Problem Set 6

Statistics 509 – Winter 2022

Due by Wednesday, March 16 by midnight EST

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. (a) Problem 1 on page 489 of Ruppert/Matteson

(b) Problem 2 on page 489 of Ruppert/Matteson.

Note: The data file Stock_Bond.csv is in Data folder under Files in Canvas.

(c) For problem 1 in **(a)**, state the equations that need to be satisfied for each of the assets in order to satisfy the Security Market Line relative to the Tangent portfolio.

(d) Verify that each of the assets does actually satisfy the Security Market Line relative to the Tangent porfolio.

2. (a) Problem 6 on page 513 in Ruppert/Matteson, but in part (c), it should read

If the variance of the return on Stock A is 250%, what percentage of this variance is due to market risk?

(b) Problem 11 on page 514 in Ruppert/Matteson, with the following adjustments of

j	β_j	$\sigma^2_{\epsilon_j}$
1	0.8	0.012
2	0.9	0.025
3	0.7	0.015

and instead of equally weighted portfolio, answer the questions for a portfolio that is 30% in 1 and 2, and 40% in 3.

3. Consider time series model of

$$X_n = U\delta_n + \epsilon_n \qquad\qquad n = 1, 2, \ldots$$

where $\delta_1, \delta_2, \ldots$ are iid rvs with mean 1 and variance of 1, $\epsilon_1, \epsilon_2, \ldots$ are iid rvs with mean 0 and variance of $\frac{1}{4}$ and are independent of $\{\delta_i\}_{i=1}^{\infty}$, and U is Uniform(0,1) rv which is independent of $\{\epsilon_i\}_{i=1}^{\infty}$ and $\{\delta_i\}_{i=1}^{\infty}$.

(a) Derive the mean function and auto-covariance function of the time series $\{X_n\}_{n=1}^{\infty}$ and validate that this time series is weakly stationary.

(b) Show that the time series $\{X_n\}_{n=1}^{\infty}$ is not ergodic. *Hint.* Consider the auto-correlation, and show that it does not satisfy a necessary property for ergodicity.