

AIMS-Rwanda

Mathematical Problem Solving- Second assignment

Choose TWO of the following exercises. Please note that your assignment should be written using latex.

Exercise 1. Let us have n different colors. Suppose we have

$$p_n = E(e(n!)) + 1$$

points on the plain and each pair of points is connected by a line colored with only one of the n colors. Show that there is at least one triangle with sides of the same color.

Exercise 2. Consider a square grid as in the figure below; the distance between consecutive parallel lines is 1. The points of intersection (which are precisely the points whose coordinates are integers) are called *lattice points*. Now we draw a polygon whose vertices are lattice points. Let p be the number of lattice points inside the polygon, and q the number of lattice points on the boundary of the polygon. In the example, $p = 18$ and $q = 9$.

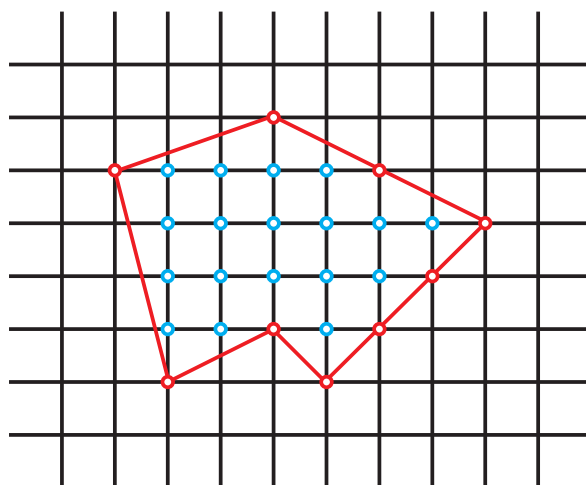


Figure 1: A polygon on a square grid.

Given p and q , what can be said about the area of the polygon?

Exercise 3.

Suppose that you have an infinite supply of coins of value p units, and an infinite supply of coins of value q units, where $p > 0$ and $q > 0$. You wish to give some money to a friend. Which positive amounts of money can you give?

hint: Be careful, you should distinguish the case where p and q are coprime or not.

Exercise 4. there are n points in the plane. they are not collinear. Show that there is a line that passes through just two of these points.

What happens if we have infinitely many points?