Lecture 7:

Introduction to Geometry, Implicit and Explicit Representation

Computer Graphics 2025

Fuzhou University - Computer Science

Course Roadmap

Rasterization Pipeline

Core Concepts
- Sampling
- Antialiasing
- Transforms

Intro

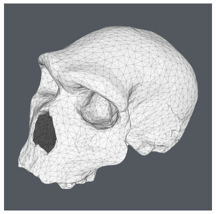
Rasterization

Transforms & Projection

Texture Mapping

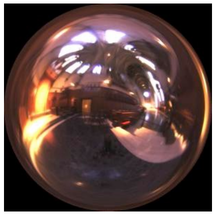
Visibility, Shading, Overall Pipeline

Geometric Modeling



Starting today

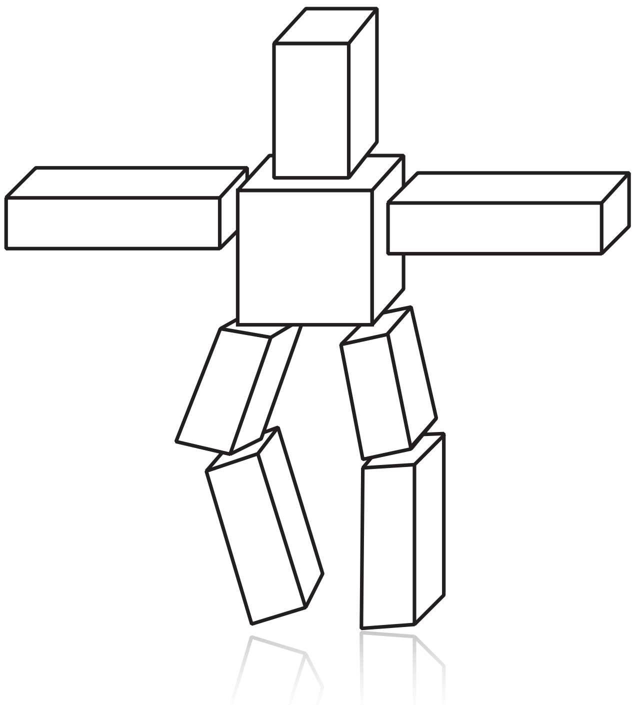
Lighting & Materials



Cameras & Imaging

Increasing the Complexity of Our Models

Transformations



Geometry

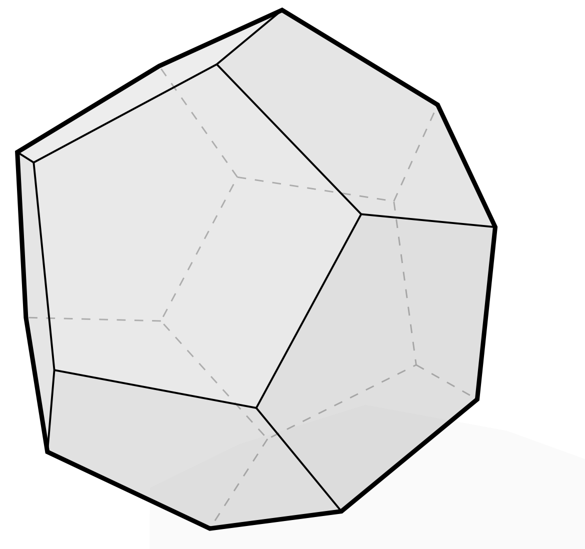
Materials, lighting, ...

What Is Geometry?

"Earth" "measure" 土地测量、测地术

ge•om•et•ry /jēˈämətrē/ *n.*

1. The study of shapes, sizes, patterns, and positions.
2. The study of spaces where some quantity (lengths, angles, etc.) can be *measured*.



Plato:"… the earth is in appearance like one of those balls which have leather coverings in twelve pieces…"

How Can We Describe Geometry?

Implicit

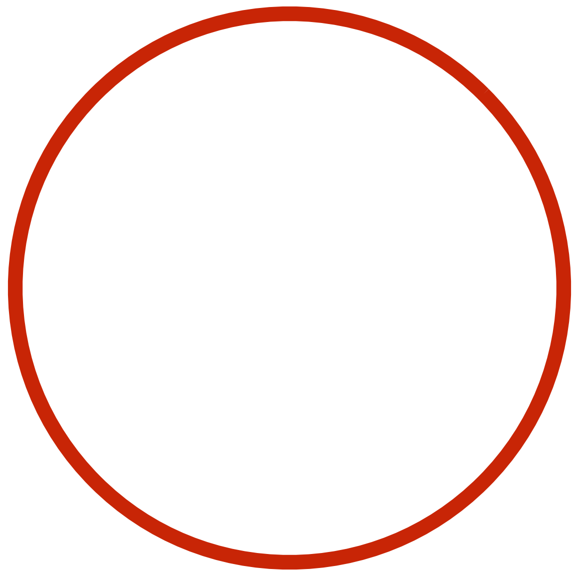
$$x^2 + y^2 = 1$$

Linguistic

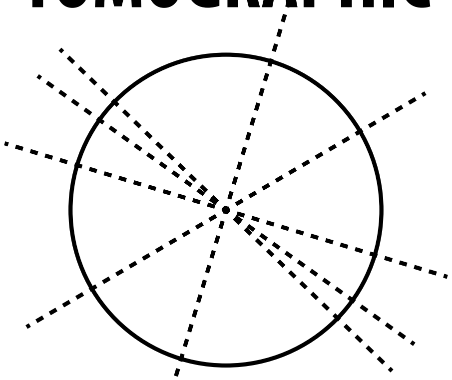
"unit circle"

Explicit

$$(\underbrace{\cos\theta}_{x}, \underbrace{\sin\theta}_{y})$$



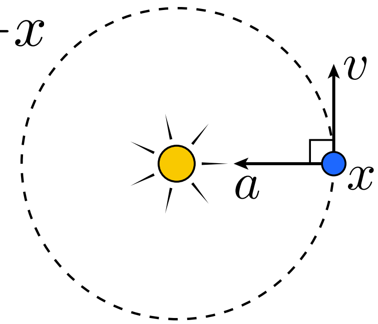
TOMOGRAPHIC



(constant density)

DYNAMIC

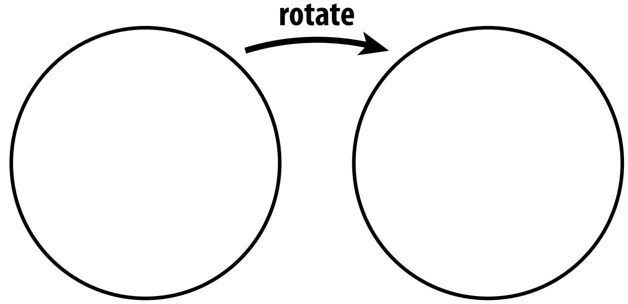
$$\frac{d^2}{dt^2}x = -x$$



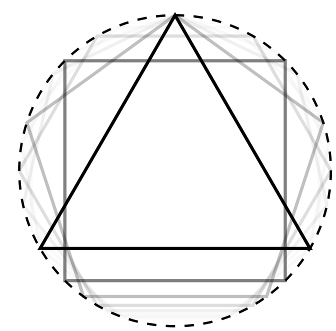
CURVATURE

$$\kappa = 1$$

SYMMETRIC



DISCRETE



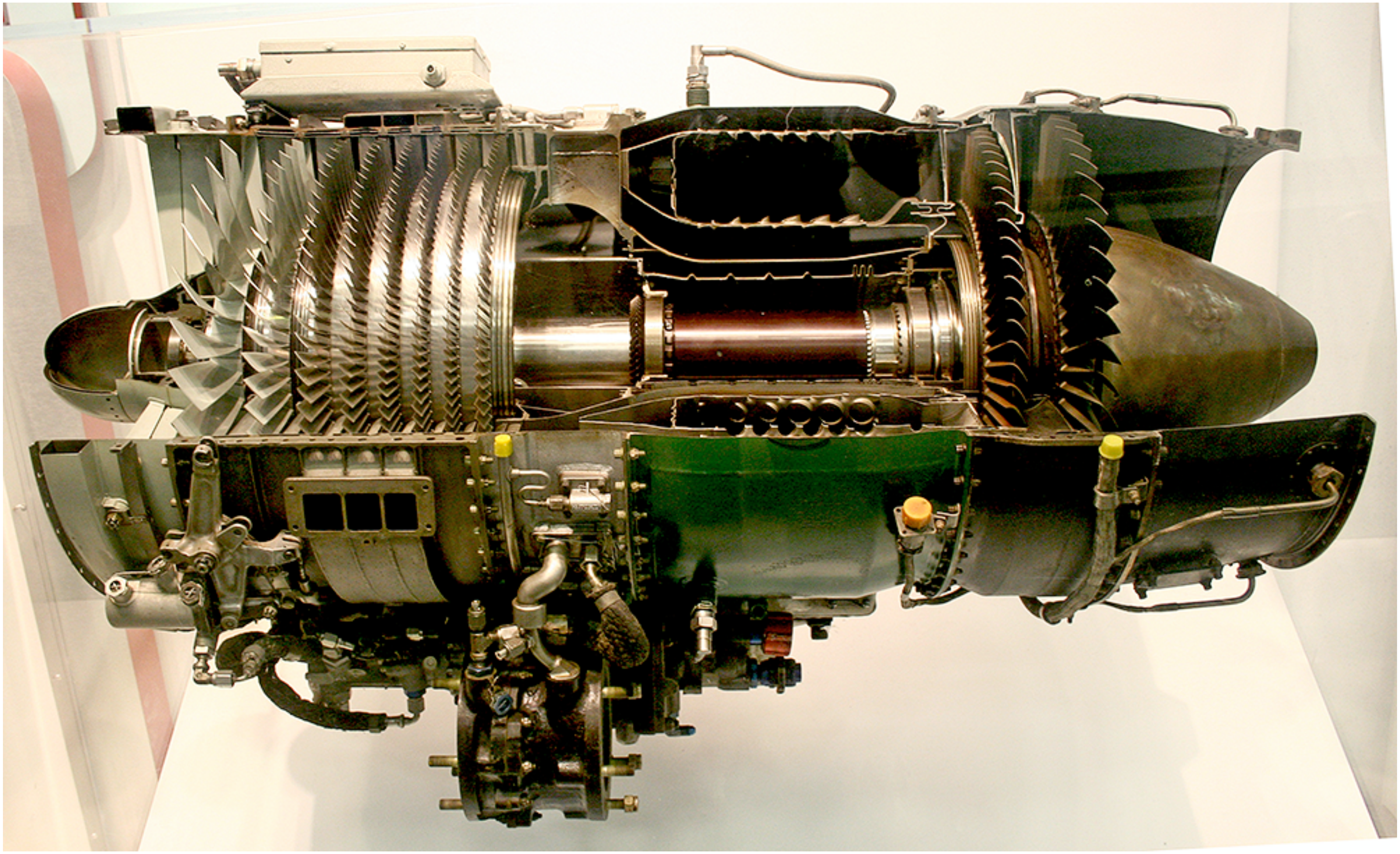
n → ∞

Given all these options, what's the best way to encode geometry on a computer?

Examples of Geometry

Examples of Geometry

Examples of Geometry



Examples of Geometry

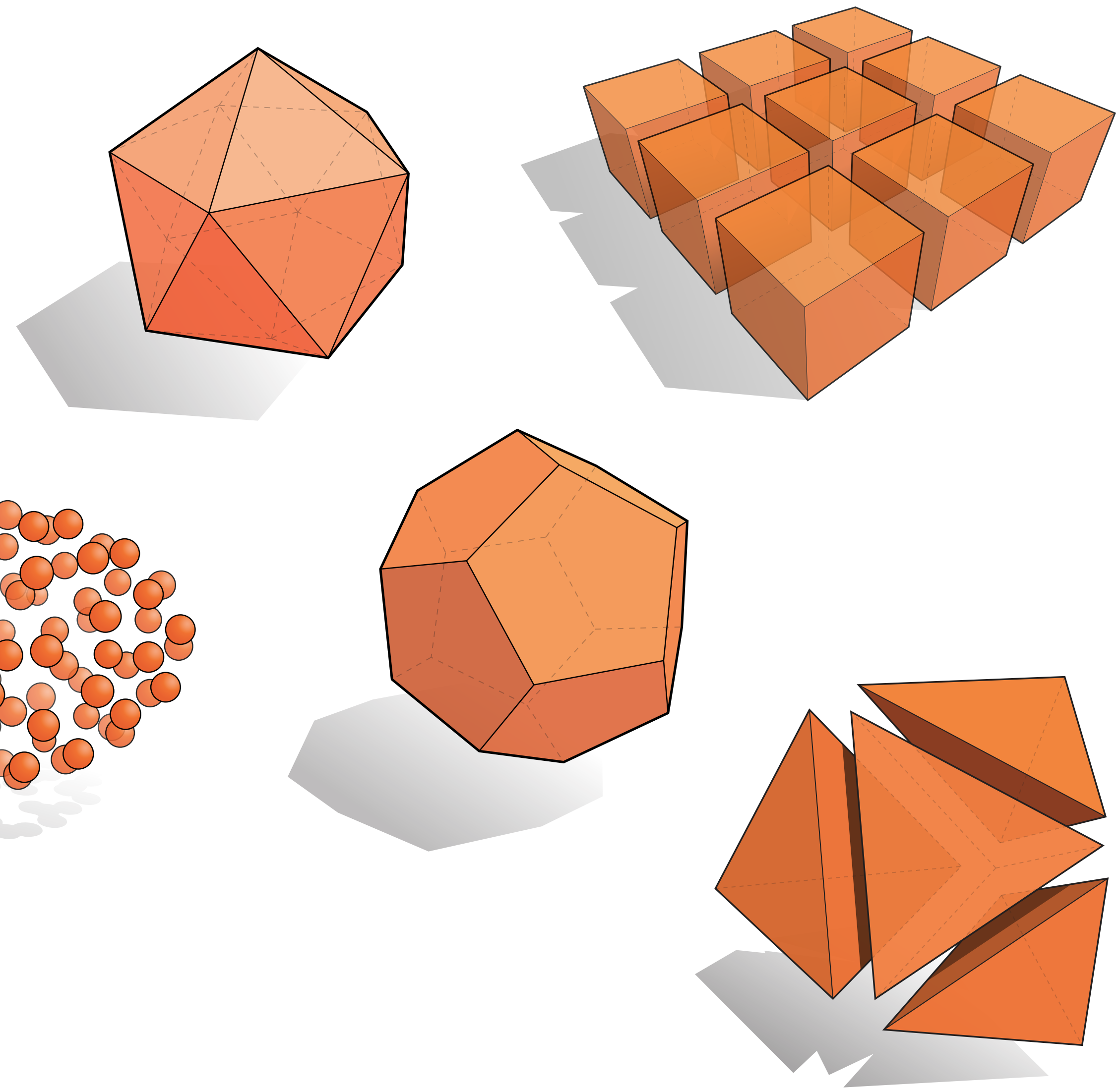
Examples of Geometry

Examples of Geometry

Examples of Geometry

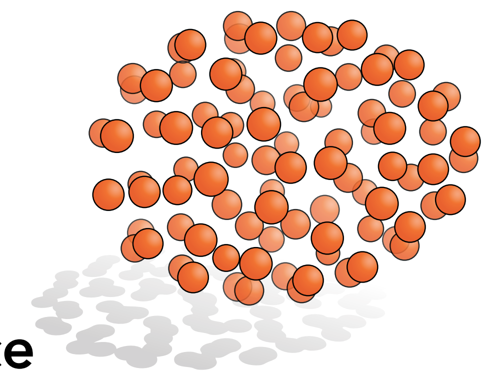
Representations

Many Ways to Represent Geometry



Explicit

- point cloud
- polygon mesh
- subdivision, NURBS
- ...



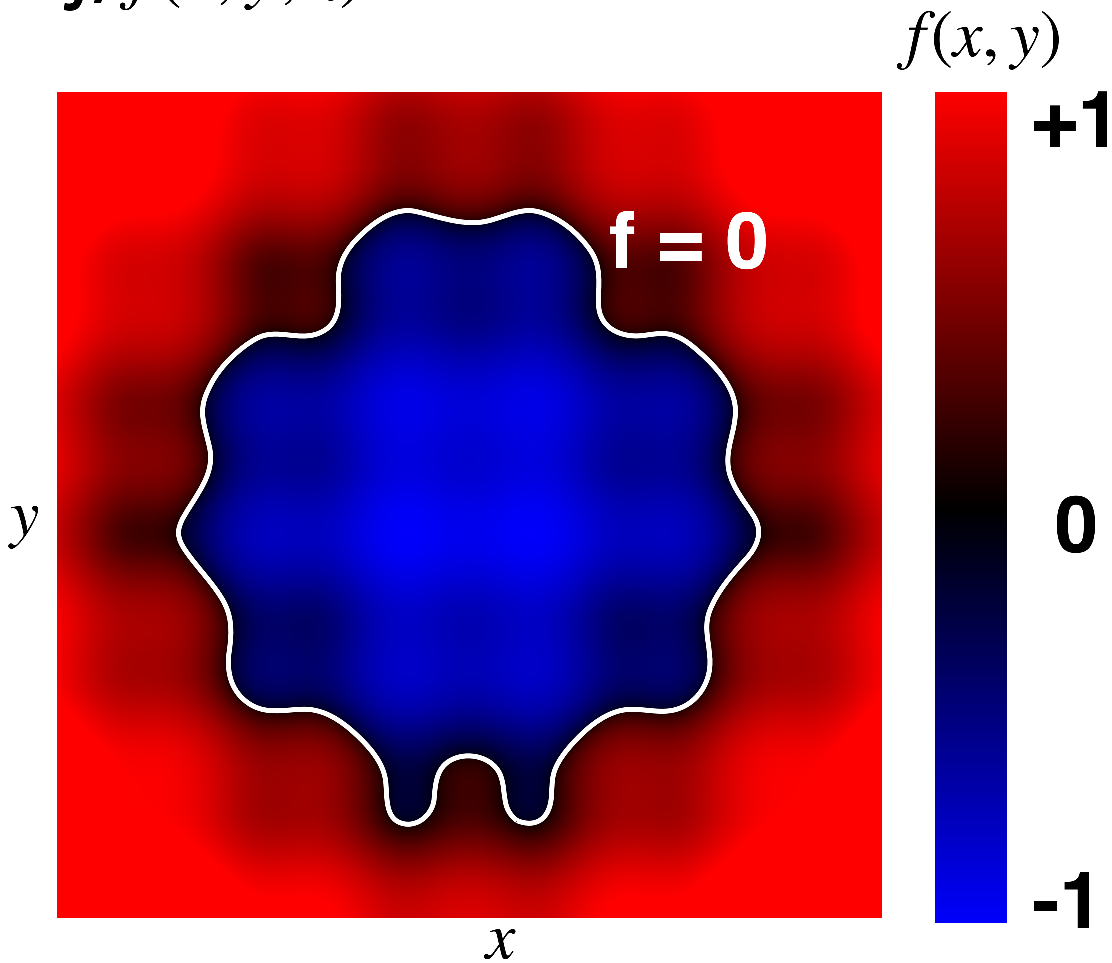
Implicit

- level sets
- algebraic surface
- distance functions
- ...

Each choice best suited to a different task/type of geometry

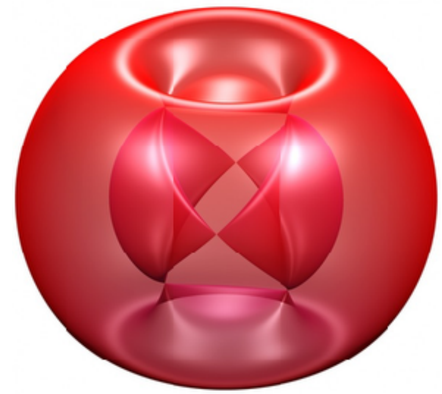
"Implicit" Representations of Geometry

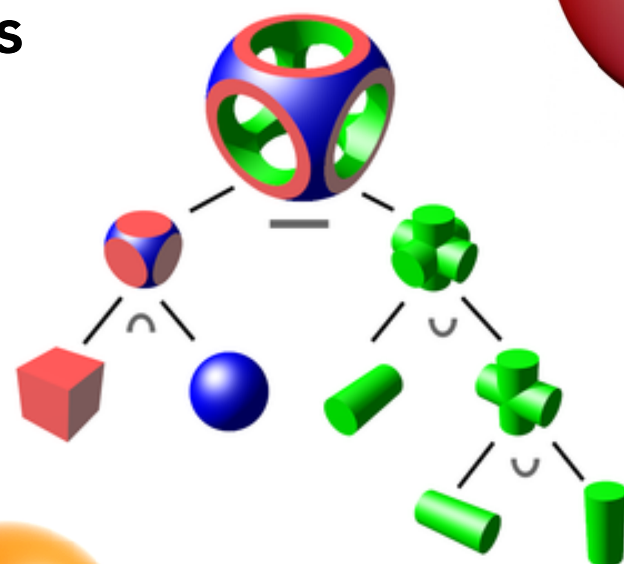
- Points are not known directly, but satisfy some relationship
- E.g., unit sphere is all points such that $x^2 + y^2 + z^2 = 1$
- More generally, $f(x, y, z) = 0$

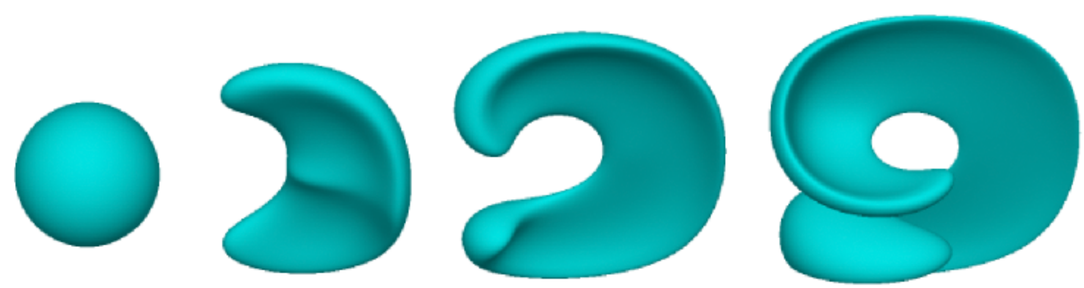


Many Implicit Representations in Graphics

- Algebraic surfaces
- Constructive Solid Geometry
- Level set methods
- Blobby surfaces
- Fractals
- …







Implicit Surface - Sampling Can Be Hard

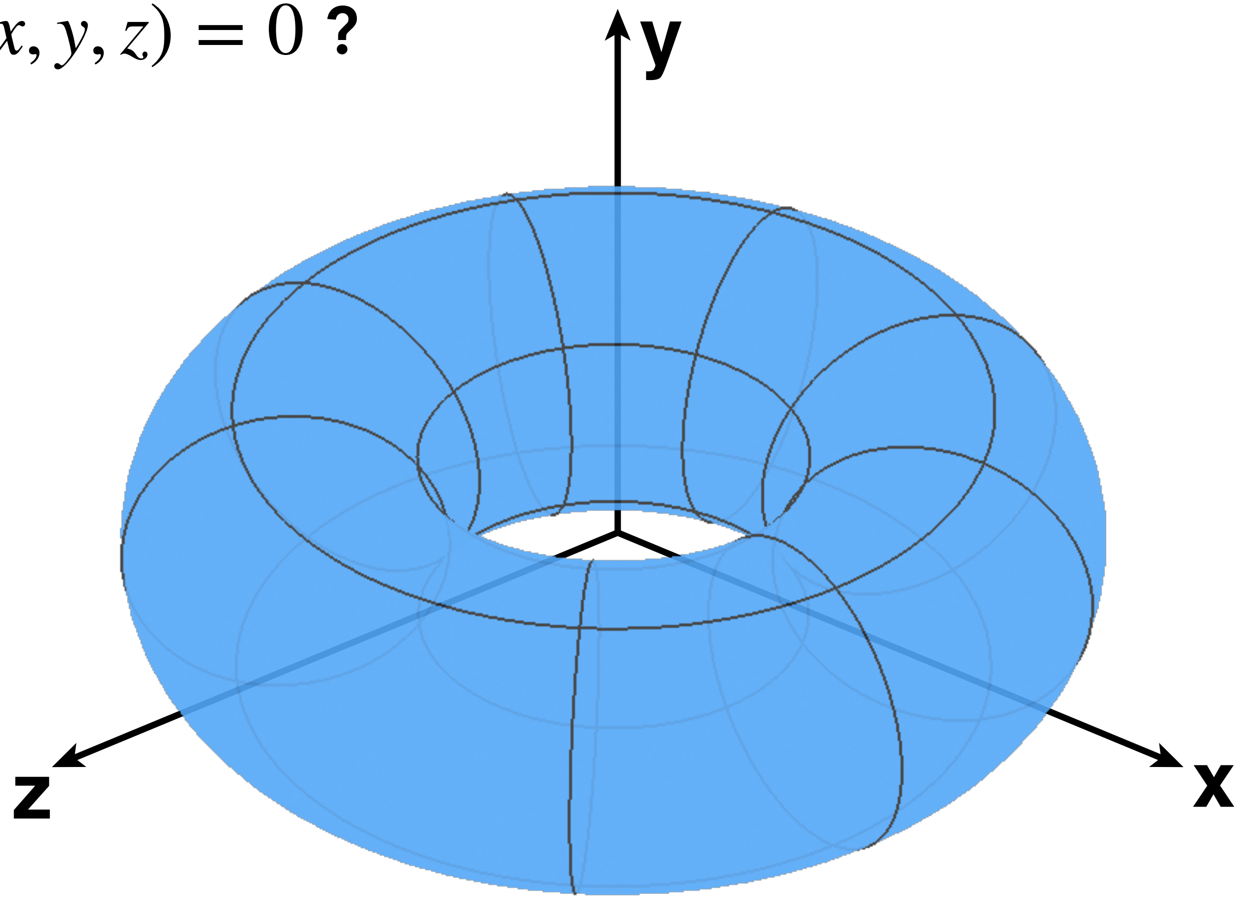
$$f(x,y,z) = (2 - \sqrt{x^2 + y^2})^2 + z^2 - 1$$

What points lie on $f(x,y,z) = 0$ **?**

Implicit Surface - Sampling Can Be Hard

$$f(x, y, z) = (2 - \sqrt{x^2 + y^2})^2 + z^2 - 1$$

What points lie on $f(x, y, z) = 0$ **?**



Torus

Implicit Surface - Inside/Outside Tests Easy

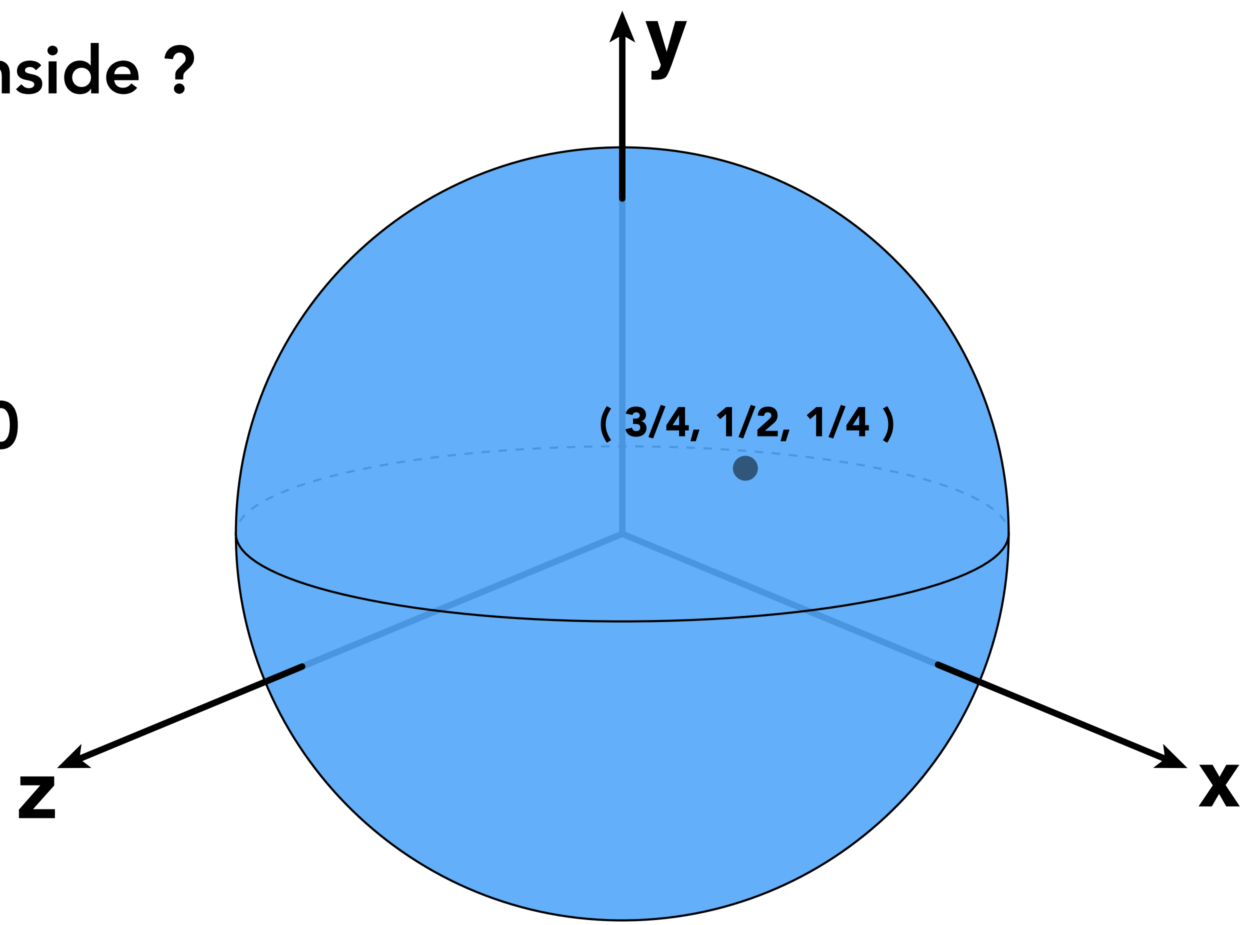
$f(x, y, z) = x^2 + y^2 + z^2 - 1$

Is (3/4, 1/2, 1/4) inside ?

Just plug it in:

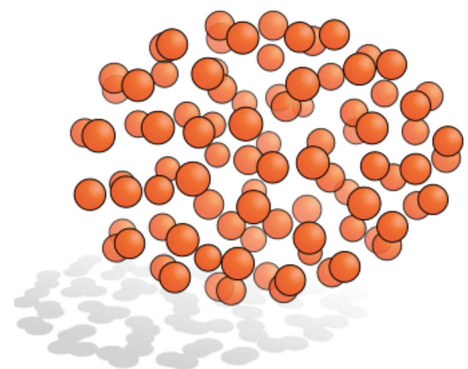
$f(x, y, z) =$ **-1/8 < 0**

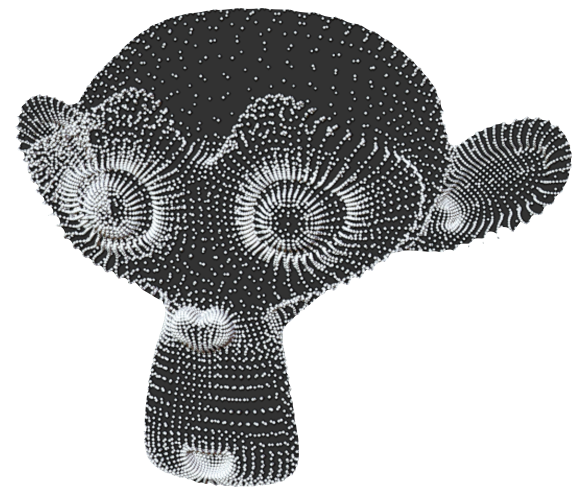
Yes, inside.



“Explicit” Representations of Geometry

All points are **given directly**

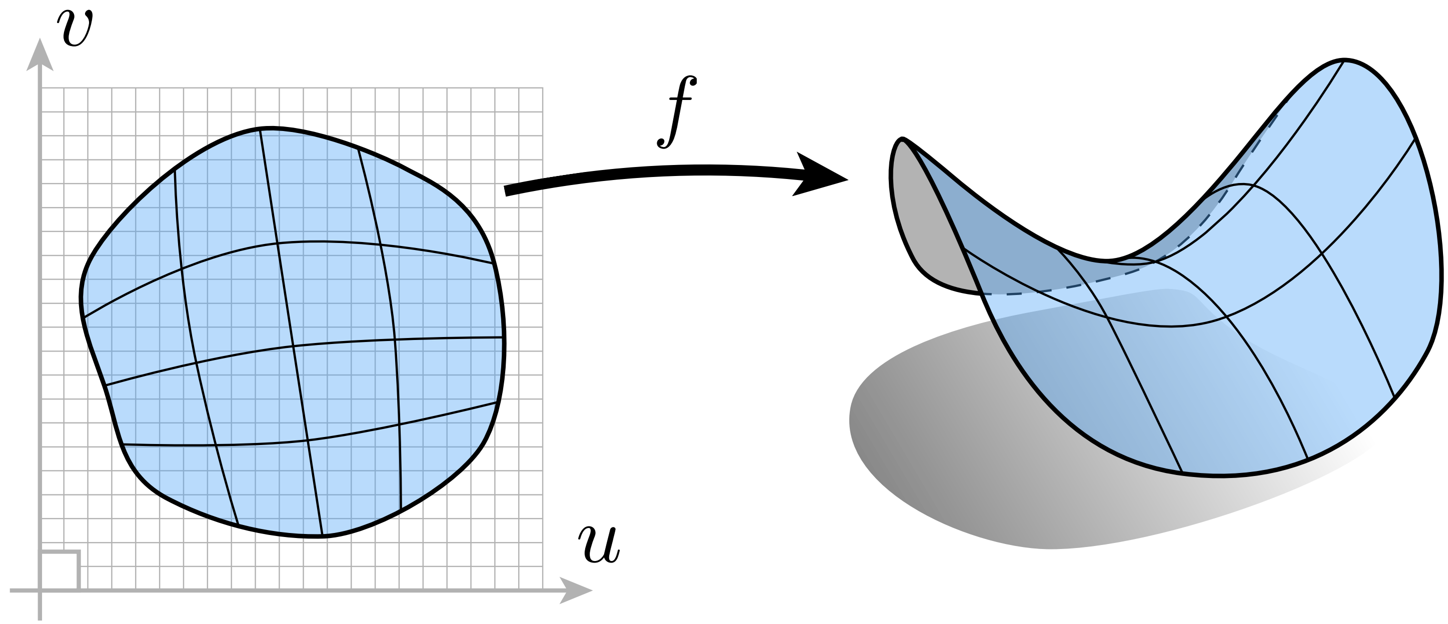




"Explicit" Representations of Geometry

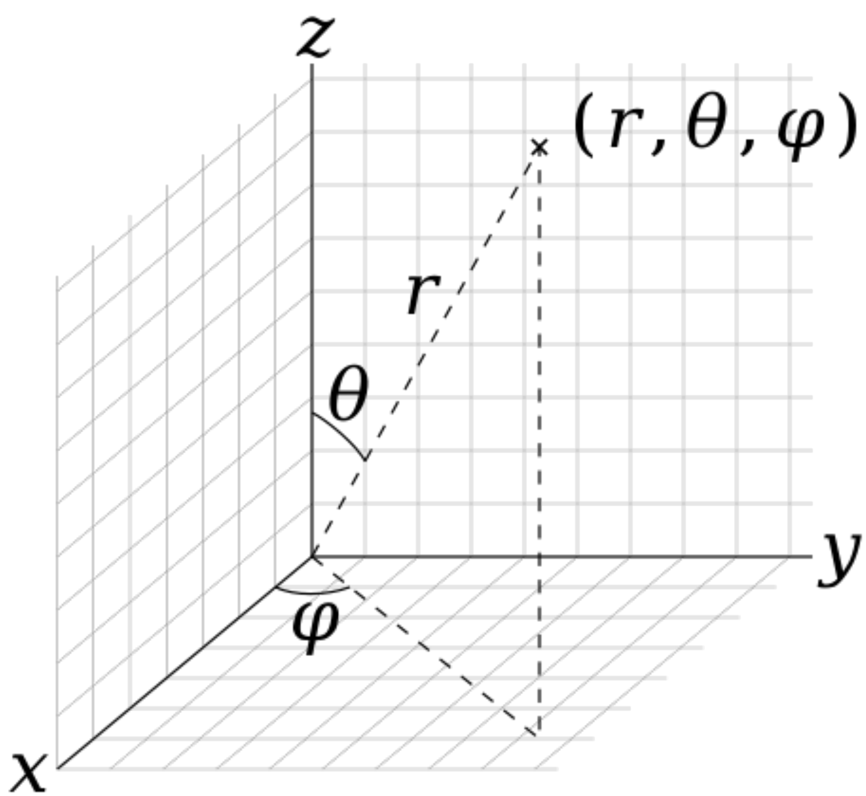
All points are **given directly** or **via parameter mapping**

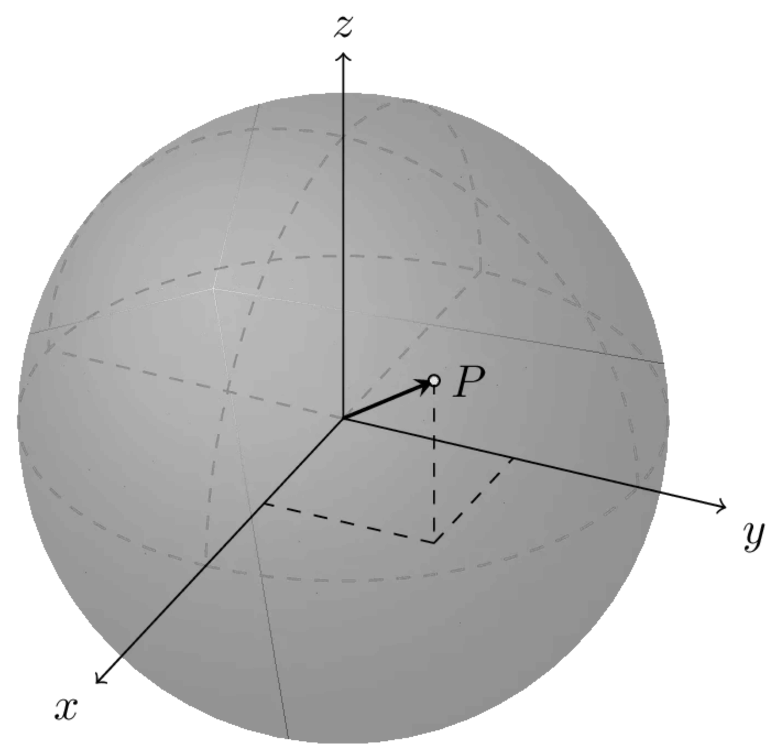
Generally: $$f : \mathbb{R}^2 \to \mathbb{R}^3; (u,v) \mapsto (x,y,z)$$



“Explicit” Representations of Geometry

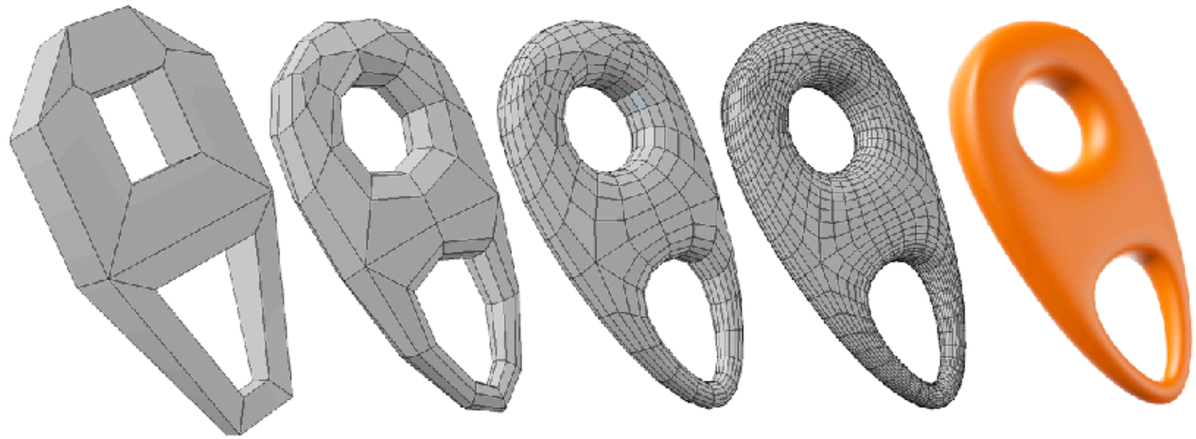
$$f(\theta, \phi) = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)$$

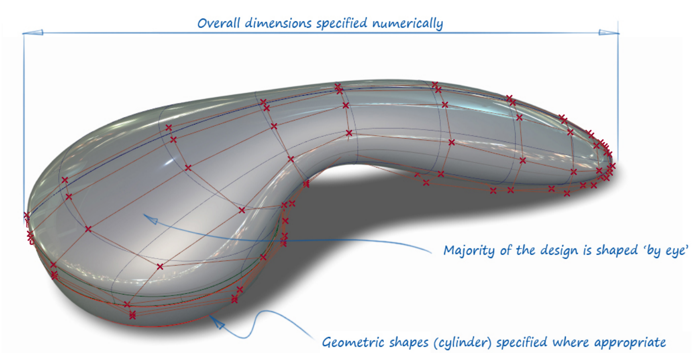


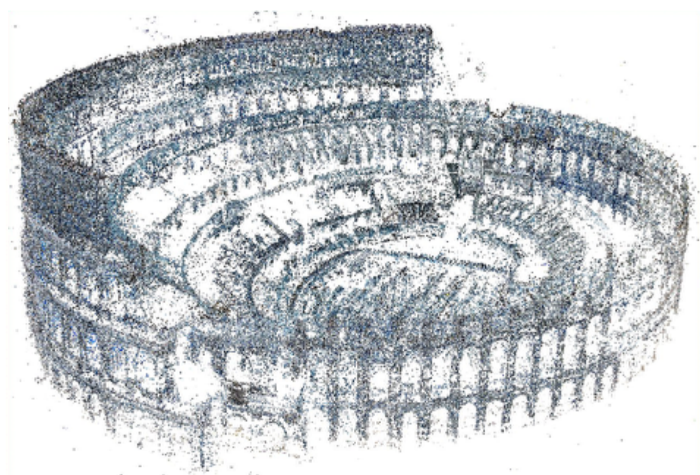


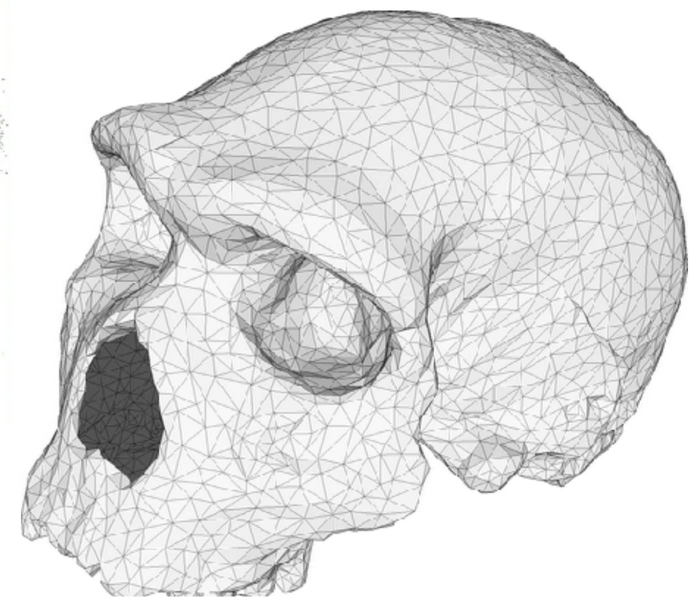
Many Explicit Representations in Graphics

- Triangle meshes
- Polygon meshes
- Subdivision surfaces
- Point clouds
- NURBS
- …







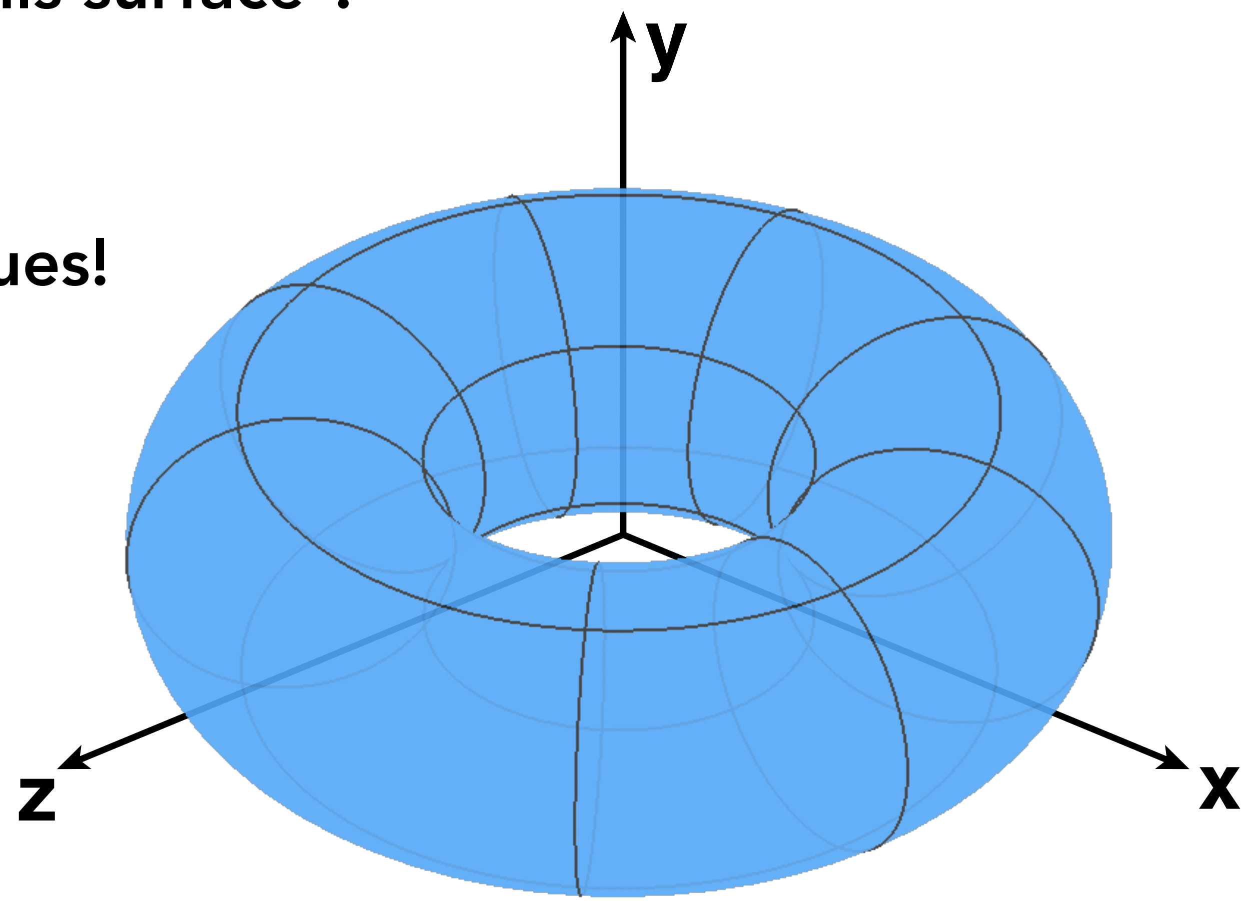


Explicit Surface - Sampling is Easy

$$f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u)$$

What points lie on this surface ?

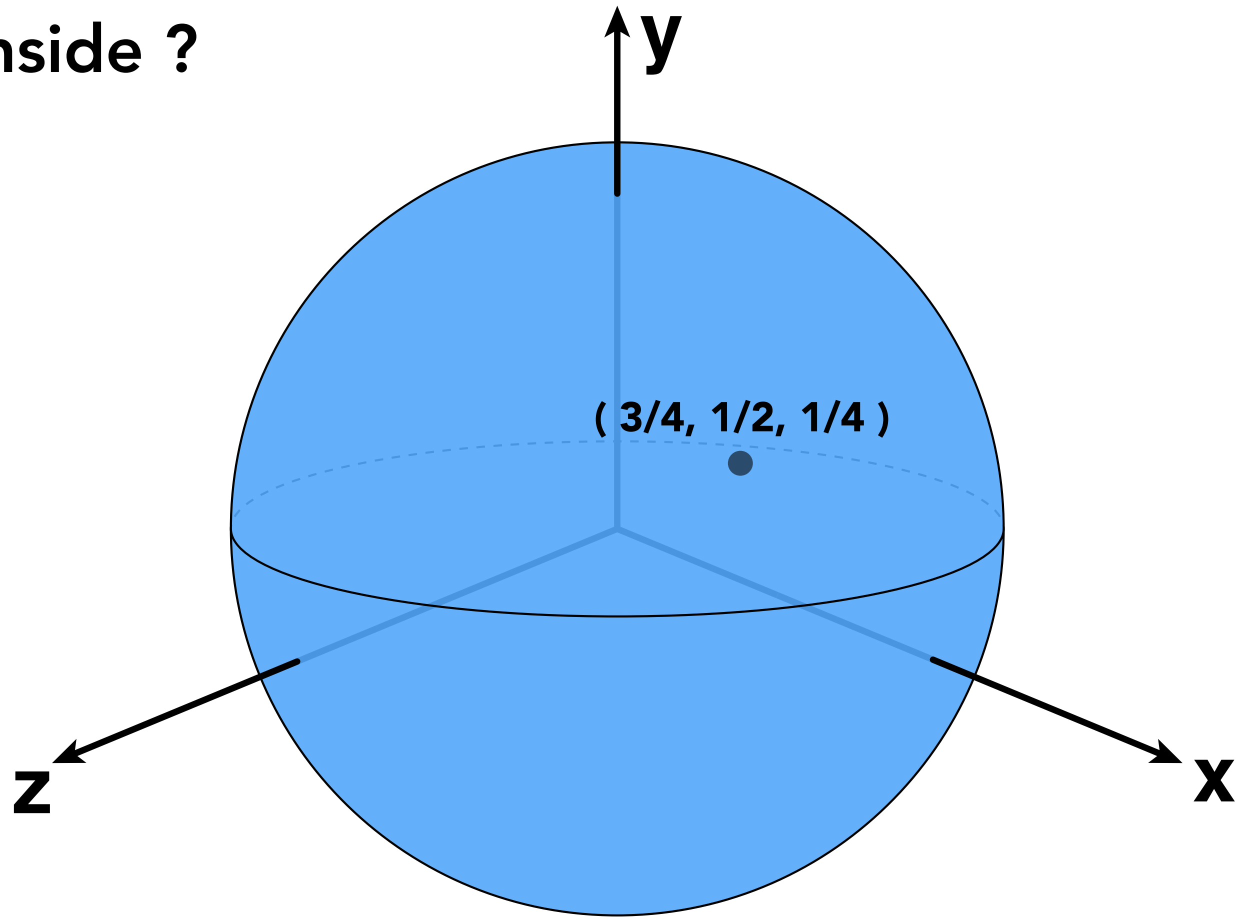
Just plug in (u,v) values!



Explicit Surface - Inside/Outside Tests Hard

$$f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u)$$

Is (3/4, 1/2, 1/4) inside ?



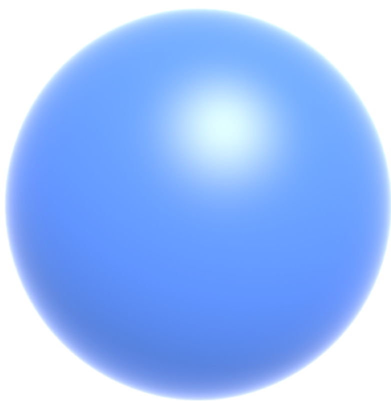
Conclusion: Some representations work better than others—depends on the task!

Different representations will also be better suited to different types of geometry.

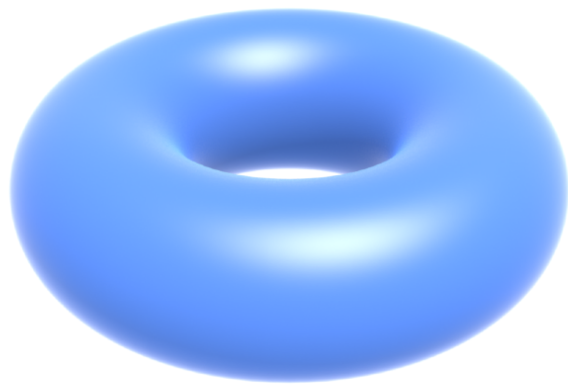
Let’s take a look at some common representations used in computer graphics.

Algebraic Surfaces (Implicit)

Surface is zero set of a polynomial in x, y, z



$$x^2+y^2+z^2=1$$

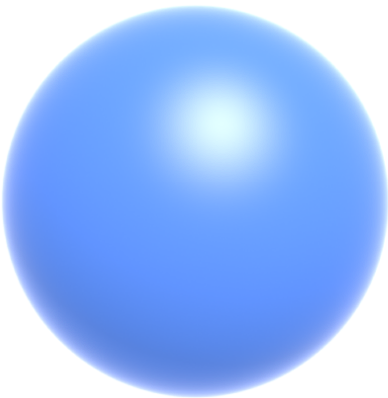


$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$

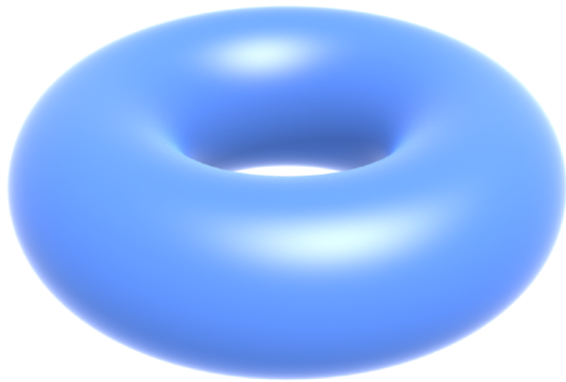
$$\left(x^2+\frac{9y^2}{4}+z^2-1\right)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

Algebraic Surfaces (Implicit)

Surface is zero set of a polynomial in x, y, z



$$x^2+y^2+z^2=1$$



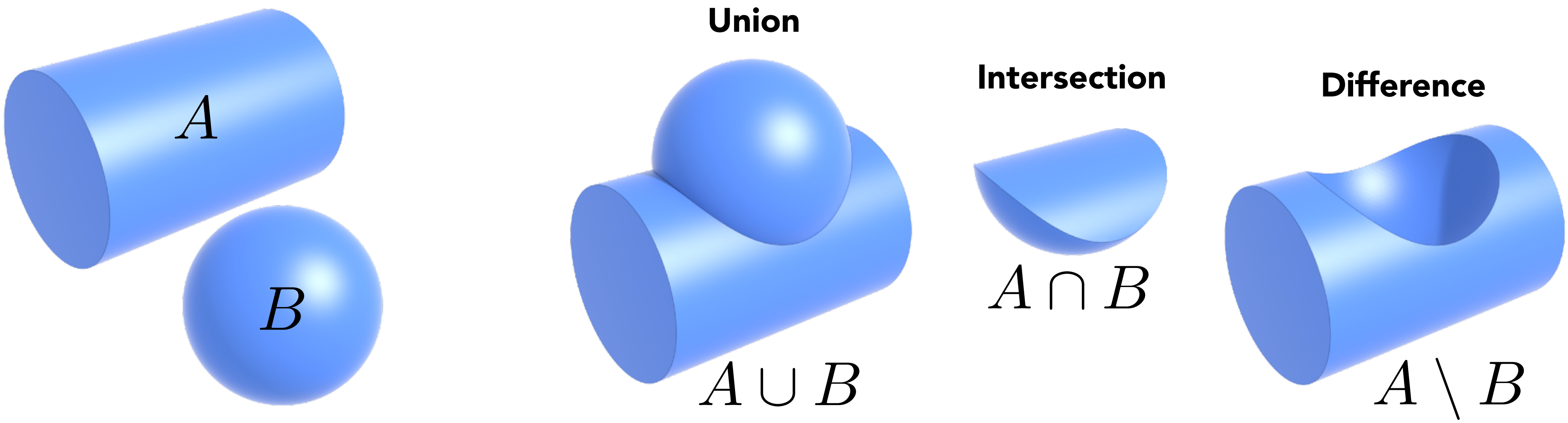
$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$

$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

What about more complicated shapes?

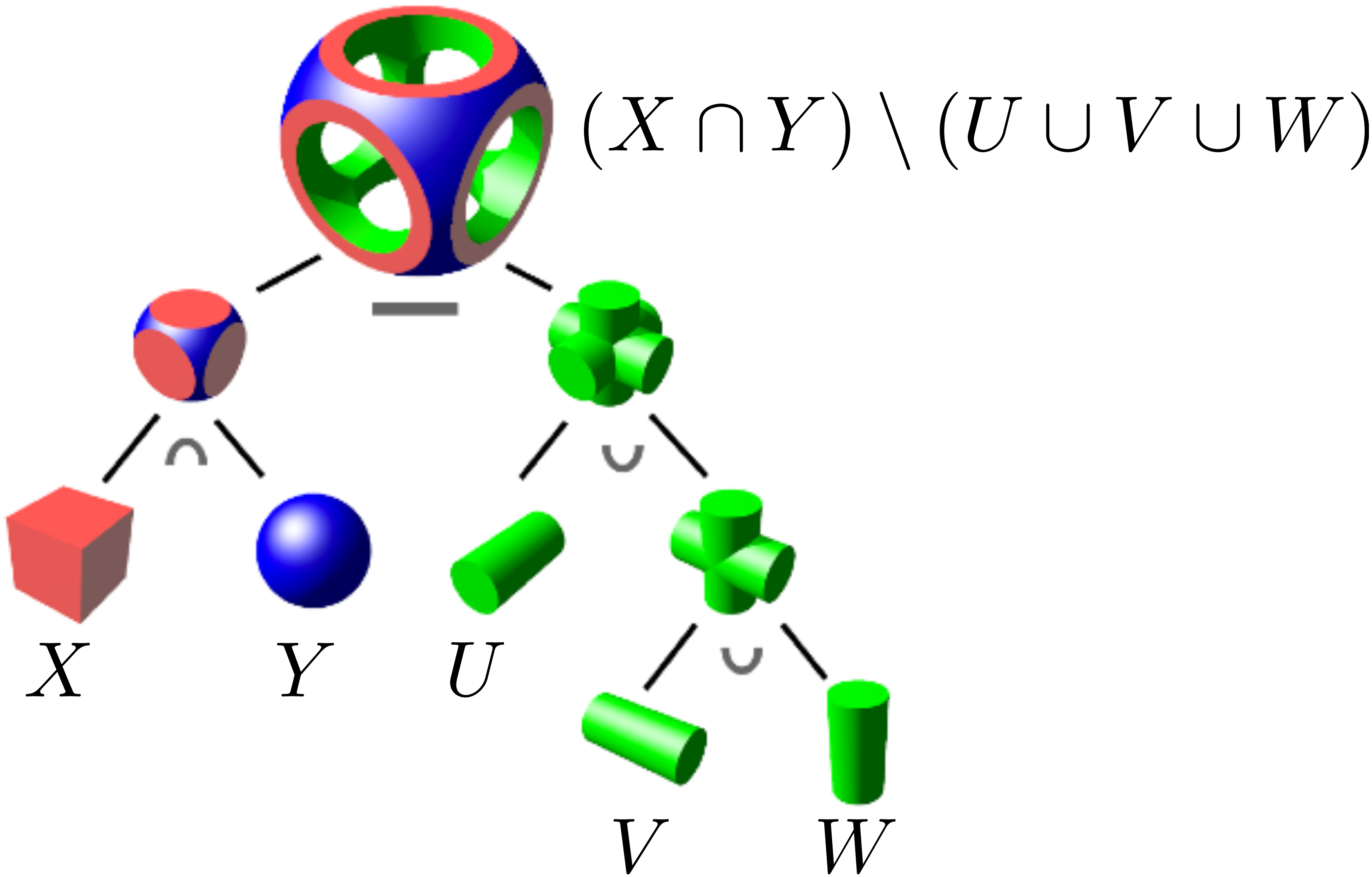
Constructive Solid Geometry (Implicit)

Combine implicit geometry via boolean operations



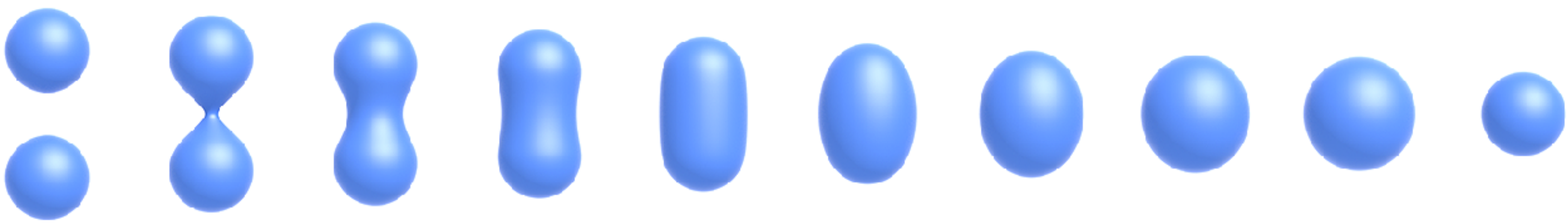
- Q: How do we implement a Boolean union of A, B
    - A: Assume A, B is represented by $d_1(x), d_2(x)$, their union is the function $f(x) = min(d_1(x), d_2(x))$

Constructive Solid Geometry (Implicit)

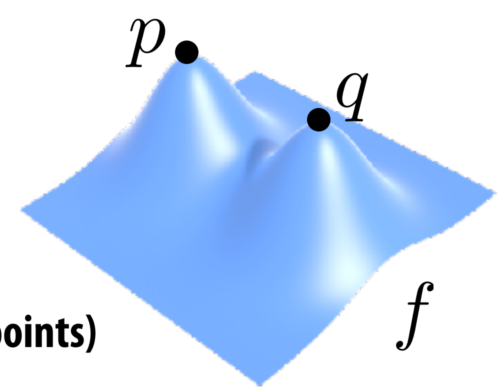


Blobby Surfaces (Implicit)

- Instead of Booleans, gradually blend surfaces together:

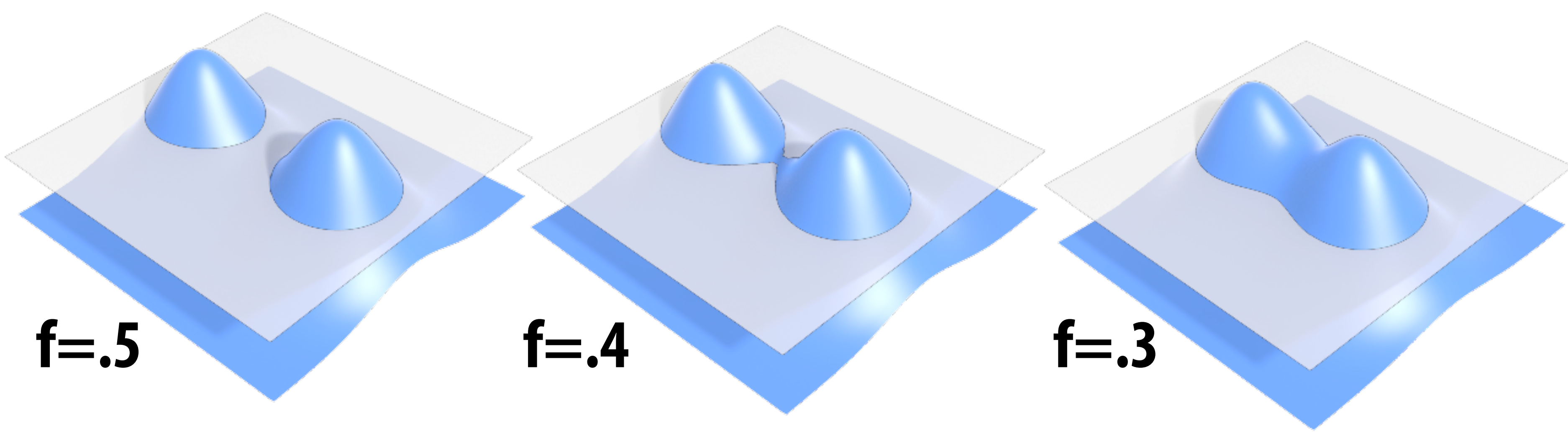


- Easier to understand in 2D:

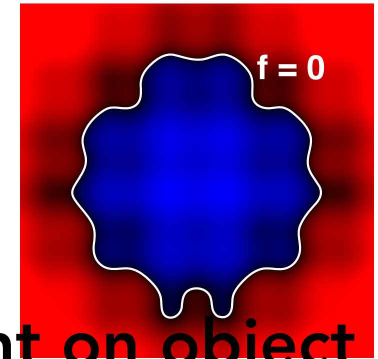


$\phi_p(x) := e^{-|x-p|^2}$ **(Gaussian centered at p)**

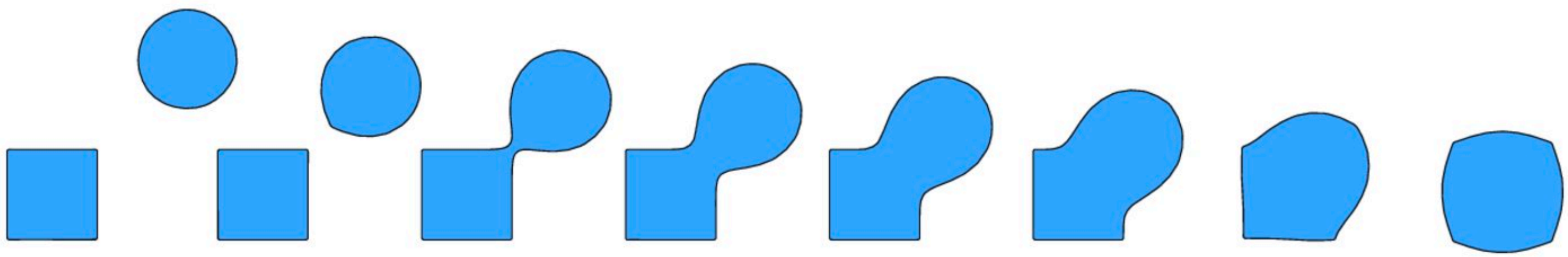
$f := \phi_p + \phi_q$ **(Sum of Gaussians centered at different points)**



Blending Distance Functions



- A distance function gives distance to closest point on object
- Can blend any two distance functions d_1, d_2



- Similar strategy to points, though many possibilities, E.g.,

$$f(x) := e^{d_1(x)^2} + e^{d_2(x)^2} - \frac{1}{2}$$

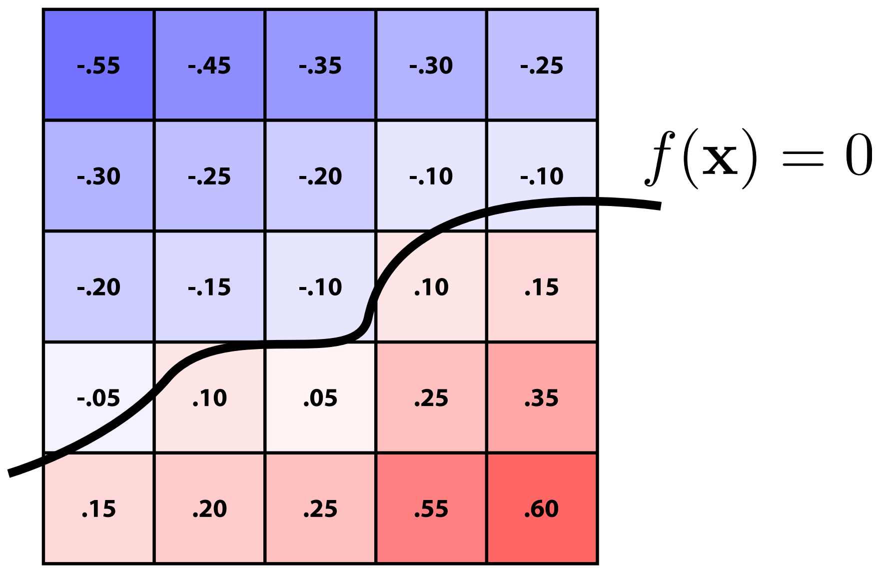
- Appearance depends on how we combine functions

Scene of Pure Distance Functions

https://iquilezles.org/www/articles/raymarchingdf/raymarchingdf.htm

Level Set Methods (Implicit)

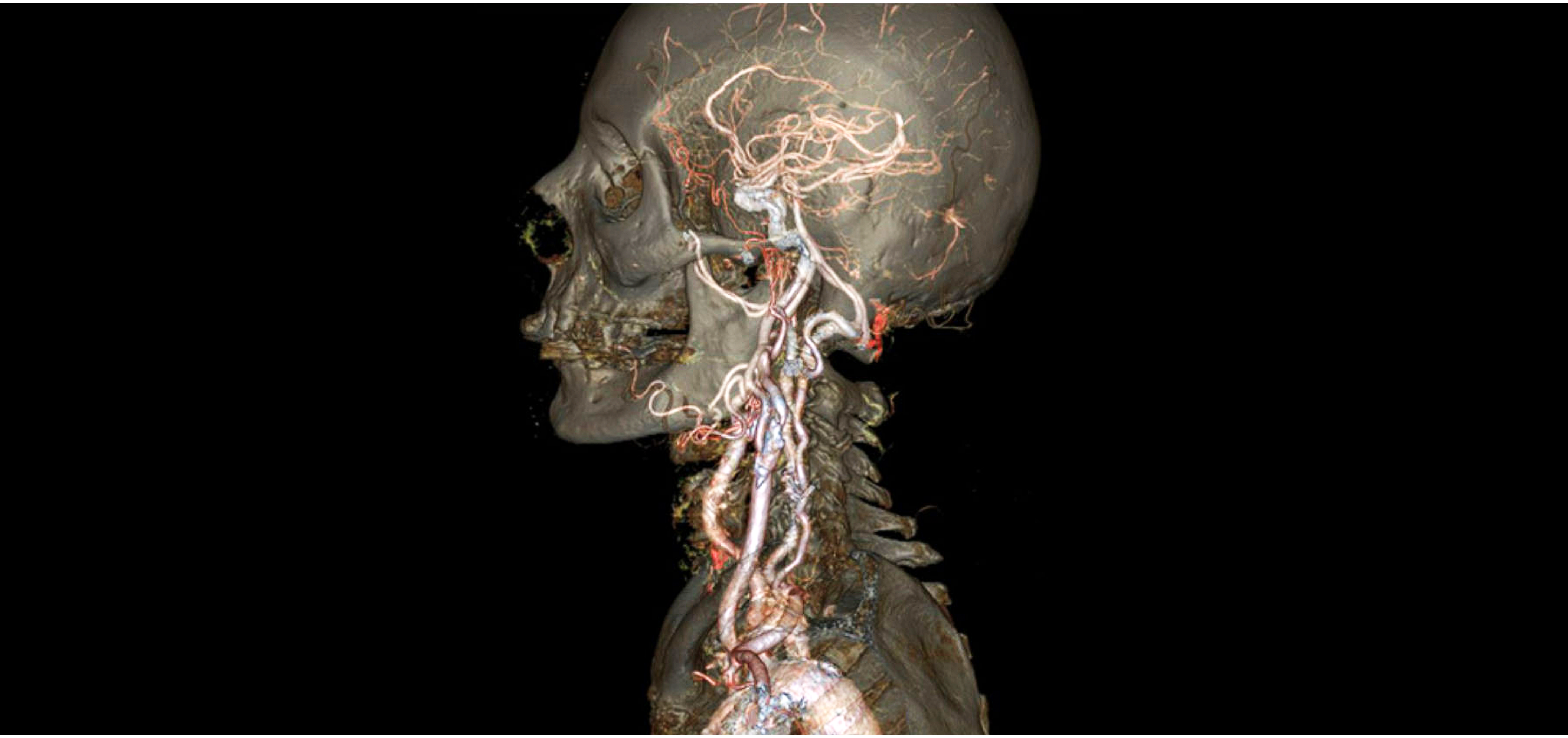
- Implicit surfaces have some nice features (e.g., merging/blending)
- But, hard to describe complex shapes in closed form
- Alternative: store a grid of values approximating function (离散化)



- Surface is found where interpolated values equal zero
- Provides much more explicit control over shapes (like a texture)

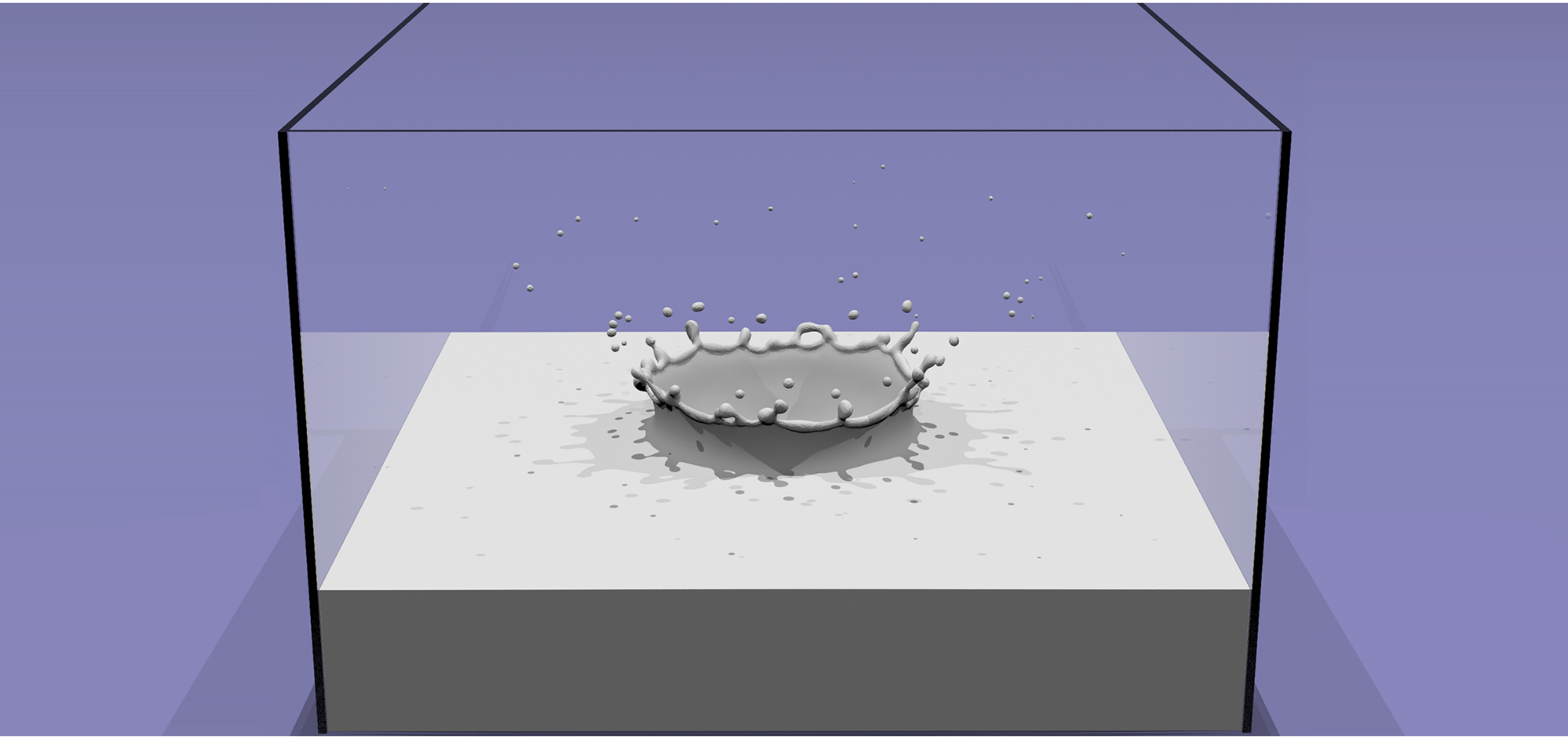
Level Sets from Medical Data

- Level sets encode, e.g., constant tissue density



Level Sets in Physical Simulation

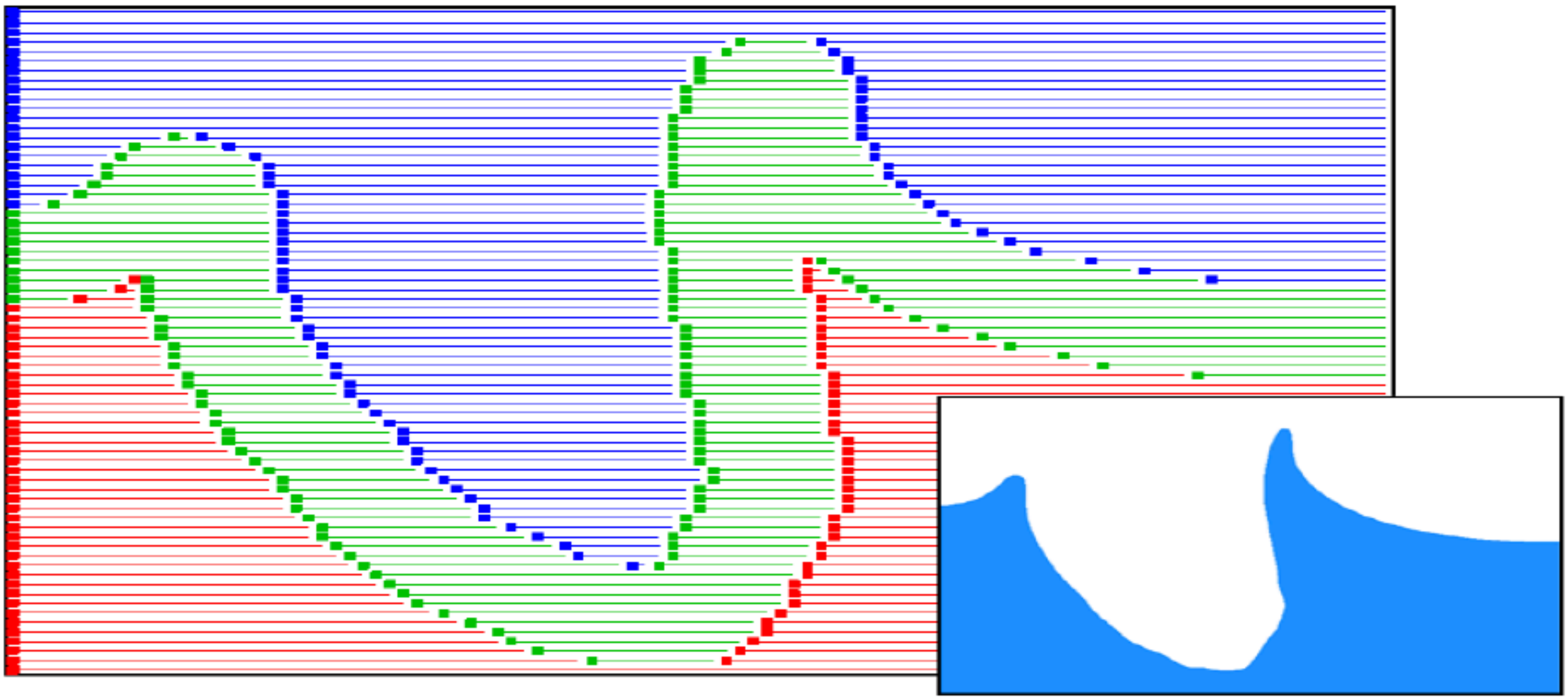
- Level sets encode distance to air-liquid boundary



http://physbam.stanford.edu

Level Sets Storage

- Drawback: storage for 2D surface is now $O(n^3)$
- Can reduce cost by storing only a narrow band around surface:



Fractals

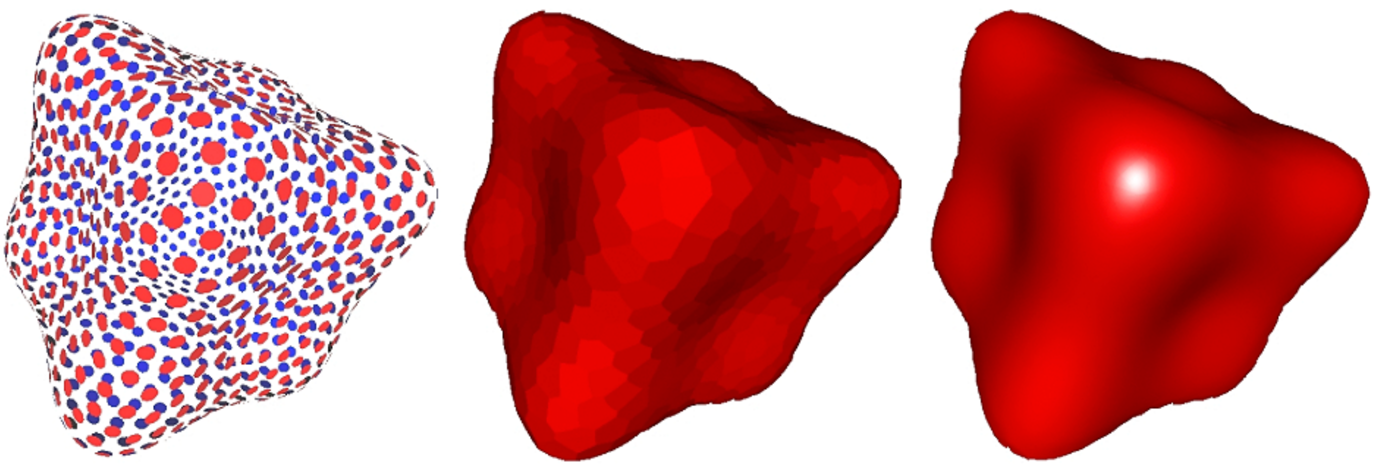
- No precise definition; exhibit self-similarity, detail at all scales
- "Language" for describing natural phenomena
- Hard to control shape


Implicit Representations - Pros & Cons

- Pros:
  - description can be very compact (e.g., a polynomial)
  - easy to determine if a point is in our shape (just plug it in!)
  - other queries may also be easy (e.g., distance to surface)
  - for simple shapes, exact description/no sampling error
- Cons:
  - expensive to find all points in the shape (e.g., for drawing)
  - very difficult to model complex shapes

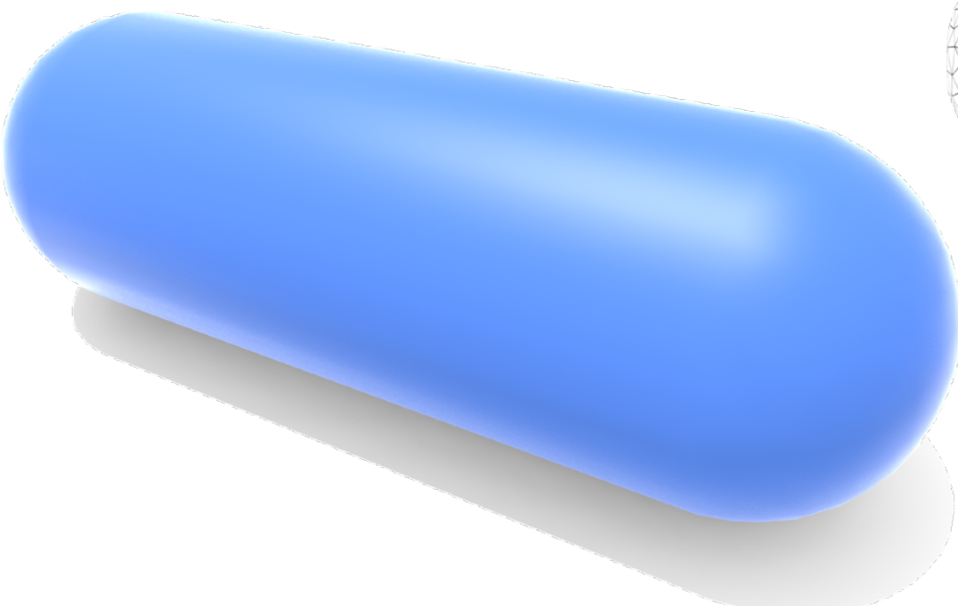
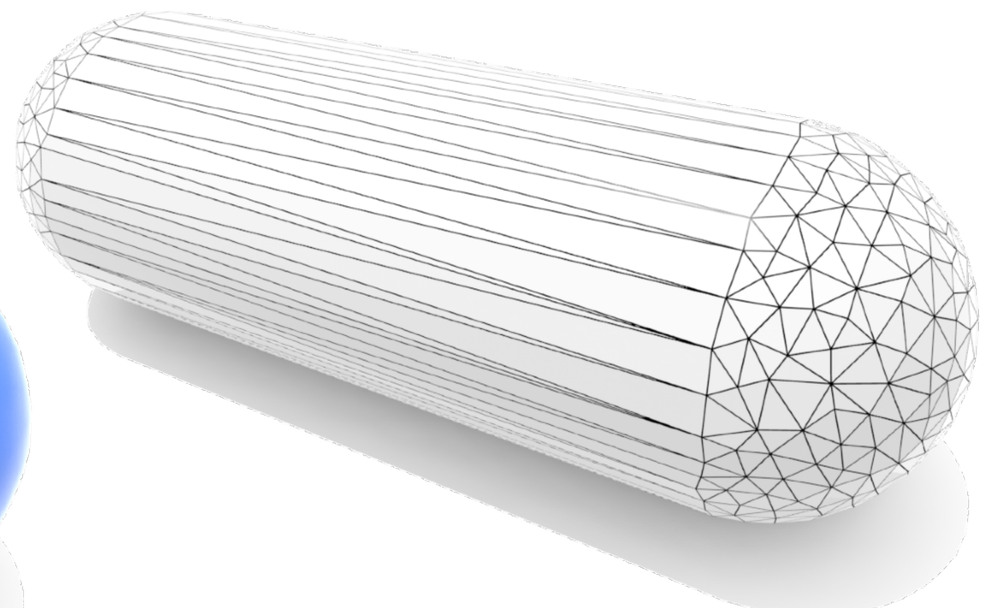
Point Cloud (Explicit)

- Easiest representation: list of points (x,y,z)
- Often augmented with normals
- Easily represent any kind of geometry
- Useful for LARGE datasets (>>1 point/pixel)
- Hard to interpolate undersampled regions
- Hard to do processing / simulation / ...



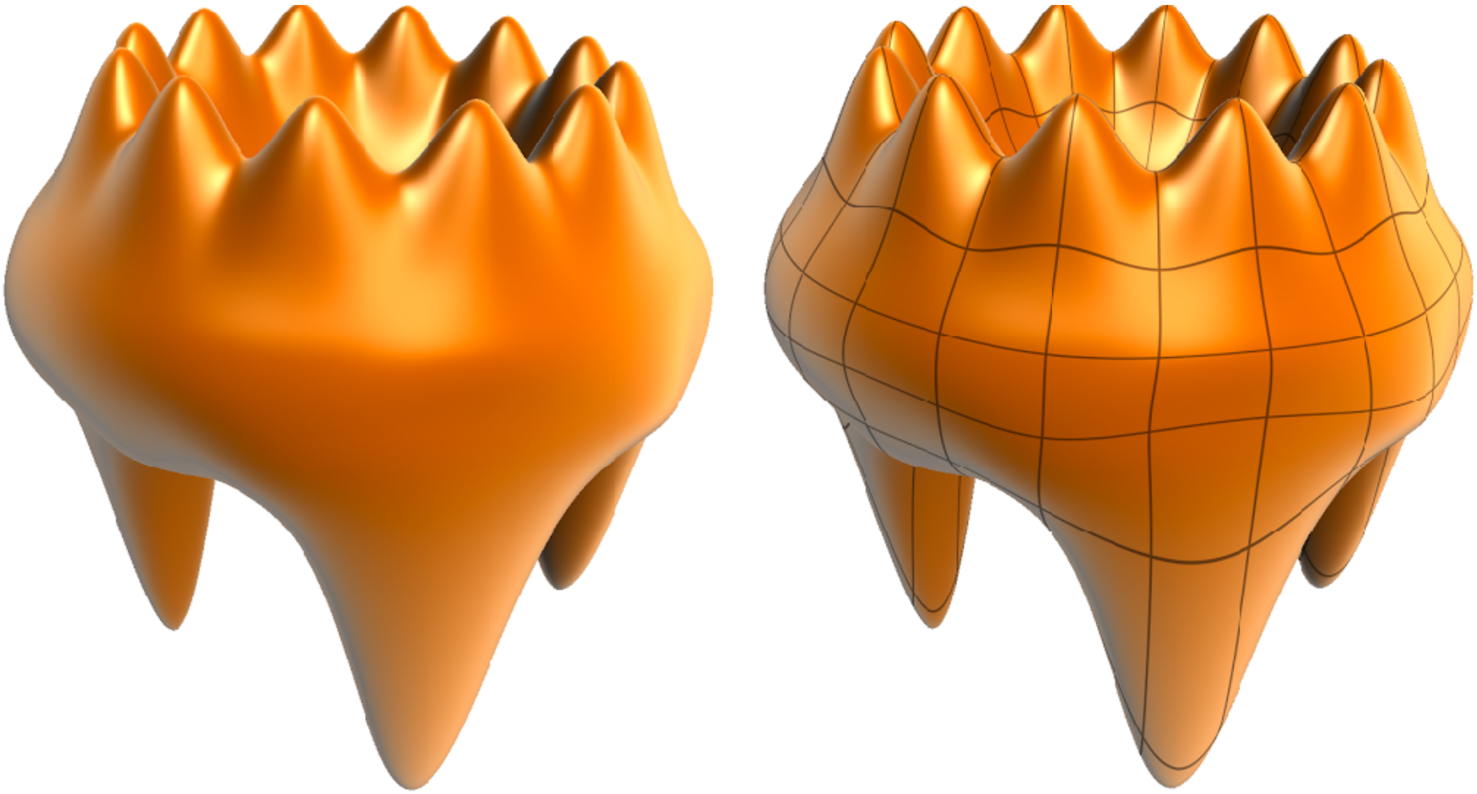
Polygon Mesh (Explicit)

- Store vertices and polygons (most often triangles or quads)
- Easier to do processing/simulation, adaptive sampling
- More complicated data structures
- Perhaps most common representation in graphics

Bézier Surface (Explicit)

- Connect Bézier patches to get a surface:



Next time: Curves, Surfaces