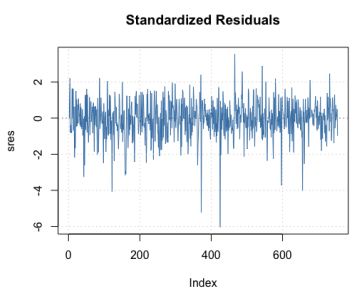
STATS 509 Winter 2022, Solution to Homework 10

1

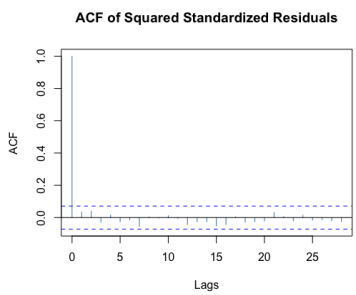
Now we look at AR(p)-GARCH(1,1) models for $p = 1, 2, 3$ for both normal distribution and the t-distribution - summary results from all of these estimations are shown below. Below is table of the AIC results.

Model	AIC
AR(1)-GARCH $(1, 1)$ - normal	-7.11771
AR(2)-GARCH $(1, 1)$ - normal	-7.11793
AR(3)-GARCH $(1, 1)$ - normal	-7.11741
AR(1)-GARCH $(1, 1)$ - t-dist	-7.18761
AR(2)-GARCH $(1, 1)$ - t-dist	-7.19181
AR(3)-GARCH $(1, 1)$ - t-dist	-7.19184

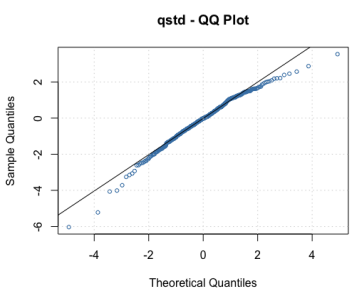
Based on these results, and looking at the AIC values, it appears that the t-distribution with the AR(2) - GARCH$(1, 1)$ model is a reasonable pick - both of the AR coefficients, though not very big, are close to being significant with p-valuese of .06 and .07 respectively, and it had close to the lowest AIC score of all of the 6 models - that AIC score was -7.191809 and this was significantly below the AR(1)-GARCH $(1, 1)$ model with t-distribution, and even more significantly below that of all the AR-GARCH results with the normal distribution. It is slightly larger than the the AR(3) − GARCH$(1, 1)$ had but it was very close. The diagnostic plots for this model are shown below - plot of st. residuals, a ACF plot for the squared residuals, and the QQ plot is shown below. They look actually somewhat similar to the GARCH model. Using this model, and the same approach as in part 2, we compute the relative VaR as 0.0115713 - all of these computations are in the R-output below.



ACF of log-returns



ACF of sq. stand. residuals



QQ plot of stand. residuals

Another reasonable alternative is that of the AR(1)-GARCH(1,1) model with a slightly higher AIC value, but a slightly simpler model.

R codes:

```
> X = read.csv("NYA-2015-2017.csv",header=TRUE)
> NYSE_lret = diff(log(X$AdjClose))
> NYSE_lret.ts <- ts(data=NYSE_lret,start=c(2015,1),frequency=252,names=c('logret'))
> library("fGarch")
Loading required package: timeDate
Loading required package: timeSeries
Loading required package: fBasics


> X.ts = NYSE_lret
> NYA_GARCH_ar1 <- garchFit(~arma(1,0) + garch(1,1), data=X.ts, cond.dist = c("norm"),
+                           include.mean = TRUE)


> summary(NYA_GARCH_ar1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = X.ts, cond.dist = c("norm"),
    include.mean = TRUE)

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x131b35f68>
 [data = X.ts]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1        omega       alpha1
 4.7484e-04  -7.3785e-02   2.9095e-06   1.7843e-01
      beta1
 7.7845e-01

Std. Errors:
 based on Hessian
```

```
Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      4.748e-04   2.116e-04    2.244  0.02485 *
ar1    -7.378e-02   4.015e-02   -1.838  0.06613 .
omega   2.909e-06   9.520e-07    3.056  0.00224 **
alpha1  1.784e-01   3.847e-02    4.638 3.52e-06 ***
beta1   7.784e-01   4.331e-02   17.974  < 2e-16 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 2688.378    normalized:  3.565488

Description:
 Sun Apr 10 15:55:30 2022 by user:


Standardised Residuals Tests:
                               Statistic p-Value
 Jarque-Bera Test   R    Chi^2  228.817   0
 Shapiro-Wilk Test  R    W      0.9718387 7.146929e-11
 Ljung-Box Test     R    Q(10)  7.924185  0.6362424
 Ljung-Box Test     R    Q(15)  13.45908  0.5668843
 Ljung-Box Test     R    Q(20)  23.39607  0.2697675
 Ljung-Box Test     R^2  Q(10)  6.58365   0.764079
 Ljung-Box Test     R^2  Q(15)  9.768791  0.834026
 Ljung-Box Test     R^2  Q(20)  11.06016  0.9446475
 LM Arch Test       R    TR^2   7.467708  0.8252231

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.117713 -7.087040 -7.117800 -7.105897

> NYA_GARCH_ar2 <- garchFit(~arma(2,0) + garch(1,1), data=X.ts, cond.dist = c("norm"),
+                           include.mean = TRUE)

> summary(NYA_GARCH_ar2)

Title:
 GARCH Modelling

Call:
```

```
 garchFit(formula = ~arma(2, 0) + garch(1, 1), data = X.ts, cond.dist = c("norm"),
    include.mean = TRUE)

Mean and Variance Equation:
 data ~ arma(2, 0) + garch(1, 1)
<environment: 0x137f27e00>
 [data = X.ts]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1          ar2        omega
 5.0278e-04  -8.0375e-02  -4.0422e-02   2.9095e-06
     alpha1        beta1
 1.7628e-01   7.8026e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      5.028e-04   2.137e-04    2.353  0.01862 *
ar1    -8.037e-02   4.051e-02   -1.984  0.04726 *
ar2    -4.042e-02   3.996e-02   -1.012  0.31174
omega   2.909e-06   9.581e-07    3.037  0.00239 **
alpha1  1.763e-01   3.825e-02    4.608 4.06e-06 ***
beta1   7.803e-01   4.350e-02   17.936  < 2e-16 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 2689.459    normalized:  3.566922

Description:
 Sun Apr 10 15:55:31 2022 by user:


Standardised Residuals Tests:
                             Statistic p-Value
 Jarque-Bera Test   R    Chi^2  238.3177  0
 Shapiro-Wilk Test  R    W      0.9709043 4.217865e-11
```

```
 Ljung-Box Test     R    Q(10)  8.543769  0.5758776
 Ljung-Box Test     R    Q(15)  14.77115  0.4680238
 Ljung-Box Test     R    Q(20)  24.09751  0.2381588
 Ljung-Box Test     R^2  Q(10)  6.089421  0.8076954
 Ljung-Box Test     R^2  Q(15)  9.303371  0.8611284
 Ljung-Box Test     R^2  Q(20)  10.54373  0.9572081
 LM Arch Test       R    TR^2   7.109923  0.8502646

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.117928 -7.081121 -7.118053 -7.103749

> NYA_GARCH_ar3 <- garchFit(~arma(3,0) + garch(1,1), data=X.ts, cond.dist = c("norm"),
+                           include.mean = TRUE)

> summary(NYA_GARCH_ar3)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(3, 0) + garch(1, 1), data = X.ts, cond.dist = c("norm"),
    include.mean = TRUE)

Mean and Variance Equation:
 data ~ arma(3, 0) + garch(1, 1)
<environment: 0x123452bc0>
 [data = X.ts]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1          ar2          ar3
 5.2511e-04  -7.7909e-02  -4.2805e-02  -4.5964e-02
      omega       alpha1        beta1
 2.8917e-06   1.7363e-01   7.8250e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
```

```
mu      5.251e-04   2.158e-04    2.433  0.01496 *
ar1    -7.791e-02   4.055e-02   -1.922  0.05467 .
ar2    -4.280e-02   4.014e-02   -1.066  0.28627
ar3    -4.596e-02   3.934e-02   -1.168  0.24263
omega   2.892e-06   9.518e-07    3.038  0.00238 **
alpha1  1.736e-01   3.708e-02    4.683 2.83e-06 ***
beta1   7.825e-01   4.281e-02   18.280  < 2e-16 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 2690.262    normalized:  3.567987

Description:
 Sun Apr 10 15:55:32 2022 by user:


Standardised Residuals Tests:
                             Statistic p-Value
 Jarque-Bera Test   R    Chi^2  238.3422  0
 Shapiro-Wilk Test  R    W      0.9707721 3.918133e-11
 Ljung-Box Test     R    Q(10)  10.53441  0.3949215
 Ljung-Box Test     R    Q(15)  16.71741  0.3360365
 Ljung-Box Test     R    Q(20)  25.88096  0.1697794
 Ljung-Box Test     R^2  Q(10)  6.377277  0.7826331
 Ljung-Box Test     R^2  Q(15)  9.419548  0.8545794
 Ljung-Box Test     R^2  Q(20)  10.78503  0.9516079
 LM Arch Test       R    TR^2   7.271708  0.839148

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.117406 -7.074465 -7.117576 -7.100864


> NYA_GARCH_ar1t <- garchFit(~arma(1,0) + garch(1,1), data=X.ts, cond.dist = c("std"),
+                            include.mean = TRUE)

> summary(NYA_GARCH_ar1t)

Title:
 GARCH Modelling
```

```
Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = X.ts, cond.dist = c("std"),
    include.mean = TRUE)

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x134cca530>
 [data = X.ts]

Conditional Distribution:
 std

Coefficient(s):
         mu          ar1        omega       alpha1
 5.3635e-04  -6.1030e-02   1.0411e-06   1.4151e-01
      beta1        shape
 8.5202e-01   5.3640e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      5.364e-04   1.919e-04    2.795 0.005190 **
ar1    -6.103e-02   3.728e-02   -1.637 0.101605
omega   1.041e-06   6.506e-07    1.600 0.109569
alpha1  1.415e-01   4.151e-02    3.409 0.000653 ***
beta1   8.520e-01   4.114e-02   20.709  < 2e-16 ***
shape   5.364e+00   1.024e+00    5.239 1.62e-07 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 2715.727    normalized:  3.60176

Description:
 Sun Apr 10 15:56:10 2022 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  368.3945  0
```

```
 Shapiro-Wilk Test  R    W      0.9655408 2.480448e-12
 Ljung-Box Test     R    Q(10)  7.557796  0.6719426
 Ljung-Box Test     R    Q(15)  12.96903  0.6046905
 Ljung-Box Test     R    Q(20)  21.231    0.3836502
 Ljung-Box Test     R^2  Q(10)  6.42774   0.7781381
 Ljung-Box Test     R^2  Q(15)  10.659    0.7763713
 Ljung-Box Test     R^2  Q(20)  13.57848  0.8512091
 LM Arch Test       R    TR^2   7.414317  0.8290643

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.187606 -7.150799 -7.187731 -7.173427

> NYA_GARCH_ar2t <- garchFit(~arma(2,0) + garch(1,1), data=X.ts, cond.dist = c("std"),
+                            include.mean = TRUE)

> summary(NYA_GARCH_ar2t)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 0) + garch(1, 1), data = X.ts, cond.dist = c("std"),
    include.mean = TRUE)

Mean and Variance Equation:
 data ~ arma(2, 0) + garch(1, 1)
<environment: 0x107b46bf0>
 [data = X.ts]

Conditional Distribution:
 std

Coefficient(s):
         mu          ar1          ar2        omega
 5.8666e-04  -6.8529e-02  -6.6366e-02   9.5398e-07
     alpha1        beta1        shape
 1.3613e-01   8.5924e-01   5.1526e+00

Std. Errors:
 based on Hessian

Error Analysis:
```

```
         Estimate  Std. Error  t value Pr(>|t|)
mu      5.867e-04   1.932e-04    3.037 0.002393 **
ar1    -6.853e-02   3.728e-02   -1.838 0.066000 .
ar2    -6.637e-02   3.741e-02   -1.774 0.076021 .
omega   9.540e-07   6.219e-07    1.534 0.125060
alpha1  1.361e-01   4.098e-02    3.322 0.000893 ***
beta1   8.592e-01   4.020e-02   21.373  < 2e-16 ***
shape   5.153e+00   9.598e-01    5.369 7.93e-08 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 2718.312    normalized:  3.605188

Description:
 Sun Apr 10 15:56:11 2022 by user:


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  404.8354  0
 Shapiro-Wilk Test  R    W      0.9634088 8.71655e-13
 Ljung-Box Test     R    Q(10)  9.211821  0.512126
 Ljung-Box Test     R    Q(15)  15.05762  0.4472751
 Ljung-Box Test     R    Q(20)  22.26301  0.3264111
 Ljung-Box Test     R^2  Q(10)  5.999422  0.8153118
 Ljung-Box Test     R^2  Q(15)  10.43818  0.7913098
 Ljung-Box Test     R^2  Q(20)  13.35604  0.8615907
 LM Arch Test       R    TR^2   7.212563  0.8432526

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.191809 -7.148867 -7.191979 -7.175267

> NYA_GARCH_ar3t <- garchFit(~arma(3,0) + garch(1,1), data=X.ts, cond.dist = c("std"),
+                            include.mean = TRUE)
> summary(NYA_GARCH_ar3t)

Title:
 GARCH Modelling

Call:
```

```
 garchFit(formula = ~arma(3, 0) + garch(1, 1), data = X.ts, cond.dist = c("std"),
    include.mean = TRUE)

Mean and Variance Equation:
 data ~ arma(3, 0) + garch(1, 1)
<environment: 0x1235af920>
 [data = X.ts]

Conditional Distribution:
 std

Coefficient(s):
         mu          ar1          ar2          ar3
 5.9420e-04  -6.7602e-02  -6.3636e-02  -2.5871e-02
      omega       alpha1        beta1        shape
 9.8287e-07   1.3734e-01   8.5810e-01   5.0970e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      5.942e-04   1.954e-04    3.041 0.002358 **
ar1    -6.760e-02   3.731e-02   -1.812 0.070012 .
ar2    -6.364e-02   3.746e-02   -1.699 0.089377 .
ar3    -2.587e-02   3.639e-02   -0.711 0.477103
omega   9.829e-07   6.383e-07    1.540 0.123588
alpha1  1.373e-01   4.138e-02    3.319 0.000903 ***
beta1   8.581e-01   4.066e-02   21.107  < 2e-16 ***
shape   5.097e+00   9.514e-01    5.357 8.45e-08 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 2719.324    normalized:  3.60653

Description:
 Sun Apr 10 15:56:12 2022 by user:


Standardised Residuals Tests:
                                Statistic p-Value
```

```
 Jarque-Bera Test   R    Chi^2  400.0432  0
 Shapiro-Wilk Test  R    W      0.9635616 9.382022e-13
 Ljung-Box Test     R    Q(10)  9.69665   0.4674981
 Ljung-Box Test     R    Q(15)  15.77517  0.3971494
 Ljung-Box Test     R    Q(20)  22.80934  0.2982529
 Ljung-Box Test     R^2  Q(10)  6.207254  0.7975604
 Ljung-Box Test     R^2  Q(15)  10.54818  0.7839161
 Ljung-Box Test     R^2  Q(20)  13.49547  0.8551288
 LM Arch Test       R    TR^2   7.367963  0.8323704

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.191840 -7.142764 -7.192062 -7.172935

> plot(NYA_GARCH_ar2t,which=9)
> plot(NYA_GARCH_ar2t,which=11)
> plot(NYA_GARCH_ar2t,which=13)
> sigma_tdistv = NYA_GARCH_ar2t@sigma.t
> sigma_tdist = sigma_tdistv[754]
> tdistquant_lret = -qstd(.005,mean = 5.867*10^(-4),sd = sigma_tdist,nu=5.153)
> VaR_tdist = exp(tdistquant_lret)-1
> VaR_tdist
[1] 0.0115713
```

2

(a)

Stability of this model corresponds to having the stochastic volatility model being stationary. Since $\log(H_n)$ is following an ARMA(p, q) model, the requirement on the coefficients for stability is that these coefficients correspond to a stationary ARMA model. One sufficient (but not necessary) condition for that is that $\sum_{j=1}^{p} |\phi_j| < 1$.

(b)

For $p = 1$ and $q = 0$, we have that the $\log(H_n)$ is an AR(1) model with normally distributed errors, and thus has a normal distribution and based on work in class it satisfies that

$$E(\log(H_n)) = \frac{\alpha_o}{1 - \alpha_1}$$
$$V(\log(H_n)) = \frac{\sigma_V^2}{1 - \alpha_1^2}$$

Thus we have mean of A_n is given by

$$\begin{aligned} E(A_n) &= E(\sqrt{H_n}\epsilon_n) \\ &= E\left(\sqrt{H_n}\right) E(\epsilon_n) \\ &= E\left(\sqrt{H_n}\right) \cdot 0 = 0 \end{aligned}$$

and thus

$$\begin{aligned} V(A_n) = E(A_n^2) &= E(H_n \epsilon_n^2) \\ &= E\left(e^{\log(H_n)}\right) E(\epsilon_n^2) \\ &= e^{E(\log(H_n)) + \frac{V(\log(H_n))}{2}} \cdot 1 \\ &= e^{\frac{\alpha_o}{1-\alpha_1} + \frac{\sigma_V^2}{2(1-\alpha_1^2)}} \end{aligned}$$