ST451 Bayesian Machine Learning
Week 9

Exercises

1. Consider the stochastic mean model. Specifically, let

$$x_t = x_{t-1} + \epsilon_t, \;\; t = 1, \dots, n, \;\; x_0 = 0,$$

where ϵ_t's are independent $\mathrm{N}(0, \sigma^2)$ random variables, and assume that we observe y_t according to the model below

$$y_t = x_t + \mu + \delta_t, \;\; t = 1, \dots, n,$$

where δ_t's are independent $\mathrm{N}(0, \omega^2)$ random variables. In other words the mean of y_t is a random walk process. Show that this is a linear Gaussian state space model by writing down the state and space equations and identifying the matrices required for the Kalman filter equations.

Answer: The Linear Gaussian State Space Models are defined as

$$\begin{aligned} z_t &= A\, z_{t-1} + B\, u_t + \epsilon_t, \quad \epsilon_t \sim N(0, Q) \\ x_t &= C\, z_t + D\, u_t + \delta_t, \quad \delta_t \sim N(0, R). \end{aligned}$$

To confirm that this models belongs to the above category we can set $z_t = x_t$, $x_t = y_t$, $A = 1$, $B = 0$, $C = 1$, $D = \mu$, $u_t = 1$, $Q = \sigma^2$, $R = \omega^2$.

2. Simulate data 100 points from the model of Exercise 1, setting $\mu = \sigma^2 = \omega^2 = 1$. Consider σ^2 and ω^2 known and find the MLE of μ by evaluating the likelihood at a grid of points between 0 and 2.