ST451 Bayesian Machine Learning
Week 8

Exercises

1. Verify that each step of a Gibbs Sampler can be viewed as a Metropolis Hastings algorithm with acceptance probability of 1. For each step you can consider the proposal that draws θ_i from $\pi(\theta_i|y, \theta_{-1})$ and keeps θ_{-i} to its current value, hence $q(\theta^{(*)}|\theta^{(t)}) = \pi(\theta_i^{(*)}|\theta_{-i}, y)$.

   Note: The notation in this exercise may be complicated. Feel free to skip it and move to the next one, the main purpose is to complement the slides.

2. Let $y = (y_1, \ldots, y_n)$ be a r.s. from a $\mathrm{N}(\mu, v)$ where v is distributed according to an $\mathrm{IGamma}(\frac{\nu}{2}, \frac{\nu}{2}\sigma^2)$. The parameter ν is assumed to be known. Finalise the model with an improper prior on μ, σ^2, $\pi(\mu, \sigma^2) \propto 1$.

   (a) Write down the posterior up to proportionality.

   (b) Specify the details needed to construct a Gibbs sampler to draw from the posterior of μ, v and σ^2.

   (c) Generate 200 numbers from the model above with $\mu = 0$, $\sigma^2 = 1$, $v = 1$ and $\nu = 20$, and set these numbers as $y = (y_1, \ldots, y_{200})$. Write a Python script to run $10,000$ iterations of the Gibbs sampler derived in the previous part based on the data you generated.

   (d) Provide posterior summaries and traceplots for μ, v and σ^2.

   (e) Repeat with PyMC3 and compare the results with the previous part.