DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

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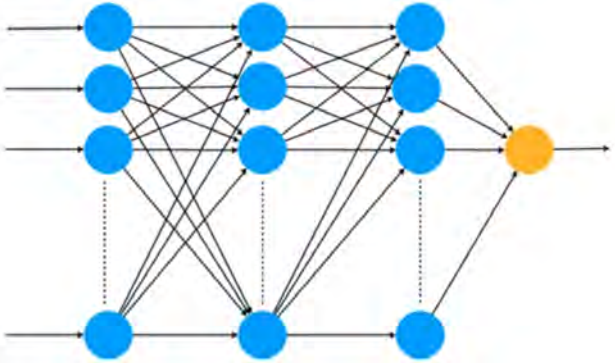
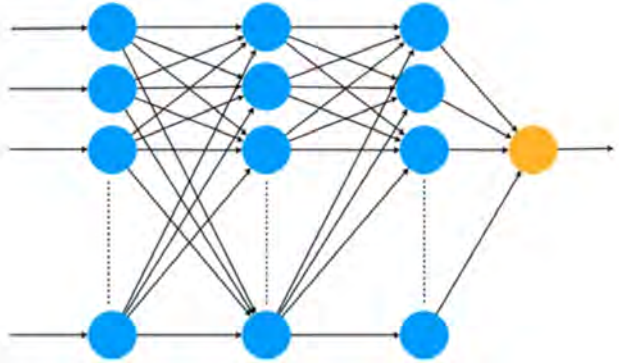
Solving System of Linear Equations

Machine learning motivation

Neural networks - Matrix operations

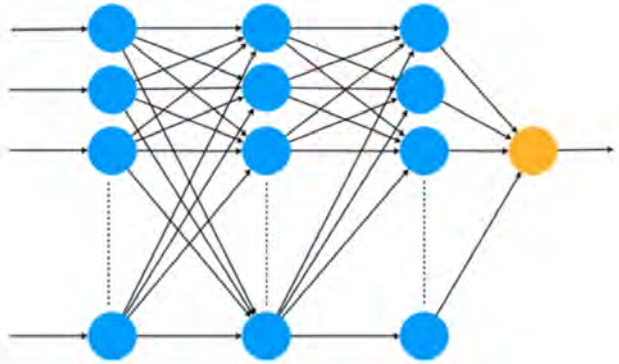


Neural networks - Matrix operations



"Hello! Welcome to Math for Machine Learning!"

Neural networks - Matrix operations



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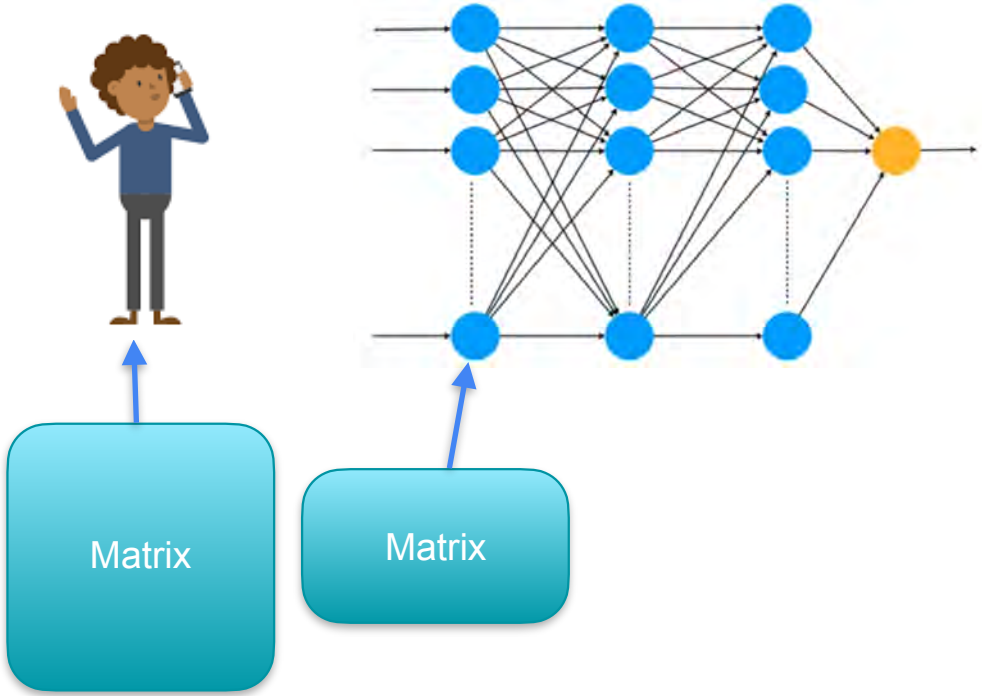
Neural networks - Matrix operations



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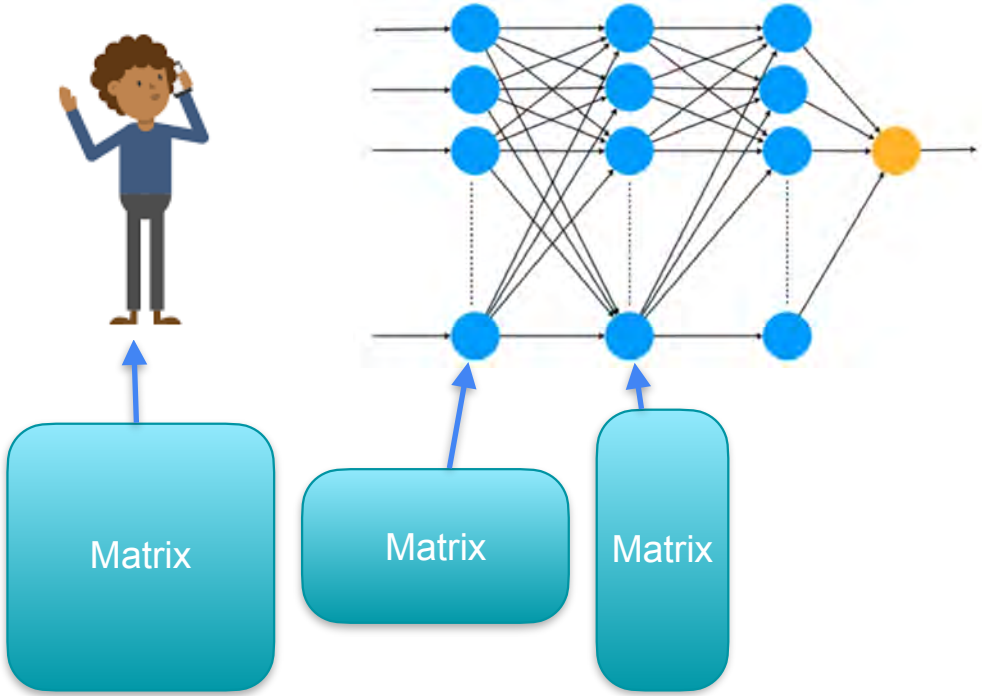
Neural networks - Matrix operations



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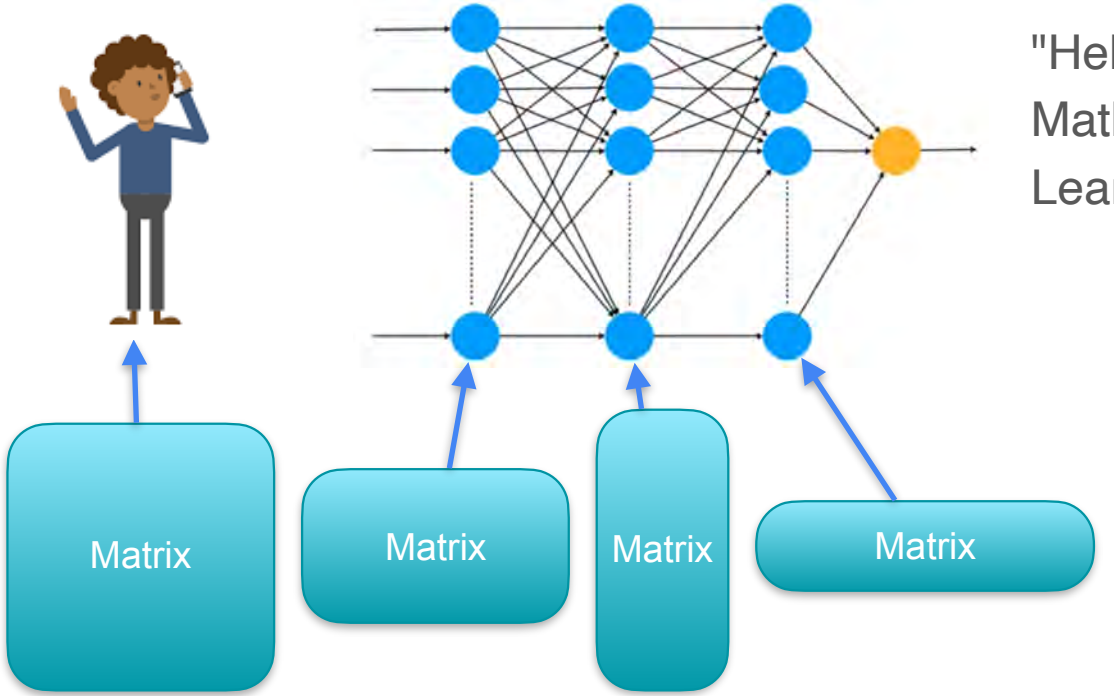
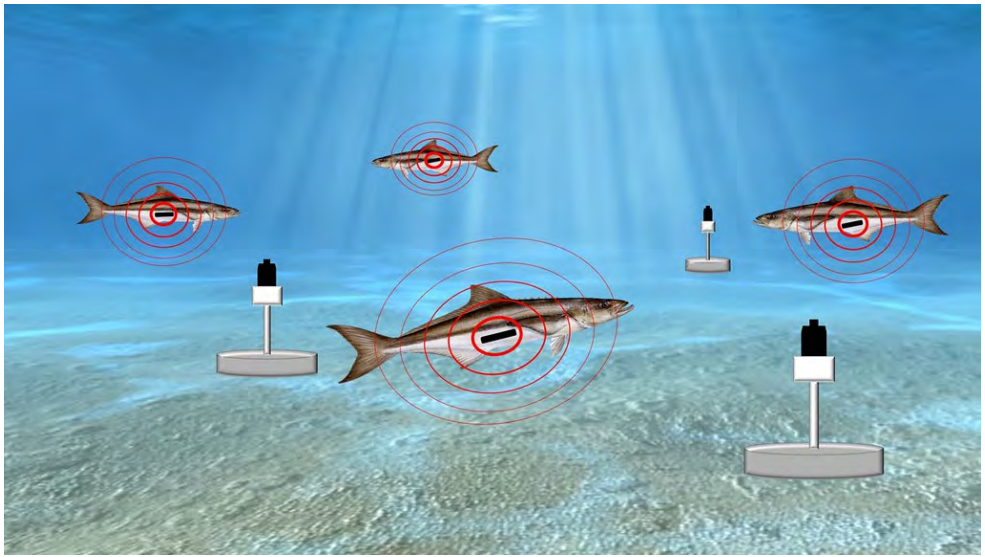
Neural networks - Matrix operations



"Hello! Welcome to Math for Machine Learning!"

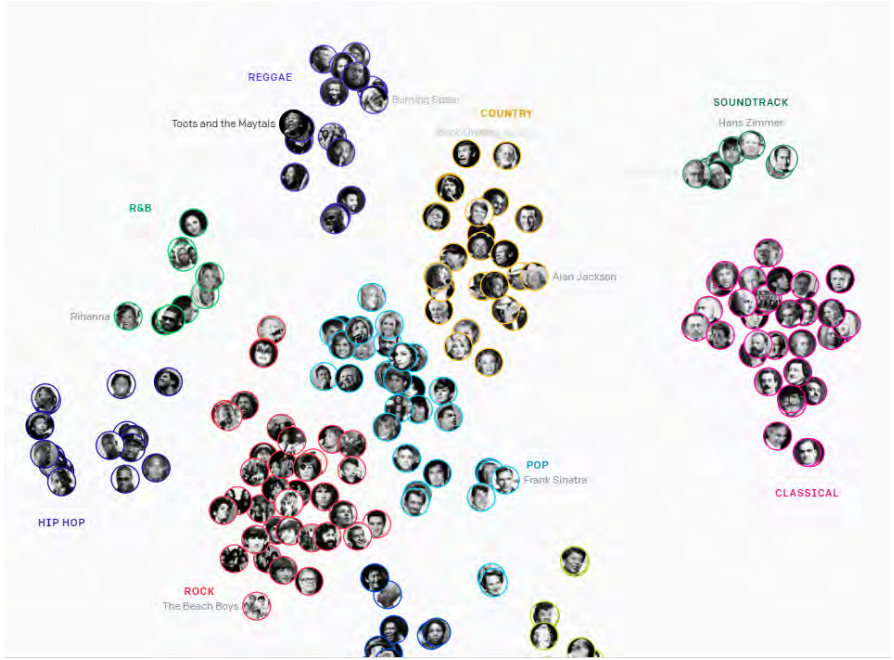
Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.

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Solving System of Linear Equations

Solving non-singular system of linear equations

Solving systems of equations

System

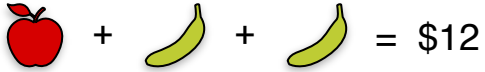
- a + b = 10

- a + 2b = 12

Solving systems of equations

System

- a + b = 10
- a + 2b = 12



Solving systems of equations

System

- a + b = 10
- a + 2b = 12

Solving systems of equations

System

- a + b = 10
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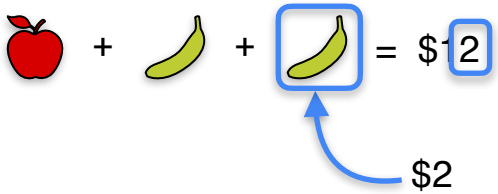
Solving systems of equations

System

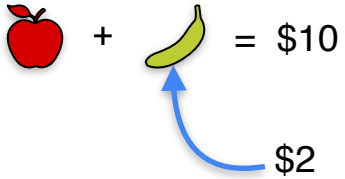
- a + b = 10
- a + 2b = 12

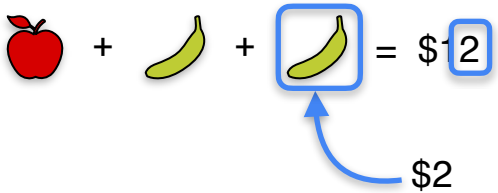


Solving systems of equations

System

- a + b = 10
- a + 2b = 12

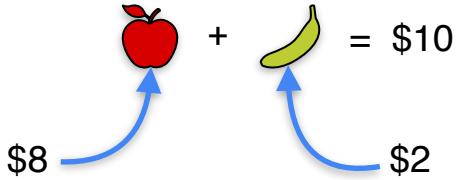


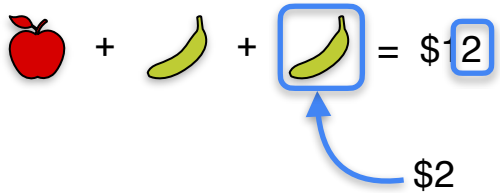


Solving systems of equations

System

- a + b = 10
- a + 2b = 12





Solving systems of equations

Solving systems of equations

System

- a + b = 10

- a + 2b = 12

Solving systems of equations

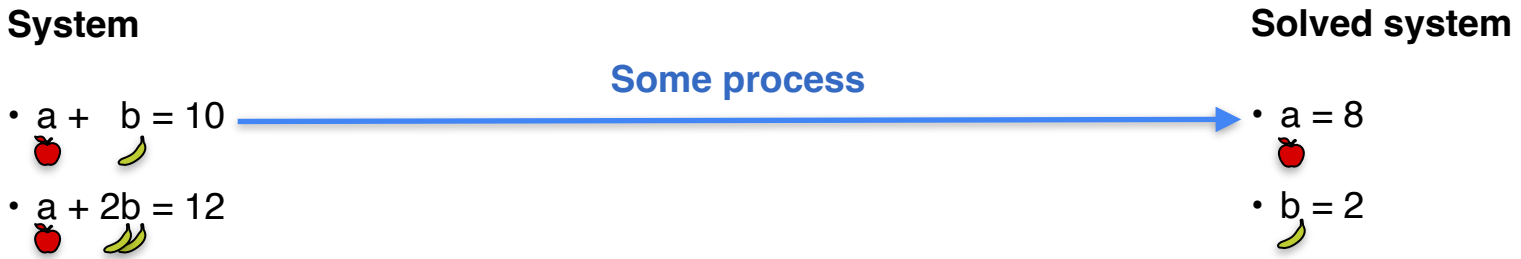
System

- a + b = 10
- a + 2b = 12

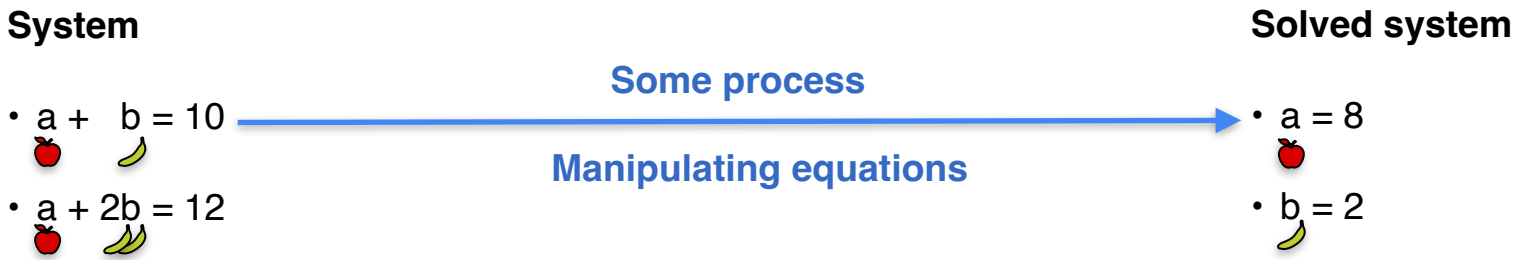
Solved system

- a = 8
- b = 2

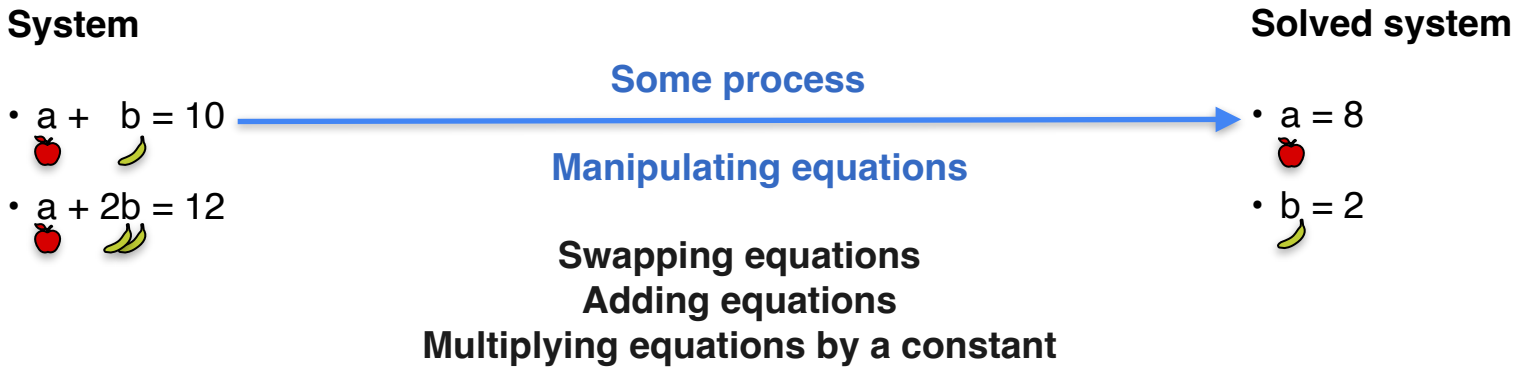
Solving systems of equations



Solving systems of equations



Solving systems of equations



Solving systems of equations

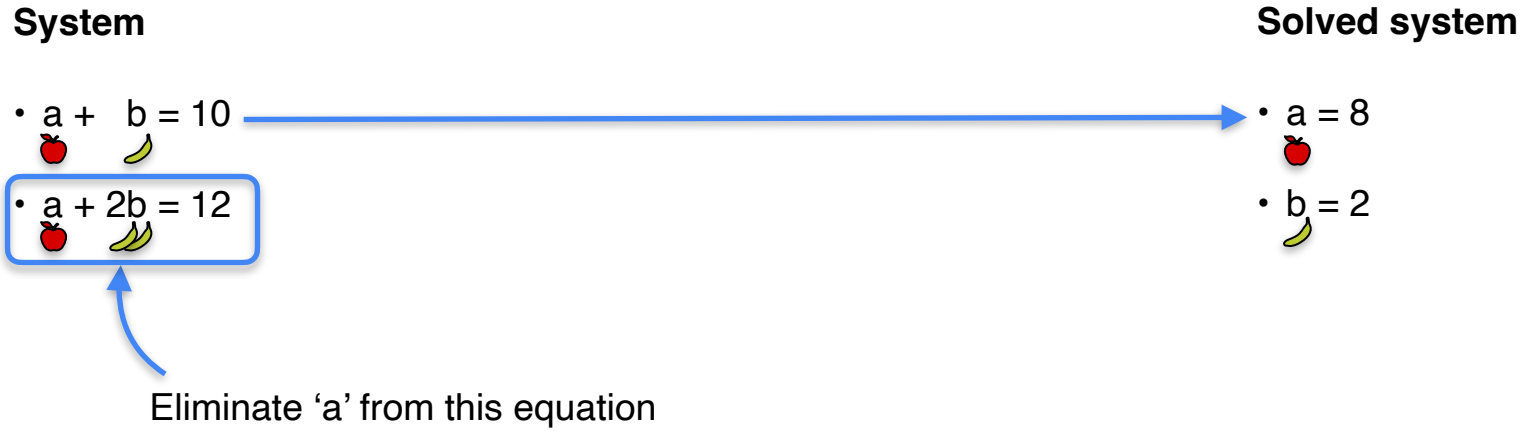
System

- a + b = 10
- a + 2b = 12

Solved system

- a = 8
- b = 2

Solving systems of equations



Manipulating equations

Manipulating equations

Multiplying by a constant

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$x \qquad 7$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\times \quad 7$$

$$7a + 7b = 70$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\times \quad 7$$

$$7a + 7b = 70$$

Adding two equations

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\times \quad 7$$

$$7a + 7b = 70$$

Adding two equations

$$a + b = 10$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\times \quad 7$$

$$7a + 7b = 70$$

Adding two equations

$$a + b = 10$$

$$2a + 3b = 26$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\times \quad 7$$

$$7a + 7b = 70$$

Adding two equations

$$a + b = 10$$

$$+ \quad 2a + 3b = 26$$

Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\times \quad 7$$

$$7a + 7b = 70$$

Adding two equations

$$a + b = 10$$

$$+ \quad 2a + 3b = 26$$

$$3a + 4b = 36$$

Let’s do a harder example

Systems of equations

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate ‘a’
from this equation

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

Subtract equation 1 from equation 2

Eliminate ‘a’ from this equation

Solved system

- $a = ?$
- $b = ?$

Systems of equations

System

- 5a + b = 17
- 4a - 3b = 6

Eliminate ‘a’ from this equation

Divide by coefficient of a

- a + 0.2b = 3.4
- a - 0.75b = 1.5

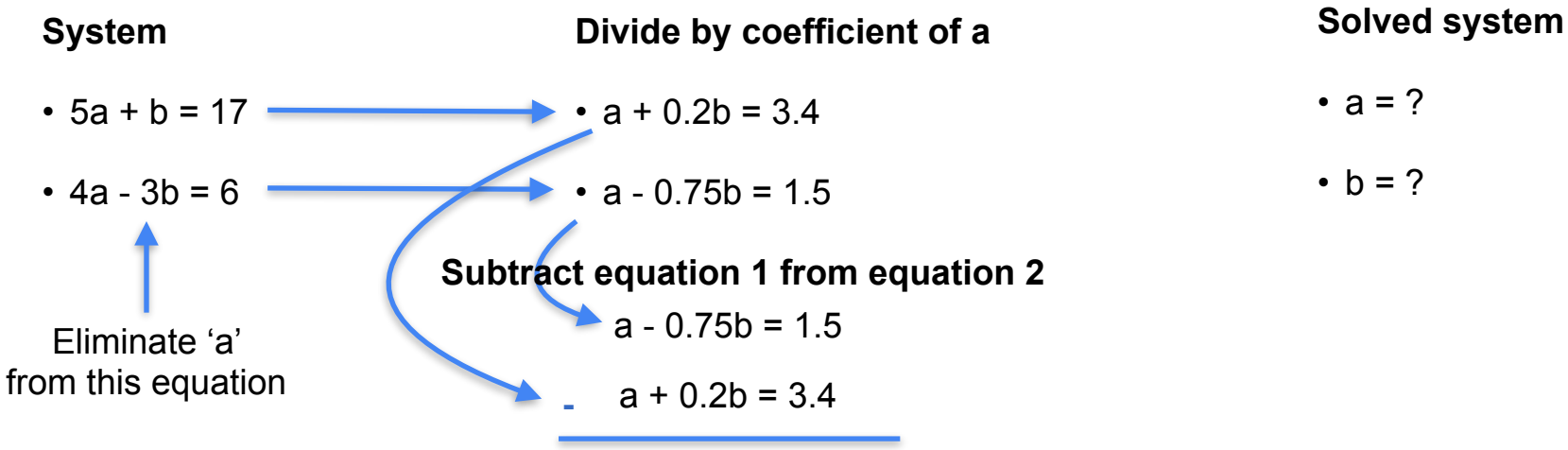
Subtract equation 1 from equation 2

a - 0.75b = 1.5

Solved system

- a = ?
- b = ?

Systems of equations



Systems of equations

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- 4a - 3b = 6

Eliminate ‘a’ from this equation

Divide by coefficient of a

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- a - 0.75b = 1.5

Subtract equation 1 from equation 2

a - 0.75b = 1.5

\- a + 0.2b = 3.4

0a - 0.95b = - 1.9

Solved system

- a = ?
- b = ?

Systems of equations

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- 5a + b = 17
- 4a - 3b = 6

Eliminate ‘a’ from this equation

Divide by coefficient of a

- a + 0.2b = 3.4
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Subtract equation 1 from equation 2

a - 0.75b = 1.5

\- a + 0.2b = 3.4

0a - 0.95b = - 1.9

-0.95b = - 1.9

Solved system

- a = ?
- b = ?

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

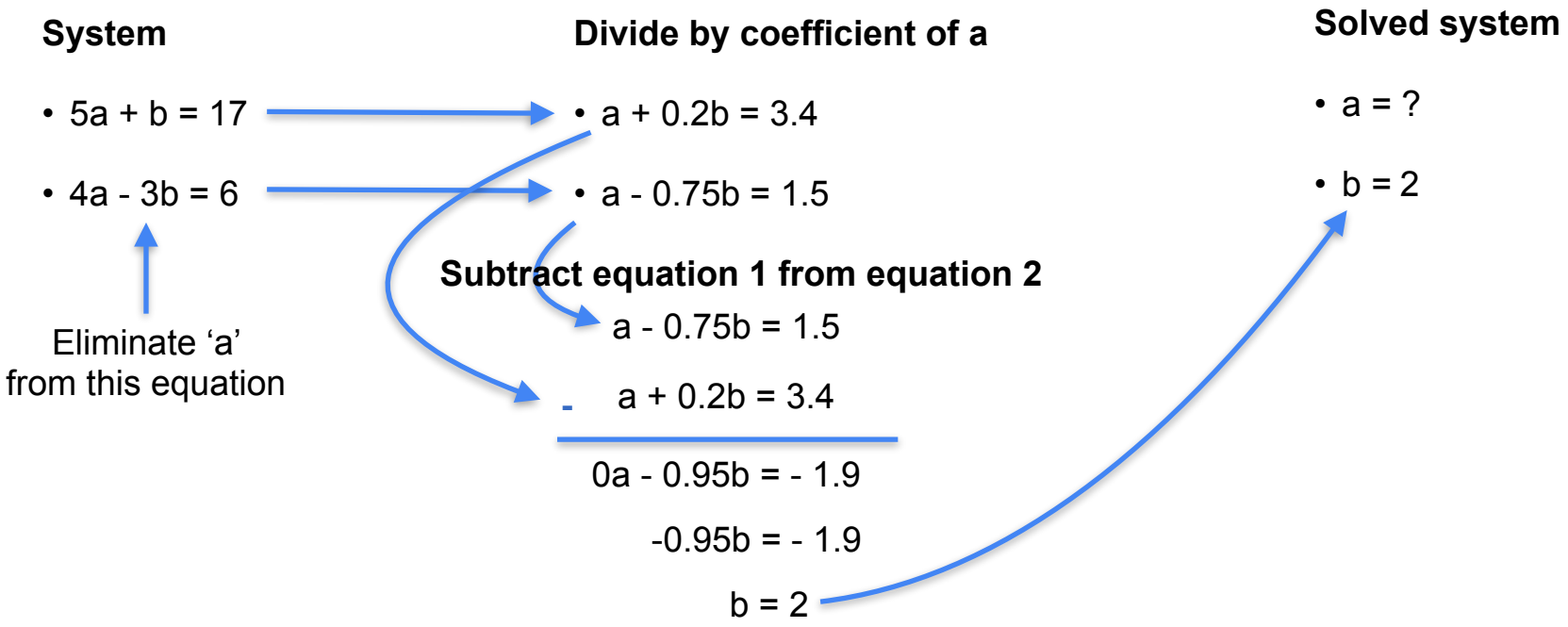
Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$
$$- \quad a + 0.2b = 3.4$$
$$0a - 0.95b = -1.9$$
$$-0.95b = -1.9$$
$$b = 2$$

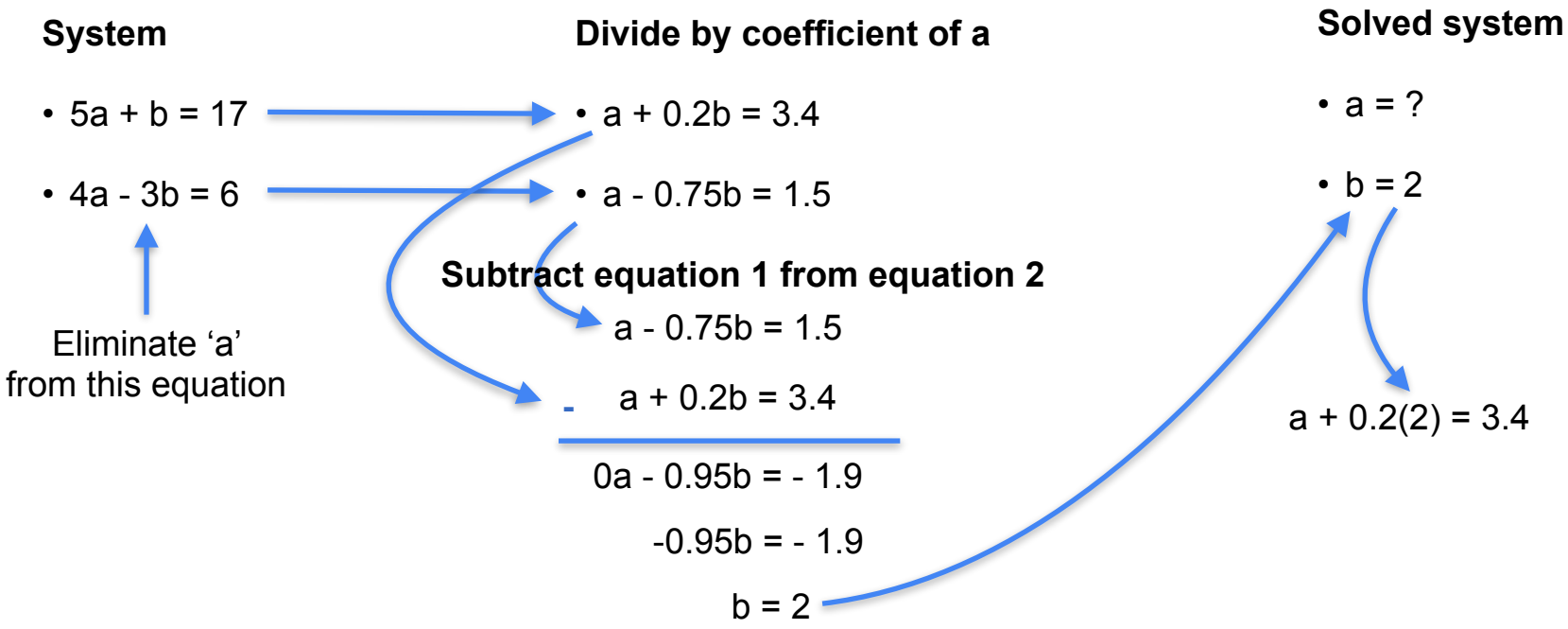
Solved system

- $a = ?$
- $b = ?$

Systems of equations



Systems of equations



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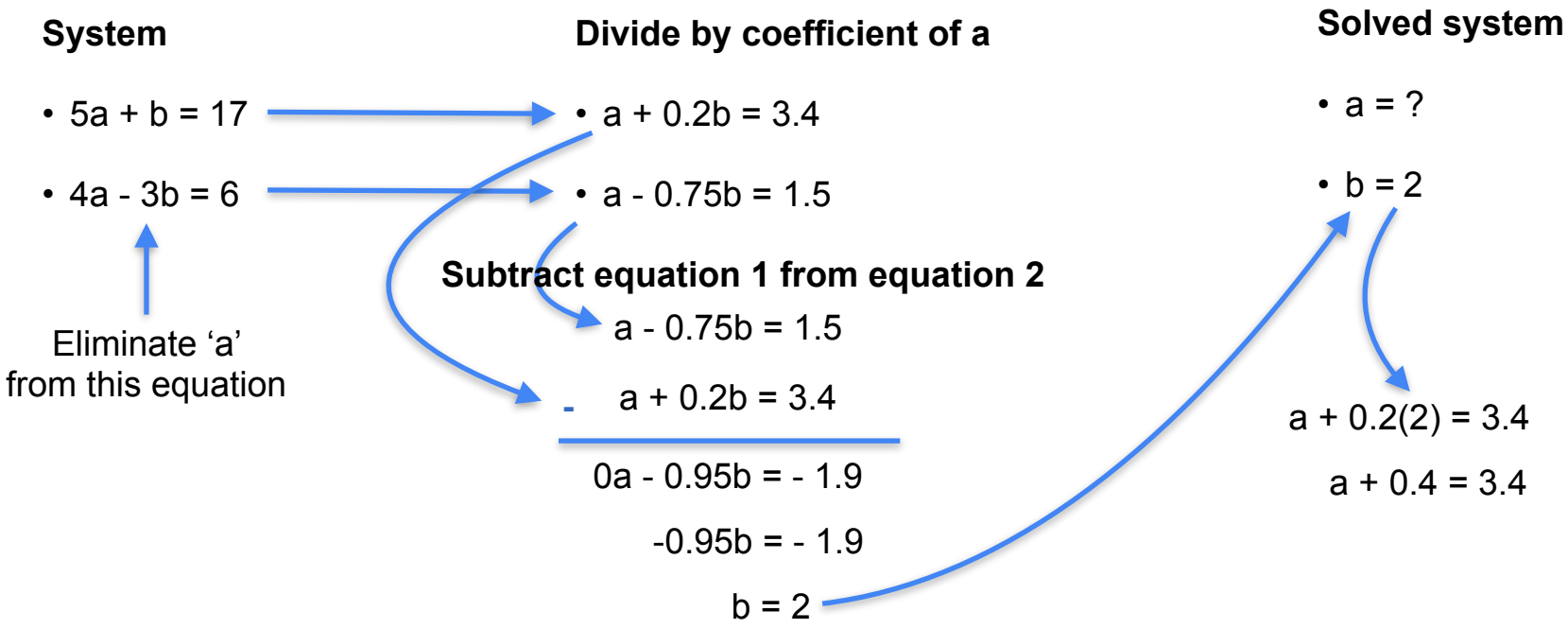

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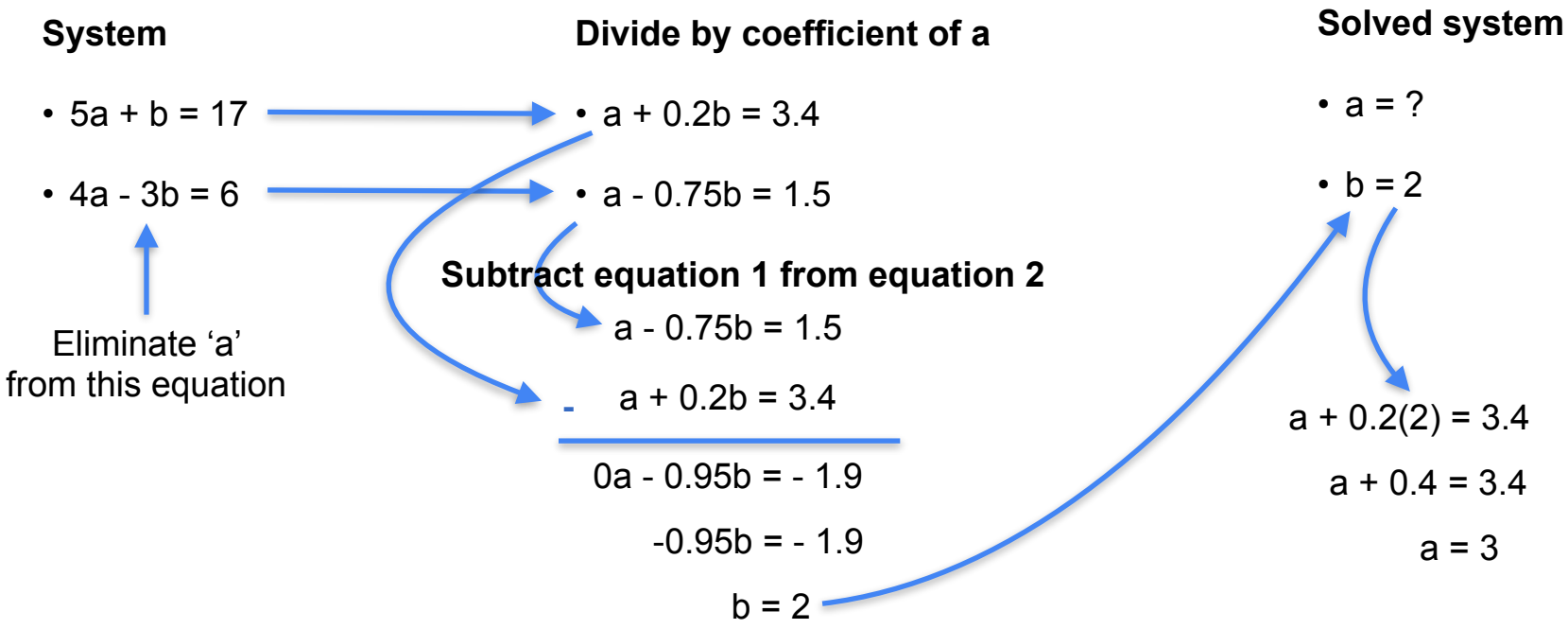
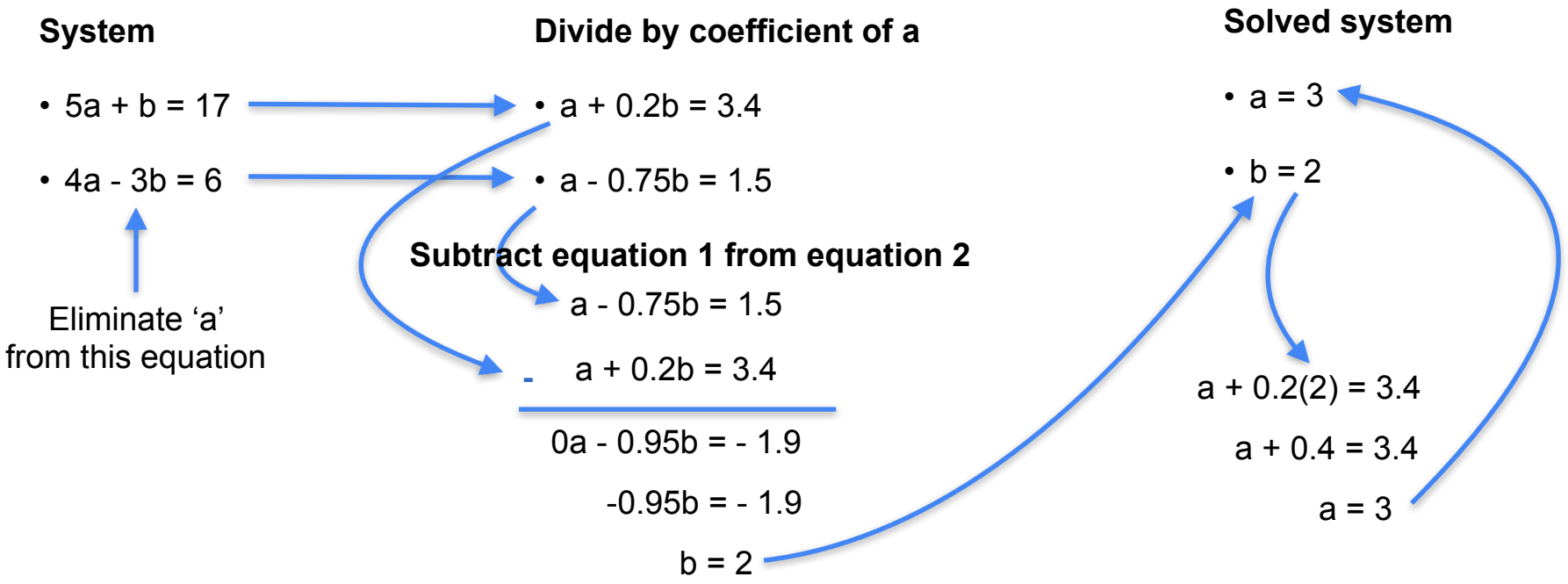


Systems of equations



What if one of the coefficients of a is zero?

System

- $5a + b = 17$
- $3b = 6$

Solved system

- $a = ?$
- $b = ?$

What if one of the coefficients of a is zero?

System

- $5a + b = 17$
- $3b = 6$

Eliminate ‘a’ from this equation

Solved system

- $a = ?$
- $b = ?$

What if one of the coefficients of a is zero?

System

- $5a + b = 17$
- $3b = 6$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + 0.2b = 3.4$
- ???

Solved system

- $a = ?$
- $b = ?$

What if one of the coefficients of a is zero?

System

- 5a + b = 17
- 3b = 6

Eliminate ‘a’ from this equation

Divide by coefficient of a

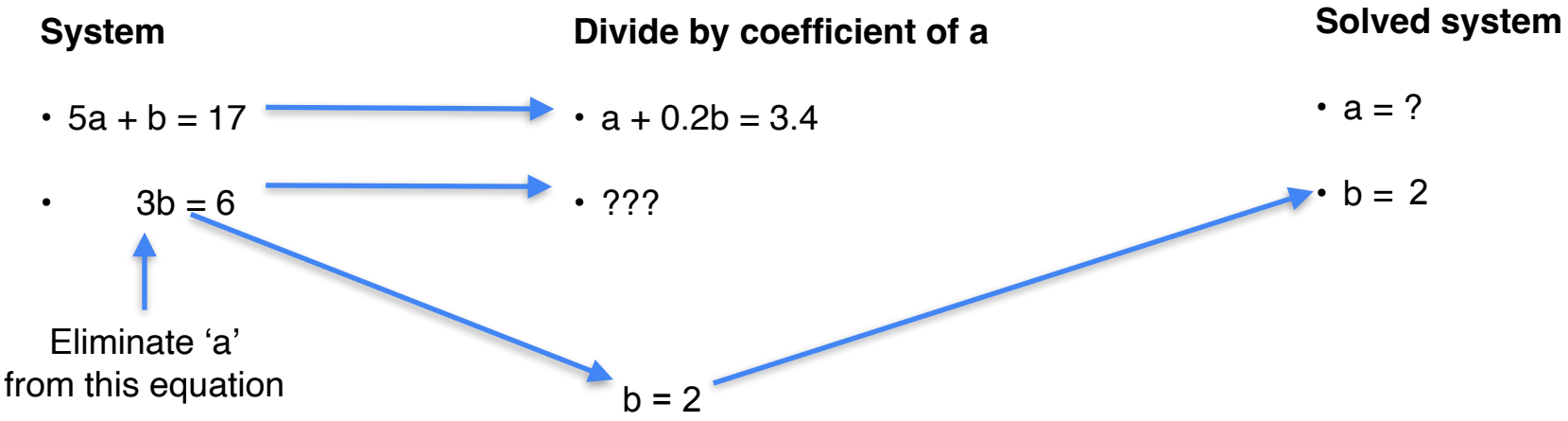
- a + 0.2b = 3.4
- ???

b = 2

Solved system

- a = ?
- b = ?

What if one of the coefficients of a is zero?



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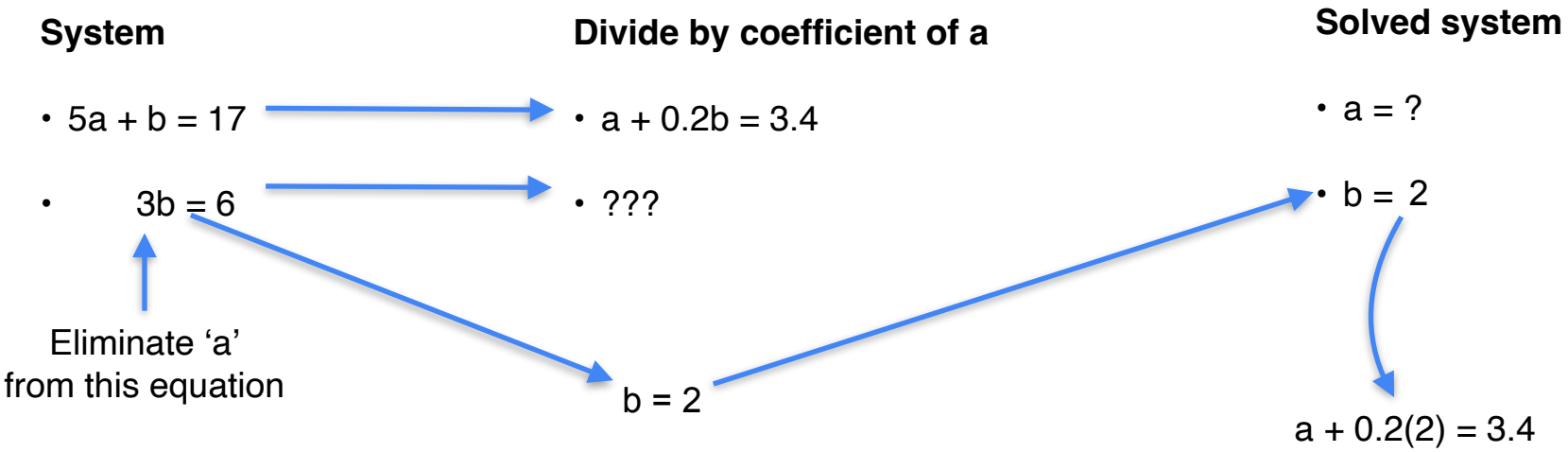

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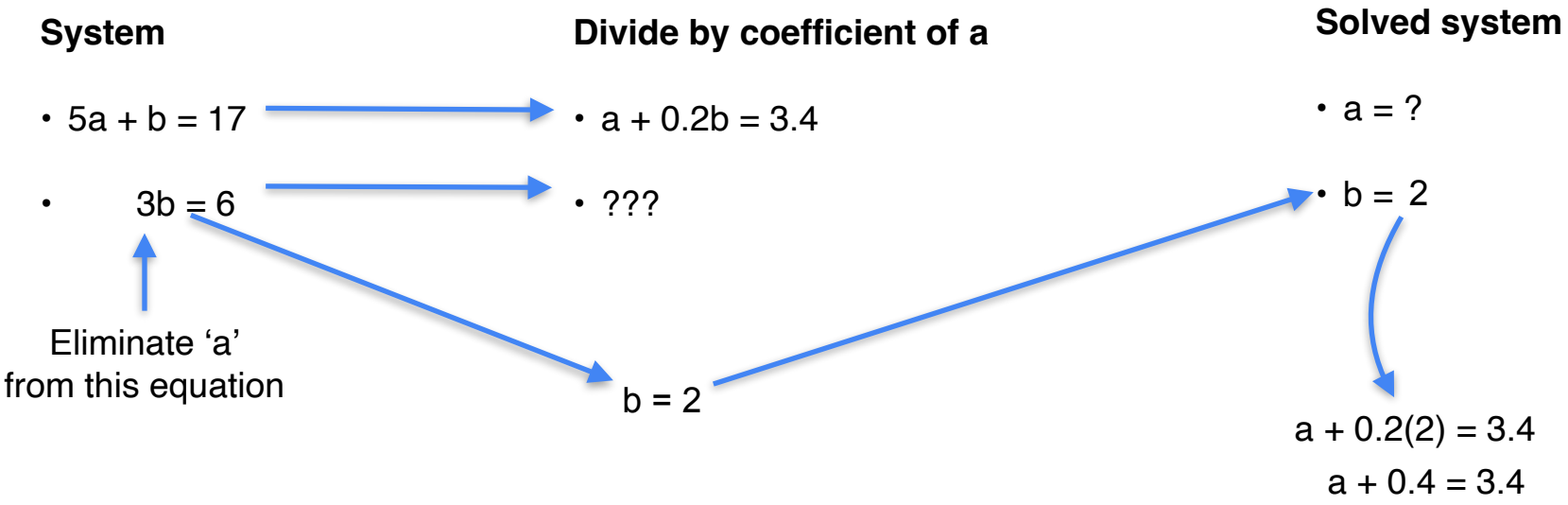

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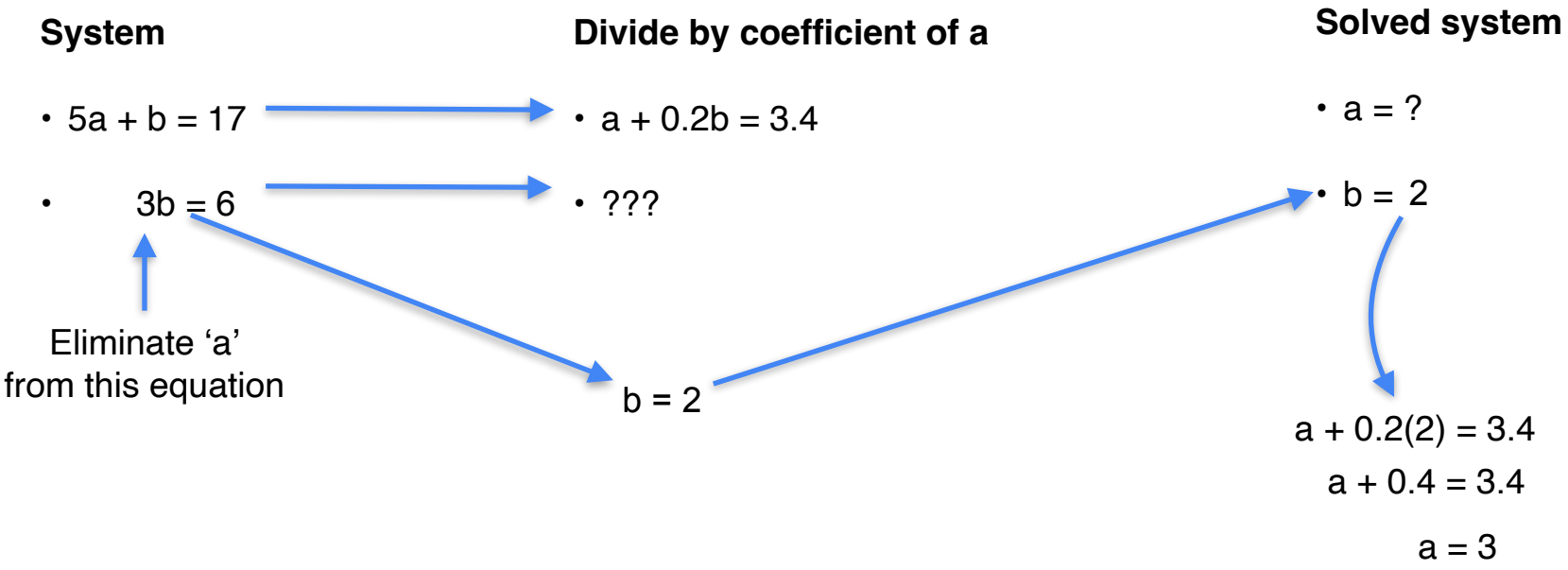

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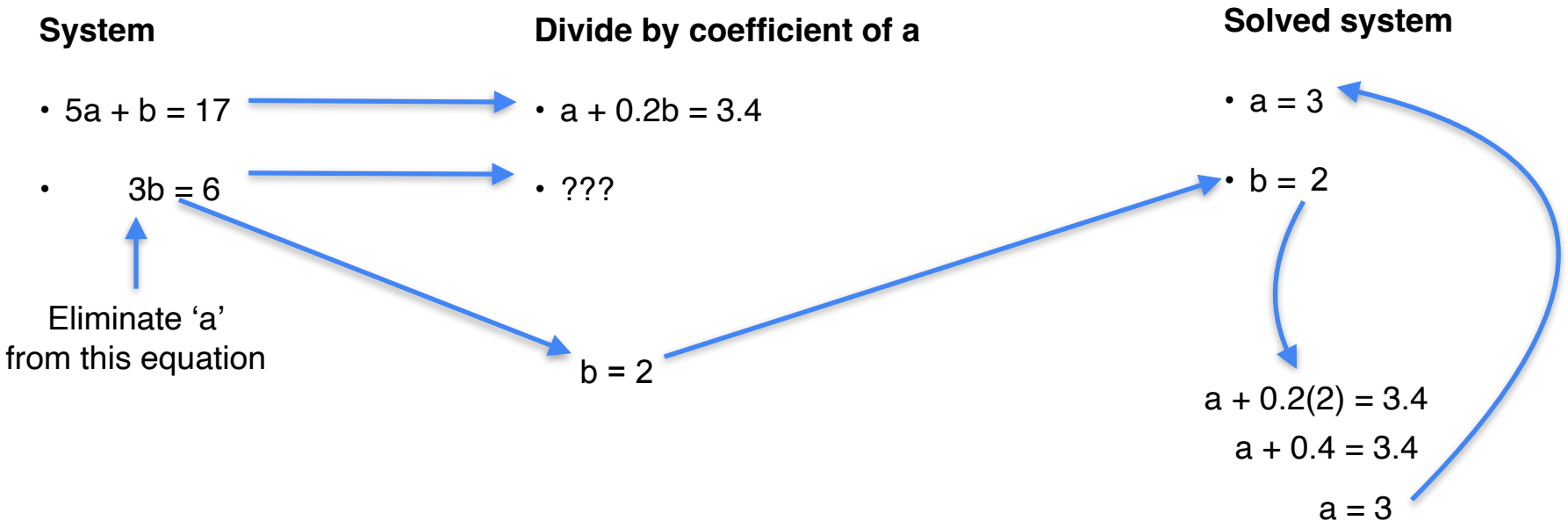

Quiz

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- $a = 3$
- $b = 8$

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Solving System of Linear Equations

Solving singular system of linear equations

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (redundant)?

System

- a + b = 10
- 2a + 2b = 20

Eliminate ‘a’ from this equation

Divide by coefficient of a

- a + b = 10
- a + b = 10

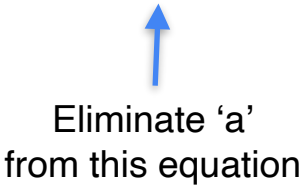
Solved system

- a = ?
- b = ?

What if the system is singular (redundant)?

System

- a + b = 10
- 2a + 2b = 20



Divide by coefficient of a

- a + b = 10
- a + b = 10

Subtract equation 1 from equation 2

Solved system

- a = ?
- b = ?

What if the system is singular (redundant)?

System

- a + b = 10
- 2a + 2b = 20

Eliminate ‘a’ from this equation

Divide by coefficient of a

- a + b = 10
- a + b = 10

Subtract equation 1 from equation 2

a + b = 10

Solved system

- a = ?
- b = ?

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$a + b = 10$

$- \quad a + b = 10$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$$\begin{array}{rl} & a + b = 10 \\ - & a + b = 10 \\ \hline & 0 = 0 \end{array}$$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$$\begin{array}{rl} & a + b = 10 \\ - & a + b = 10 \\ \hline & 0 = 0 \end{array}$$

Solved system

- $a = ?$
- $b = ?$

Solved system

- $a + b = 10$
- no other equation

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$$\begin{array}{rl} & a + b = 10 \\ - & a + b = 10 \\ \hline & 0 = 0 \end{array}$$

Solved system

- $a = x$
- $b = ?$

Solved system

- $a + b = 10$
- no other equation

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$$\begin{array}{rl} & a + b = 10 \\ - & a + b = 10 \\ \hline & 0 = 0 \end{array}$$

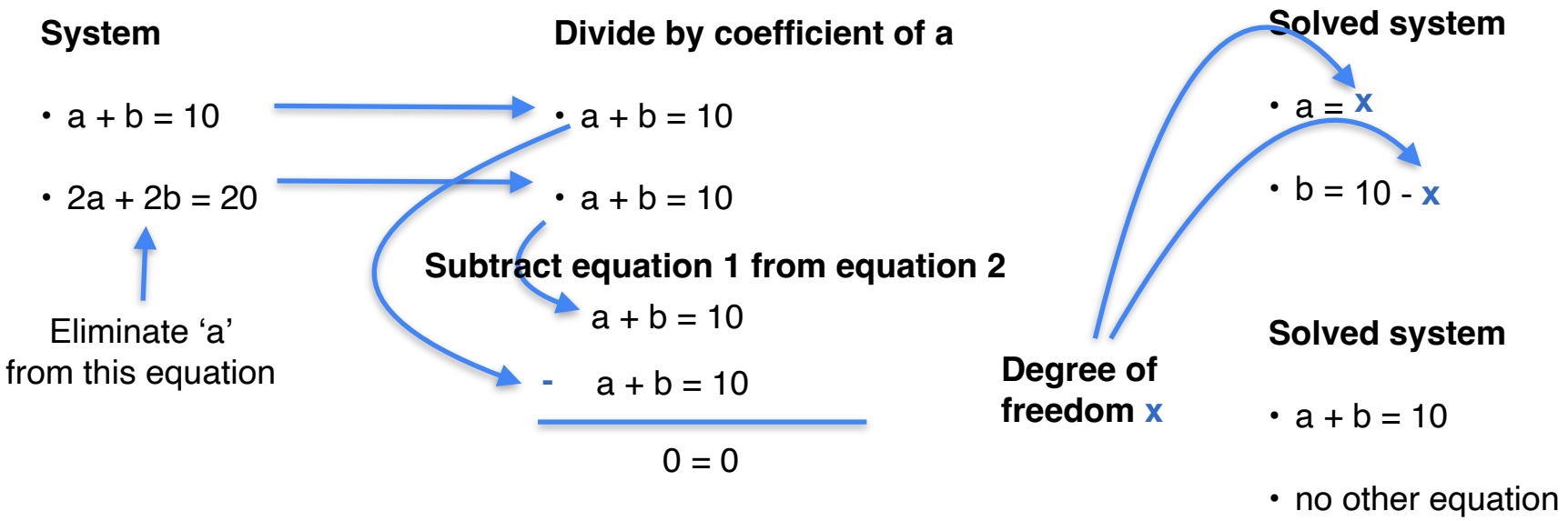
Solved system

- $a = x$
- $b = 10 - x$

Solved system

- $a + b = 10$
- no other equation

What if the system is singular (redundant)?



What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Eliminate ‘a’
from this equation

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 12$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 12$

Subtract equation 1 from equation 2

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- a + b = 10
- 2a + 2b = 24

Eliminate ‘a’ from this equation

Divide by coefficient of a

- a + b = 10
- a + b = 12

Subtract equation 1 from equation 2

a + b = 12

Solved system

- a = ?
- b = ?

What if the system is singular (contradictory)?

System

- a + b = 10
- 2a + 2b = 24

Eliminate ‘a’ from this equation

Divide by coefficient of a

- a + b = 10
- a + b = 12

Subtract equation 1 from equation 2

a + b = 12

\- a + b = 10

Solved system

- a = ?
- b = ?

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 12$

Subtract equation 1 from equation 2

$$\begin{array}{rl} & a + b = 12 \\ - & a + b = 10 \\ \hline & 0 = 2 \end{array}$$

Solved system

- $a = ?$
- $b = ?$

What if the system is singular (contradictory)?

System

- $a + b = 10$
- $2a + 2b = 24$

Eliminate ‘a’ from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 12$

Subtract equation 1 from equation 2

$$\begin{array}{rl} & a + b = 12 \\ - & a + b = 10 \\ \hline & 0 = 2 \end{array}$$

Contradiction!

Solved system

- $a = ?$
- $b = ?$

Quiz

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution: If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.

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Solving System of Linear Equations

Solving system of equations with more variables

Elimination method

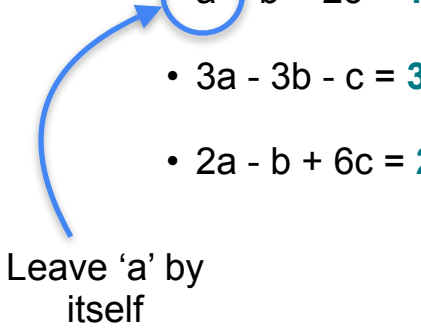
System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Elimination method

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**



Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of ‘a’

Elimination method

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**

Divide each row by the coefficient of ‘a’

- a + b + 2c = **12**
- a - b - 1/3 c = **1**
- a - b/2 + 3c = **12**

Elimination method

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**

Divide each row by the coefficient of ‘a’

- a + b + 2c = **12**
- a - b - 1/3 c = **1**
- a - b/2 + 3c = **12**

Use the first equation to remove ‘a’ from the others

Elimination method

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**

Divide each row by the coefficient of ‘a’

- a + b + 2c = **12**
- a - b - 1/3 c = **1**
- a - b/2 + 3c = **12**

Use the first equation to remove ‘a’ from the others

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Elimination method

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of ‘a’

- $a + b + 2c = 12$
- $a - b - 1/3\ c = 1$
- $a - b/2 + 3c = 12$

Use the first equation to remove ‘a’ from the others

- $a + b + 2c = 12$
- $-2b - 7/3\ c = -11$
- $-3/2\ b + c = 0$

Isolated ‘a’

Elimination method

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**

Divide each row by the coefficient of ‘a’

- a + b + 2c = **12**
- a - b - 1/3 c = **1**
- a - b/2 + 3c = **12**

Use the first equation to remove ‘a’ from the others

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Isolated ‘a’

Solve this new system of 2 equations

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3\ c = -11$
- $-3/2\ b + c = 0$

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3\ c = -11$
- $-3/2\ b + c = 0$

Divide last two rows by the coefficient of b

Elimination method

System

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Divide last two rows by the coefficient of b

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- b - 2/3 c = **0**

Elimination method

System

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Divide last two rows by the coefficient of b

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- b - 2/3 c = **0**

Use the second equation to remove ‘b’ from the third

Elimination method

System

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Divide last two rows by the coefficient of b

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- b - 2/3 c = **0**

Use the second equation to remove ‘b’ from the third

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- -11/6 c = **-11/2**

Elimination method

System

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Divide last two rows by the coefficient of b

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- b - 2/3 c = **0**

Use the second equation to remove ‘b’ from the third

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- -11/6 c = **-11/2**

Isolated ‘b’

Elimination method

System

- a + b + 2c = **12**
- -2b - 7/3 c = **-11**
- - 3/2 b + c = **0**

Divide last two rows by the coefficient of b

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- b - 2/3 c = **0**

Use the second equation to remove ‘b’ from the third

- a + b + 2c = **12**
- b + 7/6 c = **11/2**
- -11/6 c = **-11/2**

Isolated ‘b’

c = 3

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6\ c = 11/2$
- $c = 3$

Elimination method

System

- $a + b + 2c = \mathbf{12}$
- $b + 7/6\ c = \mathbf{11/2}$
- $c = \mathbf{3}$

Replace c = 3 in the second equation, get b = 2

Elimination method

System

- a + b + 2c = **12**
- b + 7/6 c = **11/2** → b + 7/2 = **11/2**, b = **2**
- c = **3**

Replace c = 3 in the second equation, get b = 2

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6\ c = 11/2$ → $b + 7/2 = 11/2$, $b = 2$
- $c = 3$

Replace c = 3 in the second equation, get b = 2

Replace c = 3 and b = 2 in the first equation, get a = 4

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6\ c = 11/2$
- $c = 3$

$a + 2 + 6 = 12$
$a = 4$

$b + 7/2 = 11/2$
$b = 2$

Replace c = 3 in the second equation, get b = 2

Replace c = 3 and b = 2 in the first equation, get a = 4

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6\ c = 11/2$
- $c = 3$

$a + 2 + 6 = 12$
$a = 4$

$b + 7/2 = 11/2$
$b = 2$

Replace c = 3 in the second equation, get b = 2

Replace c = 3 and b = 2 in the first equation, get a = 4

The solution is
a = 4
b = 2
c = 3

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Solving System of Linear Equations

Matrix row reduction

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

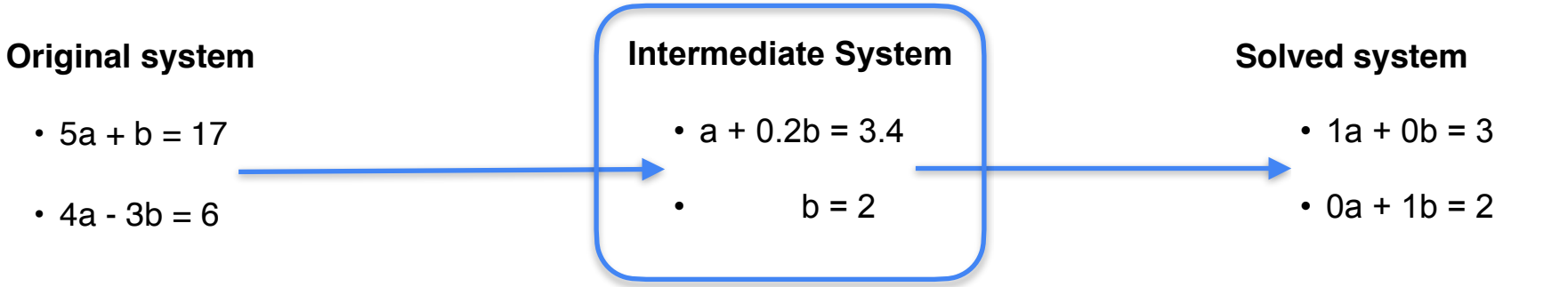
Row echelon form

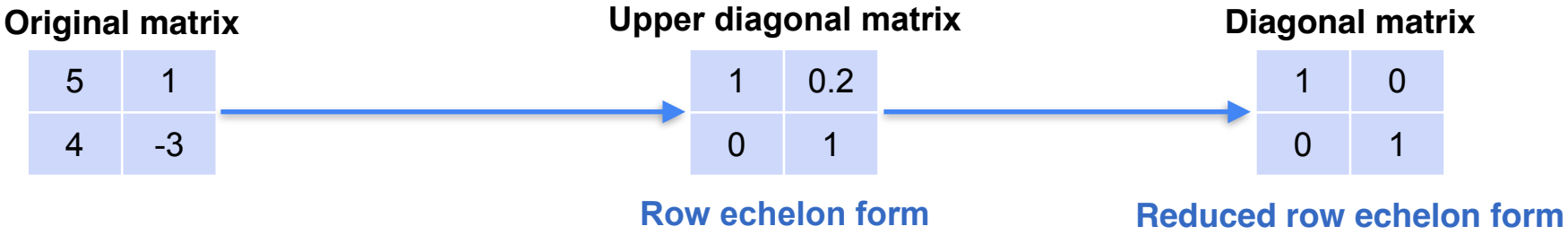
Diagonal matrix

1	0
0	1

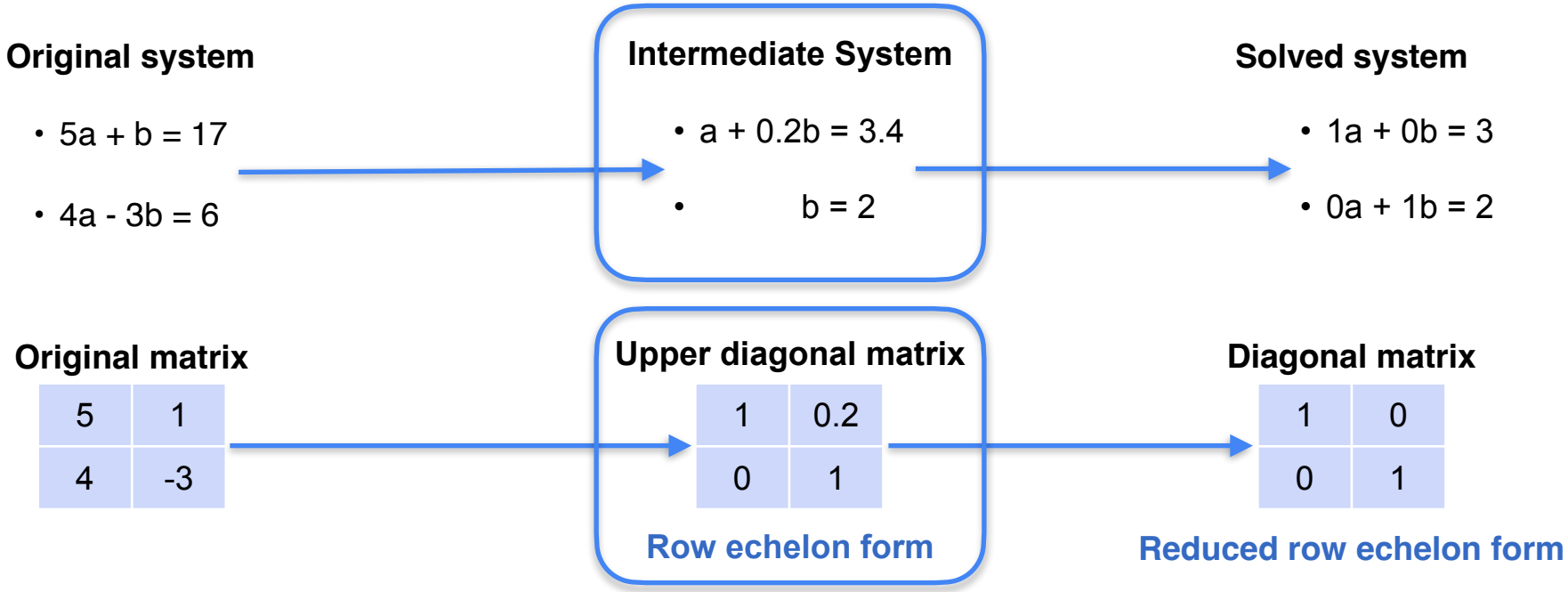
Reduced row echelon form

Systems of equations to matrices





Systems of equations to matrices



Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

1	1
2	2

Systems of equations to matrices

Original system

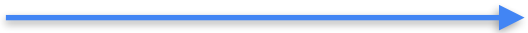
- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

1	1
2	2



Upper diagonal matrix

1	1
0	0

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

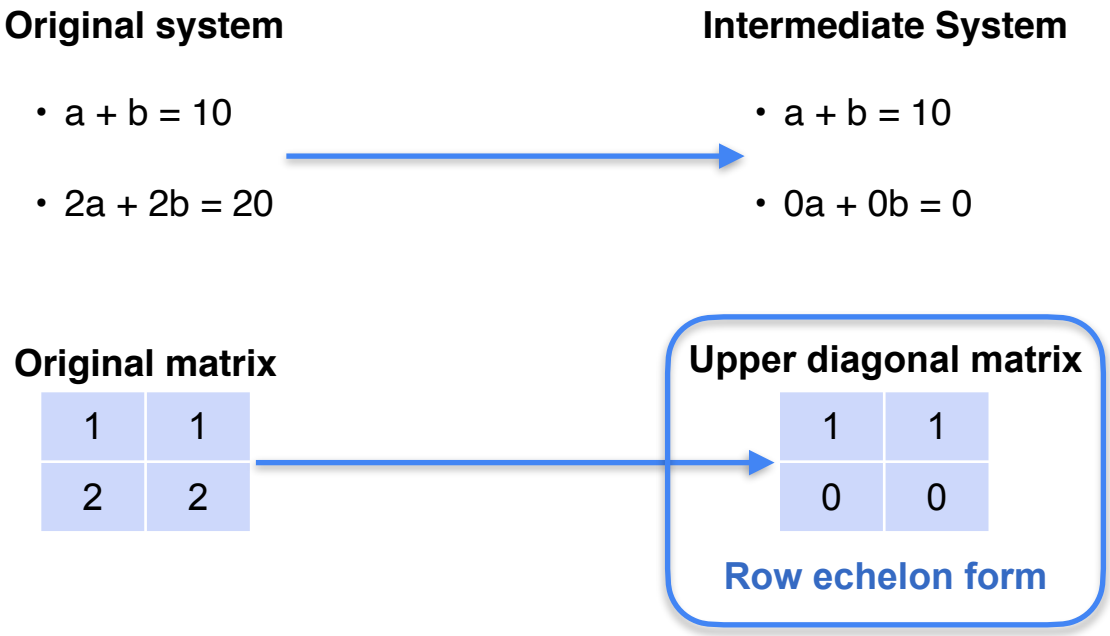
1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

Systems of equations to matrices



Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form

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Solving System of Linear Equations

Row operations that preserve singularity

Switching rows

5	1
4	3

Switching rows

5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4 = 11$

Switching rows

5	1
4	3

4	3
5	1

Determinant $= 5 \cdot 3 - 1 \cdot 4 = 11$

Switching rows

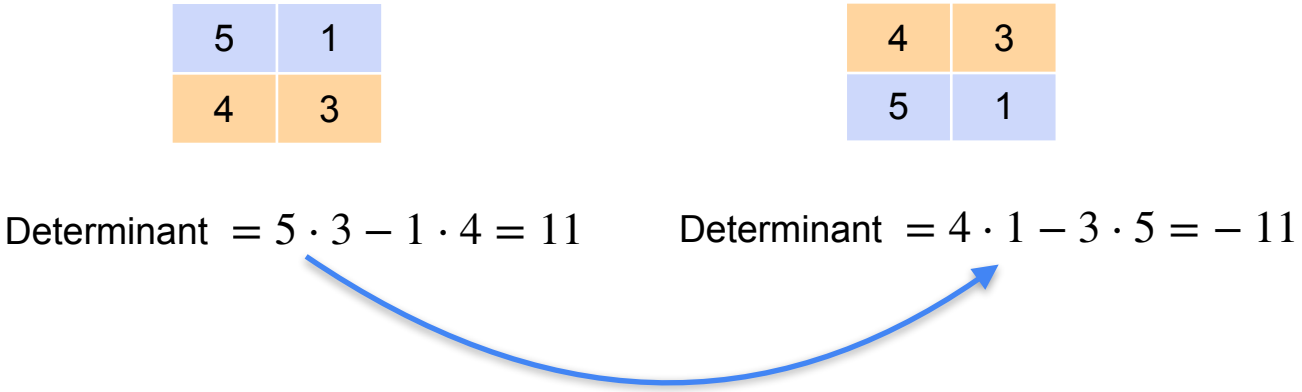
5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4 = 11$

4	3
5	1

Determinant $= 4 \cdot 1 - 3 \cdot 5 = -11$

Switching rows



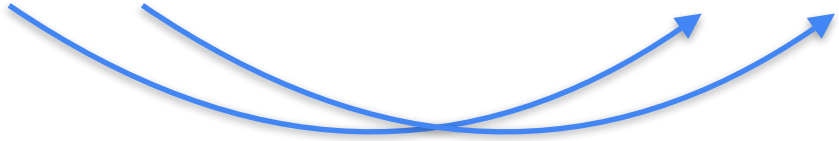
Switching rows

5	1
4	3

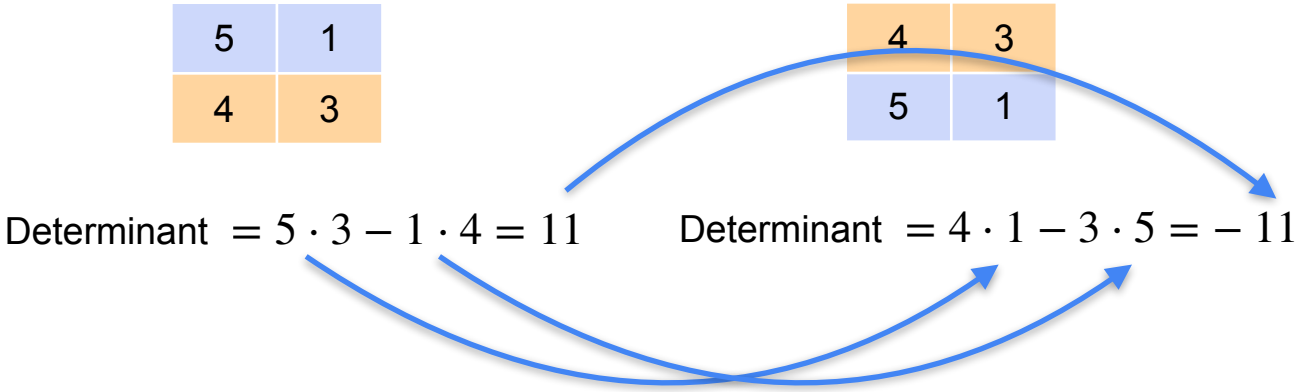
Determinant $= 5 \cdot 3 - 1 \cdot 4 = 11$

4	3
5	1

Determinant $= 4 \cdot 1 - 3 \cdot 5 = -11$



Switching rows



Multiplying a row by a (non-zero) scalar

5	1
4	3

Multiplying a row by a (non-zero) scalar

5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

Multiplying a row by a (non-zero) scalar

5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1

4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1	x 10 =

4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Multiplying a row by a (non-zero) scalar

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Multiplying a row by a (non-zero) scalar

5	1
4	-3

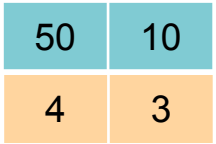
5	1	x 10 =	50	10

50	10
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Multiplying a row by a (non-zero) scalar

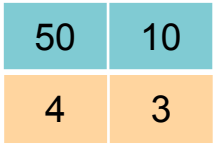


Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Determinant $= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$

Multiplying a row by a (non-zero) scalar



Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Determinant $= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$

$= 10 \cdot 11$

Adding a row to another row

5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Adding a row to another row

5	1

5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Adding a row to another row

5	1
4	3

5	1
4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Adding a row to another row

5	1
4	3

	5	1
+	4	3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Adding a row to another row

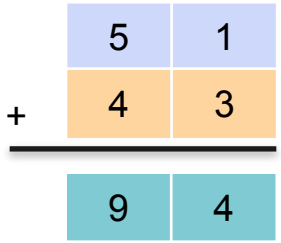
5	1
4	3

	5	1
+	4	3
	9	4

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

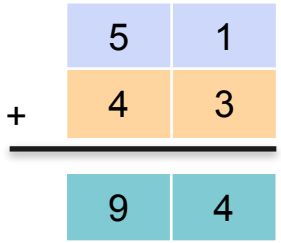
Adding a row to another row

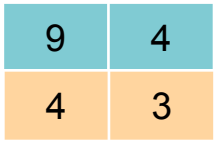


Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Adding a row to another row

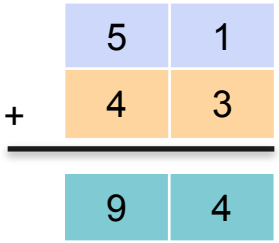


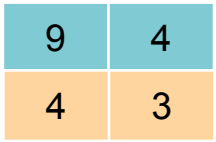


Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Adding a row to another row



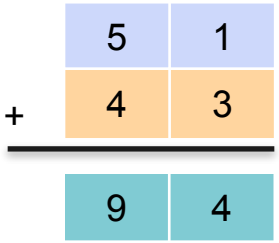


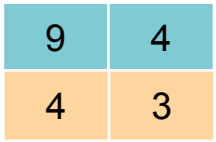
Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Determinant $= 9 \cdot 3 - 4 \cdot 4$

Adding a row to another row





Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

Determinant $= 9 \cdot 3 - 4 \cdot 4$

$= 11$

DeepLearning.AI

Solving System of Linear Equations

Rank of a matrix

Compressing Images - Reducing rank

Compressing Images - Reducing rank

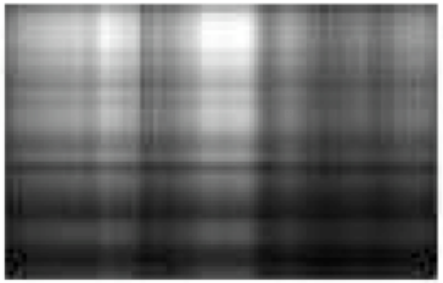
Compressing Images - Reducing rank

Original (Rank 200)

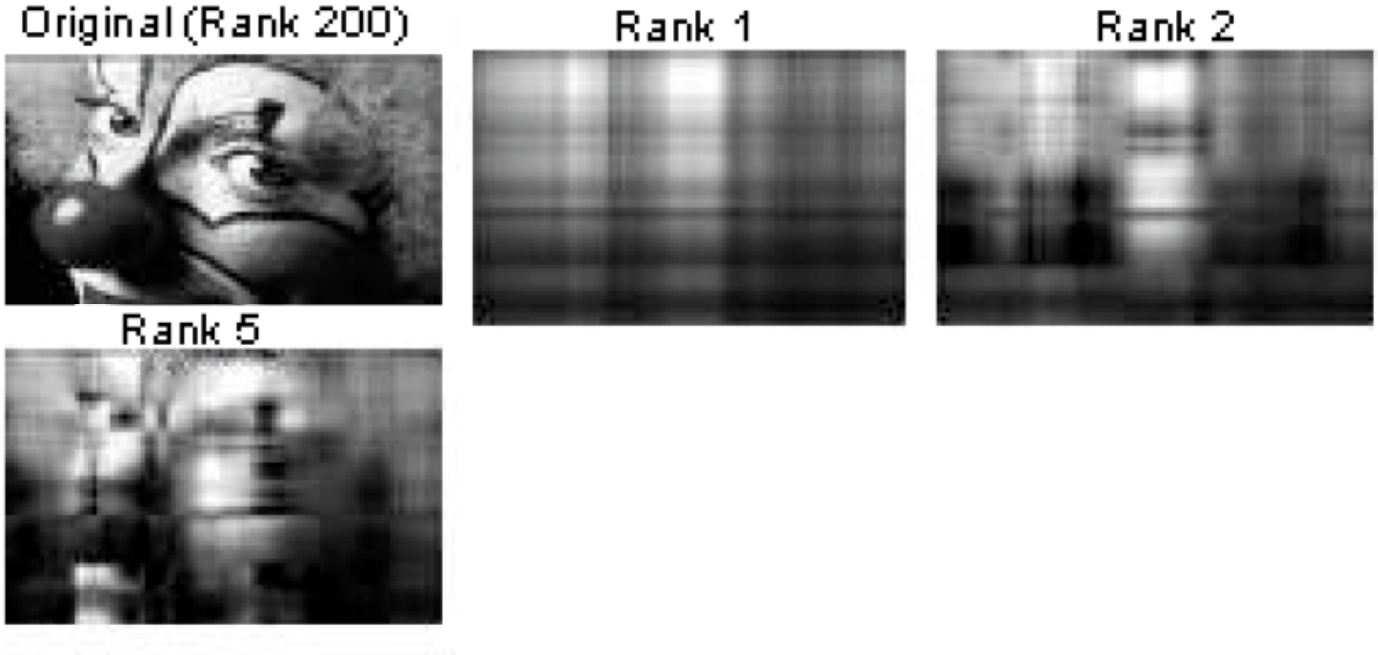
Compressing Images - Reducing rank

Original (Rank 200)

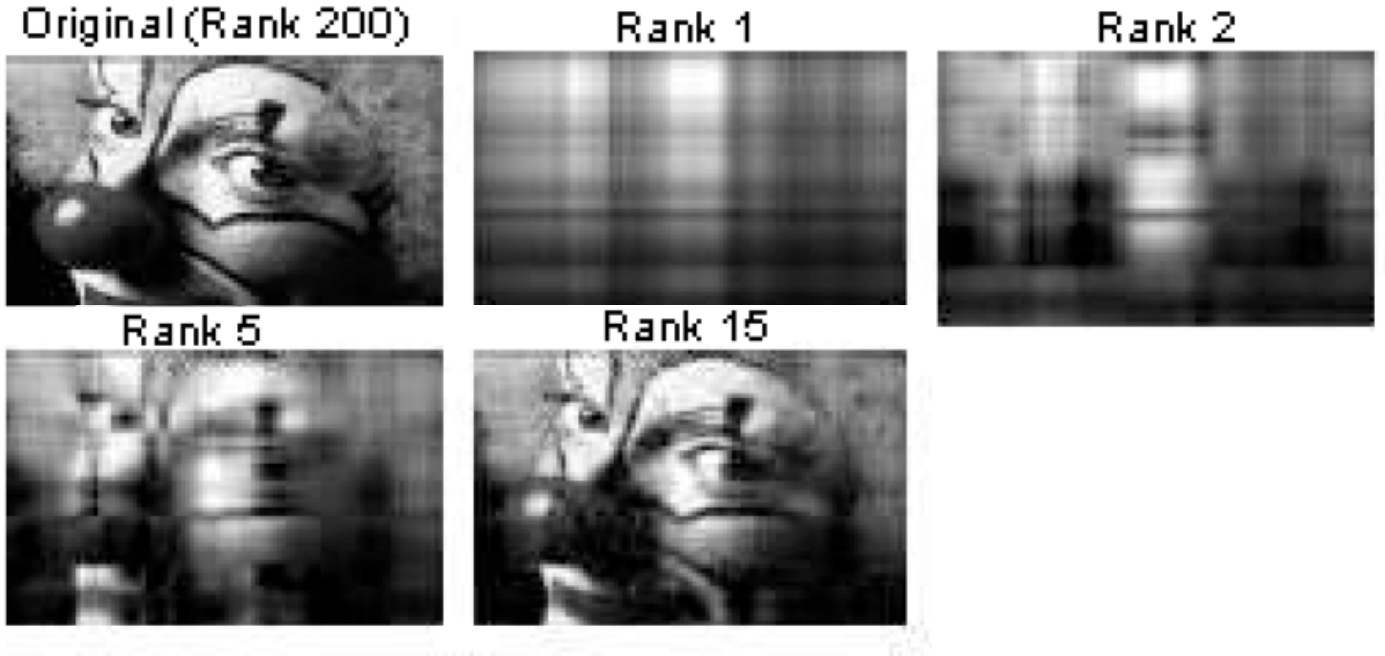
Rank 1



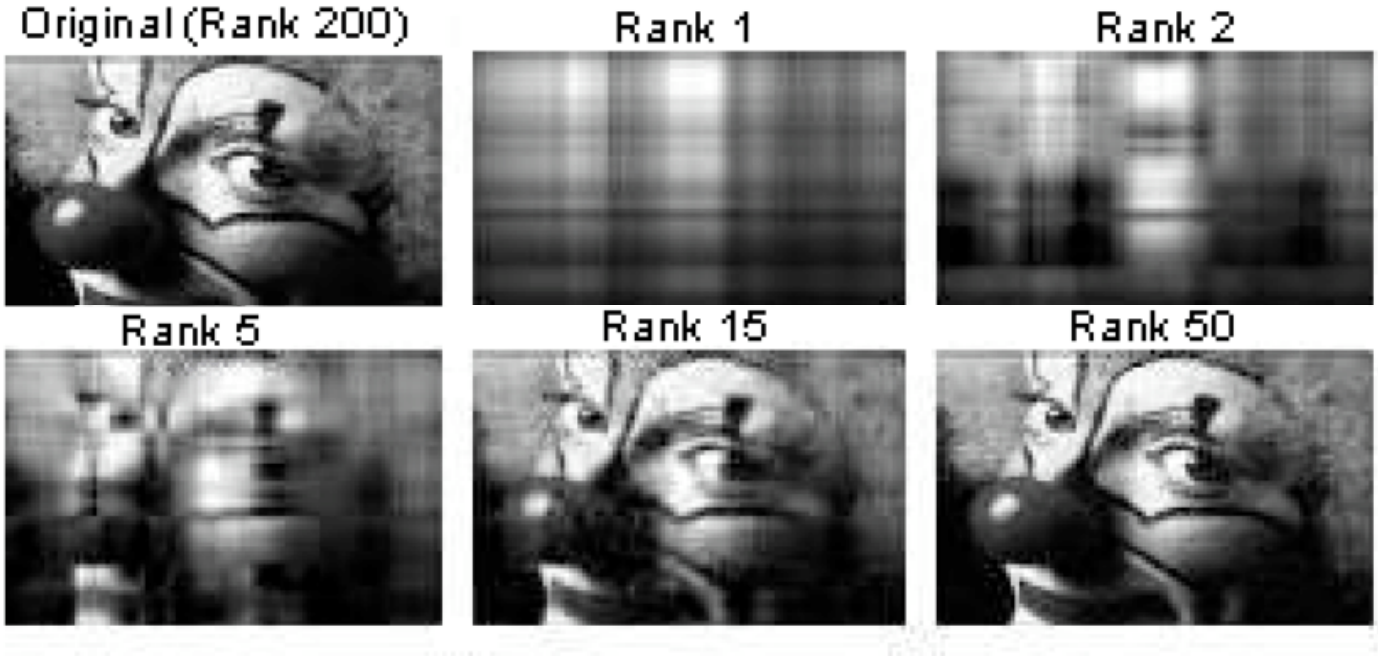
Compressing Images - Reducing rank



Compressing Images - Reducing rank



Compressing Images - Reducing rank



Systems of information

Systems of information

System 1

The dog is **black**

The cat is **orange**

Systems of information

System 1

The dog is **black**

The cat is **orange**

System 2

The dog is **black**

The dog is **black**

Systems of information

System 1

The dog is **black**

The cat is **orange**

System 2

The dog is **black**

The dog is **black**

System 3

The dog

The dog

Systems of information

System 1

The dog is **black**

The cat is **orange**

System 2

The dog is **black**

The dog is **black**

System 3

The dog

The dog

Two sentences

Systems of information

System 1

The dog is **black**
The cat is **orange**

System 2

The dog is **black**
The dog is **black**

System 3

The dog
The dog

Two sentences

Two pieces of information

Systems of information

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

System 2

The dog is **black**
The dog is **black**

Two sentences

System 3

The dog
The dog

Systems of information

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

System 2

The dog is **black**
The dog is **black**

Two sentences

One piece of information

System 3

The dog
The dog

Systems of information

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

System 2

The dog is **black**
The dog is **black**

Two sentences

One piece of information

System 3

The dog
The dog

Two sentences

Systems of information

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

System 2

The dog is **black**
The dog is **black**

Two sentences

One piece of information

System 3

The dog
The dog

Two sentences

Zero pieces of information

Systems of information

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

The dog is **black**
The dog is **black**

Two sentences

One piece of information

System 3

The dog
The dog

Two sentences

Zero pieces of information

Systems of information

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

The dog is **black**
The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

The dog
The dog

Two sentences

Zero pieces of information

Systems of information

System 1

The dog is **black**

The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

The dog is **black**

The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

The dog

The dog

Two sentences

Zero pieces of information

Rank = 0

Systems of equations

Systems of equations

System 1

a + b = 0

a + 2b = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

System 2

a + b = 0

2a + 2b = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

System 2

a + b = 0

2a + 2b = 0

System 3

0a + 0b = 0

0a + 0b = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

Two equations

System 2

a + b = 0

2a + 2b = 0

System 3

0a + 0b = 0

0a + 0b = 0

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

System 2

$a + b = 0$

$2a + 2b = 0$

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Two equations

Two pieces of information

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

System 2

$a + b = 0$

$2a + 2b = 0$

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Two equations

Two pieces of information

Rank = 2

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$

$2a + 2b = 0$

Two equations

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$

$2a + 2b = 0$

Two equations

One piece of information

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$

$2a + 2b = 0$

Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$

$2a + 2b = 0$

Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Two equations

Systems of equations

System 1

$a + b = 0$

$a + 2b = 0$

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$

$2a + 2b = 0$

Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$

$0a + 0b = 0$

Two equations

Zero pieces of information

Systems of equations

System 1

a + b = 0

a + 2b = 0

Two equations

Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

1	1
1	2

Two equations

Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

Two equations

Zero pieces of information

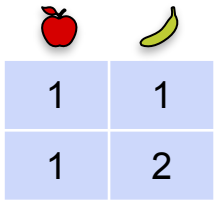
Rank = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0



1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0

1	1
2	2

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

Two equations

Zero pieces of information

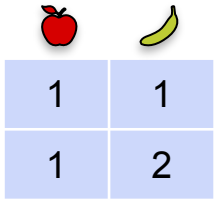
Rank = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0



Rank = 2

Two equations

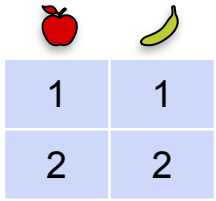
Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0



Rank = 1

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0

1	1
2	2

Rank = 1

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

0	0
0	0

Two equations

Zero pieces of information

Rank = 0

Systems of equations

System 1

a + b = 0

a + 2b = 0

1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

a + b = 0

2a + 2b = 0

1	1
2	2

Rank = 1

Two equations

One piece of information

Rank = 1

System 3

0a + 0b = 0

0a + 0b = 0

0	0
0	0

Rank = 0

Two equations

Zero pieces of information

Rank = 0

Rank and solutions to the system

1	1
1	2

Rank = 2

1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Rank and solutions to the system

1	1
1	2

Rank = 2

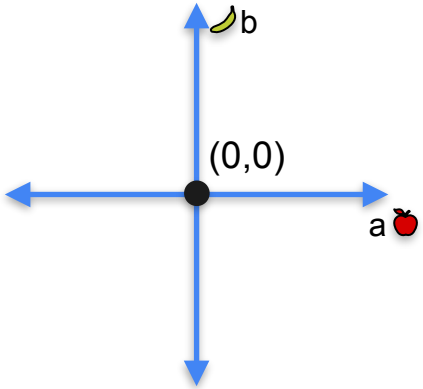
1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Dimension of solution space = 0



Rank and solutions to the system

1	1
1	2

Rank = 2

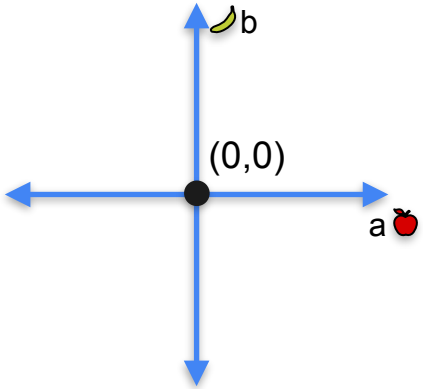
1	1
2	2

Rank = 1

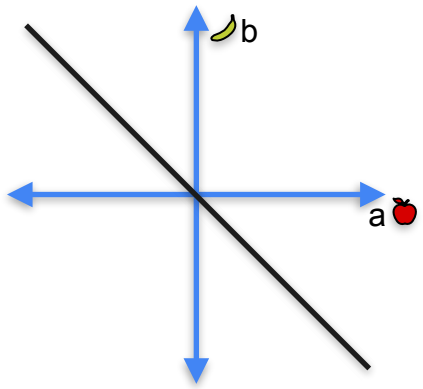
0	0
0	0

Rank = 0

Dimension of solution space = 0



Dimension of solution space = 1

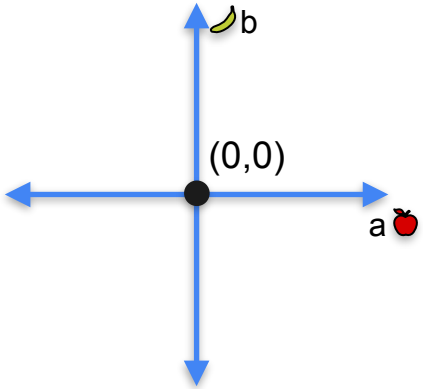


Rank and solutions to the system

1	1
1	2

Rank = 2

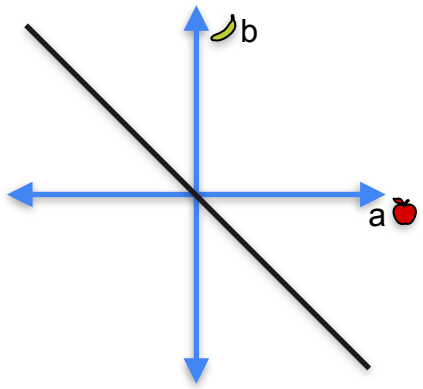
Dimension of solution space = 0



1	1
2	2

Rank = 1

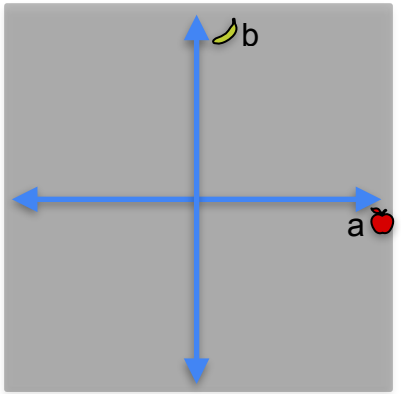
Dimension of solution space = 1



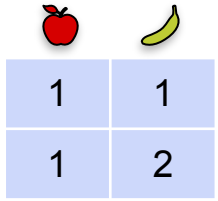
0	0
0	0

Rank = 0

Dimension of solution space = 2



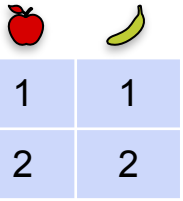
Rank of a matrix



1	1
1	2

Rank = 2

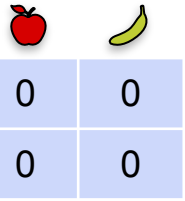
Dimension of solution space = 0



1	1
2	2

Rank = 1

Dimension of solution space = 1



0	0
0	0

Rank = 0

Dimension of solution space = 2

Rank = 2 - (Dimension of solution space)

Rank and singularity

1	1
1	2

Rank = 2

1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Rank and singularity

1	1
1	2

Rank = 2

1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Non-singular

Rank and singularity

1	1
1	2

Rank = 2

Non-singular

1	1
2	2

Rank = 1

Singular

0	0
0	0

Rank = 0

Rank and singularity

1	1
1	2

Rank = 2

Non-singular

1	1
2	2

Rank = 1

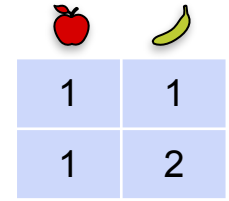
Singular

0	0
0	0

Rank = 0

Singular

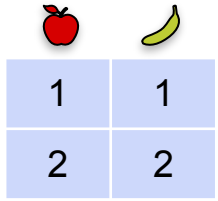
Rank and singularity



1	1
1	2

Rank = 2

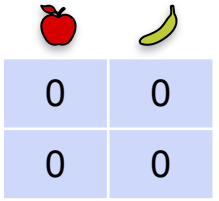
Non-singular



1	1
2	2

Rank = 1

Singular



0	0
0	0

Rank = 0

Singular

Quiz: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Solutions: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1: Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

Matrix 2: Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3

DeepLearning.AI

Solving System of Linear Equations

Rank of a matrix: General case

Rank for matrices

System 1

$$a + b + c = 0$$
$$a + 2b + c = 0$$
$$a + b + 2c = 0$$

System 2

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$
$$2a + 2b + 2c = 0$$
$$3a + 3b + 3c = 0$$

System 4

$$0a + 0b + 0c = 0$$
$$0a + 0b + 0c = 0$$
$$0a + 0b + 0c = 0$$

Rank for matrices

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✔
$a + 2b + c = 0$ ✔
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✔
$a + 2b + c = 0$ ✔
$a + b + 2c = 0$ ✔

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✔
$a + 2b + c = 0$ ✔
$a + b + 2c = 0$ ✔

3 Equations
3 Pieces of information

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✔
$a + 2b + c = 0$ ✔
$a + b + 2c = 0$ ✔

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

3 Equations
3 Pieces of information

Rank 3

Rank for matrices

System 1

$a + b + c = 0$ ✔
$a + 2b + c = 0$ ✔
$a + b + 2c = 0$ ✔

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$
$a + b + 3c = 0$ ✓

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank 3

1	1	1
1	2	1
1	1	2

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

System 3

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗

3 Equations
0 Pieces of information

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗

3 Equations
0 Pieces of information

Rank 0

Rank for matrices

System 1

$a + b + c = 0$ ✓
$a + 2b + c = 0$ ✓
$a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
$a + b + 2c = 0$ ✗
$a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
$2a + 2b + 2c = 0$ ✗
$3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗
$0a + 0b + 0c = 0$ ✗

3 Equations
0 Pieces of information

Rank 0

0	0	0
0	0	0
0	0	0

Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.

DeepLearning.AI

Solving System of Linear Equations

Row echelon form

Row echelon form of a matrix

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

Row echelon form of a matrix

Original matrix

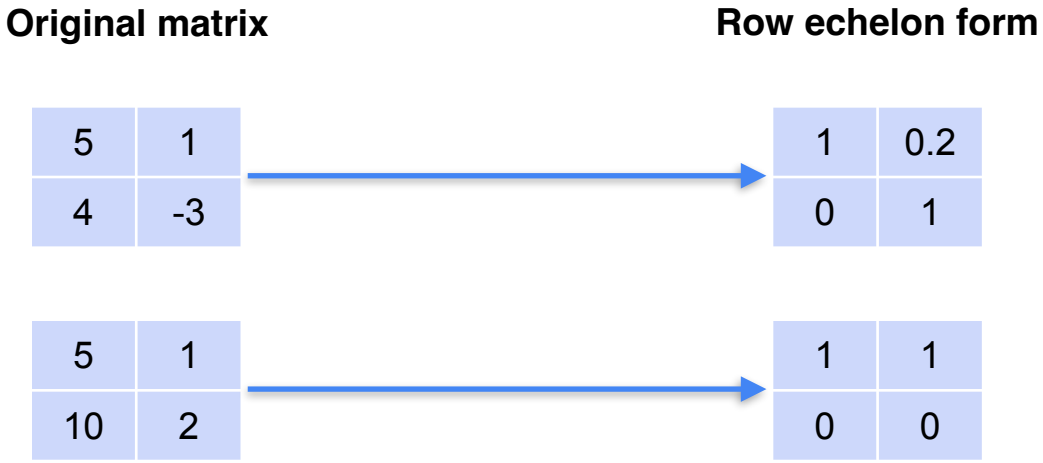
5	1
4	-3

Row echelon form

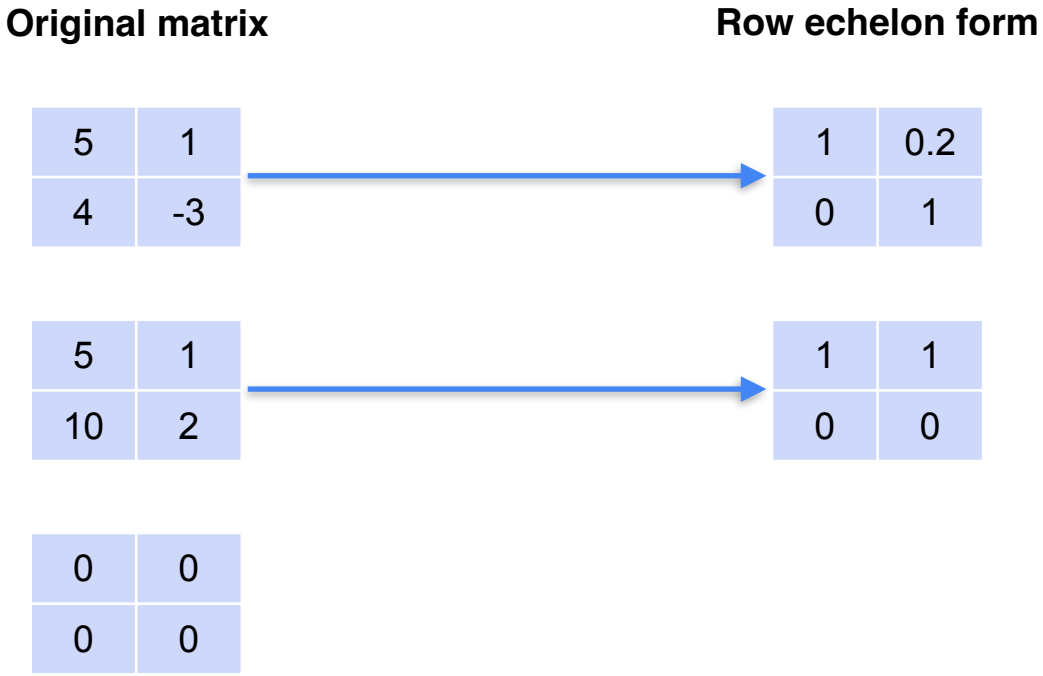
1	0.2
0	1

5	1
10	2

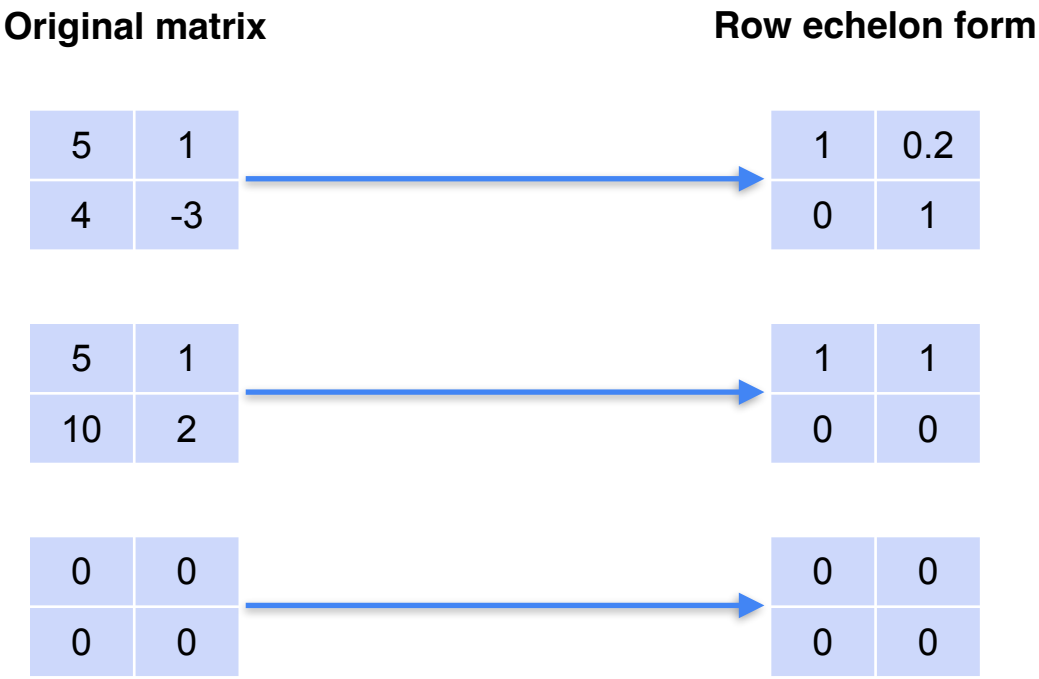
Row echelon form of a matrix



Row echelon form of a matrix



Row echelon form of a matrix



Row echelon form

Original matrix

5	1
4	-3

Row echelon form

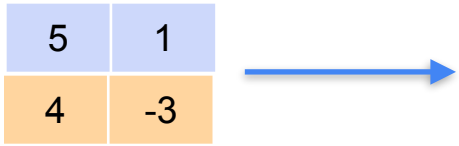
Original matrix

5	1
4	-3

Divide each row by
the leftmost coefficient

Row echelon form

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form

Original matrix

Divide each row by
the leftmost coefficient

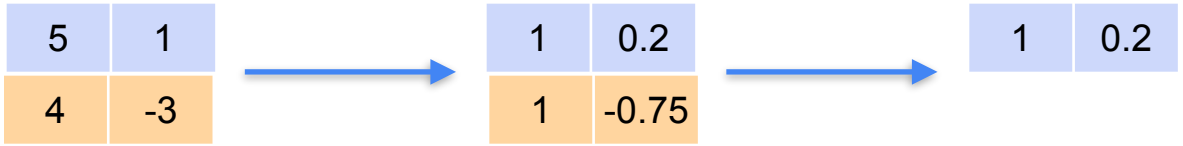
Row echelon form

Original matrix

Divide each row by
the leftmost coefficient

Row echelon form

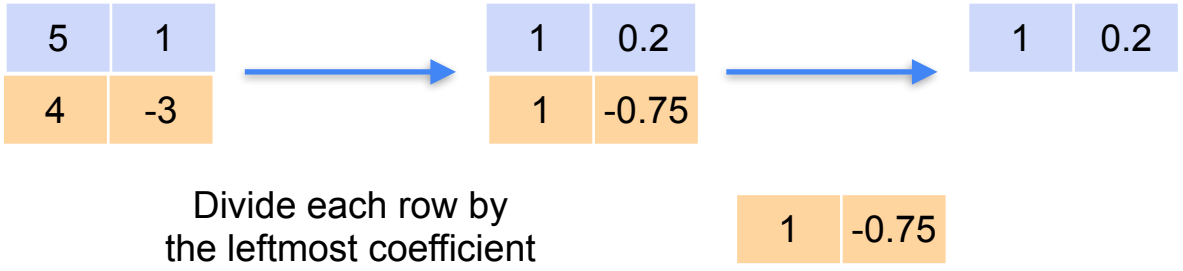
Original matrix



Divide each row by
the leftmost coefficient

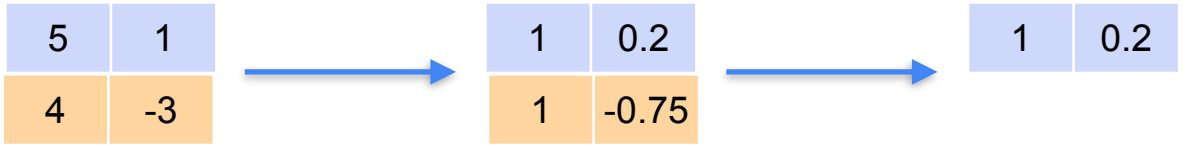
Row echelon form

Original matrix



Row echelon form

Original matrix

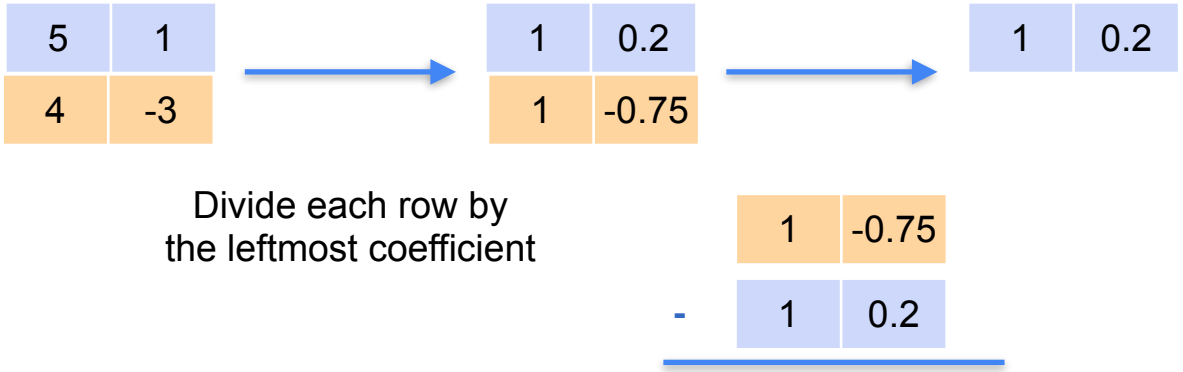


Divide each row by
the leftmost coefficient

1	-0.75
1	0.2

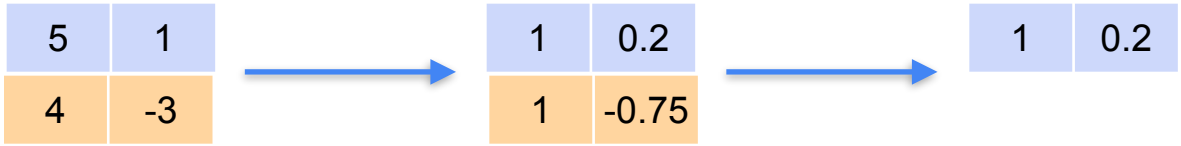
Row echelon form

Original matrix

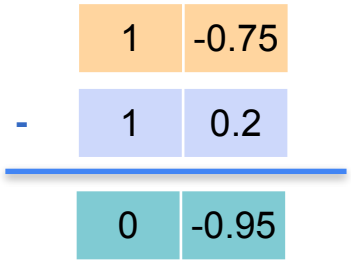


Row echelon form

Original matrix

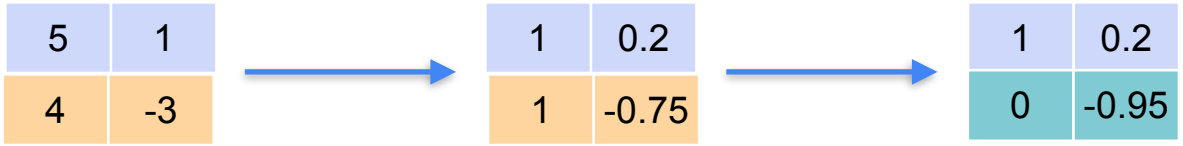


Divide each row by
the leftmost coefficient

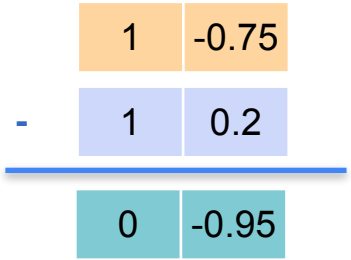


Row echelon form

Original matrix

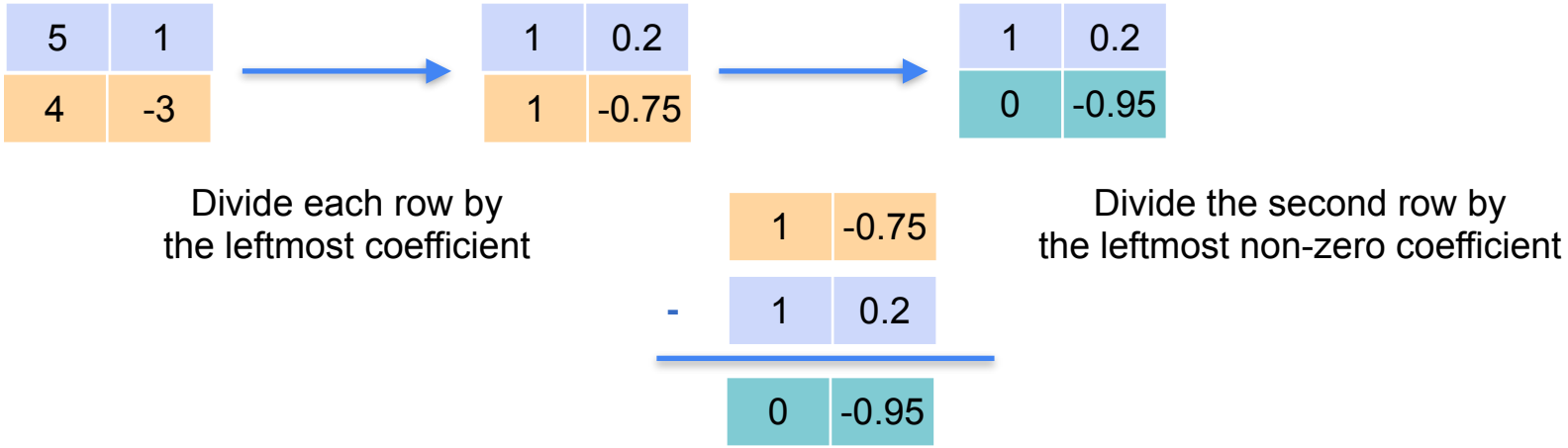


Divide each row by
the leftmost coefficient



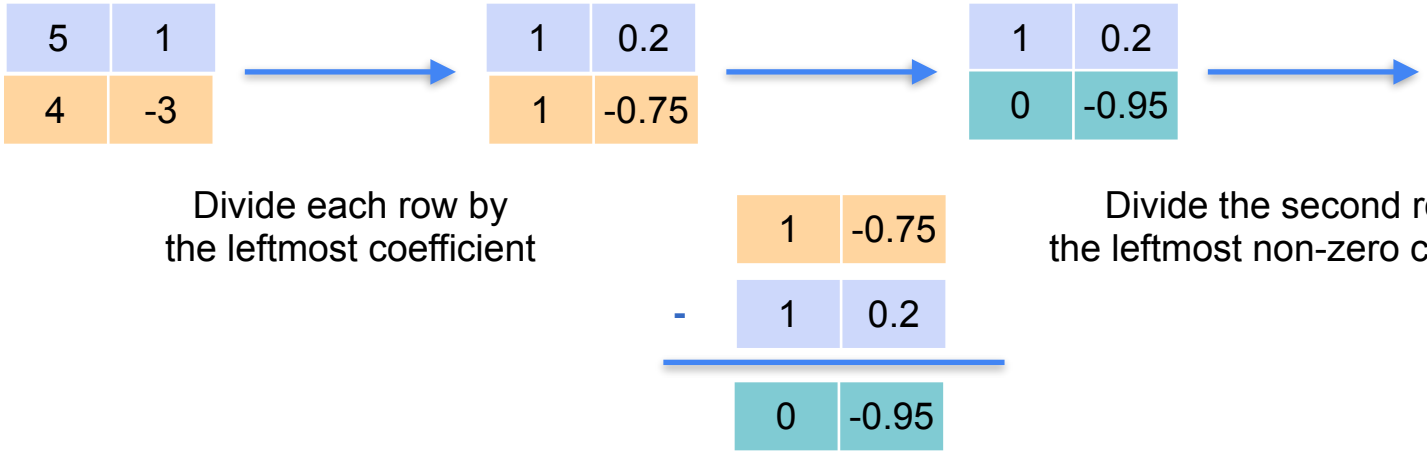
Row echelon form

Original matrix



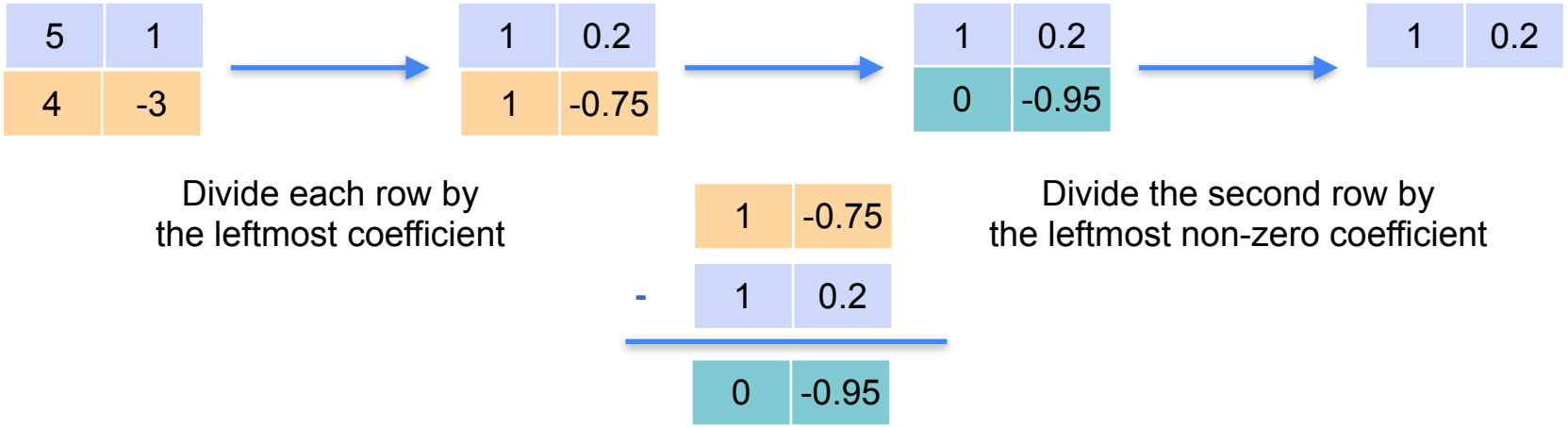
Row echelon form

Original matrix



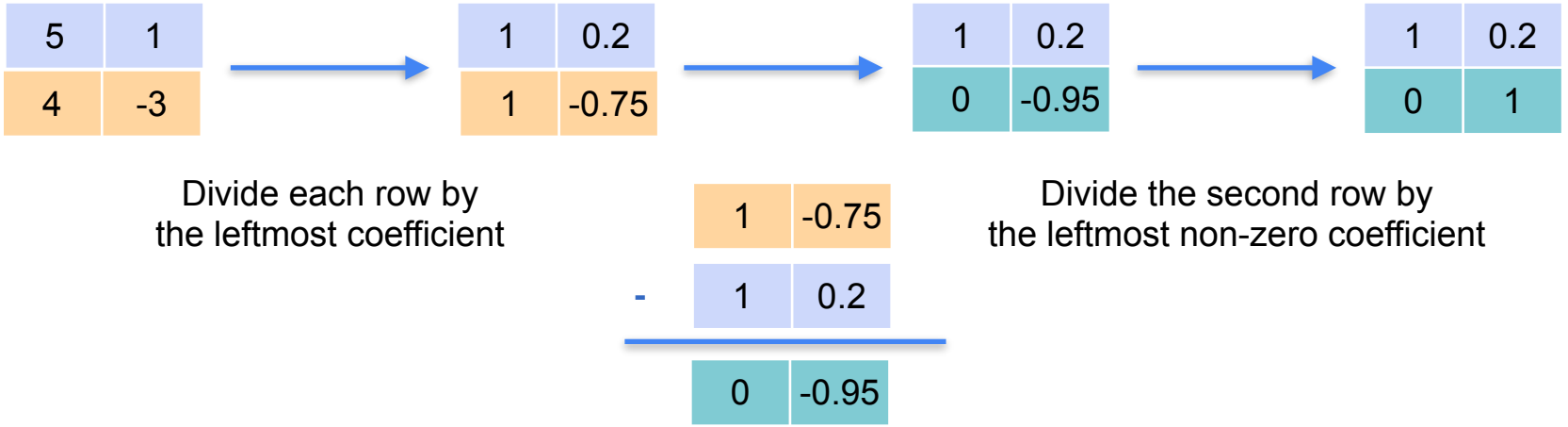
Row echelon form

Original matrix

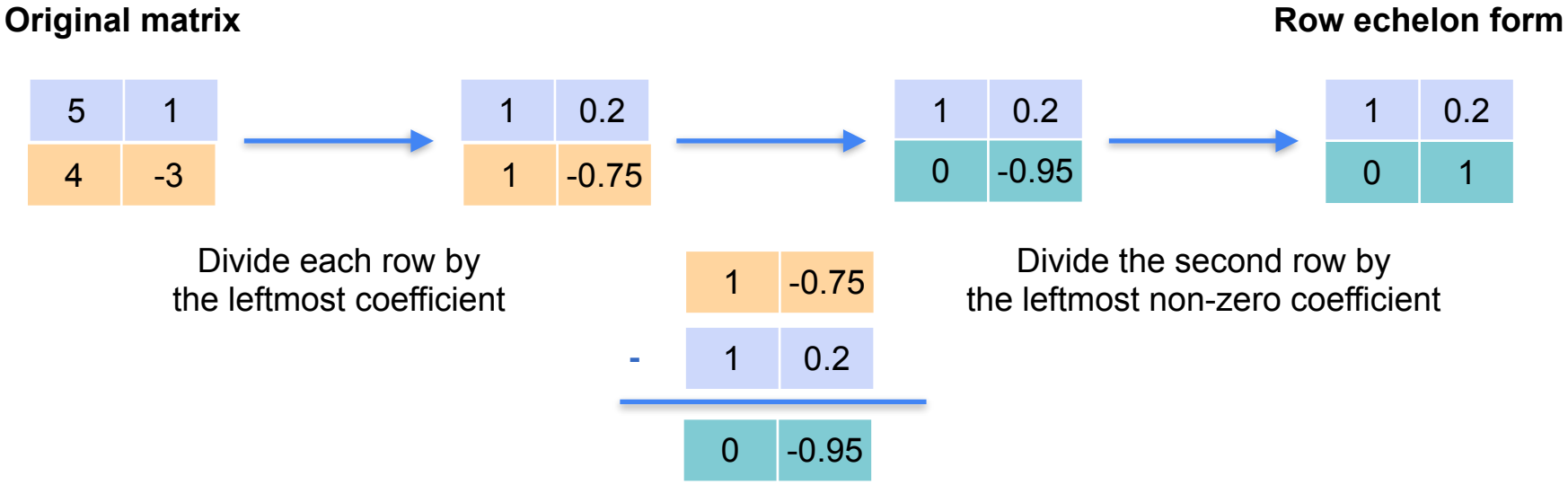


Row echelon form

Original matrix



Row echelon form



Row echelon form for singular matrices

Original matrix

5	1
10	2

Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

Divide each row by
the leftmost coefficient

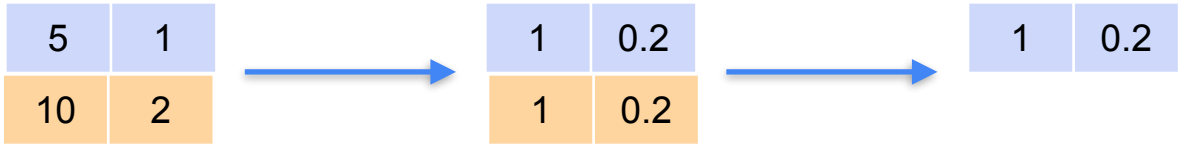
Row echelon form for singular matrices

Original matrix

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix



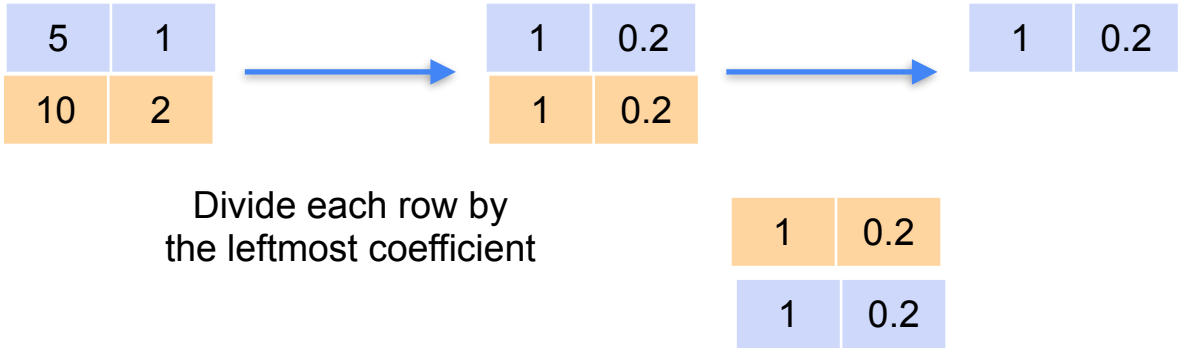
Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

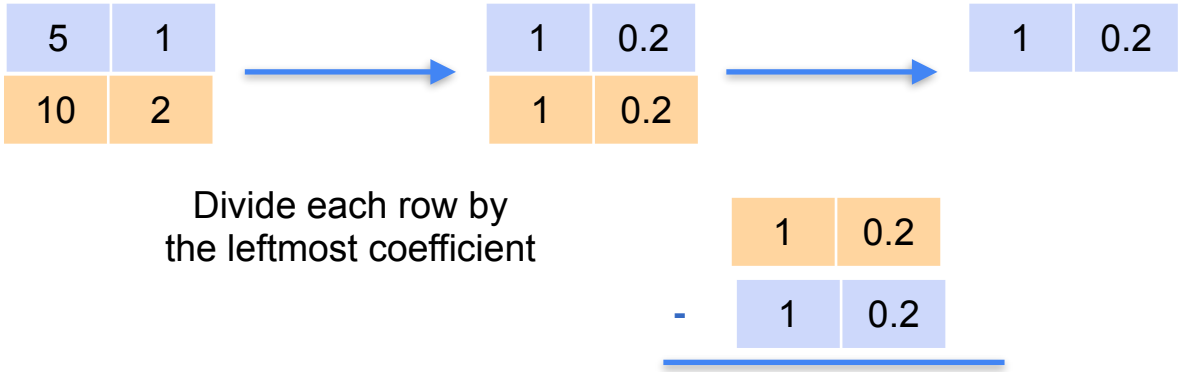
Row echelon form for singular matrices

Original matrix



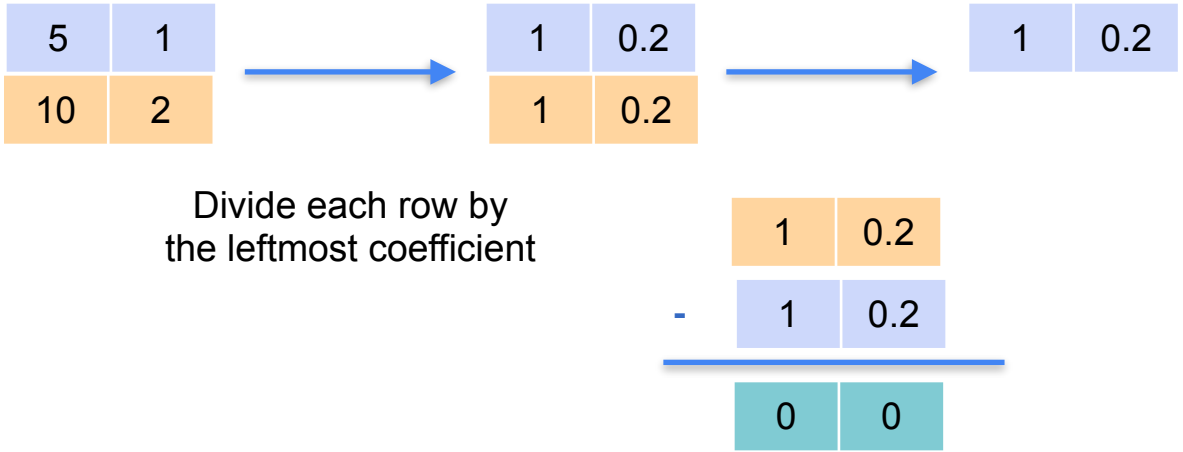
Row echelon form for singular matrices

Original matrix



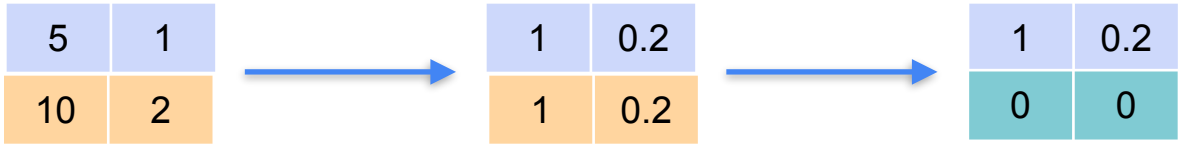
Row echelon form for singular matrices

Original matrix

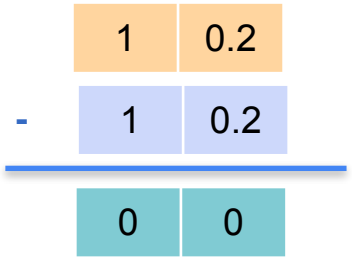


Row echelon form for singular matrices

Original matrix

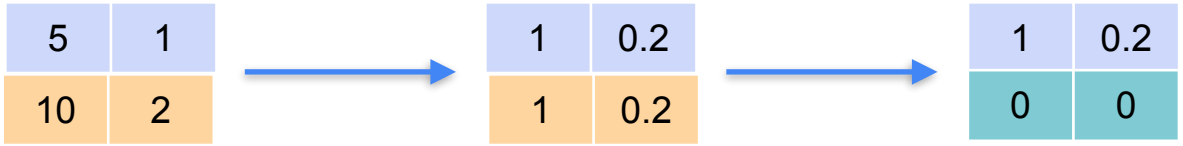


Divide each row by
the leftmost coefficient

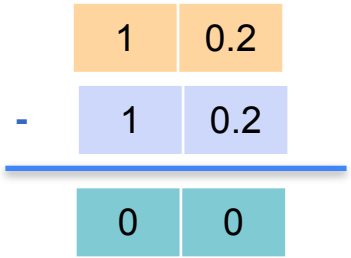


Row echelon form for singular matrices

Original matrix



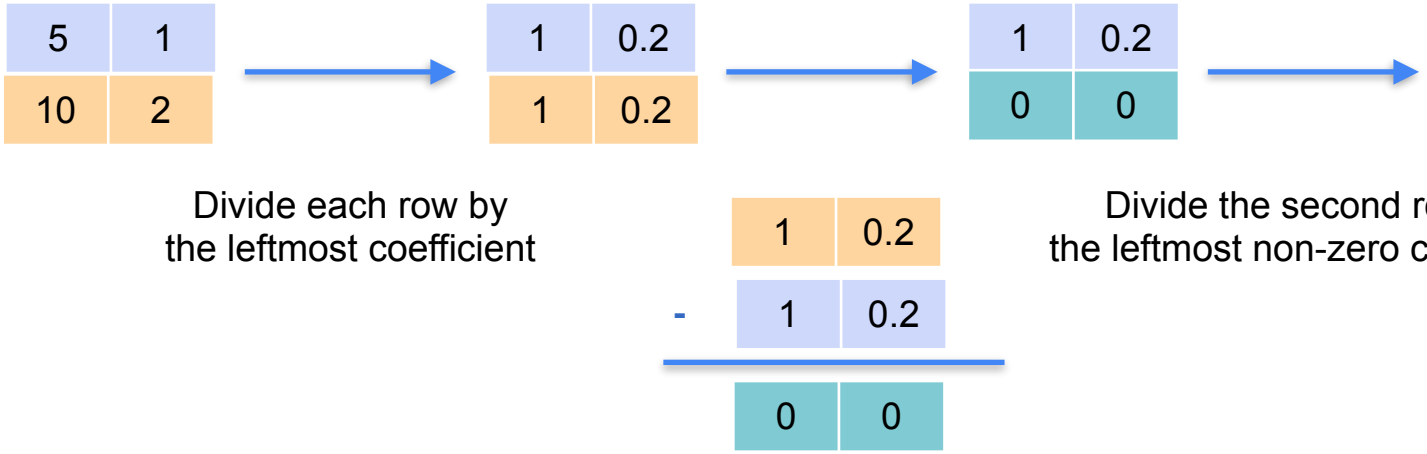
Divide each row by the leftmost coefficient



Divide the second row by the leftmost non-zero coefficient

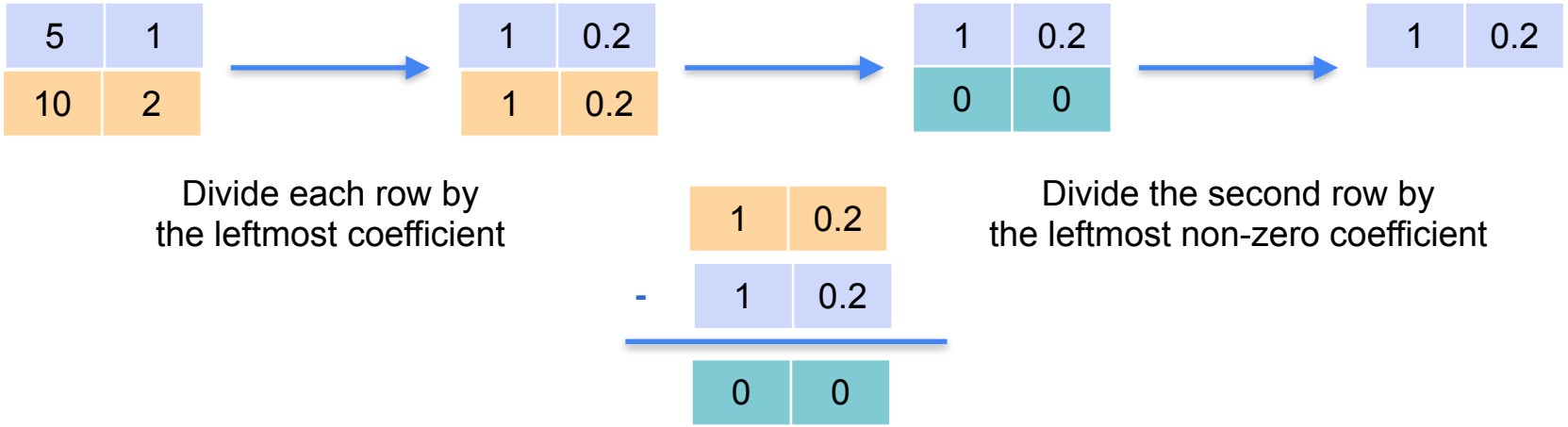
Row echelon form for singular matrices

Original matrix



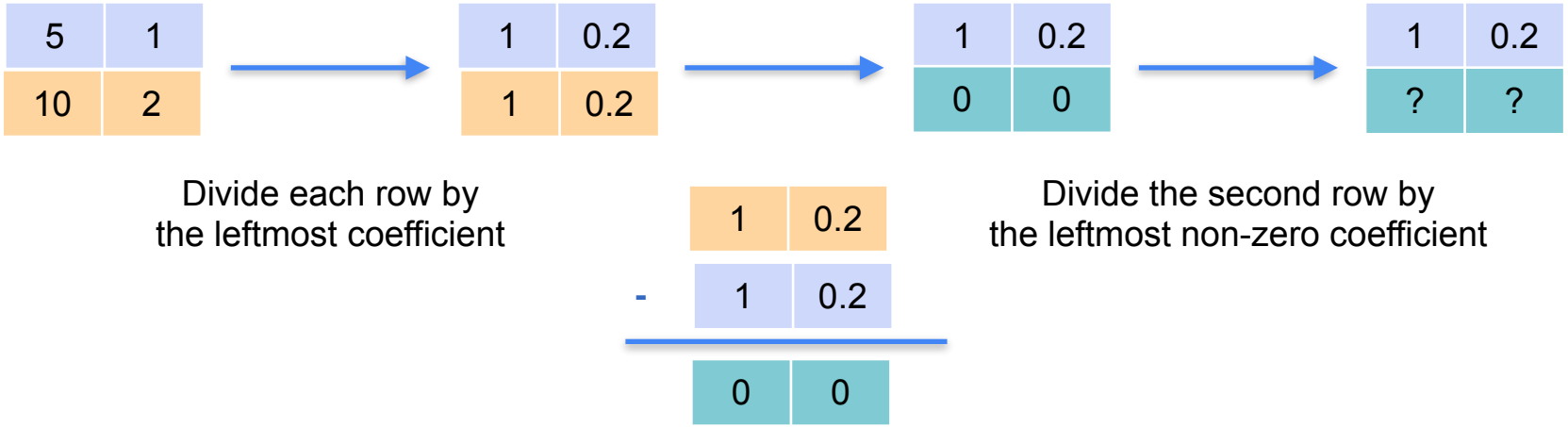
Row echelon form for singular matrices

Original matrix

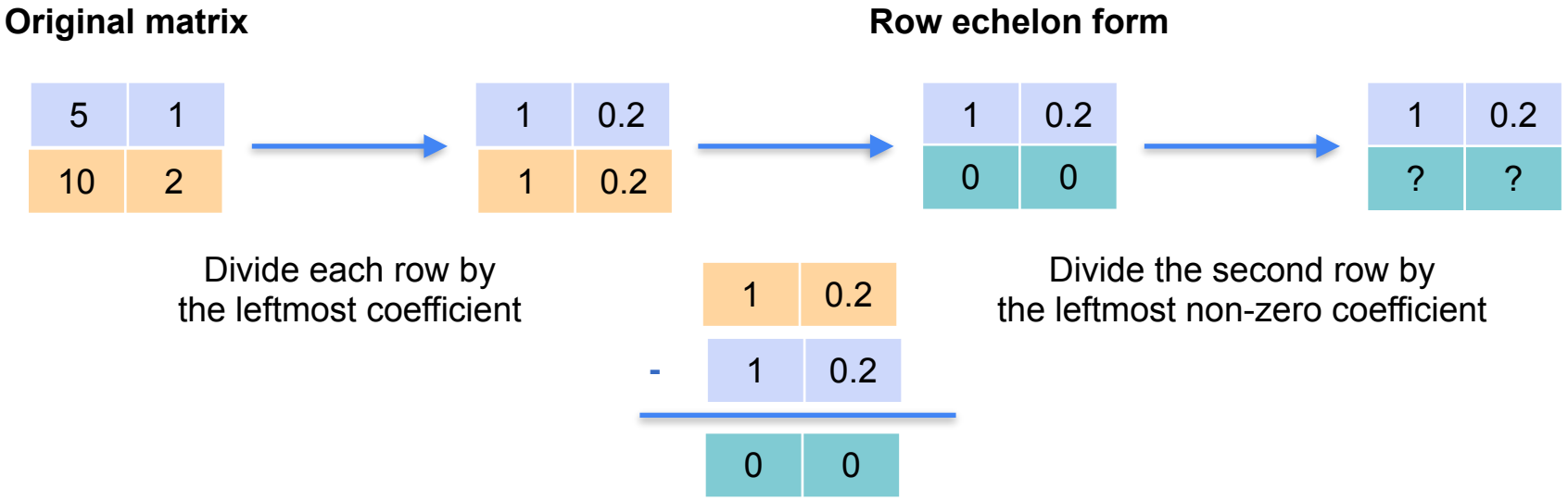


Row echelon form for singular matrices

Original matrix



Row echelon form for singular matrices



Row echelon form for singular matrices

Original matrix

0	0
0	0

Row echelon form for singular matrices

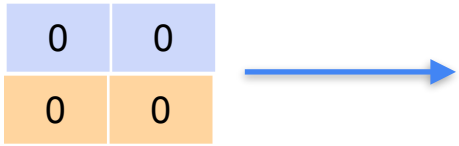
Original matrix

0	0
0	0

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Original matrix

Divide each row by
the leftmost coefficient

Row echelon form for singular matrices

Row echelon form

Original matrix

Divide each row by
the leftmost coefficient

Row echelon form, singularity, and rank

5	1
4	-3

5	1
10	2

0	0
0	0

Row echelon form, singularity, and rank

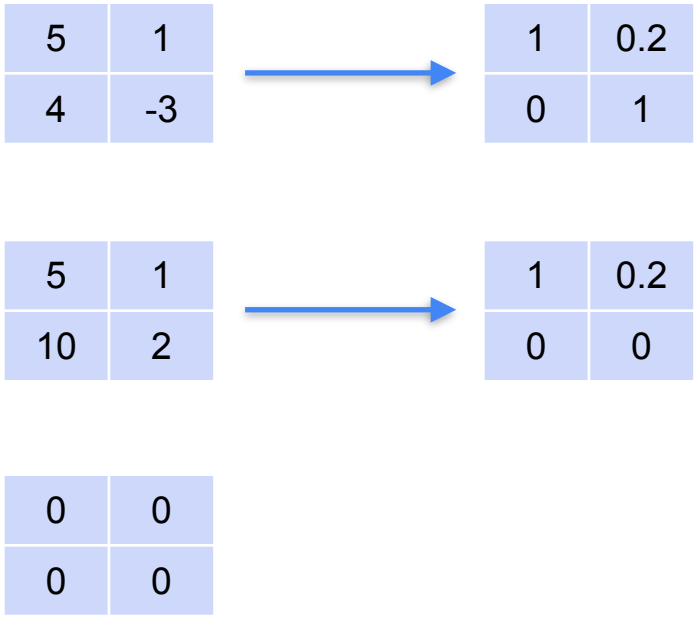
5	1
4	-3

1	0.2
0	1

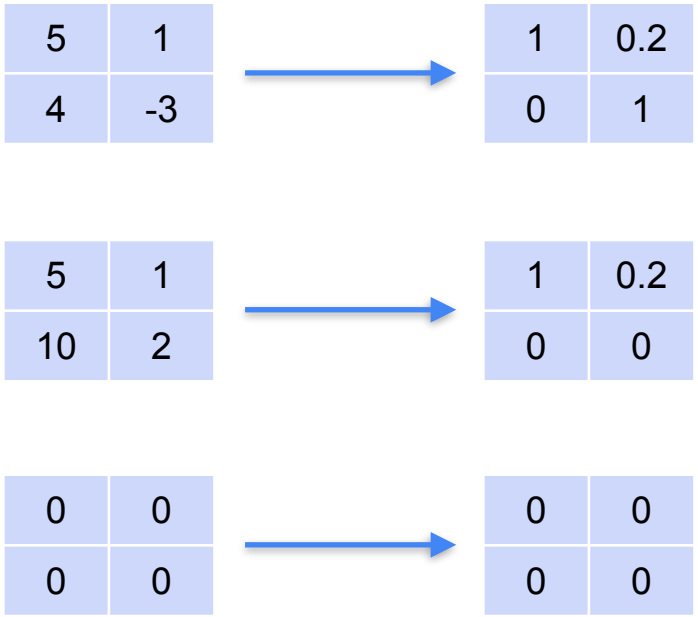
5	1
10	2

0	0
0	0

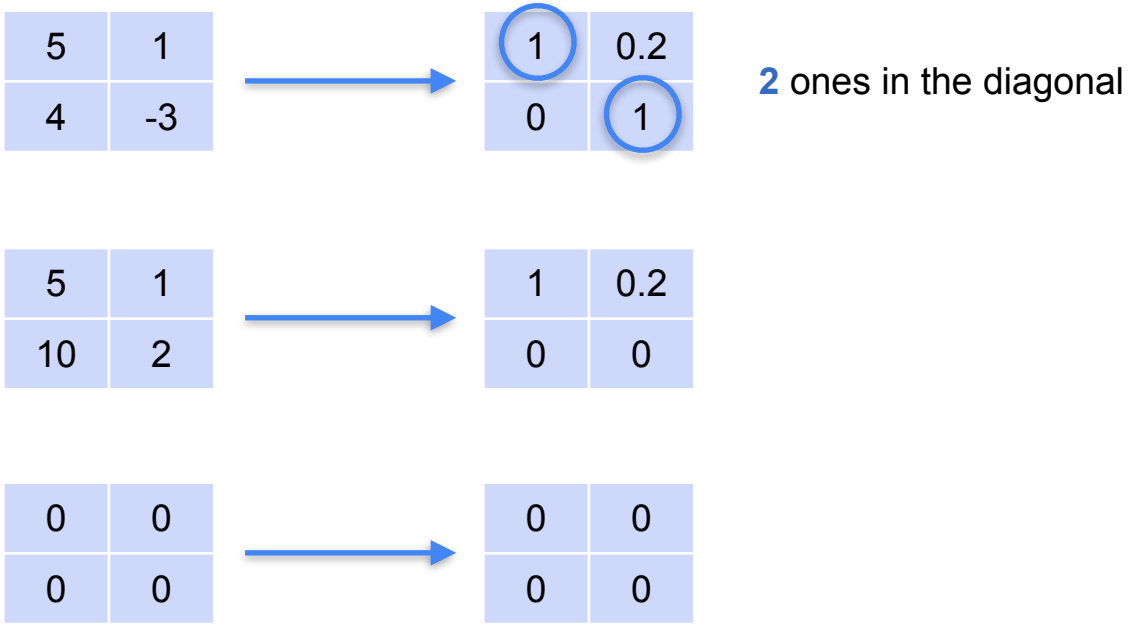
Row echelon form, singularity, and rank



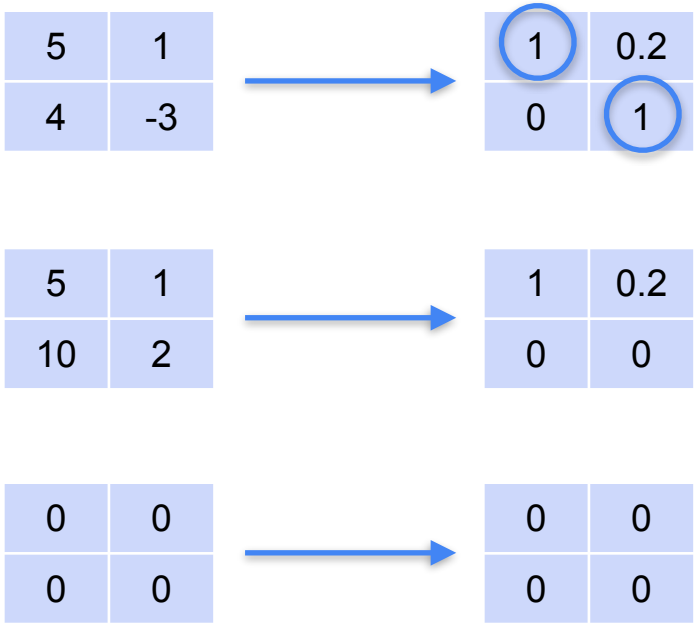
Row echelon form, singularity, and rank



Row echelon form, singularity, and rank



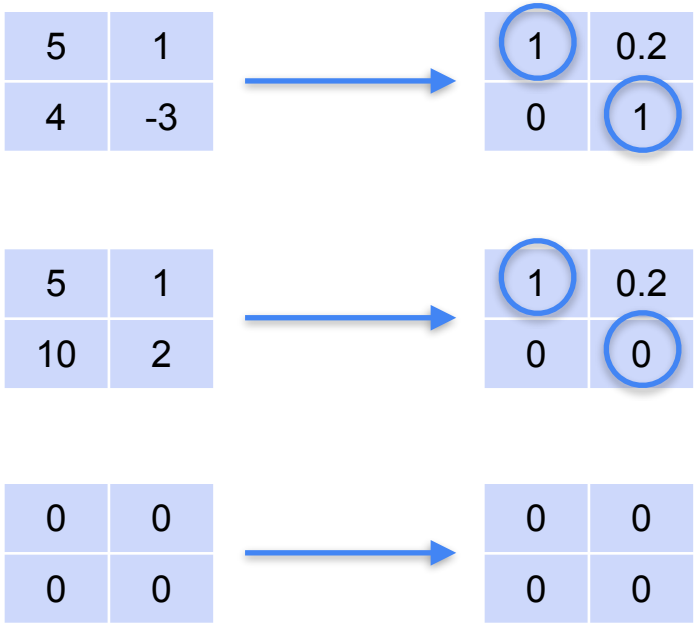
Row echelon form, singularity, and rank



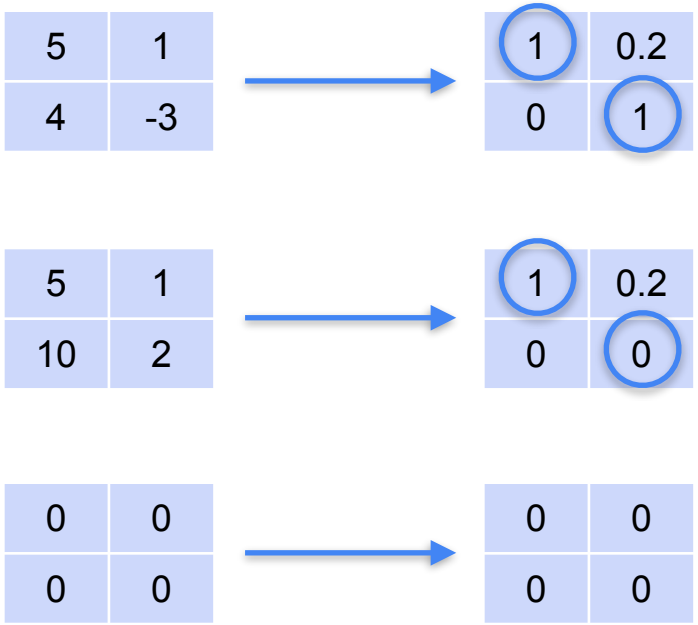
Rank 2

2 ones in the diagonal

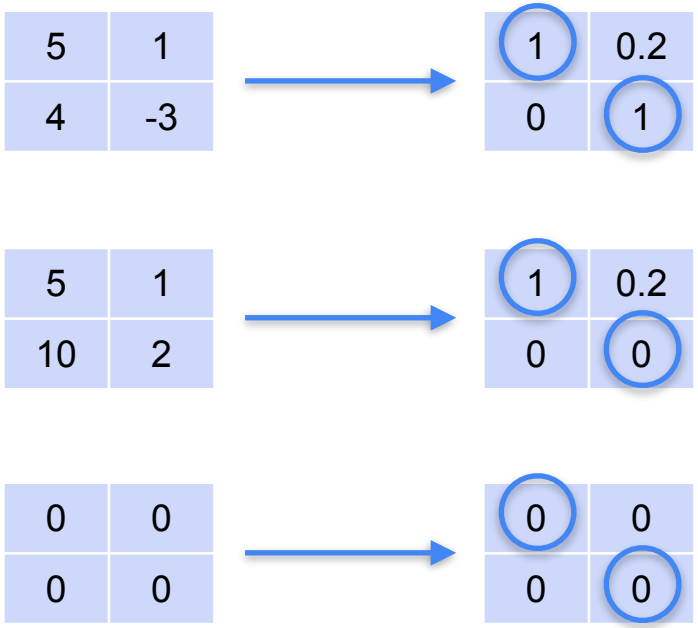
Row echelon form, singularity, and rank



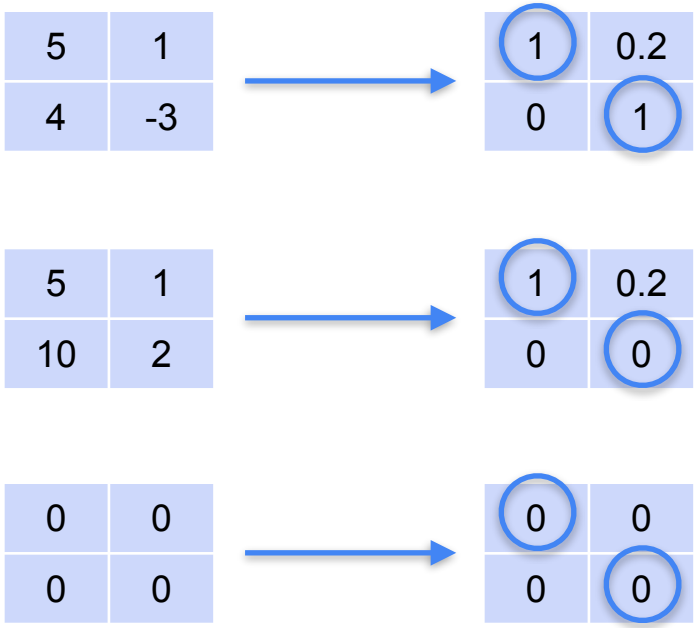
Row echelon form, singularity, and rank



Row echelon form, singularity, and rank



Row echelon form, singularity, and rank



Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3

1	0.2
0	1

Rank 2

2 ones in the diagonal

5	1
10	2

1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0

0	0
0	0

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3

1	0.2
0	1

Rank 2

2 ones in the diagonal

Singular matrix

5	1
10	2

1	0.2
0	0

Rank 1

1 one in the diagonal

0	0
0	0

0	0
0	0

Rank 0

0 ones in the diagonal

Row echelon form, singularity, and rank

Non-singular matrix

Rank 2

2 ones in the diagonal

Singular matrix

Rank 1

1 one in the diagonal

Singular matrix

Rank 0

0 ones in the diagonal

DeepLearning.AI

Solving System of Linear Equations

Row echelon form: General case

Row echelon form

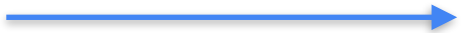
System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Row echelon form

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**

System

- a + b + 2c = **12**
- -6b - 7c = **-33**
- 6c = **18**

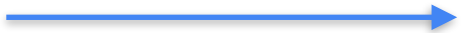
Matrix

1	1	2
3	-3	-1
2	-1	6

Row echelon form

System

- a + b + 2c = **12**
- 3a - 3b - c = **3**
- 2a - b + 6c = **24**

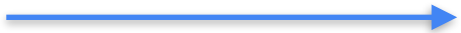


System

- a + b + 2c = **12**
- -6b - 7c = **-33**
- 6c = **18**

Matrix

1	1	2
3	-3	-1
2	-1	6



Row echelon form matrix

1	1	2
0	-6	7
0	0	6

Row echelon form

2	*	*	*	*
0	**1**	*	*	*
0	0	**3**	*	*
0	0	0	**-5**	*
0	0	0	0	**1**

3	*	*	*	*
0	0	**1**	*	*
0	0	0	**-4**	*
0	0	0	**0**	0
0	0	0	0	**0**

Row echelon form

2	*	*	*	*
0	**1**	*	*	*
0	0	**3**	*	*
0	0	0	**-5**	*
0	0	0	0	**1**

3	*	*	*	*
0	0	**1**	*	*
0	0	0	**-4**	*
0	0	0	**0**	0
0	0	0	0	**0**

- Zero rows at the bottom

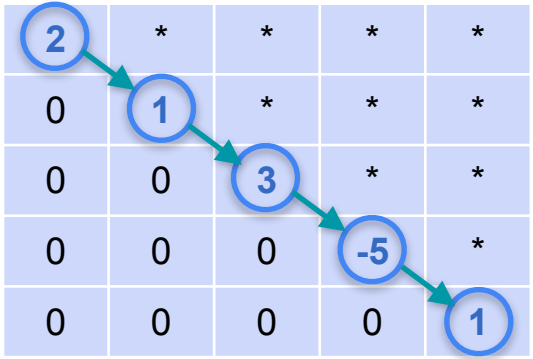
Row echelon form

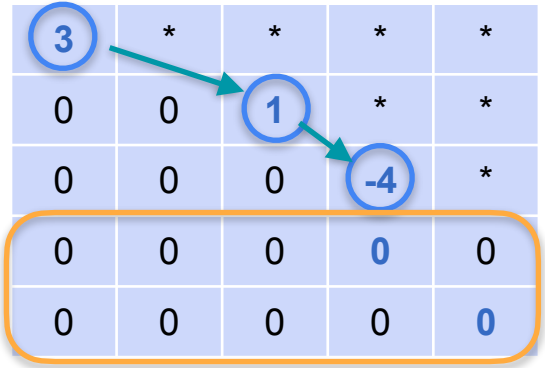
2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)

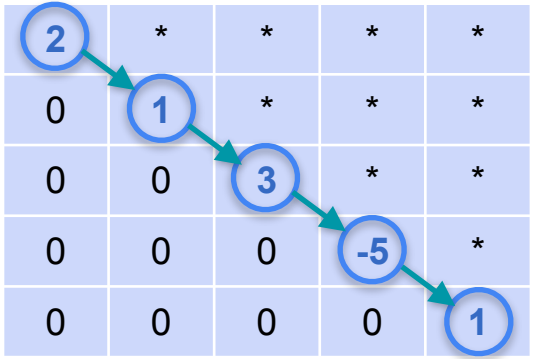
Row echelon form



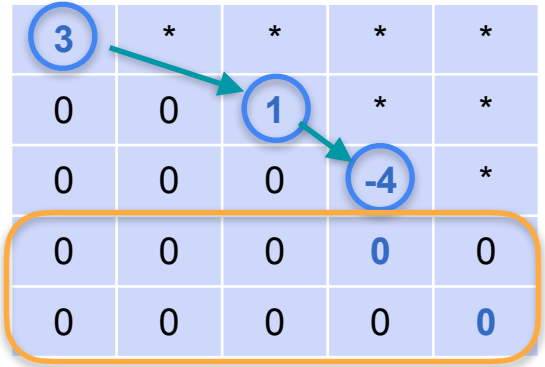


- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above

Row echelon form



Rank 5



Rank 3

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

Another example

Matrix

1	1	1
1	2	1
1	1	2

Another example

Matrix

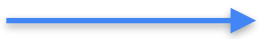
1	1	1
1	2	1
1	1	2

Subtract the first row from the second and the third ones

Another example

Matrix

1	1	1
1	2	1
1	1	2



Row echelon form

1	1	1
0	1	0
0	0	1

Subtract the first row from the second and the third ones

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3

Subtract the first row from the second and the third ones

What if the matrix is singular?

Matrix

1	1	1
1	1	2
1	1	3

1	1	1
0	0	1
0	0	2

Subtract the first row from the second and the third ones

What if the matrix is singular?

Matrix

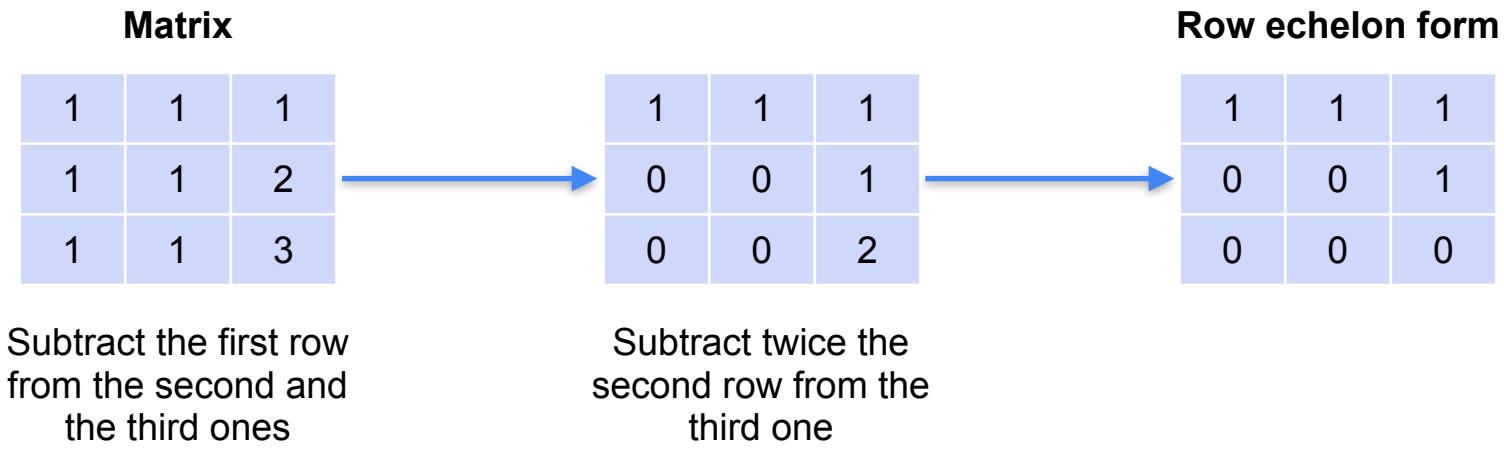
1	1	1
1	1	2
1	1	3

1	1	1
0	0	1
0	0	2

Subtract the first row from the second and the third ones

Subtract twice the second row from the third one

What if the matrix is singular?



What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

Subtract twice the first row from the second row

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

1	1	1
0	0	0
3	3	3

Subtract twice the first row from the second row

What if the matrix is singular?

Matrix

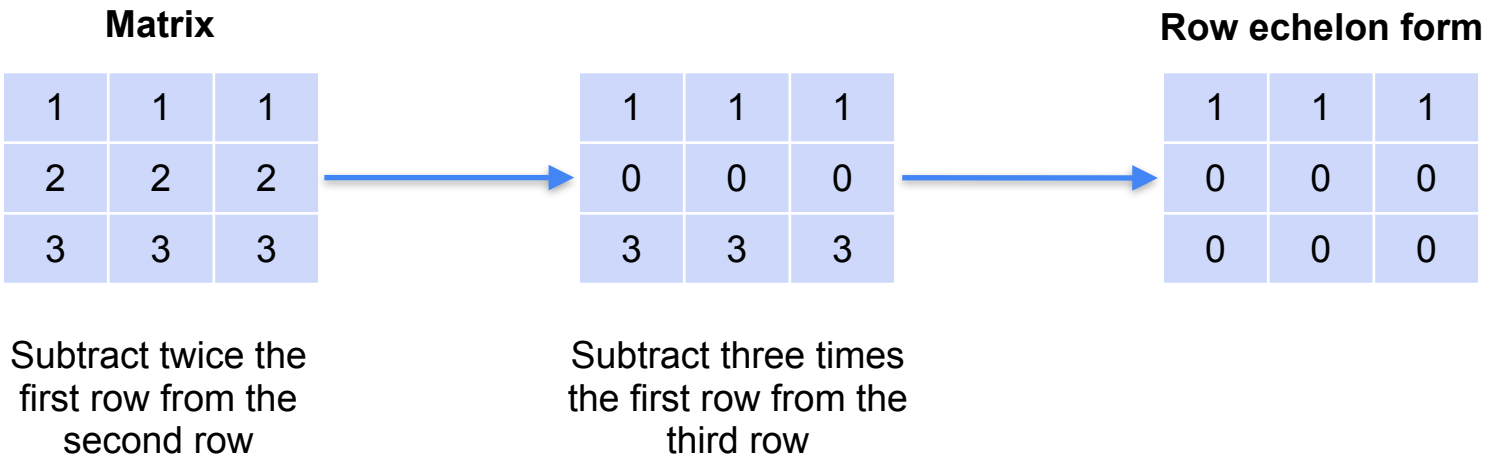
1	1	1
2	2	2
3	3	3

1	1	1
0	0	0
3	3	3

Subtract twice the first row from the second row

Subtract three times the first row from the third row

What if the matrix is singular?



Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Number of pivots = 3

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

DeepLearning.AI

Solving System of Linear Equations

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

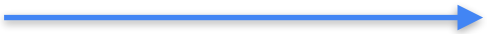
Solved system

- $a = 3$
- $b = 2$

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$



Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $a = 3$
- $b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Diagonal matrix

1	0
0	1

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Reduced row echelon form

Row echelon form

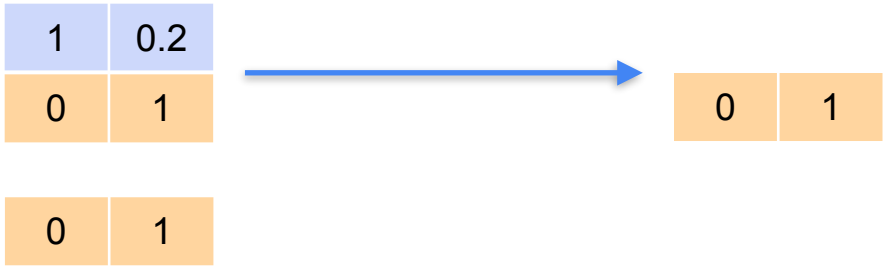
1	0.2
0	1

Reduced row echelon form

Row echelon form

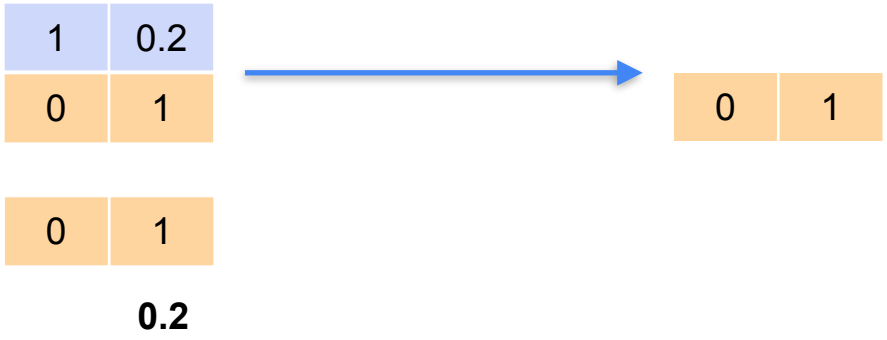
Reduced row echelon form

Row echelon form

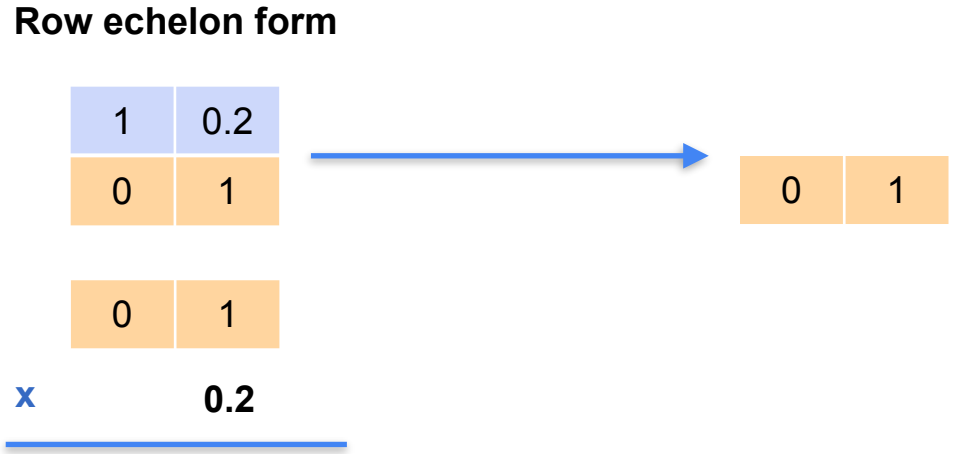


Reduced row echelon form

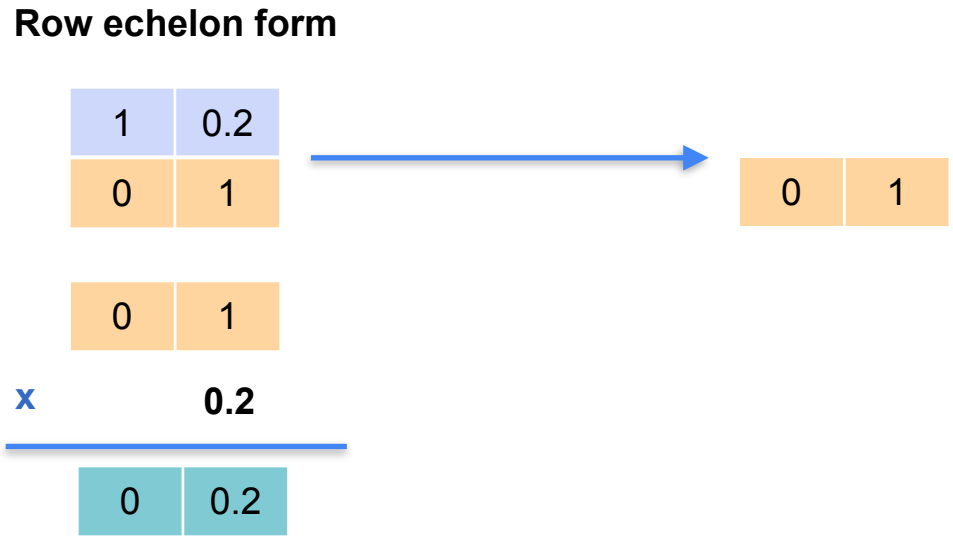
Row echelon form



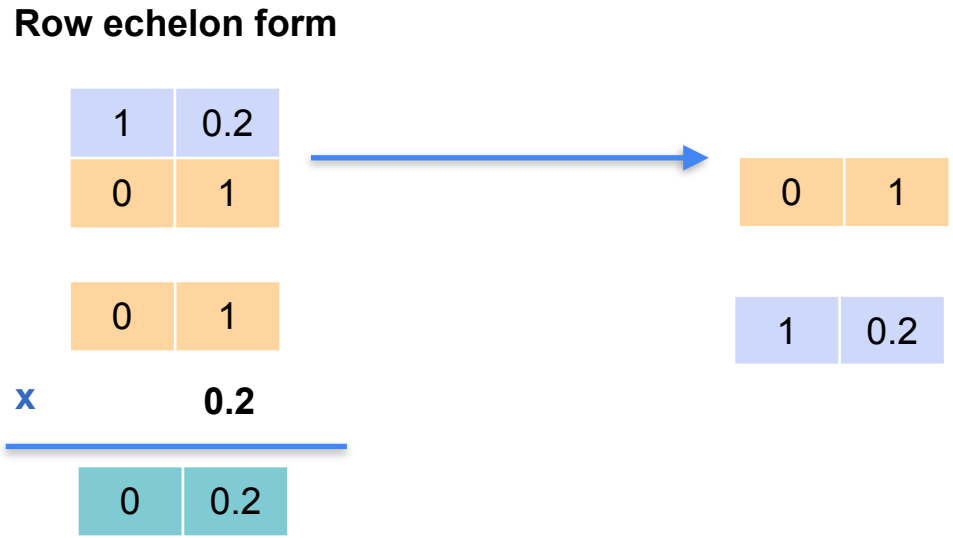
Reduced row echelon form



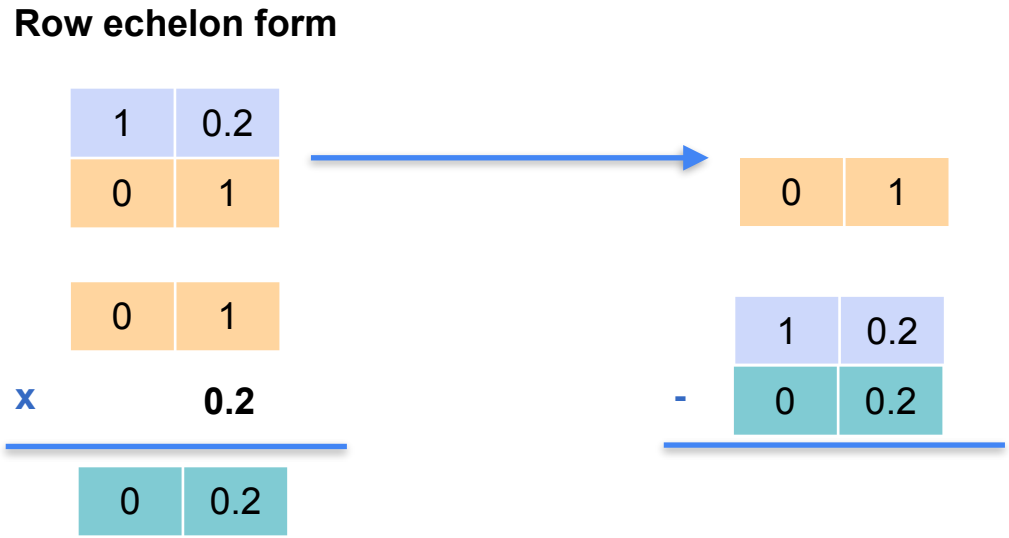
Reduced row echelon form



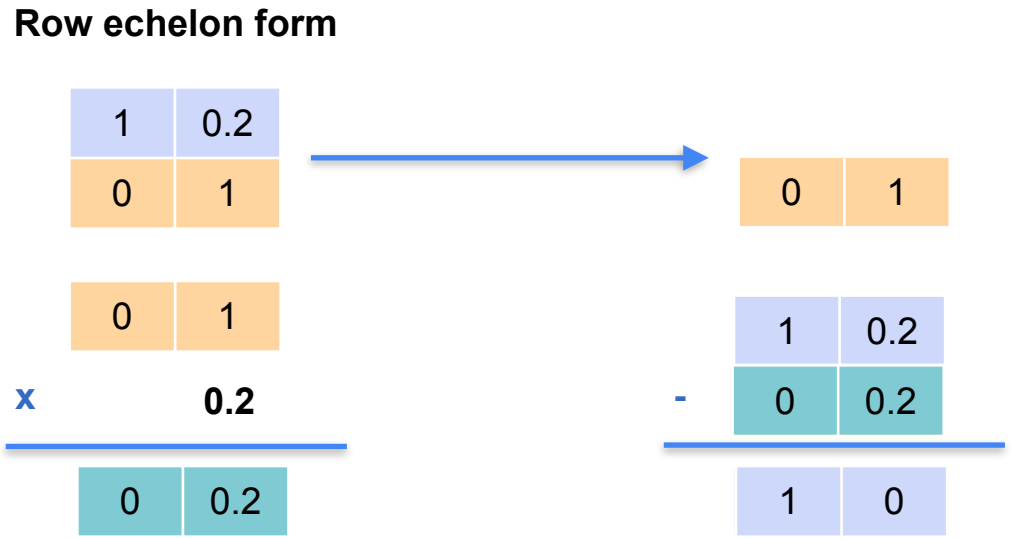
Reduced row echelon form



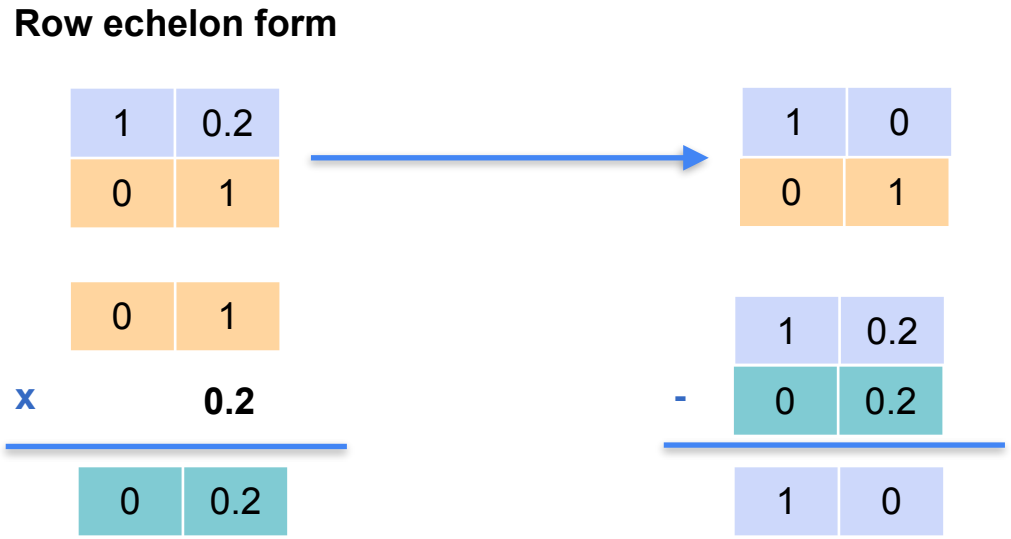
Reduced row echelon form



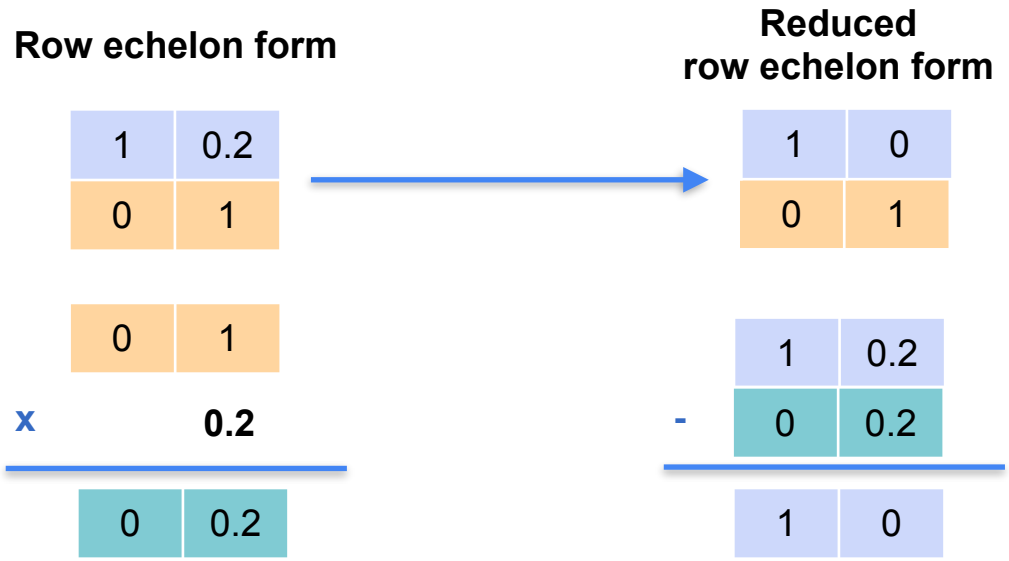
Reduced row echelon form



Reduced row echelon form



Reduced row echelon form



Reduced row echelon form

1	0	0	0	0
0	**1**	0	0	0
0	0	**1**	0	0
0	0	0	**1**	0
0	0	0	0	**1**

1	*	0	0	*
0	0	**1**	0	*
0	0	0	**1**	*
0	0	0	**0**	0
0	0	0	0	**0**

Reduced row echelon form

1	0	0	0	0
0	**1**	0	0	0
0	0	**1**	0	0
0	0	0	**1**	0
0	0	0	0	**1**

1	*	0	0	*
0	0	**1**	0	*
0	0	0	**1**	*
0	0	0	**0**	0
0	0	0	0	**0**

- Is in row echelon form

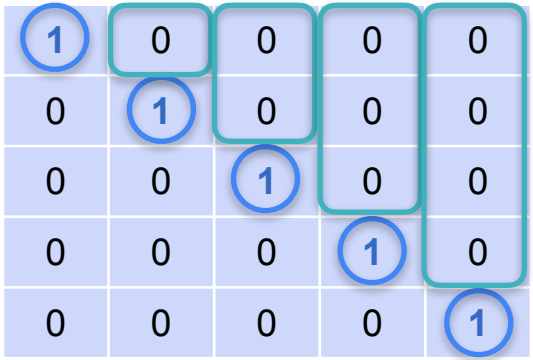
Reduced row echelon form

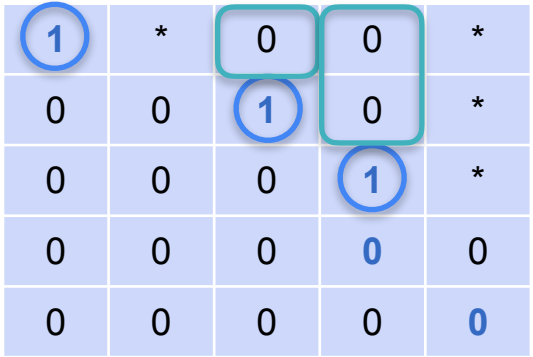
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1

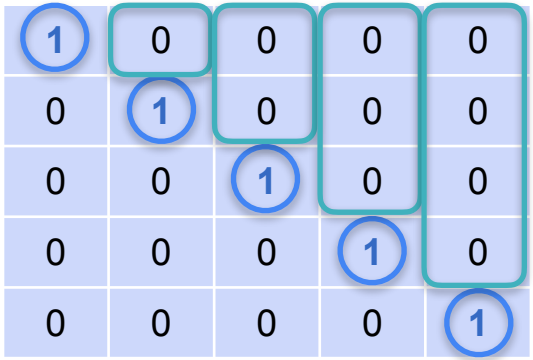
Reduced row echelon form



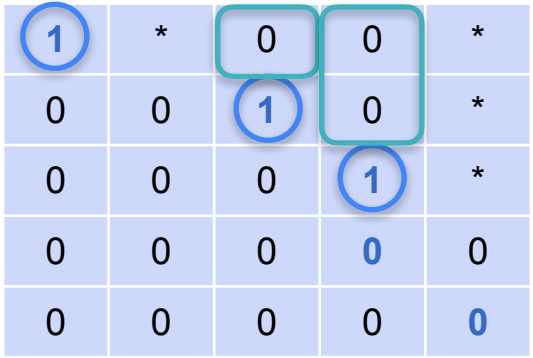


- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0

Reduced row echelon form



Rank 5



Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

Reduced row echelon form

Reduced row echelon form

3	*	*	*	*
0	0	**2**	*	*
0	0	0	**-4**	*
0	0	0	**0**	0
0	0	0	0	**0**

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the value of the pivot

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the value of the pivot

Reduced row echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a pivot to 0

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Reduced row echelon form

Row echelon form

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one

1	0	-5
0	1	4
0	0	1

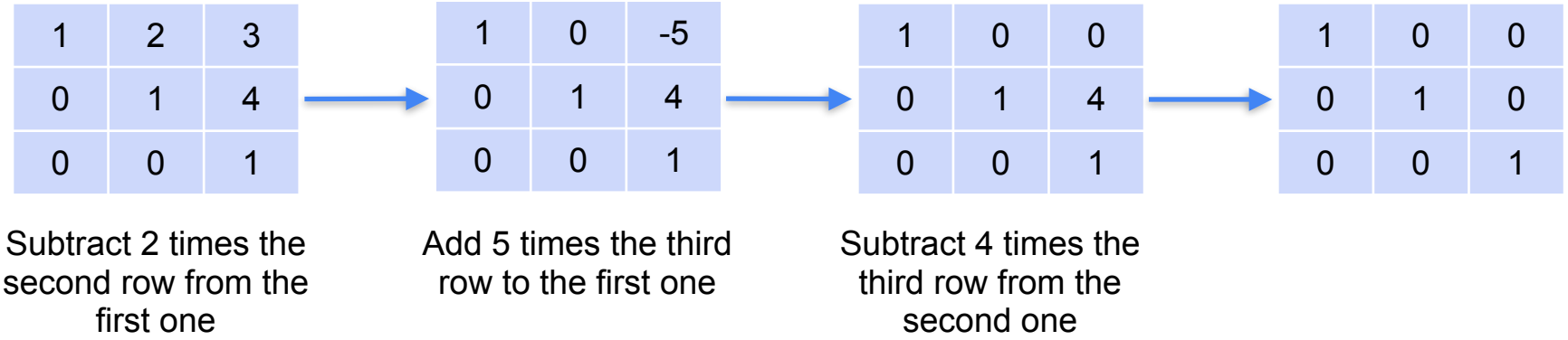
Add 5 times the third row to the first one

1	0	0
0	1	4
0	0	1

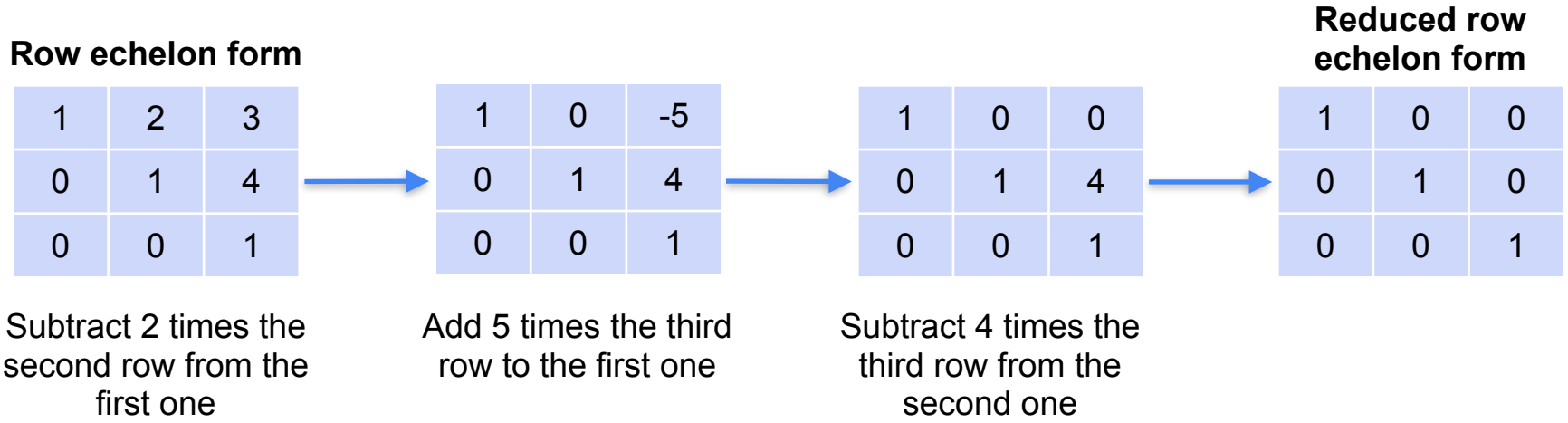
Subtract 4 times the third row from the second one

Reduced row echelon form

Row echelon form



Reduced row echelon form



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Solving System of Linear Equations

Conclusion