Diode

VE311 Electronic Circuits (Summer 2019)

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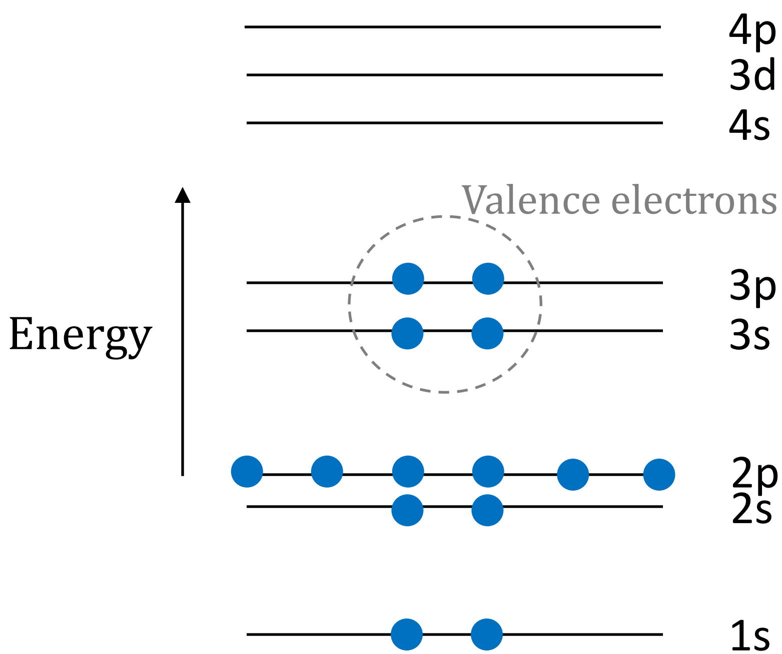
Brief Introduction of Semiconductor Physics

Although not going to be covered in the exams, this part lays a foundation for our understandings of the working principles of semiconductor devices.

Silicon Atom

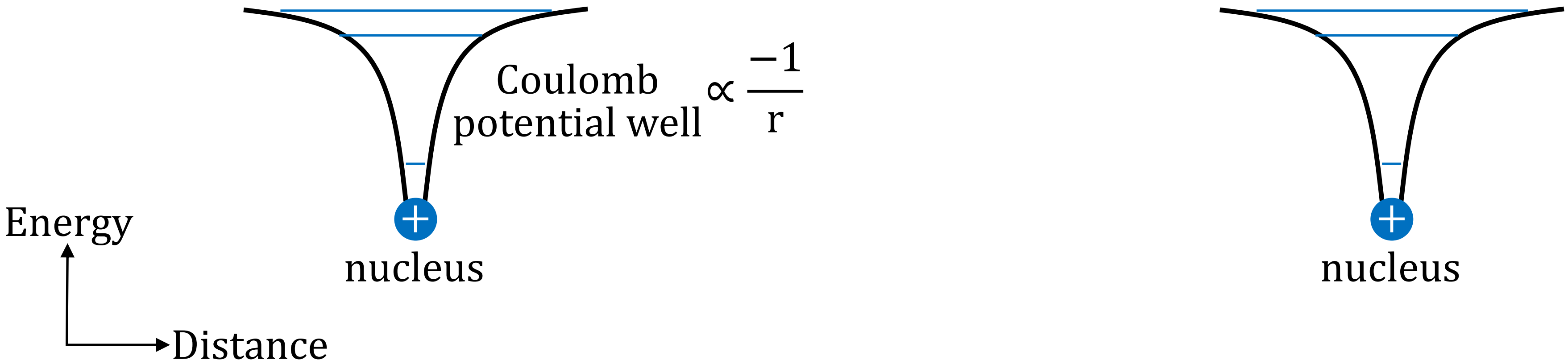
- **Pauli exclusion principle** requires that each electron must have a distinct energy state defined by an unique set of quantum numbers.
- The allowed states and associated wavefunctions can be described by 4 quantum numbers: principle quantum number (n), angular momentum quantum number (l), magnetic quantum number (m) and electron spin (s).



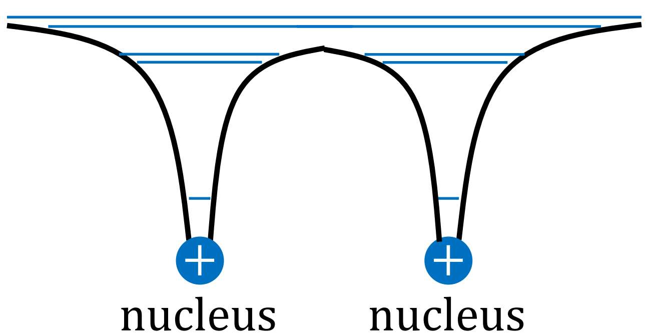
Silicon has its two inner shells (1s, 2s and 2p subshells) totally filled with electrons. It's outer shell has 2 valence electrons in two 3s orbitals and 2 valence electrons in six 3p orbitals.

Effect of Lattice (I)

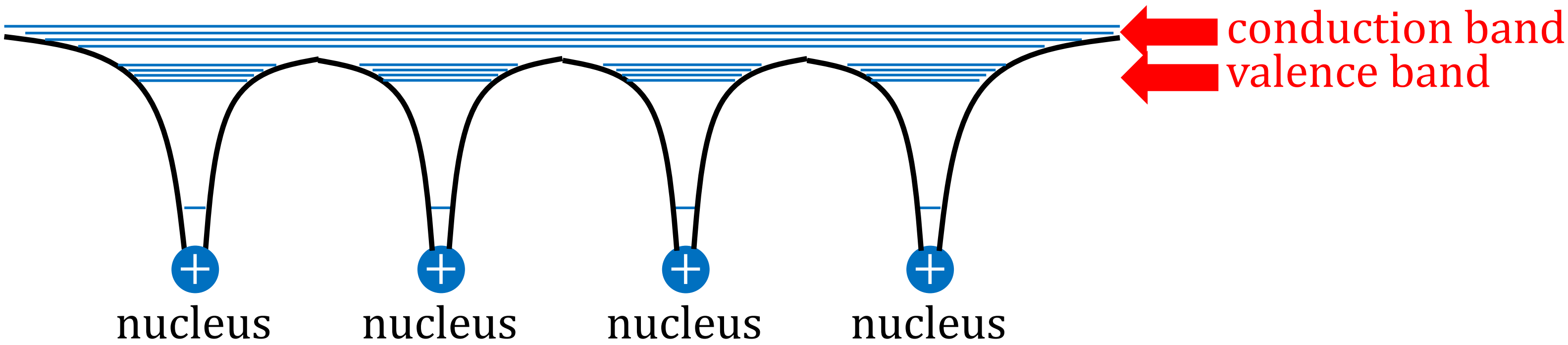
When two atoms are far away from each other, there is no interaction.



When two atoms are close to each other, like in a lattice structure: (1) wavefunctions overlap. (2) Potential wells are influenced by neighbor nucleus. (3) The valence electrons become delocalized (e.g. through tunneling). (4) By Pauli exclusion principle, **the states split into N substates, where N is number of atoms.**



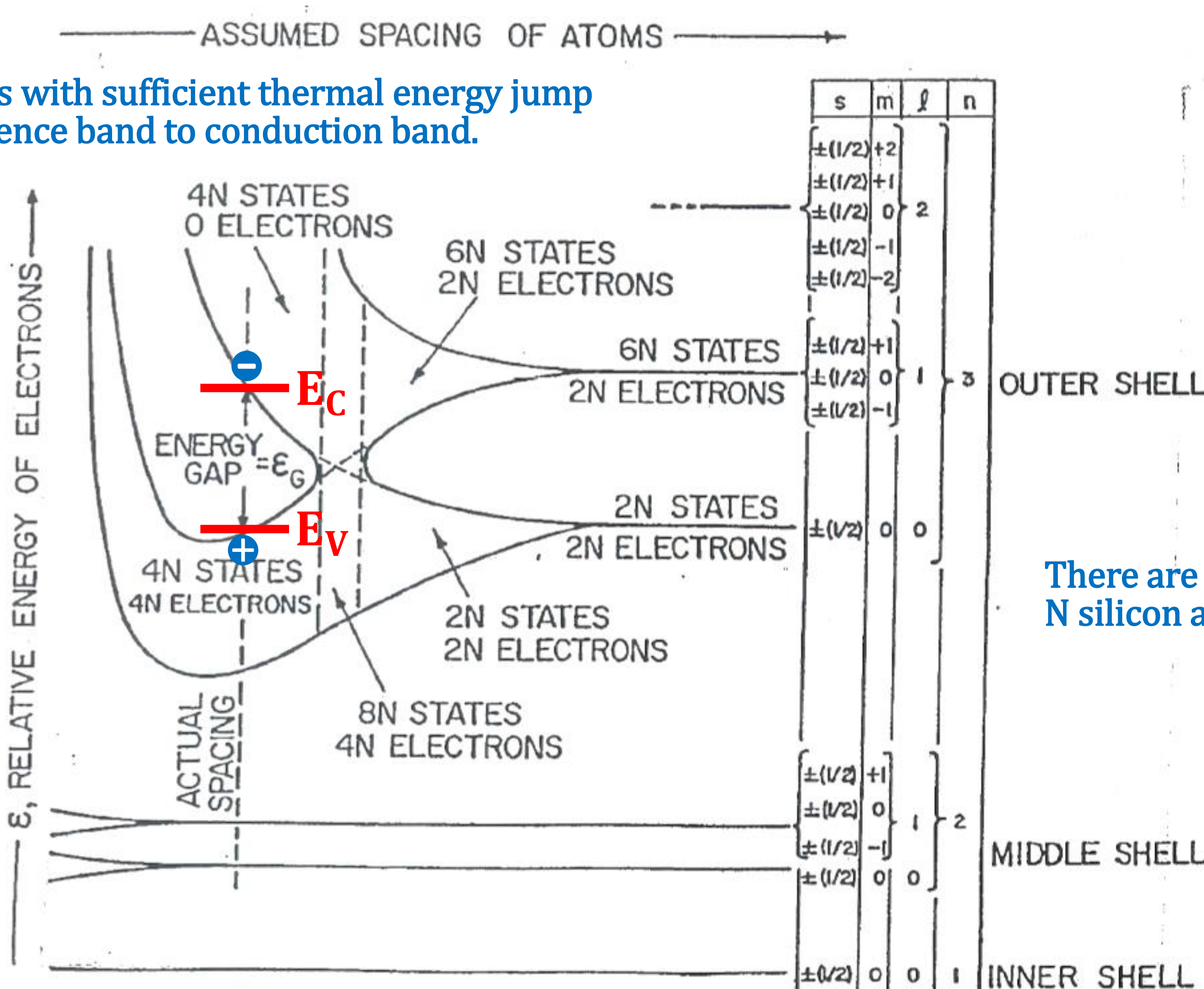
Effect of Lattice (II)



- **Semiconductor:** bandgap is small enough that electrons with adequate thermal energy can jump from valence band to conduction band.
- **Insulator:** bandgap is too large for electrons to jump from valence band to conduction band.
- **Conductor:** bandgap is very small or nearly zero.

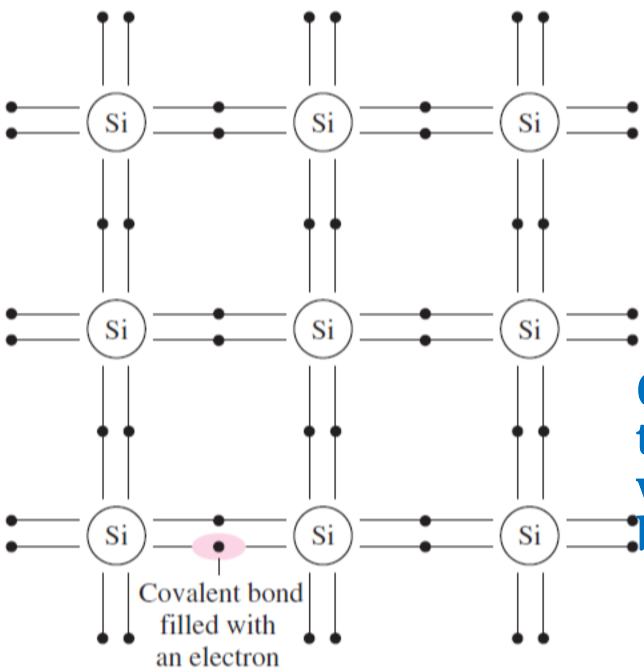
Band Formation for Si

Electrons with sufficient thermal energy jump from valence band to conduction band.

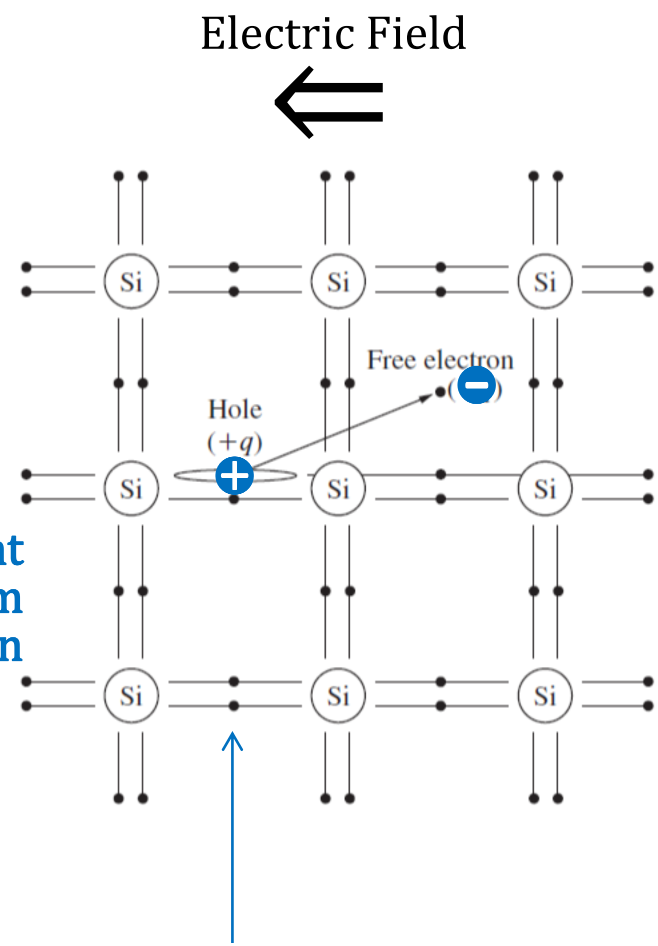


There are N silicon atoms

n and p for **Intrinsic** (i.e. no impurity) **Si** (I)

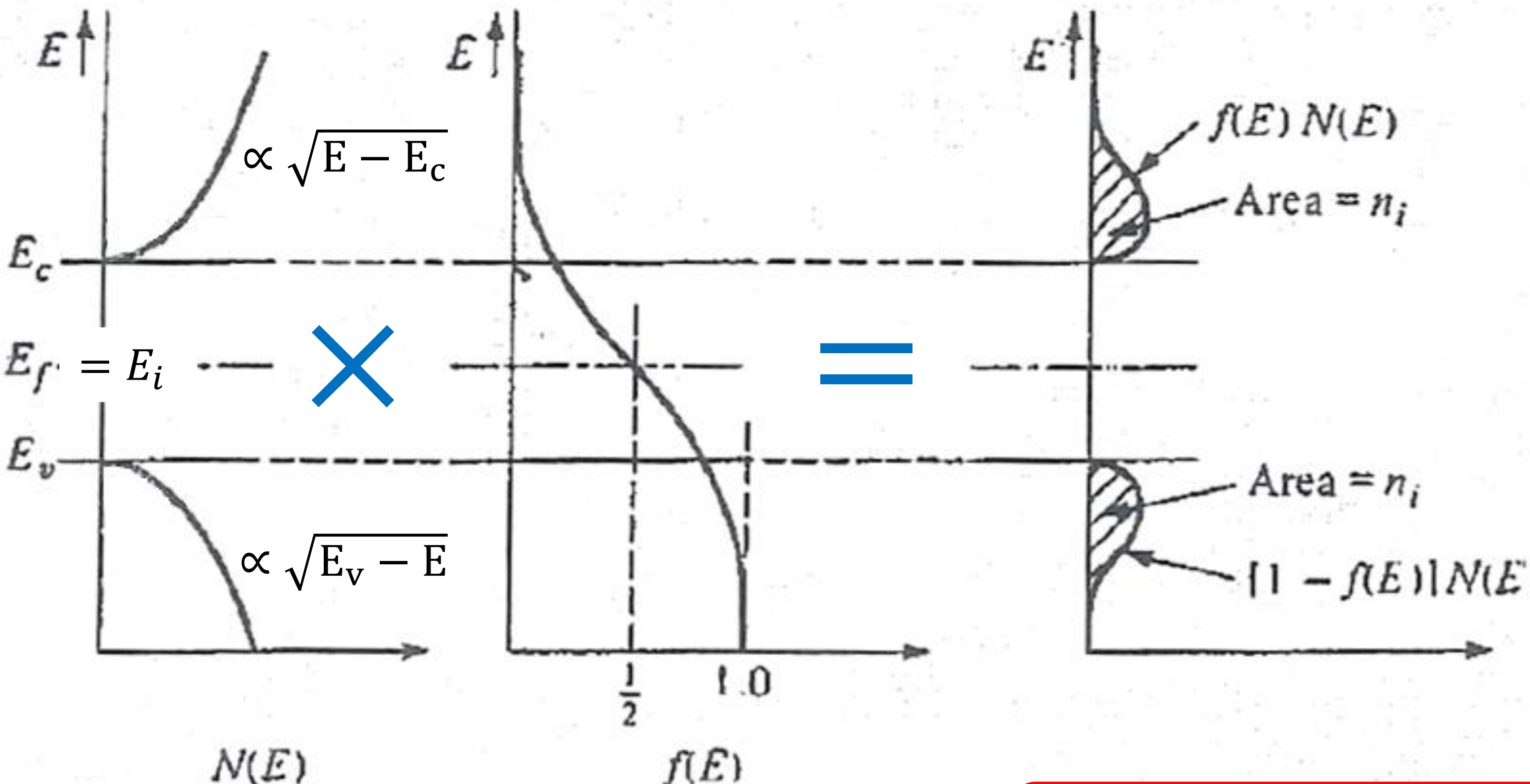


One electron with sufficient thermal energy jumps from valence band to conduction band.



Covalent Bond

n and p for **Intrinsic Si** (II)



Density of States
(# of states / energy)

Fermi-Dirac Distribution
(the probability of the state occupied with an electron)

$$\mathbf{f(E) = \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1}}$$

$$\mathbf{n = \int_{E_C}^{\infty} f(E)N(E)dE = n_i}$$

$$\mathbf{p = \int_{-\infty}^{E_V} [1 - f(E)]N(E)dE = n_i}$$

n and p for **Intrinsic Si** (III)

$$\boxed{np = n_i^2 = BT^3 \exp\left(-\frac{E_G}{kT}\right) = \text{constant}}$$

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

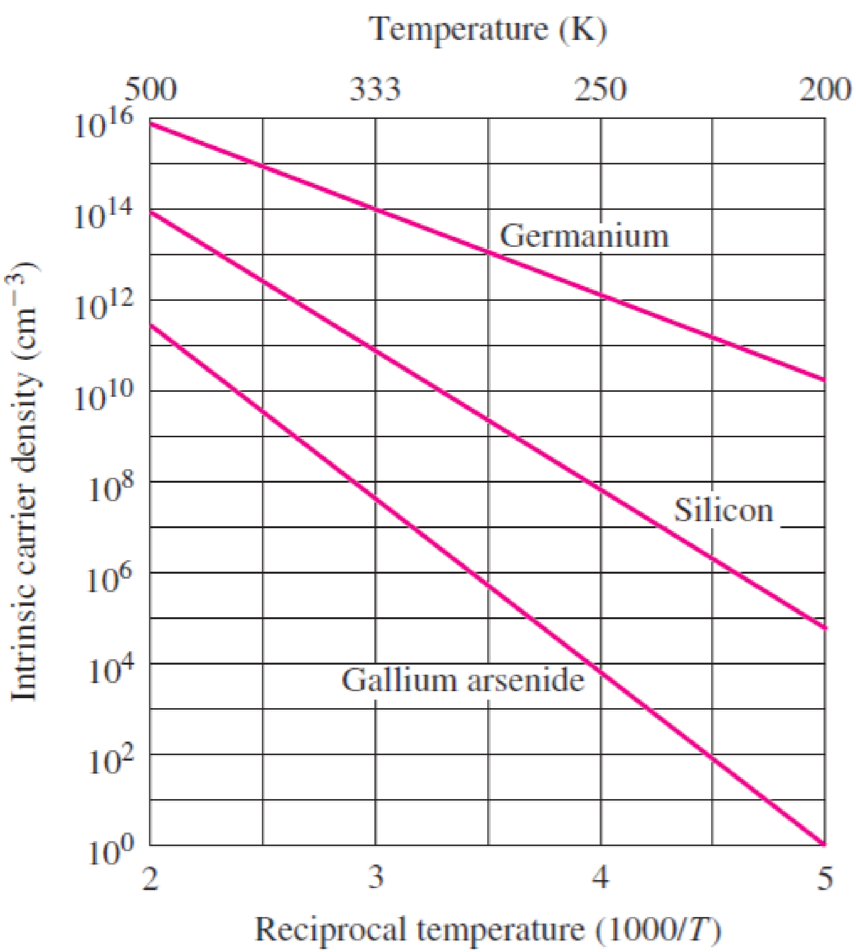


Figure 2.4 Intrinsic carrier density versus temperature from Eq. (2.1).

k (Boltzmann's Constant)
$= 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K

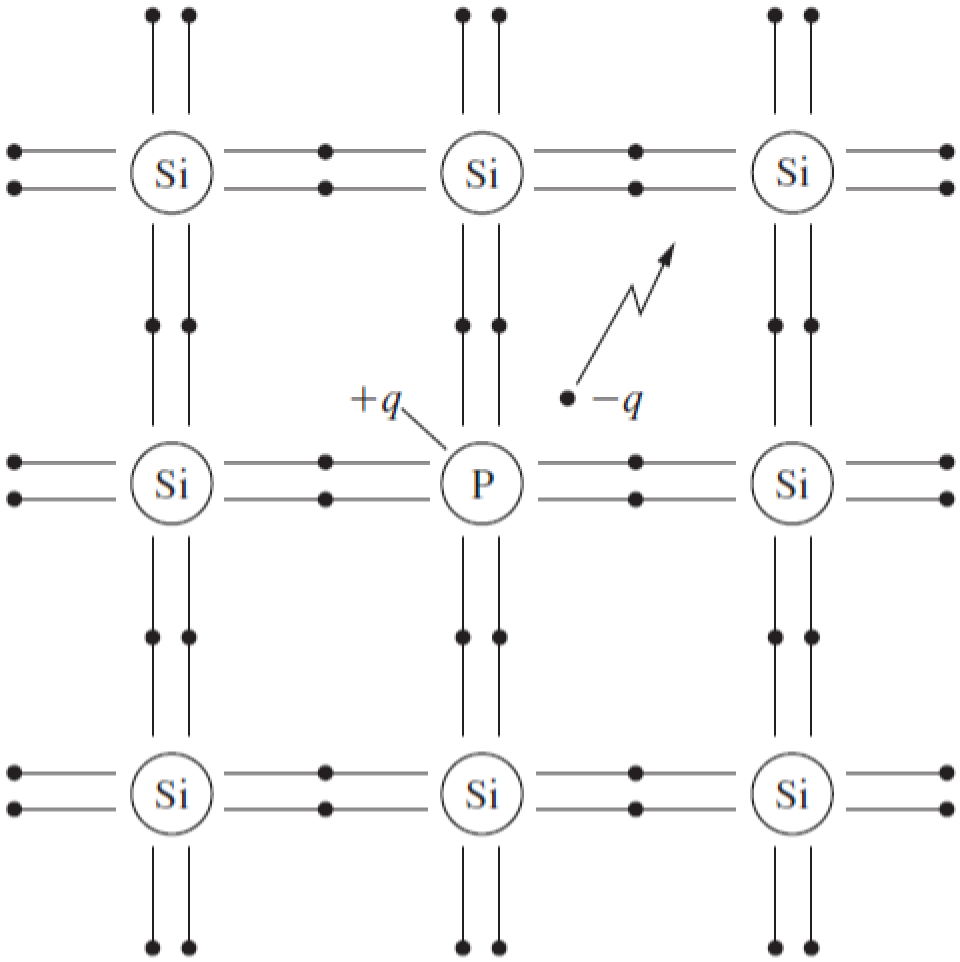
At 300 K:

$$n_i^2 = (1.08 \times 10^{31})300^3 e^{\frac{-1.12}{(8.62\times10^{-5})\times 300}}$$
$$= 4.52 \times 10^{19}\ (1/cm^6)$$

$$\boxed{n_i = 6.73 \times 10^9\ (1/cm^3) \cong 10^{10}\ (1/cm^3)}$$

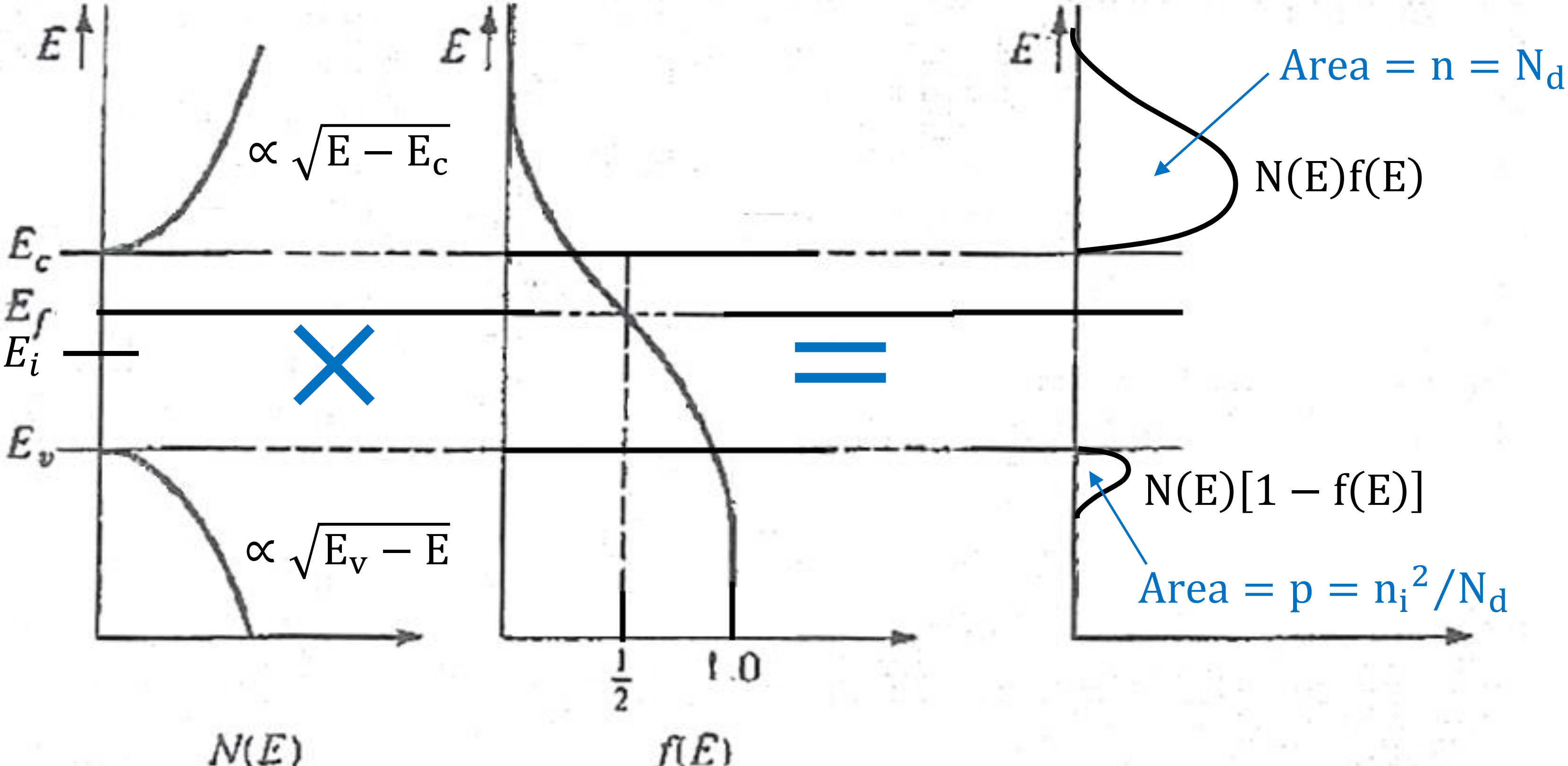
	B ($K^{-3} \cdot cm^{-6}$)	E_G (eV)
Si	1.08×10^{31}	1.12
Ge	2.31×10^{30}	0.66
GaAs	1.27×10^{29}	1.42

n and p for **n-type Si** (I)



If the n-type dopant (e.g. **phosphorous**) concentration $N_d \gg n_i$,
$n = N_d$ and $p = n_i{}^2/N_d$

n and p for **n-type Si** (II)



Density of States
(# of states / energy)

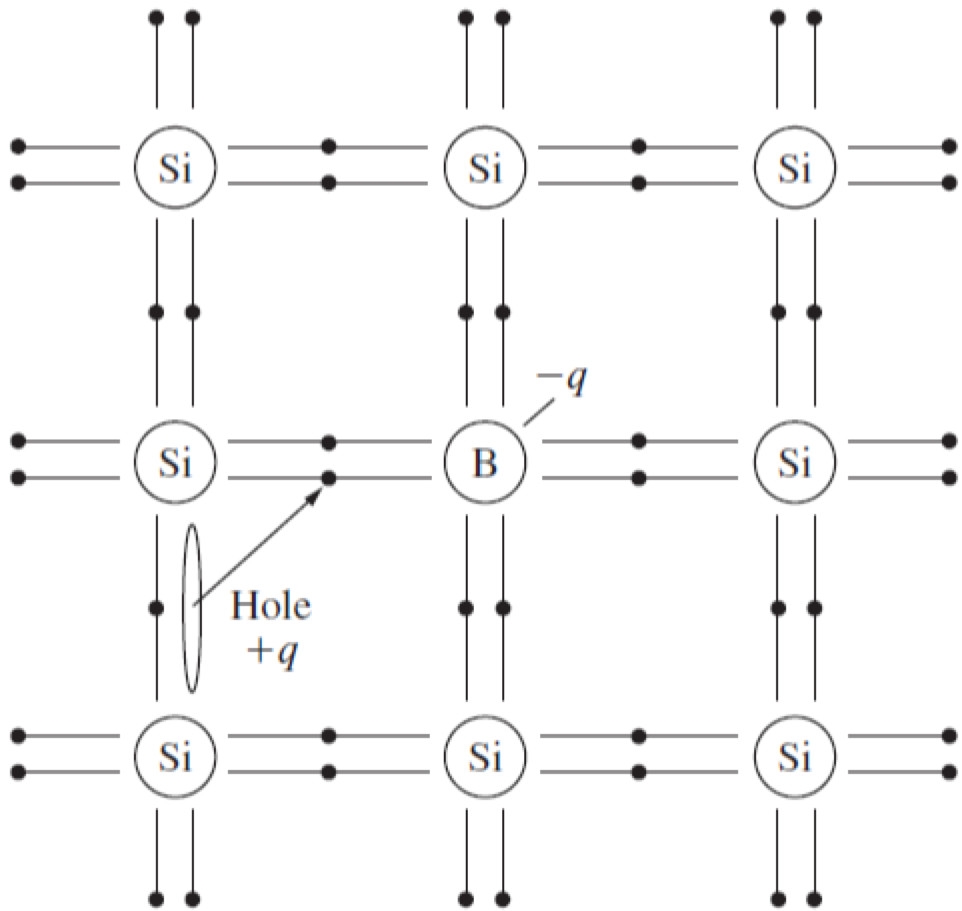
Fermi-Dirac Distribution
(the probability of the state occupied with an electron)

$$\mathbf{f(E) = \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1}}$$

$$\mathbf{n = \int_{E_C}^{\infty} f(E)N(E)dE = n_i\, e^{\frac{E_f - E_i}{kT}}}$$

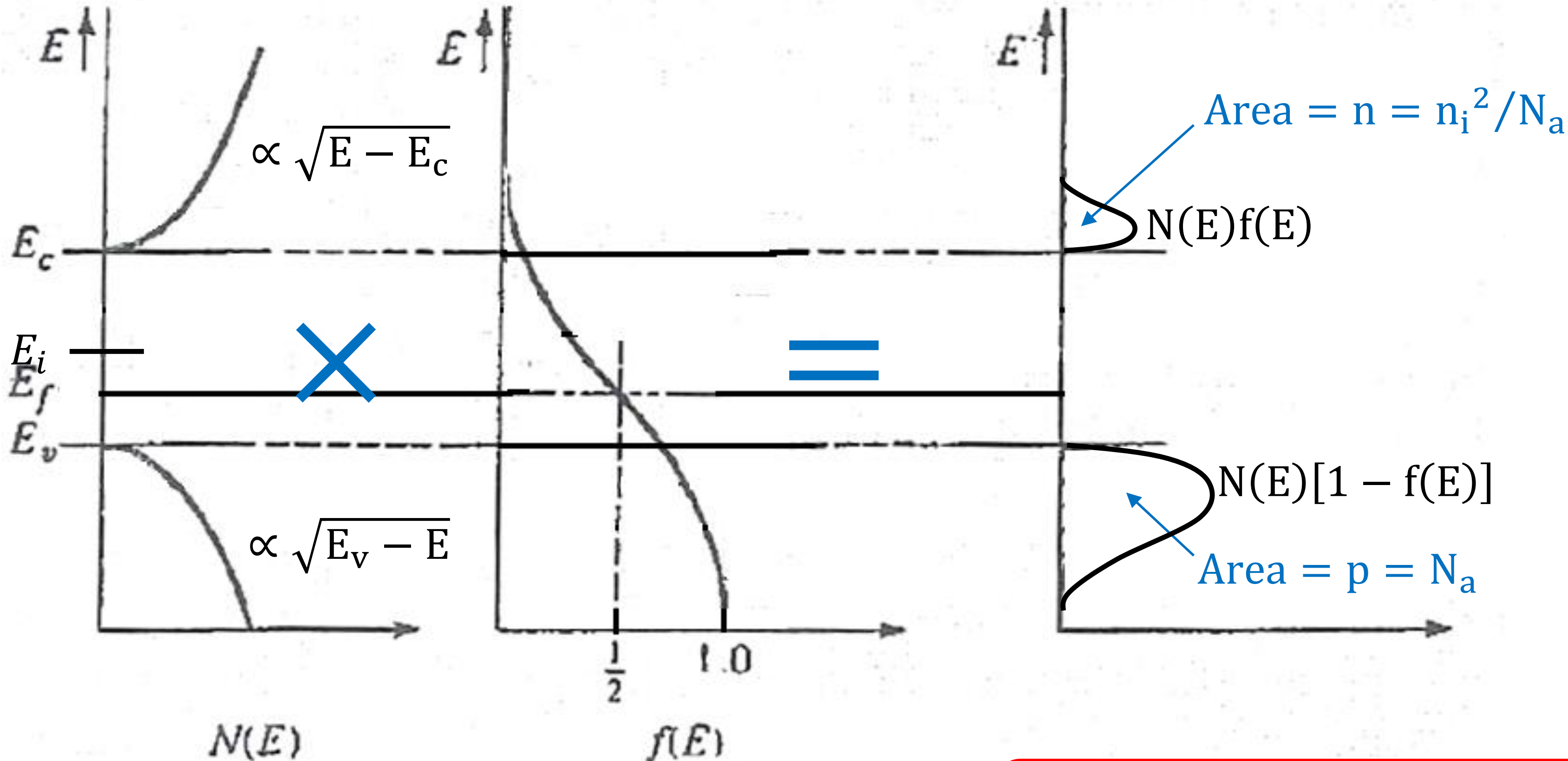
$$\mathbf{p = \int_{-\infty}^{E_V} [1 - f(E)]N(E)dE = n_i\, e^{\frac{E_i - E_f}{kT}}}$$

n and p for **p-type Si** (I)



If the p-type dopant (e.g. **boron**) concentration $N_a \gg n_i$,
$p = N_a$ and $n = n_i^2/N_a$

n and p for **p-type Si** (II)



Density of States

# of states/$(\mathrm{cm}^3 \cdot \mathrm{J})$

Fermi-Dirac Distribution

(the probability of the state occupied with an electron)

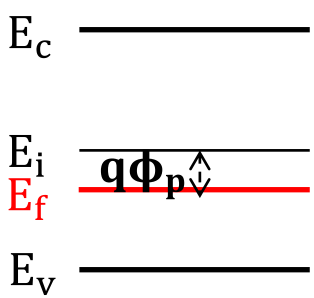
$$\mathbf{f(E) = \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1}}$$

$$\mathbf{n = \int_{E_C}^{\infty} f(E)N(E)dE = n_i\, e^{\frac{E_f - E_i}{kT}}}$$

$$\mathbf{p = \int_{-\infty}^{E_V} [1 - f(E)]N(E)dE = n_i\, e^{\frac{E_i - E_f}{kT}}}$$

Summary

p-type

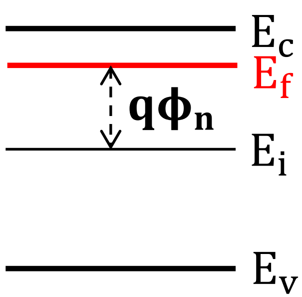


$$p = N_a = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{q\phi_p}{kT}}$$

$$n = \frac{n_i^2}{N_a} = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{-q\phi_p}{kT}}$$

$$\begin{cases} np = n_i^2 \\ n + N_a^- = p \ \text{(neutrality)} \end{cases}$$

n-type



$$n = N_d = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{q\phi_n}{kT}}$$

$$p = \frac{n_i^2}{N_d} = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{-q\phi_n}{kT}}$$

$$\begin{cases} np = n_i^2 \\ n = N_d^+ + p \ \text{(neutrality)} \end{cases}$$

Charge Carriers Flow in Semiconductor

Current density equations in 1D:

$$\begin{cases} J_p = q\left(\mu_p p E_x - D_p \dfrac{dp}{dx}\right) \\ J_n = q\left(\mu_n n E_x + D_n \dfrac{dn}{dx}\right) \end{cases}$$

μ: charge carrier mobility $[cm^2/(V \cdot sec)]$

D: charge carrier diffusion coefficient $[cm^2/sec]$

Continuity equations (steady state) in 1D:

$$\begin{cases} \dfrac{\partial p}{\partial t} = \dfrac{-1}{q}\dfrac{dJ_p}{dx} + G - R = 0 \\ \dfrac{\partial n}{\partial t} = \dfrac{1}{q}\dfrac{dJ_n}{dx} + G - R = 0 \end{cases}$$

G: charge carrier generation rate $[1/(cm^3 \cdot sec)]$

R: charge carrier recombination rate $[1/(cm^3 \cdot sec)]$

Generation and Recombination Rate

Recombination rate
unit: $1/(\mathrm{cm}^3 \cdot \mathrm{sec})$

$$\boxed{\mathbf{R = Knp\ (K\ is\ a\ constant)}}$$

When in thermal equilibrium:

Generation rate
unit: $1/(\mathrm{cm}^3 \cdot \mathrm{sec})$

$$G_o = R_o = Kn_op_o$$

When under constant light illumination (steady state):

$$G_o + G_{light} = K(n_o + \Delta n)(p_o + \Delta p)$$

When light illumination just off:

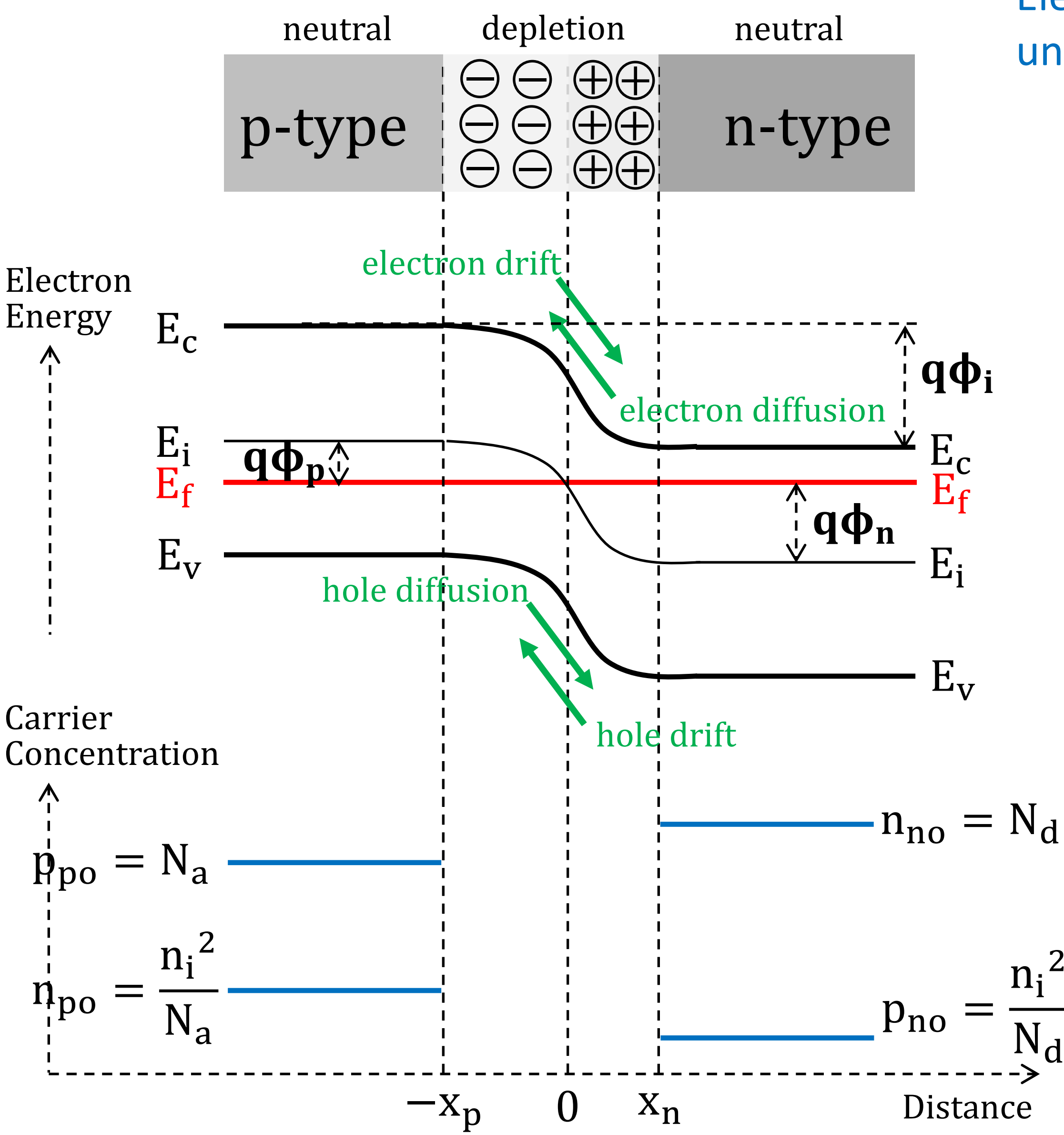
Net recombination rate
unit: $1/(\mathrm{cm}^3 \cdot \mathrm{sec})$

$$U = K(n_o + \Delta n)(p_o + \Delta p) - G_o = \frac{\Delta n}{\tau_n} = \frac{\Delta p}{\tau_p}$$

Electron or hole excess carrier lifetime
unit: sec

Diode I-V Equation

Si PN Junction in **Thermal Equilibrium**



Electron concentration in the n-type Si under thermal equilibrium condition

$$\begin{cases} n_{no} = N_d = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{q\phi_n}{kT}} \\ p_{po} = N_a = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{q\phi_p}{kT}} \end{cases}$$

$$\boxed{\phi_i = \phi_n + \phi_p = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}}$$

$$\begin{cases} N_d = \frac{n_i^2}{N_a} e^{\frac{q\phi_i}{kT}} \\ N_a = \frac{n_i^2}{N_d} e^{\frac{q\phi_i}{kT}} \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} n_{no} = n_{po} e^{\frac{q\phi_i}{kT}} \\ p_{po} = p_{no} e^{\frac{q\phi_i}{kT}} \end{cases}}$$

Electron Energy and Fermi Level

- **Electron energy level** (E_C, E_V, E_i) bending means there is **electric field.**

$$\text{electric field} \equiv -\frac{dV}{dx} = \frac{1}{q}\frac{dE_C}{dx} = \frac{1}{q}\frac{dE_V}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

- **Fermi level** (E_f) bending means there is **current.**

$$J_n = \mu_n n \frac{dE_{fn}}{dx} \qquad J_p = \mu_p p \frac{dE_{fp}}{dx}$$

Proof:

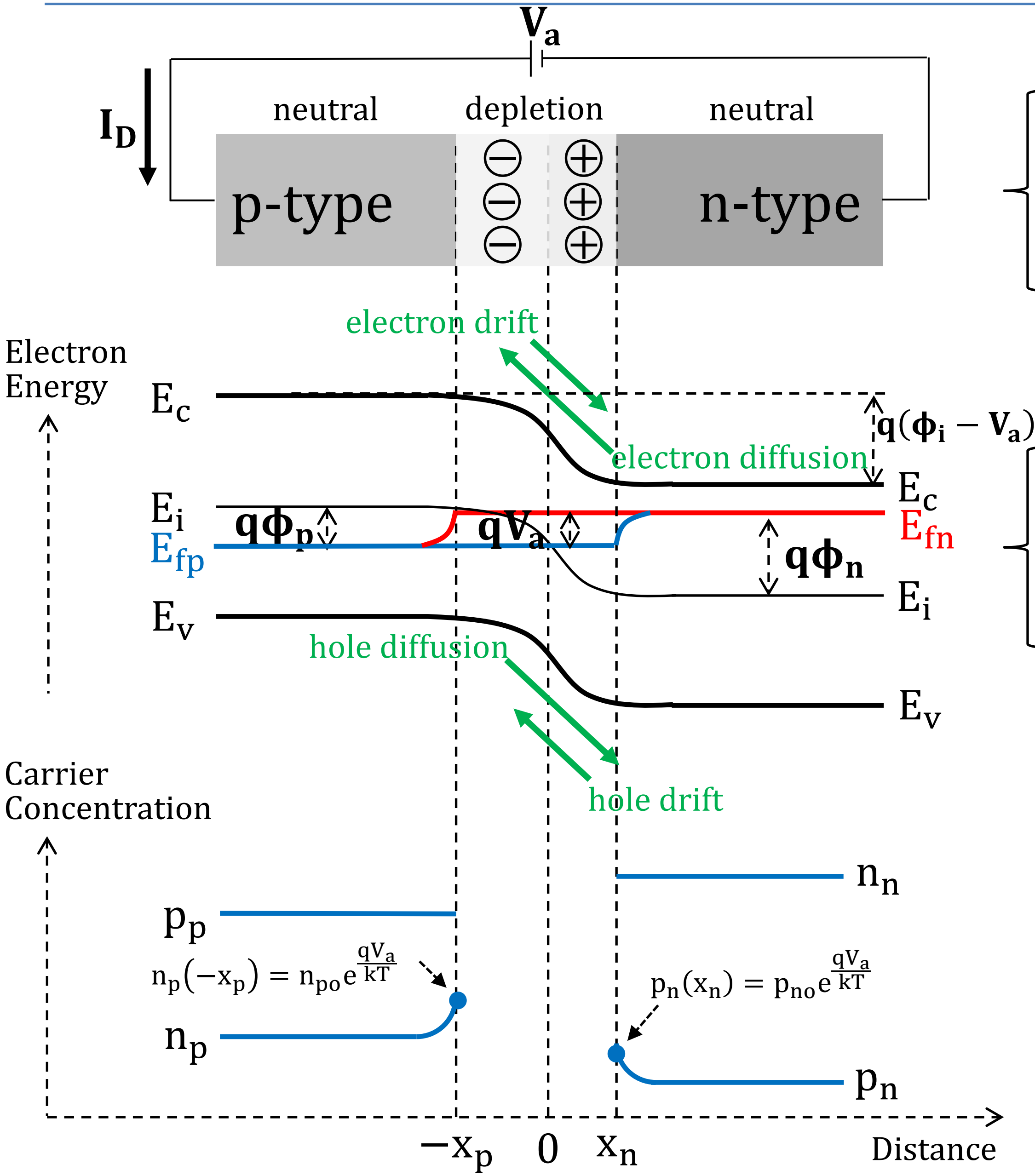
$$n = n_i e^{\frac{E_{fn}-E_i}{kT}} \Rightarrow E_{fn} = E_i + kT\ln\left(\frac{n}{n_i}\right)$$

Einstein Relations: $\frac{\mu_n}{q} = \frac{D_n}{kT}$

$$\mu_n n \frac{dE_{fn}}{dx} = \mu_n n\left(\frac{dE_i}{dx} + kT\frac{n_i}{n}\frac{1}{n_i}\frac{dn}{dx}\right) = q\mu_n n E_x + \mu_n kT\frac{dn}{dx} = q\mu_n n E_x + qD_n\frac{dn}{dx}$$

electric field

Si PN Junction in **Forward Bias (I)**



$$
\begin{cases}
n_n(x_n) = n_p(-x_p)e^{\frac{q(\phi_i - V_a)}{kT}} \\
p_p(-x_p) = p_n(x_n)e^{\frac{q(\phi_i - V_a)}{kT}}
\end{cases}
$$

Assume low-level injection $\Rightarrow$

$$
\begin{cases}
n_p(-x_p)e^{\frac{q(\phi_i - V_a)}{kT}} = N_d = n_{po}e^{\frac{q\phi_i}{kT}} \\
p_n(x_n)e^{\frac{q(\phi_i - V_a)}{kT}} = N_a = p_{no}e^{\frac{q\phi_i}{kT}}
\end{cases}
$$

$$
\boxed{\begin{cases}
n_p(-x_p) = n_{po}e^{\frac{qV_a}{kT}} \\
p_n(x_n) = p_{no}e^{\frac{qV_a}{kT}}
\end{cases}}
$$

B.C. #1

$$
\boxed{\begin{cases}
n_p(-\infty) = n_{po} \\
p_n(\infty) = p_{no}
\end{cases}}
$$

B.C. #2

Si PN Junction in **Forward Bias** (II)

Current density equations in 1D:

$$\begin{cases} J_p = q\left(\mu_p p E_x - D_p \dfrac{dp}{dx}\right) \\ J_n = q\left(\mu_n n E_x + D_n \dfrac{dn}{dx}\right) \end{cases}$$

Put J_p or J_n into here

Continuity equations (steady-state) in 1D:

$$\begin{cases} \dfrac{\partial p}{\partial t} = \dfrac{-1}{q}\dfrac{dJ_p}{dx} + G - R = 0 \\ \dfrac{\partial n}{\partial t} = \dfrac{1}{q}\dfrac{dJ_n}{dx} + G - R = 0 \end{cases}$$

Genergation rate: G

Recombination rate: R

Excess carrier lifetime: τ_p or τ_n

(see next page)

By depletion approximation
(i.e. $E = 0$ outside the depletion region)

$$\Rightarrow \boxed{\begin{cases} J_p = -qD_p \dfrac{d\Delta p_n}{dx} \\ J_n = qD_n \dfrac{d\Delta n_p}{dx} \end{cases}}$$

$$\Rightarrow \begin{cases} \dfrac{d^2\Delta p_n}{dx^2} = \dfrac{\Delta p_n}{D_p\tau_p} = \dfrac{\Delta p_n}{{L_p}^2} \\ \dfrac{d^2\Delta n_p}{dx^2} = \dfrac{\Delta n_p}{D_n\tau_n} = \dfrac{\Delta n_p}{{L_n}^2} \end{cases}$$

Diffusion legngth: $L_n = \sqrt{D_n\tau_n}$

Apply B.C. #1 and B.C. #2 $\Rightarrow$

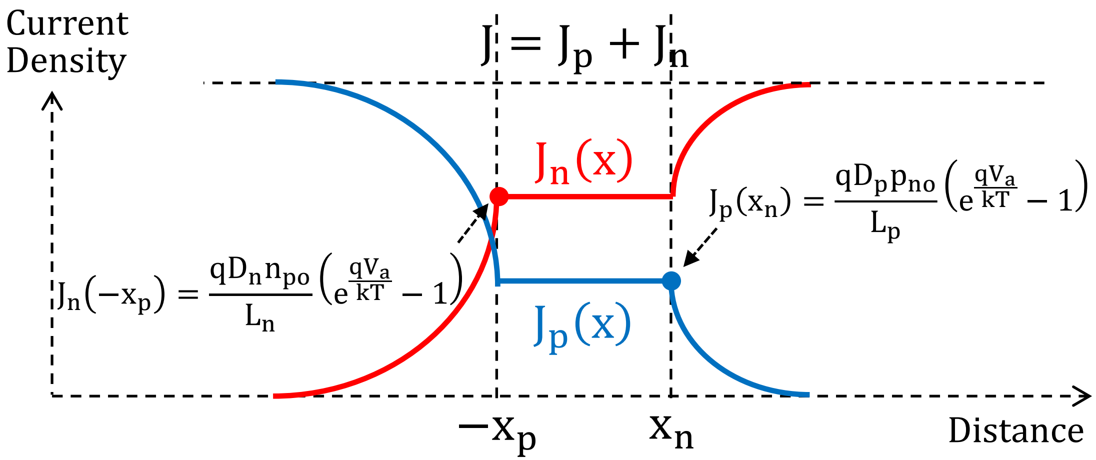
$$\boxed{\begin{cases} \Delta p_n(x) = p_{no}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} & (x \geq x_n) \\ \Delta n_p(x) = n_{po}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} & (x \leq -x_p) \end{cases}}$$

Put Δp_n or Δn_p back
into here

Si PN Junction in **Forward Bias** (III)

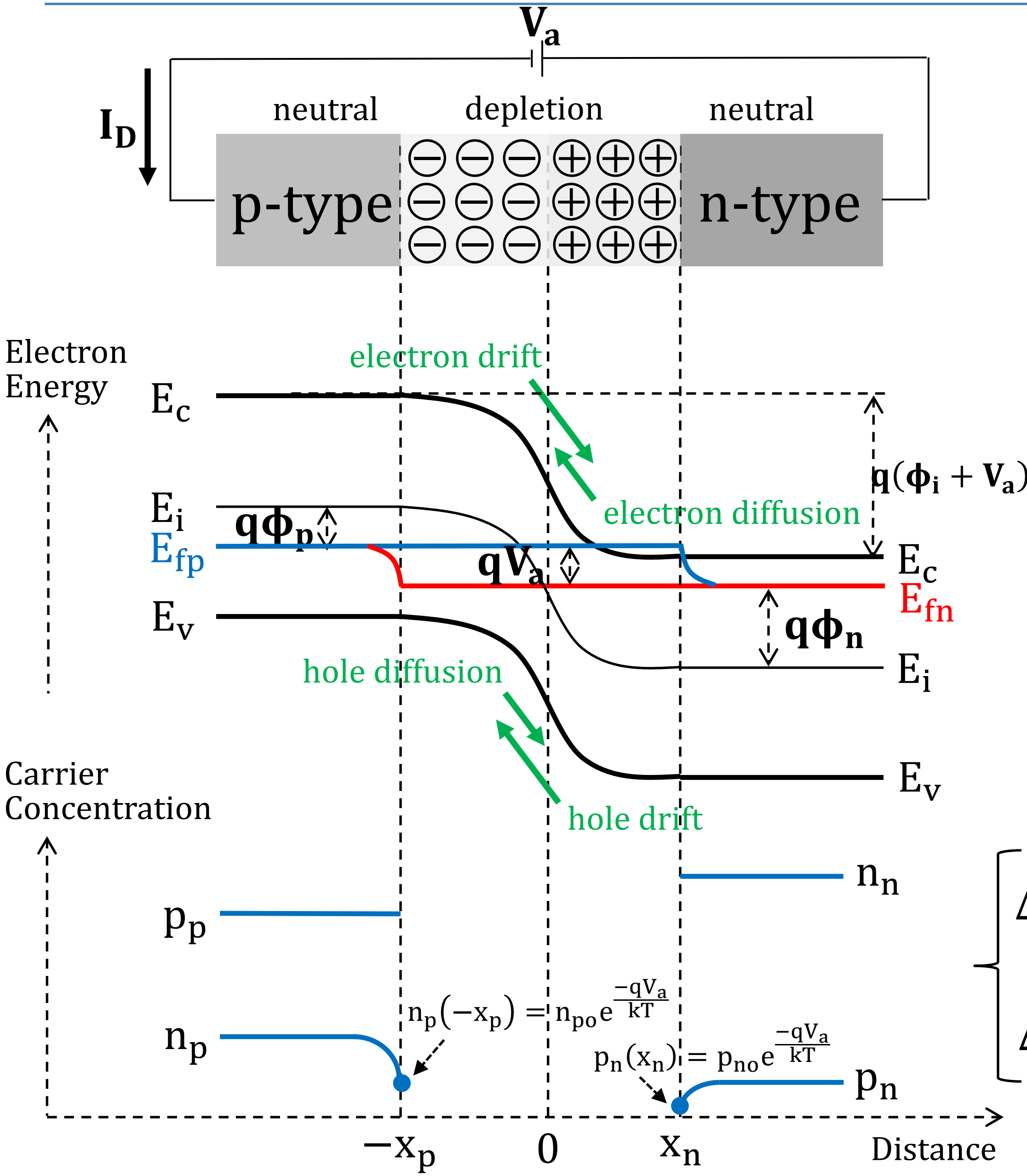
$$\begin{cases} J_p(x) = \dfrac{qD_p p_{no}}{L_p}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} & (x \geq x_n) \\ J_n(x) = \dfrac{qD_n n_{po}}{L_n}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} & (x \leq -x_p) \end{cases}$$

$$\boldsymbol{I_D = qAn_i{}^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)\left(e^{\frac{qV_a}{kT}} - 1\right) = I_S\left(e^{\frac{qV_a}{kT}} - 1\right)}$$



- Minority current gradually transformed into majority current
- Total current across PN junction is a constant

Si PN Junction in **Reverse Bias (I)**



Replace V_a with $-V_a$

$$\begin{cases} n_p(-x_p) = n_{po}e^{\frac{-qV_a}{kT}} \\ p_n(x_n) = p_{no}e^{\frac{-qV_a}{kT}} \end{cases}$$

$$\begin{cases} n_p(-\infty) = n_{po} \\ p_n(\infty) = p_{no} \end{cases}$$

$$\begin{cases} \Delta p_n(x) = p_{no}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} < 0 \\ \Delta n_p(x) = n_{po}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} < 0 \end{cases}$$

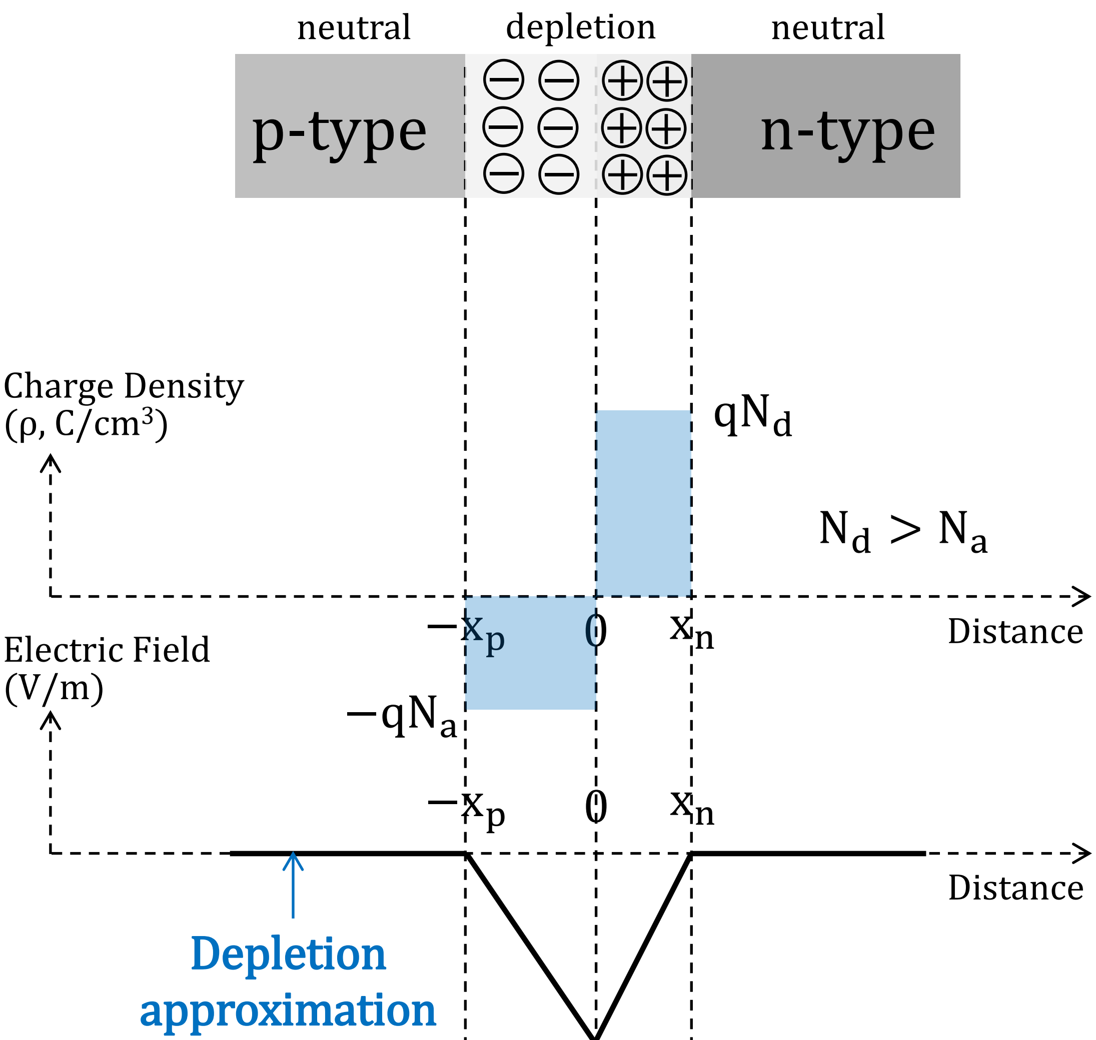
Si PN Junction in **Reverse Bias** (II)

$$\begin{cases} J_p(x) = \dfrac{qD_p p_{no}}{L_p}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} < 0 & (x \geq x_n) \\ J_n(x) = \dfrac{qD_n n_{po}}{L_n}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} < 0 & (x \leq -x_p) \end{cases}$$

$$\boxed{\mathbf{I_D = qA{n_i}^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)\left(e^{\frac{-qV_a}{kT}} - 1\right) = I_S\left(e^{\frac{-qV_a}{kT}} - 1\right)}} < 0$$

Diode Depletion Width

Depletion Width for **Thermal Equilibrium**



$$E_{max} = \frac{-qN_d x_n}{\epsilon_{Si}} = \frac{-qN_a x_p}{\epsilon_{Si}}$$

Gauss's law:

Electric field   Charge density

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \xrightarrow{\text{1D}} \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

Electric permittivity

- For $-x_p \leq x \leq 0$

$$E(x) = \frac{-qN_a}{\epsilon_{Si}}(x + x_p)$$

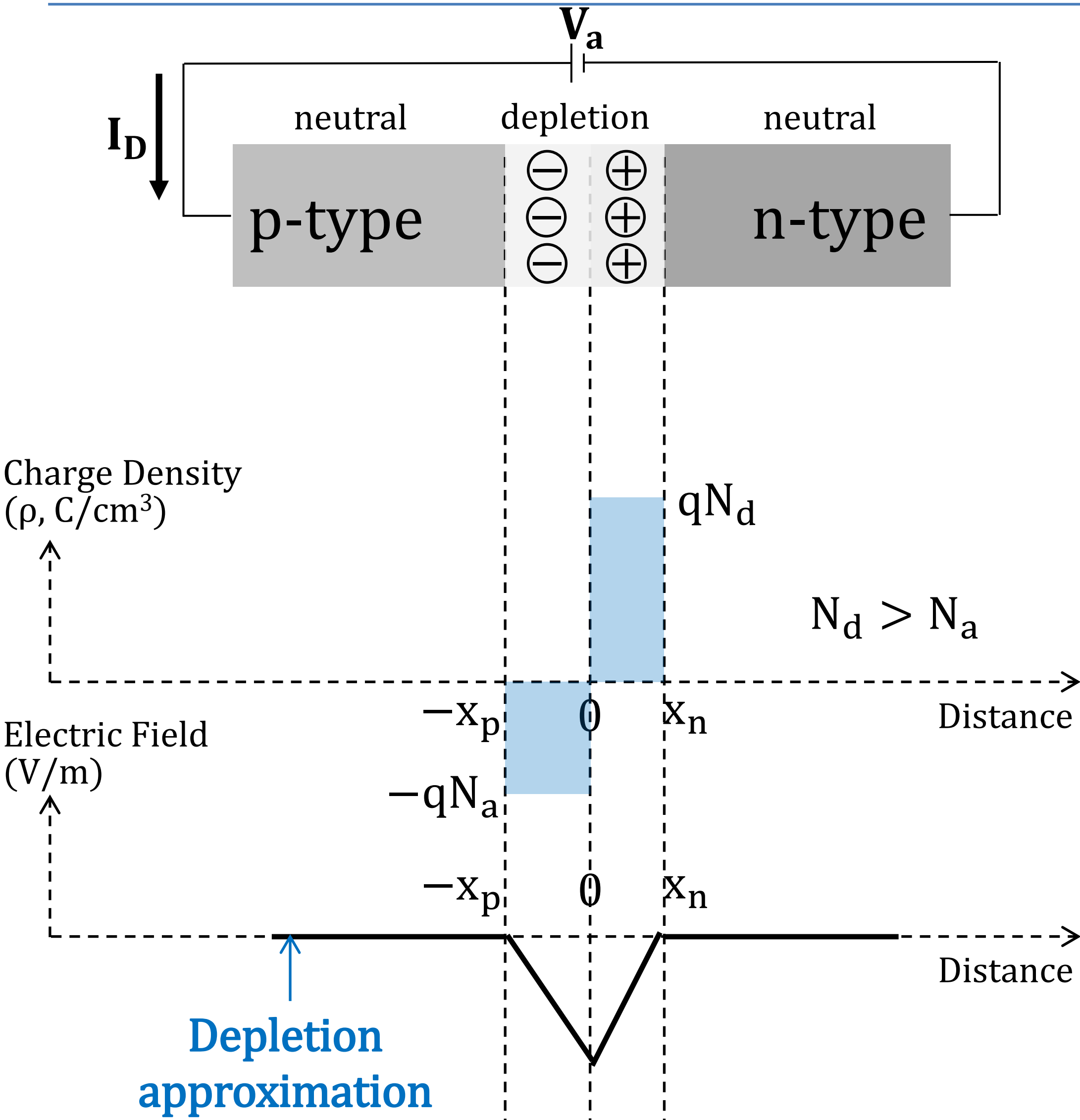
- For $0 \leq x \leq x_n$

$$E(x) = \frac{qN_d}{\epsilon_{Si}}x + \frac{-qN_a x_p}{\epsilon_{Si}}$$

$$\begin{cases} \phi_i = \frac{1}{2}E_{max}(x_n + x_p) \\ qN_a x_p = qN_d x_n \end{cases}$$

$$\boxed{\begin{aligned} x_d &= x_n + x_p \\ &= \left[\frac{2\epsilon_{Si}}{q}\phi_i\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} \end{aligned}}$$

Depletion Width for **Forward Bias**



$$E_{max} = \frac{-qN_d x_n}{\epsilon_{Si}} = \frac{-qN_a x_p}{\epsilon_{Si}}$$

Gauss's law:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \xrightarrow{\text{1D}} \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

- For $-x_p \leq x \leq 0$

$$E(x) = \frac{-qN_a}{\epsilon_{Si}}(x + x_p)$$

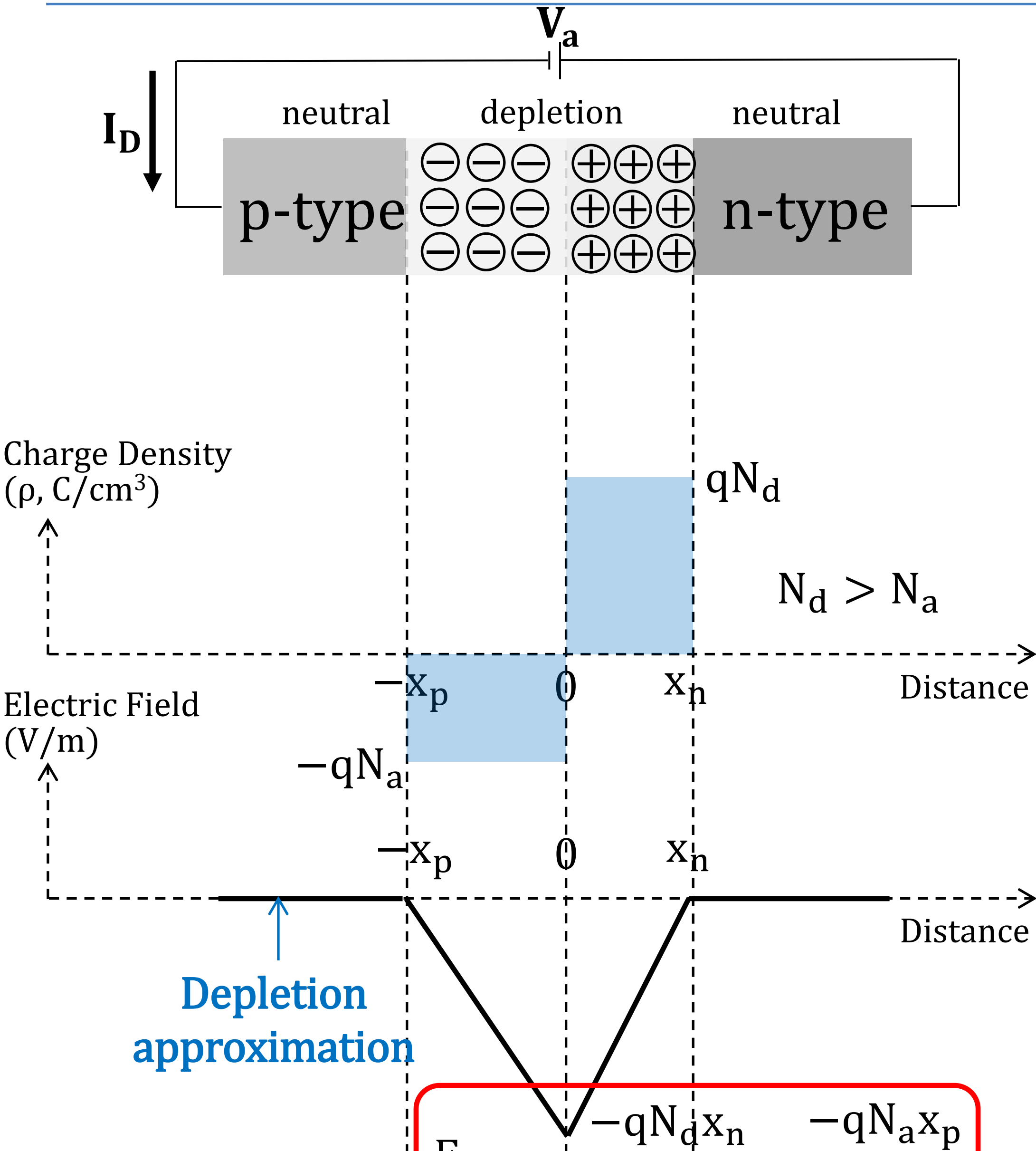
- For $0 \leq x \leq x_n$

$$E(x) = \frac{qN_d}{\epsilon_{Si}} x + \frac{-qN_a x_p}{\epsilon_{Si}}$$

$$\begin{cases} (\phi_i - V_a) = \dfrac{1}{2} E_{max}(x_n + x_p) \\ qN_a x_p = qN_d x_n \end{cases}$$

$$x_d = x_n + x_p$$
$$= \left[\frac{2\epsilon_{Si}}{q}(\phi_i - V_a)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

Depletion Width for **Reverse Bias**



$$E_{max} = \frac{-qN_d x_n}{\epsilon_{Si}} = \frac{-qN_a x_p}{\epsilon_{Si}}$$

Gauss's law:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \xrightarrow{\text{1D}} \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

- For $-x_p \leq x \leq 0$

$$E(x) = \frac{-qN_a}{\epsilon_{Si}}(x + x_p)$$

- For $0 \leq x \leq x_n$

$$E(x) = \frac{qN_d}{\epsilon_{Si}}x + \frac{-qN_a x_p}{\epsilon_{Si}}$$

$$\begin{cases} (\phi_i + V_a) = \dfrac{1}{2}E_{max}(x_n + x_p) \\ qN_a x_p = qN_d x_n \end{cases}$$

$$x_d = x_n + x_p$$

$$= \left[\frac{2\epsilon_{Si}}{q}(\phi_i + V_a)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

Example

For a silicon diode with $N_a = 10^{17}\ 1/cm^3$, $N_d = 10^{20}\ 1/cm^3$ and $V_a = 0$, calculate the ϕ_i, x_n, x_p and E_{max}. ($k = 1.38 \times 10^{-23}$ J/K, $n_i = 10^{10}\ 1/cm^3$ at 300 K, $q = 1.6 \times 10^{-19}$ C, $\epsilon_{Si} = 11.7 \times 8.85 \times 10^{-14}$ F/cm)

$$\phi_i = \phi_n + \phi_p = \frac{kT}{q}\ln\frac{N_d N_a}{{n_i}^2} = \frac{(1.38 \times 10^{-23})(300)}{(1.6 \times 10^{-19})}\ln\frac{10^{20} \times 10^{17}}{(10^{10})^2} = 1.01\ (\mathrm{V})$$

$$x_d = x_n + x_p = \left[\frac{2\epsilon_{Si}}{q}\phi_i\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19}} 1.01\left(\frac{1}{10^{17}} + \frac{1}{10^{20}}\right)}$$

$$= 1.144 \times 10^{-5}\ (\mathrm{cm}) = 0.1144\ (\mu\mathrm{m})$$

$$\begin{cases} x_p = \dfrac{10^{20}}{10^{17} + 10^{20}} \times 0.1144 = 0.1143\ (\mu\mathrm{m}) \\[2ex] x_n = \dfrac{10^{17}}{10^{17} + 10^{20}} \times 0.1144 = 0.0001\ (\mu\mathrm{m}) \end{cases}$$

Almost all depletion region on the p-side

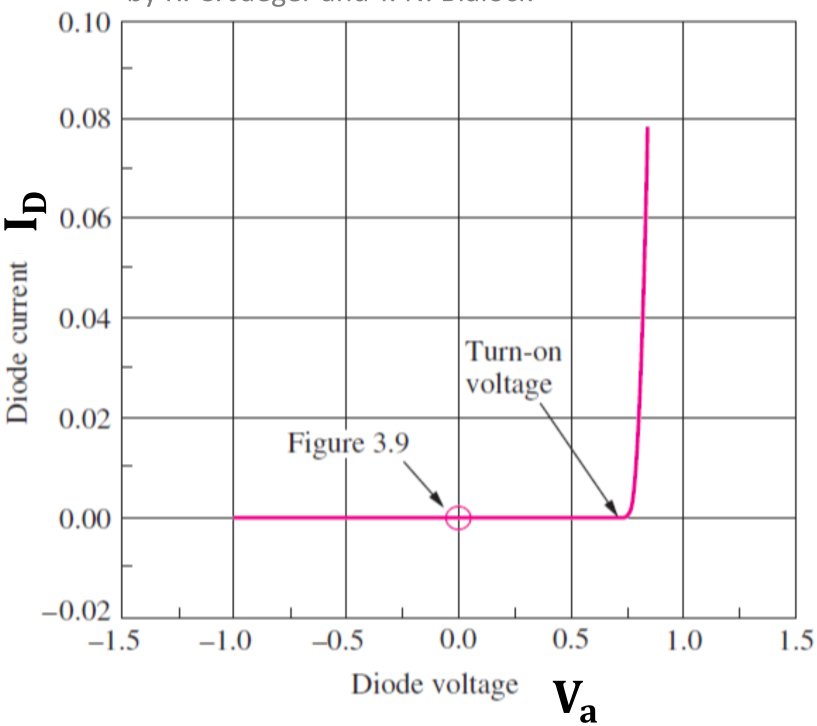
$$E_{max} = \frac{-qN_a x_p}{\epsilon_{Si}} = \frac{-(1.6 \times 10^{-19}) \times 10^{17} \times (1.143 \times 10^{-5})}{11.7 \times 8.85 \times 10^{-14}} = -1.77 \times 10^5\ (\mathrm{V/cm})$$

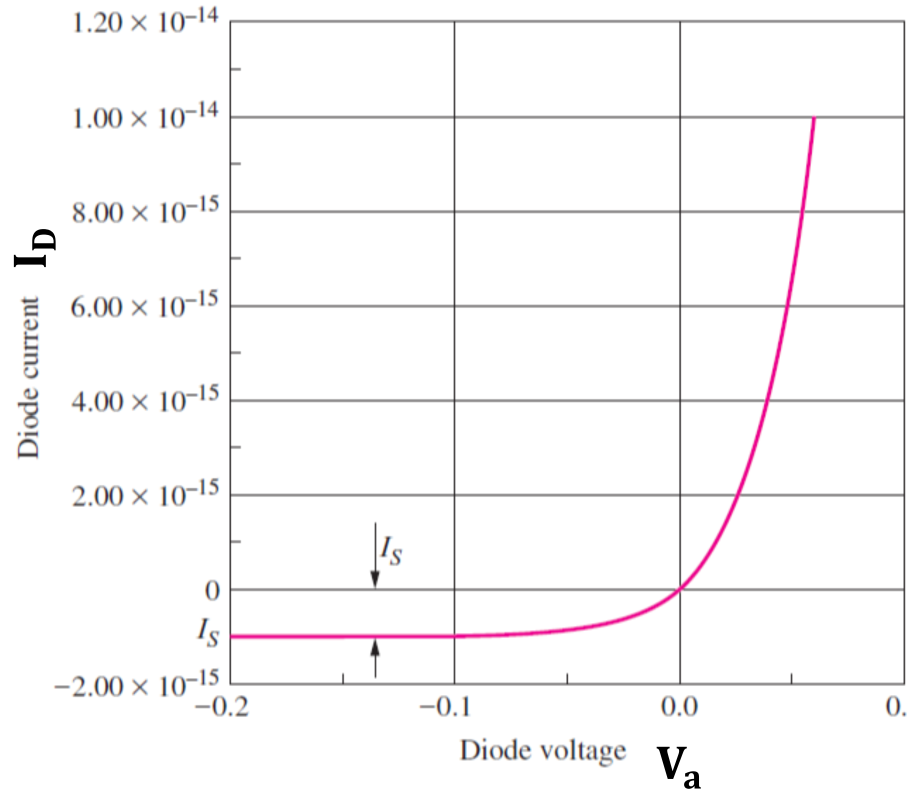
Diode I-V Characteristics

Si Diode I-V Characteristics

$$\mathbf{I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right)}$$

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock





- Turn-on voltage typically 0.5 to 0.7 V
- Saturation current (I_S) typically 10^{-18} to 10^{-9} A
- $kT/q = 0.025875$ V at 300 K
- Check of Results: $\mathbf{qV_a} < \mathbf{q\phi_i} < \mathbf{E_g}(= 1.1 \text{ eV})$

Example

(a) Calculate V_a for a silicon diode with $I_S = 0.1$ fA and I_D increasing from 300 μA to 10 mA at 300 K.

$$300 \times 10^{-6} = (0.1 \times 10^{-15})\left(e^{\frac{V_a}{0.025875}} - 1\right) \qquad V_a = 0.743\ (\text{V})$$

$$10 \times 10^{-3} = (0.1 \times 10^{-15})\left(e^{\frac{V_a}{0.025875}} - 1\right) \qquad V_a = 0.834\ (\text{V})$$

(b) Calculate I_S for a silicon diode with $I_D = 2.5$ mA and $V_a = 0.736$ V at 50°C.

$$2.5 \times 10^{-3} = I_S\left(e^{\frac{(1.6\times10^{-19})\times0.736}{(1.38\times10^{-23})(323)}} - 1\right) \qquad I_S = 8.4 \times 10^{-15}\ (\text{A})$$

Example

Calculate the required V_a for I_D of a silicon diode to increase by a factor 10 at 300 K. Assume $I_D \gg I_S$.

$$\begin{cases} I_{D1} = I_S\left(e^{\frac{qV_{a1}}{kT}} - 1\right) \approx I_S e^{\frac{qV_{a1}}{kT}} \\ I_{D2} = I_S\left(e^{\frac{qV_{a2}}{kT}} - 1\right) \approx I_S e^{\frac{qV_{a2}}{kT}} \end{cases}$$

$$\frac{I_{D2}}{I_{D1}} = 10 = \frac{I_S e^{\frac{qV_{a2}}{kT}}}{I_S e^{\frac{qV_{a1}}{kT}}} = e^{\frac{V_{a2}-V_{a1}}{0.025875}}$$

$$V_{a2} - V_{a1} = 0.025875 \times \ln 10$$
$$= 0.05958\ (V) \approx 60\ (mV)$$

The diode voltage changes by about 60 mV per decade change in diode current.

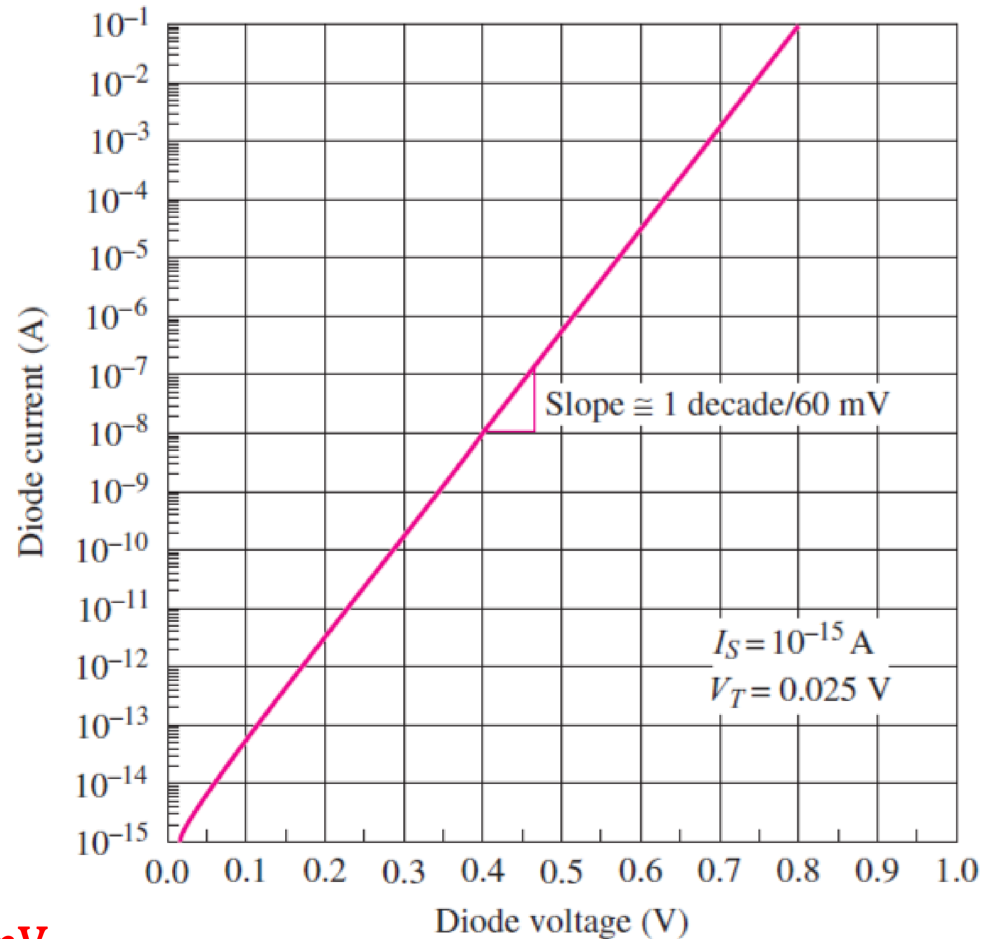
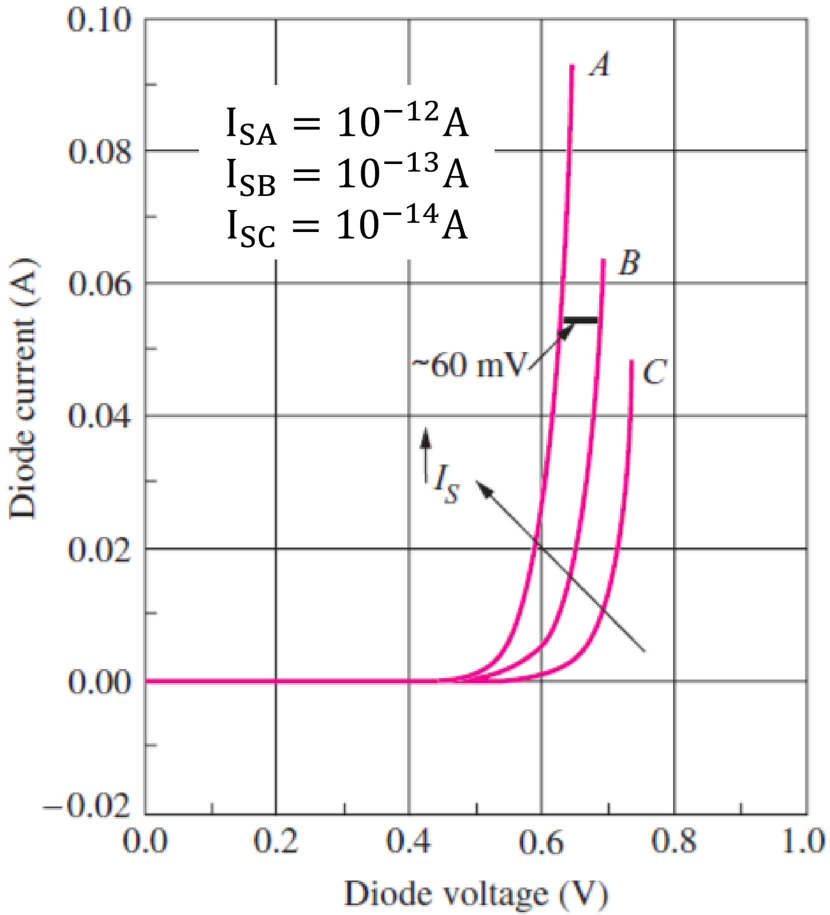


Figure 3.11 Diode i-v characteristic on semilog scale.

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

I_D and I_S versus V_a



Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

$$\mathbf{I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right)}$$

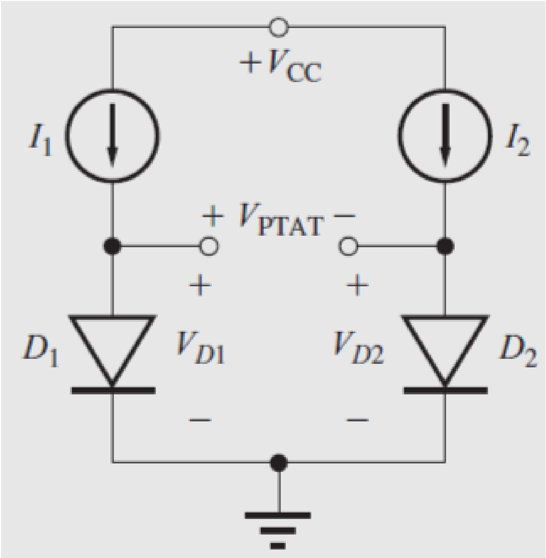
- **At the same I_D, when I_S increases by 10, V_a decreases by 60 mV.**
- **At the same I_S, when I_D increases by 10, V_a increases by 60 mV.**

Diode Temperature Dependence

A Voltage Proportional to Absolute Temperature

- For a fixed $I_D \gg I_S$:

$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) \Rightarrow V_a = \frac{kT}{q}\ln\left(\frac{I_D}{I_S} + 1\right) \cong \frac{kT}{q}\ln\frac{I_D}{I_S}$$



Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

$$\begin{cases} V_{D1} = \dfrac{kT}{q}\ln\dfrac{I_1}{I_S} \\ V_{D2} = \dfrac{kT}{q}\ln\dfrac{I_2}{I_S} \end{cases}$$

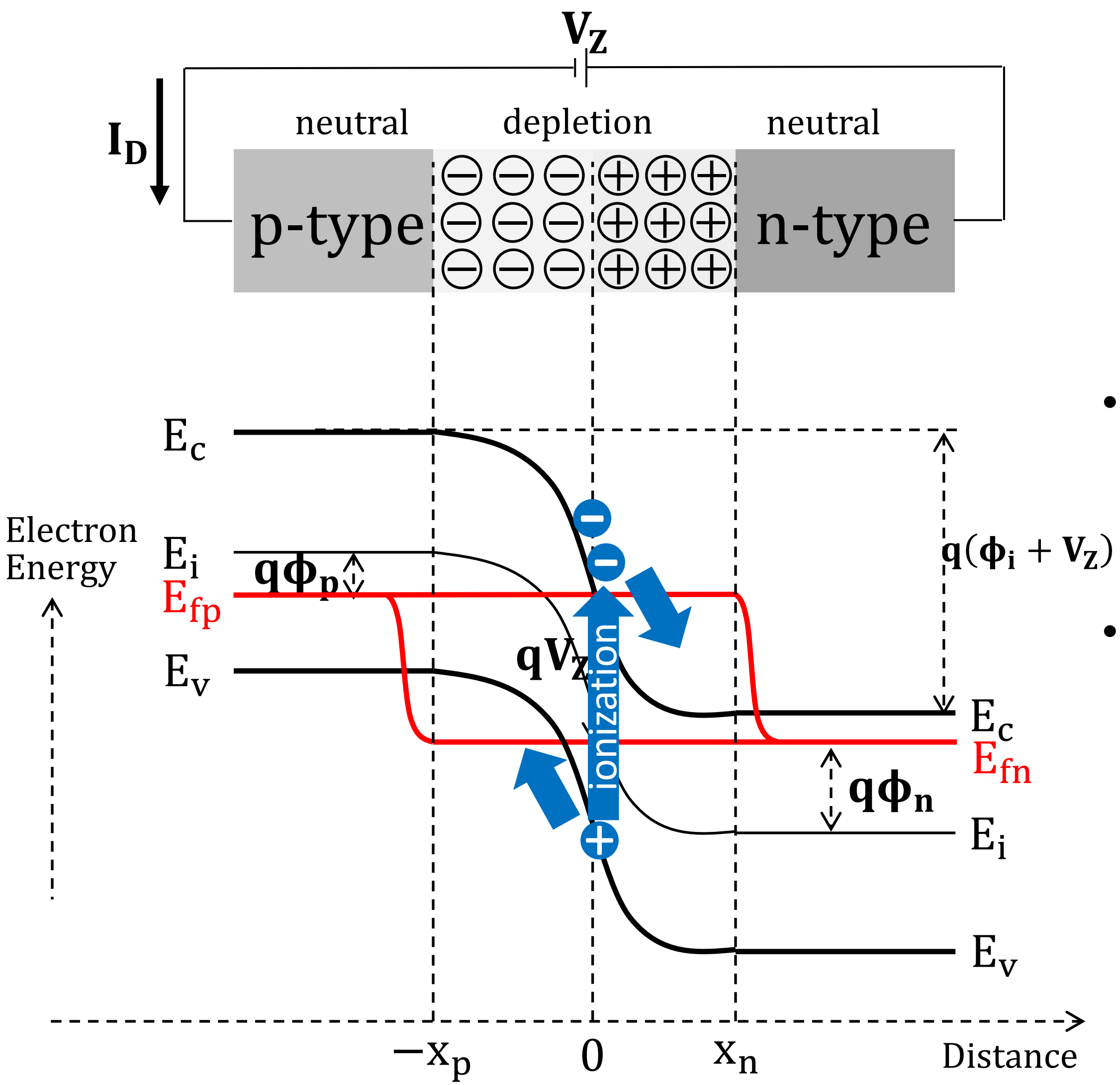
I_1 and I_2 are ideal current source.

$$V_{PTAT} = V_{D1} - V_{D2} = \frac{kT}{q}\ln\frac{I_1}{I_2}$$

$$\frac{dV_{PTAT}}{dT} = \frac{k}{q}\ln\frac{I_1}{I_2} \Longrightarrow \text{a constant}$$

Diode in Reverse Bias

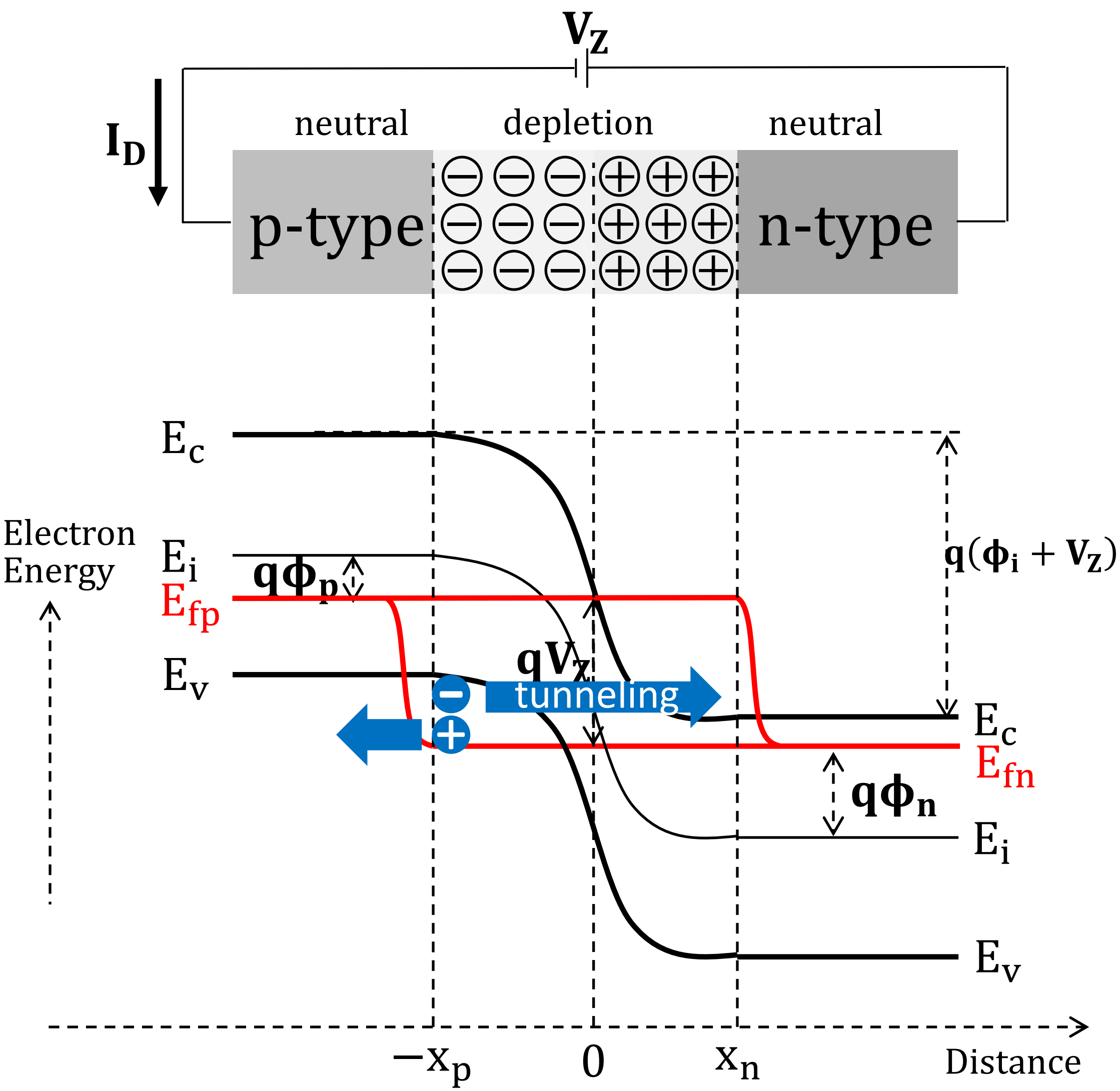
Avalanche Breakdown in Reverse Bias



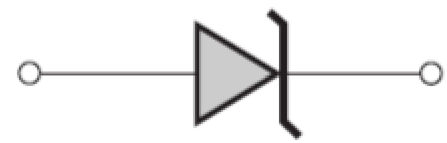
Temperature higher, V_Z higher.

- Si diode with breakdown voltages greater than about 5.6 V enter breakdown through an avalanche mechanism.
- Carriers accelerated by electric field gain sufficient energy to break covalent bonds upon impact, thereby creating electron-hole pairs.

Zener Breakdown in Reverse Bias



Temperature higher, V_Z lower.



- **Si diode with very heavy doping (i.e. very narrow depletion region)** easily enter into Zener breakdown under reverse bias.
- Electrons tunnel directly between valence and conduction bands.

Diode Spice Model and Layout

Diode Spice Model

$$I_D = \mathbf{IS}\left[\exp\left(\frac{qV_a}{\mathbf{N}kT}\right) - 1\right]$$

$$\boxed{C_D = \mathbf{TT}\frac{I_D}{\mathbf{N}(kT/q)} \qquad C_j = \frac{\mathbf{CJO}}{(1 - \frac{V_a}{\mathbf{VJ}})^{\mathbf{M}}}\mathbf{RAREA}}$$

Not covered in Ve311

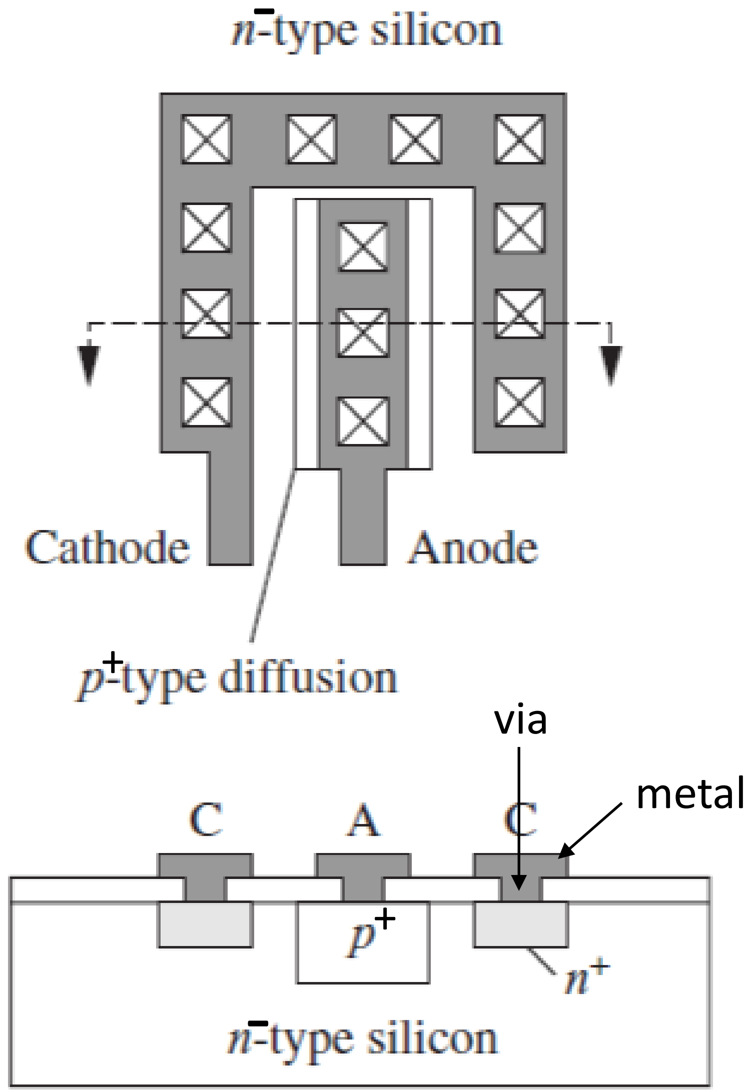
TABLE 3.1

SPICE Diode Parameter Equivalences

PARAMETER	SPICE	TYPICAL DEFAULT VALUES
Saturation current	IS	10 fA
Ohmic series resistance	RS	0 Ω
Ideality factor or emission coefficient	N	1
Transit time	TT	0 sec
Zero-bias junction capacitance for a unit area diode RAREA = 1	CJO	$0\ F/_{m^2}$
Built-in potential	VJ	1 V
Junction grading coefficient	M	0.5
Relative junction area	RAREA	1 m^2

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

Diode Layout

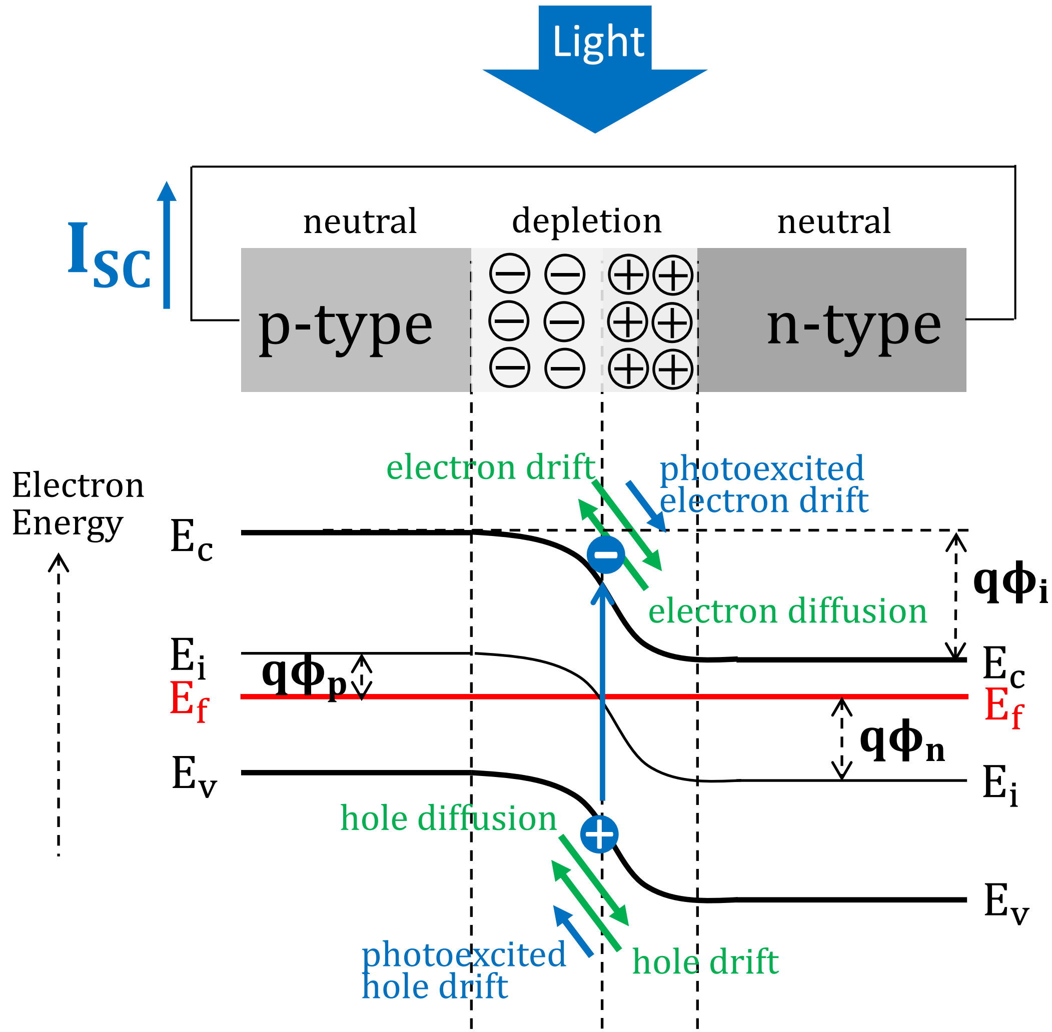


pn junction diode

Source: Microelectronic Circuit Design, 4th Edition,
by R. C. Jaeger and T. N. Blalock

Photodiode / Solar Cell

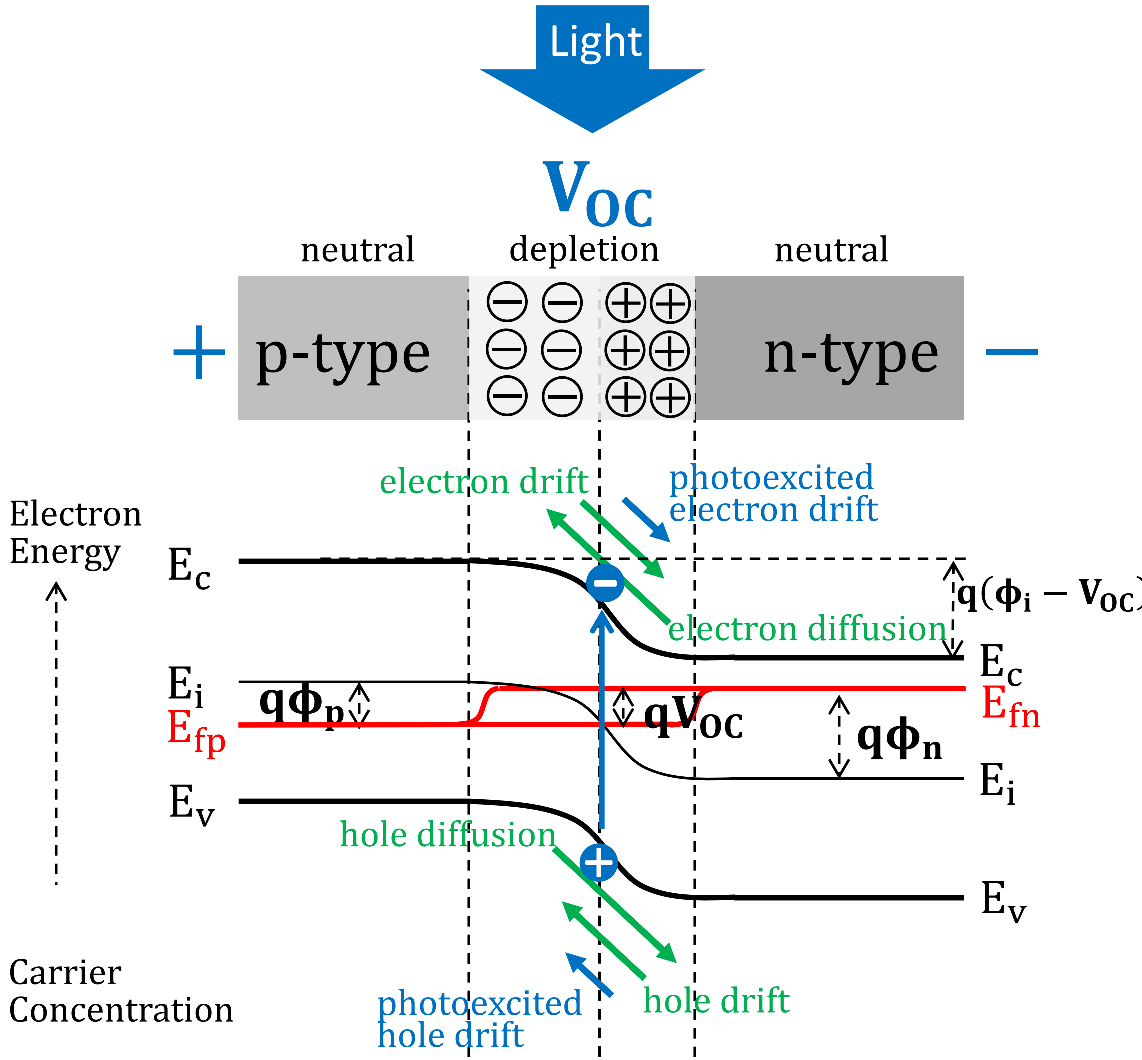
Short Circuit Current



The percentage of incident photons absorbed and converted into electron-hole pairs

$$I_{SC} = \frac{\textbf{Incident Light Power on Depletion Region (W)} \cdot \textbf{Absorption}}{\textbf{Photon Energy (J)}}$$

Open Circuit Voltage

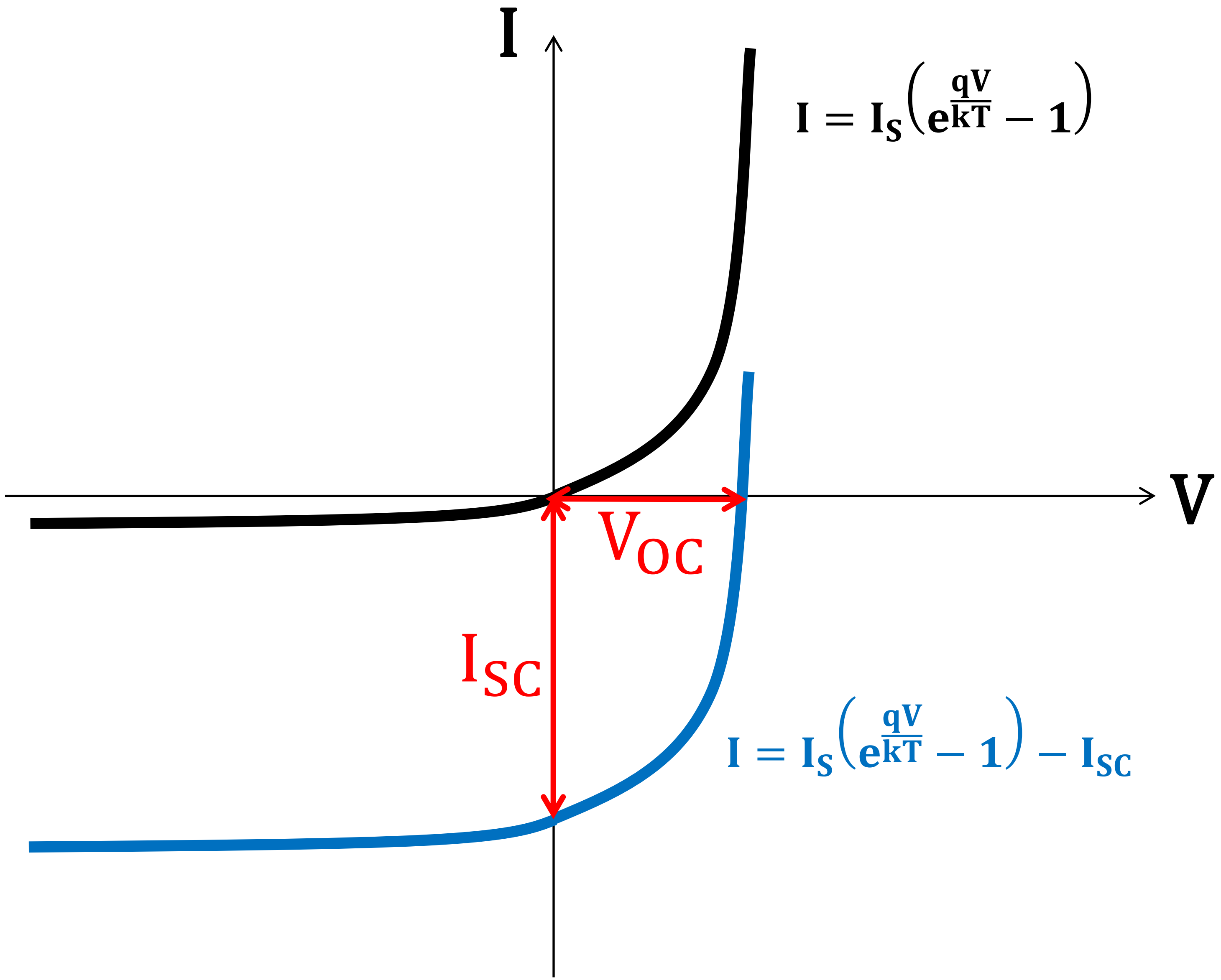


electron drift + photoexcited electron drift = electron diffusion

hole drift + photoexcited hole drift = hole diffusion

$$I_{SC} = I_S\left(e^{\frac{qV_{OC}}{kT}} - 1\right)$$

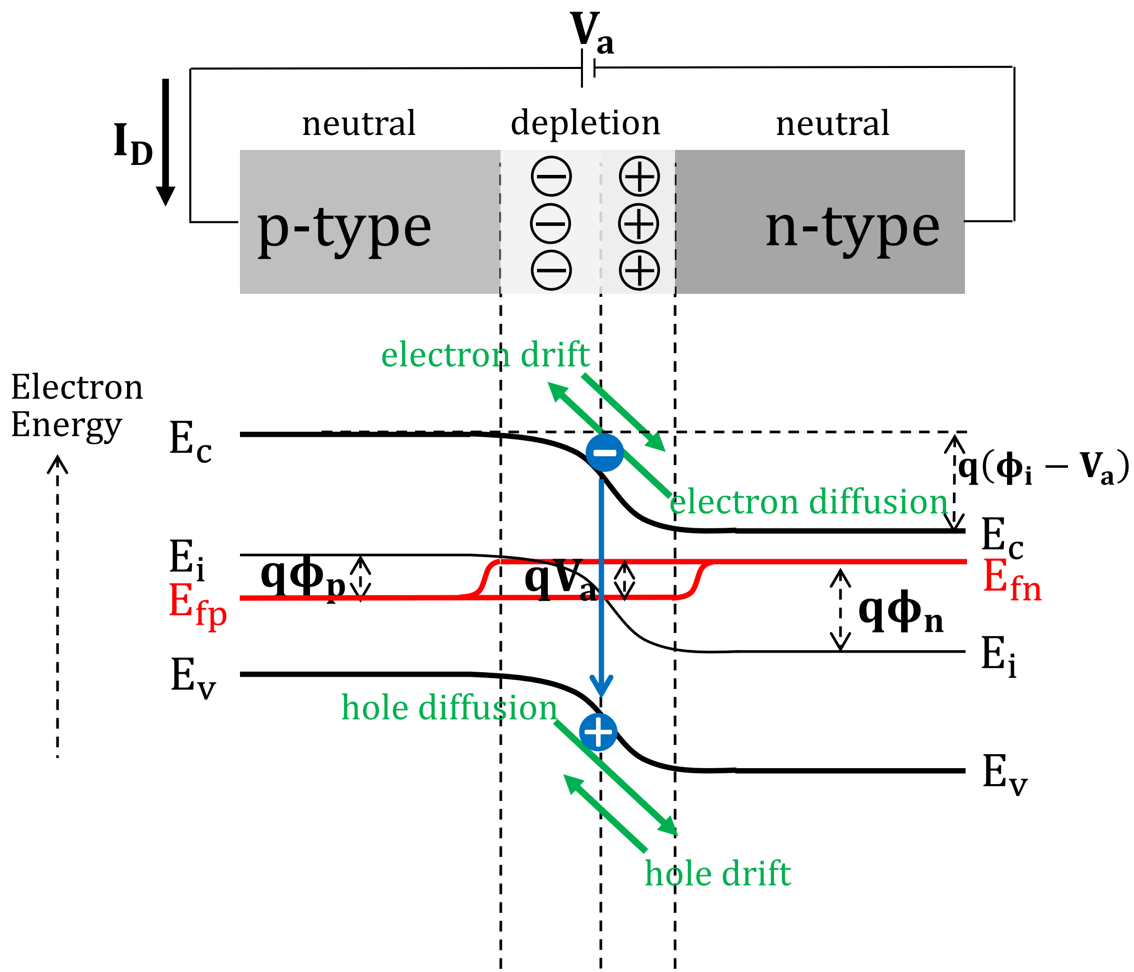
I-V Curve of Photodiode / Solar Cells



What are fill factor and power conversion efficiency?

Light Emitting Diode

Charge Injection



- **Direct bandgap** semiconductor is required. Note that silicon is indirect bandgap semiconductor.
- Under forward bias, charge carriers are **injected and recombined** in the depletion region to emit photons.