Ve215 Electric Circuits

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Chapter 13

Magnetically Coupled Circuits

13.1 Introduction

The circuits we have considered so far may be regarded as *conductively coupled*, because one loop affects the neighboring loop through current conduction.

When two loops (with or without contacts between them) affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled*. In this chapter, we study magnetically coupled circuits.

13.2 Mutual Inductance

When two coils are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as *mutual inductance*.

Let us first consider a single coil with N turns. When current i flows through the coil, a magnetic flux ϕ is produced around it (Fig. 13.1). According to Faraday's law, the voltage v induced in the coil is

$$v = N\frac{d\phi}{dt}$$

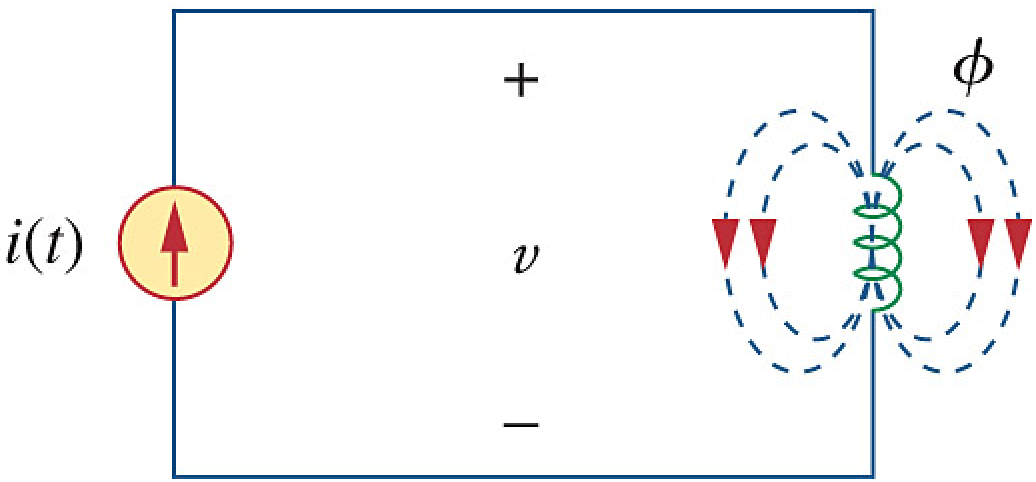


Figure 13.1 Magnetic flux produced by a single coil with N turns.

But the flux is produced by current i so that any change in ϕ is caused by a change in i. Hence,

$$v = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

where $L = N\frac{d\phi}{di}$ is commonly called the *self - inductance* of the coil.

Now consider two coils that are in close proximity with each other (Fig. 13.2). Assume that coil 2 carries no current. The magnetic flux ϕ_1 emanating from coil 1 has two components, i.e.,

$$\phi_1 = \phi_{11} + \phi_{12}$$

where ϕ_{11} links only coil 1, and ϕ_{12} links both coils.

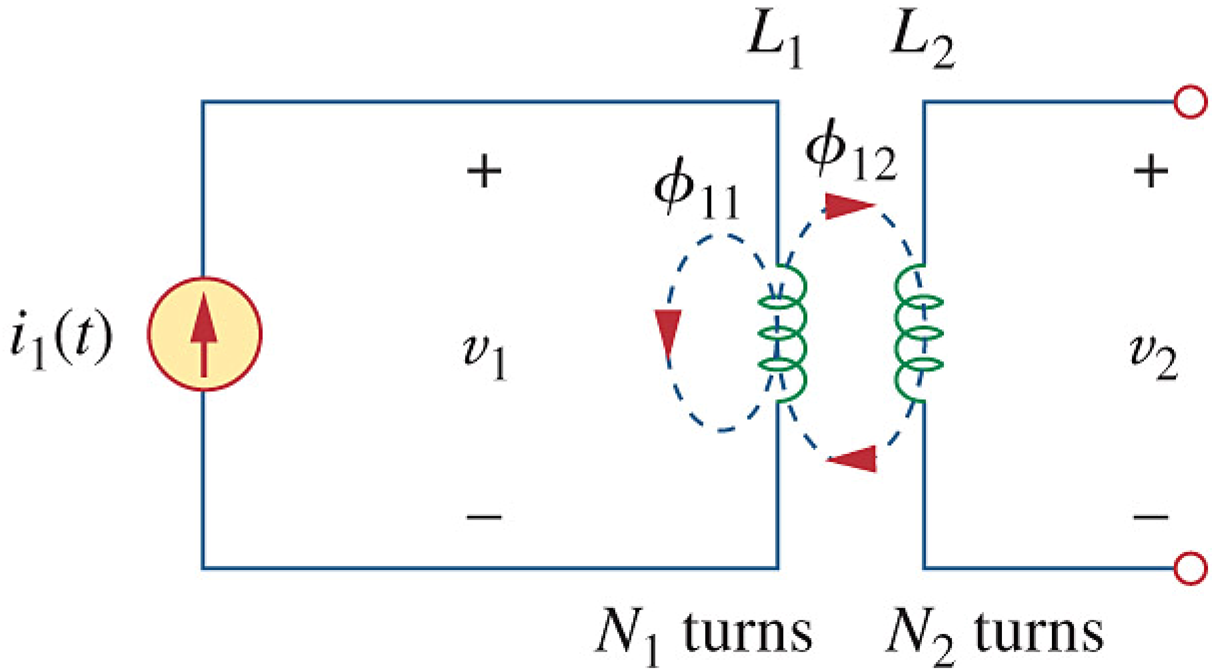


Figure 13.2 Mutual inductance M_{21} of coil 2 with respect to coil 1.

Hence,

$$v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

where $M_{21} = N_2 \dfrac{d\phi_{12}}{di_1}$ is known as the *mutual inductance* of coil 2 with respect to coil 1, measured in henrys (H).

Suppose we now let current i_2 flow in coil 2, while coil 1 carries no current (Fig. 13.3).

We have

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

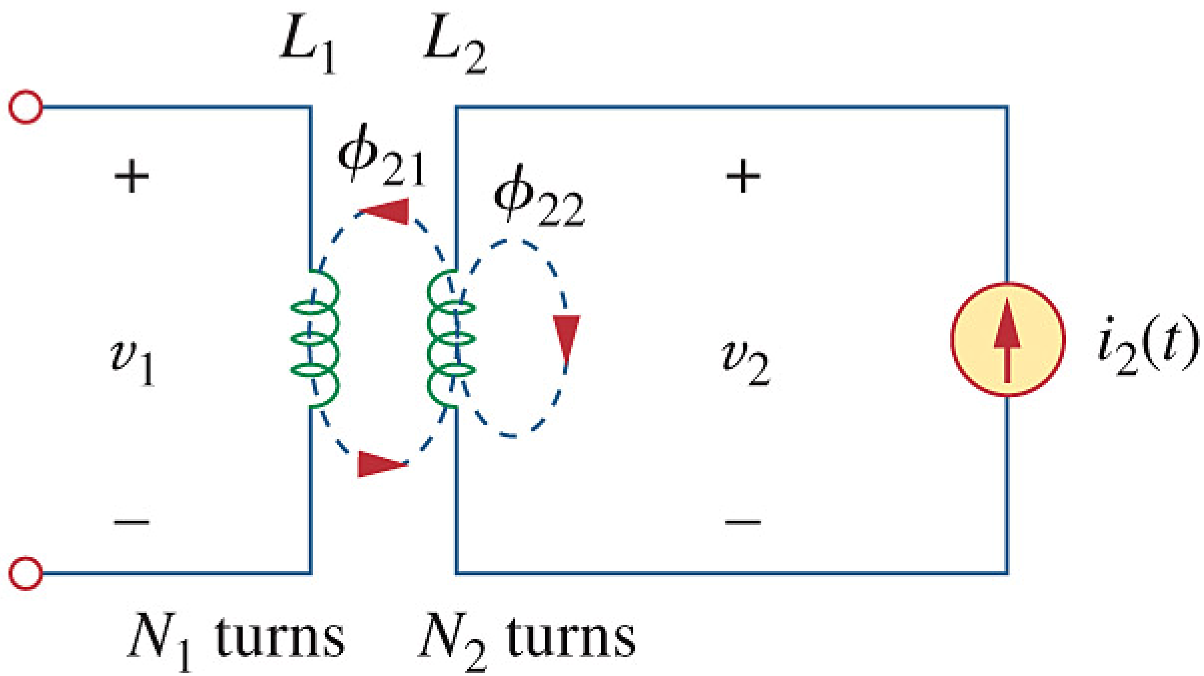


Figure 13.3 Mutual inductance M_{12} of coil 1 with respect to coil 2.

We will see in the next section that $M_{12} = M_{21} = M$, and we refer to M as the mutual inductance between the two coils.

Although mutual inductance M is always a positive quantity, the mutual voltage may be negative or positive, just like the self-induced voltage.

However, the polarity of the mutual voltage is determined by examining the orientation in which both coils are physically wound and applying Lenz's law in conjunction with the right-hand rule. Since it is inconvenient to show the construction details of coils on a circuit schematic, we apply the *dot convention* in circuit analysis.

By this convention, a dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal. This is illustrated in Fig. 13.4.

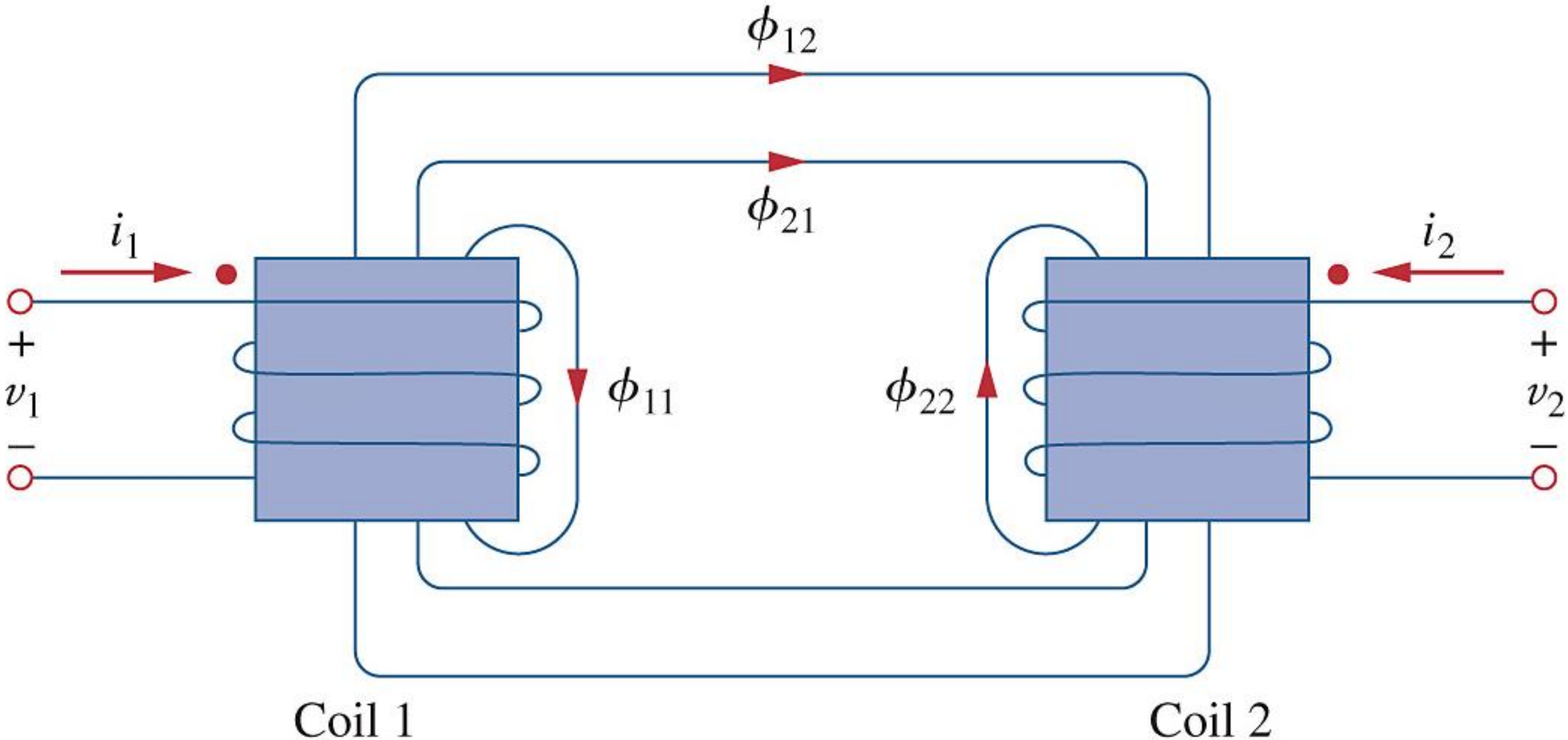


Figure 13.4 Illustration of the dot convention.

The dot convention is stated as follows:
If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage of the second coil is positive at the dotted terminal of the second coil.
Application of the dot convention is illustrated in Fig. 13.5.

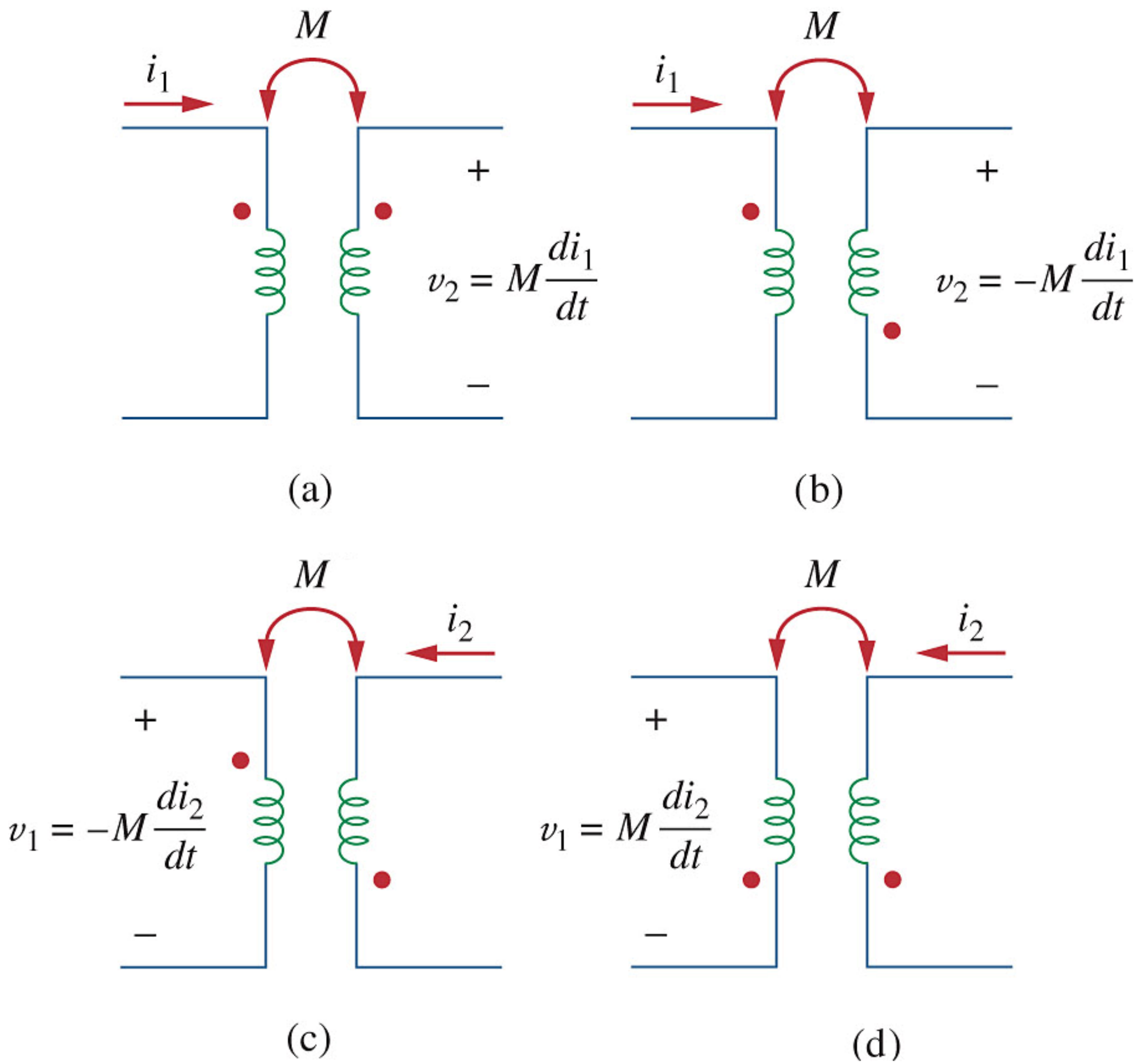


Figure 13.5 Examples illustrating how to apply the dot convention.

Figure 13.6 shows two coupled coils in series. For the coils in Fig. 13.6(a), the total inductance is

$$L = L_1 + L_2 + 2M$$

(series-aiding connection)

For the coils in Fig. 13.6(b),

$$L = L_1 + L_2 - 2M$$

(series-opposing connection)

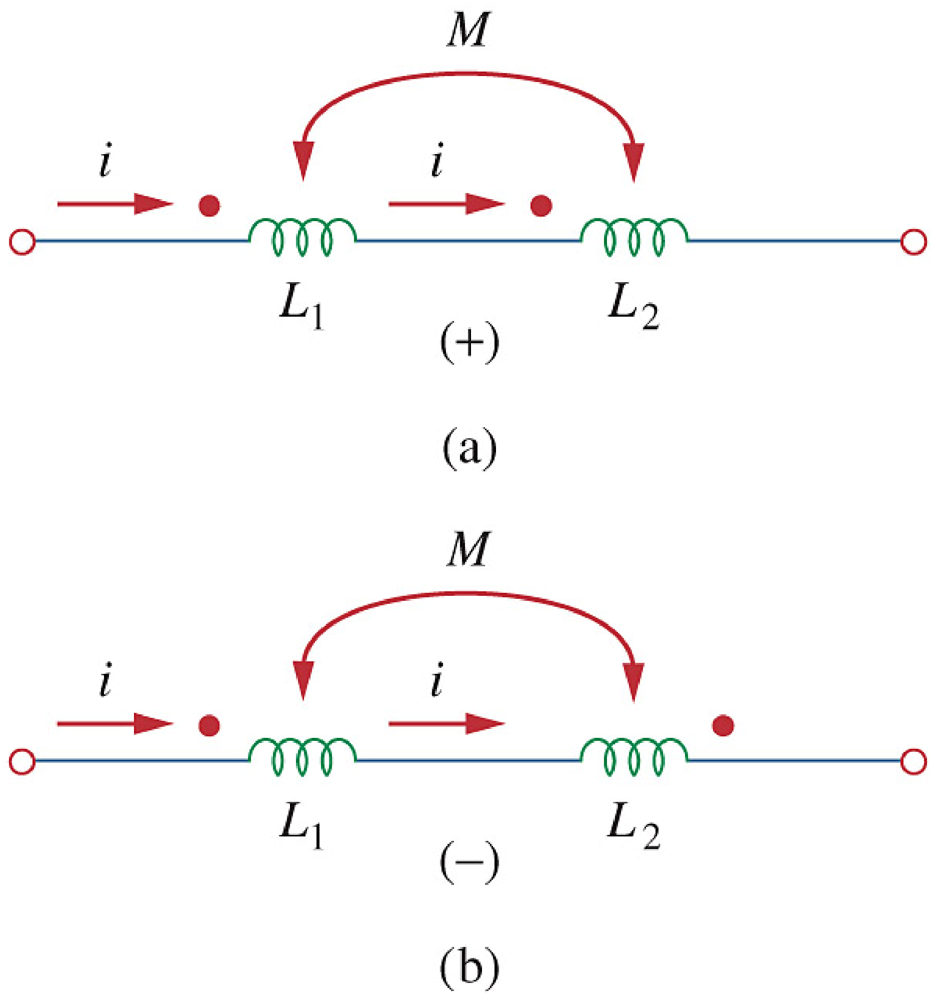


Figure 13.6 Dot convention for coils in series; the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

13.3 Energy in a Coupled Circuit

Consider the circuit in Fig. 13.14. We assume that currents i_1 and i_2 are zero initially, so that the energy stored in the coils is zero.

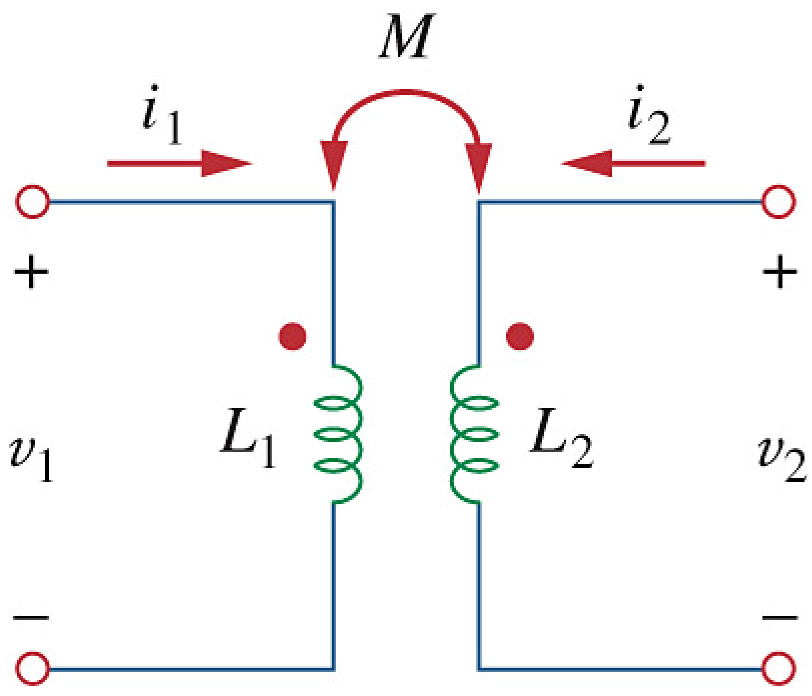


Figure 13.14 The circuit for deriving energy stored in a coupled circuit.

If we let i_1 increase from zero to I_1 while maintaing $i_2 = 0,$ the power in the circuit is

$$p_1 = v_1 i_1 = L_1 \frac{di_1}{dt} i_1$$

and the energy stored in the circuit is

$$w_1 = \int p_1 dt = L_1 \int_0^{I_1} i_1 di_1 = \frac{1}{2} L_1 I_1^2$$

If we now maintain $i_1 = I_1$ and increase i_2 from zero to I_2, the power in the coils is now

$$p_2 = \left(M_{12} \frac{di_2}{dt} \right) I_1 + \left(L_2 \frac{di_2}{dt} \right) i_2$$

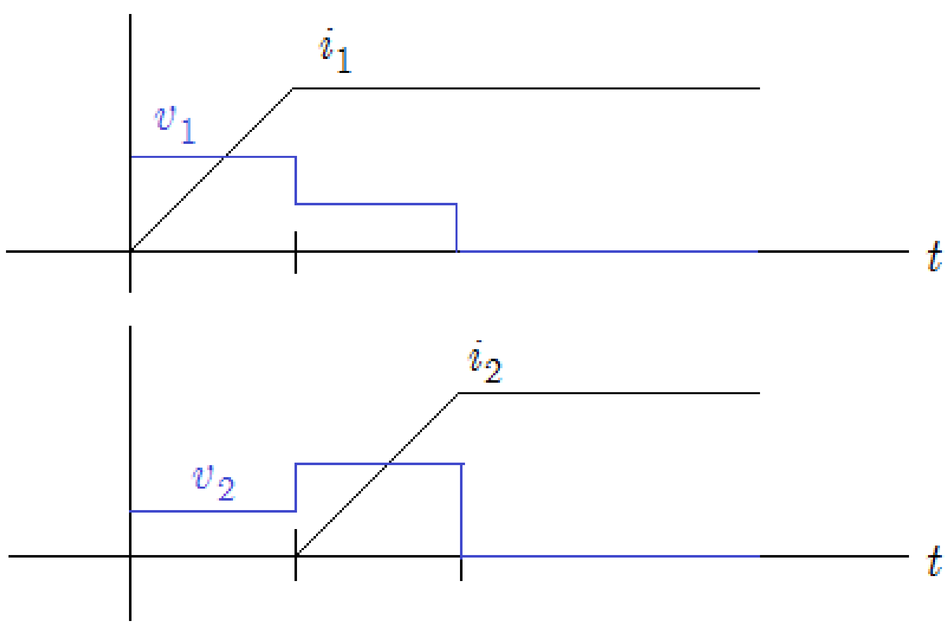
and the energy stored in the circuit is

$$w_2 = \int p_2 dt = M_{12} I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2 di_2$$

$$= M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2$$

The total energy stored in the coils when both i_1 and i_2 have reached constant values is

$$w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2$$



If we reverse the order by which the currents reach their final values, that is, if we first increase i_2 from zero to I_2 and later increase i_1 from zero to I_1, the total energy stored in the coils is

$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{21} I_1 I_2$$

Since the total energy stored should be the same regardless of how we reach the final conditions, comparing the two total energy expressions leads us to conclude that

$$M_{12} = M_{21} = M$$

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

This equation was derived based on the assumption that the coil currents both entered the dotted terminals. If one current enters one dotted terminal while the other current leaves the other dotted terminal, the total energy is

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2$$

Also, since I_1 and I_2 are arbitrary values, they may be replaced by i_1 and i_2, which gives

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

The positive sign is selected for the mutual term if both currents enter or leave the dotted terminals of the coils; the negative sign is selected otherwise.

We use $w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$ to show that M cannot exceed $\sqrt{L_1 L_2}$.

The magnetically coupled coils are passive elements, so the total energy stored can never be negative; specifically,

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$

when i_1 and i_2 are either both positive or both negative.

$$\frac{1}{2}\left(\sqrt{L_1}i_1 - \sqrt{L_2}i_2\right)^2 + i_1 i_2\left(\sqrt{L_1 L_2} - M\right) \geq 0$$

The square term can never be negative, but it can be zero. Therefore, $w(t) \geq 0$ only if $\sqrt{L_1 L_2} \geq M$.

The extent to which M approaches $\sqrt{L_1 L_2}$ is specified by the *coefficient of coupling* k, given by

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$(0 \le k \le 1)$$

Coils are said to be *loosely coupled* when $k < 0.5$. If $k > 0.5$, they are *tightly coupled*.

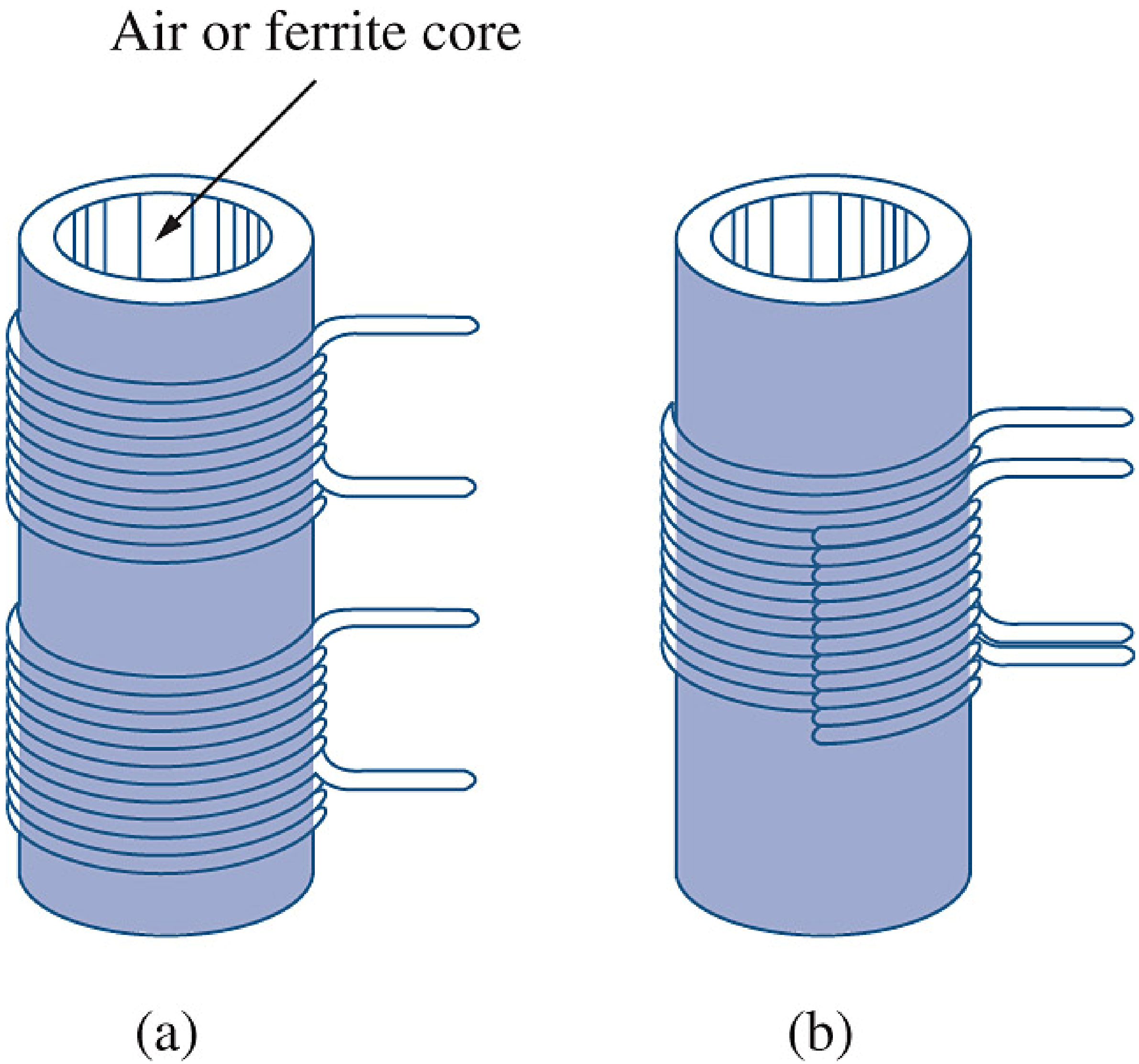


Figure 13.15 Windings: (a) loosely coupled, (b) tightly coupled.

The coefficient of coupling can be defined in another fashion. For the magnetic circuit in Fig. 13.2,

$$\phi_1 = \phi_{11} + \phi_{12}$$

The flux ϕ_1 and its components ϕ_{11} and ϕ_{12} are related to the coil current i_1 as follows:

$$\phi_1 = \mathscr{P}_1 N_1 i_1$$

$$\phi_{11} = \mathscr{P}_{11} N_1 i_1$$

$$\phi_{12} = \mathscr{P}_{12} N_1 i_1$$

where $\mathscr{P}_1$ is the permeance of the space occupied by the flux ϕ_1, $\mathscr{P}_{11}$ and $\mathscr{P}_{12}$ are the permeances corresponding to ϕ_{11} and ϕ_{12}, respectively. $\mathscr{P}_1 = \mathscr{P}_{11} + \mathscr{P}_{12}$.

$$v_1 = N_1 \frac{d\phi_1}{dt} = N_1^2 \mathscr{P}_1 \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt} = N_1 N_2 \mathscr{P}_{12} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

For the circuit in Fig. 13.3,

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$\phi_2 = \mathscr{P}_2 N_2 i_2$$

$$\phi_{21} = \mathscr{P}_{21} N_2 i_2$$

$$\phi_{22} = \mathscr{P}_{22} N_2 i_2$$

$$\mathscr{P}_2 = \mathscr{P}_{21} + \mathscr{P}_{22}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 N_2 \mathscr{P}_{21} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2^2 \mathscr{P}_2 \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$M_{12} = M_{21} \Rightarrow \mathscr{P}_{21} = \mathscr{P}_{12}$$

$$
\begin{aligned}
L_1 L_2 &= N_1^2 N_2^2 \mathcal{P}_1 \mathcal{P}_2 \\
&= (N_1 N_2 \mathcal{P}_{12})(N_1 N_2 \mathcal{P}_{21}) \frac{\mathcal{P}_1}{\mathcal{P}_{12}} \frac{\mathcal{P}_2}{\mathcal{P}_{21}} \\
&= M_{21} M_{12} \frac{\mathcal{P}_1}{\mathcal{P}_{12}} \frac{\mathcal{P}_2}{\mathcal{P}_{21}} \\
&= M^2 \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{21}}\right) \qquad (1)
\end{aligned}
$$

Replacing the two terms involving permeances by a single constant expresses Eq. (1) in a more meaningful form:

$$\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{21}}\right) \qquad (2)$$

Substituting Eq. (2) into Eq. (1) yields

$$M^2 = k^2 L_1 L_2 \text{ or } M = k\sqrt{L_1 L_2}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$k = \sqrt{\frac{\mathcal{P}_{12}}{\mathcal{P}_{11} + \mathcal{P}_{12}} \frac{\mathcal{P}_{21}}{\mathcal{P}_{21} + \mathcal{P}_{22}}}$$

$$k = \sqrt{\frac{\phi_{12}}{\phi_{11} + \phi_{12}} \frac{\phi_{21}}{\phi_{21} + \phi_{22}}}$$

13.4 Linear Transformer

Here we introduce the transformer as a new circuit element. A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.

As shown in Fig. 13.19, the coil that is directly connected to the voltage source is called the *primary winding*. The coil connected to the load is called the *secondary winding*. The resistances R_1 and R_2 are included to account for the losses (power dissipation) in the coils.

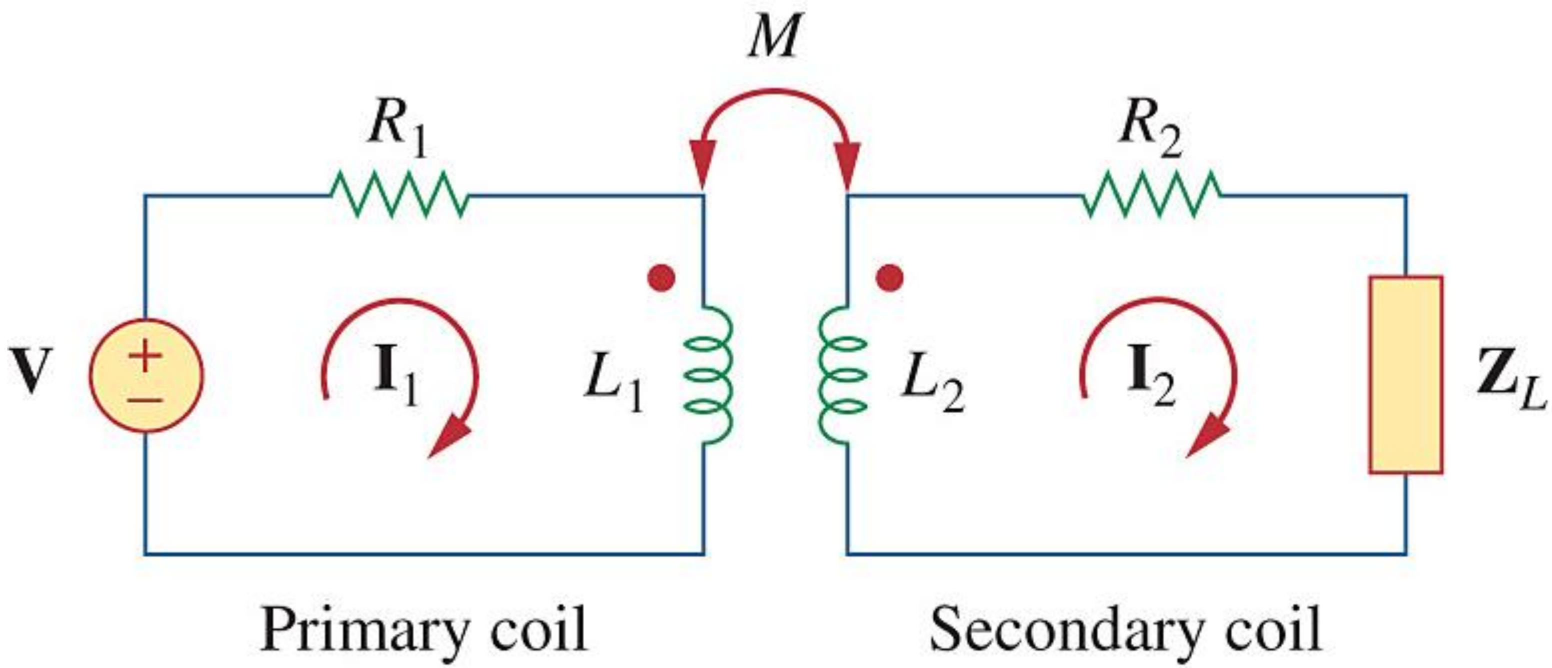


Figure 13.19 A linear transformer.

The transformer is said to be *linear* if the coils are wound on a magnetically linear material – a material for which the magnetic permeability is constant. Such materials include air, plastic, Bakelite, and wood. Linear transformers are sometimes called *air - core transformers*, although not all of them are necessarily air-core.

For the linear transformer in Fig. 13.19,

$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} - M \dfrac{di_2}{dt} \\ v_2 = M \dfrac{di_1}{dt} - L_2 \dfrac{di_2}{dt} \end{cases} \Rightarrow \begin{cases} \dot{V}_1 = j\omega L_1 \dot{I}_1 - j\omega M \dot{I}_2 \\ \dot{V}_2 = j\omega M \dot{I}_1 - j\omega L_2 \dot{I}_2 \end{cases}$$

where v_1 and v_2 denote the primary and secondary voltages, respectively.

We would like to obtain the input impedance Z_{in} as seen from the source, because Z_{in} governs the behavior of the primary circuit.

$$Z_{in} = \frac{\dot{V}}{\dot{I}_1}$$

$$\begin{cases} \dot{V} = \dot{I}_1 R_1 + \dot{V}_1 = \dot{I}_1 R_1 + j\omega L_1 \dot{I}_1 - j\omega M \dot{I}_2 \\ \dot{V}_2 = -j\omega L_2 \dot{I}_2 + j\omega M \dot{I}_1 = \dot{I}_2 R_2 + \dot{I}_2 Z_L \end{cases}$$

$$\dot{I}_2 = \frac{j\omega M \dot{I}_1}{R_2 + j\omega L_2 + Z_L}$$

$$\dot{V} = \dot{I}_1 R_1 + j\omega L_1 \dot{I}_1 - j\omega M \frac{j\omega M \dot{I}_1}{R_2 + j\omega L_2 + Z_L}$$

$$Z_{in} = \frac{\dot{V}}{\dot{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

Notice that Z_{in} comprises two terms. The first term, $R_1 + j\omega L_1$, is the primary impedance. The second term, known as the *reflected impedance* Z_R, is due to the coupling between the primary and secondary windings.

$$Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

This equation is not affected by the location of the dots on the transformer.

It is sometimes convenient to replace a magnetically coupled circuit by an equivalent circuit with no magnetic coupling. The linear transformer in Fig. 13. 21 can be replaced by an equivalent T circuit (Fig. 13.22) or Π circuit (Fig. 13.23).

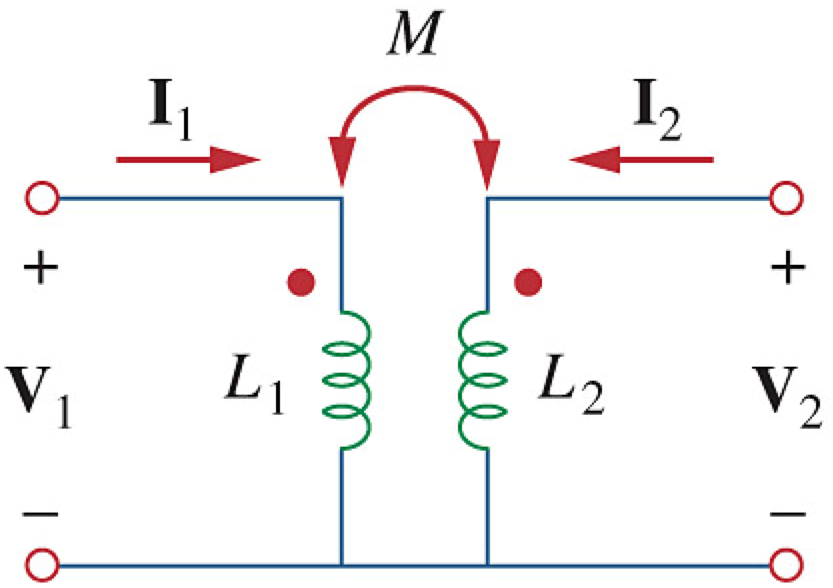


Figure 13.21 Determining the equivalent circuit of a linear transformer.

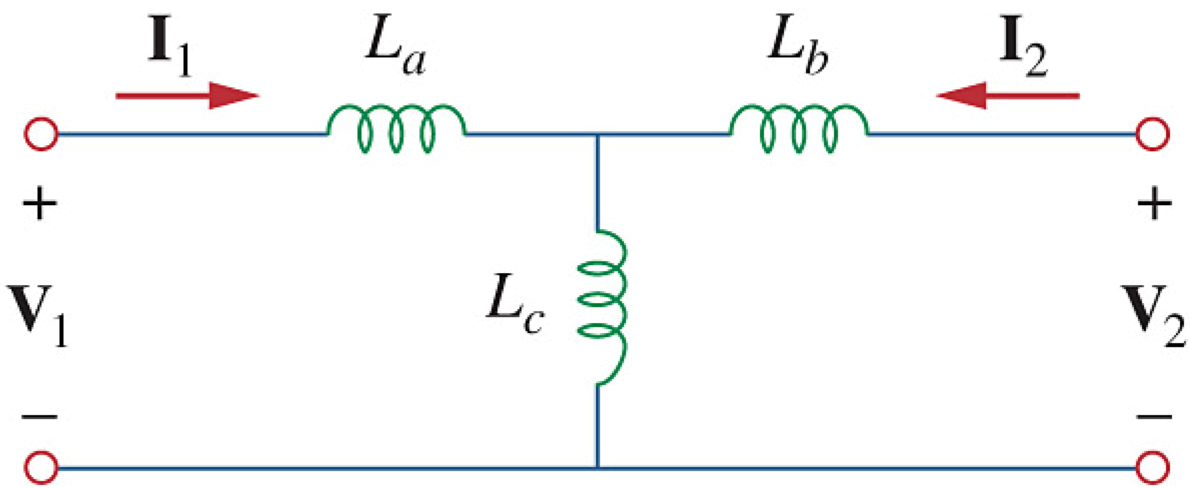


Figure 13.22 An equivalent T circuit.

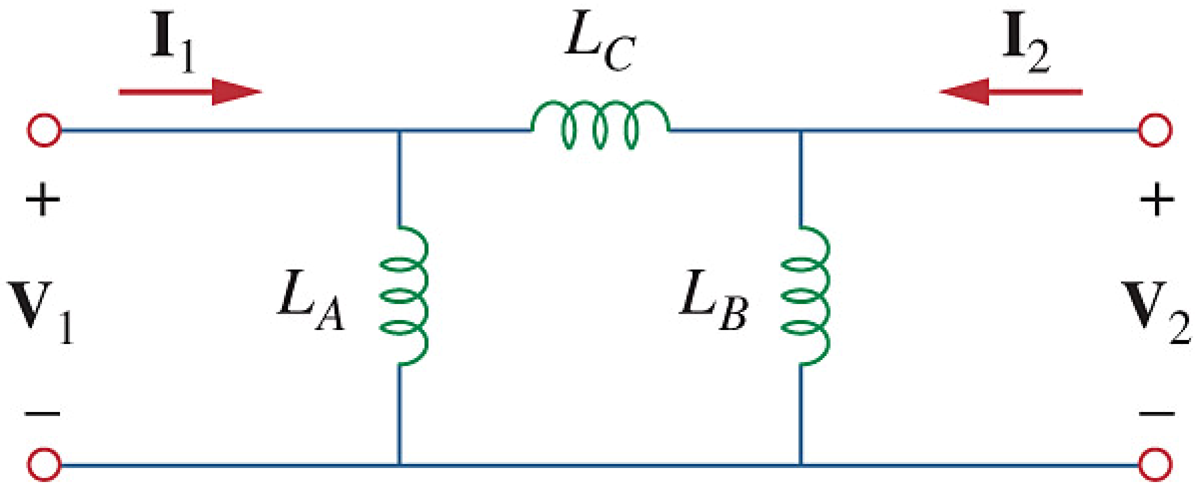


Figure 13.23 An equivalent Π circuit.

It can be shown that

$$L_a = L_1 - M, \; L_b = L_2 - M, \; L_c = M$$

and

$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M}, \; L_B = \frac{L_1 L_2 - M^2}{L_1 - M}$$

$$L_C = \frac{L_1 L_2 - M^2}{M}$$

13.5 Ideal Transformers

An ideal transformer is a unity-coupled ($k=1$), lossless ($R_1 = R_2 = 0$) transformer in which the primary and secondary coils have infinite inductances ($L_1 = \infty$, $L_2 = \infty$, $M = \infty$). Iron-core transformers are close approximations to ideal transformers.

Figure 13.30(a) shows a typical ideal transformer; the circuit symbol is in Fig. 13.30(b). The vertical lines between the coils indicate an iron core as distinct from the air core used in linear transformers. The primary winding has N_1 turns; the secondary winding has N_2 turns.

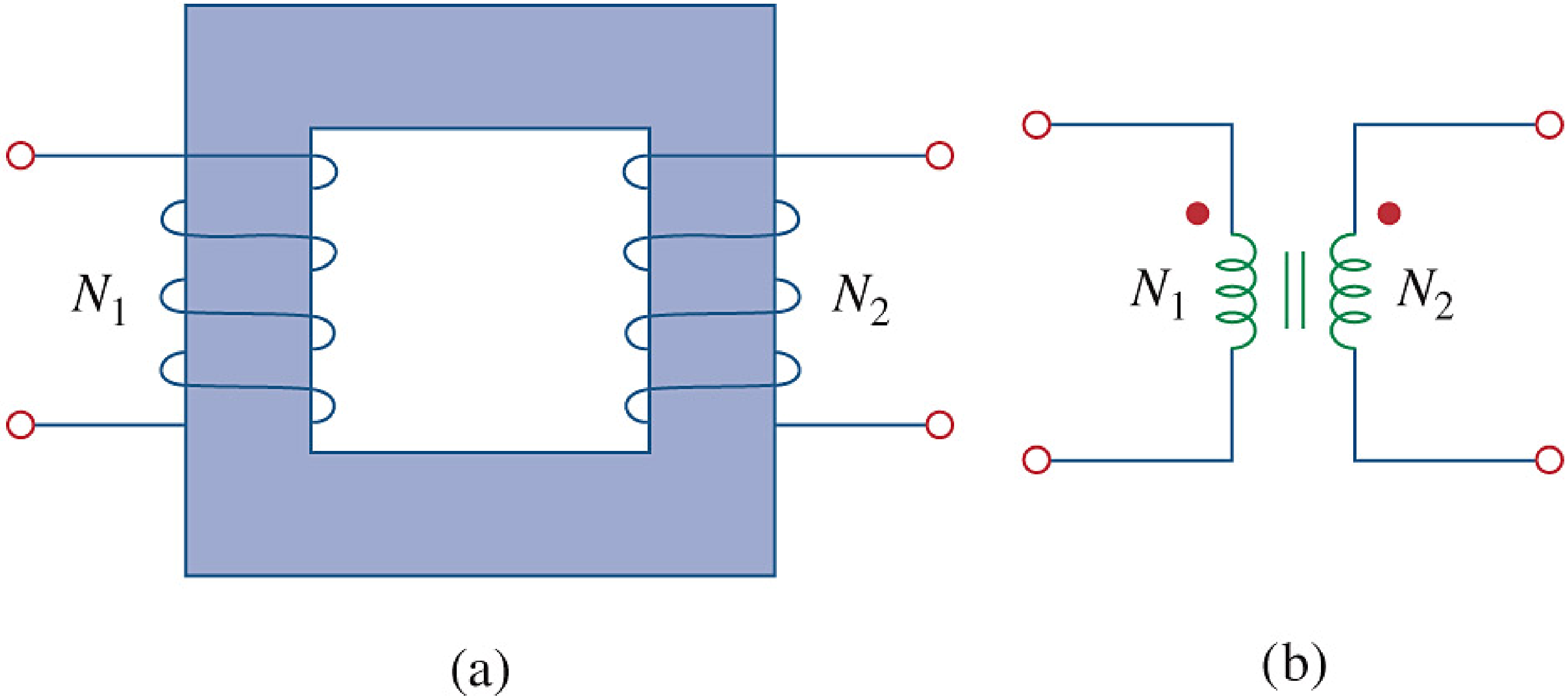


Figure 13.30 (a) Ideal transformer, (b) circuit symbol for ideal transformers.

When a sinusoidal voltage is applied to the primary winding as shown in Fig. 13.31, the same magnetic flux ϕ goes through both windings. According to Faraday's law,

$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

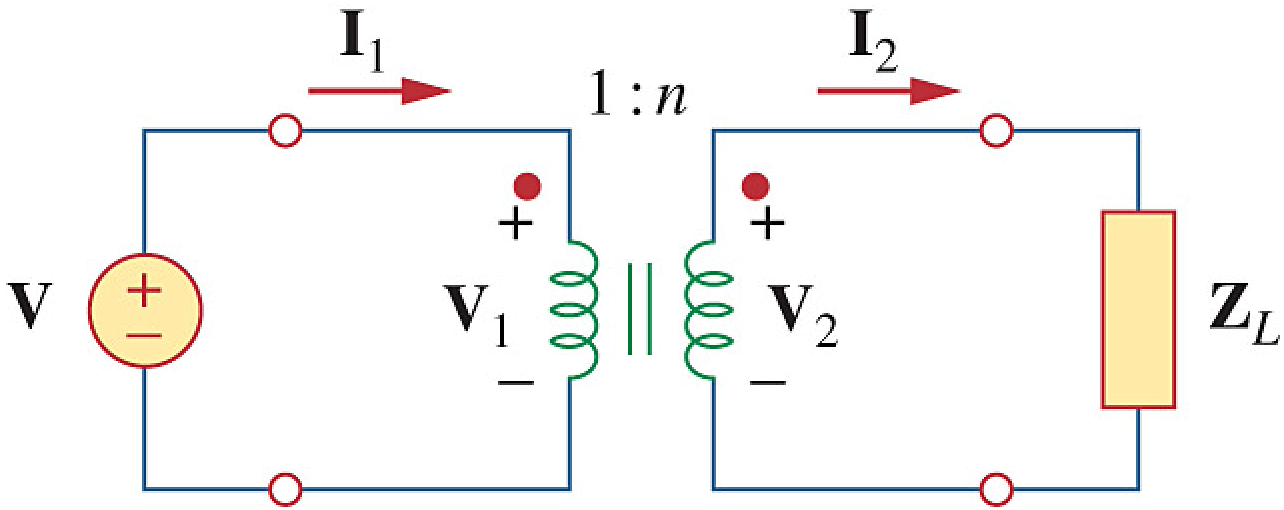


Figure 13.31 Relating primary and secondary quantities in an ideal transformer.

We have

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

where n is the *turns ratio* or *transformation ratio*. Using the phasor voltages,

$$\frac{\dot{V}_2}{\dot{V}_1} = \frac{N_2}{N_1} = n$$

For the reason of power conservation,

$$v_1 i_1 = v_2 i_2$$

$$\frac{i_1}{i_2} = \frac{v_2}{v_1} = n$$

$$\frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{V}_2}{\dot{V}_1} = n$$

If $n > 1$, we have a *step-up transformer*, as $V_2 > V_1$. On the other hand, if $n < 1$, the transformer is a *step-down transformer*, since $V_2 < V_1$. The ratings of transformers are usually specified as V_1 / V_2.

When $n = 1$, the transformer is generally called an *isolation transformer* (Section 13.9).

If the polarity of $\dot{V}_1$ or $\dot{V}_2$ or the direction of $\dot{I}_1$ or $\dot{I}_2$ is changed, n in the above equations may need to be replaced by $-n$. Two simple rules to follow are:

1. If $\dot{V}_1$ and $\dot{V}_2$ are *both* positive or negative at the dotted terminals, $\dot{V}_2 / \dot{V}_1 = n$. Otherwise, $\dot{V}_2 / \dot{V}_1 = -n$.

2. If $\dot{I}_1$ and $\dot{I}_2$ *both* enter into or both leave the dotted terminals, $\dot{I}_1 / \dot{I}_2 = -n$. Otherwise, $\dot{I}_1 / \dot{I}_2 = n$.

The rules are demonstrated with the four circuits in Fig. 13.32.

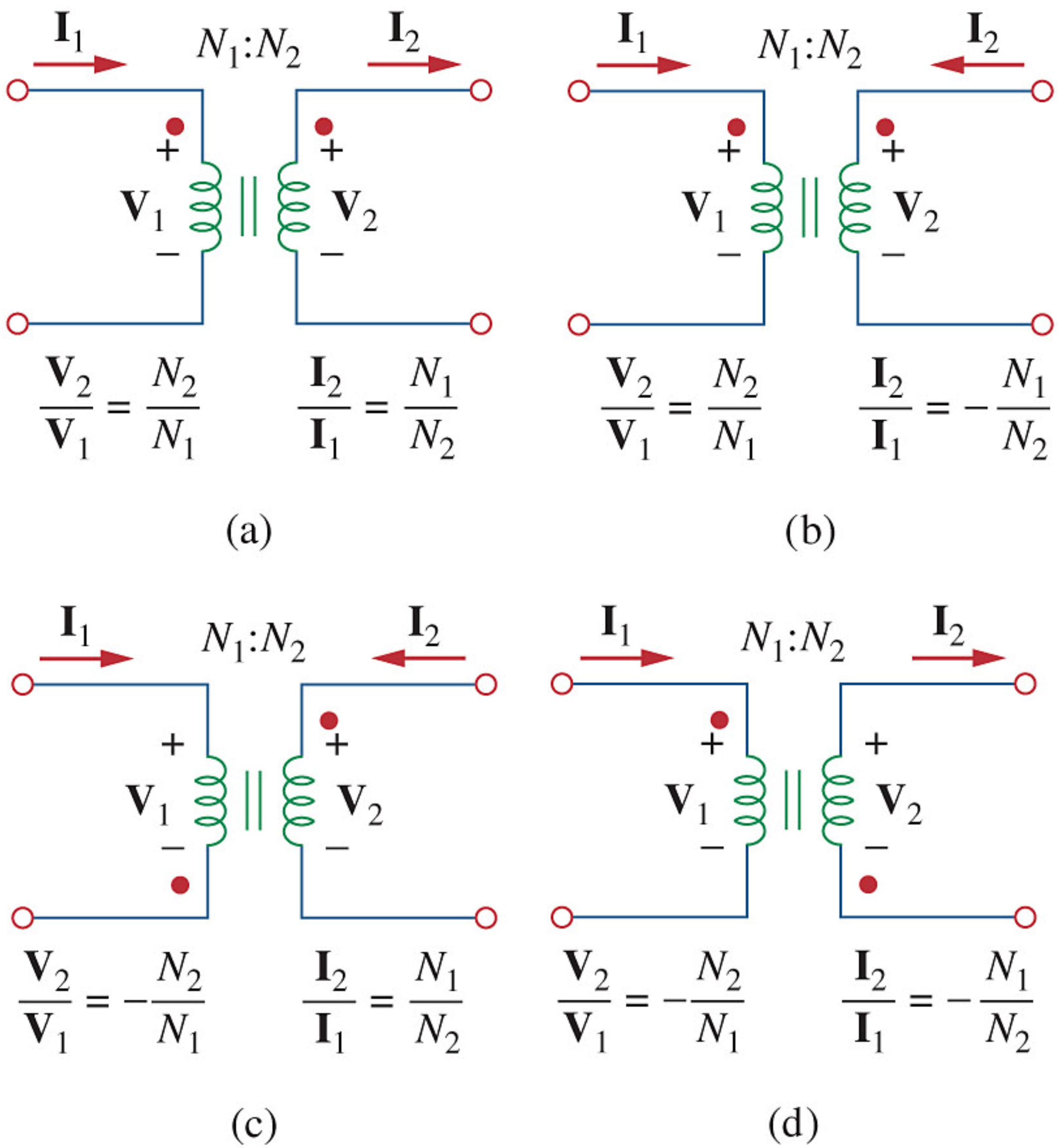


Figure 13.32 Typical circuits illustrating proper voltage polarities and circuit directions in an ideal transformer.

The complex power in the primary winding is equal to the complex power in the secondary winding:

$$S_1 = \dot{V}_1 \dot{I}_1^* = \frac{\dot{V}_2}{n}\left(n\dot{I}_2\right)^* = \dot{V}_2 \dot{I}_2^* = S_2$$

showing that the ideal transformer absorbs no power.

The input impedance as seen by the source is

$$Z_{in} = \frac{\dot{V}_1}{\dot{I}_1} = \frac{\dot{V}_2 / n}{n\dot{I}_2} = \frac{1}{n^2}\frac{\dot{V}_2}{\dot{I}_2} = \frac{1}{n^2} Z_L$$

It is also called the *reflected impedance*. This ability of the transformer provides us a means of *impedance matching* to ensure maximum power transfer (Section 13.9).

In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other. Figures 13.35 and 13.36 show the equivalent circuits for Fig. 13.33.

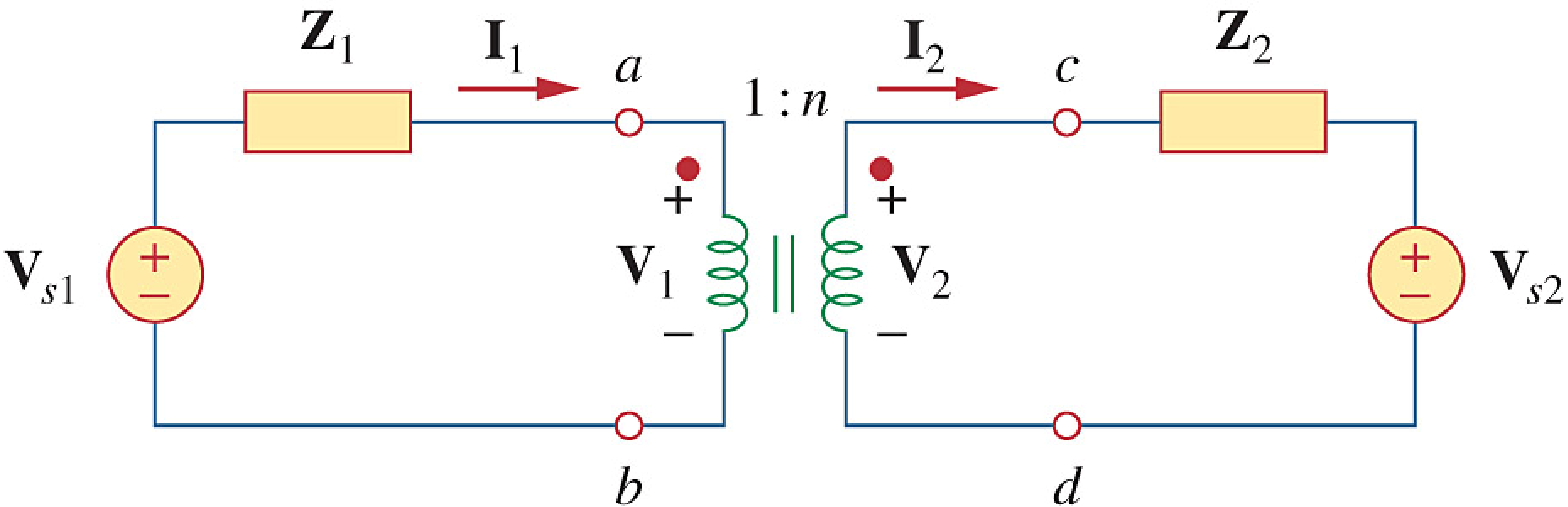


Figure 13.33 Ideal transformer circuit whose equivalent circuits are to be found.

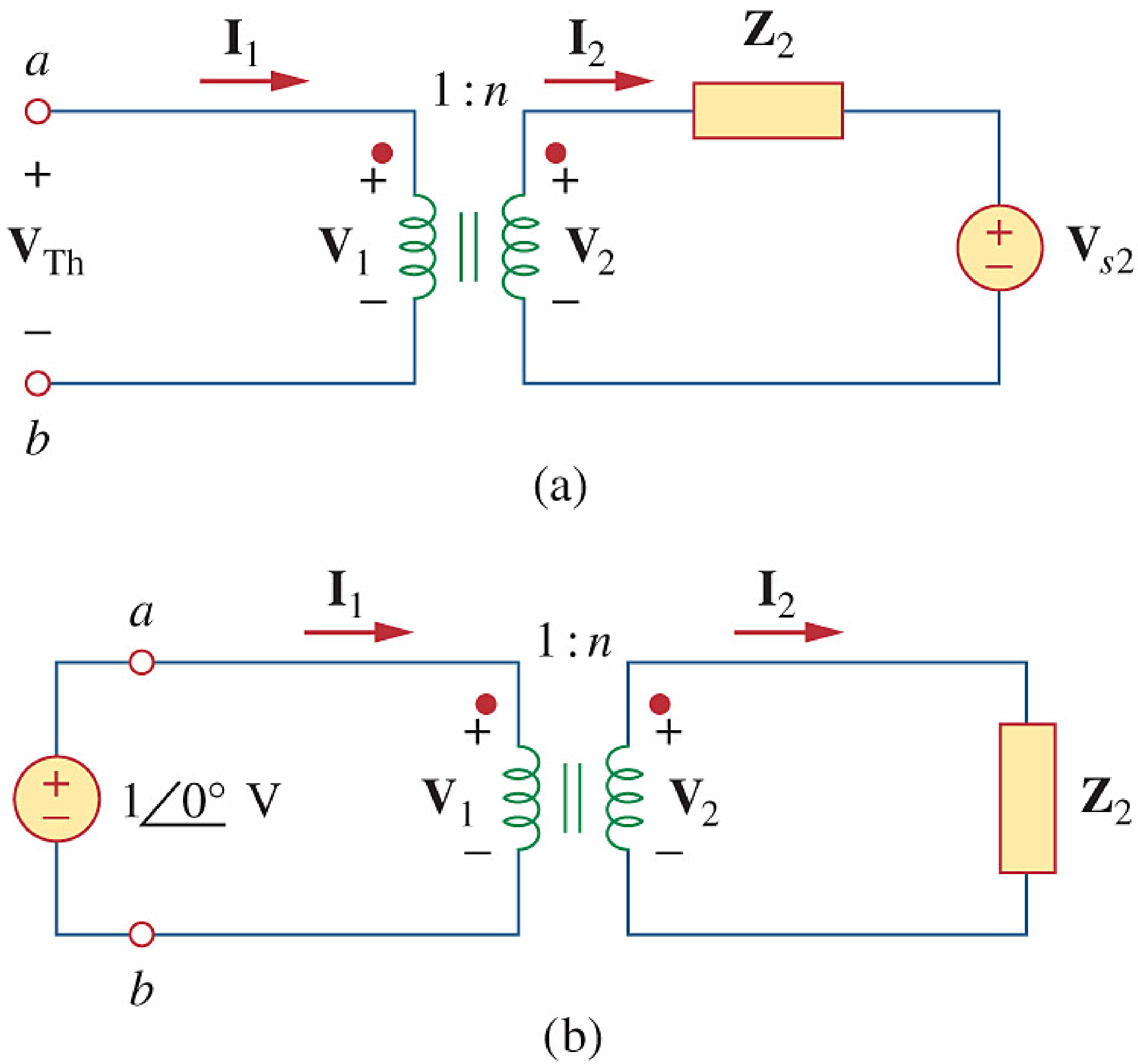


Figure 13.34 Obtaining the Thevenin equivalent for the circuit in Fig. 13.33.

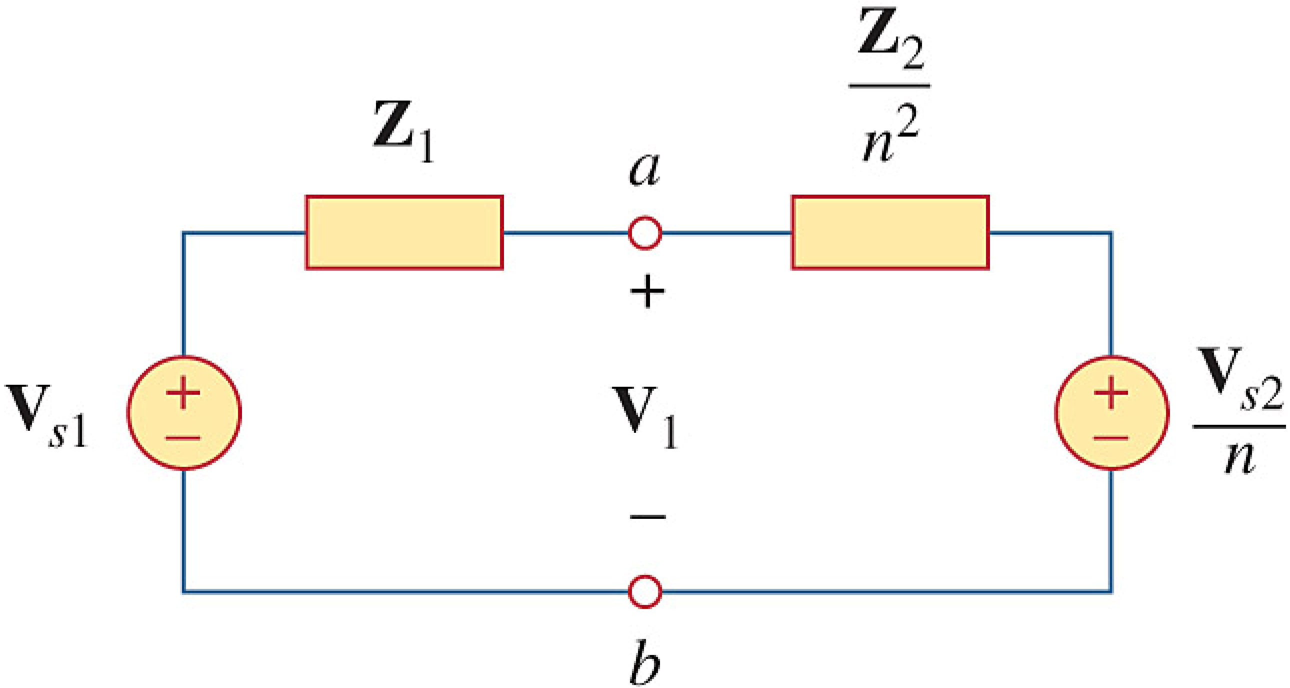


Figure 13.35 Equivalent circuit for Fig. 13.33 obtained by reflecting the secondary circuit to the primary side.

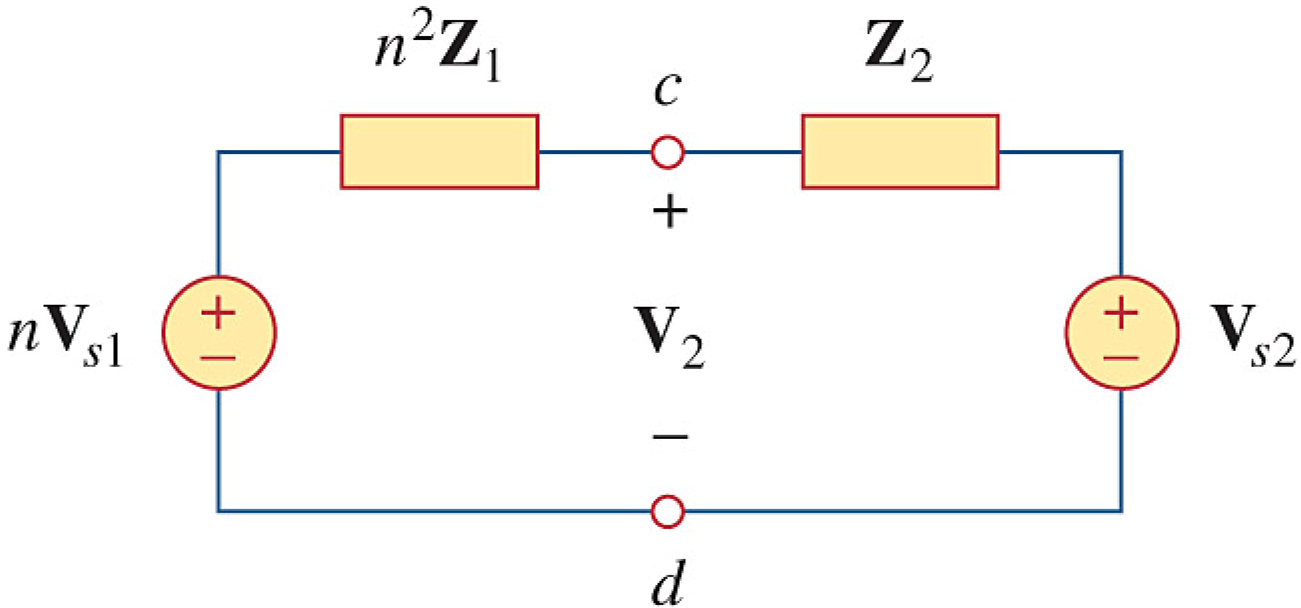


Figure 13.36 Equivalent circuit for Fig. 13.33 obtained by reflecting the primary circuit to the secondary side.

The general rule for eliminating the transformer by reflecting the secondary circuit to the primary side is: divide the secondary impedance by n^2, divide the secondary voltage by n, and multiply the secondary current by n.

The general rule for eliminating the transformer by reflecting the primary circuit to the secondary side is: multiply the primary impedance by n^2, multiply the primary voltage by n, and divide the primary current by n.

Practice Problem 13.7 The primary current to an ideal transformer rated at 3300/110 V is 5 A. Calculate: (a) the turns ratio, (b) the kVA rating, (c) the secondary current.

Solution :

$$\text{(a) } n = \frac{V_2}{V_1} = \frac{110}{3300} = \frac{1}{30}$$

$$\text{(b) } \left|S\right| = V_1 I_1 = 3300 \times 5 = 16500 \text{ (VA)}$$

$$= 16.5 \text{ kVA}$$

$$\text{(c) } \frac{I_1}{I_2} = n \Rightarrow I_2 = \frac{I_1}{n} = \frac{5}{1/30} = 150 \text{ (A)}$$

Practice Problem 13.8 In the ideal transformer circuit of Fig. 13.38, find $\dot{V}_o$ and the complex power supplied by the source.

Solution :

$$Z_R = \frac{16 - j24}{4^2} = 1 - j1.5\ (\Omega)$$

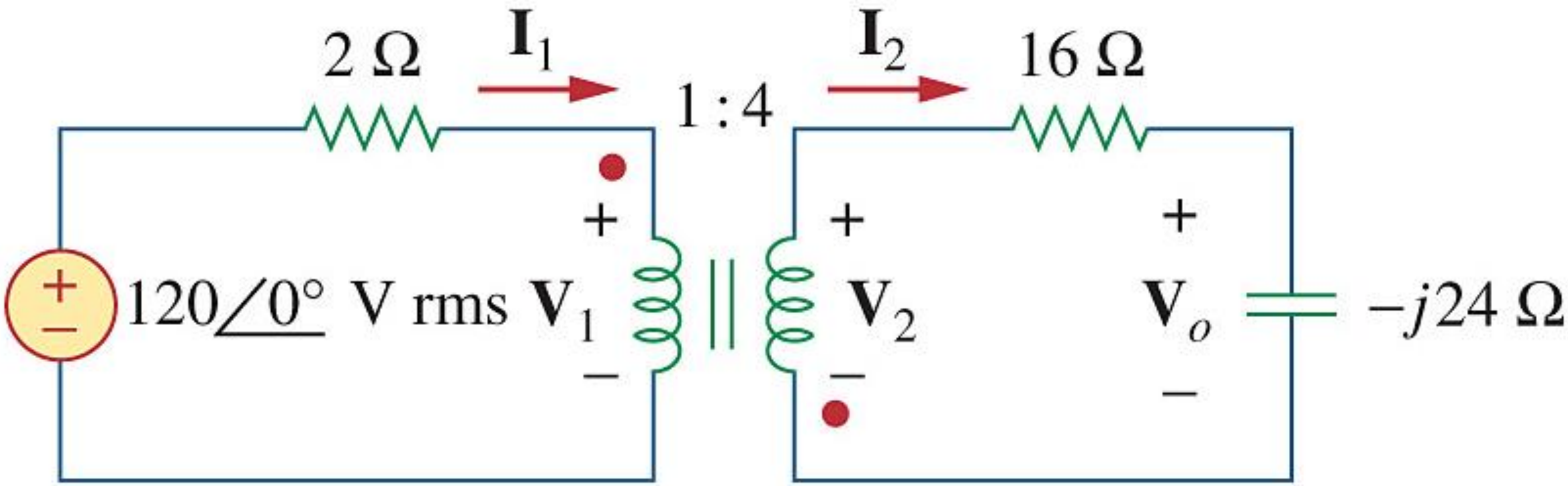

2 Ω
$\mathbf{I}_1$
$\mathbf{I}_2$
16 Ω
1 : 4
+
−
$120\angle 0^\circ$ V rms
$\mathbf{V}_1$
+
−
$\mathbf{V}_2$
+
−
$\mathbf{V}_o$
+
−
$-j24\ \Omega$

Figure 13.38

$$\dot{I}_1 = \frac{120\angle 0^\circ}{2+Z_R} = \frac{120\angle 0^\circ}{2+(1-j1.5)}$$

$$\approx \frac{120\angle 0^\circ}{3.3541\angle -26.57^\circ}$$

$$\approx 35.7771\angle 26.57^\circ \text{ (A)}$$

$$\dot{I}_2 = -\frac{\dot{I}_1}{n} = -\frac{35.7771\angle 26.57^\circ}{4}$$

$$\approx 8.9443\angle 206.57^\circ \text{ (A)}$$

$$
\begin{aligned}
\dot{V}_o &= \dot{I}_2 \times (-j24) \\
&= 8.9443\angle 206.57^\circ \times 24\angle -90^\circ \\
&= 214.6632\angle 116.57^\circ \text{ (V)}
\end{aligned}
$$

$$
\begin{aligned}
S &= 120\angle 0^\circ \times \dot{I}_1^* \\
&= 120\angle 0^\circ \times 35.7771\angle -26.57^\circ \\
&\approx 4293.252\angle -26.57^\circ \text{ (VA)} \\
&\approx 4.2933\angle -26.57^\circ \text{ kVA}
\end{aligned}
$$

Practice Problem 13.9 Find $\dot{V}_o$ in the circuit of Fig. 13.40.

Solution :

$$\frac{\dot{V}_1 - 240\angle 0^\circ}{4} + \frac{\dot{V}_1 - \dot{V}_3}{8} + \dot{I}_1 = 0 \qquad (1)$$

$$\frac{\dot{V}_3 - \dot{V}_1}{8} + \dot{I}_2 + \frac{\dot{V}_3}{8} = 0 \qquad (2)$$

$$\dot{I}_2 = \frac{\dot{V}_3 - \dot{V}_2}{2} \qquad (3)$$

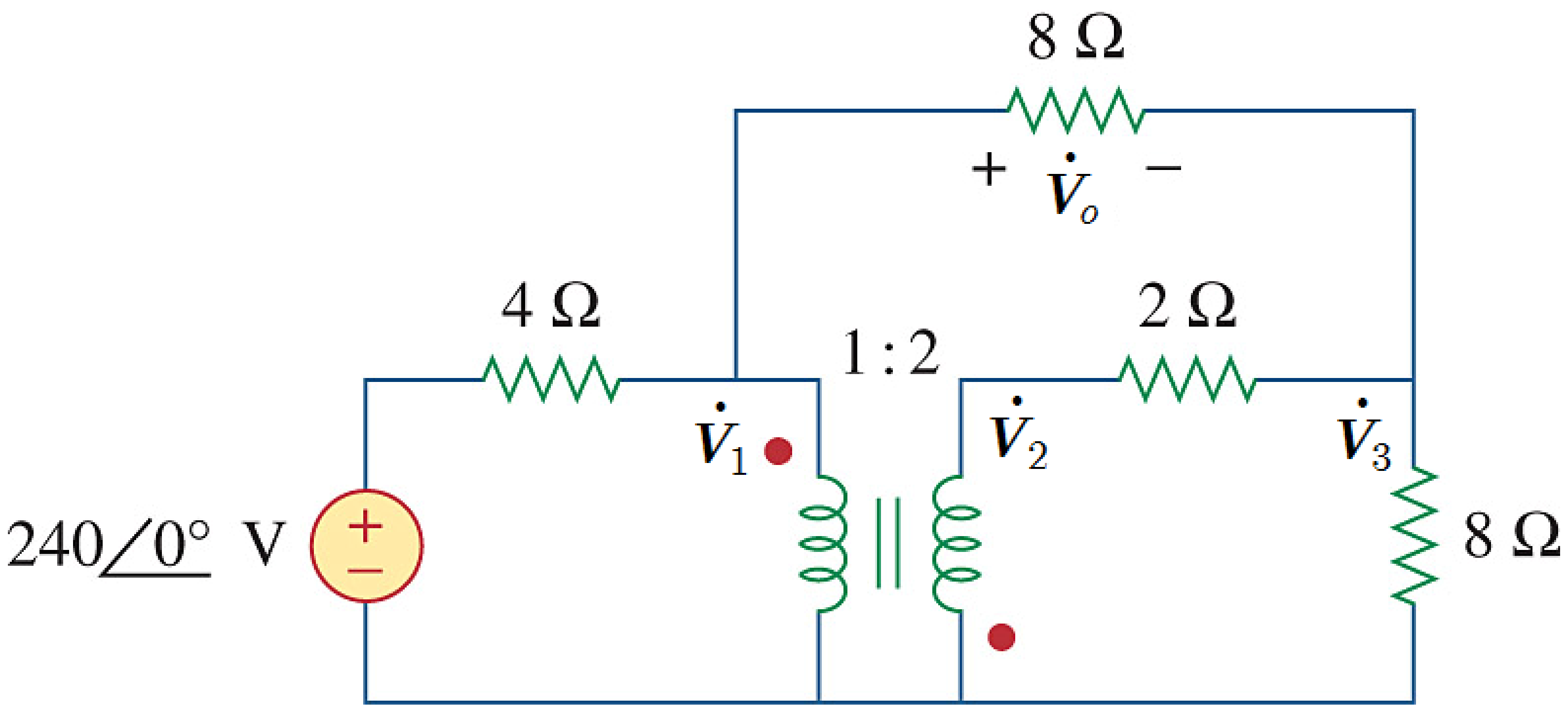


Figure 13.40

$$\dot{V}_2 = -2\dot{V}_1 \qquad (4)$$

$$\dot{I}_1 = 2\dot{I}_2 \qquad (5)$$

From (3) and (4),

$$\dot{I}_2 = \frac{\dot{V}_3}{2} - \frac{\dot{V}_2}{2} = \frac{\dot{V}_3}{2} + \dot{V}_1 \qquad (6)$$

From (5) and (6),

$$\dot{I}_1 = \dot{V}_3 + 2\dot{V}_1 \qquad (7)$$

Substitute (7) in (1),

$$\frac{\dot{V}_1 - 240}{4} + \frac{\dot{V}_1 - \dot{V}_3}{8} + \dot{V}_3 + 2\dot{V}_1 = 0 \Rightarrow$$

$$19\dot{V}_1 + 7\dot{V}_3 = 480 \qquad (8)$$

Substitute (6) in (2),

$$7\dot{V}_1 + 6\dot{V}_3 = 0 \qquad (9)$$

From (8) and (9),

$$\dot{V}_1 = \frac{576}{13}\ (\mathrm{V}),\ \dot{V}_3 = -\frac{672}{13}\ (\mathrm{V}) \Rightarrow$$

$$\dot{V}_o = \dot{V}_1 - \dot{V}_3 = 96\ (\mathrm{V}) = 96\angle 0^\circ\ (\mathrm{V})$$

13.6 Ideal Autotransformers

Unlike the conventional two-winding transformer we have considered so far, an autotransformer has a single continuous winding with a connection point called a *tap* between the primary and secondary sides, as shown in Fig. 13.42.

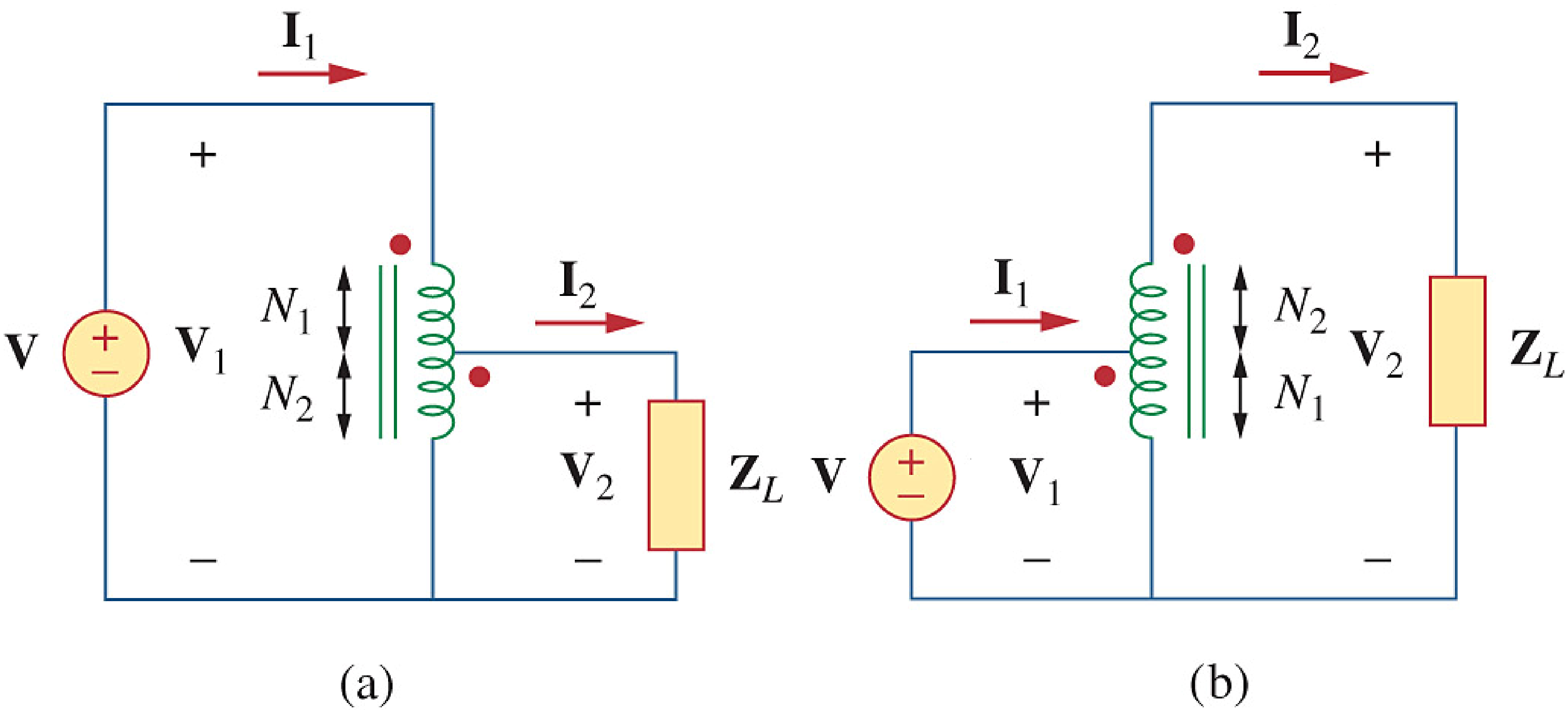


Figure 13.42 (a) Step-down autotransformer, (b) step-down autotransformer.

The autotransformer is a type of power transformer. Its major advantage over the two-winding transformer is its ability to transfer larger apparent power. Its major disadvantage is the lack of electrical isolation between the primary and secondary sides.

For the step-down autotransformer circuit of Fig. 13.42(a),

$$\frac{\dot{V}_1}{\dot{V}_2} = \frac{N_1 + N_2}{N_2} = \frac{\dot{I}_2}{\dot{I}_1} \text{ or } \frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{I_2}{I_1}$$

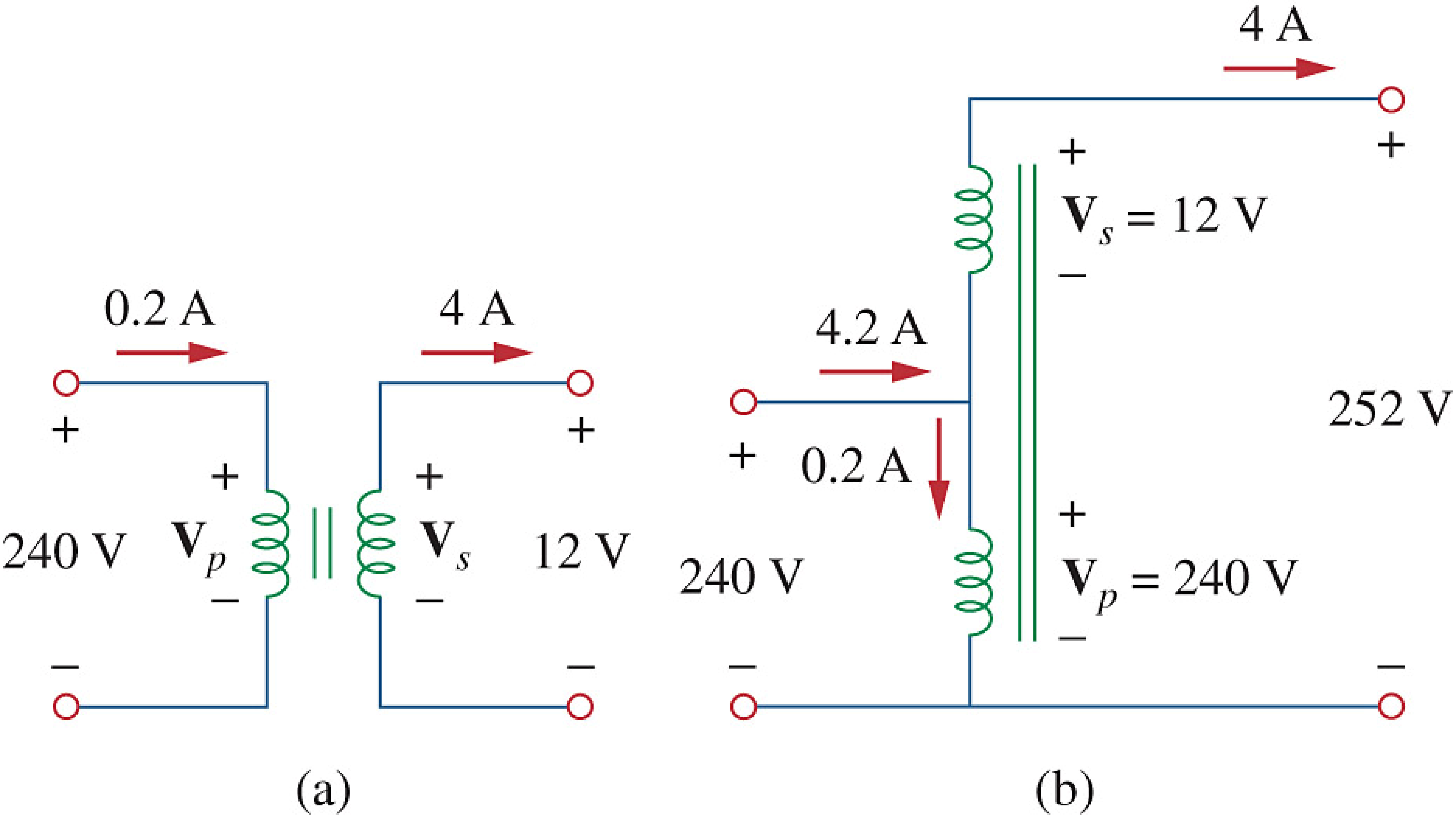
For the step-up autotransformer circuit of Fig. 13.42(b),

$$\frac{\dot{V}_1}{\dot{V}_2} = \frac{N_1}{N_1 + N_2} = \frac{\dot{I}_2}{\dot{I}_1} \text{ or } \frac{V_1}{V_2} = \frac{N_1}{N_1 + N_2} = \frac{I_2}{I_1}$$

Example 13.10 Compare the power ratings of the two-winding transformer in Fig. 13.43 (a) and the autotransformer in Fig. 13.43(b).

Solution :

We think of the continuous winding of the autotransformer as two windings with the same current and voltage as those for the two-winding transformer. This is the basis of comparing their power ratings.



0.2 A
4 A
+
+
+
+
240 V
$\mathbf{V}_p$
$\mathbf{V}_s$
12 V
–
–
–
–
(a)
4 A
+
+
$\mathbf{V}_s = 12$ V
–
4.2 A
252 V
+
0.2 A
240 V
+
$\mathbf{V}_p = 240$ V
–
–
–
(b)

Figure 13.43

For the two-winding transformer,

$$S_1 = 240 \times 0.2 = 48 \text{ (VA) or}$$

$$S_2 = 12 \times 4.2 = 48 \text{ (VA)}$$

For the autotransformer,

$$S_1 = 240 \times 4.2 = 1008 \text{ (VA) or}$$

$$S_2 = (12 + 240) \times 4 = 1008 \text{ (VA)}$$

which is 21 times the power rating of the two-winding transformer, showing that the autotransformer can transfer larger power.

13.9 Applications

Transformer as an Isolation Device The transformer in Fig. 13.59 steps up or steps down the voltage and provides electrical isolation between the ac power supply and the rectifier, thereby reducing the risk of shock hazard in handling the rectifier. (Rectification is the process of converting ac voltage/current to dc voltage/current.)

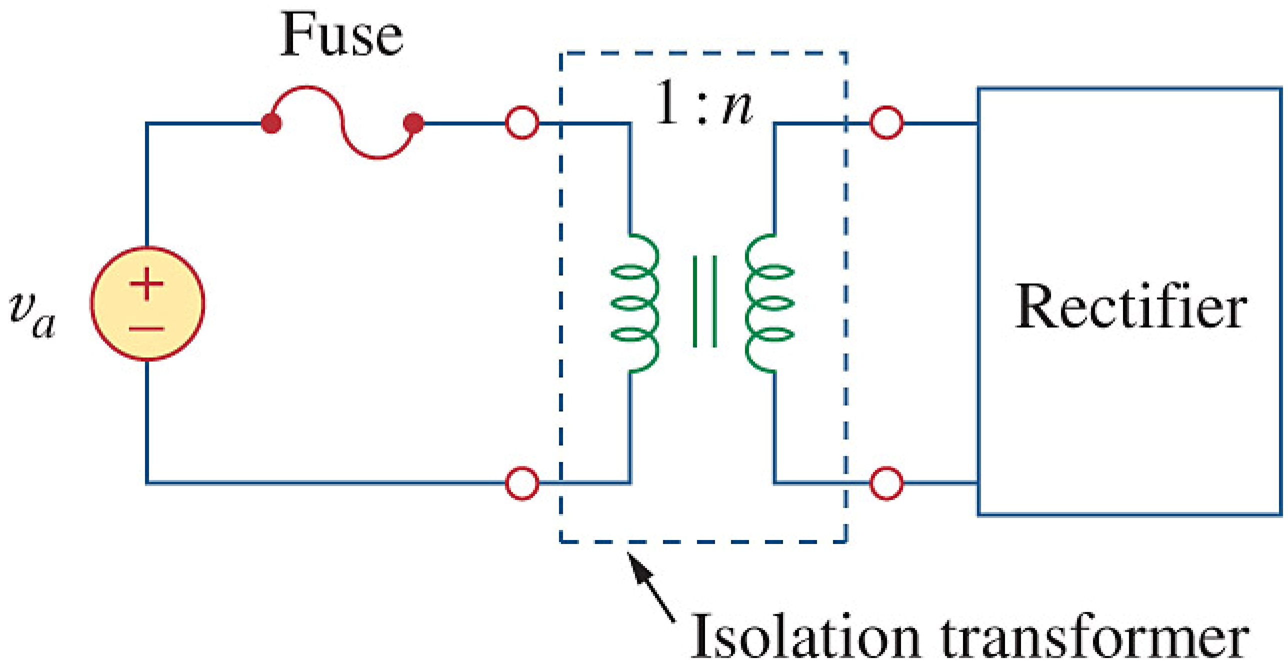


Figure 13.60 A transformer used to isolate an ac supply from a rectifier.

When the sole purpose of a transformer is to provide isolation, its turns ratio $n = 1$. In Fig. 13.60, the transformer is used to couple two stages of an amplifier, to prevent any dc voltage in one stage from affecting the dc bias of the next stage. (Biasing is the application of a dc voltage to a transistor in order to produce a desired mode of operation).

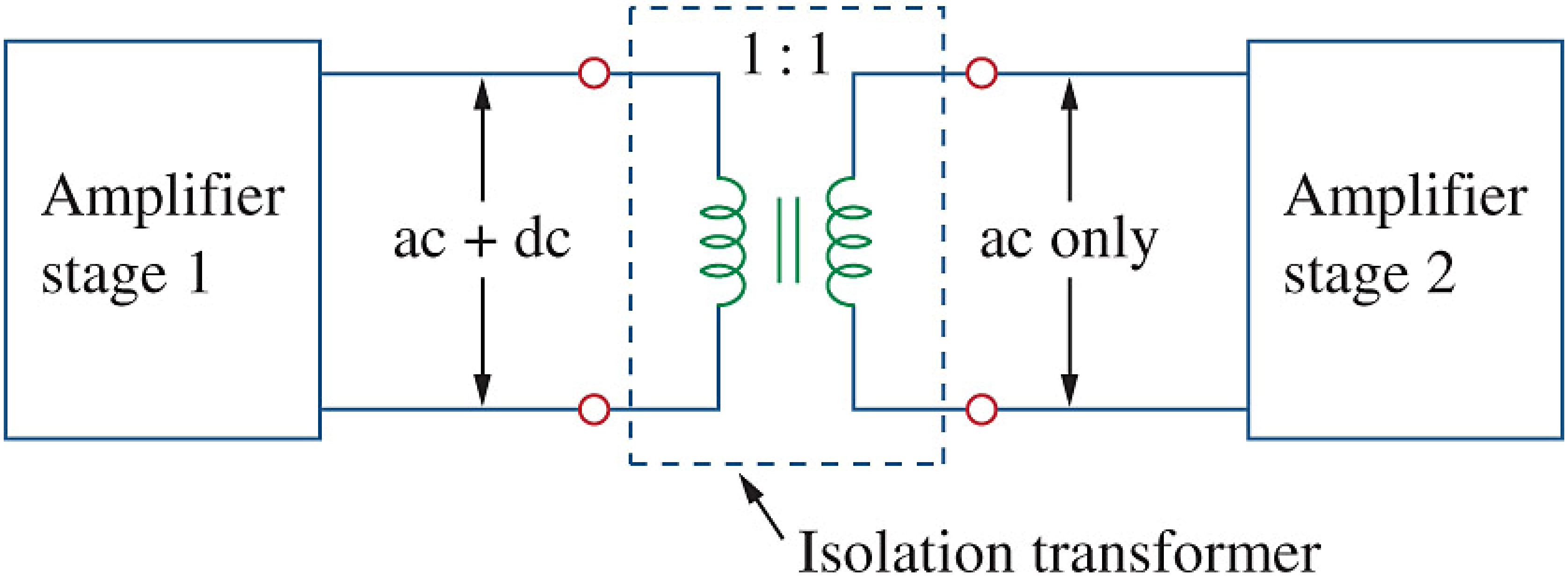


Figure 13.60 A transformer providing dc isolation between two amplifier stages.

Example 13.15 Determine the voltage across the load in Fig. 13.62.

Solution :

We use the superposition principle (Fig. 13.63).

$$v_1' = 0 \Rightarrow v_2' = 0$$

$$v_1'' \approx 120 \cos \omega t \text{ V} \Rightarrow v_2'' = \frac{1}{3} v_1'' = 40 \cos \omega t \text{ V}$$

$$v_2 = v_2' + v_2'' = 40 \cos \omega t \text{ (V)}$$

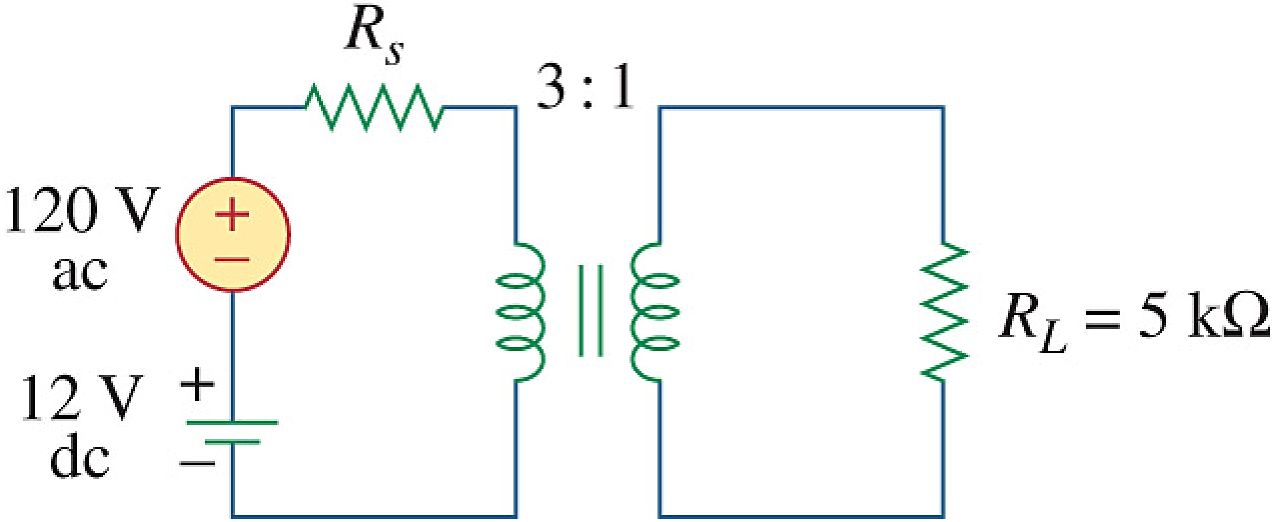


Figure 13.62

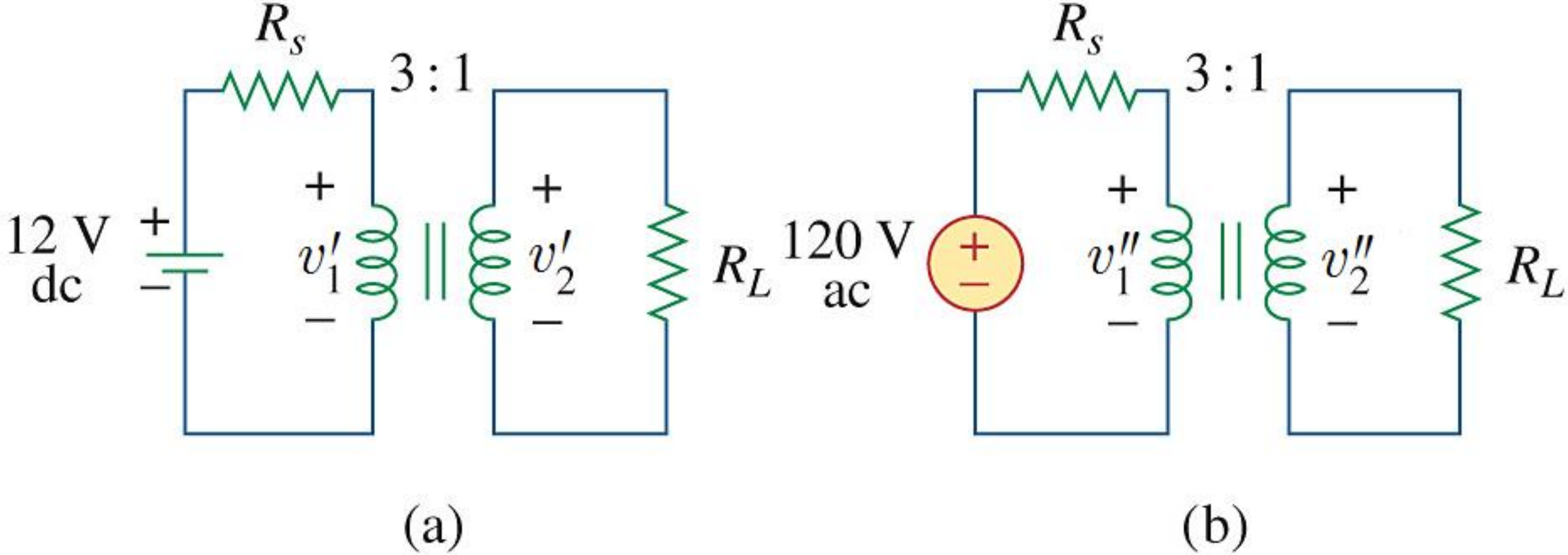


Figure 13.63

In Fig. 13.61, a transformer is used both to electrically isolate the line power from the voltmeter and to step down the voltage to a safe level.

Practice Problem 13.15 Calculate the turns ratio required to step down the 13.2-kV line voltage to a safe level of 120 V.

Answer : 110.

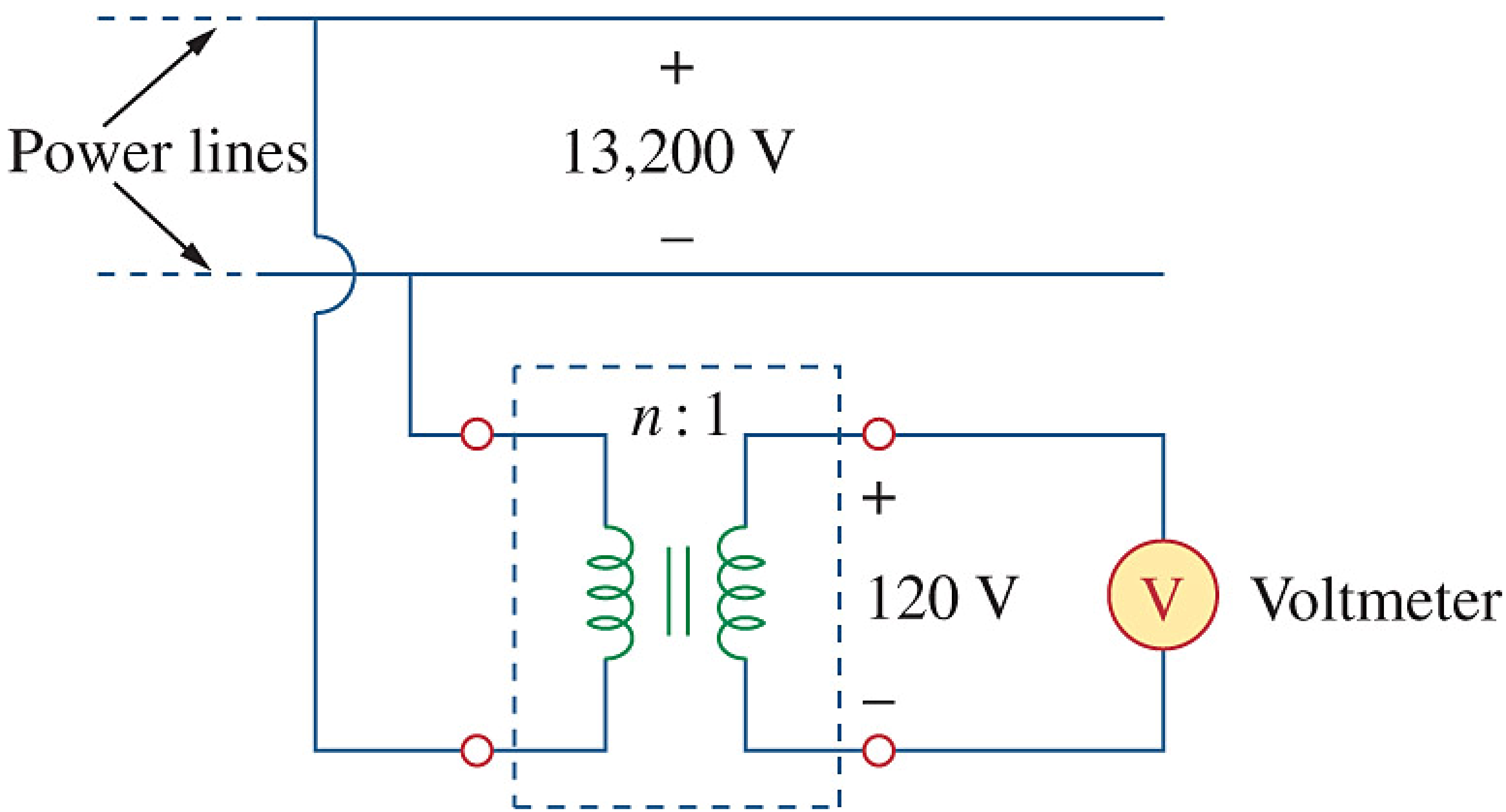


Figure 13.61 A transformer providing isolation between the power lines and the voltmeter.

Transformer as a Matching Device a transformer can be used to match the load impedance to the source impedance to achieve maximum power transfer. This is called *impedance matching*.

In Fig. 13.64, an ideal transformer is used as the matching transformer.

$$R_s = \frac{R_L}{n^2} \Rightarrow n = \sqrt{\frac{R_L}{R_s}}$$

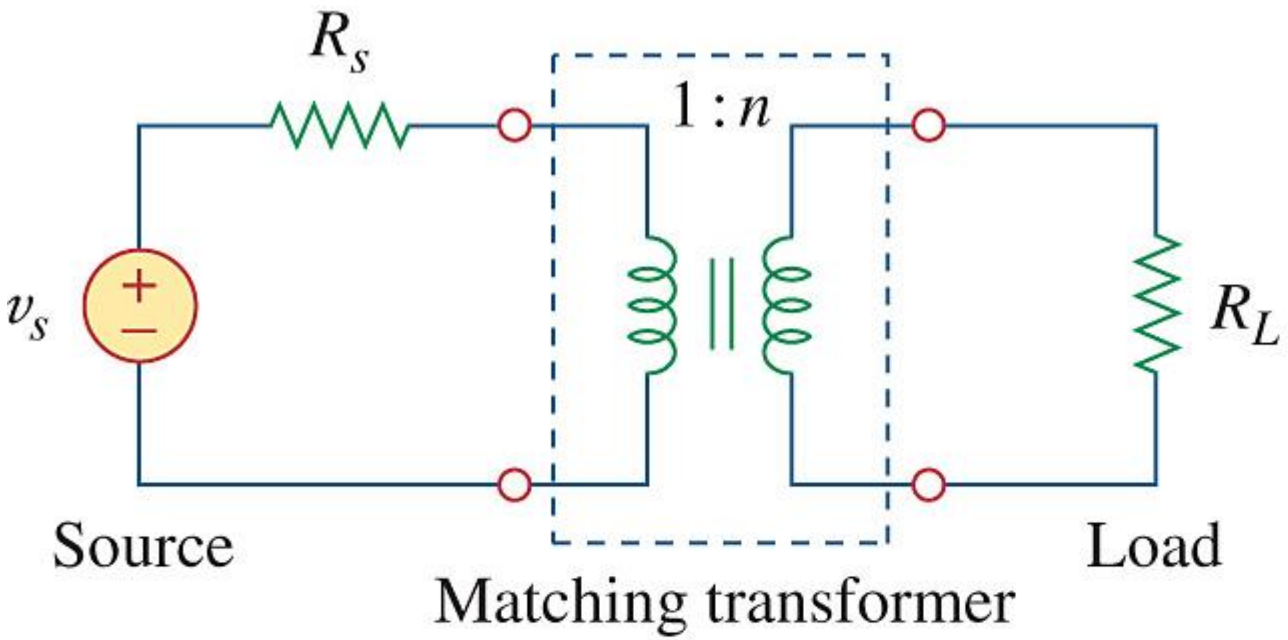


Figure 13.64 Transformer used as a matching device.

Example 13.16 Consider Fig. 13.65. The Thevenin impedance of the amplifier is 192 Ω, and the impedance of the speaker is 12 Ω. Determine the required turns ratio.

Answer : $n = 1/4 = 0.25.$

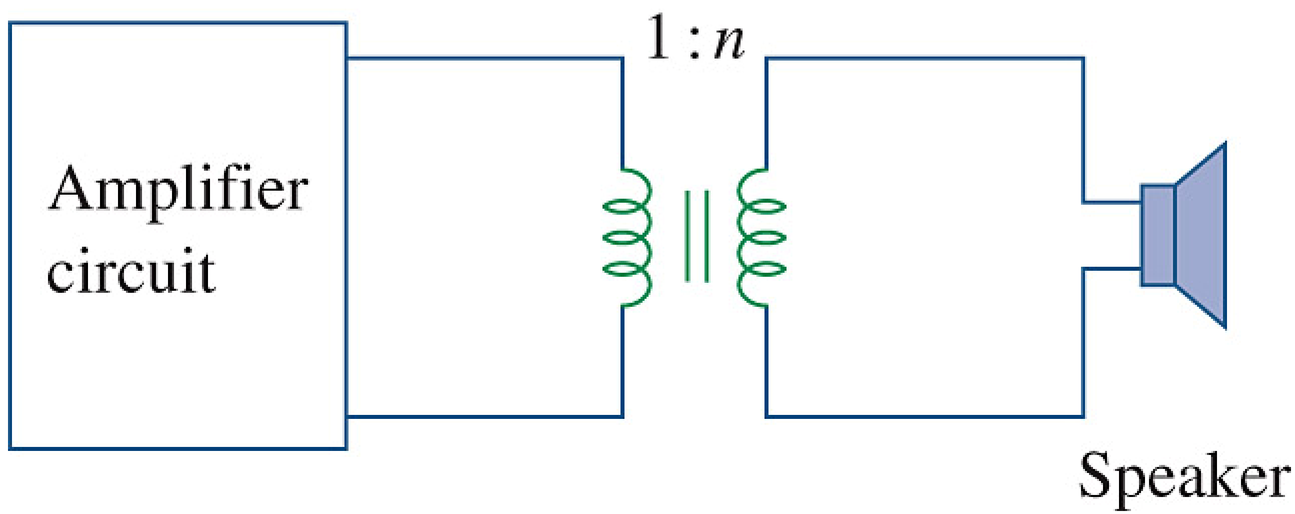


Figure 13.65 Using an ideal transformer to match the speaker to the amplifier.

Power Distribution As shown in Fig. 13.67, a power system basically consists of three components: generation, transmission, and distribution. Transformers are used to step up and step down voltage and distribute power economically.

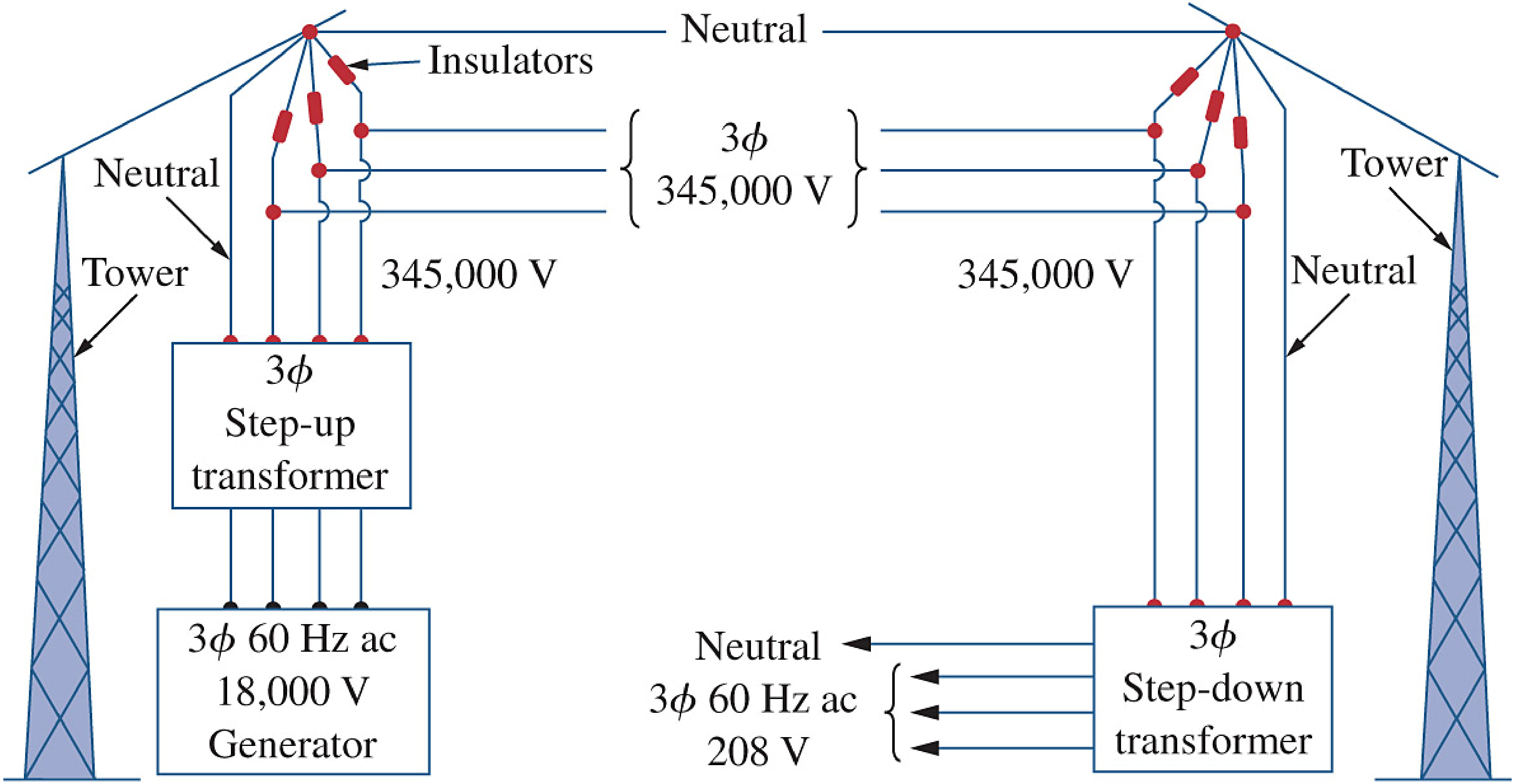


Figure 13.67 A typical power distribution system.

From $S = \sqrt{3} V_L I_L$, if V_L is increased from 18,000 V to 345,000 V, I_L will be decreased by a factor of 345,000/18,000 $\approx$19.17. The smaller current reduces the conductor size, producing considerable savings as well as minimizing tranmission line losses $I_L^2 R = V_l^2 / R$, where V_l is the potential difference between the sending and receiving ends of the line.