Unit step function

Example

Find the FT of $u(t)$.

Hint: using $\mathrm{sgn}(t)$ *and its FT.*

Solution

We have seen that

$$\mathrm{sgn}(t) = 2u(t) - 1, \quad \mathrm{sgn}(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{2}{j\omega}.$$

So $u(t) = \frac{1}{2} + \frac{1}{2}\,\mathrm{sgn}(t)$*. Thus by linearity and using*

$$1 \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta(\omega),$$

we have

$$\boxed{u(t) \overset{\mathcal{F}}{\longleftrightarrow} \pi\delta(\omega) + \frac{1}{j\omega}.}$$

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Integrating sinc to compute the FT would be painful.

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We learned the following:

- *FT pair*

$$\text{rect}(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

- *Duality property*

$$f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega) \Longrightarrow x(t) = F(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega) = 2\pi f(-\omega)$$

- *Time-scale property*

$$f(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} F\left(\frac{\omega}{a}\right), \; a \neq 0$$

Solution (1)

Method 1:
Time scale:

$$\mathrm{rect}(t/\tau) \overset{\mathcal{F}}{\longleftrightarrow} \tau\,\mathrm{sinc}\left(\tau\frac{\omega}{2\pi}\right)$$

so for $\tau = 2\pi$:

$$f(t) = \mathrm{rect}\left(\frac{t}{2\pi}\right) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega) = 2\pi\mathrm{sinc}(\omega).$$

Thus, by duality

$$2\pi\mathrm{sinc}(t) \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\mathrm{rect}\left(\frac{-\omega}{2\pi}\right) = 2\pi\,\mathrm{rect}\left(\frac{\omega}{2\pi}\right)$$

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Solution (2)

Method 2:

Duality:

$$\mathrm{rect}(t) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\Longrightarrow \quad \mathrm{sinc}\left(\frac{t}{2\pi}\right) \xleftrightarrow{\mathcal{F}} 2\pi\mathrm{rect}(-\omega) = 2\pi\,\mathrm{rect}(\omega)$$

Time scale:

$$f(t) = \mathrm{sinc}\left(2\pi\frac{t}{2\pi}\right) \xleftrightarrow{\mathcal{F}} F(\omega) = \frac{1}{2\pi}2\pi\,\mathrm{rect}\left(\frac{\omega}{2\pi}\right).$$

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