Problem Set 2

Statistics 509 – Winter 2022

Due in class on Wednesday, January 26

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Suppose R is return that can be modeled as $R \sim \text{DExp}(\mu, \lambda)$, and suppose $q < .5$ is chosen probability for the relative VaR, $\widetilde{\text{VaR}}$.

(a) Derive a general expression for relative VaR that is a function of μ, λ and q (similar to what was done for the normal distribution in class, noting that this distribution is for the return, not the loss).

(b) Derive a general expression for the expected relative shortfall as a function of μ, λ, q.

(c) For the daily price data of the NASDAQ Composite from Jan/2019 to Dec/2021, derive an estimated double exponential distribution for the returns. Based on this, derive the VaR and Expected shortfall for $q = .01$ assuming a 10 million dollar portfolio in the NASDAQ, and provide an interpretation of these values in your own words relative to risk.

(d) Repeat the problem in **(c)** for the case of normal distribution, and provide a discussion of which one you believe to more more accurate based on results from Homework 1.

2. Suppose negative return (relative loss) is a random variable X, which satisfies that it has a generalized Pareto distribution for the conditional distribution of X given $X \geq \mu$ with positive parameters $\xi, \sigma > 0$, i.e.,

$$P(X \leq x | X \geq \mu) = \begin{cases} 1 - \left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi} & x \geq \mu \\ 0 & x < \mu \end{cases}$$

and suppose $P(X \leq \mu) = .90$.

(a) Derive a general equation for $\widetilde{\text{VaR}}$ with $q = .01$.

(b) Based on **(a)** show that the shortfall distribution is still a Generalized Pareto, and identify the new parameters of this distribution.

(c) Based on **(b)**, derive an equation for the expected shortfall, assuming $\xi < 1$. *Hint:* You may use the result that for generalized Pareto distribution as in introduction of this problem,

$$E(X) = \mu + \frac{\sigma}{1 - \xi}$$