

Homework #(1)  
Hwapyeong Song

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## INSTRUCTIONS

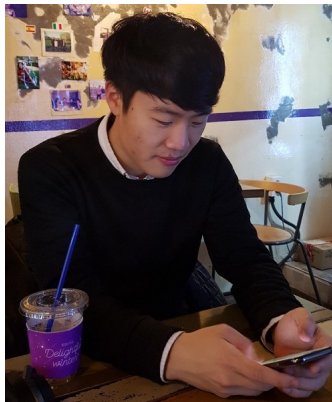
- Anything that is received after the deadline will be considered to be late and we do not receive late homeworks. We do however ignore your lowest homework grade.
- Answers to every theory questions need to be submitted electronically on ETL. Only PDF generated from LaTeX is accepted.
- Make sure you prepare the answers to each question separately. This helps us dispatch the problems to different graders.
- Collaboration on solving the homework is allowed. Discussions are encouraged but you should think about the problems on your own.
- If you do collaborate with someone or use a book or website, you are expected to write up your solution independently. That is, close the book and all of your notes before starting to write up your solution.

## 1 Q1. Learning $\text{\LaTeX}$

### 1.1 2 by 2 table

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### 1.2 Profile photo



## 2 Q2. Inverse transform sampling

### 2.1 Probability integral transform

By definition of continuous uniform distribution, if the random variable  $Y = F_X(X)$  is uniformly distributed in  $[0, 1]$ ,  $F_Y(y) = y$  for all  $y \in [0, 1]$  and vice versa. As  $F_X(x)$  is defined to be equal to  $\Pr[X \leq x]$ ,  $F_Y(y) = \Pr[Y \leq y]$ . Random variable  $Y$  is defined as  $F_X(X)$ , thus  $F_Y(y) = \Pr[Y \leq y] = \Pr[F_X(X) \leq y]$ . There may be many values such that  $F_X(X) \leq y$ . If we denote such values as  $F_X^{-1}([0, y])$ , then  $\Pr[F_X(X) \leq y]$  can be written as  $\Pr[X \in F_X^{-1}([0, y])]$ , thus  $F_Y(y) = \Pr[X \in F_X^{-1}([0, y])]$ .

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As  $F_X$  is continuous, there exists a supremum in  $F_X^{-1}([0, y])$ , and as this is a closed set, this is also a maximum. Let this maximum value be denoted as  $k$ , such that  $k = \max(F_X^{-1}([0, y]))$ . Then,  $F_Y(y) = Pr[X \in F_X^{-1}([0, y])] = Pr[X \leq k] = F_X(k) = y$  as we defined  $k$  to be a maximum element such that  $F_X^{-1}([0, y])$ .

## 2.2 Inverse transform sampling

Let a random variable  $V = F_X^{-1}(U)$ . By definition of continuous uniform distribution,  $F_V(x) = Pr[V \leq x]$ . If  $F$  is applied to the both sides in  $V = F_X^{-1}(U)$ , then we get  $U = F_X(V)$ . Therefore  $Pr[V \leq x] = Pr[U \leq F_X(x)]$ . As  $U$  is given as a uniform distribution in  $[0, 1]$ ,  $Pr[U \leq F_X(x)] = F_X(x)$ . Thus  $F_V(x) = F_{F_X^{-1}(U)}(x) = Pr[F_X^{-1}(U) \leq x] = F_X(x)$ , implying that  $F_X(x)$  is a CDF of  $F_X^{-1}(U)$ .

## 2.3 Sampling from exponential distribution

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

3 Q3

4 Q4