# CS 545 - Homework 5

Written Portion – Daniel Miller

## PROBLEM 3

### Convert **RGB** (0, 184, 160) to **HSV**

We begin by calculating the Saturation ($S_{HSV}$) as follows:

$$S_{HSV} = \begin{cases} \dfrac{C_{rng}}{C_{high}} & for\ C_{high} > 0 \\ 0 & otherwise \end{cases}$$

$$C_{max} = 255 \quad C_{high} = \max(R, G, B) \quad C_{low} = \min(R, G, B) \quad C_{rng} = C_{high} - C_{low}$$

$$\therefore \quad C_{high} = 184 \qquad C_{low} = 0 \qquad C_{rng} = 184 - 0 = 184$$

$$\boxed{S_{HSV} = \frac{C_{rng}}{C_{high}} = \frac{184}{184} = 1.000}$$

Next, the Luminance/Value is calculated using the terms above:

$$\boxed{V_{HSV} = \frac{C_{high}}{C_{max}} = \frac{184}{255} \cong 0.722}$$

Finally, the Hue is calculated as shown here:

$$R' = \frac{C_{high} - R}{C_{rng}} = \frac{184 - 0}{184} = 1.000$$

$$G' = \frac{C_{high} - G}{C_{rng}} = \frac{184 - 184}{184} = 0.000$$

$$B' = \frac{C_{high} - B}{C_{rng}} = \frac{184 - 160}{184} = \frac{24}{184} = 0.1304$$

$$H' = \begin{cases} B' - G' & \text{if } R = C_{high} \\ R' - B' + 2 & \text{if } G = C_{high} \\ G' - R' + 4 & \text{if } B = C_{high} \end{cases} \rightarrow 1.000 - 0.1304 + 2 = 2.8696$$

$$\boxed{H_{HSV} = \frac{1}{6} \cdot \begin{cases} H' + 6 & \text{for } H' < 0 \\ H' & \text{otherwise.} \end{cases} \rightarrow \frac{1}{6} \cdot 2.8696 \cong 0.4783}$$

Which yields the final answer:

$$\text{RGB}(0, 184, 160) = \boxed{\text{HSV}(0.4783, 1.0, 0.722)}$$

## PROBLEM 4

### Convert **HSV** (0.67, 0.7, 0.7) to **RGB**

The conversion from HSV to RGB is calculated as follows:

$$H' = (6 \cdot H_{HSV}) \bmod 6 = 4.02$$

$$c_1 = \lfloor H' \rfloor = 4 \qquad x = (1 - S_{HSV}) \cdot V_{HSV} = 0.21$$
$$c_2 = H' - c_1 = 0.02 \qquad y = (1 - (S_{HSV} \cdot c_2)) \cdot V_{HSV} = 0.6902$$
$$z = (1 - (S_{HSV} \cdot (1 - c_2))) \cdot V_{HSV} = 0.2198$$

$$(R', G', B') = \begin{cases} (V_{HSV}, z, x) & \text{if } c_1 = 0 \\ (y, V_{HSV}, x) & \text{if } c_1 = 1 \\ (x, V_{HSV}, z) & \text{if } c_1 = 2 \\ (x, y, V_{HSV}) & \text{if } c_1 = 3 \\ (z, x, V_{HSV}) & \text{if } c_1 = 4 \\ (V_{HSV}, x, y) & \text{if } c_1 = 5 \end{cases} = (0.2198, 0.21, 0.7)$$

The (R’, G’, B’) tuple is then normalized to the range [0, 255]:

$$R = \min(\text{round}(255 \cdot R'), 255) = 56$$
$$G = \min(\text{round}(255 \cdot G'), 255) = 54$$
$$B = \min(\text{round}(255 \cdot B'), 255) = 178$$

Which yields the final answer:

$$\text{HSV}(0.67, 0.7, 0.7) = \boxed{\text{RGB}(56, 54, 178)}$$

## PROBLEM 5

### Convert **RGB** (124, 220, 0) to **YIQ**

The R, G, B components are first normalized to the [0, 1] range:

$$R' = \frac{124}{255} = 0.4863 \qquad G' = \frac{220}{255} = 0.8627 \qquad B' = \frac{0}{255} = 0.0$$

The Luminance (Y) component is then computed, which is shared with the YUV color space.

$$Y = 0.299 * R' \;+\; 0.587 * G' \;+\; 0.114 * B'$$

$$Y = 0.6518$$

$$U = 0.492 \cdot (B' - Y) = -0.3207 \qquad V = 0.877 \cdot (R' - Y) = -0.1452$$

$$\begin{pmatrix} I \\ Q \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \cdot \begin{pmatrix} U \\ V \end{pmatrix}$$

$$\begin{pmatrix} I \\ Q \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0.8387 & 0.5446 \\ -0.5446 & 0.8387 \end{pmatrix} \cdot \begin{pmatrix} -0.3207 \\ -0.1452 \end{pmatrix} = \begin{pmatrix} 0.0526 \\ -0.3481 \end{pmatrix}$$

Which yields the final answer:

$$\mathrm{RGB}(124, 220, 0) = \boxed{\mathrm{YIQ}(0.6518, 0.0526, -0.3481)}$$

PROBLEM 6

## Convert **YIQ** (1.0, 0.3, 0.3) to **RGB**

The matrix form of the previous conversion (RGB to YIQ) is shown here:

$$\begin{pmatrix} Y \\ I \\ Q \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{pmatrix} \cdot \begin{pmatrix} R' \\ G' \\ B' \end{pmatrix}$$

The inverse transformation can be calculated using simple matrix math, as shown here:

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{pmatrix}^{-1} \cdot \begin{pmatrix} Y \\ I \\ Q \end{pmatrix}$$

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{pmatrix} 1.000 & 0.9557 & 0.6199 \\ 1.000 & -0.2716 & -0.6499 \\ 1.000 & -1.1082 & 1.7051 \end{pmatrix} \cdot \begin{pmatrix} 1.0 \\ 0.3 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 1.4727 \\ 0.7245 \\ 1.1791 \end{pmatrix}$$

The results are then scaled down to the [0, 255] range, as shown in the last step of Problem 4. The results show that this conversion has saturated the red and green channels, which are clamped to a maximum value of 255. This yields the final results as:

$$\mathrm{YIQ}(1.0, 0.3, 0.3) = \boxed{\mathrm{RGB}(255, 185, 255)}$$