Problem Set 5

Statistics 509 – Winter 2022

Due on Wednesday, February 16 by midnight – via Canvas

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Suppose $R_1.R_2$ are returns on an asset, and suppose that $E(R_1) = .03, E(R_2) = .04, \text{Var}(R_1) = (.04)^2, \text{Var}(R_2) = (.06)^2,$ and $\text{Corr}(R_1, R_2) = 0.5$.

(a) What are $E(0.6R_1 + 0.4R_2)$ and $\text{Var}(0.6R_1 + 0.4R_2)$?

(b) For what value of w is $\text{Var}(w \cdot R_1 + (1-w) \cdot R_2)$ minimized? Why would it be useful to minimize $\text{Var}(w \cdot R_1 + (1-w) \cdot R_2)$?

(c) Assuming a portfolio of a \$1 million, and a multi-variate normal distribution for R_1 and R_2, find the value w that minimizes the VaR at $q = .005$. *Hints:* Note that the random variable $[wR_1 + (1-w)R_2]$ is normally distributed for any w, and it will be easiest to use R-package – the solution can be approximate, i.e., accurate to .01.

(d) Repeat the above if have multi-variate t-distribution for R_1 and R_2, with $\nu = 6$.

2. Suppose X is a random variable and $Y = e^X$ and $Z = (-X)^3$.

(a) Derive the bivariate copula model for X, Y.

(b) Derive the bivariate copula model for X, Z.

3. In the Data directory are daily return data MidCap Stocks from the fEcofin package – it is file midcapD.csv. Carry out analysis on the returns of the 3 stocks of NYB, ALTR, and APH. *Hint.* Can utilize the following commands

```
> Data = read.csv("midcapD.csv",header=TRUE)
> Mid_Returns = Data[,c(5,6,7)]
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(a) Carry out a preliminary data analysis, including skewness, kurtosis, correlational analysis and scatter diagrams, and provide a summary discussion of your findings.

(b) Carry out a fitting of a multivariate normal distribution to the returns and carry out diagnostic plots – univariate QQ plots for each.

(c) Same as **(b)**, but now use a multivariate t distribution – also derive a confidence interval for the degrees of freedom via the method of profile likelihood.

(d) Based on results in **(b)** and **(c)**, which model do you prefer and why. Compare the two models of multivariate normal vs. multivariate t using the AIC criteria.