



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



DeepLearning.AI

Vectors and Linear Transformations

Machine Learning motivation

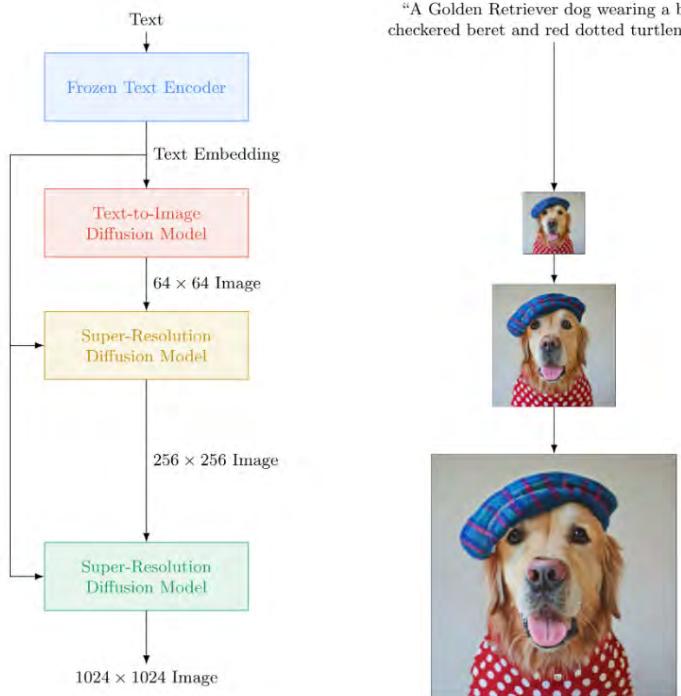
Neural Networks - AI generated images



AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."



Input

Model



Output

wall clock - wall clock.

3.0s

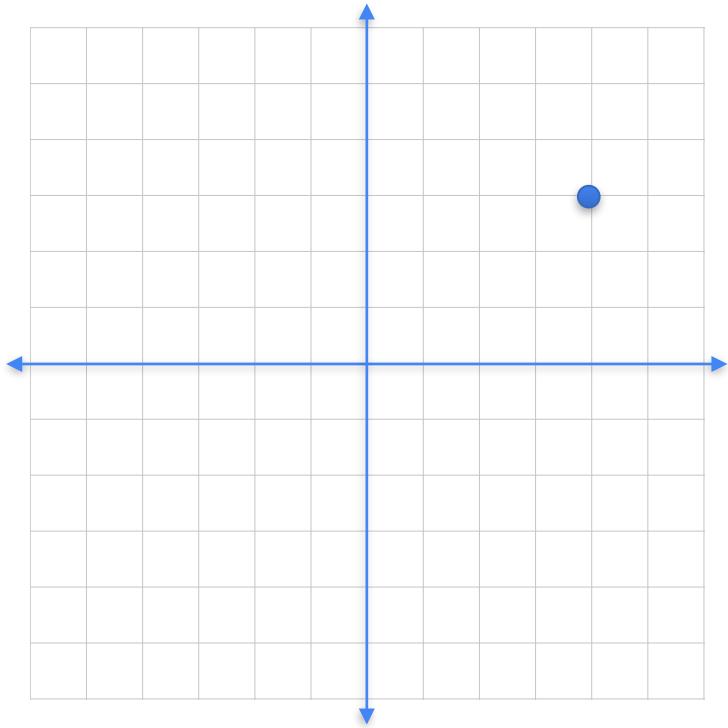


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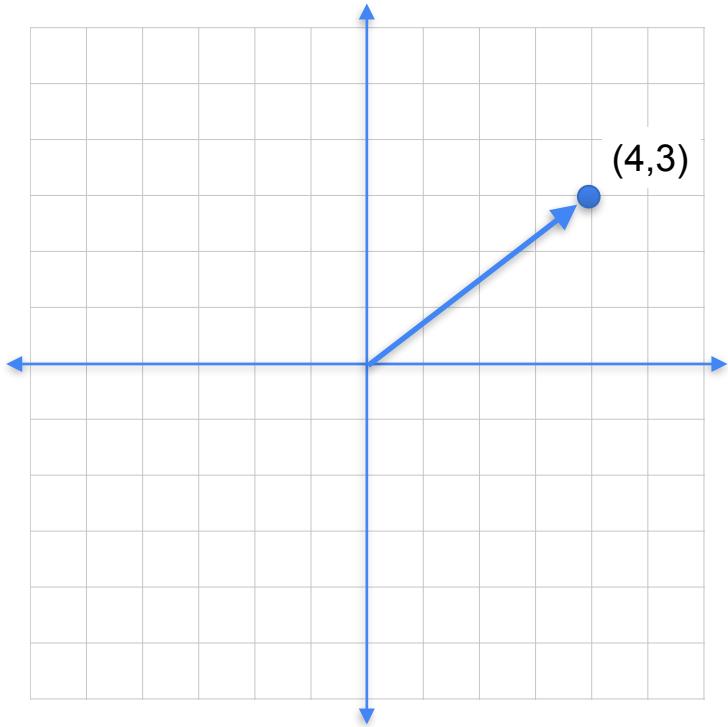
Vectors and Linear Transformations

Vectors and their properties

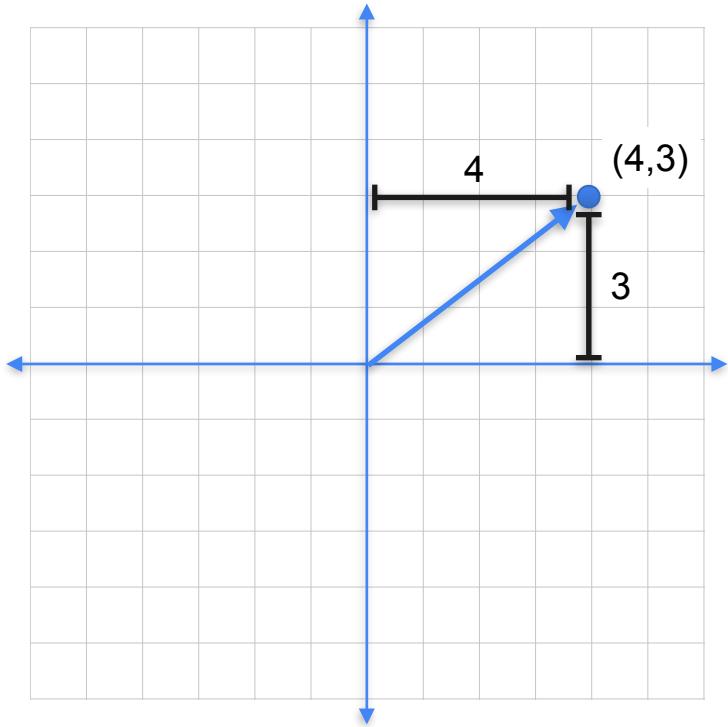
Vectors



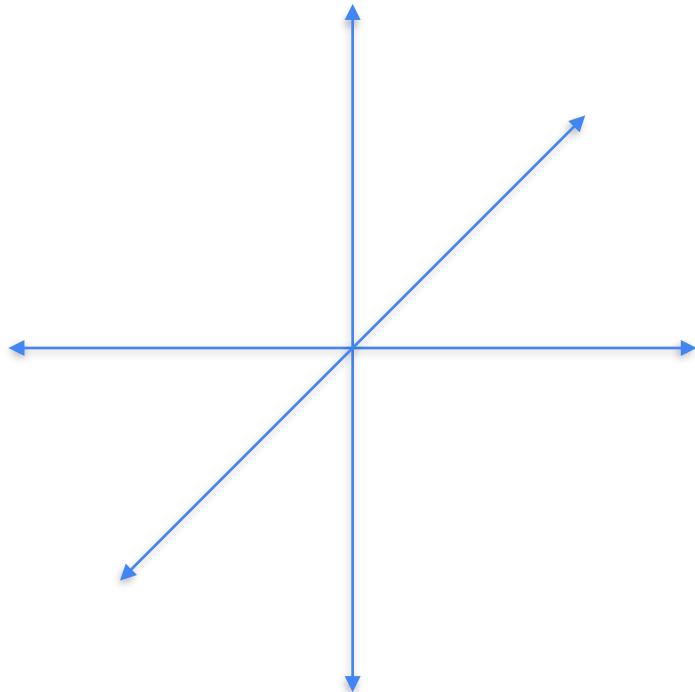
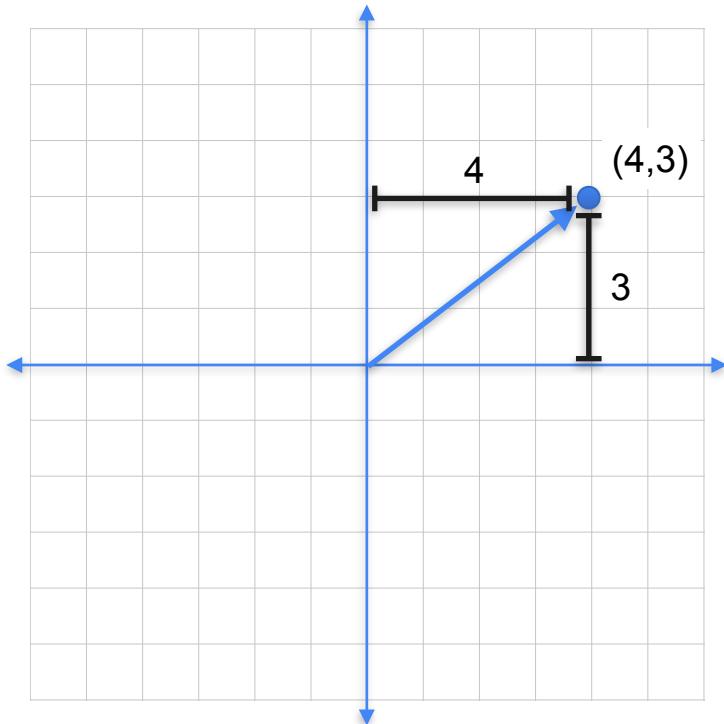
Vectors



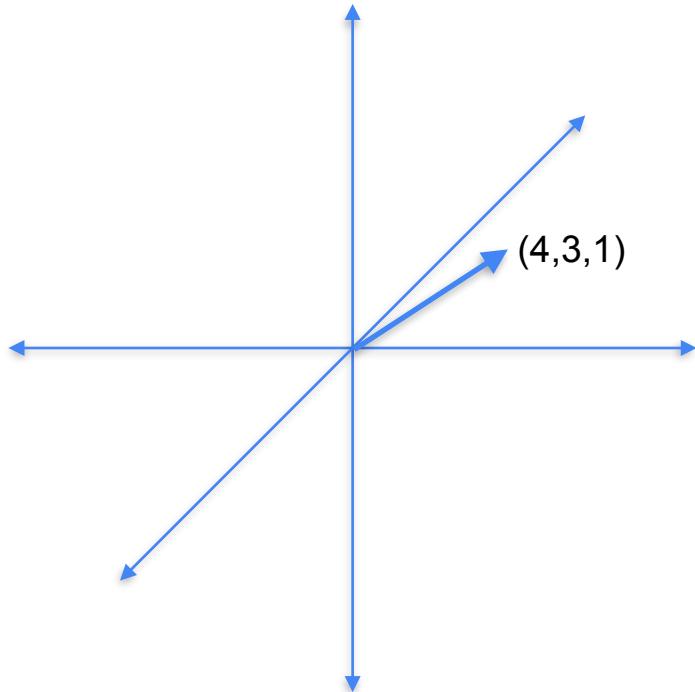
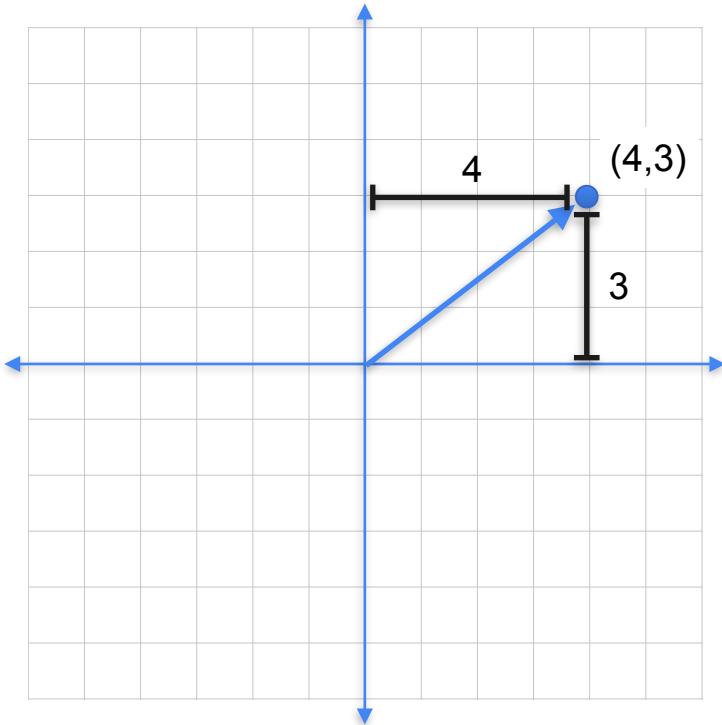
Vectors



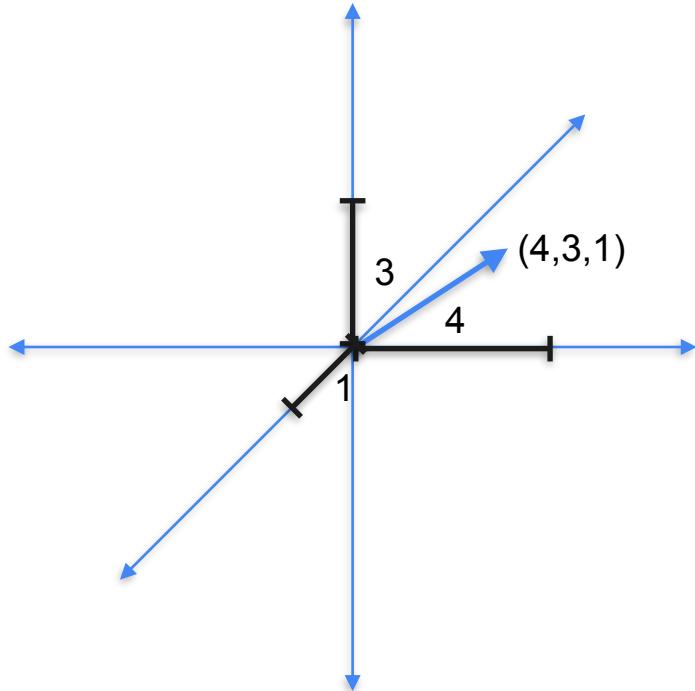
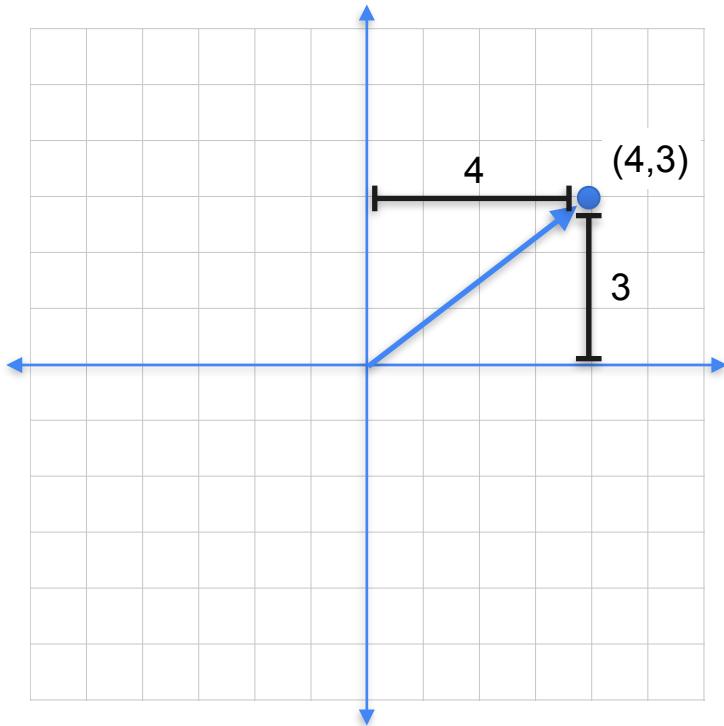
Vectors



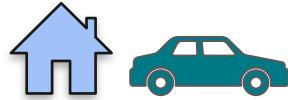
Vectors



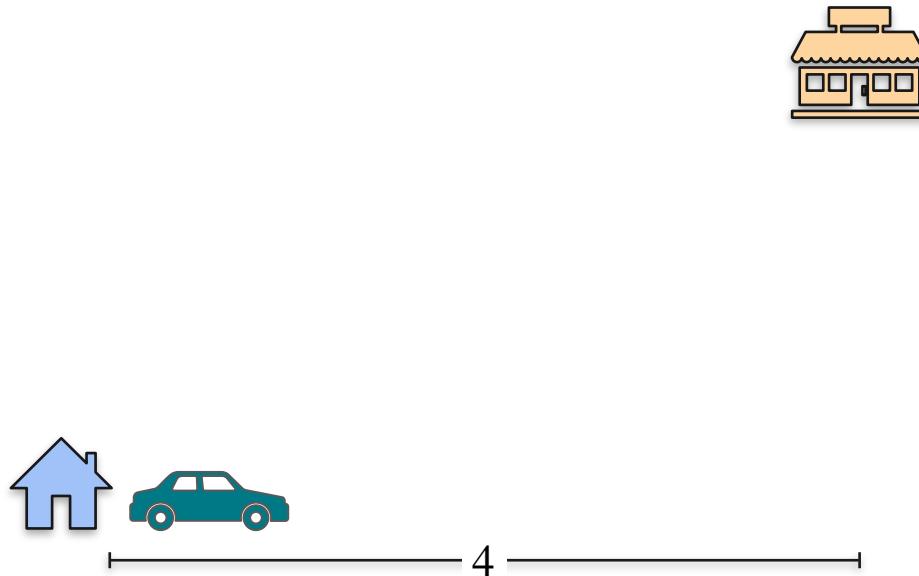
Vectors



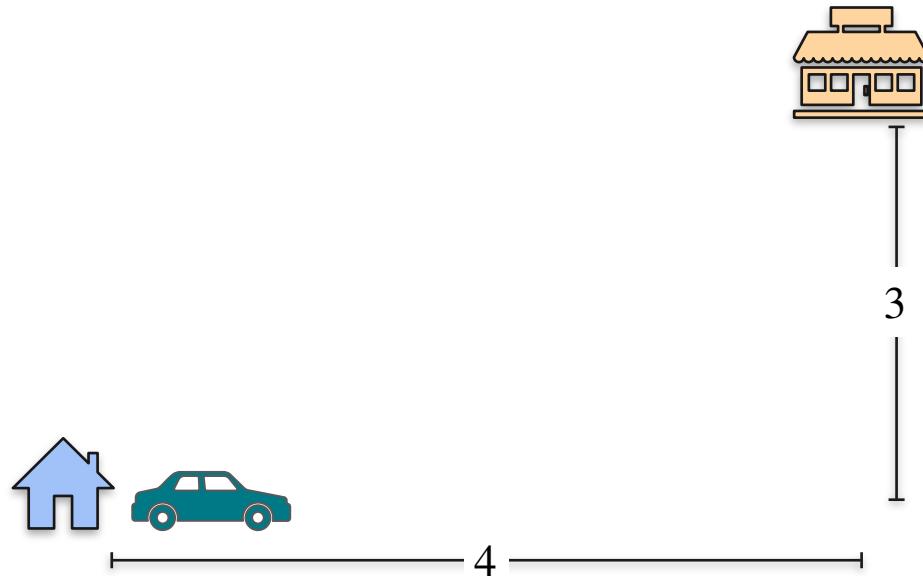
How to get from point A to point B?



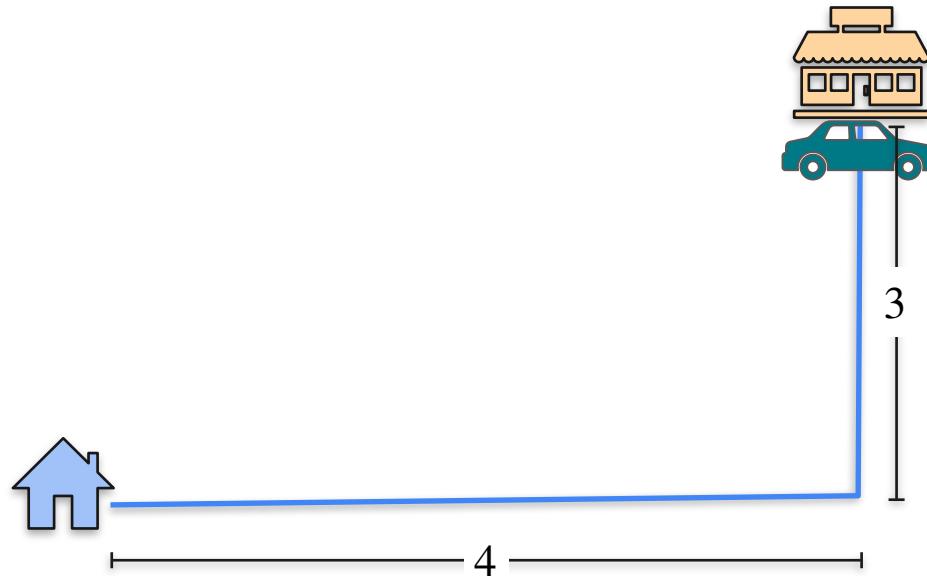
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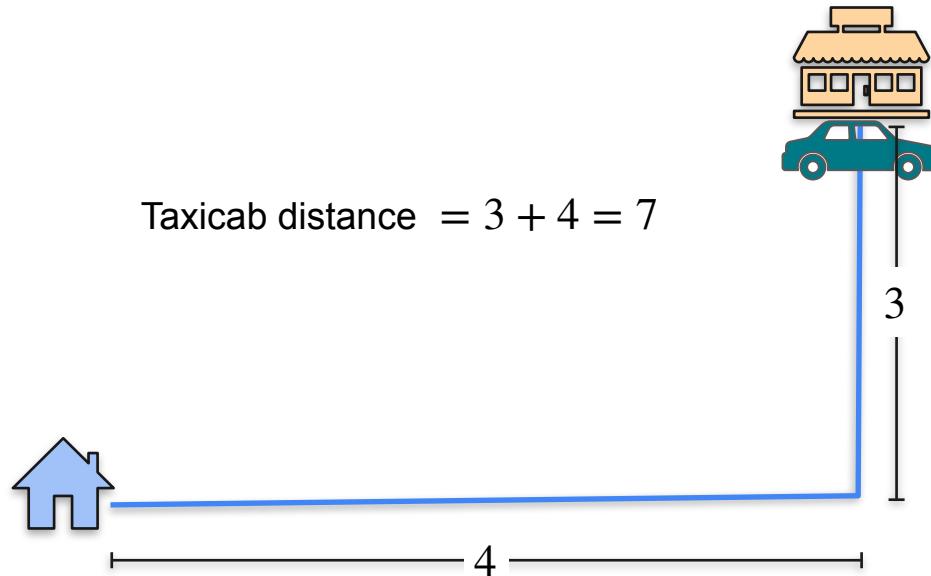
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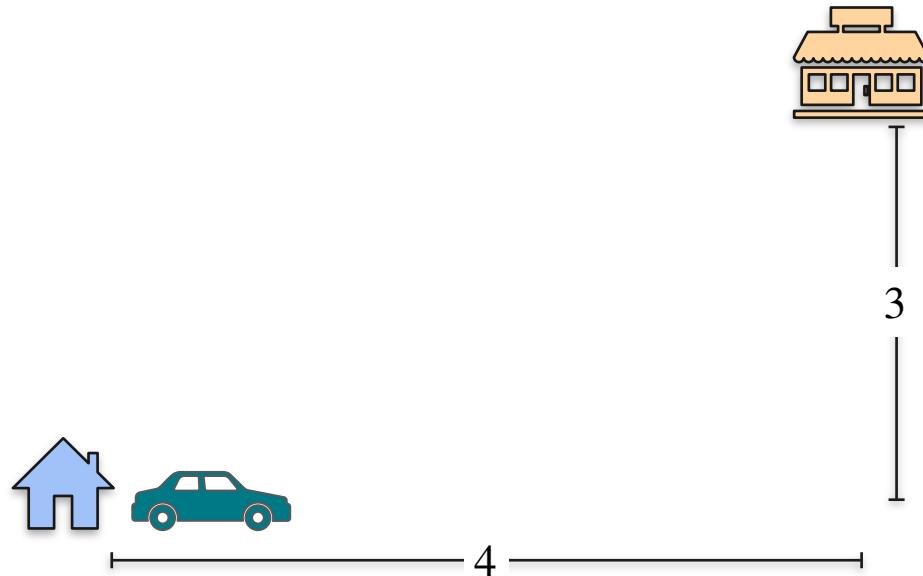
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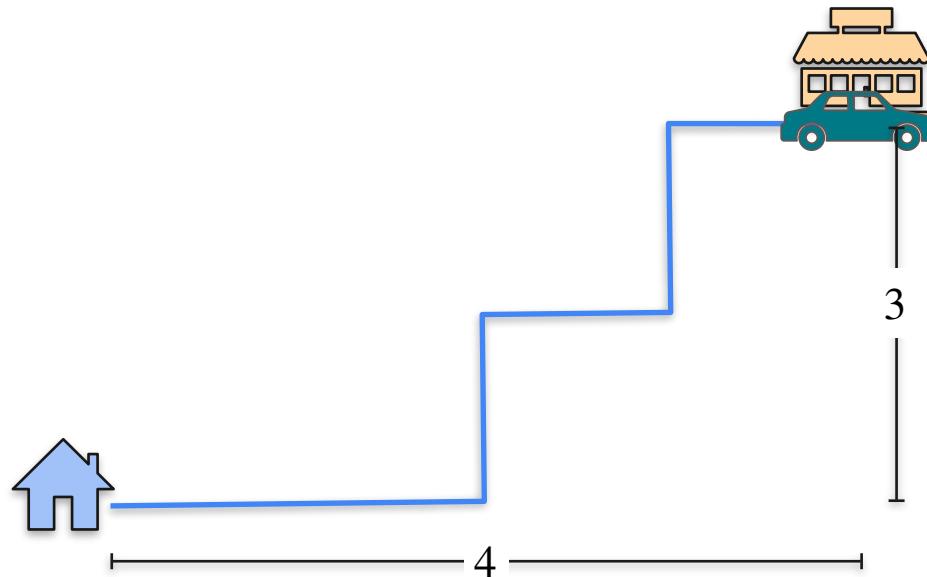
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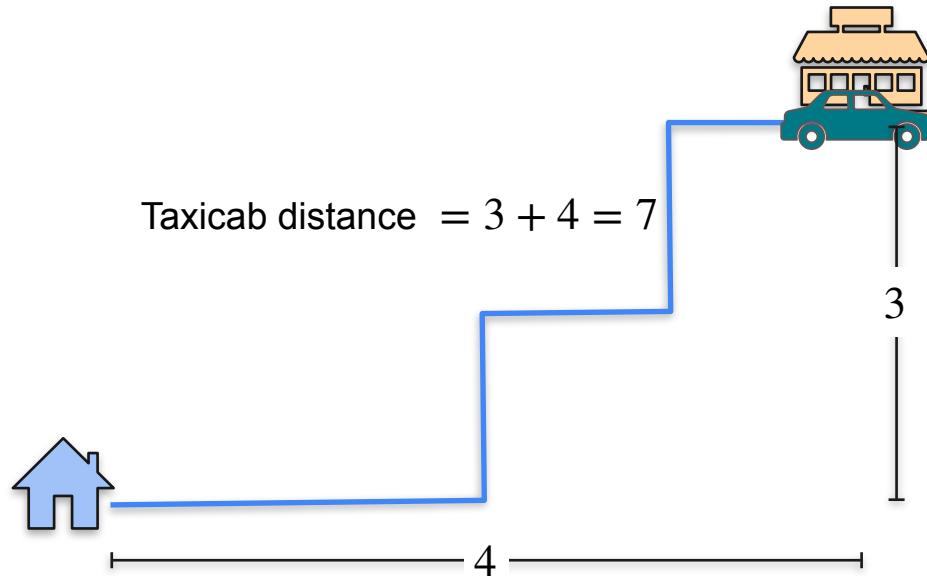
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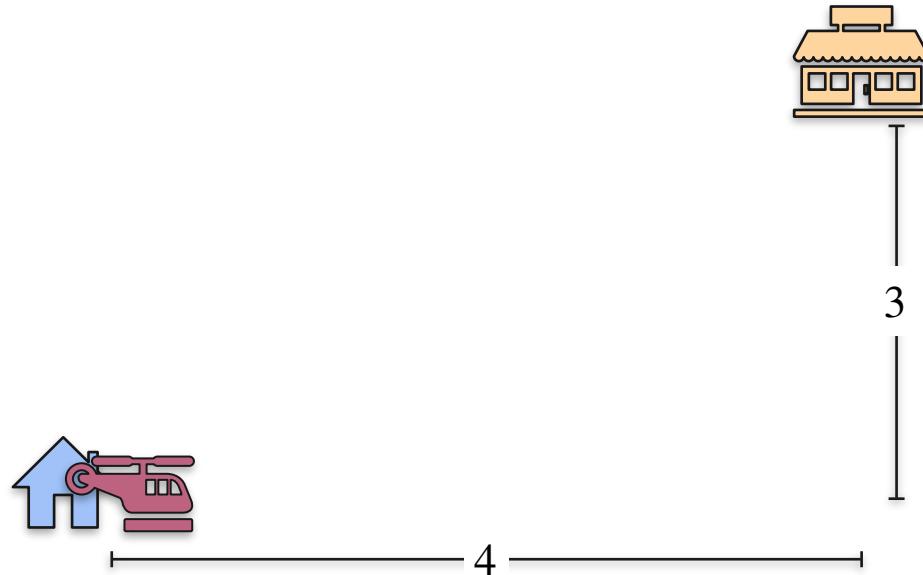
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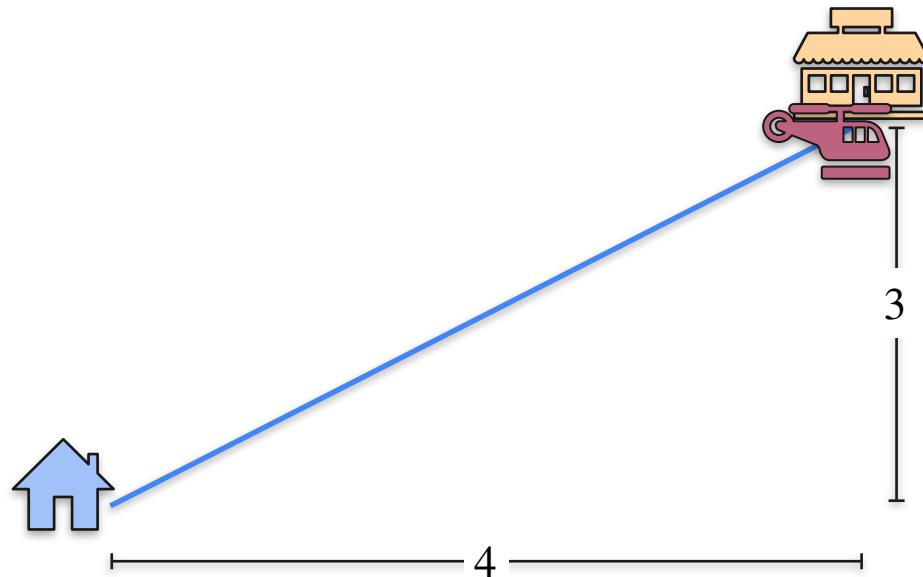
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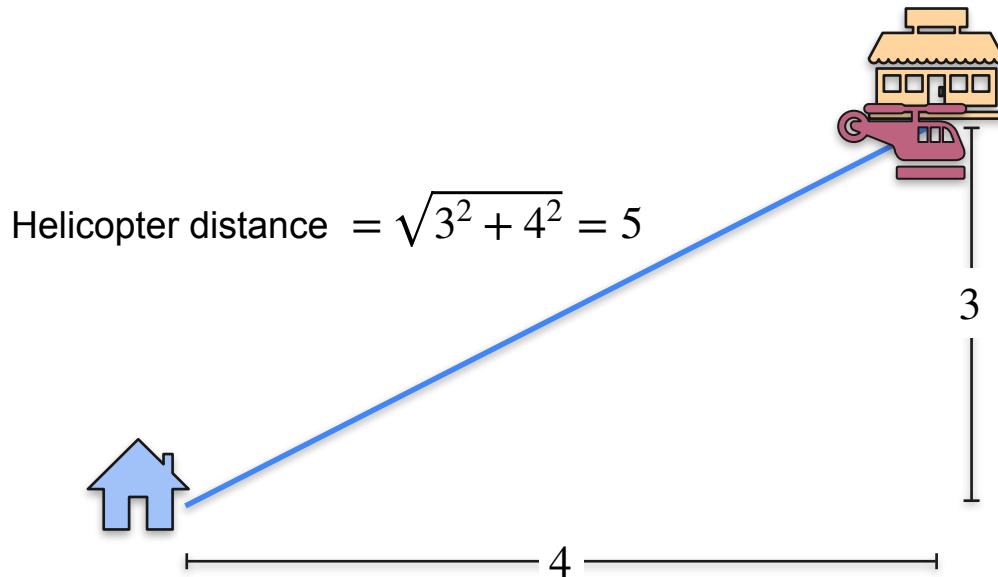
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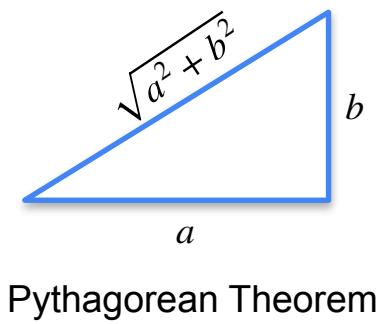
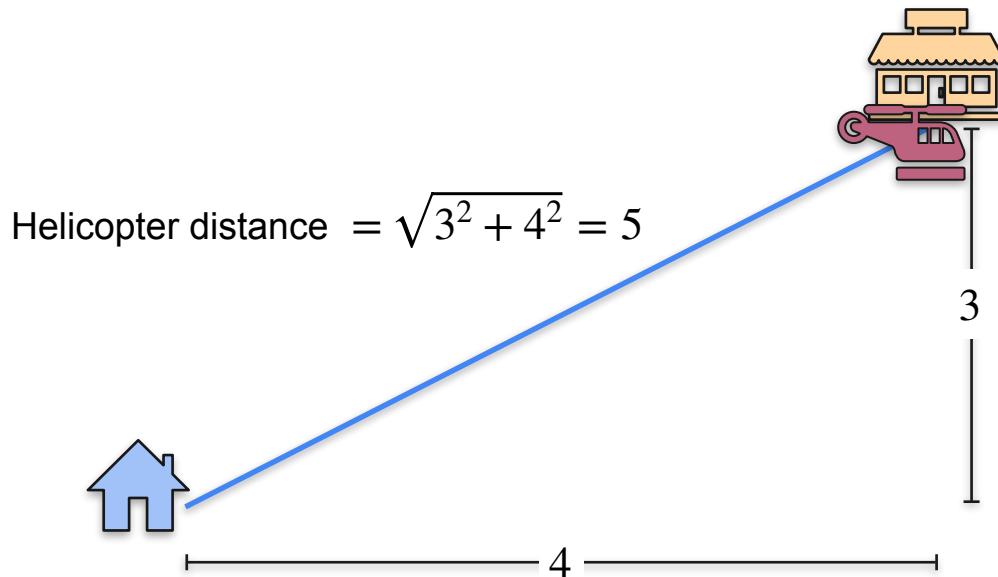
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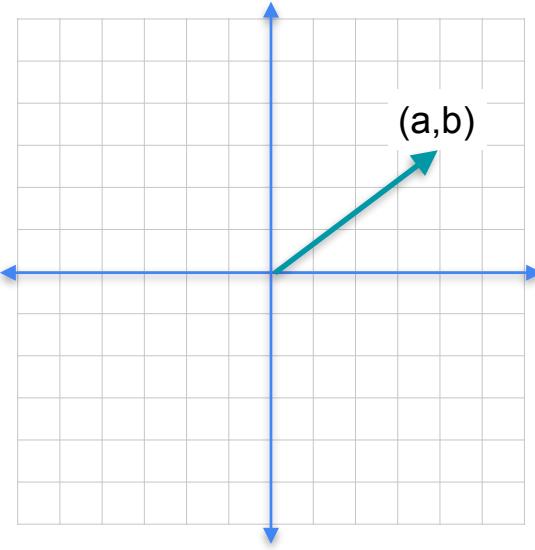
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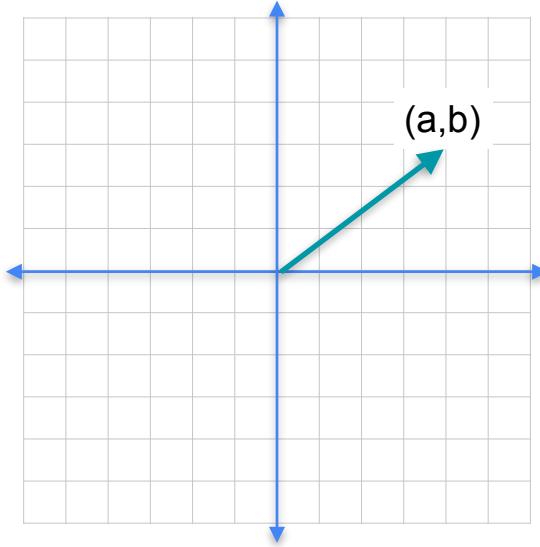
How to get from point A to point B?



Norms

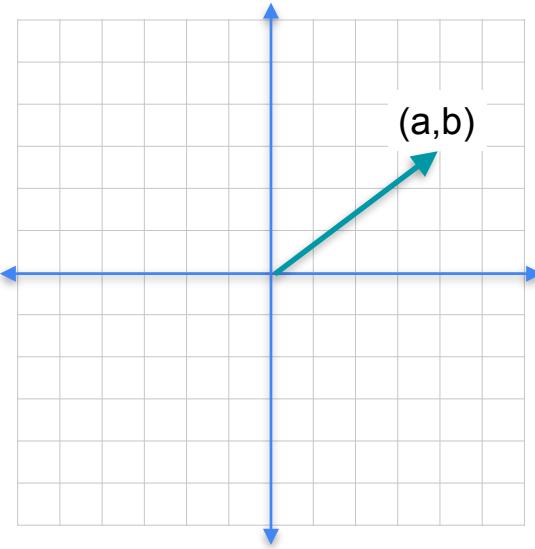


Norms

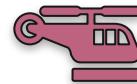


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

Norms

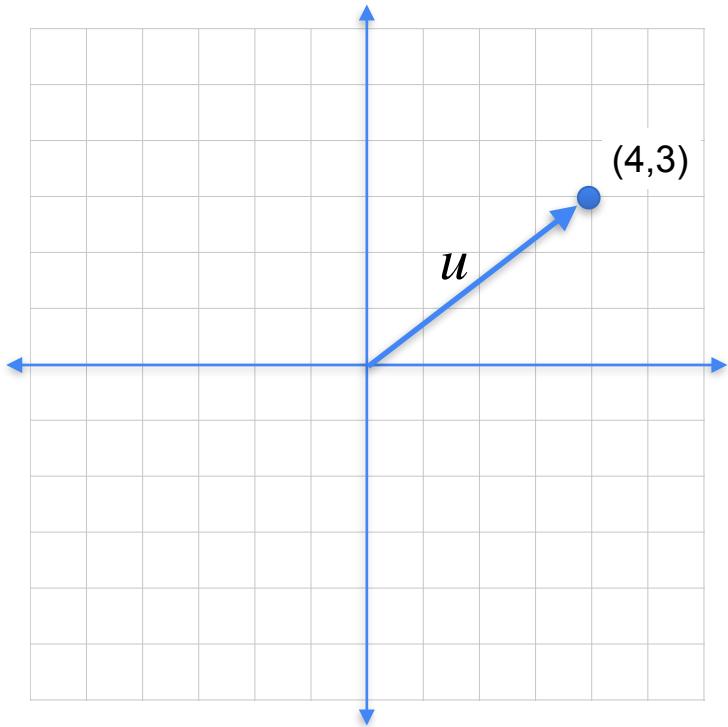


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

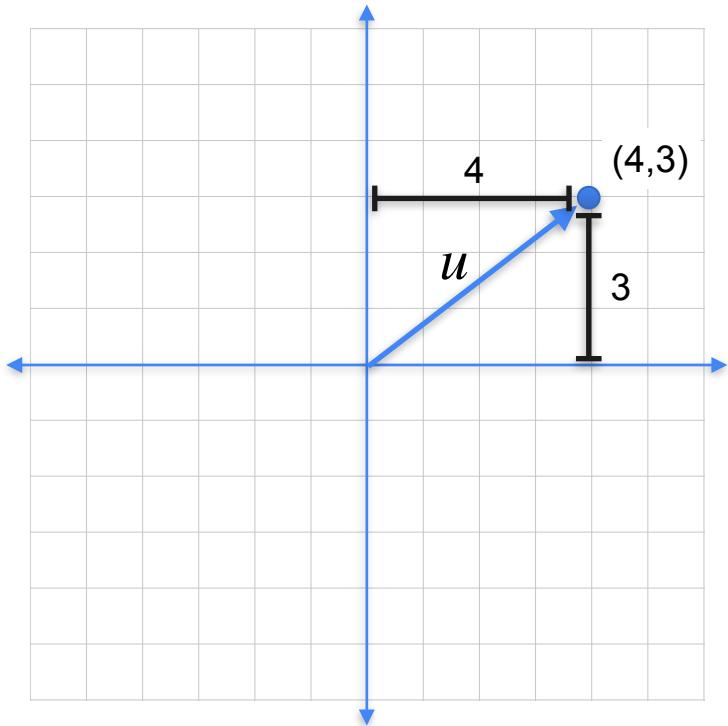


$$\text{L2-norm} = |(a, b)|_2 = \sqrt{a^2 + b^2}$$

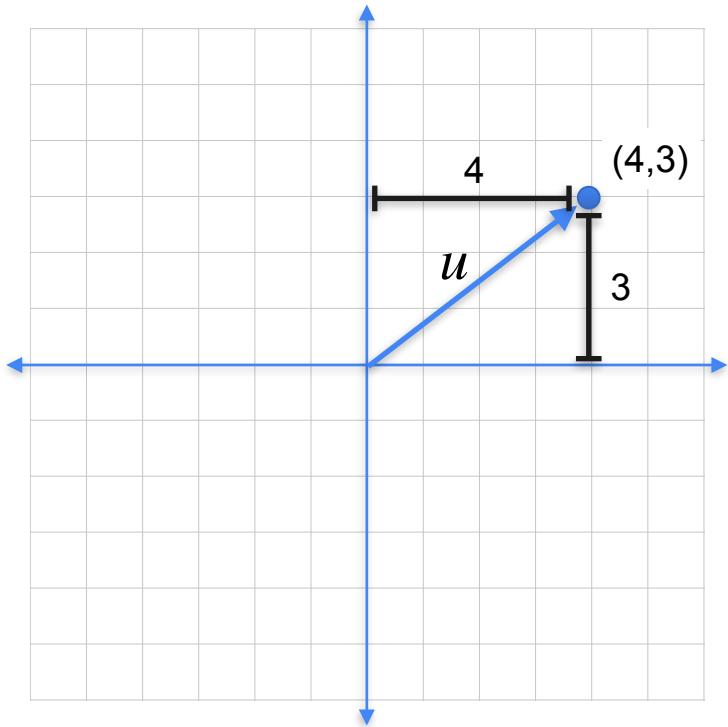
Norm of a vector



Norm of a vector

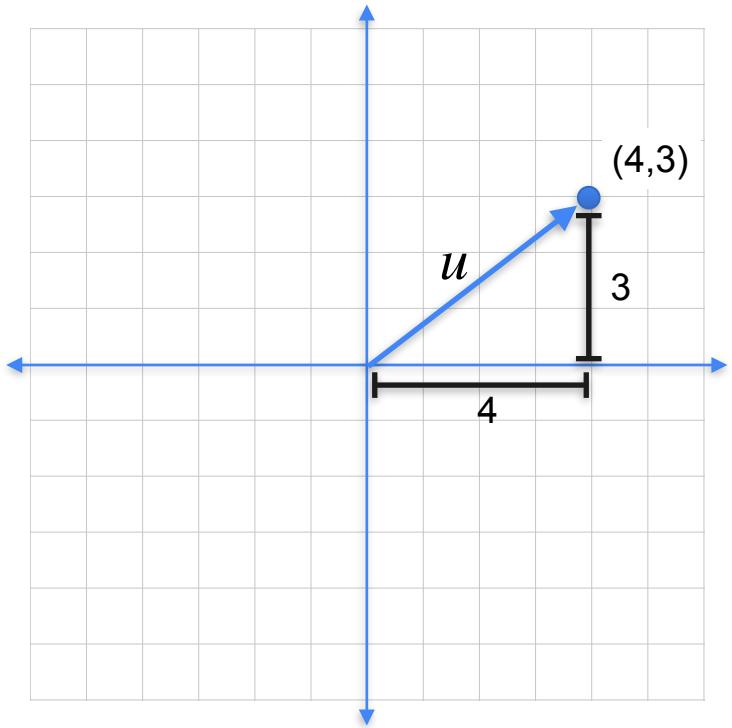


Norm of a vector

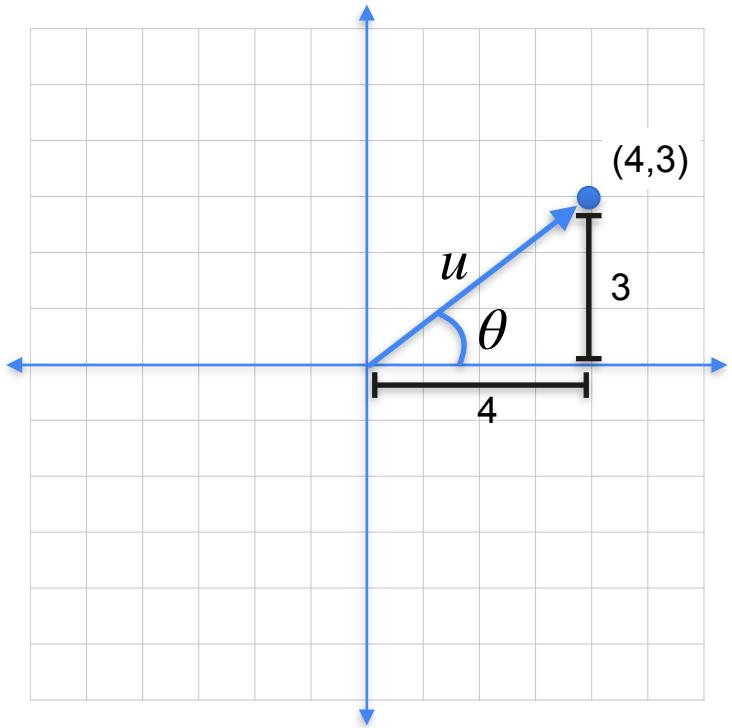


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

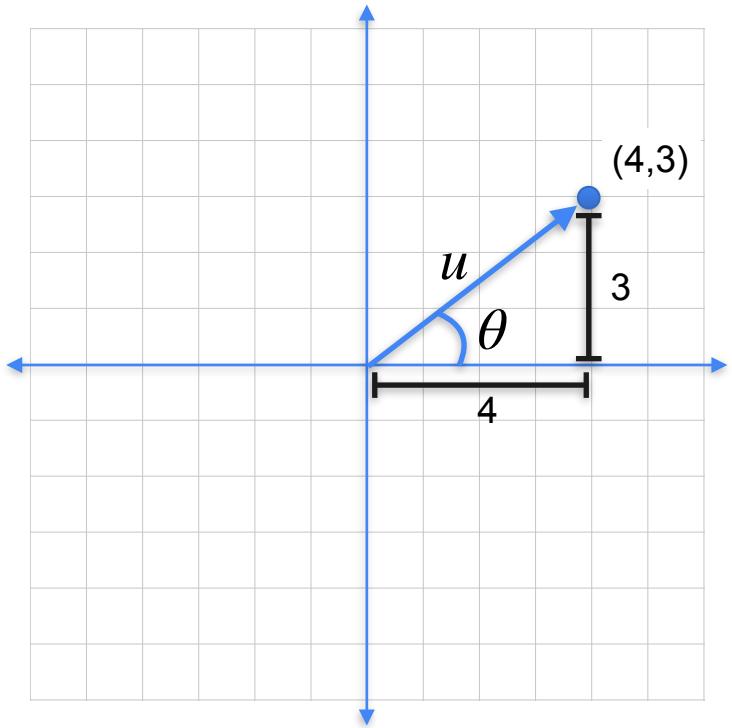
Direction of a vector



Direction of a vector

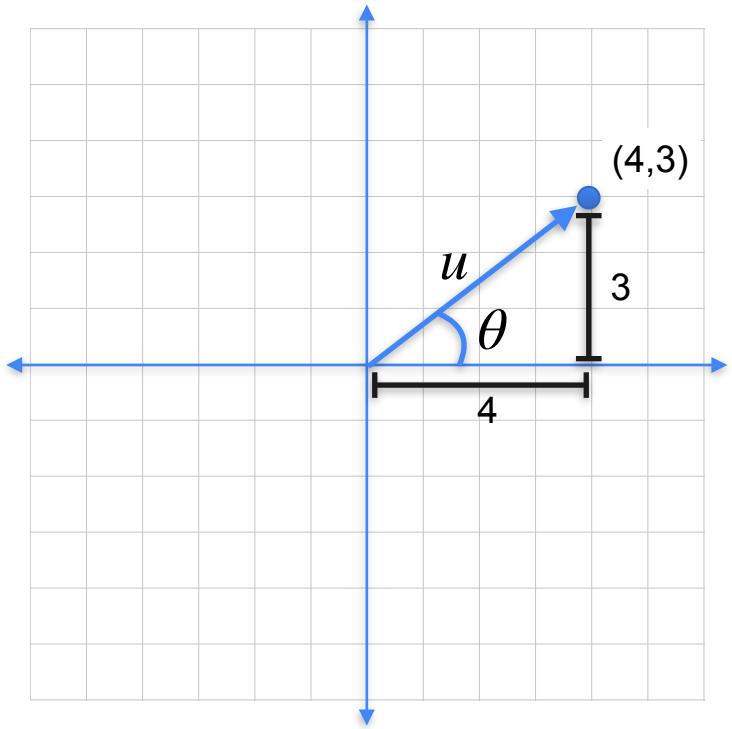


Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

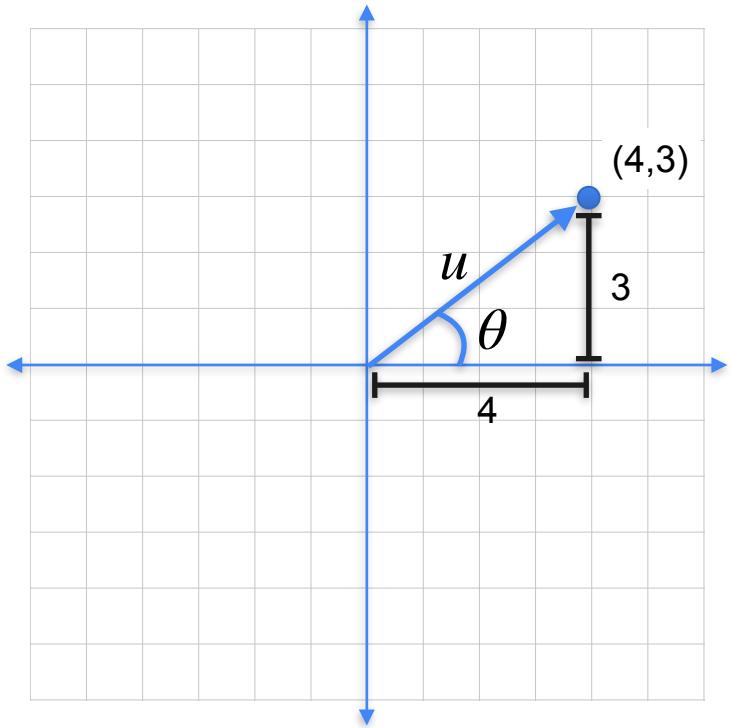
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

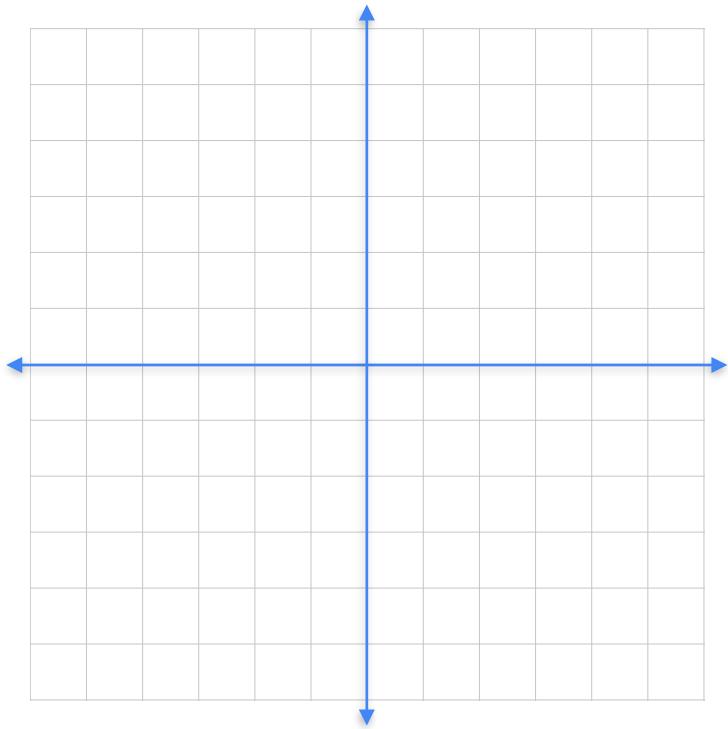


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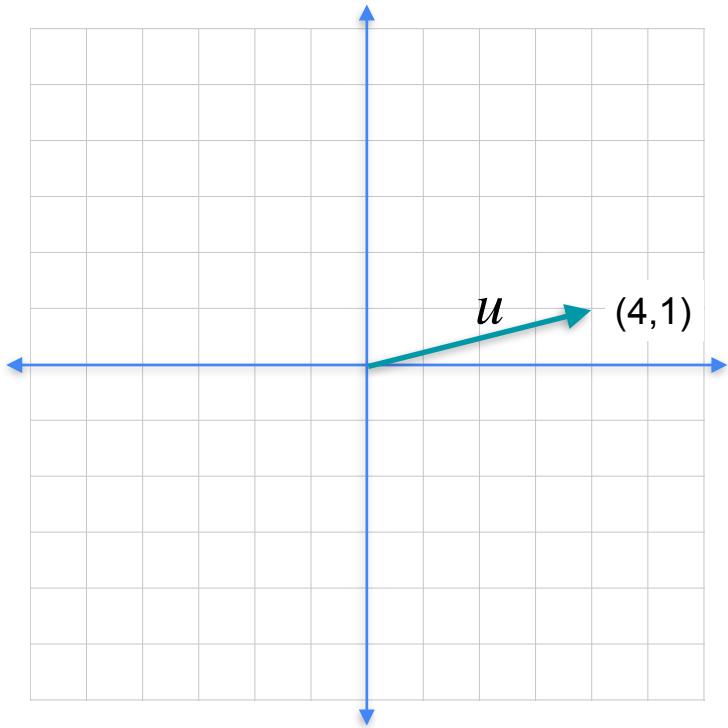
Vectors and Linear Transformations

**Sum and difference of
vectors**

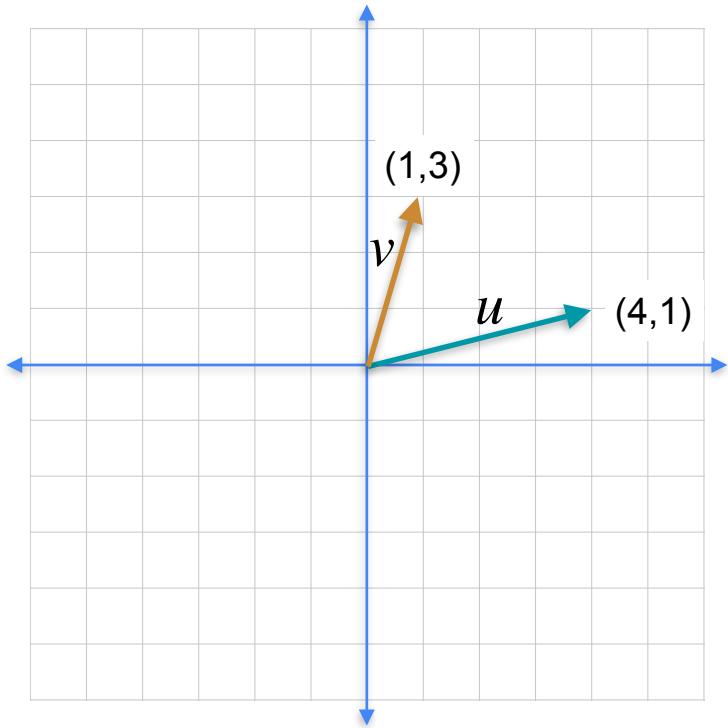
Sum of vectors



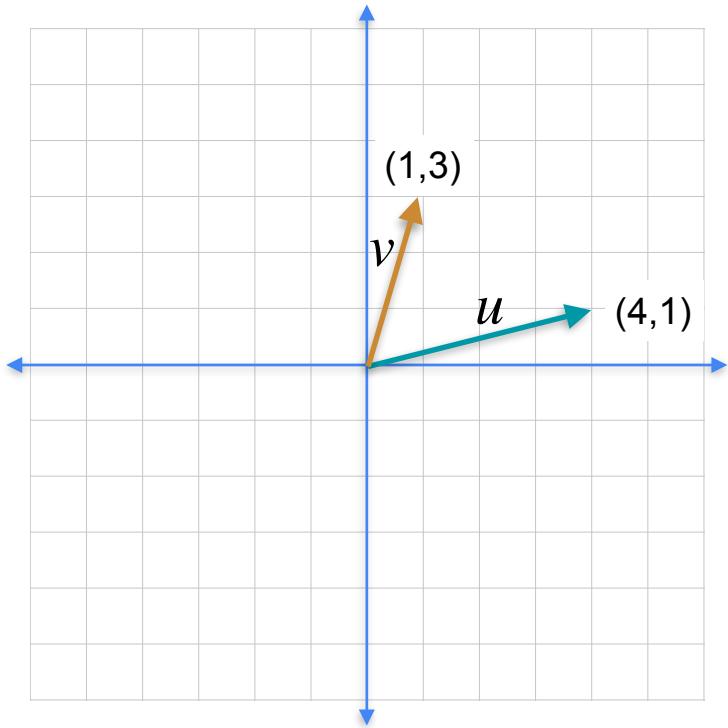
Sum of vectors



Sum of vectors

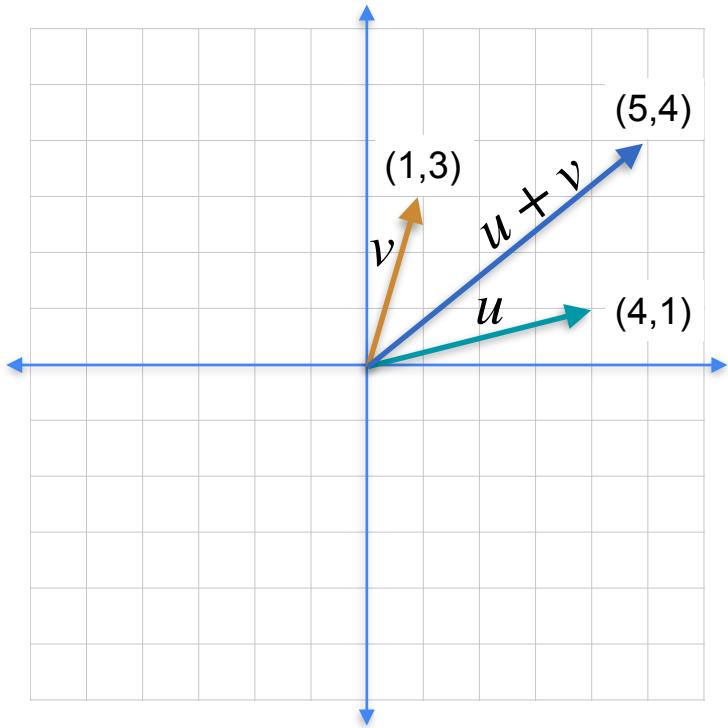


Sum of vectors



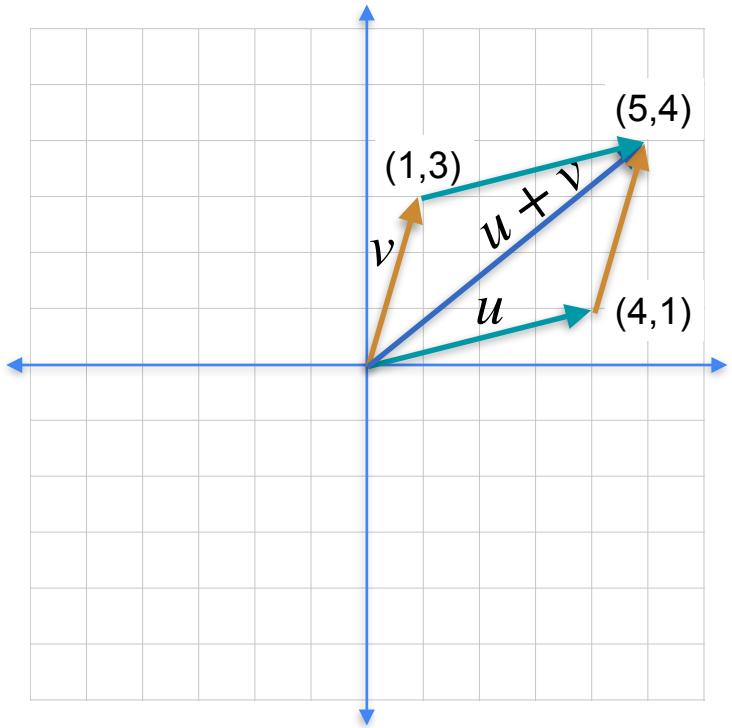
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors



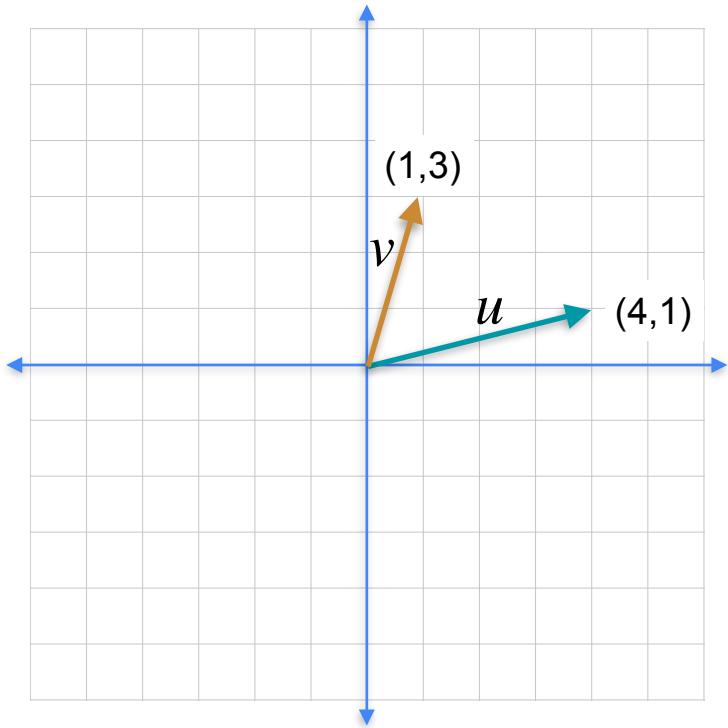
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Sum of vectors

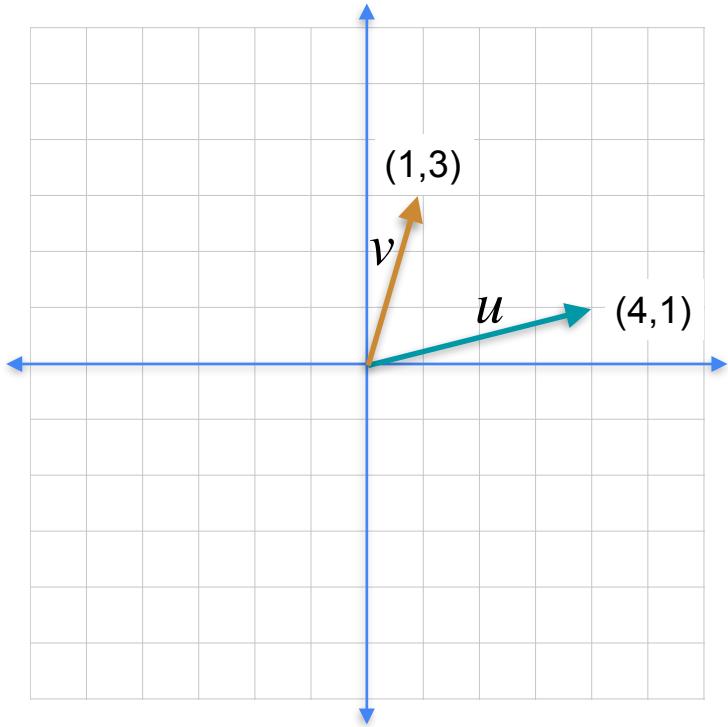


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Difference of vectors

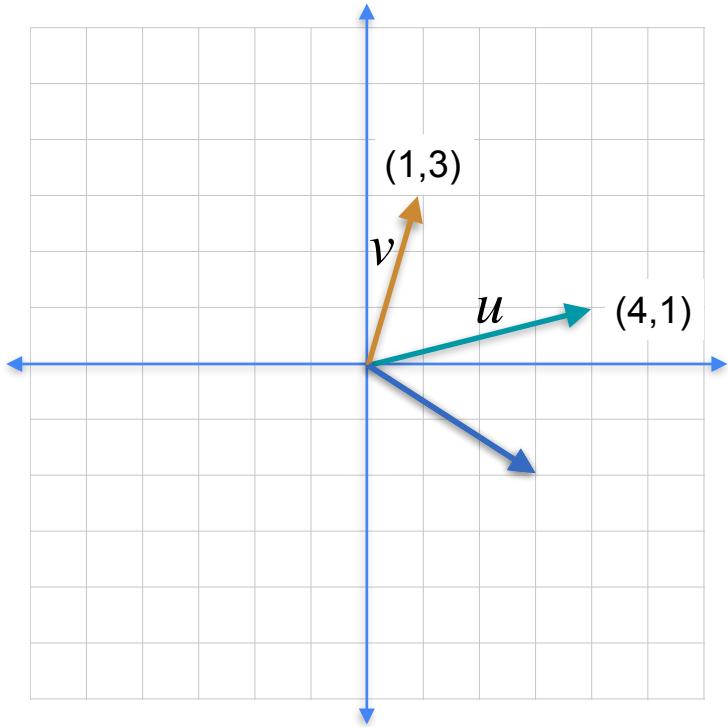


Difference of vectors



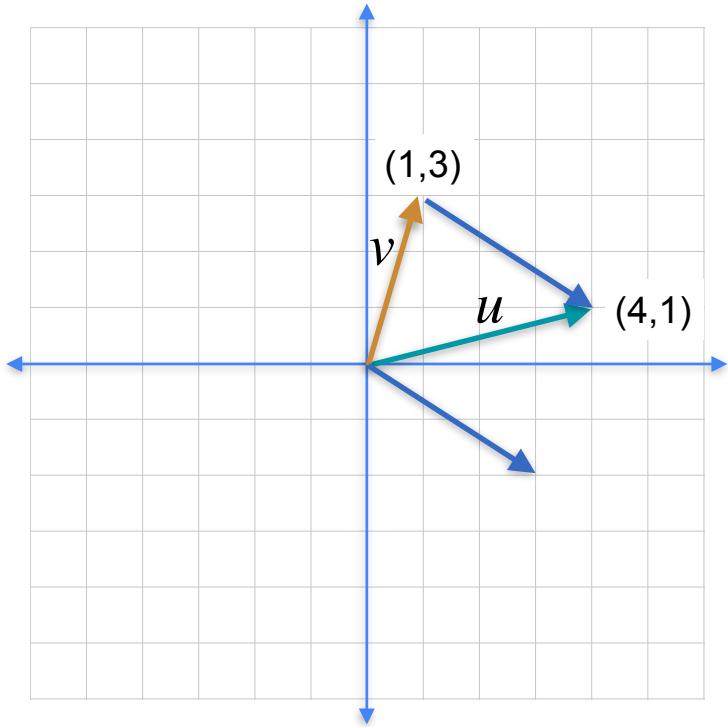
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



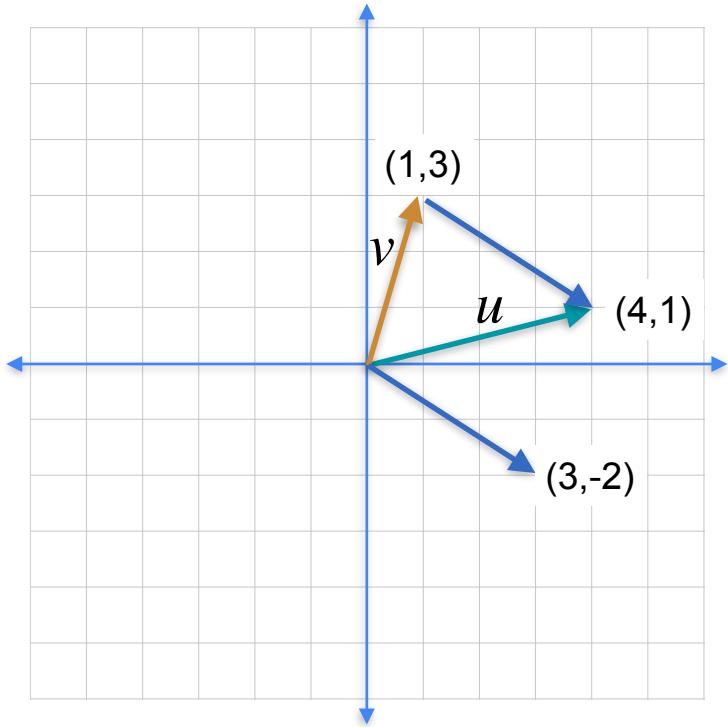
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Difference of vectors



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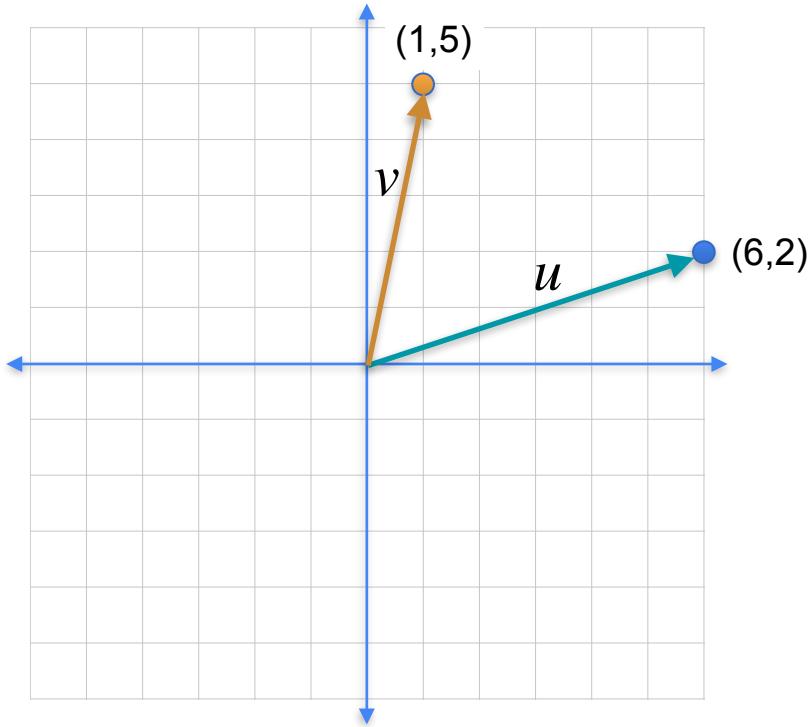


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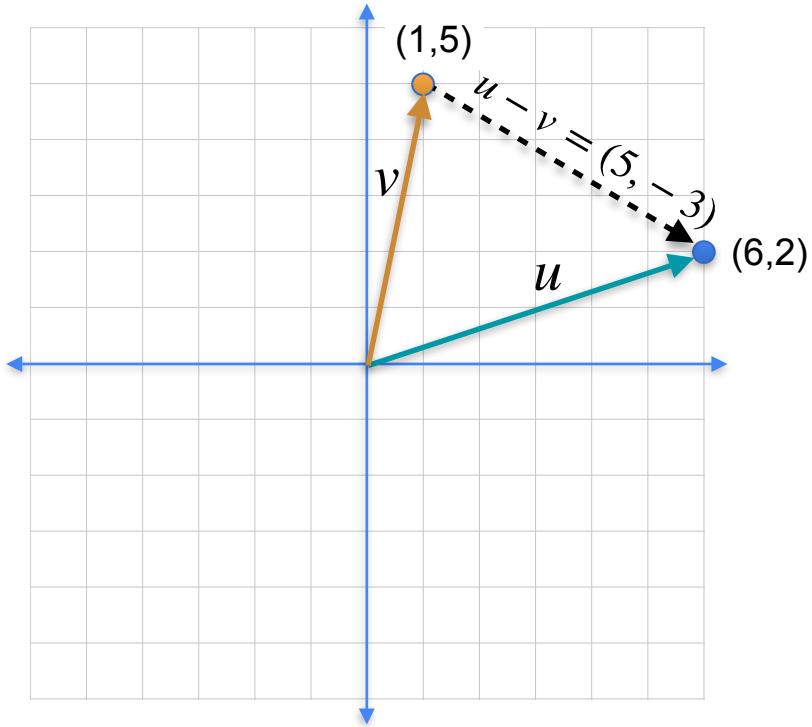
Vectors and Linear Transformations

Distance between vectors

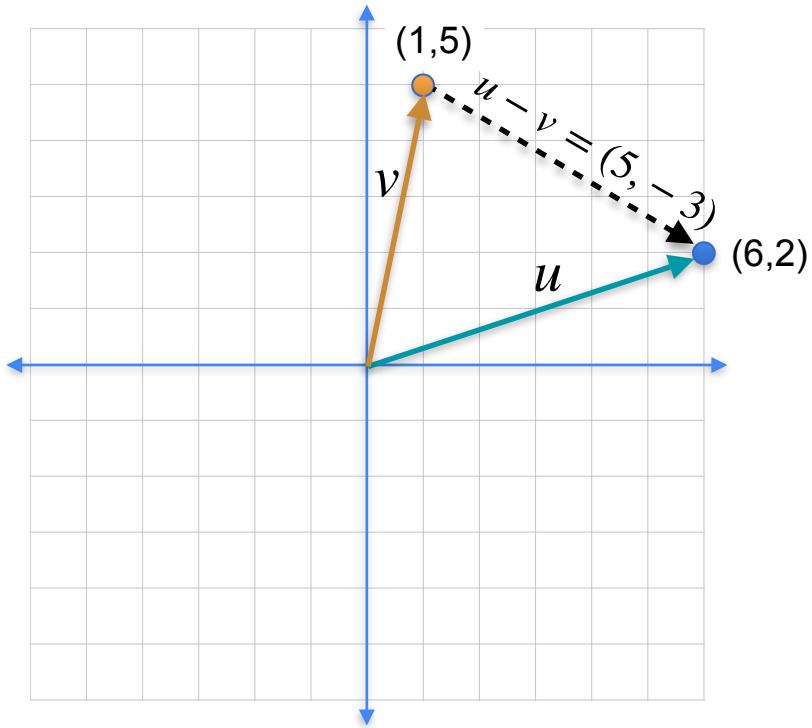
Distances



Distances

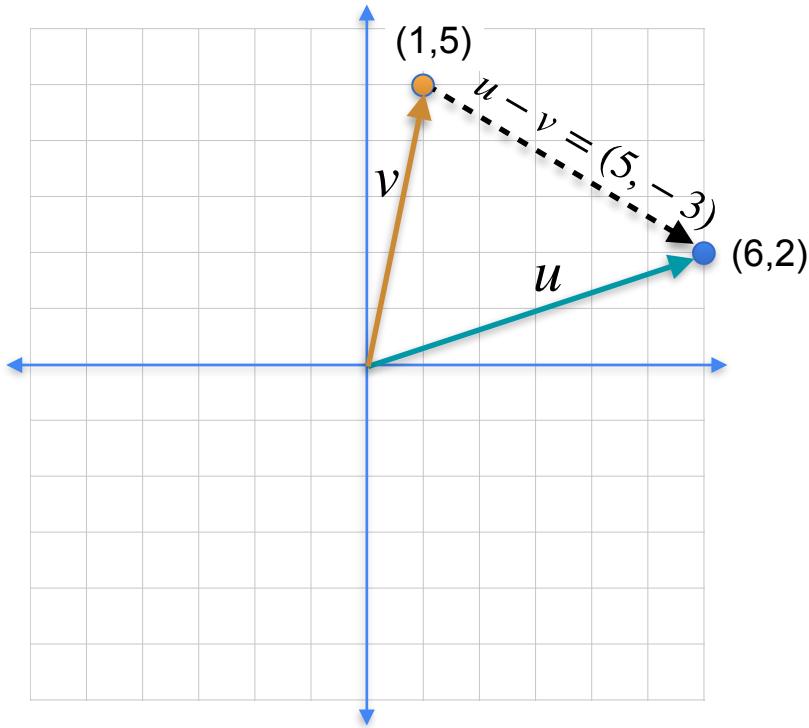


Distances



$$|u - v|_1 = |5| + |-3| = 8$$

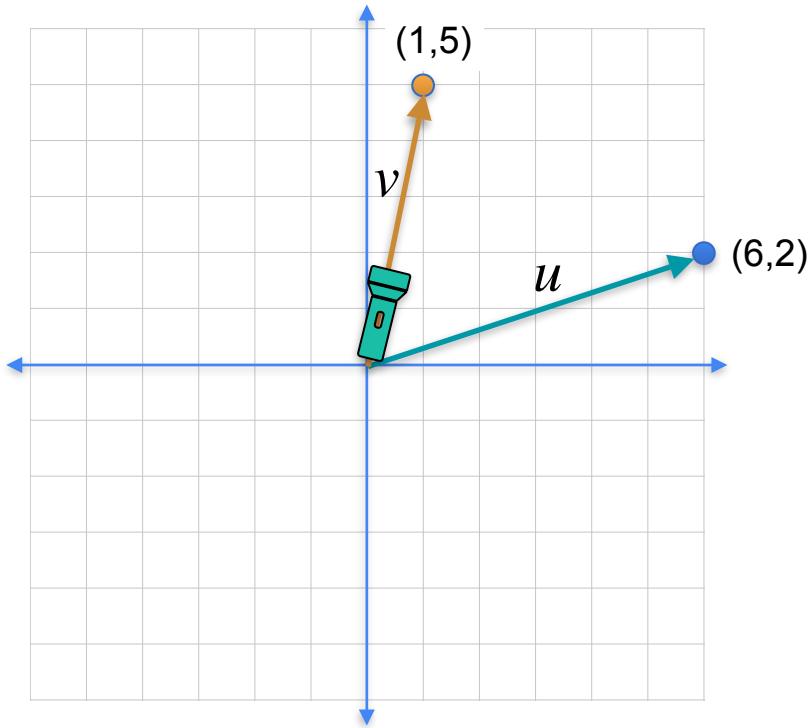
Distances



$$|u - v|_1 = |5| + |-3| = 8$$

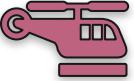
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



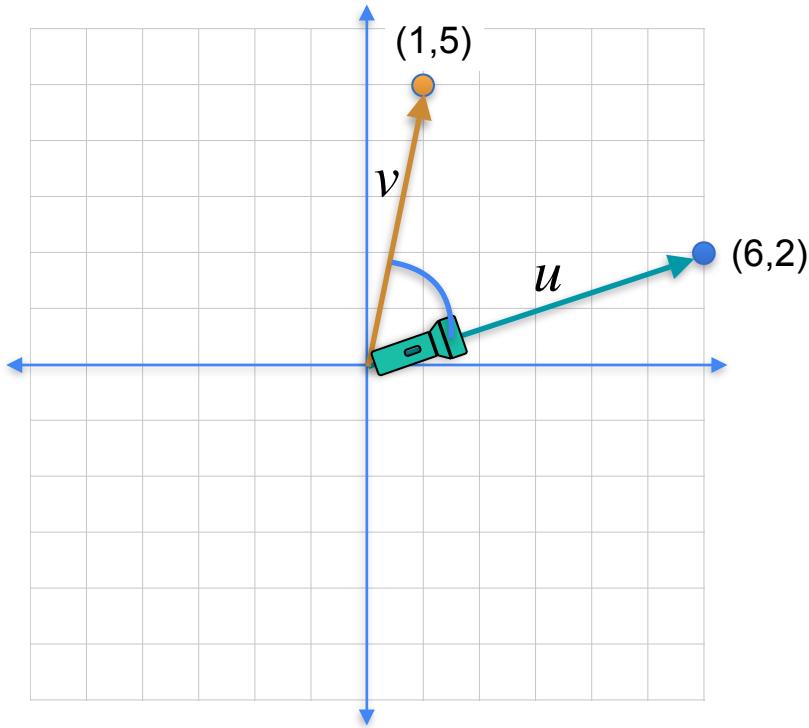


L1-distance $|u - v|_1 = |5| + |-3| = 8$



L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

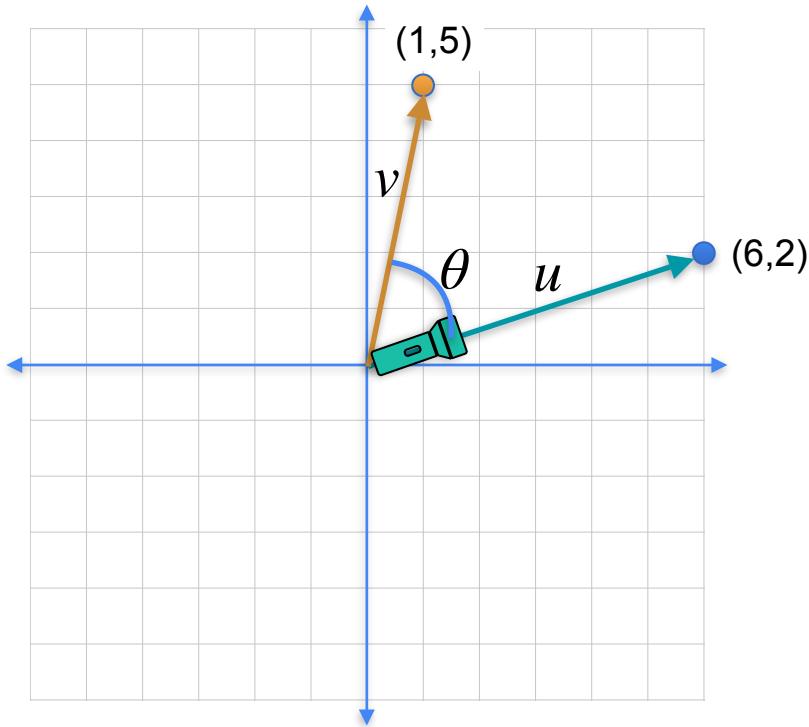
Distances



L1-distance $|u - v|_1 = |5| + |-3| = 8$

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Distances



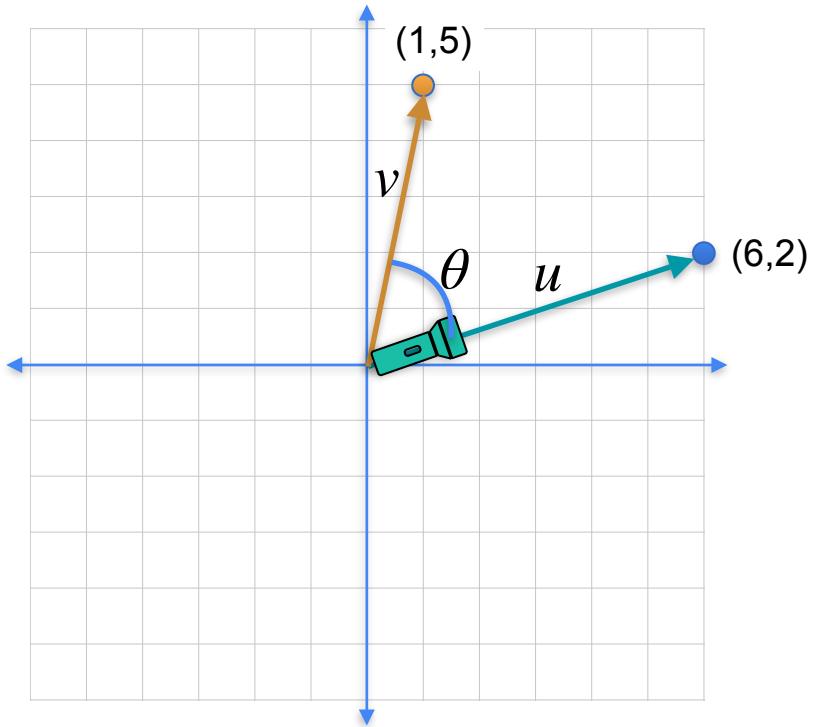


L1-distance $|u - v|_1 = |5| + |-3| = 8$



L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



L1-distance

$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$



Cosine distance

$$\cos(\theta)$$

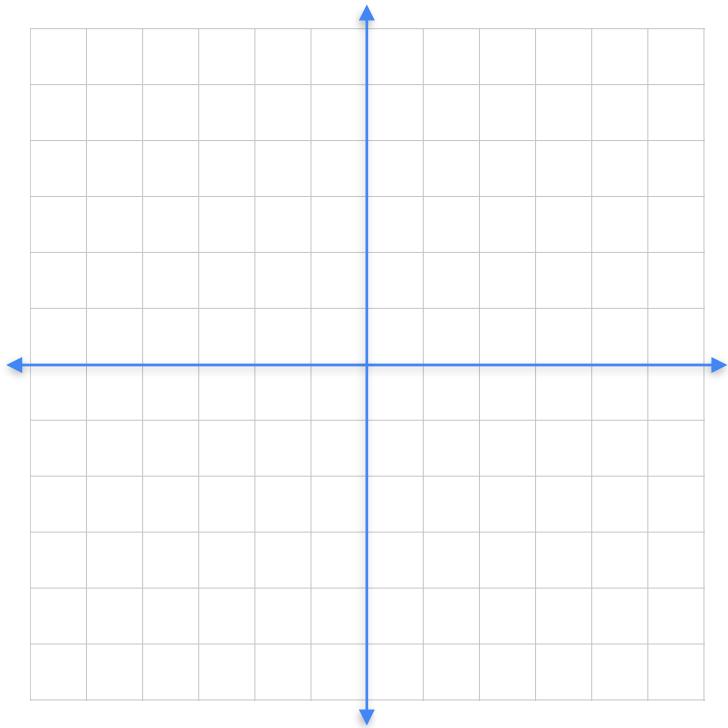


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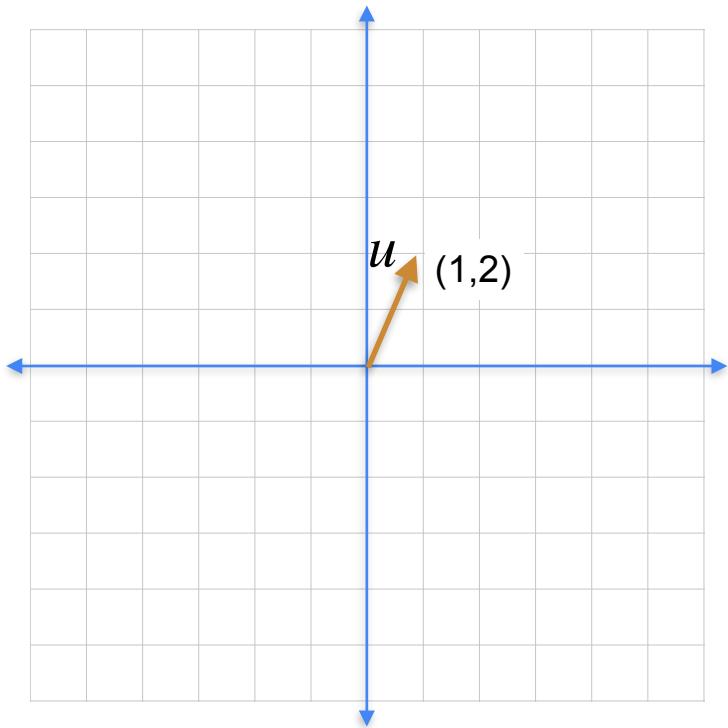
Vectors and Linear Transformations

Multiplying a vector by a scalar

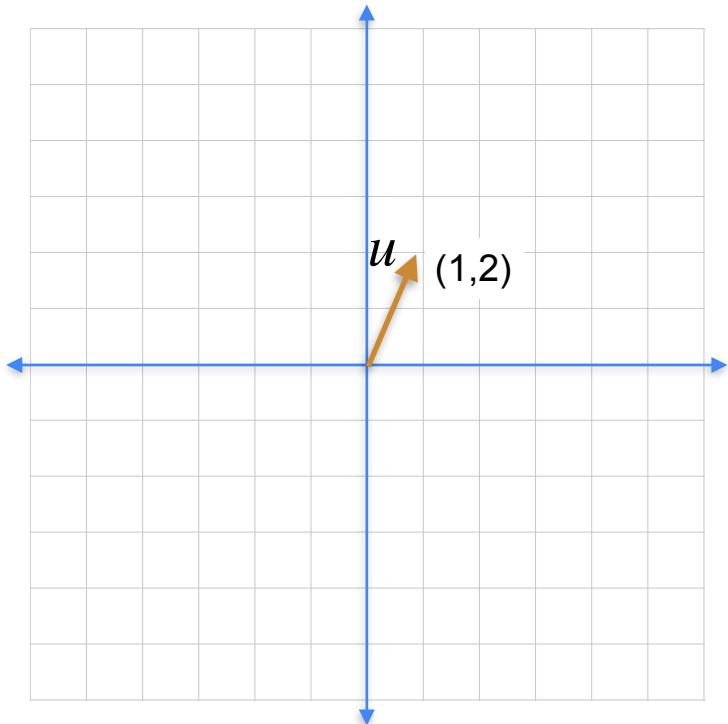
Multiplying a vector by a scalar



Multiplying a vector by a scalar

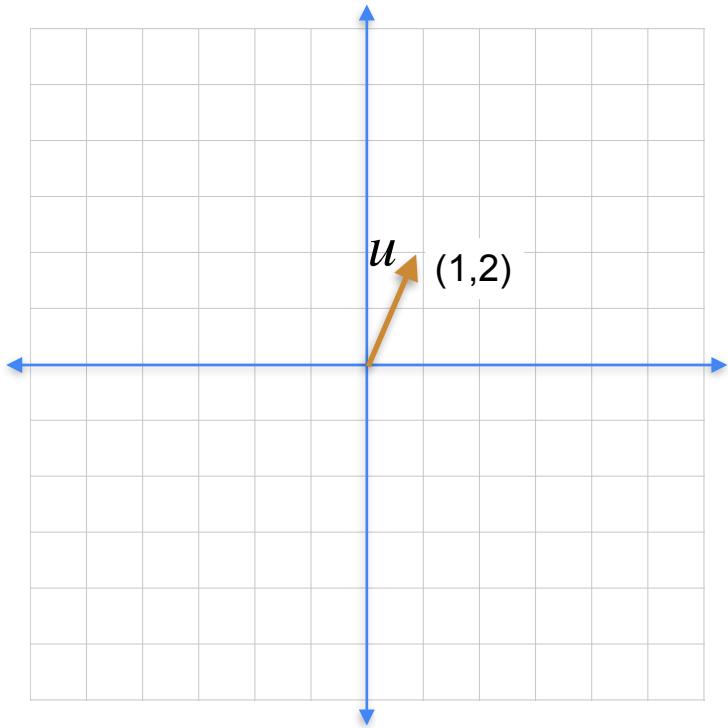


Multiplying a vector by a scalar



$$u = (1, 2)$$

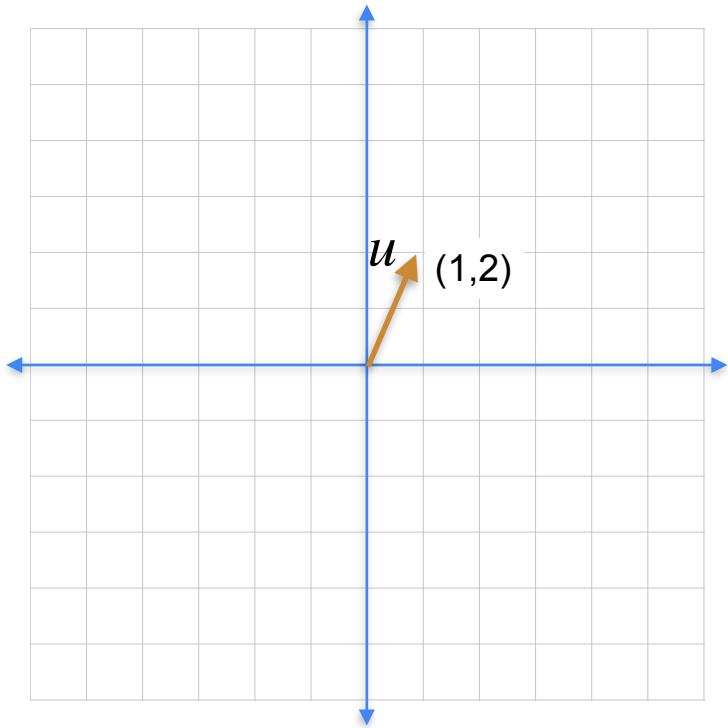
Multiplying a vector by a scalar



$$u = (1, 2)$$

$$\lambda = 3$$

Multiplying a vector by a scalar

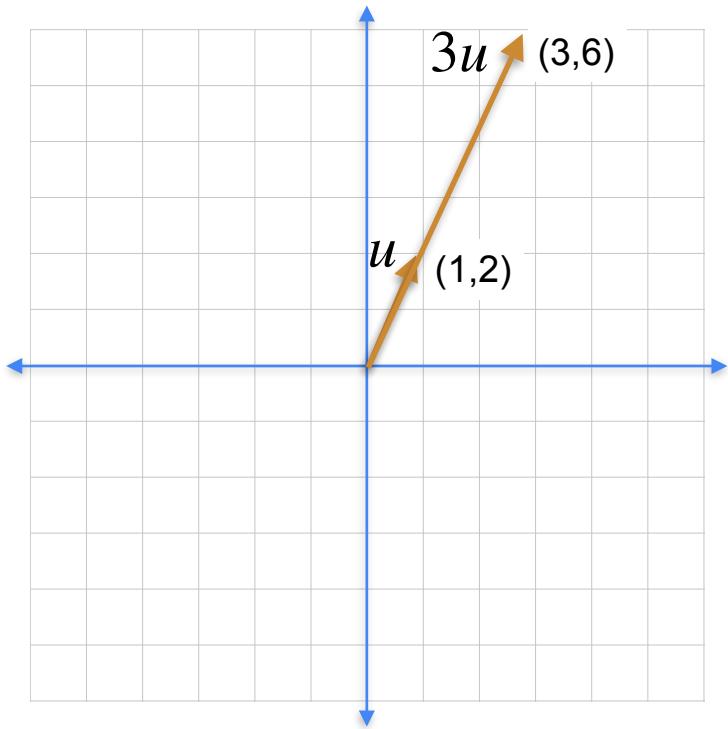


$$u = (1, 2)$$

$$\lambda = 3$$

$$\lambda u = (3, 6)$$

Multiplying a vector by a scalar

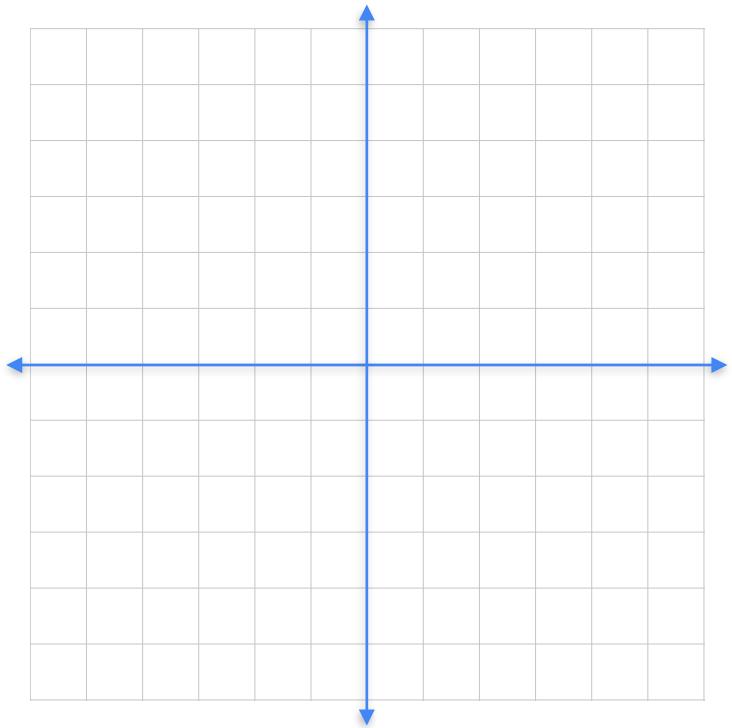


$$u = (1, 2)$$

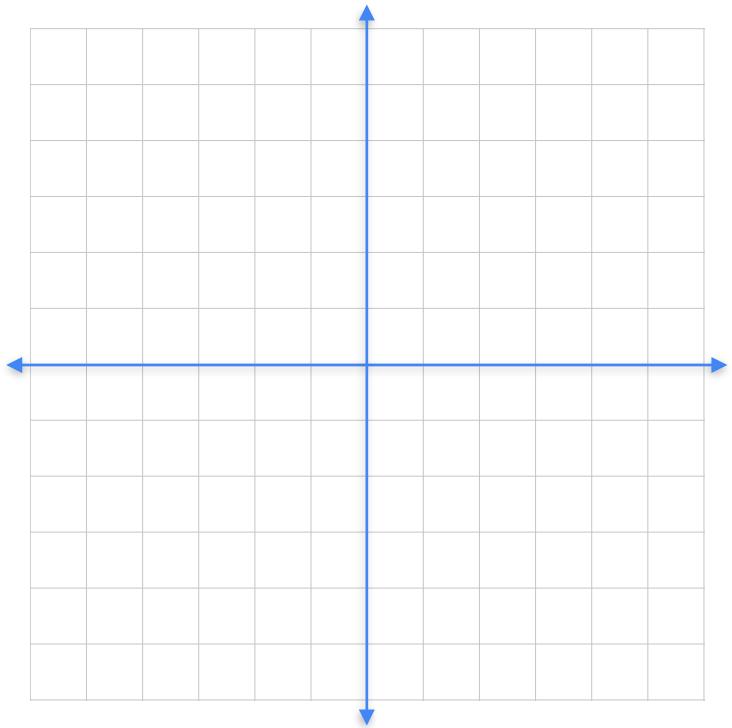
$$\lambda = 3$$

$$\lambda u = (3, 6)$$

If the scalar is negative

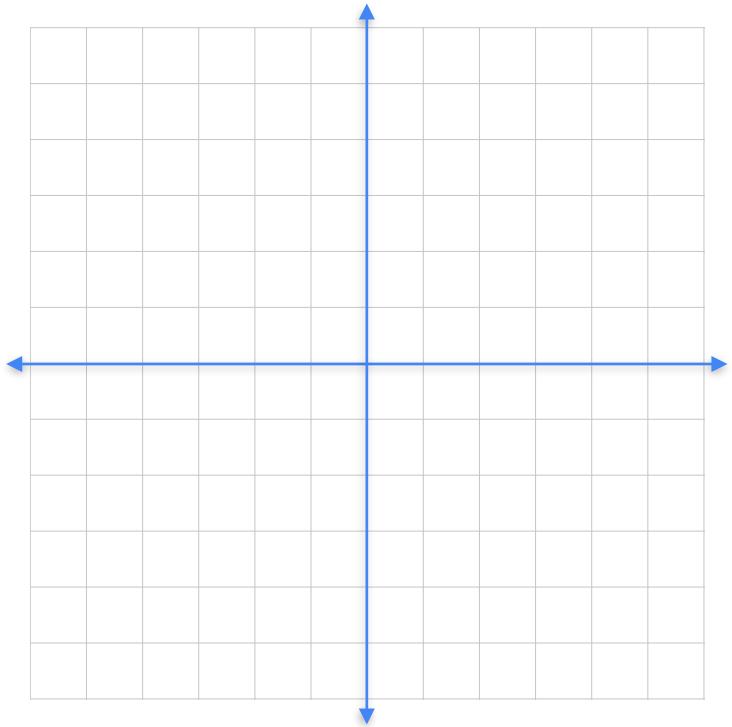


If the scalar is negative



$$u = (1,2)$$

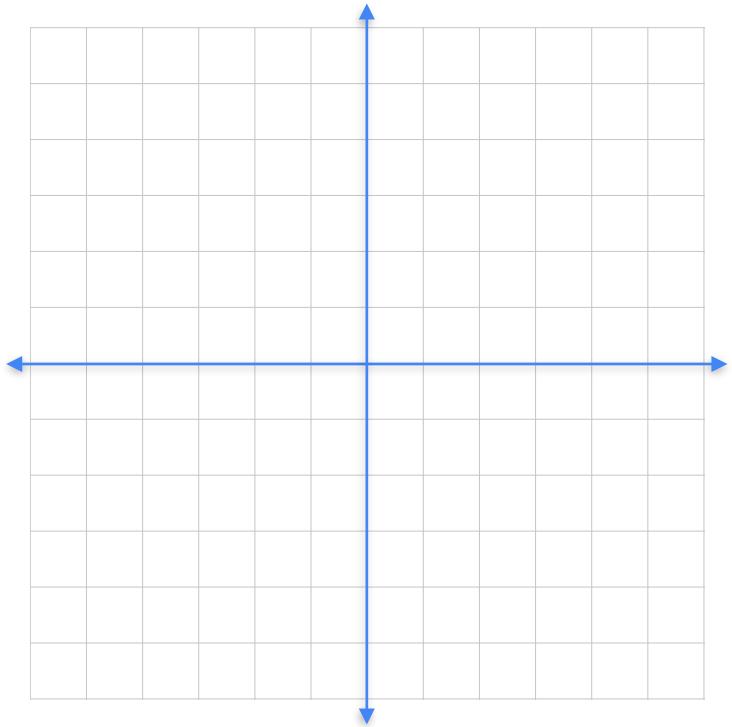
If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

If the scalar is negative

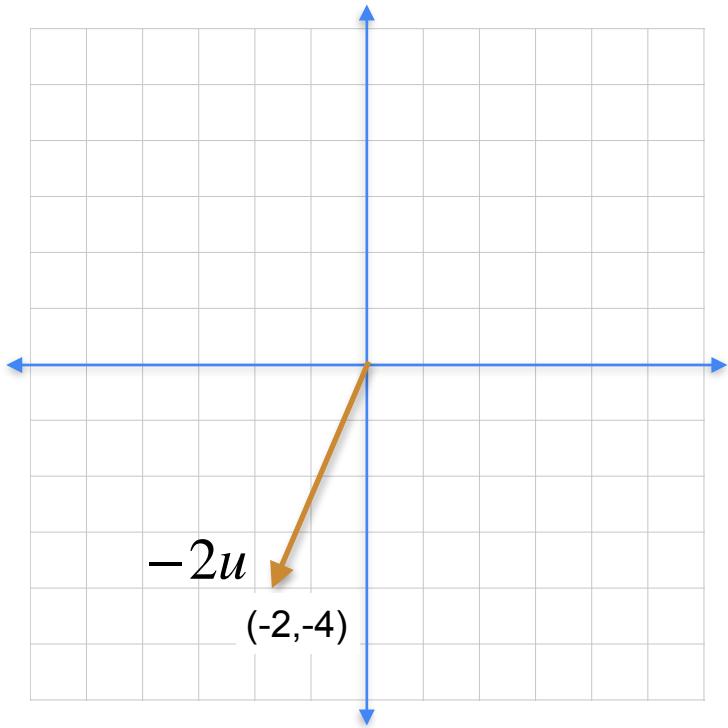


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative

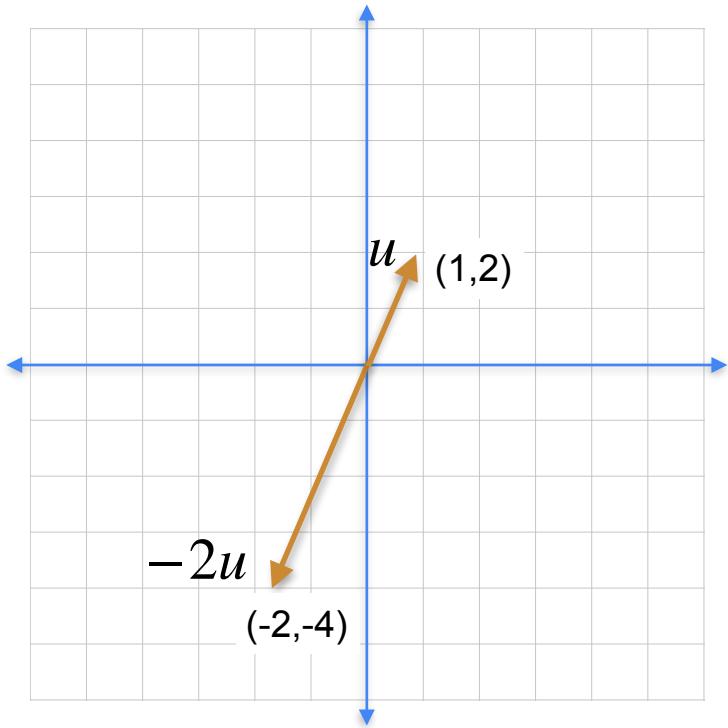


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



DeepLearning.AI

Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

	2
	4
	1

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

	2
	4
	1

Prices

apples: \$3
bananas: \$5
cherries: \$2

\$ 	3
\$ 	5
\$ 	2

Total price

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$



\$	3
\$	5
\$	2

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

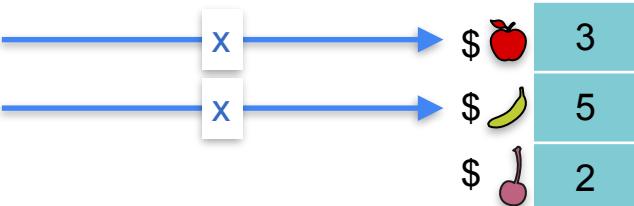
bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$



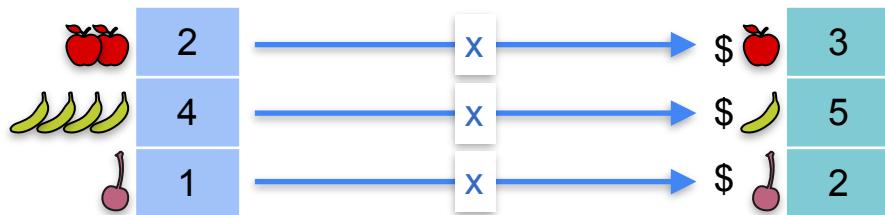
A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry



Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

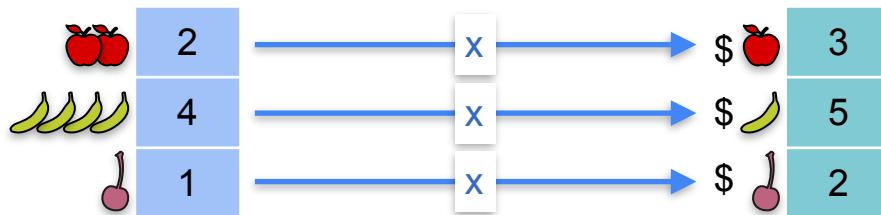
A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry



Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$

$$1 \times 2 = 2$$

$$6 + 20 + 2 = 28$$

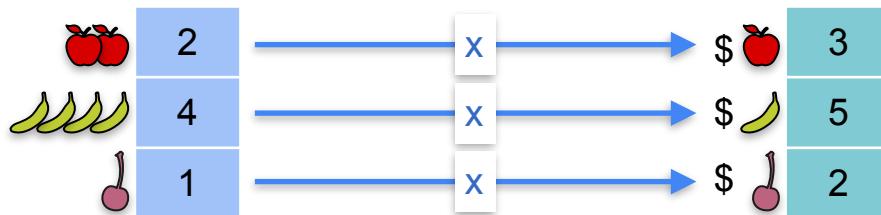
A shortcut for linear operations

Quantities

2 apples

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1 cherry



Prices

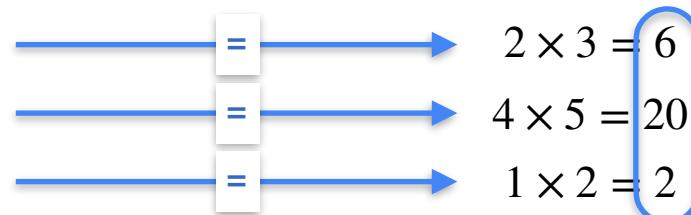
apples: \$3

bananas: \$5

cherries: \$2

Total price

\$28



The dot product

$$\begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} \begin{array}{c|c} & 2 \\ \hline 4 & \\ \hline 1 & \end{array} \cdot \begin{array}{c} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{array} \begin{array}{c|c} 3 \\ \hline 5 \\ \hline 2 \end{array} = \$28$$

The dot product

The diagram shows two vectors being multiplied. The first vector, on the left, is represented by a blue grid with three rows. The top row contains two red apples, the middle row contains four yellow bananas, and the bottom row contains one red cherry. To the right of this grid are the numbers 2, 4, and 1, representing the count of each fruit. The second vector, on the right, is represented by a teal grid with three rows. The top row contains one red apple, the middle row contains one yellow banana, and the bottom row contains one red cherry. To the right of this grid are the numbers 3, 5, and 2, representing the price of each fruit. A multiplication sign between the two grids indicates the dot product operation.

$$\begin{matrix} \text{apple} \\ \text{banana} \\ \text{cherry} \end{matrix} \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} \$ \text{apple} \\ \$ \text{banana} \\ \$ \text{cherry} \end{matrix} \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = \$28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

The diagram shows the dot product of two vectors. The first vector, on the left, represents the quantity of three types of fruit: 2 apples, 4 bananas, and 1 cherry. The second vector, on the right, represents the price per fruit: \$3 for an apple, \$5 for a banana, and \$2 for a cherry. The result of the dot product is \$28.

2	4	1
•	\$	3
\$	5	
\$	2	

= \$28

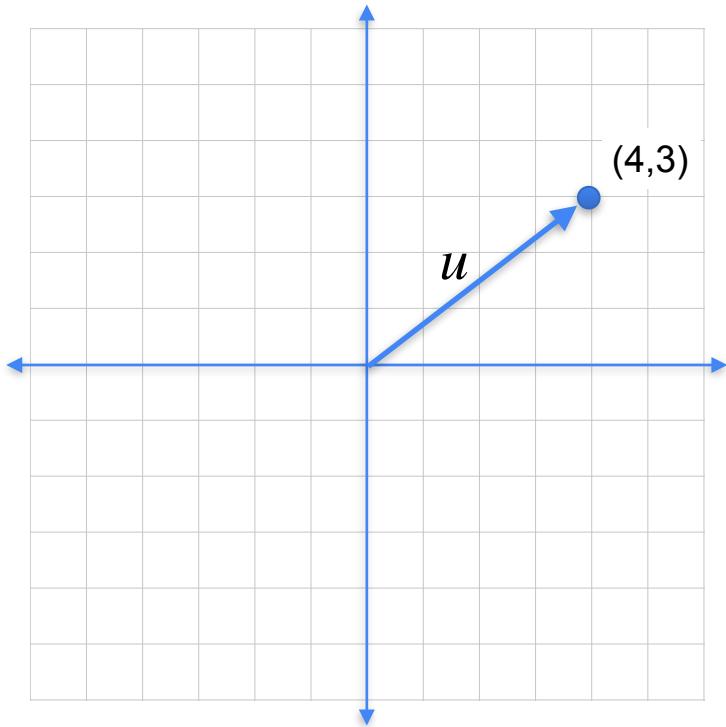
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

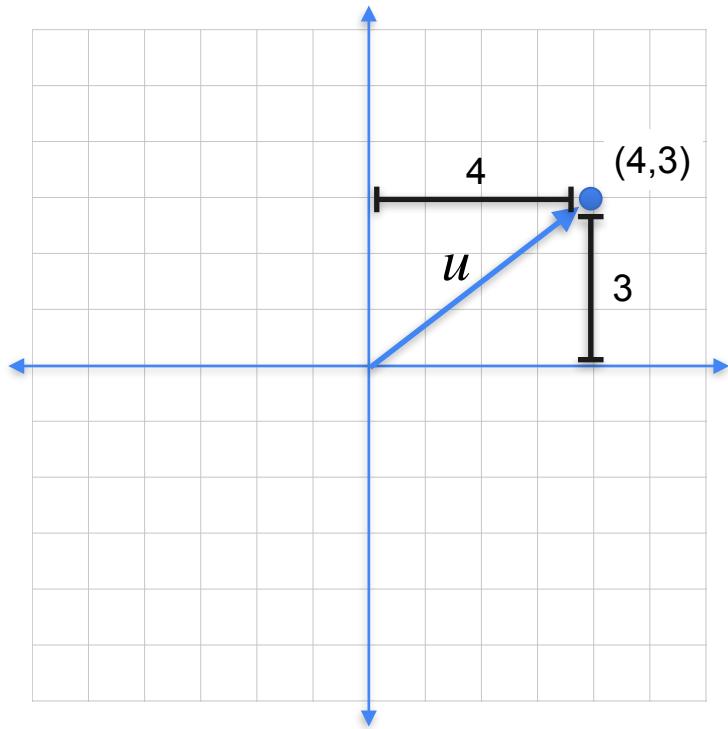
$$\begin{matrix} 2 & | & 4 & | & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

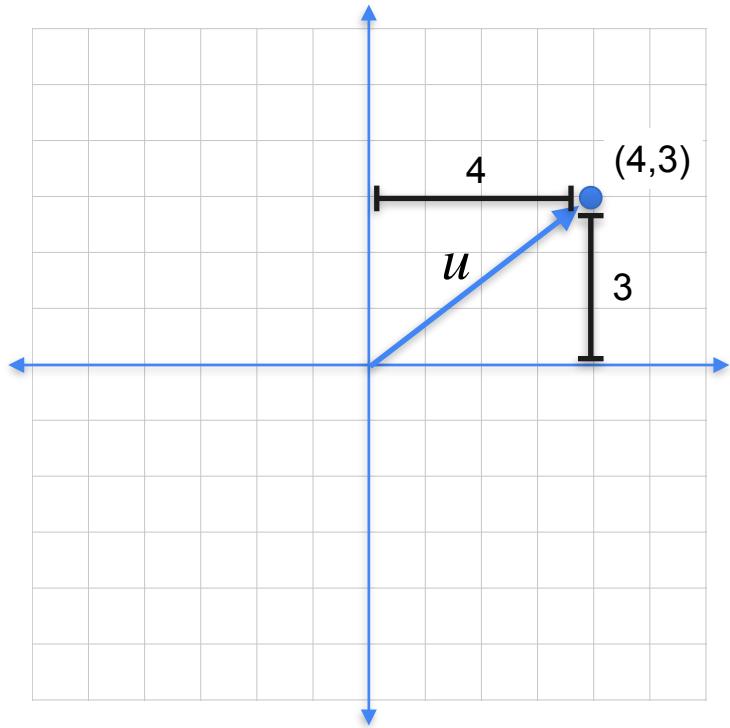
Norm of a vector using dot product



Norm of a vector using dot product

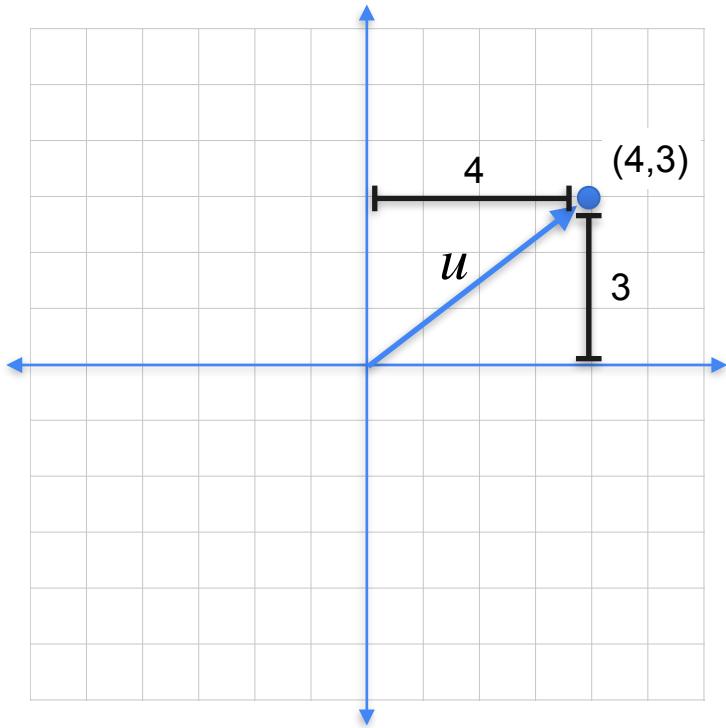


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

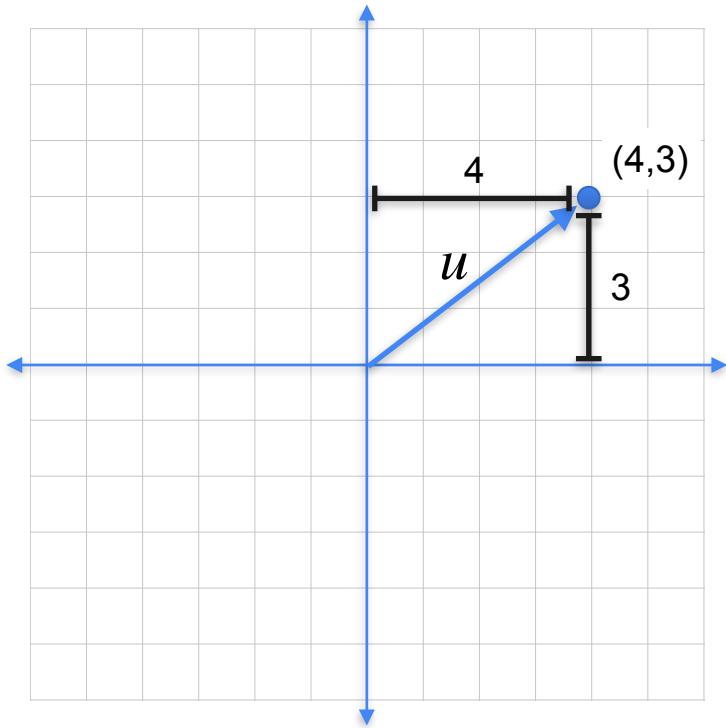
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 & \\ \hline 3 & \\ \hline \end{array} = 25$$

Norm of a vector using dot product

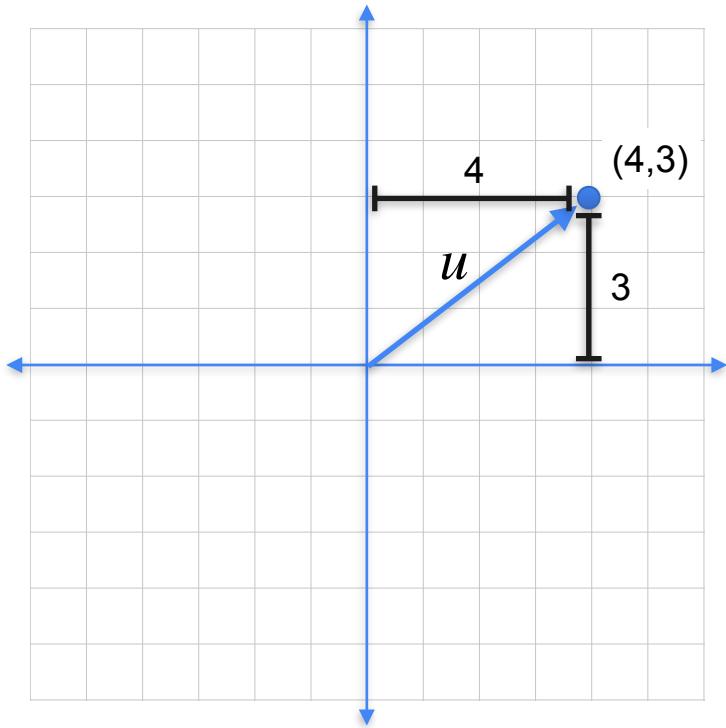


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{dot\ product(u, u)}$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{matrix} 4 & 3 \end{matrix} \quad \begin{matrix} 4 \\ 3 \end{matrix} = \begin{matrix} 25 \end{matrix}$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$\| u \|_2 = \sqrt{\langle u, u \rangle}$$

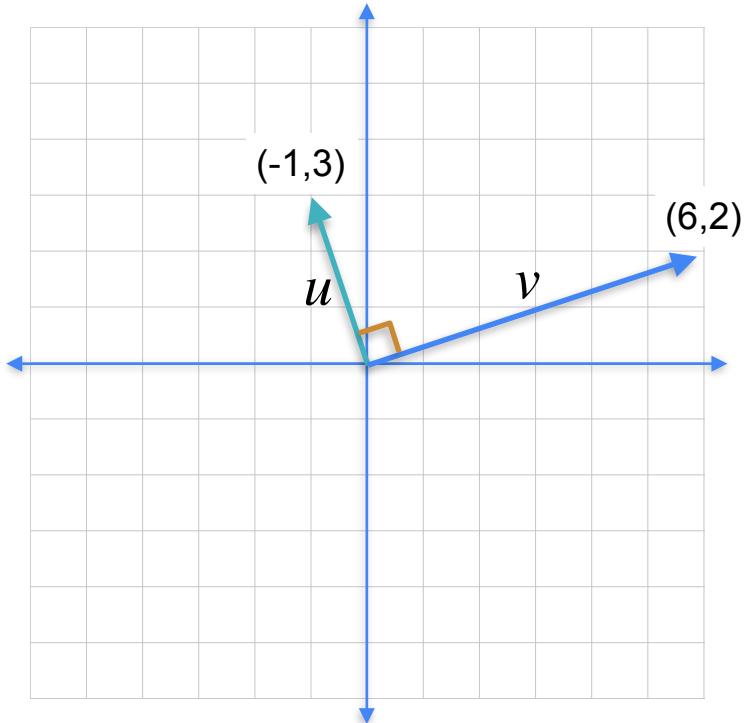


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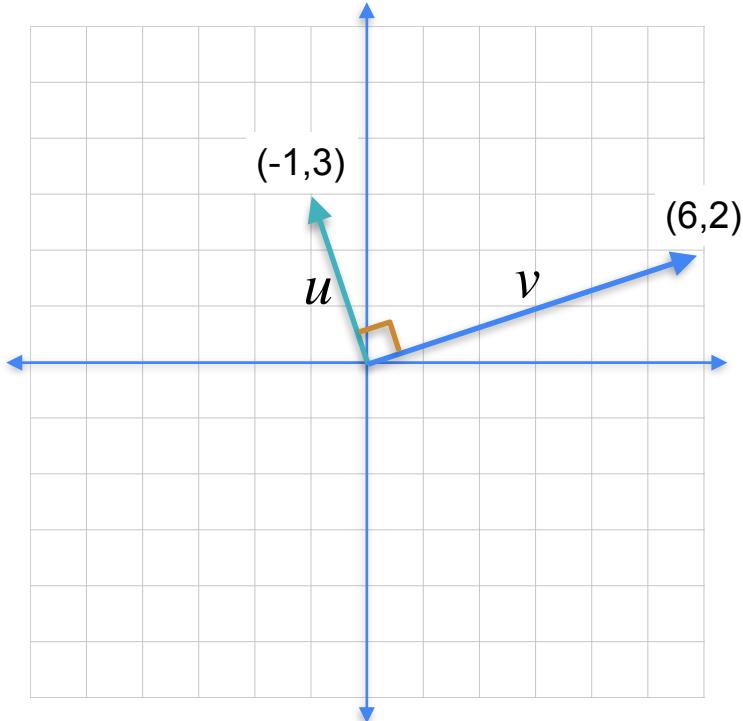
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0

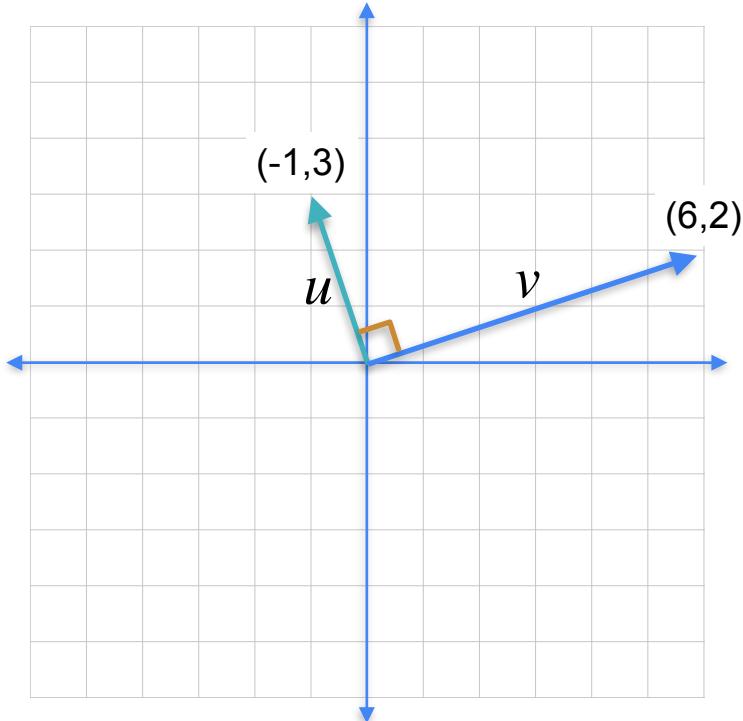


Orthogonal vectors have dot product 0



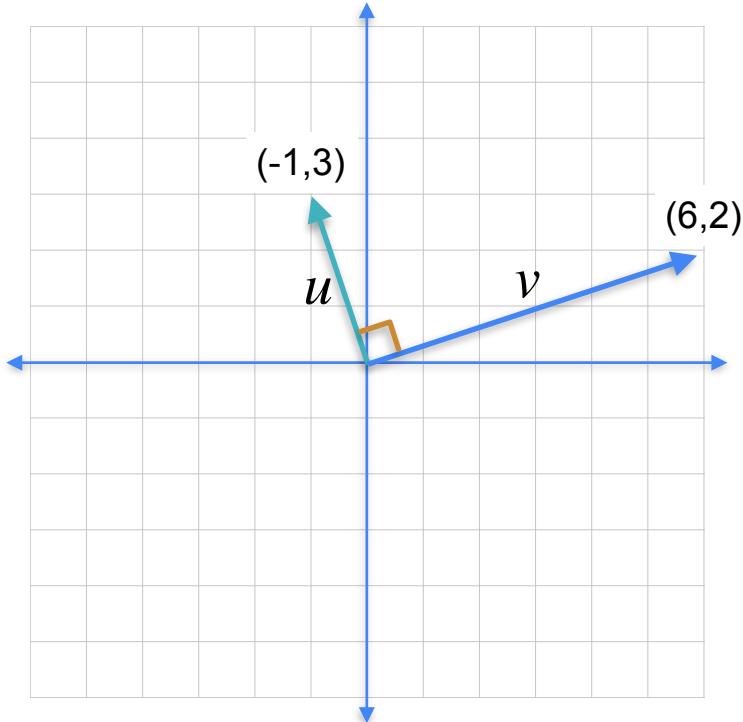
6	2
---	---

Orthogonal vectors have dot product 0



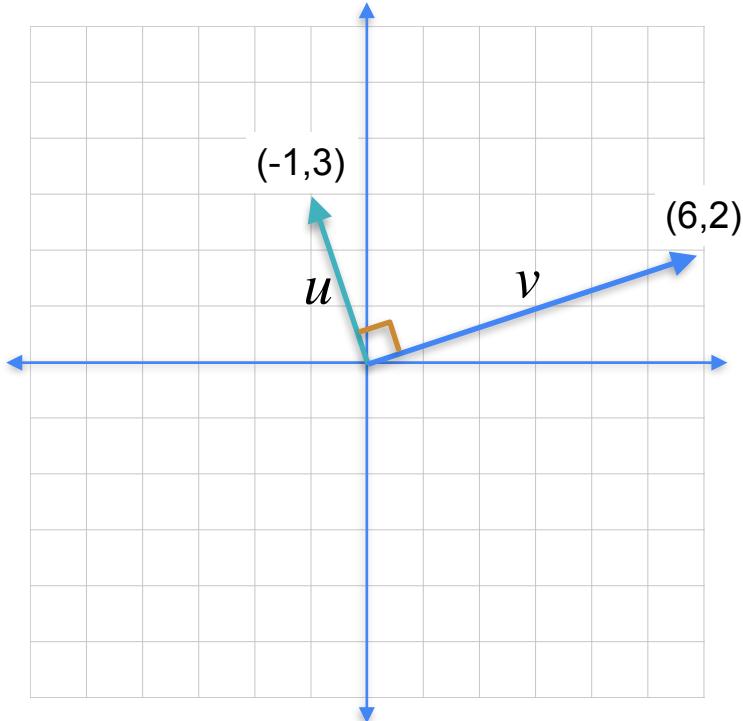
6	2	-1
3		

Orthogonal vectors have dot product 0



$$\begin{matrix} 6 & 2 \end{matrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

Orthogonal vectors have dot product 0

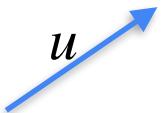


$$\begin{matrix} 6 & 2 \end{matrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

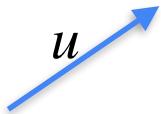
$$\langle u, v \rangle = 0$$

The dot product

The dot product

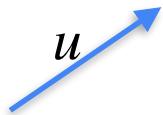


The dot product

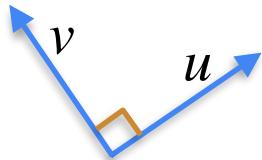


$$\langle u, u \rangle = |u|^2$$

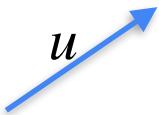
The dot product



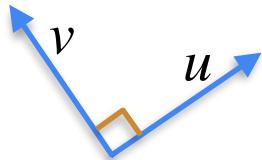
$$\langle u, u \rangle = |u|^2$$



The dot product

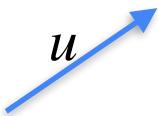


$$\langle u, u \rangle = |u|^2$$

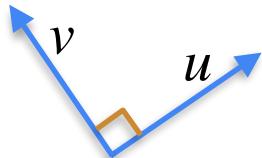


$$\langle u, v \rangle = 0$$

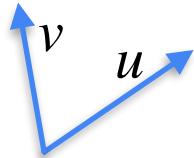
The dot product



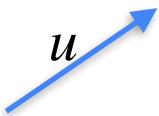
$$\langle u, u \rangle = |u|^2$$



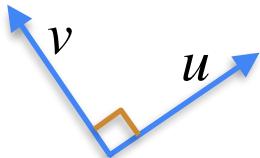
$$\langle u, v \rangle = 0$$



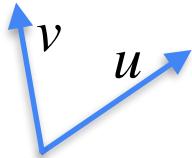
The dot product



$$\langle u, u \rangle = |u|^2$$



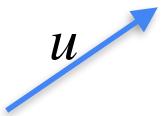
$$\langle u, v \rangle = 0$$



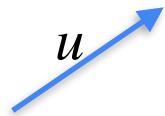
$$\langle u, v \rangle = ?$$

The dot product

The dot product

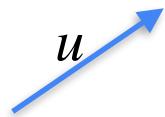


The dot product



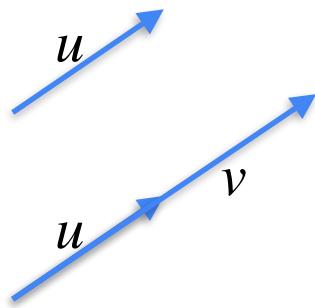
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



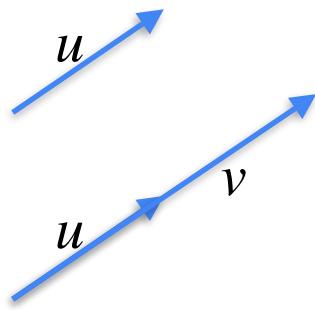
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

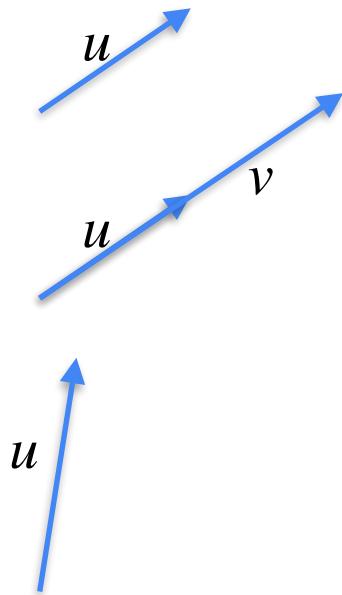
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

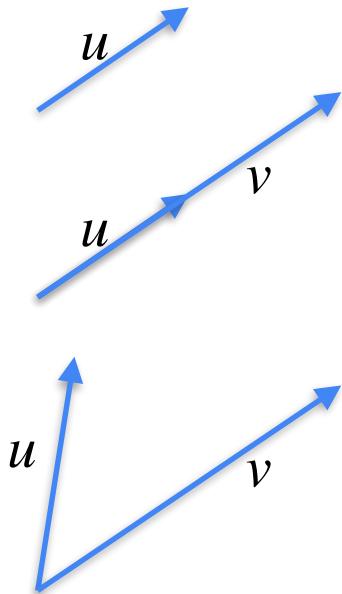
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

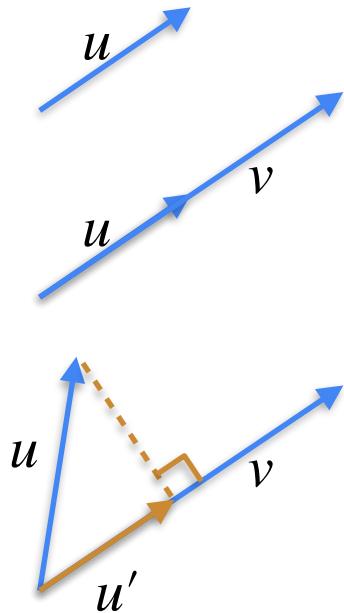
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

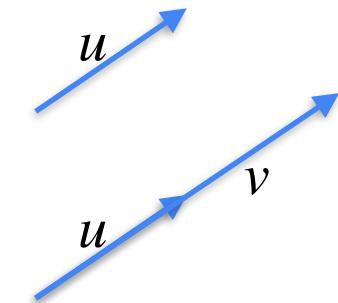
The dot product



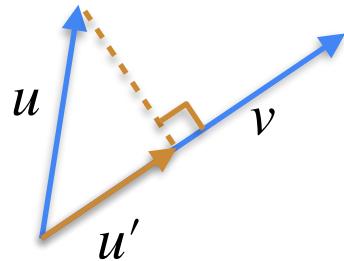
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product



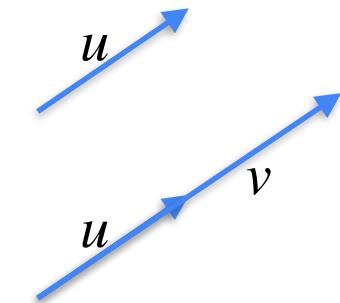
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



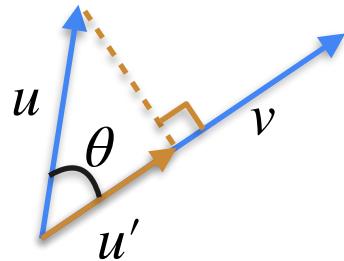
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



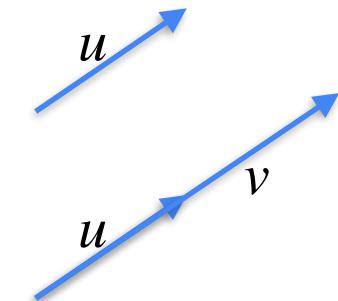
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



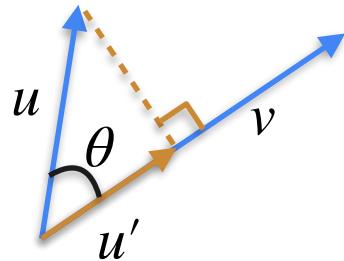
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



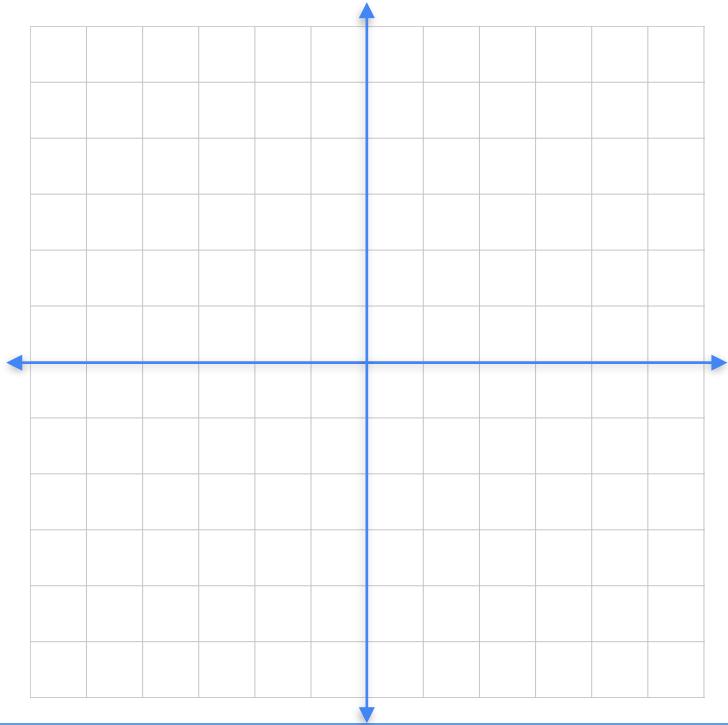
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



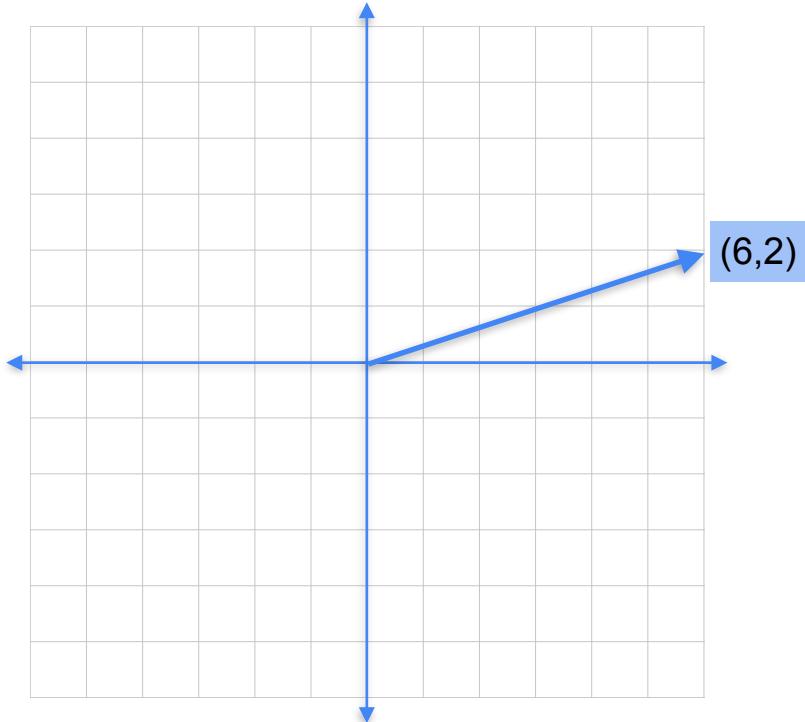
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

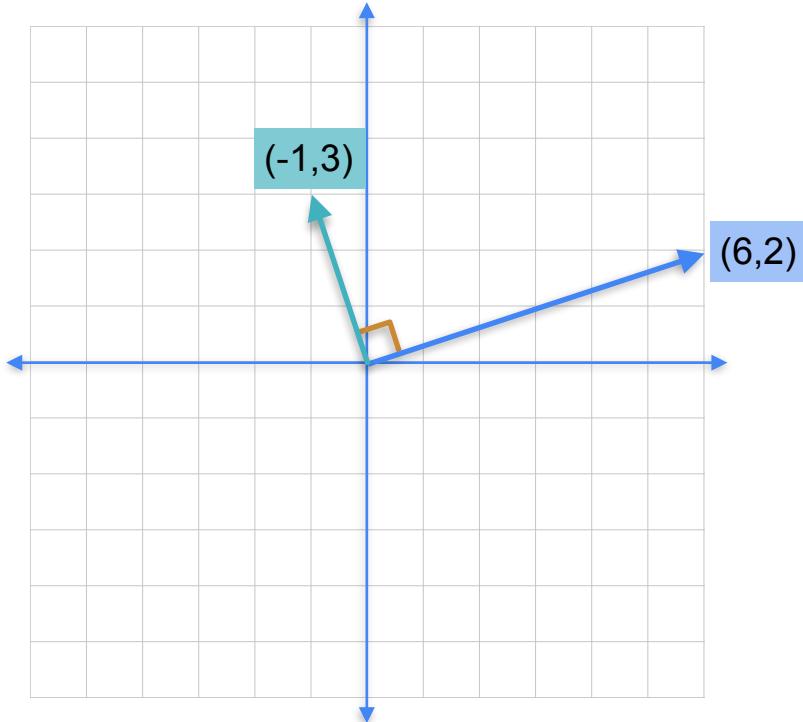
Geometric dot product



Geometric dot product

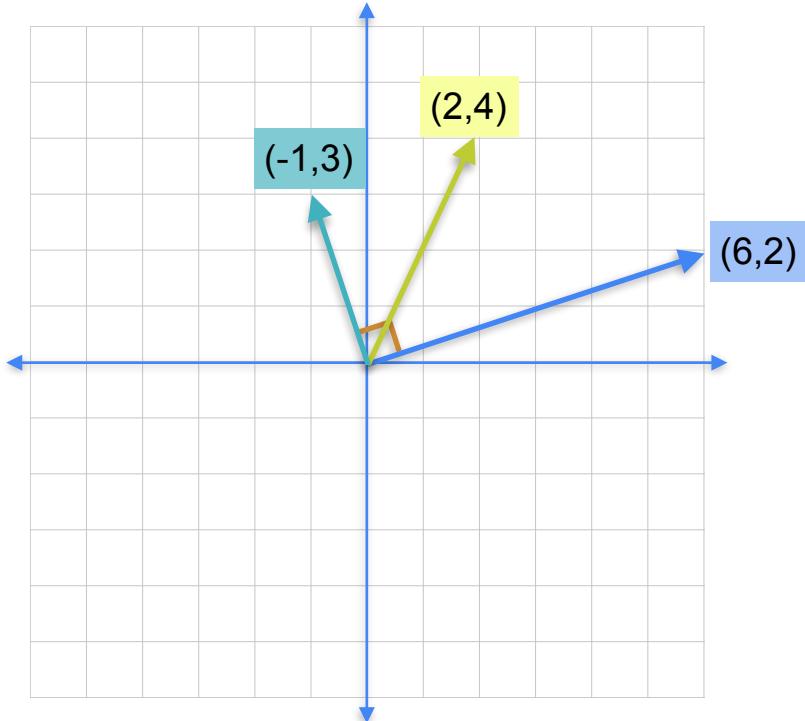


Geometric dot product



$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = 0$$

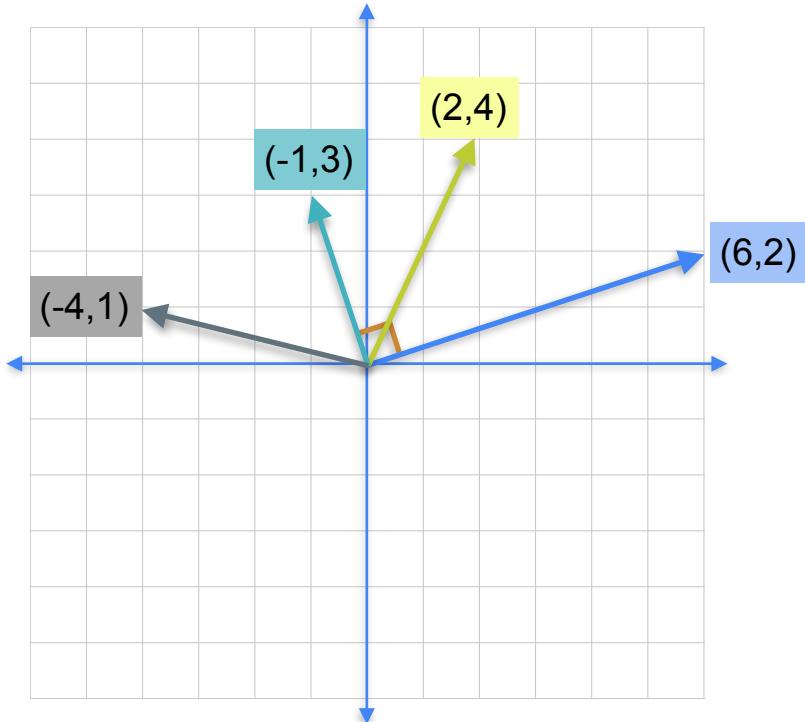
Geometric dot product



$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Geometric dot product

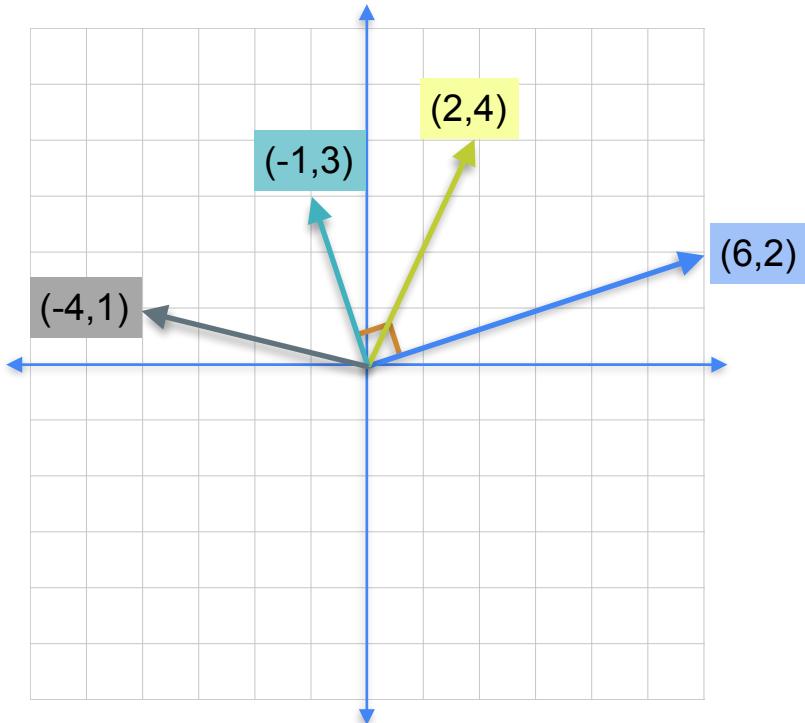


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = 20$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = 0$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = -22$$

Geometric dot product

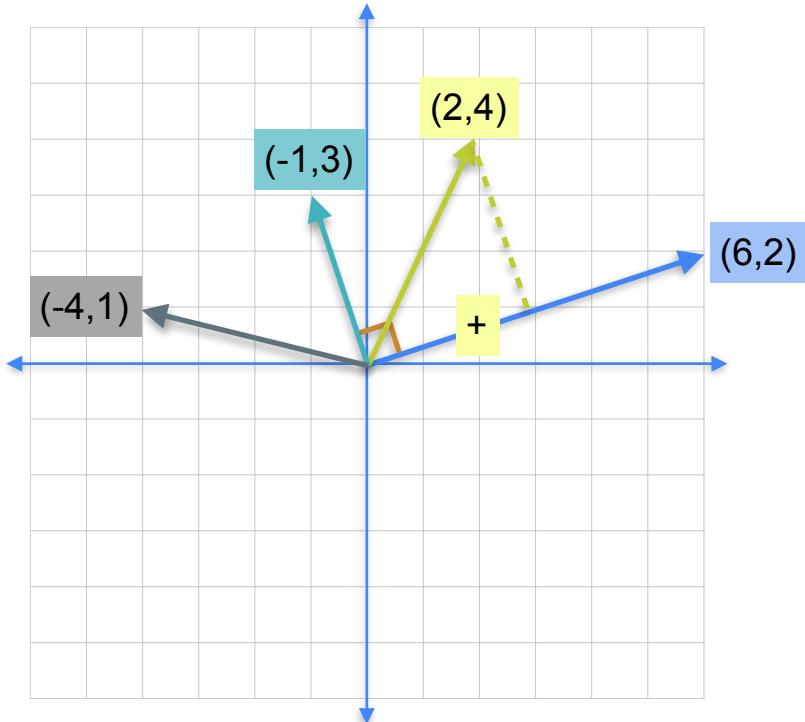


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \quad \text{Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

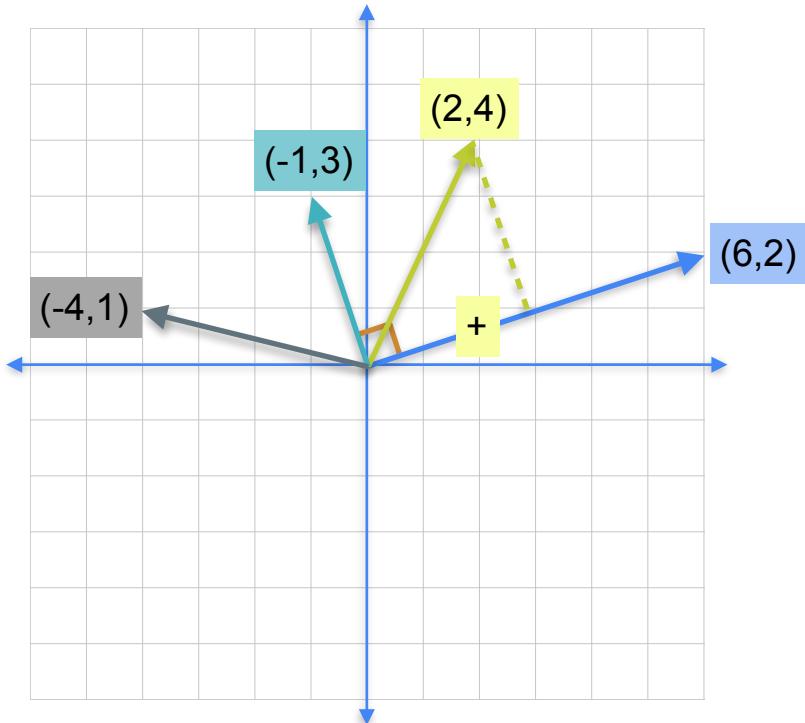


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array}$$

Geometric dot product

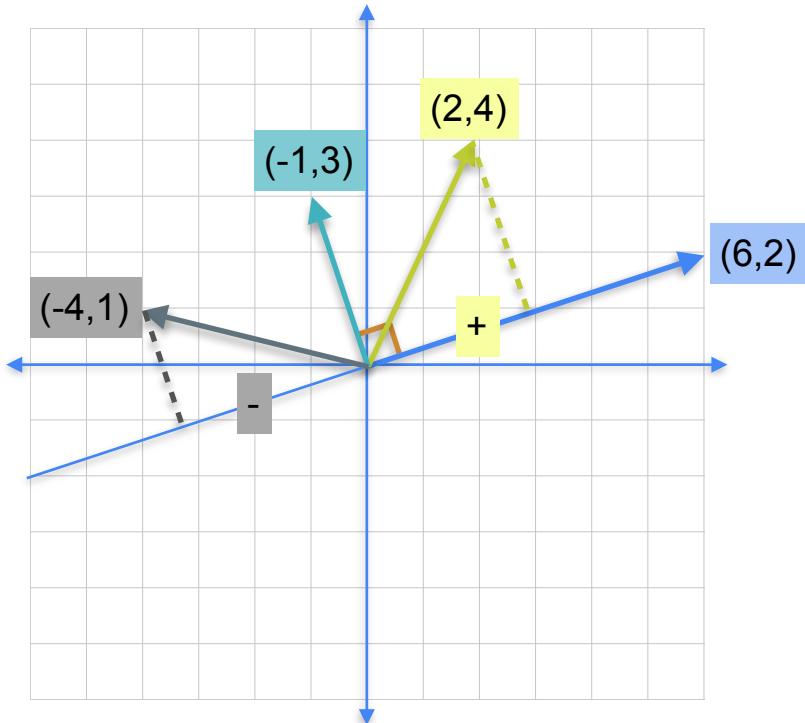


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

Geometric dot product

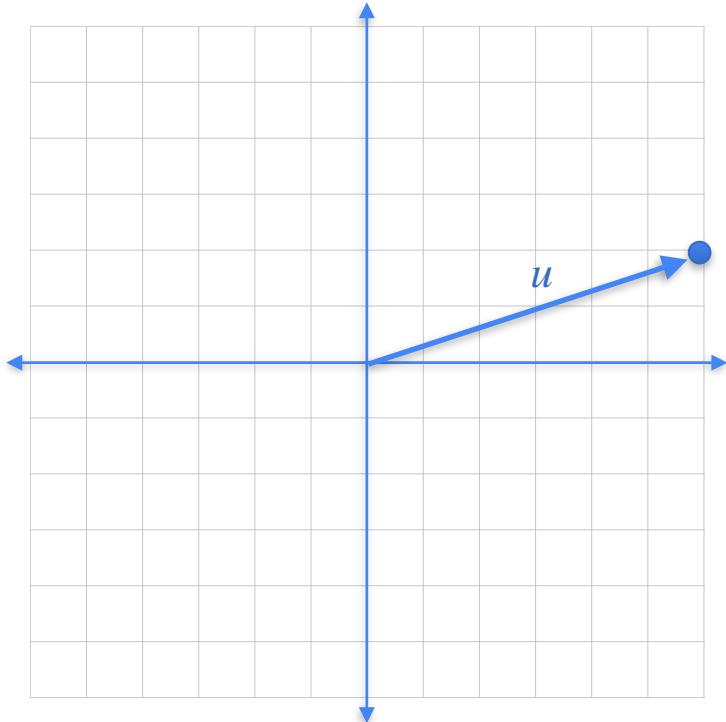


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \text{ Positive}$$

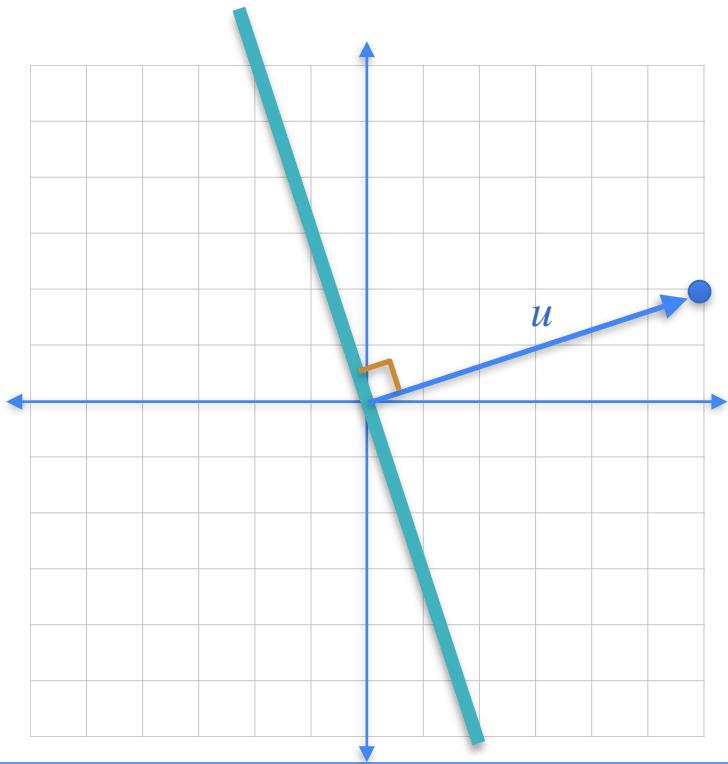
$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \text{ Negative}$$

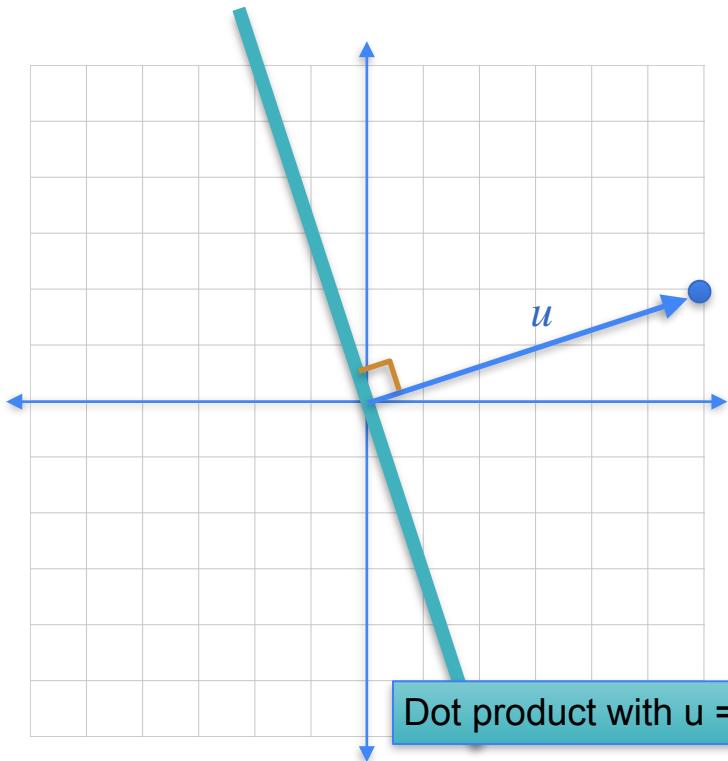
Geometric dot product



Geometric dot product

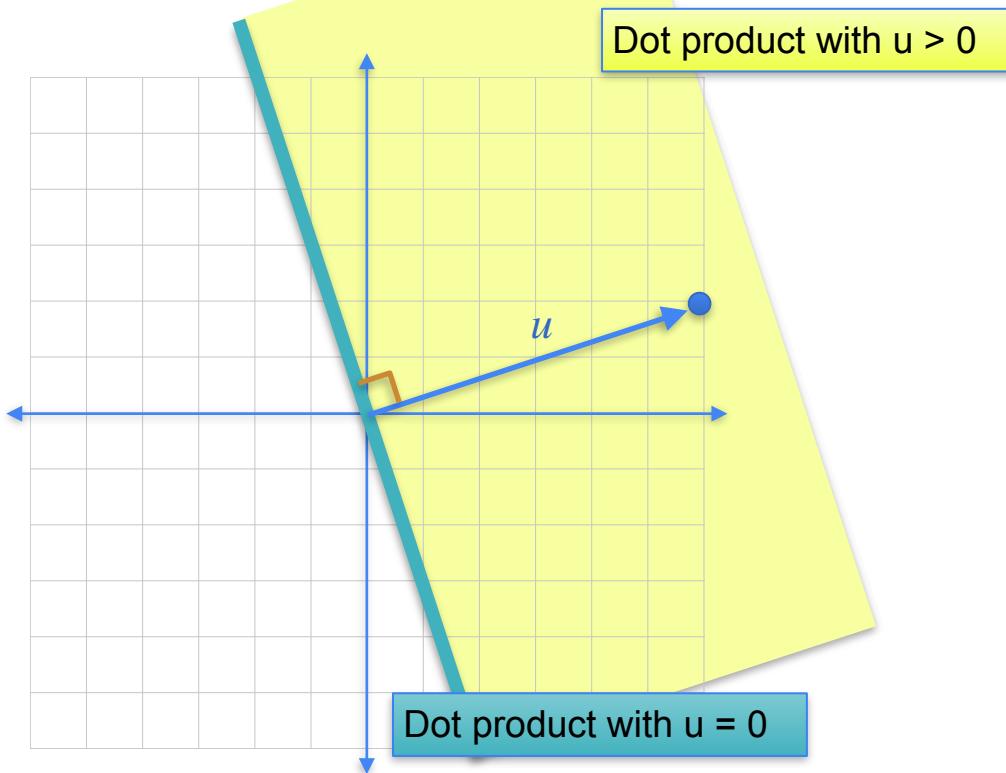


Geometric dot product



$$\langle u, v \rangle = 0$$

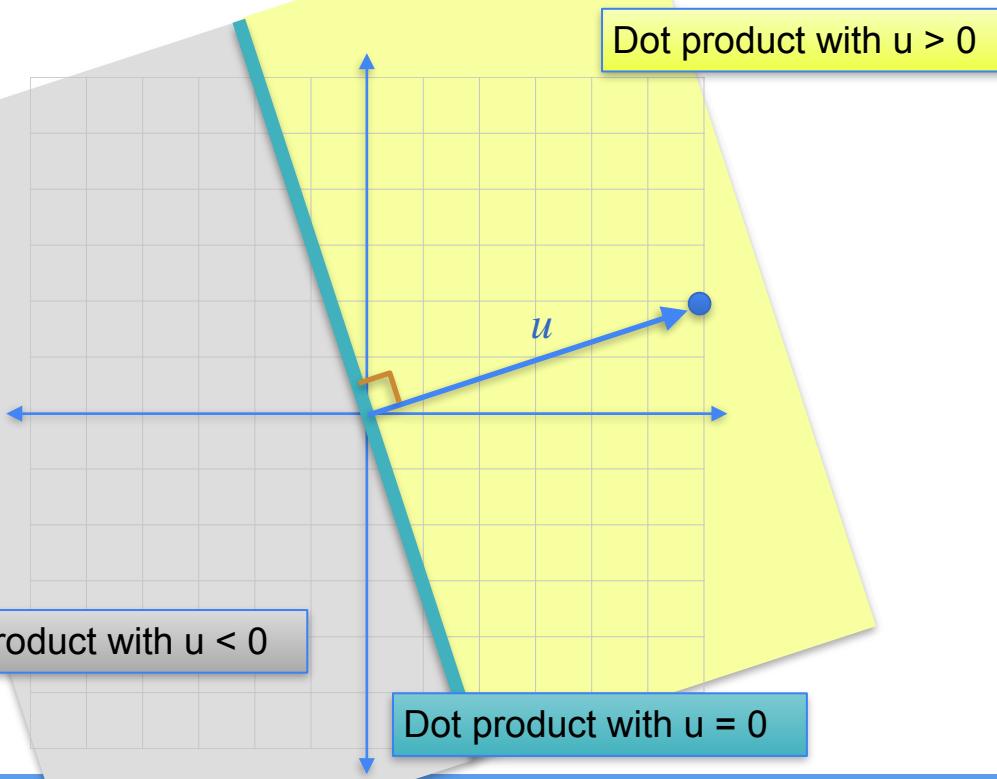
Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$



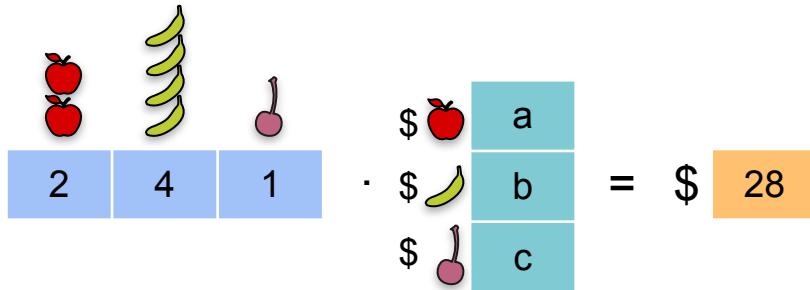
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Vectors and Linear Transformations

**Multiplying a matrix by a
vector**

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

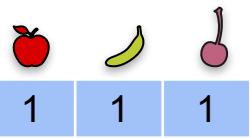
$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

				a
1	1	1		b
				c

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates the first equation $a + b + c = 10$ as a dot product. On the left, there is a row vector with three entries: an apple icon, a banana icon, and a cherry icon, followed by a blue row vector with entries 1, 1, 1. To the right of this is a multiplication symbol ·. To the right of the multiplication symbol is another row vector with three entries: a dollar sign and an apple icon, a dollar sign and a banana icon, and a dollar sign and a cherry icon. This is followed by a teal column vector with entries a, b, and c. To the right of this column vector is a dollar sign and the number 10 in an orange box.

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates three linear equations using fruit icons as vectors. Each fruit is assigned a value: Apple (\$a\$) = 1, Banana (\$b\$) = 2, and Cherry (\$c\$) = 1.

The first equation, $a + b + c = 10$, is shown as the dot product of the fruit vector $\begin{bmatrix} \text{apple} \\ 1 \\ \text{banana} \\ 1 \\ \text{cherry} \\ 1 \end{bmatrix}$ and the price vector $\begin{bmatrix} \$ \\ a \\ \$ \\ b \\ \$ \\ c \end{bmatrix} = \10 .

The second equation, $a + 2b + c = 15$, is shown as the dot product of the fruit vector $\begin{bmatrix} \text{apple} \\ 1 \\ \text{banana} \\ 2 \\ \text{cherry} \\ 1 \end{bmatrix}$ and the price vector $\begin{bmatrix} \$ \\ a \\ \$ \\ b \\ \$ \\ c \end{bmatrix}$.

The third equation, $a + b + 2c = 12$, is implied by the third column of the diagram.

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: one apple, one banana, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where each component corresponds to the price of the respective fruit. The result is a scalar value of \$10.

1	1	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$10$$

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It shows a vector of fruits: one apple, two bananas, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where each component corresponds to the price of the respective fruit. The result is a scalar value of \$15.

1	2	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: one apple, one banana, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where each component is labeled with a dollar sign (\$) and a fruit icon. The result is equal to the scalar value 10.

1	1	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\cdot \$ \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$$

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It shows a vector of fruits: one apple, two bananas, and one cherry. This vector is multiplied by a scalar vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where each component is labeled with a dollar sign (\$) and a fruit icon. The result is equal to the scalar value 15.

1	2	1
\$	apple	a
\$	banana	b
\$	cherry	c

$$\cdot \$ \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each represented by a red icon. Below this vector is a blue horizontal bar divided into three equal segments, each labeled with the number 1. To the right of the fruit vector is a multiplication sign followed by a matrix multiplication symbol ($\cdot \$$). The matrix has three columns, each corresponding to one of the fruit icons. The first column is labeled 'a' and contains the fruit icons. The second column is labeled 'b' and contains the blue segments. The third column is labeled 'c' and contains the fruit icons again. To the right of the matrix is an equals sign followed by a yellow box containing the number 10.

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. It follows a similar structure to the first diagram. On the left, there is a vector of fruits: an apple, a banana, and a cherry. Below it is a blue horizontal bar divided into three segments, with the middle segment labeled 2 and the others labeled 1. To the right of the fruit vector is a multiplication sign followed by a matrix multiplication symbol ($\cdot \$$). The matrix has three columns: the first column 'a' contains the fruit icons; the second column 'b' contains the blue segments with the middle one doubled; the third column 'c' contains the fruit icons again. To the right of the matrix is an equals sign followed by a yellow box containing the number 15.

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + 2c = 12$ using vectors. It follows the same structure as the previous diagrams. On the left, there is a vector of fruits: an apple, a banana, and a cherry. Below it is a blue horizontal bar divided into three segments, with the last segment labeled 2 and the others labeled 1. To the right of the fruit vector is a multiplication sign followed by a matrix multiplication symbol ($\cdot \$$). The matrix has three columns: the first column 'a' contains the fruit icons; the second column 'b' contains the blue segments; the third column 'c' contains the fruit icons with the last one doubled. To the right of the matrix is an equals sign followed by a yellow box containing the number 12.

Equations as dot product

$$a + b + c = 10$$

The diagram illustrates the equation $a + b + c = 10$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and a cherry, each labeled with a value of 1. To the right of this vector is a dot product expression: $\begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ 1 \\ \$ \end{matrix} \begin{matrix} \text{banana} \\ 1 \\ \$ \end{matrix} \begin{matrix} \text{cherry} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ a \\ \$ \end{matrix} \begin{matrix} \text{banana} \\ b \\ \$ \end{matrix} \begin{matrix} \text{cherry} \\ c \\ \$ \end{matrix} = \$ \begin{matrix} 10 \end{matrix}$. The vectors are represented as horizontal arrays of three elements, and the dot product is shown as a scalar value \$10.

$$a + 2b + c = 15$$

The diagram illustrates the equation $a + 2b + c = 15$ using vectors. On the left, there is a vector of fruits: an apple, two bananas, and a cherry, each labeled with values 1, 2, and 1 respectively. To the right of this vector is a dot product expression: $\begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ 1 \\ \$ \end{matrix} \begin{matrix} \text{banana} \\ 2 \\ \$ \end{matrix} \begin{matrix} \text{cherry} \\ 1 \\ \$ \end{matrix} \cdot \begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ a \\ \$ \end{matrix} \begin{matrix} \text{banana} \\ b \\ \$ \end{matrix} \begin{matrix} \text{cherry} \\ c \\ \$ \end{matrix} = \$ \begin{matrix} 15 \end{matrix}$. The vectors are represented as horizontal arrays of three elements, and the dot product is shown as a scalar value \$15.

$$a + b + 2c = 12$$

The diagram illustrates the equation $a + b + 2c = 12$ using vectors. On the left, there is a vector of fruits: an apple, a banana, and two cherries, each labeled with values 1, 1, and 2 respectively. To the right of this vector is a dot product expression: $\begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ 1 \\ \$ \end{matrix} \begin{matrix} \text{banana} \\ 1 \\ \$ \end{matrix} \begin{matrix} \text{cherry} \\ 2 \\ \$ \end{matrix} \cdot \begin{matrix} \$ \end{matrix} \begin{matrix} \text{apple} \\ a \\ \$ \end{matrix} \begin{matrix} \text{banana} \\ b \\ \$ \end{matrix} \begin{matrix} \text{cherry} \\ c \\ \$ \end{matrix} = \$ \begin{matrix} 12 \end{matrix}$. The vectors are represented as horizontal arrays of three elements, and the dot product is shown as a scalar value \$12.

Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation $a + b + c = 10$. It shows a vector of fruit counts (1 apple, 1 banana, 1 cherry) multiplied by a vector of fruit prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$10.

1	1	1
\$	apple	a
1	banana	b
\$	cherry	c

$$1 \cdot \$1 + 1 \cdot \$2 + 1 \cdot \$1 = \$10$$

$$a + 2b + c = 15$$

A diagram illustrating the equation $a + 2b + c = 15$. It shows a vector of fruit counts (1 apple, 2 bananas, 1 cherry) multiplied by a vector of fruit prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$15.

1	2	1
\$	apple	a
1	banana	b
\$	cherry	c

$$1 \cdot \$1 + 2 \cdot \$2 + 1 \cdot \$1 = \$15$$

$$a + b + 2c = 12$$

A diagram illustrating the equation $a + b + 2c = 12$. It shows a vector of fruit counts (1 apple, 1 banana, 2 cherries) multiplied by a vector of fruit prices (\$1 for apple, \$2 for banana, \$1 for cherry) resulting in a total value of \$12.

1	1	2
\$	apple	a
1	banana	b
\$	cherry	c

$$1 \cdot \$1 + 1 \cdot \$2 + 2 \cdot \$1 = \$12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

A game board illustrating a multiplication challenge. On the left, three icons represent items: an apple, a banana, and a cherry. Below each icon is a blue box containing the number 1. To the right of the icons is a grid. The first column contains the symbols '\$' and the three fruit icons. The second column contains the letters 'a', 'b', and 'c'. To the right of the grid is an equals sign followed by a large orange box containing the number 10.

1	2	1	
.	\$	\$	
	a	b	
		c	
			= \$

			\$	a
1	1	2	\$	b
			\$	c

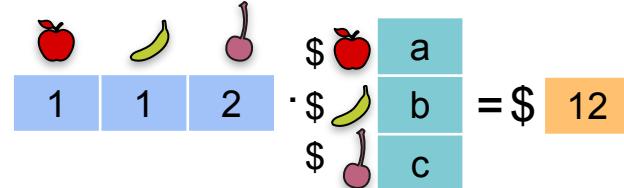
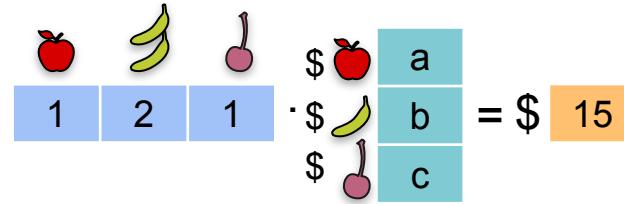
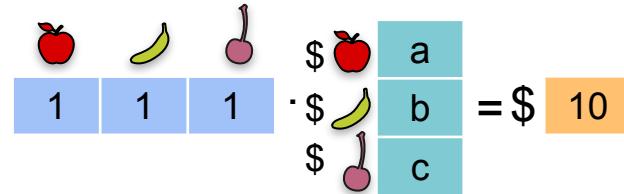
= \$ 12

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

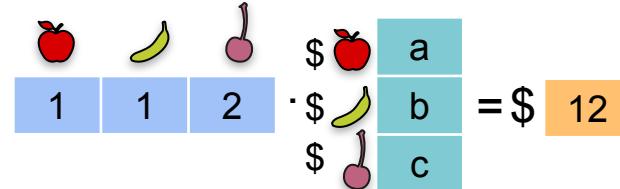
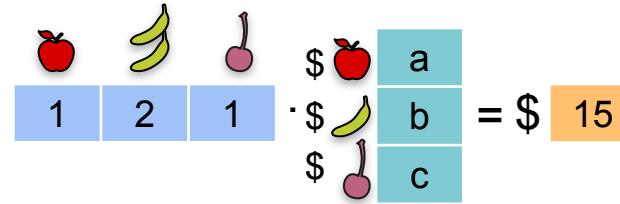
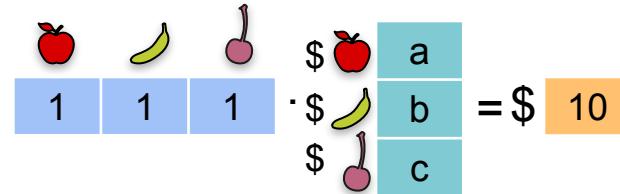


Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{matrix} \text{apple} & \text{banana} & \text{cherry} \\ \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline 1 & 1 & 2 \\ \hline \end{array} & \begin{array}{|c|c|c|} \hline \$\text{apple} & a \\ \hline \$\text{banana} & b \\ \hline \$\text{cherry} & c \\ \hline \end{array} & = \$ \begin{array}{|c|c|c|} \hline 10 \\ \hline 15 \\ \hline 12 \\ \hline \end{array} \end{matrix}$$

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$



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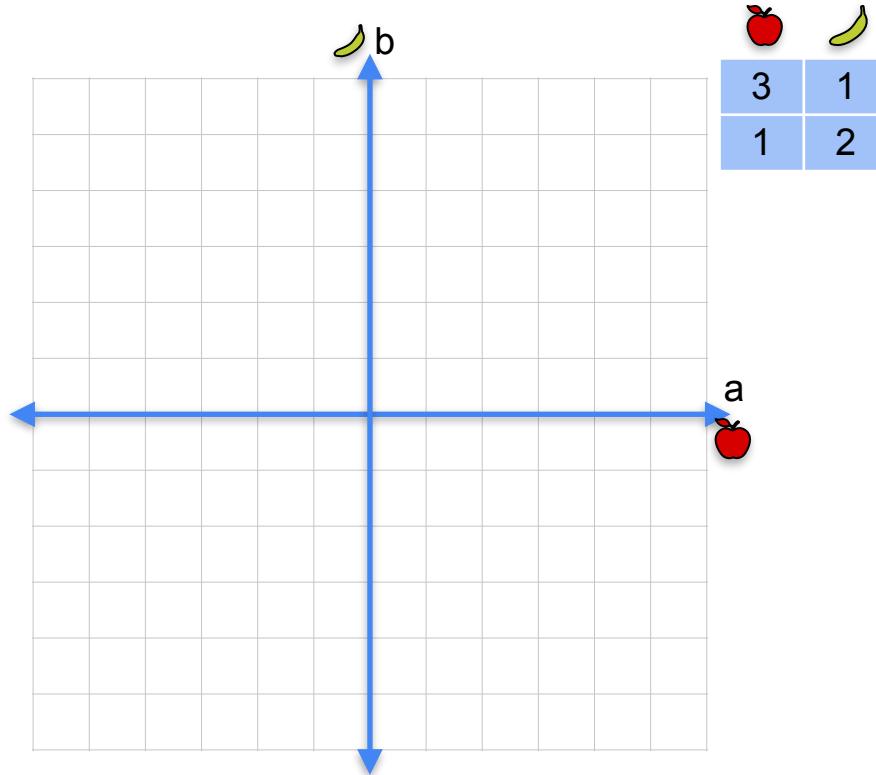
Vectors and Linear Transformations

**Matrices as linear
transformations**

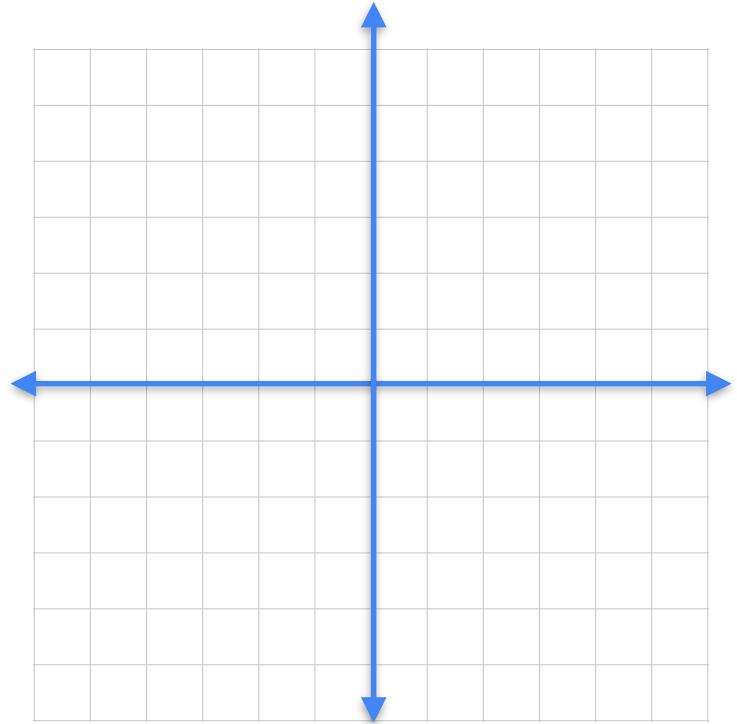
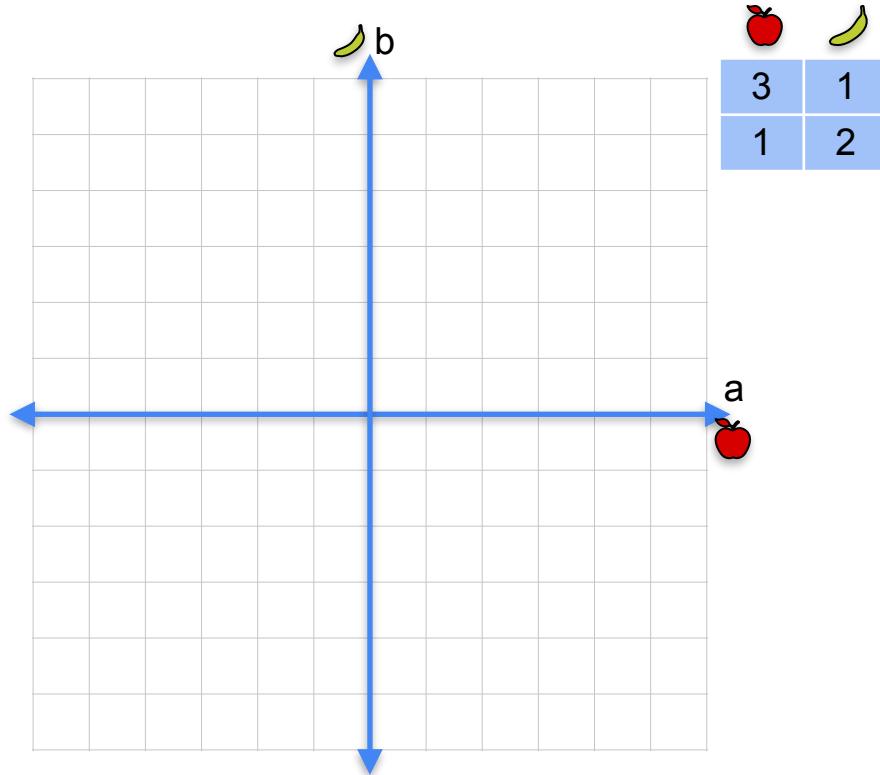
Matrices as linear transformations

		
3	1	
1	2	

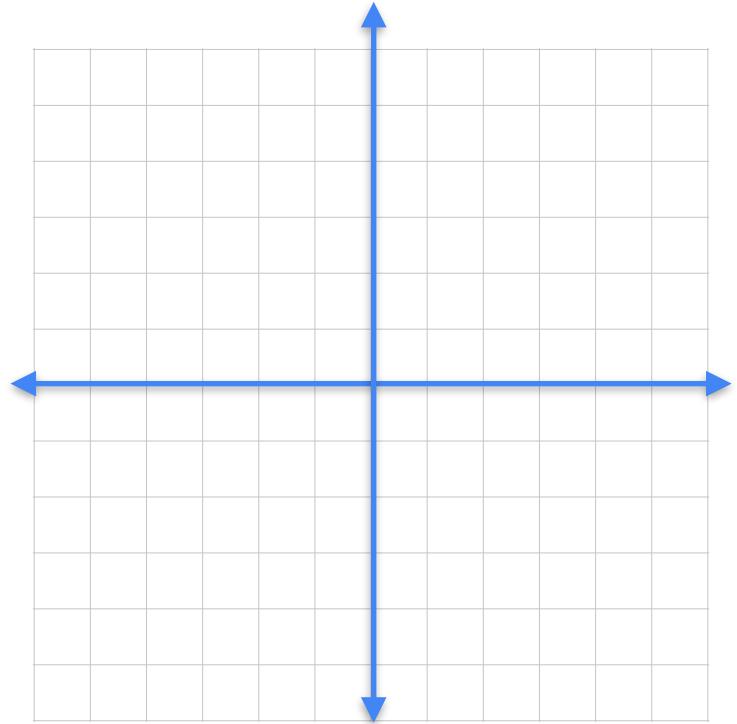
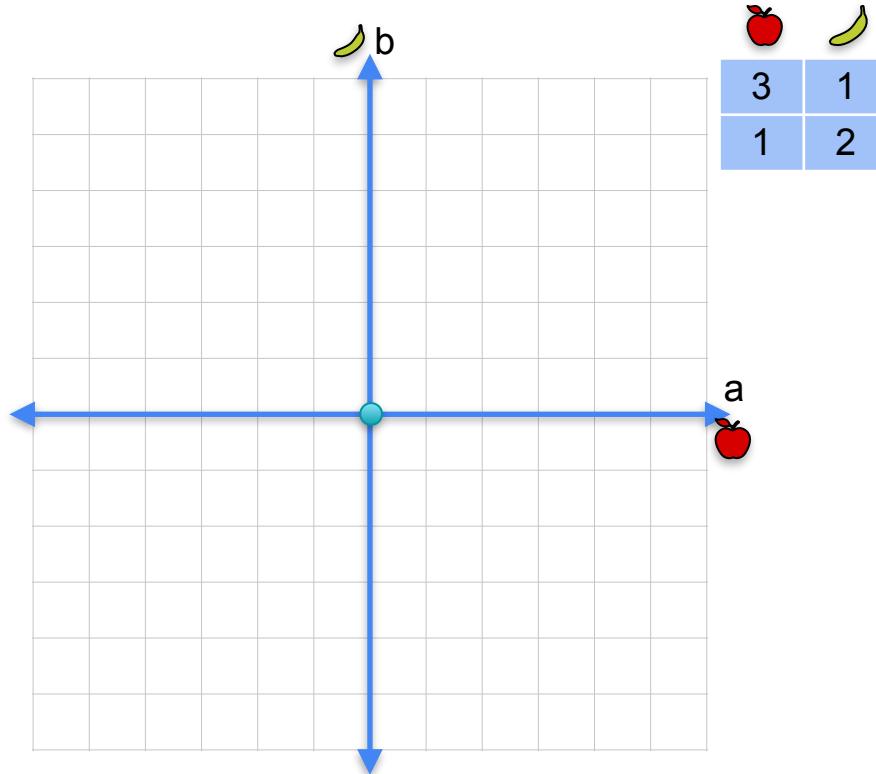
Matrices as linear transformations



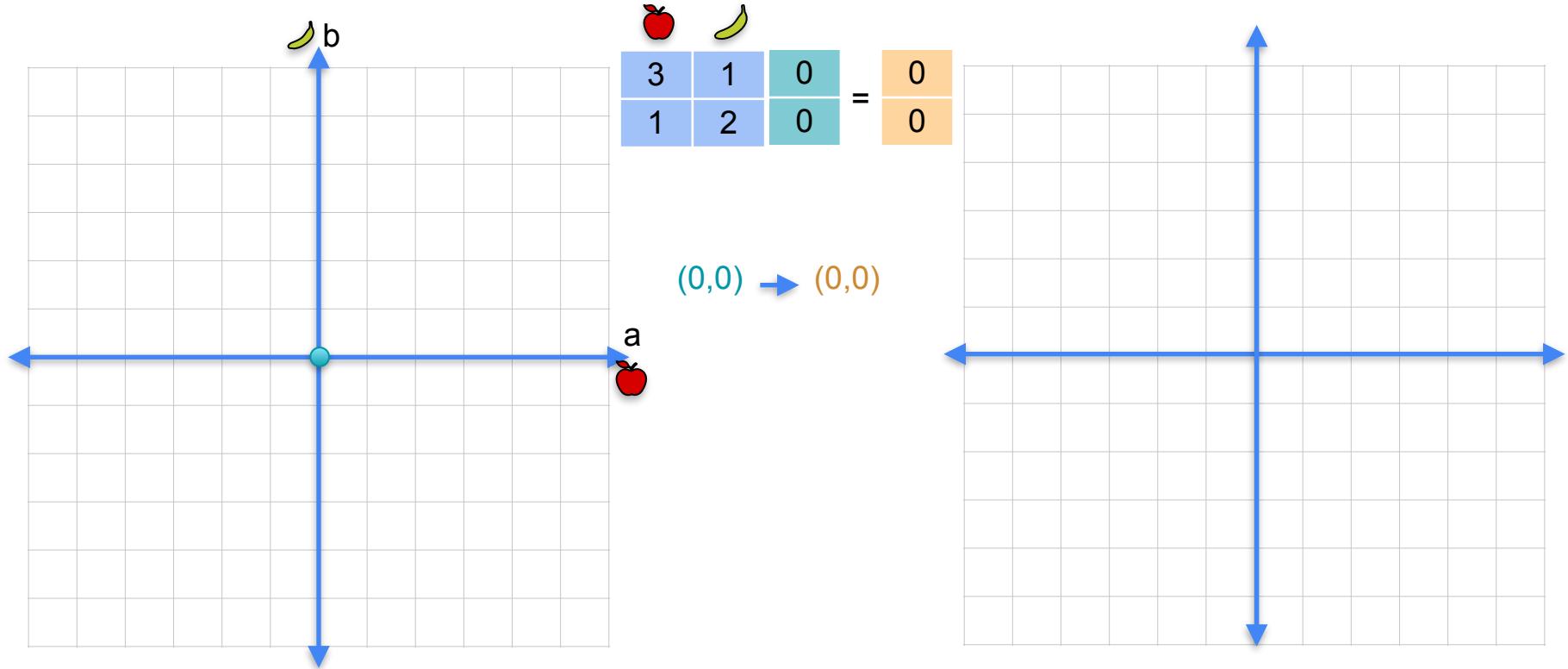
Matrices as linear transformations



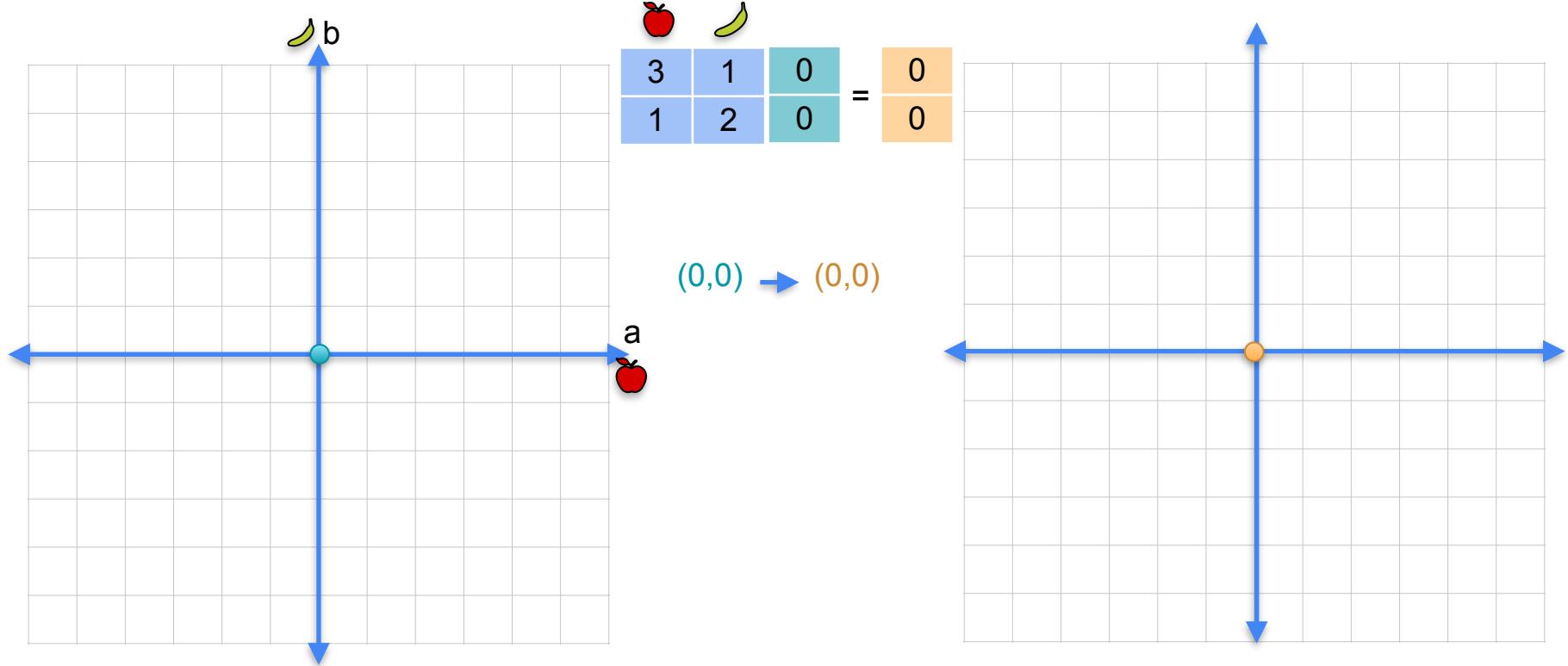
Matrices as linear transformations



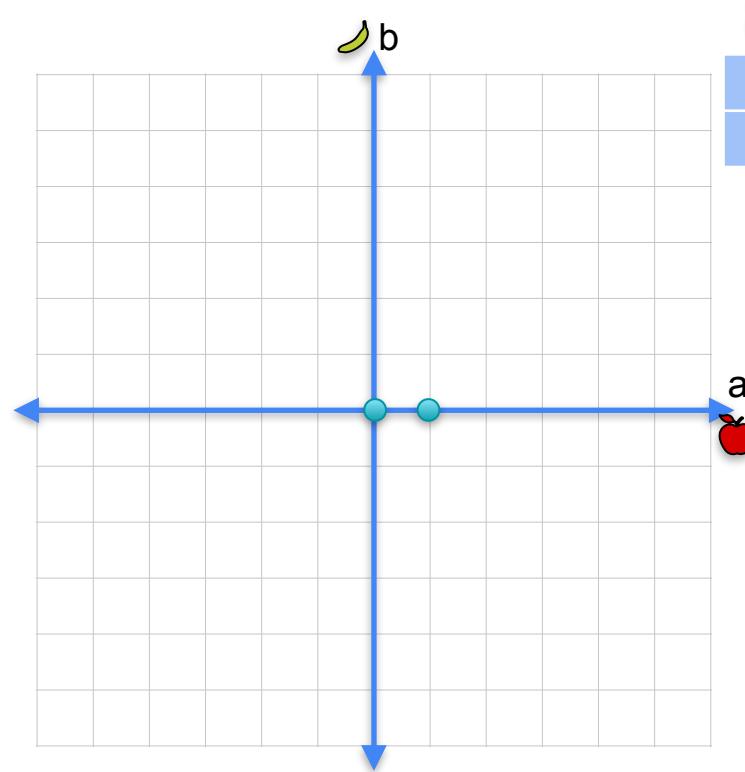
Matrices as linear transformations



Matrices as linear transformations



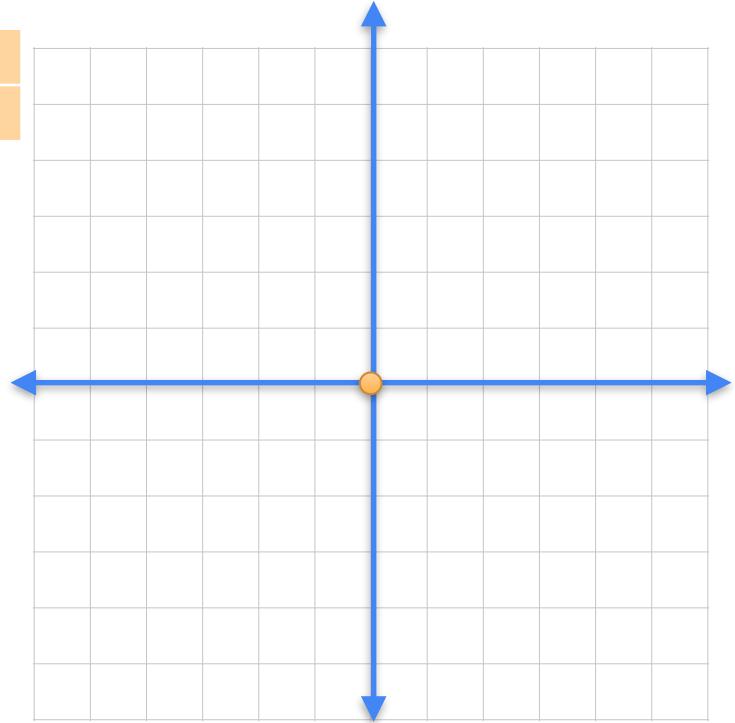
Matrices as linear transformations



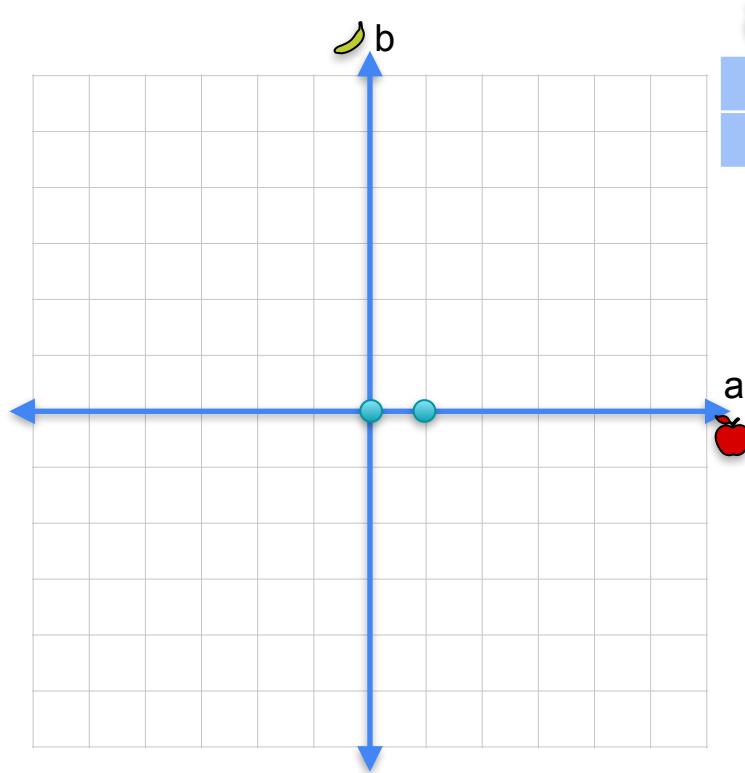
A matrix equation illustrating a linear transformation:

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$

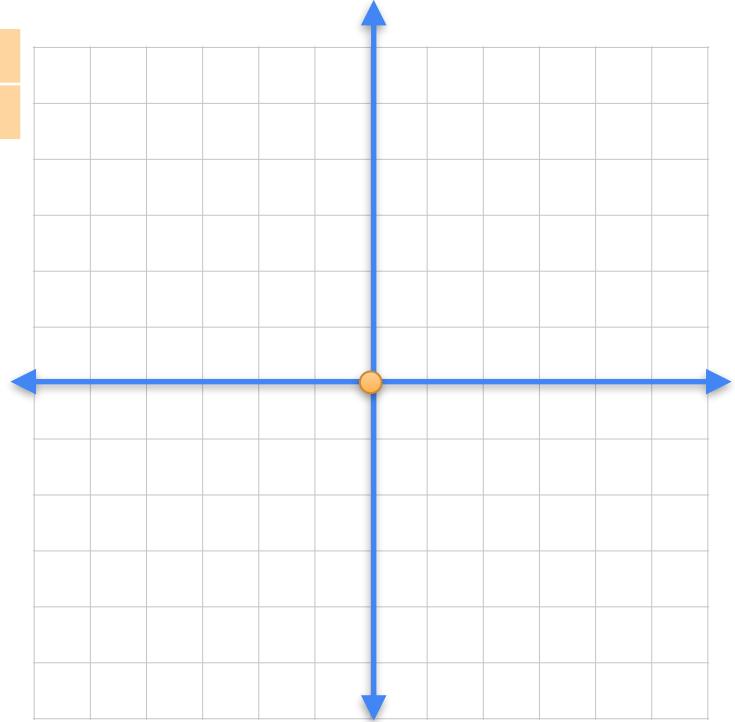


Matrices as linear transformations

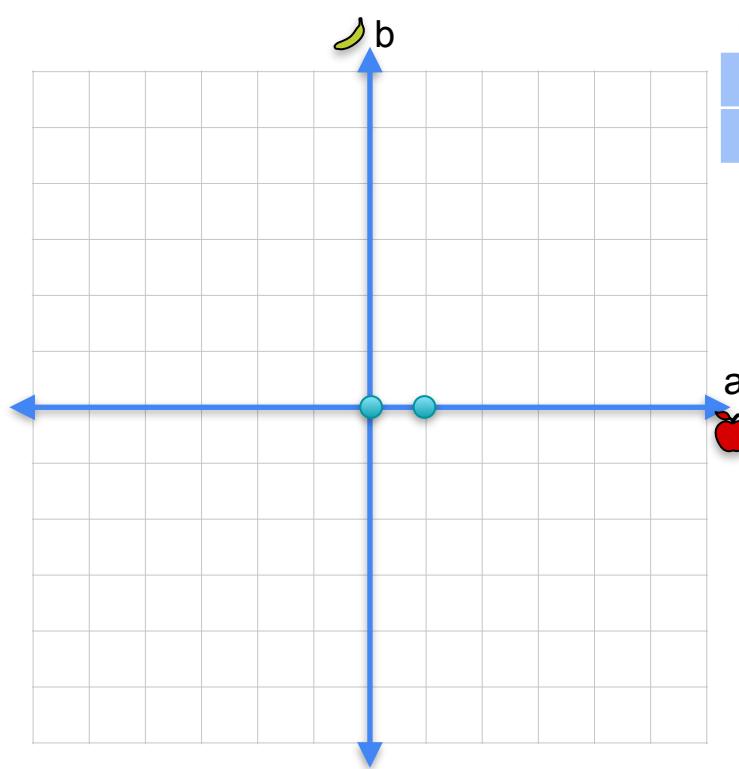


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$

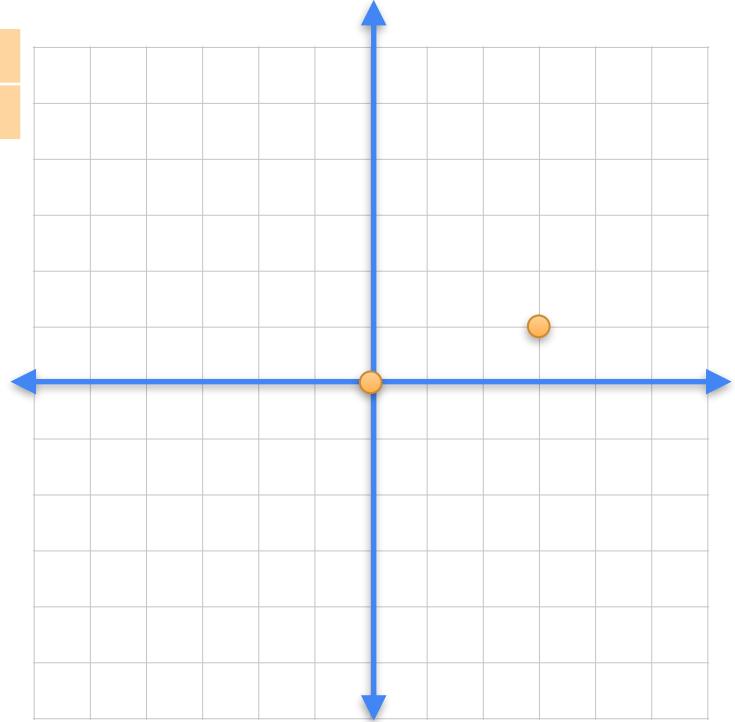


Matrices as linear transformations

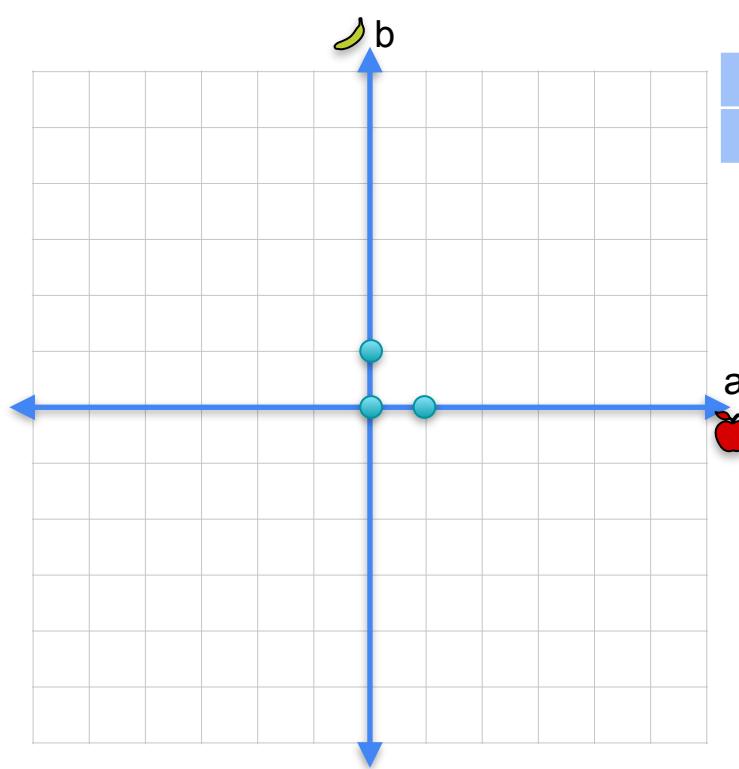


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$

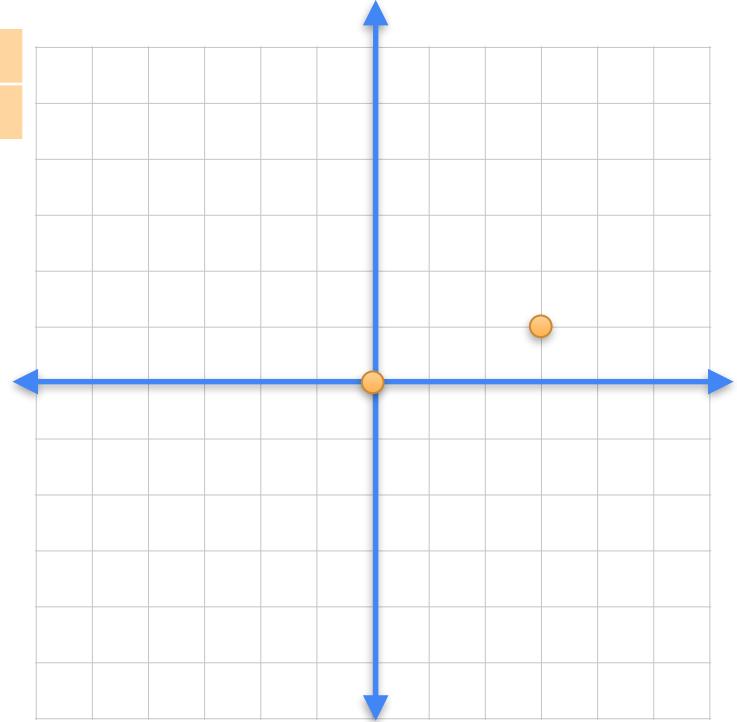


Matrices as linear transformations

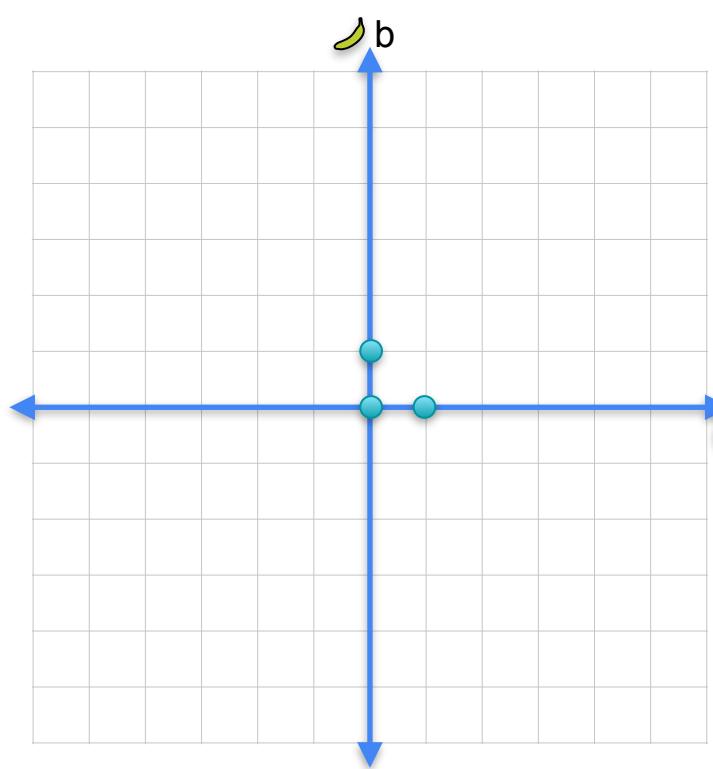


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

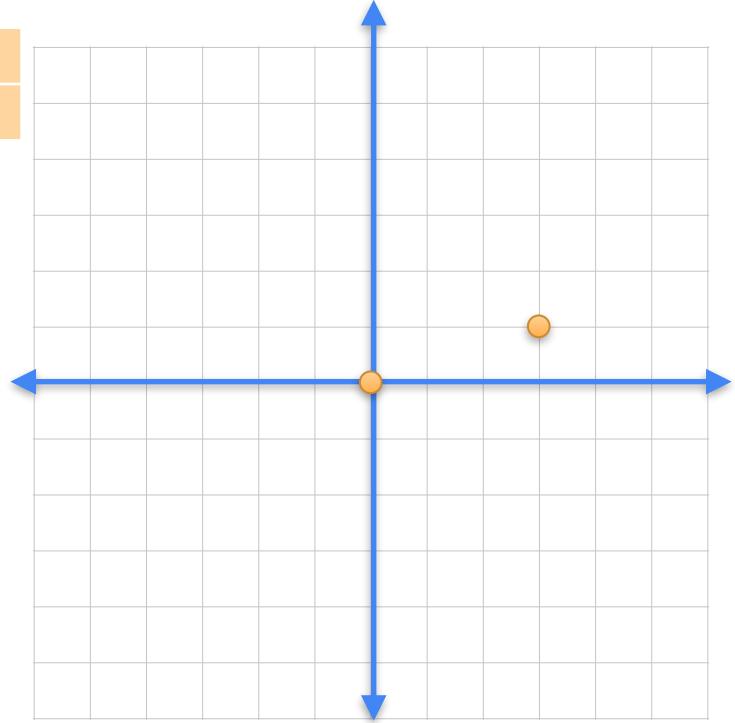


Matrices as linear transformations

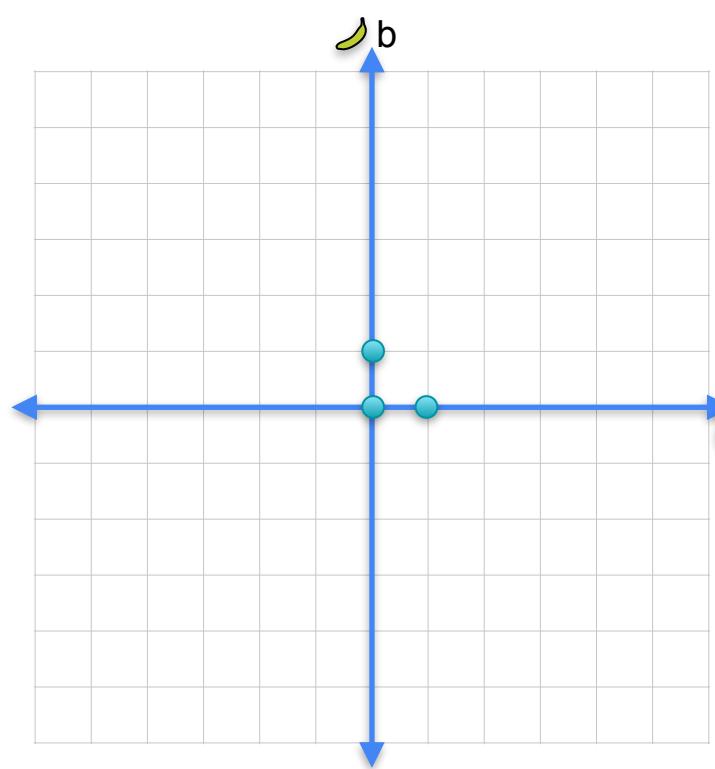


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \\ = & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

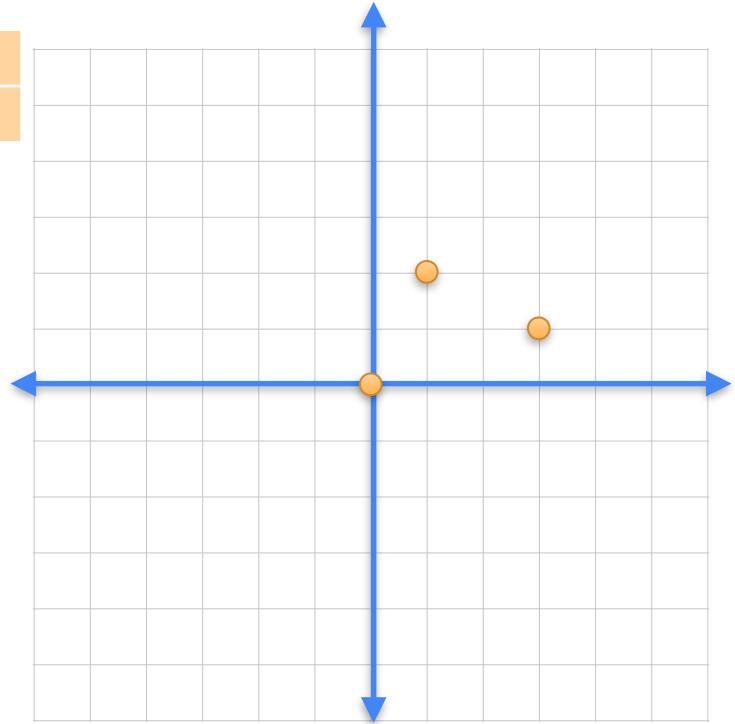


Matrices as linear transformations

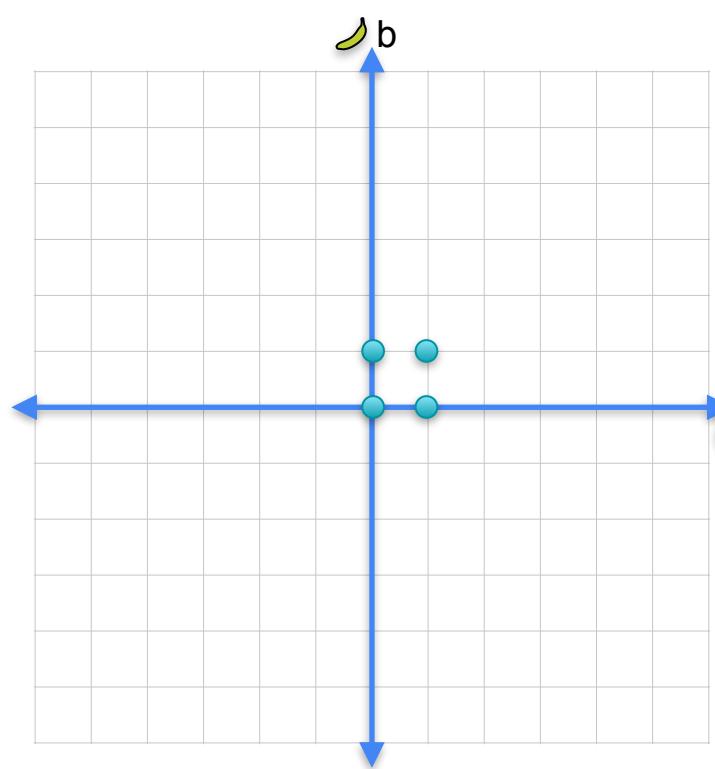


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \\ = & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

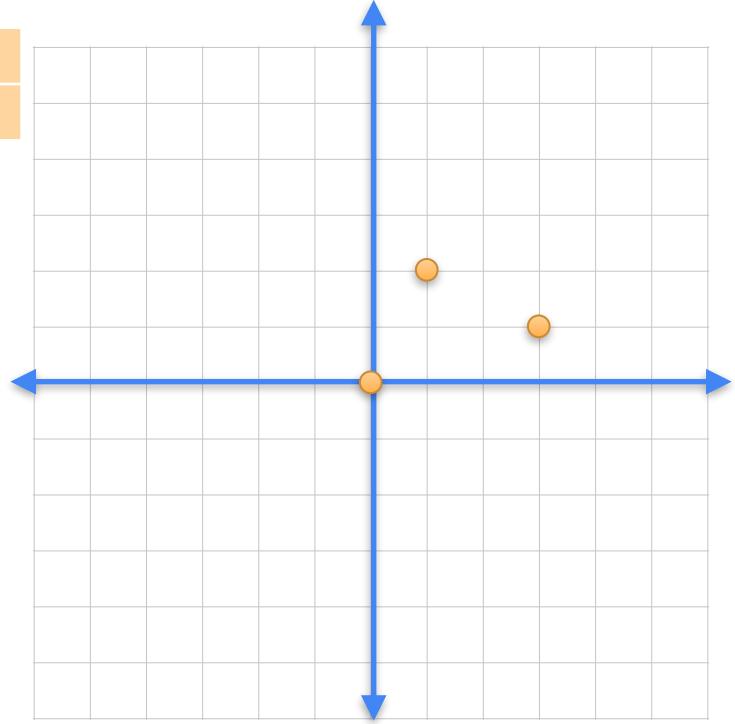


Matrices as linear transformations

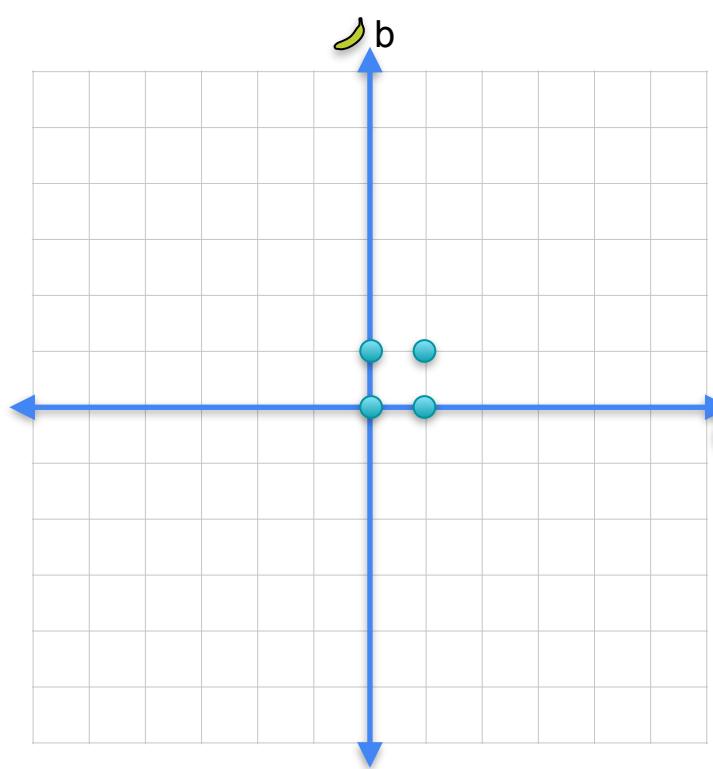


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \\ = & \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$

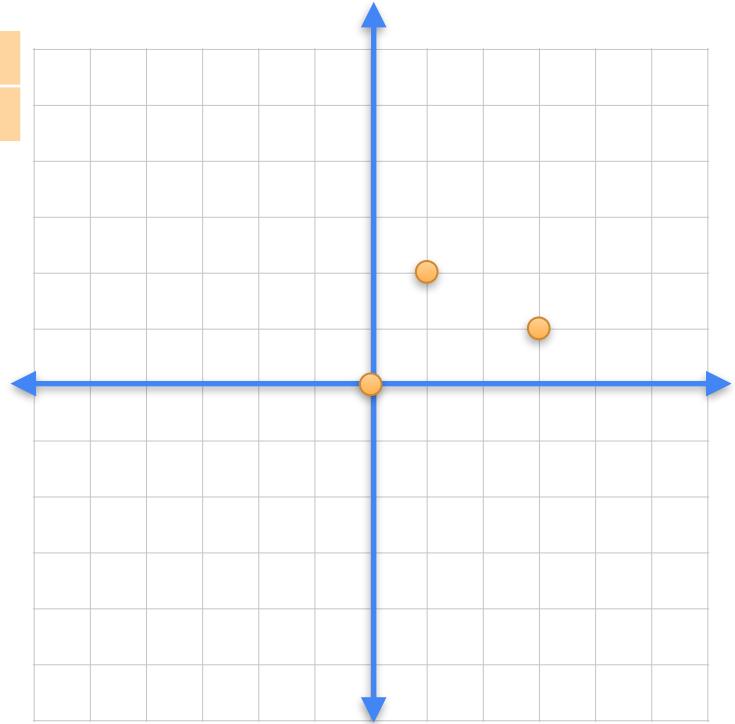


Matrices as linear transformations

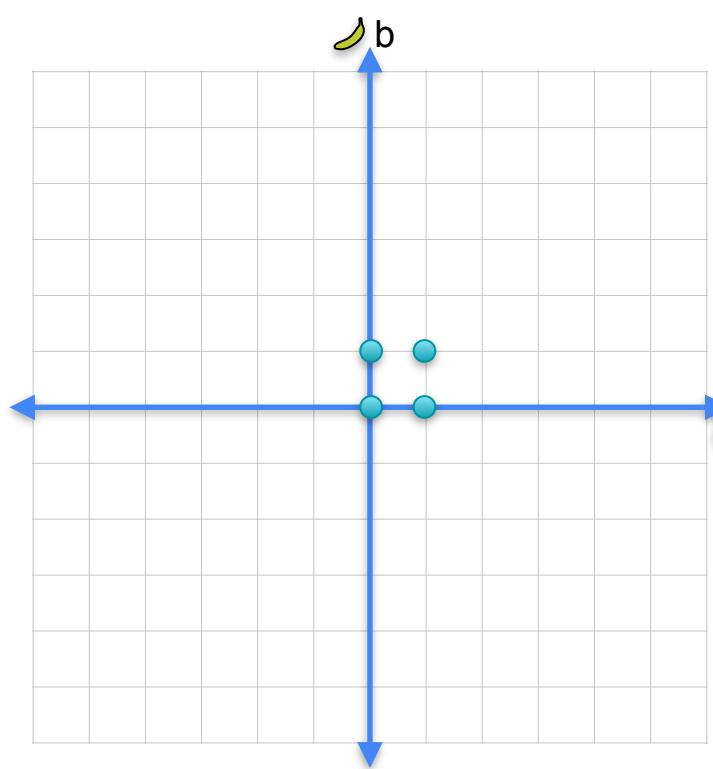


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

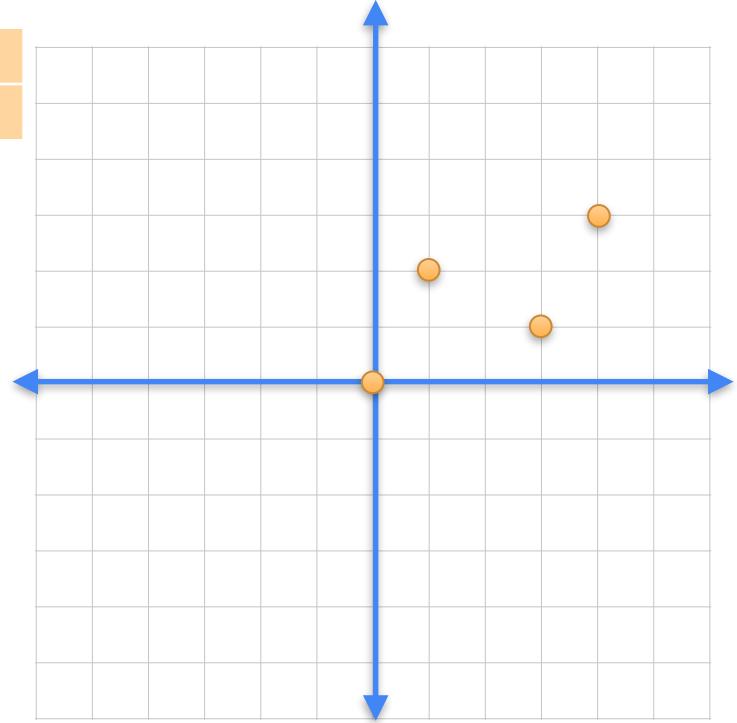


Matrices as linear transformations

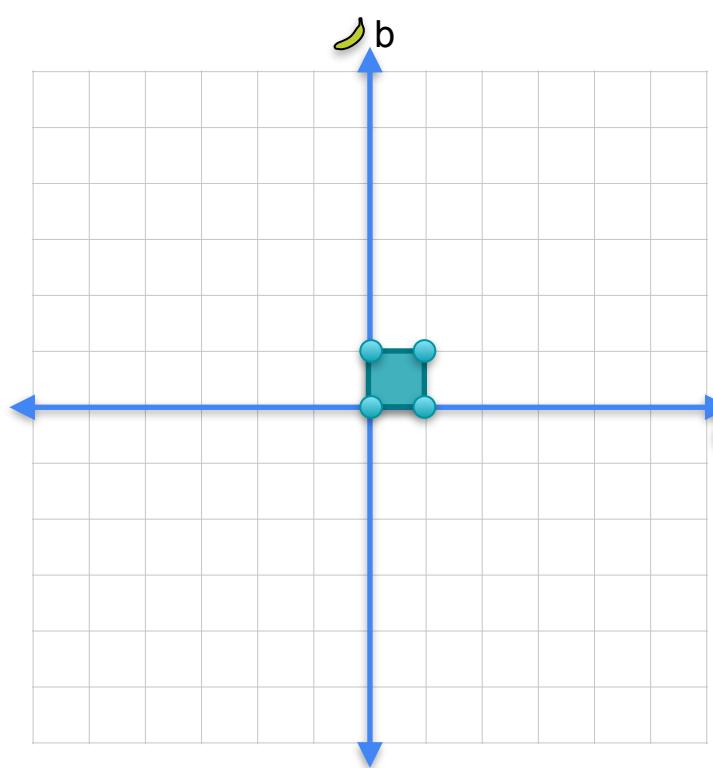


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

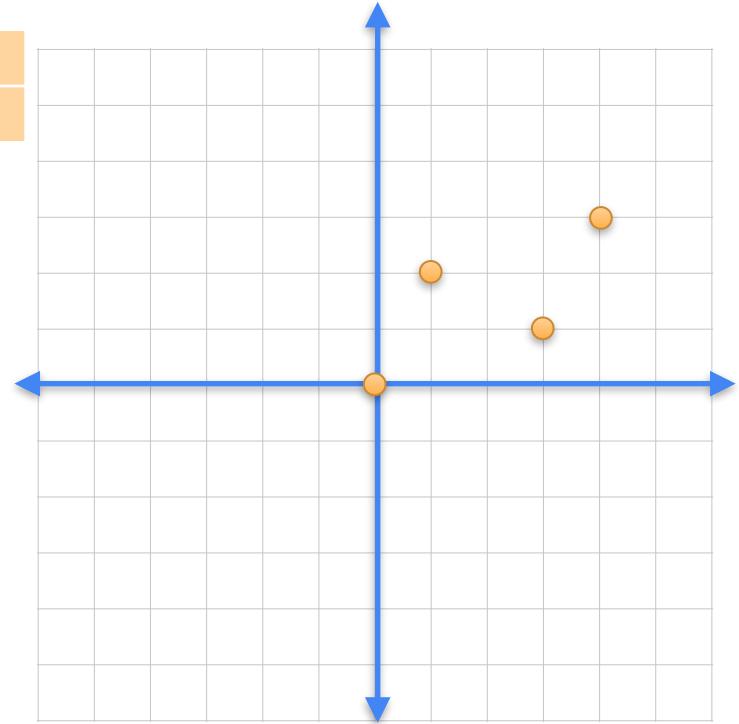


Matrices as linear transformations

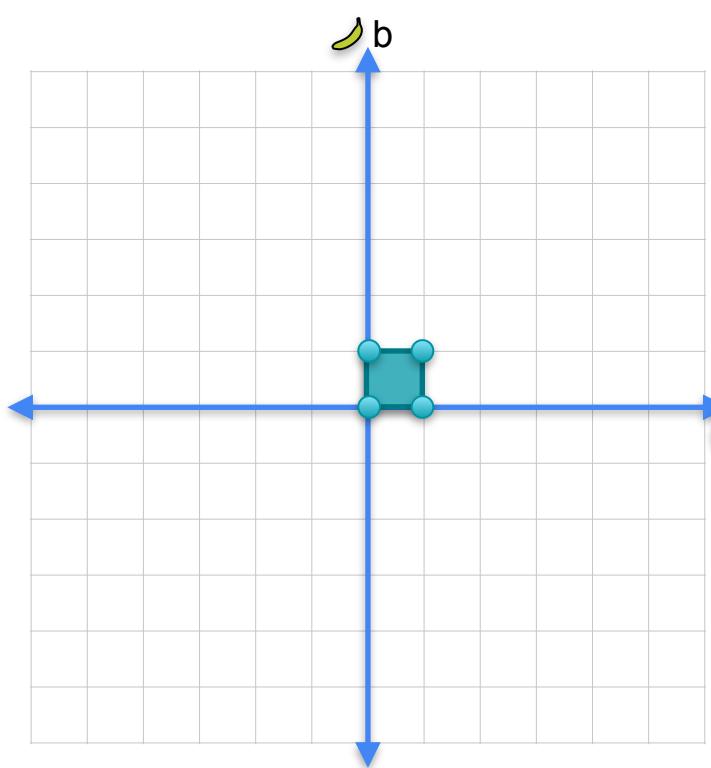


$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|c|} \hline 3 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 & \\ \hline 3 & \\ \hline \end{array} \end{array}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

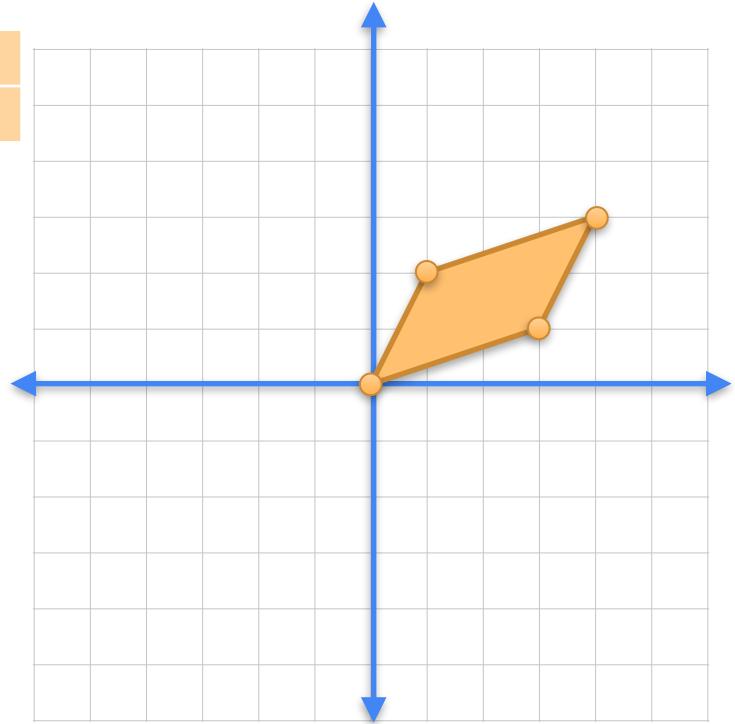


Matrices as linear transformations

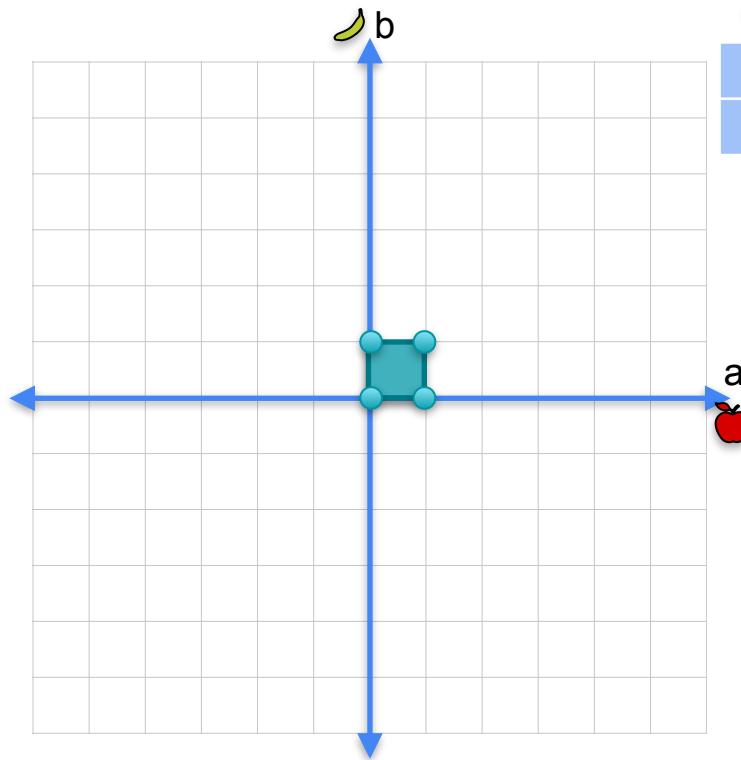


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

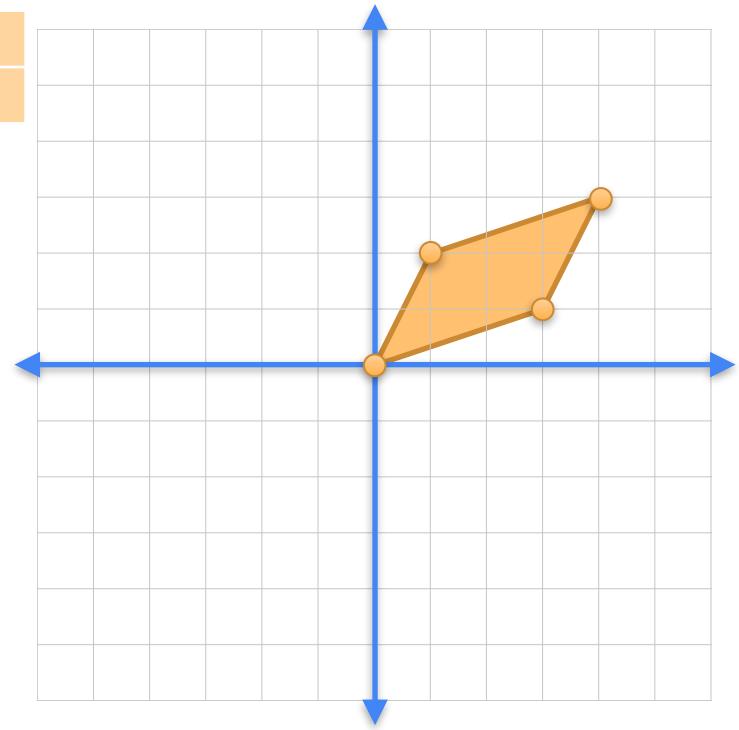
$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (4,3) \end{aligned}$$



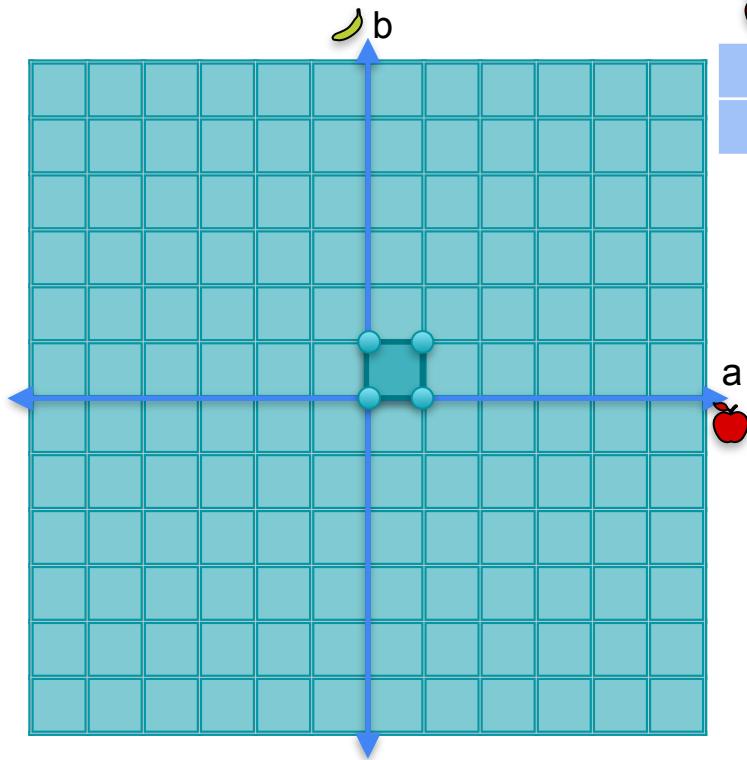
Matrices as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} & = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$

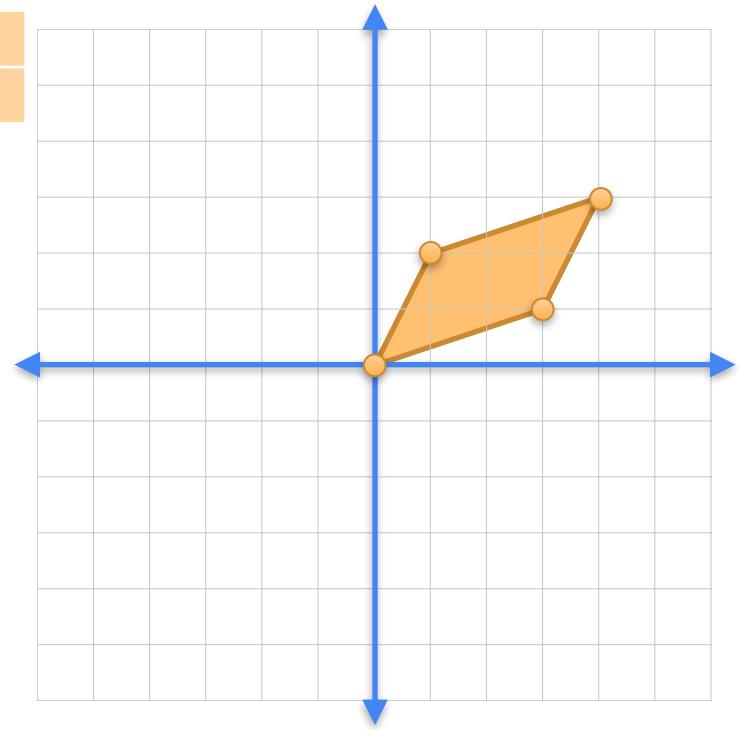


Matrices as linear transformations

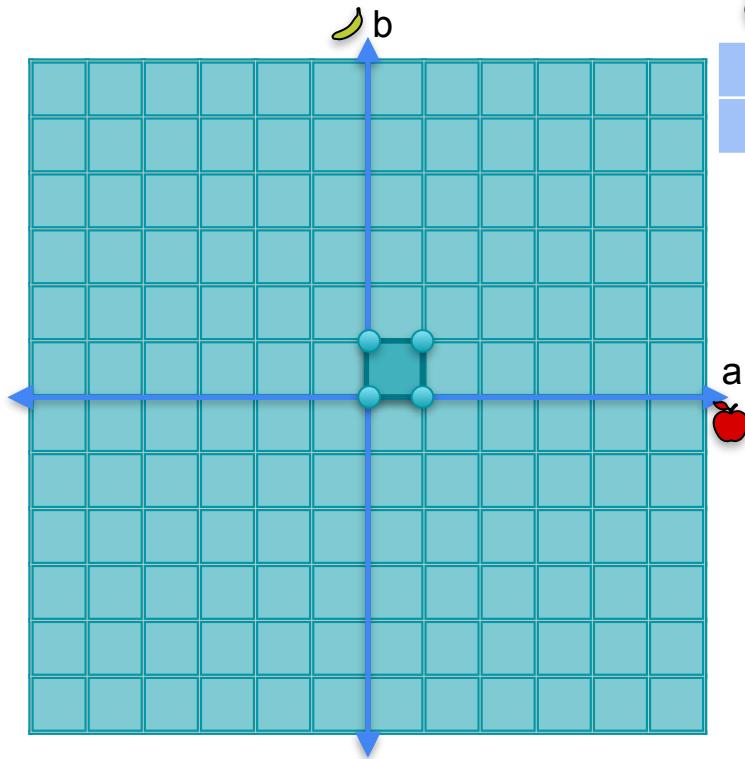


A diagram showing two fruit icons (an apple and a banana) above a mathematical equation. The equation is:

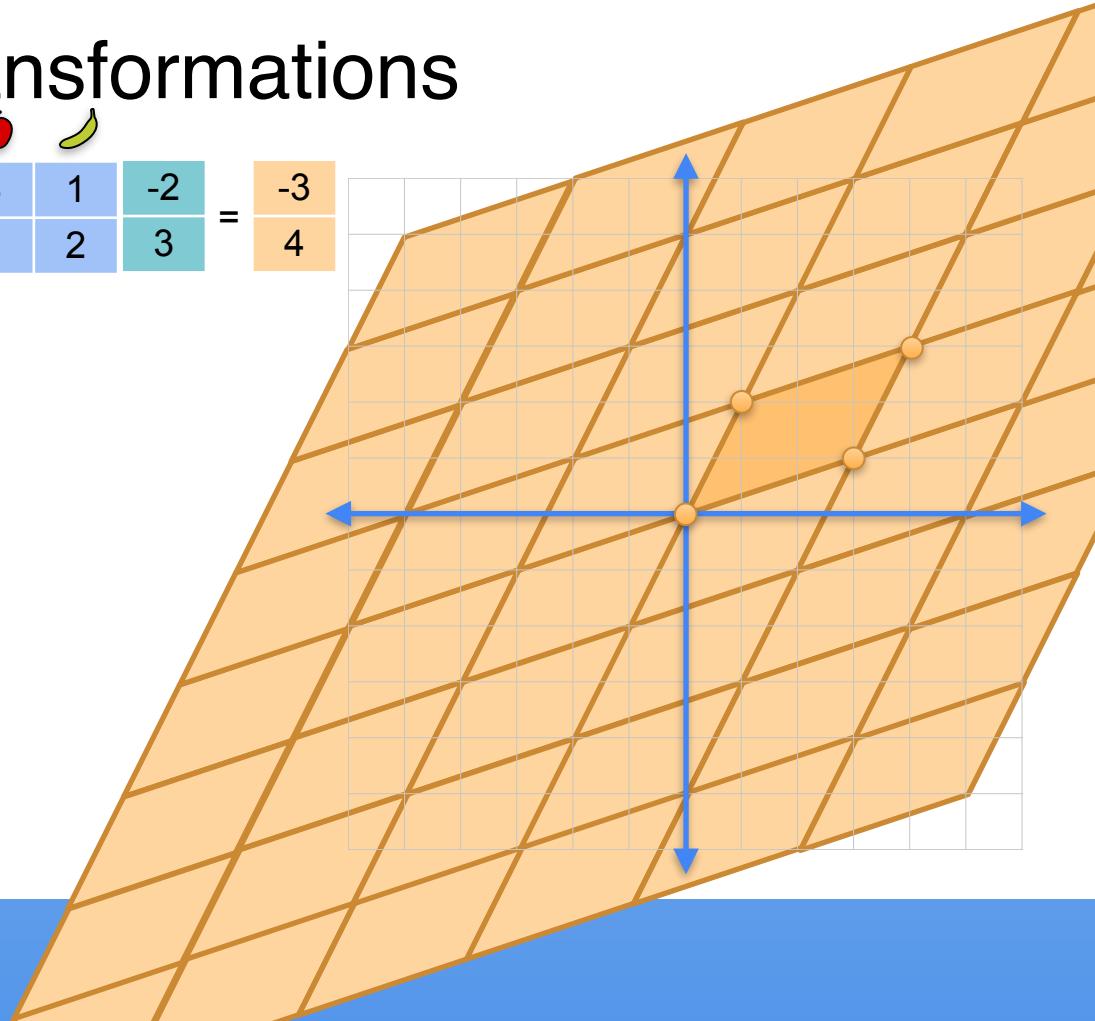
$$\begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix}$$



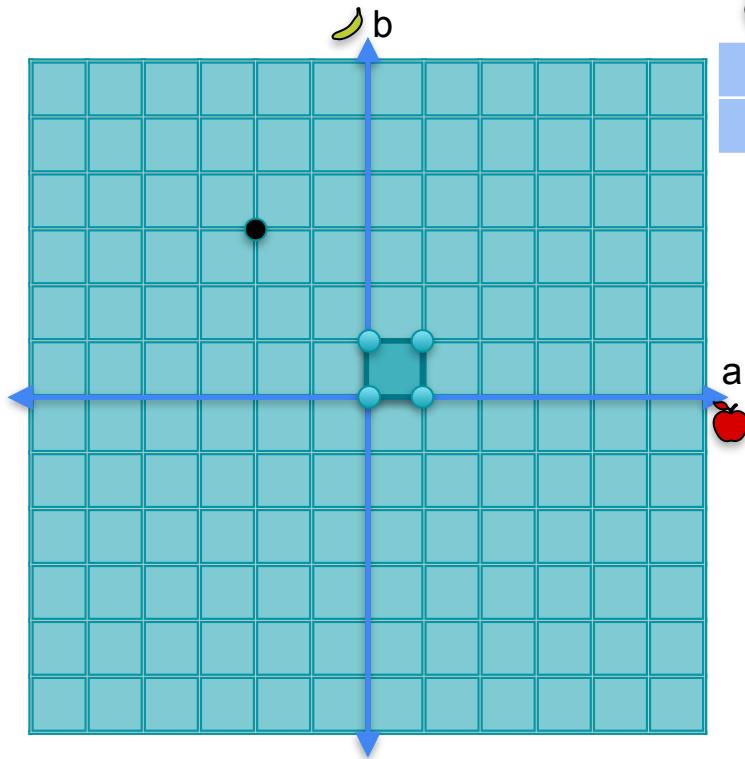
Matrices as linear transformations



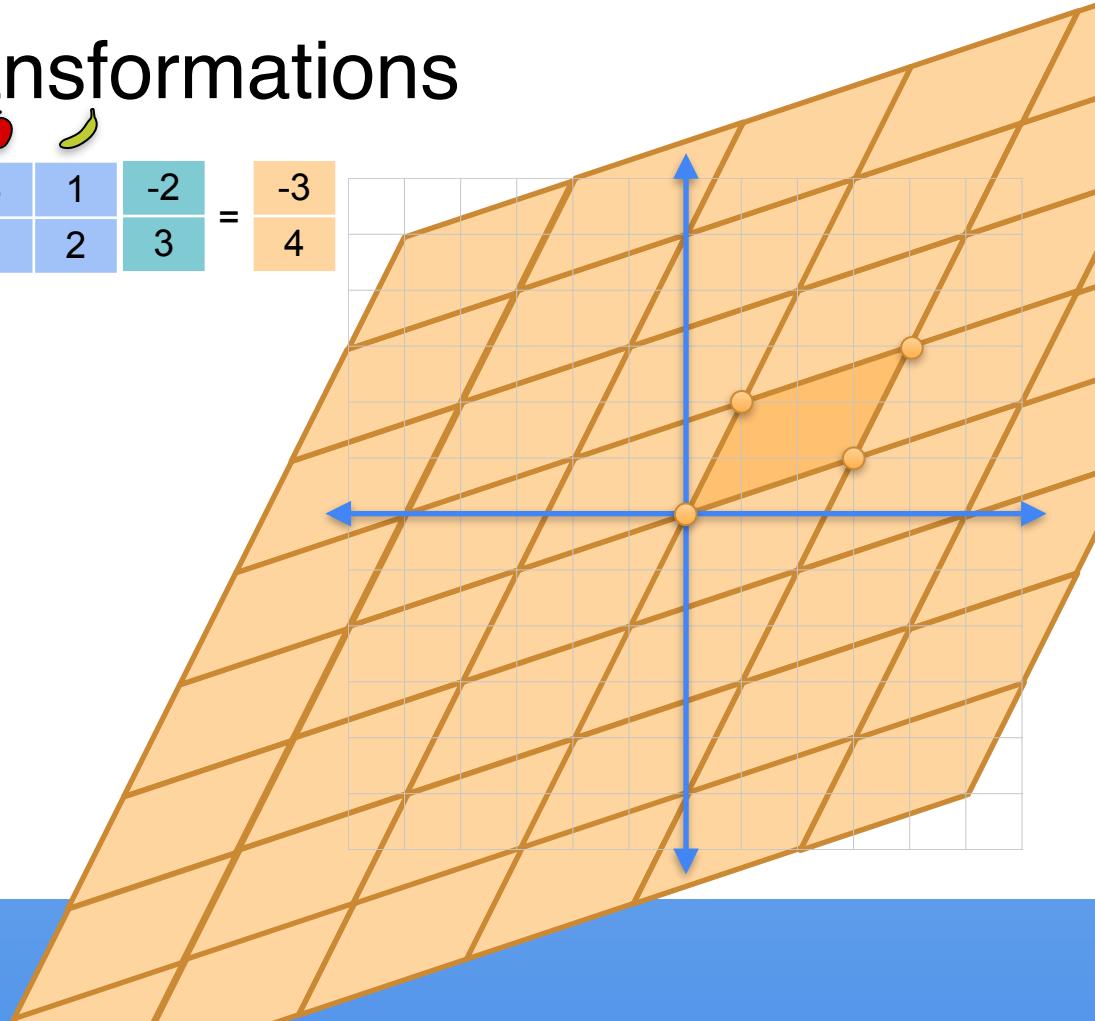
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} & = \begin{matrix} -3 \\ 4 \end{matrix} \end{matrix}$$



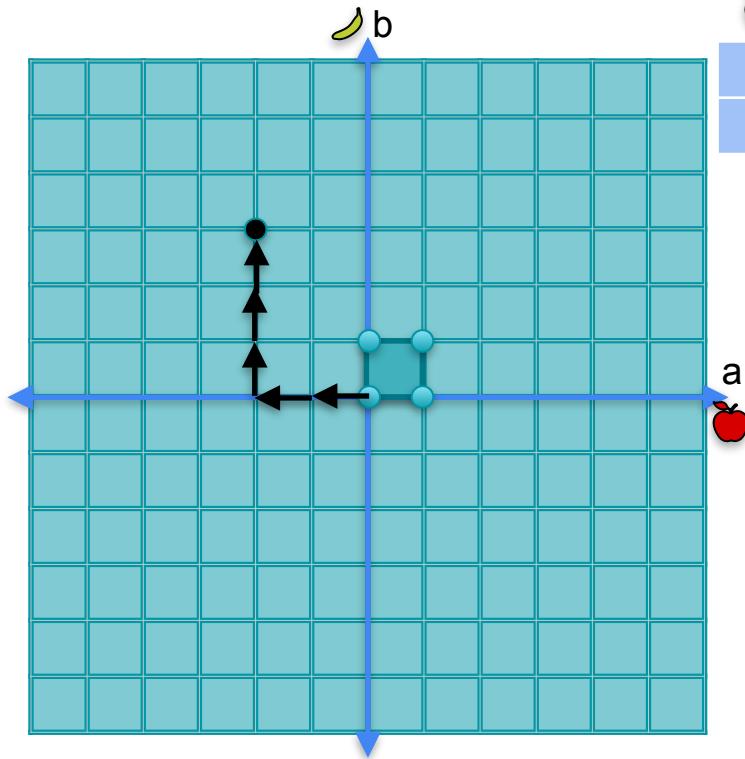
Matrices as linear transformations



$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} \times \begin{matrix} -2 & -3 \\ 3 & 4 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix}$$

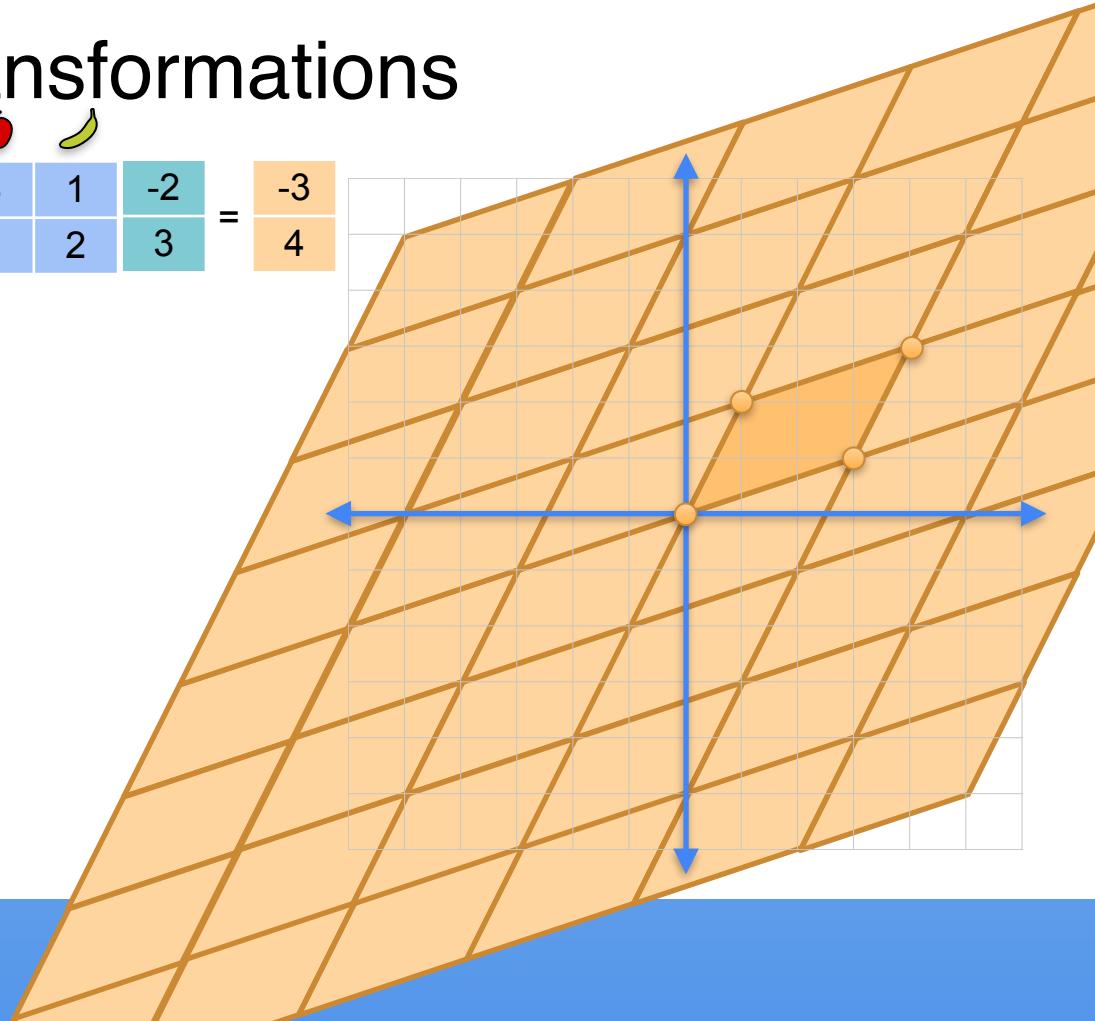


Matrices as linear transformations

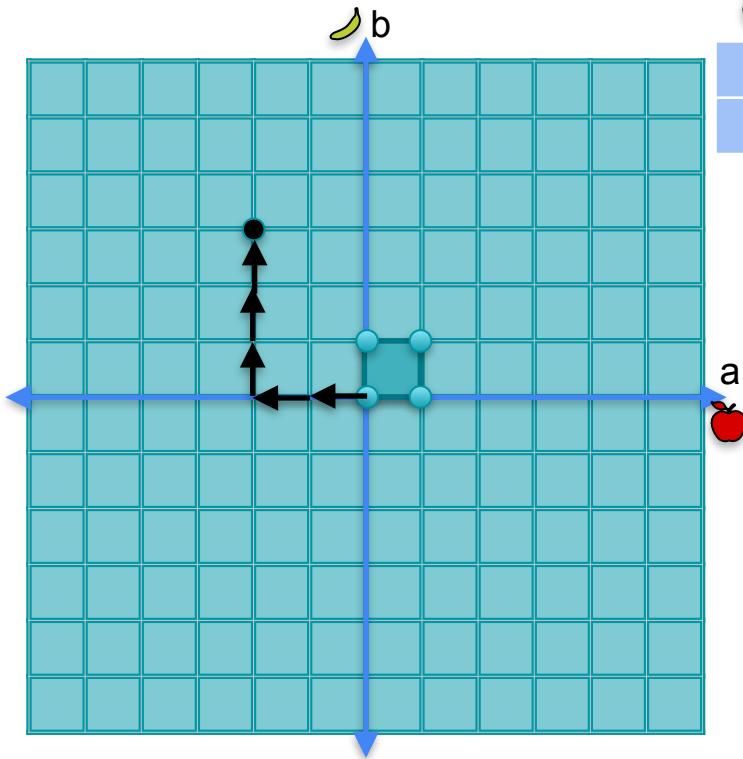


A diagram showing a red apple and a yellow banana above a matrix multiplication equation. The equation is:

$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

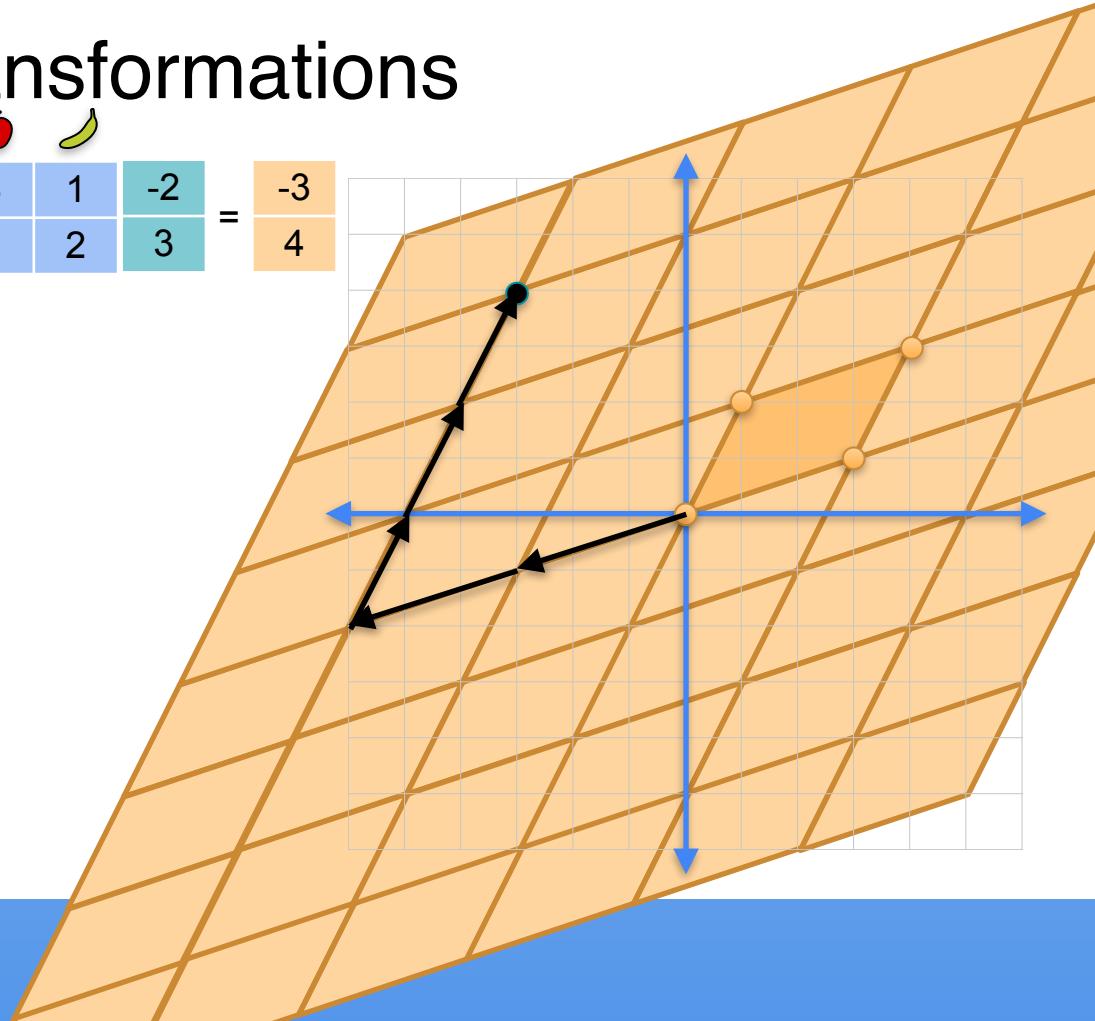


Matrices as linear transformations

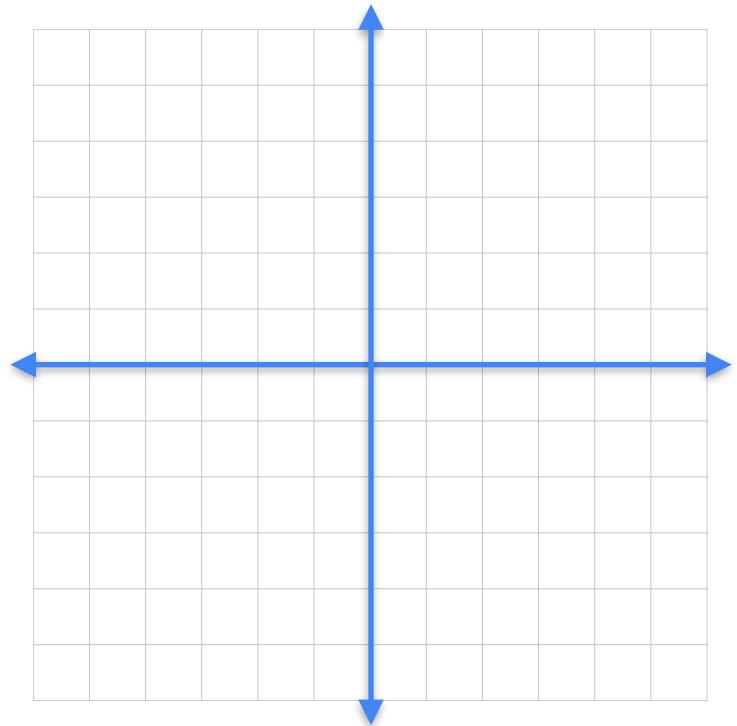
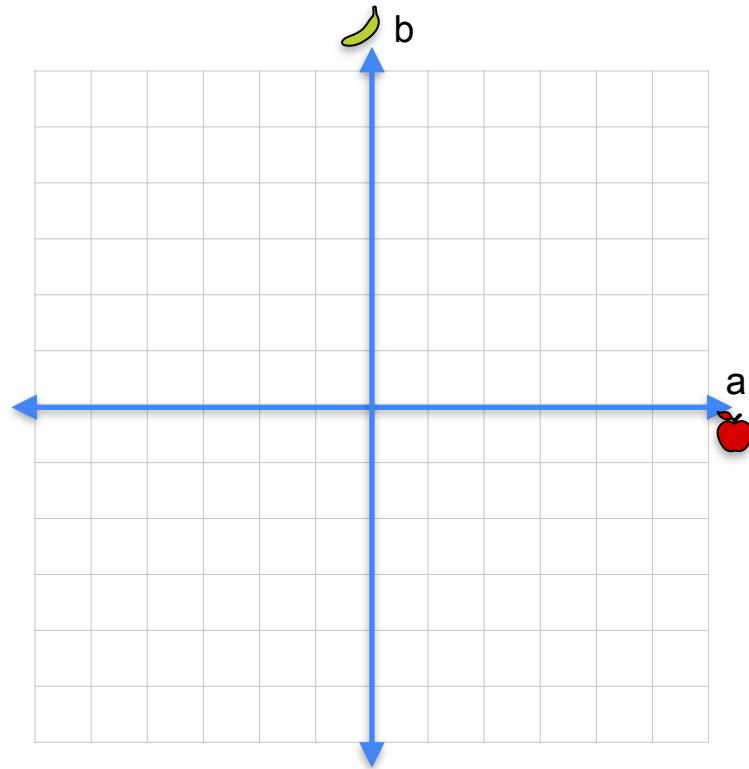


A diagram showing a red apple and a yellow banana above a matrix multiplication equation. The equation is:

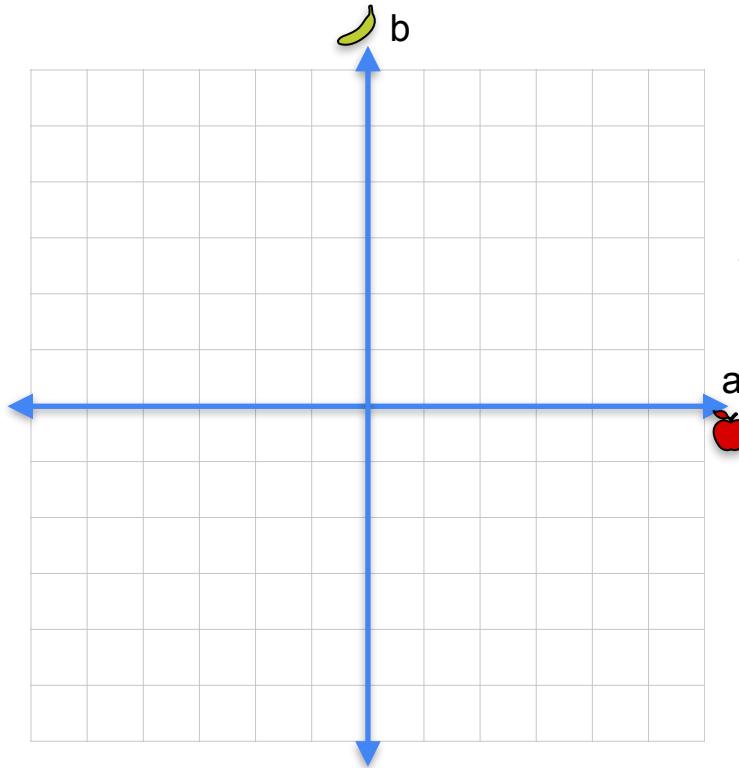
$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



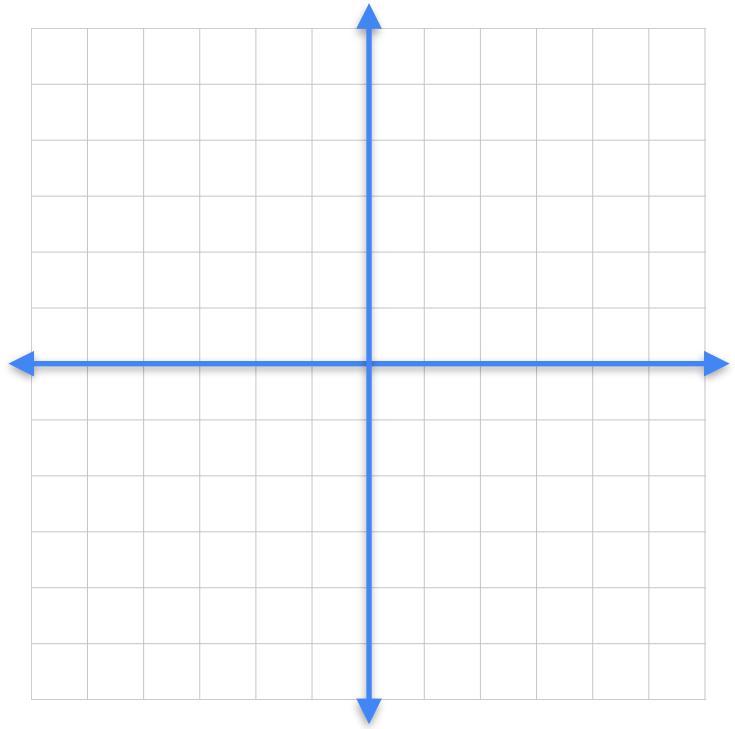
Systems of equations as linear transformations



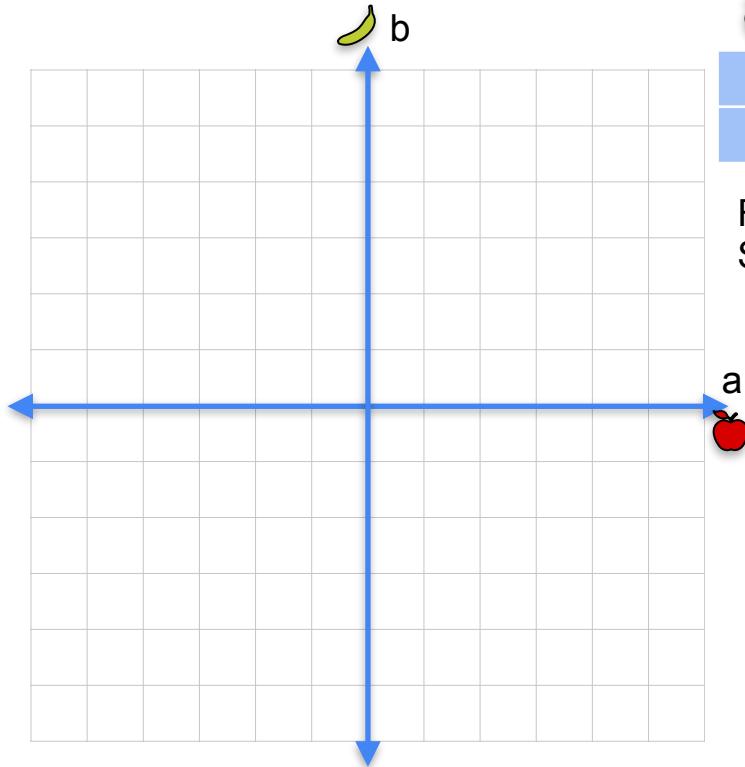
Systems of equations as linear transformations



First day: $3a + b$
Second day: $a + 2b$

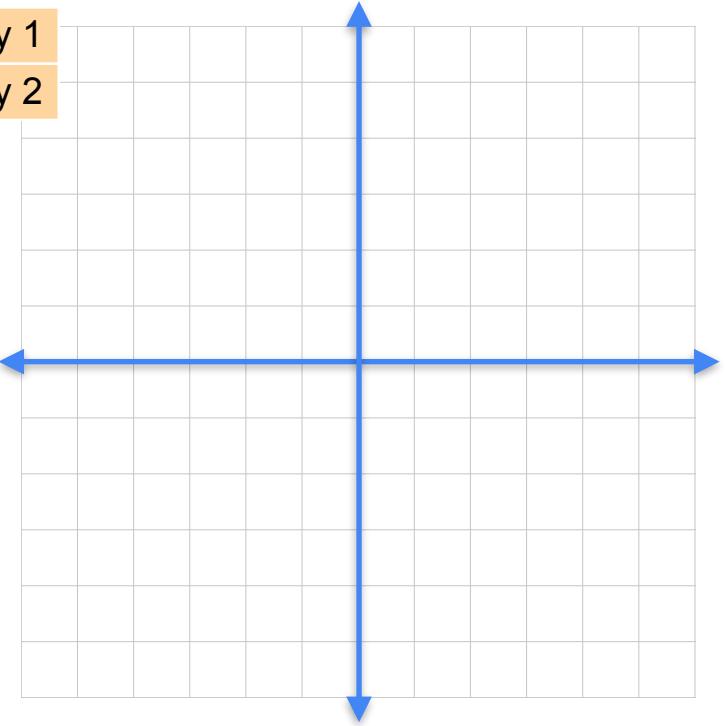


Systems of equations as linear transformations

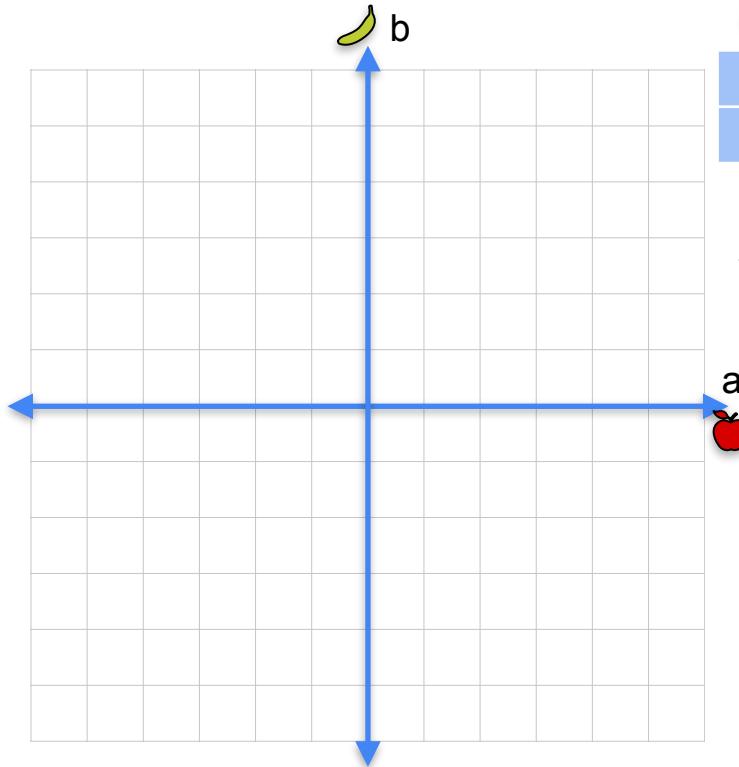


$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ b \end{matrix} = \begin{matrix} \text{Day 1} \\ \text{Day 2} \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$

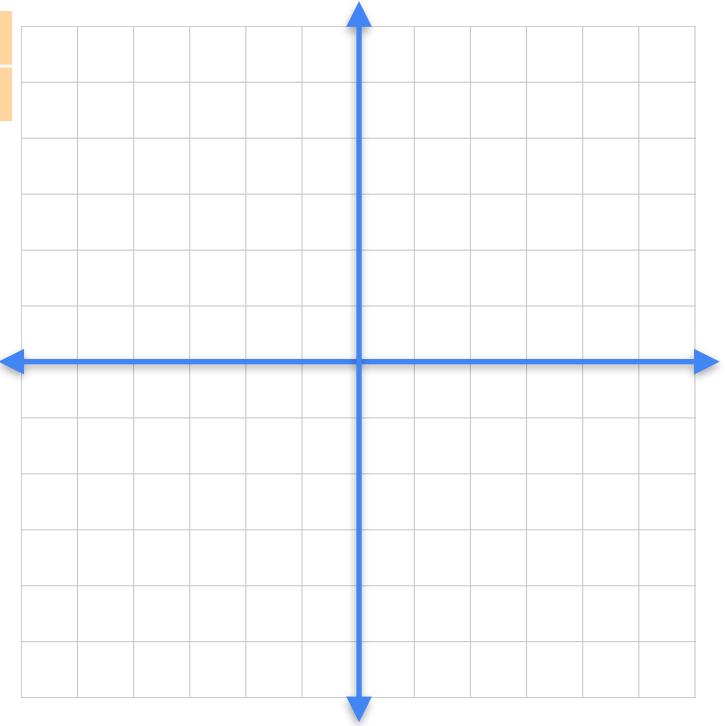


Systems of equations as linear transformations

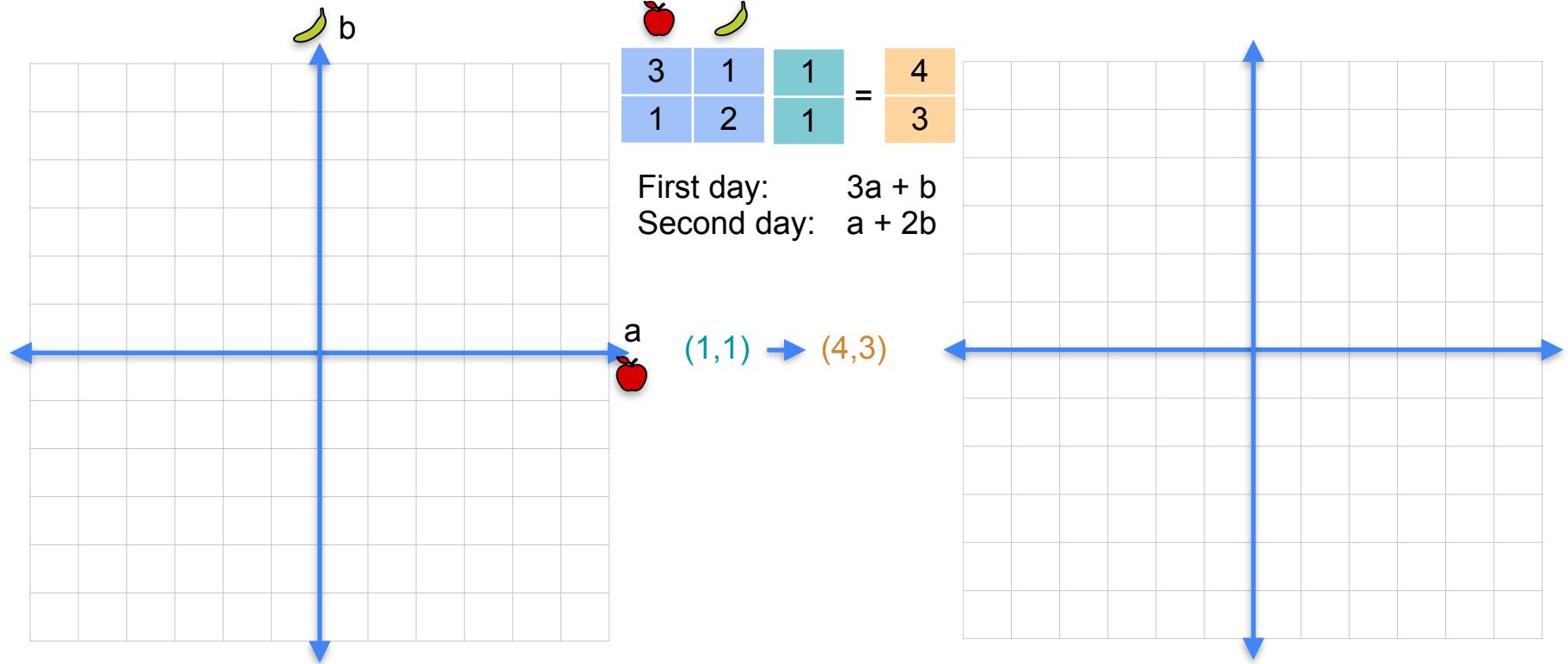


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

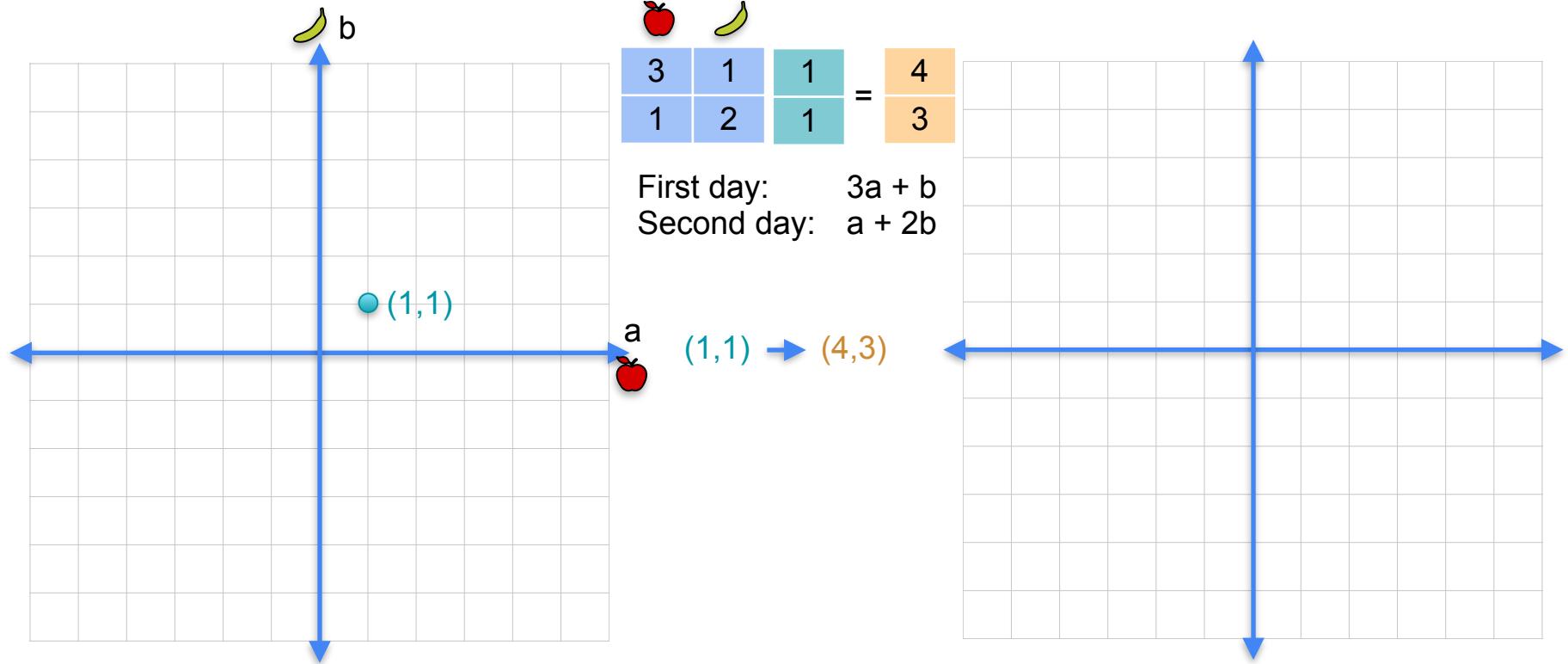
First day: $3a + b$
Second day: $a + 2b$



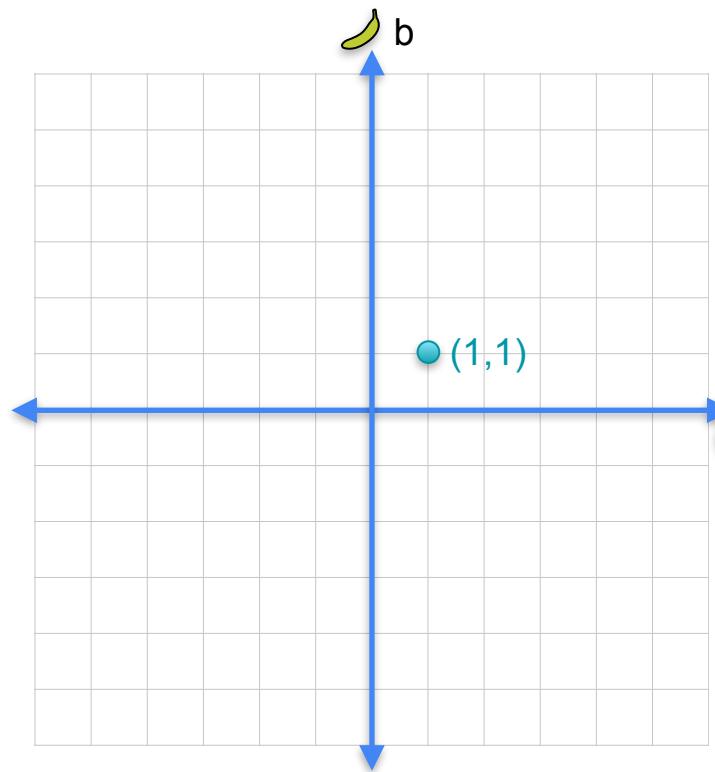
Systems of equations as linear transformations



Systems of equations as linear transformations

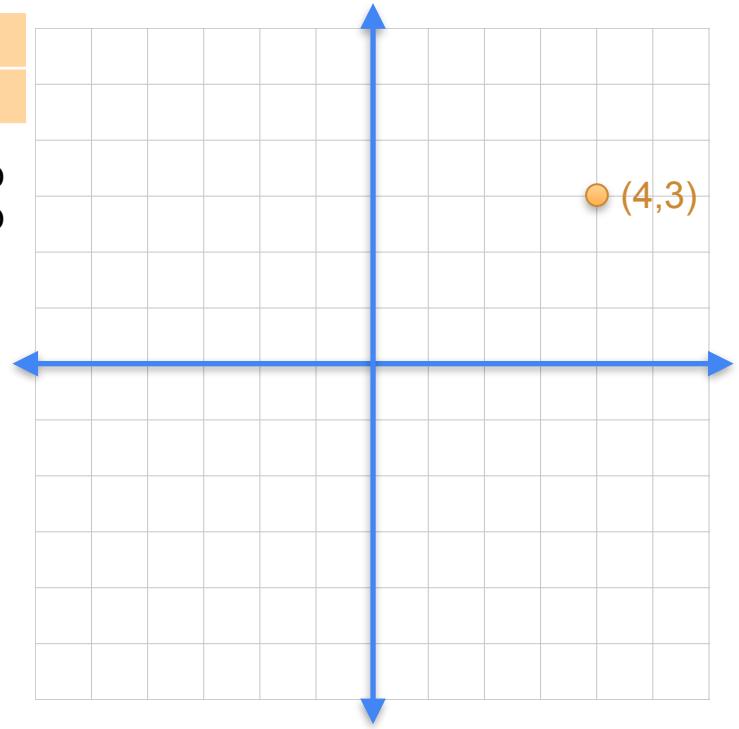


Systems of equations as linear transformations

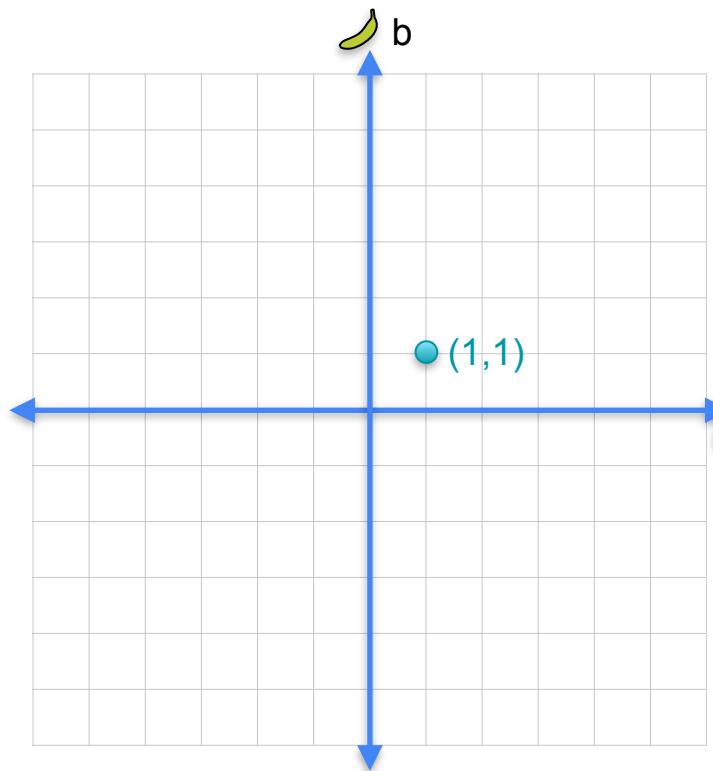


		apple	banana	
3	1			4
1	2			3

First day: $3a + b$
Second day: $a + 2b$

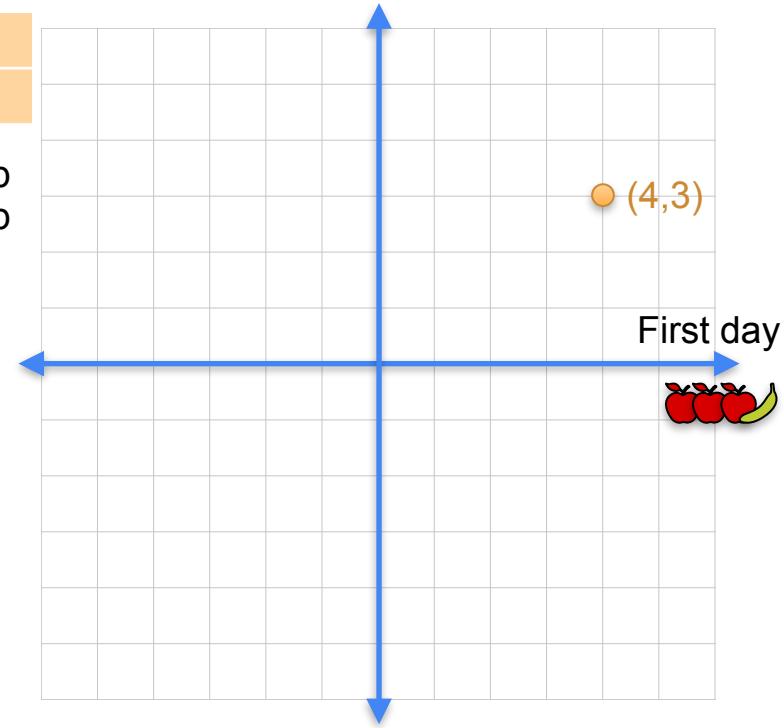


Systems of equations as linear transformations

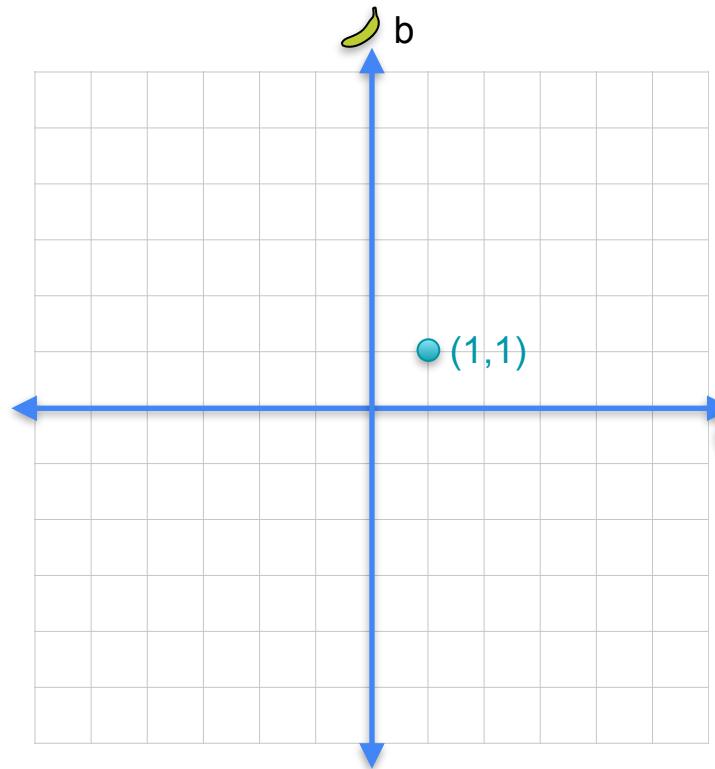


$$\begin{matrix} \text{apple} & \text{banana} \\ 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$



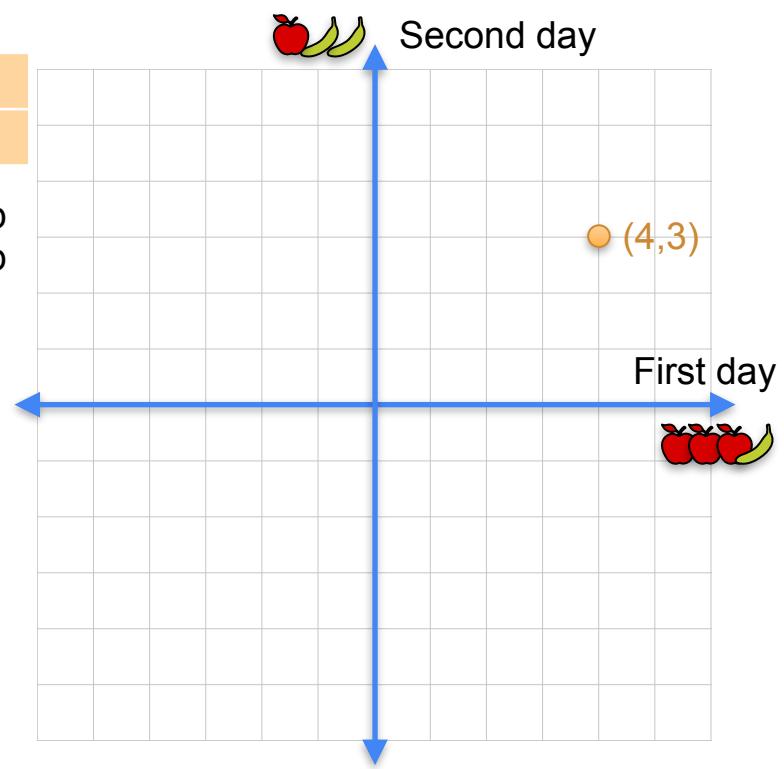
Systems of equations as linear transformations



3	1		1	=
1	2		1	

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

First day: $3a + b$
Second day: $a + 2b$



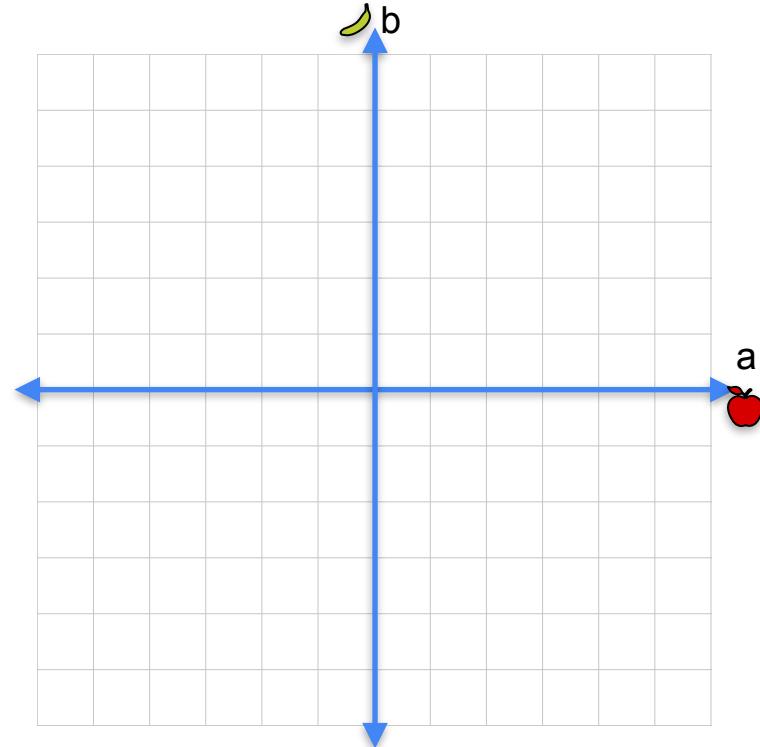
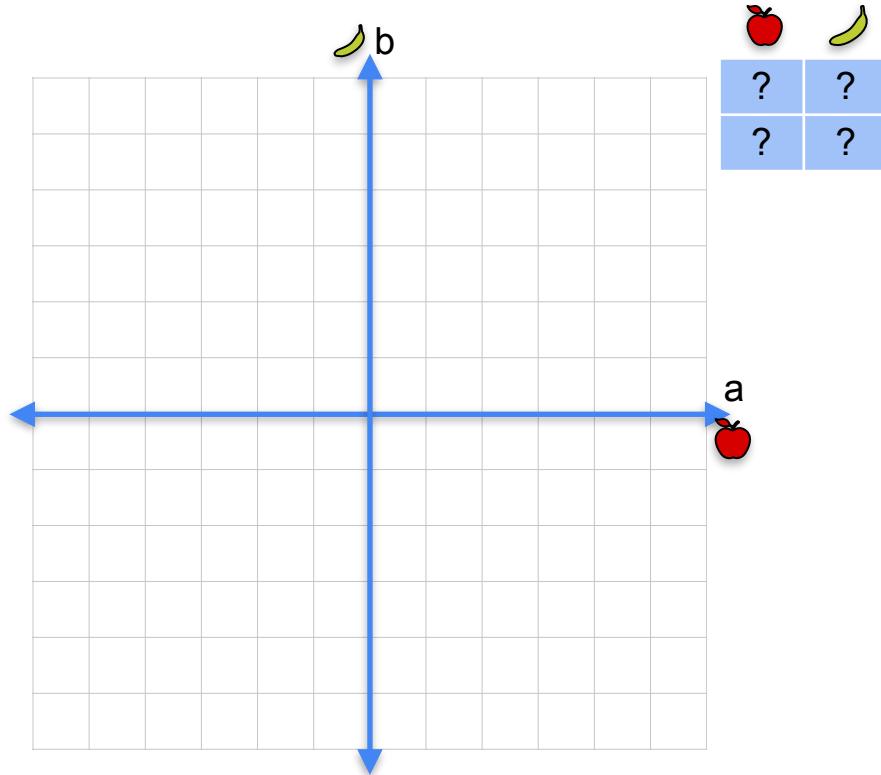


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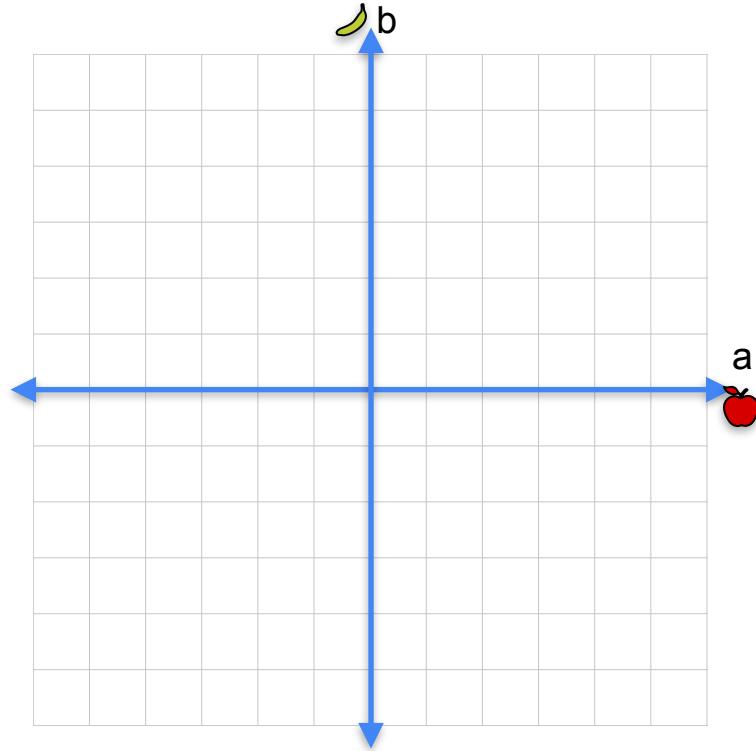
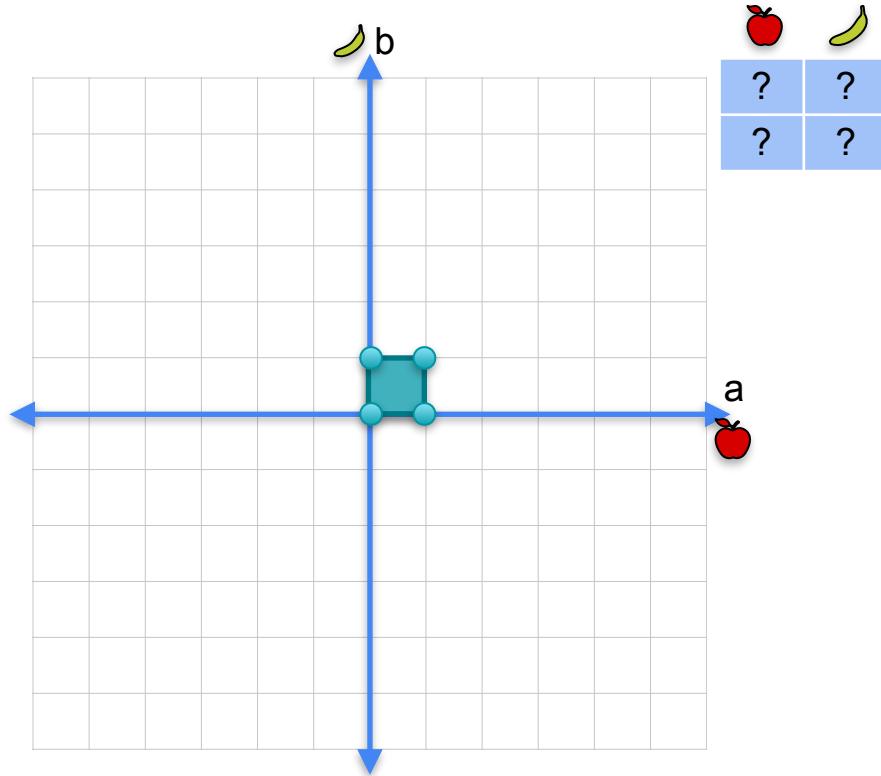
Vectors and Linear Transformations

**Linear transformations as
matrices**

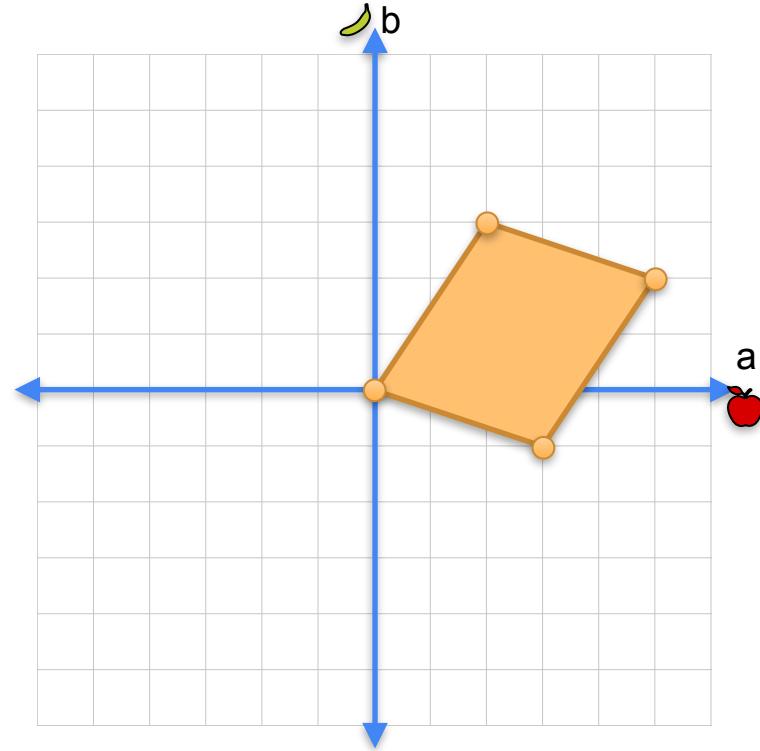
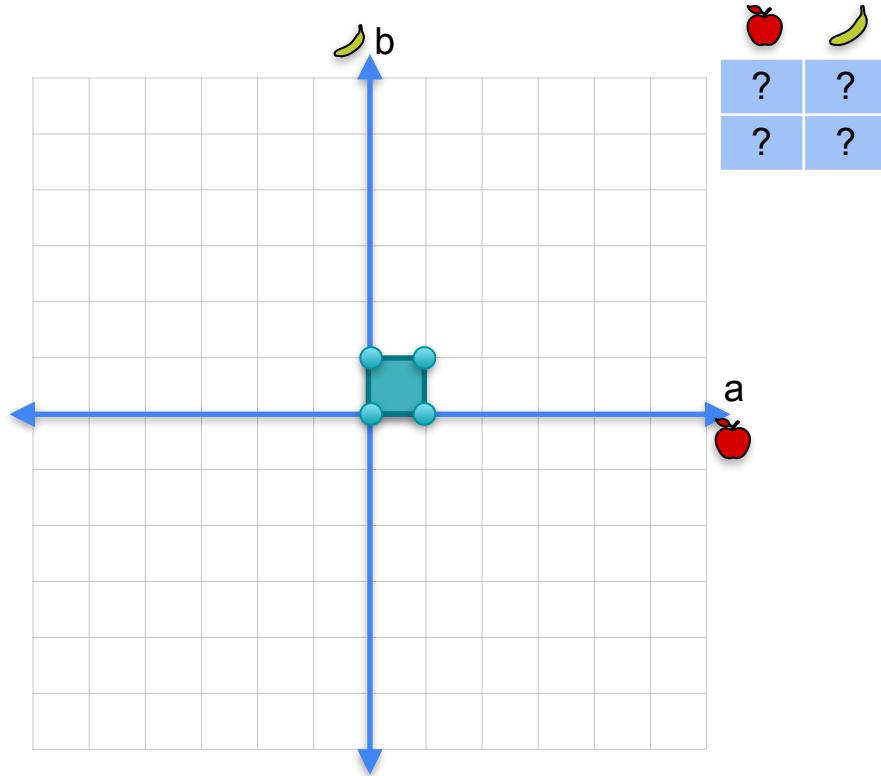
Linear transformations as matrices



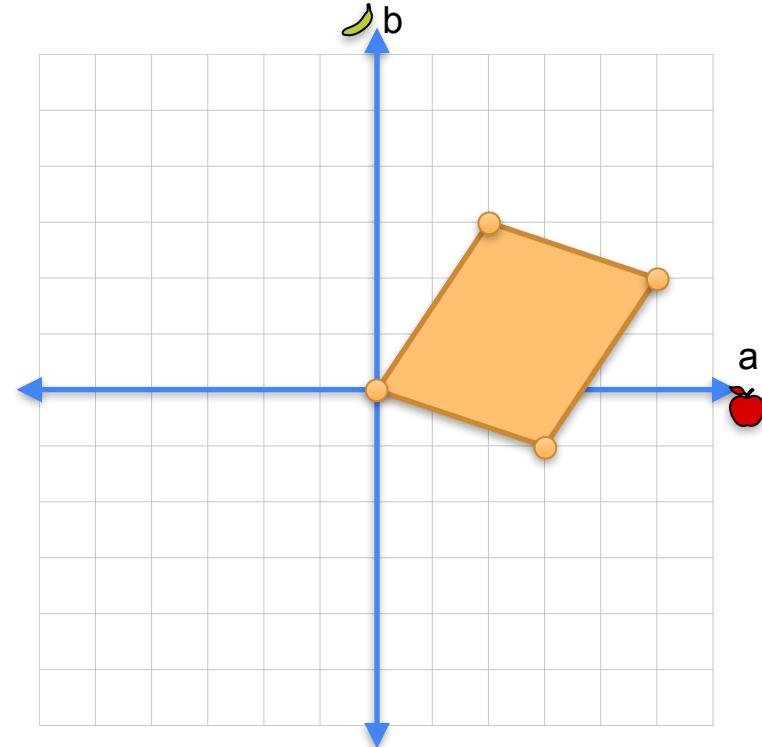
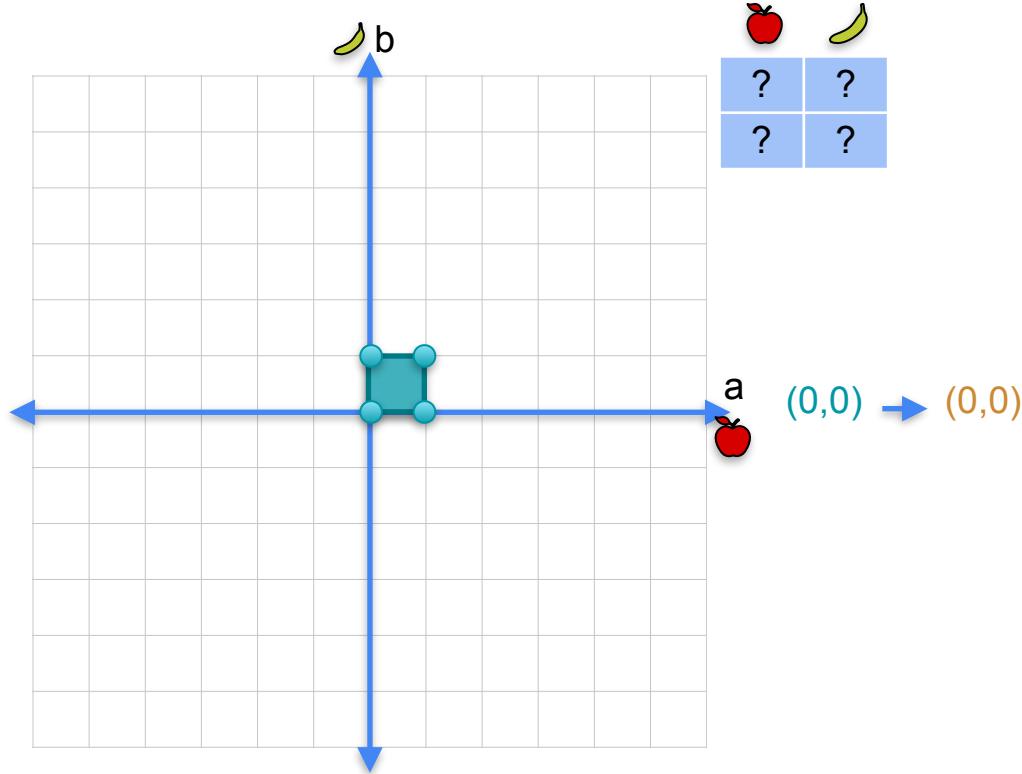
Linear transformations as matrices



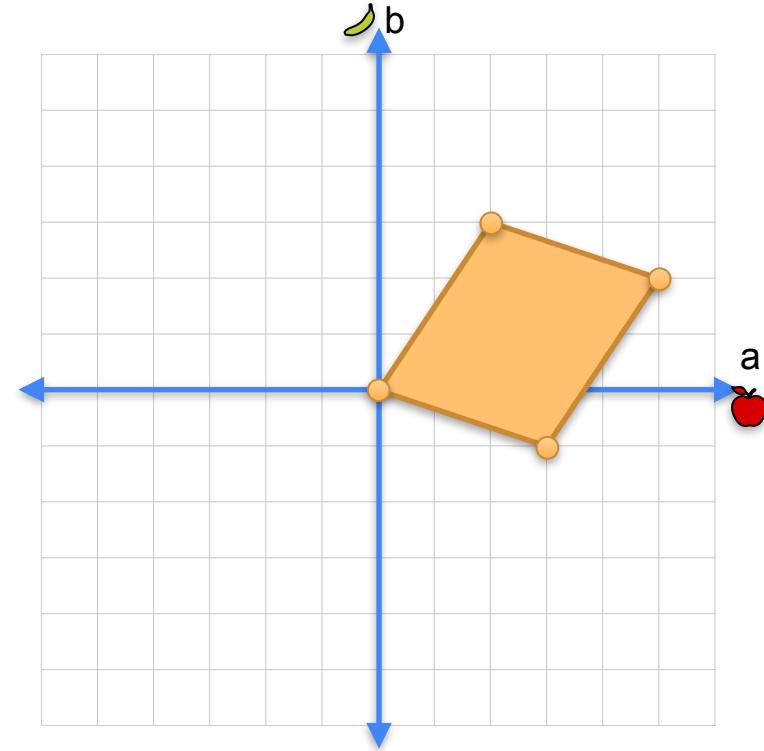
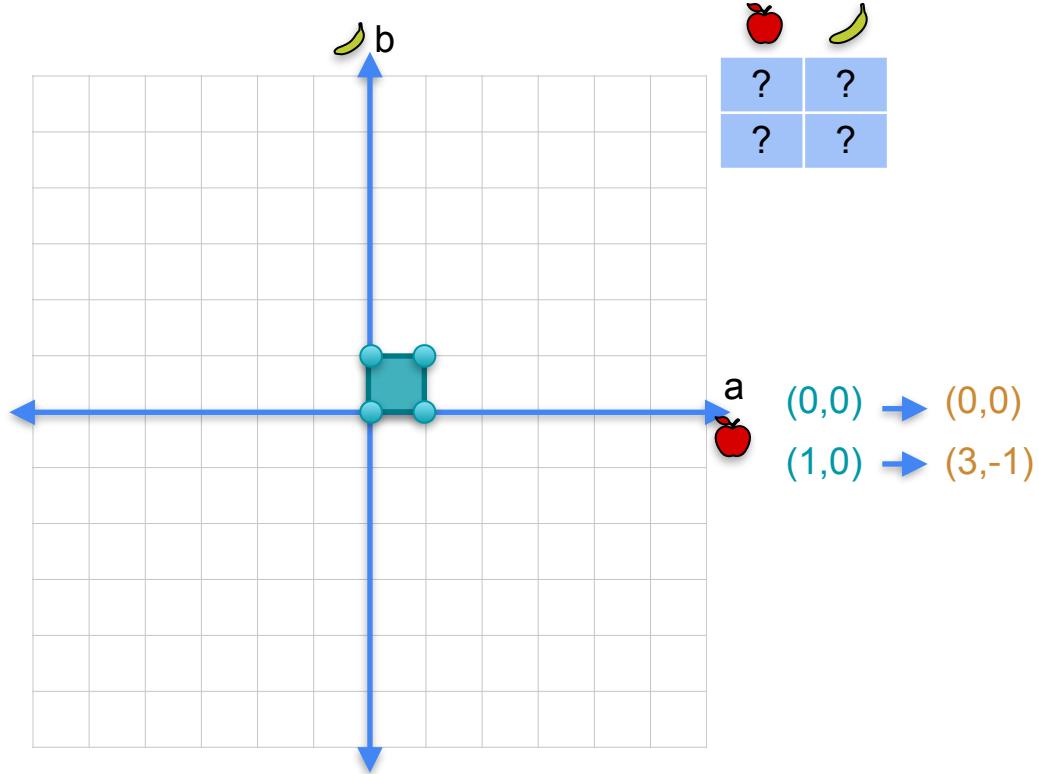
Linear transformations as matrices



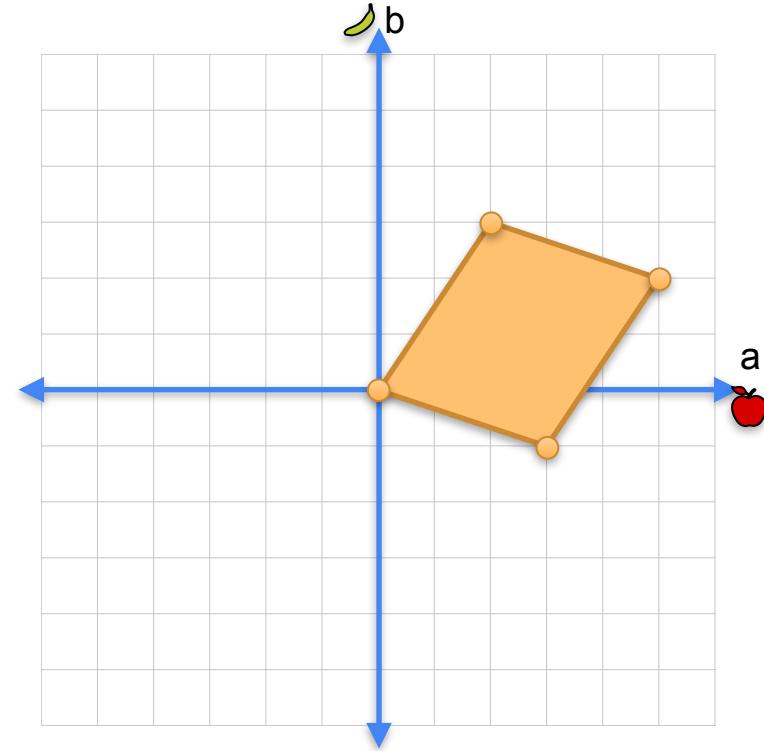
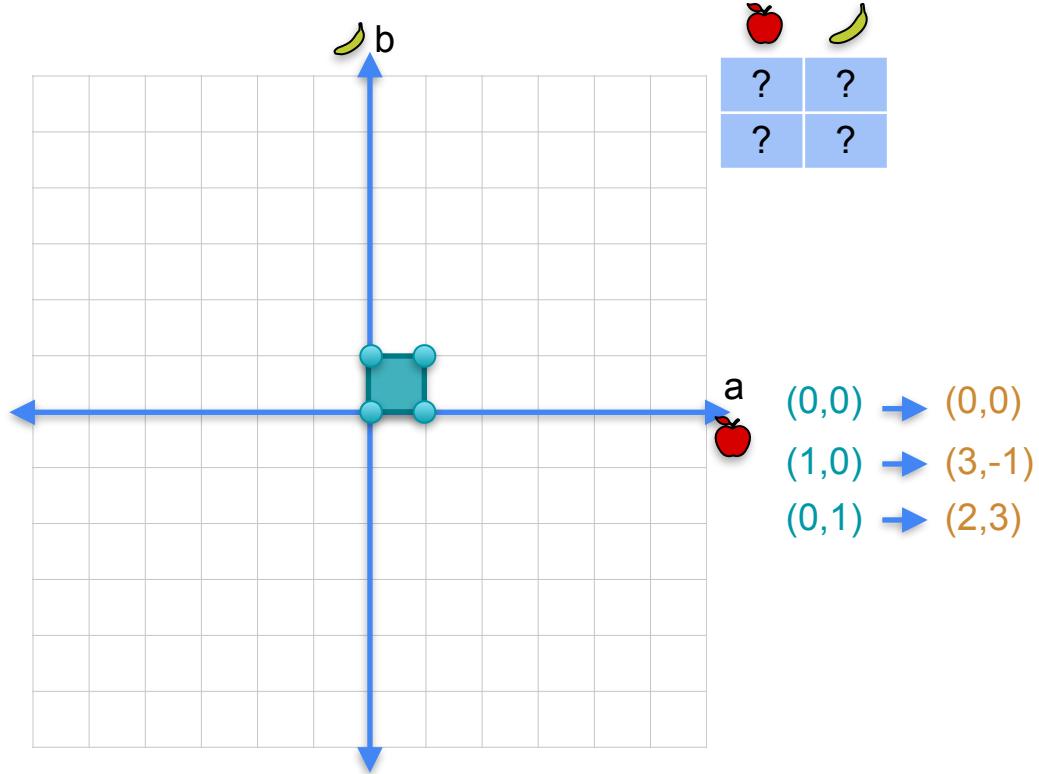
Linear transformations as matrices



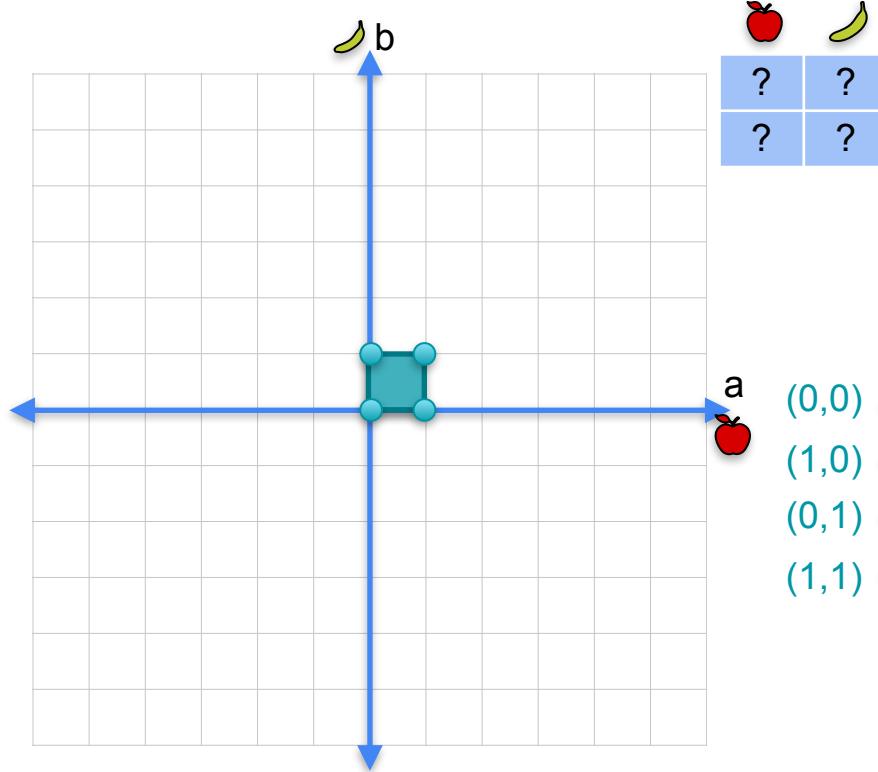
Linear transformations as matrices



Linear transformations as matrices

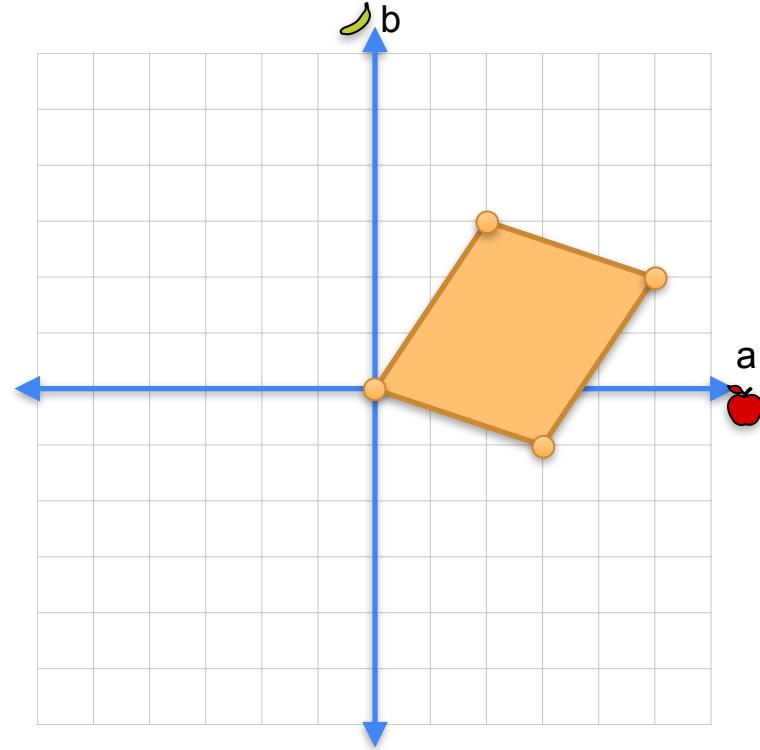


Linear transformations as matrices

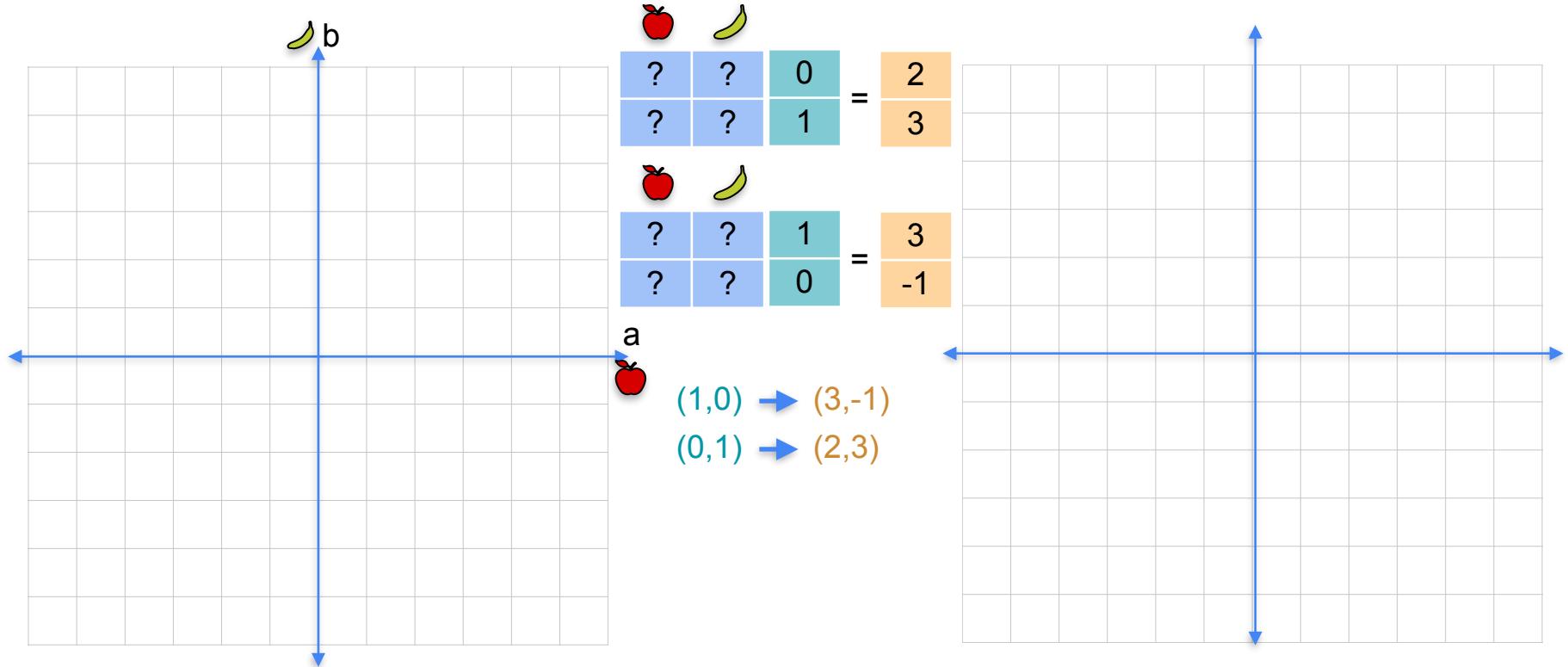


?	?
?	?

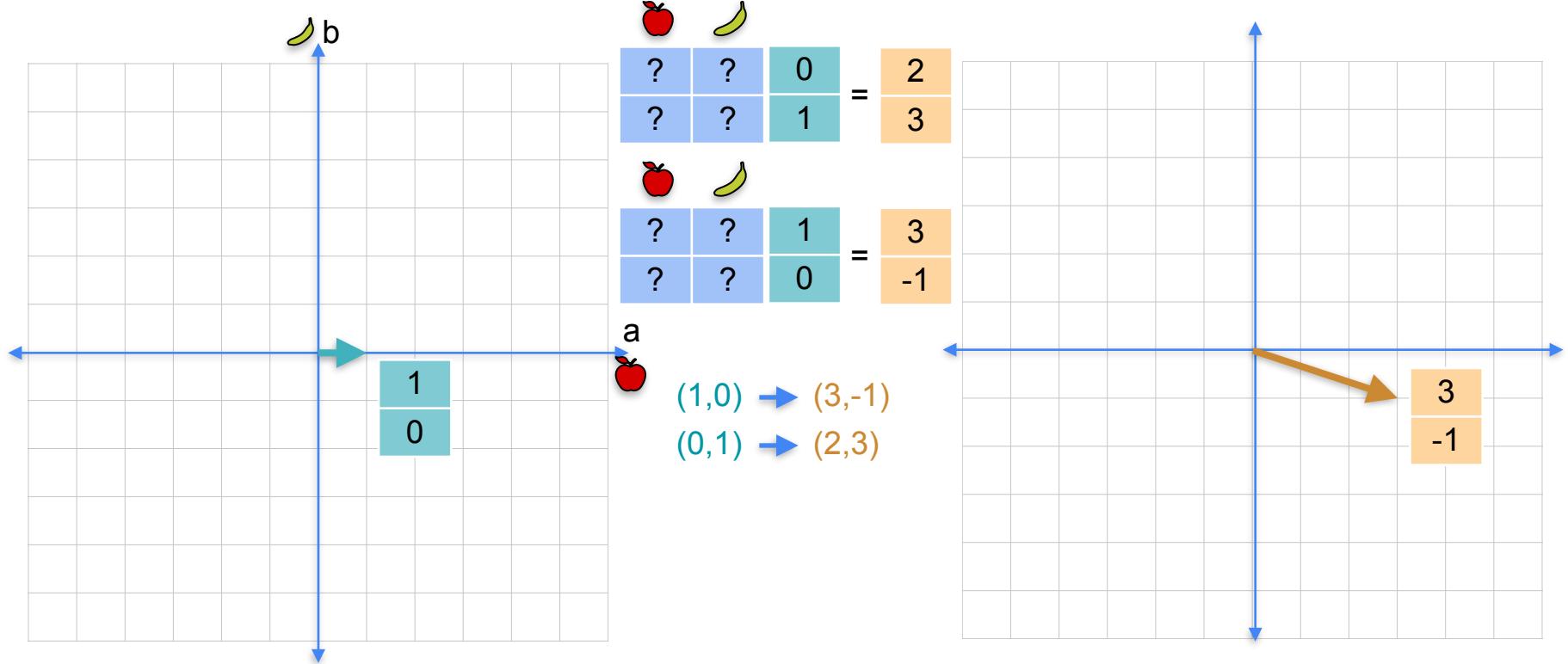
- (0,0) \rightarrow (0,0)
- (1,0) \rightarrow (3,-1)
- (0,1) \rightarrow (2,3)
- (1,1) \rightarrow (5,2)



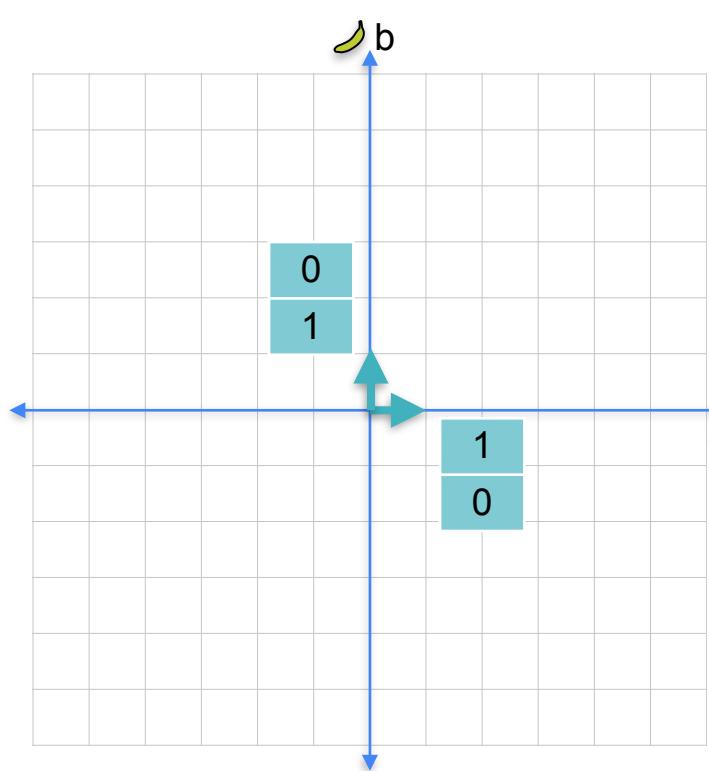
Linear transformations as matrices



Linear transformations as matrices



Linear transformations as matrices

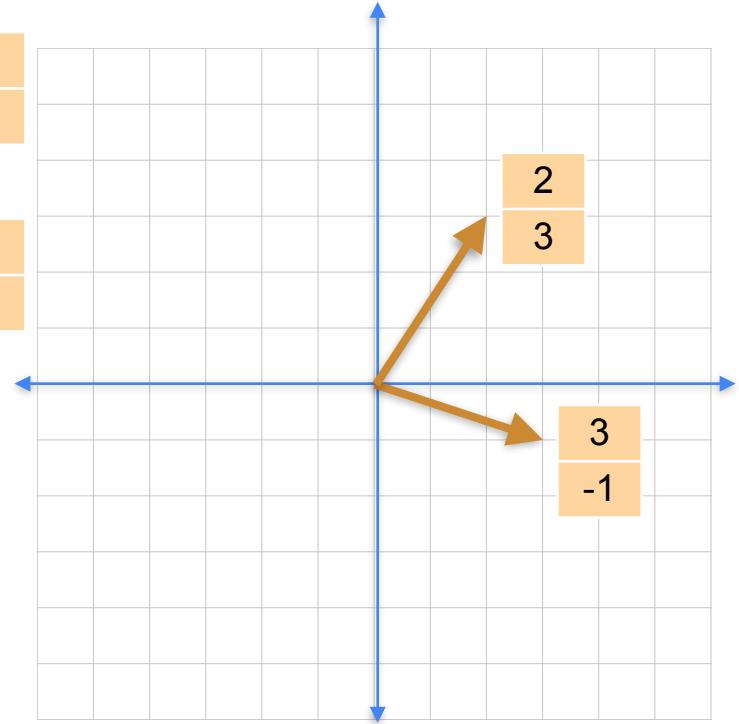


Two matrix equations illustrating the linear transformation:

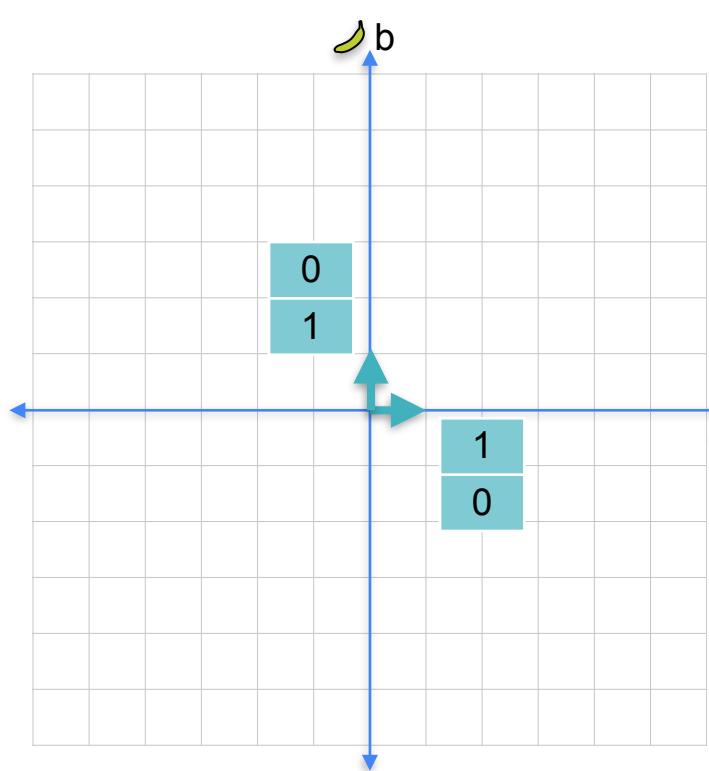
$$\begin{matrix} \text{apple} & \text{banana} \\ ? & ? \\ ? & ? \end{matrix} \begin{matrix} 0 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 3 \end{matrix}$$
$$\begin{matrix} \text{apple} & \text{banana} \\ ? & ? \\ ? & ? \end{matrix} \begin{matrix} 1 \\ 0 \end{matrix} = \begin{matrix} 3 \\ -1 \end{matrix}$$

Below the equations, the transformation of the basis vectors is summarized:

$$(1,0) \rightarrow (3,-1)$$
$$(0,1) \rightarrow (2,3)$$



Linear transformations as matrices

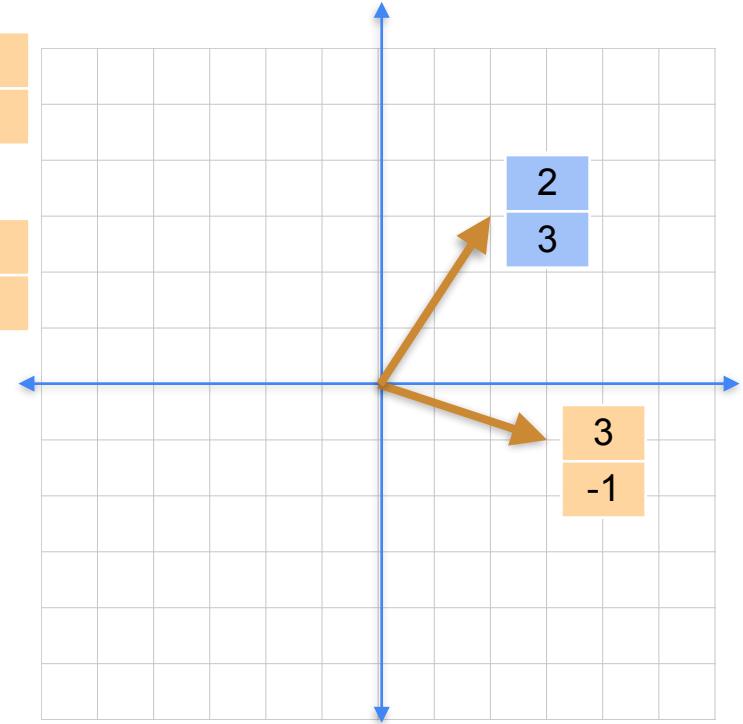


Two matrix equations illustrating the transformation of basis vectors a and b :

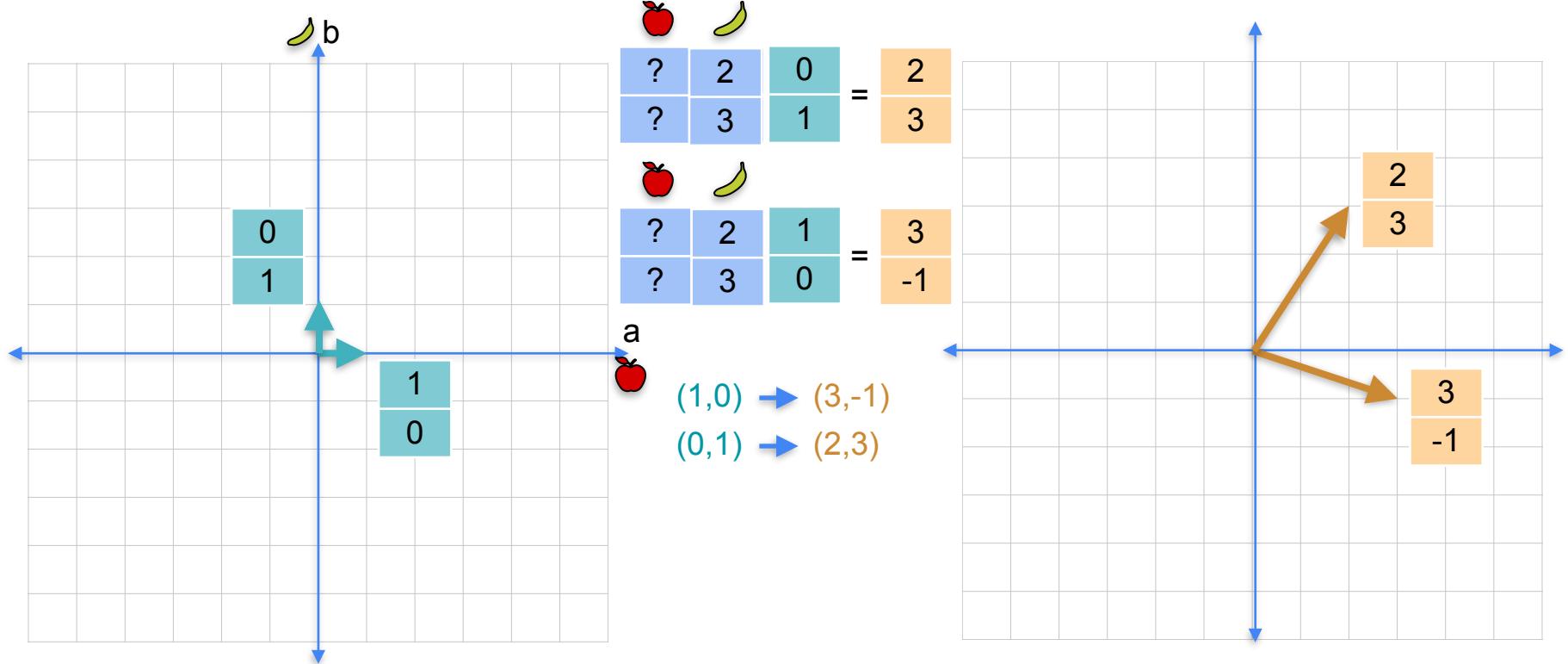
$$\begin{matrix} \text{apple} & \text{banana} \\ ? & ? & 0 & 2 \\ ? & ? & 1 & 3 \end{matrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$
$$\begin{matrix} \text{apple} & \text{banana} \\ ? & ? & 1 & 3 \\ ? & ? & 0 & -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Below the equations, the transformed vectors are listed:

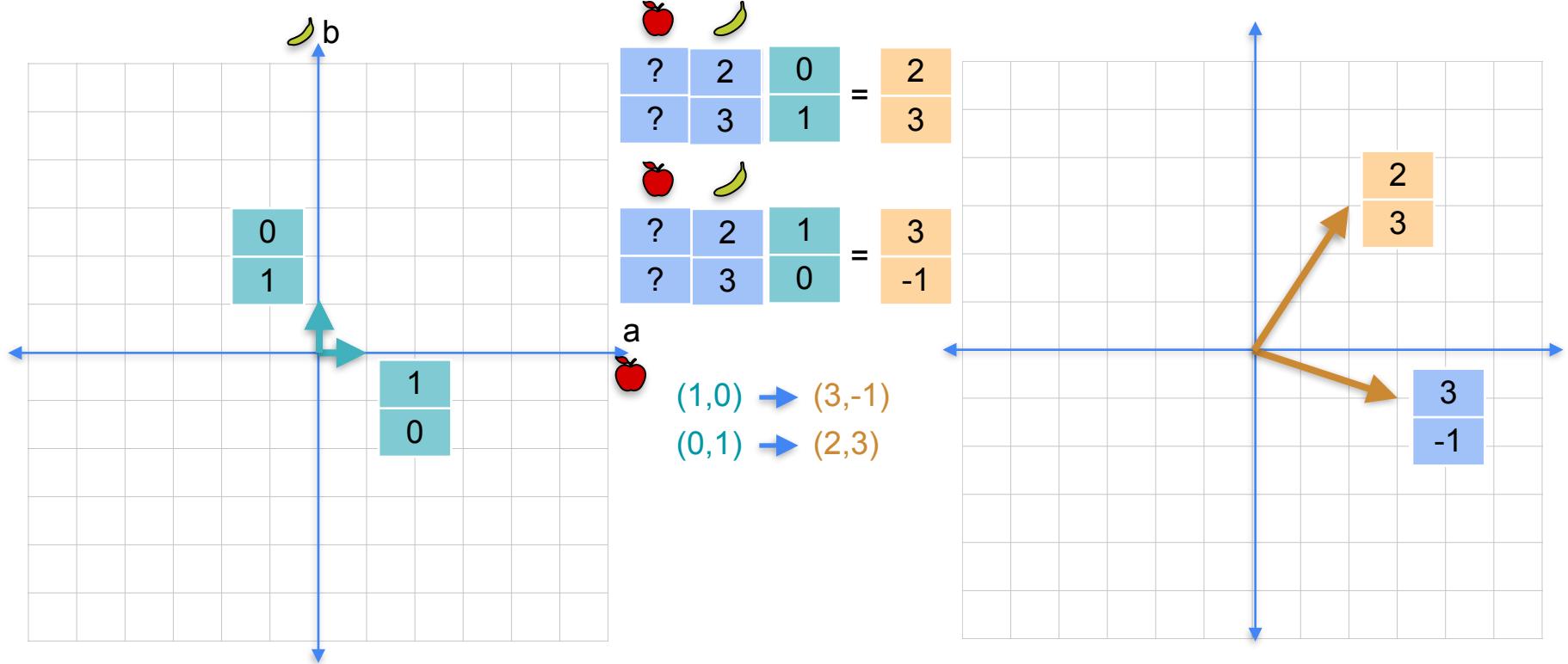
$$(1,0) \rightarrow (3,-1)$$
$$(0,1) \rightarrow (2,3)$$



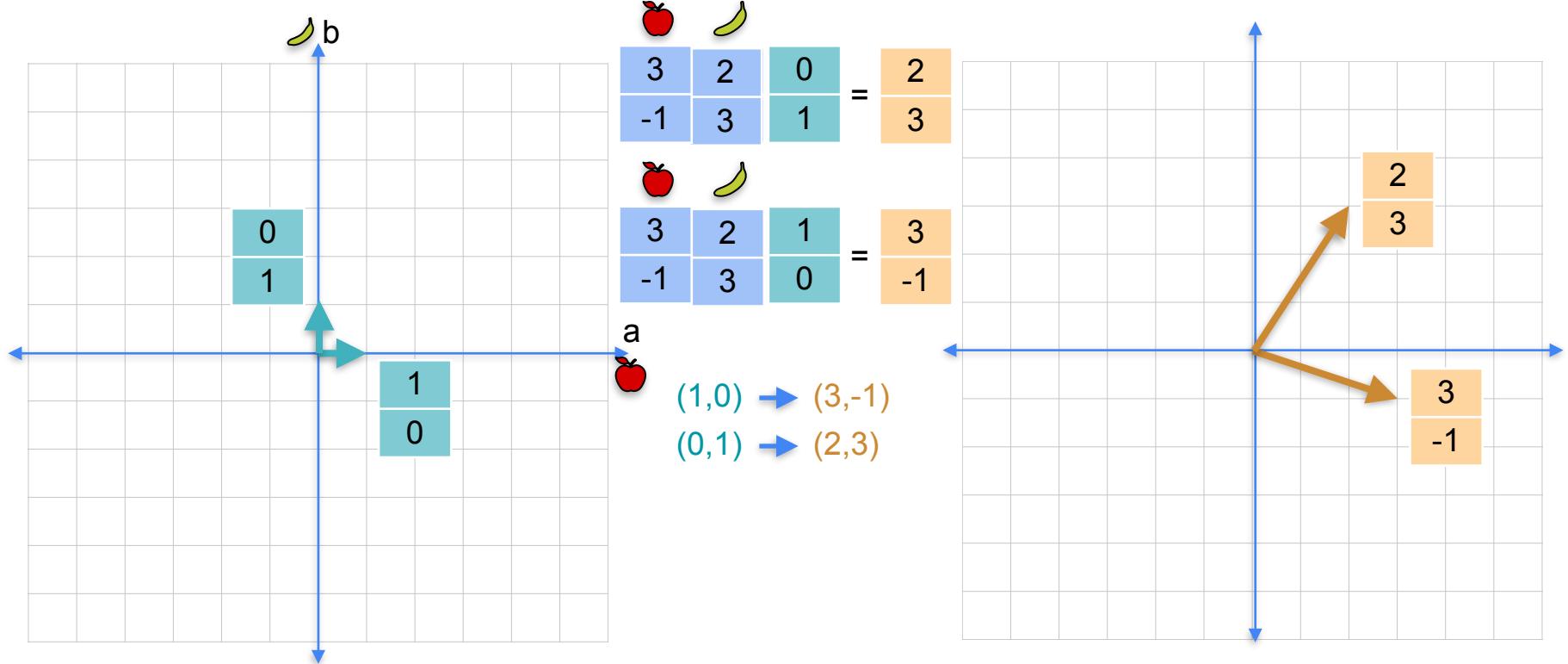
Linear transformations as matrices



Linear transformations as matrices



Linear transformations as matrices



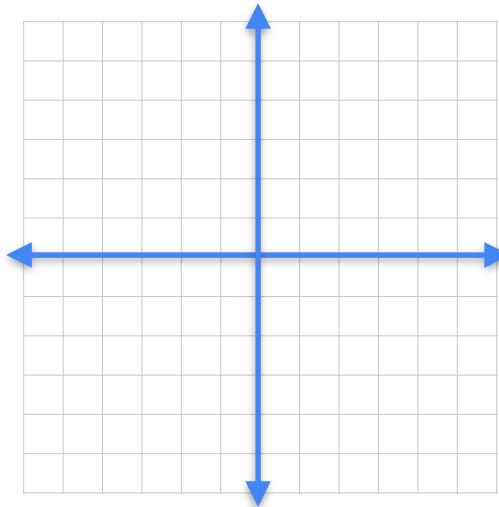
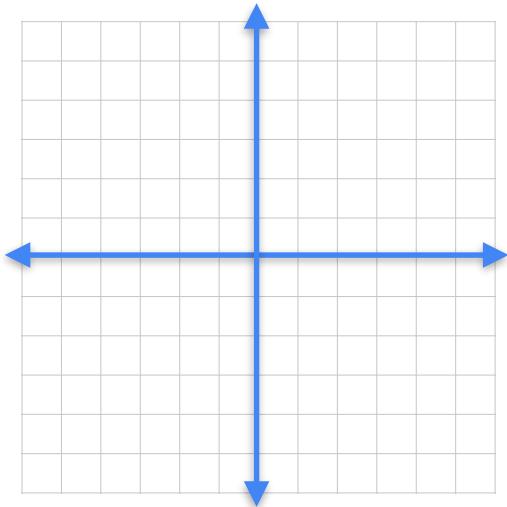


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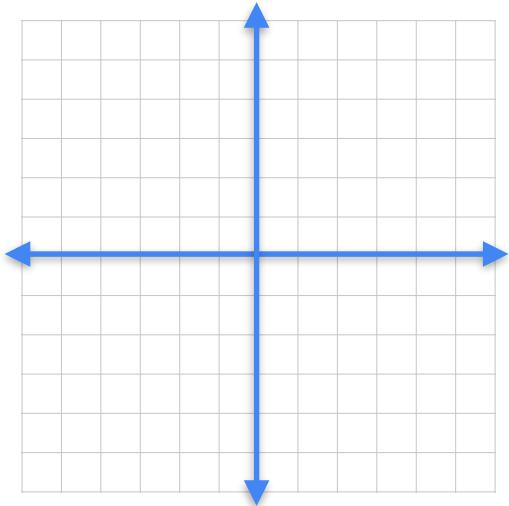
Vectors and Linear Transformations

Matrix multiplication

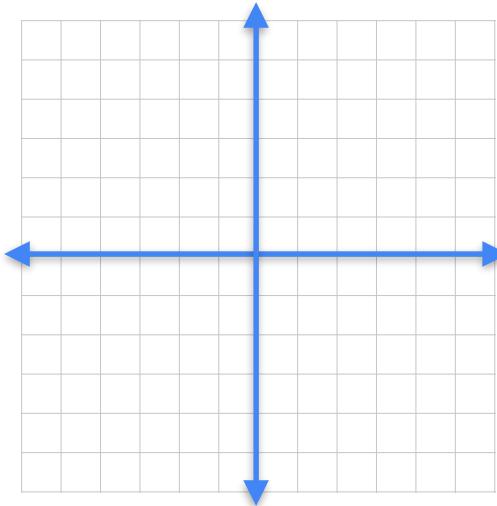
Combining linear transformations



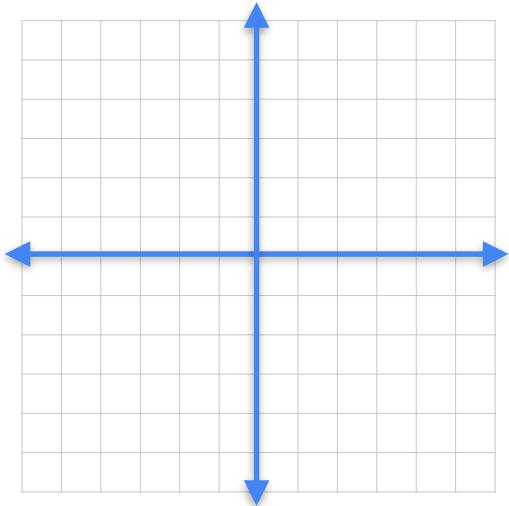
Combining linear transformations



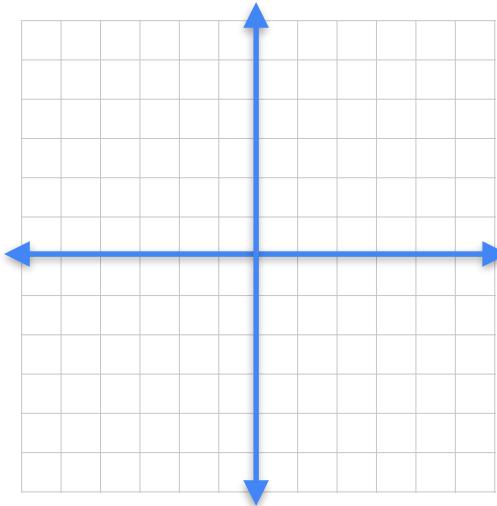
3	1
1	2



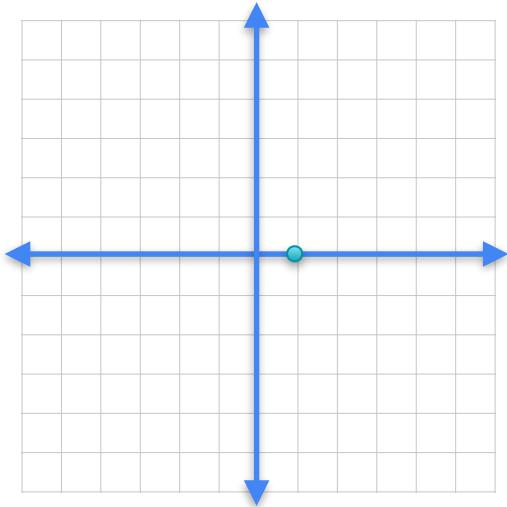
Combining linear transformations



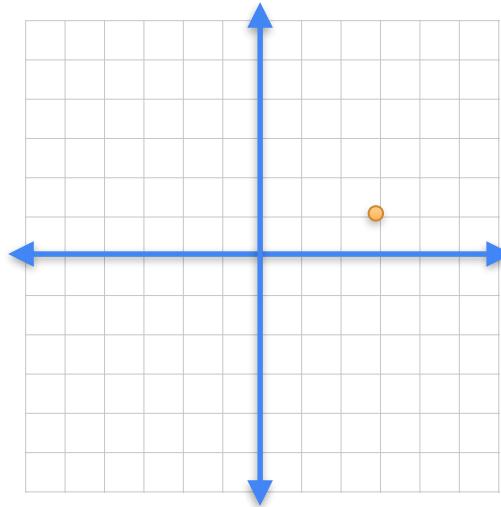
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



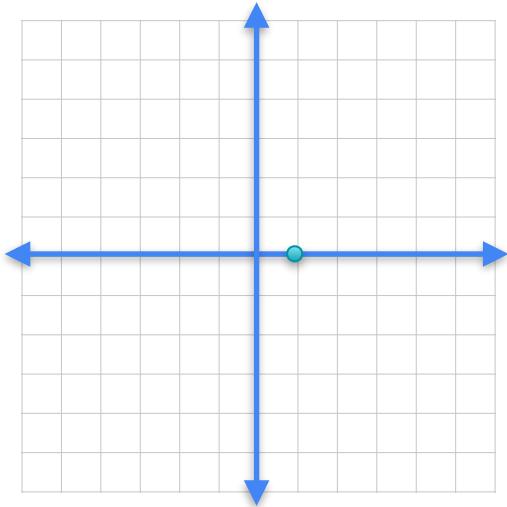
Combining linear transformations



$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

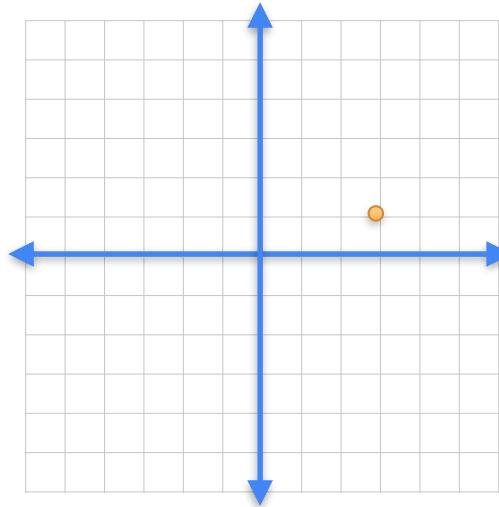


Combining linear transformations

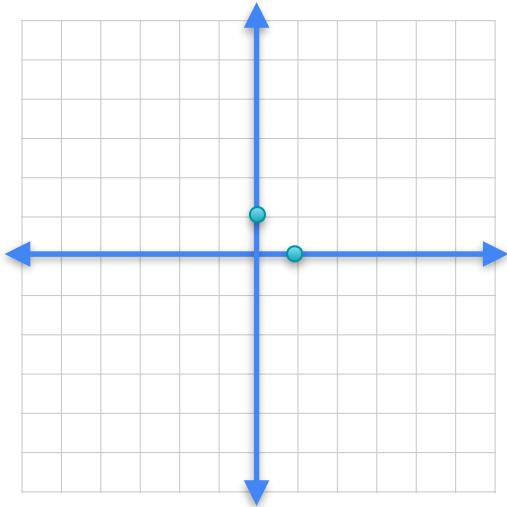


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

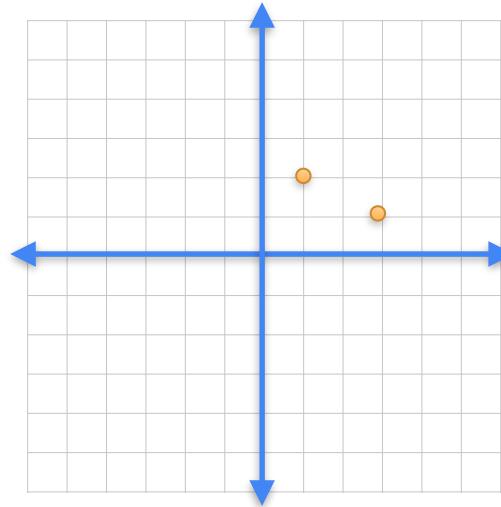


Combining linear transformations

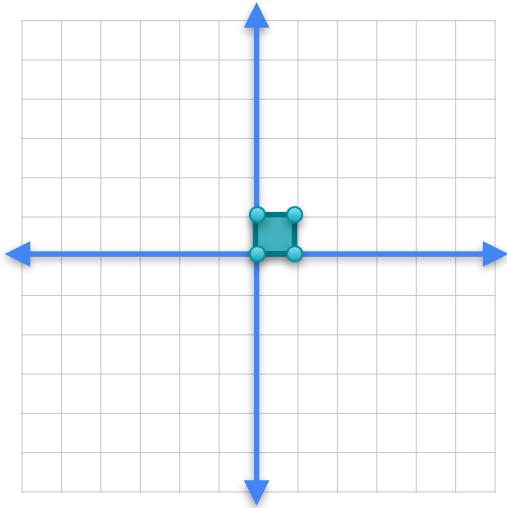


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

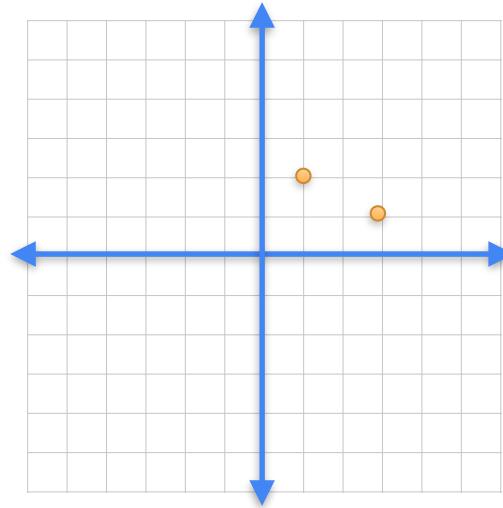


Combining linear transformations

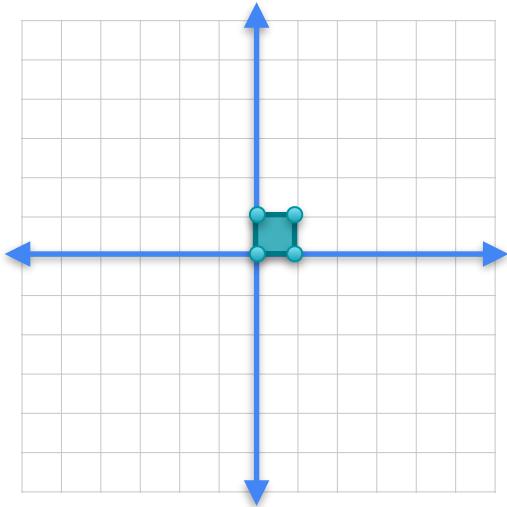


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

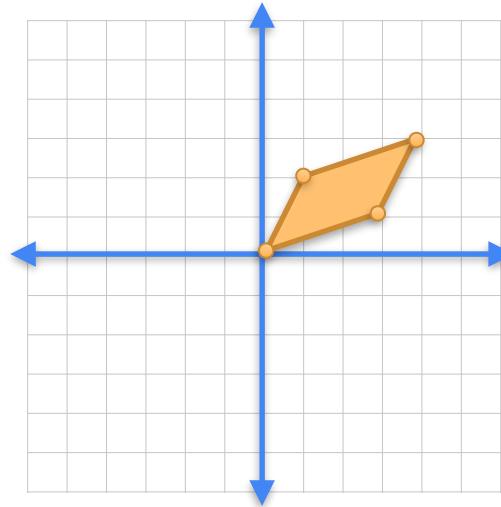


Combining linear transformations

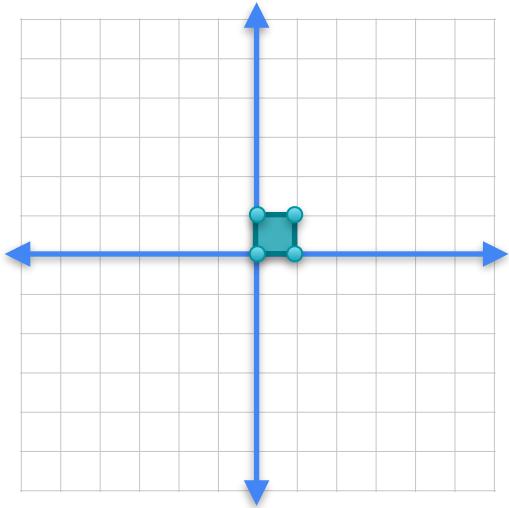


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

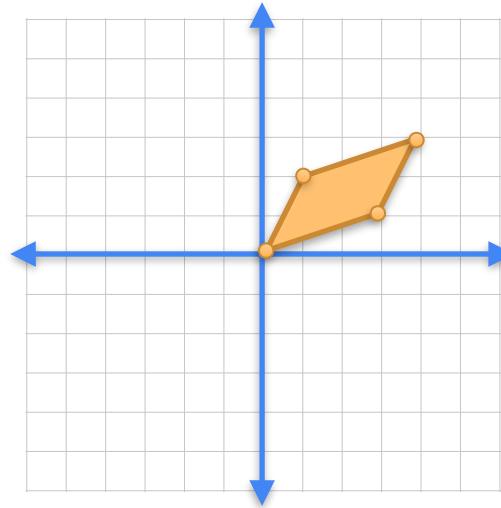


Combining linear transformations

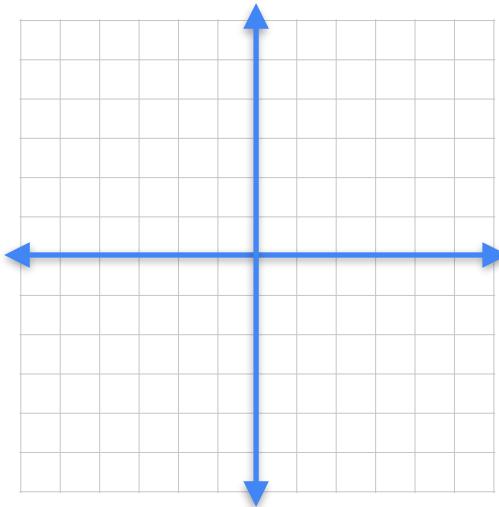
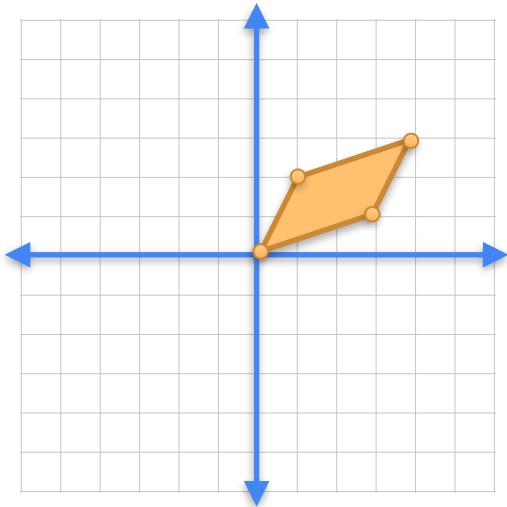


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

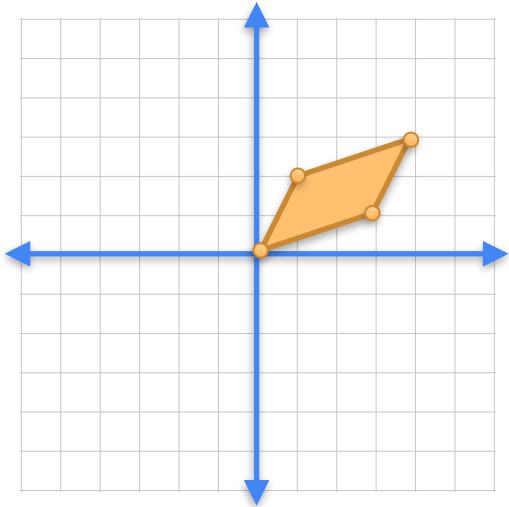
$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



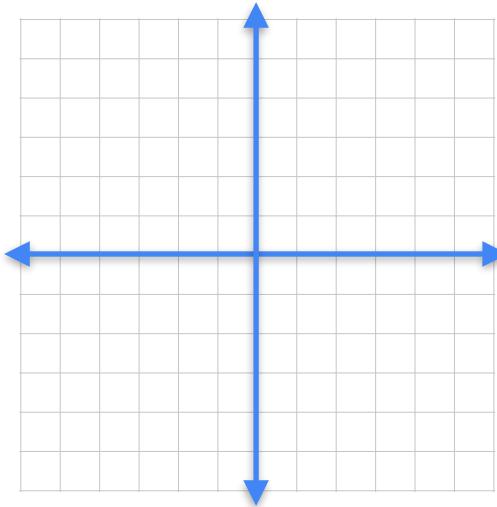
Combining linear transformations



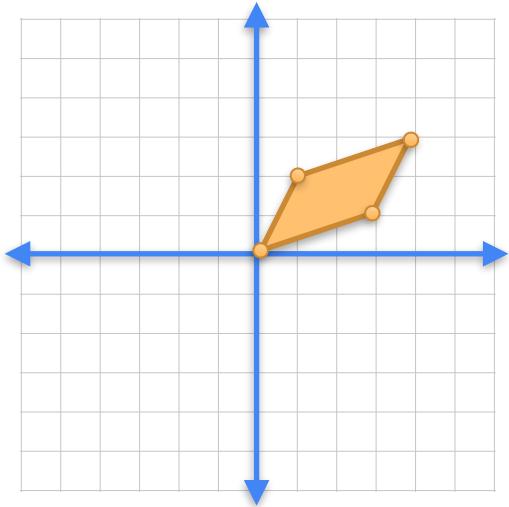
Combining linear transformations



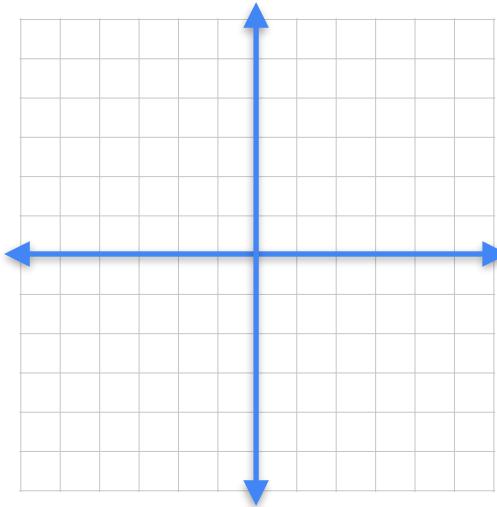
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$



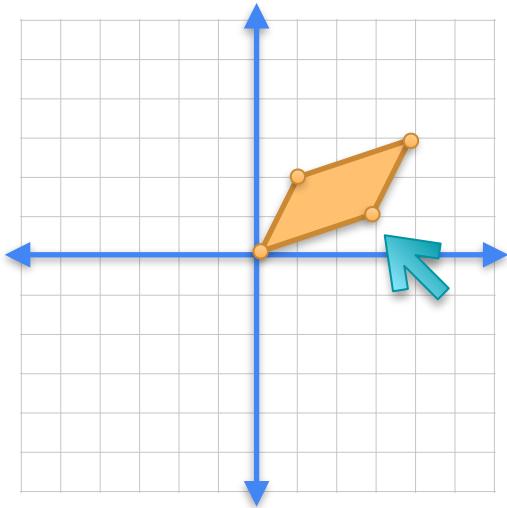
Combining linear transformations



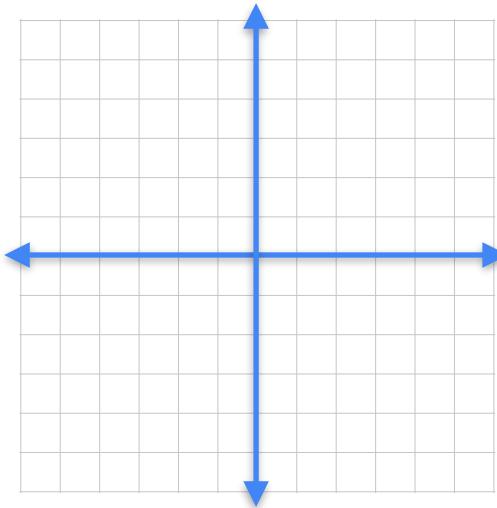
$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$



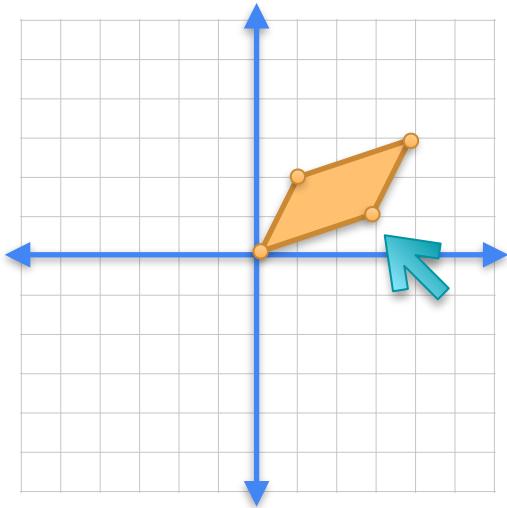
Combining linear transformations



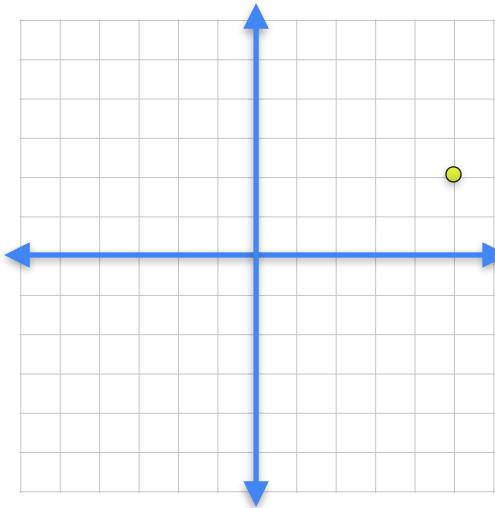
$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$



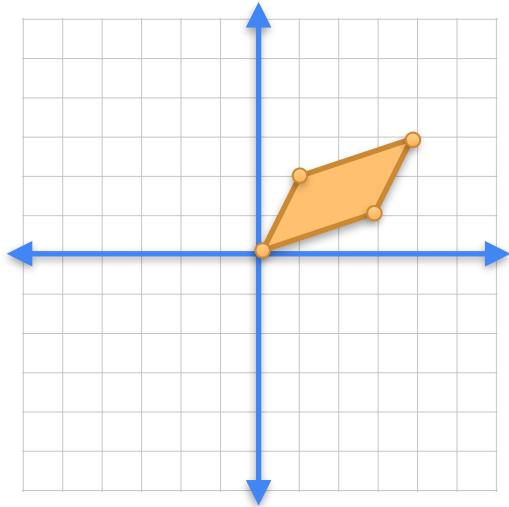
Combining linear transformations



$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

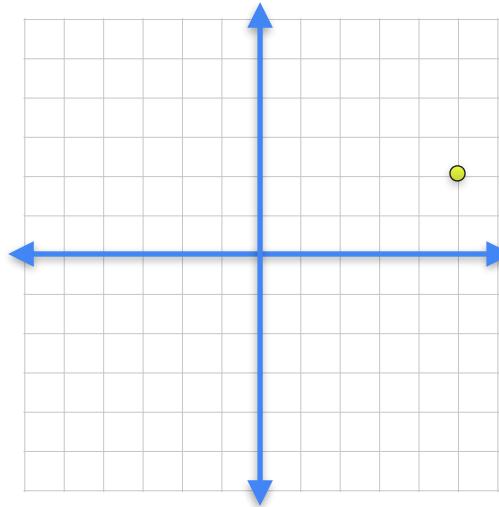


Combining linear transformations

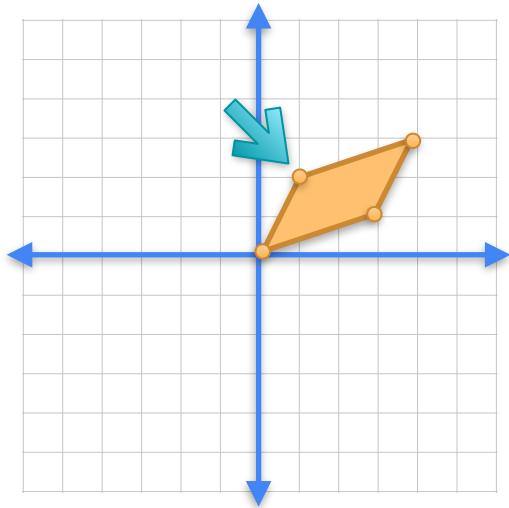


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

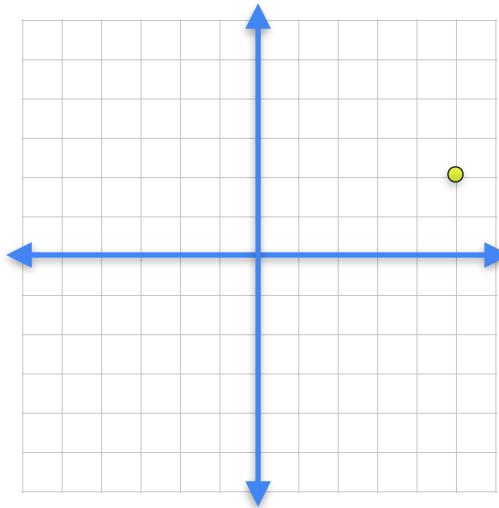


Combining linear transformations

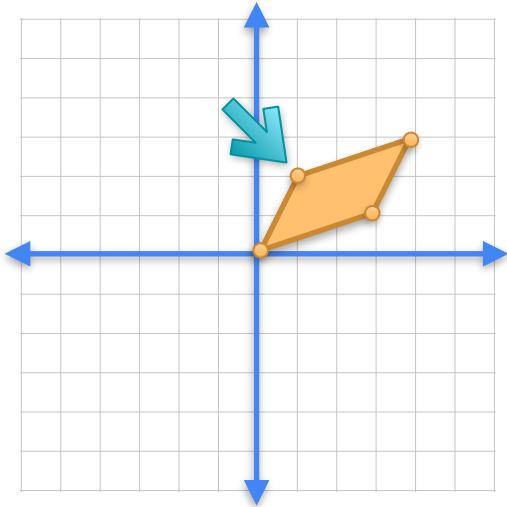


$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

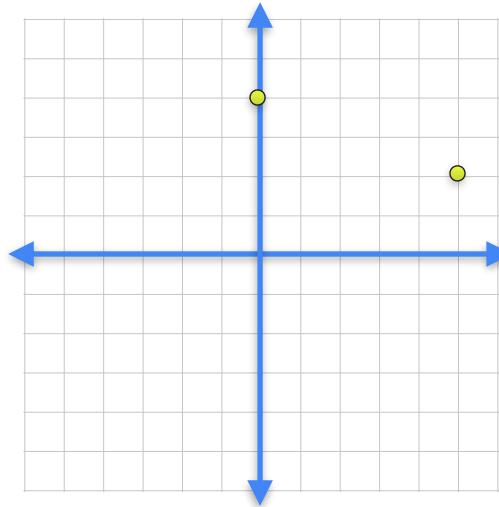


Combining linear transformations

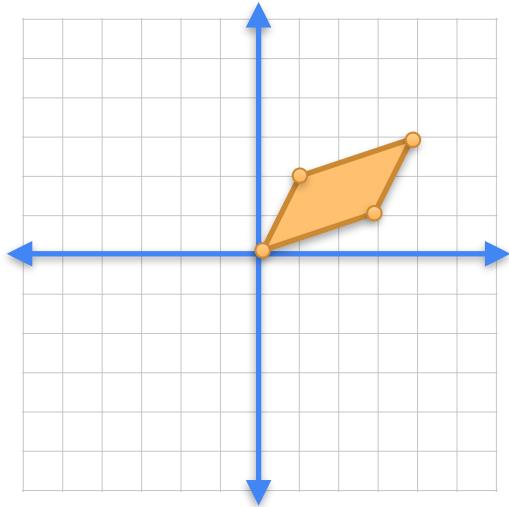


$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

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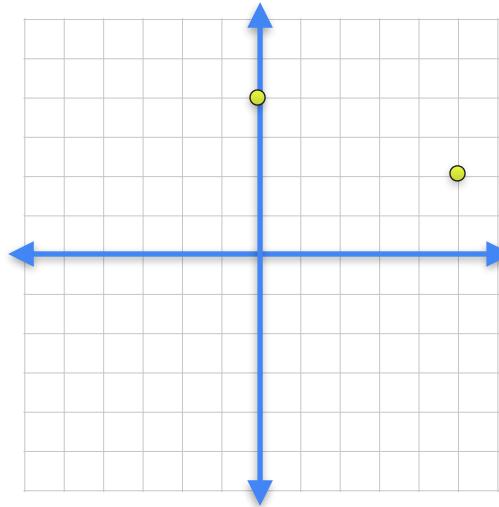


Combining linear transformations

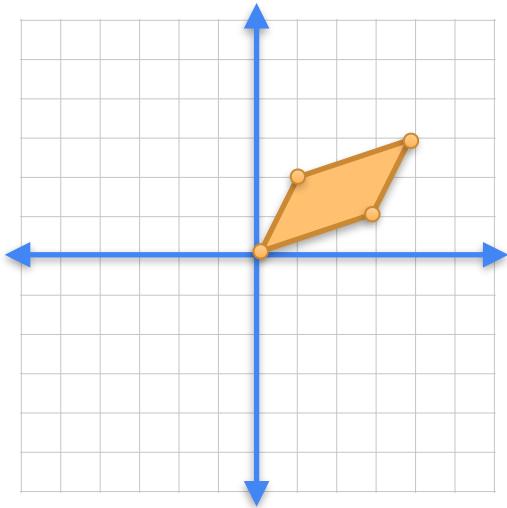


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

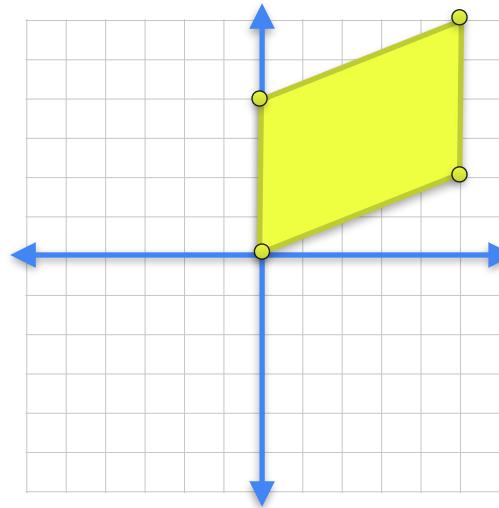


Combining linear transformations

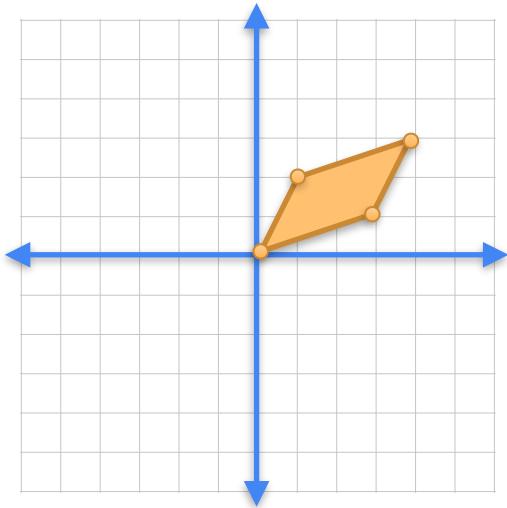


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

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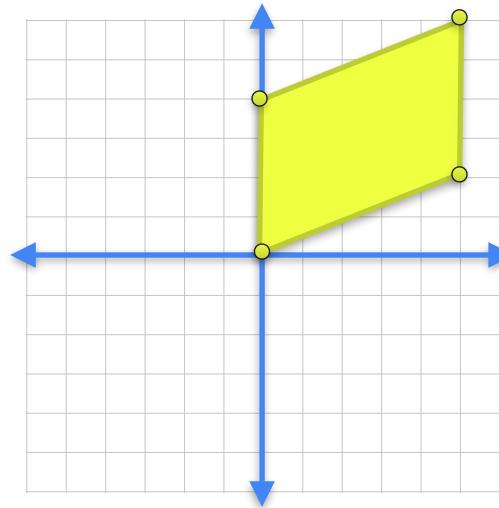


Combining linear transformations

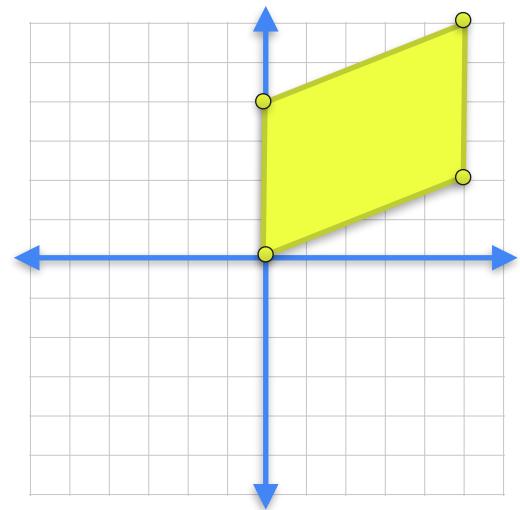
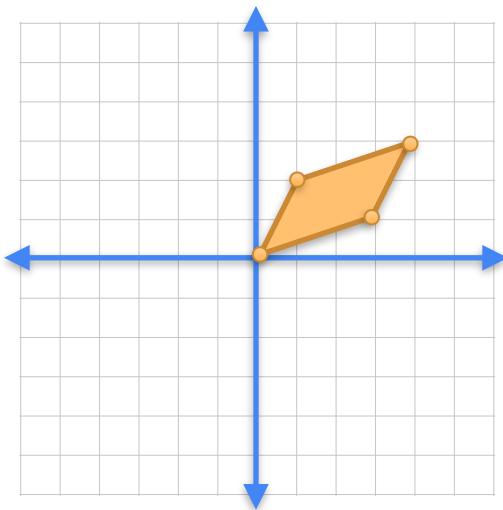
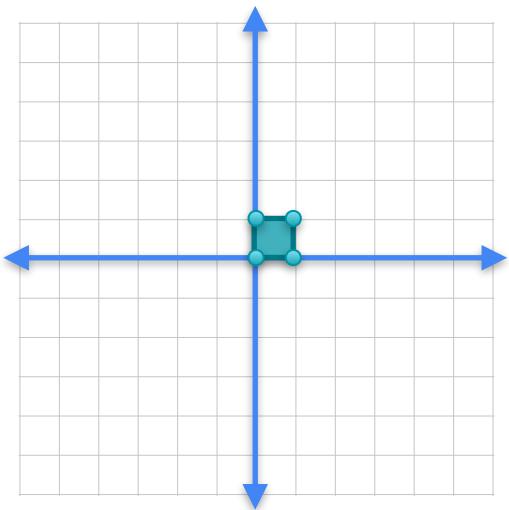


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

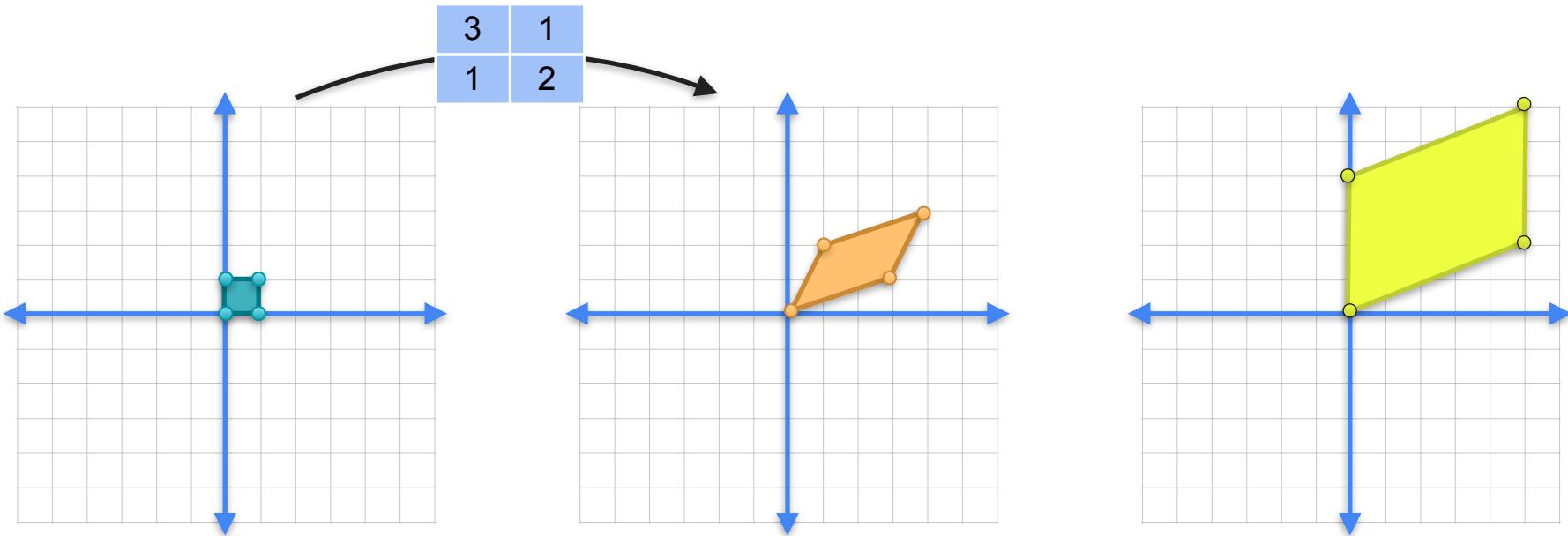
$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$



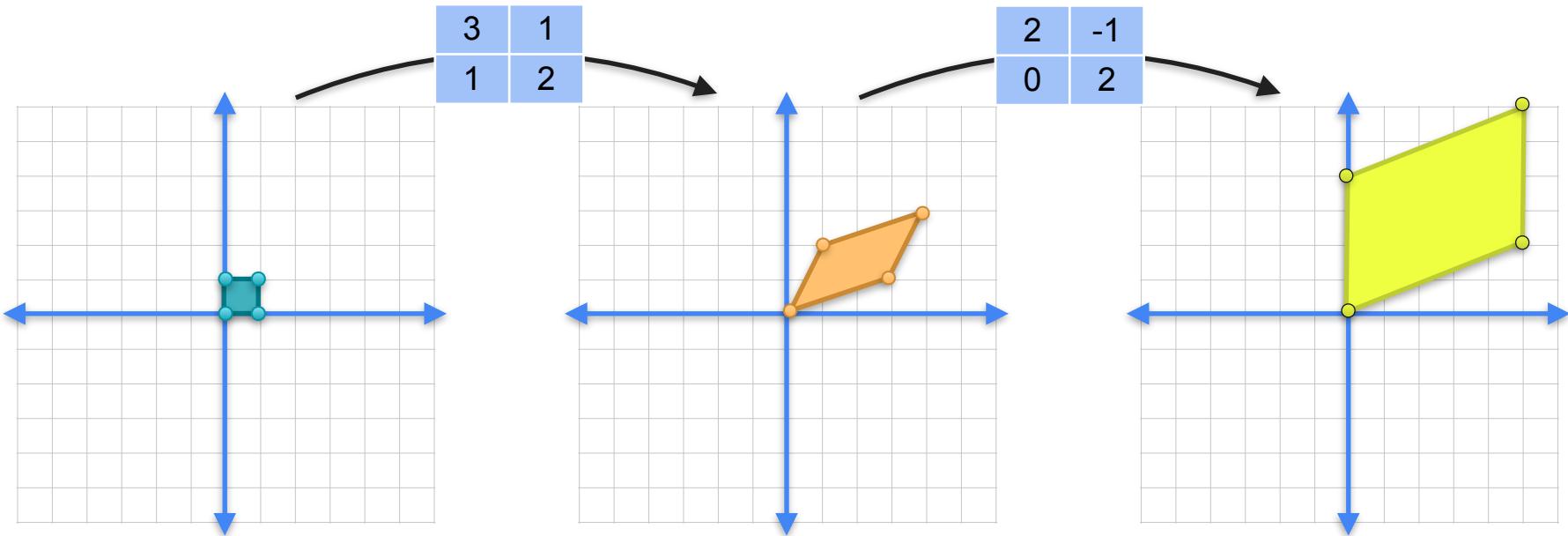
Combining linear transformations



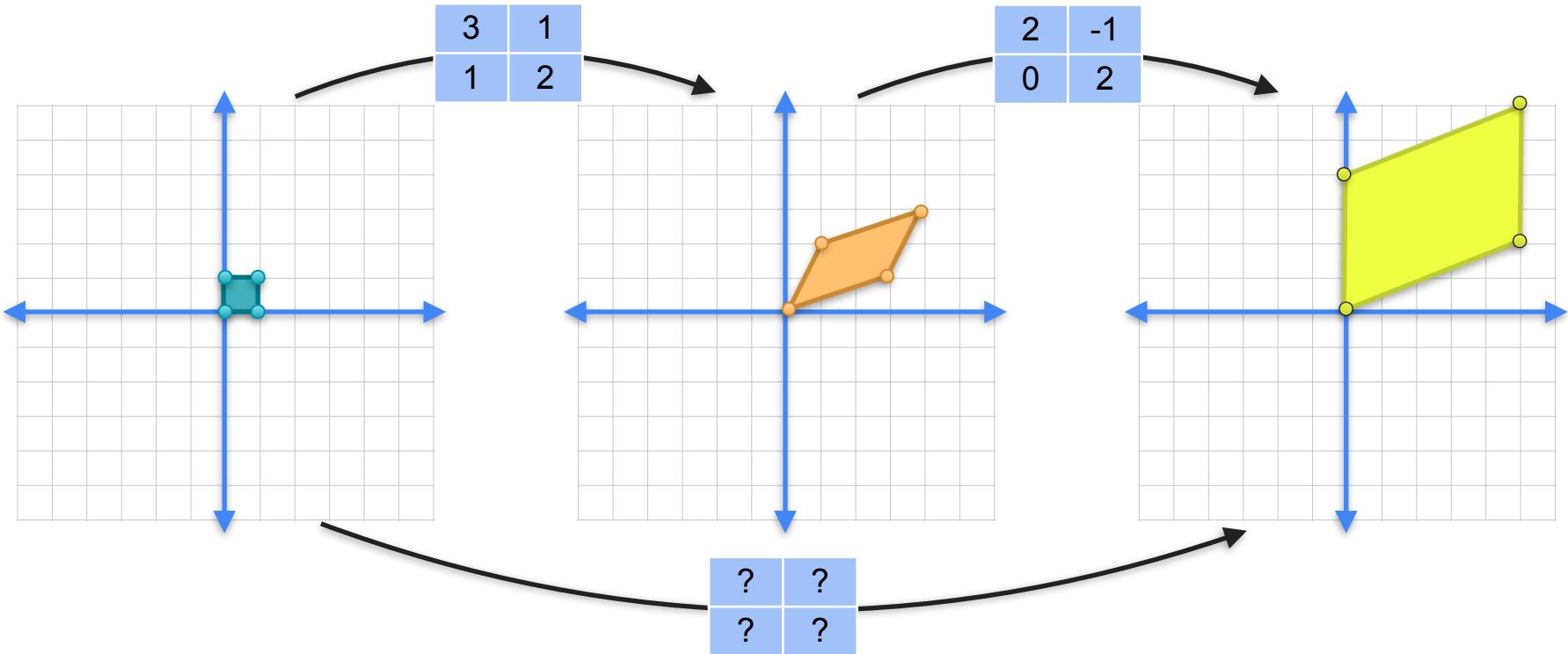
Combining linear transformations



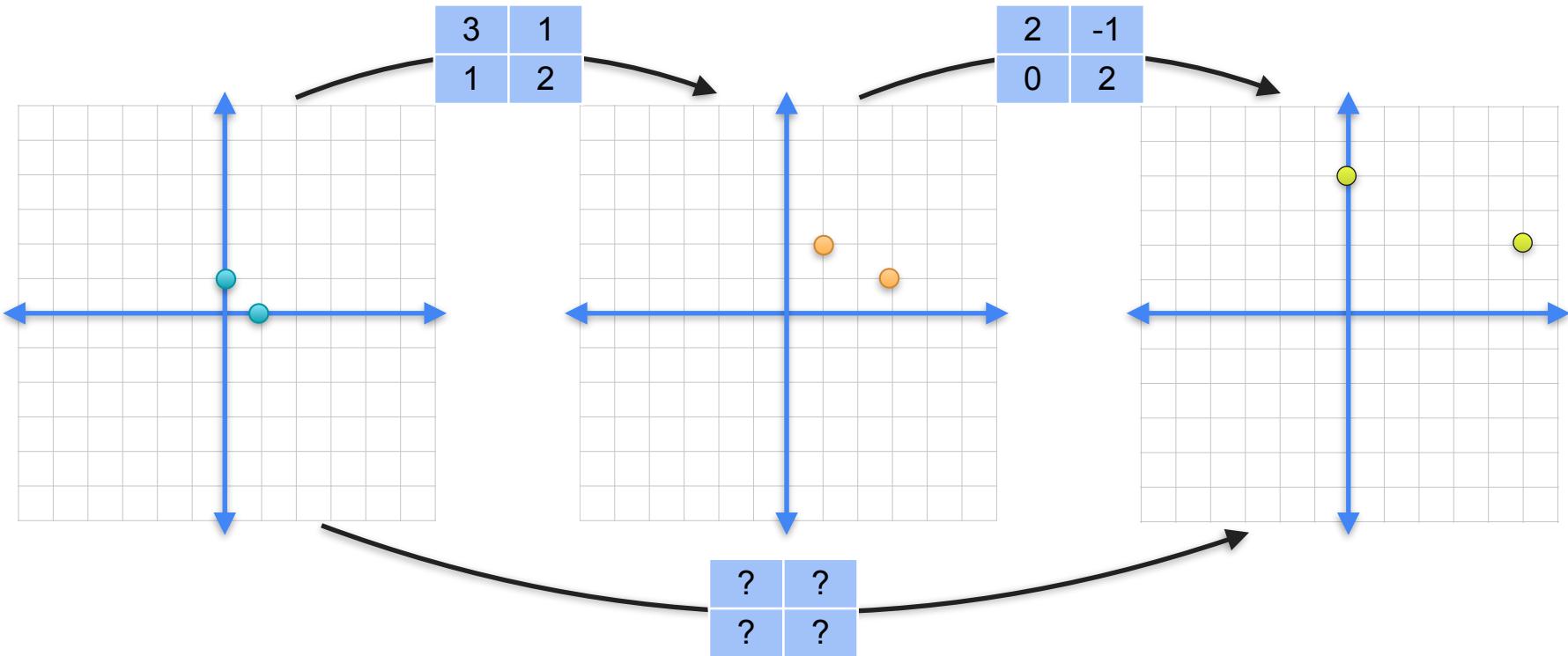
Combining linear transformations



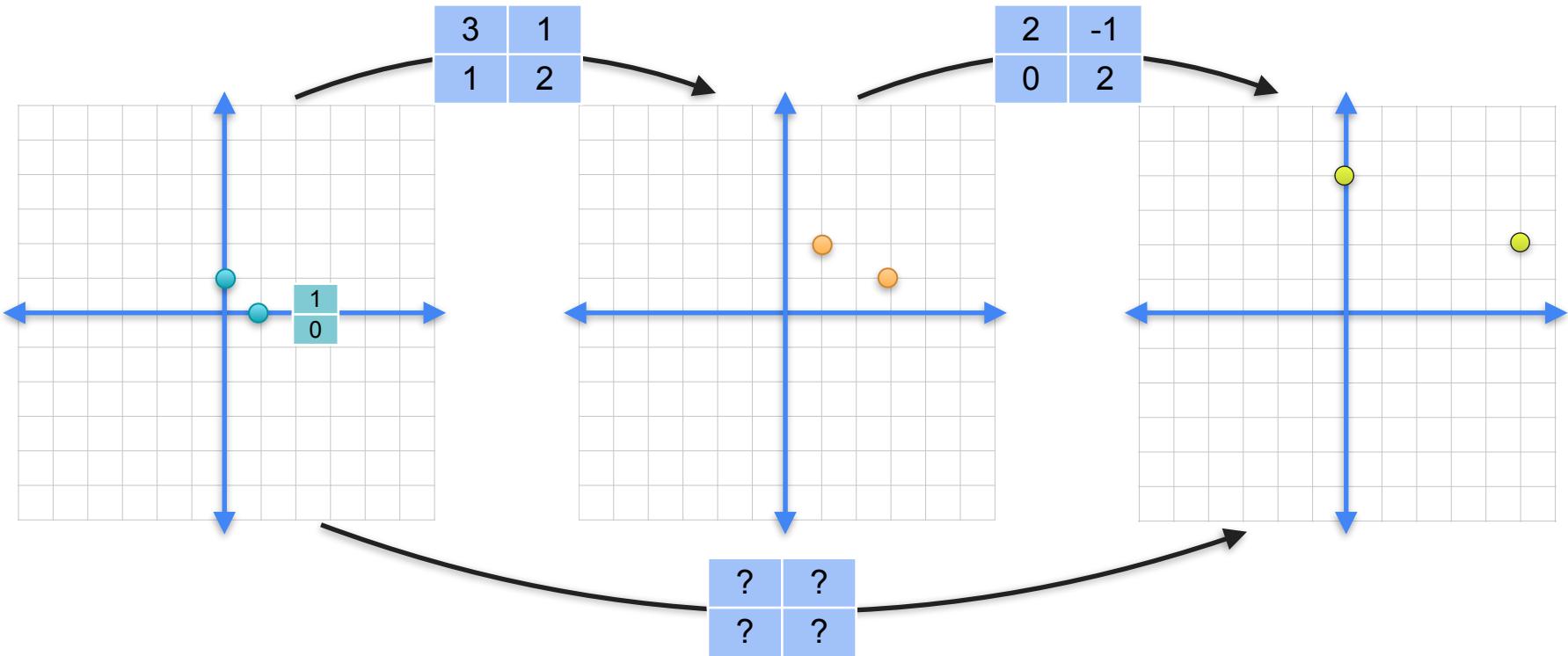
Combining linear transformations



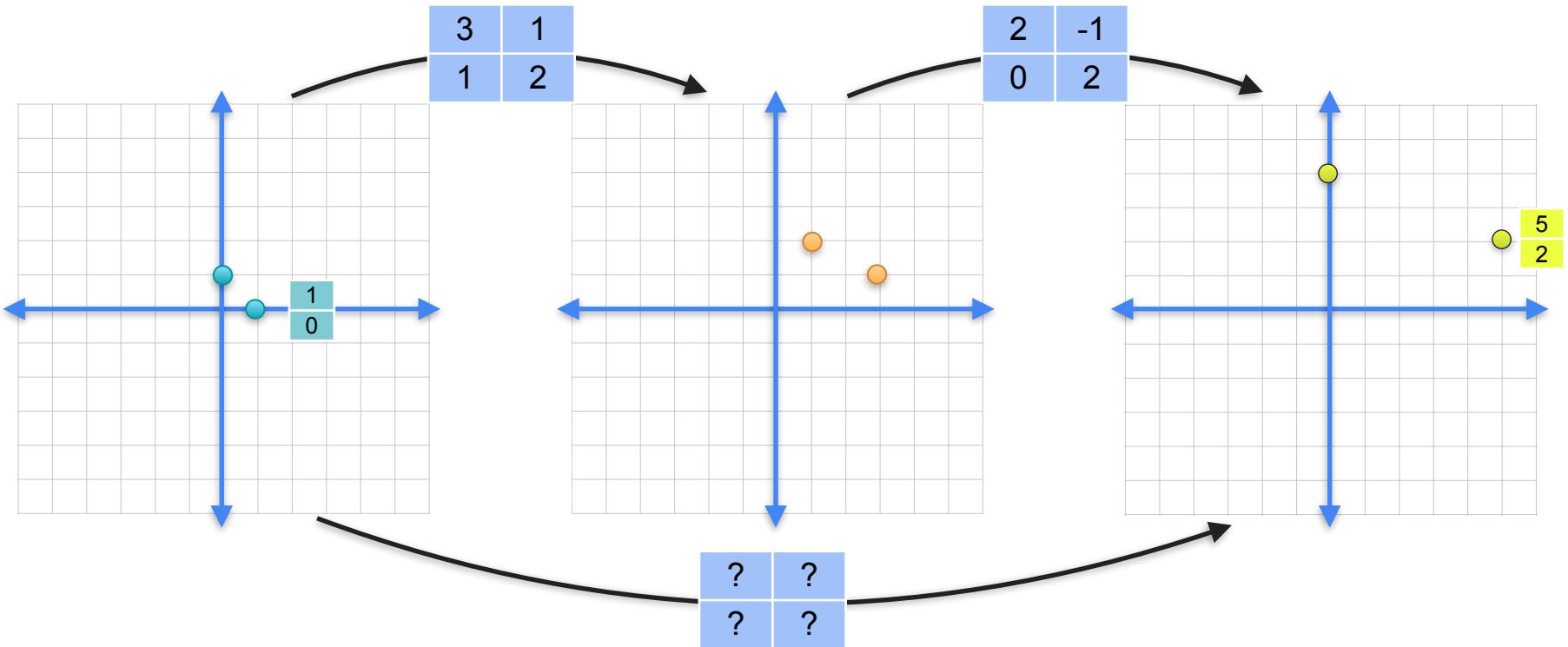
Combining linear transformations



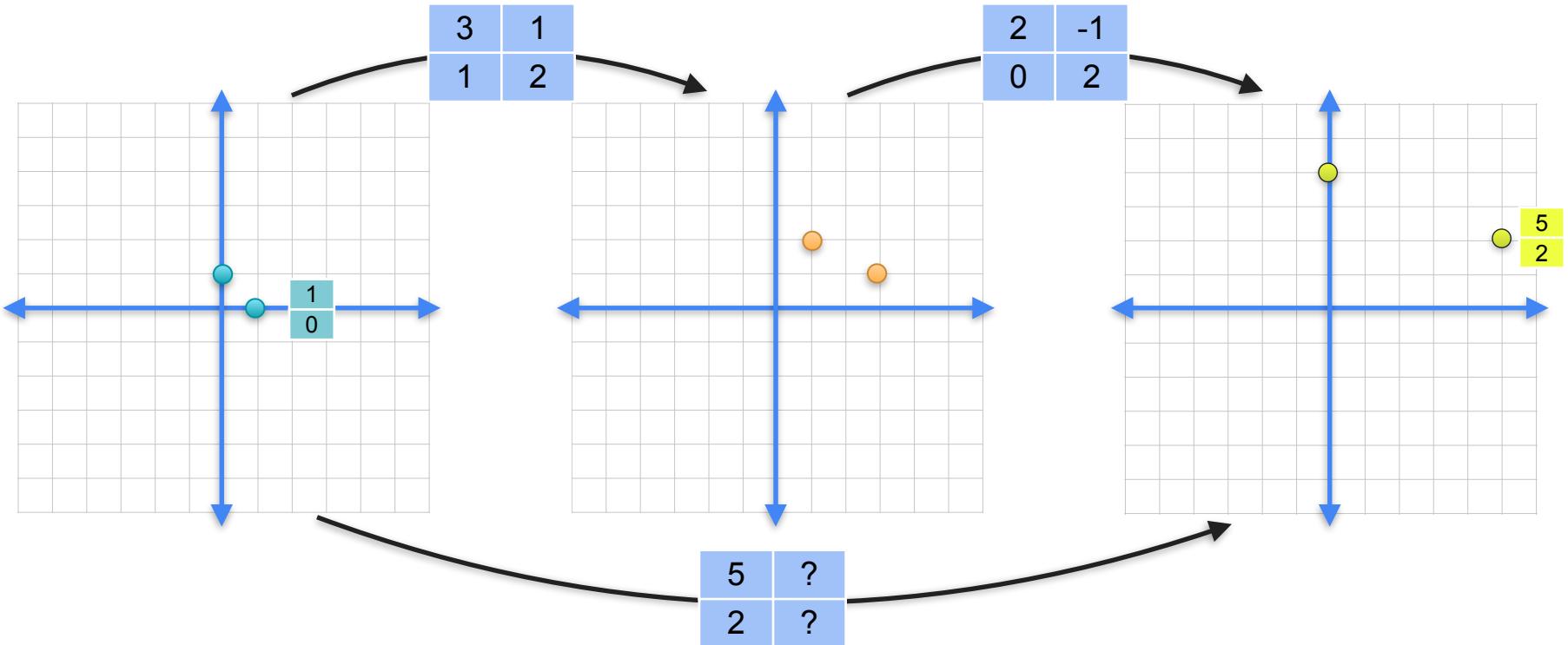
Combining linear transformations



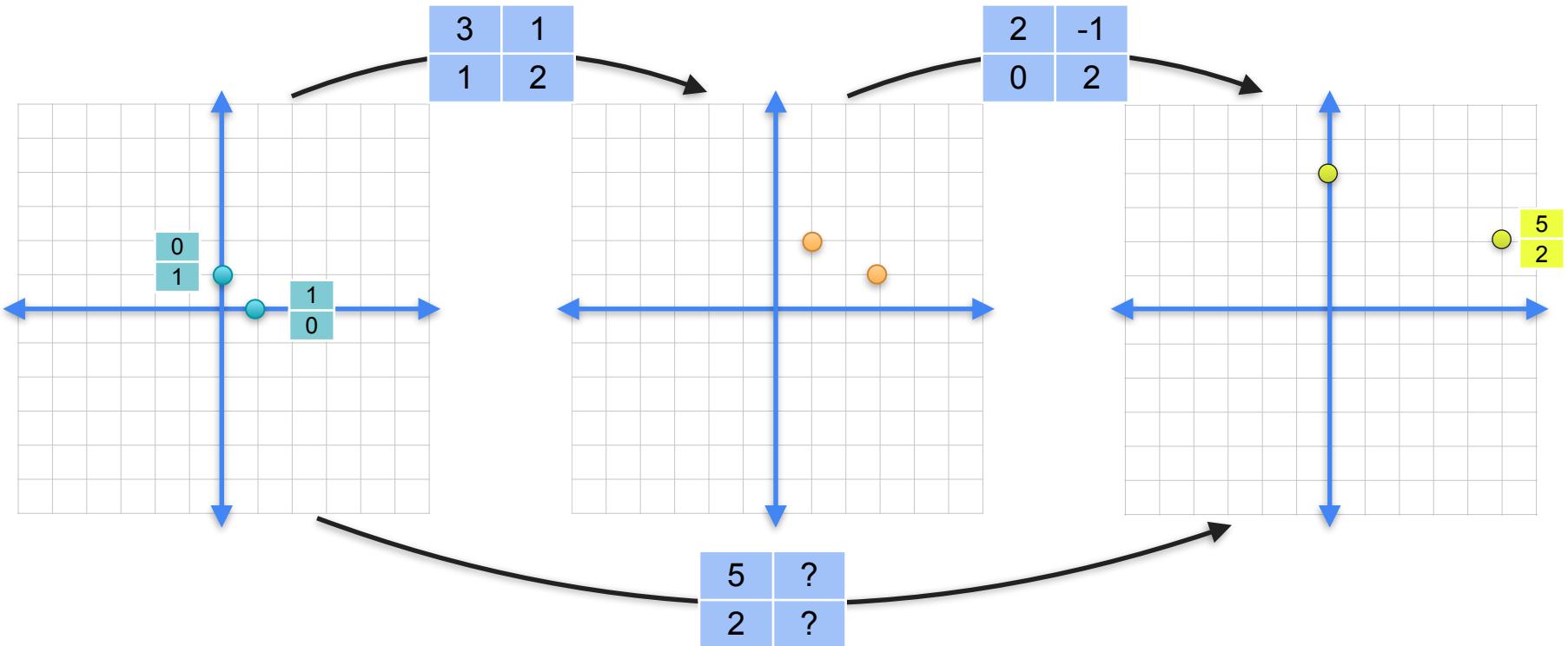
Combining linear transformations



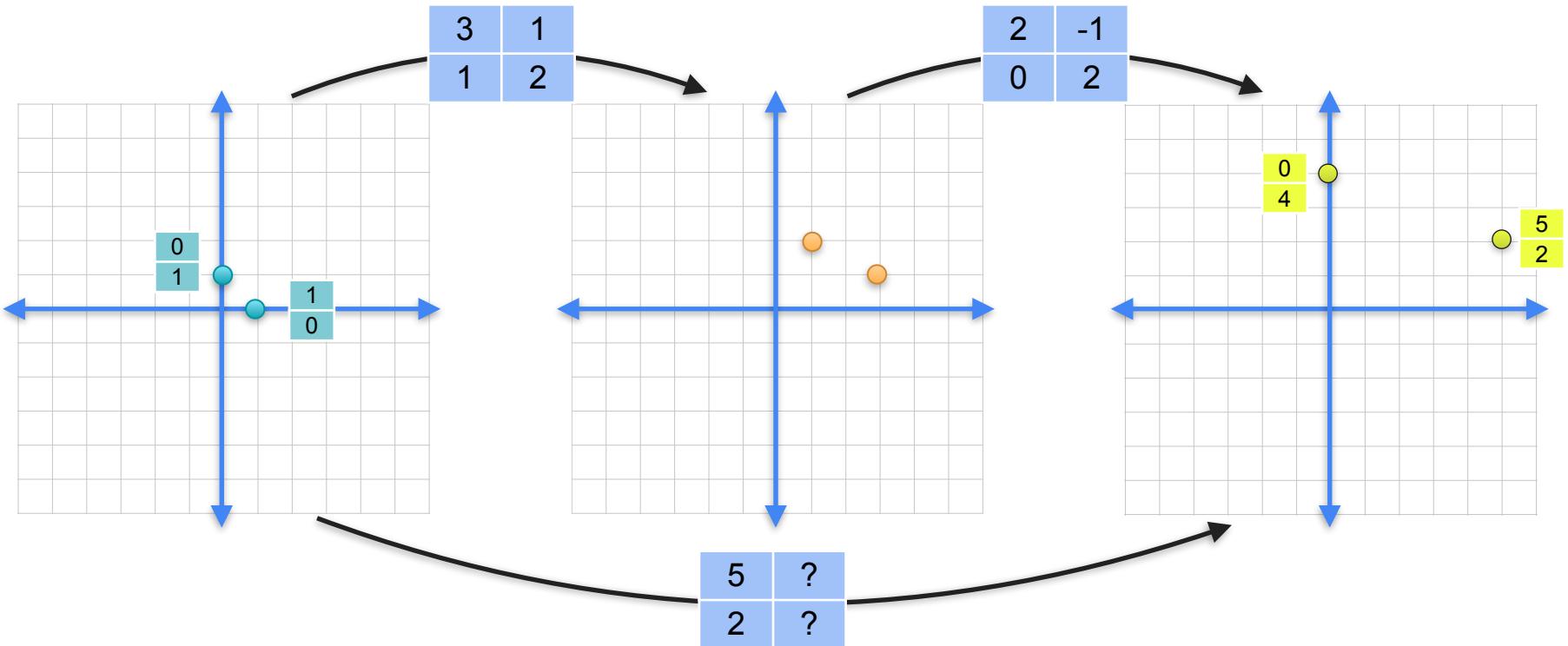
Combining linear transformations



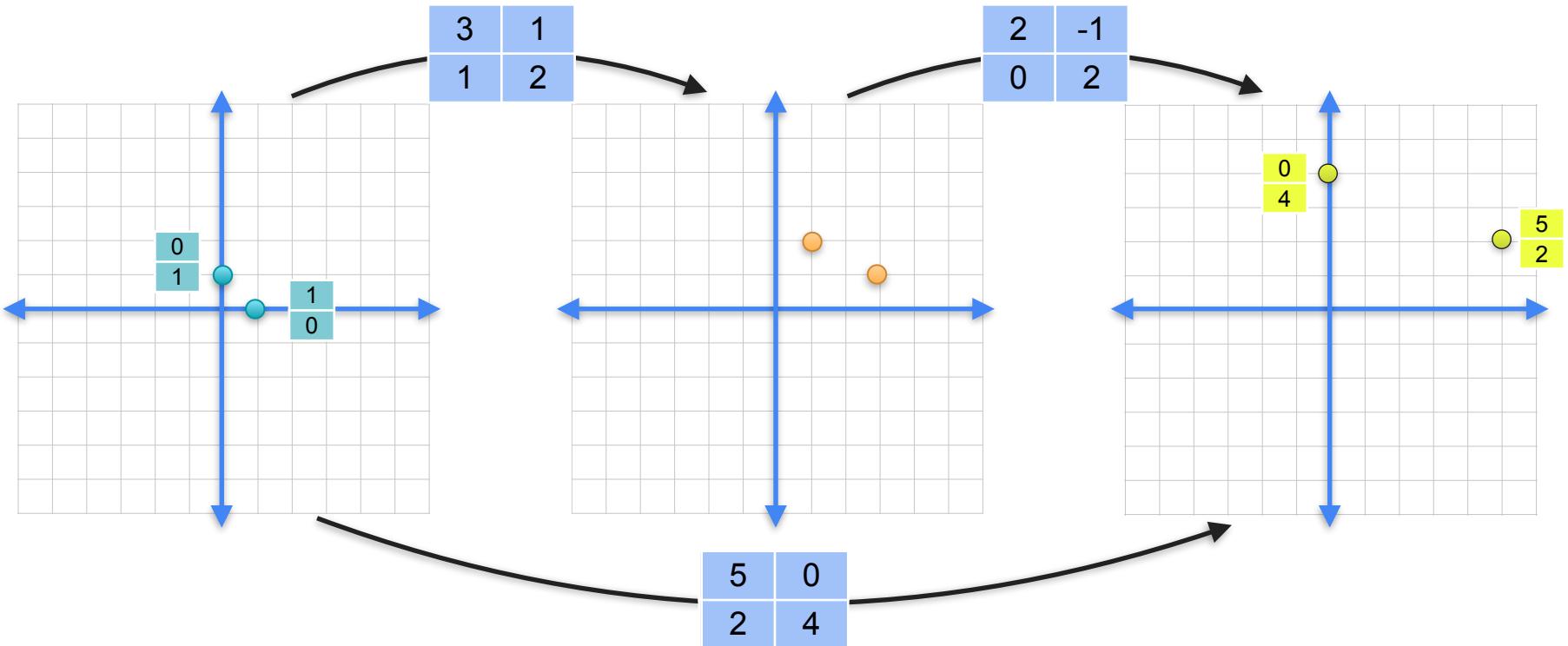
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$

Combining linear transformations

First
↓

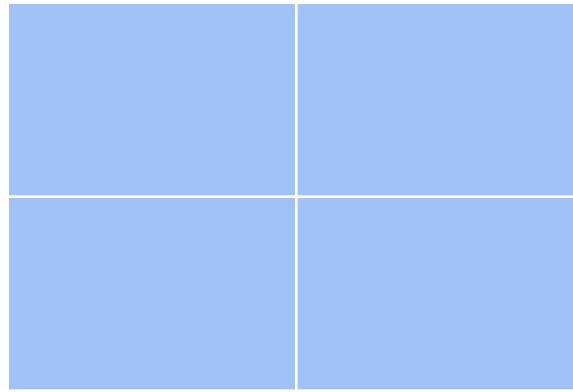
$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$

Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 5 & 0 \\ 0 & 4 \end{matrix} =$$



Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} & & 3 & 1 \\ 2 & -1 & | & 3 & 1 \\ & & 1 & 2 \\ \hline & & 0 & 2 & | & 3 & 1 \\ & & 0 & 2 & | & 1 & 2 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & & & \\ & \begin{matrix} 2 & -1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 2 \end{matrix} & \\ & \begin{matrix} 0 & 2 \\ 1 & 2 \end{matrix} & \begin{matrix} 3 \\ 1 \end{matrix} & \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 0 & 2 \end{matrix}$$

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix has cyan cells at positions (1,1), (1,2), and (2,1), and an orange cell at (2,2). The second matrix has orange cells at (1,1), (1,2), (2,1), and a cyan cell at (2,2). The result is a 2x2 matrix where the top-left cell is 5 (cyan), the top-right cell is 0 (orange), the bottom-left cell is 0 (cyan), and the bottom-right cell is 2 (orange). The result matrix is shown with a vertical line separating the columns and a horizontal line separating the rows.

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & \begin{matrix} 0 & 2 \\ 1 & 2 \end{matrix} \end{matrix}$$

Multiplying matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 5 & 0 \\ 2 & 4 \end{matrix}$$



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Vectors and Linear Transformations

The identity matrix

The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

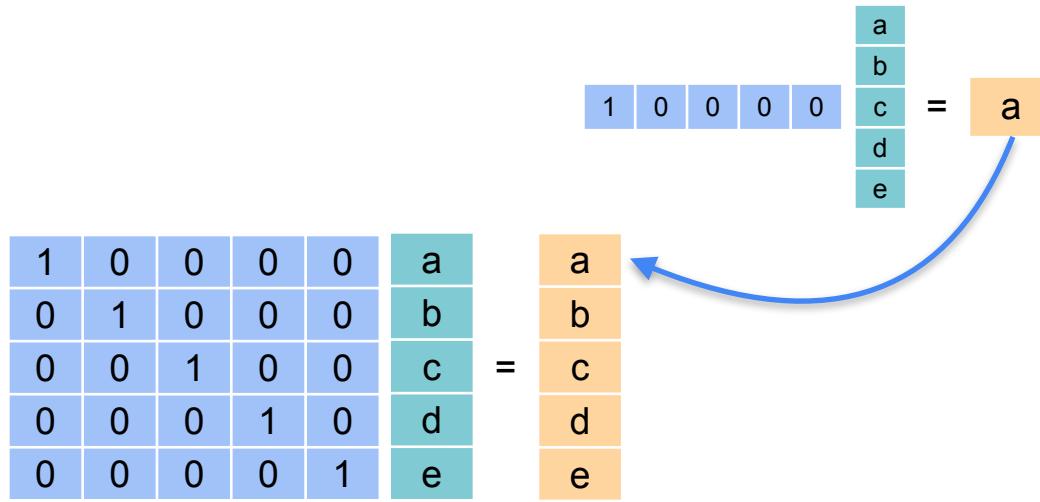
The identity matrix

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

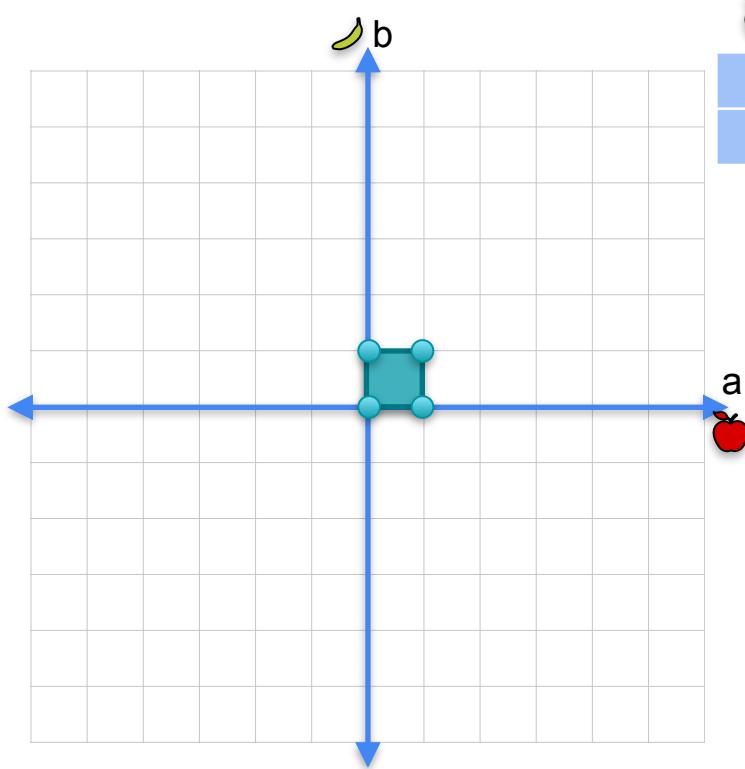
The identity matrix

$$\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & a \\ 0 & 1 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & 0 & 0 & c \\ 0 & 0 & 0 & 1 & 0 & d \\ 0 & 0 & 0 & 0 & 1 & e \end{array} = \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array}$$

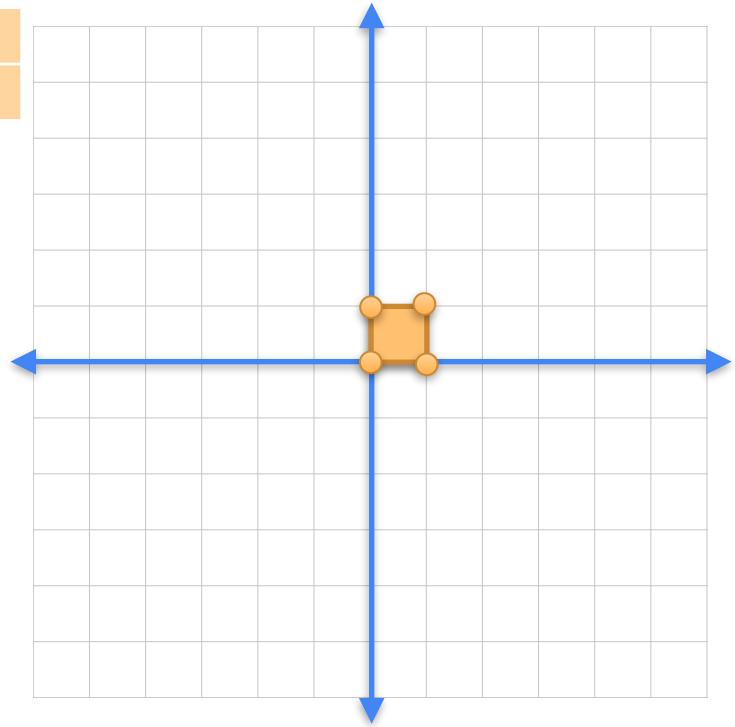
The identity matrix



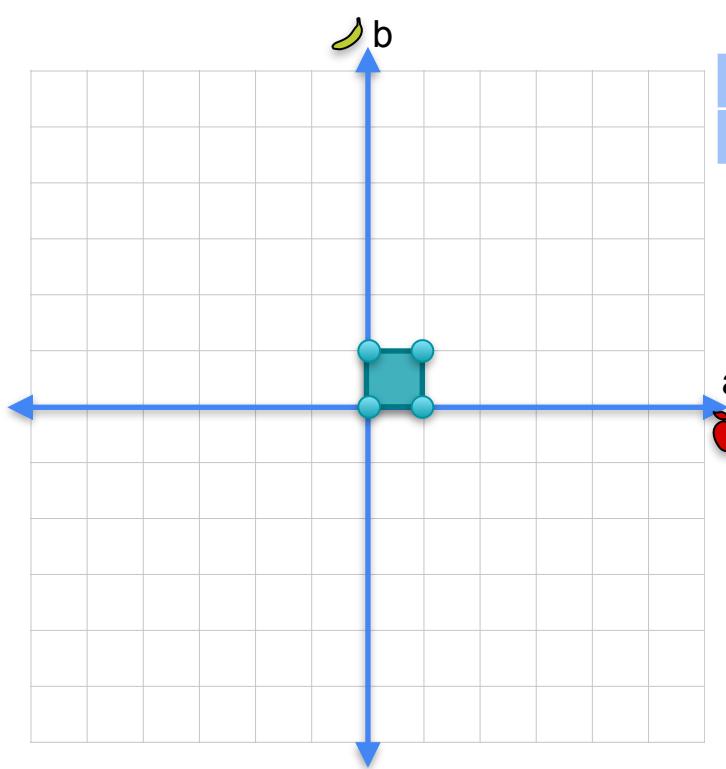
The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ 1 & 0 \\ 0 & 1 \end{matrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} \text{apple} & \text{banana} \\ 1 & 0 \\ 0 & 1 \end{matrix} \times \begin{pmatrix} x \\ y \end{pmatrix}$$

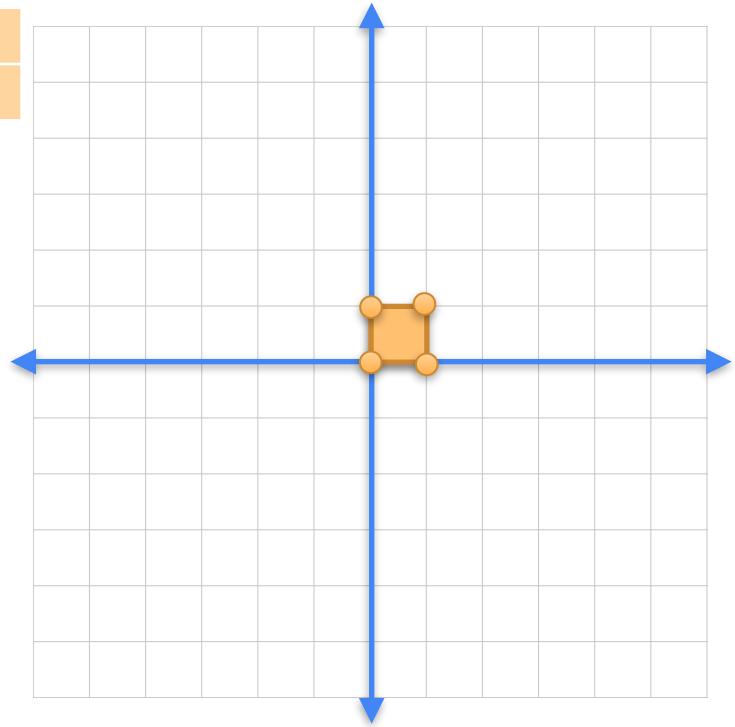


The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} x \\ y \end{matrix} = \begin{matrix} x \\ y \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,0) \\ (0,1) &\rightarrow (0,1) \\ (1,1) &\rightarrow (1,1) \end{aligned}$$





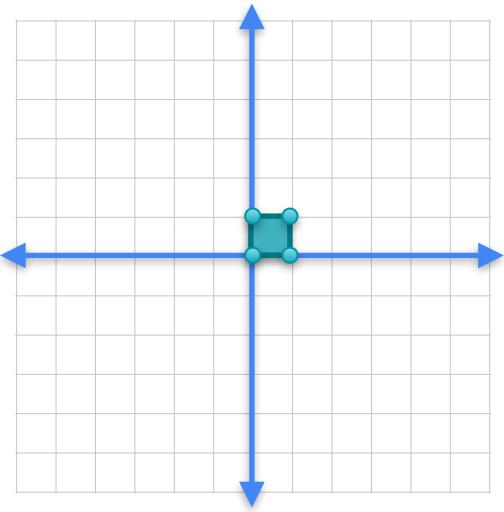
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Vectors and Linear Transformations

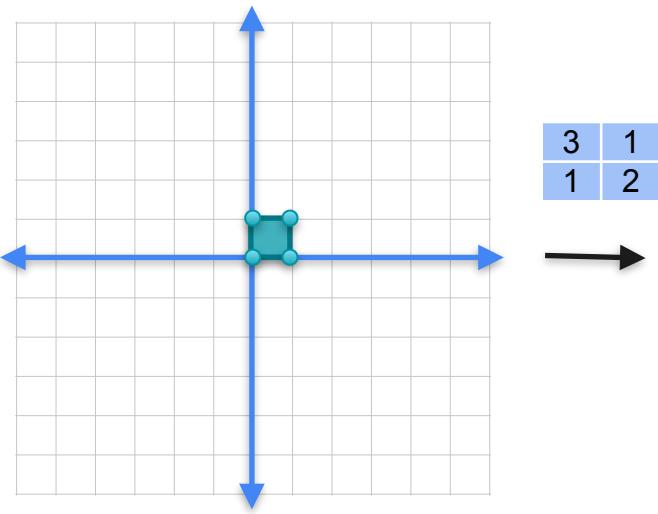
Matrix inverse

Matrix inverses

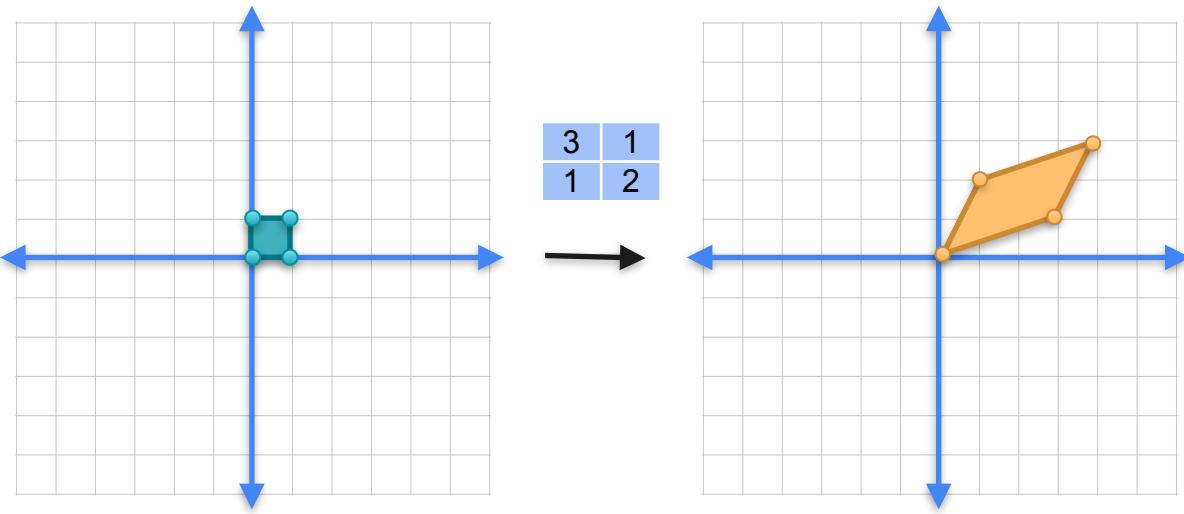
Matrix inverses



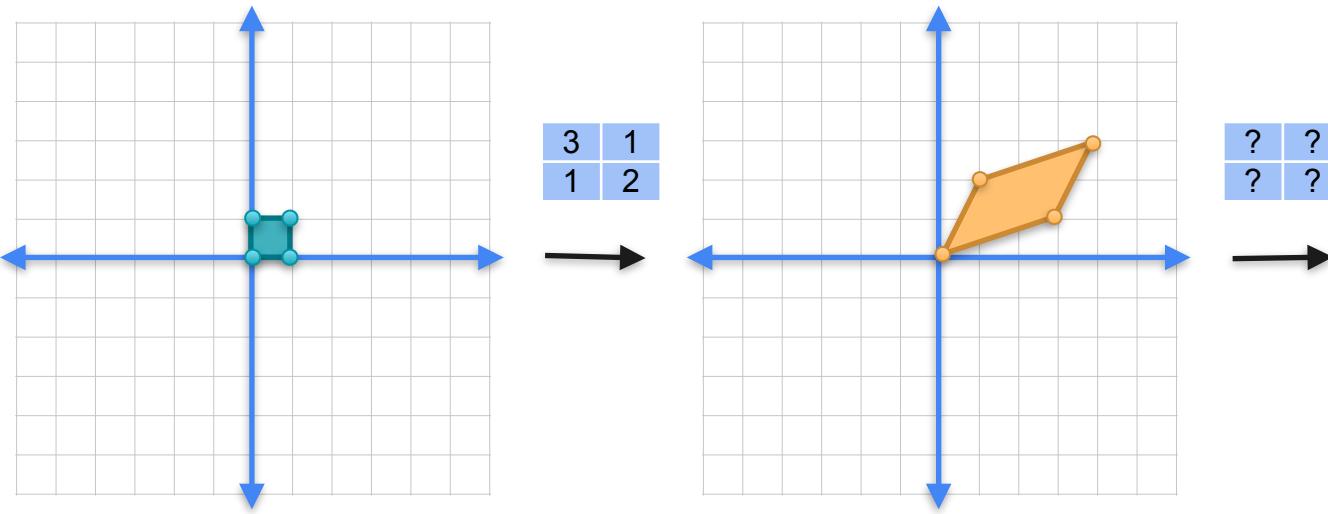
Matrix inverses



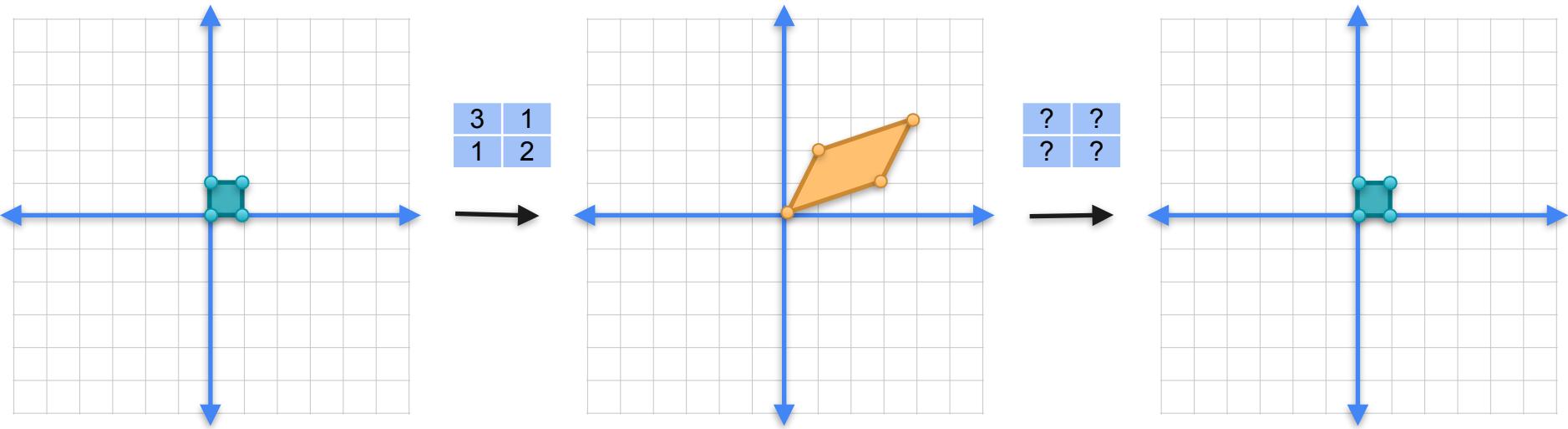
Matrix inverses



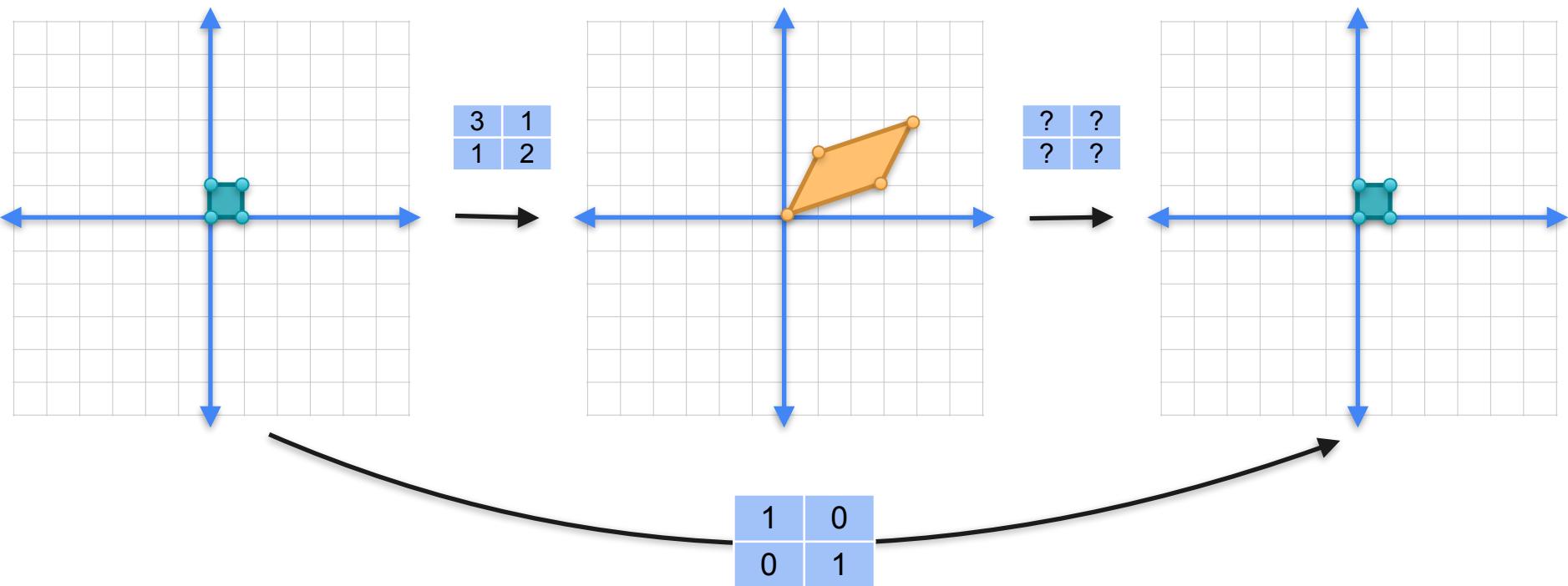
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

a	b
c	d

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1}$$

Multiplying matrices

$$\begin{matrix} a & b \\ c & d \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$
$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{matrix}$$


How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 3 \\ 1 \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left| \begin{array}{c} 1 \\ 2 \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1b = 1$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2b = 0$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3c + 1d = 0$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1c + 2d = 1$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1b = 1 \quad a = \frac{2}{5}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2b = 0 \quad b = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3c + 1d = 0 \quad c = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \left[\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \right] = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1c + 2d = 1 \quad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \\ 0 \end{matrix} \quad \bullet \quad 5a + 2c = 1$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \bullet \quad 5b + 2d = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \\ 1 \end{matrix} \quad \bullet \quad a + 2c = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \\ 0 \end{matrix} \quad \bullet \quad b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{matrix} 5 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} a \\ c \end{matrix} = \begin{matrix} 0 \end{matrix}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix} \begin{matrix} b \\ d \end{matrix} = \begin{matrix} 1 \end{matrix}$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.



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Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array}^{-1} = \begin{array}{|c|c|} \hline 0.25 & -0.25 \\ \hline -0.125 & 0.625 \\ \hline \end{array}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Which matrices have inverses?

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$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

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Non-singular matrix

Non-singular matrix

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix

Non-singular matrix

Singular matrix

Which matrices have inverses?

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

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$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

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$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

Det = 5

Non-singular matrix
Invertible

Det = 8

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 5$$

Non-singular matrix
Invertible

$$\text{Det} = 8$$

Singular matrix
Non-invertible

$$\text{Det} = 0$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Det = 0 ← Zero determinant →



DeepLearning.AI

Vectors and Linear Transformations

**Neural networks and
matrices**

AI , ML , DL , RL



Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____,
then the email is spam.

Goal: Find the best points and threshold

Lottery: ____ point

Win: ____ point

Threshold: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

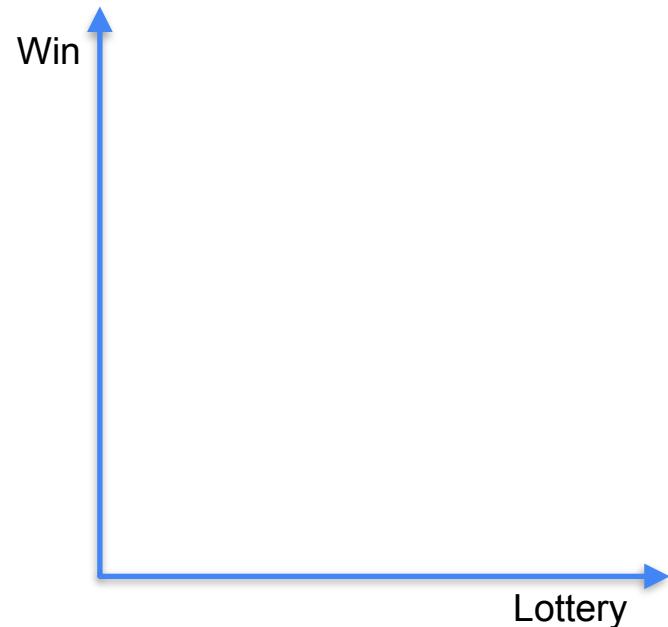
Solution:
Lottery: 1 point
Win: 1 point
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

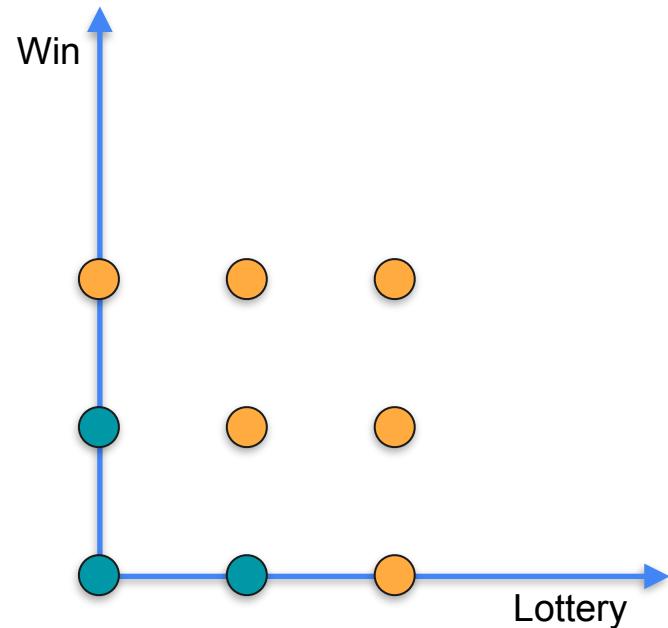
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



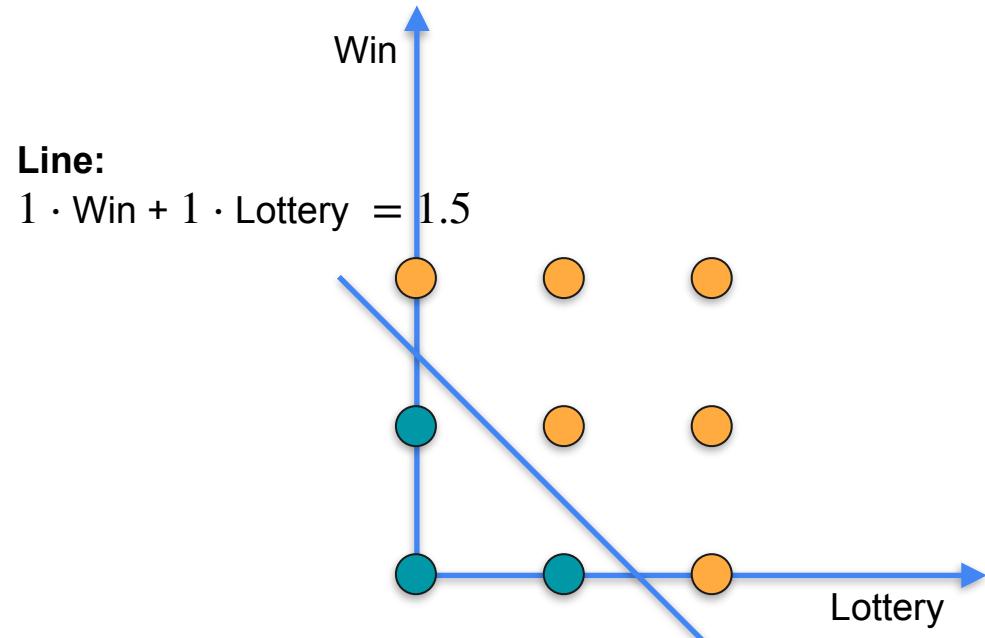
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



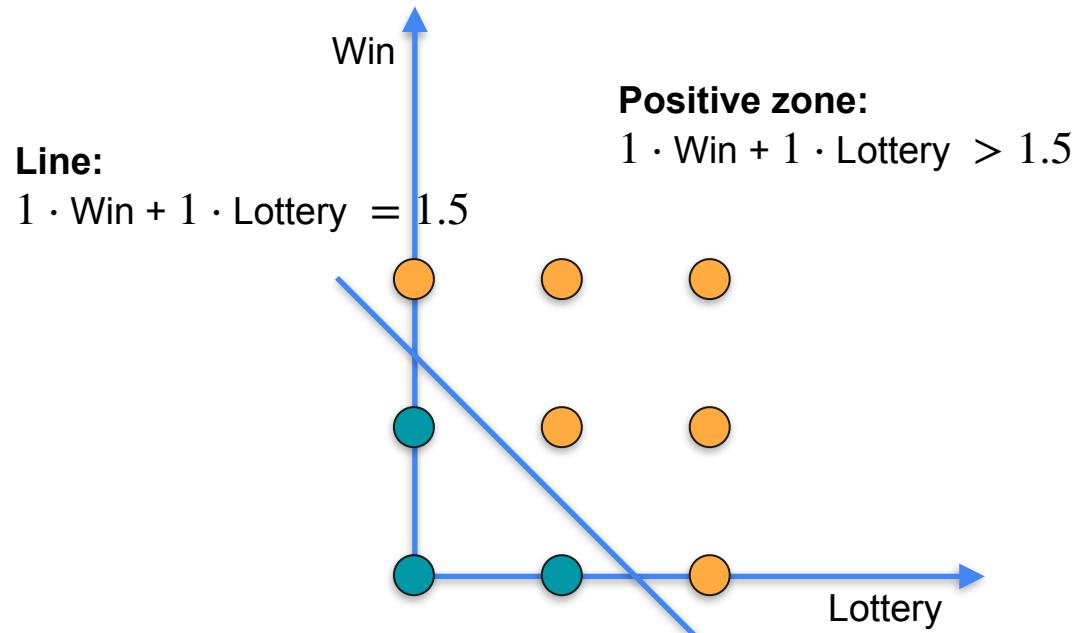
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



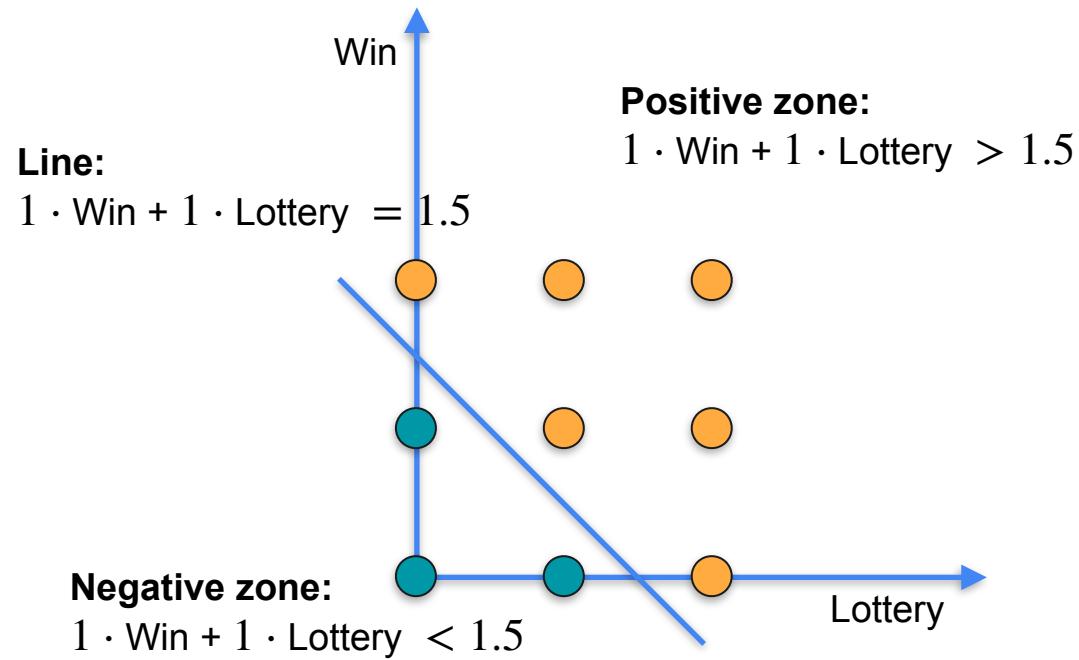
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

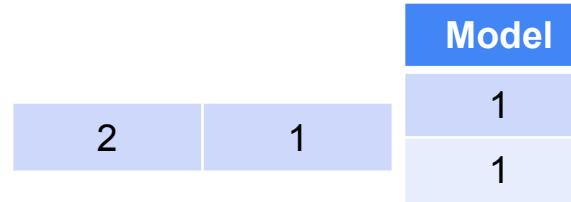
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: $> 1.5?$

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Dot product between vectors

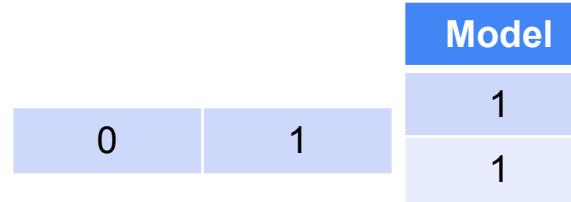
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: $> 1.5?$

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check: $> 1.5?$

$$0 \cdot 1 + 1 \cdot 1 = 1$$

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{matrix} & & \text{Model} \\ & 0 & 1 \\ \hline & 1 & 1 \end{matrix} = 1$$

Check: $> 1.5?$



Not spam

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{matrix} & \text{Model} \\ \begin{matrix} & 1 \\ & 1 \end{matrix} & = \begin{matrix} \text{Prod} \\ 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix} \end{matrix}$$

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{matrix} \text{Model} \\ 1 \\ 1 \end{matrix} = \begin{matrix} \text{Prod} \\ 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix}$$

Check: >1.5?



Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{matrix} \text{Model} \\ \hline 1 \\ 1 \end{matrix} = \begin{matrix} \text{Prod} \\ \hline 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix}$$

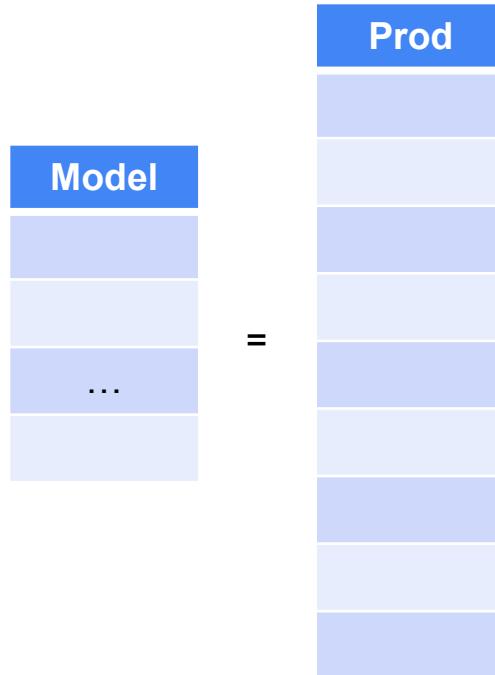
Check: >1.5?



Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Perceptrons

Spam	Word1	Word2	...	WordN
Yes				
Yes				
No				
Yes				
No				
No				
Yes				
Yes				
Yes				



Check:
→

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Model
1
1

Check: $> 1.5?$

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

Model
1
1

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

Model
1
1

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: $> 1.5?$

Model
1
1
-1.5

Bias

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check: > 0 ?

Model
1
1
-1.5

Bias

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model = Dot prod

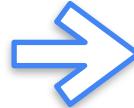
1	0
1	1
1	1
2	

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

Check: >1.5?



The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

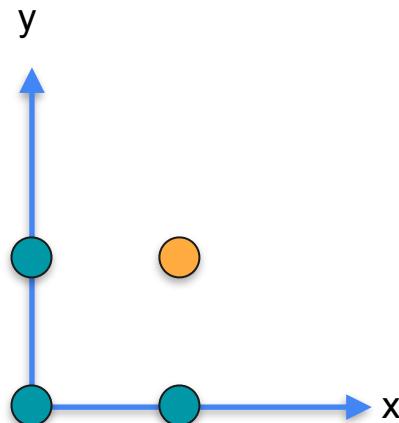
Check: $>1.5?$



Check
No
No
No
Yes

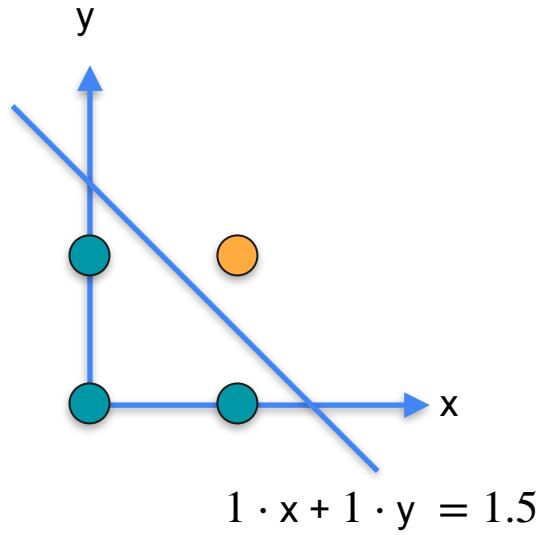
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

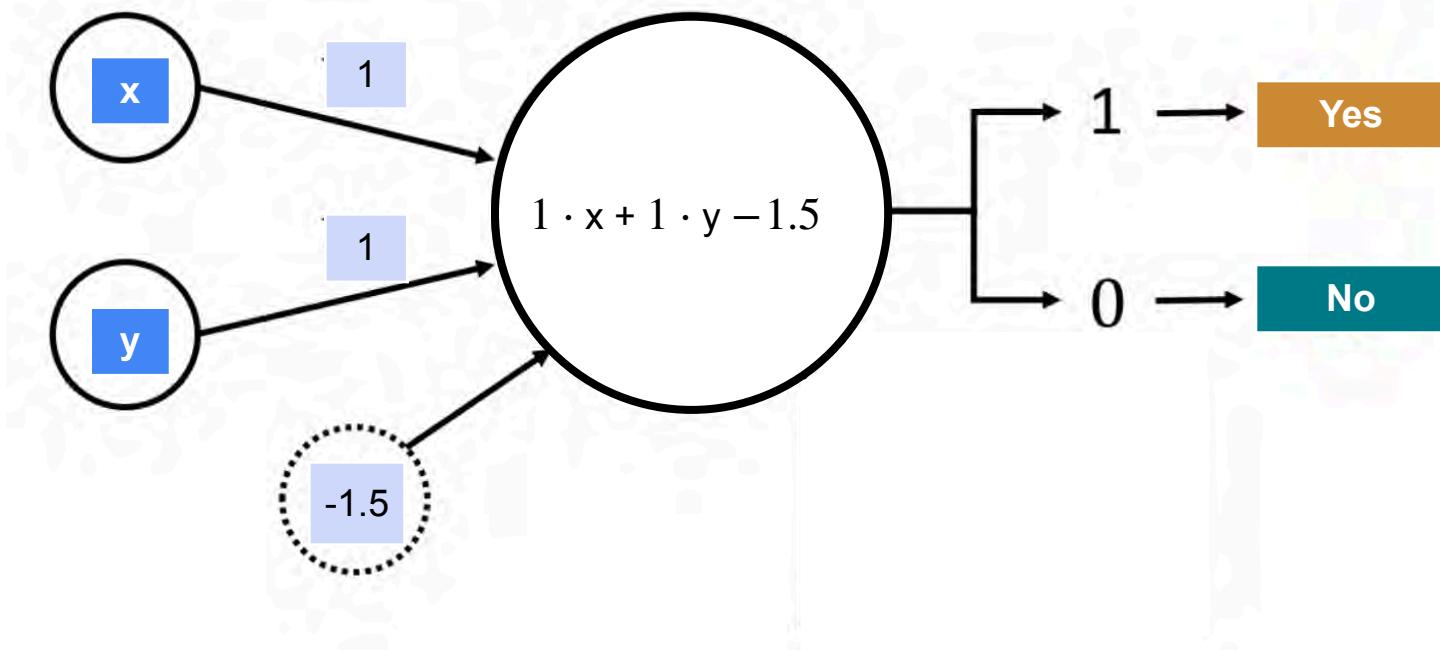


The AND operator

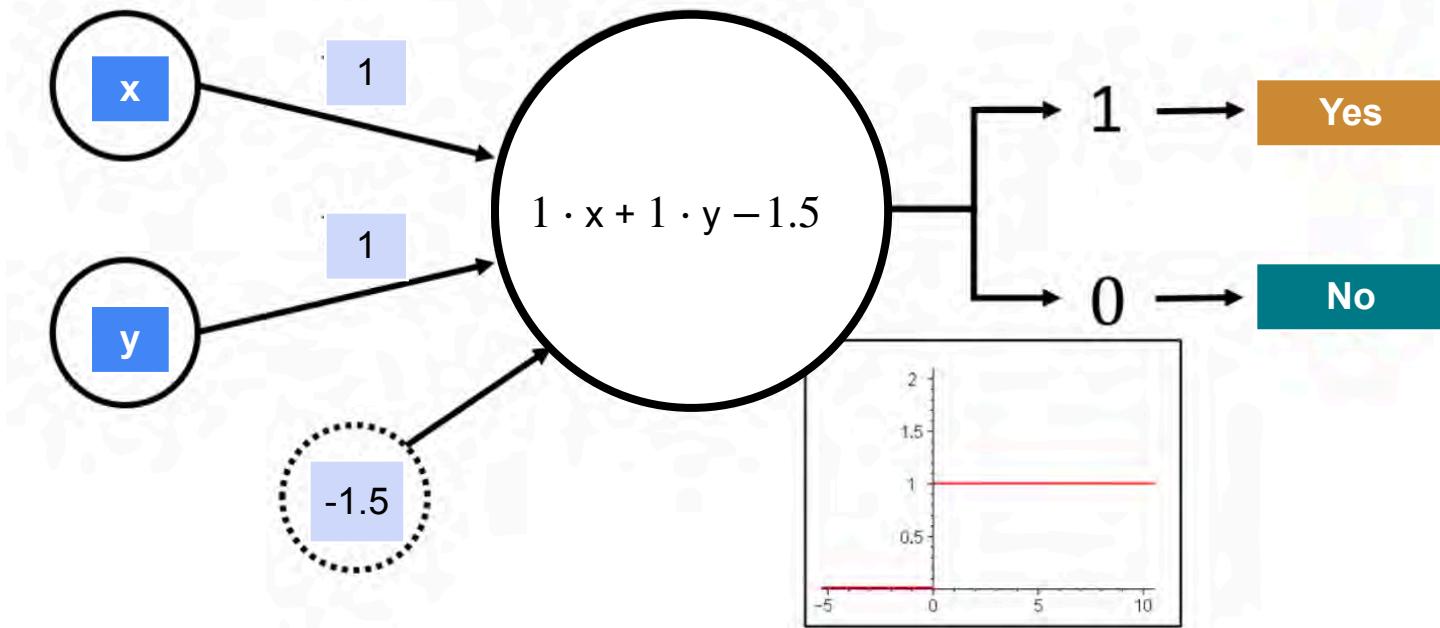
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

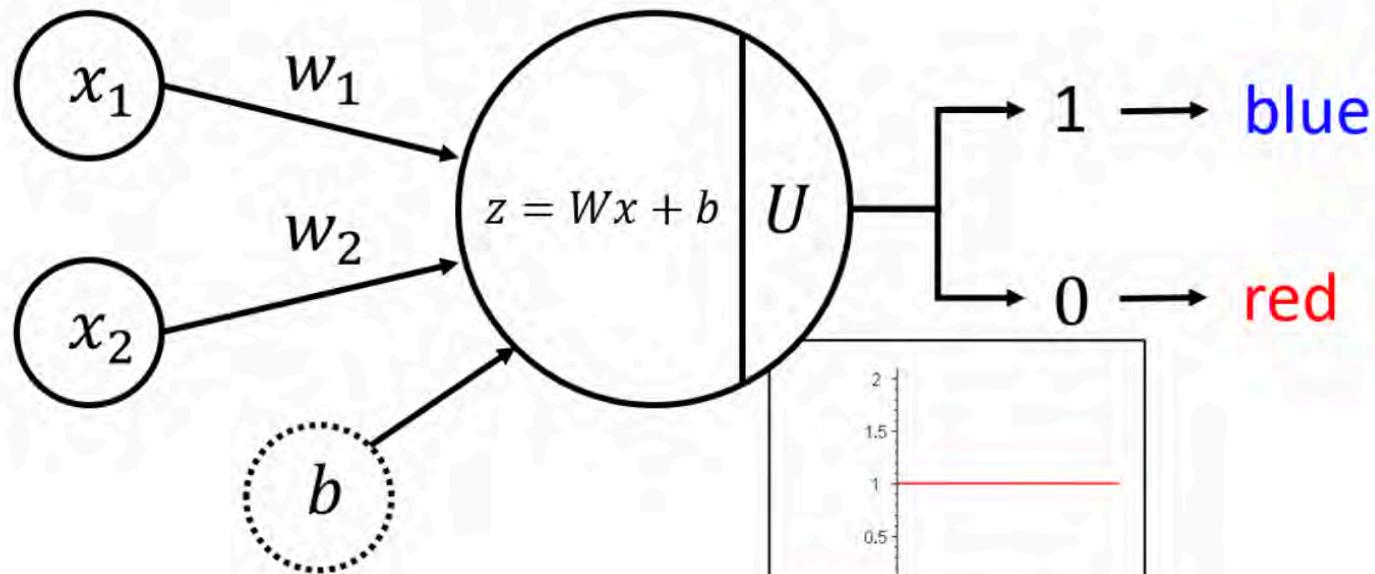


The perceptron



The perceptron







DeepLearning.AI

Vectors and Linear Transformations

Conclusion