ST451 Bayesian Machine Learning
Week 5

Exercises

1. Consider the model where we have independent observations $y = (y_1, \ldots, y_n)$ from the $\mathrm{N}(\mu, \sigma^2)$ with the $\mathrm{N}(\mu_0, \omega^2\sigma^2)$ and $\mathrm{IGamma}(\alpha_0, \beta_0)$ being the priors for μ and σ^2 respectively. The aim is to perform variational inference for this model under the mean field approximation framework. Derive the explicit algorithm for this problem and present its steps.

2. Following the previous exercise, simulate data from this model, using parameters of your preference, and apply the algorithm for variational inference for the parameters you used. Provide evidence of agreement between the estimated and true values of the parameters used.

3. Consider the model for the VIX index mentioned in the lecture slides

   $$Y_t = Y_{t-1} + \kappa(\mu - Y_{t-1})\delta + \epsilon_t,$$

   where Y_t is VIX at time t, and ϵ_t are independent error terms following the student-t distribution with scale σ and 3 degrees of freedom. Download the VIX series (file vix_201518.csv) and fit this model using automatic Variational Bayes via RStan.

   Hint: In RStan the code $x \sim$ student_t(df,m,s) indicates that x follows the student-t distribution with df degrees of freedom, location mu and scale s.