

# ST451 Bayesian Machine Learning

## Week 6

### Exercises

1. Consider a Naive Bayes classifier with binary  $y$  and a binary feature  $x$ . In other words assume that  $y, x|y = 0, x|y = 1$  are Bernoulli random variables with parameters  $\theta = (\theta_y, \theta_0, \theta_1)$  respectively. Assume that the data consist of  $n$  points  $D = (y_i, x_i)_{i=1}^n$ .
  - (a) Find the maximum likelihood estimates of  $\theta_y, \theta_0, \theta_1$  based on  $D$ .
  - (b) Fix  $\theta_y$  to its  $\hat{\theta}_y$  and assign independent  $\text{Beta}(\alpha, \alpha)$  priors to  $\theta_0$  and  $\theta_1$ . Derive the posterior of  $\theta_0$  and  $\theta_1$  and use the posterior mean as Bayes estimator of  $\theta_0, \theta_1$ . Compare with the MLE of  $\theta_0, \theta_1$ .
2. Repeat the image processing example with an image of your choice. Find a black and white bmp image distort with noise and see if you can restore it using variational inference.
3. **Optional:** In the text classification exercise of the computer workshop, explore whether the predictive performance of the naive Bayes classifier can be improved further by using the NLTK library (<https://www.nltk.org/>) to perform tasks such as lemmatising words.