BJT and BJT Circuit

Ve311 Electronic Circuits (Summer 2019)

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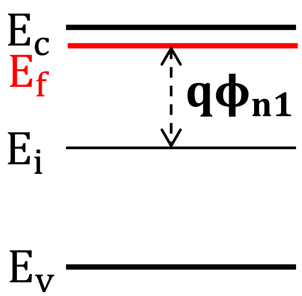
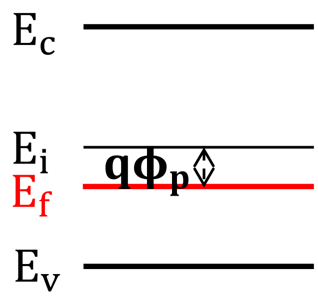
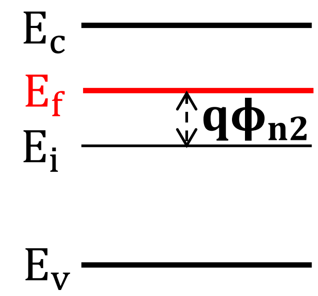
Last Updated on **June 17, 2019**

BJT (Before Contact)

Emitter n-type	**Base** p-type	**Collector** n-type







$$n \cong N_{d1} = n_i e^{\frac{q\phi_{n1}}{kT}}$$

$$p \cong \frac{{n_i}^2}{N_{d1}} = n_i e^{\frac{-q\phi_{n1}}{kT}}$$

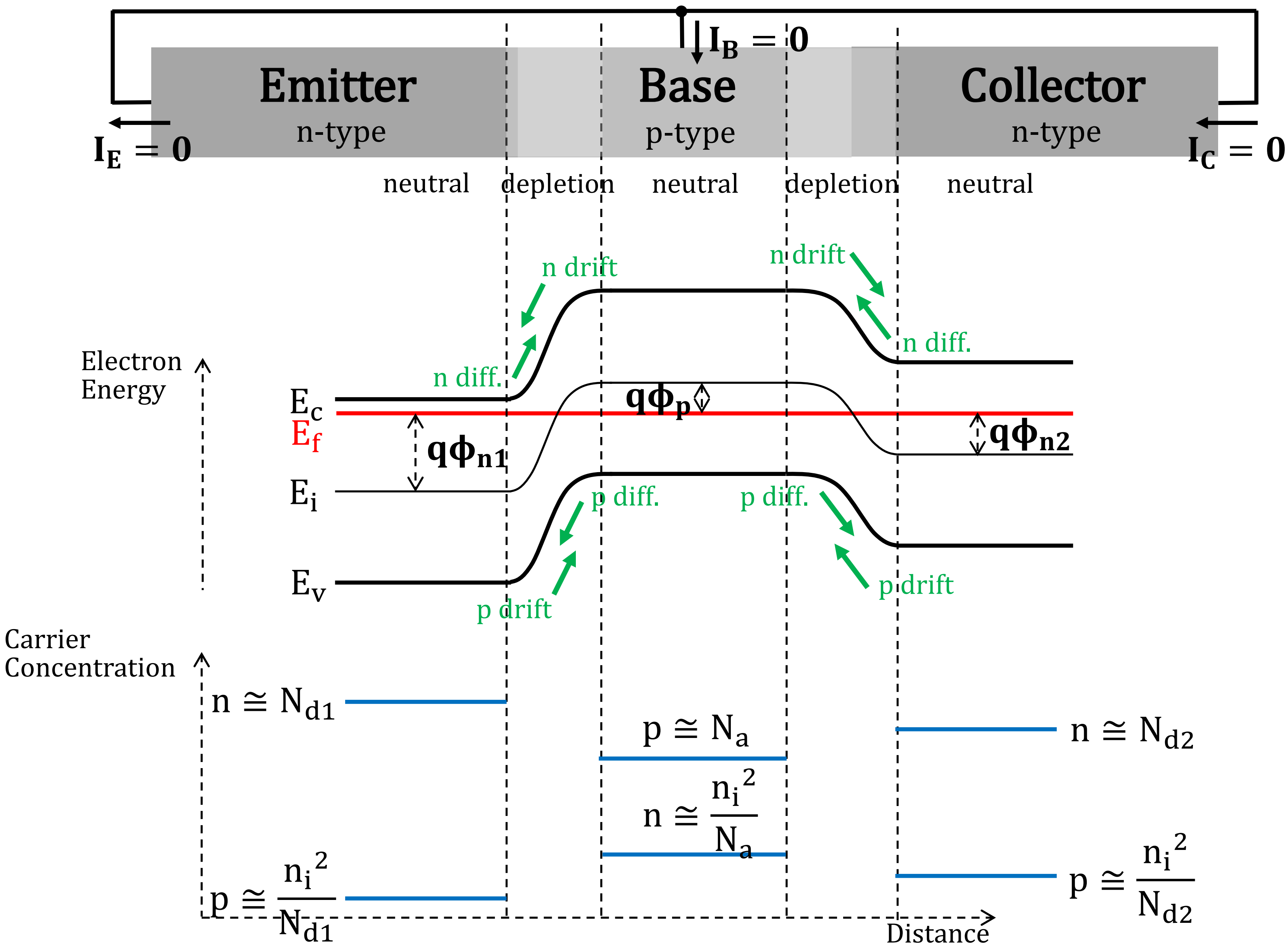
$$p \cong N_a = n_i e^{\frac{q\phi_p}{kT}}$$

$$n \cong \frac{{n_i}^2}{N_a} = n_i e^{\frac{-q\phi_p}{kT}}$$

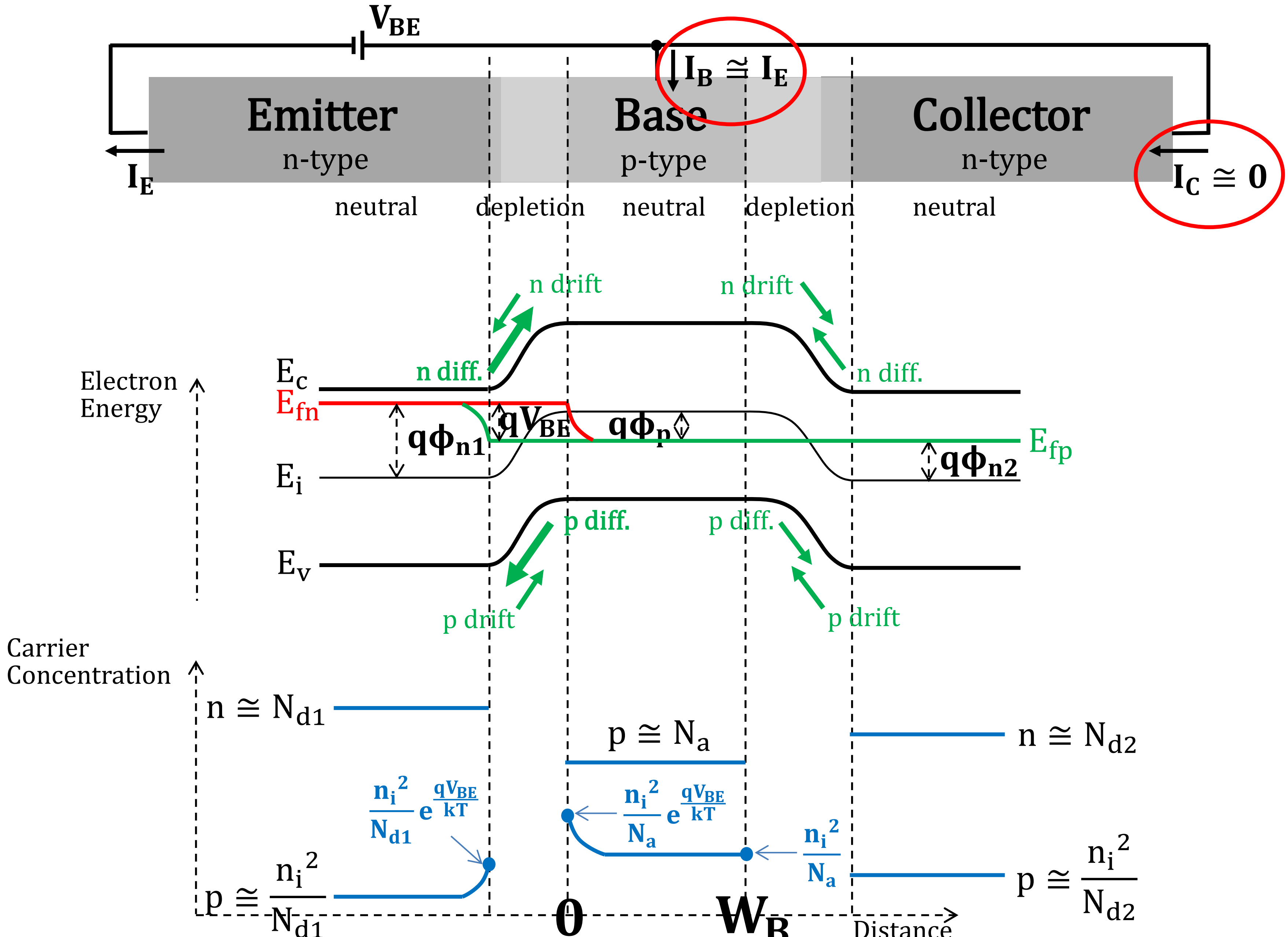
$$n \cong N_{d2} = n_i e^{\frac{q\phi_{n2}}{kT}}$$

$$p \cong \frac{{n_i}^2}{N_{d2}} = n_i e^{\frac{-q\phi_{n2}}{kT}}$$

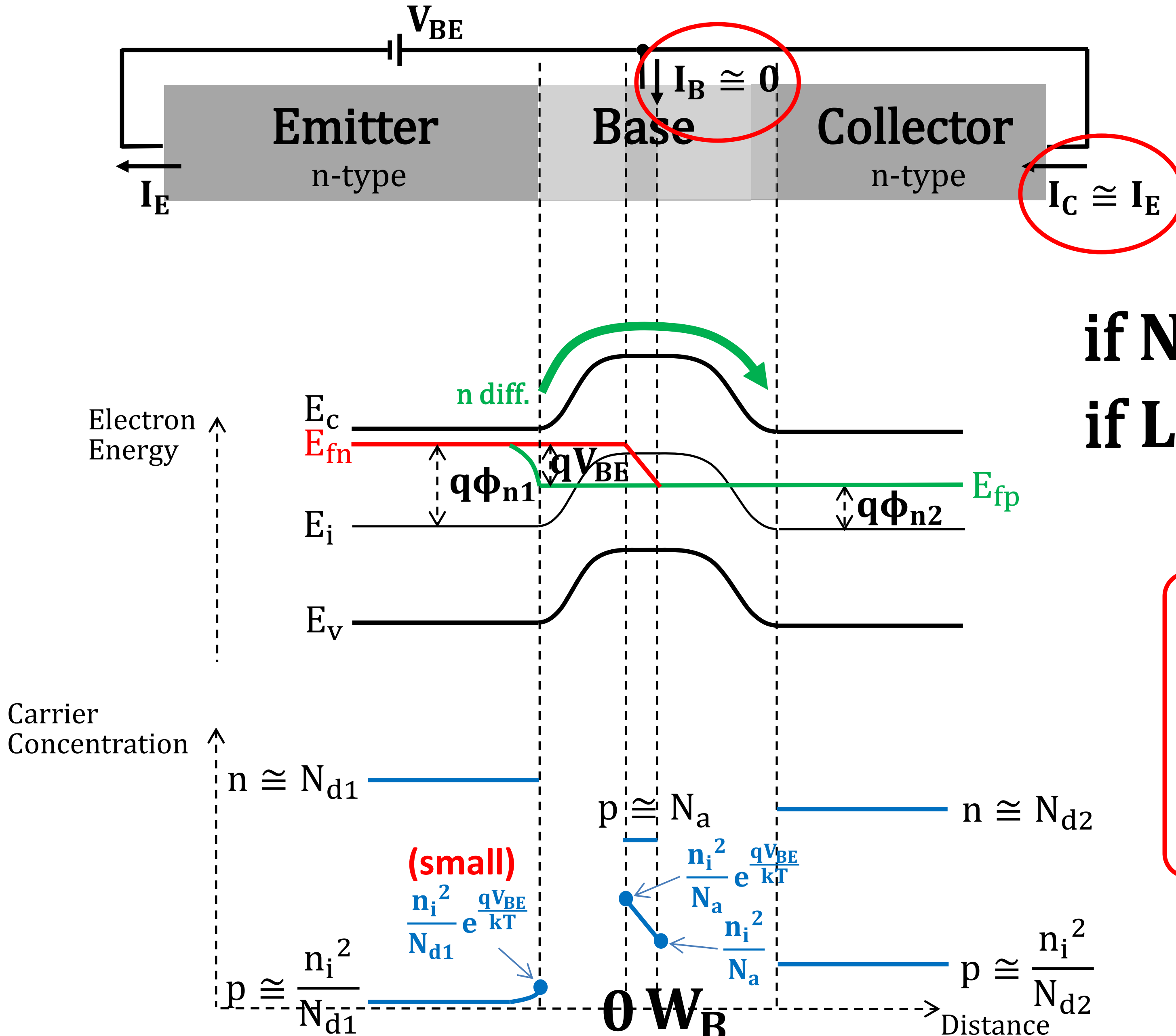
$V_{BE} = 0$ and $V_{CB} = 0$



$V_{BE} > 0$ and $V_{CB} = 0$



$V_{BE} > 0$ and $V_{CB} = 0$ (W_B Short and N_{d1} Large)



if $N_{d1} \gg N_a$

if $L_n \gg W_B$

$$I_E = I_C + I_B$$

$$\frac{I_C}{I_E} = \alpha$$

$$\frac{I_C}{I_B} = \beta$$

I-V Characteristic

I-V Characteristic (I)

- **Assume $E_x = 0$**

$(n = n_o + \Delta n)$

$$J_n(\text{electron current in the base neutral region}) = q\mu_n n E_x + qD_n\frac{dn}{dx} = qD_n\frac{dn}{dx} = qD_n\frac{d\Delta n}{dx}$$

In steady − state $\Rightarrow \dfrac{\partial n}{\partial t} = \dfrac{1}{q}\dfrac{dJ_n}{dx} + G - R = 0$

$$\Rightarrow \frac{d^2\Delta n}{dx^2} = \frac{\Delta n}{D_n\tau_n} = \frac{\Delta n}{{L_n}^2}$$

Put Δn back into here

$$\Rightarrow \Delta n(x) = K_1 e^{\frac{-x}{L_n}} + K_2 e^{\frac{x}{L_n}}$$

$$\text{B.C.}\begin{cases}\Delta n(0) = \dfrac{{n_i}^2}{N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\\[2ex] \Delta n(W_B) = 0\end{cases}$$

$$\Rightarrow \Delta n(x) = \frac{{n_i}^2}{N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\frac{\sinh\left(\frac{W_B - x}{L_n}\right)}{\sinh\left(\frac{W_B}{L_n}\right)}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

I-V Characteristic (II)

$$I_n(x=0) = I_E = \frac{AqD_n{n_i}^2}{L_nN_a}\left(e^{\frac{qV_{BE}}{kT}}-1\right)\coth\left(\frac{W_B}{L_n}\right) = \frac{AqD_n{n_i}^2}{W_BN_a}\left(e^{\frac{qV_{BE}}{kT}}-1\right) \quad \text{if } L_n \gg W_B$$

$$I_n(x=W_B) = I_C = \frac{AqD_n{n_i}^2}{L_nN_a}\left(e^{\frac{qV_{BE}}{kT}}-1\right)\operatorname{csch}\left(\frac{W_B}{L_n}\right) = \frac{AqD_n{n_i}^2}{W_BN_a}\left(e^{\frac{qV_{BE}}{kT}}-1\right) \quad \text{if } L_n \gg W_B$$

$$\boldsymbol{\alpha = \frac{I_C}{I_E} = \operatorname{sech}\left(\frac{W_B}{L_n}\right) \cong 1 - \frac{{W_B}^2}{2{L_n}^2}} \quad \text{if } L_n \gg W_B$$

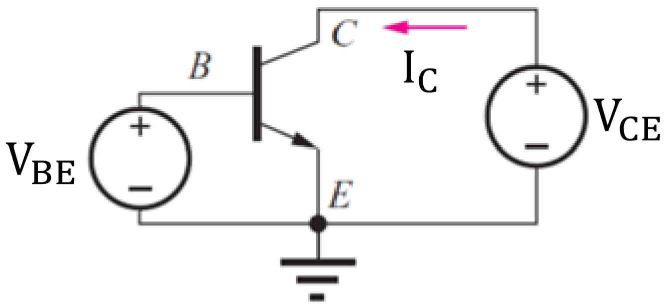
$$\boldsymbol{\beta = \frac{I_C}{I_B} = \frac{\alpha I_E}{I_E - \alpha I_E} = \frac{\alpha}{1-\alpha}}$$

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} \cong \frac{1}{x} \text{ if x small}$$

$$\operatorname{csch}(x) = \frac{2}{e^x - e^{-x}} \cong \frac{1}{x} \text{ if x small}$$

$$\operatorname{sech}(x) = \frac{2}{e^x + e^{-x}}$$

Summary



$V_{CE} \geq V_{BE}$

$\Rightarrow$ **Forward − Active**

For $L_n \gg W_B$

$$I_C \cong \frac{AqD_n n_i^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$

$$\alpha = \frac{I_C}{I_E} \cong 1 - \frac{W_B^2}{2L_n^2}$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$$

Ideal

$$I_C = \frac{AqD_n n_i^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$

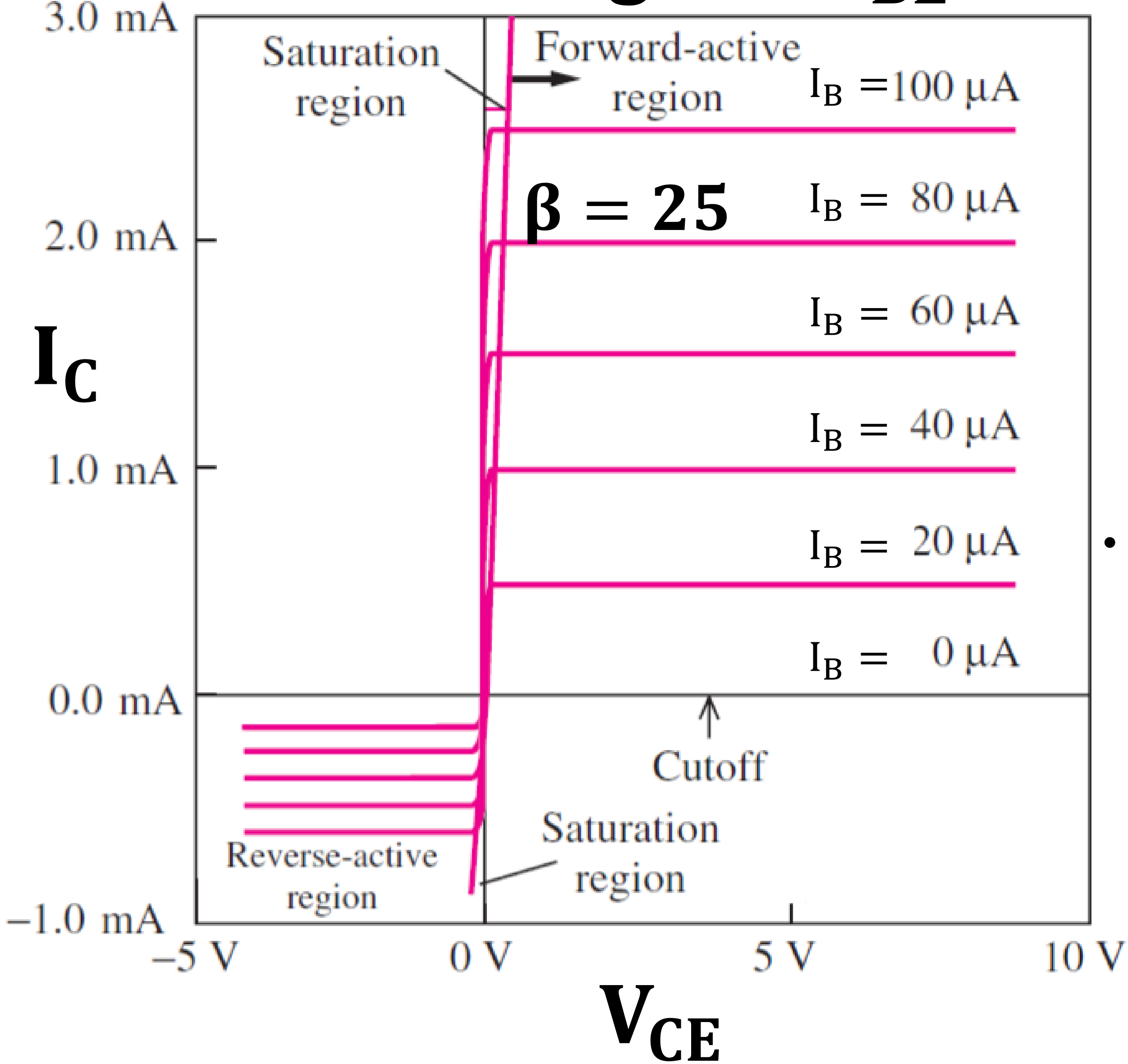
$$\alpha = \frac{I_C}{I_E} = 1$$

$$\beta = \frac{I_C}{I_B} = \infty$$

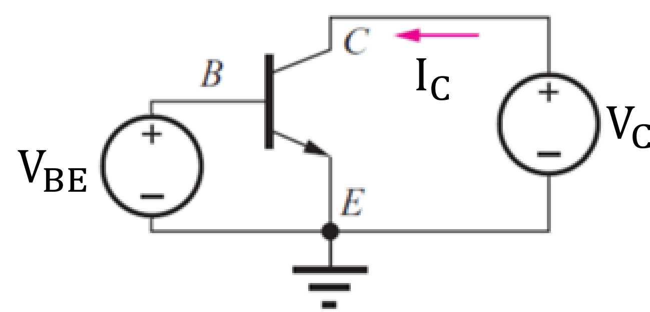
I_C vs V_{CE} and I_C vs V_{BE} in Forward-Active Region

I_C vs V_{CE}

For each given V_{BE}



$V_{CE} \geq V_{BE}$
$\Rightarrow$ Forward − Active

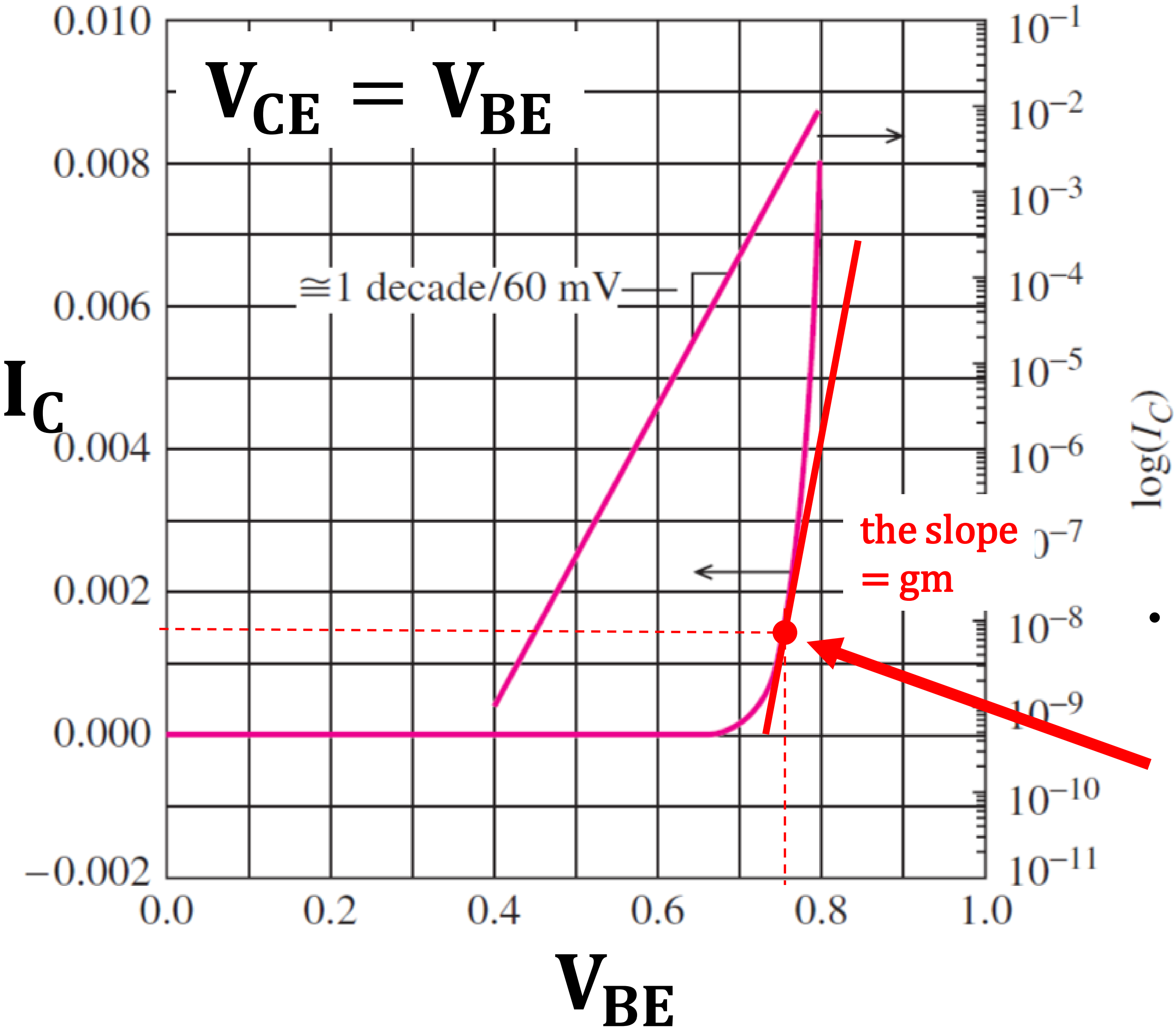


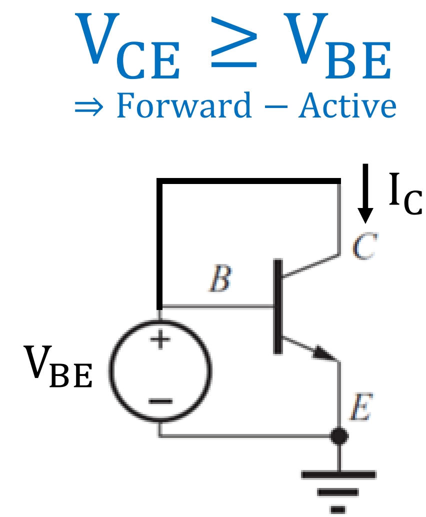
- **At certain V_{BE} and V_{CE} biasing condition, small-signal output impedance (r_o):**

$$\frac{1}{r_o} = \frac{dI_C}{dV_{CE}} = 0$$

$$r_o = \infty$$

I_C vs V_{BE}





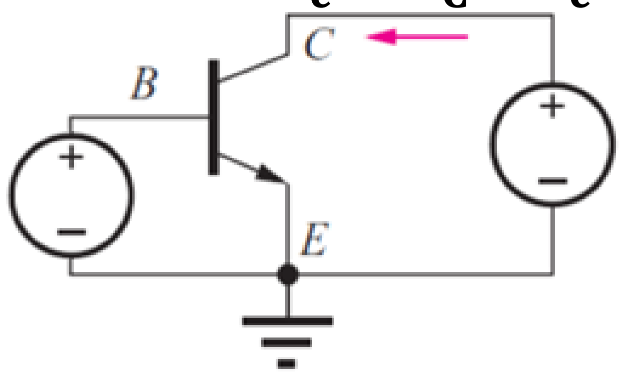
- At certain V_{BE} and V_{CE} biasing condition, small-signal transconductance gain (gm):

$$\boxed{gm = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}}$$

Small-Signal Model

Hybrid-π Model (without Early Effect)

$$\mathbf{I_c = I_C + i_c}$$



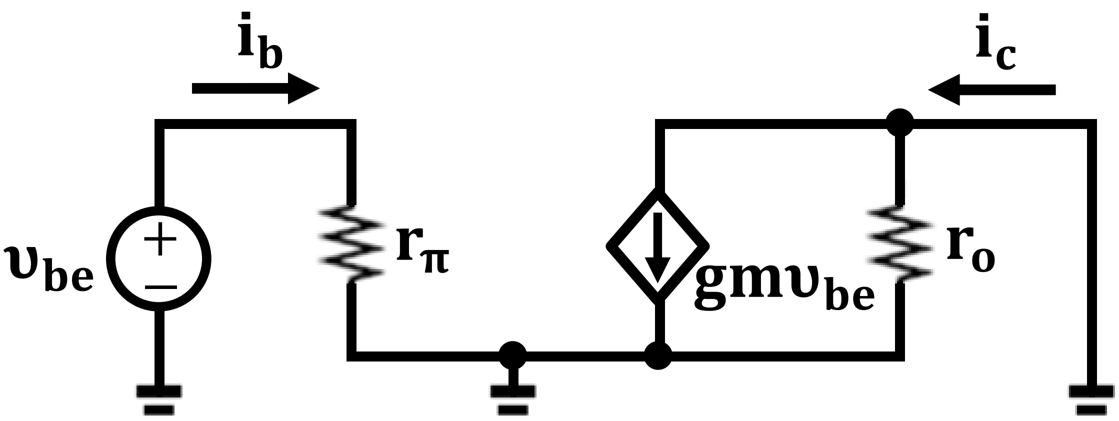
$\mathbf{V_{BE} + \upsilon_{be}}$

$\mathbf{V_{CE}}$

$$V_{CE} \geq V_{BE}$$
$\Rightarrow$ Forward – Active

$$\boxed{\mathbf{I_C \cong \frac{AqD_n n_i{}^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)}}$$

In small-signal analysis:

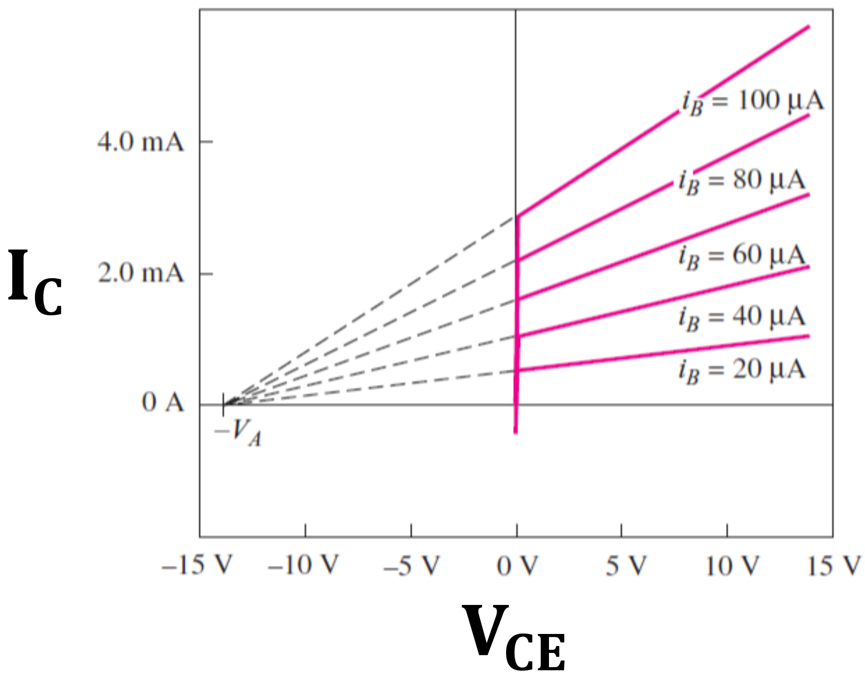


$$\mathbf{r_\pi = \frac{1}{\frac{dI_B}{dV_{BE}}} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} = \frac{1}{\frac{gm}{\beta}} = \frac{\beta}{gm}}$$

$$\mathbf{gm = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}}$$

$$\mathbf{r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} = \infty}$$

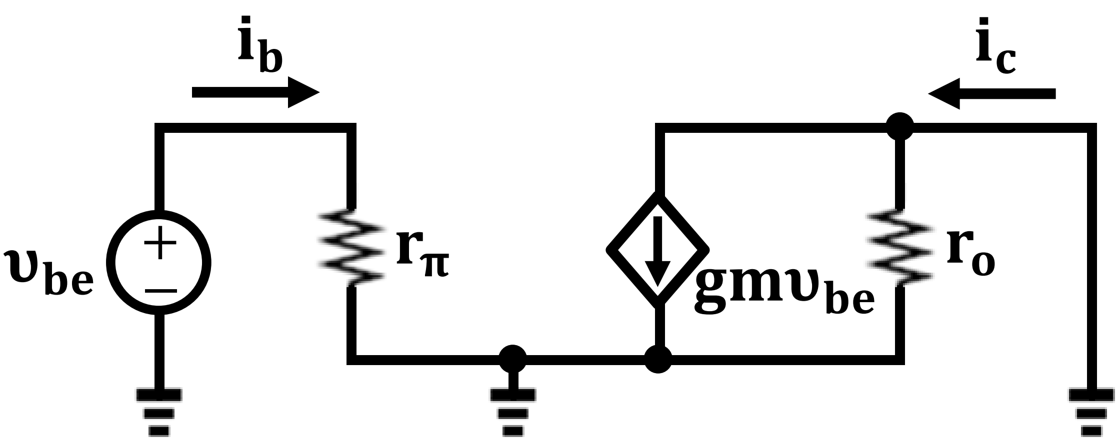
Hybrid-π Model (with Early Effect)



$$V_{CE} \geq V_{BE}$$

⇒ Forward – Active

$$I_C \cong \frac{AqD_n n_i^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$



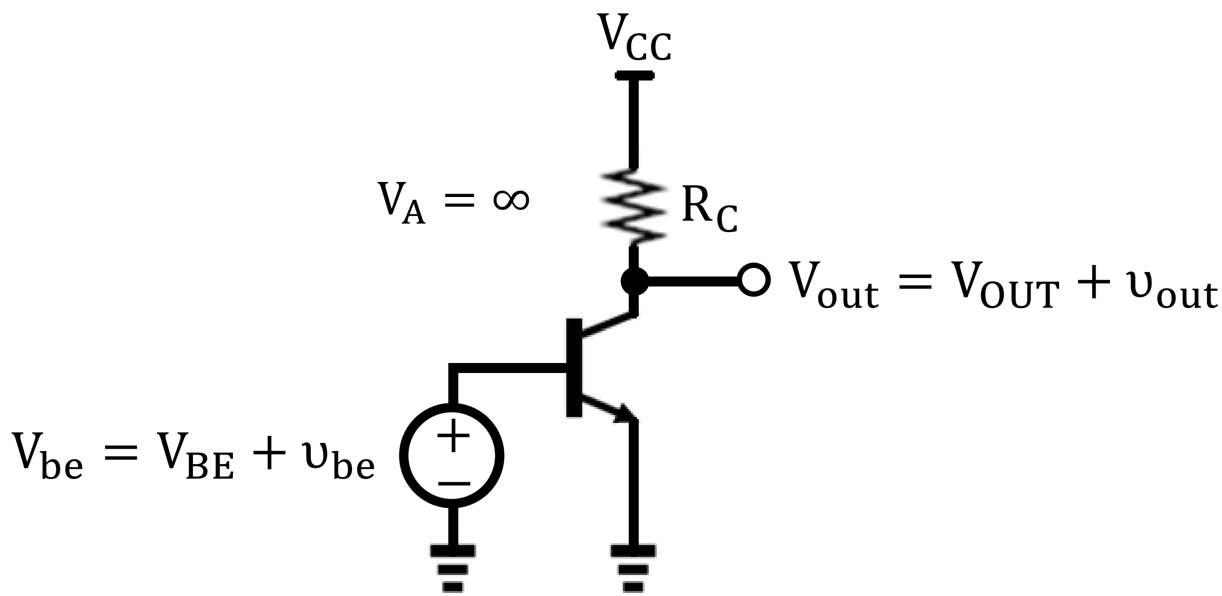
$$r_\pi = \frac{1}{\frac{dI_B}{dV_{BE}}} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} = \frac{1}{\frac{gm}{\beta}} = \frac{\beta}{gm}$$

$$gm = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}$$

$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C}$$

Common-Emitter Amplifier

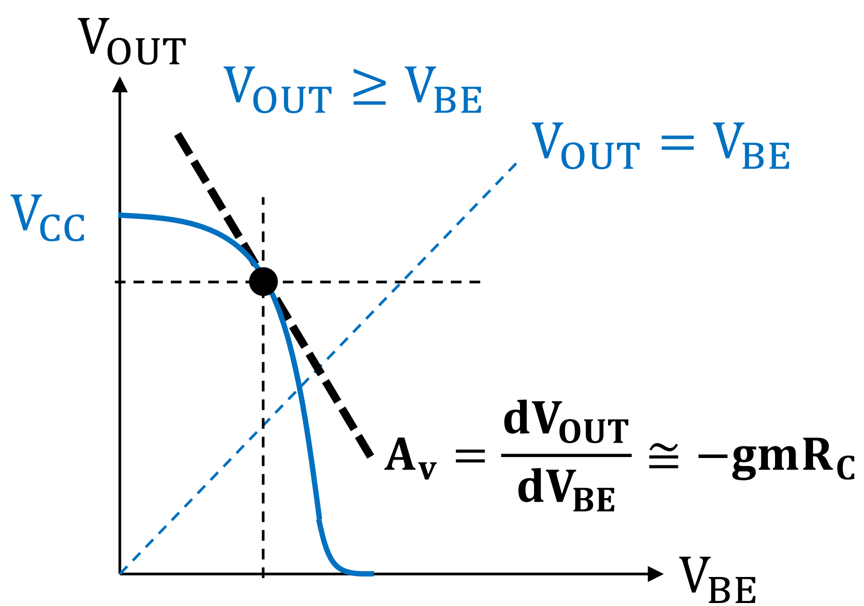
Common-Emitter Amplifier ($V_A = \infty$)



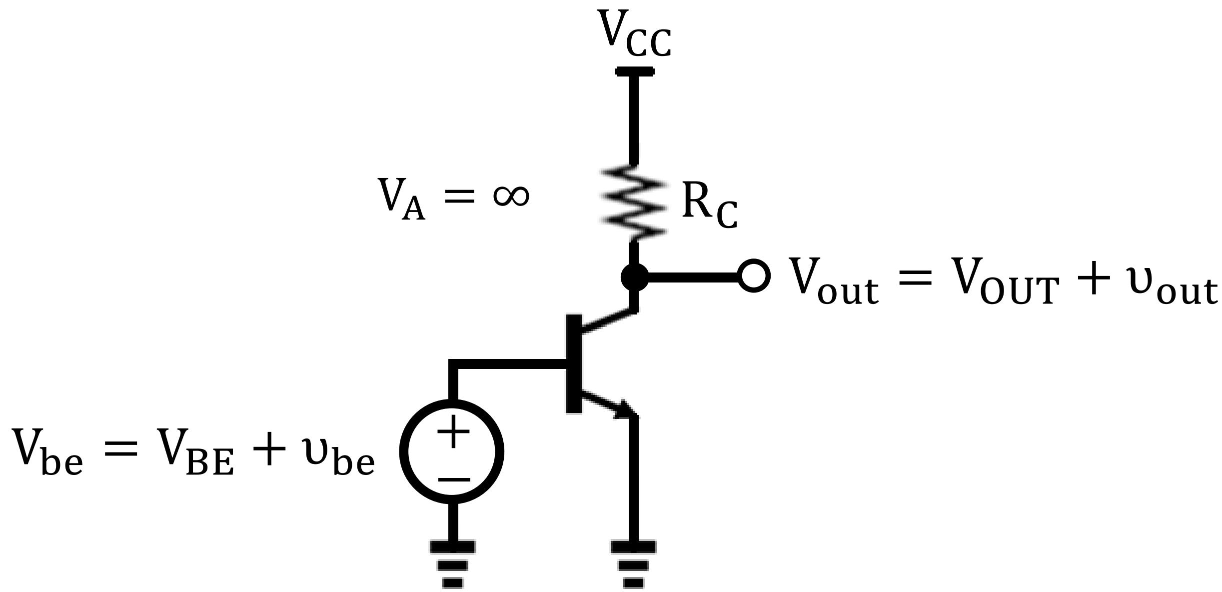
- **DC Analysis**

$$V_{OUT} = V_{CC} - I_C R_C$$
$$= V_{CC} - \frac{AqD_n n_i^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right) R_C$$

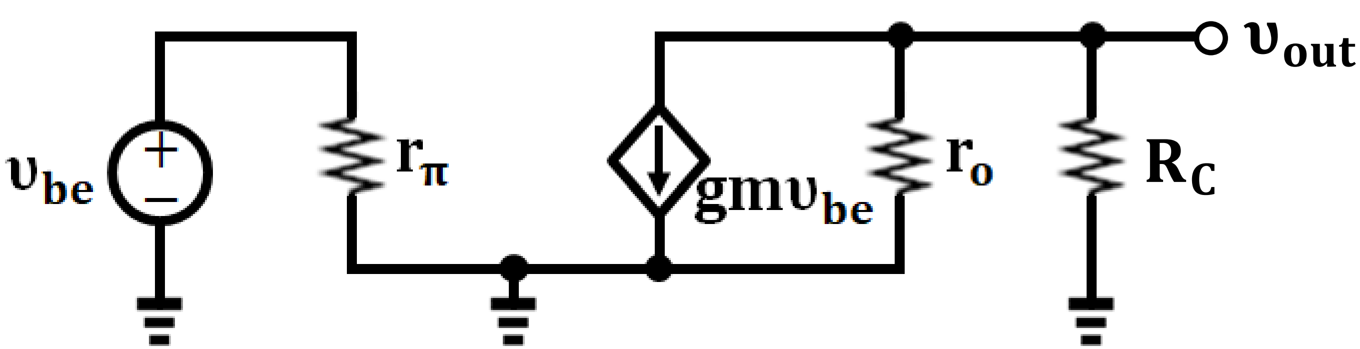
$$\boxed{A_v = \frac{dV_{OUT}}{dV_{BE}} \cong -\frac{I_C}{kT/q} R_C = -gmR_C}$$



Common-Emitter Amplifier ($V_A = \infty$)

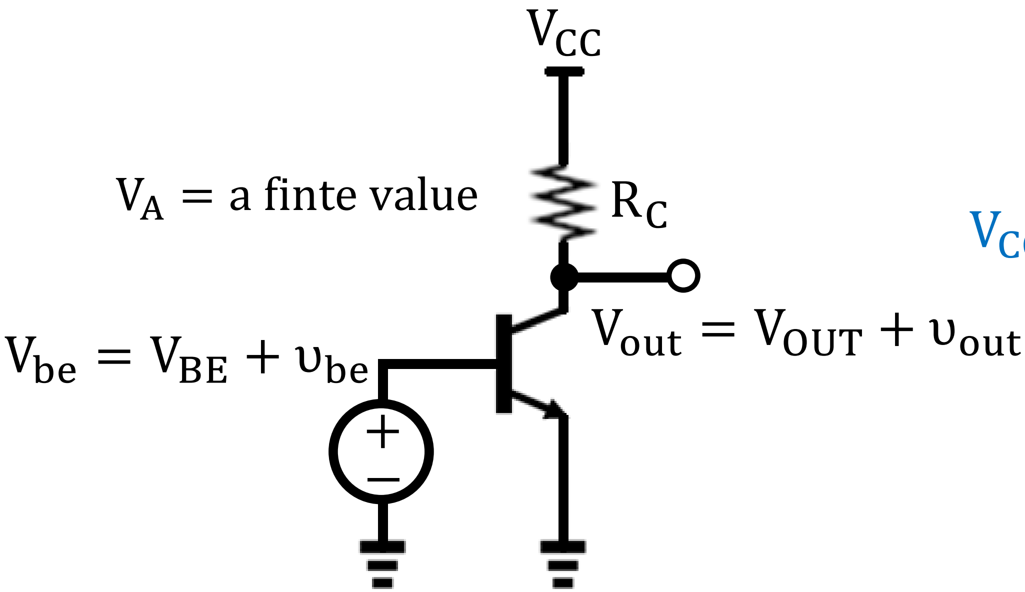


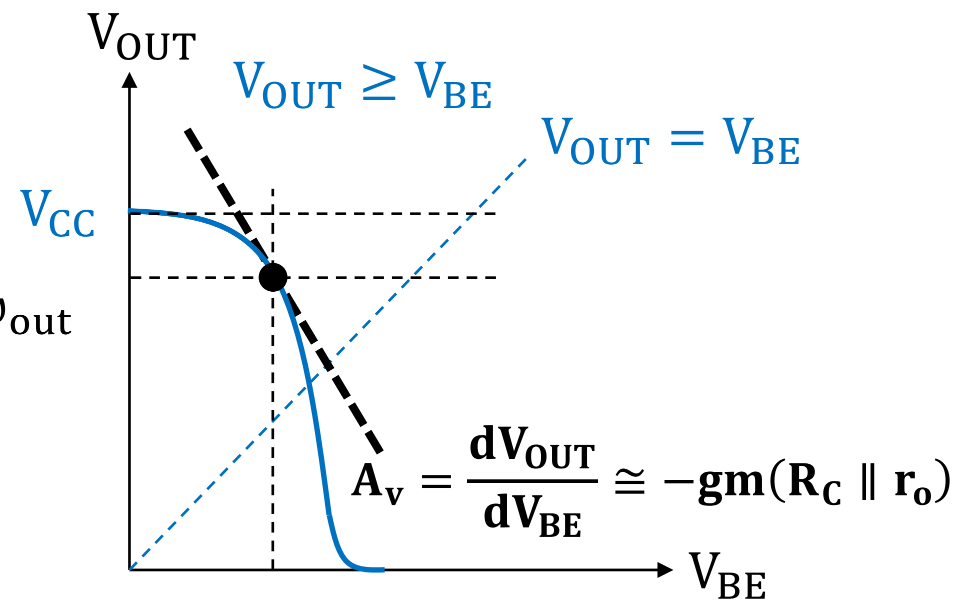
- **Small-Signal Analysis**



$$\boxed{A_v = \frac{\upsilon_{out}}{\upsilon_{be}} = -g_m(R_C \parallel r_o) = -g_mR_C}$$ (since $r_o = \infty$)

Common-Emitter Amplifier (V_A = a finite value)





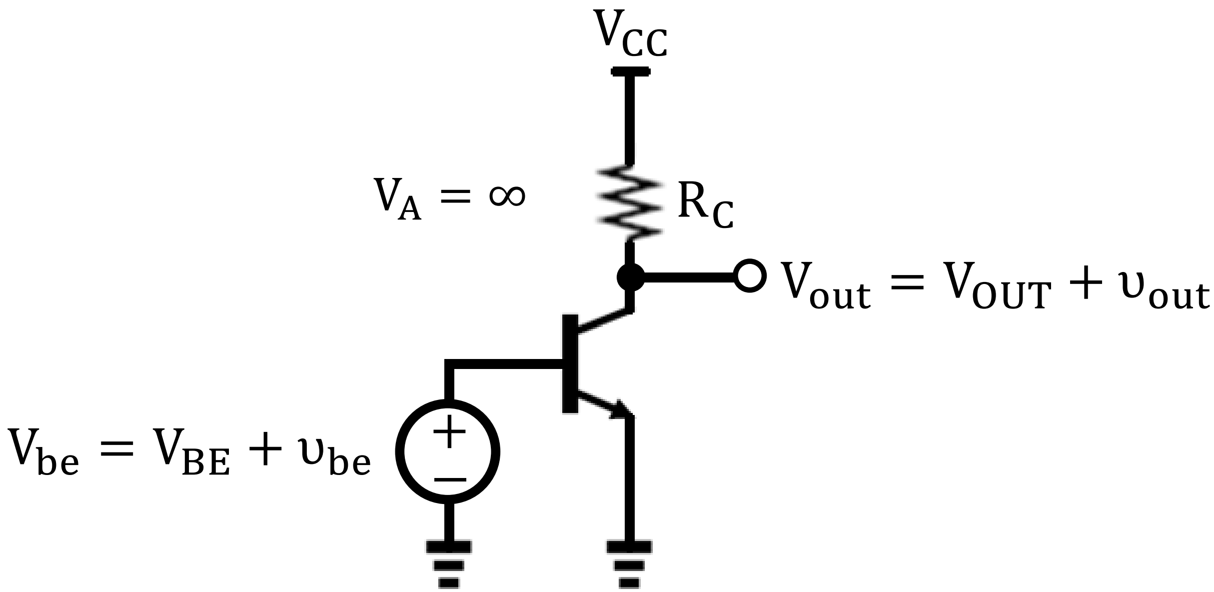
- **DC Analysis**

$$V_{OUT} = V_{CC} - I_C R_C = V_{CC} - I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{OUT}}{V_A}\right)R_C$$

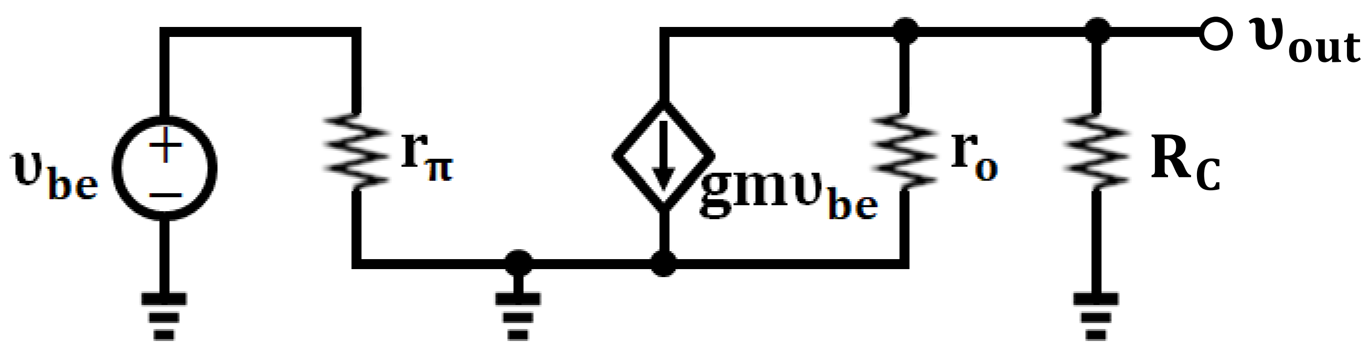
$$\frac{dV_{OUT}}{dV_{BE}} = -\frac{q}{kT} I_S e^{\frac{qV_{BE}}{kT}}\left(1 + \frac{V_{OUT}}{V_A}\right)R_C - I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\frac{1}{V_A}\frac{dV_{OUT}}{dV_{BE}}R_C \cong -gmR_C - \frac{1}{r_o}\frac{dV_{OUT}}{dV_{BE}}R_C$$

$$\boxed{A_v = \frac{dV_{OUT}}{dV_{BE}} \cong -gm(R_C \parallel r_o)}$$

Common-Emitter Amplifier (V_A = a finite value)



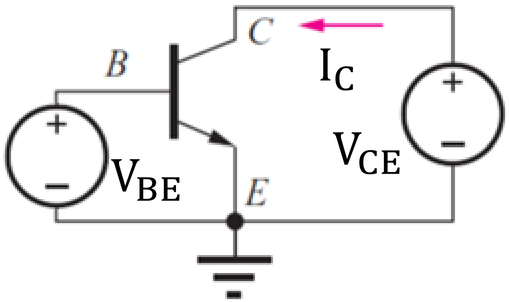
- **Small-Signal Analysis**

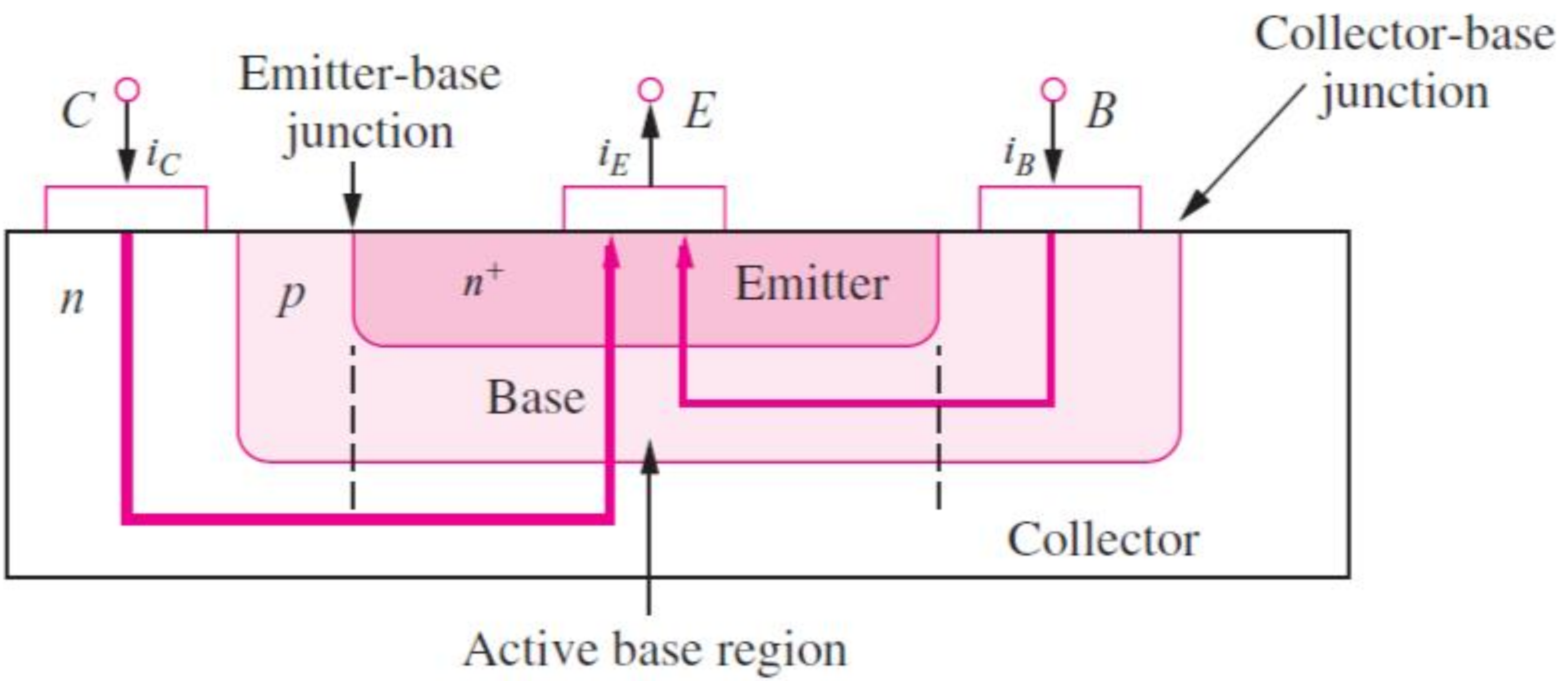


$$\boxed{A_v = \frac{\upsilon_{out}}{\upsilon_{be}} = -gm(R_C \parallel r_o)}$$

BJT Structure and Pspice Model

npn BJT Cross Section





npn BJT Pspice Model

.model Qbreakn NPN IS=1e-18 BF=100 VAF=100

$$I_C = \mathbf{IS}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{\mathbf{VAF}}\right)$$

$$\mathbf{BF} = \frac{I_C}{I_B} \qquad I_E = I_C + I_B$$

$$gm \cong \frac{I_C}{kT/q} \qquad r_\pi = \frac{\mathbf{BF}}{gm} \qquad r_o \cong \frac{\mathbf{VAF}}{I_C}$$