VE 320 Summer 2019

Introduction to Semiconductor Devices

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Lecture 4

The semiconductor in equilibrium
(Chapter 4)

Charge carrier concentration

Carriers in semiconductors: electron and hole

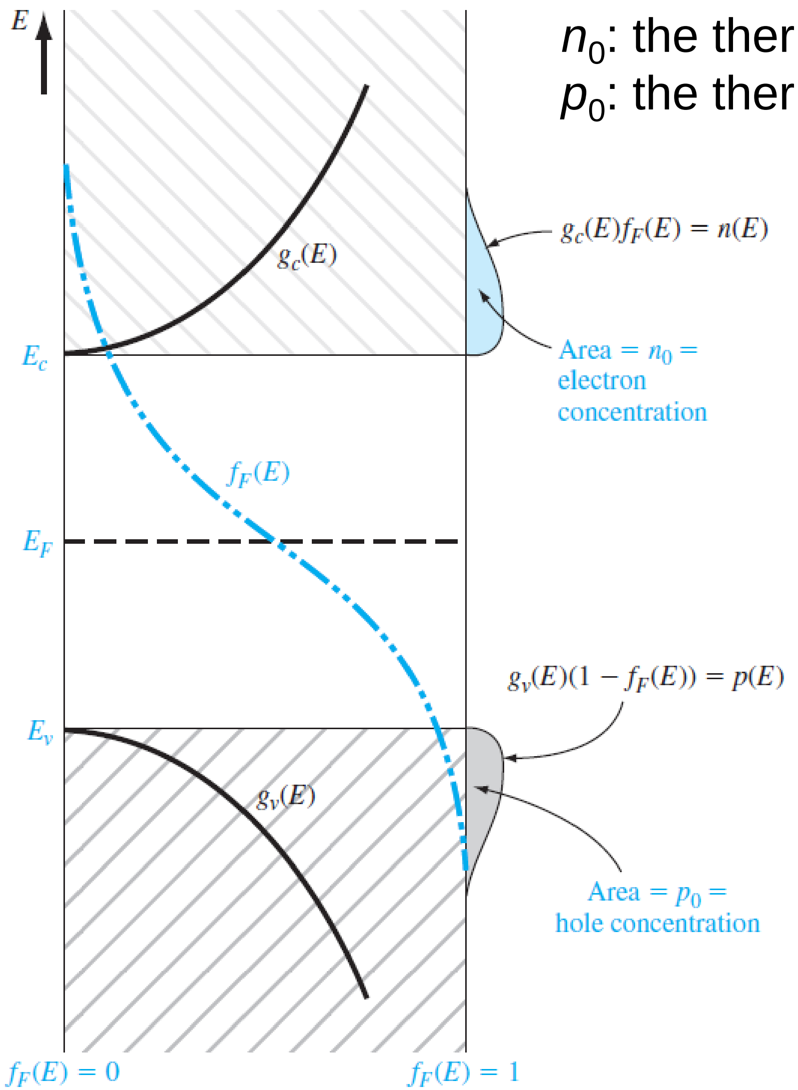
The distribution of electrons in the conduction band (per unit volume and energy) = the density of allowed quantum states × the probability that a state is occupied by an electron

$$n(E) = g_c(E) f_F(E)$$

The distribution of hole in the valence band (per unit volume and energy) = the density of allowed quantum states × the probability that a state is *not* occupied by an electron (or, occupied by a hole)

$$p(E) = g_v(E)[1 - f_F(E)]$$

Charge carrier concentration



n_0: the thermal-equilibrium concentration of electrons
p_0: the thermal-equilibrium concentration of holes

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1 - f_F(E)]dE$$

Charge carrier concentration

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= \int_{E_c}^{+\infty} \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} 4\pi \frac{(2m_n^*)^{\frac{3}{2}}}{h^3} (E - E_c\ )^{\frac{1}{2}} dE$$

Charge carrier concentration

$$\boxed{if\ \frac{E-E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

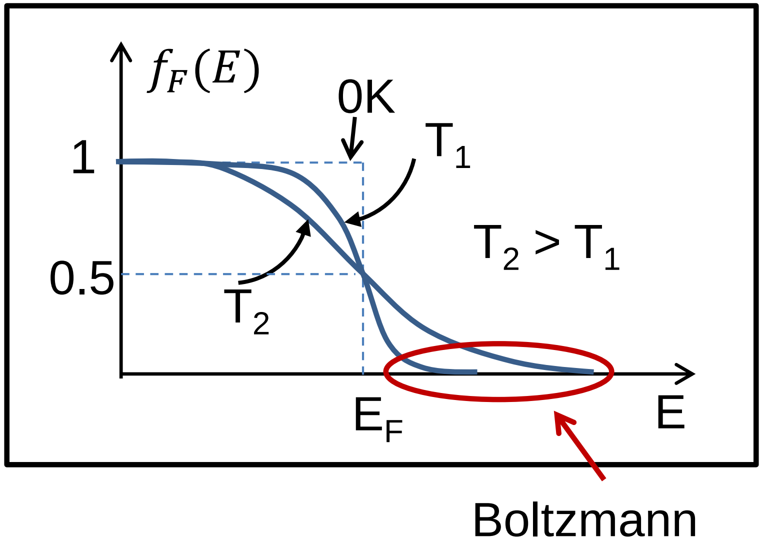
$$f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}$$

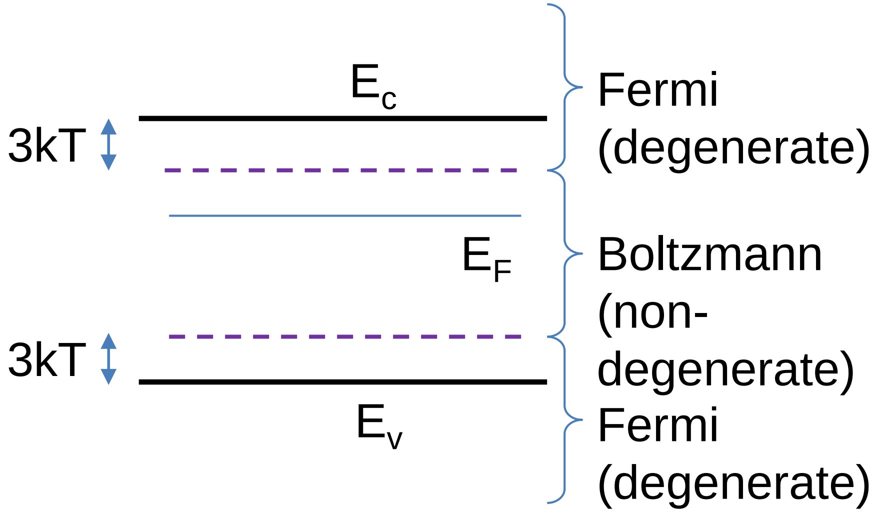
Fermi-Dirac Distribution

$$f_F(E) = \exp(\frac{E_F - E}{kT})$$

Boltzmann approximation

Fermi-Dirac





Charge carrier concentration

$$f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}$$

Fermi-Dirac Distribution

$$f_F(E) = \exp(\frac{E_F-E}{kT})$$

Boltzmann approximation

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE \ = 4\pi \frac{(2m_n^*)^{\frac{3}{2}}}{h^3} \int_{E_c}^{+\infty} (E-E_c\ )^{\frac{1}{2}} \exp\left(\frac{E_F-E}{kT}\right) dE$$

$$\eta = \frac{E-E_c}{kT} \qquad = 4\pi \frac{(2m^* kT)^{\frac{3}{2}}}{h^3} \exp(\frac{E_F-Ec}{kT}) \int_0^{+\infty} \sqrt{\eta} \exp(-\eta) dx$$

Charge carrier concentration

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}}$$

Fermi-Dirac Distribution

$\Rightarrow$

$$\boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Boltzmann approximation

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE \ = 4\pi \frac{(2m_n^*)^{\frac{3}{2}}}{h^3} \int_{E_c}^{+\infty} (E - E_c\ )^{\frac{1}{2}} \exp\left(\frac{E_F - E}{kT}\right) dE$$

$$\eta = \frac{E - E_c}{kT}$$

$$= 4\pi \frac{(2m^* kT)^{\frac{3}{2}}}{h^3} \exp(\frac{E_F - Ec}{kT}) \int_0^{+\infty} \sqrt{\eta} \exp(-\eta) dx$$

gamma function, $= \frac{\sqrt{\pi}}{2}$

Charge carrier concentration

$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

Fermi-Dirac Distribution

$$f_F(E) = \exp(\frac{E_F - E}{kT})$$

Boltzmann approximation

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE \ = 4\pi \frac{(2m_n^*)^{\frac{3}{2}}}{h^3} \int_{E_c}^{+\infty} (E - E_c\ )^{\frac{1}{2}} \exp\left(\frac{E_F - E}{kT}\right) dE$$

$$\eta = \frac{E - E_c}{kT}$$

$$= 4\pi \frac{(2m^* kT)^{\frac{3}{2}}}{h^3} \exp(\frac{E_F - Ec}{kT}) \int_0^{+\infty} \sqrt{\eta} \exp(-\eta) dx$$

$$n_0 = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

Charge carrier concentration

$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

Fermi-Dirac Distribution

$\Rightarrow$

$$f_F(E) = \exp(\frac{E_F - E}{kT})$$

Boltzmann approximation

$$n_0 = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

N_c: the effective density of states function in the conduction band

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

Charge carrier concentration

$$\boxed{if\ \frac{E-E_F}{kT} > 3 \Leftrightarrow E-E_F > 3kT}$$

Electron:

$$\boxed{n_0 = \frac{(2\pi m_n{}^* kT)^{\frac{3}{2}}}{\hbar^3}\exp\left(\frac{E_F-E_c}{kT}\right) = N_c \exp\left(\frac{E_F-E_c}{kT}\right)}$$

Hole:

$$\boxed{p_0 = \frac{(2\pi m_p{}^* kT)^{\frac{3}{2}}}{\hbar^3}\exp\left(\frac{E_v-E_F}{kT}\right) = N_v \exp\left(\frac{E_v-E_F}{kT}\right)}$$

Hole: $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

N_v: the effective density of states function in the valence band.

Intrinsic semiconductor:

$$\boxed{n_i = p_i}$$

n_i and p_i : intrinsic electron and hole concentration

Charge carrier concentration

$$\boxed{if\ \frac{E-E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

Intrinsic Fermi level (E_{Fi}) position:

$$\boxed{n_i = p_i}$$

$$N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kT\ln\left(\frac{N_v}{N_c}\right)$$

Or $$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Since $$\frac{1}{2}(E_c + E_v) = E_{\text{midgap}}$$

Then $$E_{Fi} - E_{\text{midgap}} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Charge carrier concentration

$$\boxed{\frac{E-E_F}{kT} > 3 \Leftrightarrow E-E_F > 3kT \Leftrightarrow N_D, N_A \le 10^{18} cm^{-3}}$$

$$n_0 = n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

$$p_0 = p_i = n_i = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

For Si at 300K

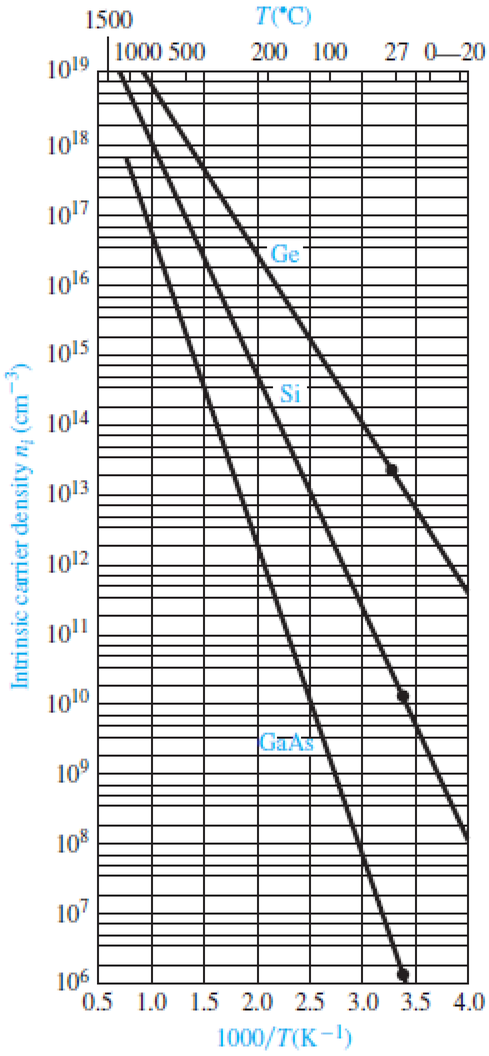
$N_c \approx 10^{19} cm^{-3}$

$N_v \approx 10^{19} cm^{-3}$

$n_i \approx 10^{10} cm^{-3}$

$$\boxed{n_0 \times p_0 = {n_i}^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v} \exp(-\frac{E_g}{2kT})}$$

- For a given semiconductor material at a constant temperature, the value of n_i is a constant, and independent of the Fermi energy
- The equations are universal for doped and undoped semiconductors

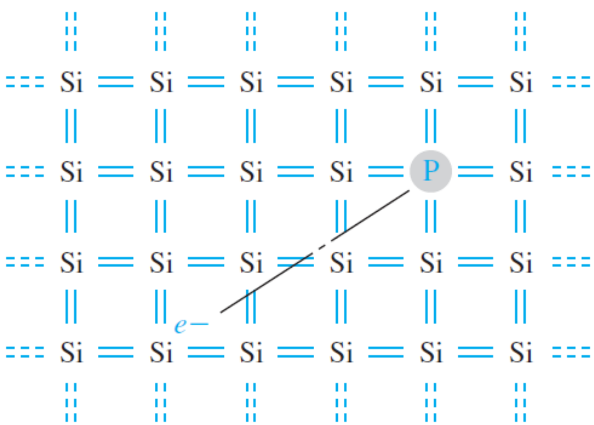


n_i vary over several orders of magnitude as the temperature changes

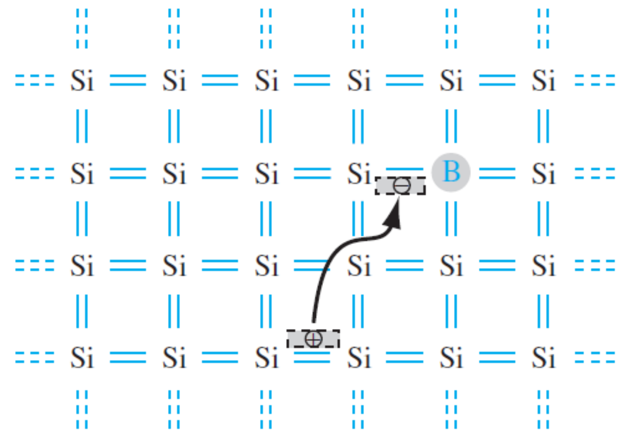
Extrinsic semiconductor

Review of doping

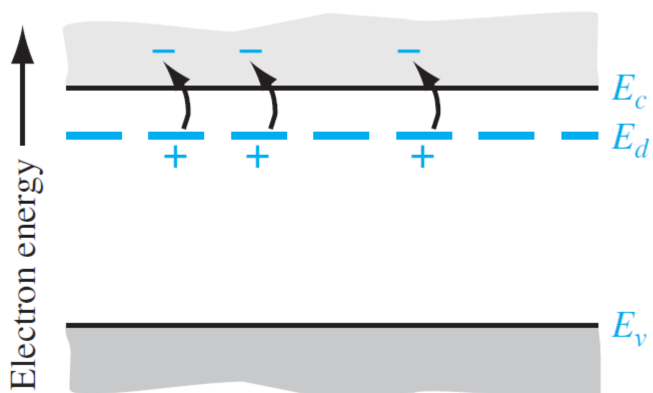
n-type: donor impurity, electrons



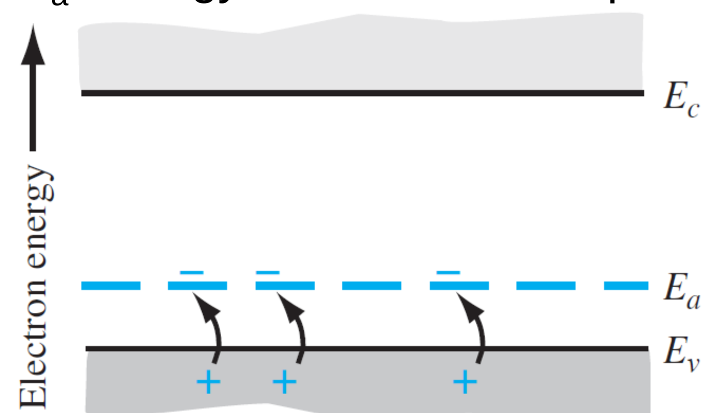
p-type: acceptor impurity, electrons



E_d: energy state of the donor electron



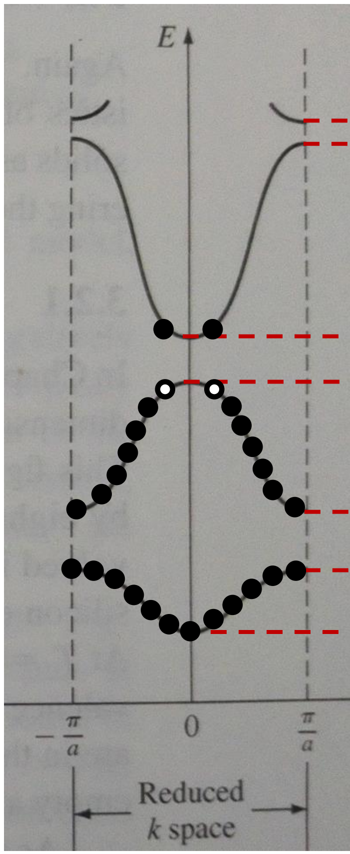
E_a: energy state of the acceptor

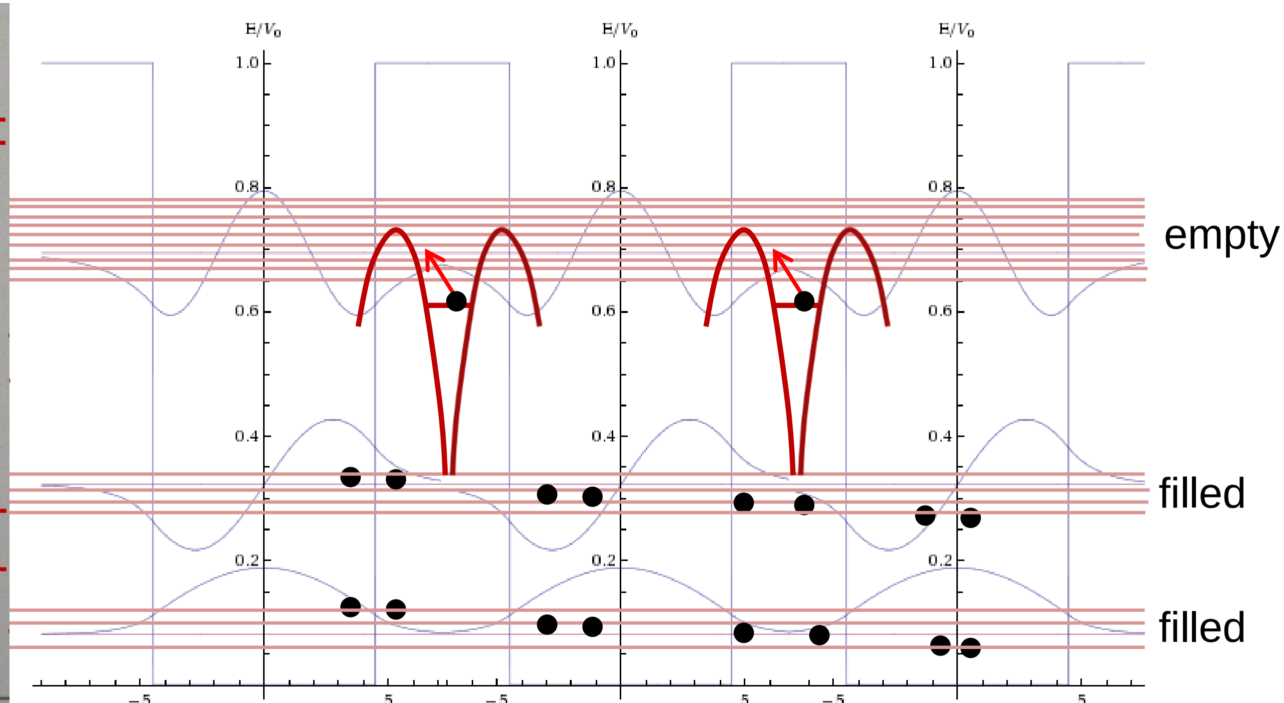


Extrinsic semiconductor

Extrinsic semiconductor: controlled amounts of specific dopant or impurity atoms have been added so that the thermal-equilibrium electron and hole concentrations are different from the intrinsic carrier concentration.

n-type doping





Where is the Fermi level?

In k space

Extrinsic semiconductor

- When $E_F > E_{Fi}$, the electron concentration is larger than the hole concentration
- When $E_F < E_{Fi}$, the hole concentration is larger than the electron concentration

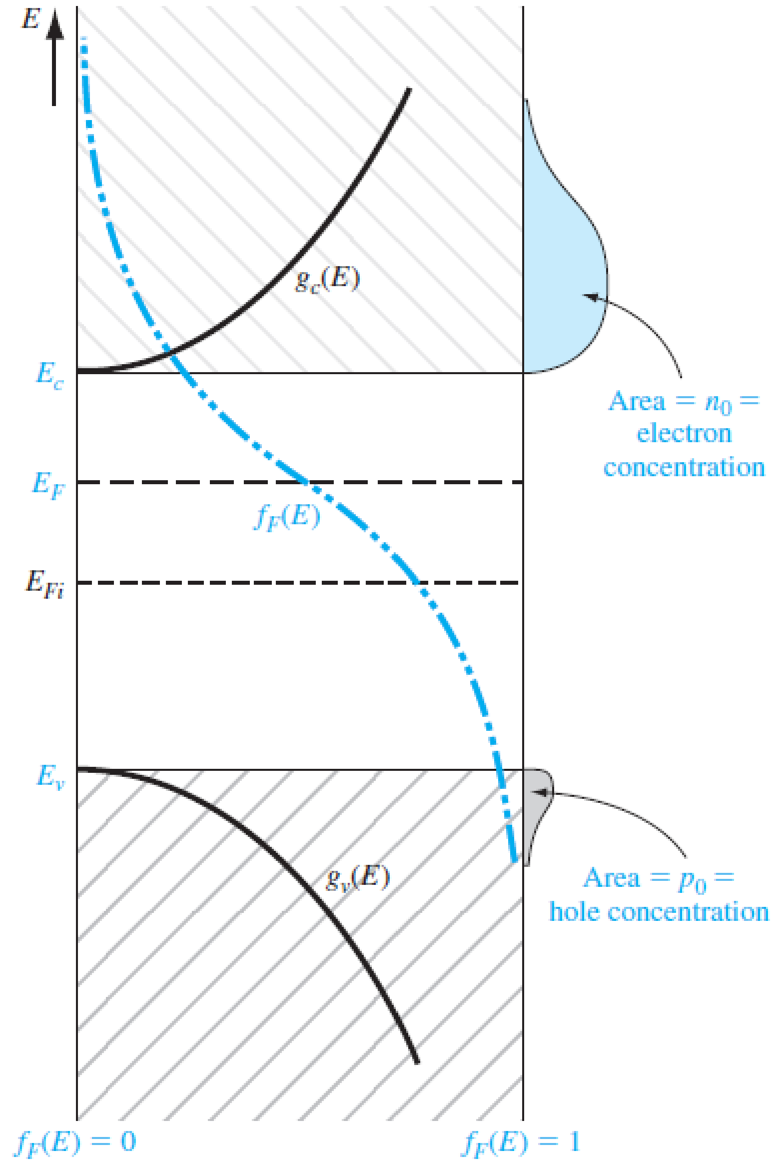
- When the density of electrons is greater than the density of holes, the semiconductor is n type.
- When the density of holes is greater than the density of electrons, the semiconductor is p type.

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

If: 1. Doping concentration: N_D, 100% ionized
2. Electrons from thermal are negligible

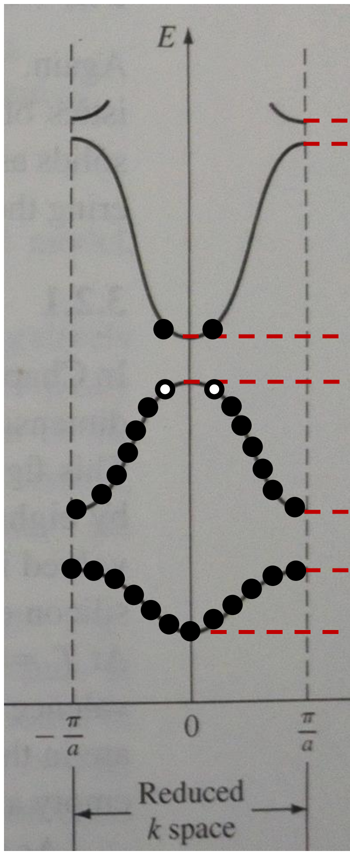
$$n_0 = N_D$$

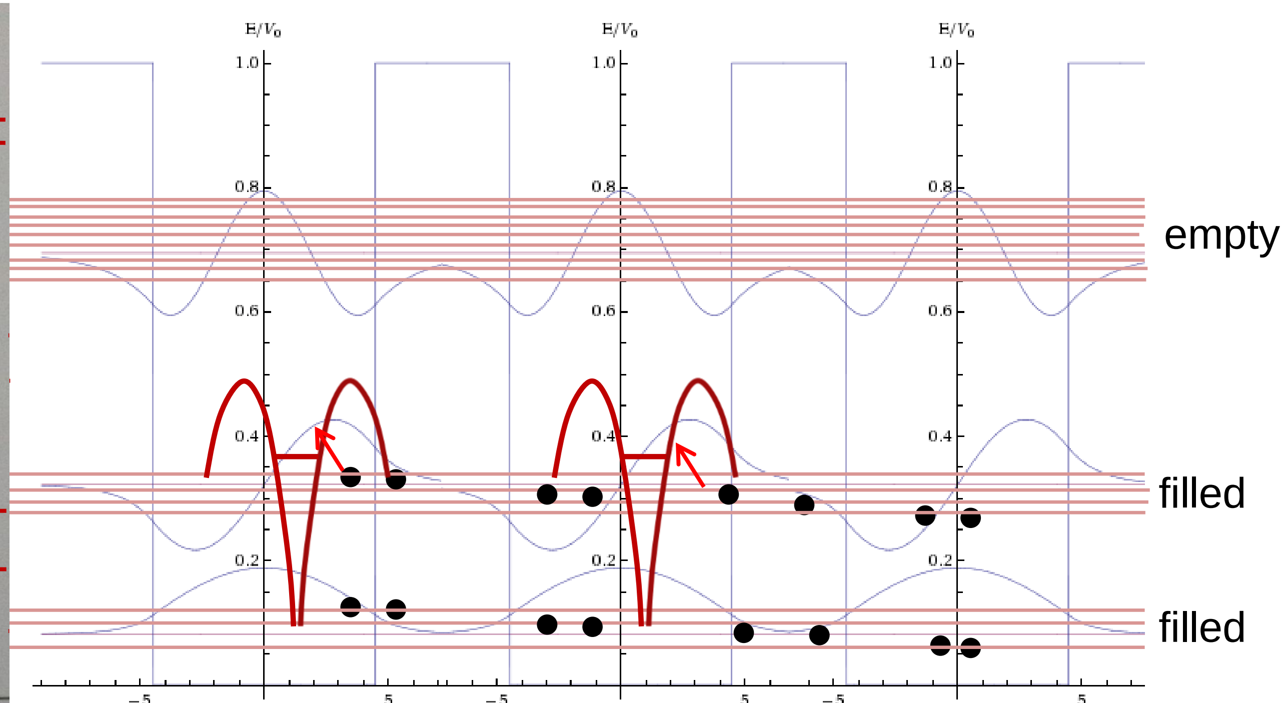
$$\Rightarrow \; E_F = E_c - kT\ln\left(\frac{N_c}{N_D}\right)$$



Compensate doping

p-type doping





In k space

Where is the Fermi level?

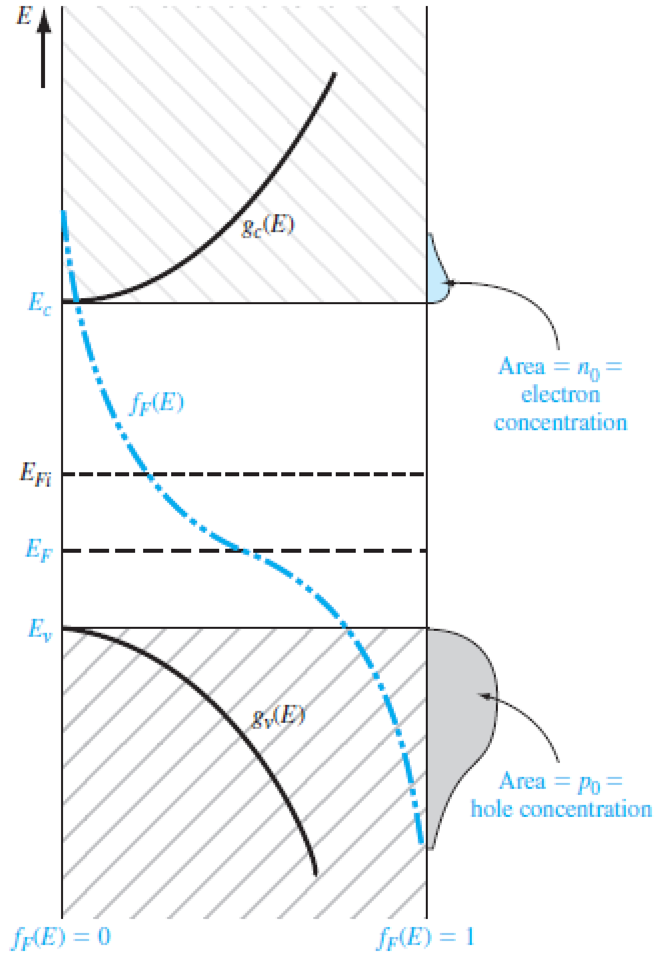
Extrinsic semiconductor

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

If: 1. Doping concentration: N_A, 100% ionized
2. Electrons from thermal are negligible

$$p_0 = N_A$$

$$\Rightarrow \quad E_F = E_v + kT ln(\frac{N_v}{N_A})$$



Extrinsic semiconductor

- $n_0 > p_0$, the semiconductor is n type
- In an n-type semiconductor, electrons are referred to as the majority carrier and holes as the minority carrier

$$n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT}\right] \Rightarrow n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

Remember: $$n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

Then $$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

Similarly $$p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

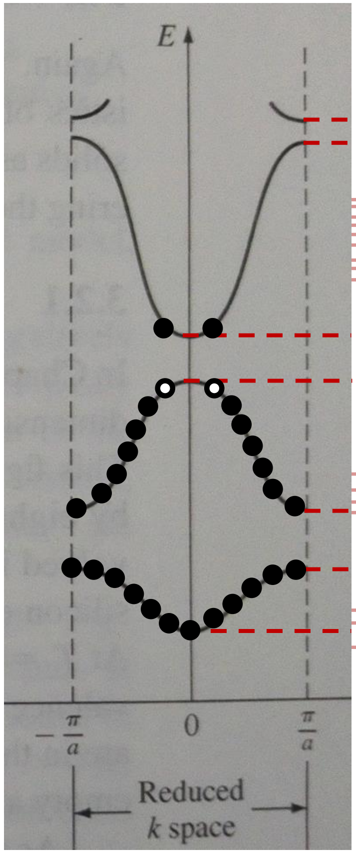
$$n_0 p_0 = n_i^2$$

$$\boxed{if \ \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

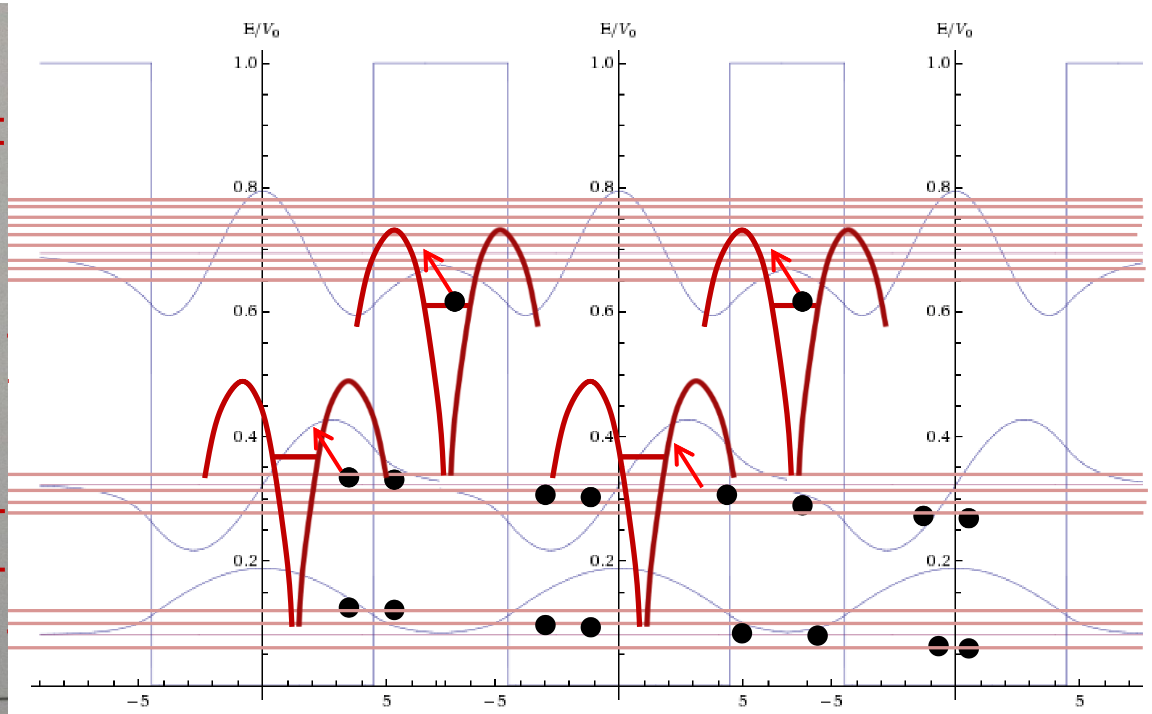
The product of n_0 and p_0 is always a constant for a given semiconductor material at a given temperature in thermal equilibrium

Extrinsic semiconductor

n-type if $N_D > N_A$, $n = N_D - N_A$
p-type if $N_D < N_A$, $p = N_A - N_D$



In k space



Where is the Fermi level?