Lecture 11:

The Rendering Equation

Computer Graphics 2025

Fuzhou University - Computer Science

Today

表面点沿ω方向
辐射出去的能量

The Rendering Equation/ Light transport Equation $L_o(p, \omega)$

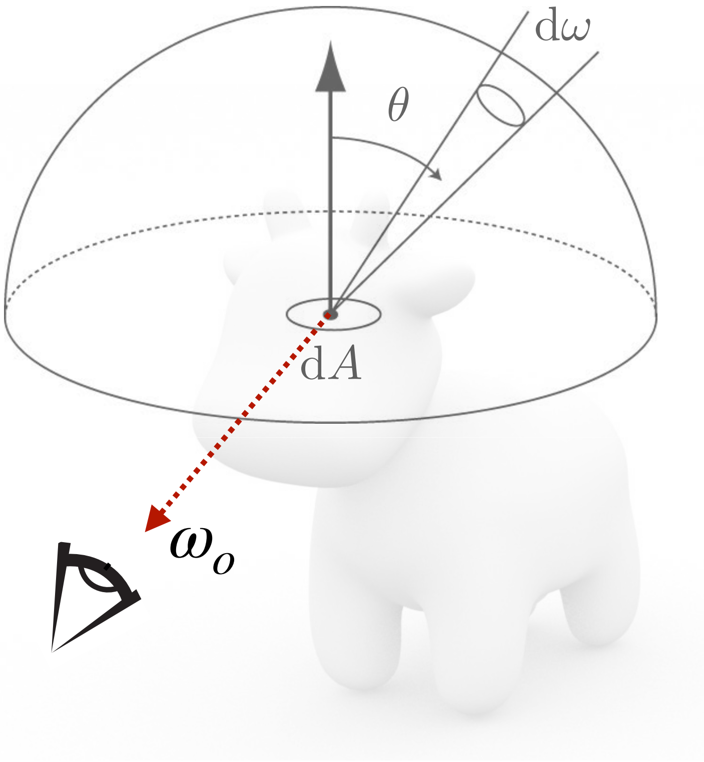
The Rendering Equation

- Core functionality of photorealistic renderer is to estimate radiance at a given point p, in a given direction ω_o

$$L_o(p,\omega_o) = L_e(p,\omega_o) + L_r(p,\omega_o)$$

本身是光源，直接向外辐射

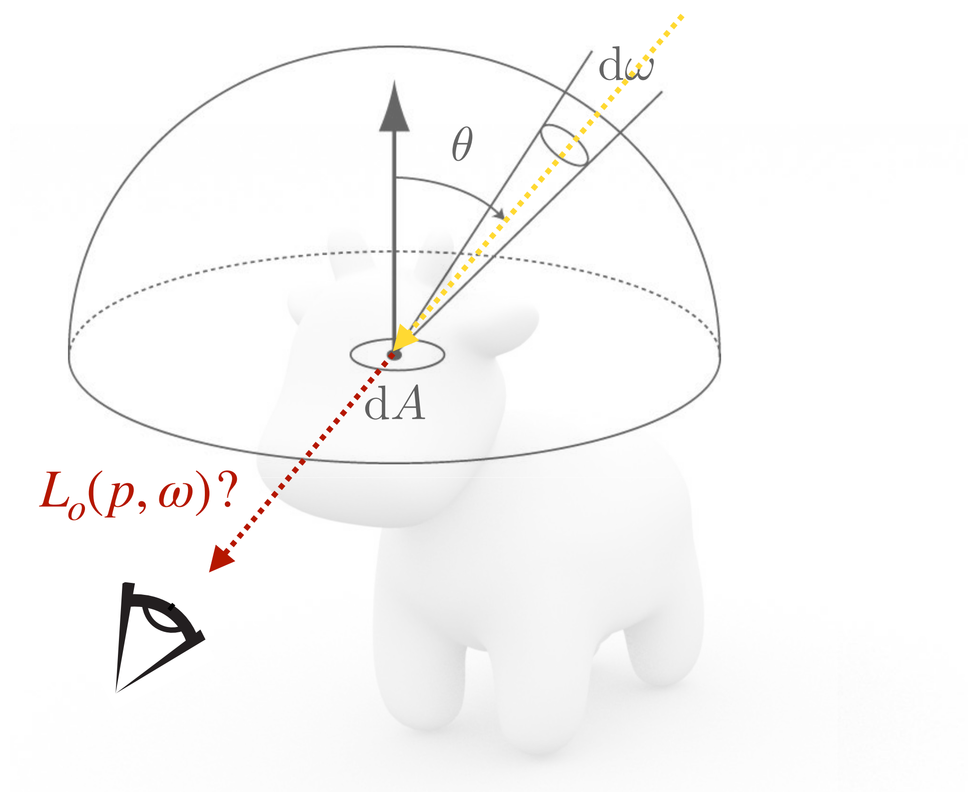
接收到外界的光辐射，再反射出去



The Rendering Equation

$$E(\mathrm{p}, \omega) = \int_{H^2} L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

表面点接收到的辐射能量

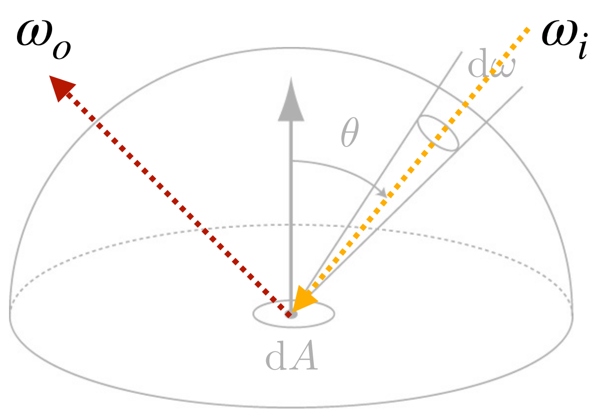


The Rendering Equation

- Core functionality of photorealistic renderer is to estimate radiance at a given point p, in a given direction ω_o

$$dE(\mathrm{p},\omega) = L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega \longleftarrow$$

Contribution to irradiance from light arriving from direction ω



$$f_r(p,\omega_i \rightarrow \omega_o) \cdot L_i(p,\omega_i)\cos\theta d\omega_i$$

点p的入射光线ω_i从ω_o方向反射出去的能量

$$L_r(p,\omega_o) = \int_{H^2} f_r(p,\omega_i \rightarrow \omega_o)L_i(p,\omega_i)\cos\theta d\omega_i$$

点p的所有入射光线ω_i从ω_o方向反射出去的能量

The Rendering Equation

- Core functionality of photorealistic renderer is to estimate radiance at a given point p, in a given direction ω_o
- Summed up by the rendering equation (Kajiya, 1986):

outgoing/observed radiance

emitted radiance(e.g., light source)

incoming radiance

angle between incoming direction and normal

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{H^2} f_r(p, \omega_i \to \omega_o) L_i(p, \omega_i) \cos\theta \, d\omega_i$$

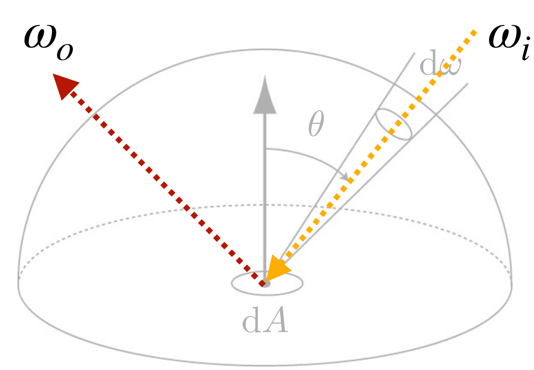
point of interest

direction of interest

all directions in hemisphere

scattering function

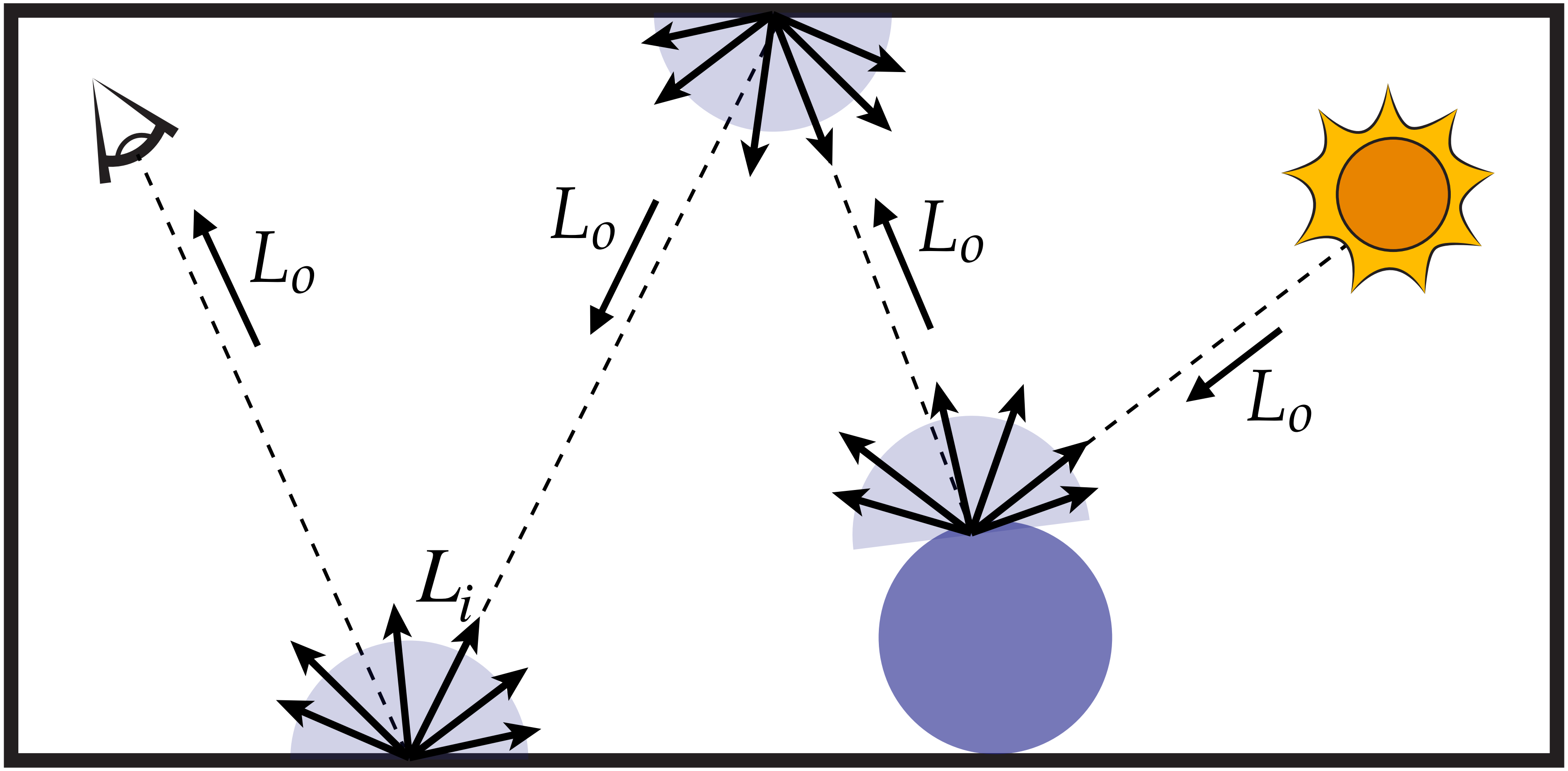
incoming direction



Key challenge: to evaluate incoming radiance, we have to compute yet another integral. I.e., rendering equation is recursive.

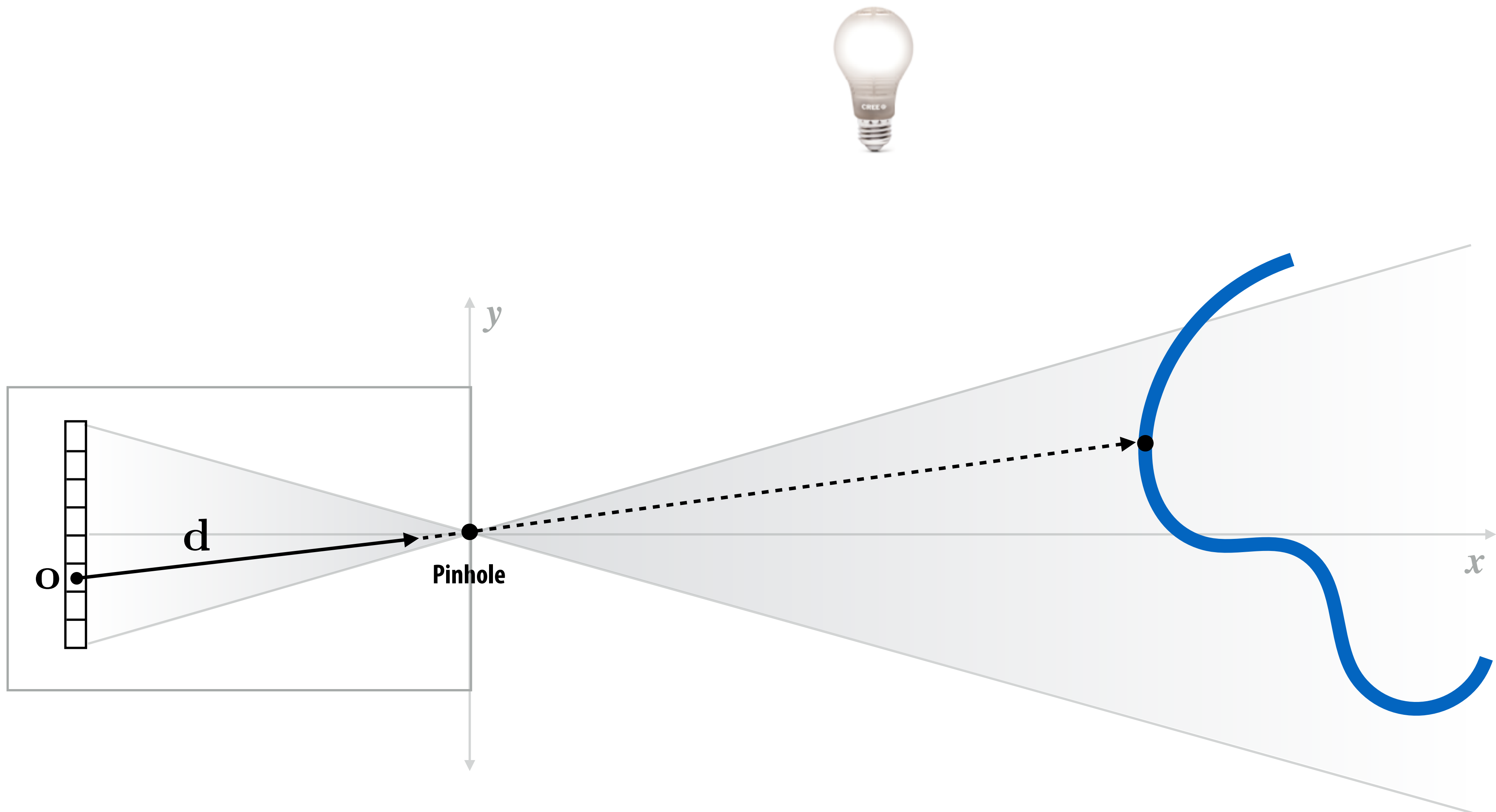
Recursive Raytracing

- Basic strategy: recursively evaluate rendering equation!



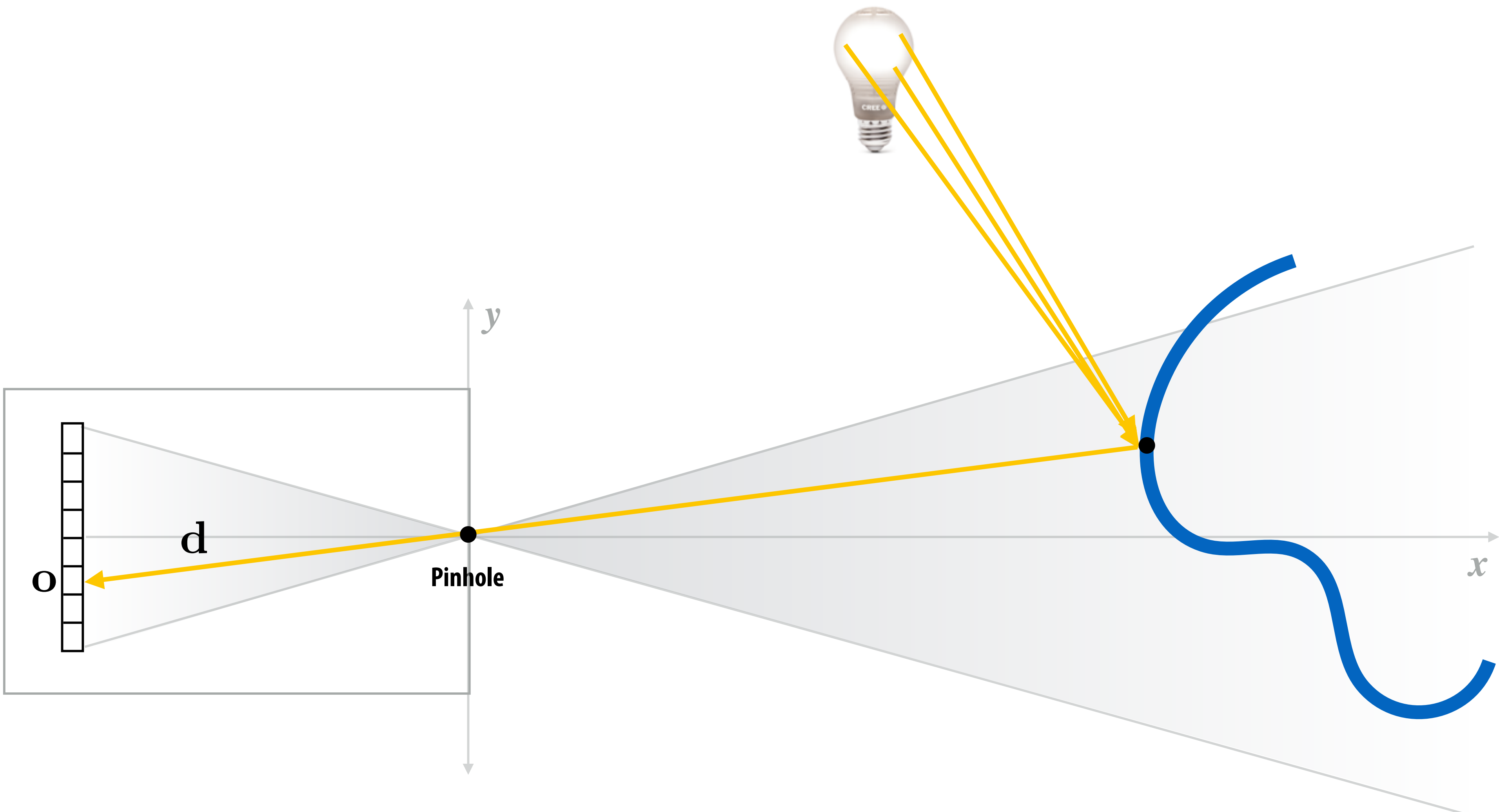
(This is why you're writing a ray tracer—rasterizer isn't enough!)

Renderer measures radiance along a ray



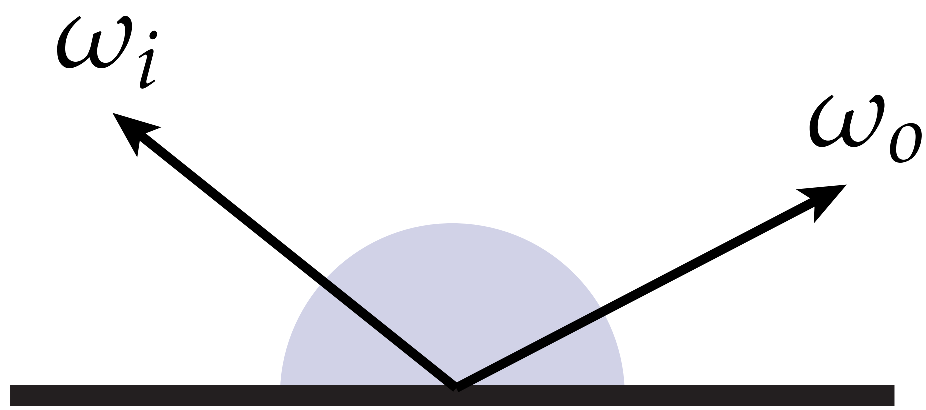
At each “bounce”, want to measure radiance traveling in the direction opposite the ray direction.

Renderer measures radiance along a ray



Radiance entering camera in direction d = light from scene light sources that is reflected off surface in direction d.

The Rendering Equation



How does reflection of light affect the outgoing radiance?

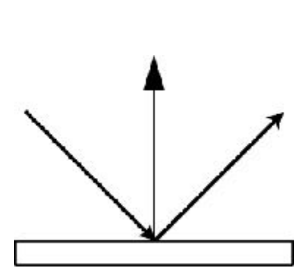
$$L_r(p,\omega_o) = \int_{H^2} \underbrace{f_r(p,\omega_i \to \omega_o)}_{\text{反射率?}} L_i(p,\omega_i)\cos\theta d\omega_i$$

Reflection Models

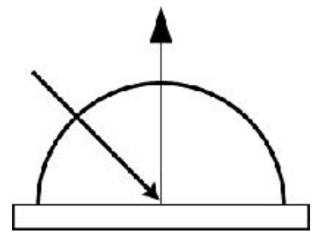
- Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency
- Choice of reflection function determines surface appearance

Some basic reflection functions

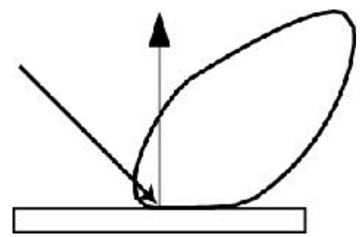
- Ideal specular
  - Perfect mirror



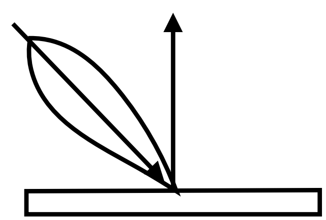
- Ideal diffuse
  - Uniform reflection in all directions

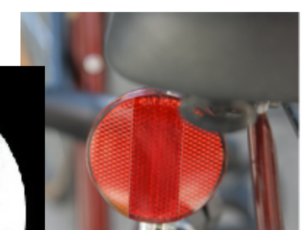
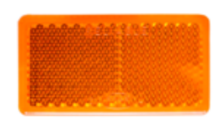



- Glossy specular
  - Majority of light distributed in mirror direction



- Retro-reflective
  - Reflects light back toward source



Diagrams illustrate how incoming light energy from given direction is reflected in various directions.

Materials: Diffuse

Materials: Mirror

Materials: Plastic

Materials: Red Semi-gloss Paint

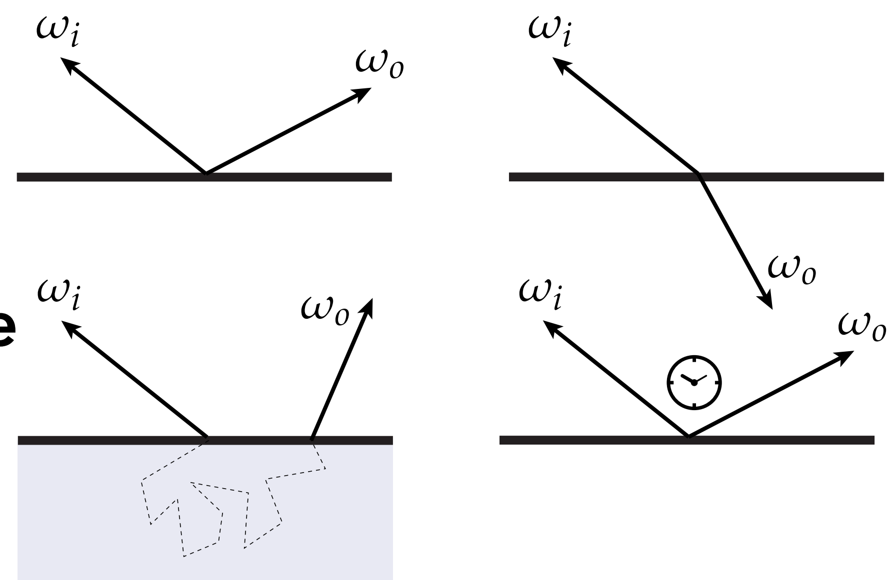
Materials: Ford Mystic Laquer Paint

Materials: Gold

Materials

Models of Scattering 散射模型

- How can we model "scattering" of light?
- Many different things that could happen to a photon:
    - Bounces off surface
    - Transmitted through surface
    - Bounces around inside surface
    - Absorbed & re-emitted

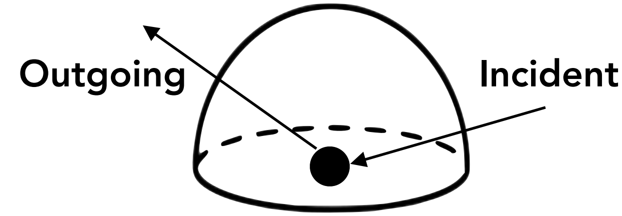


- What goes in must come out! (Total energy must be conserved)
- In general, can talk about "probability" a particle arriving from a given direction is scattered in another direction

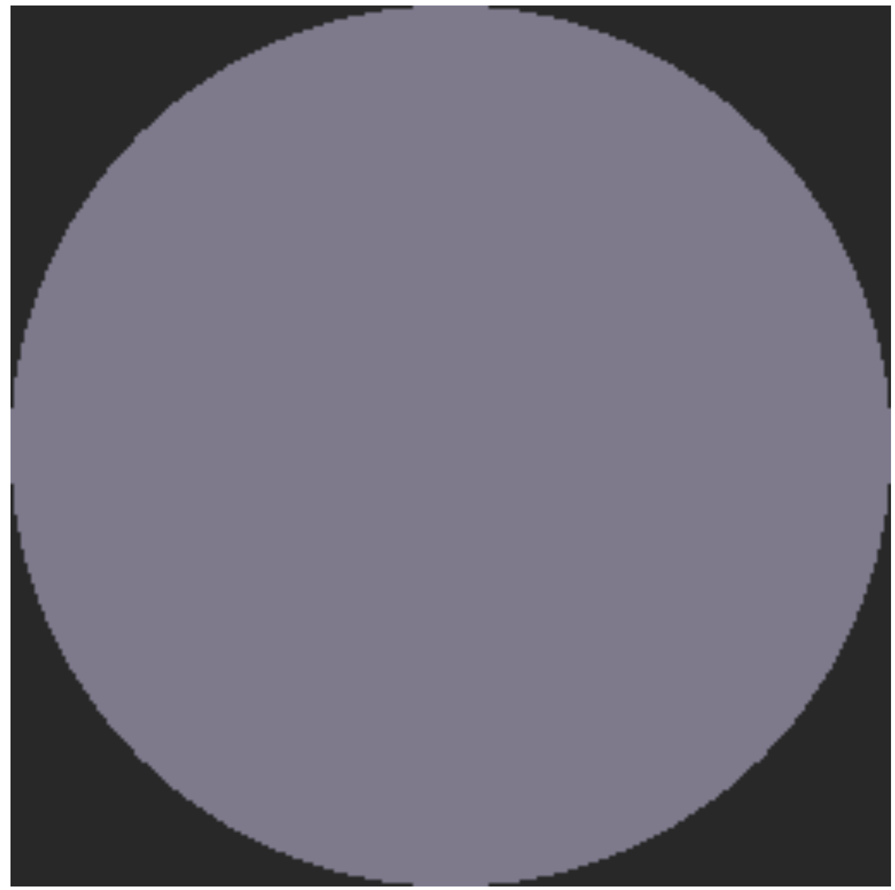
Hemispherical Incident Radiance

At any point on any surface in the scene, there's an incident radiance field that gives the directional distribution of illumination at the point

Diffuse Reflection



Exitant radiance is the same in all directions



Incident radiance

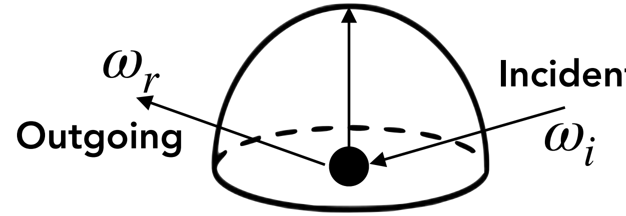
$$L_i(p, \omega_i)$$

Exitant radiance

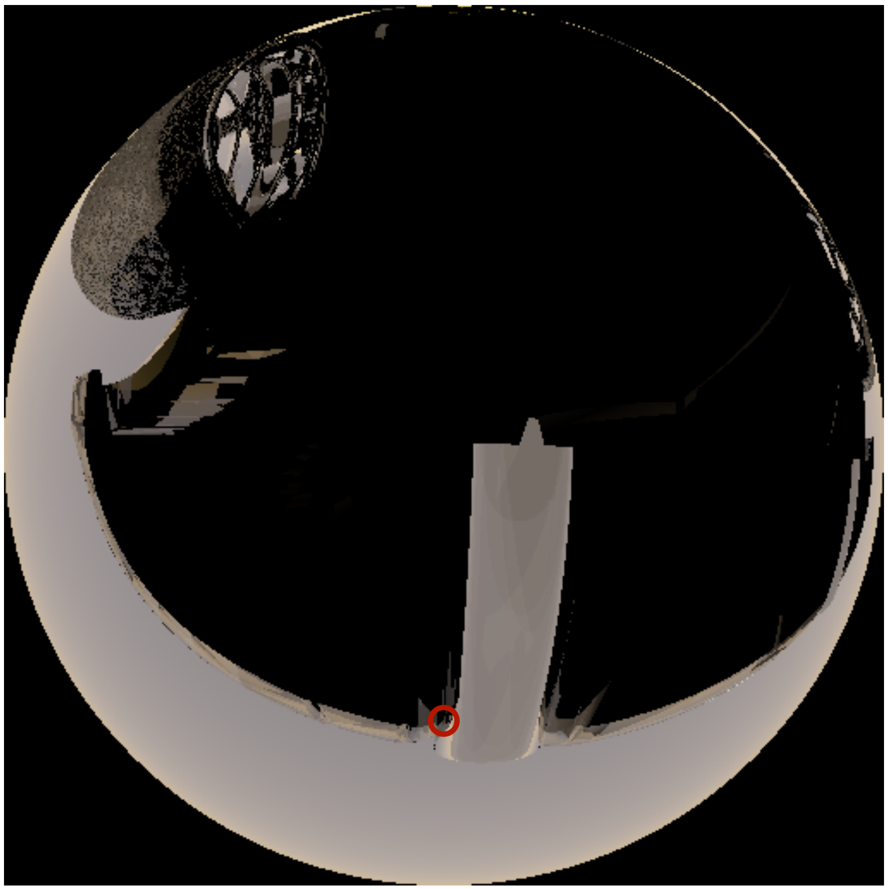
$$\int_{H^2} \underbrace{f_r(p, \omega_i \to \omega_o)}_{\text{Constant}} L_i(p, \omega_i) \cos\theta \, d\omega_i$$

$$= c \int_{H^2} L_i(p, \omega_i) \cos\theta \, d\omega_i$$

Ideal Specular Reflection

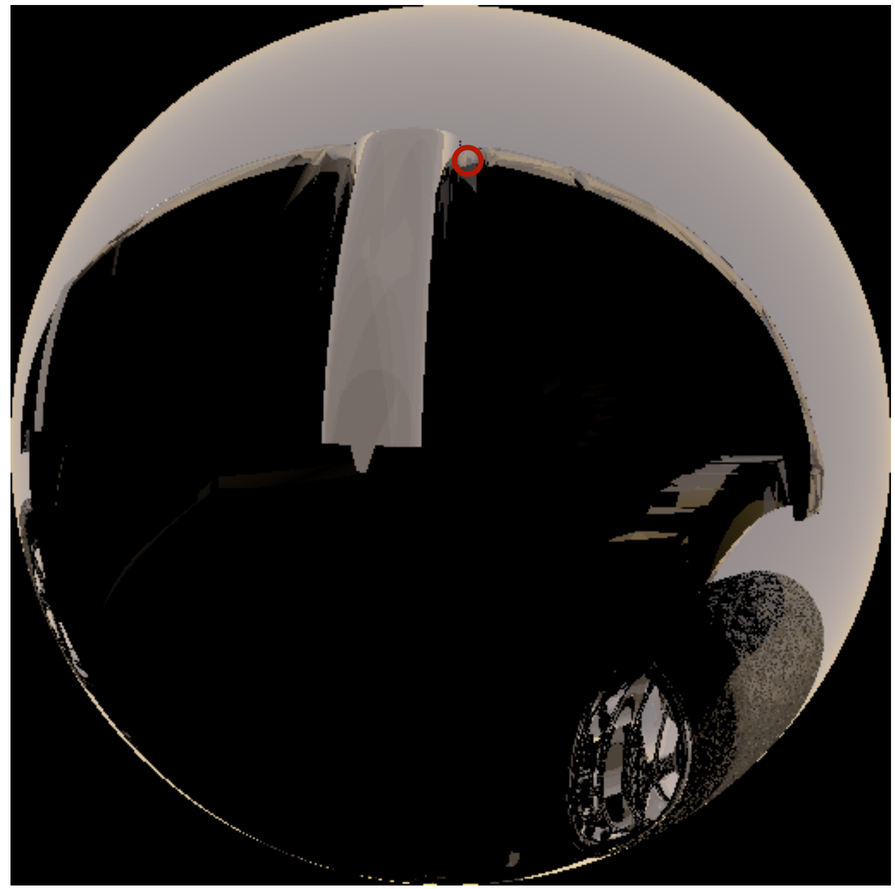


Incident radiance is "flipped around normal" to get exitant radiance



Incident radiance

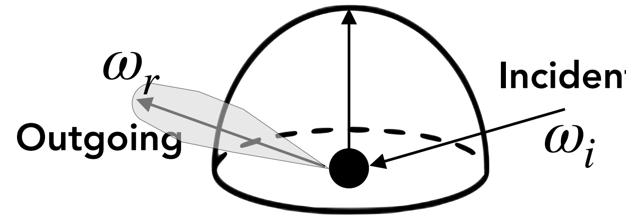
$$L_i(p, \omega_i)$$



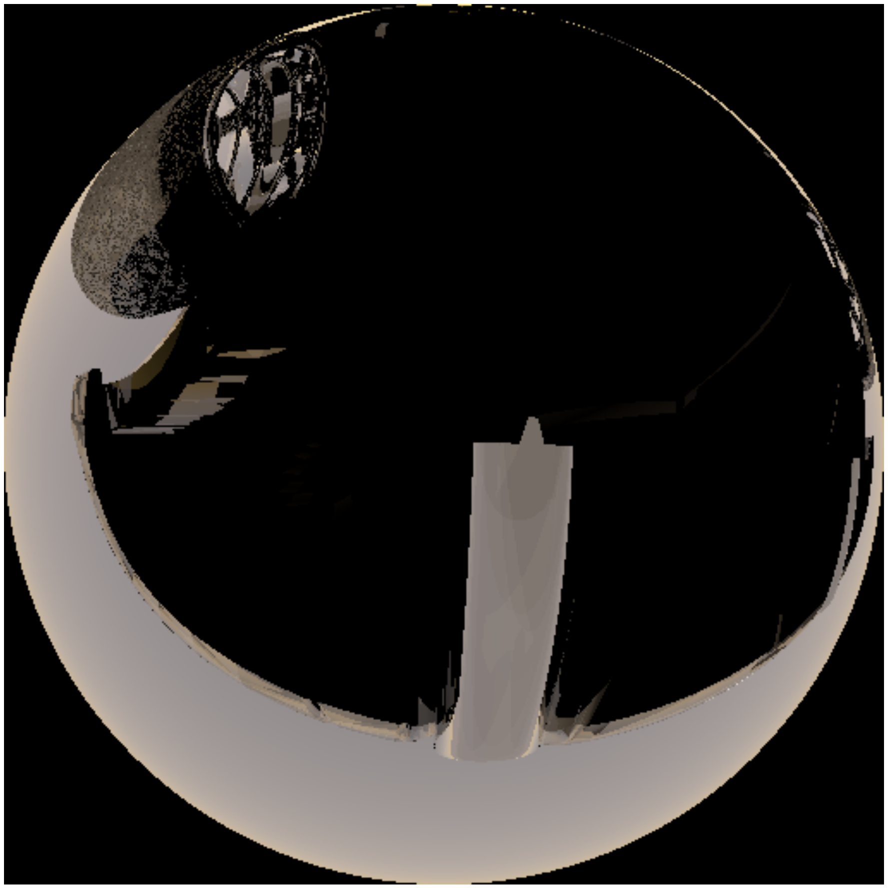
Exitant radiance

$$\int_{H^2} f_r(p, \omega_i \to \omega_o) L_i(p, \omega_i) \cos\theta d\omega_i$$
$$= f_r(p, \omega_i \to \omega_r) L_i(p, \omega_i) \cos\theta d\omega_i$$

Plastic

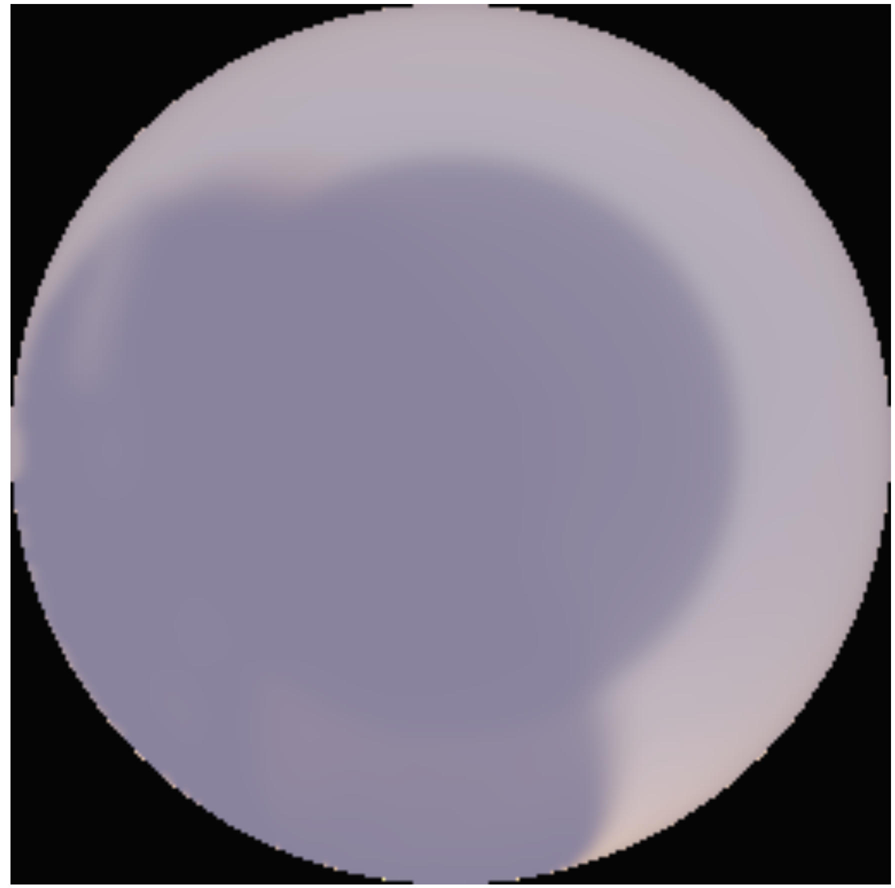


Incident radiance gets “flipped and blurred”



Incident radiance

$$L_i(p, \omega_i)$$

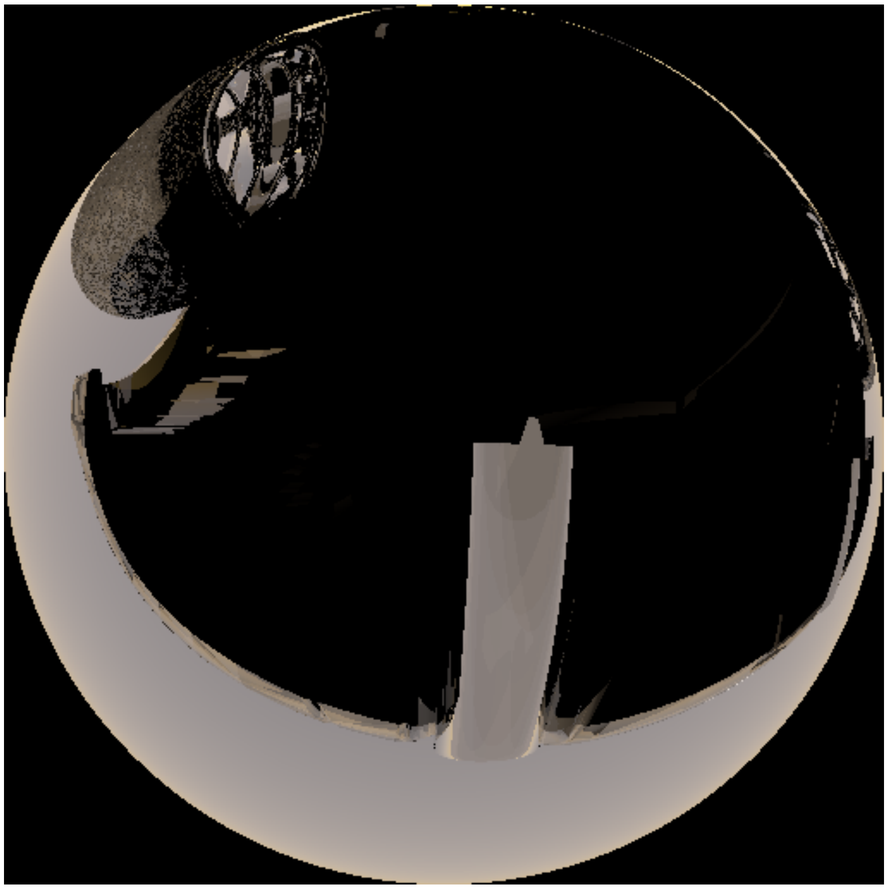


Exitant radiance

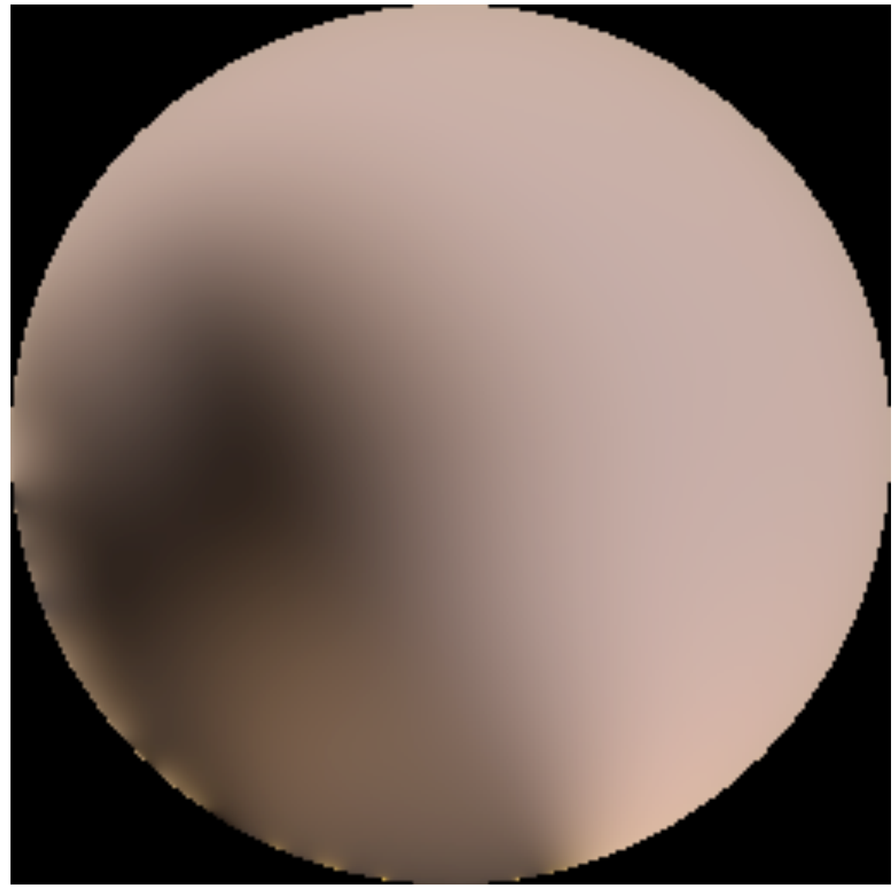
$$\int_{H^2} f_r(p, \omega_i \to \omega_o) L_i(p, \omega_i) \cos\theta \, d\omega_i$$

Copper

More blurring, plus coloration (nonuniform absorption across frequencies)



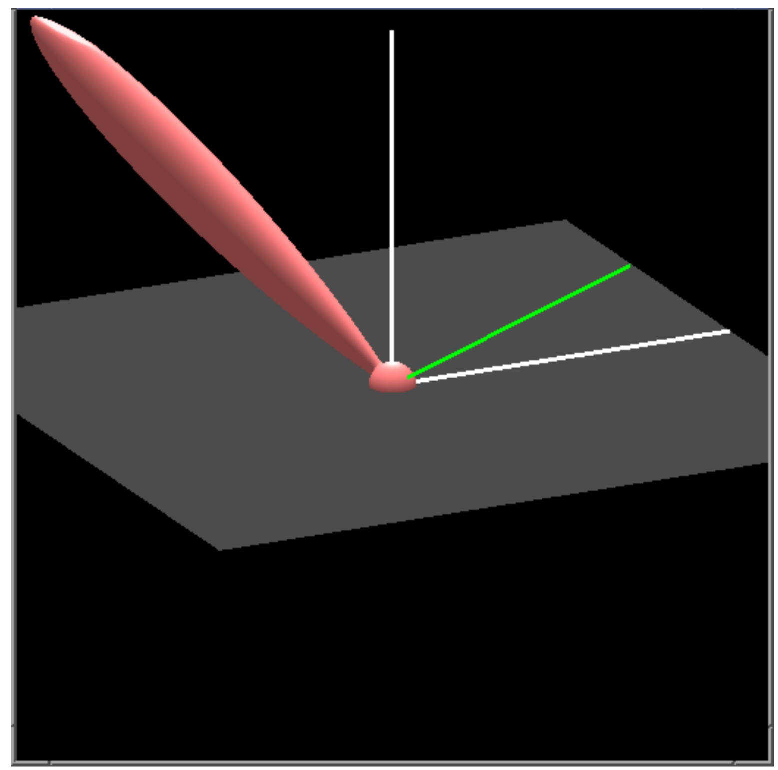
Incident radiance



Exitant radiance

Scattering Off A Surface: The BRDF

- "Bidirectional reflectance distribution function"
- Encodes behavior of light that "bounces off" surface
- Given incoming direction ω_i, how much light gets scattered in any direction outgoing direction ω_o?
- Describe as distribution $f_r(\omega_i - \omega_o)$



$$f_r(\omega_i \to \omega_o) \geq 0$$ 反射率相关的量

why less than or equal?

$$\int_{H^2} f_r(\omega_i \to \omega_o) cos\theta d\omega_i \leq 1$$ 能量守恒

Where did the rest of the energy go?

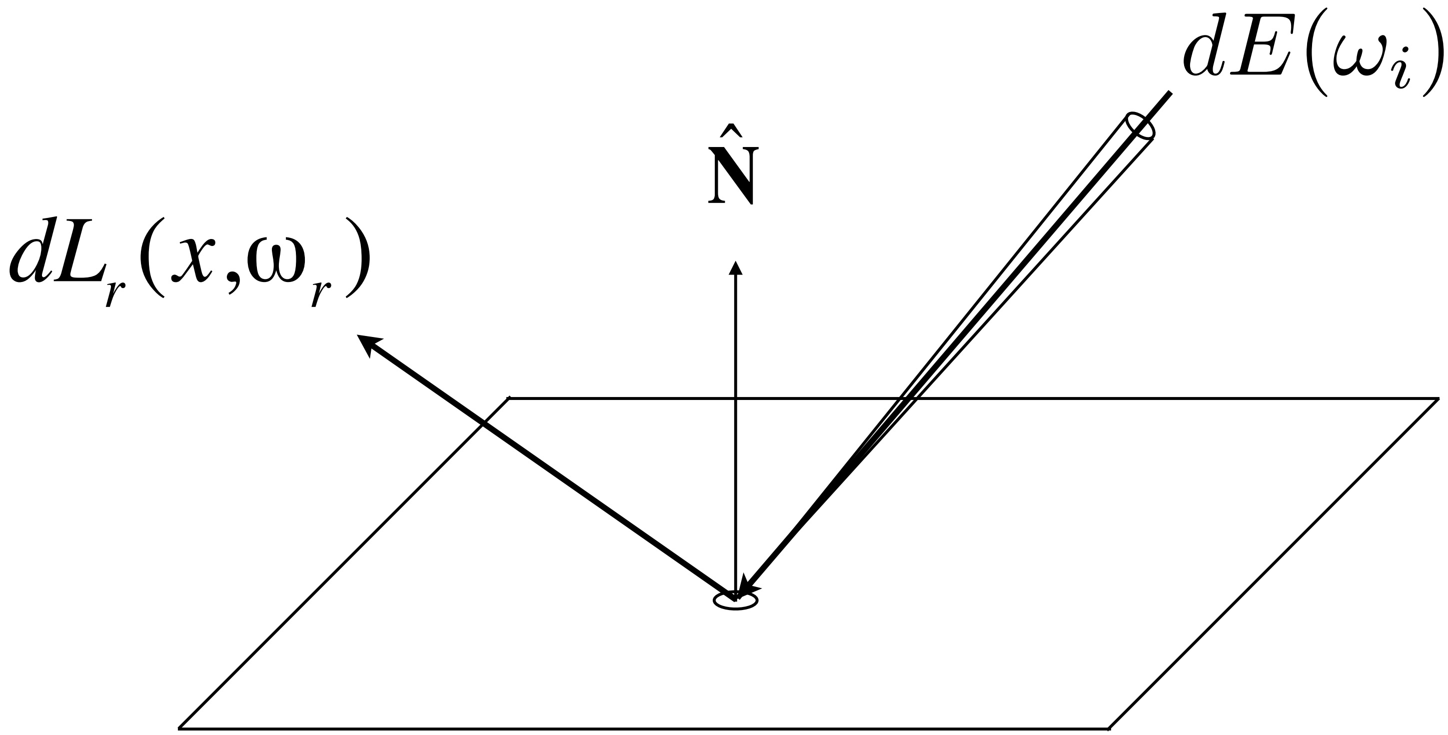
$$f_r(\omega_i \to \omega_o) = f_r(\omega_o \to \omega_i)$$ 对称性

"Helmholtz reciprocity":

Light behaves the same way if you reverse its path. (光线可逆)

Why should Helmholtz reciprocity hold? Think about little mirrors...

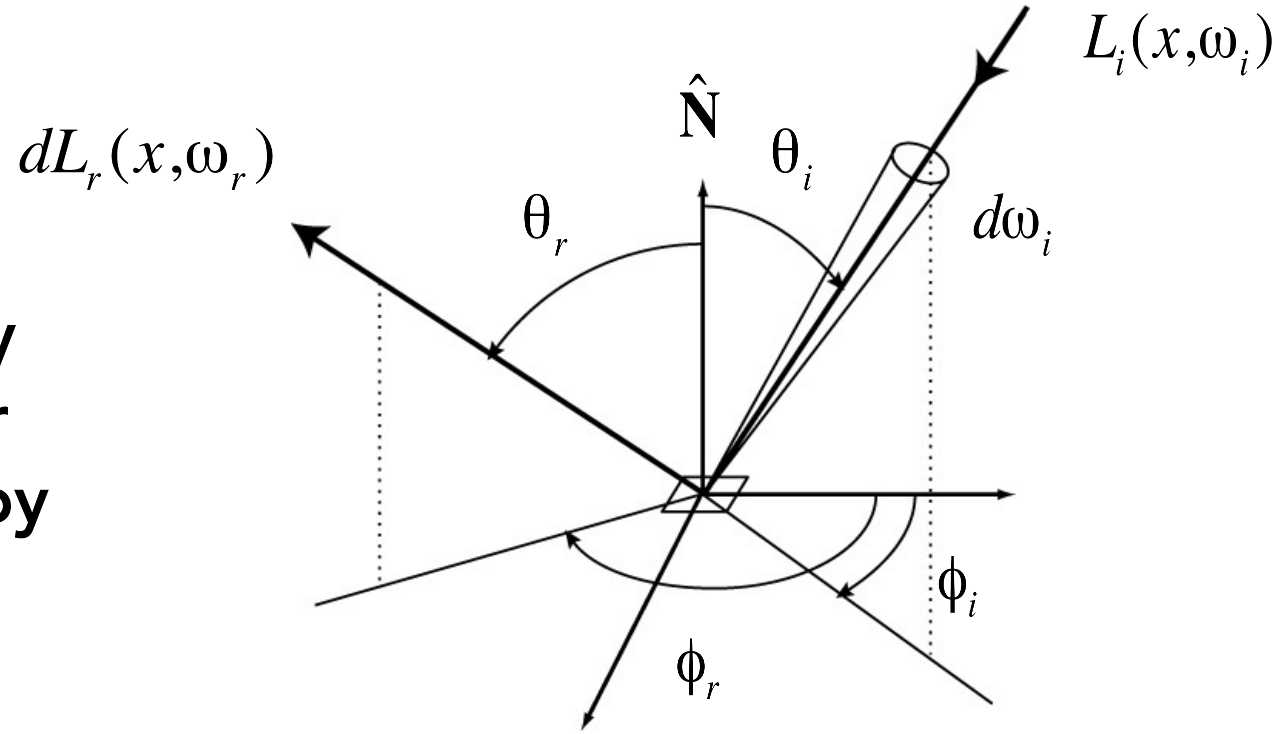
Reflection at a Point



Differential irradiance incoming: $dE(\omega_i) = L(\omega_i)\, cos\, \theta_i\, d\omega_i$

Differential radiance exiting (due to $dE(\omega_i)$**)** $dL_r(\omega_r)$

Radiometric description of BRDF



NB: ω_i points away from surface rather than into surface, by convention.

$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

"For a given change in the incident irradiance, how much does the exitant radiance change?"

为什么不是简单地定义成"出射辐亮度除以入射辐亮度?

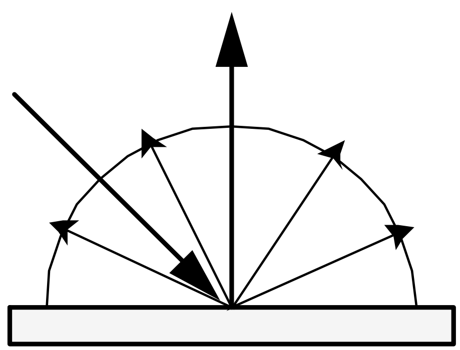
既 $\frac{L_o(\omega_o)}{L_i(\omega_i)}$ **解释：** $L_o(\omega_o)$ 并不是只由某一个 $L_i(\omega_i)$ 决定；它是整个入射场景 $L_i(\omega_i)$ 的积分结果。

我只看来自一个**极窄立体角** $d\omega_i$ 的入射光，并只观察这束光引起的**微小变化** dL_o。

这样我们才能**分辨出**"来自方向 ω_i"的光，对"出射方向 ω_o"的光亮贡献是多少。

Example: Lambertian Reflection

Assume light is equally likely to be reflected in each output direction



$$f_r = c$$

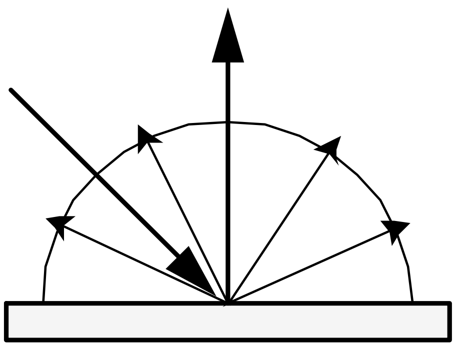
$$L_o(\omega_o) = \int_{H^2} f_r \, L_i(\omega_i) \, \cos\theta_i \, \mathrm{d}\omega_i$$
$$= f_r \int_{H^2} L_i(\omega_i) \, \cos\theta_i \, \mathrm{d}\omega_i$$
$$= f_r E$$

"albedo" (between 0 and 1)
反照率; 反射率; 漫反射系数;

$$f_r = \frac{\rho}{\pi}$$

Example: Lambertian Reflection

Assume light is equally likely to be reflected in each output direction



$$f_r = c$$

Suppose the incident lighting is uniform:

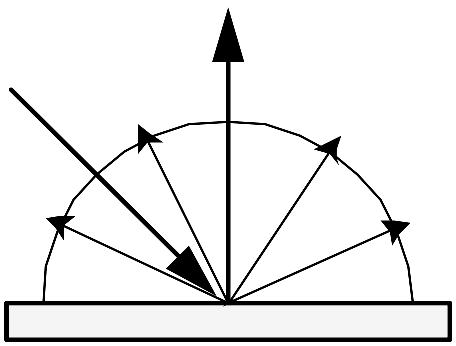
$$\begin{aligned} L_o(\omega_o) &= \int_{H^2} f_r L_i(\omega_i) \cos\theta_i \, \mathrm{d}\omega_i \\ &= f_r L_i \int_{H^2} \cancel{(\omega_i)} \cos\theta_i \, \mathrm{d}\omega_i \\ &= \pi f_r L_i \end{aligned}$$

$$f_r = \frac{\rho}{\pi}$$

"albedo" (between 0 and 1)
反照率; 反射率; 漫反射系数;

Example: Lambertian Reflection

Assume light is equally likely to be reflected in each output direction



$$f_r = c$$

$$f_r = \frac{\rho}{\pi}$$

"albedo" (between 0 and 1)
反照率; 反射率; 漫反射系数;

f_r 是否需要满足 ≤ 1？

根据表面反射的总能量 ≤ 入射能量，有下式：

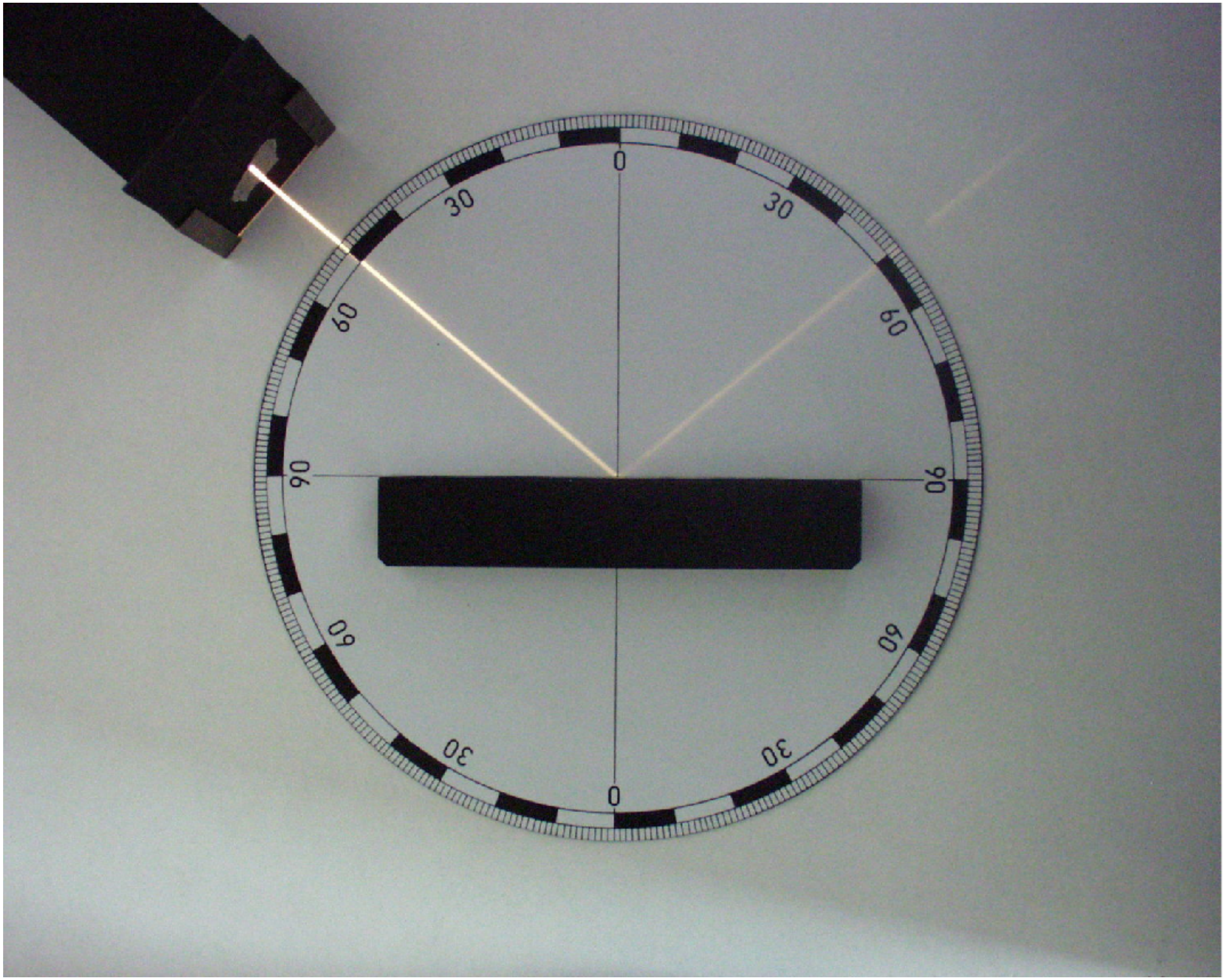
$$\int_\Omega f_r \cos\theta_i d\omega_i = \rho \leq 1$$

对球面积分得到：

$$\int_\Omega \frac{\rho}{\pi} \cos\theta_i d\omega_i = \frac{\rho}{\pi} \cdot \pi = \rho$$

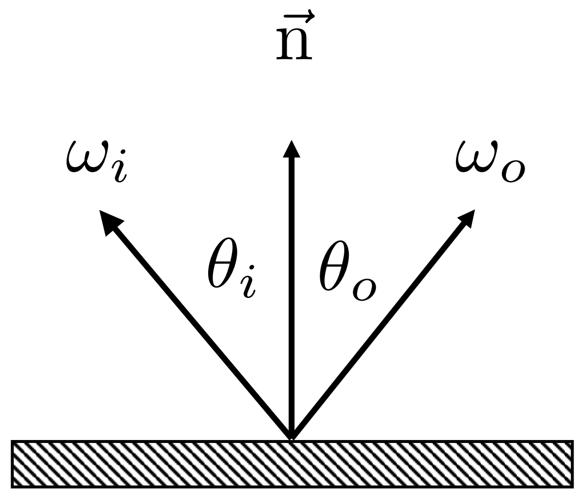
BRDF 的数值范围不必限制在 0~1，它只需要满足能量守恒条件。

Example: Perfect Specular Reflection



[Zátonyi Sándor]

Geometry of Specular Reflection

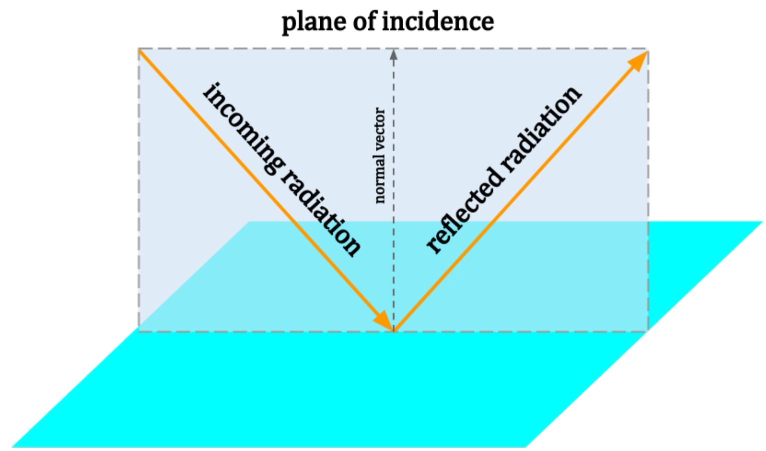


$$\theta = \theta_o = \theta_i$$

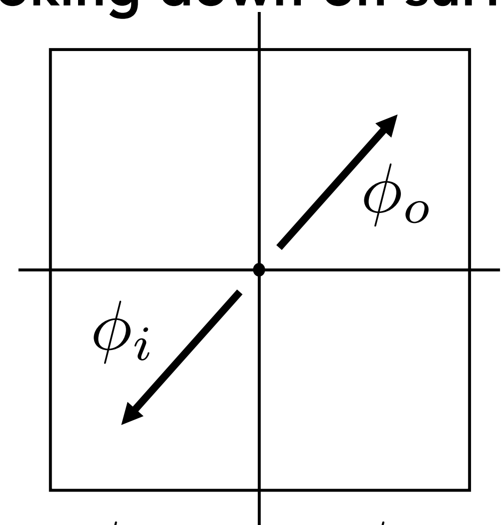
$$w_o + w_i = 2\cos\theta \cdot \vec{n}$$

$$\omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n}$$

方向角，与法向夹角



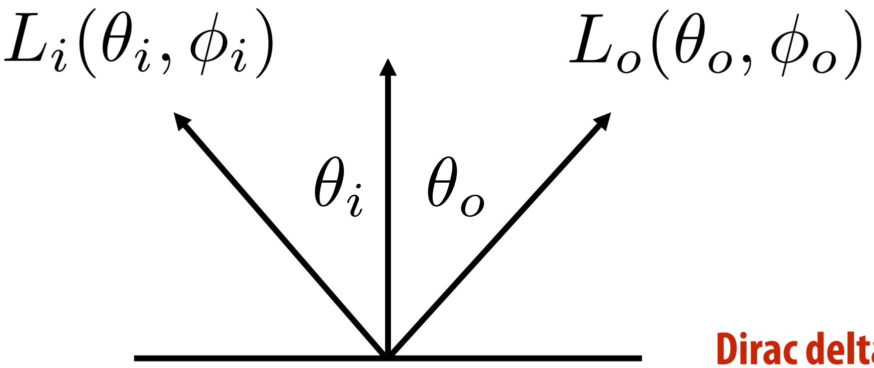
Topdown view
(looking down on surface)



$$\phi_o = -\phi_i$$

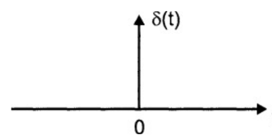
方位角，绕法向旋转

Specular Reflection BRDF



$$L_o(\theta_o, \phi_o) = L_i(\theta_i, \phi_i)$$

Dirac delta

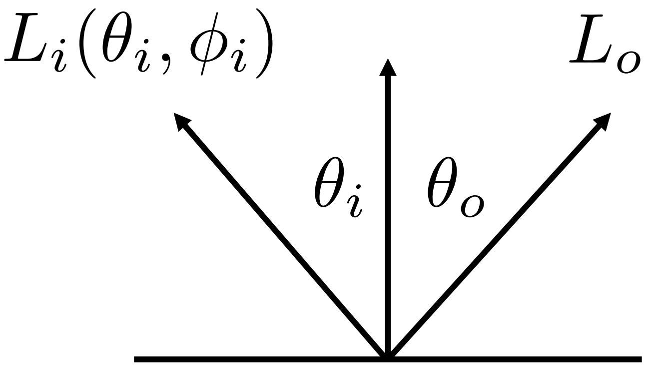


$$f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i}\delta(\phi_i - \phi_o \pm \pi)$$

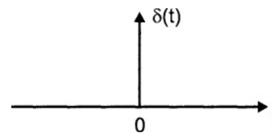
验证是否满足能量守恒:

$$\int_\Omega f_r(\theta_i, \phi_i; \theta_o, \phi_o)\ \cos\theta_o\, d\omega_o = 1$$

Specular Reflection BRDF



$$L_o(\theta_o, \phi_o) = L_i(\theta_i, \phi_i)$$



Dirac delta

$$f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i}\delta(\phi_i - \phi_o \pm \pi)$$

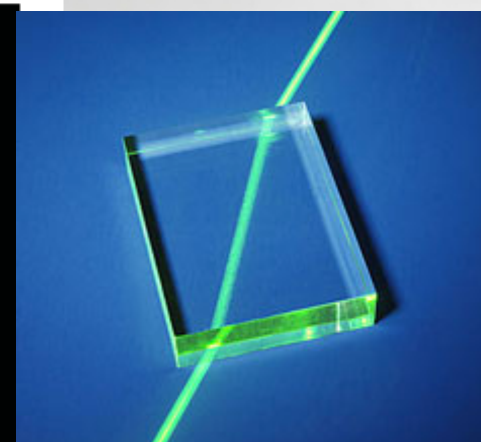
- **Strictly speaking, f_r is a distribution, not a function**
- **In practice, no hope of finding reflected direction via random sampling; simple pick the reflected direction!**

Scattering Through A Surface: The BTDF

"Bidirectional transmittance distribution function"

In addition to reflecting off surface, light may be transmitted through surface.

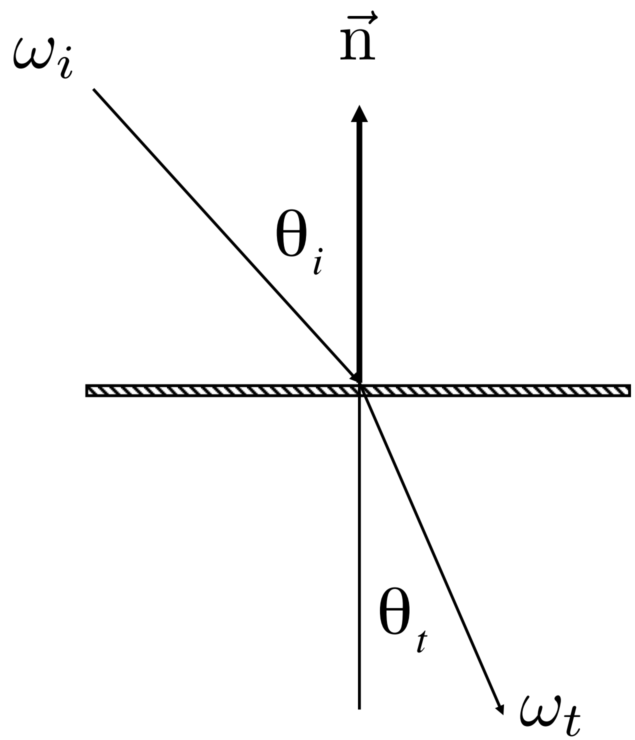
Light refracts when it enters a new medium.



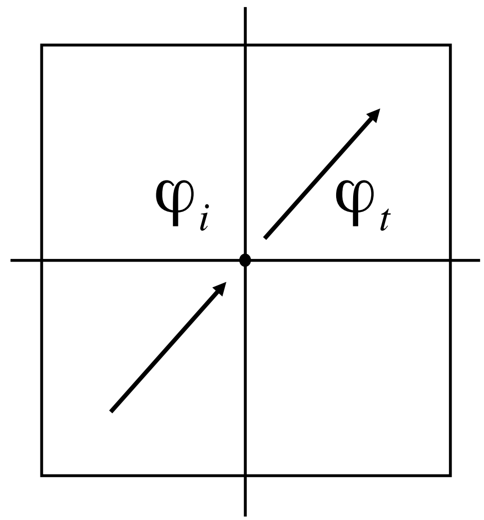
"Bidirectional scattering distribution function" (BSDF) = BRDF + BRTF…

Snell's Law

Transmitted angle depends on relative index of refraction of material ray is leaving/ entering.

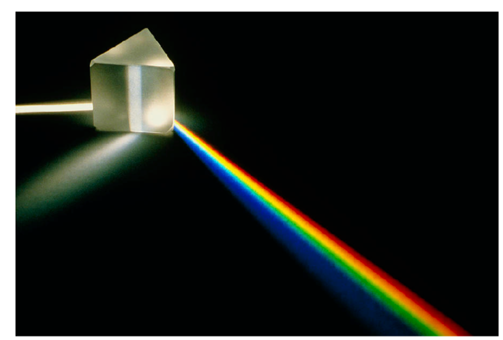


$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

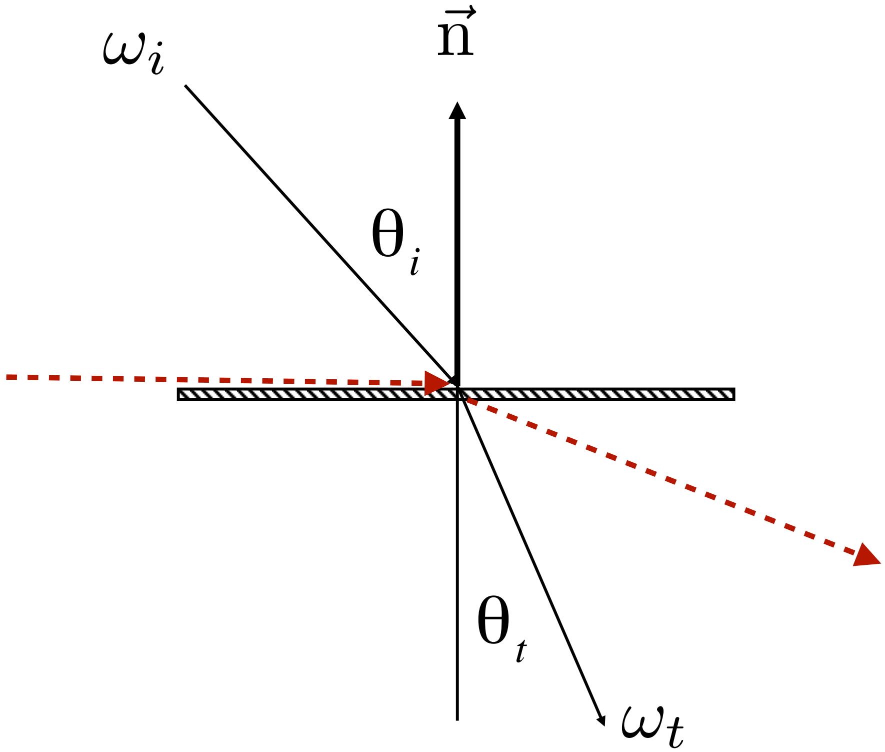


Medium	η*
Vacuum	1.0
Air (sea level)	1.00029
Water (20°C)	1.333
Glass	1.5-1.6
Diamond	2.42

*index of refraction is wavelength dependent (these are averages)



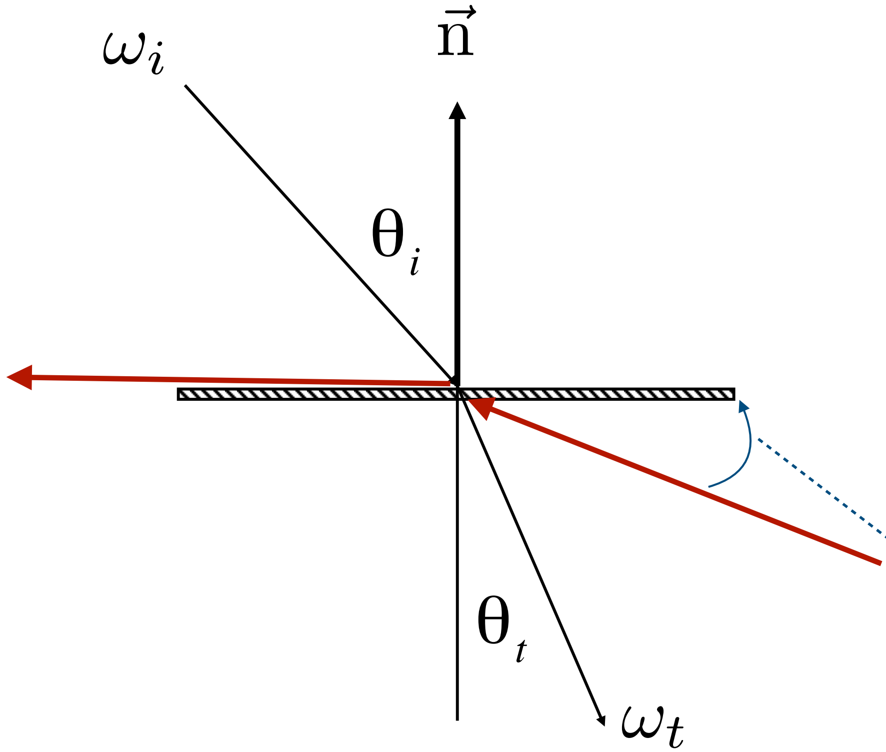
Law of Refraction



$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

$$\begin{aligned}
\cos\theta_t &= \sqrt{1-\sin^2\theta_t} \\
&= \sqrt{1-\left(\frac{\eta_i}{\eta_t}\right)^2 \sin^2\theta_i} \\
&= \sqrt{1-\left(\frac{\eta_i}{\eta_t}\right)^2 (1-\cos^2\ \theta_i)}
\end{aligned}$$

Law of Refraction



$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

$$= \sqrt{1-\left(\frac{\eta_i}{\eta_t}\right)^2 \sin^2\theta_i}$$

$$= \sqrt{1-\left(\boxed{\frac{\eta_i}{\eta_t}}\right)^2 (1-\cos^2\theta_i)}$$

$$\boxed{1-\left(\frac{\eta_i}{\eta_t}\right)^2 (1-\cos^2\theta_i) < 0}$$

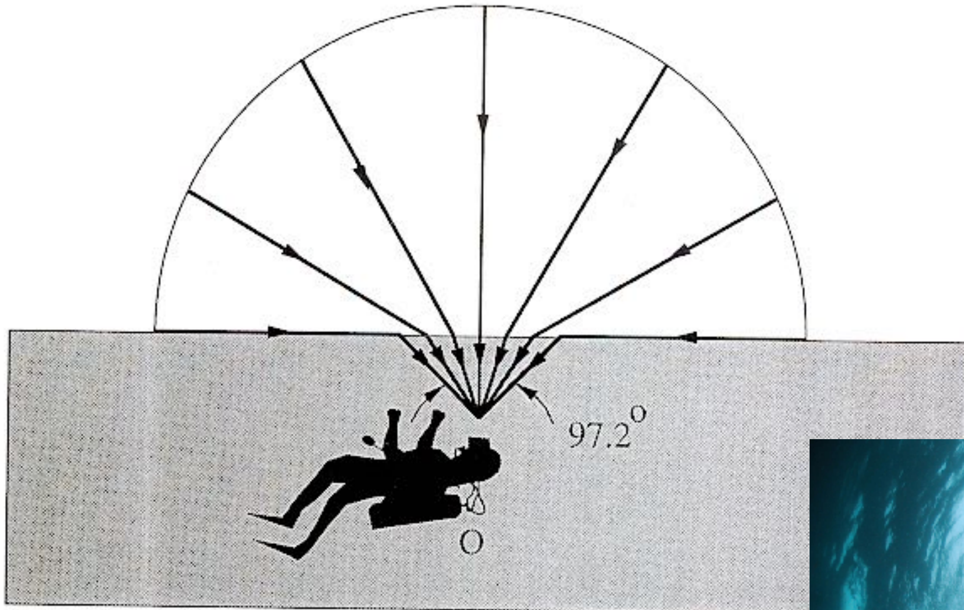
Total internal reflection: (全内反射)

When light is moving from a more optically dense medium to a less optically dense medium: $\frac{\eta_i}{\eta_t} > 1$

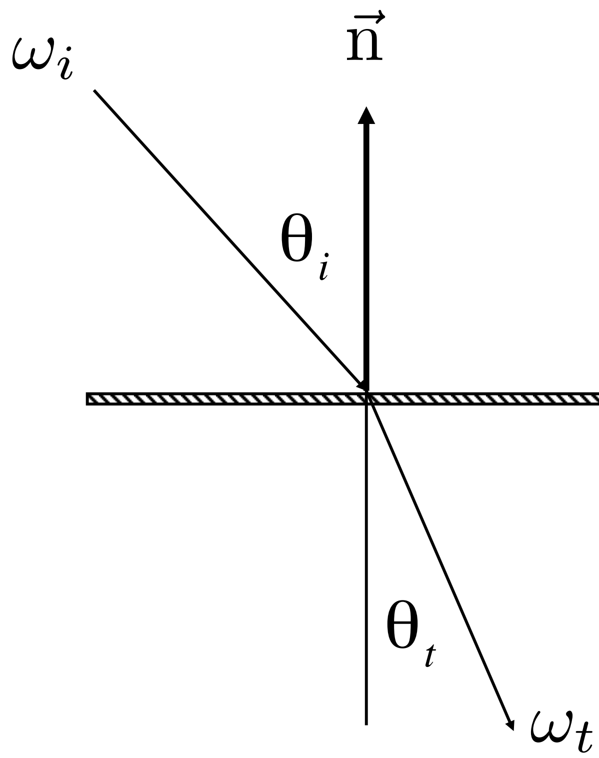
Light incident on boundary from large enough angle will not exit medium

Optical Manhole

Only small "cone" visible, due to total internal reflection (TIR)



BTDF



$$f_t(\omega_i, \omega_o) = \frac{dL_t(\omega_o)}{dE_i(\omega_i)}$$

对称性：

$$f_t(\omega_i, \omega_o) = f_t(\omega_o, \omega_i)$$

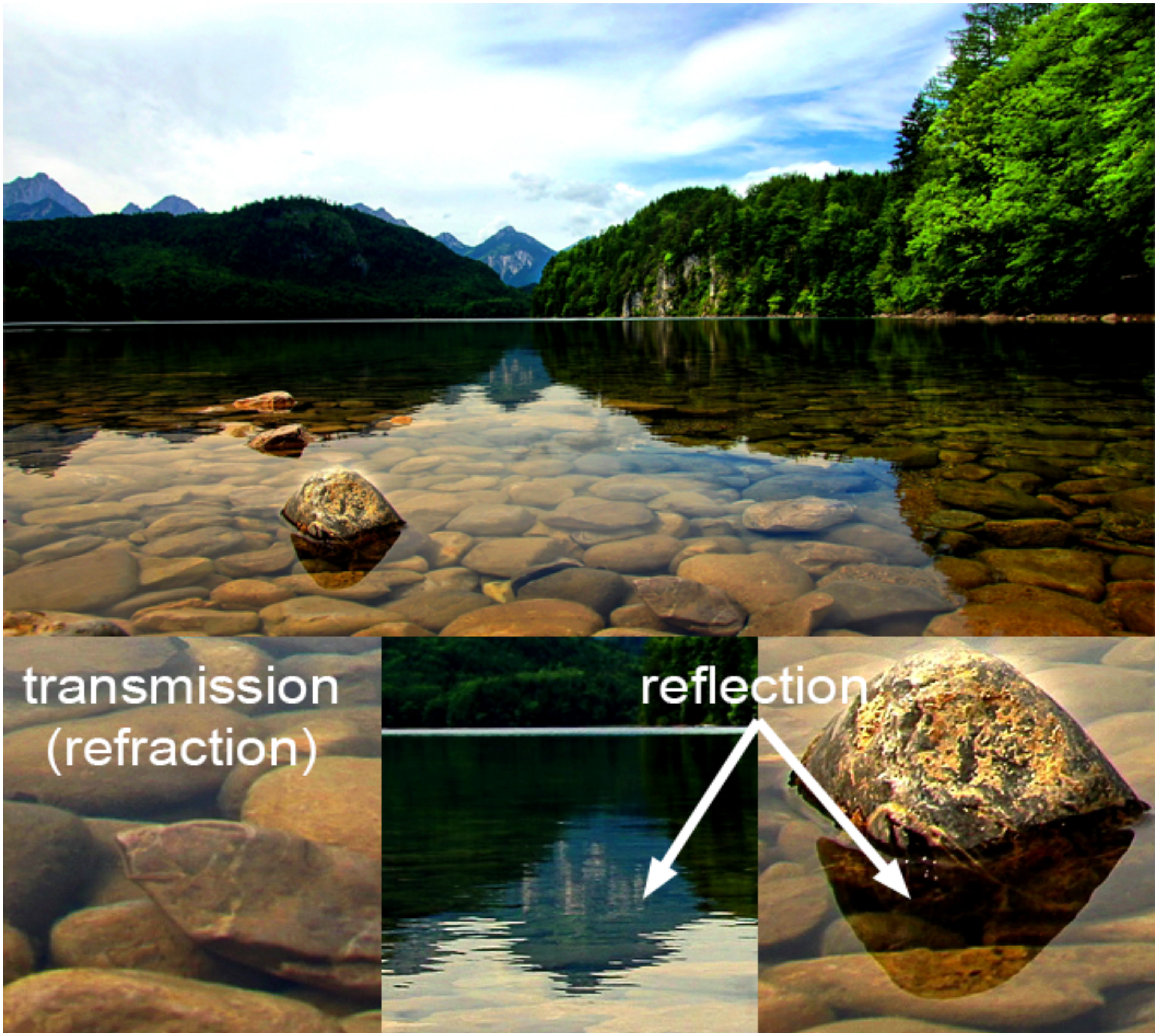
Ideal refraction (Snell's law)：

$$f_t(\omega_i, \omega_o) = \frac{(1 - F(\omega_i))}{|\cos\theta_i|} \delta(\omega_o - \mathrm{refract}(\omega_i))$$

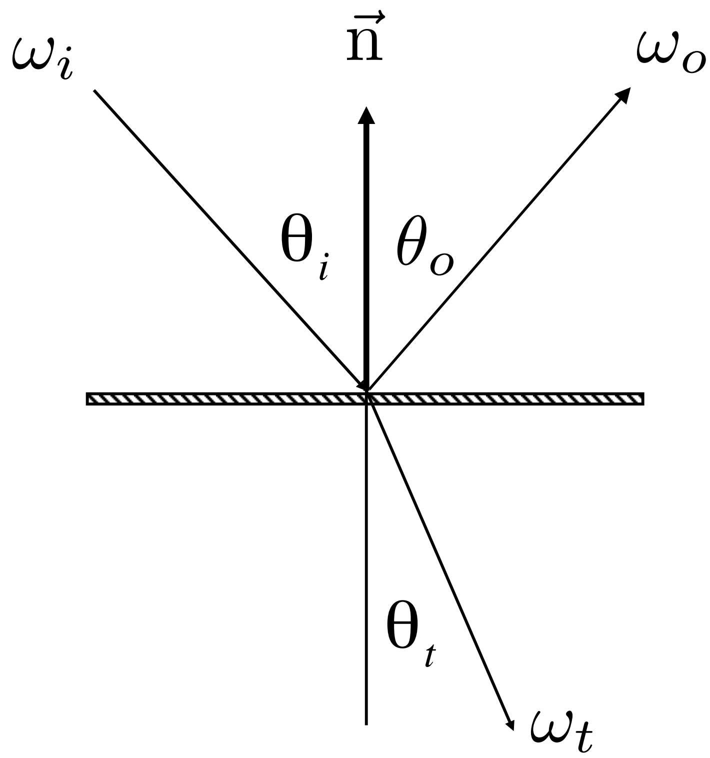
$F(\omega_i)$ 代表反射相关系数

BSDF: BTDF + BRDF

When a ray of light hits the interface between two media (e.g. air and glass), some light is reflected, and some is transmitted (refracted).



BSDF: BTDF + BRDF



For a dielectric surface (like glass or water):

$$f_r(\omega_i, \omega_o) = \frac{F(\omega_i)}{|\cos\theta_i|}\,\delta(\omega_o - \mathrm{reflect}(\omega_i))$$

$$f_t(\omega_i, \omega_o) = \frac{(1 - F(\omega_i))}{|\cos\theta_i|}\,\delta(\omega_o - \mathrm{refract}(\omega_i))$$

$F(\omega_i)$ is constant for ideal specular reflection and refraction.

$F(\omega_i)$: Fresnel reflection term

满足能量守恒：（BRDF+BTDF）

$$\int_{\mathrm{hemisphere}} (f_r + f_t)\cos\theta_o\, d\omega_o \leq 1$$

Fresnel Reflection

菲涅尔

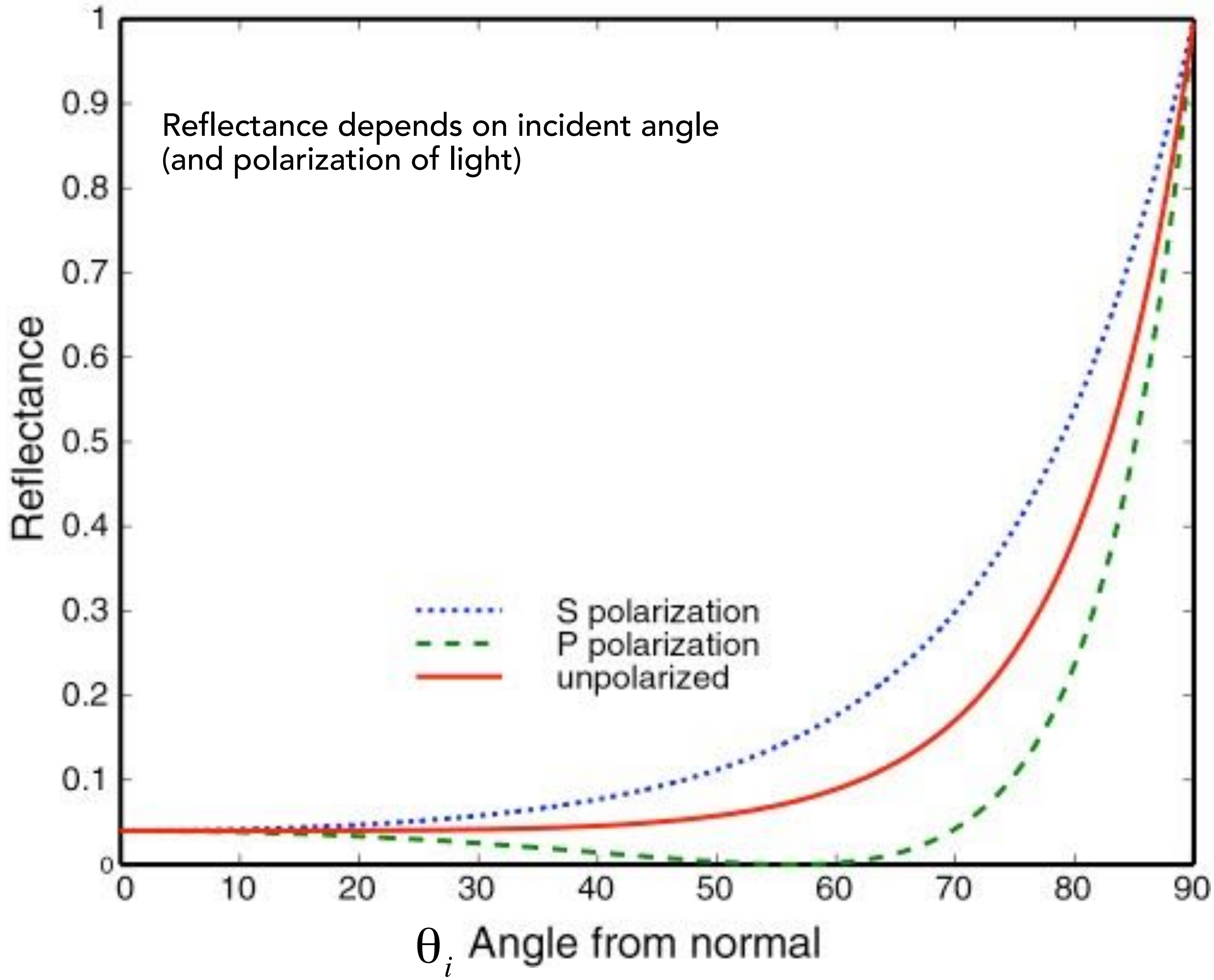
More reflective when viewed at grazing angles

Many real materials: Reflectance increase
w/ Viewing angle

θ_i

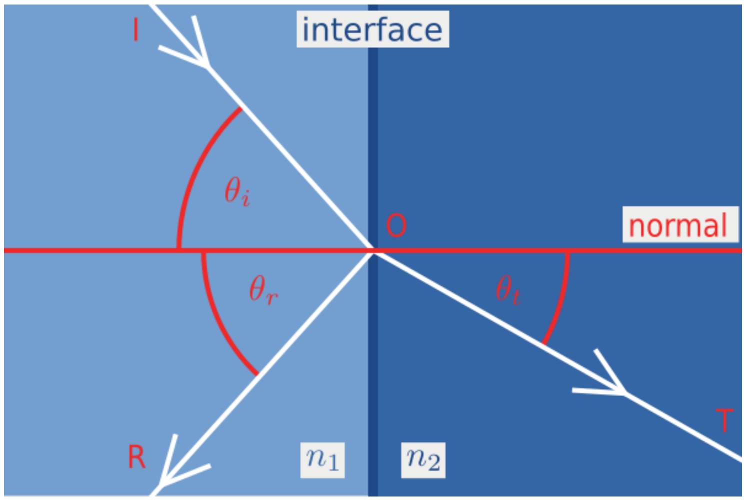
Fresnel Reflection 菲涅尔

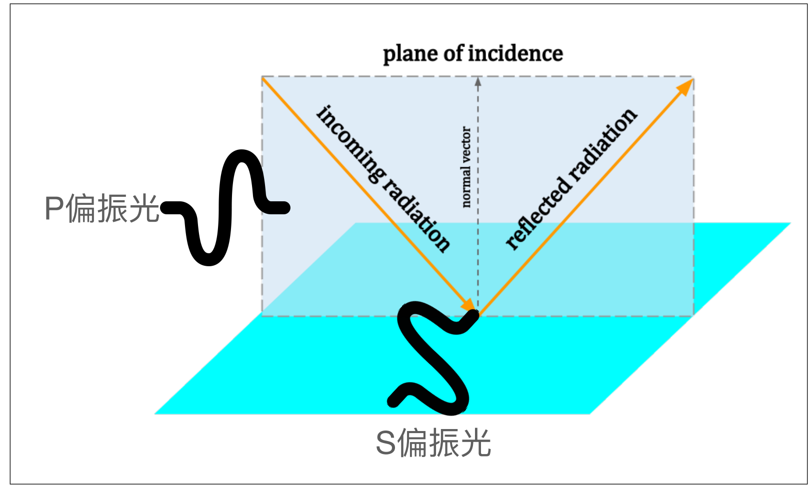
$F(\omega_i)$ **Fresnel Term (Dielectric, η =1.5, 介电常数)**



Fresnel Equation

电磁理论 → **波动光学** → 几何光学





通过满足 Maxwell 边界条件，可以得到两个偏振分量各自的反射系数：

$$R_{\mathrm{s}}=\left|\frac{n_1\cos\theta_{\mathrm{i}}-n_2\cos\theta_{\mathrm{t}}}{n_1\cos\theta_{\mathrm{i}}+n_2\cos\theta_{\mathrm{t}}}\right|^2=\left|\frac{n_1\cos\theta_{\mathrm{i}}-n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_{\mathrm{i}}\right)^2}}{n_1\cos\theta_{\mathrm{i}}+n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_{\mathrm{i}}\right)^2}}\right|^2,$$

$$R_{\mathrm{p}}=\left|\frac{n_1\cos\theta_{\mathrm{t}}-n_2\cos\theta_{\mathrm{i}}}{n_1\cos\theta_{\mathrm{t}}+n_2\cos\theta_{\mathrm{i}}}\right|^2=\left|\frac{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_{\mathrm{i}}\right)^2}-n_2\cos\theta_{\mathrm{i}}}{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_{\mathrm{i}}\right)^2}+n_2\cos\theta_{\mathrm{i}}}\right|^2.$$

$$F(\theta_i)=\frac{R_s+R_p}{2}$$

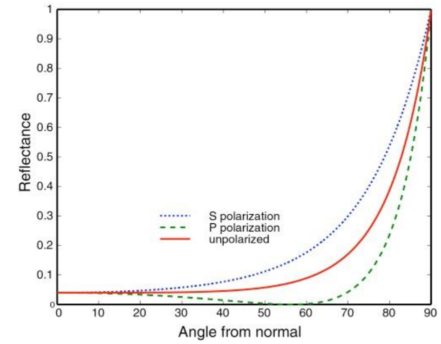
通过能量守恒定律，折射系数：

$$T_{\mathrm{s}}=1-R_{\mathrm{s}}\qquad T_{\mathrm{p}}=1-R_{\mathrm{p}}$$

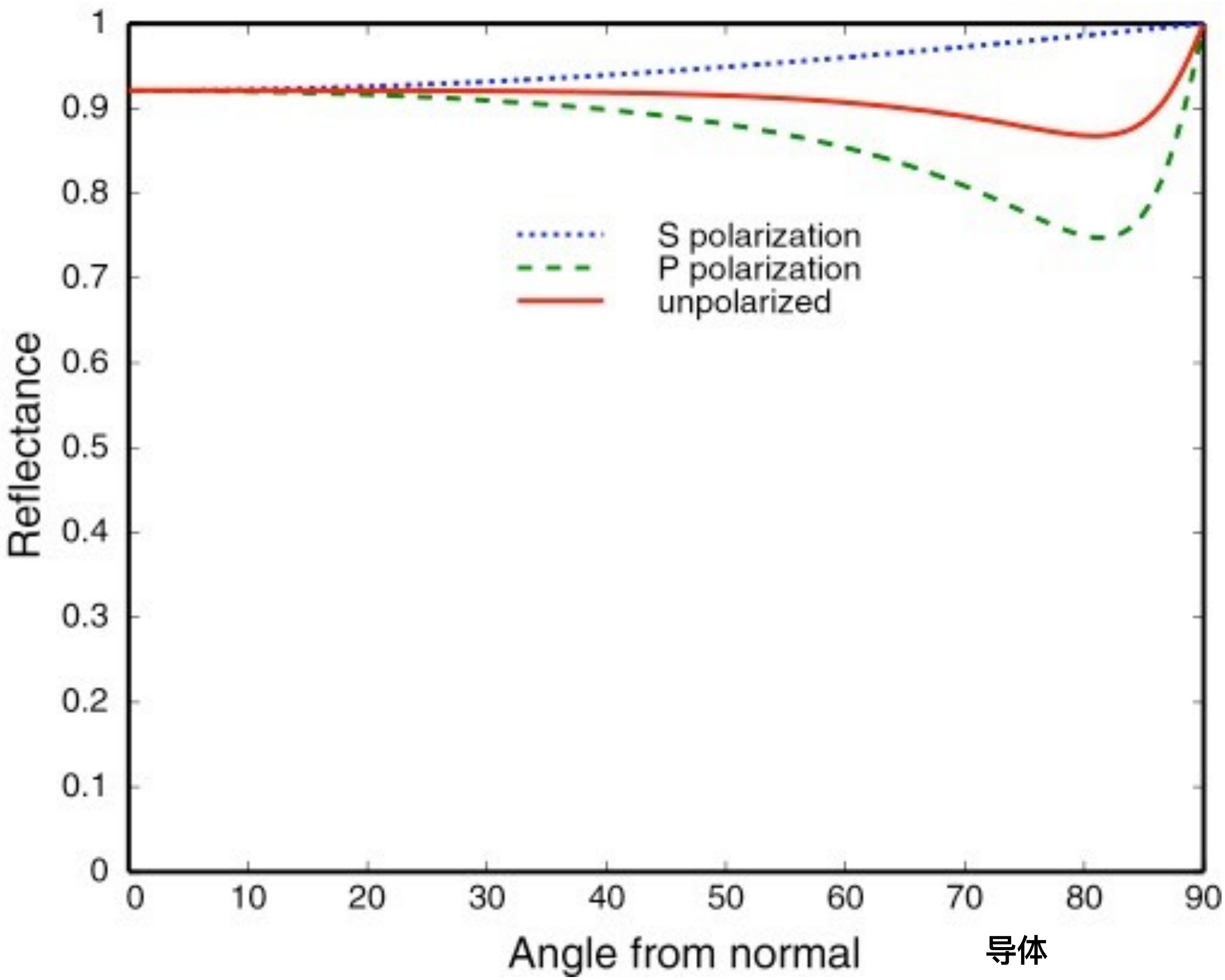
https://en.wikipedia.org/wiki/Fresnel_equations

Fresnel Reflection (Conductor)

Are Fresnel equations valid in conductors?

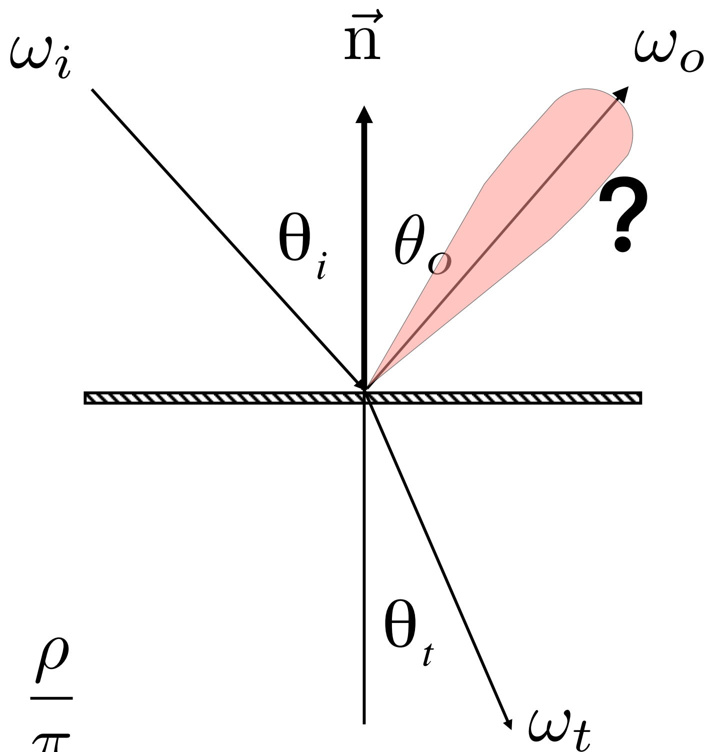


绝缘体



导体

Advance BSDF



$$f_r = \frac{\rho}{\pi}$$

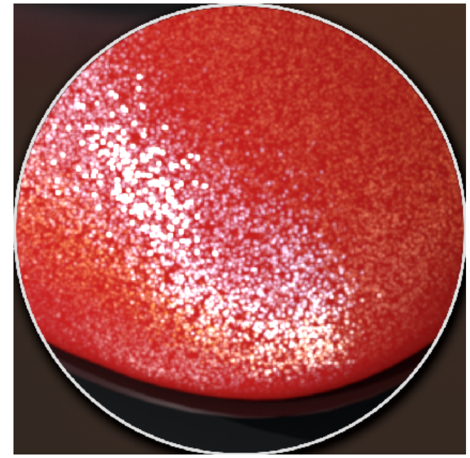
$$f_r(\omega_i, \omega_o) = \frac{F(\omega_i)}{|\cos\theta_i|}\,\delta(\omega_o - \text{reflect}(\omega_i))$$

$$f_t(\omega_i, \omega_o) = \frac{(1 - F(\omega_i))}{|\cos\theta_i|}\,\delta(\omega_o - \text{refract}(\omega_i))$$

Microfacet Reflection 微表面反射

https://twitter.com/Cmdr_Hadfield/status/318986491063828480/photo/1

Microfacet Theory "粗糙镜面反射"

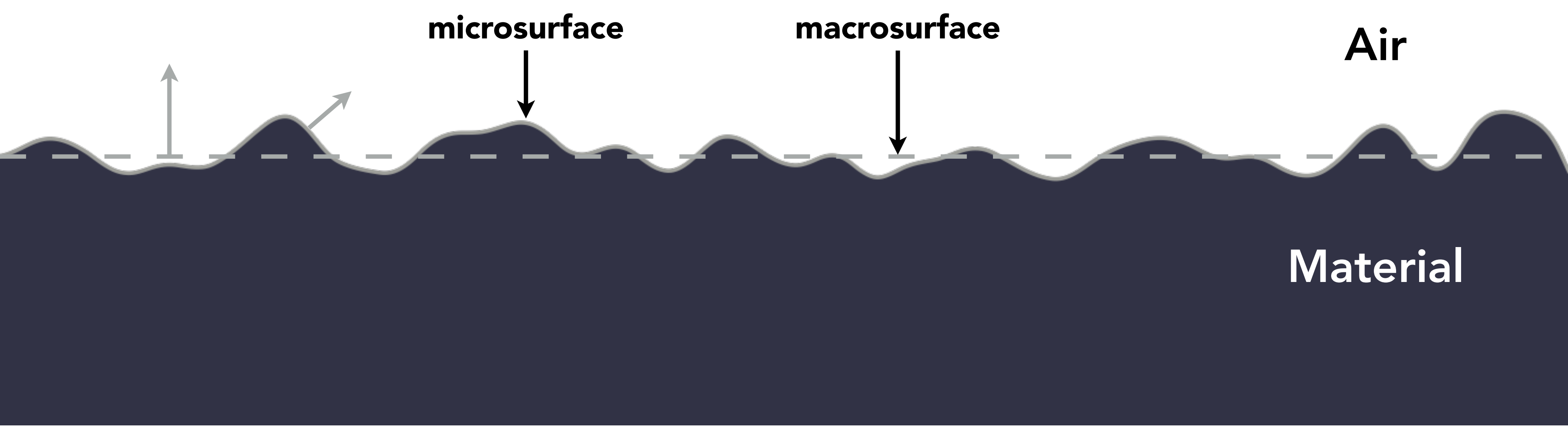


Rough surface

- Macroscale: flat & rough
- Microscale: bumpy & specular

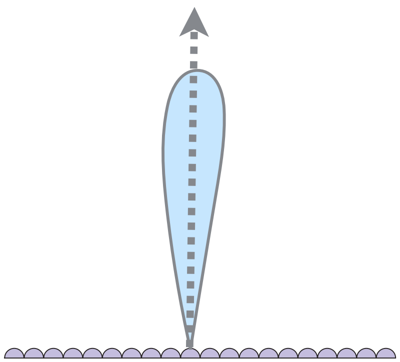
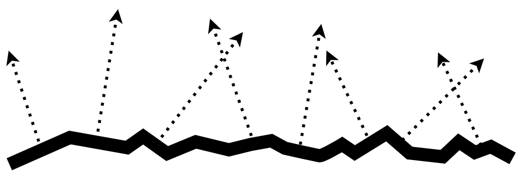
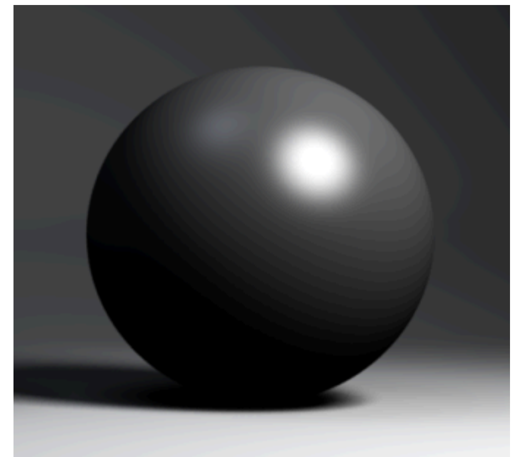
Individual elements of surface act like mirrors

- Known as "microfacets"
- Each microfacet has its own normal vector

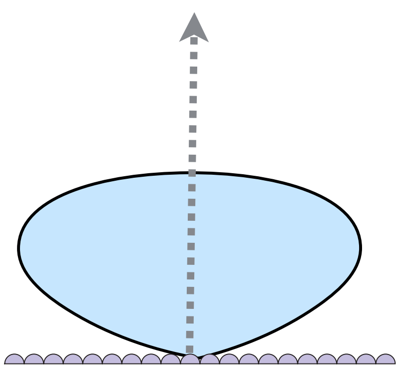
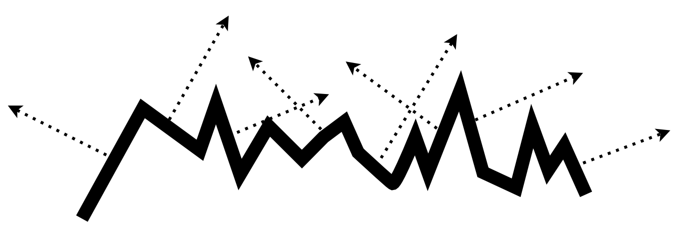
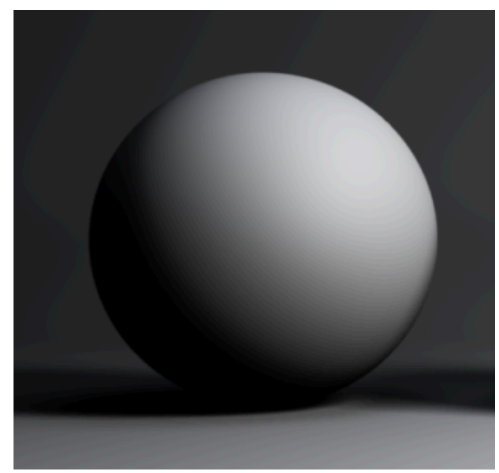


Microfacet BRDF

- Key: the distribution of microfacets' normals
    - Concentrated <==> glossy

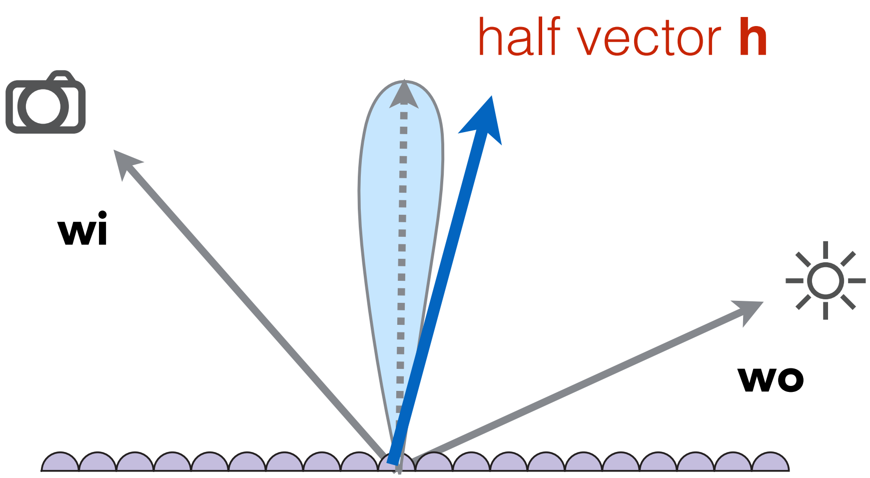




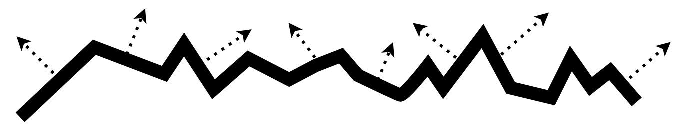
    - Spread out <==> diffuse

Microfacet BRDF

- **What kind of microfacets reflect wi to wo? (hint: microfacets are mirrors)**





$$f(\mathbf{i},\mathbf{o}) = \frac{\overset{\text{Fresnel term}}{\boxed{\mathbf{F}(\mathbf{i},\mathbf{h})}}\;\overset{\text{shadowing-masking term}}{\boxed{\mathbf{G}(\mathbf{i},\mathbf{o},\mathbf{h})}}\;\overset{\textbf{distribution of normals}}{\boxed{\mathbf{D}(\mathbf{h})}}}{4(\mathbf{n},\mathbf{i})(\mathbf{n},\mathbf{o})}$$

Microfacet BRDF: Examples

[Autodesk Fusion 360]

Anisotropic BRDFs

Isotropic vs Anisotropic Reflection

- So far, Point light + Metal = Round / Elliptical highlight
- But some reflection highlights look very different

Isotropic vs Anisotropic Reflection

Isotropic

Anisotropic

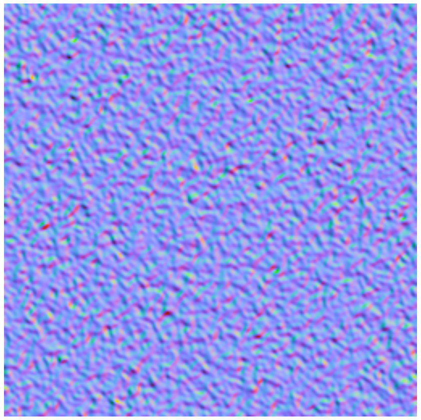
Anisotropic BRDF: Brushed Metal

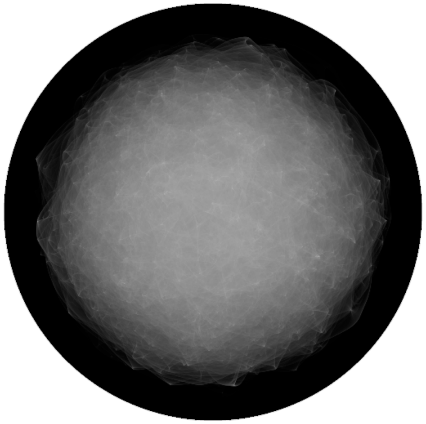
- How is the pan brushed?

Isotropic / Anisotropic Materials (BRDFs)

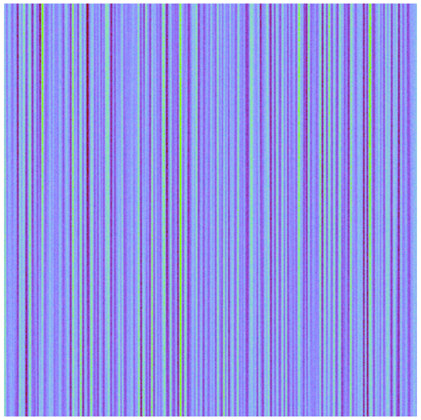
- Key: directionality of underlying surface

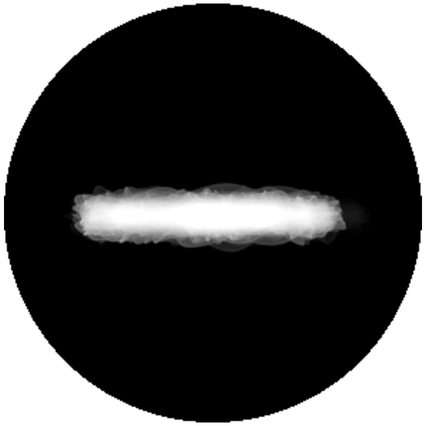
Isotropic





Anisotropic





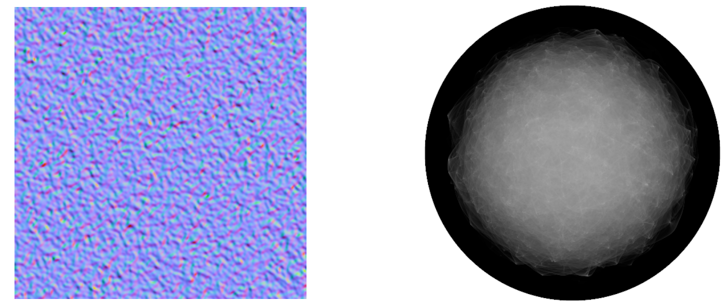
Surface (normals)

BRDF (fix wi, vary wo)

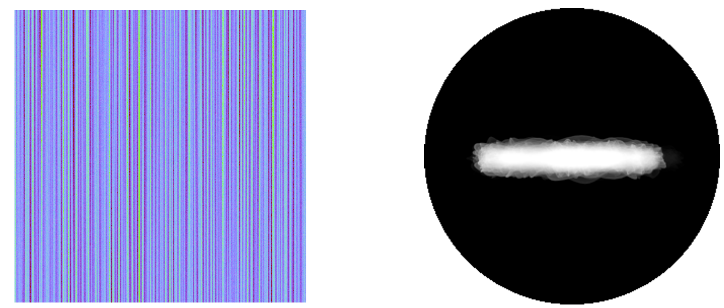
Isotropic / Anisotropic Materials (BRDFs)

- Imagine the surface is made up of tiny mirror-like facets (each facet is a perfect specular reflector).
- Isotropic Microfacet Surface: The micro-mirrors (facets) are randomly oriented in all directions around the surface normal.
- Anisotropic Microfacet Surface: The micro-mirrors are not oriented randomly — their normals tend to align more along certain tangent directions.

Isotropic



Anisotropic



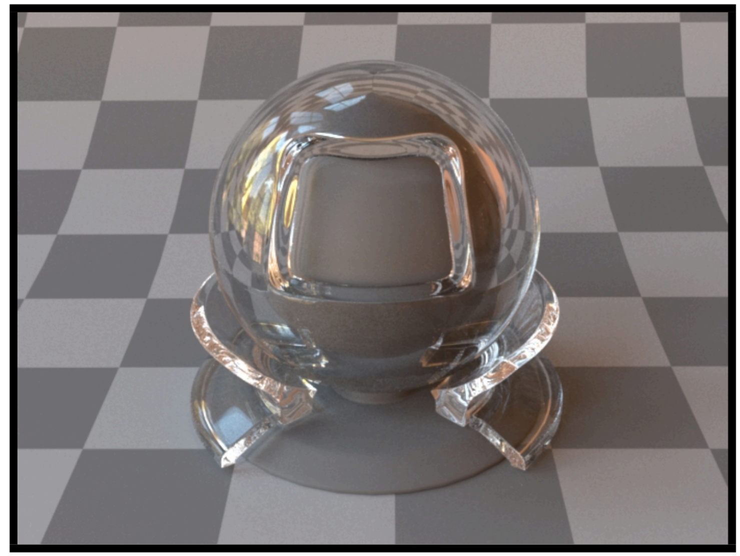
Depending on how the microfacet normal distribution function (NDF) is defined.

$$f(\mathbf{i},\mathbf{o}) = \frac{\boxed{\mathbf{F}(\mathbf{i},\mathbf{h})}\boxed{\mathbf{G}(\mathbf{i},\mathbf{o},\mathbf{h})}\boxed{\mathbf{D}(\mathbf{h})}}{4(\mathbf{n},\mathbf{i})(\mathbf{n},\mathbf{o})}$$

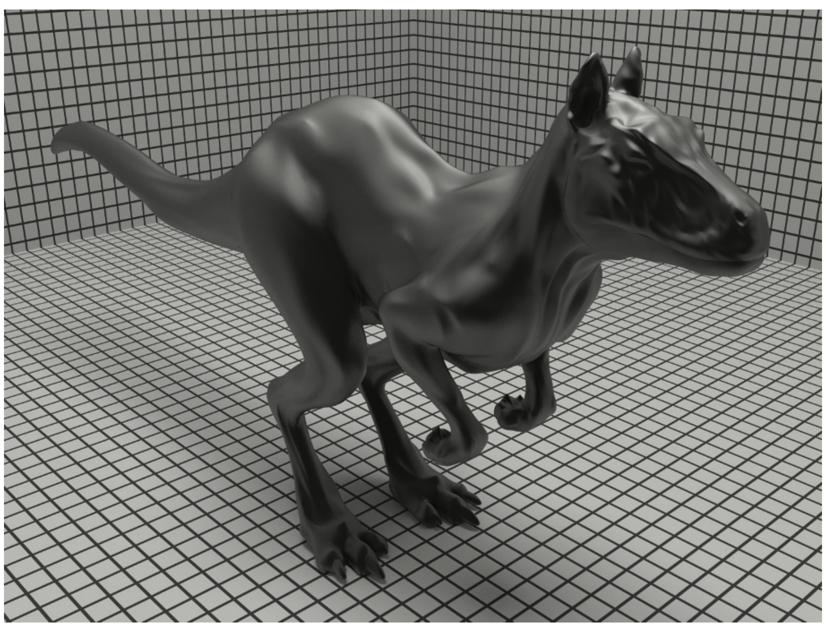
Isotropic BRDFs

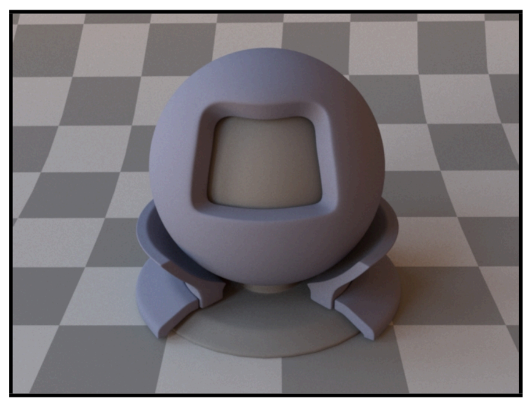
Reflection independent of azimuthal angle ϕ

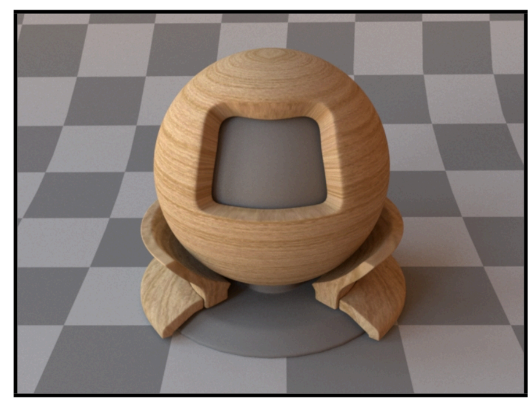
$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$



Results from surface microstructure that lacks directional structure





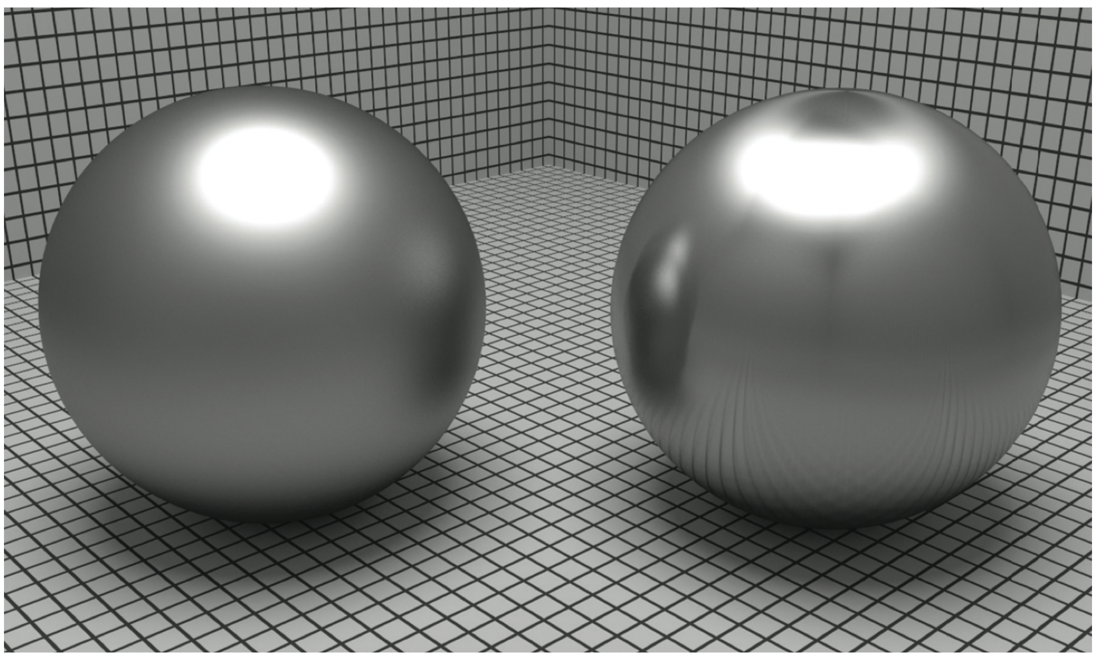


Anisotropic BRDFs

Reflection depends on azimuthal angle ϕ

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) \neq f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$

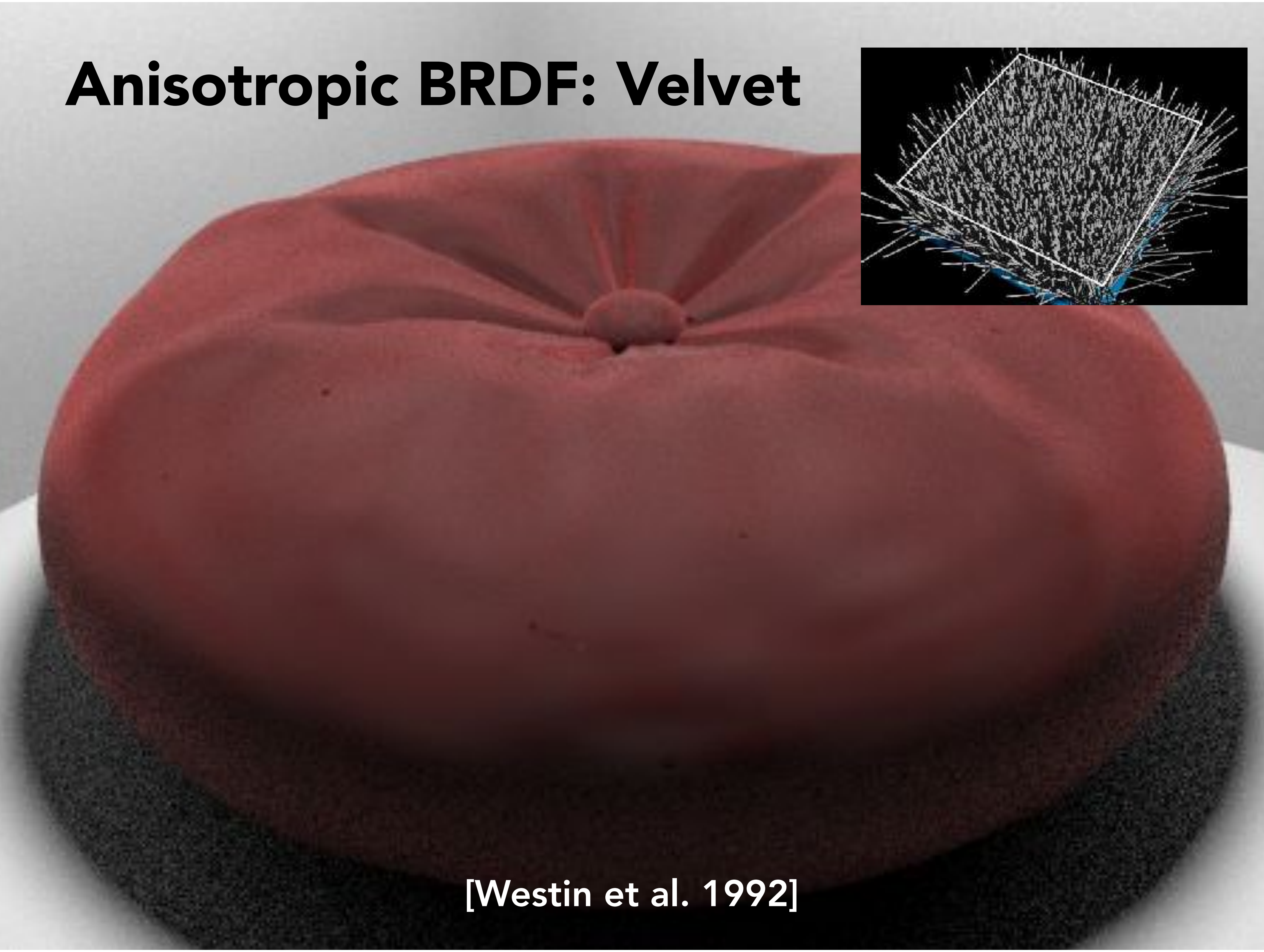
Results from oriented microstructure of surface, e.g., brushed metal



Anisotropic BRDF: Nylon

[Westin et al. 1992]

Anisotropic BRDF: Velvet



[Westin et al. 1992]

Translucent materials: Jade

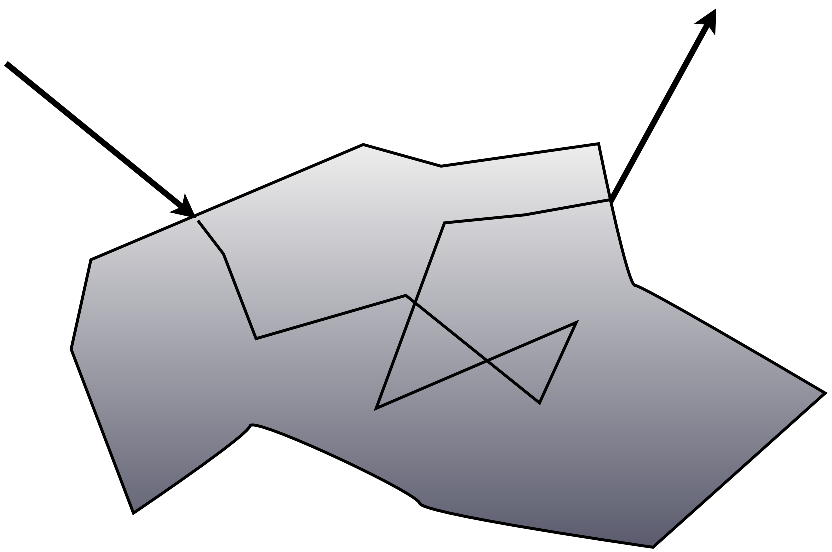
Translucent materials: Skin

Translucent materials: Leaves

Subsurface Scattering

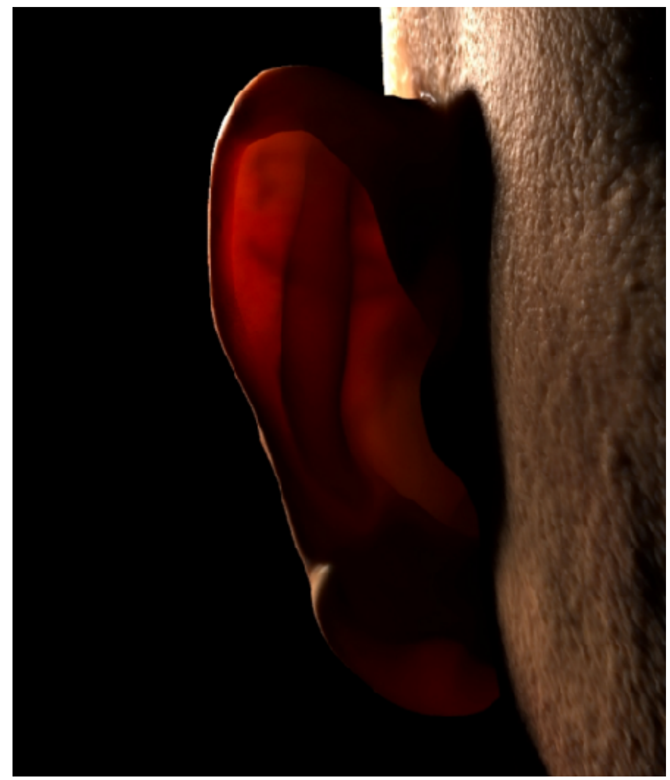
次表面

- Visual characteristics of many surfaces caused by light entering at different points than it exits
  - Violates a fundamental assumption of the BRDF
  - Need to generalize scattering model (BSSRDF)



The light does not bounce off immediately — it goes inside the object, bounces around between particles, and then exits somewhere else.

[Jensen et al 2001]



[Donner et al 2008]

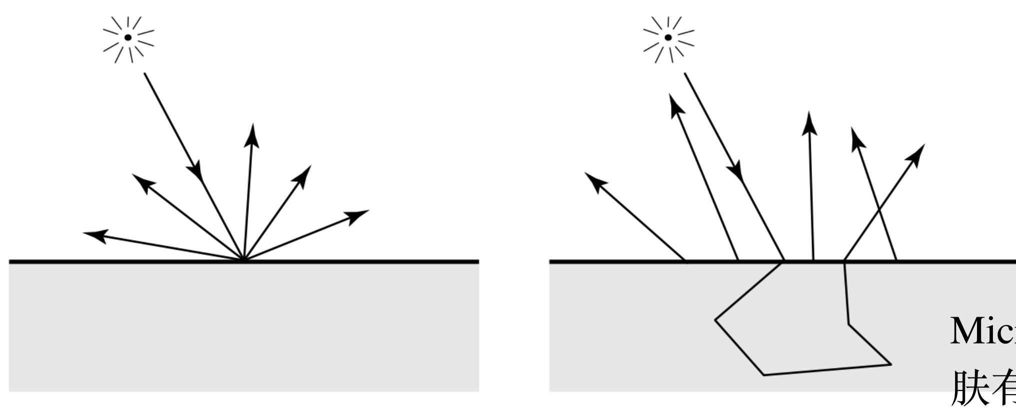
Scattering Functions

- Generalization of BRDF; describe exitant radiance at one point due to incident differential irradiance at another point:

$$S(x_i, \omega_i, x_o, \omega_o)$$

- Generalization of reflection equation integrates over all points on the surface and all directions!

$$L(x_o, \omega_o) = \int_A \int_{H^2} S(x_i, \omega_i, x_o, \omega_o)\, L_i(x_i, \omega_i) \cos\theta_i \,\mathrm{d}\omega_i \,\mathrm{d}A$$



BRDF　　BSSRDF

Microstructure几何结构：如人类皮肤有多层结构（角质层、表皮层、真皮层），含有色素细胞与血管

BRDF

BSSRDF

BRDF vs BSSRDF

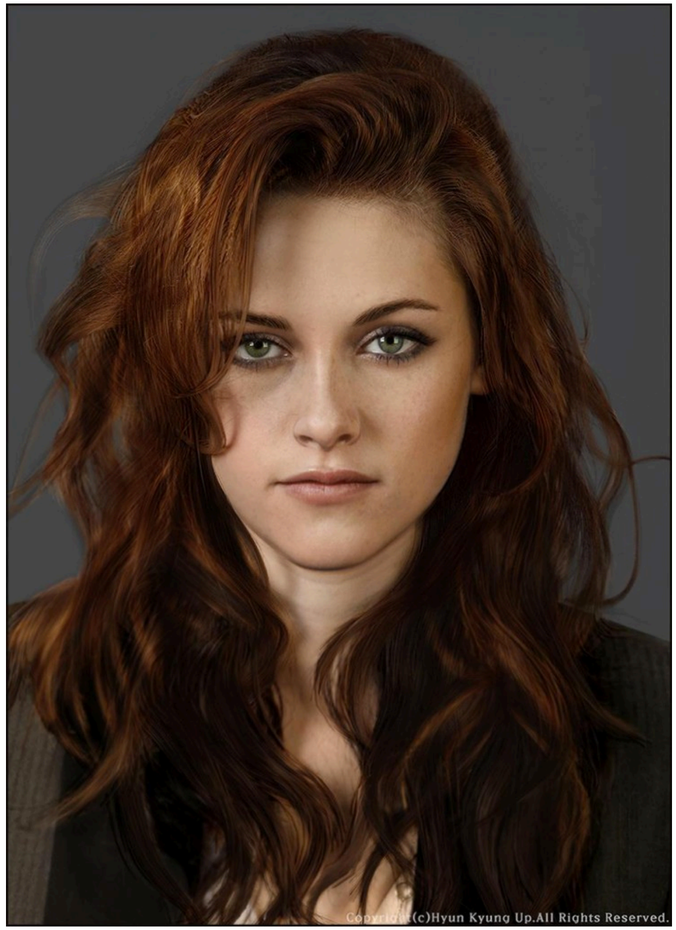
BRDF

BSSRDF

[Jensen et al. 2001]

BSSRDF: Application

[Artist: Teruyuki and Yuka]



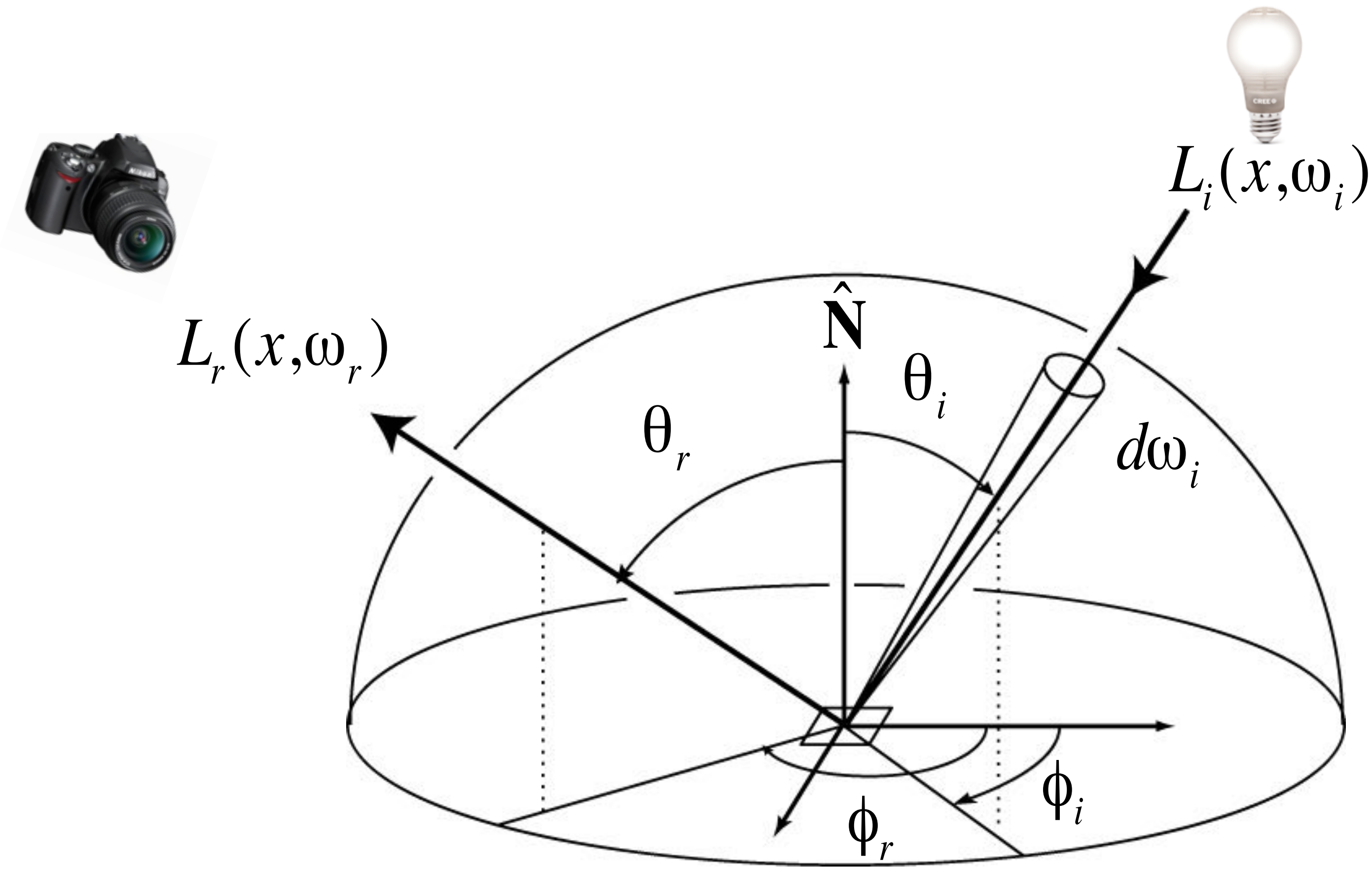
[Artist: Hyun Kyung]

[Artist: Dan Roarty]

https://cgelves.com/10-most-realistic-human-3d-models-that-will-wow-you/

So Far...

Get 光的传播规律、量化光辐射能量、光传输方程、反射、折射方程（既材质模型）...



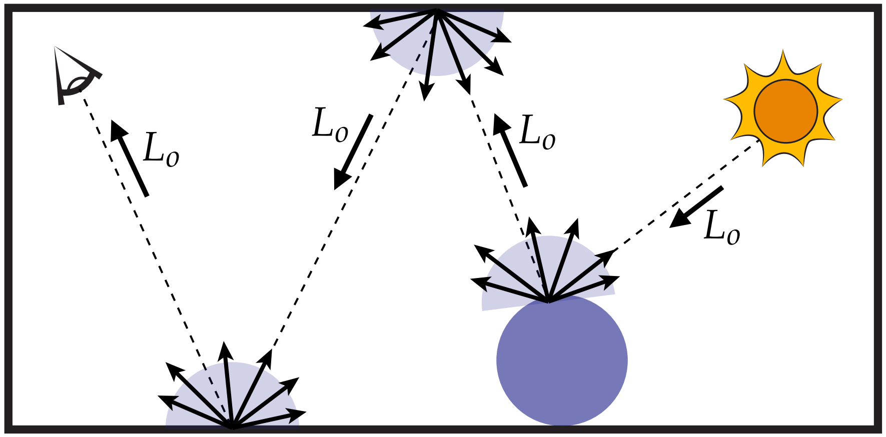
$$\mathrm{d}L_r(\omega_r) = f_r(\omega_i \to \omega_r)\,\mathrm{d}L_i(\omega_i)\,\cos\theta_i$$

$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \to \omega_r)\,L_i(\mathrm{p},\omega_i)\,\cos\theta_i\,\mathrm{d}\omega_i$$

Next time

计算像素明暗，既光传输方程的数值求解和算法实现

- Ray-surface intersection (light geometry)



光线追踪

- Monte Carlo Integration (light radiance)

$$\int_{\Omega} f(p)\ dp \quad \approx \quad \mathrm{vol}(\Omega) \frac{1}{N} \sum_{i=1}^{N} f(X_i)$$

蒙托卡洛积分