HW04

Chongdan Pan

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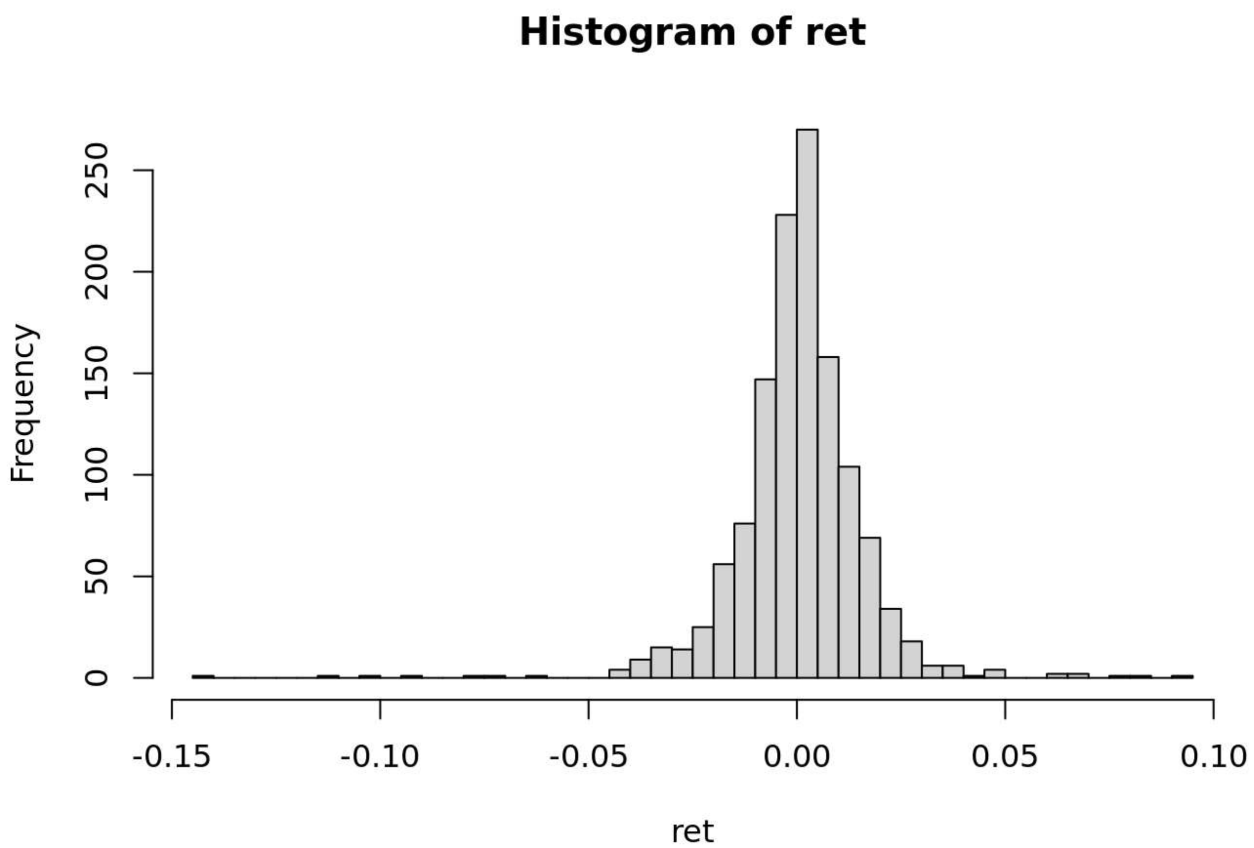
Problem 1

(a)

```
df <- read.csv("../Rus2000_daily_Feb3_2017-Feb3_2022.csv", header=TRUE)
ret = df$Adj.Close[2:nrow(df)] / df$Adj.Close[1:nrow(df)-1] - 1
hist(ret, breaks=50)
```



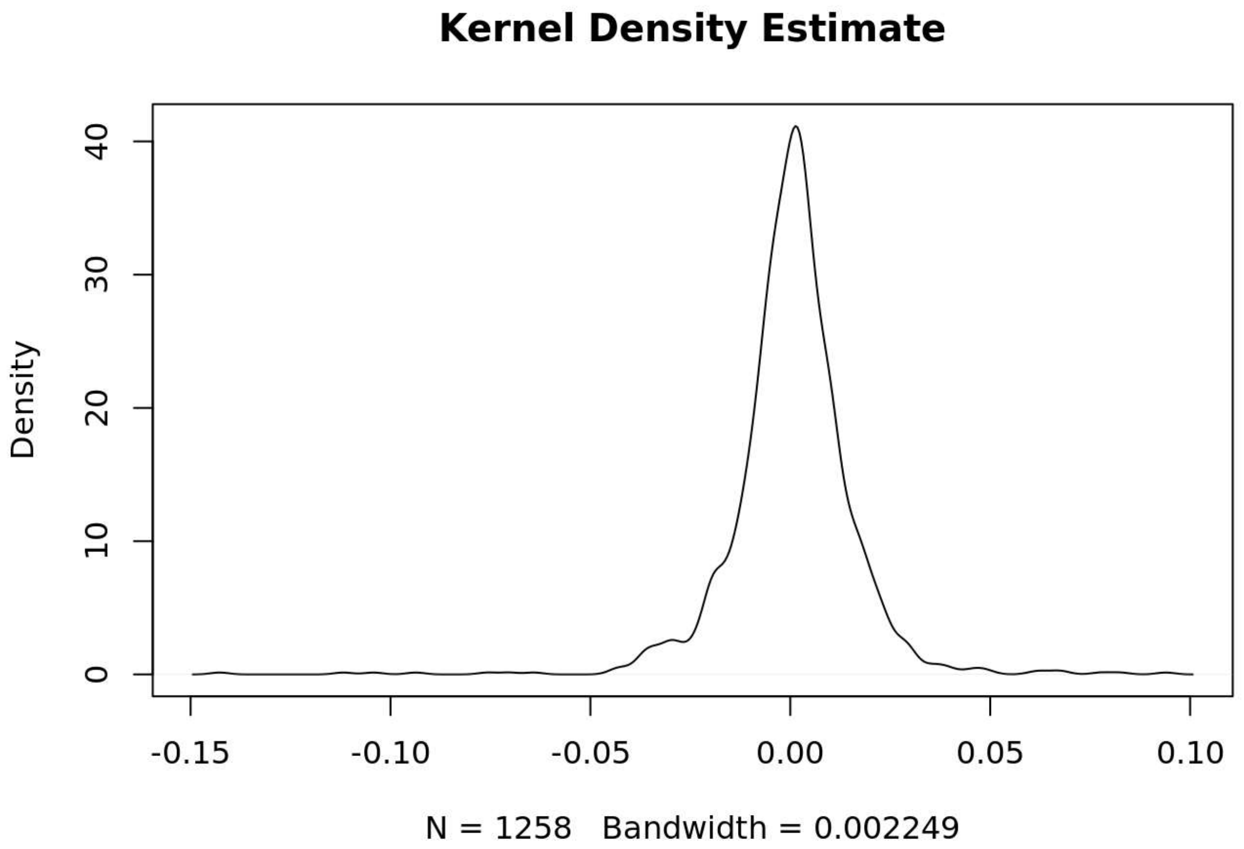
The return seems to be symmetric to the 0, and it has a high kurtosis since there a lot of data in near 0, therefore it distribution may have a heavy tail although we don't have a lot of samples with high absolute value.

(b)

median	mean	variance	skewness	kurtosis
0.0008414	0.0004222	0.0002433763	-0.9844468	13.37257

The mean, median, and skewness are close to zero, so the data is symmetric to 0. The kurtosis is very high, implying that the data may have a heavy tail.

```
plot(density(ret), main="Kernel Density Estimate")
```



The estimate result is quite good since it's smooth and similar to the original histogram. In addition, it's clearly showing the high kurotsis and low skewness.

(c)

```
-1e6 * qnorm(0.005, mean(ret), sd(ret))
```

```
## [1] 39762.09
```

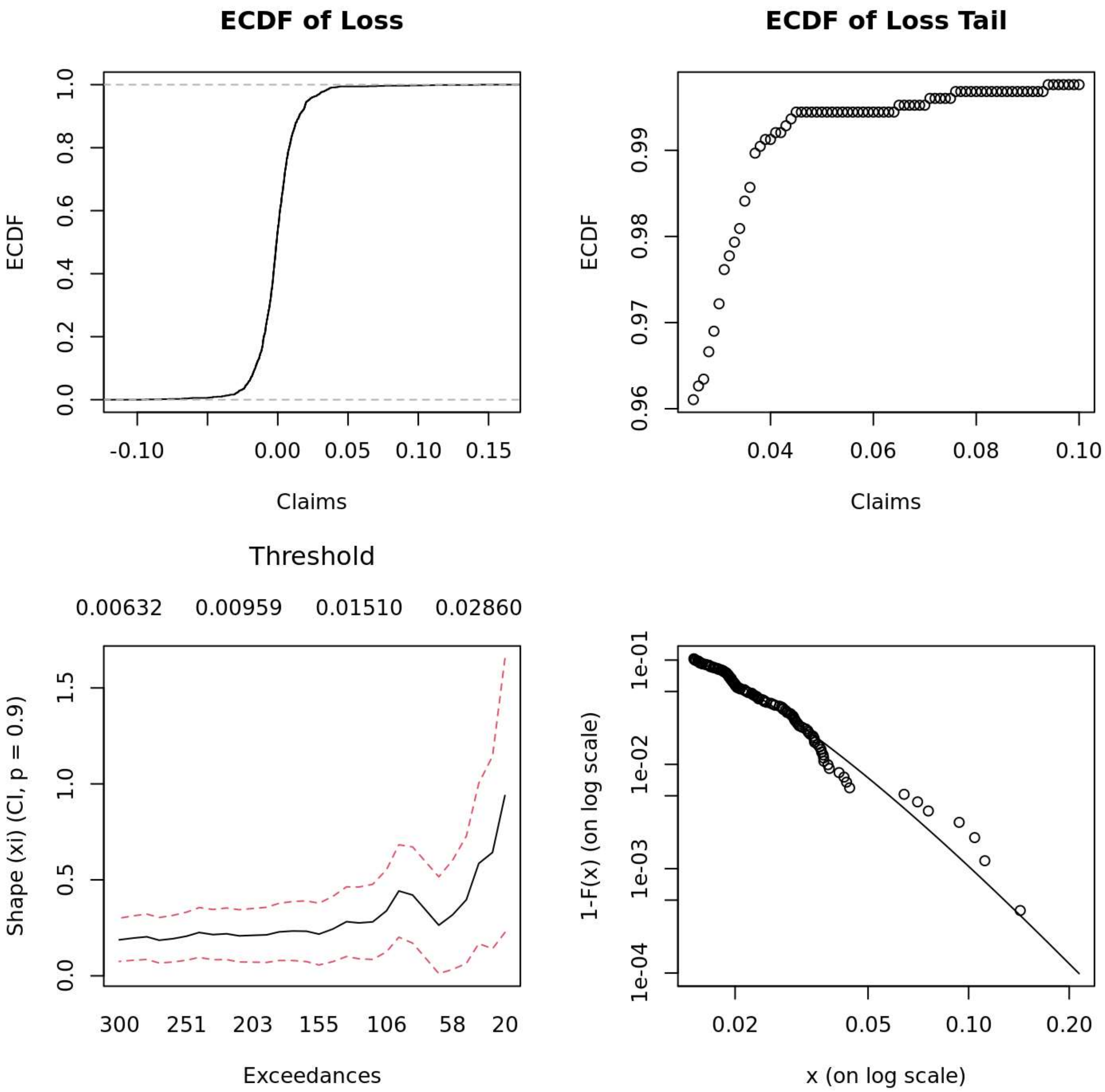
Therefore, the VaR from normal distribution estimation is 39762.09 dollars

(d)

```
library(evir)
par(mfrow=c(2,2))
loss = -ret
eecdf = ecdf(loss)
plot(eecdf, main="ECDF of Loss", xlab="Claims", ylab="ECDF")
uv = seq(from = 0.025,to = 0.1, by = .001)
plot(uv,eecdf(uv), main="ECDF of Loss Tail", xlab="Claims", ylab="ECDF")
shape(loss, models=30, start=300, end=20, ci=0.9, reverse = TRUE, auto.scale=TRUE)
mu = 0.015
gpd_out = gpd(loss, threshold = mu)
gpd_out$par.ests
```

```
##          xi        beta
## 0.29026972 0.00885722
```

```
tailplot(gpd_out)
```



It turns out when I set the threshold to be 0.015, the result from tail plot is linear, and the shape parameter is around 0.29

(e)

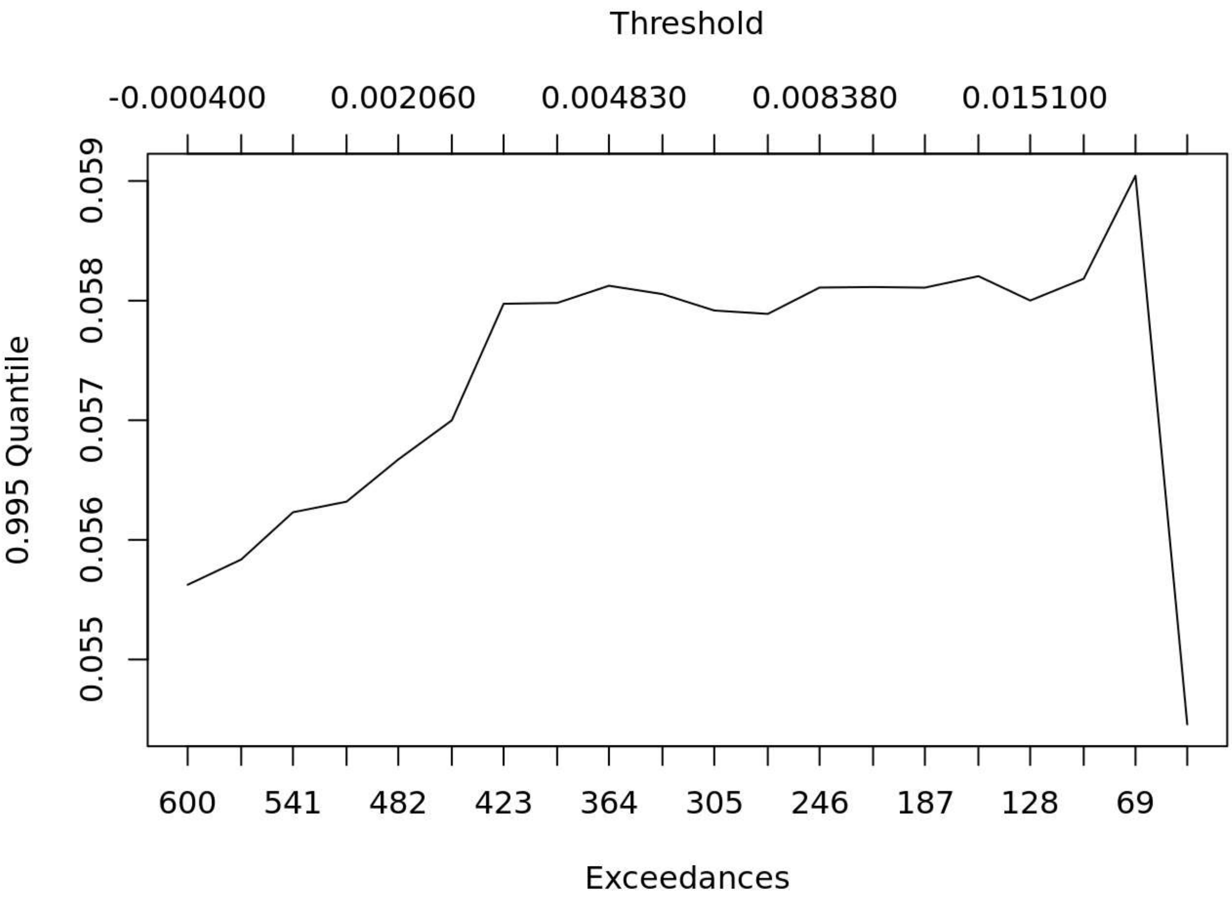
```
qt = 1-.005/(1-eecdf(mu))
xi = gpd_out$par.ests[1]
scale = gpd_out$par.ests[2]
VaR = qgpd(qt, xi, mu, scale)
1e6 * VaR
```

```
##     beta
## 57985.9
```

The VaR result is 57985.9 dollars, which is much larger than the result from normal distribution. Therefore, it's inappropriate to estimate the tail through empirical distribution when q is small.

(f)

```
quant(loss, p = 0.995, models = 20, start = 600, end = 40, reverse =TRUE, ci = FALSE, auto.scale
 = TRUE, labels = TRUE)
```



It turns out that the VaR is quite stable in the range from 0.003 to 0.018, and setting threshold to be 0.015 can work well for tail estimation.

(g)

- For ES in part(c)

```
1e6 * (mean(ret) + sd(ret) * dnorm(qnorm(0.005)) / 0.005)
```

```
## [1] 45538.1
```

The expected shortfall is 45538.1 dollars

- For ES in part(e)

```
qt = 1-.005/(1-eecdf(mu))
xi = gpd_out$par.ests[1]
scale = gpd_out$par.ests[2]
1e6 * (VaR + (scale + xi * (VaR - mu)) / (1 - xi))
```

```
##     beta
## 88046.23
```

The expected shortfall is 88046.23 dollars

2

(a)

$$COV[X,Y] = COV[X,X^2] = E[(X-E[X])(X^2-E[X^2])] = E[X^3] = 0$$

The covariance of X and Y is 0, so they're not correlated. However, since $Y = X^2$, then they're not independent.

(b)

Based on the definition, we have

$$\hat{\rho}_S = \frac{12}{n(n^2-1)}[\sum_{i=0}^{\lfloor n/2 \rfloor}(2*i+1-\frac{n+1}{2})(n+1-n-1) - (\lfloor n/2 \rfloor - \frac{n+1}{2})(1-\frac{n+1}{2})]$$
$$\hat{\rho}_S = 0$$

(c)

$$\text{Let} \quad w = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$\text{Then} \quad Var(w^TY) = w^TCov(Y)w = x^2+y^2+z^2+1.8(xy+yz+axz)$$
$$\text{Assume} \quad a=0 \quad \text{and} \quad x=z=1, y=-1.8$$
$$\text{Then} \quad Var(w^TY) = -1.24$$

```
w = as.vector(c(1, -1.8, 1))
x =matrix(c(1, 0.9, 0, 0.9, 1, 0.9, 0, 0.9, 1), nrow=3, ncol=3)
t(w)%*%x%*%w
```

```
##       [,1]
## [1,] -1.24
```

Since the variance should be always larger than 0, then a can't be 0

(d)

Since the matrix is covariance, then all its eigenvalues must greater than 0. To calculate the eigenvalues, we have

$$\begin{bmatrix} 1-\lambda & 0.9 & a \\ 0.9 & 1-\lambda & 0.9 \\ a & 0.9 & 1-\lambda \end{bmatrix} = 0$$

$$\text{Hence,it's determinant is} \quad (1-\lambda-a)[\lambda^2-(2+a)\lambda+a-0.62]=0$$

$$\text{Then we have three eigenvalues} \quad \lambda_1 = 1-a, \lambda_2 = \frac{a+2+\sqrt{a^2+6.48}}{2}, \lambda_3 = \frac{a+2-\sqrt{a^2+6.48}}{2}$$

$$\text{Since all eigenvalues must be greater or equal to 0, we have } 0.62 \le a \le 1$$

Therefore, the lower limit on a is 0.62