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Cover Page

Summary of Client Mission

The Center for the Study of Higher and Postsecondary Education (CSHPE), within the University of Michigan School of Education, offers a master's degree in Higher Education, whose goal is to train leaders in higher education research and practice, with the aim of landing roles in university administration, public policy, accreditation bodies, or professional associations.

Summary of Client Problem

The Higher Education program requires completion of an approved internship. The internship application process takes place in mid-March to mid-April, as many applicants determine whether or not to accept admission at University of Michigan based on the internship they have landed. The internship placement process is labor-intensive for CSHPE staff and frustrating for both applicants and placement sites who may not obtain their ideal match.

Questions

1. How has game theory been applied in the placement matching process in other contexts?
2. Could the "stable matching theory" be applied to CSHPE matching problem?

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Introduction to Game Theory

Game theory began as a branch of applied mathematics to analyze how individuals make decisions to maximize their welfare, when their welfare is dependent on the choices made by other individuals. Game theory expanded beyond the field of mathematics to computer science and artificial intelligence, economics, political science, psychology, philosophy, and ethics.

One real world application of game theory is the matching of medical students to hospitals for their residency. Under the National Residency Matching Program (NRMP), medical students and hospitals rank their preferences of each other and submit their ranking to the national clearinghouse. The clearinghouse then uses a modified form of the "deferred acceptance algorithm" to match residents to hospitals.[1] The NMRP is extremely successful and over 29,000 positions were filled in 2016. The fill-rate, which measures not just positions filled but also how well the algorithm matches the preferences of students and hospitals, was 96.2% for 2016.[2]

When the deferred acceptance algorithm is used, where there is an equal number of members of two groups that must be matched, given their ranking preferences, there is at least one solution that is stable, meaning that there is no alternative where both members of a pair would prefer each other over their existing match.[3] That is, there is no solution where a medical student and a hospital would both prefer each other over their current match.

This report will examine the mechanics of the deferred acceptance algorithm, the application of the algorithm to other contexts, and how the algorithm might apply to the CHSPE internship placement process.

Mechanics of the Deferred Acceptance Algorithm

The "deferred acceptance algorithm" was developed by David Gale and Lloyd Shipley in a 1962 paper studying what they called the stable marriage problem.[4] They posited that a stable solution can be found to match an equal number of men and women given their preferred rankings of each other. Stable, in this sense, means that there is no pair where both members of a pair would prefer each other over their existing match. In other words, there is no pair that would elope, which would make the matching unstable.

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[1] The residency matching algorithm has been modified over the years to account for married couples seeking residencies in the same city and other variables. *See* Roth, A. E. (1984). "The evolution of the labor market for medical interns and residents: A case study in game theory," *Journal of Political Economy* 92: 991–1016.

[2] "National Resident Matching Program, Results and Data: 2016 Main Residency Match®." National Resident Matching Program, Washington, DC. 2016, available at www.nrmp.org.

[3] Roth, A. E. (2008), "Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions," *International Journal of Game Theory*, 36, 537-569.

[4] Gale, D. and L. Shapley (1962), College Admissions and the Stability of Marriage, *American Mathematical Monthly*, 69, 9-15.

In the following example, the table shows the rankings of each man and woman below their name. For example, take the solution: A1, B2, C3, and D4. Male 3 is matched to his second choice, and Female B is matched to her third choice, but if both were matched to each other, they would both get their first choice. This is an example of a "blocking pair" – where the man and woman prefer each other to their current mates – that would make this arrangement an unstable solution. The blocking pairs for this solution are identified below by 'xxx.'

Unstable solution:

Name Ranking	Female A 2341	Female B 3421	Female C 4213	Female D 3241
Male 1 BDAC	Match			
Male 2 DABC	xxx	Match		xxx
Male 3 BCDA		xxx	Match	
Male 4 BCDA		xxx		Match

A stable matching is a solution with no blocking pairs. The stable matching theorem posits that for an equal number of men and women, no matter how they rank each other, there is always at least one stable matching solution.[5] The deferred acceptance algorithm can find the stable solution like this: The first man proposes to his highest rank woman. If she is not engaged, she tentatively accepts. If she is already engaged, she picks the man she prefers, and the rejected man then proposes to the woman he ranked next, and so on. This continues until each man is engaged.[6] Each woman has deferred her final acceptance of the proposal until all the men have made their choices. In a stable solution, any given man or woman might be better off with a different match than another, but the solution overall is stable because there is no blocking pair. A stable solution is shown in the following table.

[5] Roth, A. E. (2008), "Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions," *International Journal of Game Theory*, 36, 537-569.

[6] A demonstration of the algorithm can be found at mathsite.math.berkeley.edu/smp/smp.html.

Stable solution:

Name Ranking	Female A 2341	Female B 3421	Female C 4213	Female D 3241
Male 1 BDAC			Match	
Male 2 DABC	Match			
Male 3 BCDA		Match		
Male 4 BCDA				Match

The order in which the men propose is not important when using the deferred acceptance algorithm. However, it is important whether the men do the proposing or whether the women do the proposing.[7] If the women propose, the solution is known as women-optimal because each woman is better off when the women propose than when the men propose. In a women-optimal solution, each woman gets the best possible man from all possible stable solutions. It may not be one of her top choices, but it is still the best partner she can have in a stable matching. For men, a women-optimal solution means that he is matched with his worst possible partner; not necessarily his least favorite partner, but his least favorite possible partner. The stable solution above is a men-optimal solution.

Applications of the Deferred Acceptance Algorithm

No dating website has yet used the deferred acceptance algorithm, but it has been used in a variety of other applications, especially in markets where money does not change hands. For example, the algorithm has been used to place students in secondary schools in cities that offer school choice and to develop a kidney exchange program. Work on the deferred acceptance algorithm is so influential that Alvin Roth and Lloyd Shapley were awarded the Nobel Prize in Economics in 2012.[8]

[7] Roth, A. E. (2008), "Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions," *International Journal of Game Theory*, 36, 537-569.

[8] "The Prize in Economic Sciences 2012." Nobelprize.org. Nobel Media AB 2014 (22 Oct 2016), available at www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2012 (awarded jointly to Alvin E. Roth and Lloyd S. Shapley "for the theory of stable allocations and the practice of market design").

In 2003, the New York City public school system adopted a variation of the deferred acceptance algorithm to place students in public schools based on their rankings.[9] Under the old manual system, parents and students could rank only five schools, and students had an incentive to falsify their preferences and schools had an incentive to hold back on the number of seats they had to fill.[10] The use of the algorithm helped reduce the percentage of unplaced students by 90%, from 30,000 to 3,000 students.[11] Another variation is now used by the Boston public school system to place students in public schools.[12]

The algorithm has also been used in kidney-donation schemes.[13] Kidney transplant patients need a matching donor, but their relatives may not be a compatible match by blood type. It is illegal to buy and sell kidneys, but exchanges are permitted. By applying the deferred acceptance algorithm, the medical community has found a way to match unrelated donors and patients for kidney transplants in kidney exchange networks. In 2008, doctors successfully performed a six-way kidney transplant.[14] Six patients simultaneously received new kidneys from six donors, none of whom was a relative of any of the six patients.

As noted above, the deferred acceptance algorithm has been successfully used by the National Residency Matching Program (NRMP). By participating in the NRMP, both hospitals and medical students enter into a binding commitment to accept a position if a match results.[15] Waivers are possible in limited circumstances. Therefore, students should only rank those hospitals where they are truly willing to work and hospitals should only rank those students they are truly willing to hire. The hospitals may not ask students how they plan to rank the hospitals because hospitals are meant to submit their rankings based on credentials and practice fit rather than the likelihood of the student also ranking the hospital. Both sides are meant to rank according to their true preference and not how they think the other party will rank them.

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[9] Abdulkadiroglu, Atila, , et al., (May 2005), "The New York City High School Match," *American Economic Review* 95, no. 2: 364–367.

[10] Herszenhornh, David M. (October 3, 2003). "Revised Admission for High Schools." *The New York Times*, p. New York/Region.

[11] Abdulkadiroglu, Atila, et al., (May 2005), "The New York City High School Match," *American Economic Review* 95, no. 2: 364–367.

[12] Abdulkadiroglu, Atila, et. al., "The Boston Public School Match." *American Economic Review* 95, no. 2 (May 2005): 368–371.

[13] Rees, Michael A., et. al. (2009), "A Non-Simultaneous Extended Altruistic Donor Chain," *New England Journal of Medicine*, 360:11, 1096-1101.

[14] "'Six-way' kidney transplant first," BBC News, (Apr. 9, 2008) available at news.bbc.co.uk/go/pr/fr/-/2/hi/health/7338437.html.

[15] *See generally* National Resident Matching Program website at www.nrmp.org.

Potential Application to the CHSPE Internship Placement Process

The CHSPE internship placement process is not structured in a way to use the deferred acceptance algorithm. Currently, students submit resumes and cover letters to internship sites, who choose whom to interview. After interviews, the internship sites send offers directly to the students without CHSPE playing any intermediary role. Students are free to decline an internship offer, and hope to receive a better offer from another site. If a student declines an offer, the internship site can make an offer to another student, if that student has not already accepted another offer. If popular students receive multiple offers, CHSPE requires only that students hold no more than one active offer after 48 hours.

The key feature of the NRMP is the binding commitment that is not present in the CHSPE process. CHSPE candidates are not currently required to accept an internship offer, nor are they required to accept admission to University of Michigan before April 15. This can mean that internship sites whose offer is declined may find that their second and third choice applicants have already accepted another internship offer. This is similar to the problem that students had using the old New York City public schools allocation process. For CHSPE applicants, the offers come directly from the internship sites in no particular order. Applicants may have to decide whether to accept an offer, not knowing whether a better offer is on the way.

Using a student-optimal deferred acceptance algorithm with a binding commitment may resolve these issues. Students would be prevented from holding more than one offer at a time. But since they have ordered their preferences, the offer they receive would be the best (stable) offer. There would be no student or site who would prefer each other over their current placement.

A student-optimal solution prevents the students from gaming their choices. The NRMP was a hospital-optimal solution for its first few decades, and has been resident-optimal solution since 1997. Researchers found that a student-optimal NRMP meant that the best strategy for students was to rank their true preferences and not try to manipulate the ranking based on which hospital they perceived as wanting them.[16] A student-optimal algorithm would also have the advantage of giving CHSPE insight into the students' true rankings of internship sites from students over time. If a placement site is consistently ranked at the top, CHSPE might encourage the site to add a second internship slot if possible. On the other hand, if placement site is consistently ranked in the bottom, CHSPE might consider dropping that site from the program.

The deferred acceptance algorithm may still result in unplaced student interns needing manual placement, but this is no different than today's situation. A more problematic issue is that applicants need not accept admission and therefore their internship offer until April 15. The binding commitment to accept an internship is essentially a commitment to attend Michigan, which the school cannot require to be made before April 15. This would delay the internship offers, which are currently made by the end of March.

Furthermore, it changes the decision to be made by the applicant. Under the current system, the applicant can decide whether or not to attend Michigan based the internship offer he or she has

[16] Roth, A. E. (2008), "Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions," *International Journal of Game Theory*, 36, 537-569.

received. If the applicant does not like the internship placement, the student may choose to attend another school. If the deferred acceptance algorithm were applied, the student would be accepting admission to Michigan and signing the binding agreement to accept the match made by the program, without knowing exactly what internship offer he or she will receive.

In summary, while the CHSPE internship placement process has some similarities to the medical residency matching program, it has some key structural differences. A switch to using the deferred acceptance algorithm would change the placement dynamic and decision-making process for applicants.

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