Statistics Corner

Questions and answers about language testing statistics:

# Choosing the Right Type of Rotation in PCA and EFA

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**Question:** In Chapter 7 of the 2008 book on heritage language learning that you co-edited with Kimi Kondo-Brown, there is a study (Lee & Kim, 2008) comparing the attitudes of 111 Korean heritage language learners. On page 167 of that book, a principal components analysis (with varimax rotation) describes the relation of examining 16 purported reasons for studying Korean with four broader factors. Several questions come to mind. What is a principal components analysis? How does principal components analysis differ from factor analysis? What guidelines do researchers need to bear in mind when selecting "factors"? And finally, what is a varimax rotation and why is it applied?

**Answer:** This is an interesting question, but a big one, made up of at least four sets of sub-questions: (a) What are principal components analysis (PCA) and exploratory factor analysis (EFA), how are they different, and how do researchers decide which to use? (b) How do investigators determine the number of components or factors to include in the analysis? (c) What is rotation, what are the different types, and how do researchers decide which particular type of rotation to use? And, (d) how are PCA and EFA used in language test and questionnaire development?

I addressed the first two questions in previous columns (Brown, 2009a & b). I'll attend to the third one here, and address the last one in the next column.

### What Is Rotation?

In the PCA/EFA literature, definitions of *rotation* abound. For example, McDonald (1985, p. 40) defines *rotation* as "performing arithmetic to obtain a new set of factor loadings (*v-f* regression weights) from a given set," and Bryant and Yarnold (1995, p. 132) define it as "a procedure in which the eigenvectors (factors) are rotated in an attempt to achieve simple structure." Perhaps a bit more helpful is the definition supplied in Vogt (1993, p. 91): "Any of several methods in factor analysis by which the researcher attempts to relate the calculated factors to theoretical entities. This is done differently depending upon whether the factors are believed to be correlated (oblique) or uncorrelated (orthogonal)." And even more helpful is Yaremko, Harari, Harrison, and Lynn (1986), who define factor rotation as follows: "In factor or principal-components analysis, rotation of the factor axes (dimensions) identified in the initial extraction of factors, in order to obtain simple and interpretable factors." They then go on to explain and list some of the types of orthogonal and oblique procedures.

How can a concept with a goal of simplification be so complicated? Let me try defining *rotation* from the perspective of a language researcher, while trying to keep it simple. I think of rotation as any of a variety of methods (explained below) used to further analyze initial PCA or EFA results with the goal of making the pattern of loadings clearer, or more pronounced. This process is designed to reveal the simple structure.

The choices that researchers make among the orthogonal and oblique varieties of these rotation methods and the notion of simple structure will be the main topics in the rest of this column.

### What Are the Different Types of Rotation?

As mentioned earlier, rotation methods are either orthogonal or oblique. Simply put, *orthogonal rotation* methods assume that the factors in the analysis are *uncorrelated*. Gorsuch (1983, pp. 203-204) lists four different orthogonal methods: equamax, orthomax, quartimax, and varimax. In contrast, *oblique rotation* methods assume that the factors are *correlated*. Gorsuch (1983, pp. 203-204) lists 15 different oblique methods.<sup>1</sup>

Version 16 of SPSS offers five rotation methods: varimax, direct oblimin, quartimax, equamax, and promax, in that order. Three of those are orthogonal (varimax, quartimax, & equimax), and two are oblique (direct oblimin & promax). Factor analysis is not the focus of my life, nor am I eager to learn how to use a new statistical program or calculate rotations by hand (though I'm sure I could do it if I had a couple of spare weeks), so those five SPSS options serve as boundaries for the choices I make. But how should I choose which one to use?

Tabachnick and Fiddell (2007, p. 646) argue that "Perhaps the best way to decide between orthogonal and oblique rotation is to request oblique rotation [e.g., direct oblimin or promax from SPSS] with the desired number of factors [see Brown, 2009b] and look at the correlations among factors...if factor correlations are not driven by the data, the solution remains nearly orthogonal. Look at the factor correlation matrix for correlations around .32 and above. If correlations exceed .32, then there is 10% (or more) overlap in variance among factors, enough variance to warrant oblique rotation unless there are compelling reasons for orthogonal rotation."

For example, using the same Brazilian data I used for examples in Brown 2009a and b (based on the 12 subtests of the *Y/G Personality Inventory* from Guilford & Yatabe, 1957), I ran a three-factor EFA followed by a direct oblimin rotation. The resulting correlation matrix for the factors that the analysis produced is shown in Table 1. Notice that the highest correlation is .084. Since none of the correlations exceeds the Tabachnick and Fiddell threshold of .32 described in the previous paragraph, "the solution remains nearly orthogonal." Thus, I could just as well run an orthogonal rotation.

Table 1. Correlation Matrix for the Three Factors in an EFA with Direct Oblimin Rotation for the Brazilian Y/GPI Data

Factor	1	2	3
1	1.000	-0.082	0.084
2	-0.082	1.000	-0.001
3	0.084	-0.001	1.000

Moreover, as Kim and Mueller (1978, p. 50) put it, "Even the issue of whether factors are correlated or not may not make much difference in the exploratory stages of analysis. It even can be argued that employing a method of orthogonal rotation (or maintaining the arbitrary imposition that the factors remain orthogonal) may be preferred over oblique rotation, if for no other reason than that the former is much simpler to understand and interpret."

## **How Do Researchers Decide Which Particular Type of Rotation to Use?**

We can think of the goal of rotation and of choosing a particular type of rotation as seeking something called simple structure, or put another way, one way we know if we have selected an adequate rotation method is if the results achieve simple structure. But what is simple structure? Bryant and Yarnold (1995, p. 132-133) define *simple structure* as:

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<sup>&</sup>lt;sup>1</sup> FYI, the 15 oblique methods are binormamin, biquartimin, covarimin, direct oblimin, indirect oblimin, maxplane, oblinorm, oblimax, obliquimax, optres, orthoblique, orthotran, promax, quartimin, and tandem criteria.

A condition in which variables load at near 1 (in absolute value) or at near 0 on an eigenvector (factor). Variables that load near 1 are clearly important in the interpretation of the factor, and variables that load near 0 are clearly unimportant. Simple structure thus simplifies the task of interpreting the factors.

Using logic like that in the preceding quote, Thurstone (1947) first proposed and argued for five criteria that needed to be met for simple structure to be achieved:

- 1. Each variable should produce at least one zero loading on some factor.
- 2. Each factor should have at least as many zero loadings as there are factors.
- 3. Each pair of factors should have variables with significant loadings on one and zero loadings on the other.
- 4. Each pair of factors should have a large proportion of zero loadings on both factors (if there are say four or more factors total).
- 5. Each pair of factors should have only a few complex variables.

In order to understand Thurstone's five criteria, you will need to understand a few more concepts:

- 1. What's a zero loading? One rule of thumb (after Gorsuch, 1983, p. 180) is that zero loadings includes any that fall between -.10 and +.10.
- 2. What's a *significant loading*? With a sample size of say 100 participants, loadings of .30 or higher can be considered significant, or at least salient (see discussion in Kline, 2002, pp. 52-53). With much larger samples, even smaller loadings could be considered salient, but in language research, researchers typically take note of loadings of .30 or higher.
- 3. And what are *complex variables*? Simply put, these are variables with loadings of .30 or higher on more than one factor.

Now, try going back through Thurstone's five criteria with those three definitions at hand.

### **Moving Towards Simplicity...**

Is achieving simple structure important? Experts in factor analysis seem to think that an abbreviated version of simple structure is important. For example, Kline (2002, p. 66) says, "...I am in agreement with Cattell [1978] and all serious workers in factor analysis that the attainment of simple structure is essential to factor analysis. Where this has not been done there is little reason to take the results seriously." One page earlier, Kline (2002, p. 65) appears considerably more flexible when he says that "Thurstone proposed five criteria for deciding on simple structure, although two of these are of overriding importance, namely that each factor should have a few high loadings with the rest of the loadings being zero or close to zero...Certainly the strict Turstonian approach is no longer followed." To resolve the apparent contradiction in Kline's views, you need only realize that he is no doubt referring to the less strict definition of simple structure in both statements.

Other experts also appear to use less strict definitions of simple structure, especially when considering what rotation procedure to use in achieving it. For example, Kim and Mueller (1978, p. 50) argue that, "If identification of the basic structuring of variables into theoretically meaningful subdimensions is the primary concern of the researcher, as is often the case in an exploratory factor analysis, almost any readily available method of rotation will do the job." To explore their view, I tried rotating the EFA results for the Brazil data using three orthogonal methods (see Table 2) and two oblique methods (see Table 3). All of those rotations produced essentially the same pattern of loadings. Put another way, the literature indicates that the choice of rotation may not make much difference; certainly, in language research situations like the one that led to the analyses shown in Tables 2 and 3,

where the factors are not markedly correlated (as demonstrated above), the choice from among those options available in SPSS (whether orthogonal or oblique) appears to make very little difference.

Table 2. Three Orthogonal Rotations of the Brazil Data

	Varimax Rotation			Quar	Quartimax Rotation			Equamax Rotation		
	Factor				Factor			Factor		
Trait	1	2	3	1	2	3	1	2	3	
social extraversion	-0.135	0.665	-0.118	-0.146	0.662	-0.122	-0.129	0.666	-0.115	
ascendance	-0.109	0.548	-0.098	-0.118	0.545	-0.100	-0.104	0.549	-0.095	
thinking extraversion	-0.072	-0.026	0.530	-0.072	-0.024	0.530	-0.073	-0.028	0.530	
rhathymia	0.391	0.606	0.206	0.381	0.613	0.203	0.396	0.602	0.209	
general activity	-0.219	0.680	-0.070	-0.230	0.676	-0.073	-0.213	0.682	-0.067	
lack of agreeableness	0.119	0.540	0.197	0.110	0.543	0.194	0.124	0.538	0.199	
lack of cooperativeness	0.466	0.029	0.060	0.465	0.037	0.060	0.466	0.025	0.060	
lack of objectivity	0.614	0.044	-0.187	0.614	0.053	-0.188	0.615	0.039	-0.187	
nervousness	0.759	-0.171	-0.021	0.762	-0.158	-0.020	0.758	-0.177	-0.021	
inferiority feelings	0.677	-0.469	0.101	0.685	-0.457	0.104	0.673	-0.476	0.099	
cyclic tendencies	0.785	0.106	-0.005	0.783	0.119	-0.006	0.785	0.099	-0.004	
depression	0.784	-0.227	-0.076	0.788	-0.214	-0.076	0.782	-0.234	-0.077	

Table 3. Two Oblique Rotations of the Brazil Data

	О	Oblimin Rotation			Promax Rotation			
		Factor			Factor			
Trait	1	2	3	1	2	3		
social extraversion	-	0.666	-	-	0.672	-		
	0.169		0.139	0.200		0.125		
ascendance	-	0.549	-	-	0.553	-		
	0.137		0.115	0.163		0.103		
thinking extraversion	-	-0.015	0.523	-	-0.014	0.492		
_	0.070			0.057				
rhathymia	0.360	0.597	0.227	0.336	0.575	0.322		
general activity	-	0.685	-	-	0.695	-		
·	0.254		0.098	0.284		0.100		
lack of agreeableness	0.092	0.540	0.198	0.072	0.531	0.241		
lack of	0.464	0.016	0.097	0.462	-0.008	0.180		
cooperativeness								
lack of objectivity	0.611	0.022	-	0.603	-0.008	-		
y y			0.138			0.019		
nervousness	0.767	-0.194	0.043	0.771	-0.231	0.172		
inferiority feelings	0.700	-0.488	0.163	0.721	-0.521	0.258		
cyclic tendencies	0.778	0.082	0.057	0.771	0.042	0.204		
depression	0.794	-0.252	-	0.800	-0.290	0.123		
•			0.010					

As Gorsuch (1983, p. 205) put it, "If the simple structure is clear, any of the more popular procedures can be expected to lead to the same interpretations." He then recommends rotating with varimax [orthogonal] or promax [oblique]. Kim and Mueller (1978, p. 50) conclude by saying, "We advise that beginners choose one of the commonly available methods of rotation, such as Varimax if orthogonal rotation is sought or Direct Oblimin if oblique rotation is sought. [For more on determining the adequacy of rotation, see Tabachnick & Fiddell, pp. 646-647]

### **Conclusion**

What should second language researchers do in selecting a rotation method for a PCA or EFA in their research? At minimum, it seems useful to try one oblique rotation method (e.g., direct oblimin or promax, while examining the factor correlation matrix for values over  $\pm 0.32$ , using the criterion explained in Tabachnick & Fiddell, 2007, p. 646) and one orthogonal rotation method (e.g., the ever-popular varimax rotation). Also consider whether there are any theoretical reasons why an orthogonal method might be preferable to an oblique method or vice versa. Above all, the rotated results should be examined for simple structure, at least following Kline's (2002, p. 65) relatively flexible definition:

"...that each factor should have a few high loadings with the rest of the loadings being zero or close to zero..." (i.e., less than  $\pm 0.10$  after Gorsuch, 1983, p. 180).

Coming back to the original question at the top of this column (about the Lee & Kim, 2008, study), did they try using at least one oblique and one orthogonal rotation method (while examining oblique rotation factor correlation matrix for values lower than  $\pm 0.32$ )? We have no way of knowing because the researchers did not discuss how or why they chose varimax rotation. Is that a capital crime? Their study would have been clearer if they had provided such an explanation. But no, it is not a capital crime, especially if they ended up successfully achieved simple structure. Table 4 shows their results. Notice that two to six variables have high loadings on each factor and that most of the rest of the loadings are zero (i.e., below  $\pm 0.10$ ) or close to zero. However, it is troubling that five of the variables are complex in the sense that they have loadings above .30 on two or three factors. These indicate that, while the pattern of loadings is strong overall, there is some complexity. More about this in the next column.

Table 4. Principal Components Analysis (with Varimax Rotation) Loadings of Motivation Items

	Instrumental		Integ		
	Factor 1 School- related	Factor 2 Career- related	Factor 3 Personal fulfillment	Factor 4 Ethnic Heritage	$h^2$
I learn Korean to transfer credits to college.	0.83	0.01	0.17	-0.11	0.72
I learn Korean because my friend recommended it.	0.82	0.14	0.06	0.22	0.74
I learn Korean because my advisor recommended it.	0.80	0.10	0.36	0.04	0.78
I learn Korean because of the reputation of the program and instructor.	0.77	0.12	0.12	0.22	0.67
I learn Korean for an easy A.	0.69	0.18	-0.29	0.16	0.61
I learn Korean to fulfill a graduation requirement.	0.63	0.11	0.19	-0.18	0.47
I learn Korean to get a better job.	0.04	0.80	0.20	0.11	0.69
I learn Korean because I plan to work overseas.	0.26	0.80	0.11	0.10	0.75
I learn Korean because of the status of Korean in the world.	0.10	0.73	0.20	0.22	0.63
I learn Korean to use it for my research.	0.38	0.48	0.44	0.10	0.58
I learn Korean to further my global understanding.	0.16	0.33	0.71	0.08	0.65
I learn Korean because I have an interest in Korean literature.	0.18	0.04	0.64	0.13	0.56
I learn Korean because it is fun and challenging.	0.04	0.04	0.63	0.46	0.63
I learn Korean because I have a general interest in languages.	0.11	0.03	0.57	0.51	0.59
I learn Korean because it is the language of my family heritage.	-0.01	0.26	0.05	0.80	0.71
I learn Korean because of my acquaintances with Korean speakers.	0.10	0.21	0.20	0.70	0.58
% of variance explained by each factor	0.23	0.15	0.14	0.12	0.64

Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization. Eigenvalue>1.

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