Problem Set 10

Statistics 509 – Winter 2022

Due via Canvas on Wednesday, April 13 by Midnight EST

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Consider the daily price data of the NYSE Composite from Jan 1, 2015 to Dec 31, 2017 which is in the file NYA-2015-2017.csv and was utilized in Homework 9. The following R-commands can be used to read in the data:

```
X = read.csv("NYA-2015-2017.csv",header=TRUE)
NYSE_lret = diff(log(X$AdjClose))
NYSE_lret.ts <- ts(data=NYSE_lret,start=c(2015,1),frequency=252,names=c('logret'))
```

(a) Investigate utilizing an AR(p)+GARCH(1,1) model on the log-returns and consider using either normal/t-distribution for the innovations in the GARCH error process. Utilize the AIC criteria for your choice of model. Also, specify the half-life of the volatility for this model based on the estimated parameters.

(b) Carry out the appropriate diagnostics on your final selected AR-GARCH model and summarize your findings.

(c) Utilize the AR-GARCH model to derive the relative VaR for the following 10 weeks following the last week of data for $q = .005$ for your final selected AR-GARCH model. Derive the expected shortfall as well for the 10 weeks.

Remark. For this problem, utilize garchFit from fGarch package to generate results, and utilize plot commands to generate all of the appropriate diagnostic plots for this.

2. The stochastic volatility model is an alternative to the GARCH model for modeling volatility. An example of a stochastic volatility model is

$$X_n = \mu + A_n$$

where

$$A_n = \sqrt{H_n}\epsilon_n$$

with

$$\log(H_n) = \alpha_o + \sum_{j=1}^{p} \phi_j \log(H_{n-j}) + \sum_{j=1}^{q} \theta_j V_{n-j} + V_n$$

where $\epsilon_1, \epsilon_2, \ldots$ is iid mean 0 with variance of 1, and $V_1, V_2, \ldots$ is iid with mean 0 and variance of σ_V^2, and these two sequences are independent of each other.

(a) What are the requirements on $\phi_1, \phi_2, \ldots, \phi_p$ and $\theta_1, \theta_2, \ldots, \theta_1$ for stability of this volatility model?

(b) For the case of a stable model, and $p = 1$ and $q = 0$, derive the (unconditional) variance of A_n, given that V_n is normally distributed.

Hint: You can use that if a random variable $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$E(e^X) = e^{\mu + \frac{\sigma^2}{2}}$$