AIMS-Rwanda

Mathematical Problem Solving- First assignment

Choose one of the following exercises. Please note that your assignment should be written using latex.

Exercice 1.

Let p be an integer and n be a positive integer. Consider the following sum:

$$S_p = \sum_{k=1}^{n} k^p.$$

1. Compute S_1, S_2 and S_3. Deduce that we have:

$$(1 + 2 + 3 + \ldots + n)^2 = 1^3 + 2^3 + \ldots + n^3.$$

2. Show by induction that S_p is a polynomial (with the variable n) of degree $p + 1$. Compute the polynomial S_5.

3. Is S_{-1} a polynomial? Justifier your answer.

Exercice 2. Find the strategy of the game we had on Tuesday in class. Give a mathematical justification of your strategies or implement an algorithm for your strategy.

Exercice 3. Using the Pigeonhole principle, prove the following claim:
`Among 10 persons, there are always 3 persons who know each other or 4 who are complete strangers.`
Is the claim still true if we reduce to 8 persons, instead of 10? Justifier your answers.