Problem Set 7

Statistics 509 – Winter 2022

Due on Wednesday, March 23 by midnight – via Canvas

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. The daily adjusted closing price data for Amazon, Apple, Google, and NASDAQ over the past 2 years are in the Data folder on Canvas.

(a) Utilizing a regression analysis, determine the α, β of Amazon, Apple, and Google returns relative to the NASDAQ index returns, and quantify what portion of the variance each of these two assets is market variance. Assume that the risk-free rate of return as 2.5% – or return per trading day is .025/253

(b) Assuming independent regression errors and assuming the regression models in **(a)**, compute the covariance between Amazon, Apple, and Google returns. Derive the empirical covariance between Apple, Apple, and Google, and compare to this regression-derived covariance.

2. Consider the time series from problem 1.

(a) Put the data into a R-time series object, and show time-series plots of the adjusted closing prices and the returns for each of assets Amazon, Apple, Google, and the NASDAQ index. Discuss what the plots show for each of these assets.

(b) Compute/plot the auto-correlation functions for the NASDAQ, Apple, Amazon, and Google adjusted closing prices over the past 2 years. Discuss the plot and what it indicates about whether the closing prices are stationary. *Hint:* Show the ACF for 25 lags.

(c) Compute/plot the auto-correlation functions for the NASDAQ, Apple, Amazon, and Google returns over the past 2 years. Discuss the plot and what it indicates about whether the returns are stationary. *Hint:* Show the ACF for 25 lags

3. Suppose $Y_1, Y_2, \ldots$ and $Z_1, Z_2, \ldots$ are 2 stationary AR(1) processes with means μ_Z, μ_Y and autocovariances $\gamma_Z(1), \gamma_Y(1)$ given by

$$\gamma_Y(1) = \frac{\sigma_Y^2 \alpha_Y}{1 - \alpha_Y^2}, \ , \gamma_Z(1) = \frac{\sigma_Z^2 \alpha_Z}{1 - \alpha_Z^2}$$

which are independent of each other.

(a) Show that $X_n = Y_n - Z_n$ is a stationary process and derive its mean, auto-covariance, and auto-correlation functions.

(b) Show that $X'_n = Y_n(1+Z_n)$ is stationary process and derive its mean and auto-covariance functions.

(c) Suppose have a time series defined as

$$X_n = Y_n + Y_{n+1} \qquad\qquad n = 1, 2, \ldots$$

Verify that this new time series is stationary, and derive the mean and autocovariance functions.