Assignment 2
Ordinary Differential Equations

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August 6, 2025

1 First order ODEs [45 pts]

A Find the general solution for each of the following first order ODE.

(a) $x\dfrac{dy}{dx} + 8y = x^2 e^x$

(b) $xy^2 - x + (x^2 y + y)\dfrac{dy}{dx} = 0$

(c) $(y \log y)dx + (x - \log y)dy = 0$

(d) $\dfrac{dy}{dx} - \dfrac{\tan y}{1+x} = (1+x)e^x \sec y$

(e) $x^3(y' - x) = y^2$

B Solve the following IVPs

(a) $y' = 3x^2(1 - e^{-y}), \quad y(0) = \dfrac{1}{2}$

(b) $\dot{x}(t) = \cos(x(t)), \quad x(0) = 0$

(c) $\dfrac{dx}{dt} + x = x^4, \qquad x(0) = 1$

(d) $y'x^6 = 1 - y', \qquad y(0) = 1$

2 Second order ODEs [30 pts]

A Find the general solution to the following ODEs.

(a) $(1+x^2)y'' + 2xy' + \dfrac{1}{1+x^2}y = 0$

(b) $y^{(iv)} + 9y'' = t$

B Solve the following IVPs

(a) $y'' - 8y' + 15y = 2e^{3t}, \quad y(0) = y'(0) = 0$

3 What is Mathematics without proofs? [10 pts]

1. Show that the differential equation below does not have a unique solution.

$$\dot{x}(t) = \sqrt{x(t)}, \quad x(0) = 0$$

. ***Hint:*** *Assume the solution is $x(t) = ct^n$, where $c \in \mathbb{R}$, and $n \in \mathbb{N}$.*

2. Show that it is impossible to find continuous functions $a_0, a_1 : \mathbb{R} \to \mathbb{R}$ so that the differential equation

$$\ddot{x}(t) + a_1\dot{x}(t) + a_0 x(t) = 0,$$

has a solution $x(t) = t^2$.

4 Let's look at a real world application [15 pts]

Consider the following mathematical model for epidemics. Assume that there is a community of N members with I infected and U uninfected individuals, so $U + I = N$. Define a ratio $x = \frac{I}{N}$ and $y = \frac{U}{N}$ and assume that N is constant and so large that X and y may be considered as continuous variables. Then we have $x, y \in [0, 1]$ and

$$x + y = 1.$$

Denoting time by t, the rate at which the disease spreads is $\frac{dx}{dt}$. If we make the assumption that the disease spreads by contact between sick and healthy members of the community, and if we further assume that both groups move freely among each other, we arrive at the differential equation

$$\frac{dx}{dt} = \beta xy,$$

where β is a real and positive constant of proportionality.

1. Combine equation and to derive a differential equation for $x(t)$.
2. Find the solution of this differential equation for $x(0) = x_0$.
3. Show that $\lim\limits_{t\to\infty} x(t) = 1$ if $x_0 > 0$ and interpret this result.
4. Is the model of epidemics studied here realistic? If not, what is missing in it?

Some freebies... [20 pts]

(a) Solve the DE

$$f''(x) + f(-x) = x + \cos x, \forall\, x \in \mathbb{R}.$$

(b) Solve the IVP

$$y' = \max\{1, y^2\}, \quad y(0) = 0.$$