



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

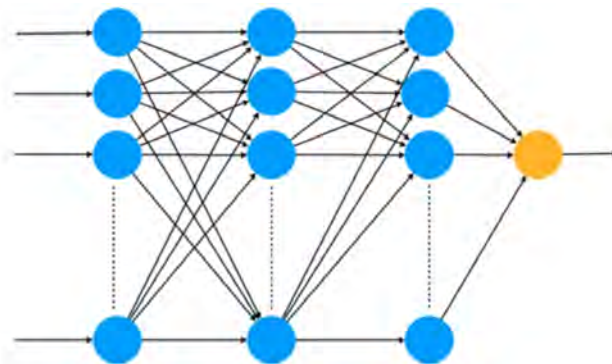


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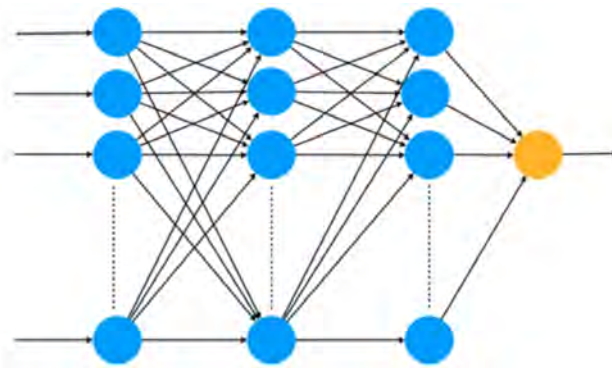
Solving System of Linear Equations

Machine learning motivation

Neural networks - Matrix operations

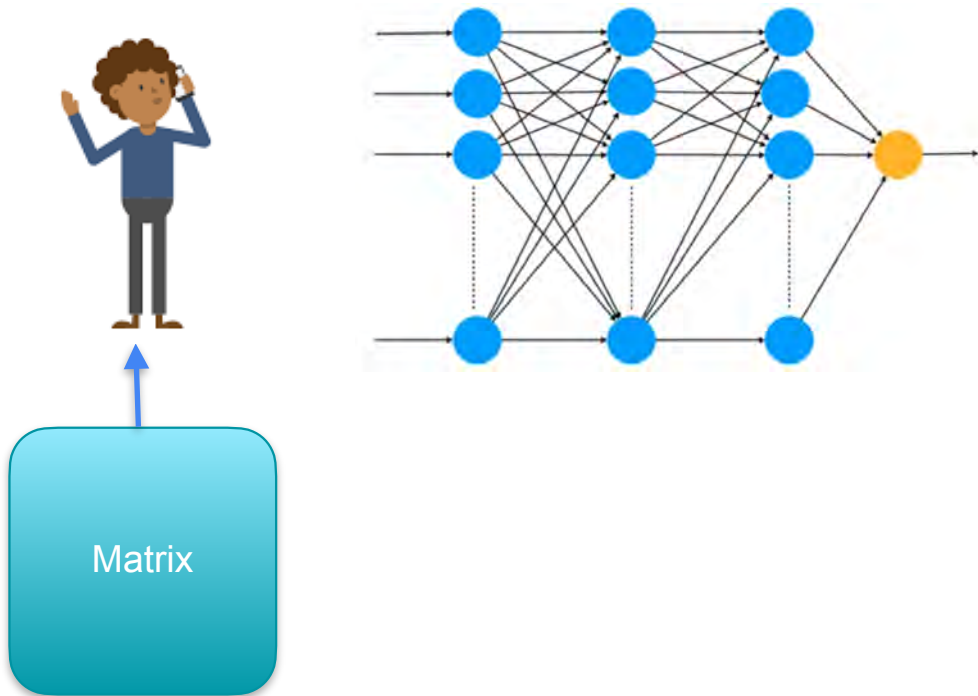


Neural networks - Matrix operations



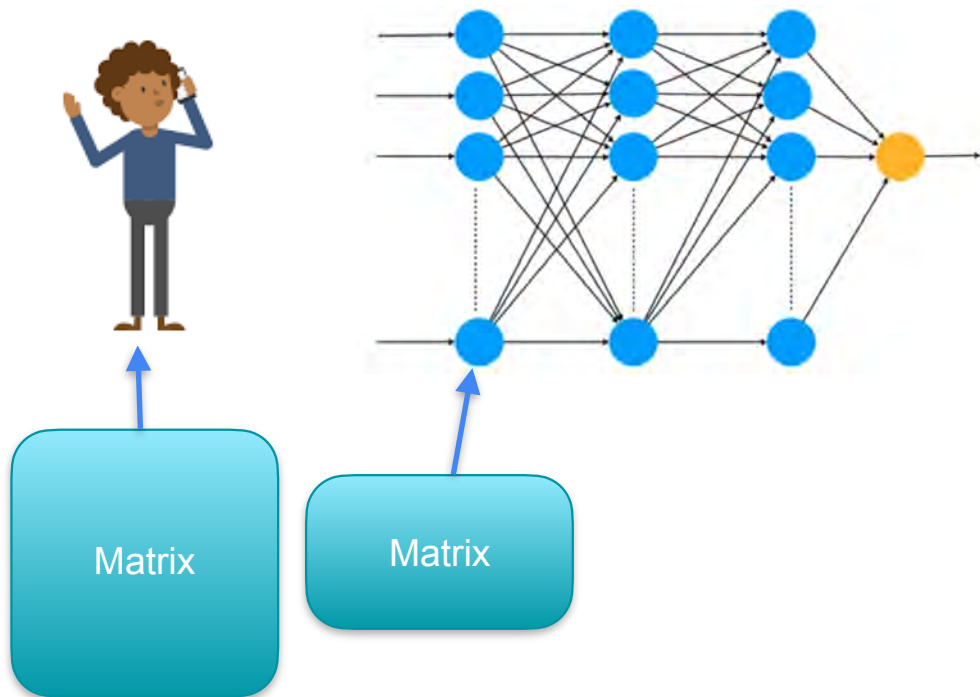
"Hello! Welcome to
Math for Machine
Learning!"

Neural networks - Matrix operations



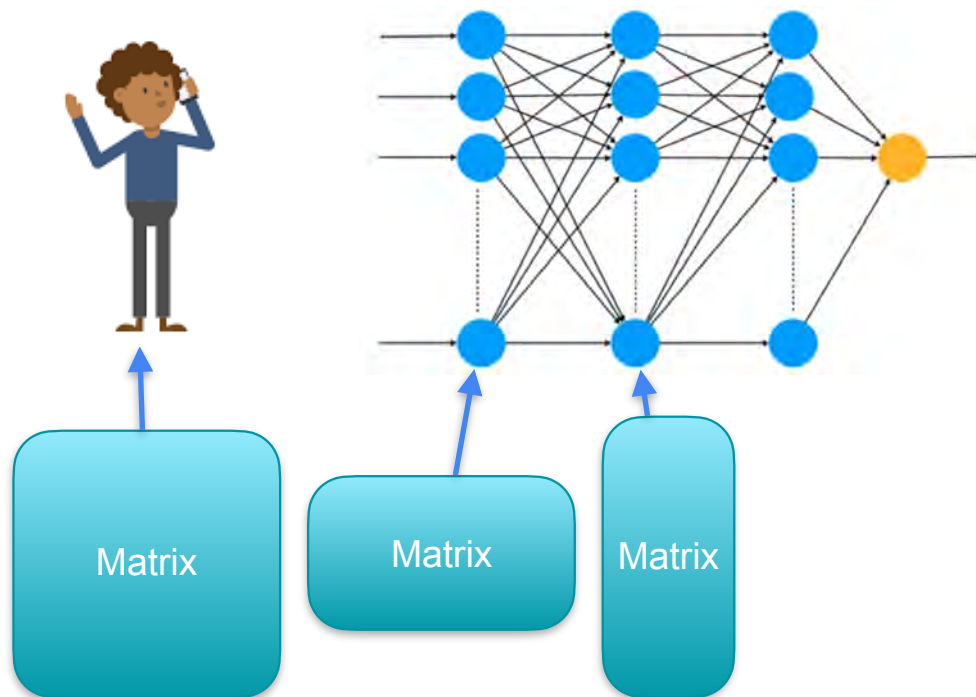
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Neural networks - Matrix operations



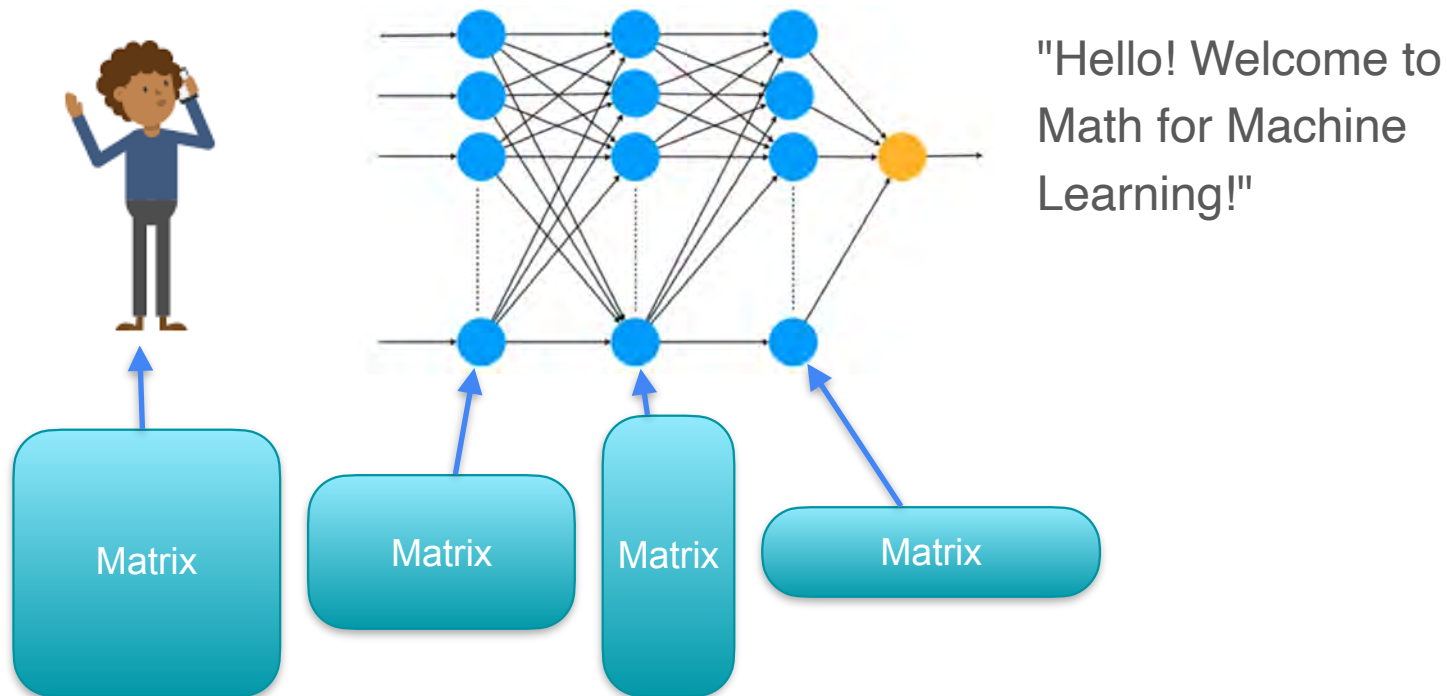
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Neural networks - Matrix operations

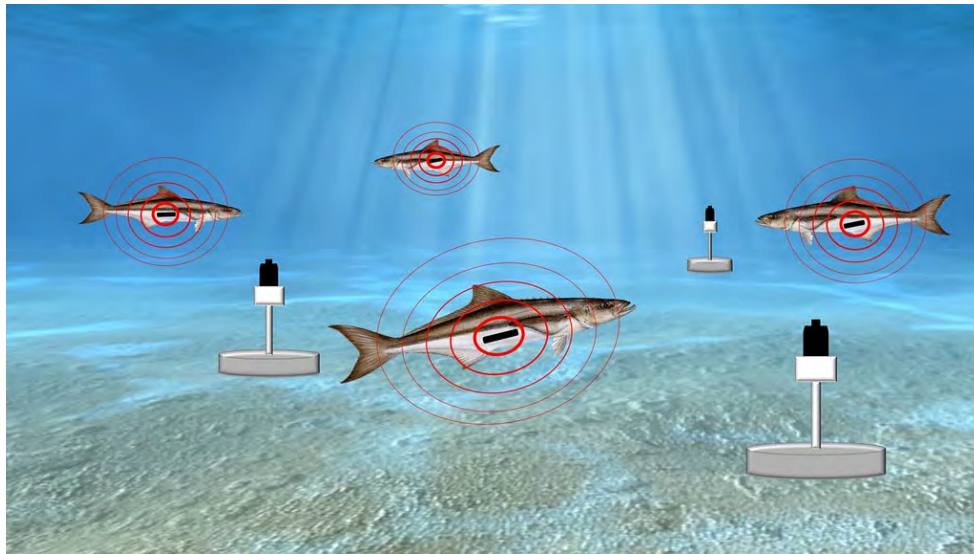


"Hello! Welcome to
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Neural networks - Matrix operations



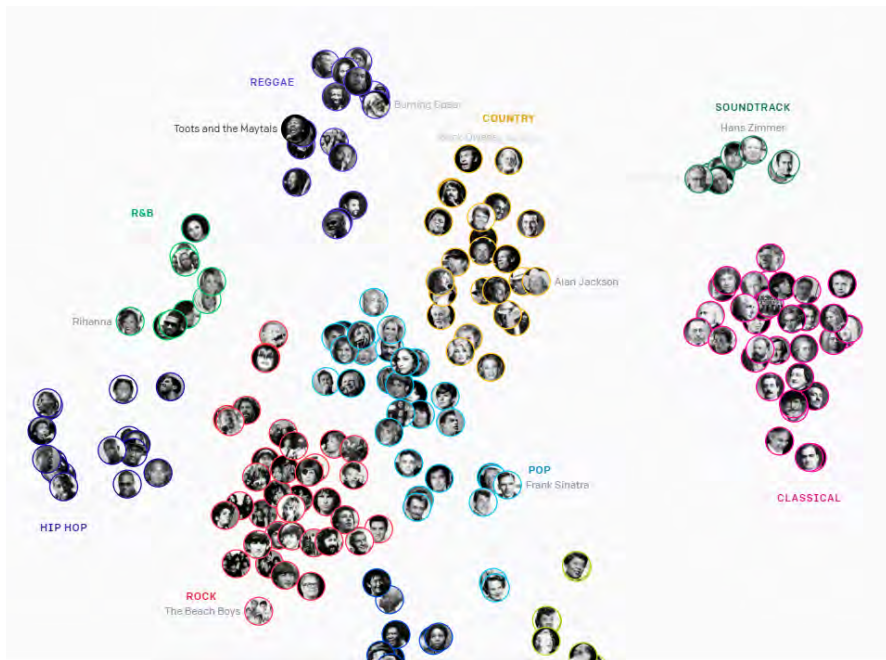
Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.





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Solving System of Linear Equations

**Solving non-singular system
of linear equations**

Solving systems of equations



System

- $a + b = 10$
 

- $a + 2b = 12$
 

Solving systems of equations

System

- $a + b = 10$
 



- $a + 2b = 12$
  

$$\text{🍏} + \text{🍌} = \$10$$

$$\text{🍏} + \text{🍌} + \text{🍌} = \$12$$

Solving systems of equations

System

- $a + b = 10$
 



- $a + 2b = 12$
  

$$\text{🍏} + \text{🍌} = \$10$$

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

Solving systems of equations

System

- $a + b = 10$
 



- $a + 2b = 12$
  


$$\text{apple} + \text{banana} = \$10$$

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$\boxed{2}$$

Solving systems of equations

System


- $a + b = 10$
 + 

- $a + 2b = 12$
 + 

$$\text{apple} + \text{banana} = \$10$$



$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$\boxed{2}$$


\$2











Solving systems of equations

System

- $a + b = 10$
 + 

- $a + 2b = 12$
 + 

 +  = \$10
 \$2

 +  +  = \$
 \$2

Solving systems of equations

System

- $a + b = 10$



- $a + 2b = 12$



$\text{Apple} + \text{Banana} = \10

$\$8$ (points to Apple) $\$2$ (points to Banana)

$\text{Apple} + \text{Banana} + \boxed{\text{Banana}} = \$10 - 2$

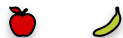
$\$2$ (points to boxed Banana)

Solving systems of equations

Solving systems of equations

System

- $a + b = 10$







- $a + 2b = 12$





Solving systems of equations

System





- $a + b = 10$
 
- $a + 2b = 12$
 

Solved system

- $a = 8$

- $b = 2$




Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 





Some process

Solved system

- $a = 8$

- $b = 2$


Solving systems of equations



System

- $a + b = 10$
 
- $a + 2b = 12$
 

Some process





Manipulating equations

Solved system

- $a = 8$

- $b = 2$


Solving systems of equations

System

- $a + b = 10$
 
- $a + 2b = 12$
 

Some process



Manipulating equations

Swapping equations

Adding equations





Multiplying equations by a constant

Solved system



- $a = 8$

- $b = 2$


Solving systems of equations

System



- $a + b = 10$
 
- $a + 2b = 12$
 

Solved system

- $a = 8$

- $b = 2$


Solving systems of equations


System


- $a + b = 10$
 

- $a + 2b = 12$
 

Eliminate 'a' from this equation

Solved system

- $a = 8$


- $b = 2$


Manipulating equations

Manipulating equations

Multiplying by a constant

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\underline{x \qquad 7}$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

$$2a + 3b = 26$$

Manipulating equations

Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 26 \\ \hline \end{array}$$

Manipulating equations

Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 26 \\ \hline 3a + 4b = 36 \end{array}$$

Let's do a harder example

Systems of equations

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$



Eliminate 'a'
from this equation

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- $5a + b = 17$

- $4a - 3b = 6$

Divide by coefficient of a

- $a + 0.2b = 3.4$

- $a - 0.75b = 1.5$

Solved system

- $a = ?$

- $b = ?$

↑
Eliminate 'a'
from this equation

Systems of equations

System

- $5a + b = 17$

- $4a - 3b = 6$

Divide by coefficient of a

- $a + 0.2b = 3.4$

- $a - 0.75b = 1.5$

Solved system

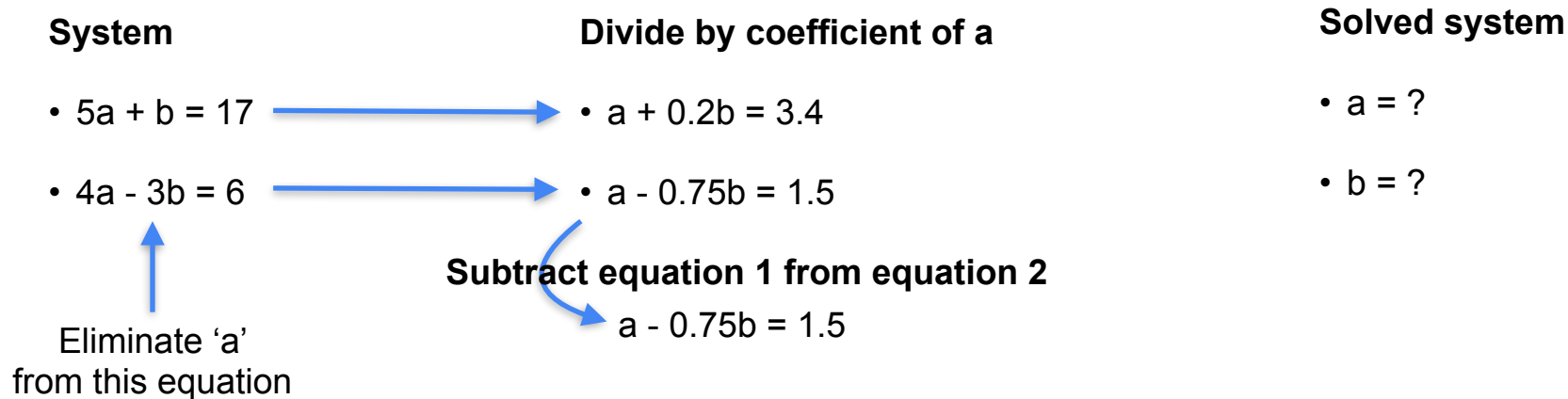
- $a = ?$

- $b = ?$

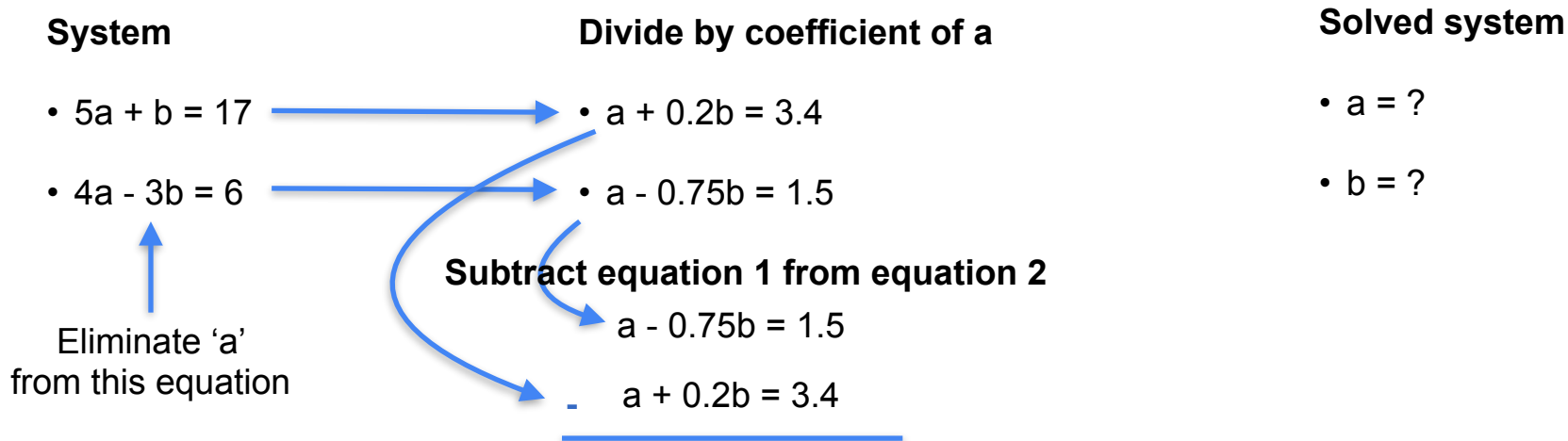
Subtract equation 1 from equation 2

Eliminate 'a'
from this equation

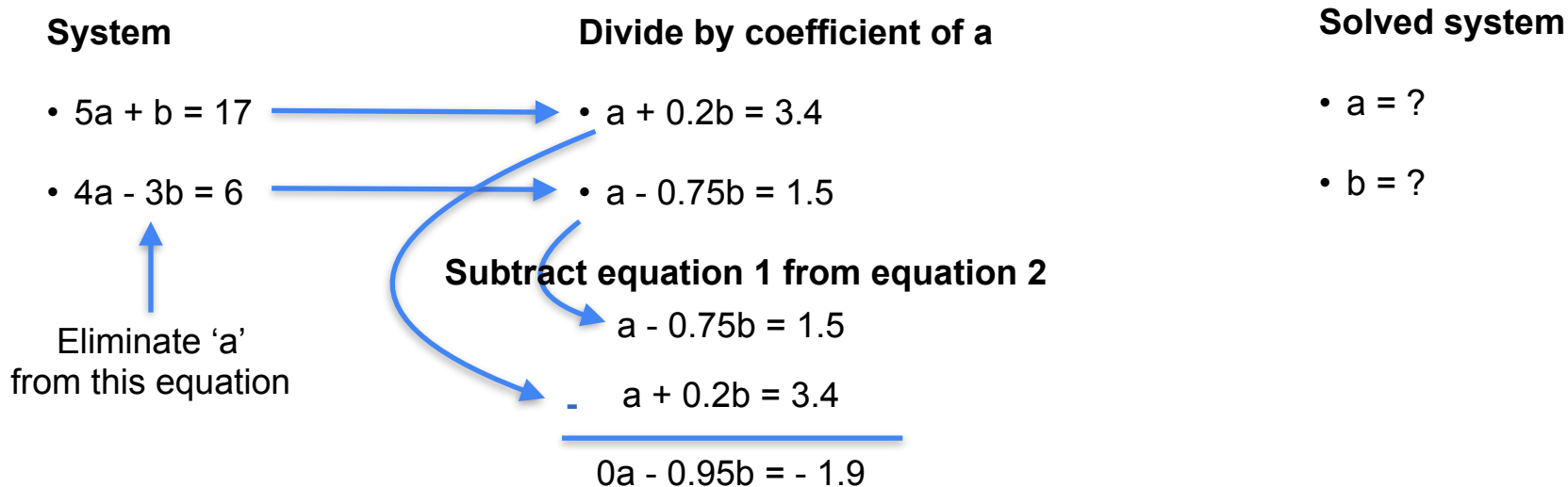
Systems of equations



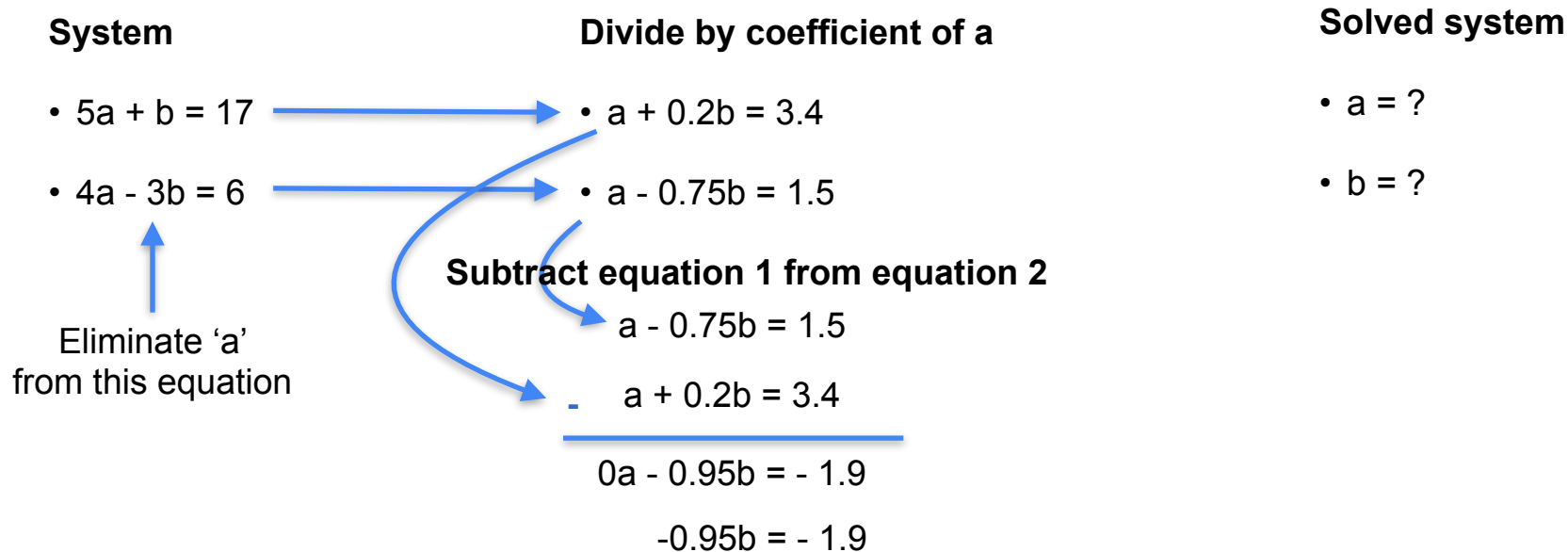
Systems of equations



Systems of equations



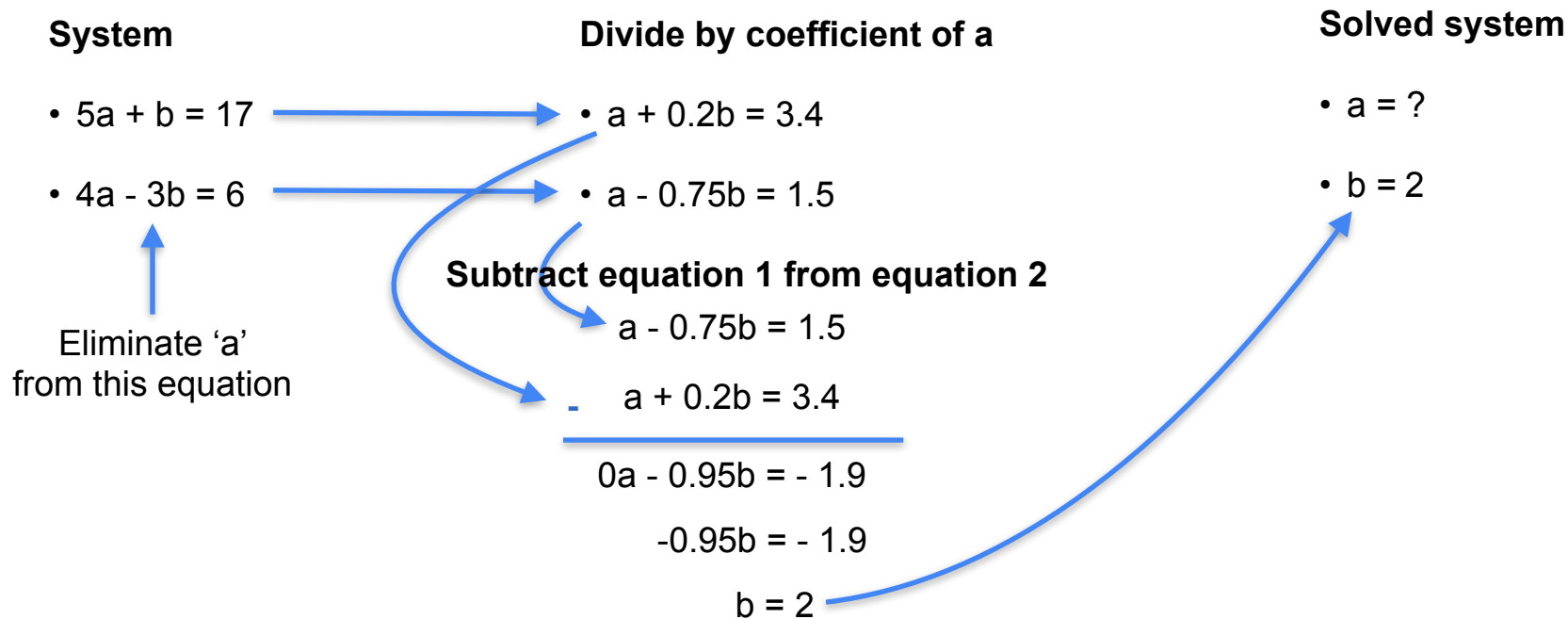
Systems of equations



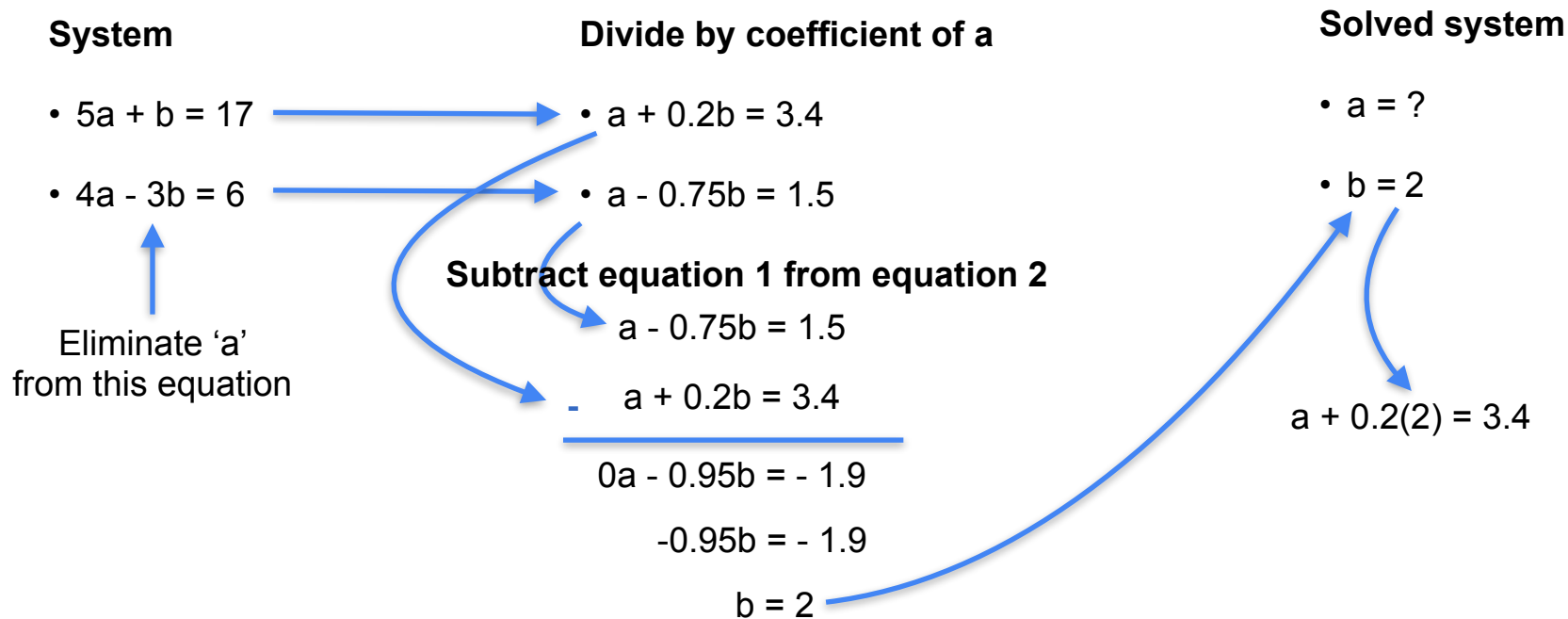
Systems of equations

| System | Divide by coefficient of a | Solved system |
|--|--|--|
| <ul style="list-style-type: none">• $5a + b = 17$ | <ul style="list-style-type: none">• $a + 0.2b = 3.4$ | <ul style="list-style-type: none">• $a = ?$ |
| <ul style="list-style-type: none">• $4a - 3b = 6$ | <ul style="list-style-type: none">• $a - 0.75b = 1.5$ | <ul style="list-style-type: none">• $b = ?$ |
| <p>↑ Eliminate 'a' from this equation</p> | <p>Subtract equation 1 from equation 2</p> $\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline 0a - 0.95b = -1.9 \\ -0.95b = -1.9 \\ b = 2 \end{array}$ | |

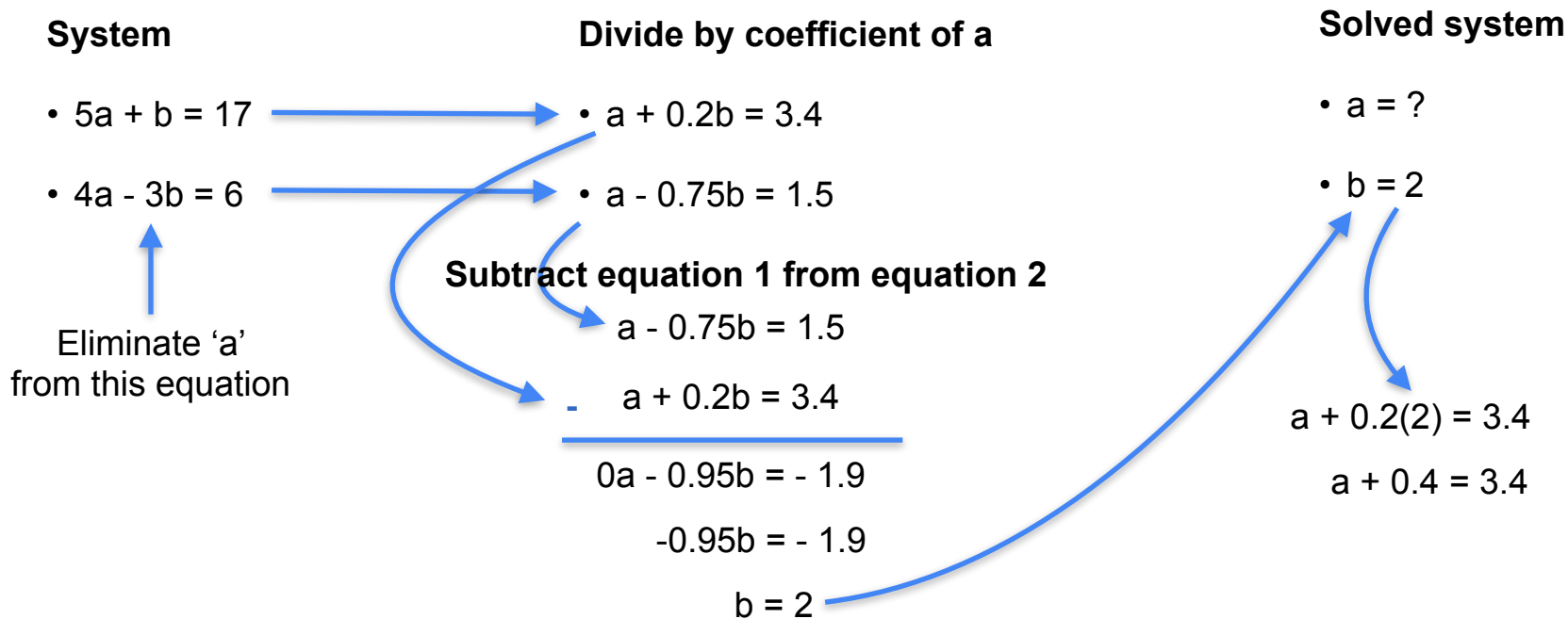
Systems of equations



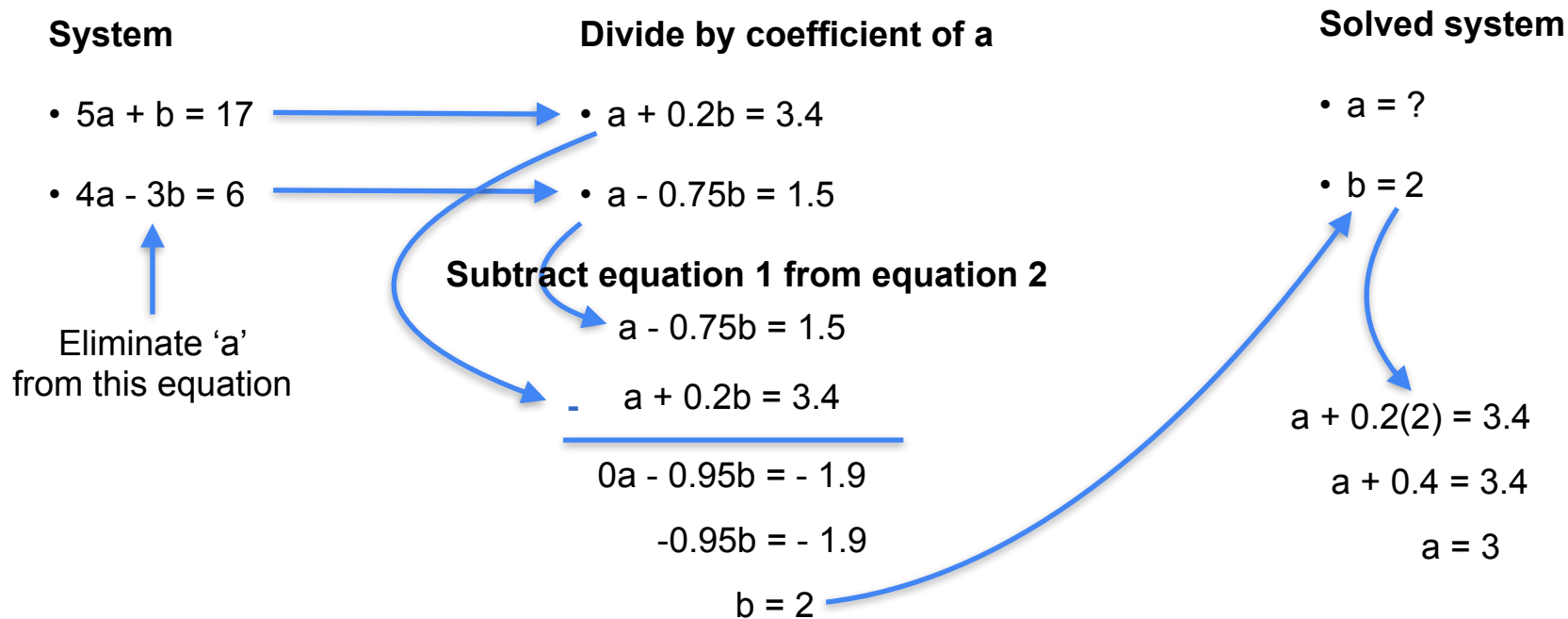
Systems of equations



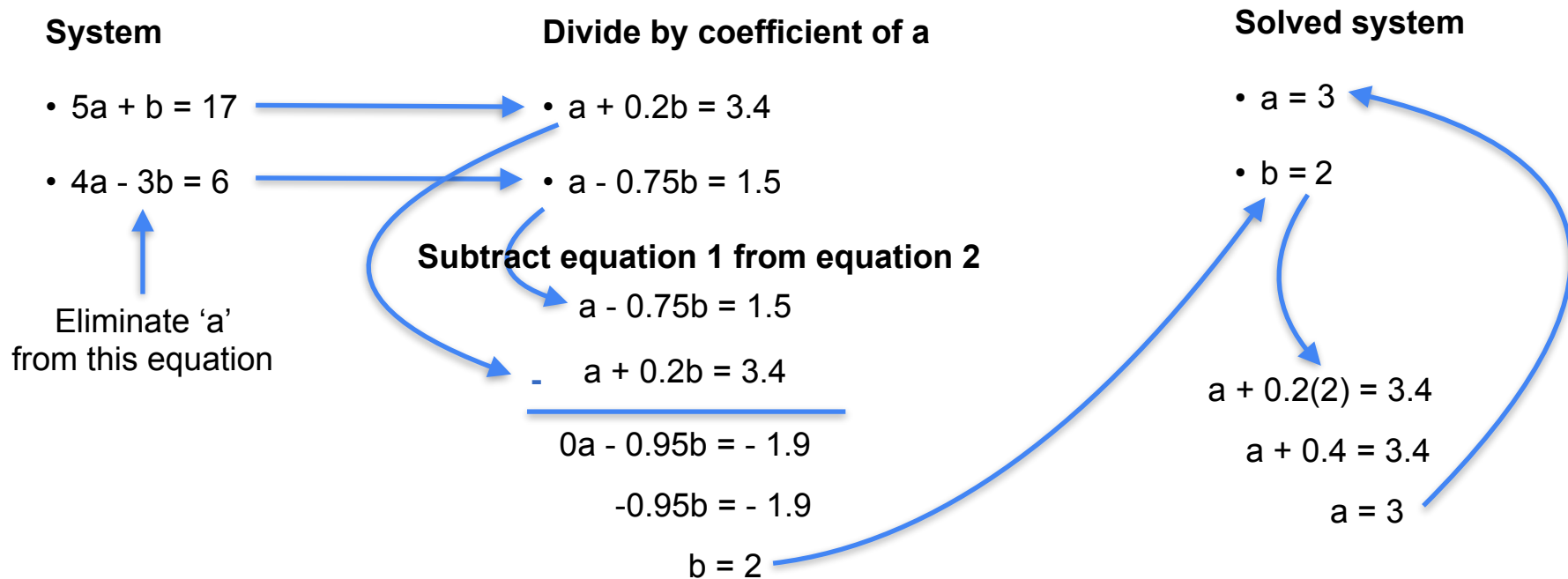
Systems of equations



Systems of equations



Systems of equations



What if one of the coefficients of a is zero?

System

- $5a + b = 17$
- $3b = 6$

Solved system

- $a = ?$
- $b = ?$

What if one of the coefficients of a is zero?

System

- $5a + b = 17$

- $3b = 6$





Eliminate 'a'
from this equation


Solved system

- $a = ?$

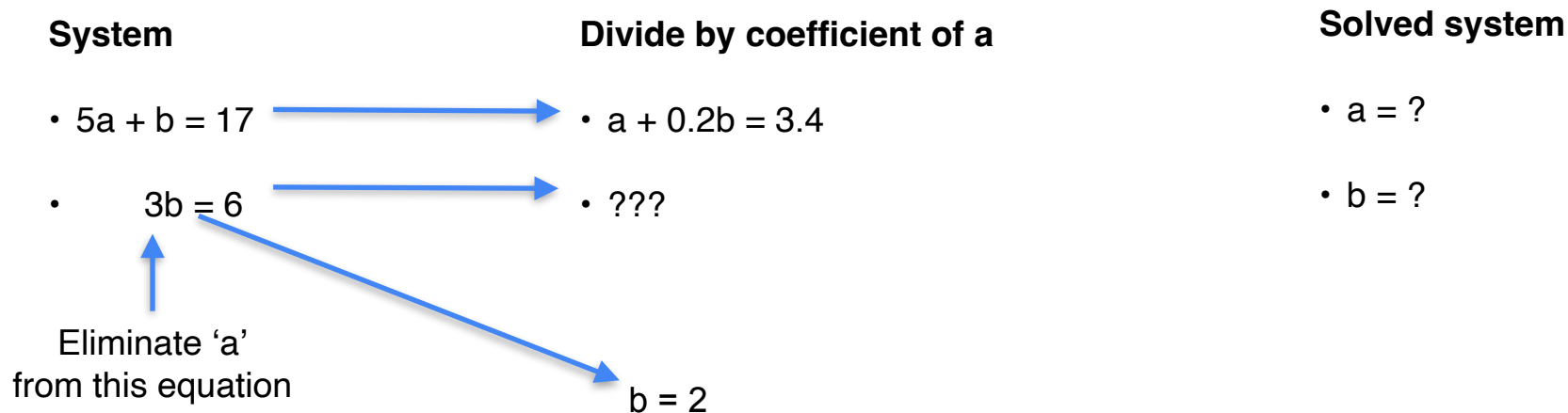
- $b = ?$

What if one of the coefficients of a is zero?

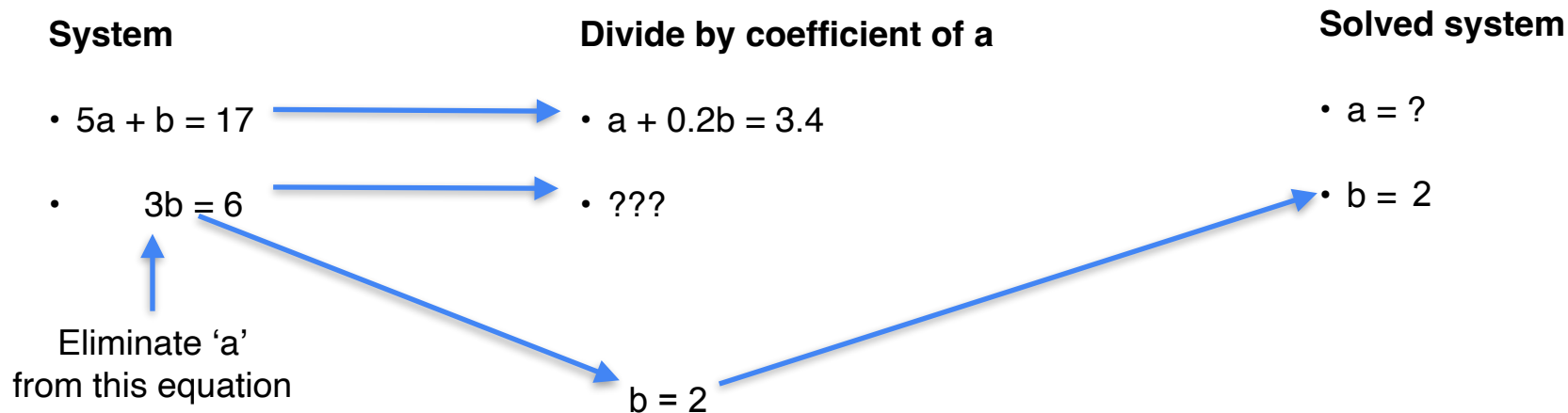
| System | | Divide by coefficient of a | Solved system |
|-----------------|---|----------------------------|---------------|
| • $5a + b = 17$ |  | • $a + 0.2b = 3.4$ | • $a = ?$ |
| • $3b = 6$ |  | • ??? | • $b = ?$ |

 Eliminate 'a' from this equation

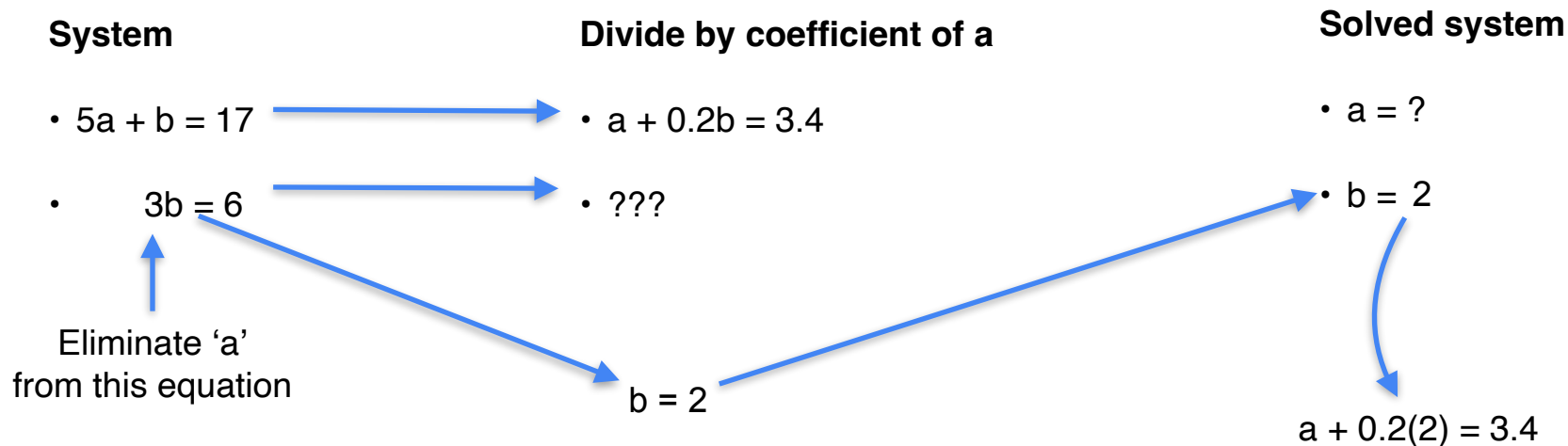
What if one of the coefficients of a is zero?



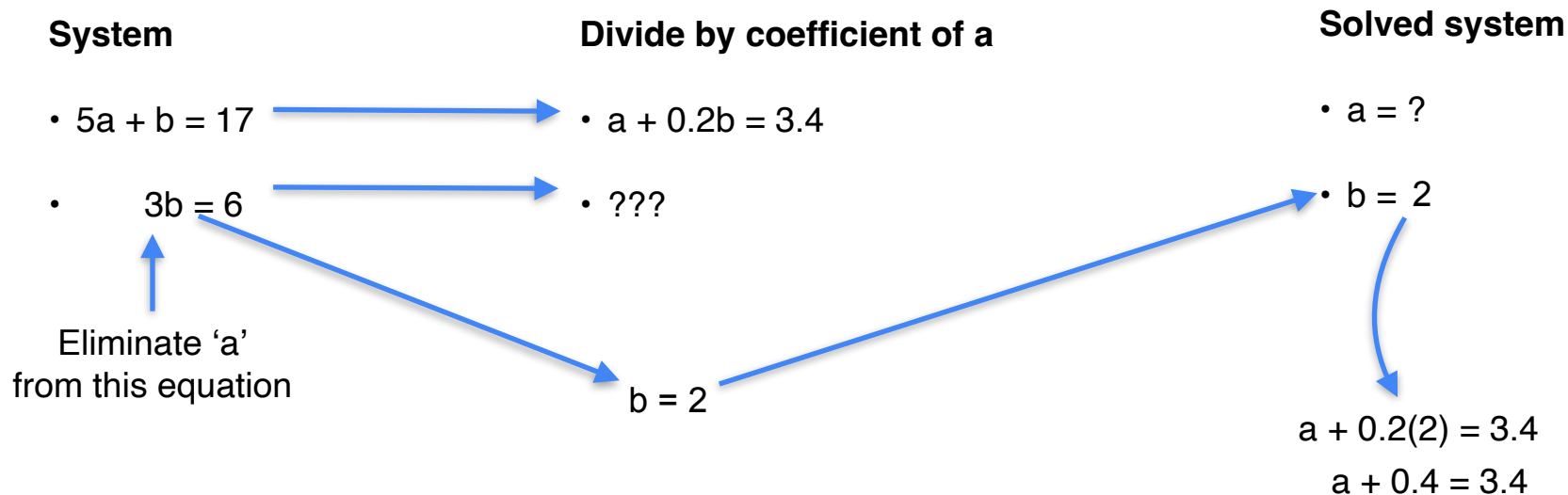
What if one of the coefficients of a is zero?



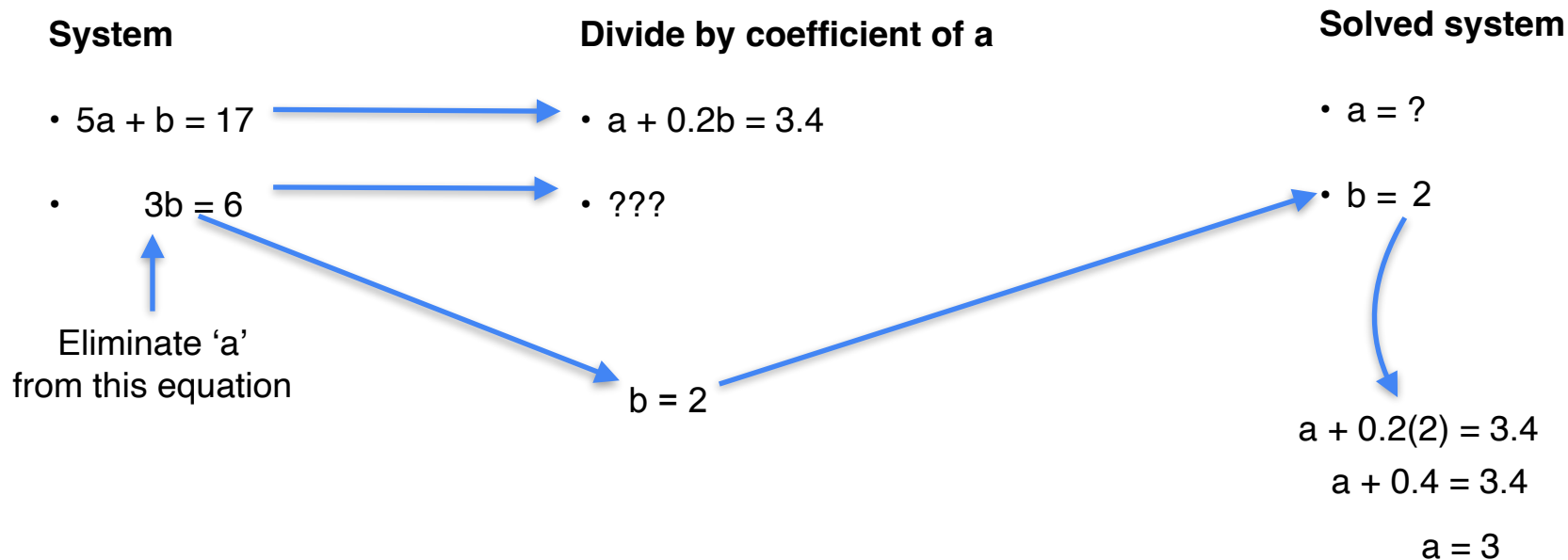
What if one of the coefficients of a is zero?



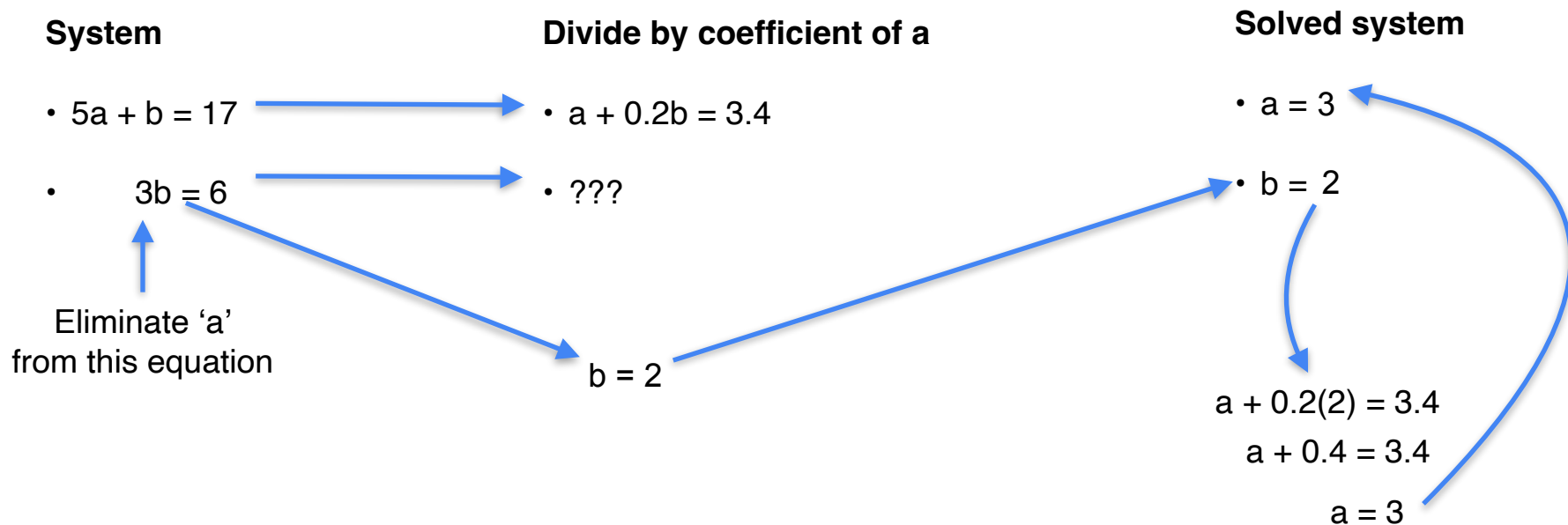
What if one of the coefficients of a is zero?



What if one of the coefficients of a is zero?



What if one of the coefficients of a is zero?



Quiz

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- $a = 3$
- $b = 8$



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Solving System of Linear Equations

**Solving singular system of
linear equations**

What if the system is singular (redundant)?



System


- $a + b = 10$
- $2a + 2b = 20$

Solved system

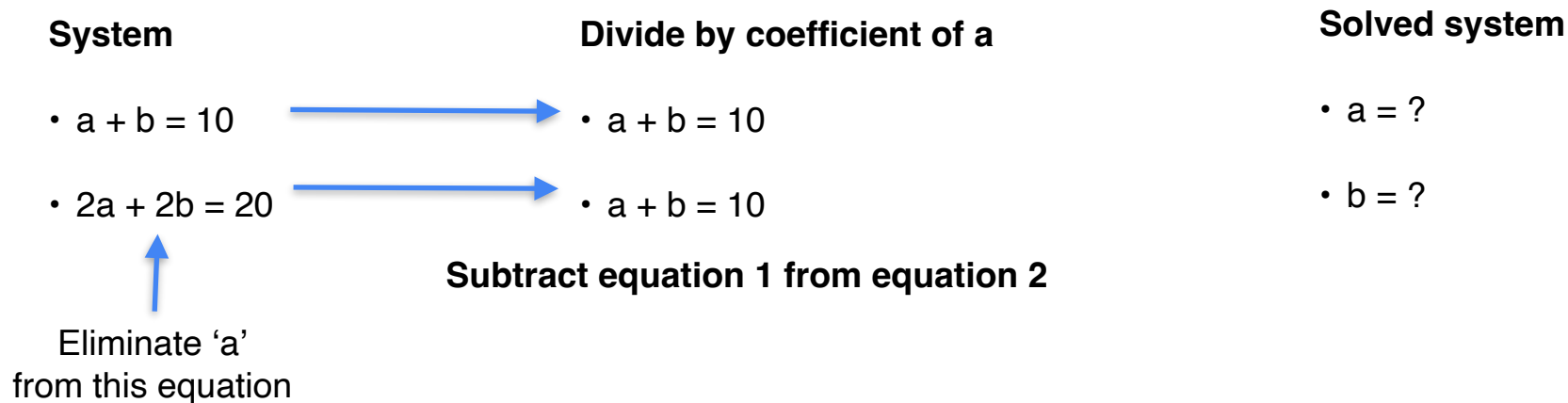
- $a = ?$
- $b = ?$

What if the system is singular (redundant)?

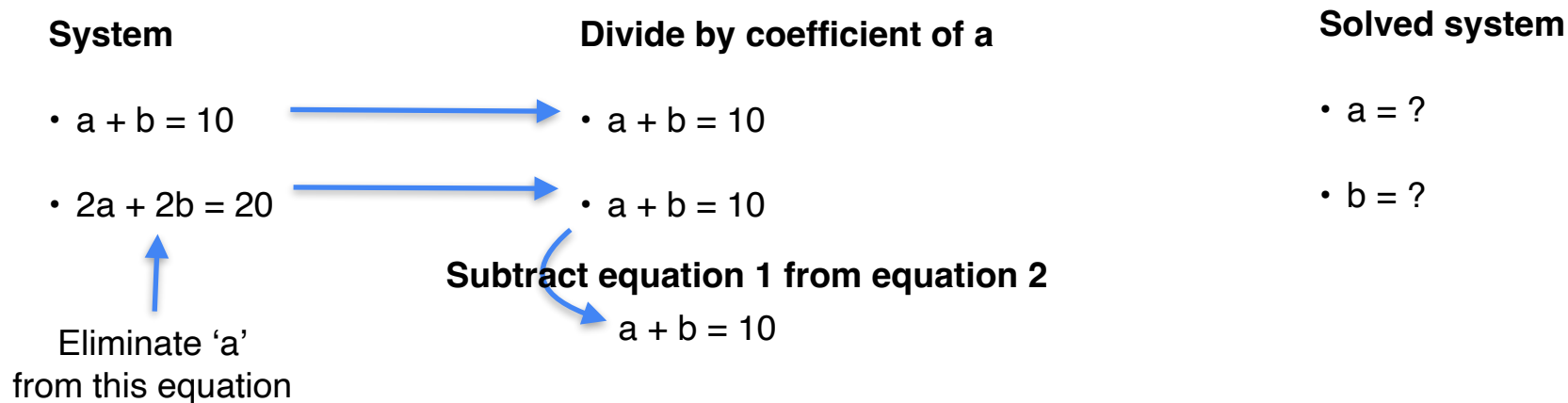
| System | | Divide by coefficient of a | Solved system |
|------------------|---|----------------------------|---------------|
| • $a + b = 10$ |  | • $a + b = 10$ | • $a = ?$ |
| • $2a + 2b = 20$ |  | • $a + b = 10$ | • $b = ?$ |

 Eliminate 'a' from this equation

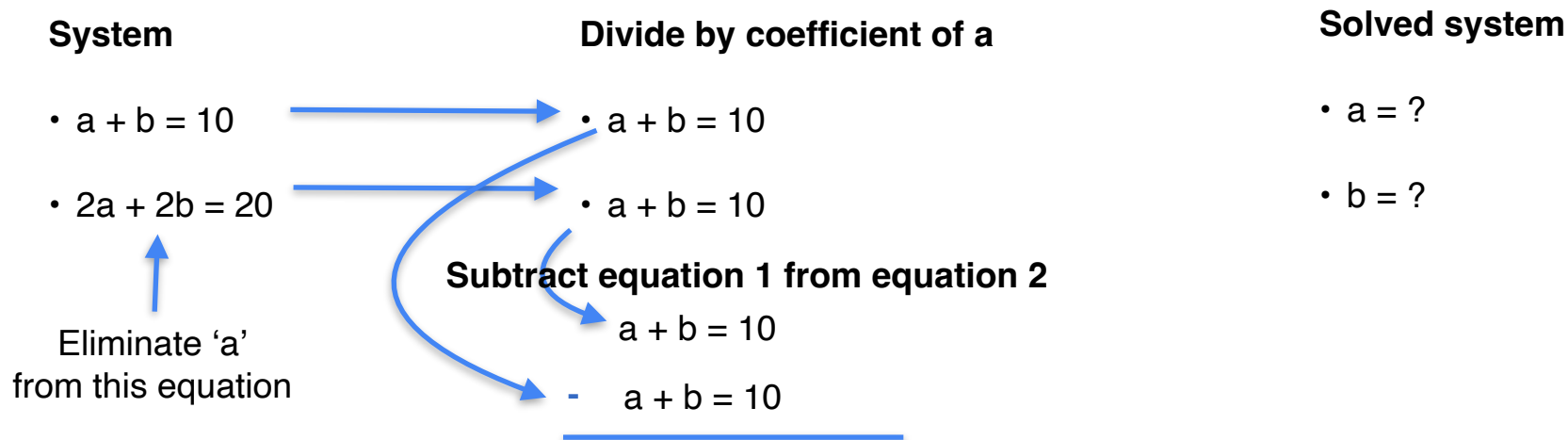
What if the system is singular (redundant)?



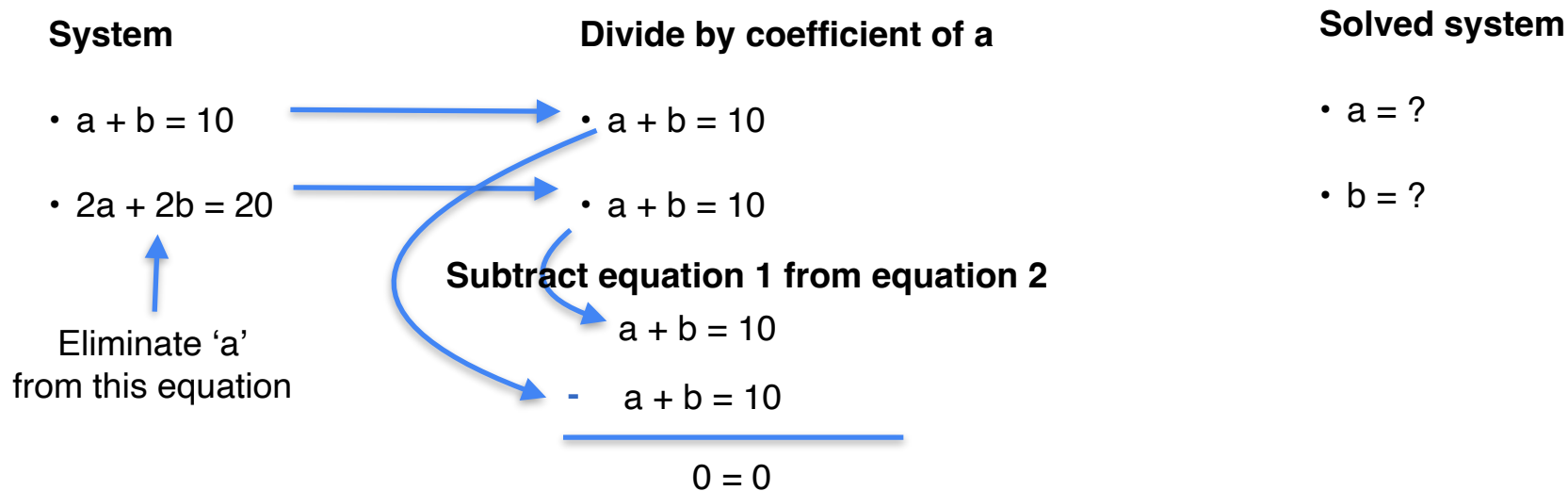
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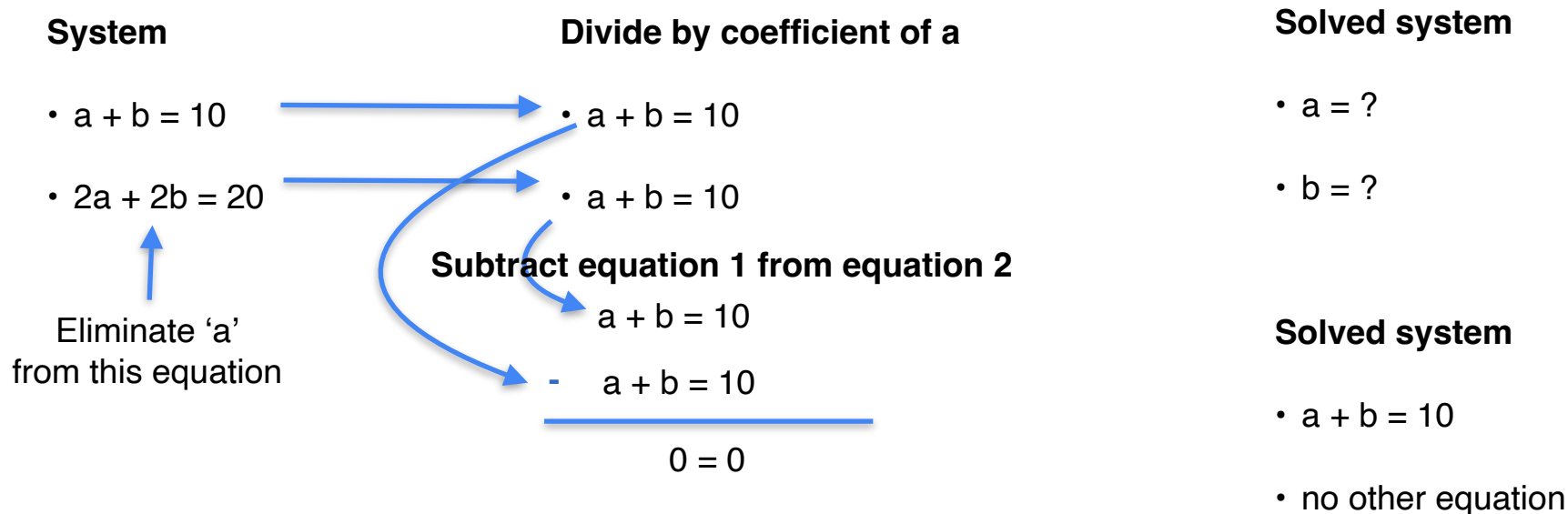
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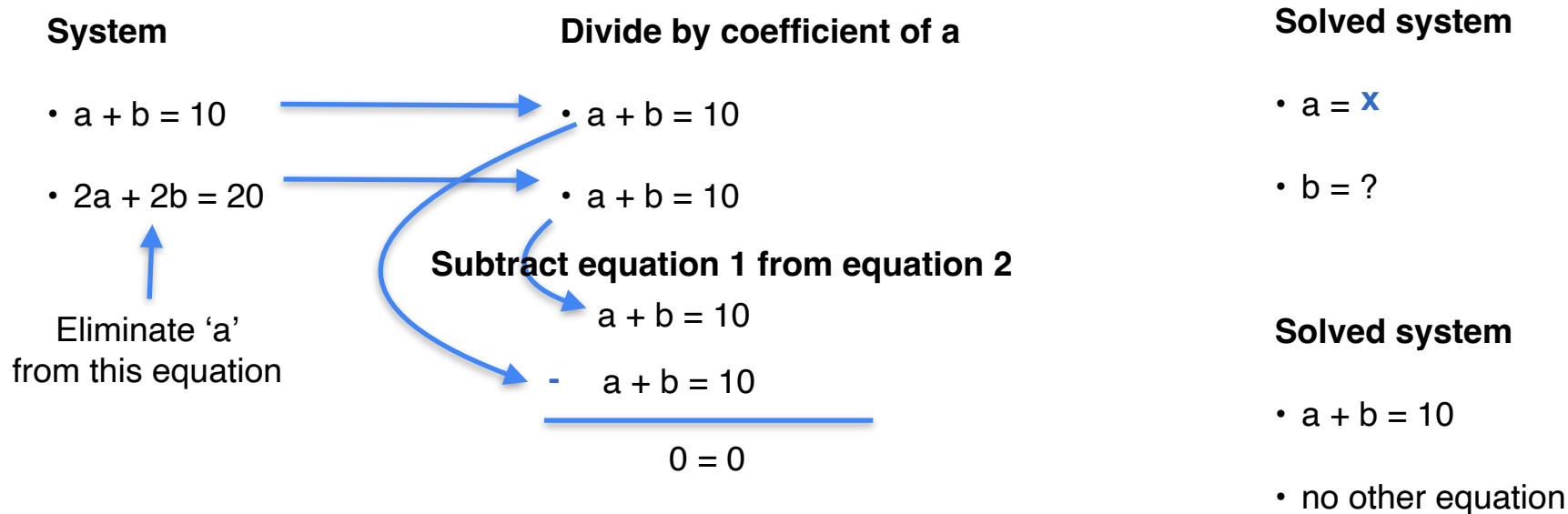
What if the system is singular (redundant)?



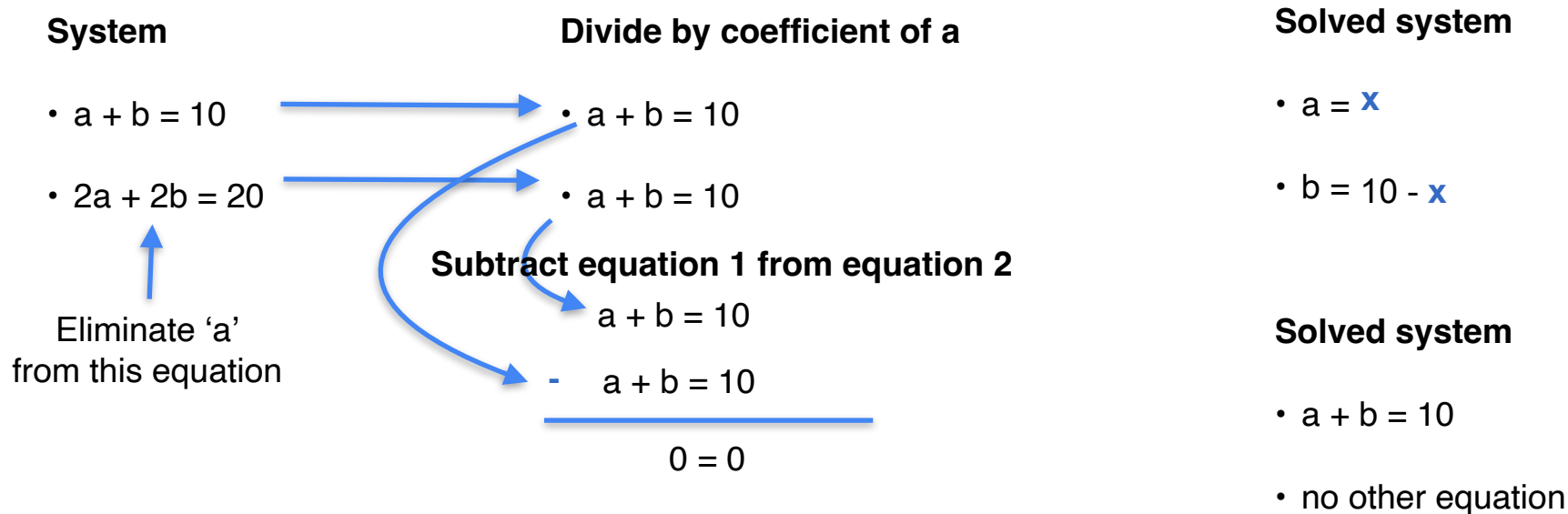
What if the system is singular (redundant)?



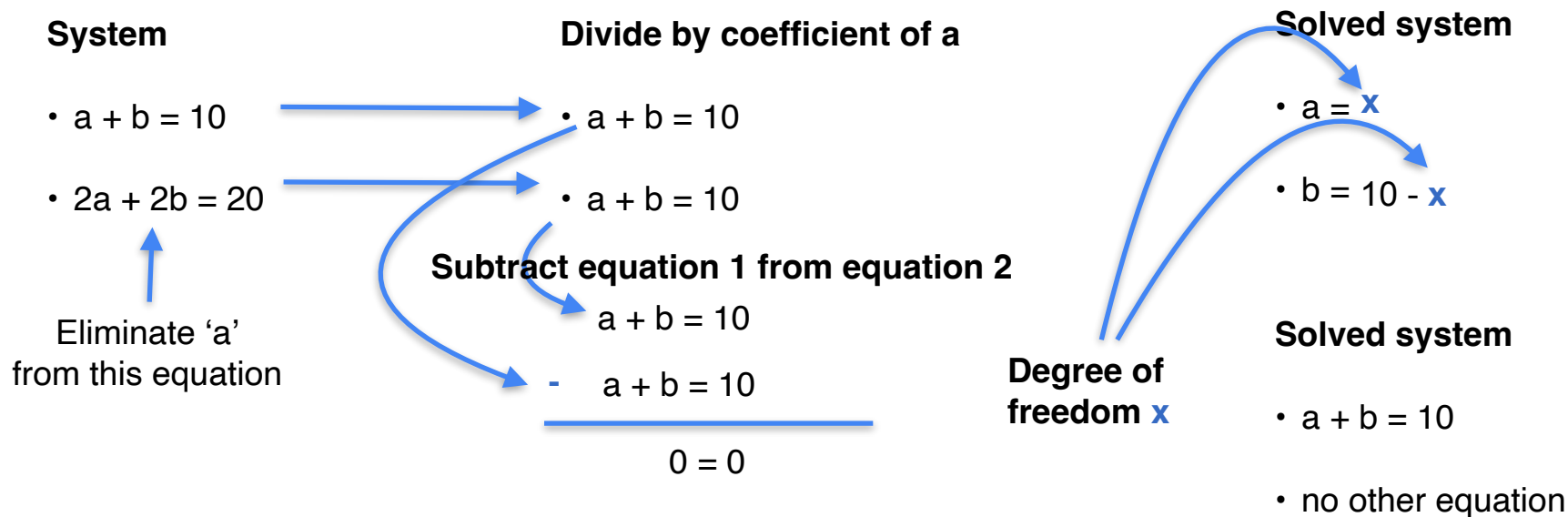
What if the system is singular (redundant)?



What if the system is singular (redundant)?



What if the system is singular (redundant)?



What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$






Eliminate 'a'
from this equation

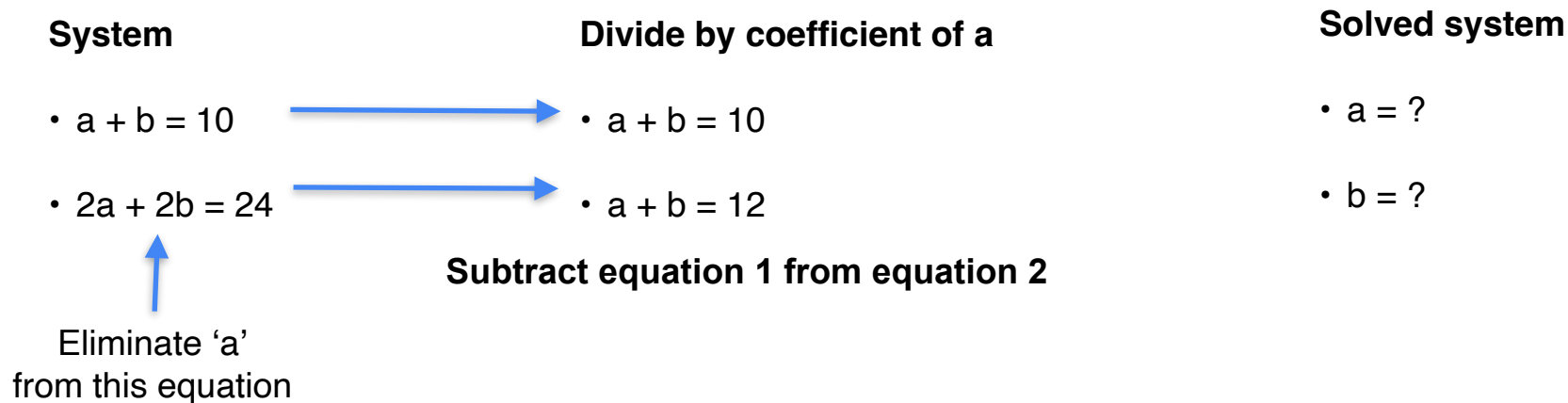
Solved system

- $a = ?$
- $b = ?$

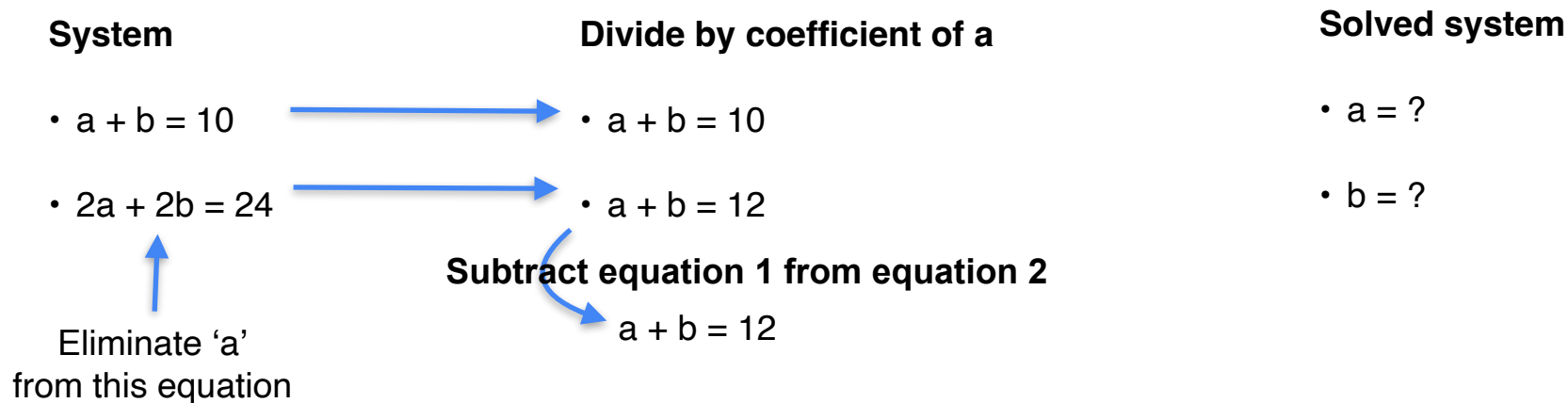
What if the system is singular (contradictory)?

| System | | Divide by coefficient of a | | Solved system |
|--|---|----------------------------|--|---------------|
| • $a + b = 10$ |  | • $a + b = 10$ | | • $a = ?$ |
| • $2a + 2b = 24$ |  | • $a + b = 12$ | | • $b = ?$ |
|  Eliminate 'a' from this equation | | | | |

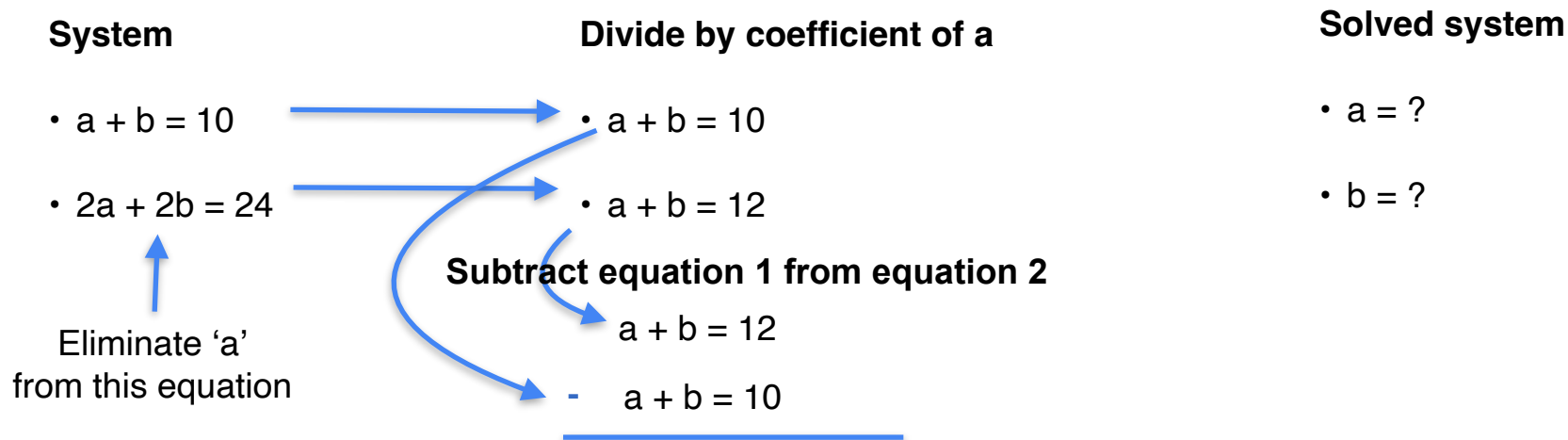
What if the system is singular (contradictory)?



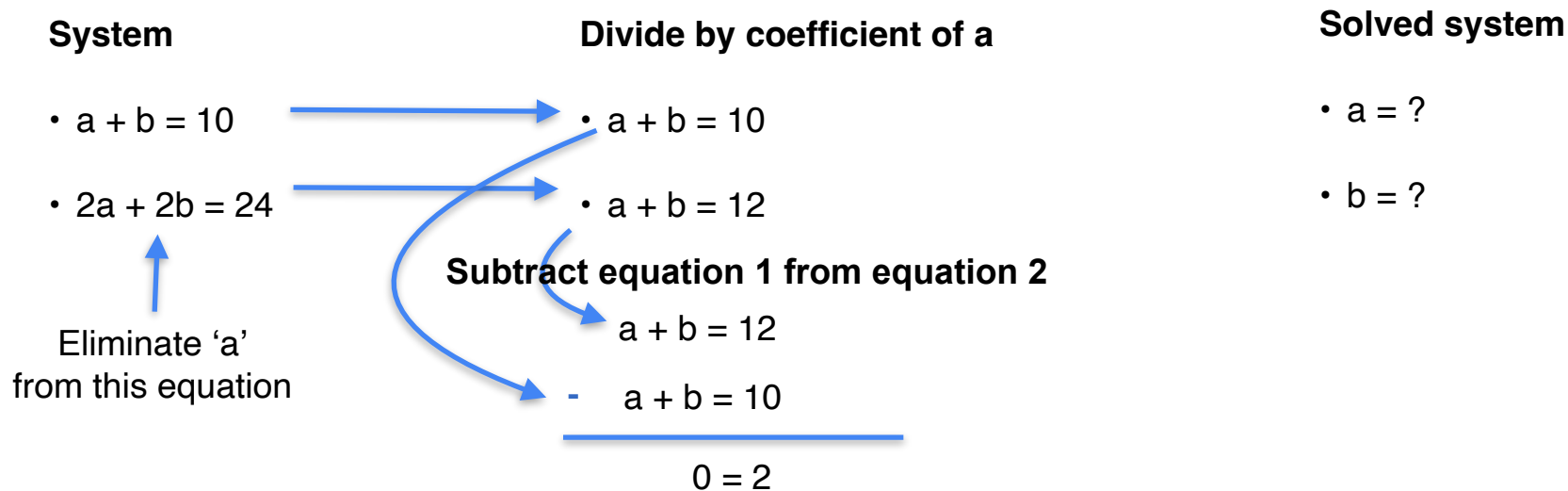
What if the system is singular (contradictory)?



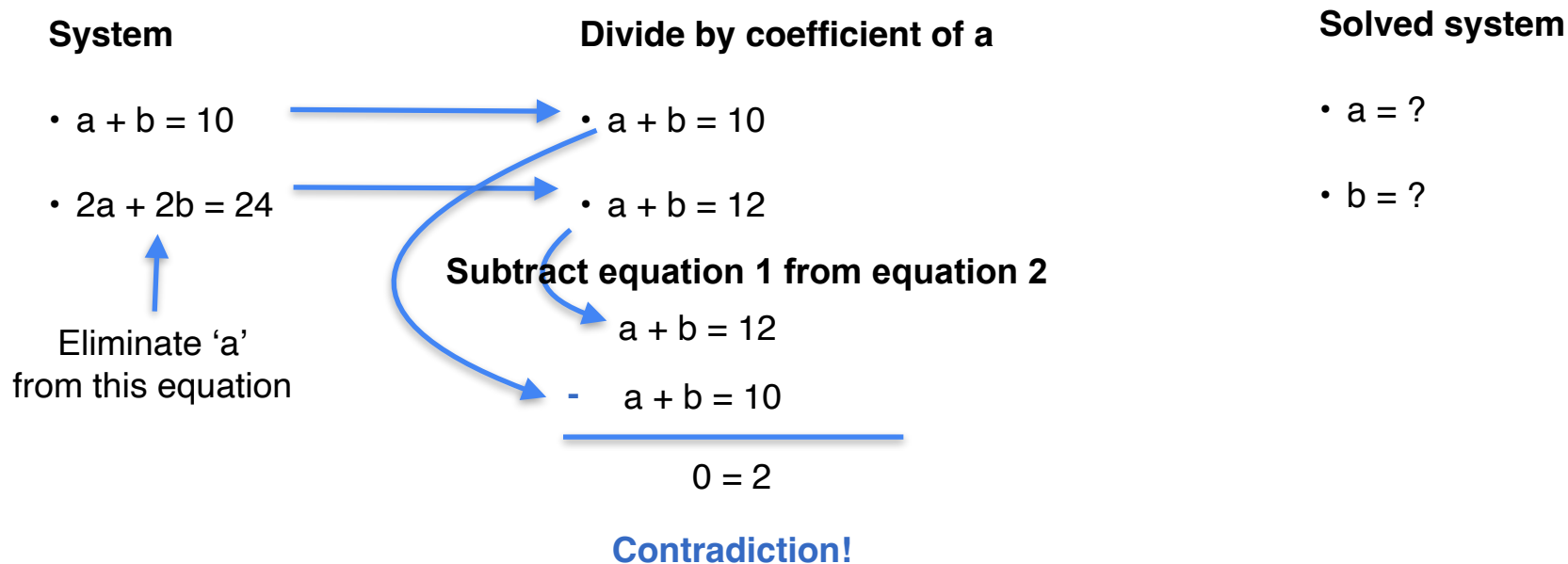
What if the system is singular (contradictory)?



What if the system is singular (contradictory)?



What if the system is singular (contradictory)?



Quiz

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution: If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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Solving System of Linear Equations

**Solving system of equations
with more variables**

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Elimination method


System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by
itself



Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each
row by the
coefficient of 'a'

Elimination method

System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

- $a + b + 2c = 12$

- $a - b - 1/3 c = 1$

- $a - b/2 + 3c = 12$

Divide each
row by the
coefficient of 'a'

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each
row by the
coefficient of 'a'

$$• a + b + 2c = 12$$

$$• a - b - \frac{1}{3}c = 1$$

$$• a - \frac{b}{2} + 3c = 12$$

Use the first
equation to
remove 'a' from
the others

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each
row by the
coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first
equation to
remove 'a' from
the others

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each
row by the
coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first
equation to
remove 'a' from
the others

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Isolated 'a'

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - \frac{1}{3}c = 1$
- $a - \frac{b}{2} + 3c = 12$

Use the first equation to remove 'a' from the others

- $a + b + 2c = 12$
- $-2b - \frac{7}{3}c = -11$
- $-\frac{3}{2}b + c = 0$

Isolated 'a'

Solve this new system of 2 equations

Elimination method

System

- $a + b + 2c = 12$
- $-2b - \frac{7}{3}c = -11$
- $-\frac{3}{2}b + c = 0$

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two
rows by the
coefficient of b

Elimination method

System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Divide last two
rows by the
coefficient of b

Elimination method

System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two
rows by the
coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second
equation to
remove 'b' from
the third

Elimination method

System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two
rows by the
coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second
equation to
remove 'b' from
the third

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $-11/6 c = -11/2$

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
 - $b + 7/6 c = 11/2$
 - $-11/6 c = -11/2$
- Isolated 'b'
- $c = 3$

Elimination method

System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

Elimination method


System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Elimination method

System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$  $b + \frac{7}{2} = \frac{11}{2}$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Elimination method

System



- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$  $b + \frac{7}{2} = \frac{11}{2}$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

Elimination method

System


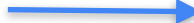
- $a + b + 2c = 12$  $a + 2 + 6 = 12$
 $a = 4$
- $b + 7/6 c = 11/2$  $b + 7/2 = 11/2$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

Elimination method

System

- $a + b + 2c = 12$  $a + 2 + 6 = 12$
 $a = 4$
- $b + 7/6 c = 11/2$  $b + 7/2 = 11/2$
 $b = 2$
- $c = 3$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

The solution is
 $a = 4$
 $b = 2$
 $c = 3$



DeepLearning.AI

Solving System of Linear Equations

Matrix row reduction

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$



Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

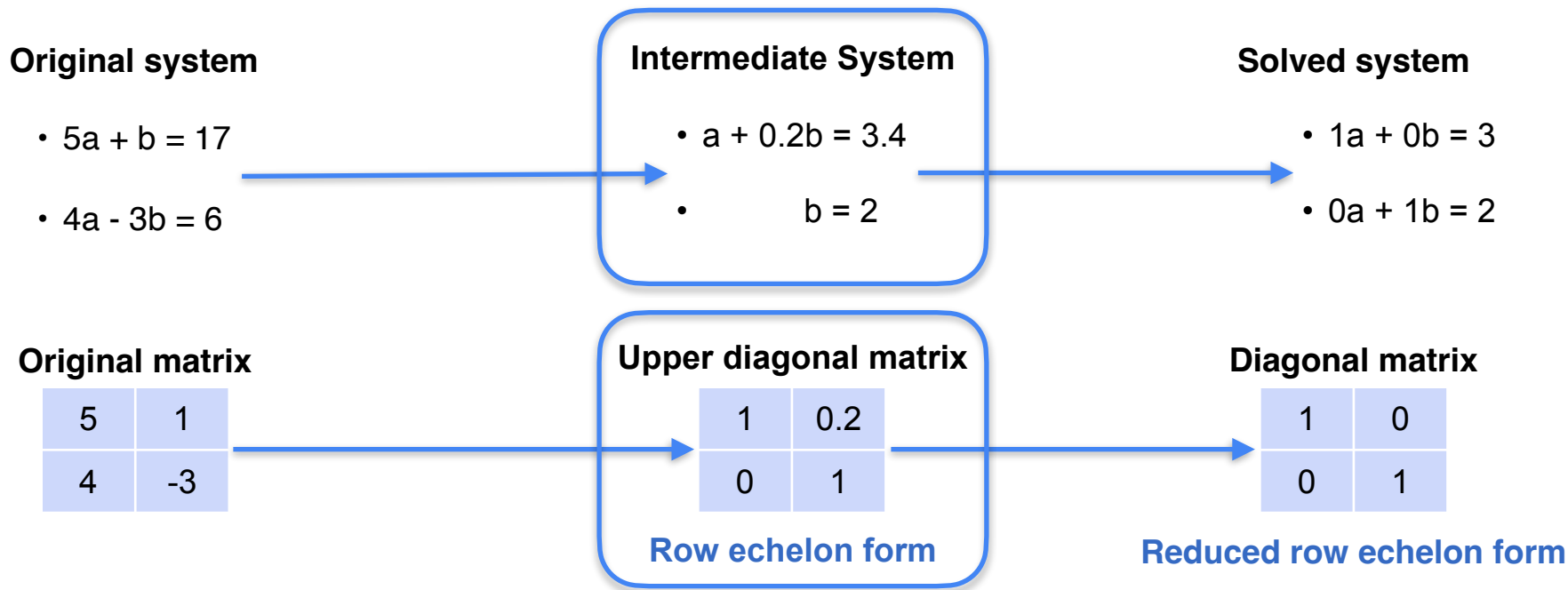
Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Row echelon form

Reduced row echelon form

Systems of equations to matrices



Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$



Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Upper diagonal matrix

| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Upper diagonal matrix

| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

Row echelon form

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Upper diagonal matrix

| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

Row echelon form

Systems of equations to matrices

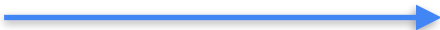
Original system

- $5a + b = 11$
- $10a + 2b = 22$

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$



Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

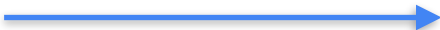
Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

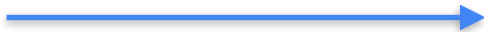
Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



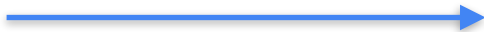
Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



Original matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Upper diagonal matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Upper diagonal matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Upper diagonal matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Row echelon form



DeepLearning.AI

Solving System of Linear Equations

**Row operations that
preserve singularity**

Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | |
|---|---|
| 4 | 3 |
| 5 | 1 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

| | |
|---|---|
| 4 | 3 |
| 5 | 1 |

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

| | |
|---|---|
| 4 | 3 |
| 5 | 1 |

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



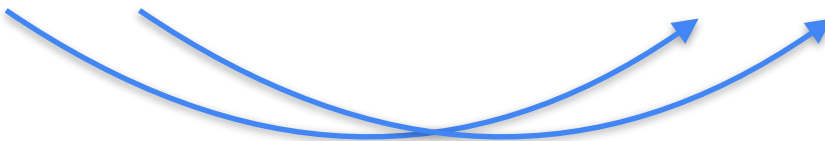
Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

| | |
|---|---|
| 4 | 3 |
| 5 | 1 |

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

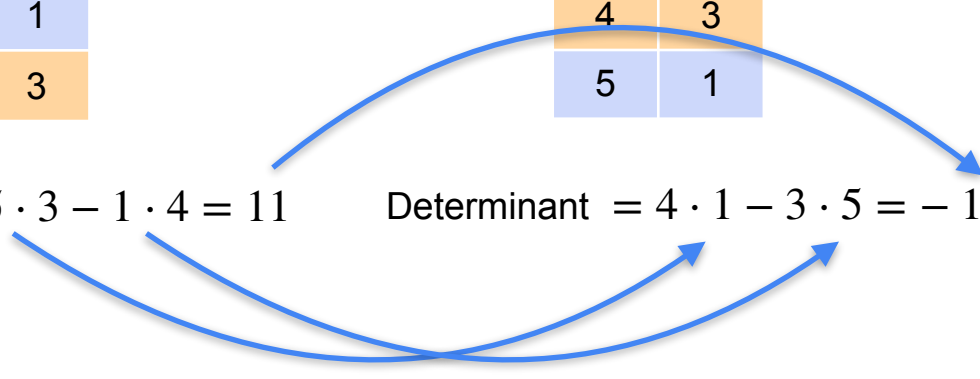


Switching rows

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

| | |
|---|---|
| 4 | 3 |
| 5 | 1 |

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$


Multiplying a row by a (non-zero) scalar

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

Multiplying a row by a (non-zero) scalar

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

Multiplying a row by a (non-zero) scalar

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

| | |
|---|---|
| 4 | 3 |
|---|---|

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

| | |
|---|---|
| 5 | 1 |
|---|---|

| | |
|---|---|
| 4 | 3 |
|---|---|

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

| | |
|---|---|
| 5 | 1 |
|---|---|

 $\times 10 =$

| | |
|---|---|
| 4 | 3 |
|---|---|

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

| | |
|---|---|
| 4 | 3 |
|---|---|

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

| | |
|----|----|
| 50 | 10 |
| 4 | 3 |

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

| | |
|----|----|
| 50 | 10 |
| 4 | 3 |

$$\text{Determinant} = 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$$

Multiplying a row by a (non-zero) scalar

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

| | |
|---|---|
| 5 | 1 |
|---|---|

 $\times 10 =$

| | |
|----|----|
| 50 | 10 |
|----|----|

| | |
|----|----|
| 50 | 10 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4) \\ &= 10 \cdot 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | |
|---|---|
| 5 | 1 |
|---|---|

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | | |
|---|-------|---|
| | 5 | 1 |
| + | 4 | 3 |
| | <hr/> | |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | | |
|-------|---|---|
| + | 5 | 1 |
| | 4 | 3 |
| <hr/> | | |
| | 9 | 4 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | | |
|-------|---|---|
| + | 5 | 1 |
| | 4 | 3 |
| <hr/> | | |
| | 9 | 4 |

| | |
|---|---|
| 9 | 4 |
|---|---|

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

| | | |
|-------|---|---|
| + | 5 | 1 |
| | 4 | 3 |
| <hr/> | | |
| | 9 | 4 |

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

| | | |
|-------|---|---|
| + | 5 | 1 |
| | 4 | 3 |
| <hr/> | | |
| | 9 | 4 |

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

$$\text{Determinant} = 9 \cdot 3 - 4 \cdot 4$$

Adding a row to another row

| | |
|---|---|
| 5 | 1 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

| | | | | | |
|---|---|---|---|---|---|
| + | <table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>3</td></tr></table> | 5 | 1 | 4 | 3 |
| 5 | 1 | | | | |
| 4 | 3 | | | | |
| | <table><tr><td>9</td><td>4</td></tr></table> | 9 | 4 | | |
| 9 | 4 | | | | |

| | |
|---|---|
| 9 | 4 |
| 4 | 3 |

$$\begin{aligned}\text{Determinant} &= 9 \cdot 3 - 4 \cdot 4 \\ &= 11\end{aligned}$$



DeepLearning.AI

Solving System of Linear Equations

Rank of a matrix

Compressing Images - Reducing rank

Compressing Images - Reducing rank



Compressing Images - Reducing rank

Original (Rank 200)

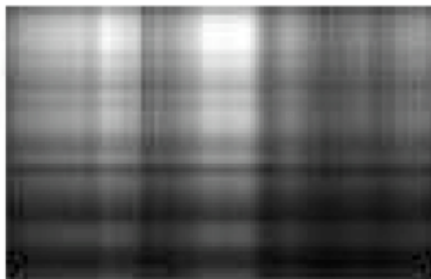


Compressing Images - Reducing rank

Original (Rank 200)



Rank 1

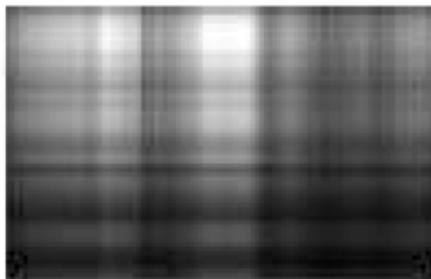


Compressing Images - Reducing rank

Original (Rank 200)



Rank 1



Rank 2



Rank 5

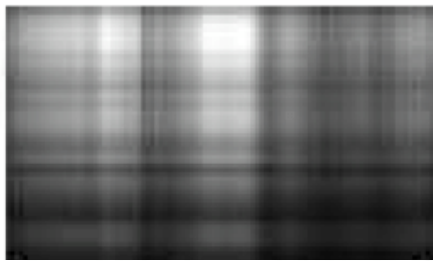


Compressing Images - Reducing rank

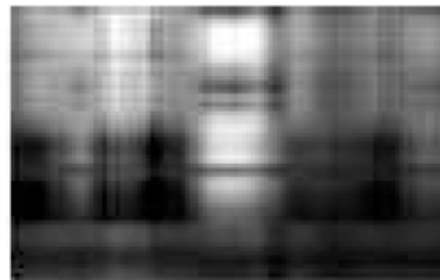
Original (Rank 200)



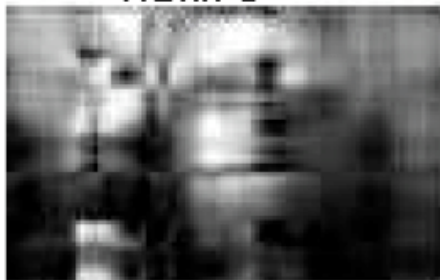
Rank 1



Rank 2



Rank 5



Rank 15

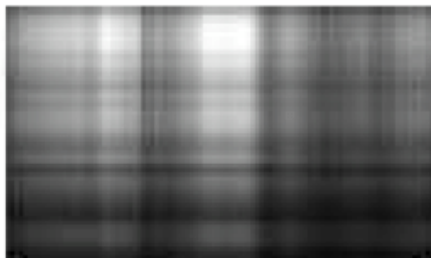


Compressing Images - Reducing rank

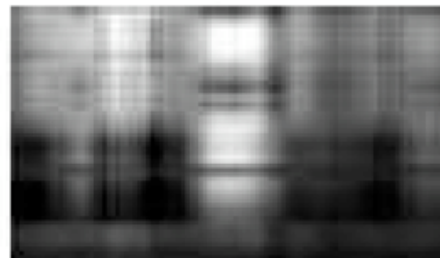
Original (Rank 200)



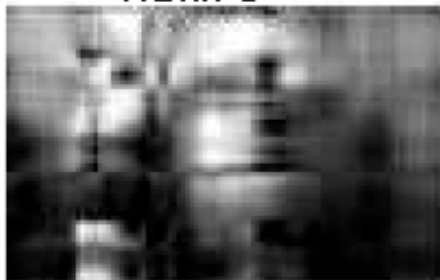
Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



Systems of information



Systems of information

System 1



 The dog is **black**
 The cat is **orange**

Systems of information

System 1



 The dog is **black**
 The cat is **orange**

System 2



 The dog is **black**
 The dog is **black**

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2



 The dog is **black**
 The dog is **black**

System 3



 The dog
 The dog

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**



System 3

 The dog
 The dog



Two sentences

Systems of information

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

System 3

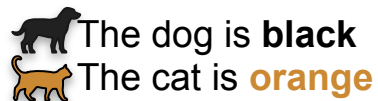
 The dog
 The dog

Two sentences

Two pieces of information

Systems of information

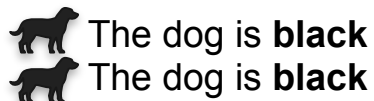
System 1



Two sentences

Two pieces of information

System 2



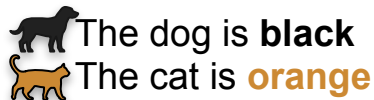
Two sentences

System 3



Systems of information

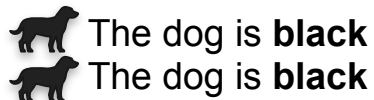
System 1



Two sentences

Two pieces of information

System 2



Two sentences

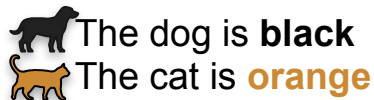
One piece of information

System 3



Systems of information

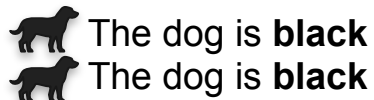
System 1



Two sentences

Two pieces of information

System 2



Two sentences

One piece of information

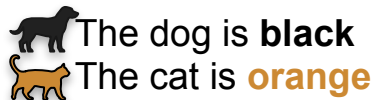
System 3



Two sentences

Systems of information

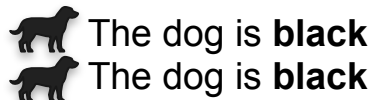
System 1



Two sentences

Two pieces of information

System 2



Two sentences

One piece of information

System 3

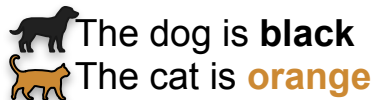


Two sentences

Zero pieces of information

Systems of information

System 1

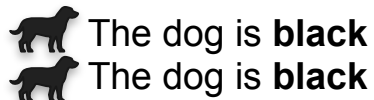


Two sentences

Two pieces of information

Rank = 2

System 2



Two sentences

One piece of information

System 3





Two sentences

Zero pieces of information

Systems of information

System 1



 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3



 The dog
 The dog

Two sentences

Zero pieces of information

Systems of information

System 1



 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

 The dog
 The dog

Two sentences

Zero pieces of information

Rank = 0

Systems of equations

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



System 2

$$a + b = 0$$







$$2a + 2b = 0$$





Systems of equations



System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 



System 3



$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations



System 1



$$a + b = 0$$
 

$$a + 2b = 0$$
 

Two equations

System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 


System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$


Systems of equations


System 1

$$a + b = 0$$


$$a + 2b = 0$$


System 2

$$a + b = 0$$


$$2a + 2b = 0$$


System 3

$$0a + 0b = 0$$



$$0a + 0b = 0$$



Two equations

Two pieces of information



Systems of equations



System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$


Two equations


Two pieces of information

Rank = 2

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations


System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information


System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$


$$0a + 0b = 0$$

Two equations

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$


$$0a + 0b = 0$$


Two equations

Zero pieces of information

Systems of equations

System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

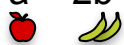
Systems of equations



System 1

$$a + b = 0$$



$$a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 1 | 2 |

Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$



$$2a + 2b = 0$$



Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

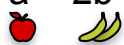
Systems of equations



System 1

$$a + b = 0$$



$$a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 1 | 2 |

Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

$$a + b = 0$$



$$2a + 2b = 0$$



Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

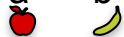
Zero pieces of information

Rank = 0

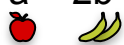
Systems of equations



System 1

$$a + b = 0$$



$$a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 1 | 2 |

Rank = 2

Two equations

Two pieces of information

Rank = 2



System 2

$$a + b = 0$$



$$2a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 2 | 2 |

Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

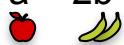
Systems of equations



System 1

$$a + b = 0$$



$$a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 1 | 2 |

Rank = 2

Two equations

Two pieces of information

Rank = 2



System 2

$$a + b = 0$$



$$2a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 2 | 2 |

Rank = 1

Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

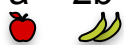
Systems of equations



System 1

$$a + b = 0$$



$$a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 1 | 2 |

Rank = 2

Two equations

Two pieces of information

Rank = 2



System 2

$$a + b = 0$$



$$2a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 2 | 2 |

Rank = 1



Two equations

One piece of information

Rank = 1

System 3

$$0a + 0b = 0$$

| | |
|---|---|
|   | |
| 0 | 0 |
| 0 | 0 |

$$0a + 0b = 0$$

Rank = 0

Two equations

Zero pieces of information

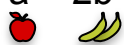
Systems of equations



System 1

$$a + b = 0$$



$$a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 1 | 2 |

Rank = 2

Two equations

Two pieces of information

Rank = 2



System 2

$$a + b = 0$$



$$2a + 2b = 0$$



| | |
|---|---|
|   | |
| 1 | 1 |
| 2 | 2 |

Rank = 1

Two equations

One piece of information



Rank = 1

System 3

$$0a + 0b = 0$$



$$0a + 0b = 0$$

| | |
|---|---|
|   | |
| 0 | 0 |
| 0 | 0 |



Rank = 0

Two equations

Zero pieces of information



Rank = 0

Rank and solutions to the system





| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |



Rank = 1



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



Rank = 0

Rank and solutions to the system





| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

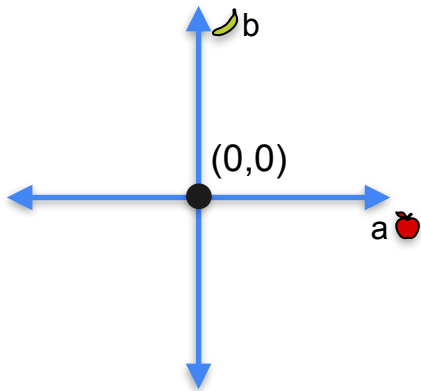
Rank = 1





| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank = 0

Dimension of solution space = 0





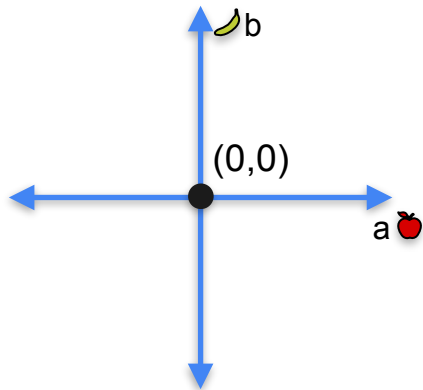
Rank and solutions to the system



| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



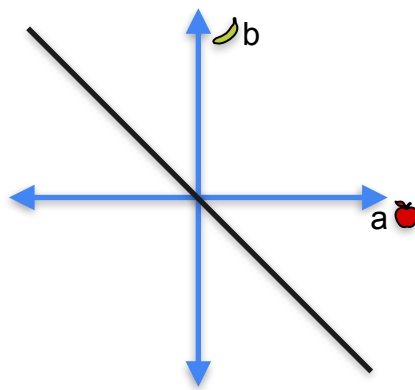
Dimension of solution space = 0



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Rank = 1



Dimension of solution space = 1



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

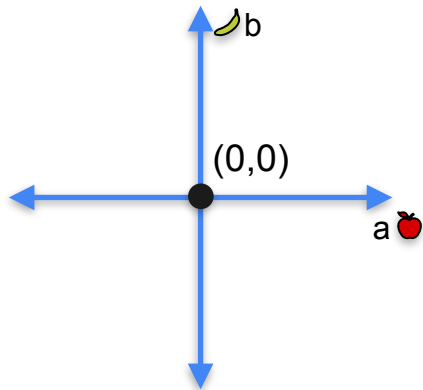
Rank = 0



Rank and solutions to the system

| | |
|---|---|
|  |  |
| 1 | 1 |
| 1 | 2 |

Rank = 2

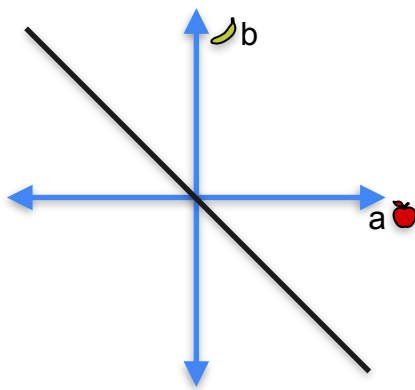
Dimension of solution space = 0





| | |
|---|---|
|  |  |
| 1 | 1 |
| 2 | 2 |

Rank = 1

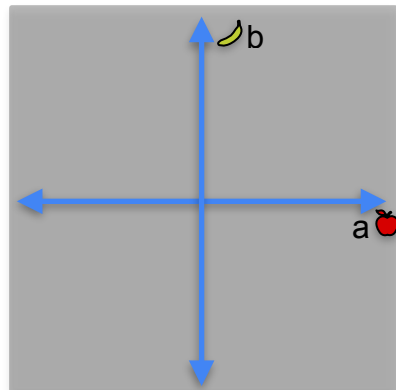
Dimension of solution space = 1





| | |
|---|---|
|  |  |
| 0 | 0 |
| 0 | 0 |

Rank = 0

Dimension of solution space = 2





Rank of a matrix





| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Rank = 1





| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank = 0

Dimension of solution space = 0 Dimension of solution space = 1 Dimension of solution space = 2



$$\text{Rank} = 2 - (\text{Dimension of solution space})$$

Rank and singularity





| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |



Rank = 1



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



Rank = 0

Rank and singularity





| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Rank = 1





| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank = 0

Non-singular



Rank and singularity



| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



Non-singular



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Rank = 1



Singular



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank = 0



Rank and singularity



| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



Non-singular



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Rank = 1

Singular





| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank = 0

Singular



Rank and singularity



| | |
|---|---|
| 1 | 1 |
| 1 | 2 |

Rank = 2



Non-singular



| | |
|---|---|
| 1 | 1 |
| 2 | 2 |

Rank = 1

Singular



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank = 0

Singular

Quiz: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1

| | |
|----|---|
| 5 | 1 |
| -1 | 3 |

Matrix 2

| | |
|----|----|
| 2 | -1 |
| -6 | 3 |

Solutions: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1: Since the solution space had dimension 0, the rank is **2**.

| | |
|----|---|
| 5 | 1 |
| -1 | 3 |

Matrix 2: Since the solution space had dimension 1, the rank is **1**.

| | |
|----|----|
| 2 | -1 |
| -6 | 3 |



DeepLearning.AI

Solving System of Linear Equations

**Rank of a matrix:
General case**

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$



System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

System 4


$$0a + 0b + 0c = 0$$

$$0a + 0b + 0c = 0$$

$$0a + 0b + 0c = 0$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$


System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank 3

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank 2

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



3 Equations
0 Pieces of information

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



3 Equations
0 Pieces of information

Rank 0

Rank for matrices

System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



3 Equations
3 Pieces of information

Rank 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



3 Equations
2 Pieces of information

Rank 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



3 Equations
1 Piece of information

Rank 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



3 Equations
0 Pieces of information

Rank 0

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



DeepLearning.AI

Solving System of Linear Equations

Row echelon form

Row echelon form of a matrix

Row echelon form of a matrix

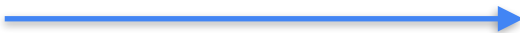
Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Row echelon form of a matrix

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

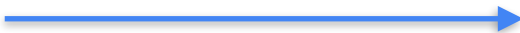
Row echelon form of a matrix

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |



| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Row echelon form of a matrix

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

Row echelon form of a matrix

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Row echelon form of a matrix

Original matrix

Row echelon form

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Row echelon form

Original matrix


| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

Divide each row by
the leftmost coefficient

Row echelon form

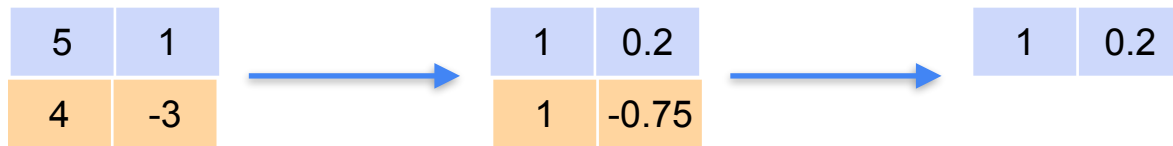
Original matrix

| | | | | |
|---|----|---|---|-------|
| 5 | 1 |  | 1 | 0.2 |
| 4 | -3 | | 1 | -0.75 |

Divide each row by
the leftmost coefficient

Row echelon form

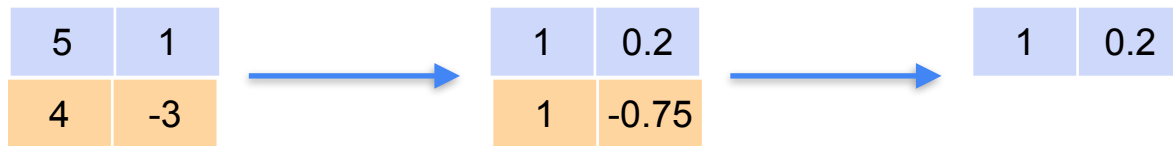
Original matrix



Divide each row by
the leftmost coefficient

Row echelon form

Original matrix



Divide each row by
the leftmost coefficient

| | |
|---|-------|
| 1 | -0.75 |
|---|-------|

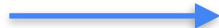
Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-------|
| 1 | 0.2 |
| 1 | -0.75 |



| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

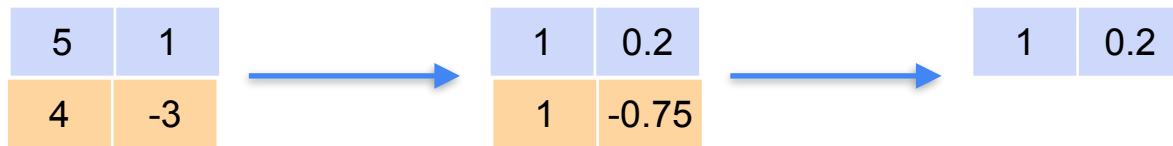
Divide each row by
the leftmost coefficient

| | |
|---|-------|
| 1 | -0.75 |
|---|-------|

| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

Row echelon form

Original matrix

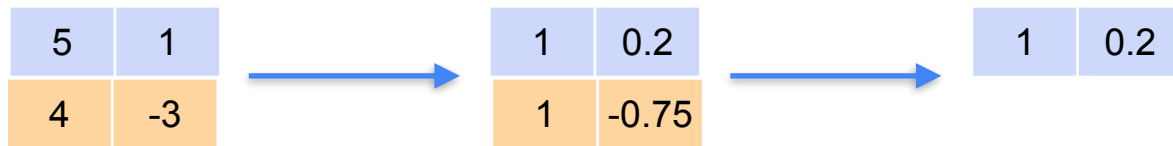


Divide each row by
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \end{array}$$

Row echelon form

Original matrix

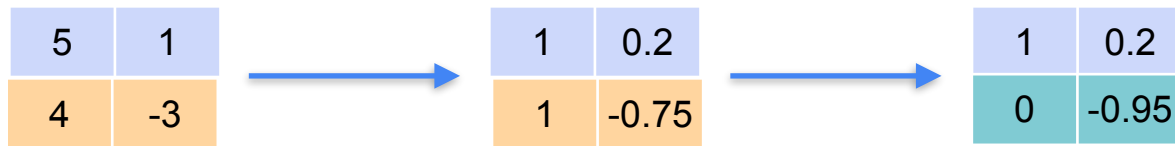


Divide each row by
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

Row echelon form

Original matrix



Divide each row by
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-------|
| 1 | 0.2 |
| 1 | -0.75 |



| | |
|---|-------|
| 1 | 0.2 |
| 0 | -0.95 |

Divide each row by
the leftmost coefficient

| | | |
|-------|---|-------|
| | 1 | -0.75 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | -0.95 |

Divide the second row by
the leftmost non-zero coefficient

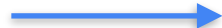
Row echelon form

Original matrix

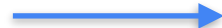
| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-------|
| 1 | 0.2 |
| 1 | -0.75 |



| | |
|---|-------|
| 1 | 0.2 |
| 0 | -0.95 |



Divide each row by
the leftmost coefficient

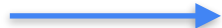
| | | |
|-------|---|-------|
| | 1 | -0.75 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | -0.95 |

Divide the second row by
the leftmost non-zero coefficient

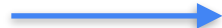
Row echelon form

Original matrix

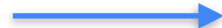
| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-------|
| 1 | 0.2 |
| 1 | -0.75 |



| | |
|---|-------|
| 1 | 0.2 |
| 0 | -0.95 |



| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

Divide each row by
the leftmost coefficient

| | | |
|-------|---|-------|
| | 1 | -0.75 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | -0.95 |

Divide the second row by
the leftmost non-zero coefficient

Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-------|
| 1 | 0.2 |
| 1 | -0.75 |



| | |
|---|-------|
| 1 | 0.2 |
| 0 | -0.95 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Divide each row by
the leftmost coefficient

| | | |
|-------|---|-------|
| | 1 | -0.75 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | -0.95 |

Divide the second row by
the leftmost non-zero coefficient

Row echelon form

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Divide each row by
the leftmost coefficient

| | |
|---|-------|
| 1 | 0.2 |
| 1 | -0.75 |

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

Divide the second row by
the leftmost non-zero coefficient

| | |
|---|-------|
| 1 | 0.2 |
| 0 | -0.95 |

Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Row echelon form for singular matrices

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Row echelon form for singular matrices

Original matrix


| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

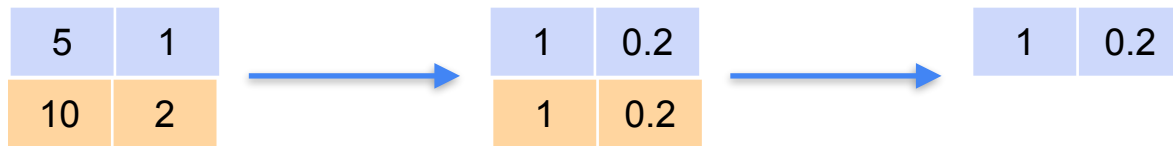
Original matrix

| | | | | |
|----|---|---|---|-----|
| 5 | 1 |  | 1 | 0.2 |
| 10 | 2 | | 1 | 0.2 |

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

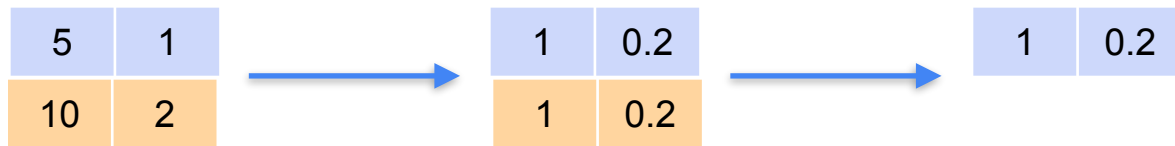


Divide each row by
the leftmost coefficient

| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

Row echelon form for singular matrices

Original matrix

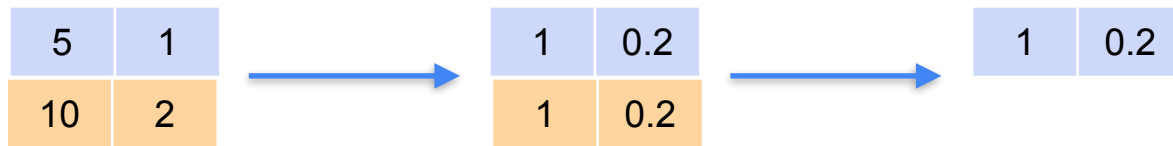


Divide each row by
the leftmost coefficient

| | |
|---|-----|
| 1 | 0.2 |
| 1 | 0.2 |

Row echelon form for singular matrices

Original matrix



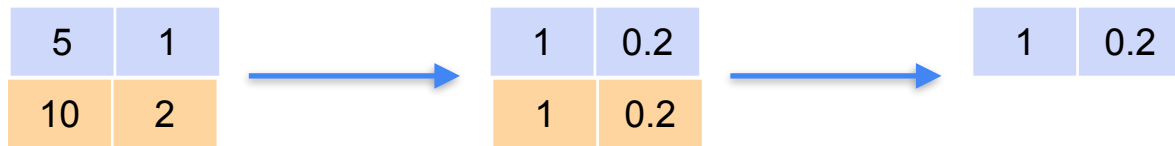
Divide each row by
the leftmost coefficient

The diagram shows the subtraction of the first row from the second row. It consists of two rows of boxes. The top row is orange and contains the values 1 and 0.2. The bottom row is blue and contains the values 1 and 0.2. A minus sign is placed to the left of the bottom row. A horizontal blue line is drawn below the bottom row.

| | | |
|---|-----|-----|
| 1 | 0.2 | |
| - | 1 | 0.2 |

Row echelon form for singular matrices

Original matrix

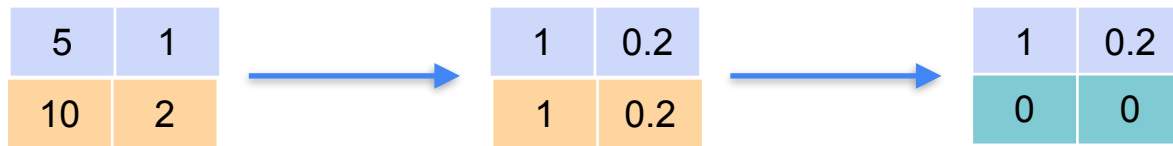


Divide each row by
the leftmost coefficient

$$\begin{array}{r|rr} & 1 & 0.2 \\ - & 1 & 0.2 \\ \hline & 0 & 0 \end{array}$$

Row echelon form for singular matrices

Original matrix



Divide each row by
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array} \end{array}$$

Row echelon form for singular matrices

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 1 | 0.2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Divide each row by
the leftmost coefficient

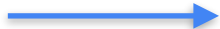
| | | |
|-------|---|-----|
| | 1 | 0.2 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | 0 |

Divide the second row by
the leftmost non-zero coefficient

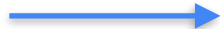
Row echelon form for singular matrices

Original matrix

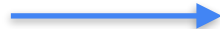
| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 1 | 0.2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |



Divide each row by
the leftmost coefficient

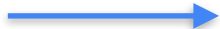
| | | |
|-------|---|-----|
| | 1 | 0.2 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | 0 |

Divide the second row by
the leftmost non-zero coefficient

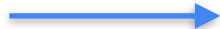
Row echelon form for singular matrices

Original matrix

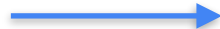
| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 1 | 0.2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |



| | |
|---|-----|
| 1 | 0.2 |
|---|-----|

Divide each row by
the leftmost coefficient

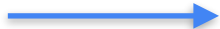
| | | |
|-------|---|-----|
| | 1 | 0.2 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | 0 |

Divide the second row by
the leftmost non-zero coefficient

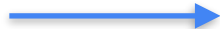
Row echelon form for singular matrices

Original matrix

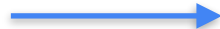
| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 1 | 0.2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |



| | |
|---|-----|
| 1 | 0.2 |
| ? | ? |

Divide each row by
the leftmost coefficient

| | | |
|-------|---|-----|
| | 1 | 0.2 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | 0 |

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

Divide each row by
the leftmost coefficient

| | |
|---|-----|
| 1 | 0.2 |
| 1 | 0.2 |

| | | |
|-------|---|-----|
| | 1 | 0.2 |
| - | 1 | 0.2 |
| <hr/> | | |
| | 0 | 0 |

Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Divide the second row by
the leftmost non-zero coefficient

| | |
|---|-----|
| 1 | 0.2 |
| ? | ? |

Row echelon form for singular matrices

Original matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Row echelon form for singular matrices

Original matrix


| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Row echelon form

Original matrix

| | | | | |
|---|---|---|---|---|
| 0 | 0 | → | ? | ? |
| 0 | 0 | | ? | ? |

Divide each row by
the leftmost coefficient

Row echelon form, singularity, and rank

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Row echelon form, singularity, and rank

| | | | | |
|---|----|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 4 | -3 | | 0 | 1 |

2 ones in the diagonal

| | | | | |
|----|---|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 10 | 2 | | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 0 | 0 | → | 0 | 0 |
| 0 | 0 | | 0 | 0 |

Row echelon form, singularity, and rank

| | | | | |
|---|----|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 4 | -3 | | 0 | 1 |

Rank 2
2 ones in the diagonal

| | | | | |
|----|---|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 10 | 2 | | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 0 | 0 | → | 0 | 0 |
| 0 | 0 | | 0 | 0 |

Row echelon form, singularity, and rank

| | | | | |
|---|----|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 4 | -3 | | 0 | 1 |

Rank 2

2 ones in the diagonal

| | | | | |
|----|---|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 10 | 2 | | 0 | 0 |

1 one in the diagonal

| | | | | |
|---|---|---|---|---|
| 0 | 0 | → | 0 | 0 |
| 0 | 0 | | 0 | 0 |

Row echelon form, singularity, and rank

| | | | | |
|---|----|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 4 | -3 | | 0 | 1 |

Rank 2

2 ones in the diagonal

| | | | | |
|----|---|---|---|-----|
| 5 | 1 | → | 1 | 0.2 |
| 10 | 2 | | 0 | 0 |

Rank 1

1 one in the diagonal

| | | | | |
|---|---|---|---|---|
| 0 | 0 | → | 0 | 0 |
| 0 | 0 | | 0 | 0 |

Row echelon form, singularity, and rank

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Rank 2

2 ones in the diagonal

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Rank 1

1 one in the diagonal

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

0 ones in the diagonal

Row echelon form, singularity, and rank

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

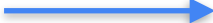


| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Rank 2

2 ones in the diagonal

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Rank 1

1 one in the diagonal

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

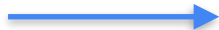


| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Rank 2

2 ones in the diagonal

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Rank 1

1 one in the diagonal

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Rank 2

2 ones in the diagonal

Singular matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Rank 1

1 one in the diagonal

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Rank 2

2 ones in the diagonal

Singular matrix

| | |
|----|---|
| 5 | 1 |
| 10 | 2 |



| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0 |

Rank 1

1 one in the diagonal

Singular matrix

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |



| | |
|---|---|
| 0 | 0 |
| 0 | 0 |

Rank 0

0 ones in the diagonal



DeepLearning.AI

Solving System of Linear Equations

**Row echelon form:
General case**

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Matrix

| | | |
|---|----|----|
| 1 | 1 | 2 |
| 3 | -3 | -1 |
| 2 | -1 | 6 |

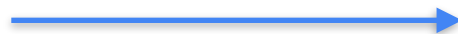
Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Matrix

| | | |
|---|----|----|
| 1 | 1 | 2 |
| 3 | -3 | -1 |
| 2 | -1 | 6 |



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Row echelon form matrix

| | | |
|---|----|---|
| 1 | 1 | 2 |
| 0 | -6 | 7 |
| 0 | 0 | 6 |

Row echelon form

| | | | | |
|---|---|---|----|---|
| 2 | * | * | * | * |
| 0 | 1 | * | * | * |
| 0 | 0 | 3 | * | * |
| 0 | 0 | 0 | -5 | * |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Row echelon form

| | | | | |
|---|---|---|----|---|
| 2 | * | * | * | * |
| 0 | 1 | * | * | * |
| 0 | 0 | 3 | * | * |
| 0 | 0 | 0 | -5 | * |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

- Zero rows at the bottom

Row echelon form

| | | | | |
|---|---|---|----|---|
| 2 | * | * | * | * |
| 0 | 1 | * | * | * |
| 0 | 0 | 3 | * | * |
| 0 | 0 | 0 | -5 | * |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)

Row echelon form

| | | | | |
|---|---|---|----|---|
| 2 | * | * | * | * |
| 0 | 1 | * | * | * |
| 0 | 0 | 3 | * | * |
| 0 | 0 | 0 | -5 | * |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above

Row echelon form

| | | | | |
|---|---|---|----|---|
| 2 | * | * | * | * |
| 0 | 1 | * | * | * |
| 0 | 0 | 3 | * | * |
| 0 | 0 | 0 | -5 | * |
| 0 | 0 | 0 | 0 | 1 |

Rank 5

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Rank 3

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

Another example

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Another example

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Subtract the first row
from the second and
the third ones

Another example

| Matrix | | | | Row echelon form | | |
|--------|---|---|---|------------------|---|---|
| 1 | 1 | 1 | → | 1 | 1 | 1 |
| 1 | 2 | 1 | | 0 | 1 | 0 |
| 1 | 1 | 2 | | 0 | 0 | 1 |

Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 2 |

Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

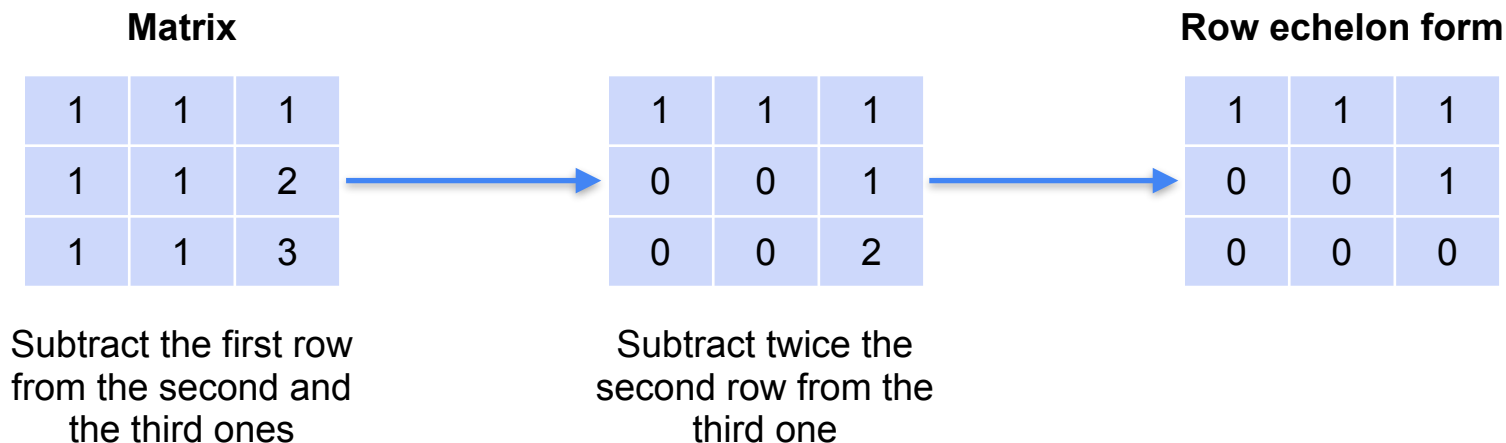
Subtract the first row
from the second and
the third ones



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 2 |

Subtract twice the
second row from the
third one

What if the matrix is singular?



What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Subtract twice the
first row from the
second row

What if the matrix is singular?

Matrix

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | 1 | 1 | 1 |
| 2 | 2 | 2 | → | 0 | 0 | 0 |
| 3 | 3 | 3 | | 3 | 3 | 3 |

Subtract twice the
first row from the
second row

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Subtract twice the
first row from the
second row



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 3 | 3 | 3 |

Subtract three times
the first row from the
third row

What if the matrix is singular?

Matrix

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Subtract twice the
first row from the
second row



| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 3 | 3 | 3 |

Subtract three times
the first row from the
third row



Row echelon form

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 3

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 1

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 1

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 0

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank = 3

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 1

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 0

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank = 3

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank = 2

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 1

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 0

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank = 3

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank = 2

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Rank = 1

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 1

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 0

Rank for matrices

Matrix 1

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Rank = 3

Matrix 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Rank = 2

Matrix 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Rank = 1

Matrix 4

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Rank = 0

Row echelon forms

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Number of pivots = 3

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Number of pivots = 2

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 1

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Number of pivots = 0



DeepLearning.AI

Solving System of Linear Equations

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$



Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Row echelon form

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

| | |
|---|----|
| 5 | 1 |
| 4 | -3 |

Upper diagonal matrix

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Row echelon form

Diagonal matrix

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

Reduced row echelon form

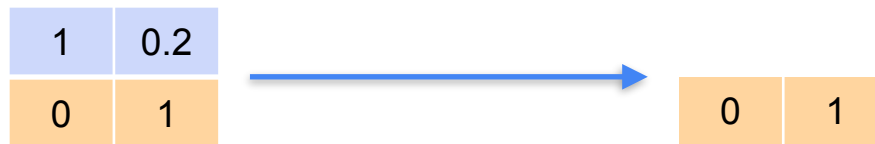
Reduced row echelon form

Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Reduced row echelon form

Row echelon form



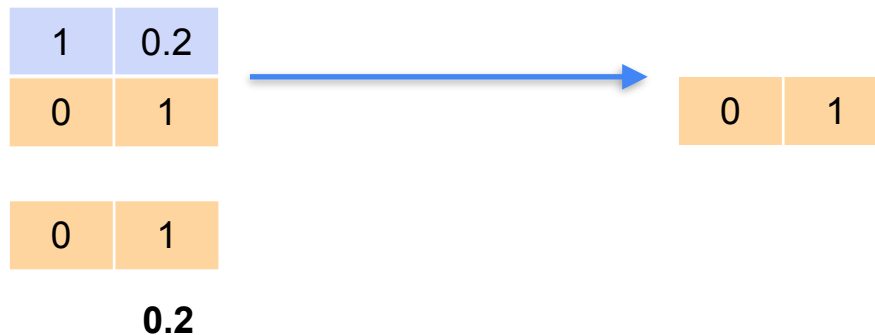
Reduced row echelon form

Row echelon form



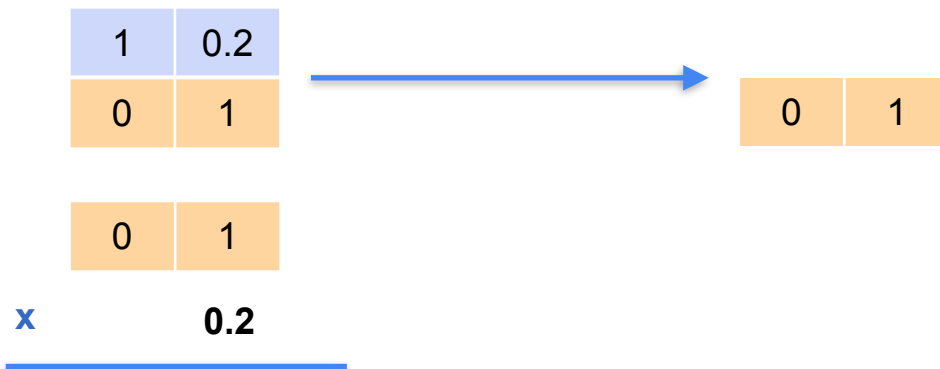
Reduced row echelon form

Row echelon form



Reduced row echelon form

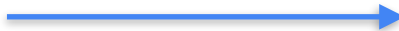
Row echelon form



Reduced row echelon form

Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |



| | |
|---|---|
| 0 | 1 |
|---|---|

| | |
|---|---|
| 0 | 1 |
|---|---|

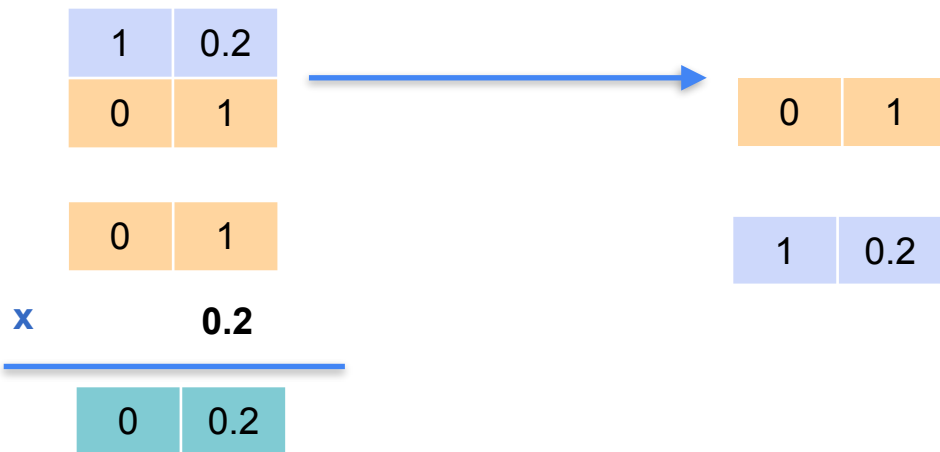
x

0.2

| | |
|---|-----|
| 0 | 0.2 |
|---|-----|

Reduced row echelon form

Row echelon form



Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form. It shows two matrices connected by a blue arrow, with the operation performed indicated below each matrix.

Left Matrix (Row Echelon Form):

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Operation: \times 0.2

Right Matrix (Reduced Row Echelon Form):

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0.2 |

Operation: $-$

Reduced row echelon form

Row echelon form

The diagram illustrates the process of converting a matrix into its reduced row echelon form (RREF) using row operations. It shows the initial matrix, the row echelon form, and the final RREF.

Initial Matrix:

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

Row Echelon Form:

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

Row Operation: $R_1 \leftarrow R_1 - 0.2R_2$

Reduced Row Echelon Form (RREF):

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form.

Initial Matrix (Row Echelon Form):

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

Operation: $\times 0.2$

Resulting Matrix (Reduced Row Echelon Form):

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

The operation shown is multiplying the first row by 0.2 to eliminate the 0.2 in the first column of the second row.

Reduced row echelon form

Row echelon form

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 1 |

| | |
|---|---|
| 0 | 1 |
|---|---|

x 0.2

| | |
|---|-----|
| 0 | 0.2 |
|---|-----|

Reduced row echelon form

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

| | |
|---|-----|
| 1 | 0.2 |
| 0 | 0.2 |

-

| | |
|---|---|
| 1 | 0 |
|---|---|

Reduced row echelon form

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Reduced row echelon form

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

● Is in row echelon form

Reduced row echelon form

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

- Is in row echelon form
- Each pivot is a 1

Reduced row echelon form

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0

Reduced row echelon form

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |

Rank 5

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

Reduced row echelon form

Reduced row echelon form

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 2 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Reduced row echelon form

Row echelon form

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 2 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 1 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Reduced row echelon form

Row echelon form

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 2 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 1 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Divide each row by
the value of the pivot

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Reduced row echelon form

Row echelon form

| | | | | |
|---|---|---|----|---|
| 3 | * | * | * | * |
| 0 | 0 | 2 | * | * |
| 0 | 0 | 0 | -4 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| 1 | * | * | * | * |
| 0 | 0 | 1 | * | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Divide each row by
the value of the pivot

Reduced row
echelon form

| | | | | |
|---|---|---|---|---|
| 1 | * | 0 | 0 | * |
| 0 | 0 | 1 | 0 | * |
| 0 | 0 | 0 | 1 | * |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Turn anything above a
pivot to 0

Reduced row echelon form

Row echelon form

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Reduced row echelon form

Row echelon form

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 2 times the
second row from the
first one

Reduced row echelon form

Row echelon form

| | | | | | | |
|---|---|---|---|---|---|----|
| 1 | 2 | 3 | → | 1 | 0 | -5 |
| 0 | 1 | 4 | | 0 | 1 | 4 |
| 0 | 0 | 1 | | 0 | 0 | 1 |

Subtract 2 times the
second row from the
first one

Reduced row echelon form

Row echelon form

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |



| | | |
|---|---|----|
| 1 | 0 | -5 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 2 times the
second row from the
first one

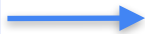
Add 5 times the third
row to the first one

Reduced row echelon form

Row echelon form

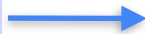
| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 2 times the
second row from the
first one



| | | |
|---|---|----|
| 1 | 0 | -5 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Add 5 times the third
row to the first one



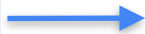
| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Reduced row echelon form

Row echelon form

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 2 times the second row from the first one



| | | |
|---|---|----|
| 1 | 0 | -5 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Add 5 times the third row to the first one



| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 4 times the third row from the second one

Reduced row echelon form

Row echelon form

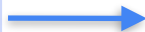
| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 2 times the second row from the first one



| | | |
|---|---|----|
| 1 | 0 | -5 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Add 5 times the third row to the first one



| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 4 times the third row from the second one



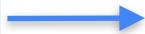
| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

Reduced row echelon form

Row echelon form

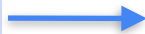
| | | |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 2 times the second row from the first one



| | | |
|---|---|----|
| 1 | 0 | -5 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Add 5 times the third row to the first one



| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 4 |
| 0 | 0 | 1 |

Subtract 4 times the third row from the second one



Reduced row echelon form

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |



DeepLearning.AI

Solving System of Linear Equations

Conclusion