FM3817B Assignment 1

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Due to Jan 28, 2025, 11:55 pm

General information Question(s) with bonus points: If you do it correctly, you will get the bonus pts (until reach the full scores). Otherwise, you will not lose any points. Hence, feel free to try it out and play with them! Enjoy.

More questions will be added progressively as the course advances. This assignment will be finalized by January 21, 2025. Students are strongly encouraged to begin working on it early, rather than waiting until the last day.

Submission rule Submissions must be done via Gradescope. You must carefully assign pages to their corresponding questions. You will lose a typical number of points in each case below:

a. Submission not in PDF format.

b. Questions with no pages assigned to them.

Please submit a single PDF file (you can create it electronically using word, latex, etc.). Here are other ways to achieve this:

a. If you write your derivation on papers, you can scan them into a pdf file

b. If they are images, paste images to a word document then save as a pdf file.

If you have difficulty in formatting your submission, please turn to TA office hours as soon as possible.

Important rule It is your responsibility to allocate some time to format your submission prior to the due time and ensure that you can work with Gradescope properly. Late submissions are NOT accepted.

Other policies You are allowed to discuss with your classmates. However, each student must submit their *own work*. Scholastic offences are taken seriously, and students are directed

to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at this link.

Question 1 (6 pts)**.** Prospect theory is a Nobel-winning theory in behavioral economics. More details can be found here. In this question, we are going to investigate an interesting utility function playing a crucial role in prospect theory:

$$U(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\alpha & x < 0 \end{cases},$$

where x represents gain or loss and the parameters are set as $\alpha = 0.9$ and $\lambda = 2$. Please answer the following questions using the analytical tools you have learned from this course so far:

a. Compute $U'(x)$ for $x > 0$ and $x < 0$. Does this function increase or decrease on $x > 0$? How about $x < 0$?

b. Compute $U''(x)$ for $x > 0$ and $x < 0$. Is this a convex or concave function on $x > 0$? How about $x < 0$?

c. Is U continuous at $x = 0$? (Yes or No) Is U differentiable at $x = 0$? (Yes or No)

Question 2 (12 pts)**.** Consider a set $S = [1, 2) \cup (2, 3]$ and answer the following questions in detail:

a. Is S a closed set? (Yes or No) Why?

b. Is S an open set? (Yes or No) Why? (Please state two *different* pieces of evidence for this question.)

c. Is S a convex set? (Yes or No) Why?

d. If your answer in the previous question is No, what is the *simplest* way to modify the set S such that the new one S^* is convex? (Hint: there is a unique answer here involving only a *single* point.)

e. For this new set S^*, show that it is a convex set.

Question 3. In economic and finance, a famous class of utility functions is called Cobb-Douglas utility function. Such a function is in the form $U(x, y) = kx^a y^b$ with $a, b \in (0, 1)$ and $k > 0$. Answer the following questions:

a. Derive the Jacobian of U.

b. Derive the Hessian of U.

c. (Bonus) Is this a convex function?

Question 4 (Bonus 4 pts)**.** Suppose $f(x)$ and $g(x)$ are both convex functions. Let $h(x) \coloneqq f(g(x))$. Can we say $h(x)$ must be a convex function? (If yes, show your proof. If not, give a counter example.)

Question 5. As discussed in the first lecture, please find a real-world example involving optimization (different from the two examples mentioned in the lecture: portfolio optimization and regression, and also different from those provided in this assignment). Present your example by addressing the following components:

a. (6 pts) Clearly describe the problem's background and outline its key components, including:
    a. What is the objective that needs to be optimized? (In other words, what does the objective function represent?) Why is this optimization necessary?
    b. What are the decision variables? If known, briefly explain how each variable impacts the objective.
    c. Are there any constraints in this problem? If so, describe them. If no constraints exist, explain why.

b. (Bonus 4 pts) Simplify the problem and write it in the general form of an optimization problem. [1]

Notes

[1]Hint: Refer to the Week 1 summary for guidance on structuring an optimization problem.