SI 671/721:
Interactive Data Mining

Lecture 11

Fall 2021

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Announcements

- Anmol will hold a Zoom discussion section today evening at the usual time 530PM to go over some of the Homework questions. Zoom link: https://umich.zoom.us/j/99837588785 *[No other discussion sections today]* Link also posted via announcement on Slack and Canvas.

- Final class next week. No lab, but I will hold a Q/A session. More details soon.

- Quiz 2 (24 hour limit) to be released at the end of next class.

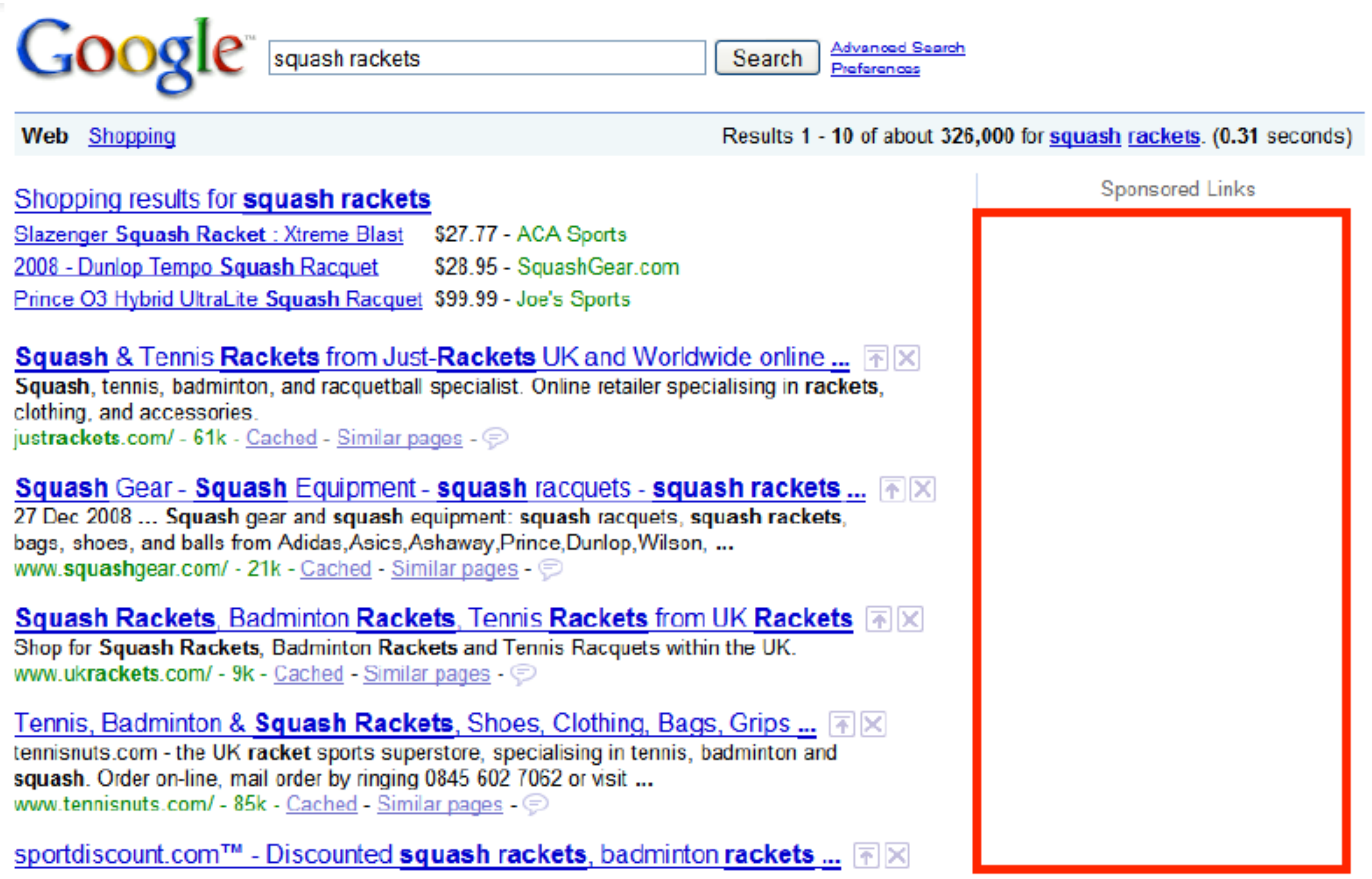
- Looking forward to your posters at the exposition! *[The final reports are due on Monday 12/13. Final grades are due by UMSI soon thereafter!]* We have posted several examples of final reports on Canvas already.

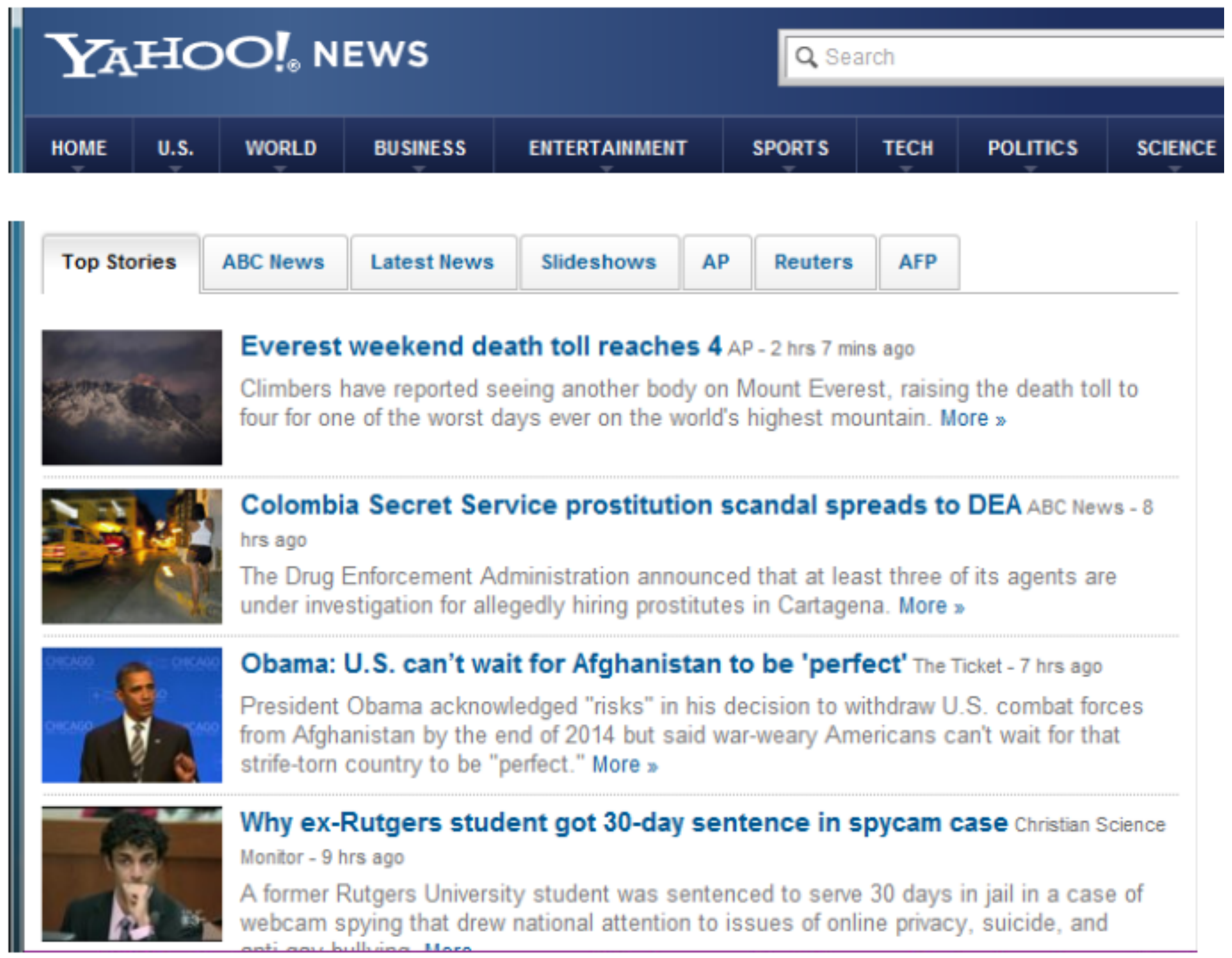
- Please complete the course evals if you haven't already. If we get >=90% response rate then everyone will get an extra 1% in their final grade.

Learning from Interactions (Experimentation)

Interactive Mining on the Web

- What do web advertising and recommender systems have in common?





- Users interact with each ad/product so we gather more data about the ad/product and also “learn” more about the user.

Let’s focus on Web Advertising

Google’s goal: Maximize revenue

The old way: Pay by impression (CPM)

Best strategy: Go with the highest bidder (but it ignores the effectiveness of an ad!)

Let's focus on Web Advertising

The new way: Pay per click! (CPC)

Best strategy: Go with expected revenue

What's the expected revenue of ad a for query q?

$$E[\text{revenue}_{a,q}] = P(\text{click}_a \mid q) * \text{amount}_{a,q}$$

Prob. user will click on ad **a** given
that she issues query **q**
(Unknown! Need to gather information)

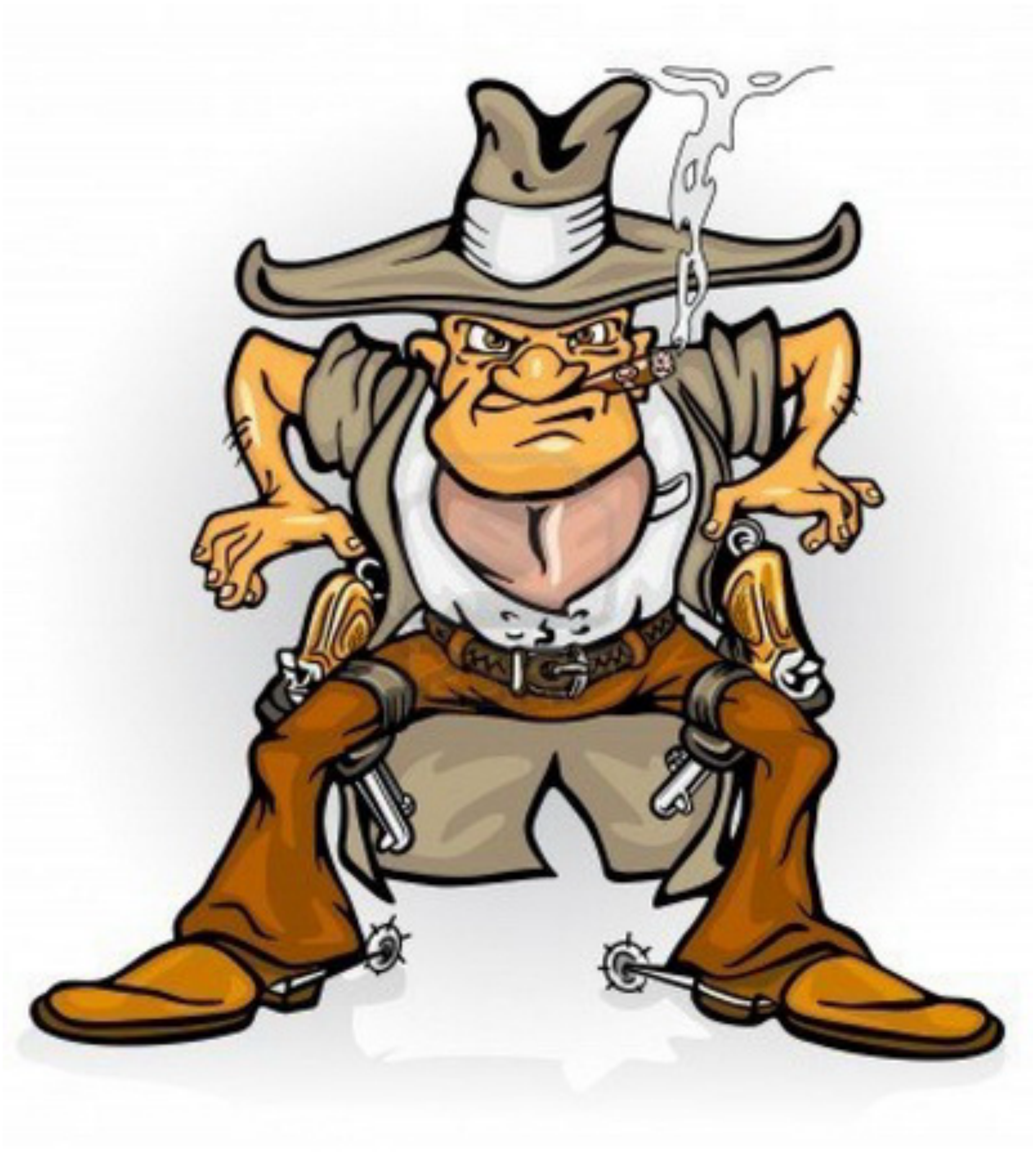
Bid amount for
ad **a** on query **q**
(Known)

But there are other applications also…

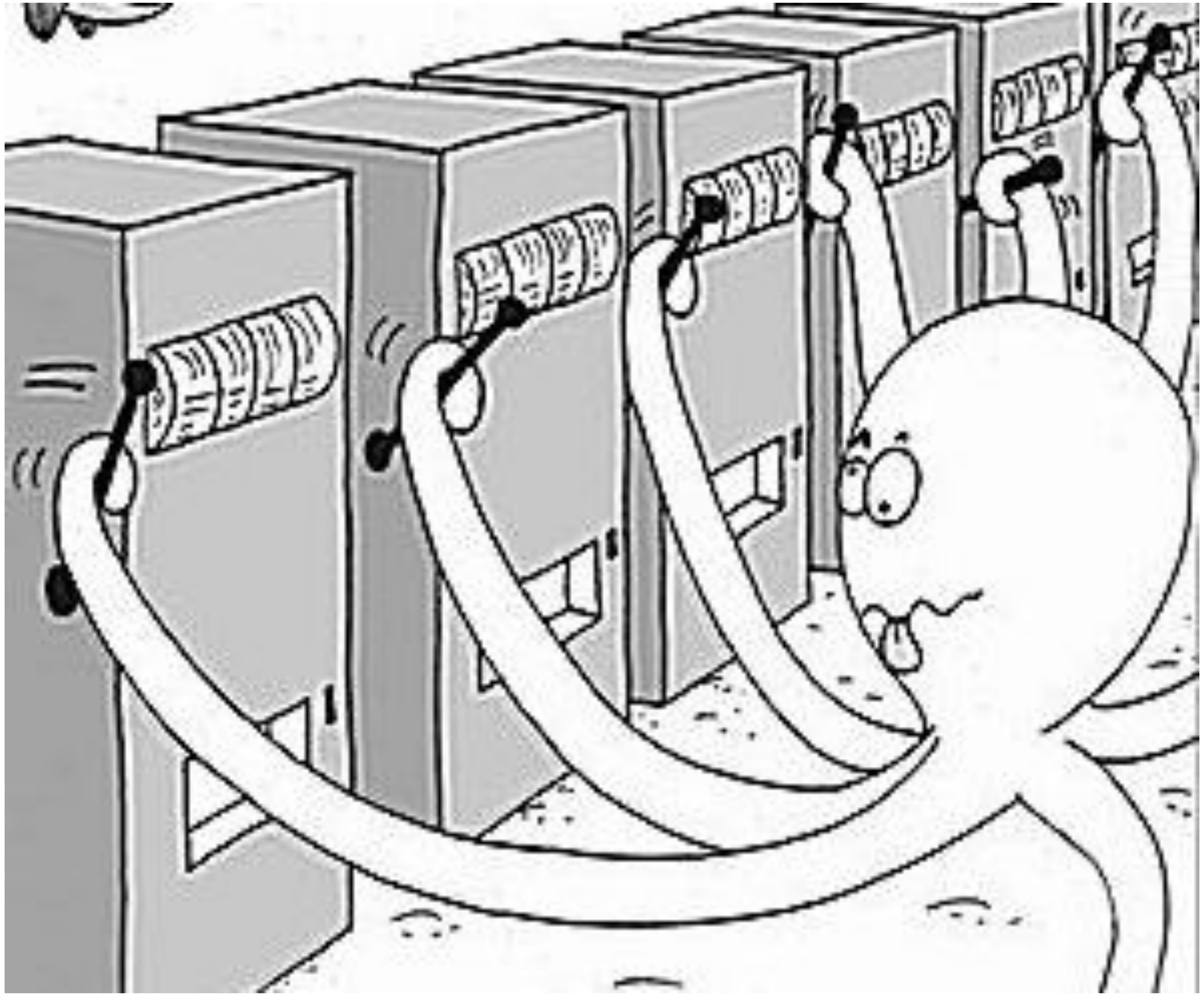
- **Clinical trials:**
  - Investigate effects of different treatments while minimizing adverse effects on patients.
- **Adaptive routing:**
  - Minimize delay in the network by investigating different routes
- **Asset pricing:**
  - Figure out product prices while trying to make most money

Bandits for Interactive Learning

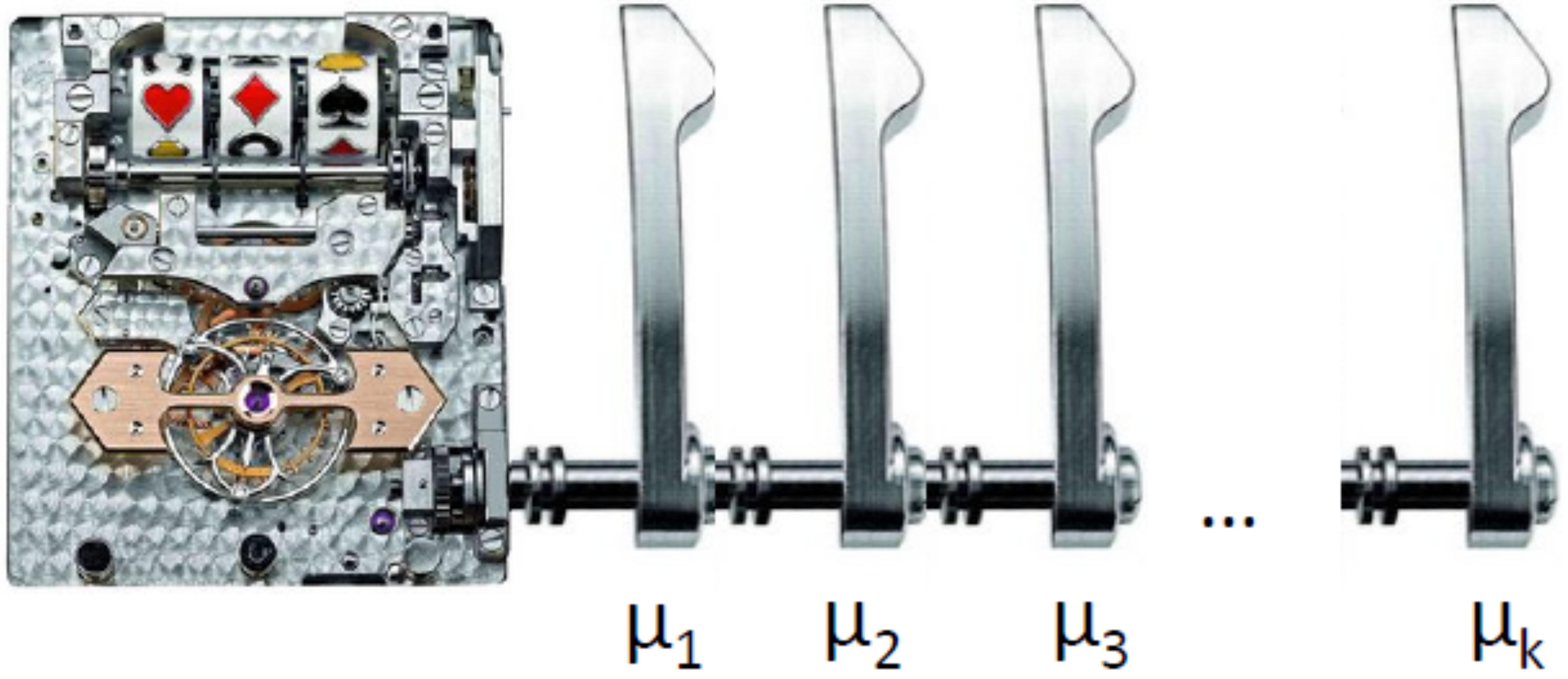
The solution: Bandits



Approach: Multiarmed Bandits

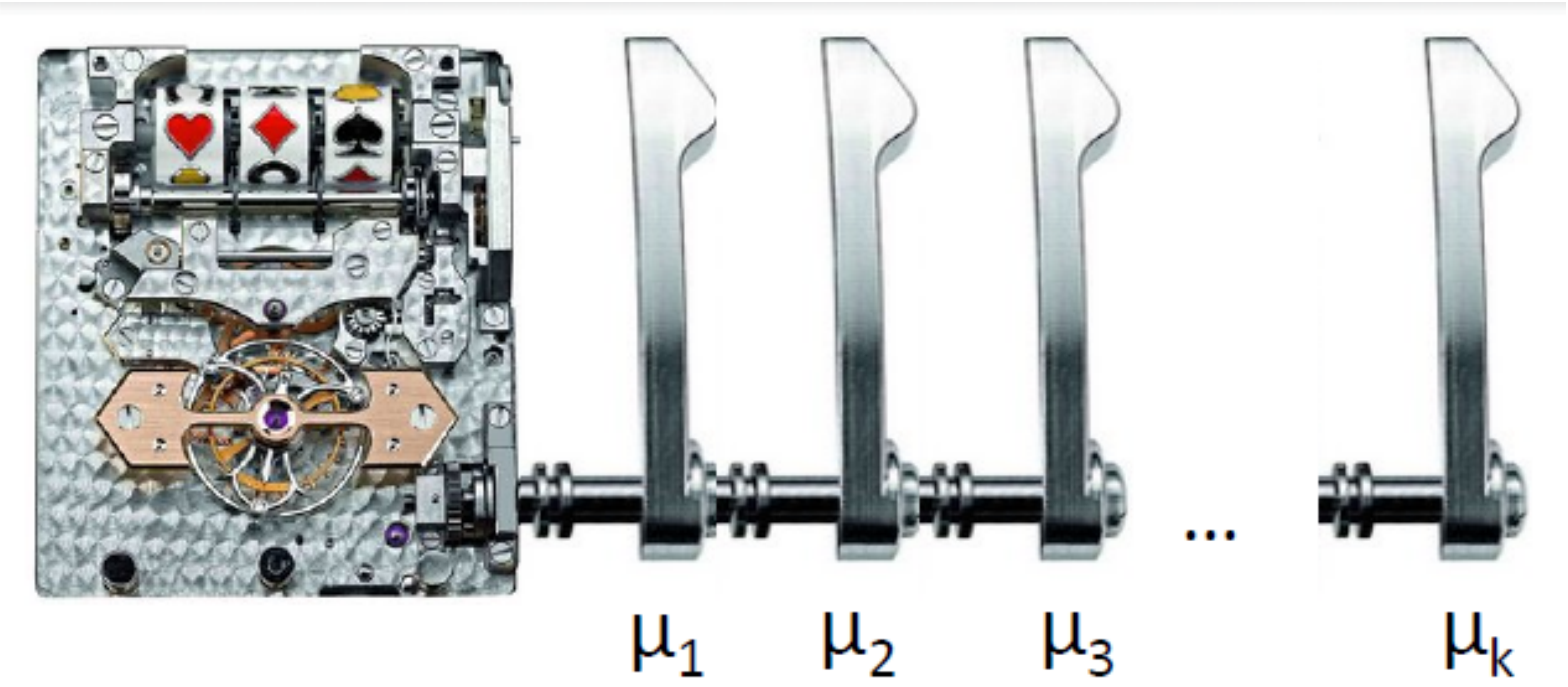


k-Armed Bandit



- **Each arm *a***
  - **Wins** (reward=**1**) with fixed (unknown) prob. $\boldsymbol{\mu_a}$
  - **Loses** (reward=**0**) with fixed (unknown) prob. $\boldsymbol{1-\mu_a}$
- All draws are independent given $\boldsymbol{\mu_1 \ldots \mu_k}$
- **How to pull arms to maximize total reward?**

k-Armed Bandit



- **How does this map to our setting?**
- Each **query** is a **bandit**
- Each **ad** is an **arm**
- **We want to estimate the arm's probability of winning μ_a (i.e., ad's CTR μ_a)**

- Every time we pull an arm we do an 'experiment'

Stochastic k-Armed Bandit

The setting:

- Set of ***k*** choices (arms)
- Each choice ***a*** is associated with unknown probability distribution P_a supported in **[0,1]**
- We play the game for ***T*** rounds
- In each round ***t***:
  - **(1)** We pick some arm ***a***
  - **(2)** We obtain random sample X_t from P_a
    - Note reward is independent of previous draws

Our goal is to maximize $\sum_{t=1}^{T} X_t$

But we don't know μ_a! But every time we pull some arm ***a*** we get to learn a bit about μ_a

Online Optimization

- Online optimization with limited feedback

Choices	X_1	X_2	X_3	X_4	X_5	X_6	…
a_1					1	1	
a_2	0		1	0			
…							
a_k		0					

Time

- Like in online algorithms:
  - Have to make a choice each time
  - But we only receive information about the chosen action

Solving the Bandit Problem

- **Policy:** a strategy/rule that in each iteration tells me which arm to pull
  - Hopefully policy depends on the history of rewards

- **How to quantify performance of the algorithm?**

  Regret!

Performance Metric: Regret

- Let μ_a be the mean of P_a
- Payoff/reward of **best arm**: $\mu^* = \max_a \mu_a$
- Let $i_1, i_2 \ldots i_T$ be the sequence of arms pulled
- Instantaneous **regret** at time t: $r_t = \mu^* - \mu_{a_t}$
- **Total regret:**

$$R_T = \sum_{t=1}^{T} r_t$$

- **Typical goal:** **Want a policy (arm allocation strategy) that guarantees:** $\frac{R_T}{T} \to 0$ **as** $T \to \infty$
  - Note: Ensuring $R_T/T \to 0$ is stronger than maximizing payoffs (minimizing regret), as it means that in the limit we discover the true best hand.

Allocation Strategies

- **If we knew the payoffs, which arm would we pull?**

  $\text{Pick } \underset{a}{\operatorname{argmax}} \mu_a$

- **What if we only care about estimating payoffs μ_a?**

  Pick each of k arms equally often: $\frac{T}{k}$

  Estimate: $\hat{\mu}_a = \frac{k}{T} \sum_{j=1}^{T/k} X_{a,j}$

  Regret: $R_T = \frac{T}{k} \sum_{a=1}^{k} (\mu^* - \hat{\mu}_a)$

$X_{a,j}$... payoff received when pulling arm a for j-th time

Bandit Algorithm: First try

- Regret is defined in terms of average reward
- So, if we can estimate avg. reward we can minimize regret
- Consider algorithm: *Greedy*

  Take the action with the highest avg. reward

Bandit Algorithm: First try

- **Consider algorithm: *Greedy***
  Take the action with the highest avg. reward

  - **Example:** Consider 2 actions
    - ✓ **A1** reward 1 with prob. 0.3
    - ✓ **A2** has reward 1 with prob. 0.7
  - Play **A1**, get reward 1
  - Play **A2**, get reward 0
  - Now avg. reward of **A1** will never drop to 0, and we will never play action **A2**

Exploration vs. Exploitation

- **The example illustrates a classic problem in decision making:**
  - We need to trade off between **exploration** (gathering data about arm payoffs) and **exploitation** (making decisions based on data already gathered)
- **The Greedy algo does not explore sufficiently**
  - **Exploration:** Pull an arm we never pulled before
  - **Exploitation:** Pull an arm a for which we currently have the highest estimate of μ_a

Optimism

- The problem with our **Greedy** algorithm is that it is **too certain** in the estimate of μ_a
  - When we have seen a single reward of 0 we shouldn't conclude the average reward is 0

- **Greedy can converge to a suboptimal solution!**

Epsilon-Greedy Algorithm

Algorithm: Epsilon-Greedy

- **For t=1:T**
  - Set $\varepsilon_t = O\left(\frac{1}{t}\right)$ (that is, ε_t decays over time t as $1/t$)
  - **With prob. ε_t: Explore** by picking an arm chosen uniformly at random
  - **With prob. $1-\varepsilon_t$: Exploit** by picking an arm with highest empirical mean payoff

- **Theorem [Auer et al. '02]**
  For suitable choice of ε_t it holds that

$$R_T = O(k \log T) \Rightarrow \frac{R_T}{T} = O\left(\frac{k \log T}{T}\right) \to 0$$

k…number of arms

Issues with Epsilon-Greedy

- What are some issues with **Epsilon-Greedy**?
  - **“Not elegant”**: Algorithm explicitly distinguishes between exploration and exploitation
  - **More importantly:** Exploration makes **suboptimal choices** (since it picks any arm equally likely)
- **Idea:** When exploring/exploiting we need to **compare** arms

Comparing Arms

- Suppose we have done experiments:
  - **Arm 1**: 1 0 0 1 1 0 0 1 0 1
  - **Arm 2**: 1
  - **Arm 3**: 1 1 0 1 1 1 0 1 1 1

- Mean arm values:
  - **Arm 1**: 5/10, **Arm 2**: 1, **Arm 3**: 8/10

- Which arm would you pick next?

- **Idea:** Don't just look at the mean (that is, expected payoff) but also the confidence!

Confidence Intervals (1)

- **A confidence interval is a range of values within which we are sure the mean lies with a certain probability**
  - We could believe μ_a is within [0.2,0.5] with probability 0.95
  - If we would have tried an action less often, our estimated reward is less accurate so the confidence interval is bigger
  - Interval shrinks as we get more information (try the action more often)

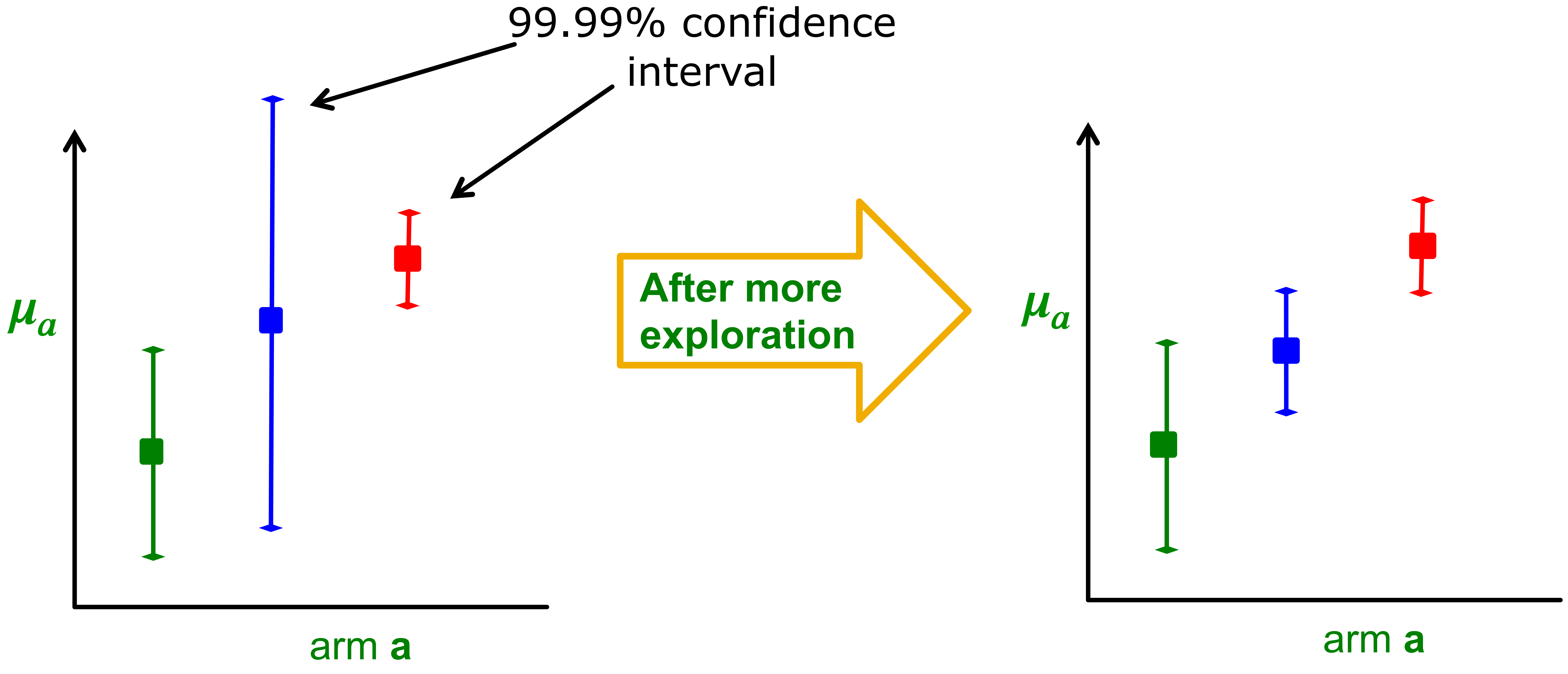
Confidence Intervals (2)

- **Assuming we know the confidence intervals**

- **Then, instead of trying the action with the highest mean we can try the action with the highest upper bound on its confidence interval**

- This is called an **optimistic policy**
  - We believe an action is as good as possible given the available evidence

Confidence Based Selection



Calculating Confidence Bounds

Suppose we fix arm *a*:

- Let $Y_{a,1} \ldots Y_{a,m}$ be the payoffs of arm ***a*** in the first ***m*** trials
  - So, $Y_{a,1} \ldots Y_{a,m}$ are **i.i.d.** random vars. taking values in **[0,1]**

Mean payoff of arm *a*: $\mu_a = E[Y_{a,\cdot}]$

Our estimate: $\hat{\mu}_{a,m} = \frac{1}{m} \sum_{\ell=1}^{m} Y_{a,\ell}$

- Want to find b such that with high probability $\left| \mu_a - \hat{\mu}_{a,m} \right| \leq b$
  - Want b to be as small as possible (so our estimate is close)

Goal: Want to bound $\mathbf{P}(\left| \mu_a - \hat{\mu}_{a,m} \right| \leq b)$

Hoeffding's Inequality (1)

Hoeffding's inequality provides an upper bound on the probability that the average deviates from its expected value by more than a certain amount:

- Let $X_1 \dots X_m$ be **i.i.d.** random vars. taking values in **[0,1]**
- Let $\mu = E[X]$ and $\hat{\mu}_m = \frac{1}{m}\sum_{\ell=1}^{m} X_\ell$
- **Then:** $P\left(\left|\mu - \hat{\mu}_m\right| \geq b\right) \leq 2\,exp\left(-2b^2 m\right) = \delta$ **(**δ **is the confidence level)**

- **To find out the confidence interval b (for a given confidence level δ) we solve:**

$2e^{-2b^2 m} \leq \delta$ **then** $-2b^2 m \leq \ln(\delta/2)$

So: $b \geq \sqrt{\frac{\ln\left(\frac{2}{\delta}\right)}{2\,m}}$

Hoeffding's Inequality (2)

- $\mathbf{P}\left(\left|\mu - \hat{\mu}_m\right| \geq b\right) \leq 2\,exp(-2b^2 m)$

  where b **is our upper bound,** m **number of times we played the action**
- Let's set $b = b(a,T) = \sqrt{2log(T)/m_a}$
- **Then:** $\mathbf{P}\left(\left|\mu - \hat{\mu}_m\right| \geq b\right) \leq 2T^{-4}$ **which converges to zero very quickly:**

Note:

- **If we don't play action** a**, its upper bound** b **increases**

  (This means we never permanently rule out an action no matter how poorly it performs)
- **Prob. our upper bound is wrong decreases with time** T

Upper Confidence Bound (UCB1) Algorithm

UCB1 Algorithm

- **UCB1 (Upper confidence sampling) algorithm**
  - **Set:** $\hat{\mu}_1 = \ldots = \hat{\mu}_k = 0$ and $m_1 = \ldots = m_k = 0$
    - $\hat{\mu}_a$ **is our estimate of payoff of arm** a
    - m_a **is the number of pulls of arm** a **so far**
  - **For *t* = *1:T***

For each arm *a* calculate: $UCB(a) = \hat{\mu}_a + \alpha\sqrt{\frac{2\ln t}{m_a}}$

Upper confidence interval (Hoeffding's inequality)

Pick arm $j = arg\ max_a\ UCB(a)$

Pull arm j **and observe** y_t

Set: $m_j \leftarrow m_j + 1$ **and** $\hat{\mu}_j \leftarrow \frac{1}{m_j}(y_t + \left(m_j - 1\right)\hat{\mu}_j)$

α...is a free parameter trading off exploration vs. exploitation

UCB1: Discussion

$$UCB(a) = \hat{\mu}_a + \alpha\sqrt{\frac{2\ln t}{m_a}}$$

$$b \geq \sqrt{\frac{\ln\left(\frac{2}{\delta}\right)}{2m}}$$

- Confidence interval **grows** with the total number of actions t we have taken
- But **shrinks** with the number of times m_a we have tried arm a
- This ensures each arm is tried infinitely often but still balances exploration and exploitation
- α plays the role of δ: $\alpha = f\left(\frac{2}{\delta}\right)$

$$\mathbf{P}\left(\left|\mu - \hat{\mu}_m\right| \geq b\right) = \delta$$

"Optimism in face of uncertainty":
The algorithm believes that it can obtain extra rewards by reaching the unexplored parts of the state space

Summary so far

- ***k***-armed bandit problem as a formalization of the exploration-exploitation tradeoff
- **Simple algorithms are able to achieve no regret (in the limit)**
  - Epsilon-greedy
  - UCB (Upper Confidence Sampling)

Use-case: Pinterest

- **Problem:** For new pins/ads we do not have enough signal on how good they are
  - How likely are people to interact with them?

- **Idea:**
  - Try to maximize the rewards from several unknown slot machines by deciding which machines and the order to play
  - Each pin is regarded as an arm, user engagement are considered as rewards
  - Making tradeoff between exploration and exploitation, avoid keep showing the best known pins and trap the system into local optima

Use-case: Pinterest

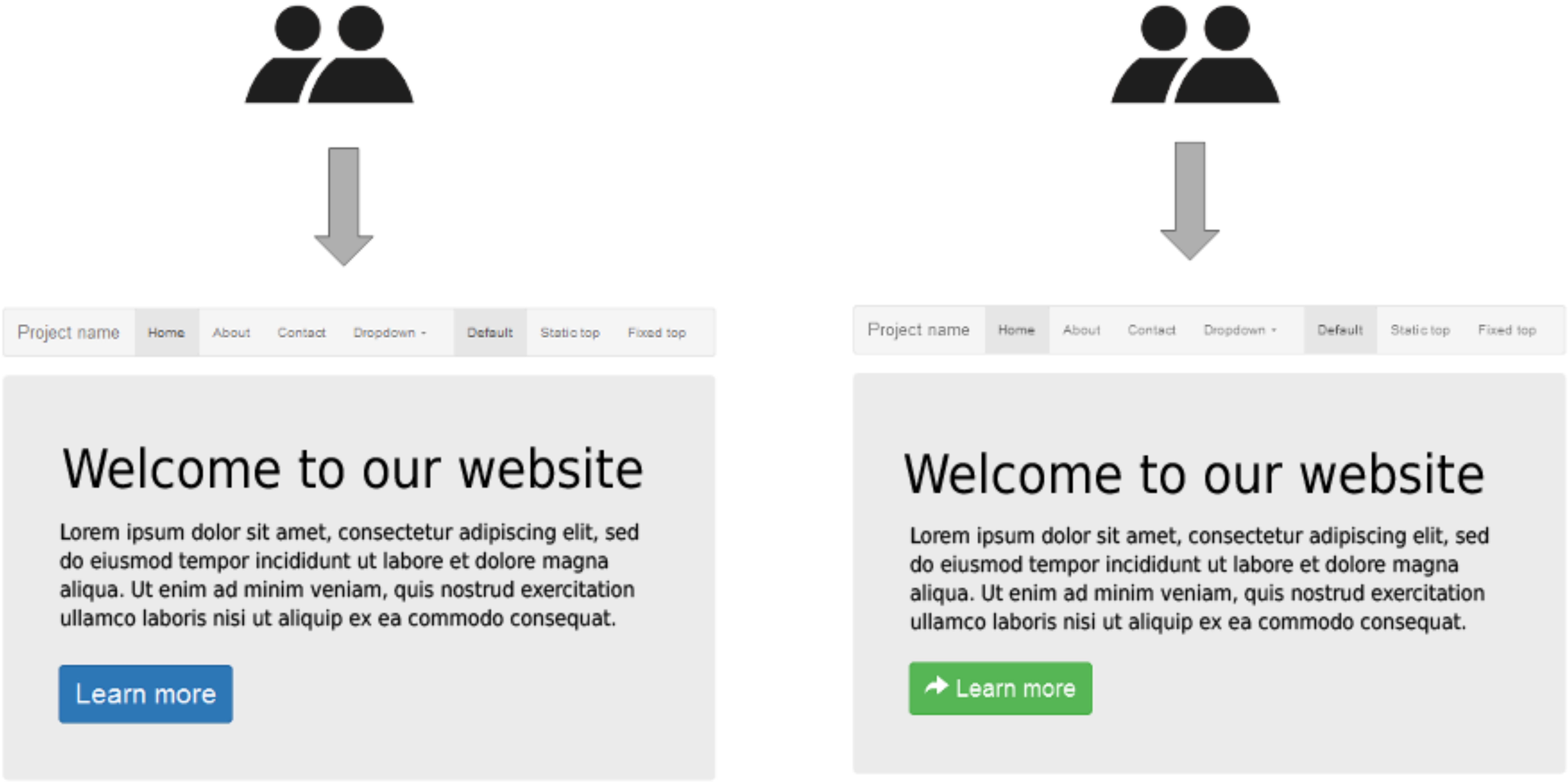
- **Solution: Bandit algorithm in round *t***
  - **(1) Algorithm** observes user a set ***A*** of pins/ads
  - **(2)** Based on payoffs from previous trials, algorithm chooses arm

    $\boldsymbol{a} \in \boldsymbol{A}$ and receives payoff $\boldsymbol{r_{t,a}}$
    - **Note only feedback for the chosen *a* is observed**
  - **(3)** Algorithm improves arm selection strategy with each observation $(a,\ r_{t,a})$

- **If the score for a pin is low, filter it out**

Use-Case: A/B testing

- **A/B testing is a controlled experiment with two variants, A and B**
- Part of the traffic sees variant **A**, part variant **B**



Click rate: 52 % 72 %

Use Case: A/B testing

- **Imagine you have two versions of the website and you'd like to test which one is better**
  - Version **A** has engagement rate of **5%**
  - Version **B** has engagement rate of **4%**
- **You want to establish with 95% confidence that version A is better**
  - You'd need 22,330 observations (11,165 in each arm) to establish that
    - Use t-test to establish the sample size
- **Can bandits do better?**

Example: Bandits vs. A/B testing

- **How long does it take to discover A > B?**
  - **A/B test:** We need 22,330 observations. Assuming 100 observations/day, we need 223 days

- **The goal is to find the best action (A vs. B)**
- The randomization distribution (traffic to A vs. B) can be updated as the experiment progresses

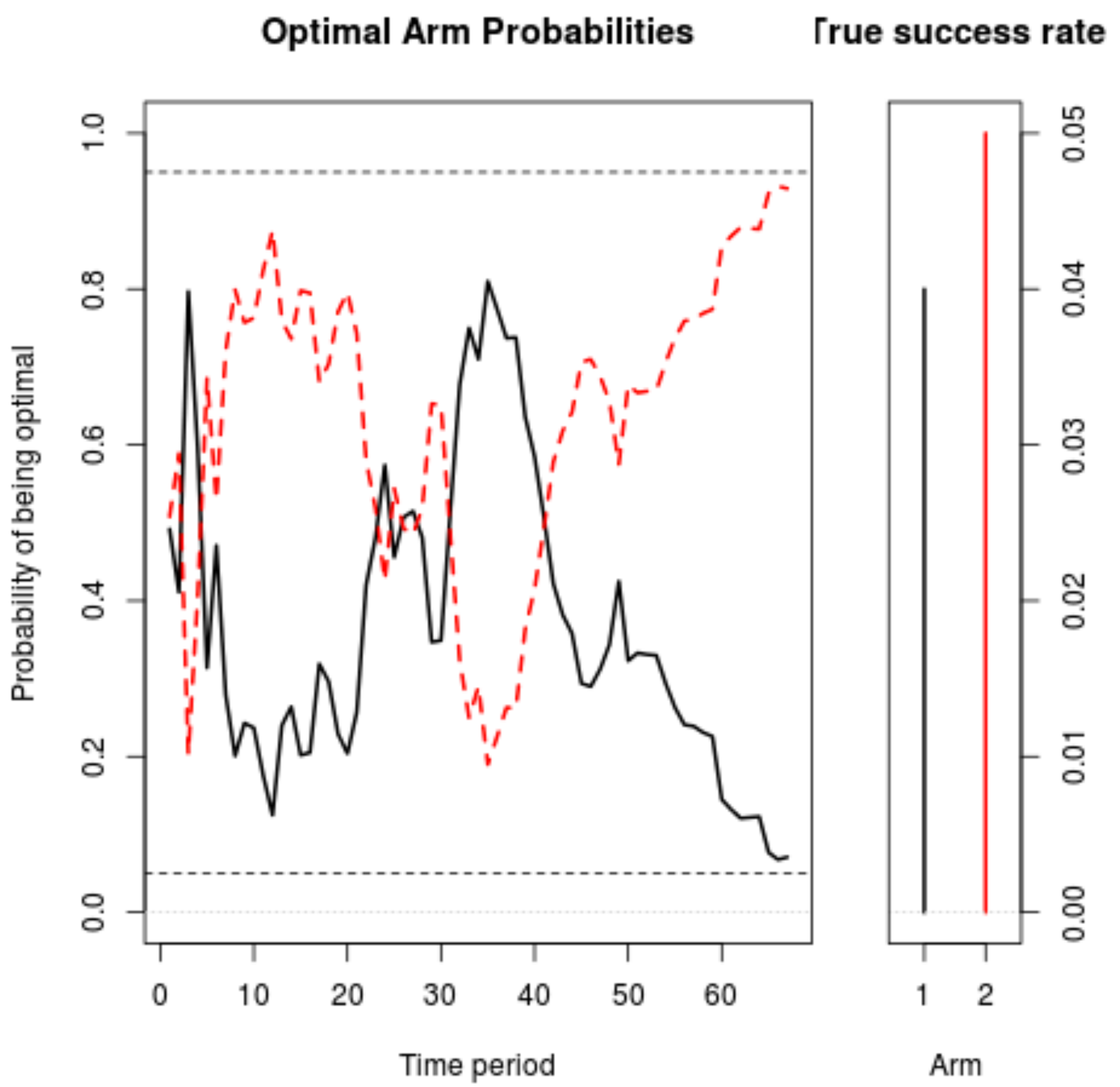
- **Idea:**
  - Twice per day, examine how each of the variations/arms has performed
  - Adjust the fraction of traffic that each arm will receive going forward
  - An arm that appears to be doing well gets more traffic, and an arm that is clearly underperforming gets less

Example

A/B test: We need 22,330 observations. Assuming 100 observations/day, we need 223 days

- On 1st day about 50 sessions are assigned to each arm
- Suppose **A** got really lucky on the first day, and it appears to have a 70% chance of being superior
- Then we assign it 70% of the traffic on the second day, and the variant B gets 30%
- At the end of the 2nd day we accumulate all the traffic we've seen so far (over both days), and recompute the probability that each arm is best

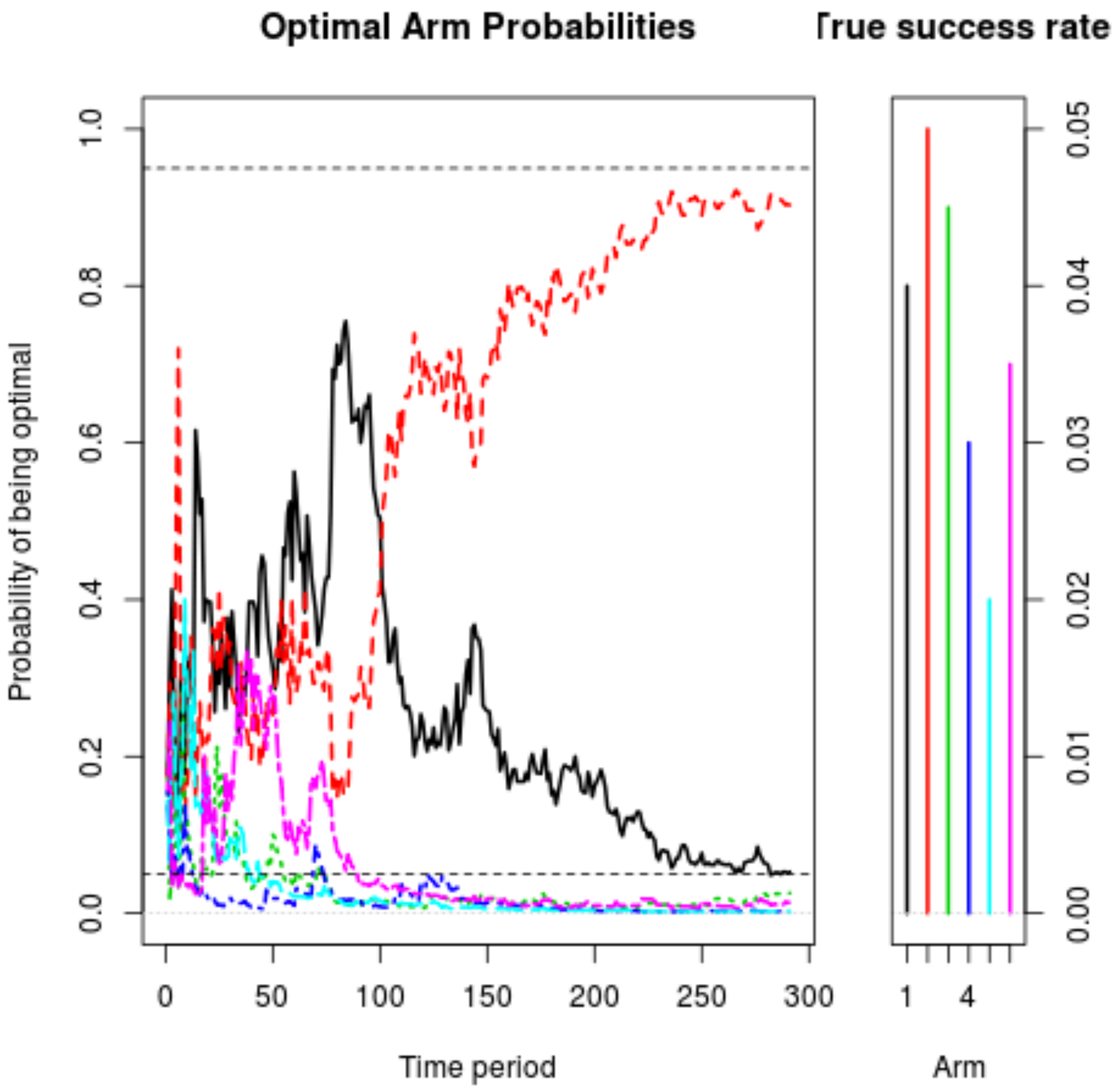
Simulation



The experiment finished in 66 days, so it saved us 157 days of testing (66 vs 223)

Generalization to multiple arms

Easy to generalize to multiple arms:



Thank You

Questions?