Lecture 11: Brief Intro to Stochastic Volatility Models

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Reference: Chapter 20 of Ruppert/Matteson

Overview

- Review of ARMA-GARCH Model
- Stochastic Volatility Models
- Brief Comparison

ARMA/GARCH Models

Recall. The ARMA(p,q) model for X_n corresponds to

$$X_n = \sum_{i=1}^{p} \alpha_i X_{n-i} + \sum_{j=1}^{q} \phi_j \epsilon_{n-j} + \epsilon_n$$

where $\{\epsilon_n\}$ is mean-zero white noise process. A more general model is to allow the noise process ϵ_n be a GARCH(p,q) process, i.e.,

$$\epsilon_n = \sigma_n \delta_n$$

where δ_n is iid $\mathcal{N}(0,1)$ (or more generally a t-distribution), and

$$\sigma_n^2 = \alpha_{g,o} + \sum_{i=1}^{p} \alpha_{g,i} \epsilon_{n-j}^2 + \sum_{j=1}^{q} \beta_{g,j} \sigma_{n-i}^2$$

Example: The AR(p)-GARCH$(1,1)$ model for X_n corresponds to

$$X_n = \alpha_o + \sum_{i=1}^{p} \alpha_i X_{n-i} + \epsilon_n$$

where $\{\epsilon_n\}$ is mean-zero white noise process. A more general model is to allow the noise process ϵ_n be a GARCH$(1,1)$ process, i.e.,

$$\epsilon_n = \sigma_n \delta_n$$

where δ_n is iid $\mathcal{N}(0,1)$ (or more generally a t-distribution), and

$$\sigma_n^2 = \alpha_{g,o} + \alpha_{g,i}\epsilon_{n-1}^2 + \beta_{g,1}\sigma_{n-1}^2$$

Stochastic Volatility Models

Remark. Primary focus of GARCH models is to accurately model the "apparent" changes in volatility as accurate as possible. A widely used alternative to GARCH are teh stochastic volatility models.

Model:

$$X_n = \alpha_o + \sum_{i=1}^{p} \alpha_i X_{n-i} + \epsilon_n$$

where $\{\epsilon_n\}$ is mean-zero white noise process which is modeled via

$$\epsilon = \sqrt{h_n}\delta_n$$

where δ_n is iid $\mathcal{N}(0,1)$ (or more generally a t-distribution), and

$$\log(h_n) = \beta_o + \sum_{j=1}^{n} \phi_j \log(h_{n-j}) + \sum_{j=1}^{q} \theta_j \nu_{n-j} + \nu_n$$

with $\{\nu_n\}$ being iid mean 0 process independent of $\{\delta_n\}$

Stochastic Volatility Models: Comments

- GARCH has one process δ_n and SV has 2 independent processes δ_n and ν_n
- Positive for SV: More flexibility in modeling volatility
- Negative for SV: More difficulty/complications for estimation in SV
    - There is not a closed form for the log-likelihood (or even conditional log-likelihood)
    - Utilize Monte Carlo Markov Chain sampling techniques to sample from the log-likelihood
    - For more info/examples see Ruppert/Matteson