Problems:

1.

	E	O	A	R	T	H	L	S
E	0	5	3	INF	INF	INF	INF	INF
O	INF	0	INF	2	7	INF	INF	INF
A	INF	6	0	1	6	INF	INF	INF
R	INF	INF	INF	0	2	9	5	INF
T	INF	INF	INF	INF	0	3	4	INF
H	INF	INF	INF	INF	INF	0	INF	2
L	INF	INF	INF	INF	INF	5	0	2
S	INF	INF	INF	INF	INF	INF	INF	0

E → A → R → T → H

Step 1:

V	E	O	A	R	T	H	L	S
D[v]	0	INF	INF	INF	INF	INF	INF	INF
Prev[v]	null	null	null	null	null	null	null	null
Visited	False	False	False	False	False	False	False	False

Step 2:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	INF	INF	INF	INF	INF
Prev[v]	null	E	E	null	null	null	null	null

Visited	True	False	False	False	False	False	False	False

Step 3:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	7	12	INF	INF	INF
Prev[v]	null	E	E	O	O	null	null	null
Visited	True	True	False	False	False	False	False	False

Step 4:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	4	9	INF	INF	INF
Prev[v]	null	E	E	A	A	null	null	null
Visited	True	True	True	False	False	False	False	False

Step 5:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	4	6	13	9	INF
Prev[v]	null	E	E	A	R	R	R	null
Visited	True	True	True	True	False	False	False	False

Step 6:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	4	6	9	9	INF
Prev[v]	null	E	E	A	R	T	R	null

Visited	True	True	True	True	True	False	False	False

Step 7:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	4	6	9	9	11
Prev[v]	null	E	E	A	R	T	R	H
Visited	True	True	True	True	True	True	False	False

Step 8:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	4	6	9	9	11
Prev[v]	null	E	E	A	R	T	R	H
Visited	True	True	True	True	True	True	True	False

Step 9:

V	E	O	A	R	T	H	L	S
D[v]	0	5	3	4	6	9	9	11
Prev[v]	null	E	E	A	R	T	R	H
Visited	True	True	True	True	True	True	True	True

2.
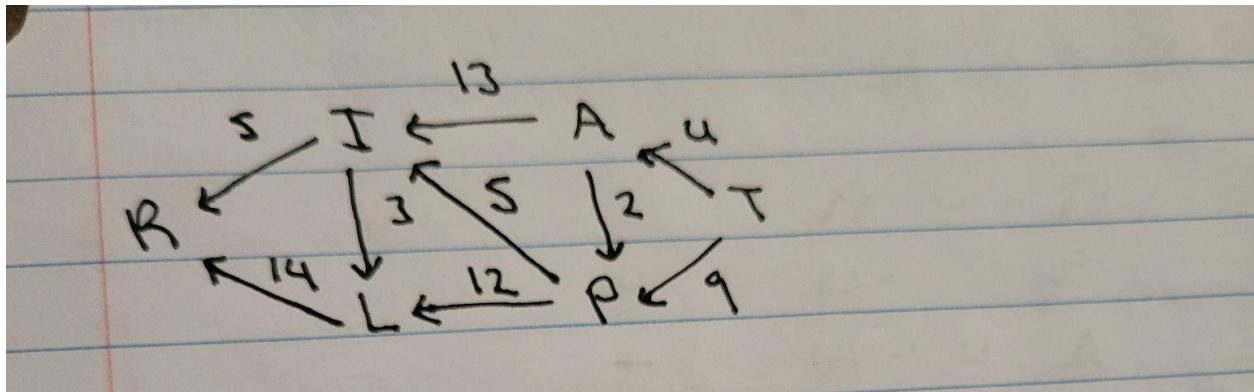
3.

T → A → P → I → R

Step 1:

V	T	A	P	I	L	R
D[v]	0	INF	INF	INF	INF	INF
Prev[v]	null	null	null	null	null	null
Visited	False	False	False	False	False	False

Step 2:

V	T	A	P	I	L	R
D[v]	0	4	9	INF	INF	INF
Prev[v]	null	T	T	null	null	null
Visited	True	False	False	False	False	False

Step 3:

V	T	A	P	I	L	R
D[v]	0	4	6	17	INF	INF
Prev[v]	null	T	A	A	null	null
Visited	True	True	False	False	False	False

Step 4:

V	T	A	P	I	L	R
D[v]	0	4	6	11	18	INF
Prev[v]	null	T	A	P	P	null
Visited	True	True	True	False	False	False

Step 5:

V	T	A	P	I	L	R
D[v]	0	4	6	11	14	16
Prev[v]	null	T	A	P	I	I
Visited	True	True	True	True	False	False

Step 6:

V	T	A	P	I	L	R
D[v]	0	4	6	11	14	16
Prev[v]	null	T	A	P	I	I
Visited	True	True	True	True	True	False

Step 7:

V	T	A	P	I	L	R
D[v]	0	4	6	11	14	16
Prev[v]	null	T	A	P	I	I
Visited	True	True	True	True	True	True

4.

	R	I	L	A	P	T
R	0	5	14	INF	INF	INF
I	5	0	3	13	5	INF
L	14	3	0	INF	12	INF
A	INF	13	INF	0	2	4
P	INF	5	12	2	0	9
T	INF	INF	INF	4	9	0

5.

The problem with modifying the above algorithm to find the longest path is there really isn't an optimal algorithm to find the longest path. You could try and modify Dijkstra's algorithm, but in general it isn't feasible to find the longest path.