Problem Set 9

Statistics 509 – Winter 2022

Due via Canvas on Wednesday, April 6 by midnight EST

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Suppose have a linear process X_n given by

$$X_n = \sum_{j=-\infty}^{\infty} \psi_j \epsilon_{n-j}$$

where $\{\epsilon_n\}_{n=-\infty}^{\infty}$ an iid mean-zero white noise process with variance σ^2. Derive an expression, in terms of the coefficients ψ_j, for the auto-covariance of X_n.

2. (a) Derive the linear process representation for a mean-zero ARMA(1,2) process.

(b) Utilize this representation and result from **1.** to derive a derive a general expression for the auto-covariance of X_n.

3. Consider the daily price data of the NYSE Composite from Jan 1, 2015 to Dec 31, 2017 which is in the file NYA-2015-2017.csv – this data file is in the Data subdirectory under Homework. The following R-commands can be used to read in the data:

```
X = read.csv("NYA-2015-2017.csv",header=TRUE)
NYSE_lret = diff(log(X$AdjClose))
NYSE_lret.ts <- ts(data=NYSE_lret,start=c(2015,1),frequency=252,names=c('logret'))
```

(a) Plot the log-returns, and summarize what the plots show relative to volatility.

(b) Fit a GARCH(1,1) model assuming iid standard normal innovations. Provide standard errors of the parameter estimates $\alpha_o, \alpha_1, \beta_1$.

(c) Generate plots of the estimated conditional volatilities $\hat{\sigma}_n$ and of the estimated innovations (residuals) $\hat{\epsilon}_n$.

(d) Carry out the appropriate diagnostics on your GARCH estimation and summarize your findings.

(e) Utilize the estimated GARCH model to derive the relative VaR for the first trading day in 2018, assuming $q = .005$. Do this assuming the white noise process is normal.

(f) Fit a t-distribution to the residuals of the GARCH model, and derive the relative VaR for the first trading day in 2018 based on the estimated t-distribution for the same $q = .005$.

(g) Repeat **(e)** and **(f)** for the day having the maximum estimated conditional volatility.

Hint. One of the outputs from the "garch" command is "fitted.values" and these correspond to the estimated standard deviations, i.e., the estimated conditional volatilities.