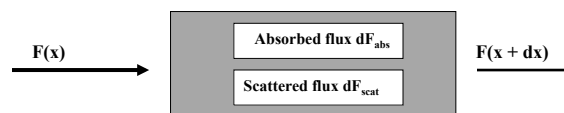


Fig. 3.2 The normalized blackbody emission spectra for the sun (6000 K) and Earth (255 K) as a function of wavelength (top). The fraction of radiation absorbed while passing from the surface to the top of the atmosphere as a function of wavelength (middle). The fraction of radiation absorbed from the tropopause to the top of the atmosphere as a function of wavelength (bottom). The atmospheric molecules contributing the important absorption features at each frequency are indicated. [Taken from Goody and Yung (1989). Reprinted with permission from Oxford University Press.]

The Beer-Lambert Law: Optical Depth

Consider the following process:



The absorption or scattering of radiation by an optically active medium (such as the atmosphere) is measured by the optical depth, τ , of the medium. For this generalized thin slab above $[x, x+dx]$, radiation can be transmitted ($F(x+dx)$), absorbed (dF_{abs}) or scattered (dF_{scat}).

$$F(x+dx) = F(x) - dF_{abs} - dF_{scat}$$

For this optically thin slab ($F(x) - F(x+dx) \approx 0$ for all wavelengths), the absorption (dF_{abs}) and scattering (dF_{scat}) will be proportional to $F(x)$, dx , and the number of absorbers or scatterers. The efficiency of a gas or particle in absorbing or scattering radiation is expressed as a "cross section":

$$dF_{abs} = n \sigma_{abs} F(x) dx,$$

$$dF_{scat} = n \sigma_{scat} F(x) dx.$$

σ_{abs} and σ_{scat} are the cross sections and typically have units of $\text{cm}^2 \text{ molecule}^{-1}$, hence the "cross section" terminology.

Beer-Lambert (cont.)

Substituting into the flux balance equation:

$$dF = F(x + dx) - F(x) = -n (\sigma_{\text{abs}} + \sigma_{\text{scat}}) F(x) dx.$$

Integrating by separation of variables we obtain the Beer-Lambert extinction equation:

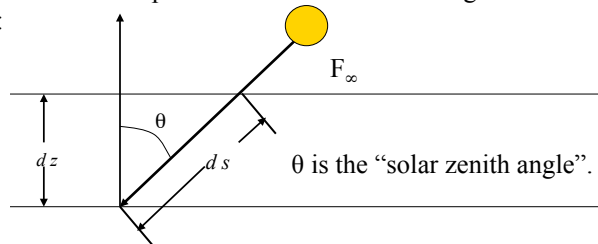
$$F(L) = F(0) \exp[-n (\sigma_{\text{abs}} + \sigma_{\text{scat}}) L] \text{ where } L \text{ is the pathlength.}$$

Spectroscopists and chemists typically use σ ($\text{cm}^2 \text{ molecule}^{-1}$) for cross sections. Most radiative transfer texts (including Hartmann) use the k formalism where k is the mass-weighted cross section ($\text{m}^2 \text{ kg}^{-1}$) and rather than number density, n , we need mass density ρ (kg/m^3). With these substitutions we have:

$$dF = -\rho_a k_{\text{abs}} F dx.$$

where ρ_a is the mass density of a (units $\text{kg a} / \text{m}^3 \text{ air}$) and k_{abs} is the absorption coefficient ($\text{m}^2 \text{ kg}^{-1}$).

Now we move to the atmosphere. Consider the following illumination geometry:



For this geometry (plane parallel atmosphere), $dz = -\cos(\theta) ds$, and solving we have:

$$\cos(\theta) dF/dz = -\rho k_{\text{abs}} F$$

We define the **optical depth** along a vertical path, $\tau = \int_z^\infty \rho k_{\text{abs}} dz$, which implies the $d\tau = -\rho k_{\text{abs}} dz$, so we can write:

$$\cos(\theta) dF/d\tau = -F$$

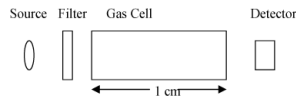
solving:

$$F = F_\infty \exp(-\tau/\cos(\theta))$$

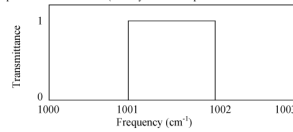
Remember τ here is defined as the vertical optical depth. F_∞ is the downward flux density at the top of the atmosphere. The incident flux thus decays exponentially along the slant path ds where the optical depth is given by $\tau/\cos(\theta)$.

Non Gray Absorbers

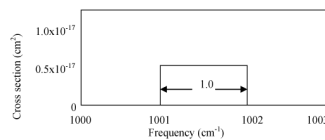
So far, we have considered in this section a gray absorber, one whose cross sections (σ_{abs} or k) do not have spectral structure. In the ticket we posed the following experiment:



The filter has the following spectral characteristics: (0 everywhere except 1 between 1001 and 1002 cm^{-1})



A gas that has the following absorption spectrum is added to the cell:

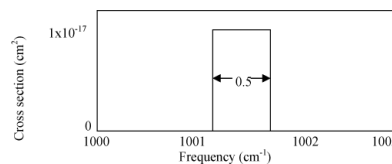


This is simply a gray atmosphere for the spectral region covered in the experiment. The flux at the detector will be:

$$F(s+ds) = F(s)\exp[-n(\sigma_{\text{abs}})L] = F(s)\exp[-\tau]$$

$$\text{for this case, } \tau = n(\sigma_{\text{abs}})L$$

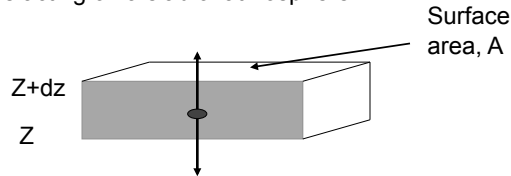
Now, consider a molecule with a narrower lineshape:



We can see instantly that the transmission will be quite different. In for $\tau = \infty$ at 1001.5 cm^{-1} , only $1/2$ of the light is absorbed and the *apparent* optical depth is only 0.7 . ($F/F_0 = 0.5$). This is what makes radiative transfer fun (or not fun depending on your view). Beer-Lambert Law only holds when we perfectly resolve the (often rapidly changing) structure of absorption in the atmosphere. Thus the importance of absorption lineshapes.

Recall - The Hydrostatic Balance

Consider the following forces acting on a slab of atmosphere:



At equilibrium (or hydrostatic balance) the weight (acting downward) must be balanced by the pressure gradient force:

$$\rho g A dz = A \times (P(z) - P(z+dz))$$

rearranging:

$$dP/dz = -\rho g$$

From the ideal gas law,

$$\rho = P M_a / RT$$

Substituting yields:

$$dP/P = - (M_a g / RT) dz$$

For isothermal atmospheres (and this is true on Earth to ~20%), we can integrate:

$$P_z = P_o \exp(- [(M_a g)/RT]z) = P_o \exp(- z / H)$$

where H, the scale height, is about 7 1/2 km.

Absorption Rate.

For an isothermal atmosphere in hydrostatic balance, the density of a gas with constant mass mixing ratio is given by:

$$\rho_a = \rho_{as} e^{-z/H} \text{ where } H = RT/g \text{ is the scale height.}$$

R in mass units is 287 J K⁻¹ kg⁻¹, g = 10 m s⁻², and ρ_{as} is the density of the absorber at the surface.

If we assume k_{abs} is constant (when is this the case?), substituting into our expression for τ , and integrating we have:

$$\tau = p_s/g M_a k_{abs} \exp[-z/H]$$

where M_a is the mass mixing ratio of a (kg_a/kg_{air}). The total optical depth is $(p_s/g) M_a k_{abs}$. The derivative of τ with respect to z is:

$$d\tau / dz = -\tau / H.$$

To calculate the energy absorbed (per unit volume), we multiply $F_z \times \rho_a \times k_{abs}$. Defining the slant column $\mu = \cos \theta$, we have:

$$\begin{aligned} \text{Absorption Rate} &= dF/dz = k_{abs} \rho_a F_z / \mu = -d\tau / dz \times F_z / \mu \\ &= F_\infty / \mu \times \exp(-\tau/\mu) \times \tau / H. \end{aligned}$$

We can see that the peak in absorption will occur where the product of the flux and the absorption mass cross sections peak.

To see where this is, we differentiate and find where the derivative is 0:

$$d [F_\infty / \mu \times \exp(-\tau/\mu) \times \tau / H] / d\tau = F_\infty / (\mu H) \times \exp(-\tau/\mu) \times [1 - \tau/\mu] = 0.$$

so, the **peak absorption occurs at $\tau/\mu = 1$** . The pressure where this occurs is:

$$P_{\max \text{ abs}}/p_s = \cos\theta / (H k_{abs} \rho_{as}).$$

So, the pressure where the max absorption occurs is proportional to $\cos\theta$ and inversely proportional to $k_{abs} \times \rho_{as}$.

The heating rate (usually expressed in $K \text{ day}^{-1}$) associated with absorption of downward flux of radiant energy is given by the flux divergence divided by the heat capacity of air ($c_p \rho$):

$$\partial T / \partial t|_{\text{rad}} = 1/(c_p \rho) \times \partial F / \partial z.$$

where c_p is the constant pressure heat capacity ($1 \text{ kJ K}^{-1} \text{ kg}^{-1} - 1/4$ that of water).

Substitution of our early expression for $\partial F / \partial z$ we have:

$$\partial T / \partial t|_{\text{rad}} = k_{abs} M_a / (c_p \mu) \times F.$$

If M_a is independent of altitude (e.g. O_2 , N_2), and the cross sections (k_a) are large then the heating rate is maximum where F is maximum (top of the atmosphere). This produces the very large heating rate in the thermosphere from UV heating. For ozone, which peaks in the stratosphere, the heating rate is maximum in the upper stratosphere. This produces the secondary maximum in temperature near 50 km.

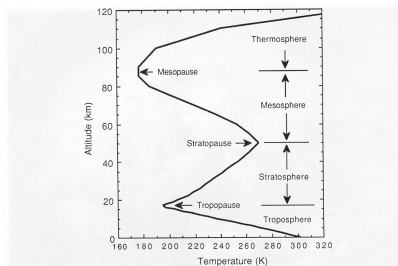
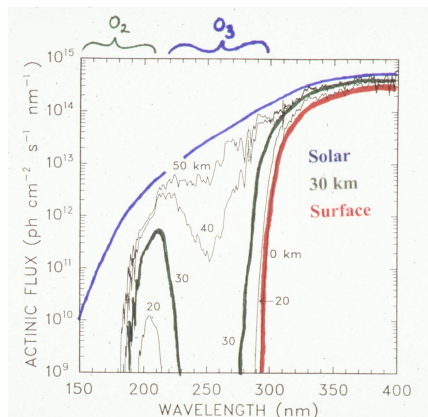
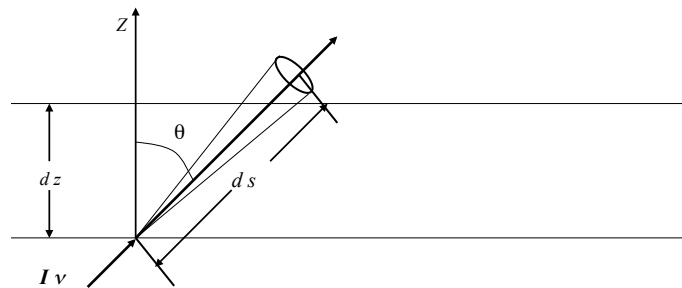


Fig. 1.2 The main zones of the atmosphere defined according to the temperature profile of the standard atmosphere profile at 15°N for annual-mean conditions. [Data from U.S. Standard Atmosphere Supplements (1966).]



Emission of photons

IR Radiative Transfer.



Simplified for plane-parallel atmosphere; properties depend only on altitude. This is generally a reasonable approximation for GH gases (including H_2O vapor). Not a great approximation for clouds. Upward directed terrestrial radiation through a plane-parallel atmosphere.

Consider the above schematic in which a beam of intensity I_v passes upward through a layer dz , making an angle ϕ with respect to the zenith. The change in I_v will equal the emission, E , along the path minus the absorption, A :

$$dI_v = E_v - A_v$$

Replacing A_v with Beer's law:

$$dI_v = E_v - \rho_a ds k_v I_v.$$

From our earlier development of thermal emission (Planck's function), $E_v = \epsilon_v B_v(T)$.

Assuming local thermodynamic equilibrium (true throughout most of the lower atmosphere), **Kirchhoff's** law holds (emissivity = absorptivity), so $\epsilon_v = \rho_a ds k_v$, thus:

$$dI_v = E_v - \rho_a ds k_v I_v = \rho_a ds k_v (B_v(T) - I_v)$$

with $dz = -\cos \theta ds$, and for small ds , we obtain:

$$\cos \theta dI_v / dz = \rho_a k_v (B_v(T) - I_v) \text{ Schwarzschild's Eqn}$$

Schwarzschild's Equation

$$I_v(\tau_v(z), \mu) = I_v(0, \mu) e^{\{-\tau_v(z)/\mu\}} + \int_0^{\tau_v(z)} \mu^{-1} B_v(T(\tau'_v)) e^{\{(\tau'_v - \tau_v(z))/\mu\}} d\tau'_v$$

Radiance at a given optical depth (z) and angle

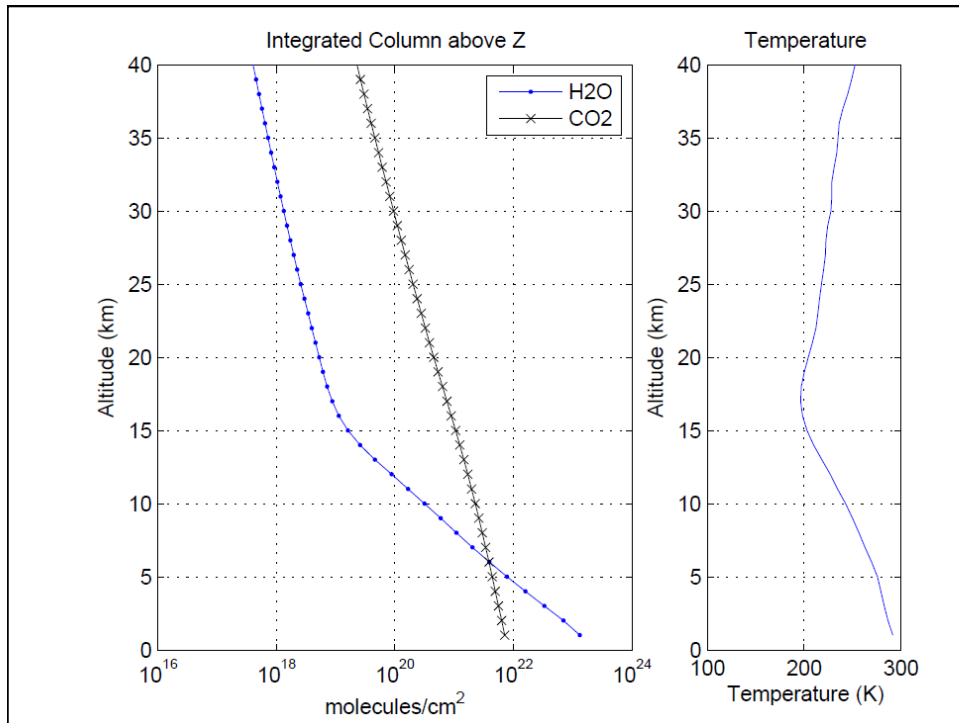
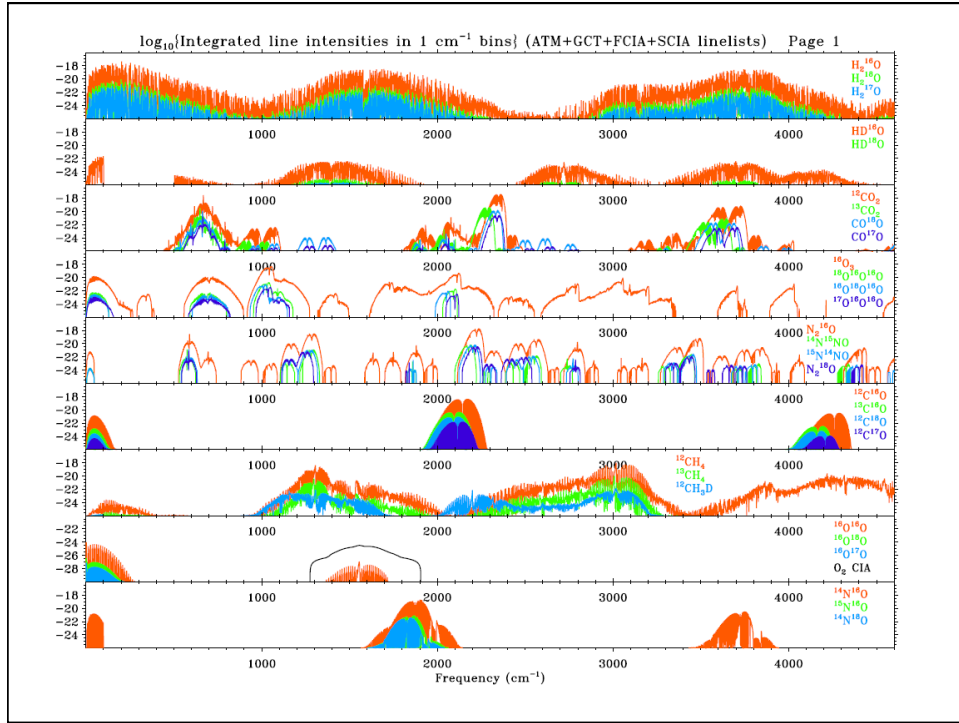
Emission from sfc

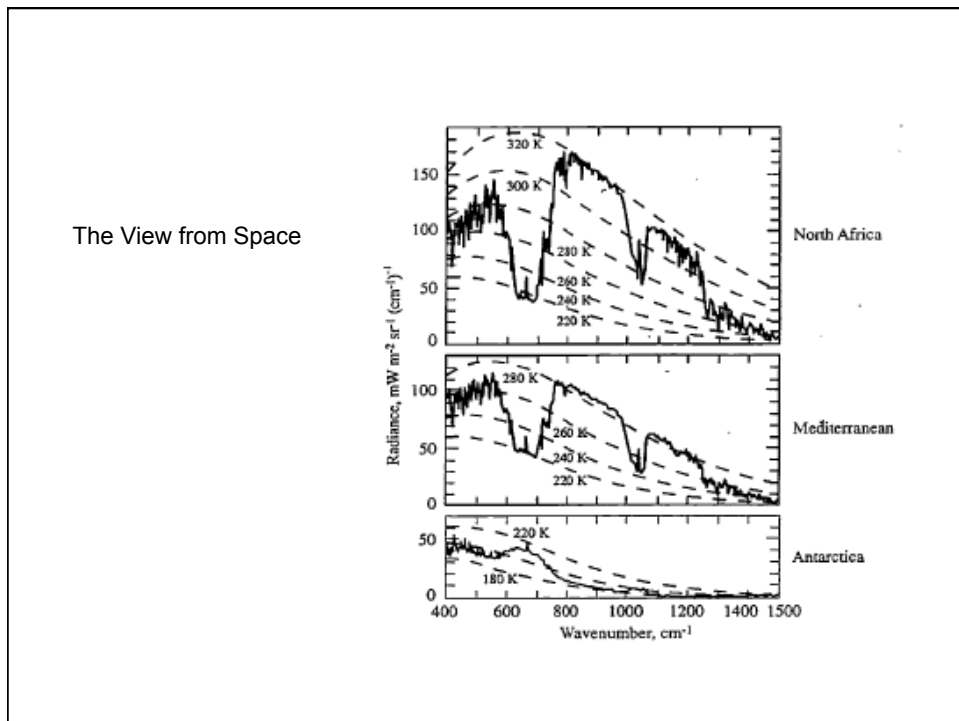
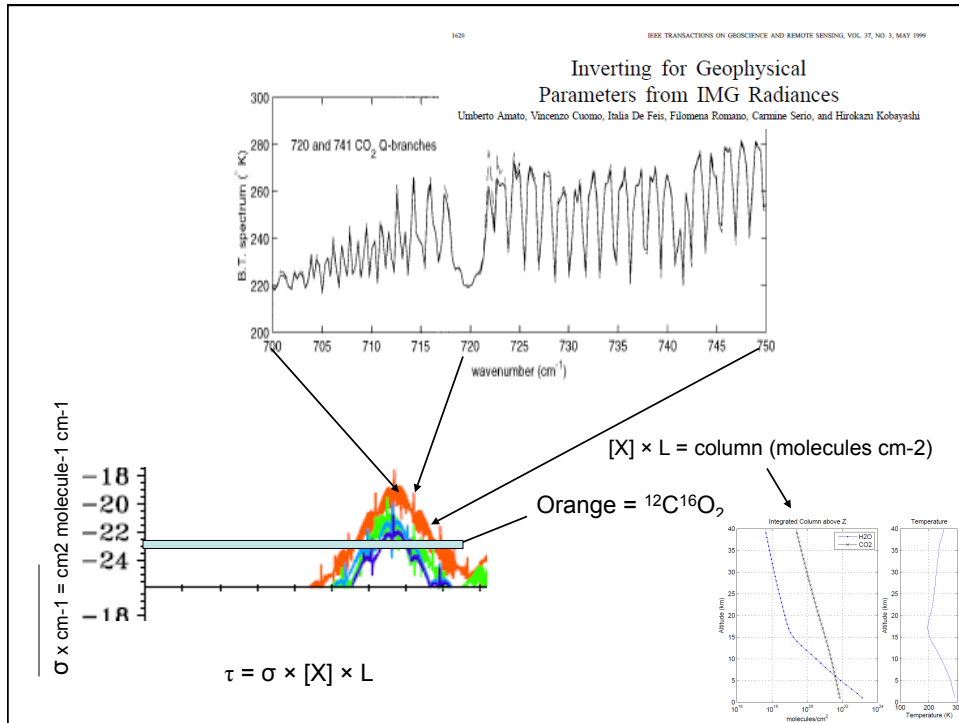
Absorption below z

Sum of emissions from each atm level

weighted by absorptivity/emissivity of each layer in between

- Upwelling radiance at a given level has **contributions from both the surface and from the atmosphere between**
- Relative contributions are controlled by vertical **profiles of temperature and absorbing gases**

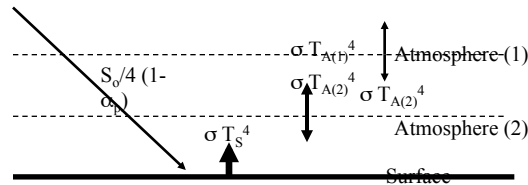




The solution of Schwarzschild's Eqn is quite complicated and requires a large amount of bookkeeping. Generally, to be computationally efficient, radiative transfer models used in climate studies are not 'line-by-line', that is they do not explicitly calculate k_ν at the resolution required to properly account for absorption by gas phase molecules ($.001 \text{ cm}^{-1}$). Instead, 'band models' are developed that provide an estimate of k_ν over an entire band as a function of optical depth. These parameterizations are tested using much more computationally intensive line-by-line calculations. The band models are then used in a computational analog of Schwarzschild's Eqn followed by iteration.

A beginning point for most of these models, is to calculate the atmosphere in 'radiative equilibrium'

Earlier, we developed a 'two layer' model of a slab atmosphere:



For the surface, $S_o/4 (1 - \alpha_p) + \sigma T_{A(2)}^4 = \sigma T_s^4$

At A(1) we have:

$$S_o/4 (1 - \alpha_p) = \sigma T_{A(1)}^4 = \sigma T_e^4$$

while that of A(1) is:

$$\sigma T_{A(2)}^4 = 2\sigma T_{A(1)}^4$$

The balance at A(2) is

$$\sigma T_{A(1)}^4 + \sigma T_s^4 = 2\sigma T_{A(2)}^4$$

solving, we have:

$$T_s^4 = 3 (S_o/4) (1 - \alpha_p) / \sigma = 3 T_e^4$$

By extension to n layers,

$$T_s = \sqrt[n+1]{3} T_e$$

The radiative equilibrium solution for a two layer model is 355 K, which is significantly warmer than the average surface temperature of Earth. However, radiative equilibrium is not a good approximation for the surface temperature (why?).

The skin temperature is a useful concept. Consider a thin layer of emissivity ϵ at the top of the atmosphere. The thermal balance of this layer is:

$$\epsilon \sigma T_e^4 = 2 \epsilon \sigma T_{\text{skin}}^4$$

We will take this layer to be the stratosphere.

Now consider a thin layer near the surface that absorbs a fraction ϵ of the emission from above and below and emits isotropically. The energy balance for this layer, T_{SA} , is:

$$\epsilon \sigma T_s^4 + \epsilon \sigma T_1^4 = 2 \epsilon \sigma T_{\text{SA}}^4$$

Now, solving for all temperatures:

$$T_{\text{skin}} = 214 \text{ K} \quad T(2) = 255 \text{ K} \quad T(1) = 303 \text{ K} \quad T_{\text{SA}} = 320 \text{ K} \quad T_s = 355 \text{ K}$$

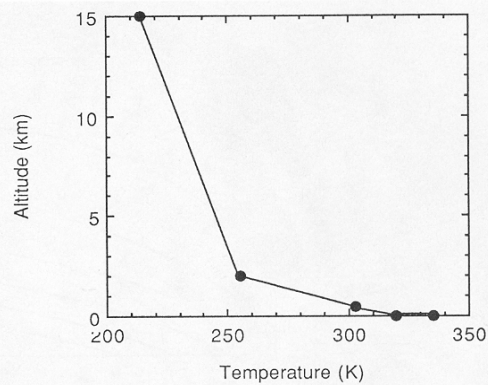


Fig. 3.11 Plot of temperature profile obtained from the simple two-level atmosphere radiative equilibrium model.