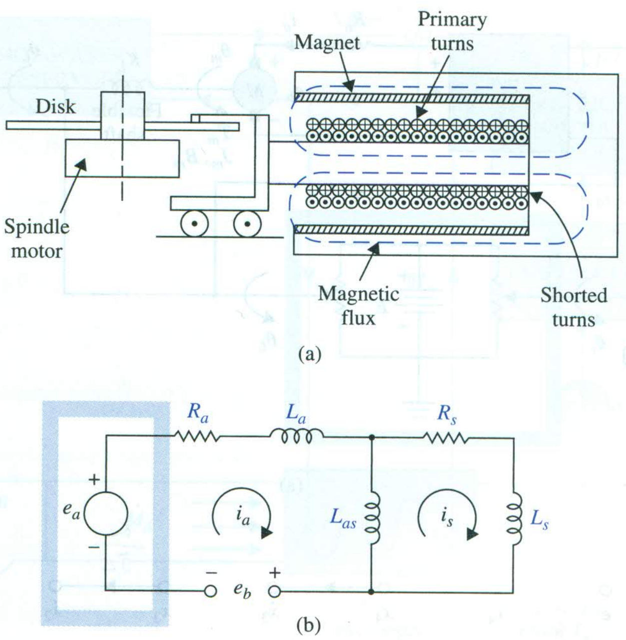


Figure 4P-15

installed for the purpose of effectively reducing the electric constant of the device. Figure 4P-15(b) shows the equivalent circuit of the coils. The following parameters and variables are defined: $e_a(t)$ is the applied coil voltage; $i_a(t)$, the primary-coil current; $i_s(t)$, the shorted-turn coil current; R_a, the primary-coil resistance; L_a, the primary-coil inductance; L_{as}, the mutual inductance between the primary and shorted-turn coils; $v(t)$, the velocity of the voice coil; $y(t)$, the displacement of the voice coil; $f(t) = K_i v(t)$, the force of the voice coil; K_i, the force constant; K_b, the back-emf constant; $e_b(t), = K_b v(t)$ the back emf; M_T, the total mass of the voice coil and load; and B_T, the total viscous-friction coefficient of the voice coil and load.

(a) Write the differential equations of the system.

(b) Draw a block diagram of the system with $E_a(s)$, $I_a(s)$, $I_s(s)$, $V(s)$, and $Y(s)$ as variables.

(c) Derive the transfer function $Y(s)/E_a(s)$.

• DC-motor control system

4-16. A dc-motor position-control system is shown in Fig. 4P-16(a). The following parameters and variables are defined: e is the error voltage; e_r, the reference input; θ_L, the load position; K_A, the amplifier gain; e_a, the motor input voltage; e_b, the back emf; i_a, the motor current; T_m, the motor torque; J_m, the motor inertia $= 0.03$ oz-in.-sec^2; B_m, the motor viscous-friction coefficient $=$ 10 oz-in.-sec; K_L, the torsional spring constant $= 50{,}000$ oz-in./rad; J_L, the load inertia $= 0.05$ oz-in.-sec^2; K_i, the motor torque constant $= 21$ oz-in./A; K_b, the back-emf constant $= 15.5$ V/1000 rpm; K_s, the error-detector gain $= E/2\pi$; E, the error-detector applied voltage $= 2\pi$ V; R_a, the motor resistance $= 1.15\ \Omega$; and $\theta_e = \theta_r - \theta_L$.

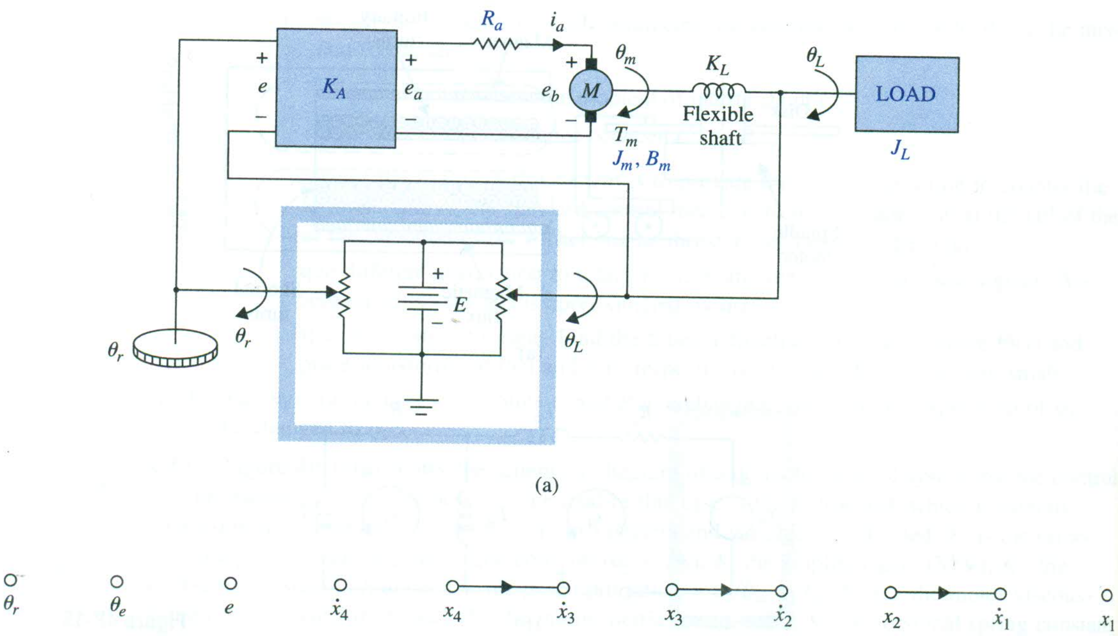
(a) Write the state equations of the system using the following state variables:

$$x_1 = \theta_L,\ x_2 = d\theta_L/dt = \omega_L,\ x_3 = \theta_3, \text{ and } x_4 = d\theta_m/dt = \omega_m$$

(b) Draw a state diagram using the nodes shown in Fig. 4P-16(b).

(c) Derive the forward-path transfer function $G(s) = \Theta_L(s)/\Theta_e(s)$ when the outer feedback path from θ_L is opened. Find the poles of $G(s)$.

(d) Derive the closed-loop transfer function $M(s) = \Theta_L(s)/\Theta_e(s)$. Find the poles of $M(s)$ when $K_A = 1$, 2738, and 5476. Locate these poles in the s-plane, and comment on the significance of these values of K_A.



(b)

Figure 4P-16

• Temperature control of air-flow system

4-17. Figure 4P-17(a) shows the setup of the temperature control of an air-flow system. The hot-water reservoir supplies the water that flows into the heat exchanger for heating the air. The temperature sensor senses the air temperature T_{AO} and sends it to be compared with the reference temperature T_r. The temperature error T_e is sent to the controller, which has the transfer function $G_c(s)$. The output of the controller, $u(t)$, which is an electric signal, is converted to a pneumatic signal by a transducer. The output of the actuator controls the water-flow rate through the three-way valve. Figure 4P-17(b) shows the block diagram of the system.

The following parameters and variables are defined: dM_w is the flow rate of the heating fluid $= K_M u$, $K_M = 0.054$ kg/sec/V; T_W, the water temperature $= K_R dM_W$; $K_R = 65$°C/kg/sec; and T_{AO} the output air temperature.

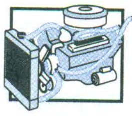
Heat-transfer equation between water and air:

$$\tau_c \frac{dT_{AO}}{dt} = T_w - T_{AO} \qquad \tau_c = 10 \text{ seconds}$$

Temperature sensor equation:

$$\tau_s \frac{dT_s}{dt} = T_{AO} - T_s \qquad \tau_s = 2 \text{ seconds}$$

(a) Draw a functional block diagram that includes all the transfer functions of the system.
(b) Derive the transfer function $T_{AO}(s)/T_r(s)$ when $G_c(s) = 1$.



Idle-speed control system

4-18. The objective of this problem is to develop a linear analytical model of the automobile engine for idle-speed control system shown in Fig. 1-2. The input of the system is the throttle position that controls the rate of air flow into the manifold (see Fig. 4P-18). Engine torque is developed from the buildup of manifold pressure due to air intake and the intake of the air/gas mixture into the cylinder. The engine variations are as follows:

$q_i(t)$ = amount of air flow across throttle into manifold
$dq_i(t)/dt$ = rate of air flow across throttle into manifold

• Broom-balancing system

4-22. Figure 4P-22(a) shows a well-known "broom-balancing" system. The objective of the control system is to maintain the broom in the upright position by means of the force $u(t)$ applied to the car as shown. In practical applications, the system is analogous to a one-dimensional control problem of balancing a unicycle or a missile immediately after launching. The free-body diagram of the system is shown in Fig. 4P-22(b), where

f_x = force at broom base in horizontal direction

f_y = force at broom base in vertical direction

M_b = mass of broom

g = gravitational acceleration

M_c = mass of car

J_b = moment of inertia of broom about center of gravity $CG = M_b L^2/3$

(a) Write the force equations in the x and the y directions at the pivot point of the broom. Write the torque equation about the center of gravity CG of the broom. Write the force equation of the car in the horizontal direction.

(b) Express the equations obtained in part (a) as state equations by assigning the state variables as $x_1 = \theta$, $x_2 = d\theta/dt$, $x_3 = x$, and $x_4 = dx/dt$. Simplify these equations for small θ by making the approximations: $\sin\theta \cong \theta$ and $\cos\theta \cong 1$.

(c) Obtain a small-signal linearized state-equation model for the system in the form of

$$\frac{d\Delta \mathbf{x}(t)}{dt} = \mathbf{A}^*\Delta\mathbf{x}(t) + \mathbf{B}^*\Delta\mathbf{r}(t)$$

at the equilibrium point $x_{01}(t) = 1$, $x_{02}(t) = 0$, $x_{03}(t) = 0$, and $x_{04}(t) = 0$.

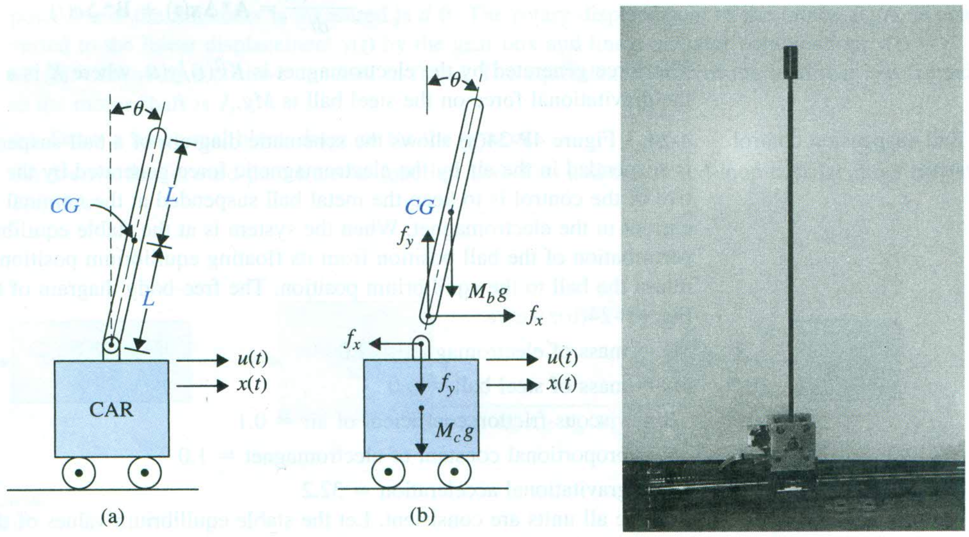


Figure 4P-22

• Ball-suspension control system

4-23. Figure 4P-23 shows the schematic diagram of a ball-suspension control system. The steel ball is suspended in the air by the electromagnetic force generated by the electromagnet. The objective of the control is to keep the metal ball suspended at the nominal equilibrium position by controlling the current in the magnet with the voltage $e(t)$. The practical application of this system is the magnetic levitation of trains or magnetic bearings in high-precision control systems.

The resistance of the coil is R, and the inductance is $L(y) = L/y(t)$, where L is a constant. The applied voltage $e(t)$ is a constant with amplitude E.