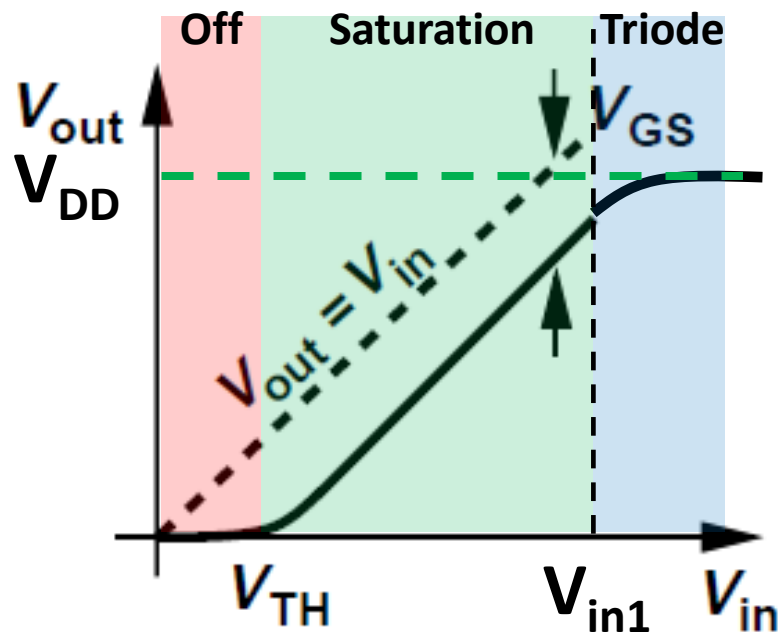
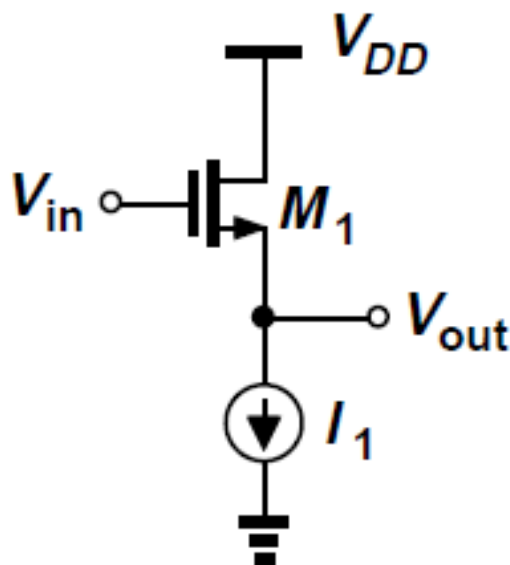
If  $(g_m + g_{mb}) r_o R_S \gg r_o$  and  $R_S$

# Source Follower with Current Source

## DC Analysis

$$\lambda = 0$$

$$\gamma \neq 0$$



$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})^2 = I_1$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_{in} - V_{out} - V_{TH}) \left( 1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}} \right) = 0$$

$$\boxed{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})} \left( 1 - \frac{\partial V_{out}}{\partial V_{in}} - \boxed{\frac{\partial V_{TH}}{\partial V_{out}}} \frac{\partial V_{out}}{\partial V_{in}} \right) = 0$$

$\quad \quad \quad = g_m \quad \quad \quad = \eta$

$$A_v = \frac{1}{1 + \eta}$$

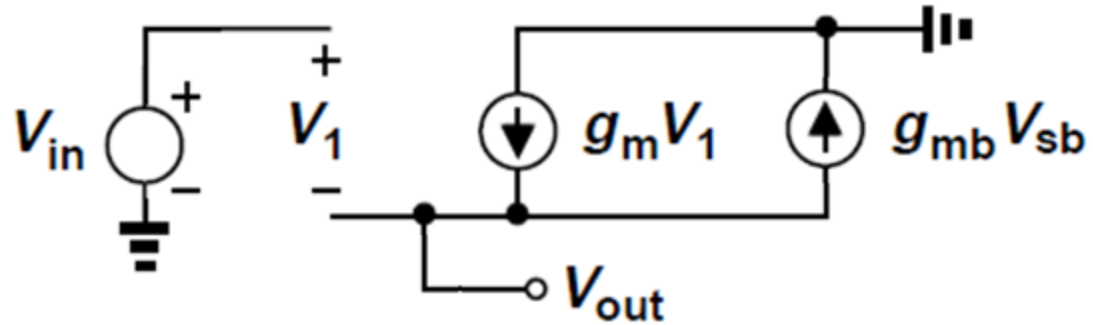
$$\text{If } \gamma = 0, A_v = 1.$$

# Source Follower with Current Source

## Small-signal Analysis

$$\lambda = 0$$

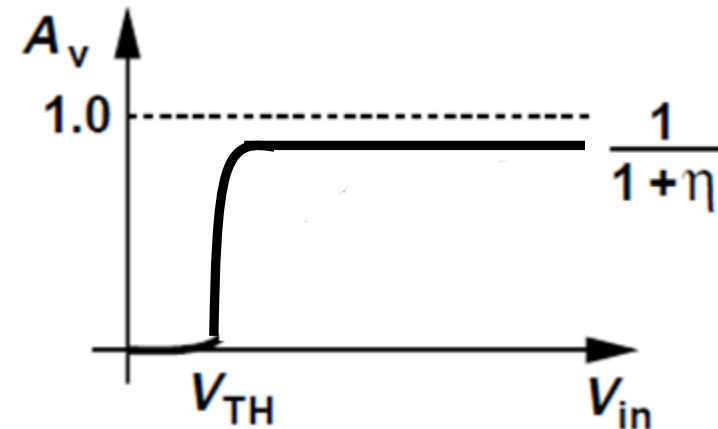
$$\gamma \neq 0$$



$$G_m = g_m$$

$$R_{out} = \frac{1}{g_m + g_{mb}}$$

$$A_v = \frac{1}{1 + \eta} \quad \text{If } \gamma = 0, A_v = 1.$$

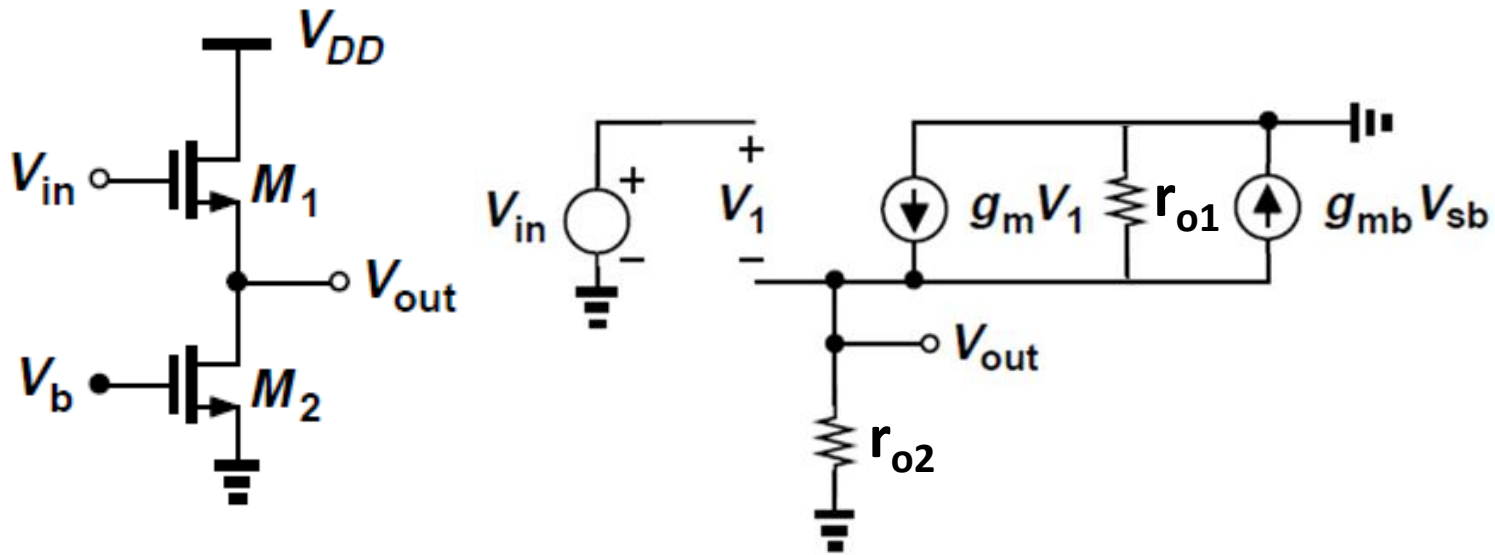


# Source Follower with Current Source

Small-signal  
Analysis

$\lambda \neq 0$

$\gamma \neq 0$



$$G_m = g_{m1}$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \left( \frac{1}{g_{m1} + g_{mb1}} \right)$$

$$A_v = \frac{g_m r_{o1} r_{o2}}{r_{o1} + r_{o2} + (g_m + g_{mb}) r_{o1} r_{o2}}$$

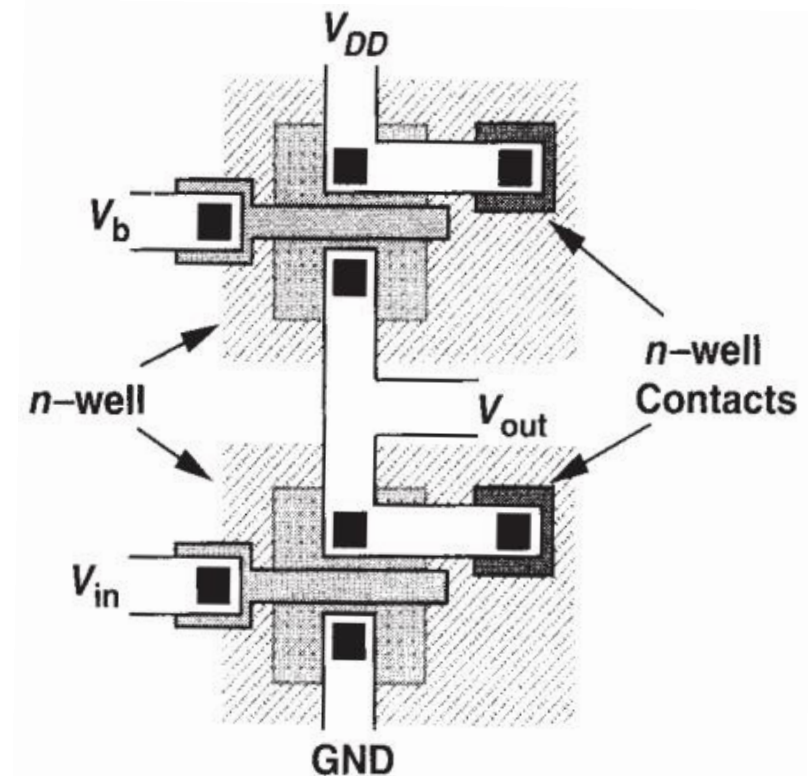
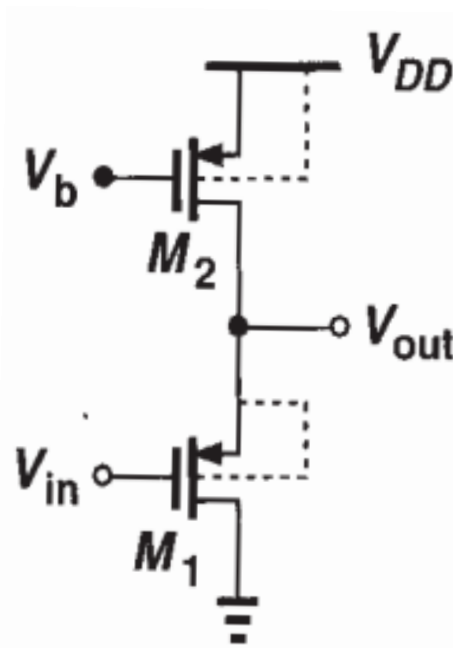
If  $r_{o1}$  and  $r_{o2}$  large,  
 $A_v$  is linear.

# Source Follower with Current Source ( $V_{SB} = 0$ ) 37

## Small-signal Analysis

$$\lambda \neq 0$$

$$\gamma \neq 0$$



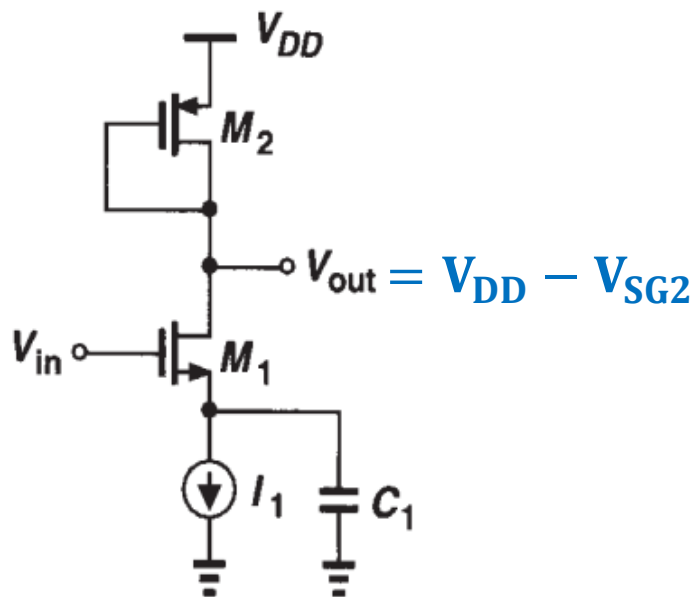
$$G_m = g_{m1}$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m1}}$$

$$A_v = \frac{g_{m1} r_{o1} r_{o2}}{r_{o1} + r_{o2} + g_{m1} r_{o1} r_{o2}}$$

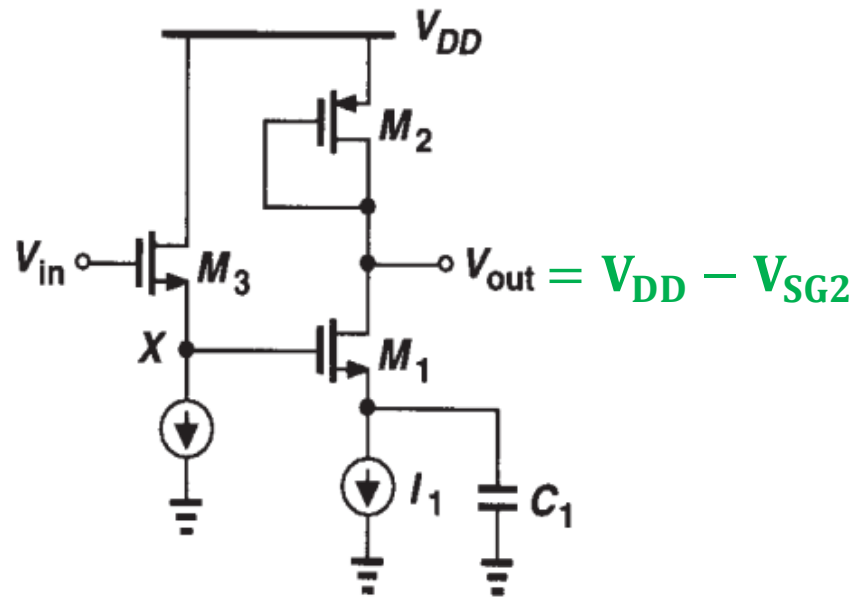
- The sacrifice here is the higher output impedance due to smaller mobility of holes relative to electrons.

# Source Follower as Level Shifter



$$V_{in} \leq V_{DD} - V_{SG2} + V_{TH1}$$

$$\begin{cases} G_m = -gm_1 \\ R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{gm_2} \end{cases}$$

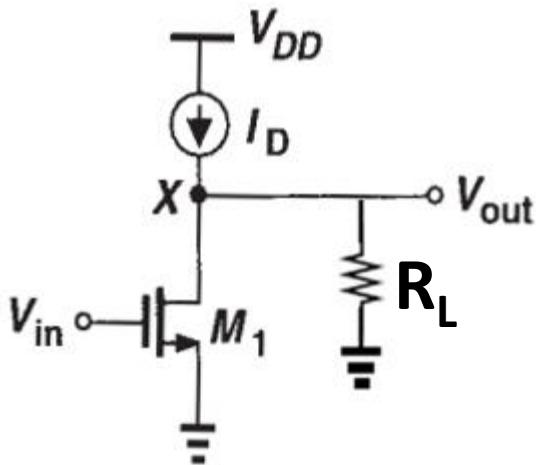


$$V_{in} - V_{GS3} \leq V_{DD} - V_{SG2} + V_{TH1}$$

$$\begin{cases} G_{m(left)} = gm_3 \\ R_{out(left)} = r_{o3} \parallel \frac{1}{gm_3 + gmb_3} \end{cases}$$

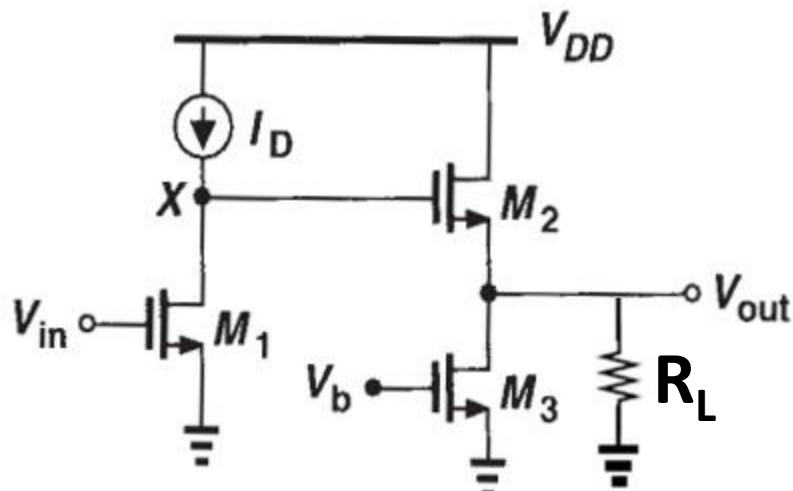
$$\begin{cases} R_{in(right)} = \infty \\ G_{m(right)} = -gm_1 \\ R_{out(right)} = r_{o1} \parallel r_{o2} \parallel \frac{1}{gm_2} \end{cases}$$

# CS + Source Follower



$$A_v = -g_{m1}(r_{o1} \parallel R_L)$$

- Voltage gain severely reduced when  $R_L$  very small



$$A_v = -g_{m1}r_{o1} \times$$

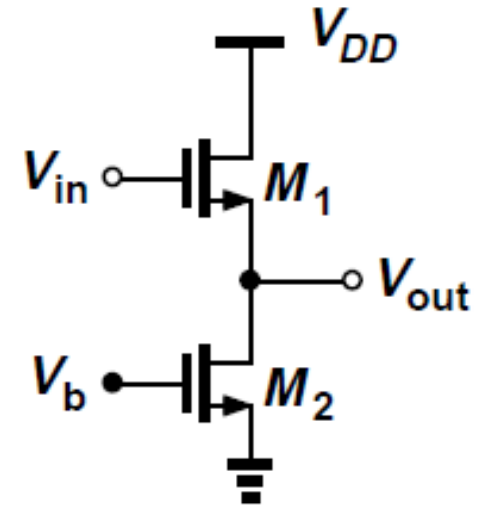
$$g_{m2} \left( r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \parallel r_{o3} \parallel R_L \right)$$

- Voltage gain maintained when  $R_L$  very small

{ Voltage Gain { Buffer

# Example

$(W/L)_1 = 20/0.5$ ,  $I_D = 0.2$  mA,  $V_{THO} = 0.6$  V,  $2\phi_F = 0.7$  V,  $\mu_n C_{ox} = 50$   $\mu\text{A}/\text{V}^2$ ,  $\gamma = 0.4$   $\text{V}^{1/2}$  and  $\lambda = 0$ . (a) Calculate  $V_{out}$  for  $V_{in} = 1.2$  V. (b) Minimum  $(W/L)_2$  for which  $M_2$  remains saturated.



Solution:

$$(a) \quad I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{THO})^2 \rightarrow V_{out} = 0.153 \text{ V}$$

$$V_{TH1} = V_{THO} + \gamma(\sqrt{2\phi_F + V_{out}} - \sqrt{2\phi_F}) = 0.635 \text{ V} \rightarrow V_{out} \approx 0.118 \text{ V}$$

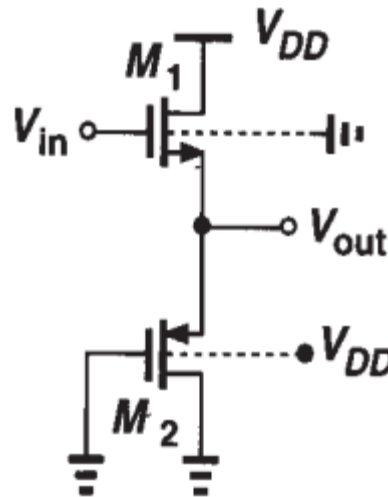
$$(b) \quad V_{out} = 0.118 \text{ V} \geq V_{GS2} - V_{TH2}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS2} - V_{TH2})^2 \rightarrow \left(\frac{W}{L}\right)_2 \geq \frac{283}{0.5}$$



# Example

Calculate the small signal voltage gain of the circuit below.



Solution:

$$G_m = g_{m1}$$

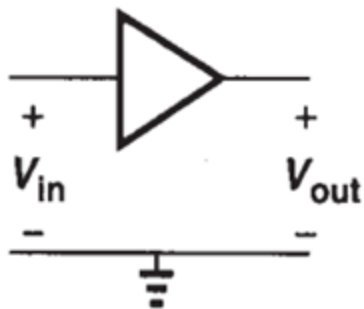
$$R_{out} = \frac{1}{g_{m1} + g_{mb1}} \parallel r_{o1} \parallel \frac{1}{g_{m2} + g_{mb2}} \parallel r_{o2}$$

$$A_v = G_m R_{out}$$

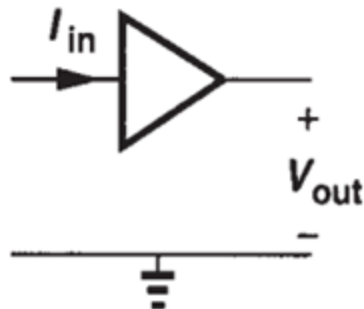
# Common-Gate

# Ideal Amplifier

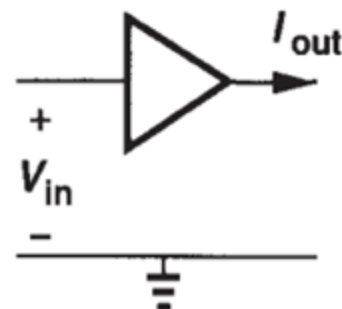
Voltage Amp.



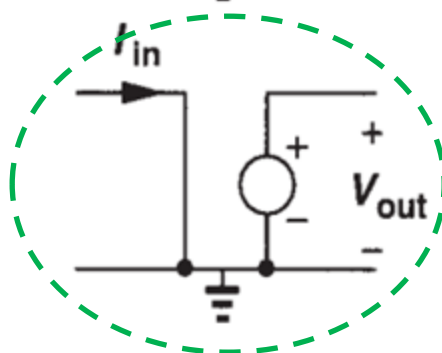
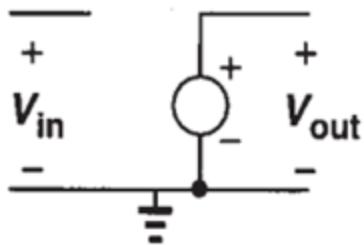
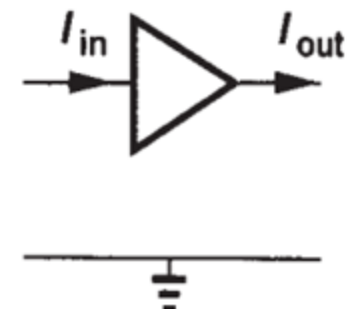
Transimpedance Amp.



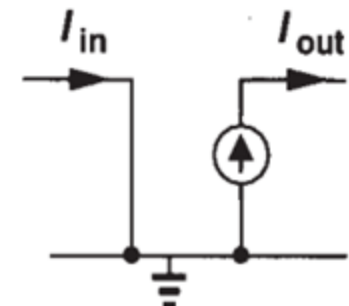
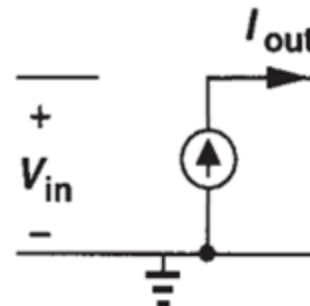
Transconductance Amp.



Current Amp.



Common-Gate  
+ Source Follower



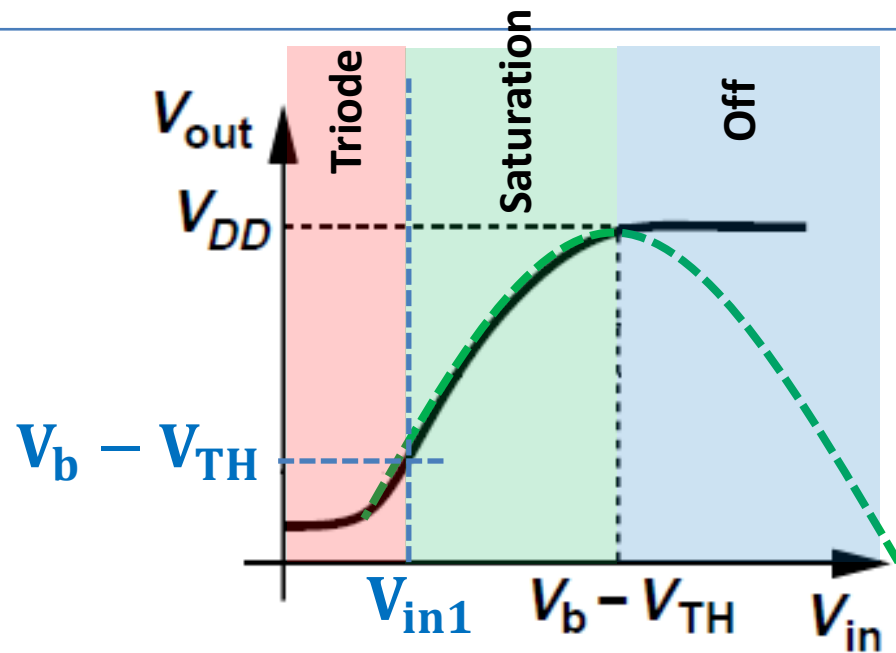
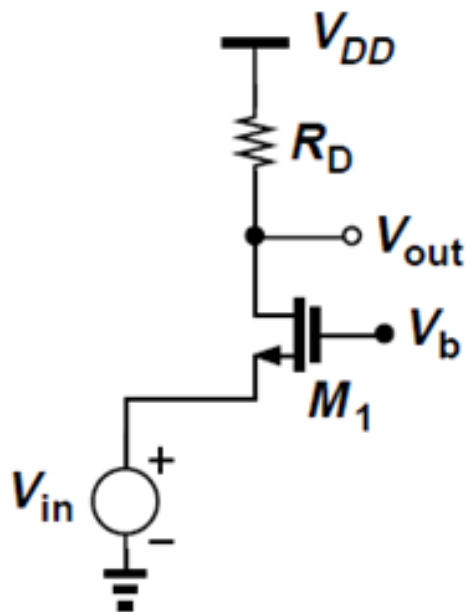
- For converting and amplifying small-signal current to voltages, common-gate provides **low input impedance** and **moderate gain**, but relatively **large output impedance**.

# Common-Gate

## DC Analysis

$$\lambda = 0$$

$$\gamma \neq 0$$



- $V_{in} > V_b - V_{TH} \rightarrow M_1$  Off

$$V_{out} = V_{DD}$$

- $V_b - V_{TH} > V_{in} > V_{in1} \rightarrow M_1$  in Saturation

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

- $V_{in} < V_{in1} \rightarrow M_1$  in Triode

$$V_{out} = V_{DD} - R_D \mu_n C_{ox} \frac{W}{L} [(V_b - V_{in} - V_{TH})(V_{out} - V_{in}) - \frac{1}{2} (V_{out} - V_{in})^2]$$

$$V_{out} = V_b - V_{TH}$$

$$= V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in1} - V_{TH})^2$$

# Common-Gate

## DC Analysis

$$\lambda = 0$$

$$\gamma \neq 0$$

- $V_b - V_{TH} > V_{in} > V_{in1} \rightarrow M_1$  in Saturation

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_b - V_{in} - V_{TH}) \left( -1 - \frac{\partial V_{TH}}{\partial V_{in}} \right)$$

$$= R_D \underbrace{\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})}_{= gm} \left( 1 + \underbrace{\frac{\partial V_{TH}}{\partial V_{in}}}_{= \eta} \right) = \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = R_D gm(1 + \eta)$$

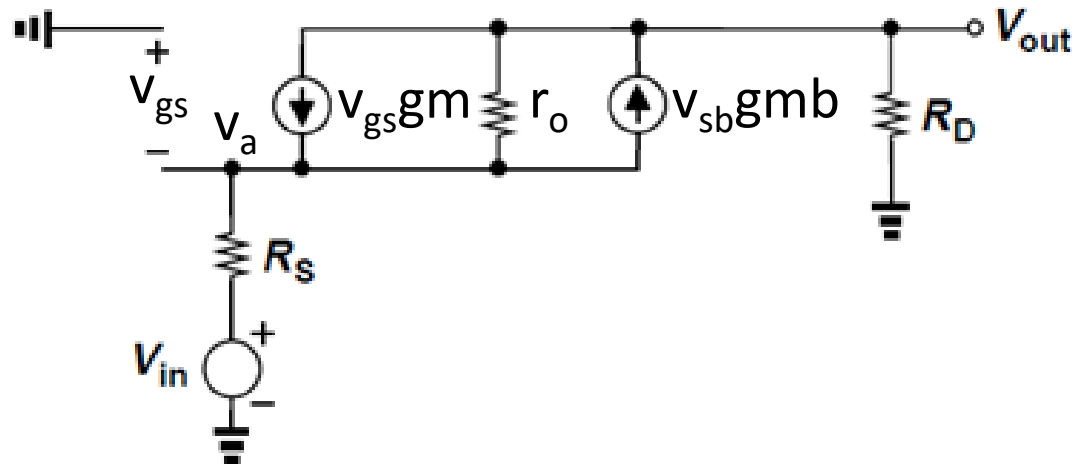
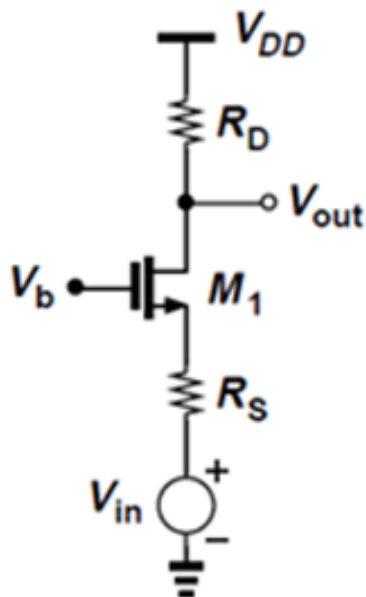
- $gm$  is a function of  $I_D$  and  $\eta$  is a function of  $V_{SB}$ .
- $A_v$  is not quite linear.

# Common-Gate

Small-signal  
Analysis

$\lambda \neq 0$

$\gamma \neq 0$



$$G_m = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_S + (g_m + g_{mb})r_o R_S}$$

$$R_{out} = R_D \parallel [r_o + R_S + (g_m + g_{mb})r_o R_S]$$

$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_S + (g_m + g_{mb})r_o R_S + R_D} R_D \approx R_D g_m (1 + \eta)$$

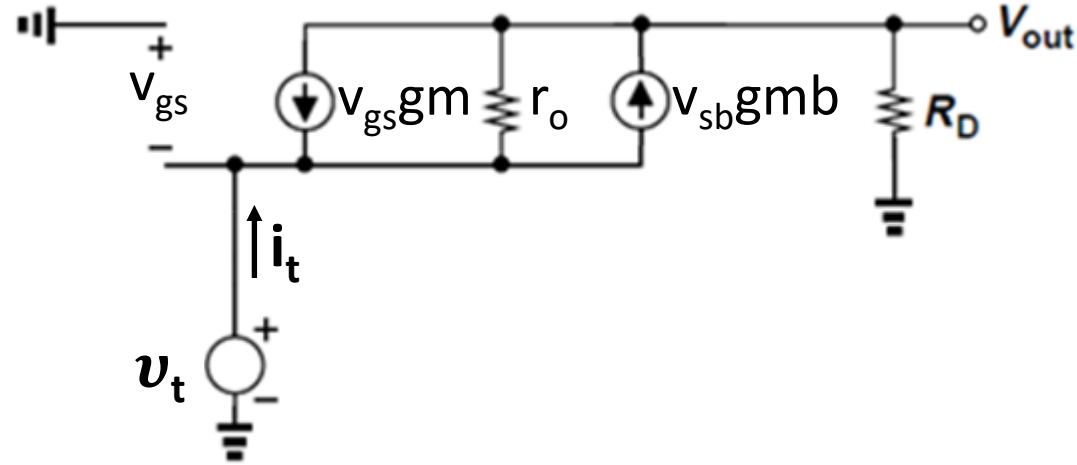
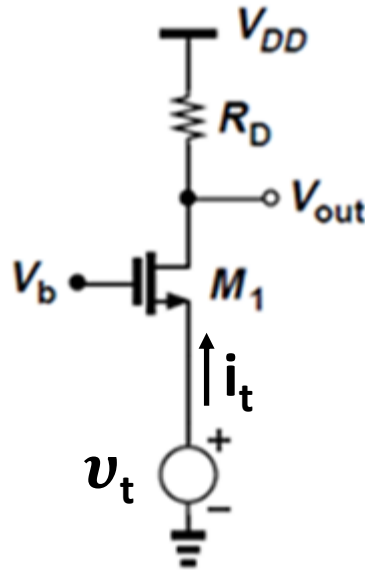
If  $R_S = 0$  and  $r_o = \infty$

# Common-Gate (Input Impedance)

Small-signal  
Analysis

$\lambda \neq 0$

$\gamma \neq 0$



$$\begin{cases} i_t = v_t(gm + gmb) + \frac{v_t - v_{out}}{r_o} \\ v_{out} = R_D i_t \end{cases}$$

$$R_{in} = \frac{R_D + r_o}{1 + (gm + gmb)r_o}$$

If  $R_D = 0$   $R_{in} = r_o \parallel \frac{1}{gm} \parallel \frac{1}{gmb}$

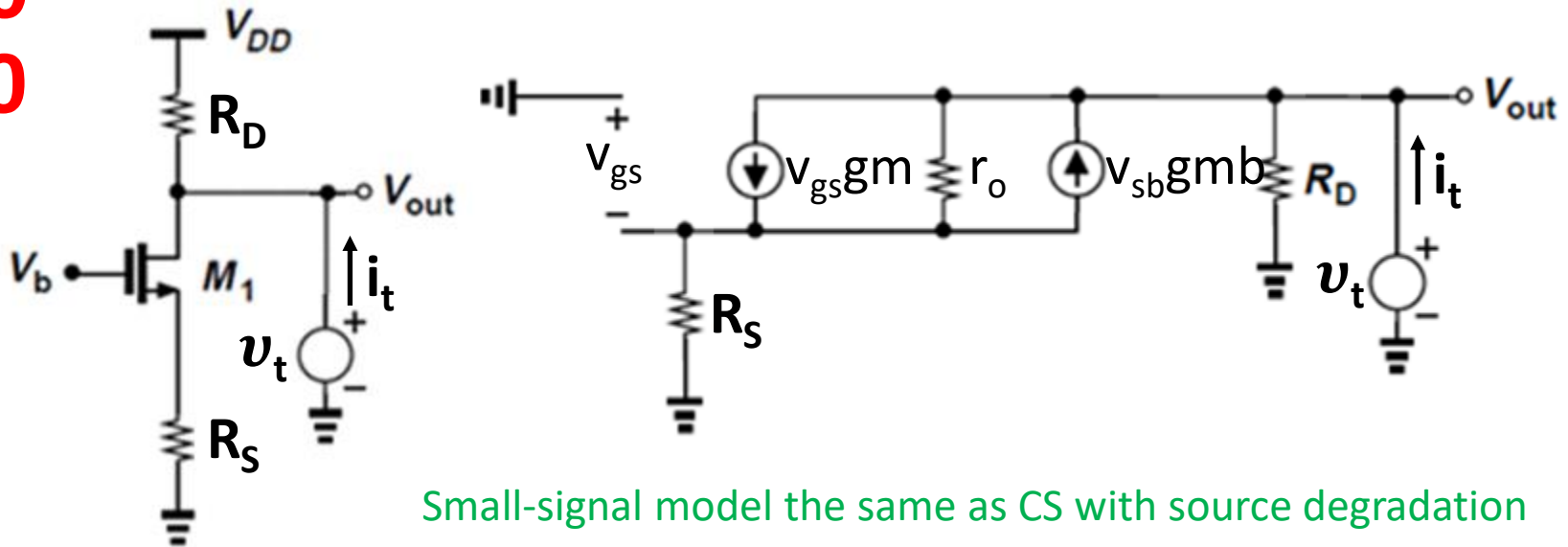
If  $R_D = \infty$   $R_{in} = \infty$

# Common-Gate (Output Impedance)

## Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$

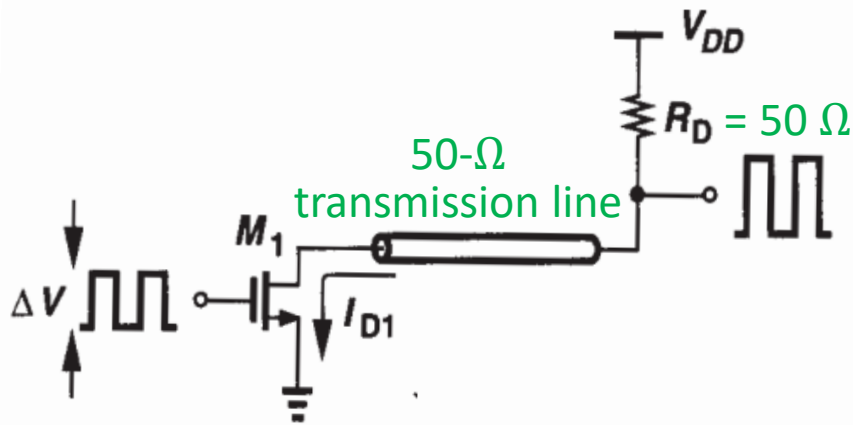


$$R_{\text{out}} = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D$$



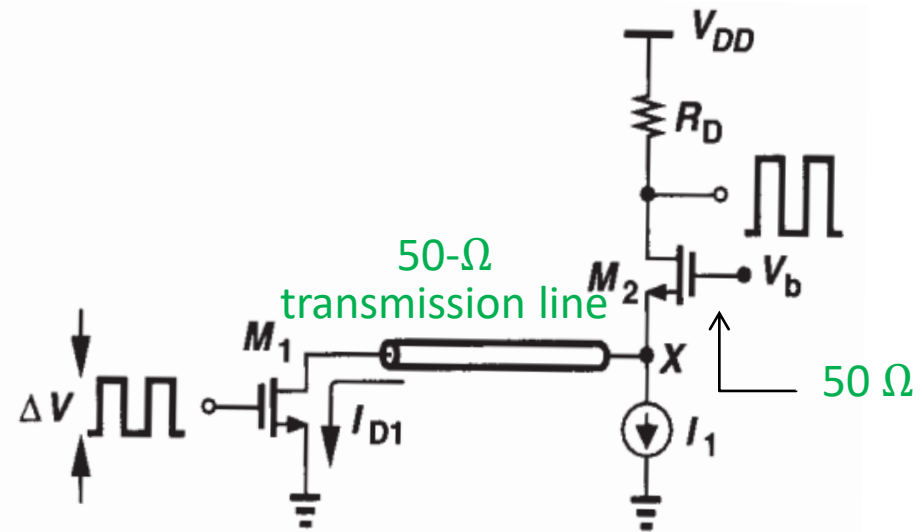
# Example

Calculate the small-signal voltage gain at low frequencies of the circuits below. To minimize wave reflection at point X, the input impedance must be equal to  $50\ \Omega$ .



$$A_v = -g_{m1} R_D$$

$= 50\ \Omega$



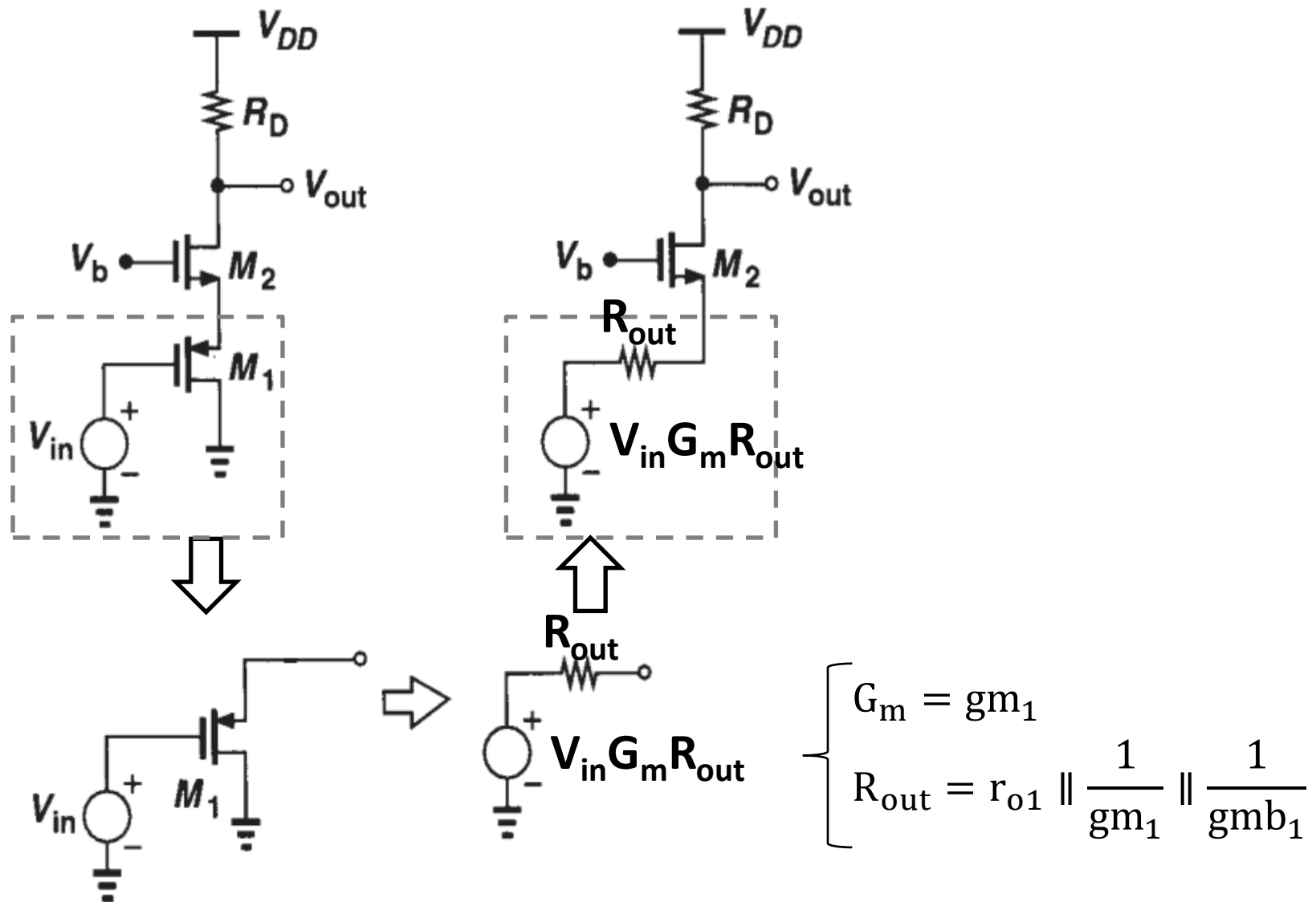
$$A_v = -g_{m1} R_D$$

can be much larger than  $50\ \Omega$ , so as to achieve a much higher gain.

$$R_{in} = \frac{R_D + r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}} = 50\ \Omega$$

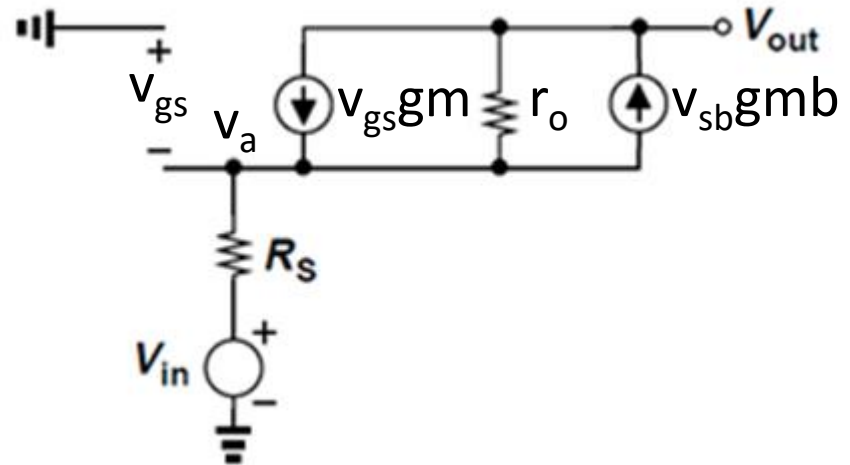
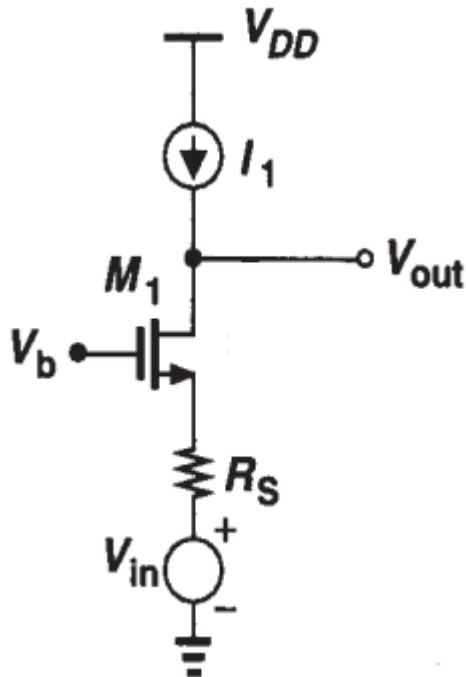
# Example

Calculate the small-signal voltage gain of the circuit below. ( $\lambda \neq 0$ ,  $\gamma \neq 0$ )



# Example

Calculate the small-signal voltage gain of the circuit below. ( $\lambda \neq 0$ ,  $\gamma \neq 0$ )



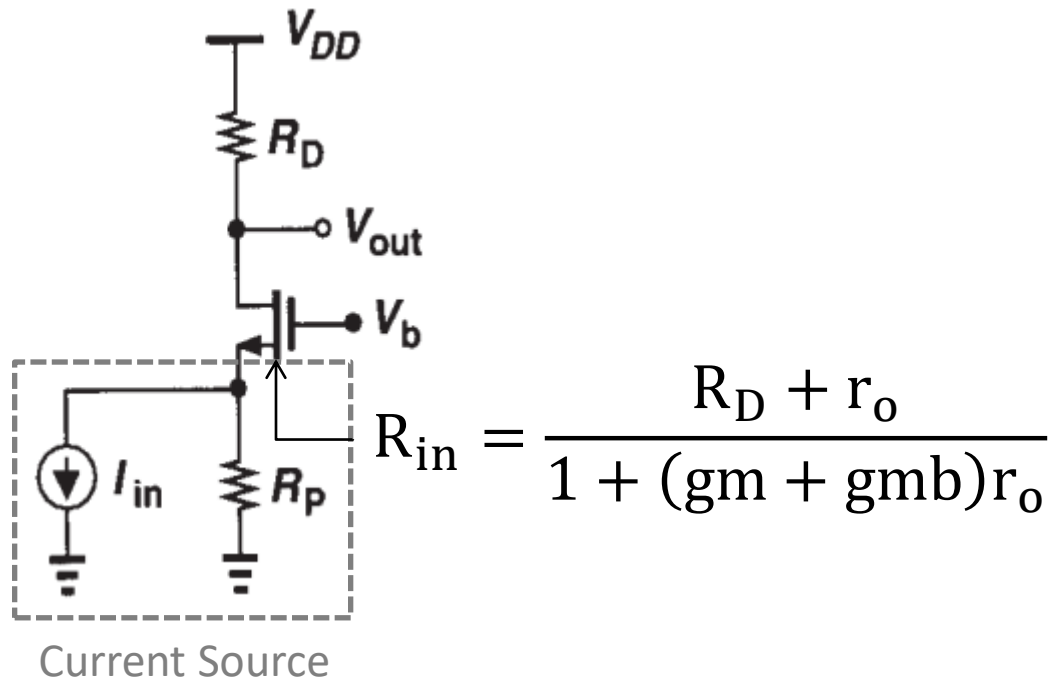
Since no current flowing in  $R_S$ ,  $v_a = v_{in}$ .

$$v_{out} - v_{in}(g_m + g_{mb})r_o = v_{in}$$

$$A_v = \frac{v_{out}}{v_{in}} = 1 + (g_m + g_{mb})r_o$$

# Example

Calculate the small-signal transimpedance gain of the circuit below. ( $\lambda \neq 0$ ,  $\gamma \neq 0$ )

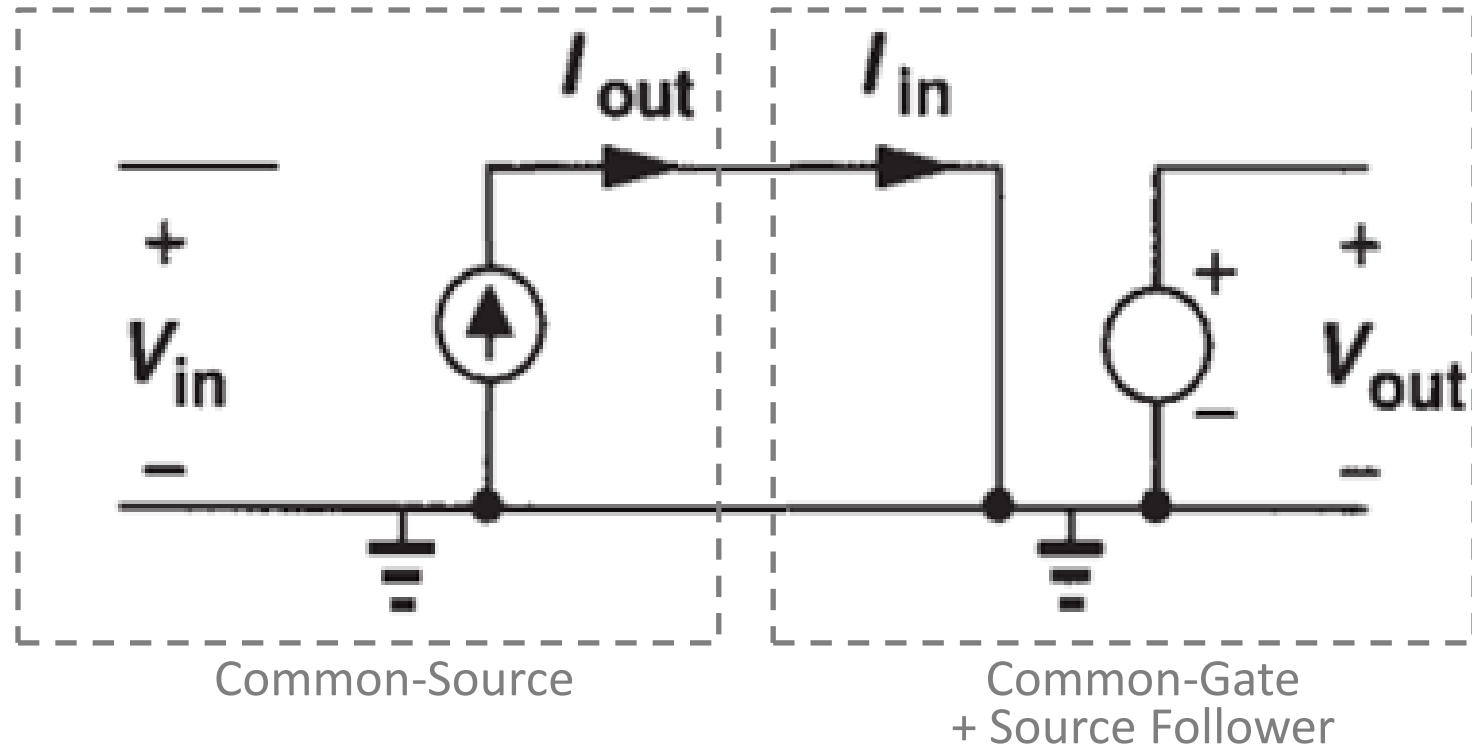


$$-i_{in} \frac{R_P}{R_{in} + R_P} R_D = v_{out}$$

$$\frac{v_{out}}{i_{in}} = -\frac{R_P}{R_{in} + R_P} R_D = \frac{-R_P R_D [1 + (g_m + g_{mb})r_o]}{R_D + r_o + R_P + (g_m + g_{mb})r_o R_P}$$

Cascode

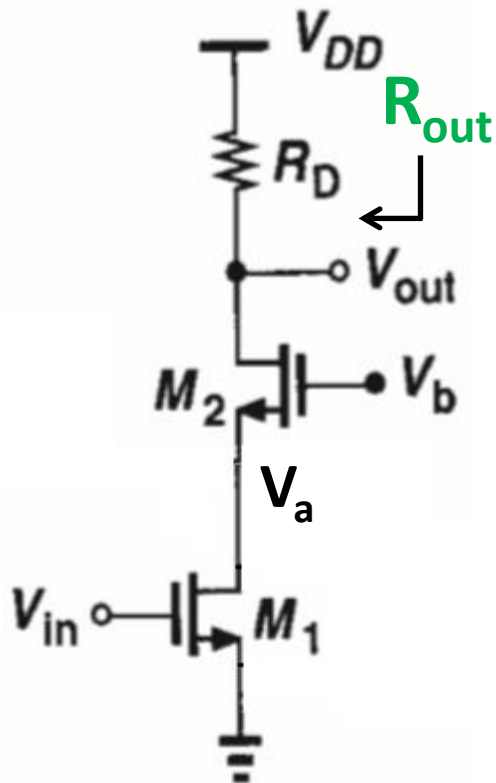
# Ideal Amplifier



# CS + CG with Resistive Load

Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$\left\{ \begin{aligned} G_m &= -g_{m1} \frac{r_{o1}}{r_{o1} + \left( r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \right)} \\ R_{out} &= [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \parallel R_D \end{aligned} \right.$$

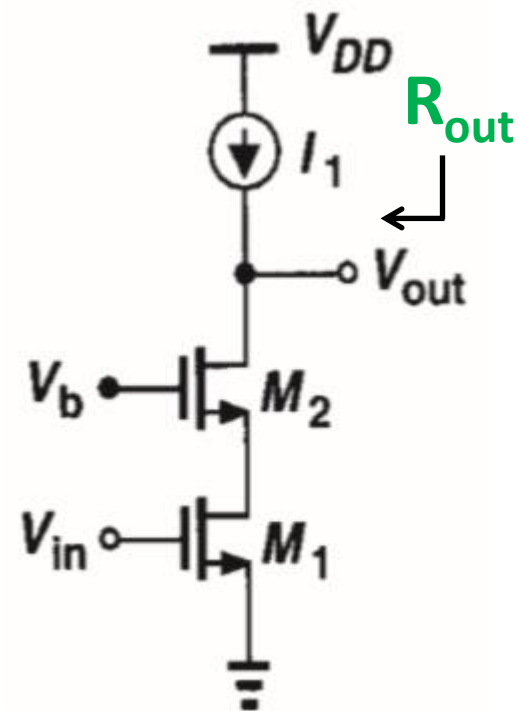
$$A_v = G_m R_{out}$$

$$\begin{aligned} V_a &\geq V_{in} - V_{TH1} \\ V_b - V_{GS2} &\geq V_{in} - V_{TH1} \\ V_b &\geq V_{in} - V_{TH1} + V_{GS2} \\ V_{out} &\geq V_b - V_{TH2} \geq (V_{in} - V_{TH1}) + (V_{GS2} - V_{TH2}) \\ V_{DD} &\geq V_{out} \geq V_{ov1} + V_{ov2} \end{aligned}$$

# CS + CG with Ideal Current Source Load

Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$\left\{ \begin{aligned} G_m &= -g_{m1} \frac{r_{o1}}{r_{o1} + \left( r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \right)} \\ R_{out} &= r_{o1} + r_{o2} + (g_{m2} + g_{mb2}) r_{o2} r_{o1} \end{aligned} \right.$$

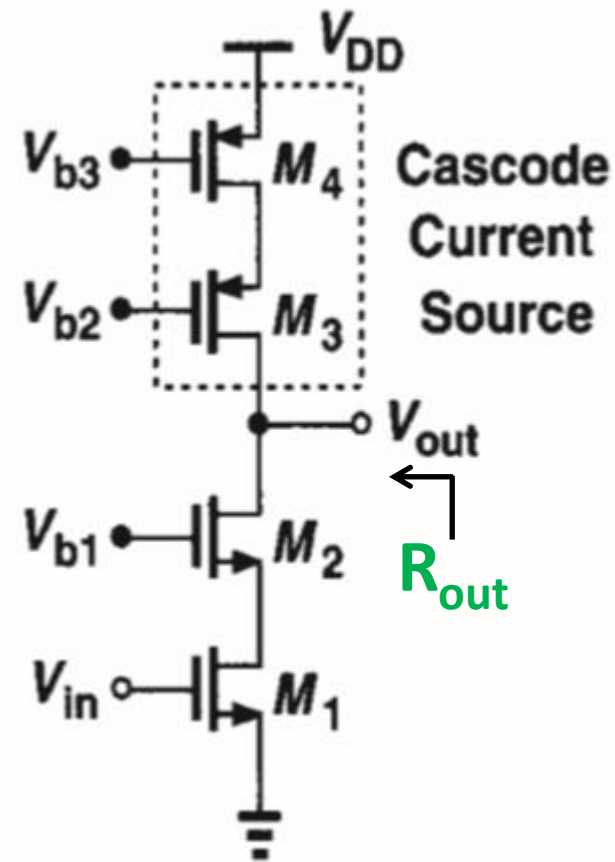
$$A_v = G_m R_{out}$$



# CS + CG with Cascode Current Source Load

Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$\left\{ \begin{aligned} G_m &= -g_{m1} \frac{r_{o1}}{r_{o1} + \left( r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \right)} \\ R_{out} &= [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \\ &\parallel [r_{o3} + r_{o4} + (g_{m3} + g_{mb3})r_{o3}r_{o4}] \end{aligned} \right.$$

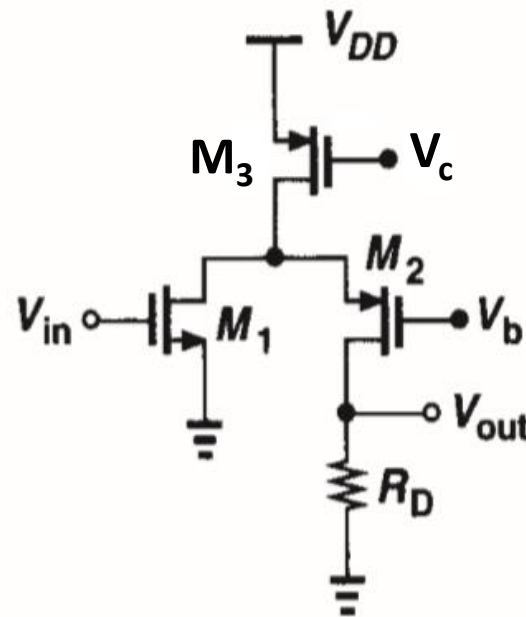
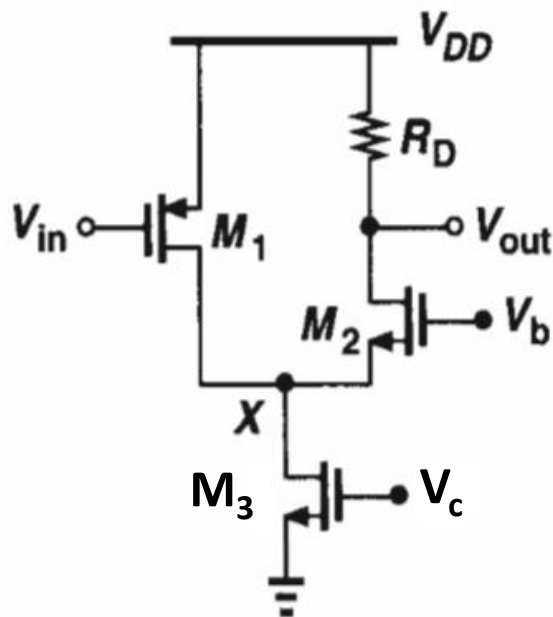
$$A_v = G_m R_{out}$$

$$V_{DD} - V_{ov3} - V_{ov4} \geq V_{out} \geq V_{ov1} + V_{ov2}$$

# Folded Cascode

Small-signal  
Analysis

$\lambda \neq 0$   $\gamma \neq 0$



$$\left\{ \begin{array}{l} G_m = -g_{m1} \frac{(r_{o1} \parallel r_{o3})}{(r_{o1} \parallel r_{o3}) + \left( r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \right)} \\ R_{out} = [(r_{o1} \parallel r_{o3}) + r_{o2} + (g_{m2} + g_{mb2})r_{o2}(r_{o1} \parallel r_{o3})] \parallel R_D \end{array} \right.$$

$$A_v = G_m R_{out}$$