Problem Set 4

Statistics 509 – Winter 2022

Due on Wednesday, February 9 by midnight – via Canvas

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. In the Data directory is historical daily price data from Russell 2000 from the past 5 years – it in file "Rus2000_daily_Feb3_2017-Feb3_2022.csv" .

(a) Derive the returns based on the adjusted closing price data and generate a plot of these returns, and summarize interesting features of this plot.

(b) Compute the median, mean, variance, skewness, and kurtosis of the returns and give a brief summary of interesting data features discovered based on these descriptive statistics. Also, generate a plot of the kernel density estimate of the probability density function of the returns, and summarize your features of interest from this as well.

(c) Compute VaR for 1 million dollars invested in Russell 2000 at the level $q = .005$ utilizing for the case of fitting a normal distribution to the returns (not using any POT).

(d) For the Russell 2000 returns, fit a Generalized Pareto Distribution (GPD for short) to the upper tail of the negative returns and give detailed plots of the fit (along the lines of what is generated in class) and discuss your results.

(e) Compute VaR for 1 million dollars invested in Russell 2000 at the level $q = .005$ utilizing the GPD distributional fits generated in part **(d)**. Discuss the comparison of this result with result from part **(c)**.

(f) Utilizing 'quant" command in evir package, give a discussion on the stability of the VaR generated from semiparametric GPD model as function of threshold.

(g) Compute expected shortfalls for models in parts **(c)** and **(e)**.

2. (a) Exercise 5 on page 181 in Ruppert/Matteson.

(b) Derive the Spearman correlation of X and Y from part **(a)**.

(c) Exercise 7 on page 181 in Ruppert/Matteson. *Hint:* Use eigen command in R to carry out eigenvalue/vector analysis of the matrix, and find a vector w so that formula for the variance of $w^\top X$ would be negative showing a contradiction if this really were a covariance matrix for $a = 0$.

(d) Determine lower limit on a in part **(c)** ensuring that the matrix is a covariance matrix.