SOLUTIONS TO STATS 509 PROBLEM SET 1

1. Note that the parameters for double exponential rv X having mean 0 and standard deviation of 2 must satisfy that $\mu = 0$, and

$$\frac{\sqrt{2}}{\lambda} = 2$$

so that

$$\lambda = \frac{1}{\sqrt{2}} = .707.$$

(a). We have

$$y_{0.1} = x_{0.1} - 2 = -4.276,$$

where the value of $x_{0.1}$ comes from the R command:

```
> source('startup.R')
> qdexp(.10,0,.707)
[1]  -2.276433
```

(b). Note that $X \sim \text{DExp}(0, \lambda)$ with $\lambda = \frac{1}{\sqrt{2}}$, and we want the value π_z such that

$$\begin{aligned}
.10 &= P(Z \leq \pi_z) \\
&= P\left(\frac{1}{(X-2)^2} \leq \pi_z\right) \\
&= P\left((X-2)^2 \geq \frac{1}{\pi_z}\right) \\
&= P\left(X \leq -\frac{1}{\sqrt{\pi_z}} + 2\right) + P\left(X \geq \frac{1}{\sqrt{\pi_z}} + 2\right) \\
&= \frac{1}{2}\left[e^{-\lambda(\frac{1}{\sqrt{\pi_z}} - 2)} + e^{-\lambda(\frac{1}{\sqrt{\pi_z}} + 2)}\right] \\
&= \left[\frac{e^{-2\lambda}}{2} + \frac{e^{2\lambda}}{2}\right] e^{-\lambda \frac{1}{\sqrt{\pi_z}}}
\end{aligned}$$

where we have used in the 5th equality that

$$-\frac{1}{\sqrt{\pi_z}} + 2 \leq 0$$

and

$$\frac{1}{\sqrt{\pi_z}} + 2 \geq 0.$$

Based on this,

$$e^{-\lambda \frac{1}{\sqrt{\pi_z}}} = \frac{.20}{e^{-2\lambda} + e^{2\lambda}}$$

or

$$-\lambda \frac{1}{\sqrt{\pi_z}} = \log\left(\frac{.20}{e^{-2\lambda} + e^{2\lambda}}\right)$$

or

$$\pi_z = \frac{\lambda^2}{\left[\log\left(\frac{.20}{e^{-2\lambda}+e^{2\lambda}}\right)\right]^2} = 0.05267$$

The R-code is shown below for generating the above.

```
> lambda = 1/sqrt(2)
> piz = (lambda^2)/(log(.2/(exp(-2*lambda)+exp(2*lambda))))^2
> piz
[1] 0.05267021
```

2.

(a). To compute this we get the .002 quantile from the log-returns which are distributed as $\mathcal{N}(0, (.025)^2)$, and from R (see code below) get result that is -0.0720, so the quantile is

$$-\{\exp(-0.0720) - 1\} = 0.0694, \text{ VaR} = 0.0694 \cdot 10^8 = 6.94 \text{ million}$$

```
qnorm(.002,0,.025)
[1] -0.07195404
```

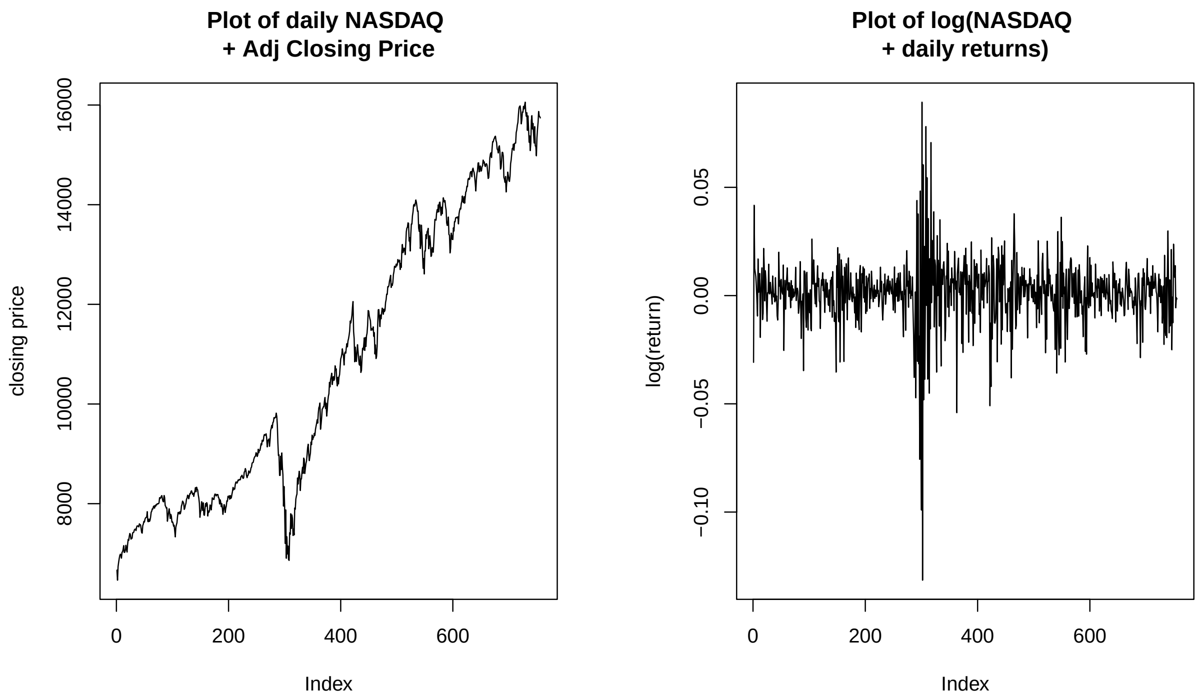
(b). Now the mean for the GED distribution is 0, and then standard deviation is always .025. The final results are in the table below for the 3 values of ν. The R-code and result that generated this are following the table. We also used the fact that the GED distribution with a mean of 0, is symmetric, and thus we have that if $x_{.002}$ is the .002-quantile of the GED distribution, then the quantile is $-(exp(x_{.002}) - 1)$. Based on this we have the following the results, and the R-code for generating these results are below that.

ν	0.002 quantile
.5	0.1260
.9	0.0976
1.4	0.0801

```
> qged(.002, mean = 0, sd = .025, nu = .5)
[1] -0.1347067
> -(exp(qged(.002, mean = 0, sd = .025, nu = .5))-1)
[1] 0.1260278
> qged(.002, mean = 0, sd = .025, nu = .9)
[1] -0.1026754
> -(exp(qged(.002, mean = 0, sd = .025, nu = .9))-1)
[1] 0.09758018
> qged(.998, mean = 0, sd = .025, nu = 1.4)
[1] 0.08350894
> -(exp(qged(.002, mean = 0, sd = .025, nu = 1.4))-1)
[1] 0.08011714
```

3.

(a). The resulting plots are:



(i). The NASDAQ adjusted closing prices seemed to have a fairly consistent positive trend over the these years thought it seem a little flatter at the beginning and possibly the end.

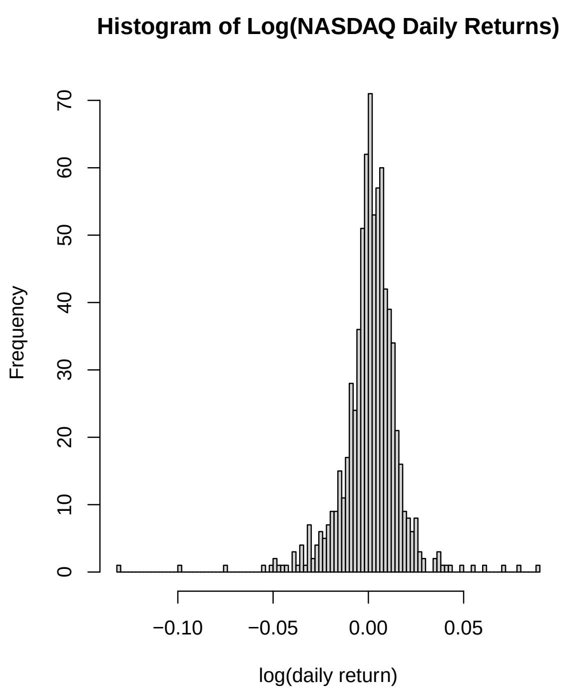
(ii). The distribution/spread of the log(daily) returns appear to be fairly uniform over period – there are couple of a serious outliers at around 420 – mostly low below -.06, but also above .04. During the period of days 420-500, it seems that there was a significant and persistent increase in the spread of the returns (volatility) for the daily data. There is a similar period of increased volatility over the approximate range of 50-120. From about 500 on to the end, it seems not to have as many volatile periods, but is more consistent – with the possible exception that there is some increased volatility in the 1420-1460 range. The R-commands for generating these plots are below:

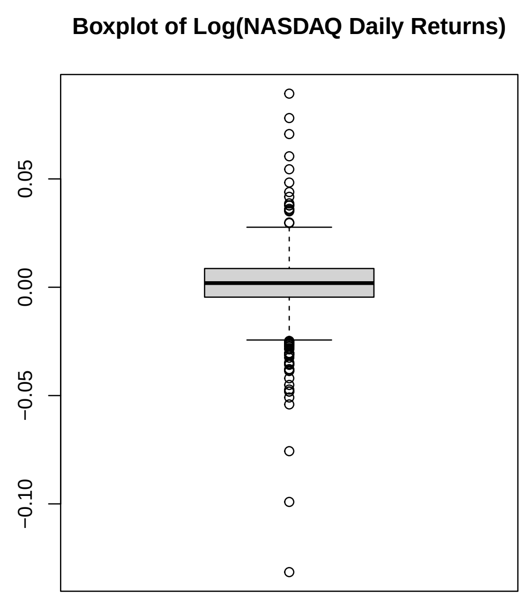
```
>Xday = read.csv("Nasdaq_daily_Jan1_2019-Dec31_2021.csv",header=TRUE)

>NASDAQday <- Xday$Adj.Close
>NASDAQday_lreturn <- diff(log(NASDAQday)) # generating log returns (daily)

>plot(NASDAQday,ylab='closing price',main='Plot of daily NASDAQ
+ Adj Closing Price',type='l')
>plot(NASDAQday_lreturn,ylab='log(return)',main='Plot of log(NASDAQ
+ daily returns)',type='l')
```

(b). The summary statistics are shown in the table below:

Statistics	Log(NASDAQ Daily Return)
Min	-0.1315
1st Quartile	-0.0046
median	0.0019
mean	0.0011
3rd Quartile	0.0086
Max	0.0893
Std. Deviation	0.0157
Skewness	-1.0470
(Excess) Kurtosis	12.3832

The histograms and boxplots for the log(NASDAQ daily returns) are given below.





Summary Discussion: These summary statistics show that the distributions for the log(daily returns) have a mean/median that is close to 0 – this is seen by comparing with the standard deviation of .011 for the log(daily returns). It seems that the daily log returns are relatively symmetric with a possible slight negative skewness being -.417. The kurtosis values for the daily log returns is significantly above 0, at the value of 3.24, and suggests that the tails of the log(daily returns) are heavier than normal, but may be able to modeled by double exponential since kurtosis is close to 3. The histogram basically confirms a slight negative asymmetry and boxplots basically show some number of outliers which correspond to heavier tails. The R commands for generating all of the above are

```
> summary(NASDAQday_lreturn)
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.131492 -0.004577  0.001912  0.001138  0.008634  0.089347
> skewness(NASDAQday_lreturn)
```

```
[1] -1.047011
> kurtosis(NASDAQday_lreturn)
[1] 12.38328
> sd(NASDAQday_lreturn)
[1] 0.01566391

>hist(NASDAQday_lreturn,breaks=100,xlab='log(daily return)', main='Histogram of Log(NASDAQ Dai

>boxplot(NASDAQday_lreturn, main = 'Boxplot of Log(NASDAQ Daily Returns)')
```

(c). Using the mean of .001138 and standard deviation of 0.015664, the corresponding double exponential distribution should correspond to

$$\mu == .001138,\ \lambda == \frac{\sqrt{2}}{0.0157} = 90.2848$$

Now using this distribution, we have that the .004 quantile is -.0348, and this can be used to compute the relative VaR as -(exp(-.0348)-1), which is .0342. In contrast based on the .004 quantile of the log-return data is -.0390, and computing the relative VaR in the same way, it is .0382, which is a little larger than the double exponential model. This is the relative VaR values, and if 1 millions dollars were invested, then the VaR at .004 is $34,800 with the double exponential model, and $38,200 using the quantile. The R-code for this below.

```
> source('startup.R')
> mu = mean(NASDAQday_lreturn)
> lambda = sqrt(2)/sd(NASDAQday_lreturn)
> mu
[1] 0.001138137
> lambda
[1] 90.28484
> qdexp(.004,mu,lambda)
[1] -0.05234054
> Vart_model = -(exp(qdexp(.004,mu,lambda))-1)
> Vart_model
[1] 0.05099436
> quantile(NASDAQday_lreturn,.004)
       0.4%
-0.05404066
> Vart_quant = -(exp(quantile(NASDAQday_lreturn,.004))-1)
> Vart_quant
      0.4%
0.05260642
```

4. Problem 10 on page 17 in Ruppert/Matteson. Now in this case the log-return after 20 trading days is normally distributed with mean of $20 * .0002 = 0.004$ and standard deviation of $\sqrt{20} * 0.03 = 0.1342$. Thus the probability is $1 - F_{0.004,0.1342}(\log(100/97)) = 0.4218$.

R command:

```
> 1 - pnorm(log(100/97), 0.004,sqrt(20)*0.03)
[1] 0.4218295
```

5. Problem 11 on page 17 in Ruppert/Matteson. Note that

$$\log\left(\frac{P_t}{P_0}\right) \sim \mathcal{N}(0.0005t, 0.012t),$$

we find the value of t using the following R command:

```
> t = 1
> while ( (1 - pnorm(log(2), 0.0005*t, sqrt(0.012*t))) < 0.9 ){
+   t = t + 1
+ }
> t
[1] 81584
```

which gives us $t = 81584$.