Problem Set 1

Statistics 509 – Winter 2022

Due in class on Wednesday, January 19

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Suppose X is double exponential with mean 0 and standard deviation of 2.

(a) Find the .10-quantile for $Y = X - 2$.

(b) Find the .10-quantile of $Z = \frac{1}{(X-2)^2}$.

2. Suppose have portfolio of 100 million dollars. Compute the $q = .002$ quantile of the total return for the following cases.

(a) Distribution of log-returns is normally distributed with a mean of 0 and standard deviation of .025.

(b) Distribution of log-returns has a GED distribution with a mean of 0 and standard deviation of .025 for the cases of $\nu = 0.5, 0.9, 1.4$.

Hint: Total return = 100 million*Return.

3. In Canvas in the Data subdirectory under files is the data set of the daily price data of the NASDAQ Composite from Jan/2019 to Dec/2021.

(a) Generate plots of adjusted closing price and log returns (based on adjusted closing price) as a function of time and give a brief summary what the plots show.

(b) Generate summary statistics, skewness, kurtosis, histogram, and boxplot of the log(adjusted closing price return) – give a brief summary of interesting data features discovered based on this analysis. *Hint:* Can get skewness and kurtosis from R-package "fBasics".

(c) Estimate the mean and standard deviation of the log-returns, and then using a double exponential distribution with these parameters, estimate the quantile of the returns at $\alpha = .004$. Also, compare this estimate with the estimate generated by simply using the .004-quantile of the log-returns.

4. Problems 10 and 11 on pages 17 in Ruppert/Matteson.

Hint: For independent and identically distributed normal random variables $X_1, X_2, \ldots, X_K \sim \mathcal{N}(\mu, \sigma^2)$, $S = \sum_{k=1}^{K} X_k \sim \mathcal{N}(n\mu, n\sigma^2)$.