

VE 320 Summer 2019

Introduction to Semiconductor Devices

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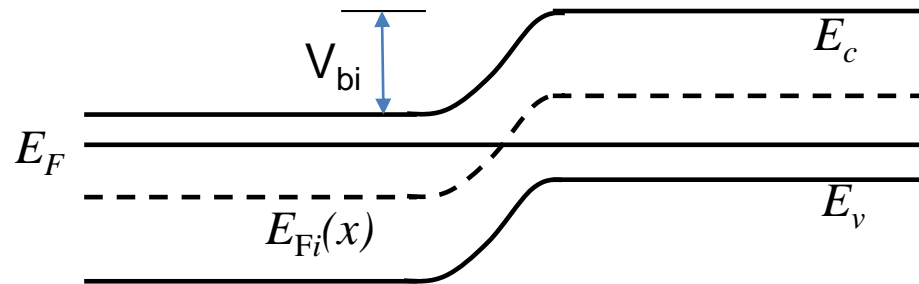
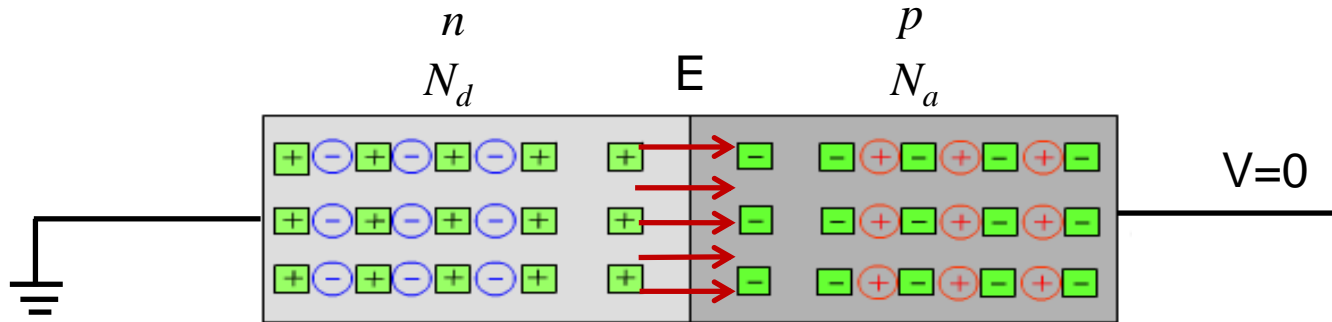


Lecture 8

pn junction diode (Chapter 8)

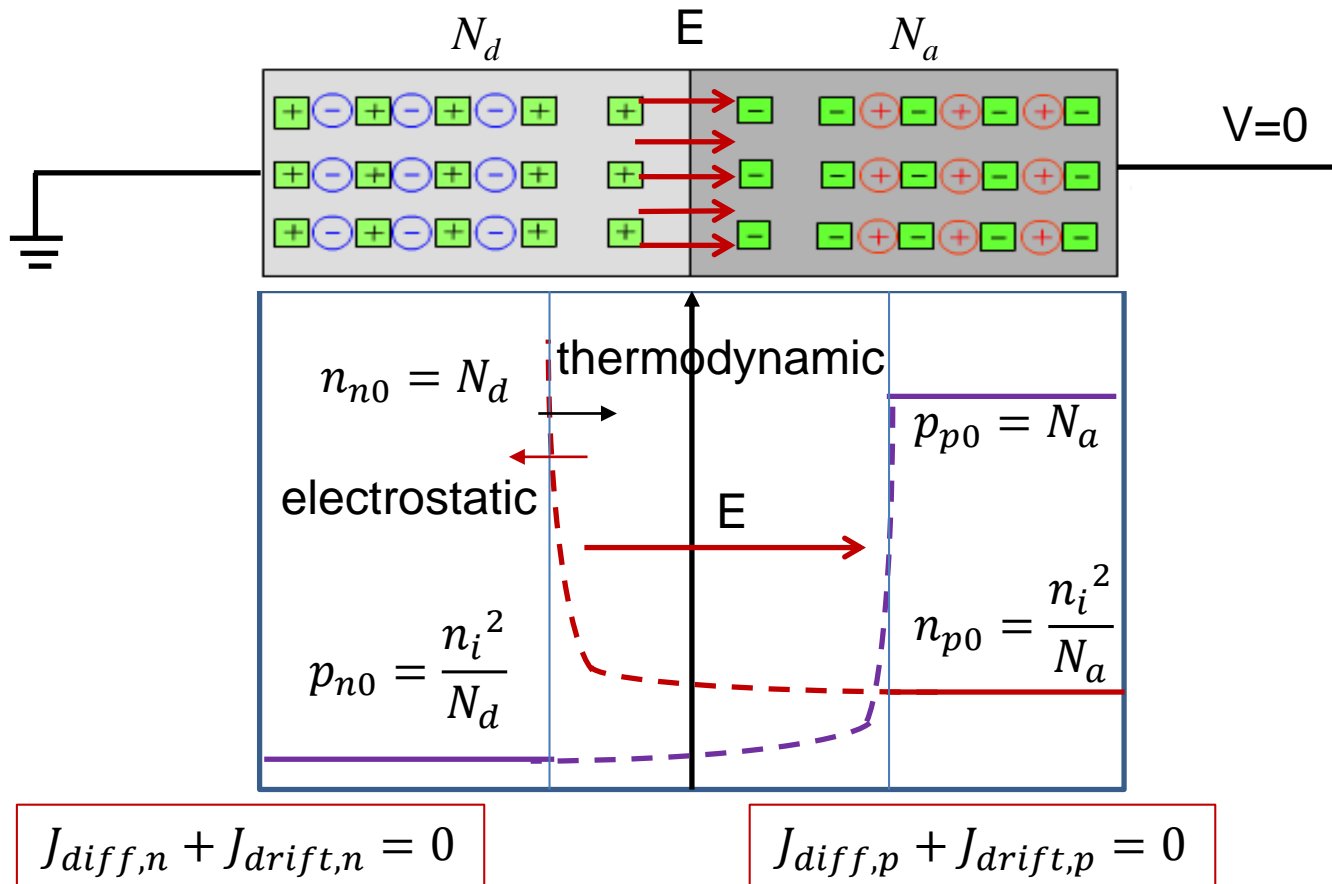
pn Junction Current

- charge carrier transport: zero bias



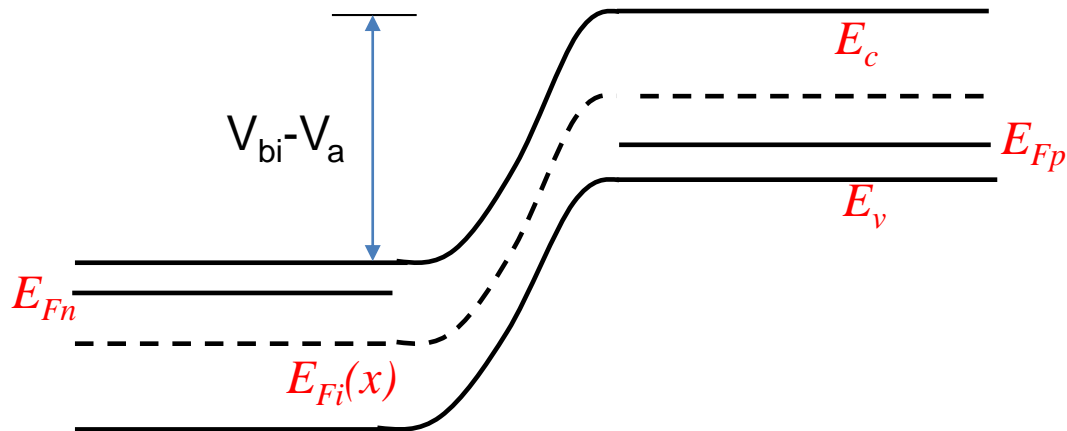
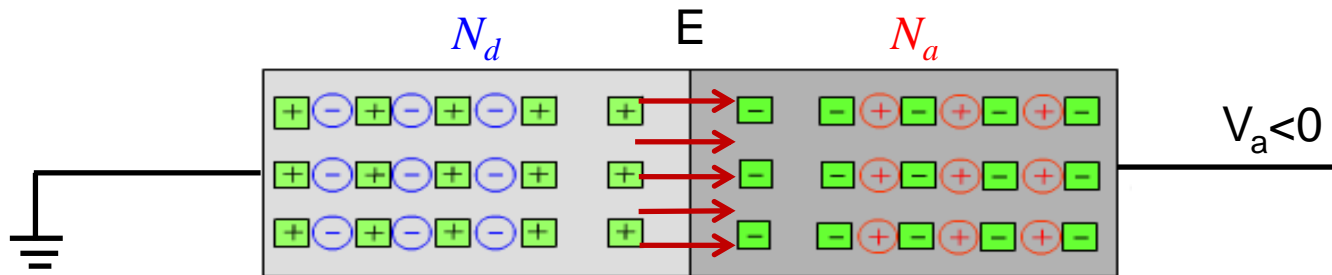
pn Junction Current

- charge carrier transport: zero bias



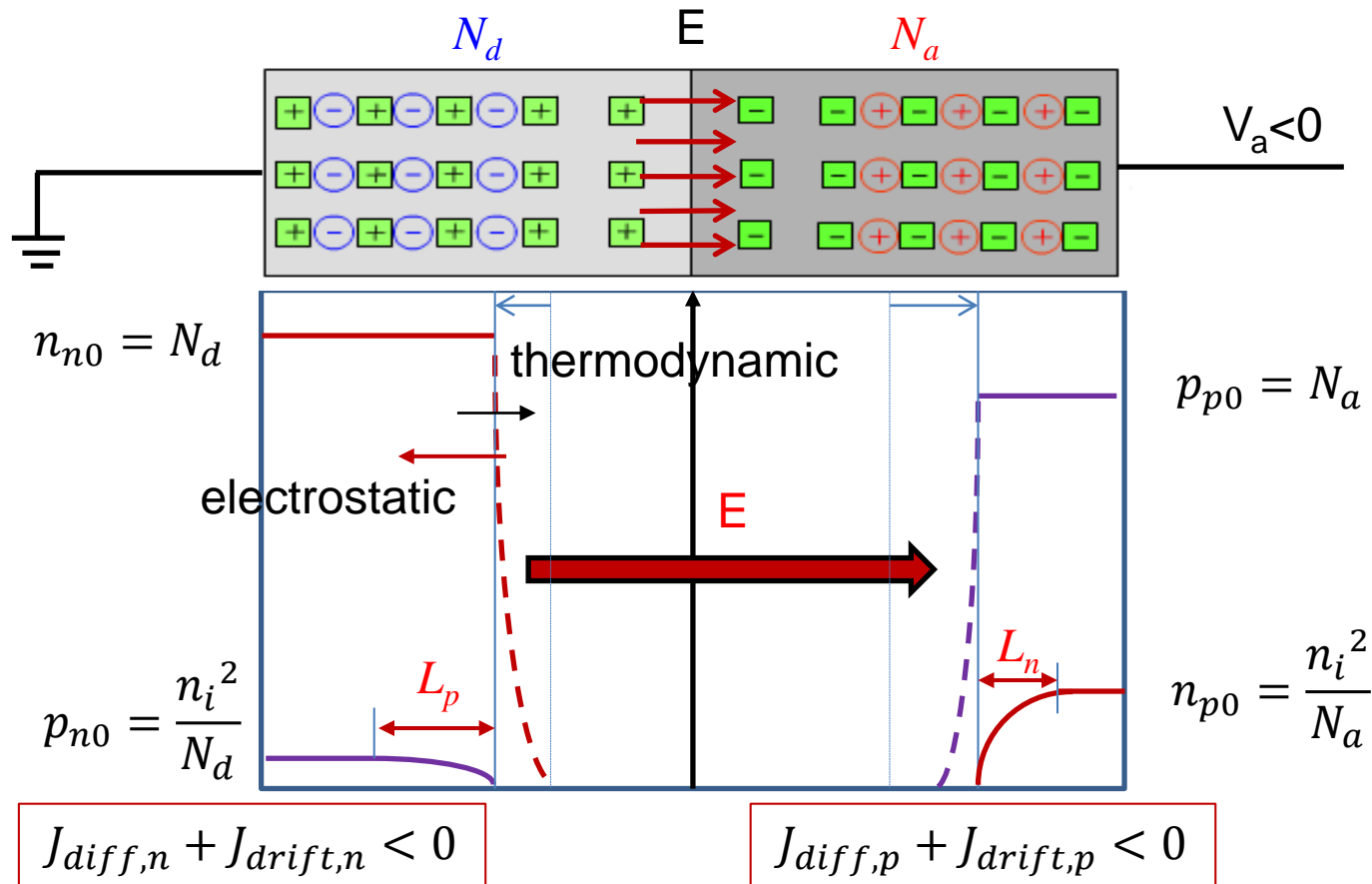
pn Junction Current

- charge carrier transport: reverse bias



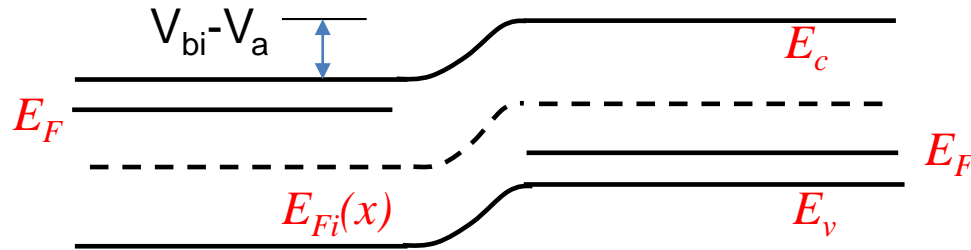
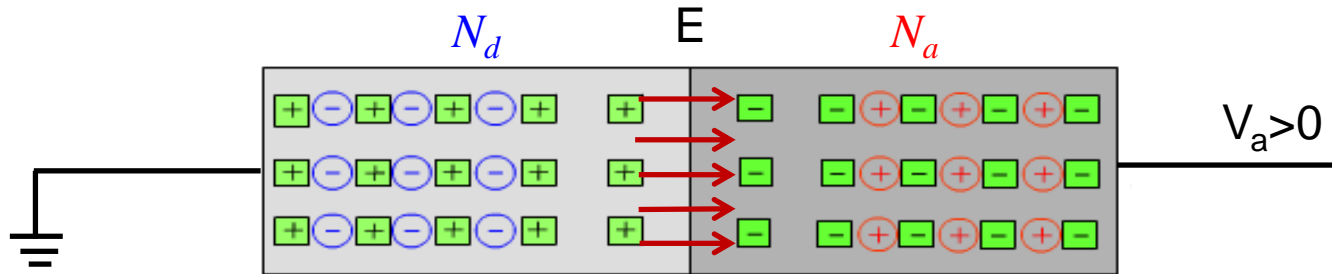
pn Junction Current

- charge carrier transport: reverse bias



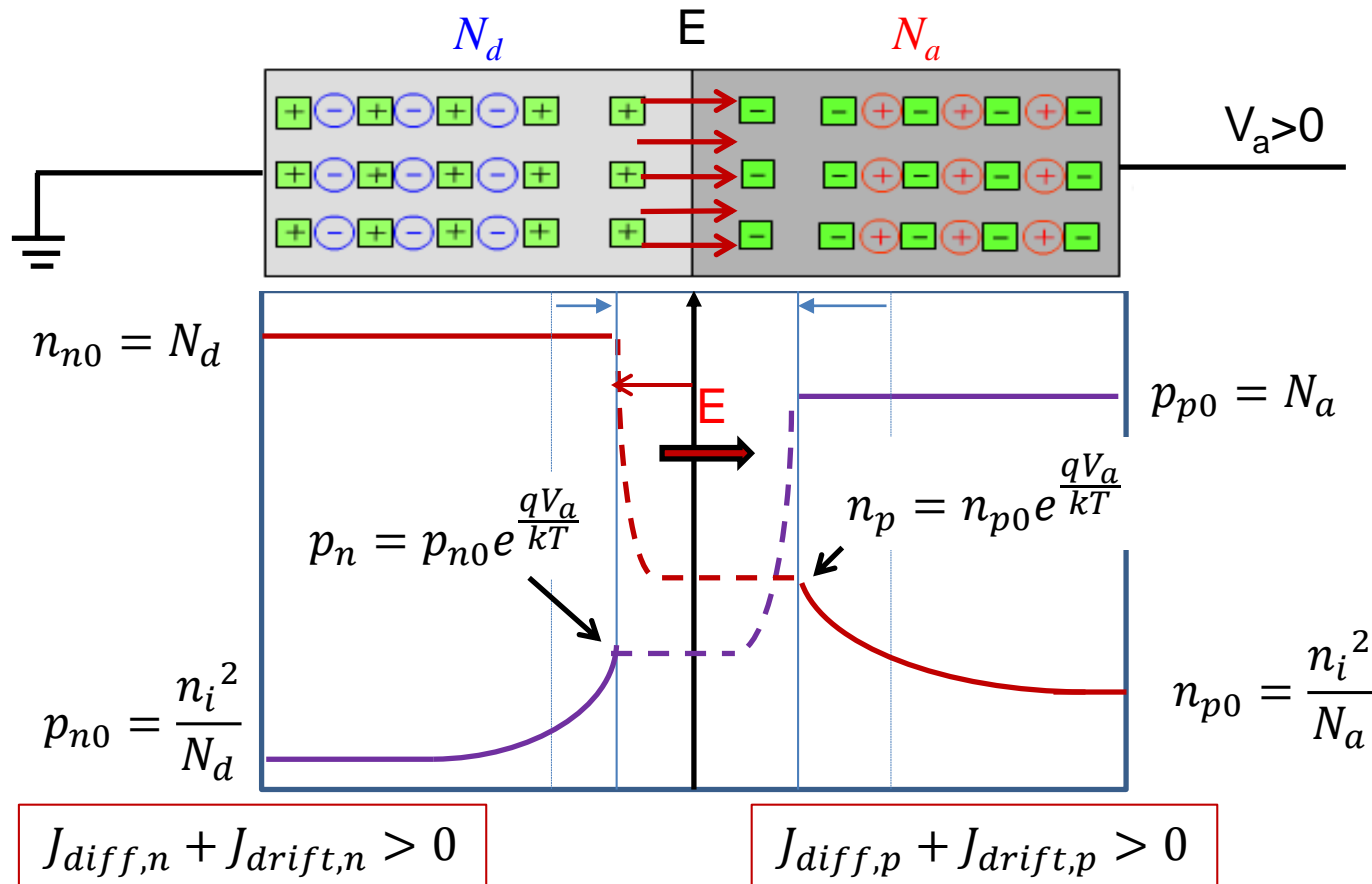
pn Junction Current

- charge carrier transport: forward bias



pn Junction Current

- charge carrier transport: forward bias



pn Junction Current

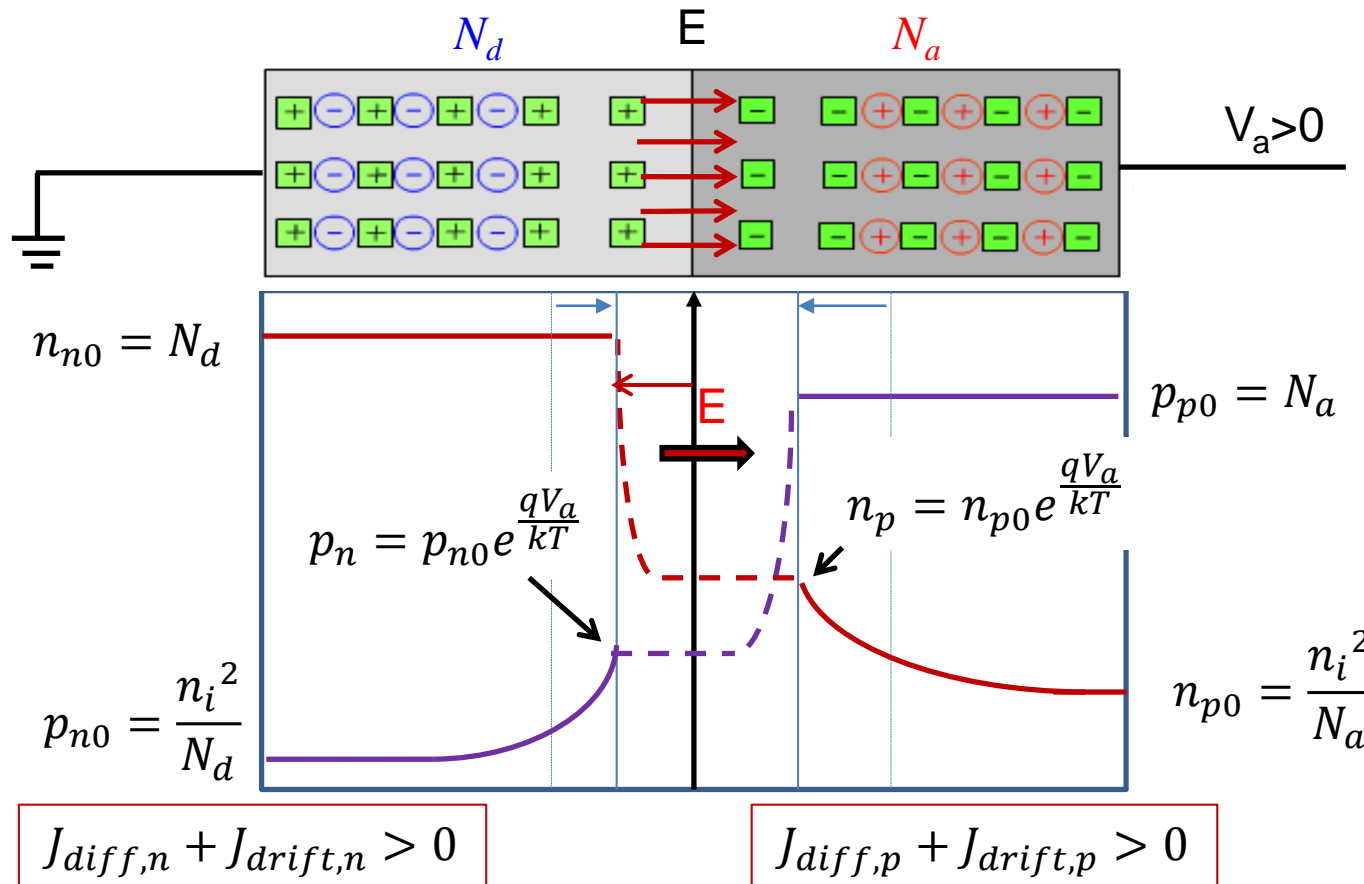
- charge carrier transport: forward bias

Electric field is reduced

Diffusion current

Minority carrier injection:

nonequilibrium excess carriers!



pn Junction Current

Ideal I - V relationship assumption:

- Abrupt depletion layer approximation: The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region
- Boltzmann approximation applies to carrier statistics
- Low injection and complete ionization apply
- Current:
 - The total current is a constant throughout the entire pn structure
 - The individual electron and hole currents are continuous functions through the pn structure
 - The individual electron and hole currents are constant throughout the depletion region

pn Junction Current

Minority carrier concentration:

Previously we have

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \quad V_t = kT/e$$

Then $\frac{n_i^2}{N_a N_d} = \exp \left(\frac{-e V_{bi}}{kT} \right)$ Complete ionization: $n_{n0} \approx N_d$ $n_{p0} \approx \frac{n_i^2}{N_a}$

So $n_{p0} = n_{n0} \exp \left(\frac{-e V_{bi}}{kT} \right)$ relates the minority carrier on the p side to the majority carrier electron concentration on the n side in thermal equilibrium

Forward-biased pn junction: bias V_a

$$n_p = n_{n0} \exp \left(\frac{-e (V_{bi} - V_a)}{kT} \right) = n_{n0} \exp \left(\frac{-e V_{bi}}{kT} \right) \exp \left(\frac{+e V_a}{kT} \right)$$

$$n_p = n_{p0} \exp \left(\frac{e V_a}{kT} \right)$$

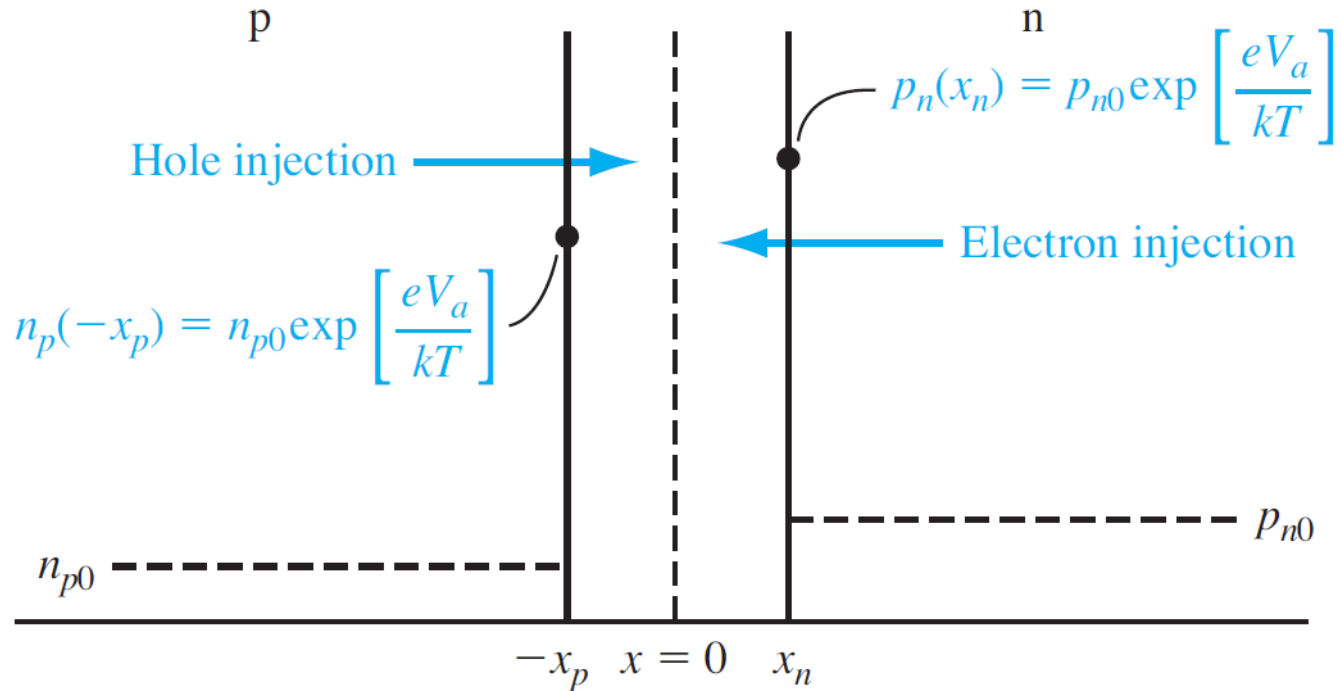
Forward-bias: **no longer in thermal equilibrium**
Exponential change, **excess minority carriers**

Concentration at the **edge**

Similarly, for holes in n-region: $p_n = p_{n0} \exp \left(\frac{e V_a}{kT} \right)$

pn Junction Current

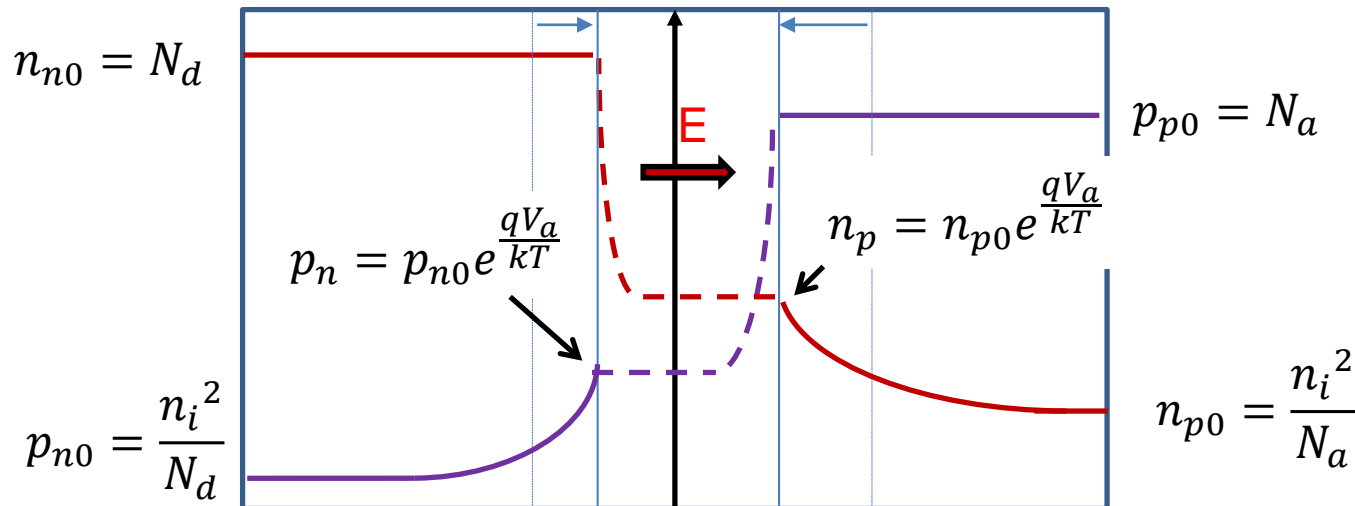
Consider Forward-biased pn junction: bias V_a



pn Junction Current

- charge carrier transport: forward bias
- In the n-region: $E=0$, and $g'=0$

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p \mu_p \frac{\partial E}{\partial x} - \frac{\delta p}{\tau} + g'$$



pn Junction Current

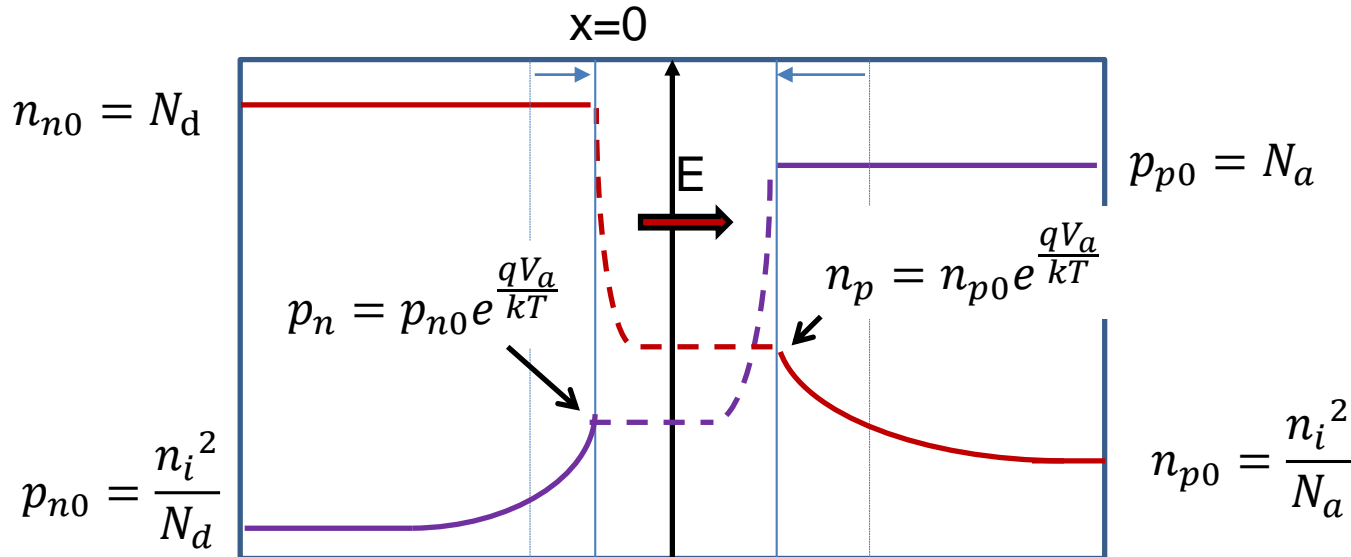
- charge carrier transport: forward bias
n-region

$$D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau} = 0$$

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

$$L_p^2 = D_p \tau_{p0}$$

$$\Rightarrow \delta p_n(x) = p_n(x) - p_{n0} = Ae^{-x/L_p} + Be^{x/L_p}$$



pn Junction Current

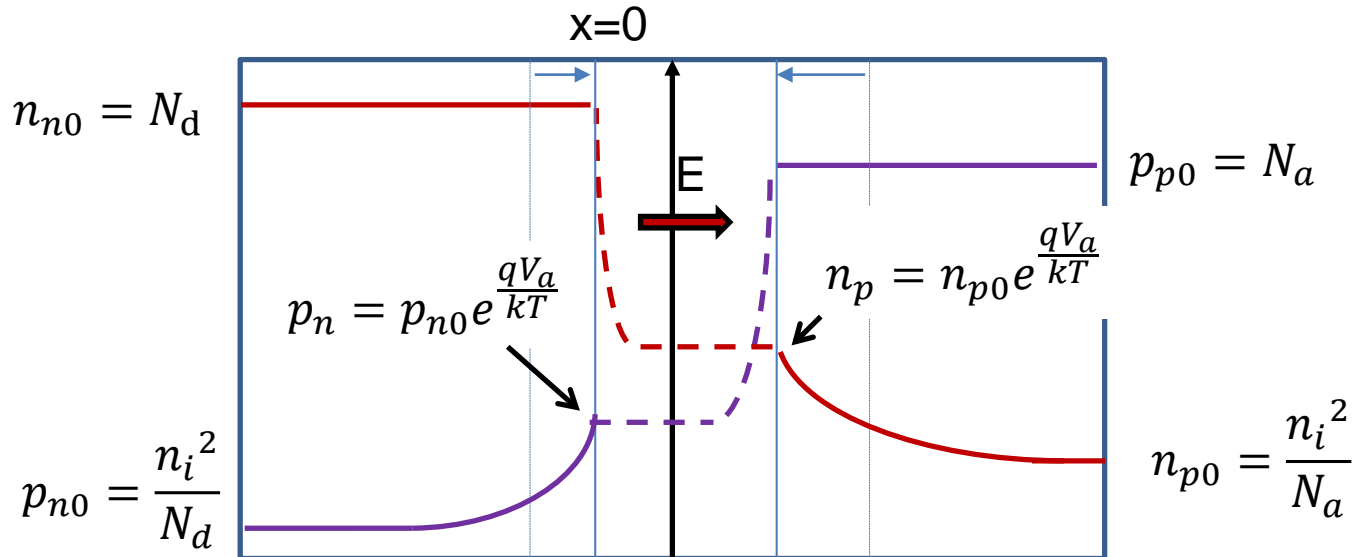
- charge carrier transport: forward bias

Similarly, p-region

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0$$

$$L_n^2 = D_n \tau_{n0}$$

$$\delta n_p(x) = n_p(x) - n_{p0} = C e^{x/L_n} + D e^{-x/L_n}$$



pn Junction Current

n on the left, p on the right

- charge carrier transport: forward bias

Boundary conditions:

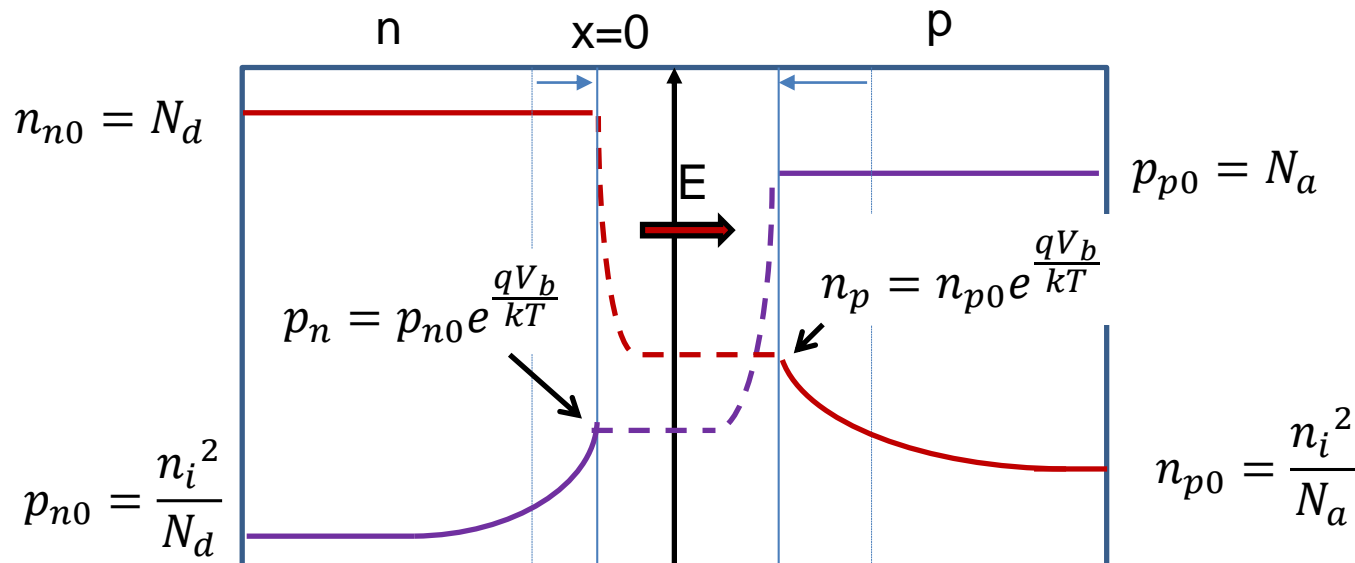
- $p_n(-x_n) = p_{n0} \exp(\frac{eV_a}{kT}), \quad n_p(x_p) = n_{p0} \exp(\frac{eV_a}{kT})$

- $p_n(x \rightarrow -\infty) = p_{n0}, \quad n_p(x \rightarrow \infty) = n_{p0}$

Or: δp must be close to 0 deep into the n region (long pn junction): recombination

δn must be close to 0 deep into the p region (long pn junction): recombination

n-region: $\Rightarrow \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}(e^{\frac{eV_a}{kT}} - 1)e^{(x+x_n)/L_p}$



pn Junction Current

n on the left, p on the right

- charge carrier transport: forward bias

Boundary conditions:

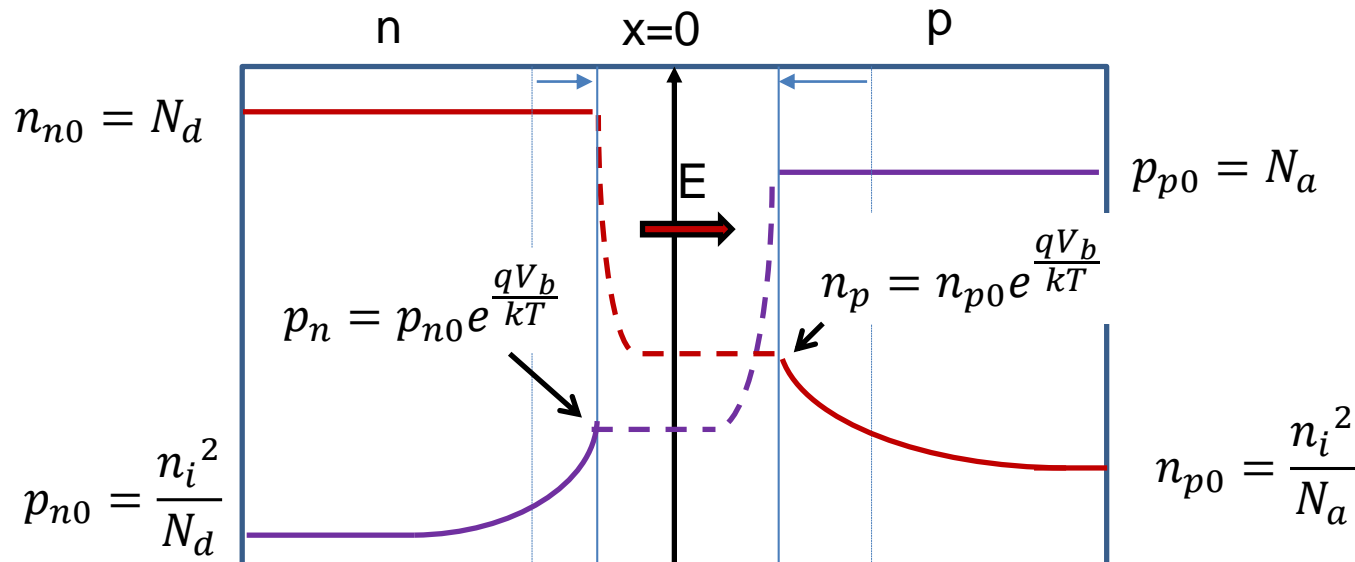
- $p_n(-x_n) = p_{n0} \exp(\frac{eV_a}{kT}), \quad n_p(x_p) = n_{p0} \exp(\frac{eV_a}{kT})$

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Or: δp must be close to 0 deep into the n region (long pn junction): recombination

δn must be close to 0 deep into the p region (long pn junction): recombination

p-region: $\Rightarrow \delta n_p(x) = n_p(x) - n_{p0} = n_{p0} (e^{\frac{eV_a}{kT}} - 1) e^{(x_p - x)/L_n}$



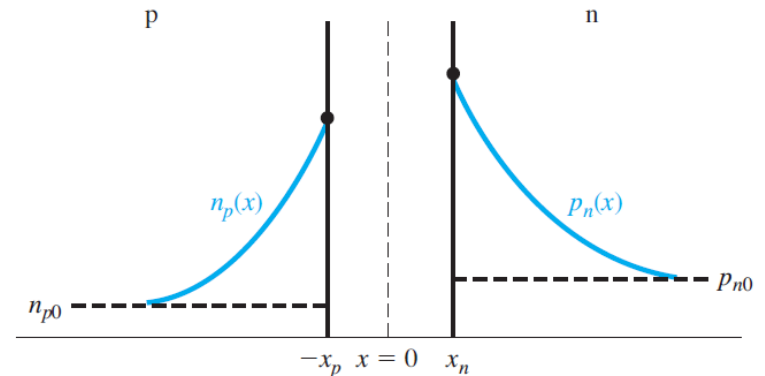
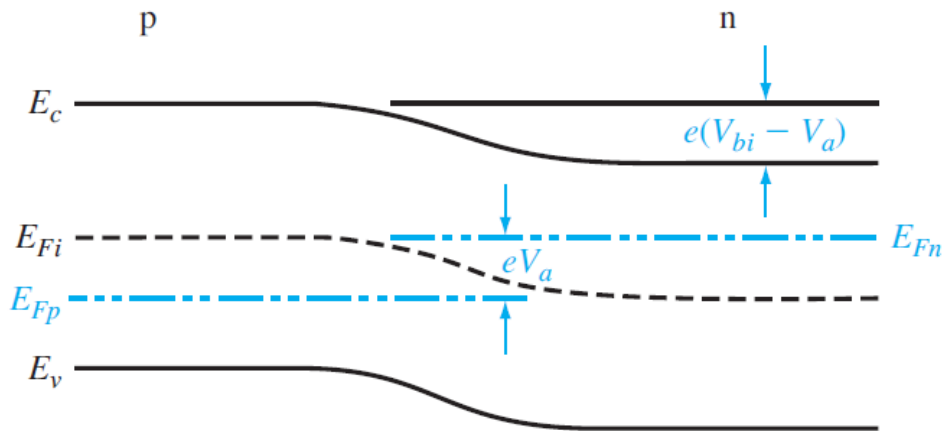
pn Junction Current

n on the right, p on the left

- charge carrier transport: forward bias

$$\text{n-region } (x \geq x_n): \quad \delta p_n(x) = p_n(x) - p_{n0} = p_{n0} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right)$$

$$\text{p-region } (x \leq -x_p): \quad \delta n_p(x) = n_p(x) - n_{p0} = n_{p0} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right)$$



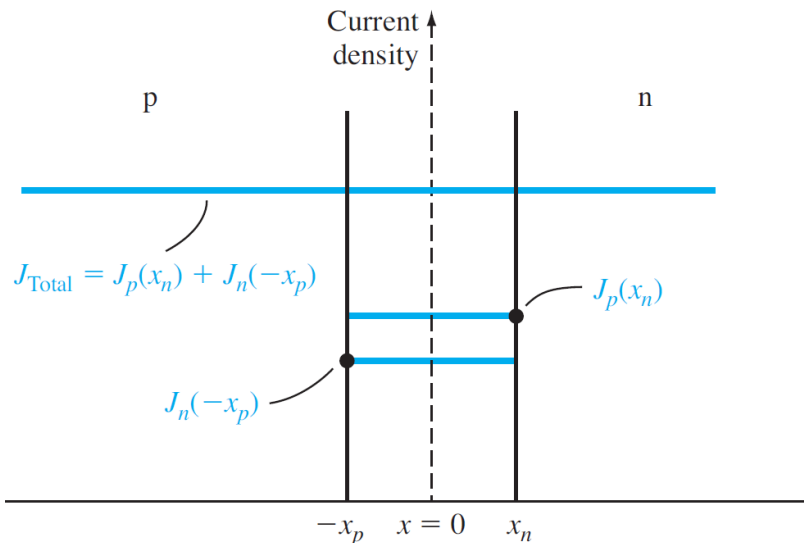
pn Junction Current

- Assumption: No generation and recombination in depletion region
- Ideal pn junction current
- charge carrier transport: forward bias

Hole diffusion: $J_p(x_n) = -eD_p \frac{dp_n(x)}{dx} \Big|_{x=x_n}$

Assume uniformly doped: $J_p(x_n) = -eD_p \frac{d(\delta p_n(x))}{dx} \Big|_{x=x_n}$

Remember $\delta p_n(x) = p_n(x) - p_{n0} = p_{n0} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right)$



Take the derivative of $\delta p_n(x)$

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

Current direction: p to n

pn Junction Current

- Assumption: No generation and recombination in depletion region
- Ideal pn junction current
- charge carrier transport: forward bias

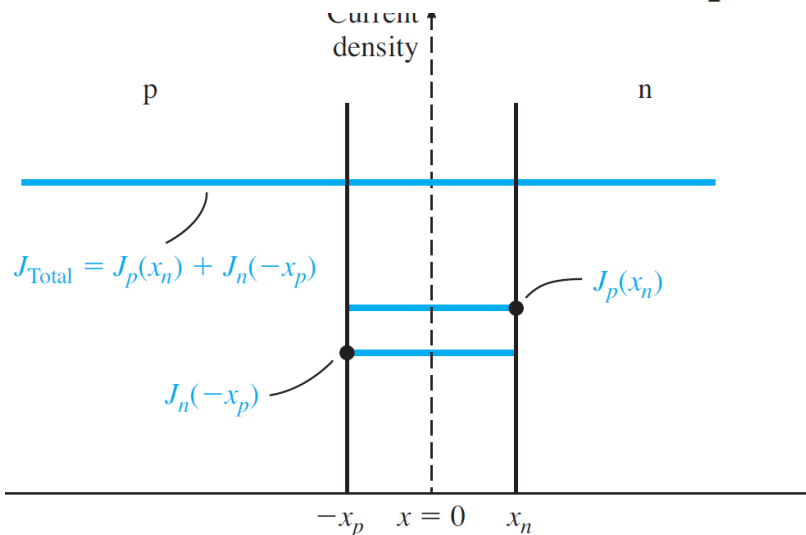
Similarly, electron diffusion: $J_n(-x_p) = eD_n \left. \frac{d(\delta n_p(x))}{dx} \right|_{x=-x_p}$

Current direction: also p to n, add together!

$$\frac{J_n}{J_p} = \frac{D_n n_{p0}/L_n}{D_p p_{n0}/L_p}$$

Total diffusion: ideal I-V relationship of a pn junction

$$J = J_p(x_n) + J_n(-x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right] \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$



Define $J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right]$

Ideal diode equation:

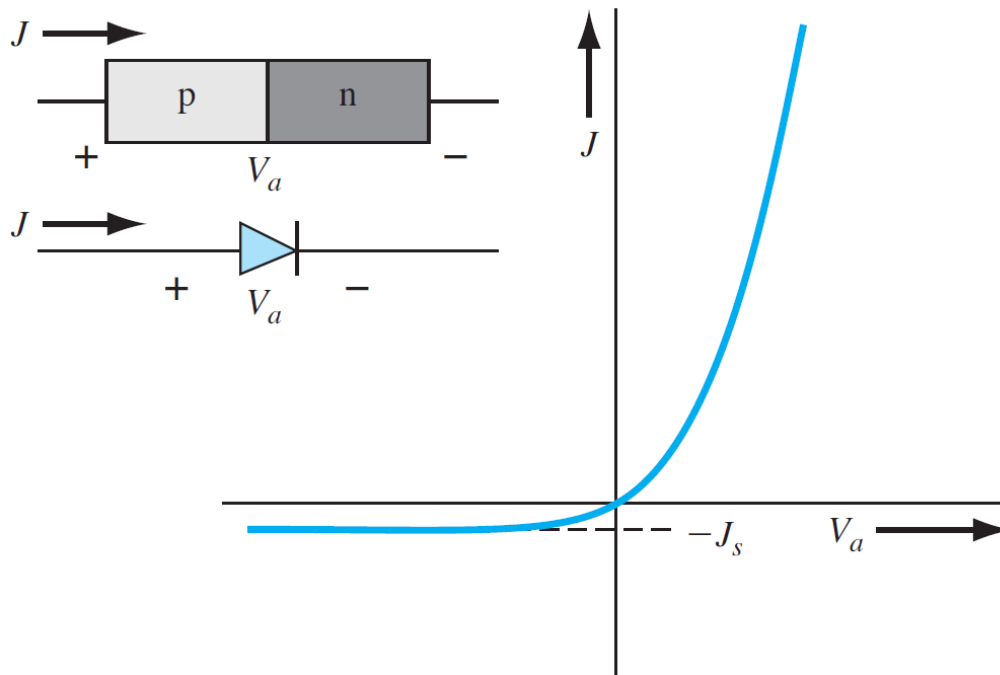
$$J = J_s \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

V_a can be negative

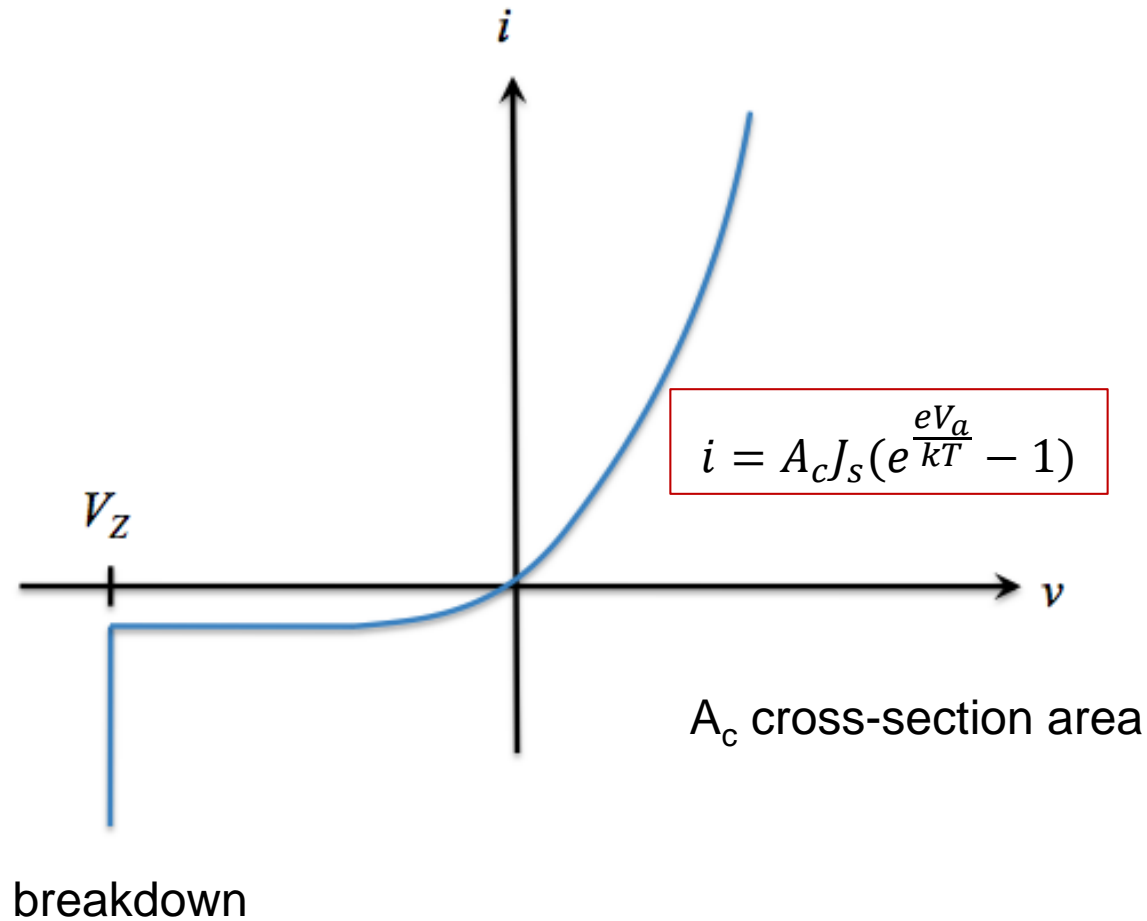
pn Junction Current

- Assumption: No generation and recombination in depletion region
- Ideal pn junction current

J_s : reverse-saturation current density



pn Junction Current



pn Junction Current

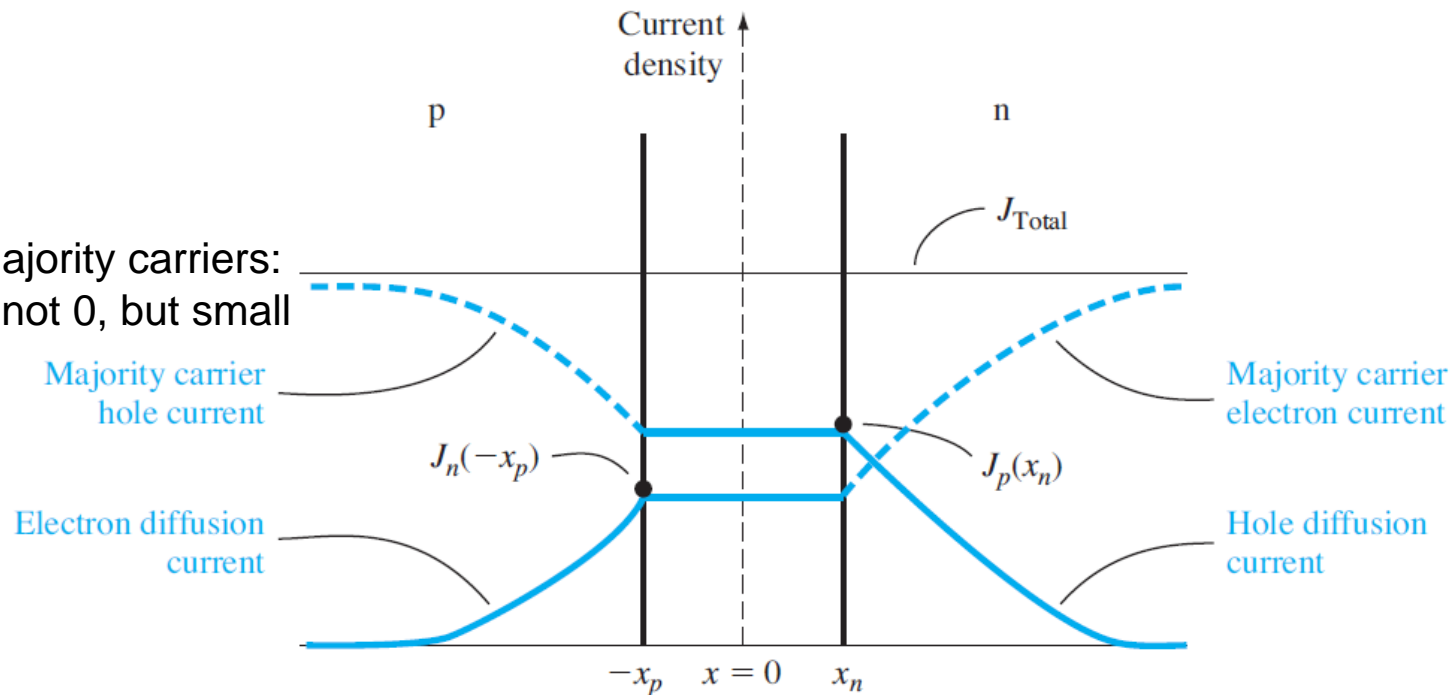
Current in p and n region:
Forward bias

$$\vec{J}_p = e\vec{F}_p = -eD_p \frac{d\delta p}{dx} \vec{x} \quad \vec{J}_n = -e\vec{F}_n = eD_n \frac{d\delta n}{dx} \vec{x}$$

$$J_p(x) = \frac{eD_p p_{n0}}{L_p} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right) \quad (x \geq x_n)$$

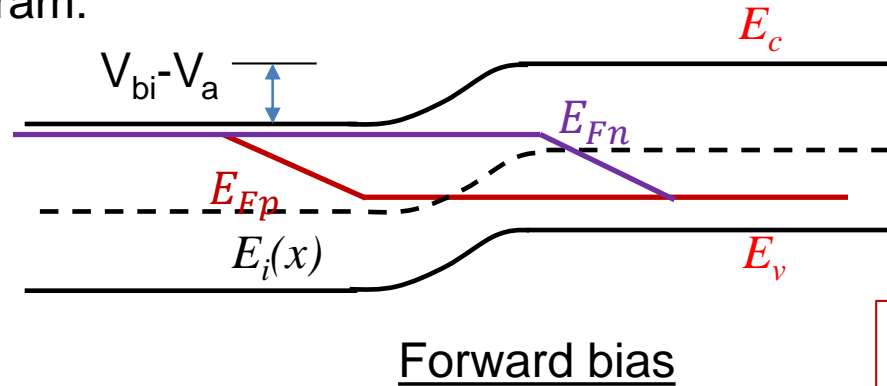
$$J_n(x) = \frac{eD_n n_{p0}}{L_n} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right) \quad (x \leq -x_p)$$

Drift of majority carriers:
E field is not 0, but small

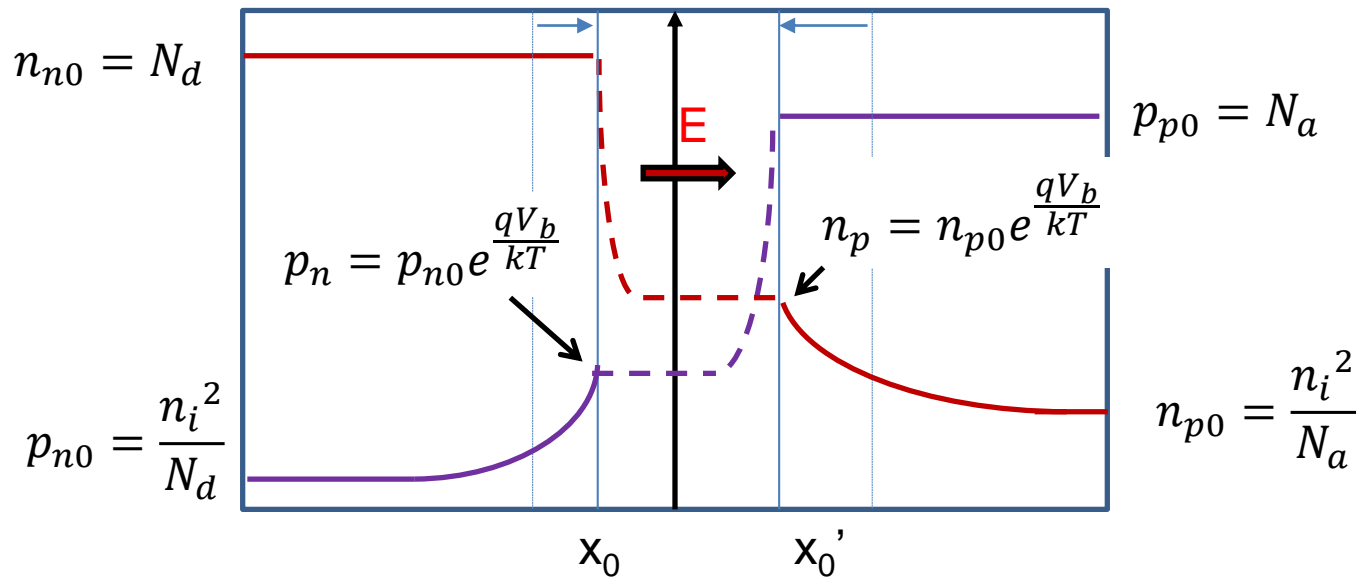


A few points about pn junction

- Energy diagram:



To understand this, see the next page.



A few points about pn junction

- Current and Fermi level

$$J_n = J_{diff,n} + J_{drift,n}$$

$$J_n = eD_n \frac{dn}{dx} + ne\mu_n|E|$$

$$D_n = kT\mu_n/e$$

$$= kT\mu_n \frac{dn}{dx} + ne\mu_n|E|$$

$$= ne\mu_n \left(\frac{kT}{ne} \frac{dn}{dx} + |E| \right)$$

$$= ne\mu_n \left[\frac{kT}{e} \frac{d \ln(n)}{dx} + |E| \right]$$

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) \Rightarrow \ln(n) = \ln(n_i) + \frac{E_F - E_{Fi}}{kT}$$

$$= ne\mu_n \left[\frac{dE_F}{edx} - \frac{dE_i}{edx} + |E| \right]$$

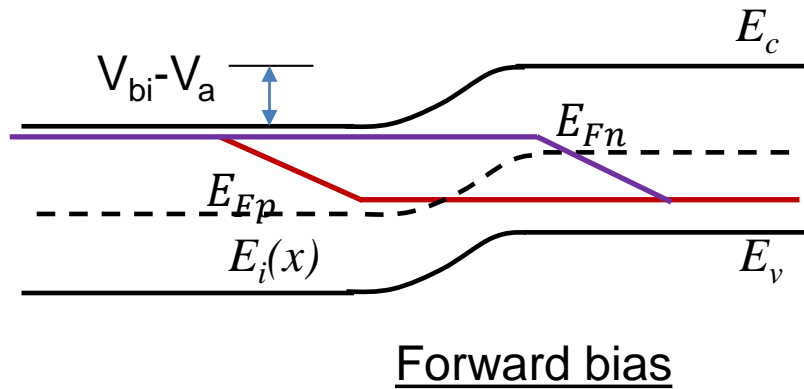
$$\Rightarrow \frac{d}{dx} \ln(n) = \frac{d}{dx} \left(\frac{E_F - E_{Fi}}{kT} \right) = \frac{1}{kT} \left(\frac{dE_F}{dx} - \frac{dE_i}{dx} \right)$$

$$J_n = n\mu_n \frac{dE_F}{dx}$$

$$\frac{dE_i}{dx} = -e \frac{dV(x)}{dx} = e|E|$$

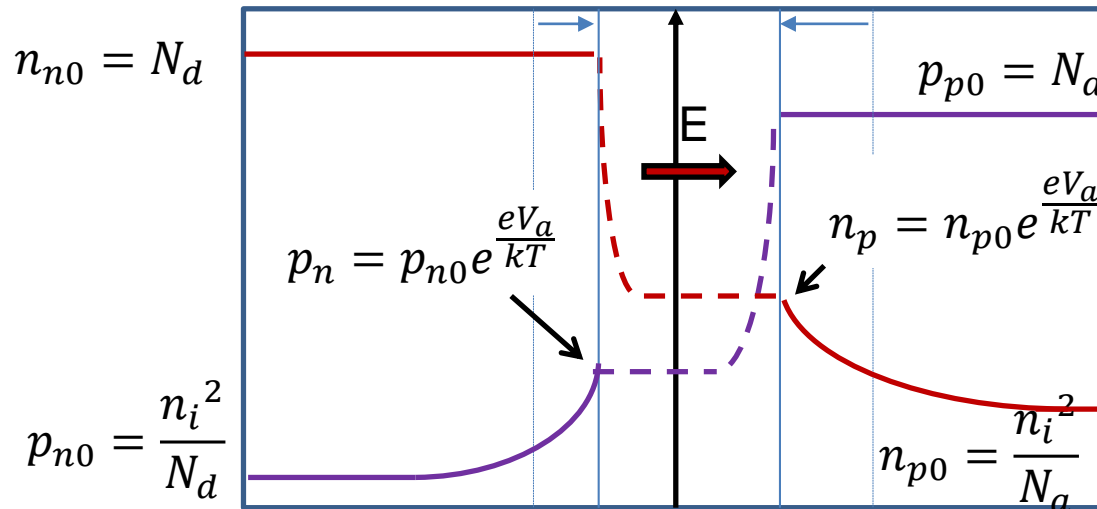
A few points about pn junction

- Energy diagram:



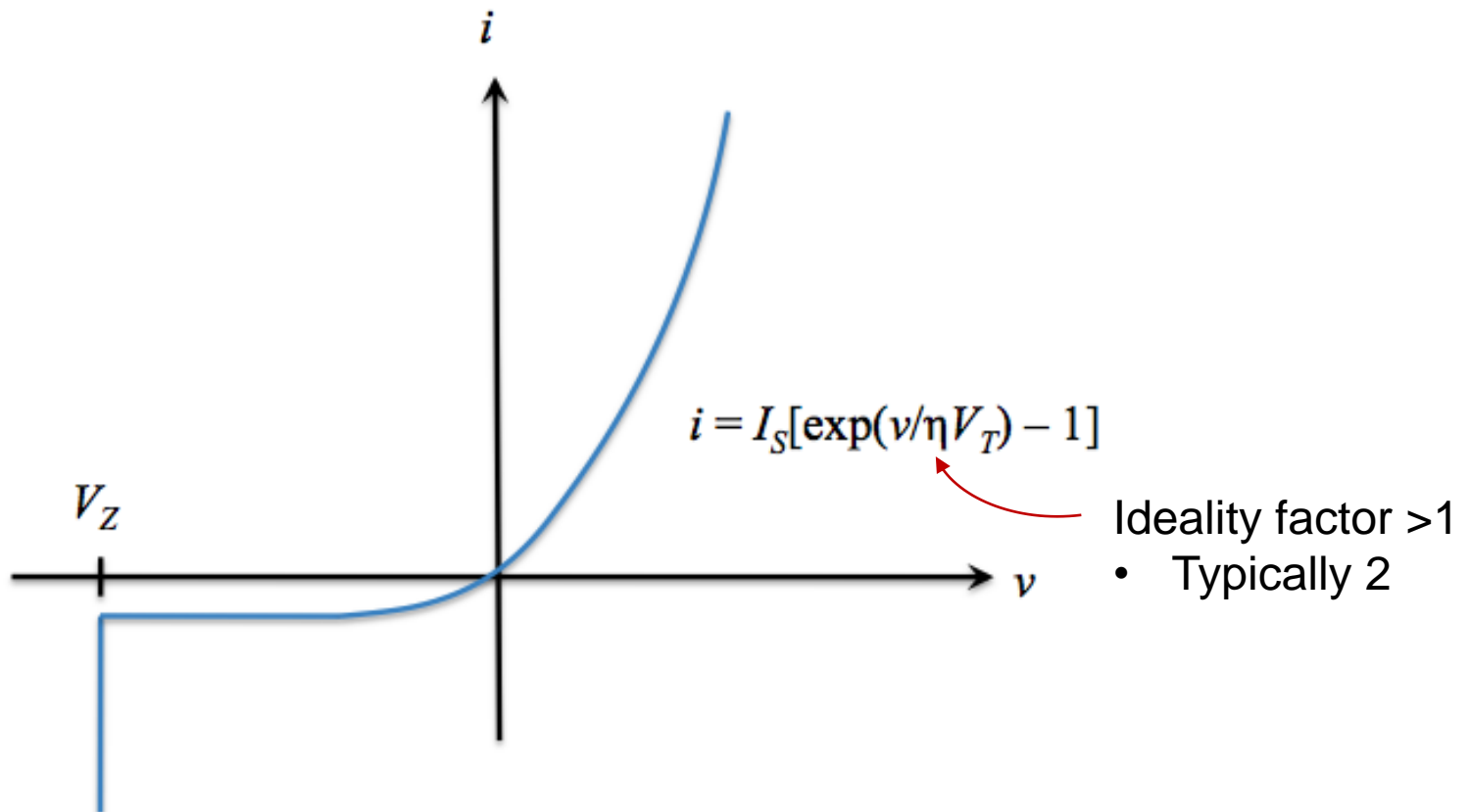
$$n_p = n_{p0} + \delta n \approx \delta n$$

$$J_n = n_p \mu_n \frac{dE_F^n}{dx}$$



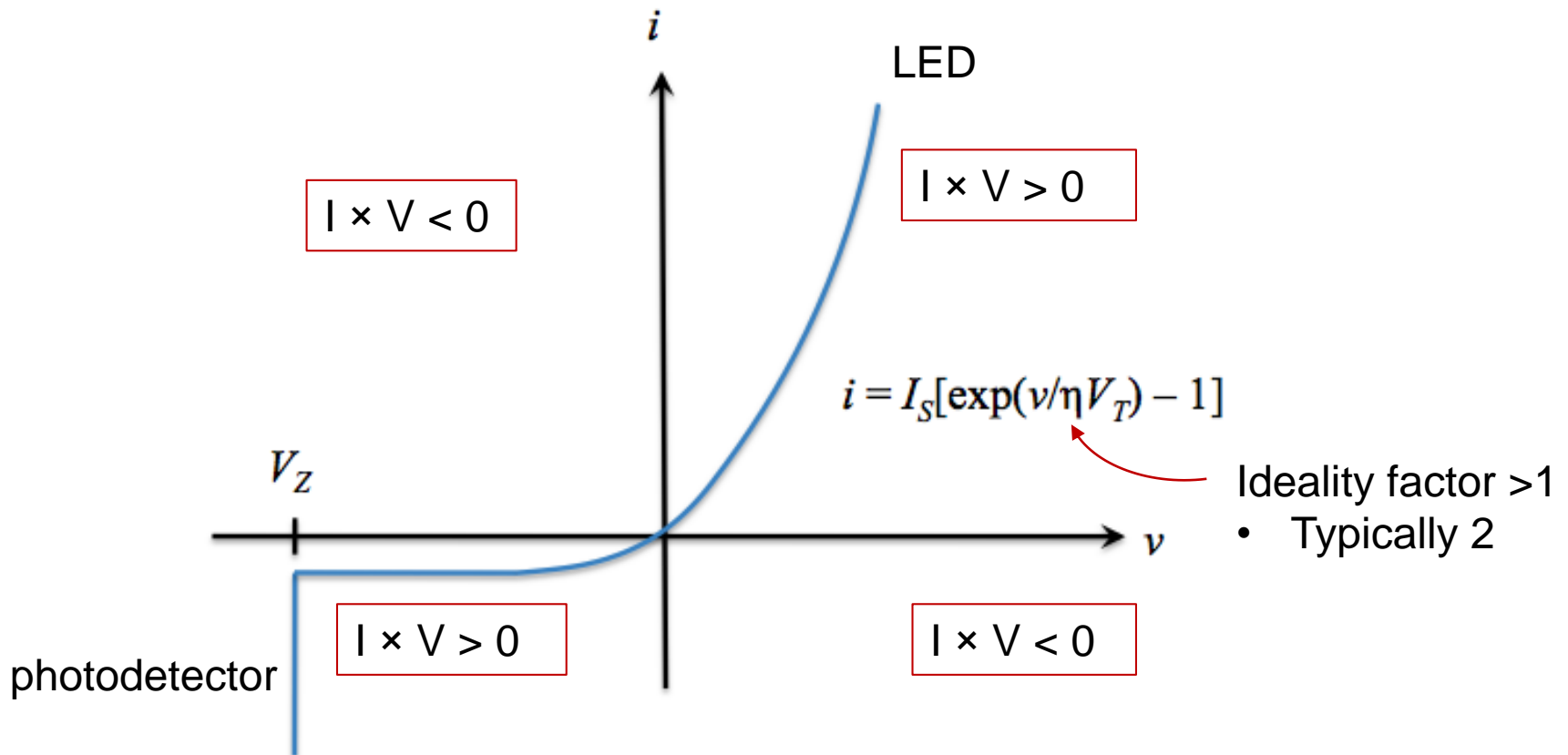
A few points about pn junction

- Ideality factor



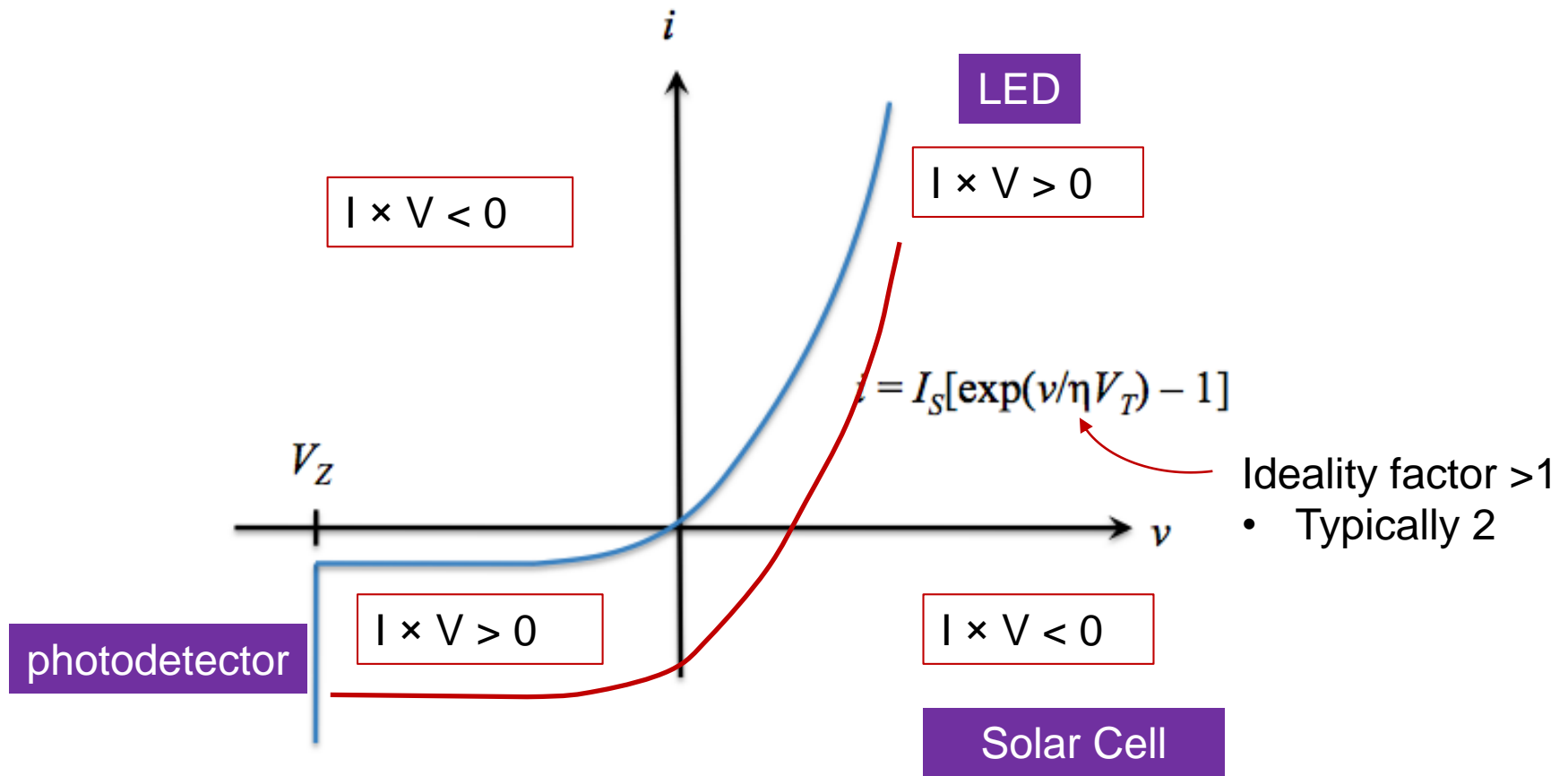
A few points about pn junction

- Energy consumption:



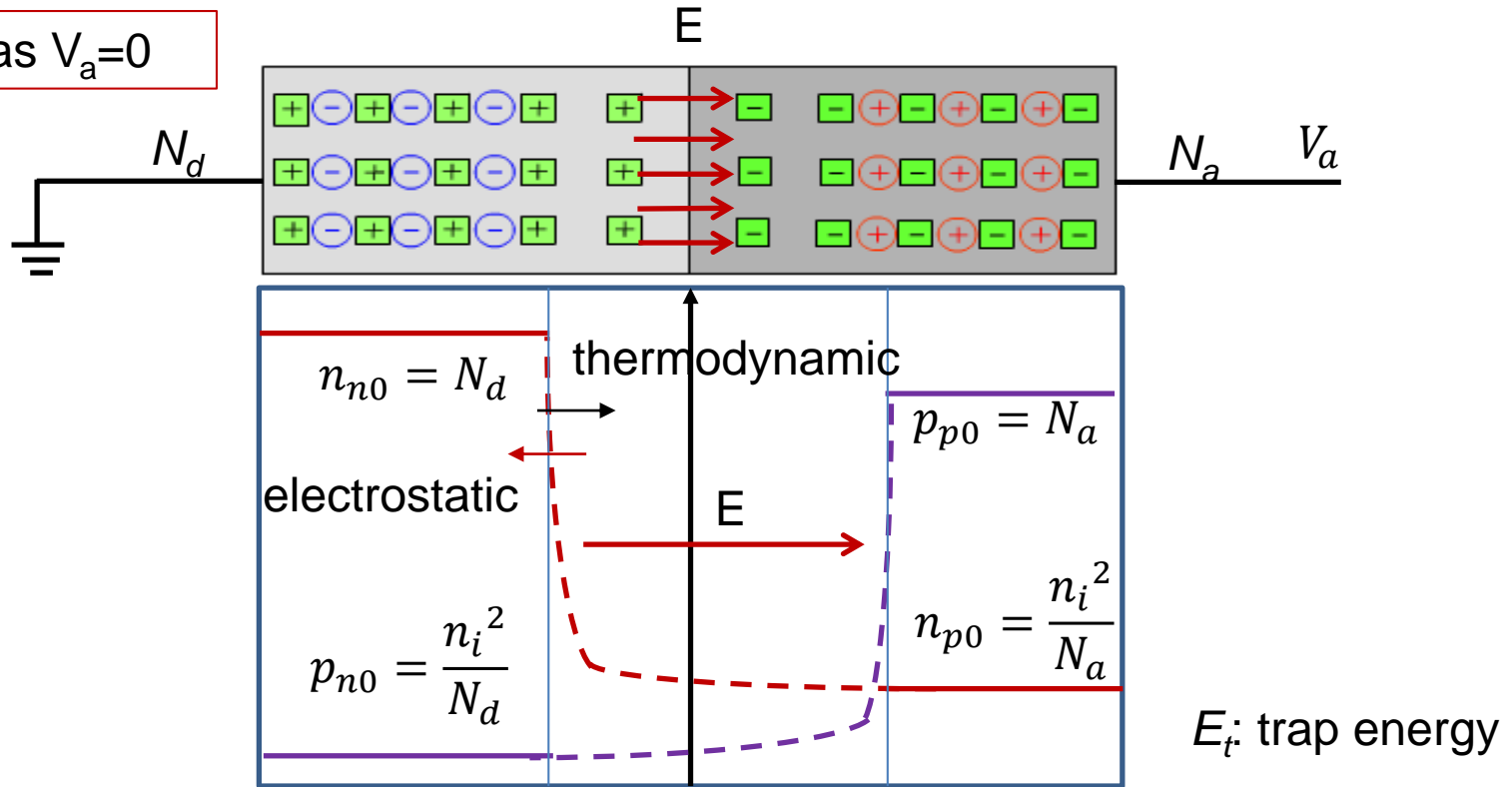
A few points about pn junction

- Energy consumption:



G-R in depletion region at zero bias

Zero Bias $V_a=0$

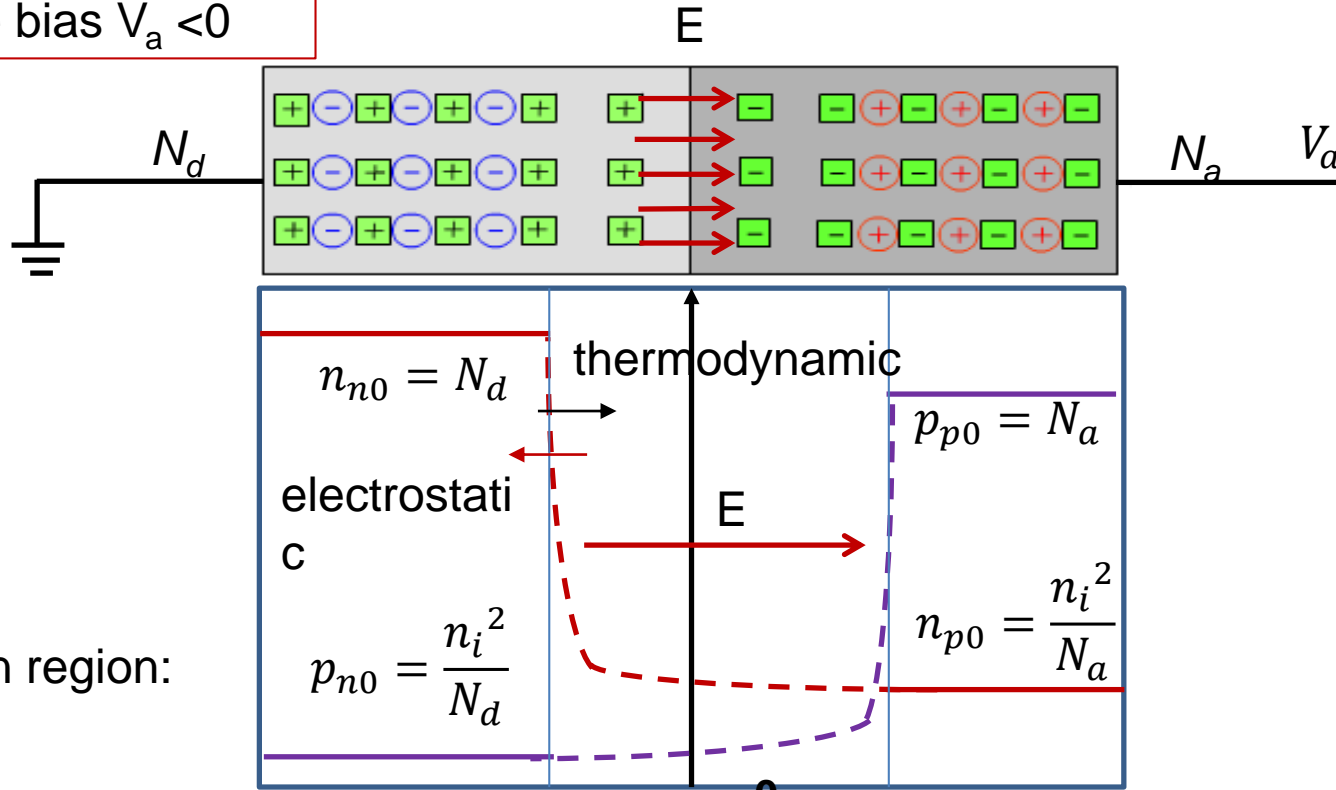


$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right) \right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right) \right]}$$

$$\text{In depletion region: } np = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$$

G-R in depletion region at Reverse Bias

Reverse bias $V_a < 0$



Depletion region:
 $n=0, p=0$

$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right) \right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right) \right]}$$

$R < 0$: generation! Generated via the trap level

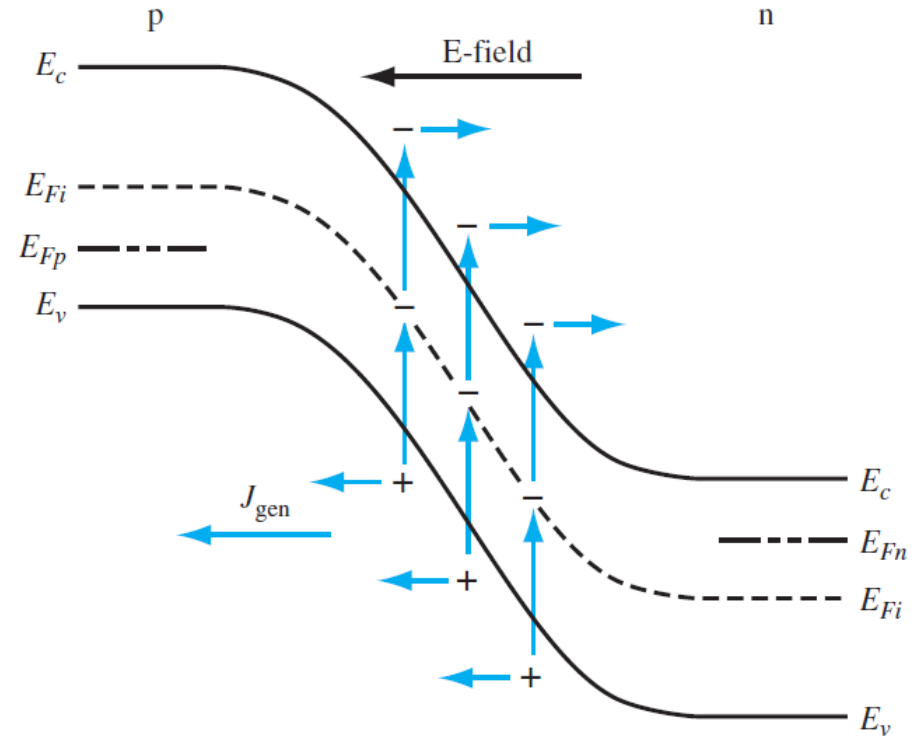
G-R in depletion region at Reverse Bias

Reverse bias $V_a < 0$

To simplify the calculation, we assume

$$E_t = E_{Fi}, \tau_n = \tau_p = \tau$$

$$R = \frac{-n_i}{2\tau} = -G_0$$



$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right) \right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fn} - E_t}{kT}\right) \right]}$$

G-R in depletion region at Reverse Bias

Reverse bias $V_a < 0$

To simplify the calculation, we assume

$$E_t = E_{Fi}, \tau_n = \tau_p = \tau$$

$$R = \frac{-n_i}{2\tau} = -G_0$$

Reverse-biased generation current,
in addition to the ideal reverse-biased
saturation current

Current density from G-R in the depletion region:

$$J_{gen} = \int_0^W q G_0 dx = \frac{q W n_i}{2\tau}$$

$$W = a + b = \sqrt{\frac{2\epsilon(V_{bi} + V_a)}{e} \frac{N_d + N_a}{N_a N_d}}$$

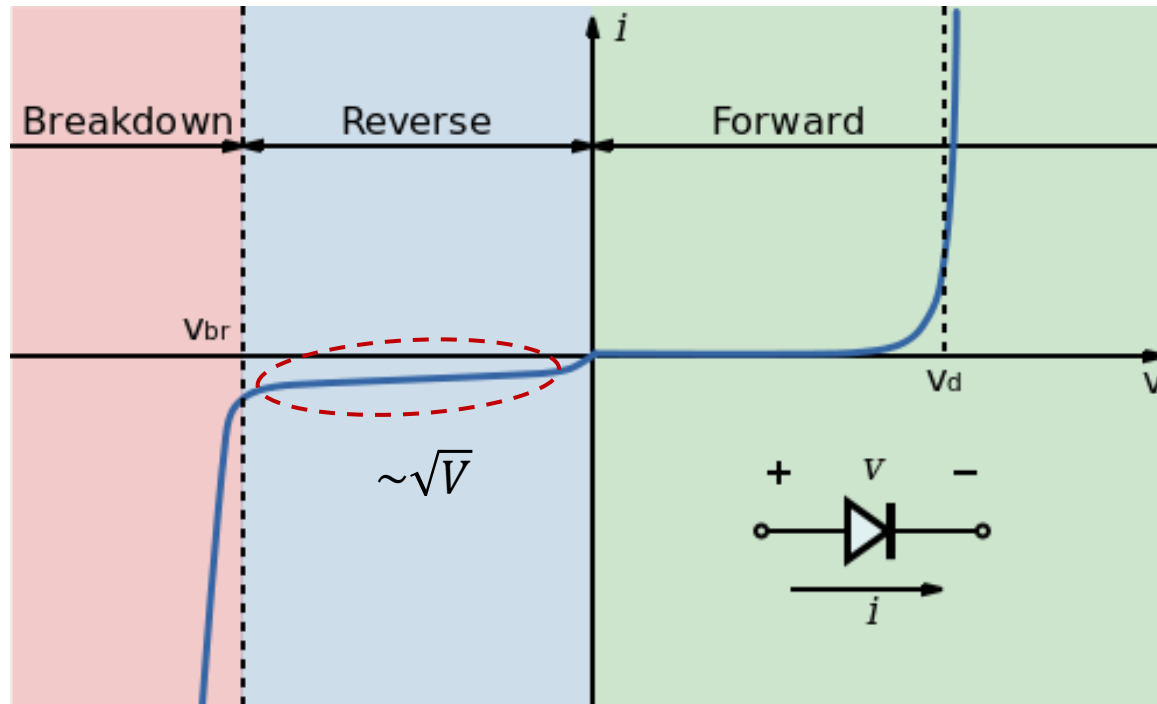
Total reverse-biased current density:

$$J_R = J_s + J_{gen}$$

Dependent on the applied voltage

G-R in depletion region at Reverse Bias

Reverse bias $V_a < 0$



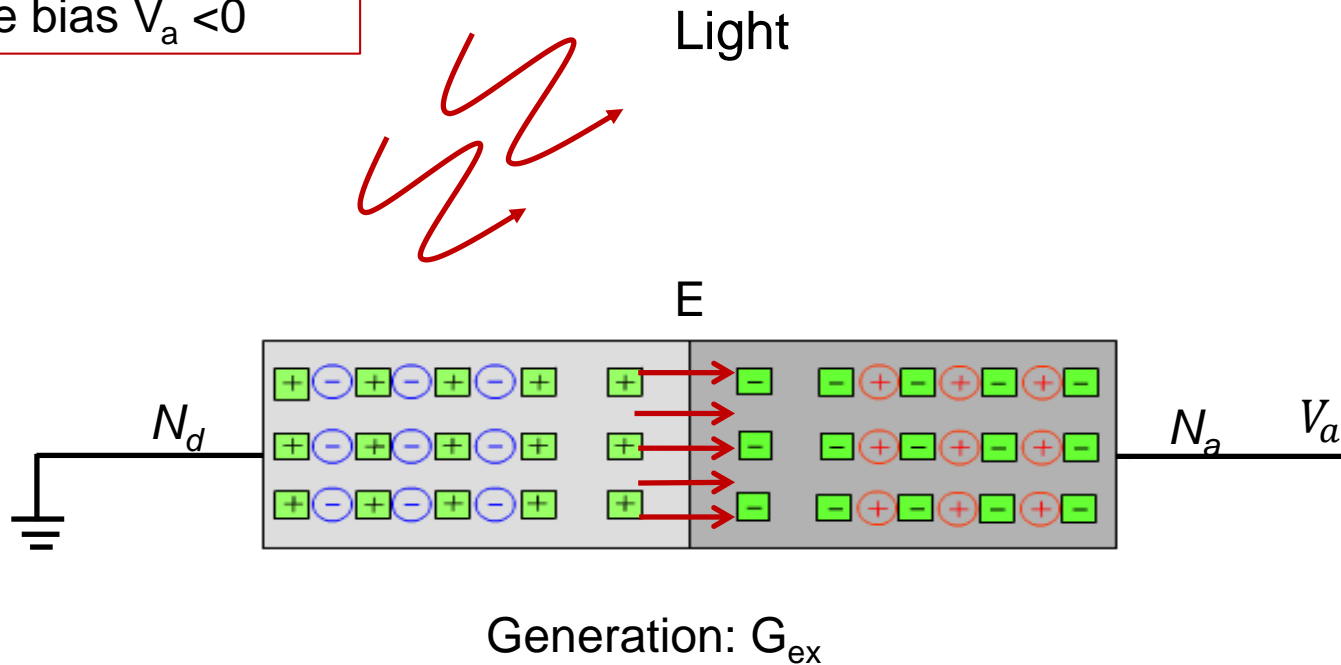
Current density from G-R in the depletion region:

$$J_r = \int_0^W eGdx = \frac{eWn_i}{2\tau}$$

$$W = a + b = \sqrt{\frac{2\epsilon(V_{bi} + V_a)}{e} \frac{N_d + N_a}{N_a N_d}}$$

Photocurrent at Reverse Bias

Reverse bias $V_a < 0$

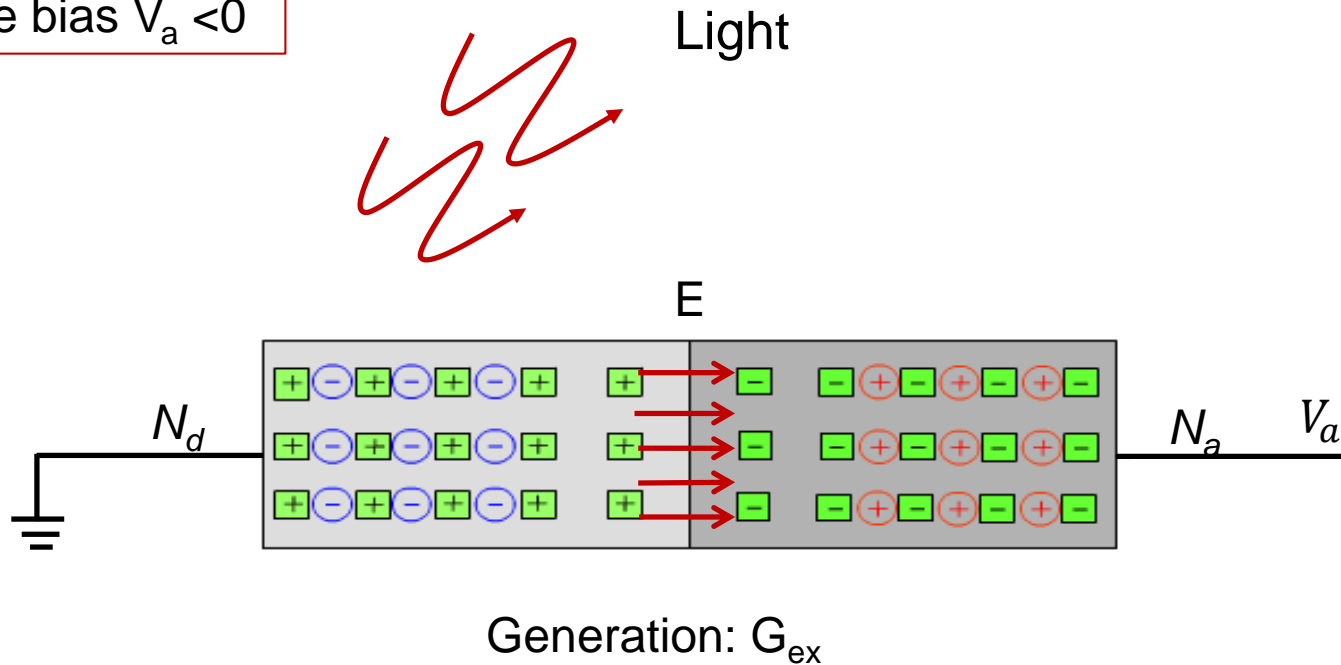


$$\text{net recombination: } R - G = R - (G_0 + G_{ex}) = -G_0 - G_{ex}$$

$n=0$ and $p=0 \rightarrow$ Recombination in the depletion is zero

Photocurrent at Reverse Bias

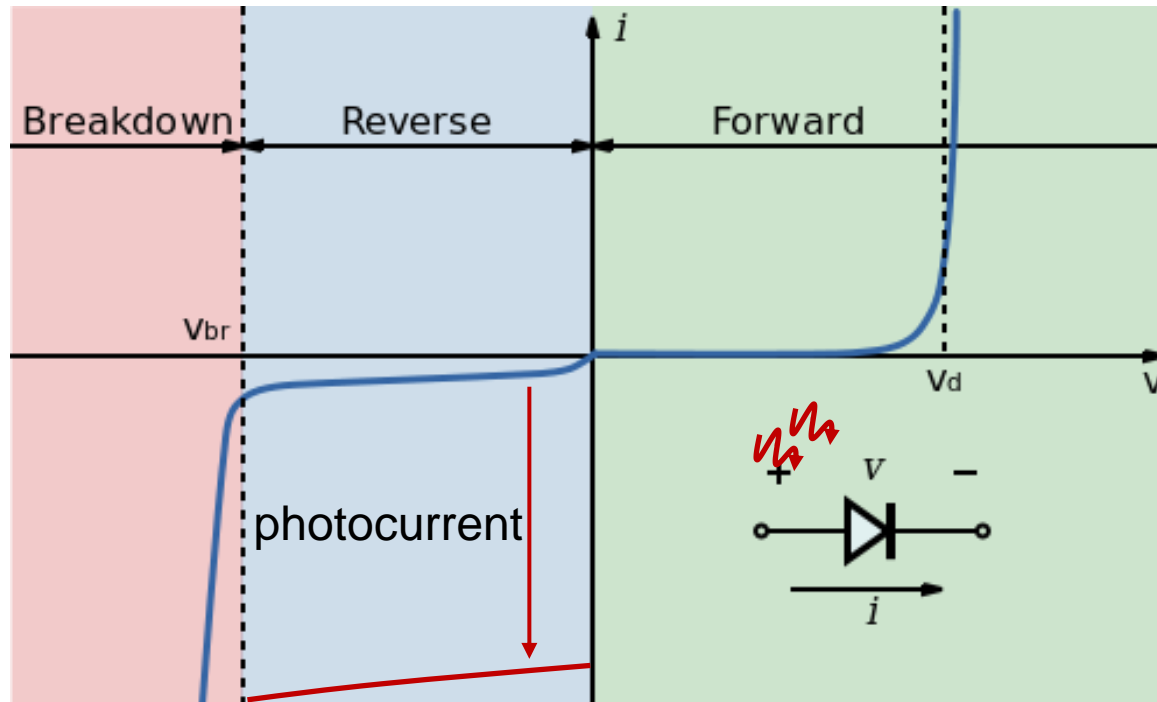
Reverse bias $V_a < 0$



$$J_{r,ph} = \int_0^W e(G_0 + G_{ex})dx = e(G_0 + G_{ex}) W$$

Photocurrent at Reverse Bias

Reverse bias $V_a < 0$



$$J_{r,ph} = \int_0^W e(G_0 + G_{ex})dx = e(G_0 + G_{ex}) W$$

G-R in depletion region at Forward Bias

Forward bias $V_a > 0$

$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right) \right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right) \right]}$$

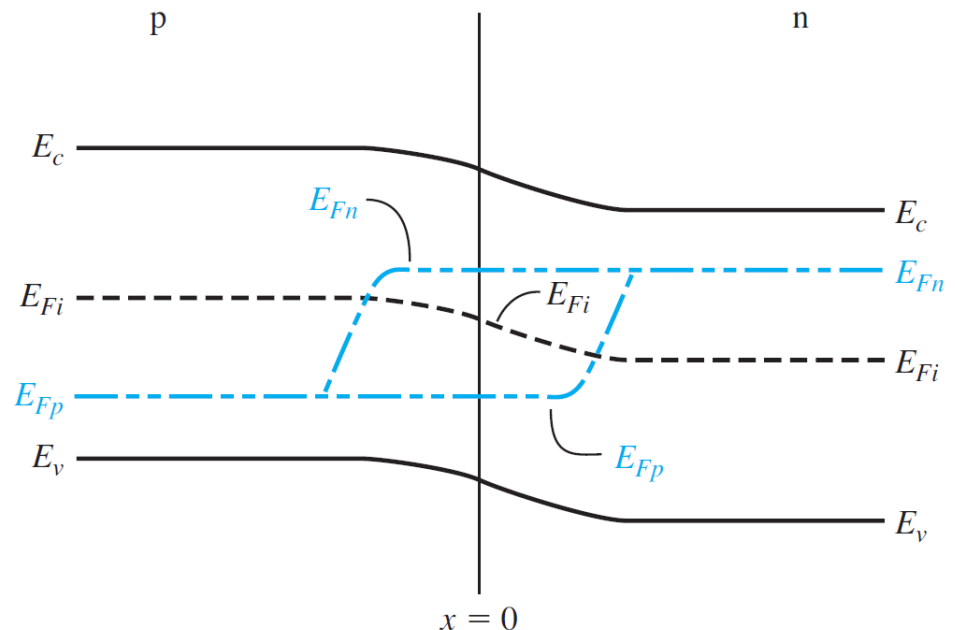
In depletion region: $np = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$

To simplify the calculation, we assume

$$E_t = E_i, \tau_n = \tau_p = \tau$$

$$R = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$

Max: $n=p$



G-R in depletion region at Forward Bias

$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right) \right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right) \right]}$$

$$\text{In depletion region: } np = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$$

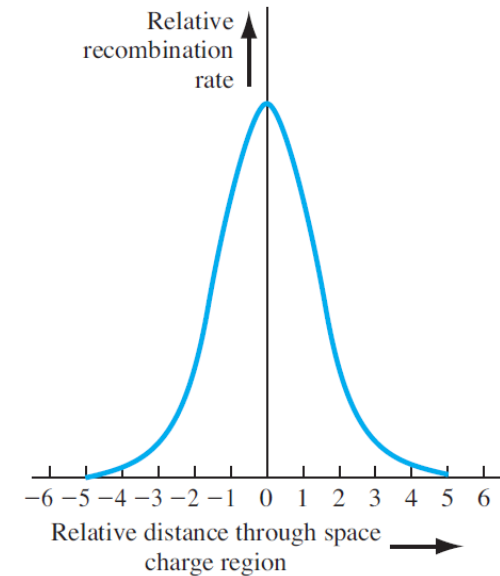
To simplify the calculation, we assume

$$E_t = E_i, \tau_n = \tau_p = \tau$$

$$R = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$

Max: $n=p$, center of space charge region

$$R_{max} = \frac{np - n_i^2}{\tau \left[n_i \exp\left(\frac{eV_a}{2kT}\right) + n_i \exp\left(\frac{eV_a}{2kT}\right) + 2n_i \right]} = \frac{n_i \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]}{2\tau \left[\exp\left(\frac{eV_a}{2kT}\right) + 1 \right]}$$



G-R in depletion region at Forward Bias

$$R_{max} = \frac{np - n_i^2}{\tau \left[n_i \exp\left(\frac{eV_a}{2kT}\right) + n_i \exp\left(\frac{eV_a}{2kT}\right) + 2n_i \right]} = \frac{n_i \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]}{2\tau \left[\exp\left(\frac{eV_a}{2kT}\right) + 1 \right]}$$

Assume $V_a \gg kT/e$

$$R_{max} = \frac{n_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

G-R in depletion region at Forward Bias

$$R_{max} = \frac{n_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

Current density from G-R in the depletion region:

$$J_r = \int_0^W eR dx = \frac{eWn_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

Assume maximum recombination rate is effective

$$J_r = J_{r0} \exp\left(\frac{eV_a}{2kT}\right)$$

For a non-ideal pn junction, the total current density:

$$J = J_F + J_r = J_s \exp\left(\frac{eV_a}{kT}\right) + \frac{eWn_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

Non-ideal IV curves

Forward bias $V > 3kT/e = 0.078V$:

$$J = J_F + J_r = J_s \exp\left(\frac{eV_a}{kT}\right) + \frac{eWn_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

At a low current density, the recombination current dominates, and at a higher current density, the ideal diffusion current dominates

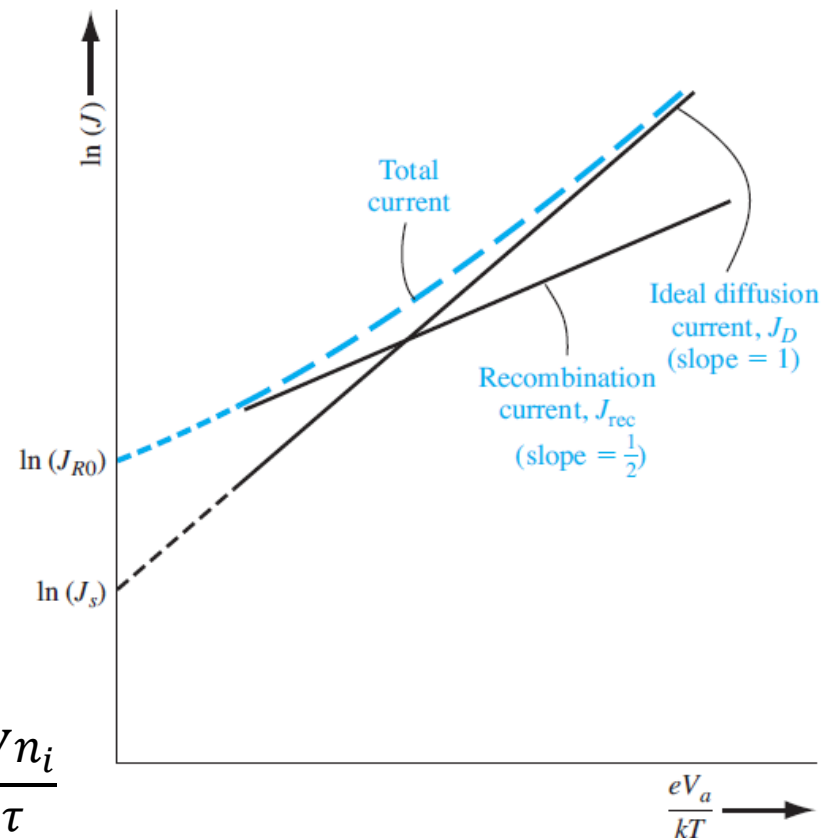
Diode I - V relationship:

$$I = I_s \left[\exp\left(\frac{eV_a}{nkT}\right) - 1 \right]$$

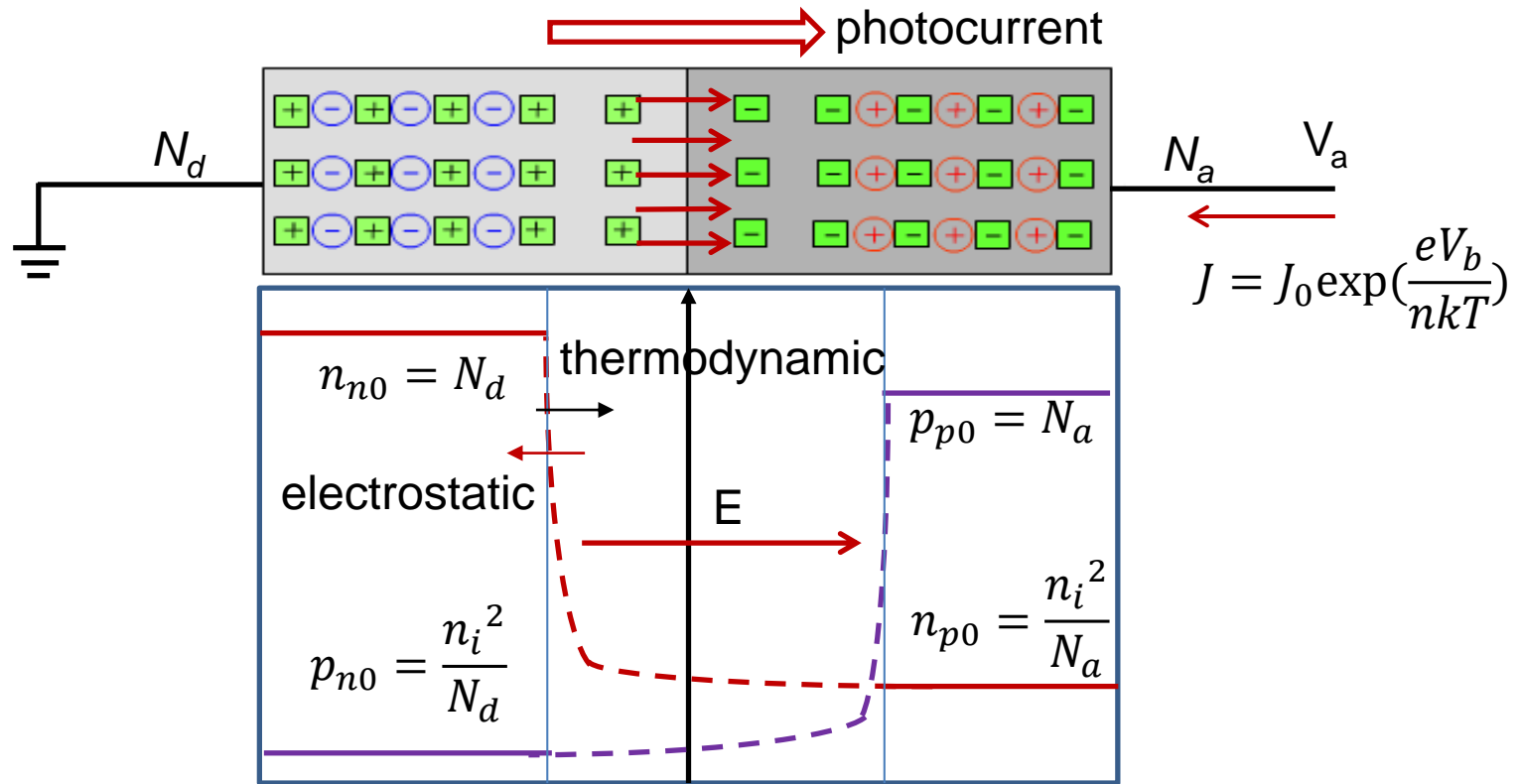
↓
the ideality factor

Reverse bias:

$$J = -J_s - \frac{eWn_i}{2\tau} = -\left(\frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}\right) - \frac{eWn_i}{2\tau}$$



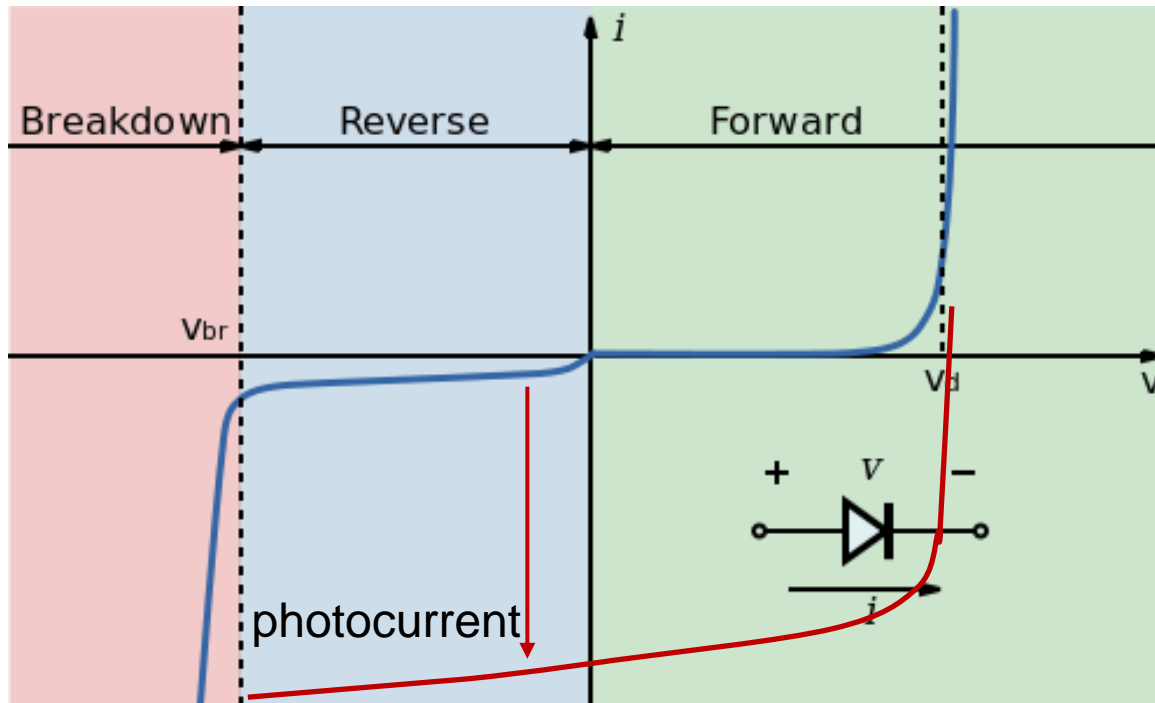
IV curve under illumination



Net current density:

$$J_{net} = J - J_{ph} = J_0 \exp\left(\frac{eV_a}{nkT}\right) - eG_{ex}W$$

IV curve under illumination

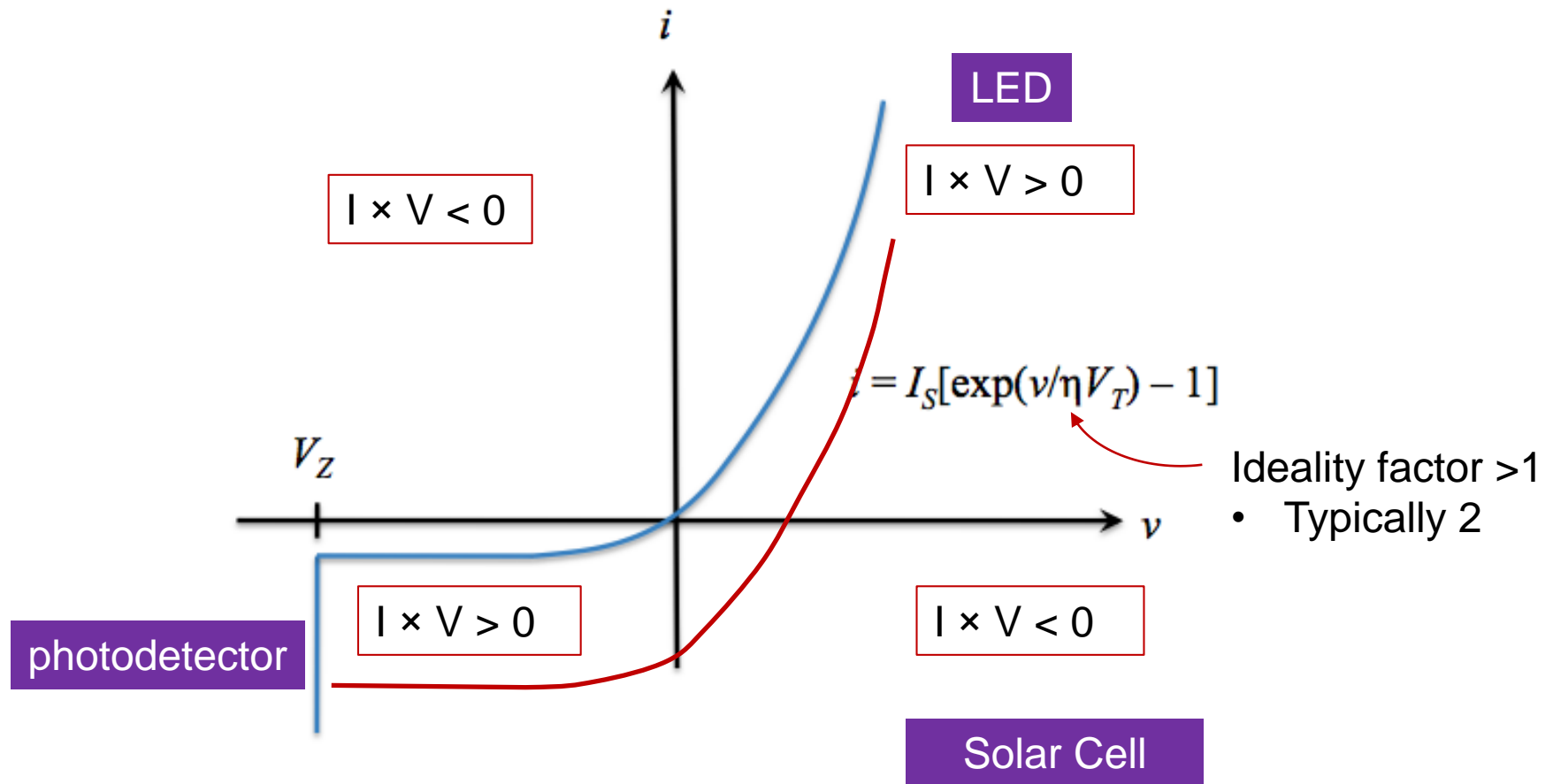


Net current density:

$$J_{net} = J - J_{ph} = J_0 \exp\left(\frac{eV_a}{nkT}\right) - eG_{ex}W$$

A few points about pn junction

- Energy consumption:



High-level injection

$$np = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

Excess carrier concentrations become comparable or even greater than the majority carrier concentration $\delta n > n_o$ and $\delta p > p_o$

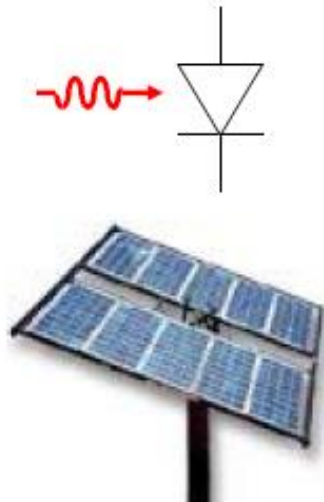
$$(\delta n)(\delta p) \cong n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$\delta n = \delta p \cong n_i \exp\left(\frac{V_a}{2V_t}\right)$$

$$I \propto \exp\left(\frac{V_a}{2V_t}\right)$$

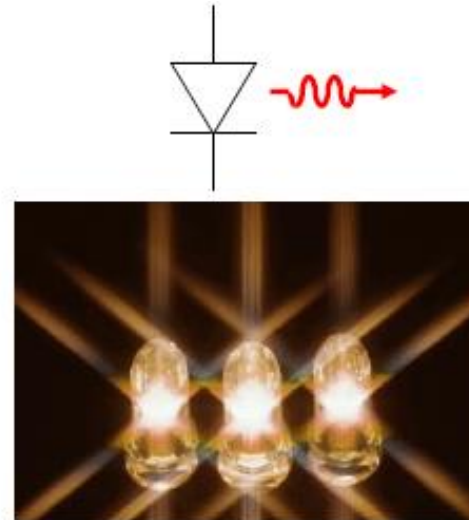
Semiconductor Optoelectronic Diodes

Detectors convert optical signals into electrical signals



- Photodetectors: primary purpose to detect photons
- Solar Cells: primary purpose is photo-to-electrical energy conversion

Emitters are a source of optical radiation



- Light-emitting diodes (LEDs)
- Lasers –may be obtained using optical cavity