(1) $L\{\int_0^t e^{-(t-\tau)} \sin(\tau)\, d\tau\} = L\{e^{-t} * \sin(t)\}$

$= L\{e^{-t}\} \cdot L\{\sin(t)\}$

$= \frac{1}{s-(-1)} \cdot \frac{1}{s^2+1} = \frac{1}{(s+1)(s^2+1)}$

$\Rightarrow \boxed{L\{\int_0^t e^{-(t-\tau)} \sin(\tau)\, d\tau\} = \frac{1}{(s+1)(s^2+1)}}$

(2) $y' = ky + f(t), \quad y(0) = y_0$

PELO TEOREMA DA TRANSFORMADA DA DERIVADA

$L\{y'\} = sY(s) - y(0) = sY(s) - y_0 \Rightarrow$

$sY(s) - y_0 = kY(s) + F(s) \Rightarrow$

$sY(s) - y_0 - kY(s) = F(s) \Rightarrow$

$(s-k)Y(s) - y_0 = F(s) \Rightarrow$

$Y(s) = \frac{F(s)+y_0}{s-k} = \frac{F(s)}{s-k} + \frac{y_0}{s-k}$

$L^{-1}\{Y(s)\} = y(t) = L^{-1}\left\{\frac{F(s)}{s-k} + \frac{y_0}{s-k}\right\} \Rightarrow$

$y(t) = L^{-1}\left\{F(s)\cdot\frac{1}{s-k}\right\} + y_0 L^{-1}\left\{\frac{1}{s-k}\right\} \Rightarrow$

$y(t) = L^{-1}\{F(s)\} * L^{-1}\left\{\frac{1}{s-k}\right\} + y_0 \cdot e^{kt} \Rightarrow$

$\boxed{y(t) = \int_0^t f(\tau)\, e^{k(t-\tau)}\, d\tau + y_0 e^{kt}}$

PROBLEMA DO VALOR INICIAL (P.V.I.):

$y'(t) = \frac{d}{dt} y(t) = \frac{\partial}{\partial t} \int_0^t f(\tau)\, e^{k(t-\tau)}\, d\tau + \frac{\partial}{\partial t} y_0 e^{kt}$

$= \frac{d}{dt}\, e^{kt} \int_0^t f(\tau)\, e^{-k\tau}\, d\tau + y_0 \frac{d}{dt} e^{kt}$

$= \underbrace{k e^{kt} \int_0^t f(\tau) e^{-k\tau}\, d\tau + \cancel{e^{kt}}\, f(t)\, \cancel{e^{-kt}}}_{\text{REGRA DE DERIVAÇÃO DO PRODUTO}} + y_0 k e^{kt}$

$\Rightarrow y'(0) = k e^{k\cdot 0} \int_0^0 f(\tau) e^{-k\tau}\, d\tau + f(0) + y_0 k e^{k\cdot 0}$

$\Rightarrow \boxed{y'(0) = f(0) + y_0 k}$