ST451 Bayesian Machine Learning
Week 9

Exercises

1. Consider the stochastic mean model. Specifically, let

$$x_t = x_{t-1} + \epsilon_t, \quad t = 1, \dots, n, \quad x_0 = 0,$$

   where ϵ_t's are independent $\mathrm{N}(0, \sigma^2)$ random variables, and assume that we observe y_t according to the model below

$$y_t = x_t + \mu + \delta_t, \quad t = 1, \dots, n,$$

   where δ_t's are independent $\mathrm{N}(0, \omega^2)$ random variables. In other words the mean of y_t is a random walk process. Show that this is a linear Gaussian state space model by writing down the state and space equations and identifying the matrices required for the Kalman filter equations.

2. Simulate data 100 points from the model of Exercise 1, setting $\mu = \sigma^2 = \omega^2 = 1$. Consider σ^2 and ω^2 known and find the MLE of μ by evaluating the likelihood at a grid of points between 0 and 2.