Graduate Course INTR-6000P
Week 7 - Lecture 14

Changhao Chen
Assistant Professor
HKUST (GZ)



Definition: An internal model describing the structure, semantics, and dynamics of the environment.

Represents geometry, topology, and meaning.

Must support localization, obstacle avoidance, and decision-making.

Desired Properties:

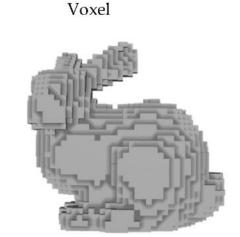
- Accuracy: Faithful geometric reconstruction
- Compactness: Efficient memory use
- Consistency: Coherent over time
- Interpretability: Supports semantic reasoning
- •Real-time update: Suitable for dynamic scenes

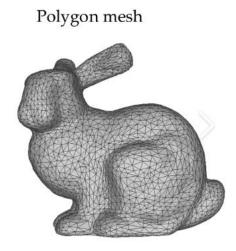
Geometric representations describe the **spatial structure** of the scene.

Common forms:

- 1.Point clouds
- 2.Meshes
- 3. Voxel grids
- 4. Signed distance fields (SDFs)
- 5. Neural Scene Representations

Point cloud





Point Clouds

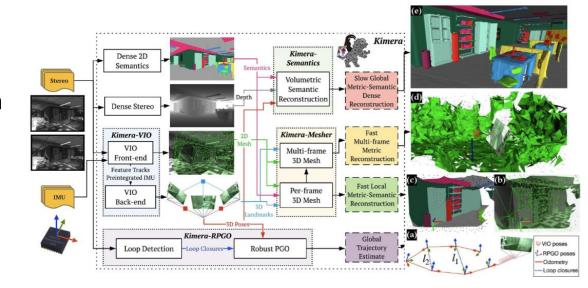
- Collection of 3D points sampled from scene surfaces.
- Directly obtained from LiDAR or stereo triangulation.
- Pros: Simple, flexible, accurate.
- •Cons: Unstructured, memory-intensive, lacks topology.

Mesh-Based Representations

- •Define surfaces using **vertices**, **edges**, and **faces**.
- •Generated from point clouds via **Delaunay triangulation** or **Poisson reconstruction**.
- Pros: Compact, visually interpretable.
- •Cons: Difficult to update dynamically; limited scalability.

Volumetric Representations

- •Environment represented as voxel grid or **Signed Distance Field (SDF)**.
- Used in KinectFusion, ElasticFusion, TSDF-SLAM.
- •Each voxel stores occupancy or distance to nearest surface.
- Enables continuous surface extraction via zero-crossing.



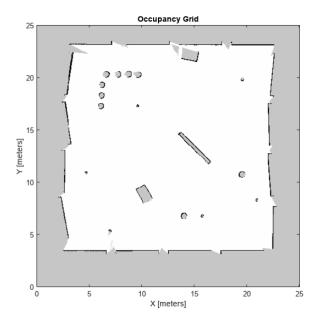
The Mapping Problem

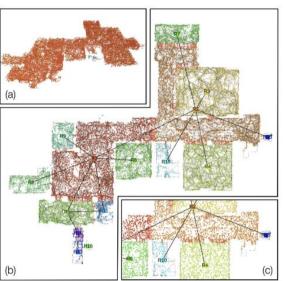
Occupancy Grids

- •Divide space into discrete cells; each cell stores occupancy probability.
- •Key in 2D/3D mapping and path planning.
- •Update rule: Bayesian fusion of sensor measurements.
- •Used in autonomous driving for free-space estimation.

Topological Maps

- •Represent scene as a **graph** of nodes (places) and edges (connectivity).
- •Compact representation suitable for large-scale navigation.
- •Example: Pose graph in SLAM, lane graph in HD maps.



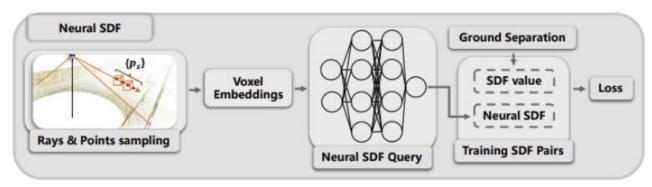


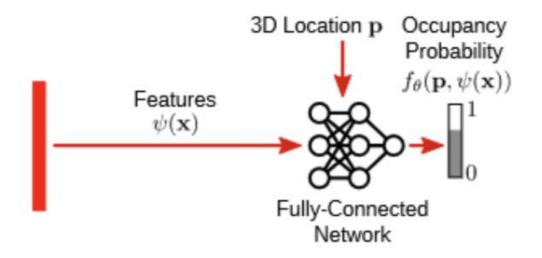
Neural Implicit Representations

Encode 3D scenes as continuous neural fields. Implicit function

$$f_{ heta}(\mathbf{x})
ightarrow \,$$
 color, density, occupancy.

Smooth, memory-efficient, and differentiable.









NeRF: Represent scenes as continuous radiance and density fields.

Learn by minimizing photometric loss across views.

Enable high-fidelity rendering from new viewpoints.

$$F_{ heta}: (\mathbf{x}, \mathbf{d}) o (\mathbf{c}, \sigma)$$

where

- x: 3D location
- d: viewing direction
- c: RGB color
- σ: volume density

1. Volume Rendering Principle

Each pixel's color is computed by integrating the emitted radiance and transmittance along the camera ray:

$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t)\, \sigma(\mathbf{r}(t))\, c(\mathbf{r}(t),\mathbf{d})\, dt$$

2. Training Objective

•Rendered color is compared with ground truth image pixel color

•Loss:

$$T(t) = \exp\left(-\int_{t_n}^t \sigma(\mathbf{r}(s)) ds
ight)$$

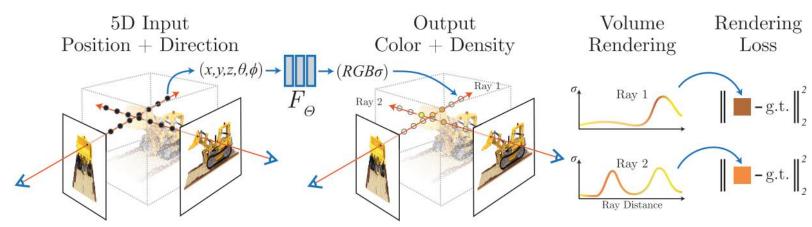
$$\mathcal{L} = \sum_{\mathbf{r}} \|C(\mathbf{r}) - C^*(\mathbf{r})\|^2$$

3. Optimization

•Train MLP parameters θ using multiple images with known camera poses.

•The network learns to encode scene geometry (via density) and

appearance (via color).



Extensions:

NeRF-SLAM (2022): Jointly optimizes camera poses and radiance field \rightarrow implicit dense mapping.

iMAP, NICE-SLAM: Real-time incremental NeRF-based SLAM systems.

NeRF++ / Instant-NGP: Improve rendering efficiency with spatial decomposition and hash encoding.

Dynamic NeRF (D-NeRF): Models temporal variations for moving scenes.





Mapping and Tracking Framework

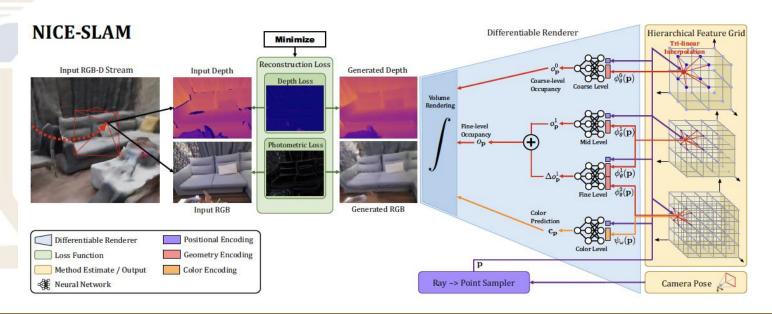
The SLAM system alternates between **mapping** and **tracking**:

Mapping Phase

- Uses the current RGB and depth images as ground truth.
- Predicts RGB and depth from the current feature grids via rendering.
- Constructs **photometric** and **geometric loss functions** between predicted and measured data.
- Jointly optimizes camera poses and feature grid values.

Tracking Phase

- Given the current feature grids and an estimated pose, renders predicted RGB and depth.
- Compares them with the actual camera observations.
- Optimizes camera pose by minimizing rendering errors.



香港科技大学(广州)
THE HONG KONG
UNIVERSITY OF SCIENCE
AND TECHNOLOGY (GUANGZHOU)

系统枢纽 SYSTEMS HUB 智能交通 INTR INTELLIGENT TRANSPORTATION





