

Probability and Uncertainty

Vocabulary

Event - A collection of outcomes of a procedure.

Simple Event - A single outcome.

Compound Event – A combination of simple event.

Sample Space – All simple events (all possible outcomes).

Example

Procedure	Event	Sample Space
Flip a coin once	H	{ H, T }
Flip a coin 3 times	1 Head & 2 Tails	{HHH, HHT, HTH,HTT, TTT, TTH, THT, THH}

Probability

Def: The likelihood of an event occurring.

Probability : P

Event : A, B, C, \dots Etc

$P(A)$ = the probability of event 'A' occurring

3 types of probability

Observed probability – its estimated probability based on observations.

$$P(A) = \frac{\text{\textit{\# of times A occurred}}}{\text{\textit{\# of times procedure was repeated}}}$$

Classical Probability - Its based on the chance of an event occurring (each simple event must have an equal chance of occurring)

$$P(A) = \frac{\text{\textit{\# of ways A could occur}}}{\text{\textit{\# of simple events (outcomes)}}$$

Subjective Probability (educative guess)- based on past experience and education

Examples

Q1. Find the probability of selecting a Heart  from a deck of cards.

$$P(\text{♥}) = \frac{13}{52} = 0.25 = 25\%$$

Q2. If you flip a coin 100 times, and you get 64 Tails.

$$P(T) = \frac{64}{100} = 0.64 = 64\%$$

Q3. Find the probability that if a couple has 3 kids, two will be boys (Assuming equal chance of Boy/Girl).

Procedure : Having 3 children

Event : 2 boys, 1 girl

Sample space: {BBB, BBG, BGB, BGG, GGG, GGB, GBG, GBB}

$$\begin{aligned} P(2B,1G) &= \frac{3}{8} \\ &= 0.375 \\ &= 37,5\% \end{aligned}$$

Rules of Probability

$P(E)$: The probability that event E occurs.

$$0 \leq P(E) \leq 1 \quad \text{or} \quad 0\% \leq P(E) \leq 100\%$$

Complementary Events

Events which are mutually exclusive (can't happen at the same time).

For example: When you roll a 6 sided dice once can you get both a 2 and a 5 at the same time ? No because they cannot happen at the same time.



Complement

The complement of an event A is denoted by A^c (or A')

If we have an event A , then A^c (or A') is all the outcomes that do not accomplish event A or all the outcomes when event A does not occur.

Example

Event: Rolling a 5

Complement: not rolling a 5

Q1. What is the probability of getting a 5 after rolling a dice once

Q2. What is the probability of not getting a 5 after rolling a dice once

$$P(5) = \frac{1}{6}$$

$$P(5^c) = \frac{5}{6}$$

Note: The probability of an event + the probability of the complement must equal to 1

$$P(A) + P(A^c) = 1$$

Exercises on simple probability

1. When you roll a 6 sided die, which number are you most likely to get?
2. If you roll a die 600 times, how many sixes would you expect to get?
3. Use the following to describe the statements bellow: certain, very likely, likely, unlikely, very unlikely, impossible. [Zimbabwean events]
 - a) It will snow tomorrow
 - b) It will rain tomorrow
 - c) The sun will not rise tomorrow
 - d) Jesus will come next week
 - e) You will win a car in a competition today
 - f) You will pass all part one courses at NUST

cont

4. In February 1995 it rained on 18 days. Calculate the probability that it will rain on a day in February. [Relative frequency of rain]
5. A six-sided die and a coin are tossed. List all the possible outcomes.
6. In a class of 30 Computer Science students at NUST, 16 are ladies, 4 wear glasses and 3 are left – handed. A student is chosen at random from the class. What is the probability that this student is:
 - a) A lady
 - b) A gentleman
 - c) Right – handed
 - d) Left - handed
 - e) Wearing glasses
 - f) Not wearing glasses
7. A card is taken at random from a full pack of playing cards with no jockers. What is the probability that the card:
 - a) Is an ace
 - b) is Black
 - c) is a heart
 - d) has an even number on it