(c) $M(s) = \dfrac{K}{s^3 + 5s + 5}$ (d) $M(s) = \dfrac{100(s-1)}{(s+5)(s^2+2s+2)}$

(e) $M(s) = \dfrac{100}{s^3 - 2s^2 + 3s + 10}$ (f) $M(s) = \dfrac{10(s+12.5)}{s^4 + 3s^3 + 50s^2 + s + 10^6}$

• Routh-Hurwitz criterion

6-2. Using the Routh-Hurwitz criterion, determine the stability of the closed-loop system that has the following characteristic equations. Determine the number of roots of each equation that are in the right-half s-plane and on the $j\omega$-axis.

(a) $s^3 + 25s^2 + 10s + 450 = 0$ (b) $s^3 + 25s^2 + 10s + 50 = 0$

(c) $s^3 + 25s^2 + 250s + 10 = 0$ (d) $2s^4 + 10s^3 + 5.5s^2 + 5.5s + 10 = 0$

(e) $s^6 + 2s^5 + 8s^4 + 15s^3 + 20s^2 + 16s + 16 = 0$ (f) $s^4 + 2s^3 + 10s^2 + 20s + 5 = 0$

• Routh-Hurwitz criterion on margin of stability

6-3. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

(a) $s^4 + 25s^3 + 15s^2 + 20s + K = 0$ (b) $s^4 + Ks^3 + 2s^2 + (K+1)s + 10 = 0$

(c) $s^3 + (K+2)s^2 + 2Ks + 10 = 0$ (d) $s^3 + 20s^2 + 5s + 10K = 0$

(e) $s^4 + Ks^3 + 5s^2 + 10s + 10K = 0$ (f) $s^4 + 12.5s^3 + s^2 + 5s + K = 0$

• Region of stability in the parameter plane

6-4. The loop transfer function of a single-loop feedback control system is given as

$$G(s)H(s) = \frac{K(s+5)}{s(s+2)(1+Ts)}$$

The parameters K and T may be represented in a plane with K as the horizontal axis and T as the vertical axis. Determine the regions in the T-versus-K parameter plane where the closed-loop system is asymptotically stable and where it is unstable. Indicate the boundary on which the system is marginally stable.

• Margin of stability, sustained oscillation

6-5. Given the forward-path transfer function of unity-feedback control systems,

(a) $G(s) = \dfrac{K(s+4)(s+20)}{s^3(s+100)(s+500)}$ (b) $G(s) = \dfrac{K(s+10)(s+20)}{s^2(s+2)}$

(c) $G(s) = \dfrac{K}{s(s+10)(s+20)}$ (d) $G(s) = \dfrac{K(s+1)}{s^3 + 2s^2 + 3s + 1}$

Apply the Routh-Hurwitz criterion to determine the stability of the closed-loop system as a function of K. Determine the value of K that will cause sustained constant-amplitude oscillations in the system. Determine the frequency of oscillation.

• Stability of system in state equation form, state feedback control

6-6. A controlled process is modeled by the following state equations.

$$\frac{dx_1(t)}{dt} = x_1(t) - 2x_2(t) \qquad \frac{dx_2(t)}{dt} = 10x_1(t) + u(t)$$

The control $u(t)$ is obtained from state feedback such that

$$u(t) = -k_1x_1(t) - k_2x_2(t)$$

where k_1 and k_2 are real constants. Determine the region in the k_1-versus-k_2 parameter plane in which the closed-loop system is asymptotically stable.

• Stability of system in state equation form, State feedback control

6-7. A linear time-invariant system is described by the following state equations.

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The closed-loop system is implemented by state feedback, so that $u(t) = -\mathbf{K}\mathbf{x}(t)$, where $\mathbf{K} = [k_1 \quad k_2 \quad k_3]$; and k_1, k_2, and k_3 are real constants. Determine the constraints on the elements of $\mathbf{K}$ so that the closed-loop system is asymptotically stable.

• Stablizability with state feedback

6-8. Given the system in state equation form,

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Can the system be stabilized by state feedback $u(t) = -\mathbf{K}\mathbf{x}(t)$, where $\mathbf{K} = [k_1 \quad k_2 \quad k_3]$?

• System with tachometer feedback

6-9. The block diagram of a motor-control system with tachometer feedback is shown in Fig. 6P-9. Find the range of the tachometer constant K_t so that the system is asymptotically stable.

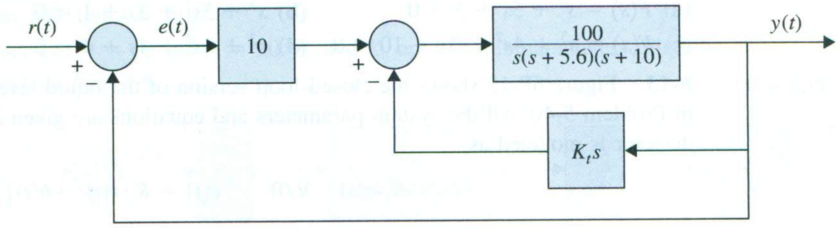

Figure 6P-9

• Stability in parameter plane

6-10. The block diagram of a control system is shown in Fig. 6P-10. Find the region in the K-versus-α plane for the system to be asymptotically stable. (Use K as the vertical and α as the horizontal axis.)

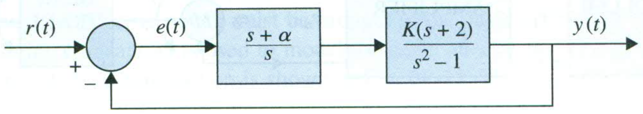

Figure 6P-10

• Stability of attitude control of a missile

6-11. The attitude control of a missile shown in Fig. 4P-13 is accomplished by thrust vectoring. The transfer function between the thrust angle $\Delta(s)$ and the angle of attack $\Theta(s)$ is represented by

$$G_p(s) = \frac{\Theta(s)}{\Delta(s)} = \frac{K}{s^2 - \alpha}$$

where K and α are positive real constants. The block diagram of the control system is shown in Fig. 6P-11.

(a) In Fig. 6P-11, consider that only the attitude-sensor loop is in operation, but $K_t = 0$. Determine the relationship between K, K_s, and α so that the missile will oscillate back and forth (marginally stable). Find the condition when the missile will tumble end over end (unstable).

(b) Consider that both loops are in operation. Determine relationships among K, K_s, K_t, and α so that the missile will oscillate back and forth. Repeat the problem for the missile tumbling end over end.

PROBLEMS

s-domain specifications

7-1. A pair of complex-conjugate poles in the s-plane is required to meet the various specifications that follow. For each specification, sketch the region in the s-plane in which the poles should be located.

(a) $\zeta \geq 0.707 \qquad \omega_n \geq 2$ rad/sec (positive damping)

(b) $0 \leq \zeta \leq 0.707 \qquad \omega_n \leq 2$ rad/sec (positive damping)

(c) $\zeta \leq 0.5 \qquad 1 \leq \omega_n \leq 5$ rad/sec (positive damping)

(d) $0.5 \leq \zeta \leq 0.707 \qquad \omega_n \leq 5$ rad/sec (positive and negative damping)

System type

7-2. Determine the type of the following unity-feedback systems for which the forward-path transfer functions are given.

(a) $G(s) = \dfrac{K}{(1+s)(1+10s)(1+20s)}$ (b) $G(s) = \dfrac{10e^{-0.2s}}{(1+s)(1+10s)(1+20s)}$

(c) $G(s) = \dfrac{10(s+1)}{s(s+5)(s+6)}$ (d) $G(s) = \dfrac{100(s-1)}{s^2(s+5)(s+6)^2}$

(e) $G(s) = \dfrac{10(s+1)}{s^3(s^2+5s+5)}$ (f) $G(s) = \dfrac{100}{s^3(s+2)^2}$

Error constants

7-3. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given.

(a) $G(s) = \dfrac{1000}{(1+0.1s)(1+10s)}$ (b) $G(s) = \dfrac{100}{s(s^2+10s+100)}$

(c) $G(s) = \dfrac{K}{s(1+0.1s)(1+0.5s)}$ (d) $G(s) = \dfrac{100}{s^2(s^2+10s+100)}$

(e) $G(s) = \dfrac{1000}{s(s+10)(s+100)}$ (f) $G(s) = \dfrac{K(1+2s)(1+4s)}{s^2(s^2+s+1)}$

Steady-state error of unity-feedback systems

7-4. For the unity-feedback control systems described in Problem 7-2, determine the steady-state error for a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$. Check the stability of the system before applying the final-value theorem.

Steady-state error of nonunity-feedback systems

7-5. The following transfer functions are given for a single-loop nonunity-feedback control system. Find the steady-state errors due to a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$.

(a) $G(s) = \dfrac{1}{(s^2+s+2)} \qquad H(s) = \dfrac{1}{(s+1)}$

(b) $G(s) = \dfrac{1}{s(s+5)} \qquad H(s) = 5$

(c) $G(s) = \dfrac{1}{s^2(s+10)} \qquad H(s) = \dfrac{s+1}{s+5}$

(d) $G(s) = \dfrac{1}{s^2(s+12)} \qquad H(s) = 5(s+2)$

Steady-state error of single-loop systems

7-6. Find the steady-state errors of the following single-loop control systems for a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$. For systems that include a parameter K, find its value so that the answers are valid.

(a) $M(s) = \dfrac{s+4}{s^4+16s^3+48s^2+4s+4}, \; K_H = 1$

(b) $M(s) = \dfrac{K(s+3)}{s^3+3s^2+(K+2)s+3K}, \; K_H = 1$

(c) $M(s) = \dfrac{s+5}{s^4+15s^3+50s^2+10s}, \; H(s) = \dfrac{10s}{s+5}$

(d) $M(s) = \dfrac{K(s+5)}{s^4+17s^3+60s^2+5Ks+5K}, \; K_H = 1$

• Error constants and steady-state errors

7-7. The block diagram of a control system is shown in Fig. 7P-7. Find the step-, ramp-, and parabolic-error constants. The error signal is defined to be $e(t)$. Find the steady-state errors in terms of K and K_t when the following inputs are applied. Assume that the system is stable.

(a) $r(t) = u_s(t)$ **(b)** $r(t) = tu_s(t)$ **(c)** $r(t) = (t^2/2)u_s(t)$

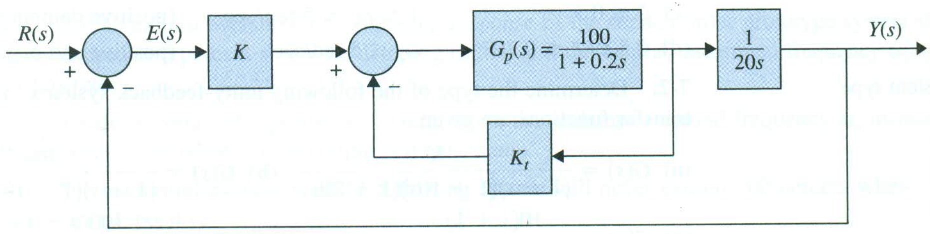

Figure 7P-7

• Error constants and steady-state errors

7-8. Repeat Problem 7-7 when the transfer function of the process is, instead,

$$G_p(s) = \frac{100}{(1 + 0.1s)(1 + 0.5s)}$$

What constraints must be made, if any, on the values of K and K_t so that the answers are valid? Determine the minimum steady-state error that can be achieved with a unit-ramp input by varying the values of K and K_t.

• Steady-state error

7-9. For the position-control system shown in Fig. 3P-18, determine the following.

(a) Find the steady-state value of the error signal $\theta_e(t)$ in terms of the system parameters when the input is a unit-step function.

(b) Repeat part (a) when the input is a unit-ramp function. Assume that the system is stable.

• Steady-state error

7-10. The block diagram of a feedback control system is shown in Fig. 7P-10. The error signal is defined to be $e(t)$.

(a) Find the steady-state error of the system in terms of K and K_t when the input is a unit-ramp function. Give the constraints on the values of K and K_t so that the answer is valid. Let $n(t) = 0$ for this part.

(b) Find the steady-state value of $y(t)$ when $n(t)$ is a unit-step function. Let $r(t) = 0$. Assume that the system is stable.

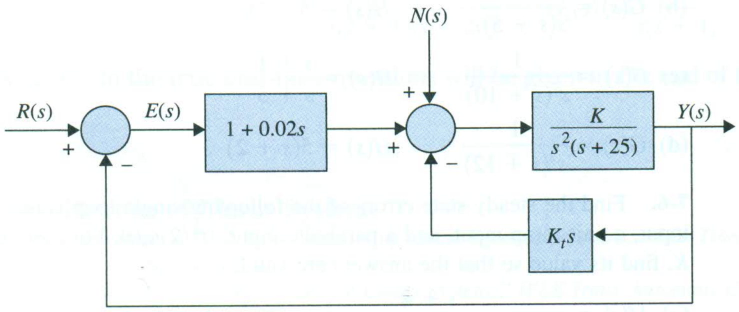

Figure 7P-10

• Steady-state error due to input and disturbance

7-11. The block diagram of a linear control system is shown in Fig. 7P-11, where $r(t)$ is the reference input and $n(t)$ is the disturbance.

(a) Find the steady-state value of $e(t)$ when $n(t) = 0$ and $r(t) = tu_s(t)$. Find the conditions on the values of α and K so that the solution is valid.

(b) Find the steady-state value of $y(t)$ when $r(t) = 0$ and $n(t) = u_s(t)$.

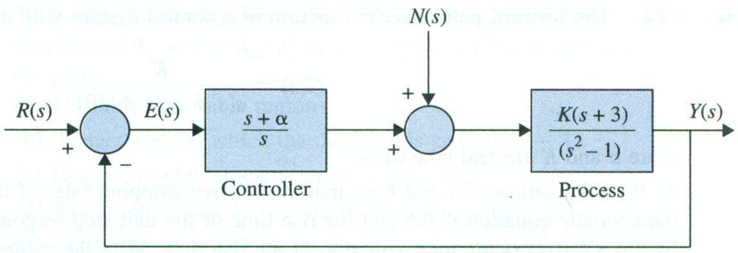

Figure 7P-11

Transfer function of second-order prototype system

7-12. The unit-step response of a linear control system is shown in Fig. 7P-12. Find the transfer function of a second-order prototype system to model the system.

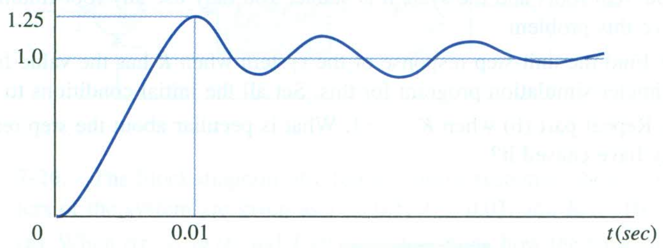

Figure 7P-12

Maximum overshoot and rise time

7-13. For the control system shown in Fig. 7P-7, find the values of K and K_t so that the maximum overshoot of the output is approximately 4.3 percent and the rise time t_r is approximately 0.2 sec. Use Eq. (7-104) for the rise-time relationship. Simulate the system with any time-response simulation program to check the accuracy of your solutions.

Maximum overshoot and rise time

7-14. Repeat Problem 7-13 with a maximum overshoot of 10 percent and a rise time of 0.1 sec.

7-15. Repeat Problem 7-13 with a maximum overshoot of 20 percent and a rise time of 0.05 sec.

Maximum overshoot and delay time

7-16. For the control system shown in Fig. 7P-7, find the values of K and K_t so that the maximum overshoot of the output is approximately 4.3 percent and the delay time t_d is approximately 0.1 sec. Use Eq. (7-102) for the delay-time relationship. Simulate the system with a computer program to check the accuracy of your solutions.

Maximum orershoot and delay time

7-17. Repeat Problem 7-16 with a maximum overshoot of 10 percent and a delay time of 0.05 sec.

7-18. Repeat Problem 7-16 with a maximum overshoot of 20 percent and a delay time of 0.01 sec.

Damping ratio and settling time

7-19. For the control system shown in Fig. 7P-7, find the values of K and K_t so that the damping ratio of the system is 0.6 and the settling time of the unit-step response is 0.1 sec. Use Eq. (7-108) for the settling time relationship. Simulate the system with a computer program to check on the accuracy of your results.

Maximum overshoot and settling time

7-20. **(a)** Repeat Problem 7-19 with a maximum overshoot of 10 percent and a settling time of 0.05 sec.

(b) Repeat Problem 7-19 with a maximum overshoot of 20 percent and a settling time of 0.01 sec.

Damping ratio and settling time

7-21. Repeat Problem 7-19 with a damping ratio of 0.707 and a settling time of 0.1 sec. Use Eq. (7-109) for the settling time relationship.