VRIJE UNIVERSITEIT AMSTERDAM

QUANTITATIVE FINANCIAL RISK MANAGEMENT

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Assignment 1: Market Risk

Authors

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January 19, 2022

1 Introduction

Value-at-Risk (VaR) is one of they key risk measures for financial institutions and is required by the Capital Requirements Directive under the Internal Models Approach for market risk. It can be defined as a forward-looking risk measure to describe, control and manage market risk. According to the Basel III framework the banks still have flexibility in the calculation of their capital requirements, but there are some minimum standards which they have to adhere.

The basis of the calculation is the VaR computed on a daily basis, using a 99th percentile, one tailed confidence interval. The 10-day-returns of the portfolio must be used, which can be approximated by using the 1-day-returns and scale these to ten days. Furthermore, the length of the sample period underlying the calculation must be at least one year. Besides that, banks are free to choose between models based on variance-covariance matrices, historical simulations or Monte Carlo simulations. Moreover, banks have to hold a minimum of k times the 10-day 99% VaR in capital, where k is at least 3.

It is well known that VaR is limited in measuring tail risk. The introduction of the Fundamental Review of the Trading Book (FRTB) by the Basel Committee on Banking Supervision (BCBS) tackles this issue by changing the capital requirements to the 10-day 97.5% Expected Shortfall (ES), which is the expected loss given that the loss is greater than the VaR level.

In this paper we will focus on the VaR and ES of our portfolio, that will be calculated with different methods. The simple Variance-Covariance and Historical Simulation method will be used, in addition to the more refined methods of Constant Conditional Correlation and Filtered Historical Simulation. The models will then be evaluated by backtesting to determine which of these is the most suitable model for accurate VaR calculations.

2 Data Description

In this report, we have set up a portfolio consisting of a combination of local and international indices and a bond to partly finance this portfolio. Table 1 shows the portfolio with its corresponding investments, totaling a startup capital of €1,000,000. Due to exposures to the foreign indices Hang Seng Index, S&P 500 Index and Nikkei 225 Index, the portfolio is also exposed to changes in the foreign currencies HKD, USD and JPY, respectively. Half of the portfolio is financed by a 20-year bond against the floating 3-Month London Interbank Offered Rate (LIBOR) rate. We download daily adjusted closing prices from Yahoo Finance for all equities and currencies and obtain the 3-Month LIBOR, based on Euro, from the

Federal Reserve Bank of St. Louis[1]. The time horizon of our analysis is from 2010-01-04 to 2021-03-26. Moreover, we synchronize the data and deleted quotes for days when there is no trading in at least one of the portfolio constituents due to continental differences.

Table 1: Portfolio of assets and liabilities

Instrument	Investment
Hang Seng Index	€500,000
AEX Index	€500,000
S&P 500 Index	€500,000
Nikkei 225 Index	€500,000
20Y floating 3M Libor bond	-€1,000,000
Total	€1,000,000

3 Linearized Loss Operator

The VaR calculations are based on the losses of the portfolio that are calculated by the approximate method of the Linearised Loss Operator. In this section we describe this theoretical approach behind our calculations. The advantage of the theoretical setup is that it easily generalizes to alternative datasets, if any wish to be applied.

We express the value of the prices of the four indices, the bond and the exchange rate as the following

$$S_{1,t}, S_{2,t}, S_{3,t}, S_{4,t}, B_{1,t}, E_{1,t}, E_{2,t}, E_{3,t} \quad \text{for} \quad t = 1, .., n$$

The value of the portfolio with absolute weights α_i then becomes

$$V(t) = \underbrace{\alpha_1 S_{1,t} \cdot E_{1,t}}_{\text{HSI}} + \underbrace{\alpha_2 S_{2,t} \cdot E_{2,t}}_{\text{S\&P500}} + \underbrace{\alpha_3 S_{3,t} \cdot E_{3,t}}_{\text{NIKKEI}} + \underbrace{\alpha_4 S_{4,t}}_{\text{AEX}} + \underbrace{\alpha_5 B_{1,t}}_{\text{BOND}}$$

We denote the risk factors as:

$$\begin{aligned} Z_{i,t} &= \log S_{i,t} \quad \text{for } i = 1, 2, 3, 4 \\ Z_{4+i,t} &= \log E_{i,t} \quad \text{for } i = 1, 2, 3 \\ Z_{8,t} &= y(t, T) \end{aligned}$$

[1]https://fred.stlouisfed.org/series/EUR3MTD156N

We construct the risk factors changes as:

$$\begin{aligned}
X_{i,t+1} &= Z_{i,t+1} - Z_{i,t} = \log \frac{S_{i,t+1}}{S_{i,t}} \quad \text{for } i = 1,2,3,4 \\
X_{4+i,t+1} &= \log \frac{E_{i,t+1}}{E_{i,t}} \quad \text{for } i = 1,2,3 \\
X_{8,t+1} &= y(t+1,T) - y(t,T)
\end{aligned}$$

Writing the portfolio value V_t as a function of the vector of risk factors $\mathbf{Z}_t$, V_{t+1} can be written as a function of $\mathbf{Z}_t$ and the vector of risk factor changes $\mathbf{Z}_{t+1}$ as is given below

$$\begin{aligned}
V_t &= \underbrace{\alpha_1 e^{Z_{1,t}+Z_{5,t}}}_{\text{HSI}} + \underbrace{\alpha_2 e^{Z_{2,t}+Z_{6,t}}}_{\text{S\&P500}} + \underbrace{\alpha_3 e^{Z_{3,t}+Z_{7,t}}}_{\text{NIKKEI}} + \underbrace{\alpha_4 e^{Z_{4,t}}}_{\text{AEX}} + \underbrace{\alpha_5 e^{-(T-t)Z_{8,t}}}_{\text{BOND}} \\
V_{t+1} &= f(\mathbf{Z}_t + \mathbf{X}_{t+1;t})
\end{aligned}$$

The portfolio loss is then denoted by

$$\begin{aligned}
L_{t+1} &= -(V_{t+1} - V_t) \\
&= -(f(t+1, \mathbf{Z}_t + \mathbf{X}_{t+1}) - f(t, \mathbf{Z}_t))
\end{aligned}$$

If the mapping (f) is differentiable, we may use the following first order approximation for the loss we arrive at the following Linearised Loss Operator

$$\begin{aligned}
L^{\Delta}_{t+1} &= -\left(f_t(t, \mathbf{Z}_t) + \sum_{i=1}^{8} f_{z_i}(t, \mathbf{Z}_t) X_{t+1,i} \right) \\
&= -V_t \left[\underbrace{\omega_{1,t}(X_{1,t} + X_{5,t})}_{\text{HSI}} + \underbrace{\omega_{2,t}(X_{2,t} + X_{6,t})}_{\text{S\&P500}} + \underbrace{\omega_{3,t}(X_{3,t} + X_{7,t})}_{\text{NIKKEI}} \right. \\
&+ \qquad \left. \underbrace{\omega_{4,t} X_{4,t}}_{\text{AEX}} + \underbrace{\omega_{5,t}(y(t,T)\Delta t - (T-t)\Delta y)}_{\text{BOND}} \right]
\end{aligned}$$

where $w_{i,t} = \alpha_i S_{i,t} / V_t$.

The linearised losses can be found in Figure 6.

4 Methodologies Value-at-Risk & Expected Shortfall

Over the years, the literature developed several methods of calculating both risk measures, Value at Risk and Expected Shortfall. We analyze, the Variance-Covariance Method, Historical Simulation, as well as Constant Conditional Correlation Method with GARCH(1,1) and Filtered Historical Simulation method with EWMA.

4.1 Variance-Covariance Method

A rather simple method for calculating the VaR is the Variance-Covariance method. The method requires knowledge of the mean and standard deviation of the losses and assumes a Normal distribution or Student's t-distribution for the returns.

4.1.1 Normal distribution

Under the normal distribution, the 1 day VaR is computed as

$$VaR_\alpha = \mu + \sigma\Phi^{-1}(\alpha) \tag{1}$$

where μ is the mean loss, σ is the estimated daily portfolio volatility and Φ denotes the standard normal density function and $\Phi^{-1}(\alpha)$ is the α-quantile of Φ. The ES is then given by

$$ES_\alpha = \mu + \sigma\frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha} \tag{2}$$

where ϕ is the density of the standard normal distribution.

In Figure 7 the sensitivities of the normal Variance-Covariance method to the amount of previous days that are used to measure μ and σ are shown. The shortest lookback period has the highest values of the risk measures due to the recent stress period related to the COVID-19 pandemic. In the rest of this report we choose to use 300 days as lookback for the Variance-Covariance method as this is close to one year of trading days and similar to market practice for large banks in the Netherlands. Moreover, using 300 days lets us adhere to the minimum requirement of one year by the BCBS.

To investigate the differences when dealing with stressed periods or not, we re-calculate the VaR by removing the worst 20 portfolio losses from the data set. This is done to keep most of the data still in place but removing the observations of more stress. Figure 8 shows that non-stressed VaR is at all times equal or below the VaR that includes stress periods, which is expected as the tails of the distribution or winsorized leading to a less extreme distribution.

To assess if these estimates are any good, we can look at Figure 9, where the first image shows the QQ-plot of our portfolio returns compared to the normal distribution quantiles.

As we can see, the portfolio returns deviate substantial from the normal distribution at both ends of the distribution. This indicates that assuming normality in returns underestimates tail risk and therefore underestimates the VaR and ES.

4.1.2 Student's t-distribution

Assuming that the risk factors change according to a normal distribution is rather naive as it lacks the fat tails losses that financial markets often display. Therefore, instead of the normal distribution we can also assume a student's t-distribution. 1 day Value at Risk is then computed as

$$VaR_\alpha = \mu + \sigma t_\nu^{-1}(\alpha) \tag{3}$$

The ES is then given by

$$ES_\alpha = \mu + \sigma \frac{g_\nu(t_\nu^{-1}(\alpha))}{1-\alpha} \left(\frac{\nu + (t_\nu^{-1}(\alpha))^2}{\nu - 1} \right) \tag{4}$$

where t_ν denotes the density function of a standard t distribution, g_ν is the density of a standard t, and σ is not the standard deviation of the distribution but is calculated as

$$\sigma = \sqrt{\frac{\nu\sigma^2}{\nu - 2}} \tag{5}$$

Compared to the normality closed formulas we have an additional parameter which is the degrees of freedom (ν). This parameter defines the fatness of the distribution tails. Figure 9 shows the goodness of the distribution assumption and that 4 degrees of freedom would be a better reasonable choice, since it fits both the right tail and the left tail best. The QQ plots also reveal the property that the greater the degrees of freedom, the more closely the t-distribution resembles the standard normal distribution.

4.2 Historical Simulation

The historical simulation approach consists of an estimation of the loss operator under the empirical distribution of the data, instead of using some explicit parametric model. To do so, one assumes that losses are i.i.d drawings from an unknown loss distribution. By applying the loss operator to each and every series of risk factor change vector in the portfolio we obtain a set of historically simulated losses generated from the proper data.

In essence, the traditional risk measures are estimated through an inherently unconditional approach, leading to rather stable numbers. In this sense, it is from the historically simulated data from where inferences are made about the loss distribution and VaR & ES.

Although this approach comes with the advantage of being a rather easy to compute measure, it comes with a number of disadvantages when measuring risk, as the implicit assumption that future returns will behave in a similar way to past returns might yield misleading information since forecasts of VaR will never exceed the biggest recorded loss, and future returns may show completely different trends.

Moreover, the use of a one-day holding period (or ten days for regulatory capital calculations) assumes that all positions in the portfolio can be liquidated or hedged in one day. In periods of when illiquidity reigns in the market this assumption may not hold. Furthermore, the use of a 99% confidence level means that Historical VaR (HVaR) does not take into account any losses beyond this confidence level. Finally, information availability and transparency may also be a big disadvantage.

Table 2 shows the VaR calculated for 2021-03-26 using different lookback windows. For the rest of this report we only consider HVaR with a 300 trading days lookback window. As new historical data is considered, the estimations will change, but at a rather slow pace and in small quantities, so it is still not considered a conditional estimation approach.

Table 2: VaR and ES historical simulation estimates for 2021-03-26 based on different window of trading days

Trading days	300	1000	2000
VaR 97.5%	29388.48	21969.31	23650.84
ES 97.5%	44895.86	33892.46	34634.45
VaR 99%	40734.10	32426.50	32595.87
ES 99%	59945.52	44431.82	45104.58

4.3 Constant Conditional Correlation (CCC)

A more sophisticated method is a conditional method, known as the Constant Conditional Correlation (CCC) method, which is given as:

$$
\begin{aligned}
Cov(t; i, j) &= \rho_{ij}\sigma_{t,i}\sigma_{t,j} \\
\sigma_{t,i}^2 &= \omega_i + \phi\sigma_{t-1,i}^2 + \alpha\epsilon_{t-1,i}^2
\end{aligned}
$$

CCC allows for daily changes in volatility of the portfolio, hence it allows for daily adjustments of VaR_t, where t is now measured in days. This time varying property assures that if we enter a stress period some time during the year, we can adjust the VaR so that it reflects more accurately of our current potential losses. For each asset a GARCH model is estimated on the first 500 data points and 1 day forecasts of volatility are then calculated rolling, where the model is re-estimated at the end of every year.[2] The constant correlation

[2]Year 1 and 2 are used to estimate the VaR for year 3, Year 2 and 3 are used to estimate the VaR for year 4 etc.

matrix is also estimated on the first 500 data points. For all volatility forecasts a covariance matrix is calculated and this is then used to estimate VaR and ES by the variance-covariance method.

4.4 Filtered Historical Simulation (FHS).

The following section covers the implementation of a conditional risk estimation known as Filtered Historical Simulation. Under this approach we estimate hypothetical series of returns for each of the risk measures. The main advantage of this approach is its dynamic nature to adapt to the market situation at each moment in time, in particular to changes in volatility. In this context, methods of historical simulation are applied to unobserved innovations rather than to the observed data. The estimation process consists on determining the aforementioned innovations for each particular asset. The following represents a case example for two different assets:

$$X_{t+1}^{i} = \sigma_{t+1}^{x} * Z_t^x \tag{6}$$

$$y_{t+1} = \sigma_{t+1}^{y} * Z_t^y \tag{7}$$

Where the sigma parameter of volatility is assumed to fluctuate over time. In both of the cases, the estimates for volatility were obtained by using an Exponentially Weighted Moving Average (EWMA) approach.

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\mu_{t-1}^2 \tag{8}$$

The estimate of current volatility is recursive, and weighted by smoothing parameter, λ. We assume that $\lambda = 0.97$ to reduce cyclicality of risk measures, following the indications of London Clearing House. We use the first 60 points of the data set to initialize the variance. Once the volatility are estimated, we generate forecasts which are multiplied by the standardized innovations, resulting in a vector of hypothetical series of returns.

$$\hat{X}_{t+1}^{(i)} = \hat{Z}_i^x \cdot \hat{\sigma}_{t+1}^x \tag{9}$$

$$\hat{Y}_{t+1}^{(i)} = \hat{Z}_i^y \cdot \hat{\sigma}_{t+1}^y \tag{10}$$

From which we build the new Portfolio Loss and its distribution in order to obtain the P&L distribution and compute the VaR_α and ES_α

$$L_p^i = \omega^x \cdot \hat{X^{(i)}}_{t+1} + \omega^y \cdot \hat{Y}_{t+1}^{(i)} \tag{11}$$

5 Backtesting

5.1 Backtest VaR-methods, total and yearly violations

All four models we have described in preceding sections will be compared on performance. The performance will be measured by comparing the number of times the VaR was exceeded (violated) by the realized Linearised Losses with the number of expected exceedances (2.5% and 1% of the cases). Figure 1 shows the difference in estimation methods. As we use two years for estimation of CCC-GARCH, we disregard the first two years for all methods. The EWMA and CCC-GARCH methods are much more sensitive to the current developments in the market, while the other measures are relatively static.

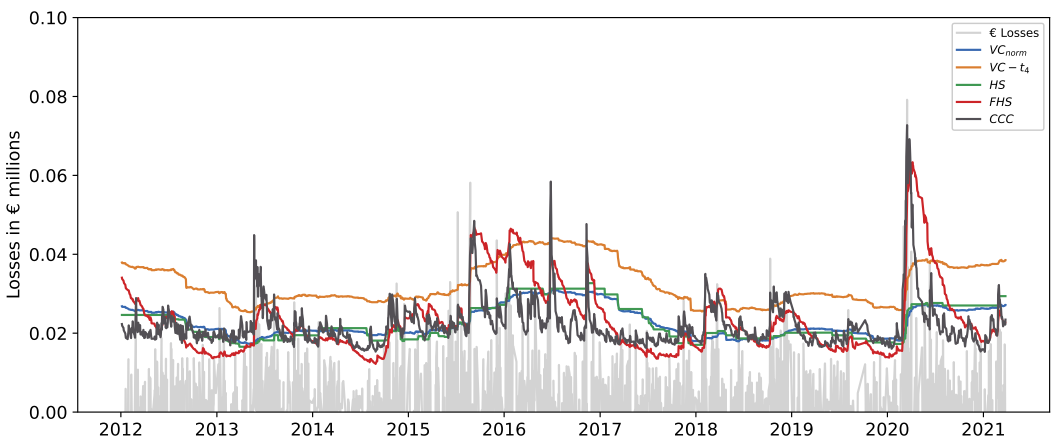

Figure 1: Backtest results of 97.5% VaR risk measures.

All the forecast models included in this report have been statistically tested using historical data at hand. A model is deemed appropriate if the number of violations is statistically equal to the expected number of violations, dependent on the confidence level considered. In this sense, we obtained p-values on the differences between the estimated risk measures VaR_α and ES_α for each of the different methods, and the observed violations along the period studied. For the case at hand, we test under the null hypothesis whether the VaR model is correct, p-values lower than 0.05 indicate that the observed number of violations significantly differ from the expected numbers.

Table 3 shows that, overall, at the 97.5% confidence level, violations of VaR are not statistically significant from the expected amount for the case of the Variance-Covariance (under Gaussian distribution), Historical Simulation approach and Filtered Historical Simulation. On the other hand, at the 99 % confidence level, violations of VaR are not statistically significant from the expected amount for the case of Historical Simulation, Filtered Historical

Table 3: The number of VaR violations for each method as a result of backtesting within the period 2012-01-05 to 2021-03-26. Figures in italics show significant discrepancies between observed and expected violation counts according to a binomial score test at the 5% level.

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	All
Trading Days	202	194	197	198	200	207	199	179	194	49	1819
Results for 97.5% VaR											
Expected Violations	6	5	5	5	6	6	5	5	5	2	50
VC Normal	0	*8*	5	*10*	4	1	*10*	1	*10*	1	50
VC t4	0	2	1	*6*	1	0	3	0	*7*	0	*20*
HS	0	*9*	5	*11*	4	1	*9*	1	*9*	1	50
FHS	1	*8*	*6*	*8*	2	4	5	1	4	1	40
CCC	0	2	5	*9*	3	3	2	1	*6*	1	*32*
Results for 99% VaR											
Expected Violations	3	2	2	2	3	3	2	2	2	1	22
VC Normal	0	*6*	3	*8*	1	1	5	1	8	0	*33*
VC t4	0	1	0	3	0	0	1	0	3	0	*8*
HS	0	*6*	2	*7*	0	1	4	0	*6*	0	26
FHS	0	3	2	4	0	3	2	1	3	0	18
CCC	0	2	1	*6*	0	1	1	0	2	0	13

Simulation and Constant Conditional Correlation approaches. A comparison of the expected shortfalls with average per year can be found in Table 5

5.2 Clustering of VaR-violations

The violations should not be clustered, but should occur randomly through time. For this we check by looking at the time between violations. Based on our results, for all models considered, and for the period between January 2012 and April 2021, VaR and ES violations show a tendency to occur in clusters of enhanced volatility periods. In particular, they tend to accumulate around the intervals ranging from 2013-14, mid 2015 until 2016 and early 2020. This suggests the existence of certain dependency in some of the violations. Visually, this relation seems less prominent for the case of the Constant Conditional Correlation approach, regardless of the confidence level.

If the violations are i.i.d., the spacing between them should follow approximately an exponential distribution. QQ-plots of the spacings of the different methods to check this can be seen in Figure 2 and 3 for a 97.5% and 99% VaR respectively. In general there is less clustering with the lower VaR. The FHS method seems to show the least clustering. As expected, the HS and all VC methods violations cluster together, because when there is a regime change and there is a period with high volatility, these methods are very slow to change their VaR estimates and there are likely to be multiple violations in a row. This effect can also be seen in Figure 1.

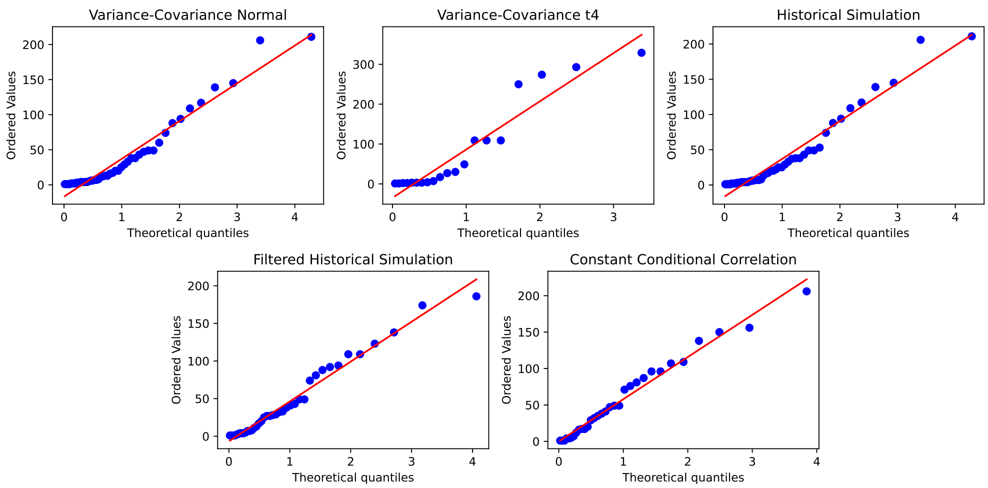

Figure 2: QQ-plots of the spacings of the VaR violations of the different methods at a confidence level $\alpha = 97.5\%$ against the exponential distribution.

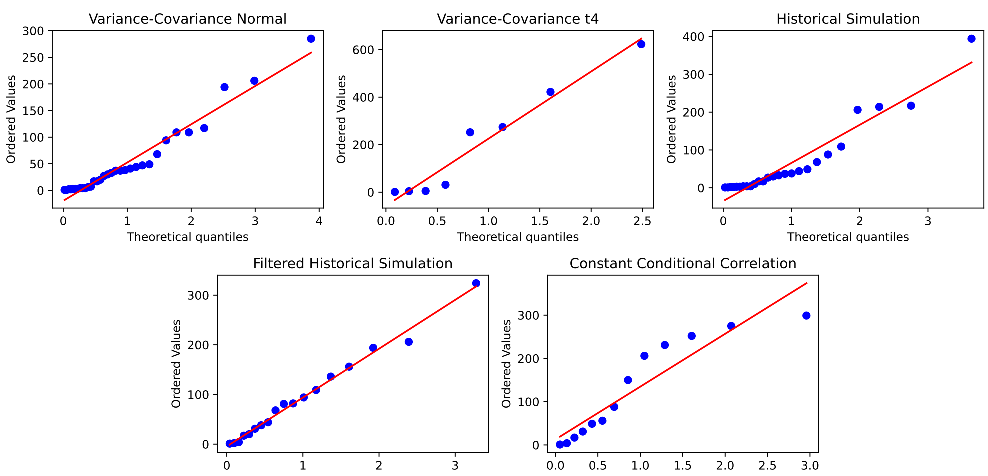

Figure 3: QQ-plots of the spacings of the VaR violations of the different methods at a confidence level $\alpha = 99\%$ against the exponential distribution.

6 5-day and 10-day VaR

Although most of risk measures estimation deals with daily horizons, trying to disentangle possible losses over the next trading day, the may be expressed over different horizons (h) such as a week or a month. This depends essentially on the expected holding period of the portfolio, since the longer positions are held, the greater the VaR. One of the most common approaches to express VaR and ES for a different horizon is to compute the daily VaR at a certain confidence level, and scale the value of VaR by the square root of time, since we know that mean losses scale linearly.

$$VaR_{h,\alpha} = VaR_{(1,\alpha)} \cdot \sqrt{h} \tag{12}$$

Despite being one of the most widely used techniques, and prominently embodied in the Basel Accord's Market Risk Amendment for horizons of up to 10 days, it is thought to be highly unreliable and to often overestimate VaR. This approach gives accurate estimates of VaR only if the expected daily return is assumed to be zero, however it is unlikely for firms to hold risky portfolios with average daily returns of zero.Furthermore, one of the main disadvantages of this approach is that it relies heavily on the assumption of a Gaussian return distribution and independent identically distributed observations.

However, one may also compute an empirical 5-days and 10-days VaR using a historical simulation approach by simply replacing the mean return and volatility over 1 day by the volatility and mean return over 5 or 10 days. By computing these two moments and plugging them into the formula, the result would be the VaR over a given period.

$$VaR_{h,\alpha} = \mu_h + Z_h \cdot \sigma_h \tag{13}$$

As mentioned, the traditional square root of time may lead to more conservative strategies since it tends to overestimate VaR. This may be the case since in case the average expected returns were positive, the square root rule typically implies an error that increases proportionally with the horizon h, and thus becomes larger and larger as the horizon lengthens. This phenomenon can also be seen in Table 4 as for the square root of time rule overestimates the 10 day VaR by almost 65%. For the 5 day we can see that the difference is much smaller.

Table 4: Table providing empirical 5-day and 10-day VaR and ES estimates including square root of time estimation for comparison

	1D scaled to 5D	5D	1D scaled to 10D	10D
VaR 97.5%	73649.65	77302.28	104156.34	63573.13
VaR 99%	99246.55	92299.35	140355.82	85361.94

7 Stress testing

We compute different stress test scenarios by modifying the prices of the last day 2021-03-26:

- Equity index values or stock prices changing by +/- 20% and +/- 40% of the current values
- Currencies moving by +/- 10%
- Interest rates shifting by +/- 2% and +/- 4%:

In these stylized scenarios we adjust only asset class at once. Figure 4 shows expected results, as negative shocks to equities increase the losses in the portfolio, since it consists mostly of equities. For the foreign currency exposures, we see that appreciations in the exchange rate lead to losses. On the other hand, changes in interest rates affect the portfolio by changing the interest installments and the bond´s valuation, since we short a bond when borrowing. All of the losses generated in these scenarios would violate the VaR for 2021-03-26. This is also the reason for which BCBS proposed to enhance the minimum capital requirements under the Pillar 1 market risk through requiring banks to calculate, in addition to the current VaR, a stressed VaR taking into account a one-year observation period relating to significant losses. The additional stressed VaR requirement is expected to help reduce the pro-cyclicality of the minimum capital requirements for market risk.

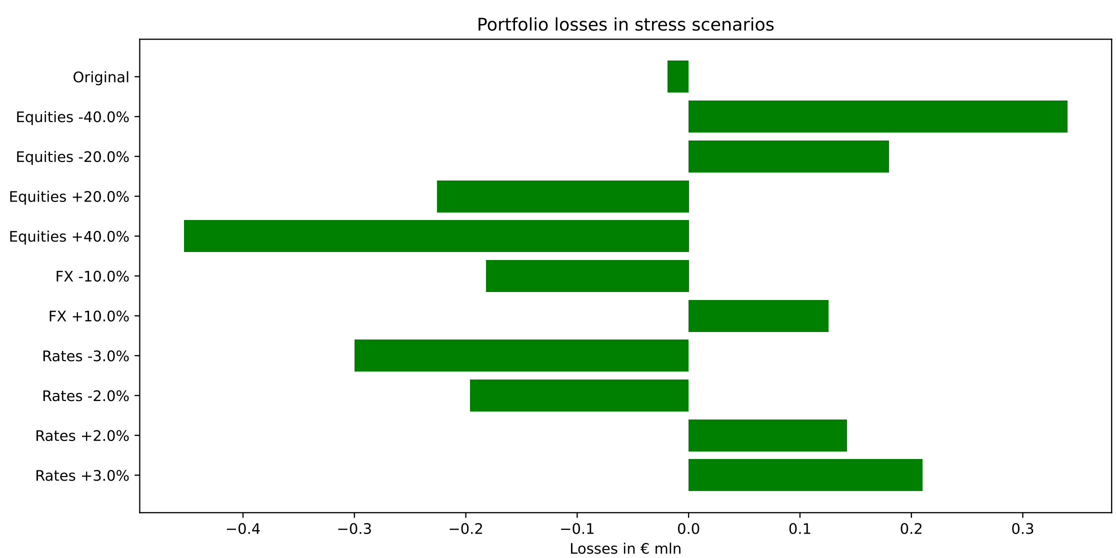

Figure 4: Stress testing analysis for the portfolio in 2021-03-26

We see that the shocks have the most impact on the filtered methods.

8 Conclusion and recommendations

The present document reviews various types of risk estimation methods applicable to financial risk management. In particular, the report focuses on the estimation of VaR and ES. Based on the empirical results obtained for the backtesting and stress testing of the models, the following section includes our recommendation on which estimation procedure deems more appropriate for the portfolio considered.

In our results we find that the VaR forecasts from the FHS method perform the best amongst all the methods, in both normal and extreme conditions. We confirm that there is volatility clustering in the portfolio returns, and when the volatility surges, the number of violations surges along with it. To reduce the potential damages caused by such a phenomenon, conditional methods should be applied. Specific model adjustments can also be made by induced modifications in the λ parameter. The forecasts from the unconditional methods are more stable than the ones from the conditional methods. However, because of their backward- looking approach, the methods are not able to capture the relatively recent changes in the risk factors.

If the bank does not have a proper modelling department, the historical simulation is also an option as it has regulatory acceptance. Historical simulation for VaR is a popular market practice as it has many advantages. Using historical data is easy to explain, conservative, assumption-free to some extent, and captures fat tails. For this method it is very important to consider the data the VaR is estimated on. If the data does not contain stressed periods the VaR estimates will not reflect periods of enhanced volatility, thus it is likely that higher losses than those estimated take place. This may be solved with stress testing if this is the preferred method. The stressed VaR could be obtained from stressed historical and/or hypothetical scenarios, e.g. using historical prices/returns from historical stress periods such as the Lehman Brothers collapse or the September 11 attacks in 2001.

A Appendix

A.1 Figures

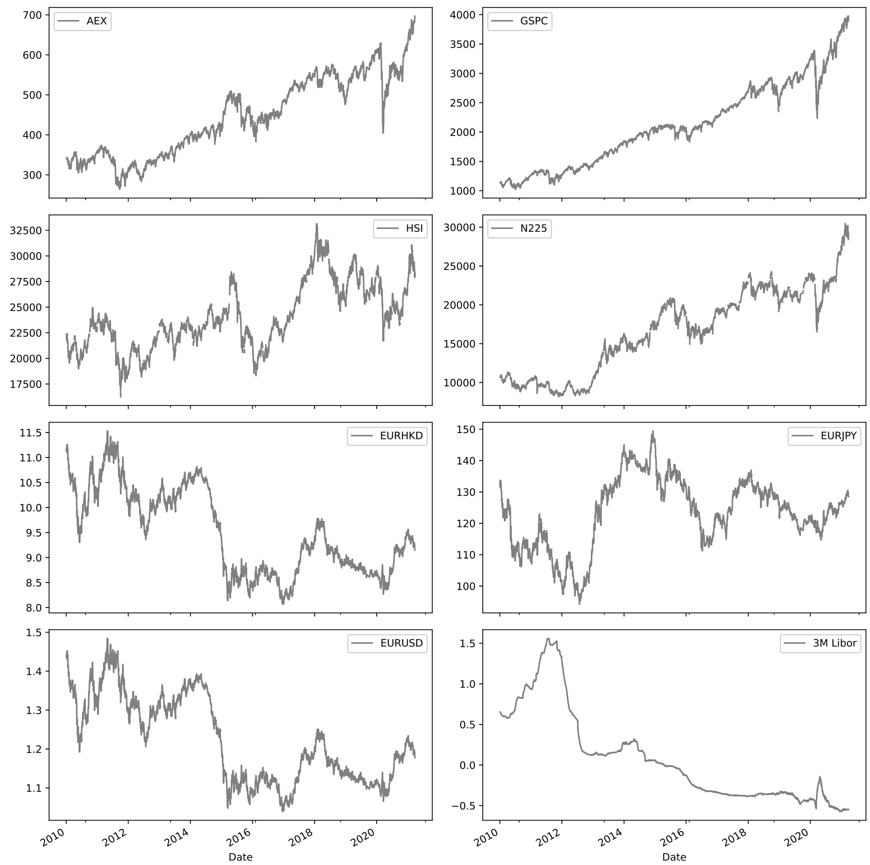

Figure 5: Levels of all financial variables used in this paper. Indices and exchange rates are shown in local currency, while 3-month Libor is shown in percentages.

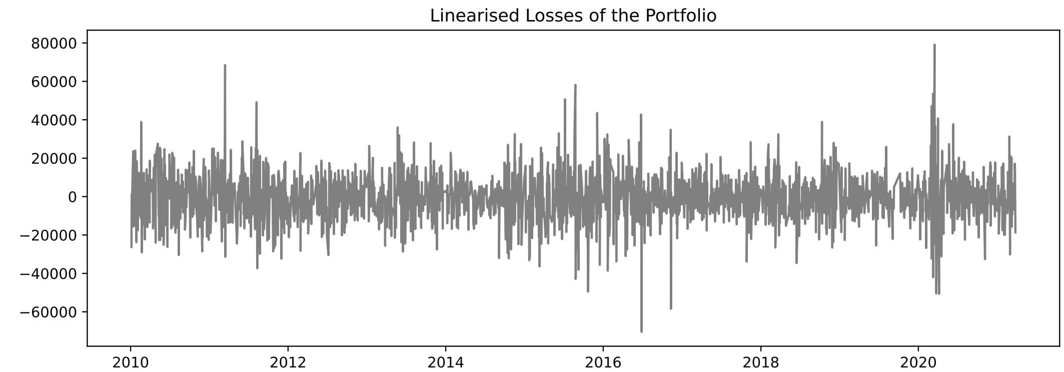

Figure 6: Linearised losses.

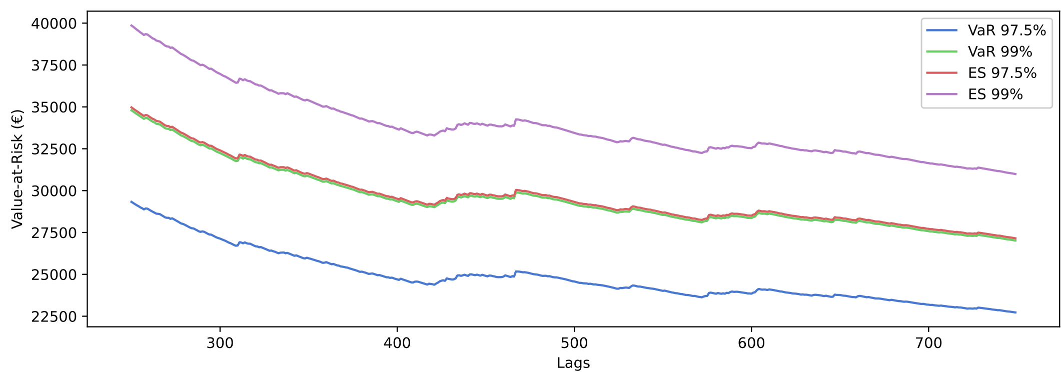

Figure 7: Value-at-Risk and Expected Shortfall amounts for the same day (2021-03-26) calculated by Variance-Covariance under a normal distribution with different amounts of past periods.

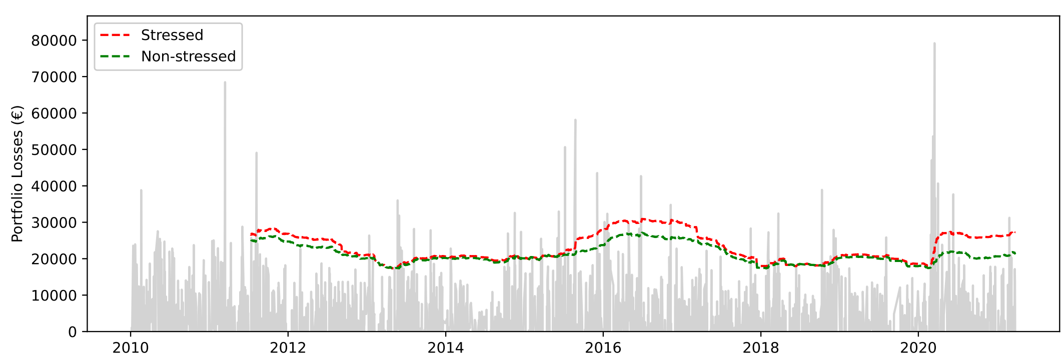

Figure 8: 97.5% Value-at-Risk for stressed and non-stressed period. For non-stressed we remove the worst 20 losses within the data set

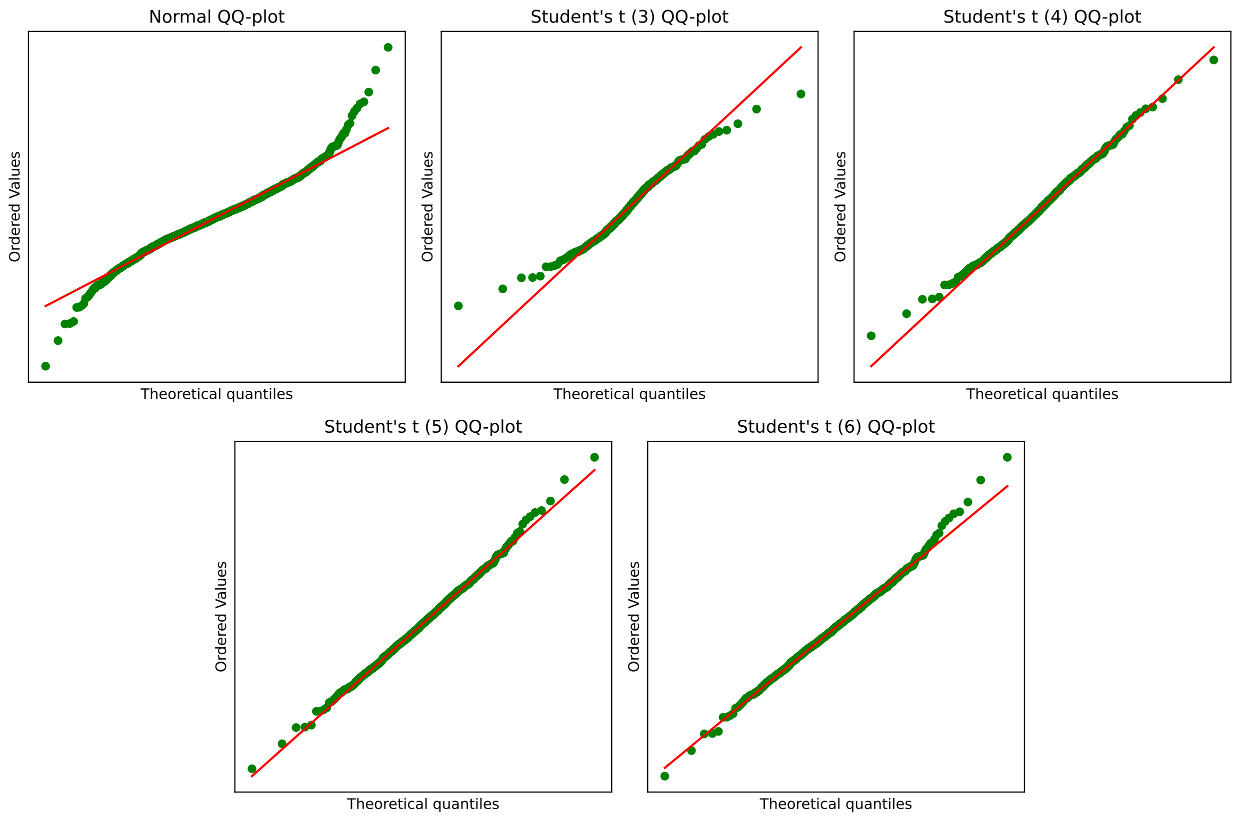

Figure 9: QQ-plot of the portfolio losses against normal and student-t distributions.

A.2 Tables

Table 5: Average expected shortfalls per year

Date	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Losses	-1676	-343	-885	-902	-569	-334	278	-1004	235	-516
Results for 97.5% ES										
VC N	28978	23425	24323	27258	35948	28334	22442	24280	29745	31777
VC t4	50070	40875	41784	47370	61805	48757	38549	41682	50918	54449
HS	32522	24203	26947	30842	42690	31544	24716	26657	40818	44605
FHS	27716	27861	24171	39373	46100	22878	30402	25922	49799	33590
CCC	24560	26432	23328	29097	30285	22448	27613	22793	31024	26633
Results for 99% ES										
VC N	33149	26875	27776	31235	41061	32373	25627	27721	33932	36260
VC t4	65700	53806	54724	62274	80967	63892	50486	54577	66609	71250
HS	45295	29418	31842	38673	53823	39388	30482	32737	54957	59945
FHS	33790	35510	29866	46931	53834	28087	36175	34506	62588	42420
CCC	28000	30134	26595	33172	34527	25592	31480	25985	35369	30363