STATS 509 Winter 2022, Solution to Homework 7

1

(a)

The results are that Apple had an alpha = .00063 and a beta = 1.0852, Amazon had an alpha = .000002 and a beta = 0.962, Google had an alpha = 0.0006 and a beta = 0.9624, Nasda had a standard deviation of 0.0156. Thus Apple has higher volatility than the NASDAQ due to market risk, Amazon and Google are lower than Apple with their beta. The standard deviations for the returns of the NASDAQ, APPLE, AMAZON and GOOGLE are in the R codes chunk. Note that the standard deviations of Apple and Amazon returns are significantly larger than the NASDAQ and GOOGLE market. Utilizing the betas, we compute systematic market variance of Apple, Amazon and GOOGLE to be 0.669, 0.513 and 0.638. The R-code for this analysis and the result is shown below.

R Codes:

```
> # 1a
> NASdat = read.csv("Nasdaq_March16_2000_March 16_2022.csv", header = TRUE)
> AAPLdat = read.csv("AAPL_March16_2020_March16_2022.csv", header = TRUE)
> AMZNdat = read.csv("AMZN_March16_2020_March16_2022.csv", header = TRUE)
> GOOGLdat = read.csv("GOOGL_March16_2020_March16_2022.csv", header = TRUE)
> NASRet = exp(diff(log(NASdat$Adj.Close)))-1
> AAPLRet = exp(diff(log(AAPLdat$Adj.Close)))-1
> AMZNRet = exp(diff(log(AMZNdat$Adj.Close)))-1
> GOOGLRet = exp(diff(log(GOOGLdat$Adj.Close)))-1
> muf = .02/250
> # Problem 1-a
> ExNASRet = NASRet - muf
> ExAAPLRet = AAPLRet - muf
> ExAMZNRet = AMZNRet - muf
> ExGOOGLRet = GOOGLRet - muf
> lm_AAPL = lm(ExAAPLRet ~ ExNASRet)
> lm_AAPL

Call:
lm(formula = ExAAPLRet ~ ExNASRet)
```

```
Coefficients:
(Intercept)     ExNASRet
  0.0006281    1.0851737

> lm_AMZN = lm(ExAMZNRet ~ ExNASRet)
> lm_AMZN

Call:
lm(formula = ExAMZNRet ~ ExNASRet)

Coefficients:
(Intercept)     ExNASRet
  2.049e-06    9.620e-01

> lm_GOOGL = lm(ExGOOGLRet ~ ExNASRet)
> lm_GOOGL

Call:
lm(formula = ExGOOGLRet ~ ExNASRet)

Coefficients:
(Intercept)     ExNASRet
  0.0005995    0.9623872

> beta_AMZN = lm_AMZN$coefficients[2]
> beta_AAPL = lm_AAPL$coefficients[2]
> beta_GOOGL = lm_GOOGL$coefficients[2]
> sigmaNAS = sd(NASRet)
> sigmaAP = sd(AAPLRet)
> sigmaAM = sd(AMZNRet)
> sigmaGOO = sd(GOOGLRet)
> c(sigmaNAS,sigmaAP,sigmaAM,sigmaGOO)
[1] 0.01560517 0.02070504 0.02095303 0.01880251
> FrAPPL_Mark = beta_AAPL^2*sigmaNAS^2/(sigmaAP^2)
> FrAMZN_Mark = beta_AMZN^2*sigmaNAS^2/(sigmaAM^2)
> FrGOOGl_Mark = beta_GOOGL^2*sigmaNAS^2/(sigmaGOO^2)
> c(FrAPPL_Mark,FrAMZN_Mark,FrGOOGl_Mark)
 ExNASRet  ExNASRet  ExNASRet
0.6689344 0.5133706 0.6379769
```

(b)

The covariance between A and B from the regression model is derived from

$$\widehat{\beta}_A \widehat{\beta}_B \sigma_N^2$$

and the empirical covariances are calculated as follows, so these are quite close.
R Codes:

```
> # Prblem 1-b
> cov(cbind(AAPLRet,AMZNRet,GOOGLRet))
              AAPLRet      AMZNRet     GOOGLRet
AAPLRet  0.0004286985 0.0002659781 0.0002432843
AMZNRet  0.0002659781 0.0004390293 0.0002323363
GOOGLRet 0.0002432843 0.0002323363 0.0003535344
> # AAPL vs AMZN
> 1.0851737 * 9.620e-01 *(sd(NASRet)^2)
[1] 0.0002542209
> # AMZN vs GOOGL
> 9.620e-01 * 0.9623872 *(sd(NASRet)^2)
[1] 0.0002254561
> # AAPL vs GOOGL
> 0.9623872 * 1.0851737 *(sd(NASRet)^2)
[1] 0.0002543233
```

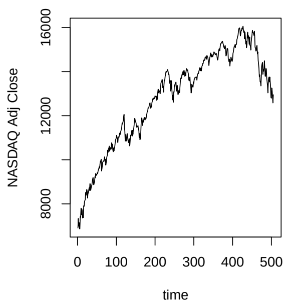
A table for comparison is as follows:

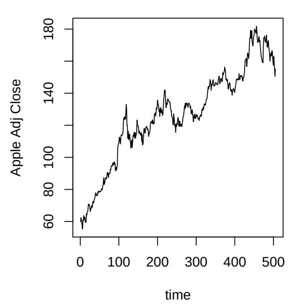
	regression	empirical
AAPL vs AMZN	0.00026	0.00025
AMZN vs GOOGL	0.00023	0.00023
AAPL vs GOOGL	0.00024	0.00025

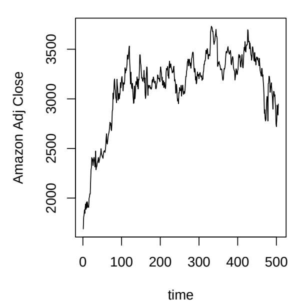
2

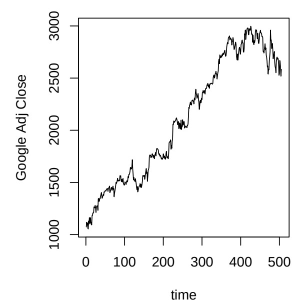
(a)

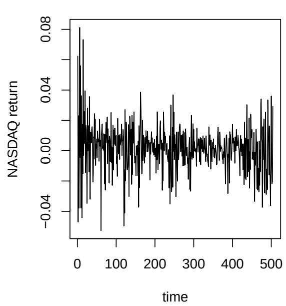
The plots are below. Note we see similarities in the Adjust closing prices - both Apple and Amazon have similar patterns in terms of going up to around day 120, and taking a fairly steep dive to a minimum around 195, and the back up. For the plots of the returns, they all seem so to have similar periods of high and low volatility with starting off with moderate volatility, and then low volatility from 25 to around 135, and then significant volatility from 135 to around 210, and then lower volatility from 210 on.

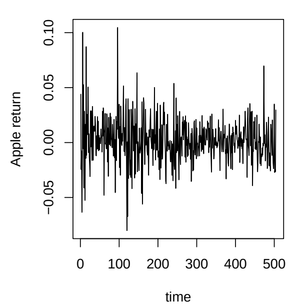


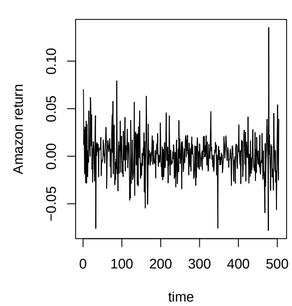


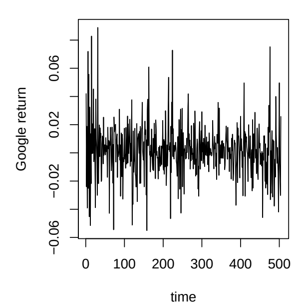










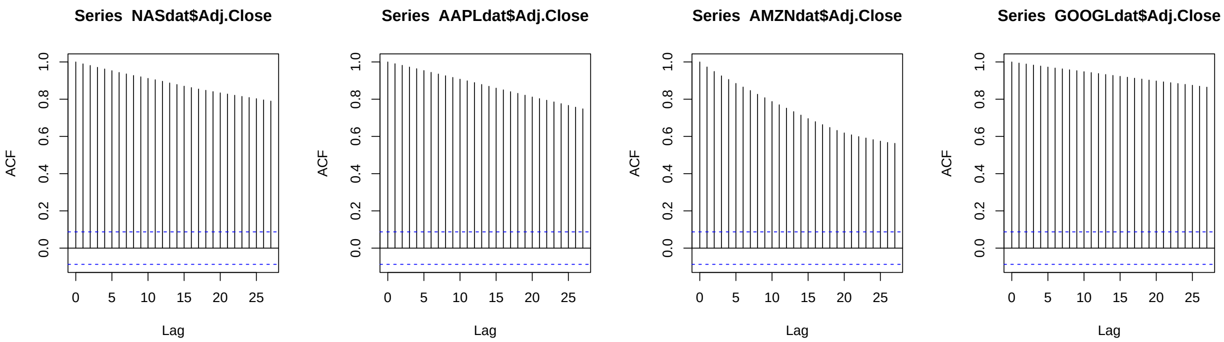


R Codes:

```
> plot(NASdat$Adj.Close,type='l',xlab='time',ylab='NASDAQ Adj Close')
> plot(AAPLdat$Adj.Close,type='l',xlab='time',ylab='Apple Adj Close')
> plot(AMZNdat$Adj.Close,type='l',xlab='time',ylab='Amazon Adj Close')
> plot(GOOGLdat$Adj.Close,type='l',xlab='time',ylab='Google Adj Close')
>
> plot(NASRet,type='l',xlab = 'time',ylab='NASDAQ return')
> plot(AAPLRet,type='l',xlab = 'time',ylab='Apple return')
> plot(AMZNRet,type='l',xlab = 'time',ylab='Amazon return')
> plot(GOOGLRet,type='l',xlab = 'time',ylab='Google return')
```

(b)

The autocorrelations for all adjusted closing prices show an auto-correlation that decreases very slowly, and this is always an indication of a process that is fundamentally not stationary, as most stationary processes have auto-covariances that tend to be smaller and go to zero much more quickly.

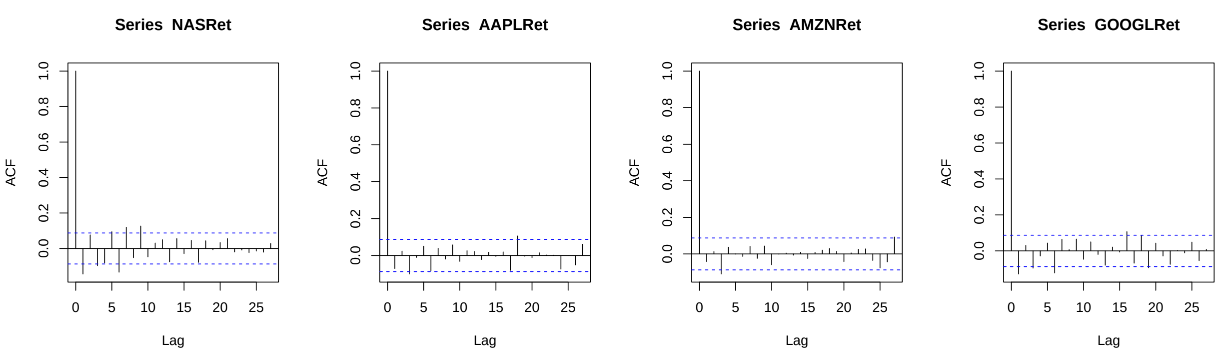


R Codes:

```
> acf(NASdat$Adj.Close)
> acf(AAPLdat$Adj.Close)
> acf(AMZNdat$Adj.Close)
> acf(GOOGLdat$Adj.Close)
```

(c)

The auto-correlations of the returns are much smaller and not many are significantly outside of the 95% confidence interval, and thus it looks like most of the non-zero lag auto- correlations are zero. NasRet has some number of ACF values outside of the 95% confidence band, as does apple and google, bu less so for amazon. Not a big pattern, but a pattern.



R Codes:

```
> acf(NASRet)
> acf(AAPLRet)
```

```
> acf(AMZNRet)
> acf(GOOGLRet)
```

3

(a)

$$
\begin{aligned}
\mathbb{E}[X_n] &= \mathbb{E}[Y_n - Z_n] = \mathbb{E}[Y_n] - \mathbb{E}[Z_n] = \mu_Y - \mu_Z; \\
\operatorname{var}(X_n) &= \operatorname{var}(Y_n - Z_n) = \operatorname{var}(Y_n) + \operatorname{var}(Z_n) \\
&= \frac{\sigma_Y^2}{1-\alpha_Y^2} + \frac{\sigma_Z^2}{1-\alpha_Z^2} \\
\operatorname{cov}(X_n, X_{n-t}) &= \operatorname{cov}(Y_n - Z_n, Y_{n-t} - Z_{n-t}) = \operatorname{cov}(Y_n, Y_{n-t}) + \operatorname{cov}(Z_n, Z_{n-t}) \\
&= \frac{\sigma_Y^2 \alpha_Y^t}{1-\alpha_Y^2} + \frac{\sigma_Z^2 \alpha_Z^t}{1-\alpha_Z^2} \quad \forall t \geq 1 \\
\operatorname{Corr}(X_n, X_{n-t}) &= \frac{\operatorname{cov}(X_n, X_{n-t})}{\sqrt{\operatorname{var}(X_n)\operatorname{var}(X_{n-t})}} = \frac{\frac{\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + \frac{\sigma_Z^2\alpha_Z^t}{1-\alpha_Z^2}}{\frac{\sigma_Y^2}{1-\alpha_Y^2} + \frac{\sigma_Z^2}{1-\alpha_Z^2}} \quad \forall t \geq 1.
\end{aligned}
$$

The mean and variance functions are constant w.r.t. n, and covariance functions only depends on the time gap t, so X_n is a stationary process.

(b)

$$
\begin{aligned}
\mathbb{E}[X_n'] &= \mathbb{E}[Y_n(1+Z_n)] = \mathbb{E}[Y_n]\mathbb{E}[(1+Z_n)] = \mu_Y(1+\mu_Z) \\
\operatorname{var}(X_n') &= \operatorname{var}(Y_n(1+Z_n)) = \mathbb{E}[Y_n^2(1+Z_n)^2] - (\mathbb{E}[Y_n(1+Z_n)])^2 \\
&= \mathbb{E}[Y_n^2]\mathbb{E}[(1+Z_n)^2] - \mu_Y^2(1+\mu_Z)^2 \\
&= \left\{\operatorname{var}(Y_n) + (\mathbb{E}[Y_n])^2\right\}\left\{\operatorname{var}(1+Z_n) + (\mathbb{E}[1+Z_n])^2\right\} - \mu_Y^2(1+\mu_Z)^2 \\
&= \left\{\frac{\sigma_Y^2}{1-\alpha_Y^2} + \mu_Y^2\right\}\left\{\frac{\sigma_Z^2}{1-\alpha_Z^2} + (1+\mu_Z)^2\right\} - \mu_Y^2(1+\mu_Z)^2 \\
&= \frac{\sigma_Y^2\sigma_Z^2}{(1-\alpha_Y^2)(1-\alpha_Z^2)} + \frac{\mu_Y^2\sigma_Z^2}{1-\alpha_Z^2} + \frac{(1+\mu_Z)^2\sigma_Y^2}{1-\alpha_Y^2} \\
\operatorname{cov}(X_n', X_{n-t}') &= \operatorname{cov}(Y_n + Y_nZ_n, Y_{n-t} + Y_{n-t}Z_{n-t}) \\
&= \operatorname{cov}(Y_n, Y_{n-t}) + \operatorname{cov}(Y_n, Y_{n-t}Z_{n-t}) + \operatorname{cov}(Y_nZ_n, Y_{n-t}) + \operatorname{cov}(Y_nZ_n, Y_{n-t}Z_{n-t}) \\
&= \frac{\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + \mathbb{E}[Y_nY_{n-t}Z_{n-t}] - \mathbb{E}[Y_n]\mathbb{E}[Y_{n-t}]\mathbb{E}[Z_{n-t}] \\
&\quad + \mathbb{E}[Y_nY_{n-t}Z_n] - \mathbb{E}[Y_n]\mathbb{E}[Z_n]\mathbb{E}[Y_{n-t}] + \mathbb{E}[Y_nZ_nY_{n-t}Z_{n-t}] - \mathbb{E}[Y_n]\mathbb{E}[Z_n]\mathbb{E}[Y_{n-t}]\mathbb{E}[Z_{n-t}] \\
&= \frac{\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + 2\mu_Z\left(\frac{\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + \mu_Y^2\right) - 2\mu_Y^2\mu_Z + \mathbb{E}[Y_nY_{n-t}]\mathbb{E}[Z_nZ_{n-t}] - \mu_Y^2\mu_Z^2 \\
&= \frac{(1+2\mu_Z)\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + \left(\frac{\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + \mu_Y^2\right)\left(\frac{\sigma_Z^2\alpha_Z^t}{1-\alpha_Z^2} + \mu_Z^2\right) - \mu_Y^2\mu_Z^2 \\
&= \frac{(1+2\mu_Z)\sigma_Y^2\sigma_Y}{1-\alpha_Y^2} + \frac{\sigma_Y^2\sigma_Z^2\alpha_Y^t\alpha_Z^t}{(1-\alpha_Y^2)(1-\alpha_Z^2)} + \frac{\mu_Y^2\sigma_Z^2\alpha_Z^t}{1-\alpha_Z^2} + \frac{\mu_Z^2\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2}. \quad \forall t \geq 1.
\end{aligned}
$$

The mean and variance functions are constant w.r.t. n, and covariance functions only depends on the time gap t, so X_n' is a stationary process.

(c)

$$\begin{aligned}
\mathbb{E}[X_n] &= \mathbb{E}[Y_n + Y_{n+1}] = \mathbb{E}[Y_n] + \mathbb{E}[Y_{n+1}] = 2\mu_Y \\
\operatorname{var}(X_n) &= \operatorname{var}(Y_n) + \operatorname{var}(Y_{n+1}) + 2\operatorname{cov}(Y_n, Y_{n+1}) = \frac{2\sigma_Y^2(1+\alpha_Y)}{1-\alpha_Y^2} \\
\operatorname{cov}(X_n, X_{n-t}) &= \operatorname{cov}(Y_n + Y_{n+1}, Y_{n-t} + Y_{n-t+1}) \\
&= \operatorname{cov}(Y_n, Y_{n-t}) + \operatorname{cov}(Y_n, Y_{n-t+1}) + \operatorname{cov}(Y_{n+1}, Y_{n-t}) + \operatorname{cov}(Y_{n+1}, Y_{n-t+1}) \\
&= \frac{2\sigma_Y^2\alpha_Y^t}{1-\alpha_Y^2} + \frac{\sigma_Y^2\alpha_Y^{t+1}}{1-\alpha_Y^2} + \frac{\sigma_Y^2\alpha_Y^{t-1}}{1-\alpha_Y^2} \quad \forall t \geq 1.
\end{aligned}$$

The mean and variance functions are constant w.r.t. n, and covariance functions only depends on the time gap t, so X_n is a stationary process.