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SynerGEE Electric Technical Reference

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General Topics

Introduction to SynerGEE Electric

SynerGEE® Electric is a software package developed by Advantica Stoner designed to aid you in the simulation, analysis, and planning of power distribution feeders, networks, and substations. The package is a modular collection of tools built on a by-phase simulation engine. The simulation engine is based on an object-oriented design consisting of highly detailed models for power system devices such as lines, transformer banks, regulator banks, switched capacitors, active generators, and others. The models for these devices are built to reflect the actual construction of real power system equipment. Likewise, the usability and capability of SynerGEE demonstrate the level of commitment involved in producing quality analysis software.

SynerGEE is very user-friendly and allows you to quickly construct and maintain distribution system models. The tools, utilities, and features such as graphical editing tools were designed to help you get feeders modeled quickly and accurately. Data requirements are clearly marked in dialog boxes and are kept to a minimum so that the models can be specified with basic nameplate parameters.

The device models and calculations used within SynerGEE conform to those methods accepted and depended upon by the power engineering community. Advantica Stoner relies on books, manuals, and technical papers describing algorithms that have been tested and well-proven. Furthermore, Advantica Stoner is advancing distribution analysis technology in many areas such as device modeling, Delta-modeling, by-phase looped load-flow and fault analysis, and generator simulation.

SynerGEE supports an enhanced load model that allows the apportionment of loads into three classifications. This model provides a better tool for you to create more accurate representations of a distribution system by having control over the way that loads respond to their voltage level. Loads can be connected between phases and to ground. A variety of tools is supplied to allocate loads and simulate growth.

More than 400 engineers and technicians at utilities around the world use SynerGEE Electric. Advantica Stoner software consistently represents your needs. SynerGEE demonstrates Advantica Stoner's commitment to provide the most accurate, reliable, and easy-to-use software designed to meet your engineering and operational needs.

SynerGEE is capable of modeling four types of systems.

- **Radial systems** – Sections and loads fed from feeders and substation transformers
- **Looped systems** – Otherwise radial systems with ten to twenty loops using "looped tie switches"
- **Wandering laterals** – Single phase takes off from trunk, feeds load, and rejoins the trunk to serve downstream load
- **Networks** – Densely meshed downtown, subtransmission, or transmission networks

The electric model for radial and looped systems

Feeder and substation models, as well as unfed areas or "islands," are built with sections and nodes. In SynerGEE, a node is a named spatial point, while a section is defined by a unique name and must also be associated with a from-node and to-node. A section's from-node and to-node never change with the direction in which a section is fed. Here is an example of a source feeder with a loop broken by a couple of switches:

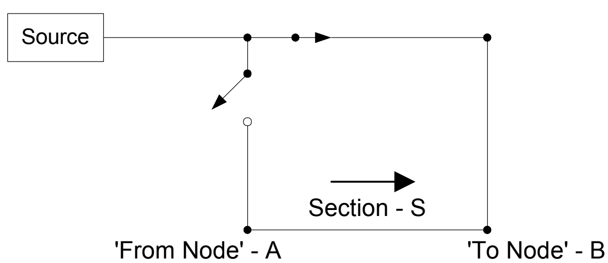


Fig. 1-1 *Section S being fed from left*

If you toggle the switches and feed the section from the other direction, you get the following.

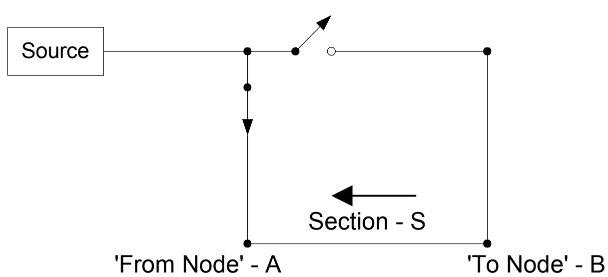


Fig. 1-2 *Section S being fed from right*

The following is the status of the nodes pertaining to section S for each of the switching arrangements:

	Feed from left	Feed from right

From-node	Node A	Node A
To-node	Node B	Node B
Source node	Node A	Node B
Load node	Node B	Node A

The from-node and to-node of the section do not change. The source node and load node may change depending on how the section is being fed.

The network topology is set by the relationships between sections and nodes. The network electrical connectivity is determined from the topology and the status and position of switches.

Nominal voltage

Nominal voltage is a key SynerGEE concept. Voltage values are nearly always listed on the voltage base that you select. Consider this feeder with voltages shown as kV values.

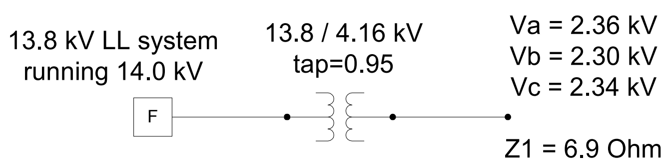


Fig. 1-3 *Feeder serving load through a transformer*

As an example, assume a voltage (user) base of 120V, with the nominal voltage at the feeder of 13.8kV. Since the feeder is running at 14.0kV, the feeder voltage is:

$$V_F = \frac{14.0}{13.8} \cdot 120V = 121.7V \tag{1-1}$$

Transformer taps

For the transformer, SynerGEE has two ways of handling the tap. SynerGEE can ignore the transformer taps or it can alter the nominal voltage past the transformer to recognize the taps.

Taps ignored

If the taps are ignored, the nominal voltage on the secondary of the transformer is:

$$V_{sec\text{-}nom} = 13.8 \frac{4.16}{13.8} = 4.16kV \tag{1-2}$$

The voltages expressed on the nominal are therefore:

$$V_a = \frac{\sqrt{3}\cdot 2.36}{4.16}\cdot 120V = 117.9V$$
$$V_b = \frac{\sqrt{3}\cdot 2.30}{4.16}\cdot 120V = 114.9V$$
$$V_a = \frac{\sqrt{3}\cdot 2.34}{4.16}\cdot 120V = 116.9V \tag{1-3}$$

Taps recognized

With taps recognized, the calculation of nominal voltage is as follows.

$$V_{sec\text{-}nom} = 13.8\frac{4.16}{13.8}\cdot\frac{1}{0.95} = 4.38kV \tag{1-4}$$

Now, the end-of-line voltages expressed on the 4.38kV base are:

$$V_a = \frac{\sqrt{3}\cdot 2.36}{4.38}\cdot 120V = 112.0V$$
$$V_b = \frac{\sqrt{3}\cdot 2.30}{4.38}\cdot 120V = 109.1V$$
$$V_a = \frac{\sqrt{3}\cdot 2.34}{4.38}\cdot 120V = 111.0V \tag{1-5}$$

The voltage in "volts" listed for the end of the line is very different between the case when transformer taps are considered and when they are not considered in nominal voltage evaluation.

Effect on fault current

When looking at voltage from load-flow, the actual kV voltage is the same in both cases. Fault current is different. You can specify the use of pre-fault voltage or nominal voltage in fault analysis. If nominal voltage (the default in SynerGEE) is selected, fault currents can vary significantly due to transformer taps.

In the case where transformer taps did not affect nominal voltage:

$$I_{f\text{-}3ph} = \frac{1000\cdot\sqrt{3}\cdot 4.16}{6.9} = 1044A \tag{1-6}$$

If the taps affect nominal voltage:

$$I_{f\text{-}3ph} = \frac{1000\cdot\sqrt{3}\cdot 4.38}{6.9} = 1099A \tag{1-7}$$

Care should be taken to ensure that transformer taps are set correctly and that the choice of using taps in nominal voltage propagation has been established.

Network models in SynerGEE

SynerGEE supports network load-flow and network fault applications. These:

- Form a network model from the distribution model
- Solve the network model
- Populate results into the distribution model

The network model is bus-based; that is, all loads are modeled on the nodes. Artificial nodes are generated for equipment like capacitors, generators, and transformers.

Loads are handled in the following manner:

- **Distributed loads** – Half of the load is applied to each of the section nodes.
- **Spot loads** – If the load is based in the center of the section, half of the load goes to each node. Otherwise, the entire load is applied to the node specified by the spot load.

Open switches that are marked as "loop tie switches" or "wandering lateral tie switches" are considered closed during network analyses. It is recommended that no load is place on sections with open switches to avoid confusion between the network representation of the load and the normal SynerGEE representation of the switched section.

Node Reduction

Often, the number of nodes (or *granularity*) of a distribution model extends beyond the detail needed for engineering analysis. An excessive number of nodes in a model can lead to a variety of problems, including:

- Data clutter
- Slow analysis
- Unwieldy reports
- Difficulty spotting trends in voltage and loading
- Hard-to-read results annotations on the map

For such a model, SynerGEE node reduction can generate a model that is nearly identical to the original but with fewer nodes. Models reduced with SynerGEE are geo-spatially identical to the full models, with power flow, voltage, and fault values that nearly match the values in the original model. In most cases, only slight variation can be expected.

Key features

Node reduction provides the following major features:

- Pre-reduction model and data validation
- User constraints and voltage limit checking
- Topologically identical model before and after
- Lookup table to map full model to reduced model

General operation

The application is designed to reduce the number of sections by combination, moving from the smallest sections to the largest. The resulting reduced model should have less clutter and the important areas should be maintained. You can customize combination constraints, including the following.

- **Total Length**—The maximum section length allowed following a combination.
- **Connected kVA & kWh**—The total connected load on a combined section.
- **Combined Nodes**—The maximum number of nodes that can be eliminated on a combined section.
- **Voltage Drop**—The maximum total voltage drop allowable on a combined section.

Following the operation, you have the option of making the reduction permanent, or simply producing a report of the findings. In either case, the reporting is comprehensive, including:

- A list of nodes maintained, and why
- A list of nodes eliminated, and why
- A lookup table to reference deleted nodes to the combined sections where they were formerly located

The lookup tables are key tools for linking results from the reduced model to the full model.

Example

Shown below is a complete feeder generated from a GIS extraction. Although this model does not have nodes for each pole, many models generated from a GIS do.

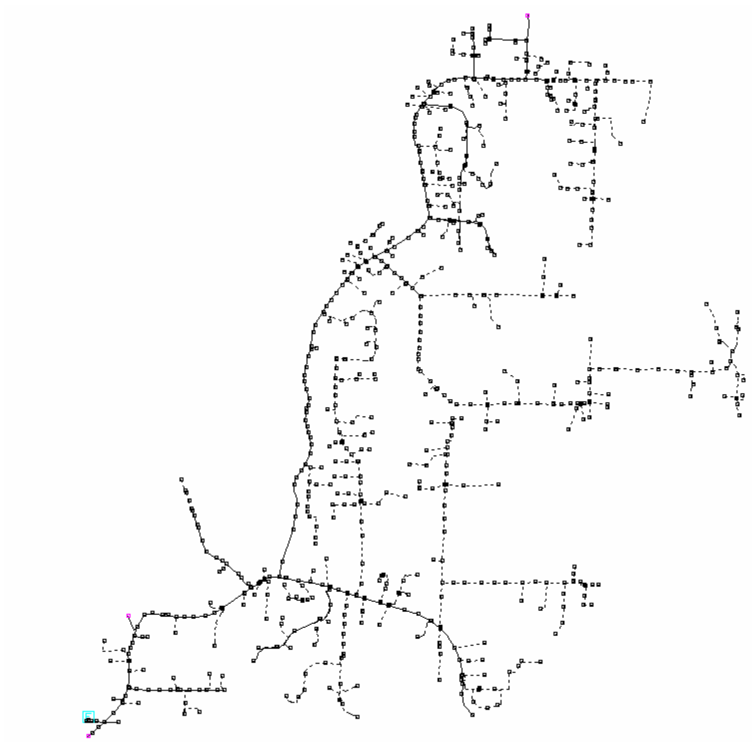


Fig. 1-4 Full model

The following is the model after node reduction eliminated 35 percent of the nodes.

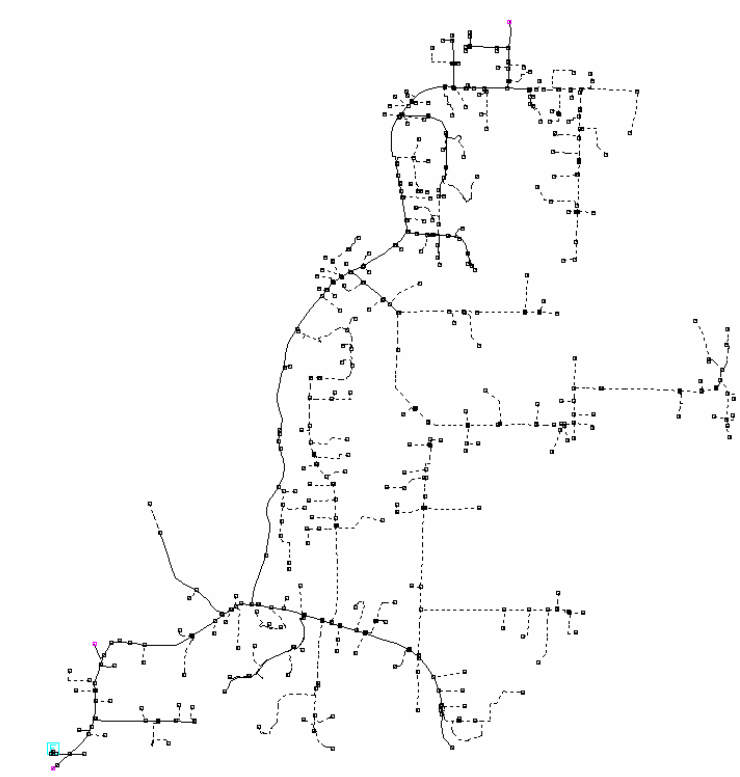


Fig. 1-5 Reduced model

Notice that the shape or spatial topology of the feeder is identical between the full and reduced model. In fact, if node display is turned off, the reduced model lays precisely over the full model, as they are indistinguishable.

SynerGEE is a map-oriented product. Node reduction is therefore designed to have no impact on the "map" of a feeder. Consistency in the spatial topology is achieved by inserting graphic points in place of eliminated nodes, as explained in the next section.

Removing nodes

Graphic points (sometimes referred to as vertices, intermediate points, contour points, or graphic nodes) are put in the place of eliminated nodes to maintain the shape of the feeder. Consider the following example, starting with two sections.

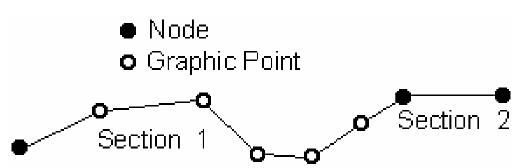


Fig. 1-6 Two sections

If the node separating the sections is eliminated, it is replaced with a graphic point.

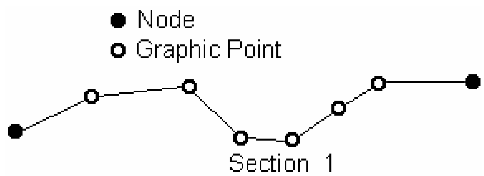


Fig. 1-7 One section resulting from reduction

The result is one section with the same shape as the original two.

Reduction rules

Node reduction is a rule-based application. When the application starts, all nodes are reduction candidates. The following rules are evaluated for each node. If the node violates any of the rules, it cannot be eliminated.

Rule	Description
NRR_01	Miscellaneous reduction constraint. SynerGEE was unable to eliminate the node for some unspecified reason.
NRR_02	Node incidence exceeds 2. Nodes with more than two connected sections can not be eliminated. Only nodes with two incident sections are reduced.
NRR_03	Node forms an edge. Nodes tied to just one section cannot be eliminated.
NRR_04	Configuration change. If the two sections connected by a node have different phasing, spacing values, conductor names, or height, the node is not eliminated.
NRR_05	Exceeding user defined length limit. If elimination of the node would form a combined section whose total length exceeds your specified constraint, the node is kept.
NRR_06	Exceeding user defined kVA limit. If elimination of the node would form a combined section whose total kVA exceeds your specified constraint, the node is kept.
NRR_07	Exceeding user defined kWh limit. If elimination of the node would form a combined section whose total kWh exceeds your specified constraint, the node is kept.
NRR_08	Exceeding user defined voltage drop limit. If elimination of the node would form a combined section having a scalar voltage drop total that exceeds your specified constraint, the node is kept.
NRR_09	Exceeding user defined combined node limit. The number of eliminated nodes used to form combined sections is tracked. If the elimination of the node would cause the count for the combined section to exceed your specified constraint, the node is kept.
NRR_10	Topology error prevents the elimination of the node.

Rule	Description
NRR_11	Source nodes – feeders and substation transformers is not eliminated.
NRR_12	If either section incident to the node holds devices like regulators or fuses, the node is not eliminated.
NRR_13	If the node forms a tie point between multiple feeders, it is not eliminated.
NRR_14	Load model I, Z, and PQ values are checked on each side of the node. If values vary by more than a fixed 1.5 percent, the node is not eliminated.
NRR_15	If the load growth rate on either of the incident sections differs by more than 0.15 percent, the node is not eliminated.

Detailed operation

The node reduction analysis performs the following steps.

- Check model data
- Run load-flow
- Evaluate all nodes against rules
- Find shortest section incident to a reducible node
- Evaluate reducible node
- If node can be reduced, combine sections and loads, eliminate the node
- Repeat from step 4 until no more reduction is possible

This algorithm is very simple and effective. The existing rule base is set up to be conservative in the reduction, and reductions of 10 to 60 percent are normal. If you experience a reduction of more than 60 percent, consider making your constraints more restrictive.

Load handling

When a section is eliminated in the process of reducing a node, the load, length, and other data for that section is added to the combined section.

The following example shows a group of sections with load values indicated.

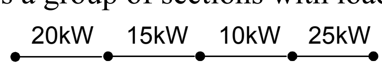


Fig. 1-8 Sections with load

If the three center nodes are eliminated, the combined section yields:

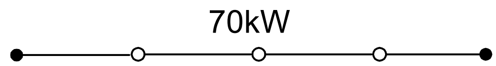


Fig. 1-9 Reduced model

Distributed loads are modeled in the center of the section. Therefore, reduction can cause minor changes to the model due to the combination of load and shifting of location. Nonetheless, extensive testing has shown that loading and losses are nearly identical between the full and reduced models. The minimum and maximum voltages on the feeder are also nearly identical between the full and reduced models.

Ideal models with evenly distributed constant current loads tend to have higher loss values in the reduced model. However, the voltage drop to the end of the last line is the same after the reduction. The issue becomes more complex when I, Z, PQ load models are introduced and when detailed by-phase analysis is performed with coupling. If you utilize SynerGEE in this manner, you should do further research before using node reduction.

Node Overlap report

This tool scans your model and reports on any nodes that are within five feet of each other. It is designed to help you find node-related connectivity errors in model data. By searching for nodes within such a close proximity, the tool can help find problems in GIS-extracted data. If two nodes are very close and one is unfed, there may be a connectivity problem in the data, with roots in the GIS extraction.

Overlapping Nodes

Node 1		Node 2		
Name	Feeder	Name	Feeder	Distance
2640	Ship - South Penn	2644	Ship - South Penn	3.0
6624	Ship - West Martin	6625	Ship - West Martin	1.0
8392	Ship - South Penn	Node_0006	Ship - South Penn	3.0
8397	Ship - South Penn	Node_0005	Ship - South Penn	1.8
Carlisle Xfmr 1	*** Source ***	Node_319285805	Carlisle Xfmr 1	2.7
Carlisle Xfmr 1	*** Source ***	Node_319286887	Carlisle Xfmr 1	3.3

Fig. 1-10 *Report showing nodes within five feet of each other*

The following is a map image representing the area circled in the previous report.

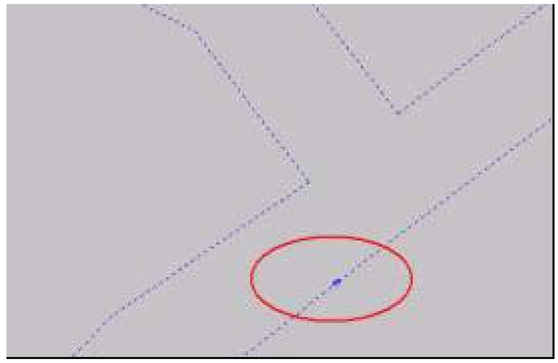


Fig. 1-11 *Map of nodes listed in the report*

In this example, the nodes, representing cable terminations, are probably correctly placed in the model, and no connectivity corrections need to be made.

However, here is an example of an area exposed by the report that may represent a connectivity problem.

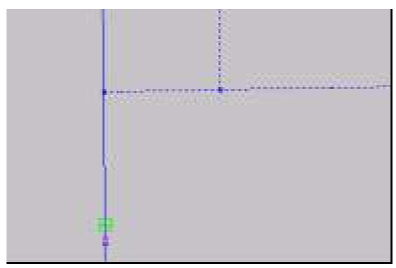


Fig. 1-12 *Proximal nodes representing connectivity problem*

Two nodes are very close and an open switch is tied to one. Operating the switch has no effect. A single node should probably replace the two nodes. This would connect the sections and allow proper modeling of the switch.

Data management within SynerGEE

SynerGEE uses Access database files for a data repository. Model instance tables, usually prefixed with "Inst," and equipment or device tables, usually prefixed with "Dev," are used to store data. The device data, often referred to as the *equipment* database, holds nameplate-type information about conductors, regulators, etc. Instance tables, or the *model* database, hold references to device types as well as the particular parameters and settings for the specific device. In addition, the model database contains all section and node spatial data, such as lengths and coordinates.

SynerGEE can also load and save data to XML files. This text file format is useful in web-based environments or in situations requiring application integration.

SynerGEE has many tools to help you manage data, some of which are explained below.

Loading data

When loading from a database, SynerGEE loads data into a temporary area for validation. If no problems are found or if you accept any existing data problems, the model in the temporary area is merged with the model in memory.

To assist you with evaluating data problems, a conflicts report is generated during the load. This report lists data read from the database that maps to a comparable entity already in memory. For example, if a section in the database already exists in the model in memory, it is reported. If you choose to accept the report and complete the load, data in memory is kept and the conflicting database data is not loaded.

Saving data

When saving data, SynerGEE starts a database transaction and saves all of the data. Any problems are reported to the user. The user is asked to commit the transaction to complete the save. If you choose not to commit the save, the transaction is rolled back and the database is unaffected.

If you commit the save, delete SQL queries are constructed based on the feeders that are going to be saved. These queries are run on the target database resulting in the deletion of any data associated with the names of the feeders that will be saved. The selected feeders are then saved. Because of the deletion, SynerGEE expects to only create new database records. If a record is found in the database that corresponds to an entity being saved, a message is generated indicating that an unexpected update was found.

Compare tool

The data comparison tool generates a report of differences between a model in memory and a database on disk. The tool can review both model and equipment tables. This tool is only useful if you are comparing a complete database with another complete database. Therefore, you should have an entire database loaded into memory before making the comparison, otherwise a lengthy list of messages for all entities not loaded could be generated.

The comparison report provides a detailed list of:

- Data elements in memory, but not on disk
- Data elements in the database on disk, but not in memory
- Data elements shared by both databases but have different field values

Defaults & Limits database

SynerGEE supports a separate, optional database that contains defaults and limits for certain parameters. Defaults are used during model building to supply missing data, and limits (or ranges) are used during analysis for validation. If a parameter exceeds the limits you assign, a warning message appears on the analysis reports.

You can decide whether or not you want to load a defaults database when loading databases. In addition, you can enable and disable individual defaults and limits from within the database, using Microsoft® Access.

Scenarios

SynerGEE supports the loading and saving of scenarios, which are stored in the model database in separate scenario tables. This function allows you to make changes to a model and save just those changes, without having to do a complete save to a different database.

SynerGEE generates scenarios based on a comparison of the model in memory with the database on disk. Any differences are assumed to be a change, and are incorporated into the scenario. Once generated, the scenario is stored in special instance tables with a "Scenario_" prefix. Consider the following diagram.

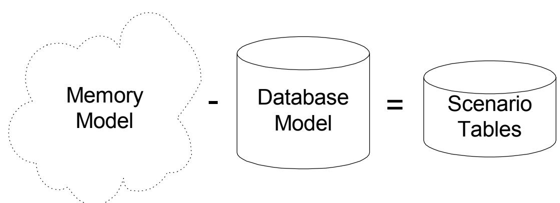


Fig. 1-13 *Creating a version*

After the scenario has been saved, the base model can be reloaded from the database (after closing the old model without saving). At this point, you are back where you started, with a scenario saved in the database. If you want to keep the scenario isolated from the base model, be sure not to save the changes to the base model after saving the scenario. Otherwise, those changes will be incorporated permanently into the base model.

Afterwards, scenarios can be loaded into the model and used for analysis. The records in the scenario table are simply loaded and added to the model. Consider the following diagram:

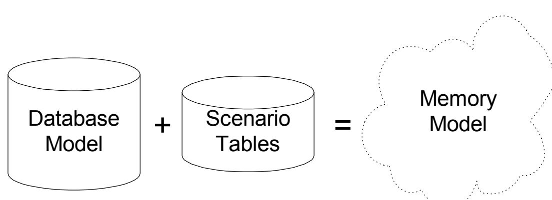


Fig. 1-14 *Loading a version*

You can load multiple scenarios. However, care should be taken when scenarios represent changes to the same facilities. If a scenario contains data that conflicts with data in memory, conflicting data is not loaded. Again, remember not to save changes to the base model after loading a scenario, unless you want that scenario to become a permanent addition to the base model.

When generating a scenario, keep in mind that the comparison is made between the memory model and the database to which the scenario is being saved, not simply the database that was originally loaded. If you try to save a scenario to a database other than the original, you may end up with a very large record count, due to compatibility problems. A scenario generated in this manner will not likely be of any use.

In addition, remember that scenarios are generated based on a simple comparison, and are not handled as individual entities. Generally, you should reload a base model after saving a scenario, and especially before creating another. For example, consider loading a model, making changes, and then saving scenario "A". Next, more changes are made and scenario "B" is saved. Scenario B will consist of all the changes made since the base model was loaded, not just the changes since Scenario A was saved. In other words, Scenario B will contain all of Scenario A and any changes made since.

Data truncation during a save

SynerGEE may need to truncate data during a save, based on maximum string lengths allowed by the target database.

Saving to an existing database

When SynerGEE saves data to an existing database, it:

- Queries the target database for the maximum text field lengths
- Truncates SynerGEE data according to those lengths, if necessary
- Sends a warning message about any truncations to a conflicts report

When truncating, string lengths that you established within SynerGEE are not considered. For example, you could select node name length values of 100 characters, but SynerGEE will still truncate according to the target database, as needed.

In rare circumstances, truncation could result in a loss of uniqueness between identifiers, if the strings differ only past the point of truncation. If you need longer maximum string lengths, you can use Microsoft® Access to manually extend the fields, then resave the database in SynerGEE.

Saving to a new database

When SynerGEE generates new tables in a database, it uses internal values to establish maximum lengths for text fields. In most cases, the maximum string length is set to 32. Some previous versions of SynerGEE typically allowed only 15 characters. If you are loading from and saving to an old database, you may see warnings if you change your text strings to exceed 15 characters. You can eliminate this problem by saving to a new database.

Device IDs and names

Devices such as capacitors and regulators have names or descriptions, which show up on reports and lists. Unlike FIDs and node names, however, they are not required to be unique and serve no function in topology or in the database. They are available simply for your convenience.

Versions

This document reviews SynerGEE versioning concepts and how versions can be used to integrate GIS updates with engineers' models.

What is a version

A version is a list of records, from the SynerGEE Electric schema, that are marked as "adds", "deletes", or "modifies". When a version is loaded into SynerGEE, the "add" records are added to the model in memory. Data from the "modify" records replaces in-memory data for the corresponding facility. Records marked as "delete" will result in the elimination of the facility from memory.

Versions are useful for capturing the "delta" between a model stored in Access, Oracle, or other sources and a model in memory.

A tool for the engineer

Versions are designed as a SynerGEE tool to be used by the engineer to capture changes and additions that were made as a part of a study. Capturing additions, modifications, and deletions made by the engineer will allow that design work to be applied to a fresh model brought in from the utility's GIS or other source.

A good approach is to build the initial engineering model from GIS data.

Periodic GIS model build example

There are a couple of ways that versions can be used to manage workstation updates from periodic GIS model builds. Here is an example where a fresh model is built from the GIS every week:

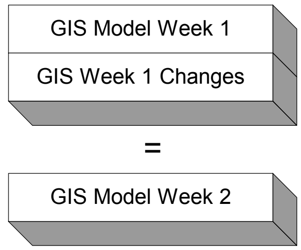


Fig. 1-15 Weekly model build

An engineer got the GIS generated model from week 1. After a week of work, the model has changed:

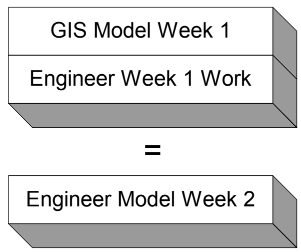


Fig. 1-16 Engineer's model after working a week

There are a couple of ways that the GIS model build for the next week can be integrated with the engineer's model.

Approach 1

This approach will work for models that can be loaded entirely into SynerGEE. Larger systems that are loaded by substation or planning area will probably not work well with this approach. To get changes into the engineer's model, we will generate a version between the GIS model generated from week 1 and the one generated for week 2.

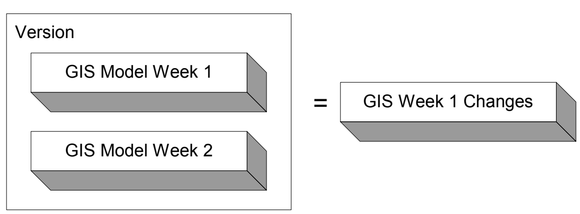


Fig. 1-17 Version capturing GIS changes

Note that the original GIS model for each week will need to be stored and untouched until the next week's model is generated.

At the beginning of week 2, the engineer can load the "GIS Week 1 Changes" version:

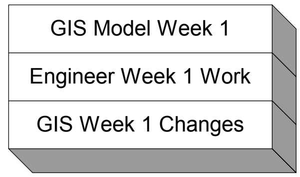


Fig. 1-18 *Updated engineering model*

Approach 2

Here, a version will be generated from the engineer's models:

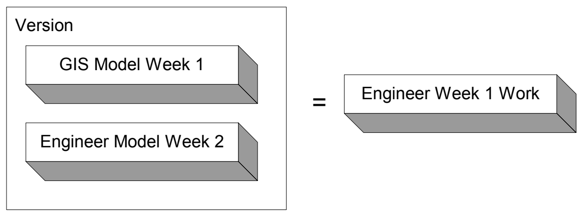


Fig. 1-19 *Version from engineer's models*

At the beginning of week 2, the engineer would save the version created between his end of week and beginning of week models. He would then load the new GIS model and then load the version:

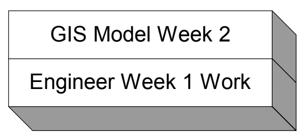


Fig. 1-20 *Week 2 model update with engineer's changes*

SynerGEE analysis options

General options

The following table lists the general options available during analyses and other engineering applications.

	Application Option	**Description**
LA	Looped Analysis	Open switches marked as "tie" switches are treated as closed switches and used to form loops during the analysis.
AG	Active Generators	During analysis, synchronous generators actively maintain the set terminal voltage and power output. Induction generators maintain the specified output power.
ES	Exclude Spot Loads	Spot loads are ignored by the analysis.
AL	Average Loads	Load values are averaged over the section phases prior to the

	Application Option	Description
		analysis.
IC	Ignore Line Charging	Line charging or line capacitance is ignored by the analysis. Use this option with caution, since line capacitance is a fundamental and significant aspect of a valid feeder model.
RH	Regulator Hold	Regulators that are not in manual mode can be held at particular taps during the analysis. Regulators can be held at their minimum taps, maximum taps, neutral tap, or current tap. You can also set them up to be controlled by SynerGEE.
CH	Capacitor Hold	Switched capacitors can be tripped or closed and held during the analysis.
SO	Start Options	Regulator and capacitors can have their initial or starting state set before the analysis runs.
IM	Ignore Motors	Motors are ignored as loads.

The following table indicates the options that are available within specific engineering applications.

Engineering Application	Options								
	LA	**AG**	**ES**	**AL**	**IC**	**RH**	**CH**	**SO**	**IM**
Load-Flow	Y	Y	Y	Y	Y	Y	Y	Y	Y
Fault	Y	Y	Y	Y	Y	Y	Y	N	Y
Fault Flow	Y	Y	Y	Y	Y	Y	Y	N	Y
Fault Voltage	Y	Y	Y	Y	Y	Y	Y	N	Y
Optimal Switching	N	N	Y	Y	Y	Y	Y	N	Y
Capacitor Placement	N	N	Y	Y	Y	Y	Y	N	Y
Allocation	N	N	Y	Y	Y	Y	Y	Y	Y
Check Coordination	N	N	Y	Y	Y	Y	Y	N	Y
Balance Improvement	N	N	Y	Y	Y	Y	Y	N	Y
Reliability	N	N	Y	Y	Y	Y	Y	N	Y
Cons. Volt. Red.	N	N	Y	Y	Y	Y	Y	Y	Y
Motor Start	N	N	Y	Y	Y	Y	Y	N	Y
Locked Rotor	Y	N	Y	Y	Y	Y	Y	Y	Y
Harmonic	N	N	Y	Y	Y	Y	Y	Y	Y

Time of day

SynerGEE has extensive capabilities for modeling the hourly variation of customer loads, based on diurnal customer load curves. For more information on customer load curves and time of day selection, see the loads chapter.

SynerGEE error codes

SynerGEE produces error messages for data problems associated with distribution system models or engineering analysis. Many of these errors are labeled with an error code number. The table below lists these errors codes.

Code	Subject	Description	Severity
D100	Database	SynerGEE tried to read from a table that did not exist or was locked. Example: *"Unable to read table InstSwitches from C:\MyData\West.MDB"*	Error
D101	Database	Switch data was unavailable. Example: *"SynerGEE was unable to obtain information about switches. We strongly recommend resolving this issue on the database side before loading into SynerGEE."*	Error
D102	Database	SynerGEE could not establish the necessary referential integrity rules. Example: *"Unable to establish referential integrity relationships to table InstSwitches from C:\MyData\West.MDB "*	Error
D103	Database	SynerGEE could not establish a table in the target database during a save. The table may be locked. Example: *"Unable to open, verify, or add necessary fields to InstSwitches from C:\MyData\West.MDB"*	Error
D104	Database	SynerGEE ran into a problem while trying to write to a field in the target database. The Microsoft Jet Engine generates the portion of this message referring to the error. Example: *"Database error writing to InstSwitches from C:\MyData\West.MDB: Database in use - 0x15AC"*	Error
D105	Database	Some data that is in the SynerGEE memory model could not be written to the database. Example: *"SynerGEE was unable to write all available data to InstSwitches from C:\MyData\West.MDB"*	Error
D106	Database	A problem was encountered running a query from SynerGEE. Example: *"Error running query: DELETE Gnodes.* FROM Gnodes LEFT JOIN InstSection ON Nodes.SectionId = InstSection.SectionId WHERE (((InstSection.FeederId)='West-239'))"*	Error
D107	Database	SynerGEE could not create the desired table during a write. Example: *"SynerGEE was unable to open database C:\MyData\WestResults.MDB for writing."*	Error

Code	Subject	Description	Severity
D108	Database	SynerGEE was unable to create specified field in target database. Example: *"SynerGEE was unable to create field SectionName in table MyResultsTable_Sect in database at C:\MyData\WestResults.MDB - 0x1020db67"*	Error
D109	Database	SynerGEE could not open curve definitions. Example: *"Database error reading protection curve definitions from C:\MyData\ProtectionCurves.MDB "*	Error
D110	Database	SynerGEE could not find the database control record. Example: *"Database at C:\MyData\West.MDB is missing SAI_Control table. This will impact unit and phasing conversions."*	Error
D111	Database	While saving (exporting) data to a database, SynerGEE updated an unexpected record. Before a save, SynerGEE deletes all model data associated with the feeders being saved. After the delete, there should be no data in the database with ID's matching those being saved. If conflicts are found, a section in SynerGEE may have been renamed to a name existing in the target database.	Warning
M100	General	A phasing problem was found between the device's data based phasing and the phasing of the device which feeds it. Example: *"Line on section 239_9823 has unfed phases."*	Warning
M101	Devices	Device has unspecified equipment type. Example: *"Unspecified type for regulator RK750 on 239_9823."*	Error
M102	Devices	Device type could not be found in loaded equipment types. Example: *"Could not find type 750kVA for regulator RK750 on 239_9823."*	Error
M103	Devices	Phasing problems were found around a feeder or substation transformer. Example: *"Phasing problem leading into Feeder 239."*	Warning
M200	Section	A phasing problem was found between a section's specified phasing and the phases feeding it. Example: *"Section 239_9823 has a specified phasing of ABN but is fed by a phasing of BCN. Missing wire(s) A."*	Warning

Code	Subject	Description	Severity
M201	Section	A section has a specified value for a load on a phase that is not fed or specified. Example: *"Distributed Load on 239_9823 has 33 kVA setup on unfed or unconfigured phase B."*	Warning
M202	Section	An ungrounded section should not have loads connected line-ground. Example: *"Distributed Load on 239_9823 is specified with a L-G connection but the section is ungrounded or has an ungrounded feed."*	Warning
M203	Section	A section's height must be greater than zero. Example: *"Height for 239_9823 must be greater than zero."*	Error
M204	Section	A section's length cannot be negative. Example: *"Length for 239_9823 cannot be negative."*	Error
M205	Section	A section's length cannot exceed 1000 miles. Example: *"Length for 239_9823 cannot be greater than 1000 miles."*	Error
M206	Section	If a section's loads are connected phase-phase, the section needs at least two-phase wires. Example: *"Load connection specified as phase-phase for 239_9823. At least two-phase wires are required."*	Error
M207	Section	If a section's loads are connected to ground, the section needs a neutral wire. Example: *"Load connection specified as phase-grnd for 239_9823. A neutral wire is required."*	Error
M208	Regulator	A valid value for reverse sensing threshold is needed. Example: *"Reverse sensing threshold for regulator RK750 needs to be between 1 and 5 percent."*	Error
M209	Regulator	Regulator tap positions need to be set to values with the tap range specified by the regulator equipment type or with the range specified by the regulator tap limiter. Example: *"Tap positions for regulator RK750 should be between -16 and 16."*	Error
M210	Regulator	If the regulator is set up for gang operation, the metering phase needs to be a valid. Example: *"Gang operation metering phase for regulator RK750 is not valid."*	Error

Code	Subject	Description	Severity
M211	Regulator	If a regulator is on, a unit should be connected to at least one phase. Example: *"Regulator RK750 has all phases disabled."*	Warning
M212	Regulator	Regulator must be fed by a voltage consistent with the regulator's rated voltage. Example: *" Regulator RK750 has a rated voltage of 12.47 kVLL but it is fed by a nominal voltage of 24.9 kVLL."*	Error
M213	Regulator	Regulator in automatic mode must have a PT ratio within 20% of expected value. Example: *"Regulator RK750 has a PT ratio of 104. We expected a value closer to 63.5."*	Warning
M214	Transformer	Transformer must be fed by a voltage consistent with its source side rated voltage. Example: *"Transformer T13C has a rated source side voltage of 13.2 kVLL but it is fed by a nominal voltage of 24.9 kVLL."*	Error
M215	Capacitor	A grounded connection cannot be used for a capacitor installed on an ungrounded section. Example: *"Capacitor C600 on 239_9823 was temporarily given a delta connection since the section is ungrounded."*	Fix
M216	Capacitor	A capacitor should not be configured for phases that are not present on the section. Example: "Capacitor on C600 on *239_9823* had A set as an active phase. That phase is not present on the section."	Warning
M217	Generator	Generator must be fed by a voltage consistent with its rated voltage. Example: *"Generator Karns has a rated voltage of 4.8 kVLL but it ties to a nominal voltage of 13.2 kVLL."*	Error
M218	Generator	Synchronous generators must have a PT ratio within 20% of expected value. Example: *"Synchronous generator Karns has a PT ratio of 63.5. We expected a value closer to 40."*	Warning
M219	Regulator	A regulator that is on should be configured for at least one phase. Example: *"Regulator RK750 has all phases disabled."*	Error
M220	Source	SynerGEE supports positive rotation only. Example: *"Feeder 239 voltage level must be set for positive rotation."*	Error
M221	Source	Voltage level should be reasonable. Example: *"Feeder 239 voltage level should be between 60 and 180 volts on a 120V base. It is currently 230 Volts."*	Error

Code	Subject	Description	Severity
M222	Source	Fault resistance used for L-G faults cannot be negative. Example: *"Feeder 239 fault resistance cannot be negative."*	Error
M223	Source	A source cannot have a negative value for load multiplier. Example: *"Feeder 239 load multiplier cannot be negative."*	Error
M224	Source	Feeder or subtran voltages must be greater than .1kV (100V). Example: *"Feeder 239 source kV is invalid."*	Error
M225	Source	Source resistance must be positive. Example: *"Feeder 239 source resistance cannot be negative."*	Error
M226	Source	Feeder demand kW must be positive. Example: *"Feeder 239 demand kW cannot be negative."*	Error
M227	Section	Section's phase conductor must be in the currently loaded conductor list. Example: *"Could not find phase conductor 336 ACSR for section 239_9823 "*	Error
M228	Section	Section must have a specified phase conductor. Example: *"Unspecified phase conductor for section 239_9823 "*	Error
M229	Section	Section is grounded. SynerGEE could not find the specified neutral conductor in the currently loaded conductor list. SynerGEE will use phase conductor for neutral. Example: *"Could not find neutral conductor 2/0 ACSR for 239_9823. Phase conductor 336 ACSR will be used."*	Warning
M230	Section	Section is grounded. The neutral conductor is unspecified. SynerGEE will use phase conductor for neutral. Example: *"Unspecified neutral conductor for 239_9823. Phase conductor 336 ACSR will be used."*	Warning
M231	Section	The section is grounded, the neutral is unspecified or could not be found, and the phase conductor is unavailable. Example: *"Unspecified neutral conductor for section 239_9823"*	Error

Code	Subject	Description	Severity
M232	Motor	Motor load curve was not found. Example: *"Could not find load torque curve for motor M500H on 239_9823."*	Error
M233	Motor	Motor distribution transformer was not found. Example: *"Could not find distribution transformer motor M500H on 239_9823."*	Error
M234	Motor	Motor torque curve was not found. Example: *"Could not find motor torque curve for motor M500H on 239_9823."*	Error
M235	Motor	Motor amp curve was not found. Example: *"Could not find motor amp curve for motor M500H on 239_9823."*	Error
M236	Motor	Motor pf curve was not found. Example: *"Could not find motor pf curve for motor M500H on 239_9823."*	Error
M237	Regulator	Regulator is set up with a phasing that is not supported by its feed path. The regulator is set up to use phases that are not present on the section or feeding device. Example: *"Regulator RK750 has phases A of its ABC phasing fed by unconfigured phases."*	Error
M238	Regulator	A three-phase regulator was found on a section that is not three-phase or is lacking a three-phase feed. Example: *"Three-phase regulator RK750 (750 kVA - 3P) has been placed on a one- or two-phase section (239_9823)."*	Warning
M239	Capacitor	A capacitor has a fixed kvar unit specified for an unconfigured phase. Example: *"Capacitor C600 on 239_9823 has a fixed 200.0 kvar on unfed phase A."*	Warning
M240	Prot.Dev	A protective device (classic protective device) has configured phases that are not fed by the feeding device. Example: *"Protective device F100 on 239_9823 has unfed phases AB. Unfed phases will be ignored."*	Warning
M241	Motor	A three-phase motor must have a three-phase service. Example: *"Three-phase motor M500H on 239_9823 needs a three-phase service. It is being fed with a phasing of ABN."*	Error

Code	Subject	Description	Severity
M242	Motor	A single-phase motor has an invalid service. Example: *"Single-phase motor M500H on 239_9823 is configured for phase AN. It is being fed with a phasing of BCN."*	Error
M243	Generator	A three-phase generator must be connected to a three-phase section. Example: *"Three-phase generator Karns on 239_9823 must have a three-phase feed."*	Error
M244	Motor	All phases of a motor service must be fed. If the service phasing is invalid, the section phasing will be used. Example: *"Motor service for M500H on 239_9823 has unfed phases AN. Section phasing of ABCN will be used."*	Warning
M245	Motor	A motor must be fed with a valid nominal voltage. Example: *"Motor M500H has a rated voltage of 4.16 kV but is fed with a nominal voltage of 13.8 kVLL."*	Error
M246	Generator	A three-phase generator must be connected to a three-phase section. Example: *"Three-phase generator Karns on 239_9823 must have a three-phase feed."*	Error
M247	Section	A serious problem exists with the section line construction model. Check conductors, configuration, spacings, etc. Example: *"A serious problem exists with the construction model for section 239_9823."*	Error
M248	Section	Section is missing conductor information. Example: *"Section 239_9823 has invalid phase conductor data."*	Error
M249	Section	Topological loops have been found in the model. Loops should be specified with open "loop-tie" switches. Example: *"Topological loops have been detected within the selected feeders."*	Warning
M250	Switch	An unfed node, isolated by open switches, was found. Example: *"Open switch PM3234 on 239_9823 ties to unfed node 434422. Possible switch problem."*	Warning
M251	Motor	Conductor from section to distribution transformer or from distribution transformer to starter is not defined. Example: *"Cannot find service conductor 4/0 ACSR for motor Aux Pump. It will be ignored."*	Error

Code	Subject	Description	Severity
M252	Section	Sections constructed with configuration models and bare conductors must have conductors above ground. That is, the reference height of the configuration plus the height to reference in the section should be more than twice the diameter of the conductor. Example: *"Bare conductors on section 239_9823 need to be above ground."*	Error
M300	Analysis	Regulator exceeded its operation limit for the analysis. It was locked at its last tap position. Example: *"Regulator RK750 on 239_9823 locked down after 12 operations."*	Warning
M301	Analysis	Switched capacitor exceeded its operation limit for the analysis. It was locked at its last state. Example: *"Switched capacitor C600 on 239_9823 locked down after 6 operations."*	Warning
M302	Analysis	A device's phasing may be augmented to allow the analysis to complete. The change is made for the run of the current analysis only. Example: *" Regulator RK750 on 239_9823 was temporarily assigned a phasing of ABCN for this run."*	Fix
M303	Analysis	Looped Fault Analysis uses an iterative load-flow technique to determine fault impedance values. Averaging loads is incompatible with the technique. Example: *"Looped Fault Analysis must not be run with the 'Use Average Loads' setting."*	Error
M900	Topology	A device has a propagated phasing that is incompatible with the feeding device. Example: *"SynerGEE has run into a severe problem tracing phasing. Please call customer support."*	Error - Call Technical Support
M901	Topology	Error was encountered while building radial topology. Example: *"Severe topology error near Regulator RK750."*	Error - Call Technical Support

Code	Subject	Description	Severity
M902	Topology	Unrecoverable error encountered building topology for a feeder or substation transformer. Example: *"Severe topology error connecting %s."*	Error - Call Technical Support

Unit handling

SynerGEE comes with a versatile system for handling units, supporting both metric and English measurements. By managing and processing data with its own internal units, SynerGEE can load, display, and save data in a variety of different units using its powerful converters.

The following diagram represents the basic data path between the database, SynerGEE's memory, and the screen, as units are concerned.

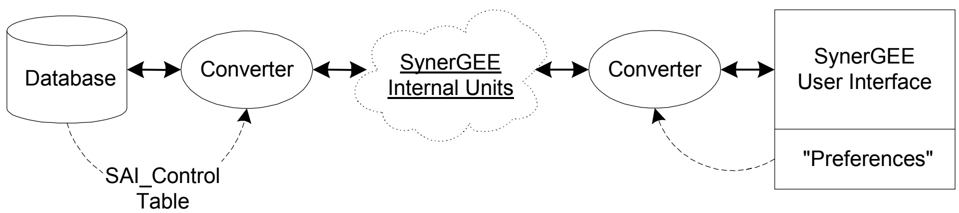


Fig. 1-21 *Data path*

A converter is used to convert units from the database units system to internal units. For display, a second converter is used to convert from internal units to the units specified in your preferences. Conversion is automatically done, based on the following rules.

- **Database loading** – SynerGEE determines the units in the source database based on settings in the SAI_Control table. It then converts them into internal units accordingly. For display on the screen, the second conversion is done according to the settings in your preferences. These settings control the data units on the map, and in editors and reports.
- **Database writing** – SynerGEE reads the SAI_Control table from the target database. It then establishes a converter to convert between the units specified in that table and the internal units system of SynerGEE. If a new database is being created, the units system currently specified in your preferences is the one written to the SAI_Control table of the new database.
- **Entering data into editors** – New data put into editors is converted to internal units, using the same converter that converts internal units for on-screen display.

The unit handling system is flexible. You could load a feeder from a metric database, load another feeder from an English kFt database, and display them both in SynerGEE as English miles.

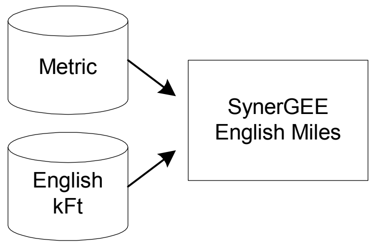


Fig. 1-22 *Loading feeders from databases with different unit systems*

Remember that SynerGEE saves units to an existing database according to the settings in the SAI_Control table. If the original units are different, SynerGEE automatically converts them for the save. Preference settings are not considered when saving to an existing database. For example:

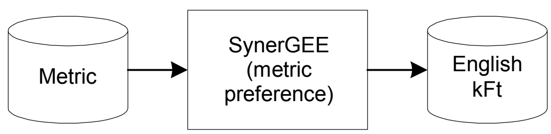


Fig. 1-23 *Converting a model from one unit system to another*

If the target database in the case above were a new database, however, SynerGEE would automatically save in metric units and establish the SAI_Control table accordingly. Before saving to new databases, you should be sure that your preferences reflect the type of database you want to create.

Tips

The following are some tips to use when working with units.

- Do not manually configure the ″SAI_Control″ or ″SAI_Equ_Control″ table contents, unless you are also prepared to manually convert every piece of measurement data in the database. If the fields indicating the units system are changed, SynerGEE assumes that the database contains data in the new units. No automatic conversion takes place if you change these tables.

- You can change the units system in your preferences at any time, and data shown in the editors and on the map updates instantly and automatically. However, analysis reports are static and do not change. If you run a needed analysis and change your units later, you should run the analysis again to regenerate the report.

Maps and unit systems

SynerGEE uses an X and Y coordinate system in units of feet or meters. If SynerGEE is set up for metric, the map units are assumed to be meters. Otherwise, the map units are assumed to be in feet.

If SynerGEE is set up in metric mode, background maps that are loaded into SynerGEE should be in units of meters. Background maps in units inconsistent with SynerGEE′s unit setting will not align properly with feeder models.

Note: Fuse exception threshold

A capability of altering the exception threshold for fuses was added during a service release of SynerGEE 3.5. Some utilities intentionally load their fuses above rated amps. A 100A fuse, for example, may be routinely run up to 150A. In these cases, the 100A fuse should not be exception flagged by SynerGEE. A setting was added to allow the exception flag threshold to be raised as a percentage of the fuse rating. Setting the value to 150% would cause loading emergency exceptions on a 100A fuse to appear with 150A or greater loading.

After the release of the new emergency exception flag threshold, some clients used the capability for derating fuses. Values below 100% were entered. A 100A fuse would have an emergency rating of 80A if an 80% value were put in.

A problem became evident with clients putting in flag threshold values below 100%. The exception rating was below the continuous rating since the flag only affected the emergency exception value. Using 80%, for example, would cause a 100A fuse to have a 100A continuous rating and an 80A emergency rating. This was confusing and corrected by setting the continuous rating equal to the emergency rating in this situation.

SynerGEE will be corrected again by handling the fuse exception flag threshold as two cases; one where the flag is below 100% and one where it is at or above 100%.

Threshold above 100%:

$$\begin{aligned} Continuous_{Fuse} &= Rating_{Fuse} \\ Emergency_{Fuse} &= Rating_{Fuse}\frac{FlagThresh}{100\%} \end{aligned} \tag{1-8}$$

Threshold below 100%

$$Continuous_{Fuse} = Rating_{Fuse} \frac{FlagThresh}{100\%} \qquad (1\text{-}9)$$
$$Emergency_{Fuse} = Rating_{Fuse}$$

Example with setting over 100%

Lets look at a the threshold set at 120% and a 100A fuse with 110A load current.

	SynerGEE Version		
	3.53	3.65	3.70
Cont.	100A	100A	100A
Emer	120A	120A	120A
Pct Load	110%	110%	110%
Warning?	Yes	Yes	Yes
Emergency?	No	No	No

Example with setting under 100%

Now the threshold is set at 80%. The fuse is 100A with a 90A load current.

	SynerGEE Version		
	3.53	3.65	3.70
Cont.	100A	80A	80A
Emer	80A	80A	100A
Pct Load	80%	100%	100%
Warning?	No	Yes	Yes
Emergency?	Yes	Yes	No

Guidelines

This approach may still not be suitable for all clients and we are very willing to look into different approaches. SynerGEE has been designed for consistency and integrity. That design drives us to use these self-imposed guidelines:

1. All devices have continuous and emergency ratings
2. Exceptions are generated from continuous and emergency loading percentages
3. Reserve amps are derived from upstream continuous or emergency loading values

Other options

Working within the guidelines listed above, the following changes could be made if they seemed suitable to most clients:

- Base reserve amps on emergency ratings instead of continuous ratings.
- List fuse nameplate amp rating on fuse report

2
Power Engineering Topics

Distribution System Characteristics

Distribution systems have unique characteristics that create diversity and complexity beyond the transmission systems typically presented in most power engineering texts. In general, distribution systems have:

Unbalanced load – Different load values are on each phase of a load or loads may be connected to only one phase. Loads are connected line-to-ground and line-to-line.

Unbalanced construction – power lines are constructed non-symmetrically and non-transposed. Different conductors are sometimes used on different phases of a line. Some phases are sometimes bundled while others are not.

Distributed and spot loads – large loads for industrial customers and commercial customers are considered loads at a particular spot in a distribution system. Contingency plans must be established to support these loads. Loads for residential and small commercial customers are also distributed along the thousands of miles of distribution line in a typical utility. Depending on the utility and the area, the total of the "distributed" loads usually match or exceed the larger spot loads. Contingency plans are also established to address the needs of the distributed loads. A few dozen (economically active) customers may represent the large commercial and industrial load of a utility. Hundreds of thousands of (economically and politically important) customers represent the distributed load of a utility.

Diverse equipment – Many distribution systems have been in operation for nearly one hundred years. Over that operation time, there were changes in service territory, power supply, utility ownership, engineers, hardware technology, design philosophy, and suppliers. Naturally, a distribution system is going to have a variety of new and old equipment of various types. Knowledge of each peace of equipment and its function is important.

Radial nature – Power distribution systems are nearly always radial in North America. Radial construction keeps costs down, safety up, and simplifies the operation of the distribution system. Urban underground or "downtown" distribution systems are an exception. They are oftentimes run in a meshed network to increase reliability and power transfer capability.

Common power system relationships

If only the kW value of a load is known, the kvar value can be determined from the kW and an estimated power factor angle. For example,

$$kvar = kW * \tan(\theta_{PF}) \tag{2-1}$$

The per-unit system

Although the per-unit (PU) system is not directly used in the radial load-flow analysis methods described herein, it is essential for a number of reasons.

- Some data is represented in PU on equipment nameplates and is required in actual values within SynerGEE.
- Voltages are always displayed on your voltage base within SynerGEE. This value is the PU voltage times the specified user-base (typically 120V).

It should be stressed that all calculations within SynerGEE are performed with actual values such as kV, amps, and kW. The PU values and your voltage base settings are interface conveniences.

$$\begin{aligned} \text{Given:} \quad & kVA_{Base} \\ & kVLL_{Base} \end{aligned} \tag{2-2}$$

$$Z_{Base} = \frac{1000 * kVLL^2_{Base}}{kVA_{Base}} \tag{2-3}$$

$$I_{Base} = \frac{kVA_{Base}}{\sqrt{3} kVLL_{Base}} \tag{2-4}$$

An impedance value can be changed from one base to another with the following:

$$Zpu_{New} = Zpu_{Old} \left(\frac{kVLL_{Old}}{kVLL_{New}} \right)^2 \left(\frac{kVA_{New}}{kVA_{Old}} \right) \tag{2-5}$$

Admittance can similarly be shifted across bases using the following equation.

$$Ypu_{New} = Ypu_{Old}\left(\frac{kVLL_{New}}{kVLL_{Old}}\right)^2\left(\frac{MVA_{Old}}{MVA_{New}}\right) \tag{2-6}$$

Symmetrical components

The method of symmetrical components is used for representing a three-phase power system that is assumed to be balanced or symmetrical up to a particular point of imbalance. Symmetrical components are based on a three-phase linear transformation that transforms a coupled three-phase model into three decoupled, single-phase models. The transformation is based on a 120-degree phase shifter:

$$a = 1\angle 120 \tag{2-7}$$

From this, the following identity can be formed.

$$a^2 + a + 1 = 0 \tag{2-8}$$

This forms the basis for a transformation matrix as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{2-9}$$

These relationships between phase-and sequence-domain quantities are generated as follows:

$$V_S = \begin{Bmatrix} V_0 \\ V_1 \\ V_2 \end{Bmatrix} I_S = \begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} V_P = \begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} I_P = \begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} \tag{2-10}$$

$$\begin{aligned} V_S &= A^{-1}V_P \quad & V_P &= AV_S \\ I_S &= A^{-1}I_P \quad & I_P &= AI_S \end{aligned} \tag{2-11}$$

$$Z_P = AZ_SA^{-1} \quad Z_S = A^{-1}Z_PA \tag{2-12}$$

$$Y_P = AY_SA^{-1} \quad Y_S = A^{-1}Y_PA \tag{2-13}$$

If you have a line described with zero sequence and positive sequence values:

$$Z_P = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} Z_0 & 0 & 0 \\ & Z_1 & 0 \\ & & Z_1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \quad \text{(2-14)}$$

$$Z_S = \left(Z_0 + 2Z_1\right)/3$$
$$Z_M = \left(Z_0 - Z_1\right)/3$$

If you have a completely balanced impedance line with equal self and mutual impedance values:

$$Z_S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ & Z_1 & 0 \\ & & Z_1 \end{bmatrix} \quad \text{(2-15)}$$

$$Z_0 = Z_S + 2Z_M$$
$$Z_{1,2} = Z_S - Z_M$$

Transformer impedances

Often, it is necessary to switch between forms of representing transformer or regulator impedance. If impedances are given as %Z and X/R Ratio, the following is true.

$$R = \frac{\%Z}{100 * \sqrt{1 + \left(\text{X/R Ratio}\right)^2}} \qquad X = \frac{\%Z * \left(\text{X/R Ratio}\right)}{100 * \sqrt{1 + \left(\text{X/R Ratio}\right)^2}} \quad \text{(2-16)}$$

Delta / Wye transformations

The following diagram shows a load connected in a delta and a wye configuration.

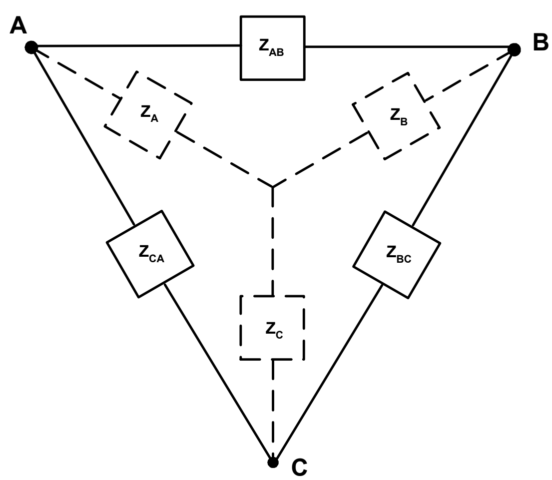


Fig. 2-1 *Wye and delta forms of load*

Values for the wye can be determed from the delta values with these equations:

$$Z_A = \frac{Z_{AC} Z_{AB}}{Z_{AB} + Z_{BC} + Z_{AC}}$$
$$Z_B = \frac{Z_{AB} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}}$$
$$Z_C = \frac{Z_{AC} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}} \tag{2-17}$$

The values for the delta can be determined from wye impedances:

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$$
$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$
$$Z_{AC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B} \tag{2-18}$$

If the impedances are balanced then:

$$Z_Y = \frac{Z_\Delta}{3} \tag{2-19}$$

And

$$Z_\Delta = 3 Z_Y \tag{2-20}$$

Current, voltage, power

Power is calculated as follows:

$$S_{Into} = V_S I_{Into}^* \tag{2-21}$$

We can deliver a given level of power by raising the voltage and lowering the current by the same factor.

Voltage drop needs to be contained

The current into a non-tapped power line must match the current out.

$$I_{Into} = I_{Out} \tag{2-22}$$

Line impedance will cause a voltage drop along the line.

$$V_{Drop} = I_{Into} \cdot Z_{Line} \tag{2-23}$$

Use the source end voltage to calculate the power into the line:

$$S_{Into} = V_S I_{Into}^* \tag{2-24}$$

Next, we can account for the voltage drop to determine the power out of the line.

$$S_{Out} = \left(V_S - I_{Into} Z_{Line}\right) I_{Into}^* \tag{2-25}$$

This can be simplified to:

$$S_{Out} = V_S I_{Into}^* - \left|I_{Into}\right|^2 Z_{Line} \tag{2-26}$$

Different voltage levels

Voltage drop is a directly affected by current flow and line impedance.

$$V_{Drop} = I \cdot Z \tag{2-27}$$

Electrical power cannot be directly stored. Therefore, the power entering a system must match the total of the power leaving the system, the power emitted as losses, and the power consumed. This is quite important because voltage levels and current levels will change but power must always be conserved.

Graph theory definitions

Non-directed graphs

<u>Vertex</u> – a point in space

<u>Edge</u> – unordered pair of vertices (a, b)

<u>Graph</u> – a set of vertices and edges

Directed graphs

Non-directed graphs are of little use in power system analysis. Power networks are analyzed to determine amps, kVA, customers, or other flow values. Voltage values cannot be determined without designating a reference direction for current flow along lines.

<u>Arc</u> – ordered pair of vertices {f, t}. An arc is incident to two vertices. The preceding arc is adjacent from ‘f’ and adjacent to ‘t’.

<u>Digraph</u> – a finite set of vertices and arcs

Relationships

If arcs of a digraph are treated as edges, the non-directed graph is called the underlying graph. The arcs distinguishing the digraph from that graph are called the orientation of the digraph.

3
Lines and Cables

Introduction

SynerGEE has been designed to provide an outstanding line model without complicated data requirements. It captures the effects of electric and magnetic field coupling by using the full set of Carson's equations, and its methods incorporate the work of Carson, Wagner and Evans, and Kersting.

SynerGEE's by-phase impedance and admittance calculations and reduction techniques can handle coupling between conductors, and between conductors and the earth. SynerGEE considers one-, two-, and three-phase lines with and without a neutral return, and can handle bare overhead lines as well as cables.

Like most device models in SynerGEE, the spatial characteristics of a line model are defined in a section instance contained in the model database (InstSection table). Each section record should also reference a conductor type in the equipment database, which defines its electrical characteristics (DevConductors table). In addition, if a section is configured to use detailed spacing, it also references a configuration type in the equipment database (DevConfig table). A visual diagram of data flow to build a section is as follows.

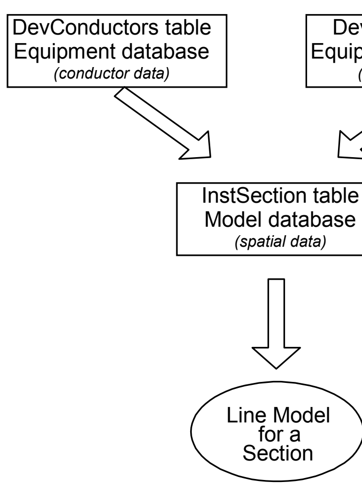


Fig. 3-1 *Tables used to specify the line model for a section*

Conductors

The conductor table

The conductor table, DevConductors, contains the required data for standard conductors and cables. One record in the conductor file contains all the required data for one type of conductor used in the system. Before you create a section, the conductor table should be set up with the conductors that you need to model your system. New conductors may be entered directly in the database, or you may use the SynerGEE Warehouse feature.

The sample equipment database provided with your software package contains a conductor table with many of the commonly used conductors. Most values were taken from the *Transmission and Distribution Reference Book*. You may use the information in the table or enter your own information.

Conductor data

The following table lists some of the conductor data used in calculations, and to which types of conductor the data applies. You can enter the following data, as applicable, using the conductor editor .

Data	Description	Actual Z	1' 1M	Sep. Neut.	Con. Neut.	Tape Shield
$R_1 + jX_1$ $R_0 + jX_0$ jB_o jB_1	Sequence impedance values in Ohms/length unit	✓				
$R_1 + jX_1$	Positive sequence impedance with conductors spaced at a 1' or 1M GMD		✓	✓		
Diameter (conductor)	Distance across the conductor		✓	✓	✓	✓
Diameter (insulation)	Diameter over insulation, under strands or tape shield				✓	✓
Diameter (outside)	Diameter across outside of conductor				✓	✓
Diameter (strand)	Diameter across one neutral strand				✓	
Core R	Resistance of conductor (or equivalent) in Ohms/length unit				✓	✓
Core GMR	Geometric mean radius of conductor (or equivalent) in feet or meters				✓	✓

Strand R	Resistance of a single strand in Ohms/length unit				✓	
Strand GMR	Geometric mean radius of a single strand in feet or meters				✓	

Conductor configurations

SynerGEE provides detailed modeling for the line portion of a section and lets you model both overhead conductors and underground cables. There are three methods for representing lines within SynerGEE.

Conductor/equivalent spacing method

This method uses conductor information from the conductor table (DevConductors) and the equivalent spacing values of an individual section to calculate the section impedance and admittance. You can specify both the conductor type and the spacing by editing a section's characteristics. Carson's equations are used as described later in this chapter. This method is most applicable to overhead lines.

Sequence method

This method allows you to directly specify impedance values. You can set positive and zero sequence values in per-length units for a particular conductor. You can also select a conductor that has been set up with sequence values to disable the fields associated with spacings and the neutral conductor. This can be attributed to the sequence values that define the impedance for a four-wire grounded section. SynerGEE uses valid spacings and a neutral conductor to determine the sequence impedances.

When setting up a section, you can turn off phase conductors *and* the neutral conductor with a sequence method conductor. In these cases, appropriate rows and columns of the 3 by 3 impedance and admittance matrices are eliminated.

Configuration method

With this method, each section references a configuration type record in the spacings table (DevConfig), which contains coordinate information about conductor positions for that configuration. These positions, labeled 1, 2, 3, and N, are relative to a reference point. When you apply a configuration to a section, you can specify which phase occupies which position.

The following is a sample configuration to demonstrate how position coordinates are determined. In this example, English units of measurement are assumed, in which case the coordinates represent feet.

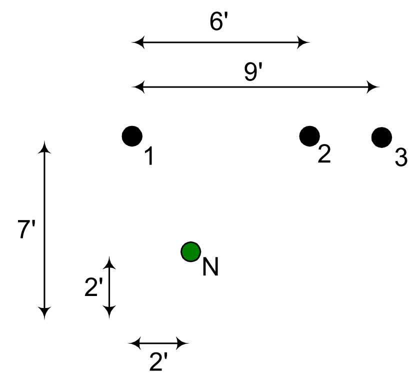


Fig. 3-2 Sample configuration

To form a set of coordinates for this cross-section of conductors, you first need a reference point, or 0,0. Since SynerGEE models self and mutual impedance and admittance values of lines, the horizontal reference point can be arbitrary. However, the distance between conductors and the ground is important, so the vertical reference point requires more consideration.

To maintain the proper conductor-to-ground distance for calculations, SynerGEE uses an additional "height-to-reference" setting in conjunction with configurations. Essentially, this height-to-reference value is added to the vertical reference point that you choose, thereby establishing the proper heights for all the positions in the configuration. Therefore, you can choose an arbitrary vertical reference point as well, provided that you set the proper height-to-reference value so SynerGEE knows the exact distance between the conductors and the ground. The height-to-reference value is specified by section and is not part of the configuration data itself.

Returning to the previous example, assume that you chose your 0,0 reference point as follows:

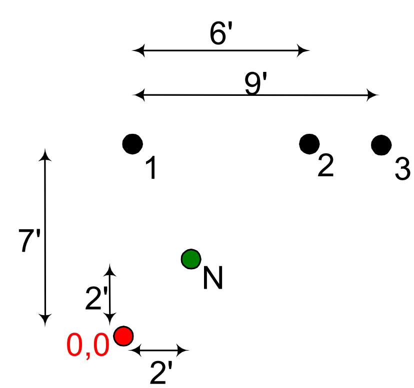


Fig. 3-3 Sample configuration with reference point

This reference point is horizontally 2 feet below the neutral conductor, and vertically in line with position 1. Using simple Cartesian coordinates, the positions in the configuration are as follows.

Sample configuration		
	X (feet)	Y (feet)
Position 1	0	7
Position 2	6	7
Position 3	9	7
Neutral	2	2

It is these coordinates that you would enter into the configurations editor when defining the configuration.

Usage of configurations

Applying a configuration to a section is as simple as selecting it in the section editor, once you have defined it in the configurations table. However, remember that for SynerGEE to use a configuration, it must also know:

- The correct height-to-reference value, which is added to all vertical coordinates
- Which phases occupy which positions

These two items are also specified in the section editor. Using the sample configuration presented earlier, assume that you have the following data applied to one of your sections.

Section 1 data	
Configuration	Sample
Height-to-reference	20 feet
Phase positions	BAC

Applying these settings, the section would look like the following.

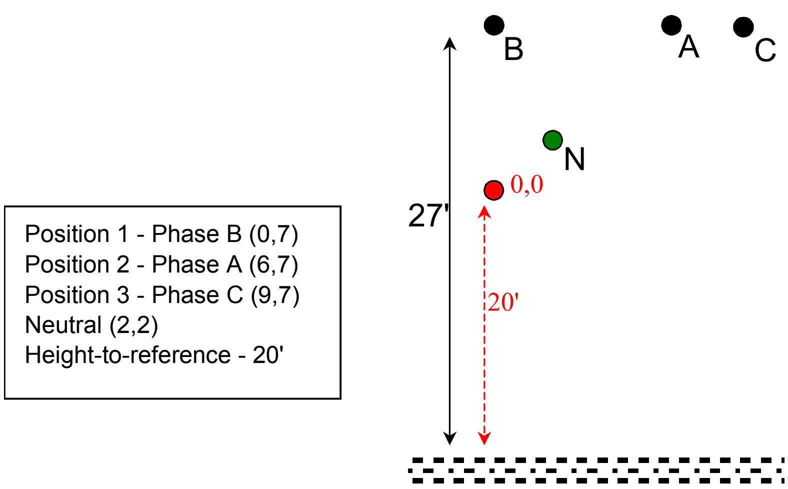


Fig. 3-4 Section using sample configuration

As another example, assume you had another two-phase section (B and C only), and set it up as follows.

Section 2 data	
Configuration	Sample
Height-to-reference	35 feet
Phase positions	BCA

In this case, the section would appear as follows.

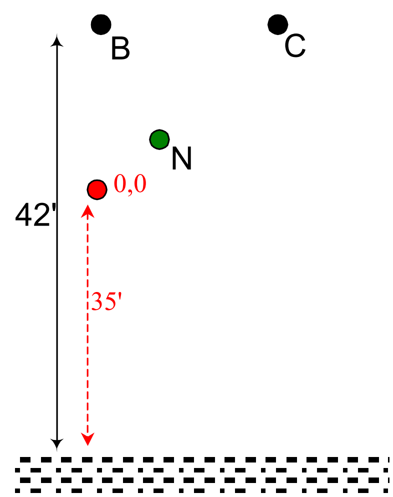


Fig. 3-5 Section 2 using same configuration

Note that the last position is simply ignored, since it is mapped to phase A, which does not exist. With the capability to change phase positions and height-to-reference values, a small set of configurations should be sufficient for a variety of constructions.

Impedance display in section characteristics

SynerGEE initially performs calculations for the line section impedance and for any changes made to the section characteristics. It uses either Carson's equations or a sequence domain transformation to determine the phase domain impedances. For convenience, the equations used to transfer sequence impedances to phase domain impedances are as follows.

$$Z_S = (Z_0 + 2Z_1)/3 \qquad Z_0 = Z_S + 2Z_M$$
$$Z_M = (Z_0 - Z_1)/3 \qquad Z_{1,2} = Z_S - Z_M \tag{3-1}$$

Line impedance calculations

Symbols

The following symbols and notations are used throughout this chapter.

(3-2)

$$
\begin{aligned}
Z_S &= \text{Self impedance} \\
Z_M &= \text{Mutual impedance} \\
\rho &= \text{Resistivity of earth in ohm-meters} \\
\mathrm{D_{ij}} &= \text{Distance between conductor i and j in feet} \\
\mathrm{D'_N} &= \text{Phase/Neutral equivalent spacing} \\
\mathrm{D'_P} &= \text{Phase/Phase equivalent spacing} \\
GMR_i &= \text{Geometric Mean Radius of conductor i} \\
f &= \text{Nominal frequency in Hz} \\
k_1 &= 0.00158837 \frac{\Omega}{Hz * mi} \\
k_2 &= 0.0020224 \frac{\Omega}{Hz * mi} \\
k_3 &= 7.67860 \\
i, j &= A,\ B,\ C
\end{aligned}
$$

Line modeling details

SynerGEE performs voltage drop calculations with the use of a 3 by 3 impedance matrix. It uses the current out of a line to determine the voltage drop across the line as follows.

(3-3)

$$
\begin{Bmatrix} V_{DropA} \\ V_{DropB} \\ V_{DropC} \end{Bmatrix} = \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix}
$$

The 3 by 3 impedance matrix may have some rows and columns of zeros for lines with fewer than three-phase conductors. For an N-phase conductor line with a neutral, reducing a matrix with rank N+1 forms the matrix. The matrix that is reduced is referred to as the primitive impedance matrix and has the following form.

(3-4)

$$
Z_{\text{Prim}} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} & Z_{AN} \\ & Z_{BB} & Z_{BC} & Z_{BN} \\ & & Z_{CC} & Z_{CN} \\ & & & Z_{NN} \end{bmatrix} \quad \begin{aligned} Z_{ii} &= \text{Self or Series Impedance} \\ Z_{ij} &= \text{Mutual Impedance} \end{aligned}
$$

Kersting summarizes the expressions for the Carson diagonal and off-diagonal terms as follows.

(3-5)

$$Z_{ii} = r_i + k_1 f + jk_2 f\left(k_3 - \ln GMR_i + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi}$$

$$Z_{ij} = k_1 f + jk_2 f\left(k_3 - \ln D_{ij} + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi} \quad i \neq j$$

These expressions can be used with the spacings and characteristics of individual conductors to determine the impedance for a line section.

Modeling with equivalent spacings

Tracking and verifying individual conductor spacings and characteristics in large feeders is difficult. As an alternative, SynerGEE can simplify the Carson line model by averaging the diagonal and the off-diagonal terms of the primitive impedance matrix. This allows line construction to be specified by geometric mean distances instead of individual conductor distances. The resulting line model is practically identical to the extensive model and requires much less data.

The Carson expressions above determine mutual impedances based on the spacings between individual conductors. The average value for off-diagonal terms and average value for diagonal terms can be calculated from these. For example:

(3-6)

$$Z_{PS} = \frac{Z_{AA} + Z_{BB} + Z_{CC}}{3} \quad Z_{PM} = \frac{Z_{AB} + Z_{BC} + Z_{CA}}{3} \quad Z_{PN} = \frac{Z_{AN} + Z_{BN} + Z_{CN}}{3}$$

The average of the diagonal terms can be found directly as follows.

(3-7)

$$Z_{PS} = r + k_1 f + jk_2 f\left(k_3 - \ln GMR + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi}$$

The off-diagonal terms require a little more work. For example:

(3-8)

$$Z_{PM} = k_1 f + jk_2 f\left(k_3 - \frac{\ln D_{AB} + \ln D_{BC} + \ln D_{CA}}{3} + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi}$$

A substitution of an equivalent spacing expression can be used to get the following:

(3-9)

$$Z_{PM} = k_1 f + jk_2 f\left(k_3 - \ln D'_P + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi} \quad \text{where } D'_P = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$

The average mutual impedance can, therefore, be calculated from the geometric mean spacing of the distribution line.

The average coupling between the phase conductors and the neutral can be found in a similar way to get the following:

$$Z_{PN} = k_1 f + jk_2 f\left(k_3 - \ln D'_N + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi} \quad \text{where}\, D'_N = \sqrt[3]{D_{AN} D_{BN} D_{CN}} \tag{3-10}$$

Primitive impedance matrix

In their general form, Carson's equations are applied to phase conductors and neutral conductors in the calculation of the primitive impedance matrix. This matrix has the following form. Keep in mind that the expressions for each of the terms were expressed in the preceding section.

$$Z_{\text{Prim}} = \begin{bmatrix} Z_{PS} & Z_{PM} & Z_{PM} & Z_{PN} \\ & Z_{PS} & Z_{PM} & Z_{PN} \\ & & Z_{PS} & Z_{PN} \\ & & & Z_{NS} \end{bmatrix} \tag{3-11}$$

Z_{PS} = Average Phase Self Impedance

Z_{PM} = Average Coupling Between Phases

Z_{PN} = Average Coupling Between Phases and Neutral

Z_{NS} = Self Impedance of Neutral

Notice that there are four distinct terms in the impedance matrix. You can write general expressions for the series and mutual terms of a grounded four-wire line as follows.

$$Z_{XS} = r_X + k_1 f + jk_2 f\left(k_3 - \ln GMR_X + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi} \tag{3-12}$$

$$Z_{XM} = k_1 f + jk_2 f\left(k_3 - \ln D'_X + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/_{mi}$$

X = P or N

These equations are applied to the phase and neutral conductors as follows.

(3-13)

r_P, r_N	Conductor resistance of Phase and Neutral	Ω/Mile
GMR_P, GMR_N	Geometric Mean Radius of Phase and Neutral	Feet
D_P, D_N	Geometric Equivalent Spacing of Phase and Neutral	Feet

The geometric equivalent spacing is a function of the number of conductors present. For example,

(3-14)

	D_P	D_N
		With Neutral
Three-Phase Wires	$\sqrt[3]{D_{AB} D_{BC} D_{CA}}$	$\sqrt[3]{D_{AN} D_{BN} D_{CN}}$
Two-Phase Wires - J,K	$\mathrm{D_{JK}}$	$\sqrt{\mathrm{D_{JN}} D_{KN}}$
One-Phase Wire - J	$\mathrm{Z_{MP}} = 0$	D_{JN}

Reduction of the primitive impedance model

The previous sections developed the primitive impedance matrix. Since Carson's equation couples the earth return with the neutral and phase conductors, the 4 by 4 primitive impedance matrix is over-specified. The matrix corresponds to a system with four unknowns. However, Kirchhoff's current law can be used to eliminate one equation for grounded lines to get a 3 by 3 impedance matrix. Although three-wire ungrounded lines only have two independent current flows, their impedance matrix is left unreduced.

SynerGEE directly uses either of these matrices in its load-flow calculations.

No neutral conductor case

If the line being modeled does not contain a neutral conductor, the fourth row and column of the primitive impedance matrix are ignored since the calculations used to determine this fourth row are based on a neutral conductor. Earth coupling for the phase conductors has been accounted for in the expressions for mutual and series impedance. The impedance terms of the line model are copied directly from their corresponding terms in the primitive impedance matrix and no reduction is performed. The following is the 3 by 3 impedance matrix for a three-wire ungrounded line.

$$Z_{Line}^{\Delta} = \begin{bmatrix} Z_{PS} & Z_{PM} & Z_{PM} \\ & Z_{PS} & Z_{PM} \\ & & Z_{PS} \end{bmatrix} \tag{3-15}$$

Neutral conductor case

In this case, return flow is divided between the neutral conductor and the earth. Since the sum of the conductor currents and the neutral/earth current must be zero, the system of equations can be reduced. This reduction is done with Kron's method. SynerGEE assumes that the neutral is grounded at intervals sufficient to keep its voltage drop along the neutral close to zero. The following equation represents the expression for the 3 by 3 impedance matrix for a 4-wire grounded line.

$$Z_{Line}^{Gnd} = \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \quad \text{where} \quad \begin{aligned} Z_S &= Z_{PS} - \frac{Z_{PN}^2}{Z_{NS}} \\ Z_M &= Z_{PM} - \frac{Z_{PN}^2}{Z_{NS}} \end{aligned} \tag{3-16}$$

Zero values occur in some positions of the impedance matrices if phases are not present on the line.

Impedance values for balanced analysis

Often, information about detailed phasing and load placement necessary for valid three-phase analysis is not available and a balanced analysis should be used. SynerGEE uses by-phase models for lines in both by-phase and balanced analysis. In balanced analysis, loads are assumed to be balanced over all phases and, therefore, a single impedance value can be used to represent line section. For example:

$$V_{Drop} = Z_{Bal} I_{Line} \tag{3-17}$$

This balanced impedance is dependent on the number of conductors present. Impedances for one-, two-, and three-phase cases are as follows.

(3-18)

Three Phase	Two Phase	One Phase
$Z_{Bal} = Z_S - Z_M$	$Z_{Bal} = Z_S - \frac{1}{2} Z_M$	$Z_{Bal} = Z_S$

It is assumed that all currents and voltages in this type of analysis are balanced quantities over the phases present. The following is a look at three-, two-, and one-phase cases.

Three-phase case

The expression for voltage drop on a three-phase line using the full 3 by 3 matrix is as follows:

$$\begin{Bmatrix} V_{DropA} \\ V_{DropB} \\ V_{DropC} \end{Bmatrix} = \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} \tag{3-19}$$

The voltage drop from three-phase currents on phase A is listed below.

$$\begin{aligned} V_{DropA} &= I_A Z_S + \left(I_B + I_C\right) Z_M \\ &= I_A \left(Z_S - Z_M\right) + \left(I_A + I_B + I_C\right) Z_M \end{aligned} \tag{3-20}$$

Similar expressions can be found for the other two phases. Since you are assuming balanced loading served by this line, the sum of line currents must be zero. For example:

$$I_A + I_B + I_C \equiv 0 \tag{3-21}$$

The line impedance for the single-phase equivalent model is the difference between the series and mutual impedances of the balanced three-phase model. For example:

$$V_{DropA} = I_A(Z_S - Z_M) \tag{3-22}$$

You have seen other derivations to show that the positive sequence impedance of the balanced three-phase model is also $Z_S - Z_M$.

Two-phase case

Consider a two-phase line with conductor A and either conductor B or conductor C present. In the first case, the line-drop can be expressed as follows.

$$\begin{Bmatrix} V_{DropA} \\ V_{DropB} \\ - \end{Bmatrix} = \begin{bmatrix} Z_S & Z_M & 0 \\ & Z_S & 0 \\ & & 0 \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ 0 \end{Bmatrix} \tag{3-23}$$

The line-drop in the second case is as follows.

$$\begin{Bmatrix} V_{DropA} \\ - \\ V_{DropC} \end{Bmatrix} = \begin{bmatrix} Z_S & 0 & Z_M \\ & 0 & 0 \\ & & Z_S \end{bmatrix} \begin{Bmatrix} I_A \\ 0 \\ I_C \end{Bmatrix} \tag{3-24}$$

You are assuming balanced loading over the phases present. The magnitudes of currents in the above expressions are, therefore, equal and rotated 120° apart.

$$I_B = a^2 I_A \quad I_C = aI_A \tag{3-25}$$

The voltage drop along conductor A in the first case is, therefore, expressed as follows.

$$\text{Case 1}: V_{DropA} = I_A Z_S + a^2 I_A Z_M \tag{3-26}$$

In the second case, it is expressed as follows.

$$\text{Case 2}: V_{DropA} = I_A Z_S + aI_A Z_M \tag{3-27}$$

Therefore, the average voltage drop along conductor A over these two cases is:

$$\begin{aligned} \text{Single Line Equivalent}: V_{DropA} &= I_A\left(Z_S + \frac{a^2 + a}{2} Z_M\right) \\ &= Z_S - \tfrac{1}{2} Z_M \end{aligned} \tag{3-28}$$

Single-phase case

This section discusses the line-drop expression for a single-phase line using the full balanced impedance matrix and phase A. For example:

$$\begin{Bmatrix} V_{DropA} \\ - \\ - \end{Bmatrix} = \begin{bmatrix} Z_S & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{bmatrix} \begin{Bmatrix} I_A \\ 0 \\ 0 \end{Bmatrix} \tag{3-29}$$

The line-drop is, therefore, expressed as follows.

$$V_{DropA} = I_A Z_S \tag{3-30}$$

There is no mutual impedance expressed in the equation because there are no conductors other than conductor A.

Lines specified by sequence values

Data for lines and more often cables is given as positive and negative sequence values. The self and mutual impedance of the line can be determined directly from these values.

$$\begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} Z_0 & 0 & 0 \\ & Z_1 & 0 \\ & & Z_1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} : \begin{matrix} Z_S = (Z_0 + 2Z_1)/3 \\ Z_M = (Z_0 - Z_1)/3 \end{matrix} \tag{3-31}$$

It is essential to realize that the development of the sequence domain is based on a three-phase model. Three-phase wires and a neutral wire or neutral return path are needed for any line having its parameters expressed in the sequence domain. It follows that when the impedance of a line is expressed in the sequence domain, the impedance represents a three-phase line. It is acceptable to allow a sequence domain representation of a one- or two-phase line by ignoring the non-present phase on the transformed phase domain impedance matrix.

Sequence impedances for cables with separate neutral

Manufacturers typically generate sequence impedance values for three-phase lines made of cables assuming that all return current passes through the cable sheath. Sequence impedances are often given in two sets labeled single-phase and three-phase. The single-phase set of impedances corresponds to three-phase lines composed of individual cable conductors lying separately. The three-phase set of impedances corresponds to three-phase lines composed of cables having three conductors (and possibly a neutral) wrapped in a common jacket.

If a separate neutral (not accounted for in the cable impedance calculations by the cable manufacturer) is to be pulled with the cable, the self-and mutual-impedance of the neutral can be augmented onto the phase domain equivalent of the sequence impedance given for the cable. Geometric mean distances should be used in this procedure since the exact position of the separate neutral with respect to the other conductors is not known after cable pulling.

$$\begin{bmatrix} Z_S & Z_M & Z_M & Z_{NP} \\ & Z_S & Z_M & Z_{NP} \\ & & Z_S & Z_{NP} \\ & & & Z_{NN} \end{bmatrix} \text{where} \begin{aligned} Z_S &= (Z_0 + 2Z_1)/3 \\ Z_M &= (Z_0 - Z_1)/3 \\ Z_{NP} &= \text{Neutral - Phase Impedance} \\ Z_{NN} &= \text{Phase Self Impedance} \end{aligned} \tag{3-32}$$

The impedances of the neutral are determined using Carson's method for calculating mutual and self-impedances of cables. A reduction can now be done, assuming that the neutral-ground voltage remains small.

$$\begin{bmatrix} Z_S^{'} & Z_M^{'} & Z_M^{'} \\ & Z_S^{'} & Z_M^{'} \\ & & Z_S^{'} \end{bmatrix} \text{where} \begin{aligned} Z_S^{'} &= Z_S - \frac{Z_{NP}^2}{Z_{NN}} \\ Z_M^{'} &= Z_M - \frac{Z_{NP}^2}{Z_{NN}} \end{aligned} \tag{3-33}$$

Now you can convert back to the sequence domain as follows.

$$\begin{aligned} Z_0^{'} &= Z_S^{'} + 2Z_M^{'} = Z_0 - 3\frac{Z_{NP}^2}{Z_{NN}} \\ Z_{1,2}^{'} &= Z_S - Z_M = Z_{12} \end{aligned} \tag{3-34}$$

If you have cable data from a manufacturer, it is probably based on all return current passing through the cable sheaths. If you have a separate neutral lying with the cables, you can modify the zero sequence impedance based on the above equation.

Getting sequence impedances from self and mutual values

You can transform the line section model from the phase domain to the sequence domain as follows.

$$Z_P = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ & Z_1 & 0 \\ & & Z_1 \end{bmatrix} \tag{3-35}$$

$$a = 1\angle 120 \qquad \begin{aligned} Z_0 &= Z_S + 2Z_M \\ Z_{1,2} &= Z_S - Z_M \end{aligned}$$

Line capacitance (admittance) calculations

If your line section is based on the conductor/spacing method of line modeling, Carson's method of images can determine the line admittance of the conductors. Admittance values are calculated between the phases and from the phases to the earth plane. SynerGEE calculates the line-ground admittance, and utilizes it in balanced and by-phase load-flows for both grounded and ungrounded sections.

SynerGEE's admittance calculations are complex. The detailed derivation and implementation of these calculations are available upon request. An overview shows that the admittance matrix is a function of a potential matrix that is reduced from a four-wire model to a three-wire equivalent.

$$[Y] = \begin{bmatrix} Y_s & Y_m & Y_m \\ & Y_s & Y_m \\ & & Y_s \end{bmatrix} = j\omega \begin{bmatrix} P_s & P_m & P_m \\ & P_s & P_m \\ & & P_s \end{bmatrix}^{-1} \text{S/m} \tag{3-36}$$

From this, values of self and mutual admittance can be found.

$$Y_S = j\omega \frac{P_s + P_m}{(P_s - P_m)(P_s + 2P_m)} * 304.7 \ {}^{\mu S}\!/_{1000\,ft}$$
$$Y_M = j\omega \frac{-P_m}{(P_s - P_m)(P_s + 2P_m)} * 304.7 \ {}^{\mu S}\!/_{1000\,ft} \tag{3-37}$$

The potential matrix relates line charge with line voltage. Its terms are given as follows.

$$P_s = P_p - \frac{P_{np}^2}{P_n}$$
$$P_m = P_{pp} - \frac{P_{np}^2}{P_n} \tag{3-38}$$

The self and mutual potential values are as follows.

$$P_{PS} = \frac{1}{2\pi\varepsilon} \ln \frac{H_{Eq}}{R_P} \ {}^{m}\!/_{F} \qquad P_{NS} = \frac{1}{2\pi\varepsilon} \ln \frac{H_{Eq}}{R_N} \ {}^{m}\!/_{F}$$
$$P_{PM} = \frac{1}{2\pi\varepsilon} \ln \frac{\sqrt{H_{Eq}^2 + D_P^2}}{D_P} \ {}^{m}\!/_{F} \qquad P_{NM} = \frac{1}{2\pi\varepsilon} \ln \frac{\sqrt{H_{Eq}^2 + D_N^2}}{D_N} \ {}^{m}\!/_{F} \tag{3-39}$$

The symbols R_P and R_N refer to phase and neutral conductor radius values. The symbol H_{Eq} refers to the equivalent height of the line section and is defined in a manner similar to the equivalent spacing. Consistent units for distance, height, and radius are implied in the above equations.

Robust by-phase derivation

In SynerGEE, admittance and capacitance values represent electromagnetic coupling between phase conductors, phase and neutral conductors, and conductors and earth. The information provided in this section deals with the phase equivalent model of one-, two-, and three-phase overhead line sections. The derivations are directed to distribution lines but are easily extended to transmission lines with multiple conductors per-phase and multiple neutrals.

The following model results from the derivations:

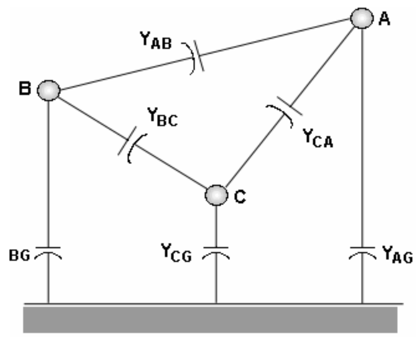


Fig. 3-6 *Line section with admittance components*

This figure represents the admittance values between the equivalent phase conductors and between the conductors and ground. The effects of line spacing, earth, and conductor height are included. The effects of the neutral conductor are included and reduced into the three phases represented above. This model is valid for both grounded and ungrounded lines as will be discussed later.

The admittance values in this model are used with the section voltages to determine the capacitive current or charging current for a particular line section. The relationship between admittance, section voltage, and charging current can generally be written as follows:

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} Y_s & Y_m & Y_m \\ & Y_s & Y_m \\ & & Y_s \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} \tag{3-40}$$

Robust admittance calculations

SynerGEE uses the method of images for admittance calculations. It replaces the earth plane with reflections of each conductor lying below the earth's surface. The following definitions are used.

(3-41)

$D_{ij'}$ = Distance from conductor i to image of j

D_{ij} = Distance from conductor i to conductor j

D_{kk} = Radius of conductor k

$D_{kk'}$ = 2*Height of conductor k (cond. to reflection)

q_k = Charge on conductor k

P_{ij} = Potential between conductors i and j

These values apply to this representation of a transmission line and its reflection. For example:

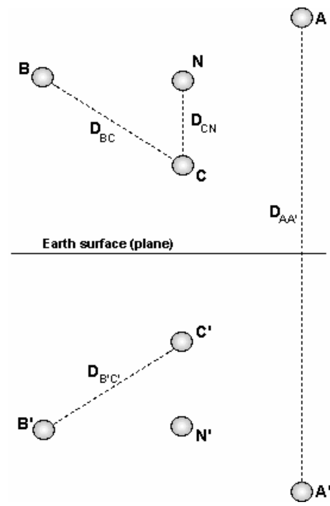


Fig. 3-7 *Four conductors and their reflections*

The voltage between two conductors due to a charge on another conductor is given by:

$$V_{ij}\Big|_k = \frac{q_k}{2\pi\varepsilon}\ln\frac{D_{jk}}{D_{ik}} \tag{3-42}$$

This is found by integrating the electric field along the path from i to k and then k to j.

To determine the admittance matrix for a line section, you are interested in the voltage from a conductor to the earth plane. The voltage from a conductor to its image can be obtained from a superposition of the above equation that results in the following.

$$V_{ii'} = \frac{1}{\pi\varepsilon}\sum_{k=a}^{N} q_k \ln\frac{D_{ik'}}{D_{ik}} \tag{3-43}$$

The valid range of an index such as i or k is over phases A, B, C, and N (the single neutral).

The voltage between a conductor and its image is given by the equation above. The voltage from conductor i to the earth plane is, therefore, one-half of this value since the earth plane is exactly halfway between a conductor and its image. Hence:

$$V_{in} = \frac{1}{2\pi\varepsilon}\sum_{k=a}^{N} q_k \ln\frac{D_{ik'}}{D_{ik}} \tag{3-44}$$

The potential is extracted from the terms of this expression as follows:

$$P_{ij} = \frac{1}{2\pi\varepsilon}\ln\frac{D_{ik'}}{D_{ik}} \quad \text{m/F} \tag{3-45}$$

Neutral conductors are periodically grounded and are assumed to have a voltage nearly that of ground, resulting in the following:

$$V_{Nn} \equiv \bar{0} \tag{3-46}$$

The following matrix equation can be written.

$$\begin{Bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \\ \bar{0} \end{Bmatrix} = \begin{bmatrix} P_{aa} & P_{ab} & P_{ac} & P_{aN} \\ P_{ba} & P_{bb} & P_{bc} & P_{bN} \\ P_{ca} & P_{cb} & P_{cc} & P_{cN} \\ P_{Na} & P_{Nb} & P_{Nc} & P_{NN} \end{bmatrix} \begin{Bmatrix} q_a \\ q_b \\ q_c \\ q_N \end{Bmatrix} \tag{3-47}$$

This matrix can be reduced with the Kron technique to get the following.

$$\{V_{kn}\} = \left[P^{\text{Red.}}\right]\{q_k\} \tag{3-48}$$

The admittance matrix is, therefore, expressed as follows.

$$[Y] = j\omega\left[P^{\text{Red.}}\right]^{-1} \text{S/m} \tag{3-49}$$

Physical model

Shown below is a representation of a line section with charging admittance components drawn in.

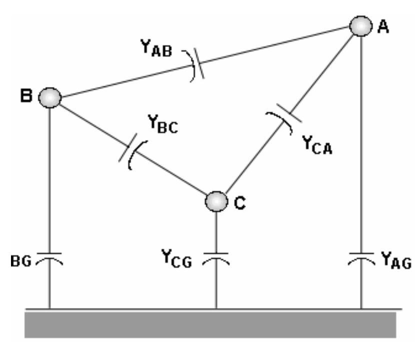


Fig. 3-8 *Line model with capacitance*

You can write three equations describing the conductor current flow based on the voltages across the conductors. Your model is based on the knowledge of line-ground voltages. For example:

$$\begin{aligned} I_A &= V_{AB}Y_{AB} + V_{AC}Y_{AC} + V_{AG}Y_{AG} \\ I_B &= V_{BA}Y_{AB} + V_{BC}Y_{BC} + V_{BG}Y_{BG} \\ I_C &= V_{CA}Y_{AC} + V_{CB}Y_{CB} + V_{CG}Y_{CG} \end{aligned} \tag{3-50}$$

You can substitute differences in line-ground voltages as follows.

$$V_{XY} = V_{XG} - V_{YG} \tag{3-51}$$

Substituting these values, you get the grounded line's admittance matrix, as follows.

$$[Y_{Gnd}] = \begin{bmatrix} Y_{AG} + Y_{AB} + Y_{AC} & -Y_{AB} & -Y_{AC} \\ & Y_{BG} + Y_{AB} + Y_{BC} & -Y_{BC} \\ & & Y_{CG} + Y_{AC} + Y_{BC} \end{bmatrix} \quad \textbf{(3-52)}$$

Ungrounded Wye- or Delta-connected lines

The line-ground admittance should be considered in lines with no neutral conductor. These effects are a little more difficult to analyze because the voltage from a conductor to the earth plane is not known. Below is a diagram of an ungrounded line section.

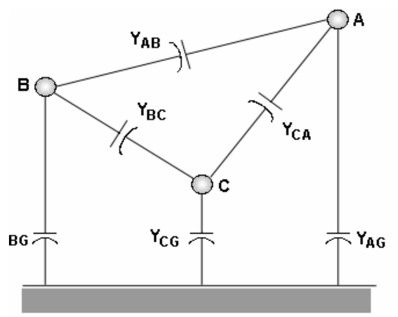


Fig. 3-9 *Ungrounded line with admittance components*

You can express the voltage and current relationship through the line admittances as four equations relating each conductor current and the earth current. For example:

$$\begin{aligned} I_A &= V_{AB}Y_{AB} + V_{AC}Y_{AC} + V_{AG}Y_{AG} \\ I_B &= V_{BA}Y_{AB} + V_{BC}Y_{BC} + V_{BG}Y_{BG} \\ I_C &= V_{CA}Y_{AC} + V_{CB}Y_{CB} + V_{CG}Y_{CG} \\ I_G &= -V_{AG}Y_{AG} - V_{BG}Y_{BG} - V_{CG}Y_{CG} \end{aligned} \quad \textbf{(3-53)}$$

Your goal is to eliminate the ground current I_G and express the voltage/current relationship of the line in terms of arbitrary reference voltages used in analyzing ungrounded lines. The first step is to reduce the references of wire-ground quantities as follows.

$$\begin{aligned} V_{BG} &= V_{AG} - V_{AB} \\ V_{CG} &= V_{AG} + V_{CA} \end{aligned} \quad \textbf{(3-54)}$$

You can rewrite the above set of equations with the following substitutions.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \\ I_G \end{Bmatrix} = \begin{bmatrix} Y_{AB} & 0 & -Y_{AC} & Y_{AG} \\ -Y_{AB} - Y_{BN} & Y_{BC} & 0 & Y_{BG} \\ 0 & -Y_{BC} & Y_{CA} + Y_{CN} & Y_{CG} \\ Y_{BG} & 0 & -Y_{CG} & -K \end{bmatrix} \begin{Bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \\ V_{AG} \end{Bmatrix} \quad \textbf{(3-55)}$$

$$K = Y_{AG} + Y_{BG} + Y_{CG}$$

You wish to achieve a ground current of zero for the ungrounded section. You can use a Kron reduction to reduce the system with $I_G = \bar{0}$ to obtain the following.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} Y_{AB} + \frac{Y_{AG}Y_{BG}}{K} & 0 & -Y_{CA} - \frac{Y_{AG}Y_{CG}}{K} \\ -Y_{AB} - Y_{BG} + \frac{Y_{BG}^2}{K} & Y_{BC} & -\frac{Y_{BG}Y_{CG}}{K} \\ \frac{Y_{BG}Y_{CG}}{K} & -Y_{BC} & Y_{CA} + Y_{CG} - \frac{Y_{CG}^2}{K} \end{bmatrix} \begin{Bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{Bmatrix} \quad \textbf{(3-56)}$$

You can choose an arbitrary reference voltages to get the following results.

$$V_{AB} = V_{AR} - V_{BR} \quad V_{BC} = V_{BR} - V_{CR} \quad V_{CA} = V_{CR} - V_{AR} \quad \textbf{(3-57)}$$

These quantities can be substituted into the previous matrix equation to get a voltage/current relationship as follows.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \left[\underline{Y}_{UnGnd}\right] \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} \quad \textbf{(3-58)}$$

In this case, the admittance matrix acquires the following form.

$$\left[\underline{Y}_{UnGnd}\right] = \begin{bmatrix} Y_{AB} + \frac{Y_{AG}Y_{BG}}{K} + Y_{CA} + \frac{Y_{AG}Y_{CG}}{K} & -Y_{AB} - \frac{Y_{AG}Y_{BG}}{K} & -Y_{CA} - \frac{Y_{AG}Y_{CG}}{K} \\ -Y_{AB} - Y_{BG} + \frac{Y_{BG}Y_{CG}}{K} + \frac{Y_{BG}^2}{K} & Y_{BC} + Y_{AB} + Y_{BG} - \frac{Y_{BG}^2}{K} & -Y_{BC} - \frac{Y_{BG}Y_{CG}}{K} \\ -Y_{CA} - Y_{CG} + \frac{Y_{BG}Y_{CG}}{K} + \frac{Y_{CG}^2}{K} & -Y_{BC} - \frac{Y_{BG}Y_{CG}}{K} & Y_{CA} + Y_{CG} + Y_{BC} - \frac{Y_{CG}^2}{K} \end{bmatrix} \quad \textbf{(3-59)}$$

You can then make the following simplifications.

$$\begin{gathered} -Y_{AB} - Y_{BG} + \frac{Y_{BG}Y_{CG}}{K} + \frac{Y_{BG}^2}{K} = -Y_{AB} - \frac{Y_{AG}Y_{BG}}{K} \\ -Y_{CA} - Y_{CG} + \frac{Y_{BG}Y_{CG}}{K} + \frac{Y_{CG}^2}{K} = -Y_{CA} - \frac{Y_{AG}Y_{CG}}{K} \\ \frac{Y_{AG}Y_{BG}}{K} + \frac{Y_{AG}Y_{CG}}{K} = Y_{AG} - \frac{Y_{AG}^2}{K} \end{gathered} \quad \textbf{(3-60)}$$

After making these substitutions, you have a symmetrical admittance matrix for an ungrounded section, formulated as follows.

$$\left[\underline{Y}_{UnGnd}\right] = \begin{bmatrix} Y_{AB} + Y_{CA} + Y_{AG} - \frac{Y_{AG}^2}{K} & -Y_{AB} - \frac{Y_{AG}Y_{BG}}{K} & -Y_{CA} - \frac{Y_{AG}Y_{CG}}{K} \\ & Y_{BC} + Y_{AB} + Y_{BG} - \frac{Y_{BG}^2}{K} & -Y_{BC} - \frac{Y_{BG}Y_{CG}}{K} \\ & & Y_{BC} + Y_{CA} + Y_{CG} - \frac{Y_{CG}^2}{K} \end{bmatrix} \quad \textbf{(3-61)}$$

Relationship of Wye ungrounded to Wye grounded

The nodal admittance matrix for a line section was presented earlier with the development of Carson's equations. The admittance values of that matrix correspond to capacitance values in the grounded line section model as follows.

$$[Y]_{Gnd} = \begin{bmatrix} Y_{AG}+Y_{AB}+Y_{AC} & -Y_{AB} & -Y_{AC} \\ -Y_{AB} & Y_{BG}+Y_{AB}+Y_{BC} & -Y_{BC} \\ -Y_{AC} & -Y_{BC} & Y_{CG}+Y_{AC}+Y_{BC} \end{bmatrix} \quad \text{(3-62)}$$

The easiest way to extract $\underline{Y}_{UnGnd}$ from this matrix is to explicitly determine the individual admittance values from the numerical instance of the above matrix and form the ungrounded line's admittance matrix.

It should be noted that a grounded model is valid for ungrounded lines. However, the problem with always using the grounded matrix is that the wire-ground voltage is not known. The formulation used to determine the ungrounded matrix accounted for the indeterminate wire-ground voltage by forcing the earth current due to line charging to be zero.

Balanced ungrounded line

If you are using a model for a line having a completely balanced construction, you have the following.

$$\begin{aligned} Y_{AB} &= Y_{BC} = Y_{CA} = Y_{LL} \\ Y_{AG} &= Y_{BG} = Y_{CG} = Y_{LG} \end{aligned} \quad \text{(3-63)}$$

You can represent your admittance matrix as follows.

$$[\underline{Y}] = \begin{bmatrix} Y_S & Y_M & Y_M \\ & Y_S & Y_M \\ & & Y_S \end{bmatrix} \quad \text{where} \quad \begin{aligned} Y_S &= 2Y_{LL} - \tfrac{2}{3}Y_{LG} \\ Y_M &= -Y_{LL} - \tfrac{1}{3}Y_{LG} \end{aligned} \quad \text{(3-64)}$$

The positive sequence admittance is as follows:

$$Y_1 = Y_S - Y_M = Y_{LL} - Y_{LG} \quad \text{(3-65)}$$

Approximate capacitance values

You can identify the capacitance between two long cylindrical conductors by solving for the following.

$$C_{xy} = \frac{\pi\varepsilon}{\ln\left(\dfrac{D_{x-y}}{radius}\right)} \quad \text{(3-66)}$$

Keep in mind that you cannot use this equation directly for the line-line capacitance values in a Wye or Delta system, since the earth plane and other conductors contribute to each capacitance value.

Equidistant case

Consider the case where minimal data is known about a line section. Assume that all conductors of a three-phase line have the same radius and that their geometric equivalent spacing and geometric equivalent height is known. The potentials along the diagonal are functions of conductor height and conductor radius. The unknown height of each conductor, *k*, in feet is given as H_k and the radius is also given in feet as R_k. You can find a height to represent all phases so that the average conductor potentials can be found, as follows.

(3-67)

$$P_{AvgPhase} = \frac{1}{3} * \frac{1}{2\pi\varepsilon} \sum_{k=1}^{3} \ln \frac{2 * H_k}{R_k}$$

The sum of the logarithms can revert to a product within. Assume that all phase conductors are identical, so that $R_k = R_c$. For example:

(3-68)

$$\overline{P}_P = \frac{1}{2\pi\varepsilon} \ln \frac{2 * \sqrt{H_1 * H_2 * H_3}}{R_C}$$

Define the equivalent height as follows.

(3-69)

$$\overline{D_{kk}} = 2 * \sqrt[3]{H_1 * H_2 * H_3}$$

The phase self-potential is, therefore, solved as follows.

(3-70)

$$\overline{P}_P = \frac{1}{2\pi\varepsilon} \ln \frac{\overline{D_{kk}}}{R_C}$$

If you assume that the equivalent height is an approximate height of any one conductor, and the equivalent spacing presented earlier represents an approximate distance between conductors, then the distance between a conductor, *j*, and the reflection of conductor *k* can be approximated as follows.

(3-71)

$$\overline{D}_{jk'} = \sqrt{\left(2 * \overline{D_{kk}}\right)^2 + D_{eq}^2}$$

Therefore, the potential between conductors is as follows.

(3-72)

$$\overline{P}_{pp} = \frac{1}{2\pi\varepsilon} \ln \frac{\sqrt{\overline{D_{kk}^2} + D_{eq}^2}}{D_{eq}}$$

Neutral self-potential can be approximated as the average conductor value, as follows.

$$\overline{P}_n = \frac{1}{2\pi\varepsilon} \ln \frac{\overline{D_{kk}}}{R_n} \tag{3-73}$$

Potential between the neutral and phases is as follows.

$$\overline{P}_{np} = \frac{1}{2\pi\varepsilon} \ln \frac{\sqrt{\overline{D_{kk}^2} + N_{eq}^2}}{N_{eq}} \tag{3-74}$$

The potential matrix equation now appears as follows.

$$\begin{Bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \\ \vec{0} \end{Bmatrix} = \begin{bmatrix} P_p & P_{pp} & P_{pp} & P_{np} \\ 0 & P_p & P_{pp} & P_{np} \\ 0 & 0 & P_p & P_{np} \\ 0 & 0 & 0 & P_n \end{bmatrix} \begin{Bmatrix} q_a \\ q_b \\ q_c \\ q_N \end{Bmatrix} \tag{3-75}$$

Kron reduction can be used to get the reduced potential matrix. Self- and mutual-potentials are expressed as follows.

$$P_s = P_p - \frac{P_{np}^2}{P_n} \tag{3-76}$$

$$P_m = P_{pp} - \frac{P_{np}^2}{P_n}$$

You can now determine the admittance matrix as follows.

$$[Y] = j\omega \begin{bmatrix} P_s & P_m & P_m \\ & P_s & P_m \\ & & P_s \end{bmatrix}^{-1} \text{ S/m} \tag{3-77}$$

The matrix inversion leads to the following.

$$[Y] = \frac{j\omega}{(P_s - P_m)(P_s + 2P_m)} \begin{bmatrix} P_s + P_m & -P_m & -P_m \\ & P_s + P_m & -P_m \\ & & P_s + P_m \end{bmatrix} \text{S/m} \tag{3-78}$$

The line-ground admittance results as follows.

$$Y_g = j\omega \frac{P_s + P_m}{(P_s - P_m)(P_s + 2P_m)} \tag{3-79}$$

$$Y_m = j\omega \frac{-P_m}{(P_s - P_m)(P_s + 2P_m)}$$

Cable impedance calculations

Cable conductors are typically bound with neutral strands or tape. Before performing cable impedance calculations, you must find the parameters for an equivalent neutral conductor associated with each phase cable. These parameters are found from the characteristics of the neutral strands or neutral shield of the cable.

Three-phase cables with three neutral return paths along with a possible separate neutral conductor lead to a 7 by 7 primitive impedance matrix. In the following examples, the matrix will be organized with the following numbering.

$$\begin{aligned}
&A-\text{Phase 'A' cable}\\
&B-\text{Phase 'B' cable}\\
&C-\text{Phase 'C' cable}\\
&N-\text{Neutral conductor}\\
&na-\text{Equivalent neutral for phase 'A'}\\
&nb-\text{Equivalent neutral for phase 'B'}\\
&nc-\text{Equivalent neutral for phase 'C'}
\end{aligned} \qquad \textbf{(3-80)}$$

You can form the primitive impedance matrix using Carson's equations, using:

$$\begin{aligned}
Z_{ii} &= r_i + k_1 f + jk_2 f\left(k_3 - \ln GMR_i + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/mi\\
Z_{ij} &= k_1 f + jk_2 f\left(k_3 - \ln D_{ij} + \ln\sqrt{\frac{\rho}{f}}\right)\Omega/mi \quad i \neq j
\end{aligned} \qquad \textbf{(3-81)}$$

to fill the elements for:

$$Z_{Prim} = \begin{bmatrix}
Z_{A-A} & Z_{A-B} & Z_{A-C} & Z_{A-N} & Z_{A-an} & Z_{A-bn} & Z_{A-cn}\\
 & Z_{B-B} & Z_{B-C} & Z_{B-N} & Z_{B-an} & Z_{B-bn} & Z_{B-cn}\\
 & & Z_{C-C} & Z_{C-N} & Z_{C-an} & Z_{C-bn} & Z_{C-cn}\\
 & & & Z_{N-N} & Z_{N-an} & Z_{N-bn} & Z_{N-cn}\\
 & & & & Z_{an-an} & Z_{an-bn} & Z_{an-cn}\\
 & & & & & Z_{bn-bn} & Z_{bn-cn}\\
 & & & & & & Z_{cn-cn}
\end{bmatrix} \qquad \textbf{(3-82)}$$

After this matrix is generated, the last four rows are reduced in a fashion similar to that for overhead lines. The result is a reduced 3 by 3 impedance matrix suitable for by-phase analysis.

Concentric neutral

The equivalent neutral conductor representing the strands of a concentric neutral cable can be found with the following definitions.

$$d_{cn} = \frac{\text{cable outside diameter - neutral strand}}{2}\ ft \tag{3-83}$$

$$GMR_c = \text{conductor GMR in feet}$$

$$GMR_s = \text{strand GMR in feet}$$

$$r_c = \text{conductor resistance in ohms/mile}$$

$$r_s = \text{strand resistance in ohms/mile}$$

$$N_s = \text{number of neutral strands}$$

The GMR of the equivalent neutral is given by:

$$GMR_{cn} = \sqrt[N_s]{GMR_S N_s d_{cn}^{N_s-1}{}_{cn}} \tag{3-84}$$

The concentric neutral resistance is found by dividing the per-strand resistance by the number of strands.

$$r_{cn} = \frac{r_s}{N_s} \tag{3-85}$$

The last two equations provide parameters sufficient for the Carson self-impedance equation pertaining to the equivalent neutral conductors. Applying the mutual equation will require distance values between the equivalent neutrals, between phases and the equivalent neutrals, and between the neutral and the equivalent neutrals.

The distance between the equivalent neutral and its associated phase conductor is defined as the distance between each strand and the center of the phase conductor. Assuming a phase "x":

$$D_{xn-x} = d_{cn} \tag{3-86}$$

The distance between equivalent neutrals is the same as the distance between their respective phase conductors.

$$D_{xn-xn} = D_{x-x} \tag{3-87}$$

Finally, the distance between an equivalent neutral and an adjacent phase or neutral conductor is given as:

$$D_{xn-y} = \sqrt[N_S]{D_{x-y}^{N_S} - d_{cn}^{N_S}} \tag{3-88}$$

These distance values can be applied directly to Carson's mutual impedance term.

Tape shield cables

W.H. Kersting in 92 EHO 361-6-PWR provides a derivation for the resistance of a tape shield. The result is:

$$r_{tape}\ \Omega / mile = \frac{3.7653}{\text{diameter over insulation (in.)}} \tag{3-89}$$

The GMR of the equivalent conductor is given as:

$$GMR_{tape}(ft) = \frac{\text{diameter over insulation (in.)}}{24} + 0.000208 \tag{3-90}$$

Distance values for use in Carson's mutual impedance equation are also listed in that reference.

Cable admittance

W.H. Kersting in 92 EHO 361-6-PWR provides the following cable admittance equations.

For concentric neutral cables:

$$Y_{ii}(\mu S / mile) = j \frac{77.582}{\ln\left(\frac{r_{cn}}{r_{conductor}}\right) - \frac{\ln\left(\frac{N_S\left(r_{cn} + r_{strand}\right)}{r_{cn}}\right)}{N_S}} \tag{3-91}$$

For tape shield conductors:

$$Y_{ii}(\mu S / mile) = j \frac{77.586}{\ln\left(\frac{r_{insulation}}{r_{phase}}\right)} \tag{3-92}$$

Conductor damage curves

Current in conductors must be kept below values that would damage the tensile or conducting properties of the conductors. In overhead lines, conductor annealing or tensile damage may be of concern (although with today's larger loads, conductors are larger). A cable may be limited by the ability of its insulation to withstand heat before being damaged, although in some cases, cable annealing may be the limiting factor instead of cable insulation damage. In all cases, conductor damage curves are generated based on conductor material, insulation, and temperature rise. For example, a conductor of a given material and insulation is allowed to have a temperature rise of a given amount before damage occurs. Current flow over time causes the temperature rise, so time relates directly to current in determining when damage may occur.

The fundamental equation for conductor damage curves is as follows.

$$I^2 t = N_1 A^2 \log\left[\frac{T_2 + T_{Mat}}{T_1 + T_{Mat}}\right] \tag{3-93}$$

$$\begin{array}{ll} I = Amps & t = seconds \\ N_1 = material\ constant & A = area\ in\ MCM \\ T_1 = ambient\ temp\ °C & T_2 = max\ temp\ °C \\ \multicolumn{2}{c}{T_{Mat} = material\ temp\ constant} \end{array}$$

You can arrange this equation into a more typical form.

$$I = \frac{1000A\sqrt{N_1 \log\left[\frac{T_2 + T_{Mat}}{T_1 + T_{Mat}}\right]}}{\sqrt{t}} \tag{3-94}$$

Now, you can define the "conductor damage factor."

$$f_D = A\sqrt{N_1 \log\left[\frac{T_2 + T_{Mat}}{T_1 + T_{Mat}}\right]} \tag{3-95}$$

Then you can reach the final form.

$$I\ (kA) = \frac{f_D}{\sqrt{t}} \tag{3-96}$$

Notice that the current is in kilo-amps.

Here are some typical values for N_1 from [9, E558]:

$$\begin{array}{r} \text{Copper: } N_1 = 0.0297 \\ \text{Aluminum: } N_1 = 0.0125 \end{array} \tag{3-97}$$

Here is an example:

Assume that you have a 250 MCM copper polyethylene cable. This cable can reach a maximum of 150 degrees Celsius during a fault before damage occurs. Typically, cable ambient temperature is assumed to be 75 degrees. The material constant for this type of conductor and insulation was looked up and found to be 0.0297. The material temperature constant for copper is 234 degrees.

$$f_D = 250 * \sqrt{0.0297 \log\left[\frac{150 + 234}{75 + 234}\right]} = 13.2 \tag{3-98}$$

The equation for the damage curve is as follows.

$$I(kA) = \frac{13.2}{\sqrt{t}} \tag{3-99}$$

The damage factor, f_D, is stored directly in the conductor tables.

MCM and circular mils

In other most countries wire size is usually specified in square-millimeters. In the United States, wire size is typically specified using the **American wire gauge** (*AWG*). For non MCM wire gauges (an example is #6 copper), you can think of the conductor size as the number of bare conductors that can be placed side by side to span one inch. A #8 conductor is about 0.125 inches and #12 would be much smaller.

Conductors not sized by wire gauge are sized by thousand-circular mils. That standard is based on a unit called circular mils. Circular mils and thousand-circular mil concepts are somewhat non-intuitive. Lets start with the circular mil.

Circular mils

The circular mil is a unit of area. It is the equivalent area of a circle with a diameter of one thousandth of an inch.

$$d = 1x10^{-3} in \tag{3-100}$$

The diameter can be used to calculate the area per mil:

$$A = \frac{\pi}{4} x10^{-6} \quad in^2 / mil \tag{3-101}$$

Or in square mm:

$$A = 161.29\pi\, x10^{-6} \quad mm^2 / mil \tag{3-102}$$

If kcmil are used:

$$A = 161.29\pi\, x10^{-3} \quad mm^2 / kcmil \tag{3-103}$$

For example, the area of a 750MCM AA conductor would be:

$$A = 750 \cdot 161.29\pi\, x10^{-3} = 380.0\ mm^2 \tag{3-104}$$

MCM

Conductors larger than 4/0 AWG are generally identified by the area in thousands of circular mils. Now, things get confusing. As we saw above:

- One circular mil = area of rod 1 mil in diameter
- One MCM (1000 kcmil) = area of rod 1000 mils in diameter

How many thousands of circular mils are there in 1 MCM? We can find out by dividing the area in square inches of 1 MCM by one-thousand mils.

$$\frac{\frac{\pi}{4}1^2}{1000 \cdot \frac{\pi}{4}\left(1x10^{-3}\right)^2} = 1000 \tag{3-105}$$

So, the MCM value represents a rod 1" in diameter

$$d = 1.0\,in \tag{3-106}$$

The diameter can be used to calculate the area per MCM:

$$A = \frac{\pi}{4} \quad in^2 / MCM \tag{3-107}$$

Converting strand parameters to MCM

Power conductors are stranded. Oftentimes strand diameter and count are given. In this case, the conductor conducting area would be:

$$A_{in^2} = N_{strands} \cdot \frac{\pi}{4} Diam_{strand}^2 \quad in^2 \tag{3-108}$$

We can divide that by inches per MCM calculated above to get:

$$A_{MCM} = N_{strands} \cdot Diam_{strand}^2 \quad MCM \tag{3-109}$$

We have a conductor with 19 strands having diameters of 0.1622". What is the MCM value of the conductor?

$$A = 19 \cdot 0.1622^2 \cdot 1000 = 500\ MCM \tag{3-110}$$

Conversions

The following sections present some discussions related to converting parameters and values for line section modeling.

Circular Mils to mm^2 area

Converting kilovar to admittance values

Since admittance is a property of the physical construction of a cable and is independent of the voltage applied to the cable, SynerGEE accepts the conductor admittance value.

You can use the following equation to convert kvar values to admittance values when the nominal voltage is known.

$$B_1(uS) = 1000 * \frac{kvar_{3Ph}}{Nominal\ kV_{LL}^2} \tag{3-111}$$

SynerGEE stores admittance values with the units of siemens x 10^{-6} or µS per kFt or km.

Converting values in pF to admittance values

Often, cable capacitance is reported in pico-Farads/ft. In the following example, assume that the value is C_{pF} and you are looking for B_{uS}.

$$\begin{aligned} Y(S) &= j\omega C \\ &so \\ B_{uS} &= C_{\rho F} \bullet 10^{-12} \bullet 2\pi \bullet Freq \bullet 10^{3} \bullet 10^{6} \\ &= \frac{2\pi F \bullet C_{\rho F}}{1000} {}^{uS}\!/\!_{kFt} \end{aligned} \tag{3-112}$$

Assume that you have a cable with a capacitance of 78pF/ft. You can calculate the admittance as:

$$\begin{aligned} B_{uS} &= \frac{377 \bullet 78}{1000} {}^{uS}\!/\!_{kFt} \\ &= 29.4 {}^{uS}\!/\!_{kFt} \end{aligned} \tag{3-113}$$

Geometric mean radius

The geometric mean radius (GMR) is a physical parameter of a conductor and is directly related to the conductor's reactance at 1 foot.

$$\begin{aligned} GMR\ (\text{in ft}) &= e^{-43.5X} \\ X &= \text{Reactance at 1' in Ohms/1000Ft} \end{aligned} \tag{3-114}$$

The GMR can also be approximated from the conductor radius. Since the GMR represents the radius of a solid conductor that exhibits the same reactive impedance properties as the actual conductor, it follows that this approximation is worse with more strands in the bundle.

$$GMR\ (\text{in ft}) = Radius * e^{-1/4} \tag{3-115}$$

Converting ohms/kFt to ohms/km

Converting an impedance from Ohms/1000 ft. at a 1-foot spacing to an impedance in Ohms/km at a 1m spacing can be achieved with the following equation.

$$Z_2 = 3.282 Z_1 + j0.0862 \tag{3-116}$$

where:

Z_1 = Line reactance in Ohms/kFt at 1 ft spacing

Z_2 = Line reactance in Ohms/km at 1 m spacing

The following steps demonstrate how the equation is derived. Taking the basic line impedance equation:

$$Z_1 = r_1 + j\frac{0.12137 * \ln \dfrac{D_{eq}}{GMR}}{5.28}\ \Omega / kft \tag{3-117}$$

You can change the equivalent spacing from 1 foot to 1 meter, and express the entire impedance value in Ohms/km.

$$Z_2 = \left(r_1 + j\frac{0.12137 * \ln \dfrac{D_{eq} * 3.28}{GMR}}{5.28} \right) \frac{5.28}{1.609}\ \Omega / km \tag{3-118}$$

After grouping and breaking up the logarithm, you can derive the conversion equation.

$$Z_2 = 3.28 Z_1 + j.0896\ \ \Omega / km \tag{3-119}$$

As another example, assume that you have line reactance values in Ohms/km with a 2m spacing. You can go through a similar series of steps.

$$X \text{ at 1m spacing} = \frac{0.12137 * \ln \dfrac{0.5 * D_{eq}}{GMR}}{5.28} \tag{3-120}$$

This yields:

$$X \text{ at 1m spacing} = X \text{ at 2m spacing - j0.052285 } \Omega\text{/kM} \tag{3-121}$$

Numerical Examples

The following are examples of actual values applied to some of the equations in this chapter.

Line impedance for an overhead line

Given:

Line	4/0 ACSR. #1 ACSR neutral. Dp = 5 feet. Dn = 3.5 feet

Desired:

Line impedance in Ohms/mile.

Calculations:

First, you need resistance and GMR for the conductor. You can use the *Transmission and Distribution Manual* to find these for 4/0 ACSR.

(3-122)

$$\begin{aligned} r_{4/0} &= 0.592\ \Omega/mile \\ GMR_{4/0} &= 0.00814\,ft \\ r_{\#1} &= 1.38\ \Omega/mile \\ GMR_{\#1} &= 0.00418\,ft \end{aligned}$$

Using equivalent spacing values leads to a simplified primitive impedance matrix. You need to get self and mutual impedance values.

(3-123)

$$\begin{aligned} Z_{PS} &= 0.592 + 0.00158837 * 60 + \\ &\quad j * 0.0020224 * 60 * \left(7.67860 - \ln(0.00814) + \ln(100/60)\right) \\ &= 0.687302 + j1.577519\ \Omega/mile \end{aligned}$$

(3-124)

$$\begin{aligned} Z_{PM} &= 0.00158837 * 60 + \\ &\quad j * 0.0020224 * 60 * \left(7.67860 - \ln(5.0) + \ln(100/60)\right) \\ &= 0.095302 + j0.798442\ \Omega/mile \end{aligned}$$

$$Z_{NS} = 1.38 + 0.00158837 * 60 + \\ j * 0.0020224 * 60 * \left(7.67860 - \ln(0.00418) + \ln(100/60)\right) \\ = 1.4753 + j1.658393 \ \Omega/_{mile} \tag{3-125}$$

$$Z_{PN} = 0.00158837 * 60 + \\ j * 0.0020224 * 60 * \left(7.67860 - \ln(3.5) + \ln(100/60)\right) \\ = 0.095302 + j0.841722 \ \Omega/_{mile} \tag{3-126}$$

We can now calculate the reduced series and mutual impedance values.

$$Z_S = 0.687302 + j1.577519 - \frac{\left(0.095302 + j0.841722\right)^2}{1.4753 + j1.658393} \\ = 0.842734 + j1.294049 \ \Omega/_{mile} \tag{3-127}$$

$$Z_M = 0.095302 + j0.798442 - \frac{\left(0.095302 + j0.841722\right)^2}{1.4753 + j1.658393} \\ = 0.250734 + j0.514972 \ \Omega/_{mile} \tag{3-128}$$

The sequence impedance values can now be calculated.

$$Z_0 = 0.842734 + j1.294049 + 2(0.250734 + j0.514972) \\ = 1.344202 + j2.323993 \ \Omega/_{mile} \tag{3-129}$$

$$Z_1 = 0.842734 + j1.294049 - (0.250734 + j0.514972) \\ = 0.592 + j0.779077 \ \Omega/_{mile} \tag{3-130}$$

Results

You now have line impedances in Ohms/mile:

$$Z_0 = 1.344202 + j2.323993 \ \Omega/_{mile} \\ Z_1 = 0.592 + j0.779077 \ \Omega/_{mile} \\ Z_S = 0.842734 + j1.294049 \ \Omega/_{mile} \\ Z_M = 0.250734 + j0.514972 \ \Omega/_{mile} \tag{3-131}$$

You may notice that these values are slightly different than those that are calculated by SynerGEE, since the GMR values came from the *Transmission and Distribution Manual*. SynerGEE calculates a GMR from the X value at 1' spacing. The GMR values are slightly different. Round off error also plays a role in the differences.

Drop on a single-phase 'simple Z' line

Given:

On a 4.16kV feeder, 26,736 ft of single-phase #2 CU serves a constant current load of 50kW, 25kvar.

Impedance given as:

Z0=2.059 +j 0.4594 Ohm/mi

Z1=1.2075 +j 0.4815 Ohm/mi

Desired:

Voltage at load.

Comments:

This is a single phase line with impedance given in the sequence domain. We could do the calculations in the sequence domain but it is easier to use the phase domain and we are less likely to make a theoretical mistake.

Calculations:

Sequence domain impedances always correspond to three-phase lines and the return path. The given sequence domain impedance accounts for a three-phase line with earth return. First, transform it to the phase domain:

$$Z_{Seq} = \begin{bmatrix} 2.0592, 0.4594 & 0 & 0 \\ 0 & 1.2075, 0.4815 & 0 \\ 0 & 0 & 1.2075, 0.4815 \end{bmatrix} \tag{3-132}$$

By using:

$$Z_{PH} = AZ_{Seq}A^{-1} \tag{3-133}$$

To get:

$$Z_{AA} = Z_{BB} = Z_{CC} = 1.4914 + j0.474 \ \Omega / mi \tag{3-134}$$

This phase domain impedance includes the earth and neutral return path. We have a single phase line and will only use Z_{AA} (and ignore any off diagonal phase coupling values that were not shown above.)

Our load is a constant current one. The load current is:

$$I_L = \frac{(50,-25)\sqrt{3}}{4.160} = 23.28\angle -26.6A \tag{3-135}$$

The voltage drop can now be calculated as:

$$V_{Drop} = \frac{26736}{5280}(1.4914, 0.474)(23.28\angle -26.6) = 184.3\angle -8.96V \tag{3-136}$$

The voltage at the end of the line is:

$$V_{End} = \left| \frac{4160}{\sqrt{3}} - 184.3\angle -8.96 \right| = 2220V \tag{3-137}$$

On a 120V base, we have:

$$V_{End} = \frac{2220\sqrt{3}}{4160} \cdot 120 = 110.9V \tag{3-138}$$

This example does not include calculations for the effect of line capacitance.

Admittance for concentric neutral cable

Given:

Core	
Conductor Diameter	0.365 in
Insulation Diameter	1.091 in
Jacket Outside Diameter	1.419 in
Core Res	1.109 Ohms / mile
Core GMR	0.0115 ft
Concentric Strands	
Count	16
Diameter	0.064 in
Res	16.32 Ohms / mile

Admittance with material in this chapter

The admittance equation for concentric neutral cables listed in DSR 410.14 is:

$$Y_{shunt}(\mu S/mile) = j\frac{77.582}{\ln\left(\frac{r_{cn}}{r_{conductor}}\right) - \frac{\ln\left(\frac{N_S\left(r_{cn}+r_{strand}\right)}{r_{cn}}\right)}{N_S}} \tag{3-139}$$

The radius of the circle of concentric neutral strands is:

$$r_{cn} = \frac{1.419-0.064in}{2} = 0.677in \tag{3-140}$$

The conductor and strand radius values are:

$$r_{conductor} = \frac{0.365in}{2} = 0.1825in$$
$$r_{strand} = \frac{0.064in}{2} = .0.320in \tag{3-141}$$

Putting in values from above leads to:

$$Y_{shunt} = \frac{77.582}{\ln\left(\frac{0.677}{0.1825}\right) - \frac{\ln\left(\frac{16\left(0.677+0.320\right)}{0.677}\right)}{16}} = 69.68 \ \mu S/mile \tag{3-142}$$

Admittance with CRC

Here is the admittance equation (5.24) from CRC Electric Power Engineering Handbook:

$$Y_{shunt}(\mu S/mile) = j\frac{77.3619}{\ln\left(\frac{r_b}{RD_c}\right) - \frac{1}{k}\ln\frac{k \cdot RD_s}{R_b}} \tag{3-143}$$

The diameter of the ring of concentric neutral strands is:

$$r_b = \frac{1.419-0.064in}{2} = 0.677in \tag{3-144}$$

Radius of the conductor is:

$$RD_c = \frac{0.365in}{2} = 0.1825in \tag{3-145}$$

Radius of the strand is:

$$RD_S = \frac{0.064in}{2} = .0.320in \tag{3-146}$$

And

$$Y_{shunt} = j\frac{77.3619}{\ln\left(\frac{0.677}{0.1825}\right) - \frac{1}{16}\ln\frac{16 \cdot 0.320}{0.677}} = 65.31\,\mu S\,/\,mile \tag{3-147}$$

Appendix 1 – Bare conductor parameters

The following table has typical values for various wire sizes and materials. These values do not correspond to any particular manufacturer or reference. Better values may be available to the engineer.

All impedances are at 50C.

AWG or MCM	Material	Diam Inch	GMR Ft	Ohm/Mile	Amp Rat	Name
#6	AA	0.184	0.00555	3.903	65	Peachbell
#4	AA	0.232	0.007	2.453	90	Rose
#2	AA	0.292	0.00883	1.541	156	Iris
#1	AA	0.328	0.00991	1.224	177	Pansy
1/0	AA	0.368	0.0111	0.97	202	Poppy
2/0	AA	0.414	0.0125	0.769	230	Aster
3/0	AA	0.464	0.014	0.611	263	Phlox
4/0	AA	0.522	0.0158	0.484	299	Oxlip
250	AA	0.567	0.0171	0.41	329	Sneezewort

AWG or MCM	Material	Diam Inch	GMR Ft	Ohm/Mile	Amp Rat	Name
250	AA	0.567	0.0171	0.4712	329	Sneezewort
266.8	AA	0.586	0.0177	0.384	320	Daisy
300	AA	0.629	0.0198	0.342	350	Peony
336.4	AA	0.666	0.021	0.305	410	Tulip
350	AA	0.679	0.0214	0.294	399	Daffodil
397.5	AA	0.724	0.0228	0.258	440	Canna
450	AA	0.77	0.0243	0.229	450	Goldentuft
477.0	AA	0.795	0.0254	0.216	510	Syringa
500	AA	0.813	0.026	0.206	483	Hyacinth
556.5	AA	0.858	0.0275	0.186	560	Mistletoe
600	AA	0.891	0.0285	0.172	520	Meadows
636.0	AA	0.918	0.0294	0.163	620	Orchid
700	AA	0.963	0.0308	0.148	580	Verbena
715.5	AA	0.974	0.0312	0.145	680	Violet
750	AA	0.997	0.0319	0.139	602	Petunia
795.0	AA	1.026	0.0328	0.131	720	Arbutus
900	AA	1.092	0.0349	0.116	650	Cockcomb
954.0	AA	1.124	0.036	0.11	840	Magnolia
1,033.5	AA	1.17	0.0374	0.102	880	Bluebell
1,272	AA	1.3	0.0418	0.0841	1000	Narcissus
1,430	AA	1.379	0.0444	0.0756	1200	Carnation
1,590	AA	1.454	0.0468	0.0688	1300	Coreopisis
1,000	AA	1.151	0.0368	0.105	698	Hawkweed
#6 6/1	ACSR	0.198	0.00394	3.98	100	#6 ACSR

AWG or MCM	Material	Diam Inch	GMR Ft	Ohm/Mile	Amp Rat	Name
#5 6/1	ACSR	0.223	0.00416	3.18	120	#5 ACSR
#4 7/1	ACSR	0.257	0.00452	2.55	140	#4 ACSR
#4 6/1	ACSR	0.25	0.00437	2.57	140	#4 ACSR
#3 6/1	ACSR	0.281	0.0043	2.07	160	#3 ACSR
#2 7/1	ACSR	0.325	0.00504	1.65	180	#2 ACSR
#2 6/1	ACSR	0.316	0.00418	1.69	180	#2 ACSR
#1	ACSR	0.355	0.00418	1.38	200	#1 ACSR
1/0	ACSR	0.398	0.00446	1.12	230	1/0 ACSR
2/0	ACSR	0.447	0.0051	0.895	270	2/0 ACSR
3/0 6/1	ACSR	0.502	0.006	0.723	300	3/0 ACSR
4/0 6/1	ACSR	0.563	0.00814	0.592	340	4/0 ACSR
266.8 26/7	ACSR	0.642	0.0217	0.385	460	Partridge
300. 26/7	ACSR	0.68	0.023	0.342	490	Ostrich
300 30/7	ACSR	0.7	0.0241	0.342	500	Piper
336.4 30/7	ACSR	0.741	0.0255	0.306	530	Oriole
336.4 26/7	ACSR	0.721	0.0244	0.306	530	Linnet
397.5 26/7	ACSR	0.783	0.0265	0.259	590	Ibis
397.5 30/7	ACSR	0.806	0.0278	0.259	600	Lark
477 30/7	ACSR	0.883	0.0304	0.216	670	Hen
477,000 26/7	ACSR	0.858	0.029	0.216	670	Hawk
556.5 30/7	ACSR	0.953	0.0328	0.1859	730	Eagle
556.5 26/7	ACSR	0.927	0.0313	0.1859	730	Dove
605 54/7	ACSR	0.953	0.0321	0.1775	750	Peacock
605 26/7	ACSR	0.966	0.0327	0.172	760	Squab

AWG or MCM	Material	Diam Inch	GMR Ft	Ohm/Mile	Amp Rat	Name
636 54/7	ACSR	0.977	0.0329	0.1688	770	Rook
636 30/19	ACSR	1.019	0.0351	0.1618	780	Egret
636 27/7	ACSR	0.99	0.0335	0.1618	780	Grosbeak
666.6 54/7	ACSR	1	0.0337	0.1601	800	Flamengo
715.5 54/7	ACSR	1.036	0.0349	0.1482	830	Crow
715.5 30/19	ACSR	1.081	0.0372	0.1442	840	Redwing
715.5 26/7	ACSR	1.051	0.0355	0.1442	840	Starling
795 26/7	ACSR	1.108	0.0375	0.1288	900	Drake
795 54/7	ACSR	1.093	0.0368	0.1378	900	Condor
795 30/19	ACSR	1.14	0.0393	0.1288	910	Mallard
874.5 54/7	ACSR	1.146	0.0386	0.1228	950	Crane
900 54/7	ACSR	1.162	0.0391	0.1185	970	Canary
954 54/7	ACSR	1.196	0.0403	0.1128	1010	Cardinal
1,033.5 54/7	ACSR	1.246	0.042	0.1035	1060	Curlew
1,113 54/19	ACSR	1.293	0.0435	0.0969	1110	Finch
1,192.5 54/19	ACSR	1.338	0.045	0.0906	1160	Grackle
1,272 54/19	ACSR	1.382	0.0465	0.0851	1200	Pheasant
1,351 54/19	ACSR	1.424	0.0479	0.0803	1250	Martin
1,431 54/19	ACSR	1.465	0.0493	0.076	1300	Polver
1,510.5 54/19	ACSR	1.506	0.0507	0.072	1340	Parrot
1,590 54/19	ACSR	1.545	0.052	0.0684	1380	Falcon
1,781 84/19	ACSR	1.602	0.0534	0.0598	1400	Chukar
2,156 84/19	ACSR	1.762	0.0588	0.0505	1500	Bluebird
2,167 72/7	ACSR	1.735	0.057	0.0511	1600	Kiwi

AWG or MCM	Material	Diam Inch	GMR Ft	Ohm/Mile	Amp Rat	Name
2,312 76/19	ACSR	1.802	0.0595	0.0482	1700	Thrasher
2,515 76/19	ACSR	1.88	0.0621	0.045	1800	Joree
#24	Cu Solid	0.0201	0.000652	151.6163	1	24 Cu
#22	Cu Solid	0.0253	0.000821	95.4835	2	22 Cu
#20	Cu Solid	0.032	0.001038	59.684	3	20 Cu
#19	Cu Solid	0.0359	0.001165	47.5103	4	19 Cu
#18	Cu Solid	0.0403	0.001308	37.6726	5	18 Cu
#16	Cu Solid	0.0508	0.001649	23.7262	10	16 Cu
#14	Cu Solid	0.0641	0.00208	14.8722	20	14 Cu
#12	Cu Solid	0.0808	0.002622	9.3747	40	12 Cu
#10	Cu Solid	0.1019	0.003307	5.9026	75	10 Cu
#9	Cu Solid	0.1144	0.003712	4.6758	80	9 Cu
#8	Cu Solid	0.1285	0.004169	3.8	90	8 Cu
#7	Cu Solid	0.1443	0.00468	3.01	110	7 Cu
#6	Cu Solid	0.162	0.00526	2.39	120	6 Cu
#5	Cu Solid	0.1819	0.0059	1.895	140	5 Cu
#4	Cu Solid	0.204	0.00663	1.503	170	4 Cu
#3	Cu Solid	0.229	0.00745	1.192	190	3 Cu
#2	Cu Solid	0.258	0.00836	0.945	220	2 Cu
#2 7 Str	Cu Strand	0.292	0.00883	0.964	230	2 Cu
#1 7 Str	Cu Strand	0.328	0.00992	0.765	270	1 Cu
#1/0 7 Str	Cu Strand	0.368	0.01113	0.607	310	1/0 Cu
2/0 7 Str	Cu Strand	0.414	0.01252	0.481	360	2/0 Cu
3/0 7 Str	Cu Strand	0.464	0.01404	0.382	420	3/0 Cu

AWG or MCM	Material	Diam Inch	GMR Ft	Ohm/Mile	Amp Rat	Name
3/0 12 Str	Cu Strand	0.492	0.01559	0.382	420	3/0 Cu
4/0 7 Str	Cu Strand	0.522	0.01579	0.303	480	4/0 Cu
4/0 19 Str	Cu Strand	0.528	0.01668	0.303	480	4/0 Cu
4/0 12 Str	Cu Strand	0.552	0.0175	0.303	490	4/0 Cu
250 12 Str	Cu Strand	0.6	0.01902	0.257	540	250 Cu
250 19 Str	Cu Strand	0.574	0.01813	0.257	540	250 Cu
300 12 Str	Cu Strand	0.657	0.0208	0.215	610	300 Cu
300 19 Str	Cu Strand	0.629	0.01987	0.215	610	300 Cu
350 12 Str	Cu Strand	0.71	0.0225	0.1845	670	350 Cu
350 19 Str	Cu Strand	0.679	0.0214	0.1845	670	350 Cu
400 19 Str	Cu Strand	0.726	0.0229	0.1619	730	400 Cu
450 19 Str	Cu Strand	0.77	0.0243	0.1443	780	450 Cu
500 19 Str	Cu Strand	0.811	0.0256	0.1303	840	500 Cu
500 37 Str	Cu Strand	0.814	0.026	0.1303	840	500 Cu
600 37 Str	Cu Strand	0.891	0.0285	0.1095	940	600 Cu
700 37 Str	Cu Strand	0.963	0.0308	0.0947	1040	700 Cu
750 37 Str	Cu Strand	0.997	0.0319	0.0888	1090	750 Cu
800 37 Str	Cu Strand	1.029	0.0329	0.0837	1130	800 Cu
900 37 Str	Cu Strand	1.092	0.0349	0.0752	1220	900 Cu
1,000 37 Str	Cu Strand	1.151	0.0368	0.0685	1300	1000 Cu
3/8"	Steel	0.375	$1.56 \cdot 10^{-5}$	4.3	150	3/8” Steel

4
Switchable Devices

Introduction

SynerGEE allows you to model switches within your feeder models. These switches can be used to reconfigure your feeder connectivity. The following rules apply for placing switches on sections.

- There can be only one switch per section.
- A switch is located at either the source or load end of the section.
- Excepting network models, a switch cannot be closed to form a loop. A tie switch, which is an open switch with tie calculation capability, must be used instead.

In SynerGEE, network topology is governed solely with nodes and sections. Topology as well as the position and status of switches govern *connectivity*, or electrical flow.

When building radial feeders, the drawing and model manipulation tools within SynerGEE should prevent the creation of loops. The tie switch model should be used in cases where a looped analysis is needed.

Switches are placed on sections like other devices such as regulators and transformers. Switches are always the most outside device. When closed, a switch has no effect on the electrical connectivity of a section. When the switch is opened, the switch prevents power flow through the source or load end of a section. The following diagram presents a high-level view of an entire section within SynerGEE. It shows the locations for device placement on the section.

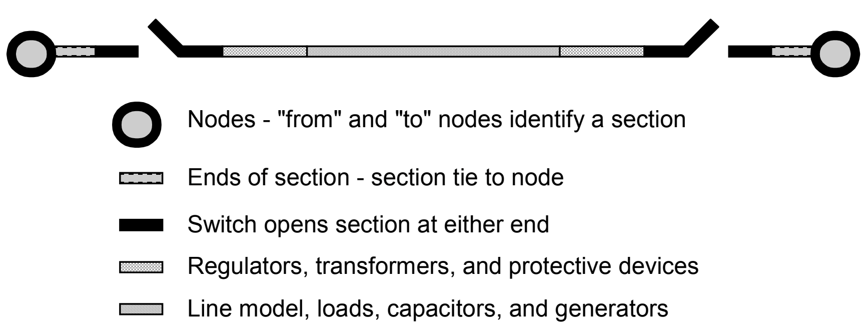


Fig. 4-1 Section broken into areas where switchable devices may be placed

This diagram represents some important concepts related to sections and switches.

- A section always lies between two nodes. Switches have no effect on the relationship between a section and its nodes
- A switch either allows or prevents current flow into or out of a section
- With respect to other devices, switches are always located closest to nodes

Visual modeling

SynerGEE is designed so that the electrical connectivity governed by switches can be represented visually in the display. This section discusses some of the configurations that may be created.

Example 1

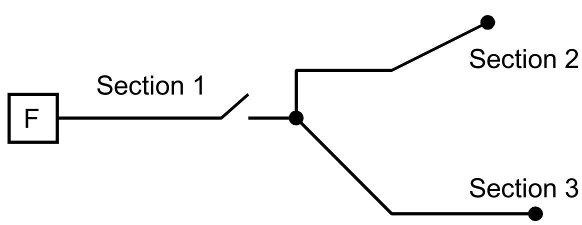


Fig. 4-2 Open switch on outward end of section

The open switch on the end of Section 1 leaves Section 2 and Section 3 as unfed islands. The sections may be fed if another feed path can be established to those sections or to the node at the outward end of Section 1. All of the devices and load on Section 1 are fed.

Example 2

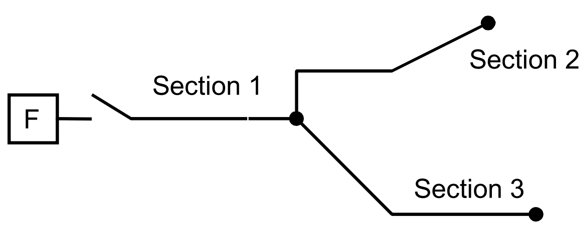


Fig. 4-3 Switch on inward end of section

Here, the switch is on the inward end of Section 1. Since the switch is open, no loads or devices on Section 1, Section 2, or Section 3 will be included in the analysis. All sections are completely unfed.

Section 1, Section 2, and Section 3 would all be fed if the outward end of Section 3 or Section 2 were connected to a fed section. For example:

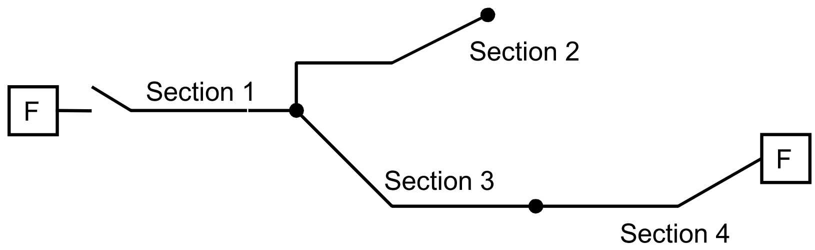


Fig. 4-4 *Connecting a new feed path*

In this case, all sections are fed. However, device feed directions and the load-flow reference directions on Section 1 and Section 2 are reversed. Other than voltage level changes, devices on Section 2 would see no change in the modified feed direction.

Example 3

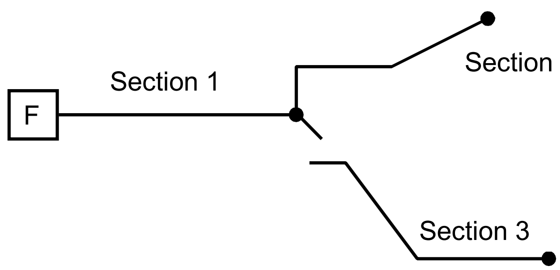


Fig. 4-5 *Inward switch isolating a branch*

In this case, the switch on Section 3 isolates the section while allowing Section 1 to feed Section 2. No devices on Section 3 receive power.

Example 4

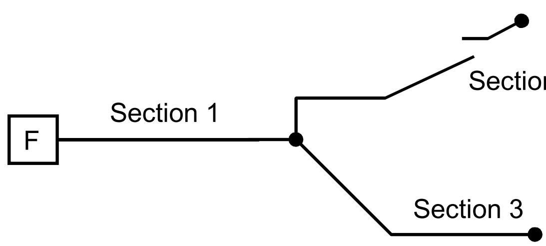


Fig. 4-6 *Open switch at isolated outward end*

The switch at the end of Section 2 has no effect on the model. The switch opens the section at its outward end, and there are no loads or devices past that point. All loads and devices on all sections receive power.

SynerGEE allows one switch to be placed on either end of every section. Complex switch gears can be effectively modeled by placing and operating these switches as shown in the switch gear installation guides.

Switch types and functions

Switches can perform several different functions in a SynerGEE model. All are initially modeled using a switch type from the equipment database (drag and drop from the Warehouse tab). Once on the model, you can configure a switch into any of the following functional types.

Basic Switch—This is a "normal" switch. Opening a switch creates an island. Closing a switch restores connectivity and typically picks up load.

Loop tie switch—This switch can be used to model loops in a radial network. It resides on the model as an open switch, but you can set SynerGEE to treat it as closed during analysis, in your model and analysis options (Use Loops). Typically, you would place one on the last section that connects to create a loop, breaking connectivity for topology purposes. Lightly meshed systems can be built and analyzed with these switches. See the following section, *Switch details*.

Wandering lateral tie switch—This switch can be used to reconnect branching single-phase lines back to a three-phase trunk, serving three-phase loads downstream. It resides on the model as an open switch, but you can set SynerGEE to treat it as closed during analysis, in your model and analysis options (Use Wandering Lateral). Typically, you would place one on the last section of a lateral before it reconnects to the three-phase line, breaking connectivity for topology purposes. When you set SynerGEE to consider these switches as closed for analysis, the phases into the switch feed all sections connected to the associated downstream node.

Automatic switch—These switches simulate real devices which automatically close to pick up load in the event of an outage or a drop in voltage. You can set your contingency analysis options to Prefer Auto Switches, in which case automatic switches are operated first, regardless of the switching objective. Also, automatic switches may have a reduced operation time in reliability analysis.

Auto-transfer switches—These switches simulate real devices which automatically transfer a critical load to an alternate feed path, in the event of an outage. In SynerGEE, these switches are modeled as a pair of regular switches, marked as Auto-Transfer, and located on either side of the node that serves a critical load. To create a valid auto-transfer switch pair, one must be open and the other closed, such that the node only receives power from one source (the load can be placed on a third section connected to that node). Once modeled, you can toggle the switches as a pair to alternate the feed source. Contingency analysis operates the switches together and reliability analysis may use the switches to reduce a sustained interruption to a momentary one for load served by the auto-transfer switch pair.

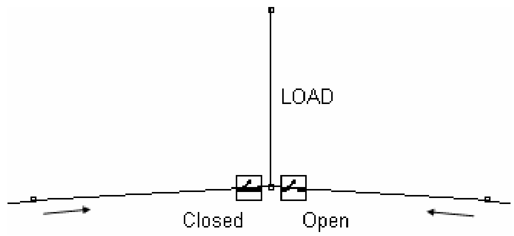


Fig. 4-7 *Auto-transfer switch pair in SynerGEE*

Switch details

Phasing

Switches only connect those phases that are present on both sides of the switch. This is the case for balanced and by-phase looped analyses.

Exception handling

During balanced or by-phase analysis, the current through a switch is compared with the amp rating listed in the database for that switch. If the current is too high, the value of current in the Results tab of the Section editor will be appropriately colored.

Switch results

Current (in amps) through switches is displayed on reports, along with the name of the switch type. Sections with switches have a special area in the Results tab of the Section editor that has either the balanced or by-phase current flow displayed. The color of the switch numbers in the listing may reflect an overloaded switch.

Radial feeders with loops

Balanced and by-phase analyses can handle radial feeders with loops formed by special switches, known as tie switches. A tie switch is an open switch that SynerGEE recognizes as a tie point for forming a loop, treating it as closed during analysis. For normal building of radial models, loop creation is prohibited without tie switches. There is no limit to the number of tie switches or loops that can be created.

With tie switches, you can also model ring buses, parallel substation transformers, and networks. A tie switch is created from a normal switch by selecting the Tie Switch option in the switch editor. In addition, to analyze loops, you should set the Loops option in your Model options, otherwise tie switches are ignored. When tie switches are considered, SynerGEE analyzes the system and determines the current flow through the tie switches such that the voltage on both sides of the switch is the same. When this flow has been determined, the solution of the radial analysis matches the solution of the network.

Invalid tie switches

A tie switch will be ignored during balanced or by-phase loop analysis if:

- The connected sections have no phases in common (for example, switching from a section with phasing AB to a section with phasing C).
- An ungrounded section is tied to a grounded section.

As you would expect, only those phases common to the connected sections will be tied during by-phase looped analysis.

Protective Devices

SynerGEE electric supports detailed models for fuses, reclosers, breakers, and sectionalizers. One or more of these devices can be placed on a section near the "from node" or "to node." These models are supported entirely with instance data; that is, they have no associated tables in the equipment database. If you are licensed to use SynerGEE Protection, however, you also received a separate protection database. This database contains time vs. current characteristics for over 10,000 devices, used with the Time vs. Current Coordination (TCC) features of SynerGEE Protection.

Detailed models for fuses, reclosers, breakers, and sectionalizers were introduced with SynerGEE Electric 3.1. They should be used in place of the general or "classic" protective device that evolved from SynerGEE's predecessor, Distributed Primary Analysis/Graphics. Within the TCC mode, all curve viewing and coordination checking requires the use of the new models.

Although loading and other factors are still considered by analyses for the classic devices, Advantica Stoner strongly encourages the use of the new models when adding new facilities within SynerGEE. Advantica Stoner also recommends converting existing classic protective device data to the new device records. SQL queries can be written to create records in the fuse, recloser, breaker, and sectionalizer tables for respective records in the protective device table migrated from DPA/G. If you require conversion assistance, contact Technical Support.

Note: For the remainder of this chapter, the term "protective device" is a general reference to the group of detailed fuse, recloser, breaker, and sectionalizer models.

Data storage

Given the overwhelming variety of protective equipment available, SynerGEE does not attempt to maintain specific profiles in the equipment database. A protective device is defined entirely by instance data in the model database, which is specified after it is placed on the model. This instance data contains settings and makes references to curves in the protection database. These references are made with manufacturer, model, rating, and setting information.

Loading of curves

Curves in the protection database are normalized on a compound index consisting of five fields. Manufacturer, model, voltage rating, family, and amp rating determine a unique curve. The table below shows the way in which curves from fuses, reclosers, and relays are listed in this database.

Device		Manufacturer	Model	Rating	Type	Curve Name
Expulsion Fuse	Min Melt	Ex: S&C	Ex: 122	-	FusExpMelt	Amp Rating
	Max Clear	Ex: S&C	Ex: 122	-	FusExpClear	Amp Rating
Current Lim. Fuse	Min Melt	Ex: Cooper	Ex: NX	Volt Rat.	FusCurLimMelt	Amp Rating
	Max Clear	Ex: Cooper	Ex: NX	Volt Rat.	FusCurLimClear	Amp Rating
Electronic Recloser	Phs Curve	Ex: Cooper	Ex: Form 4C	-	RecElecPh	Ex: 101
	Grnd Curve	Ex: Cooper	Ex: Form 4C	-	RecElecGrnd	Ex: 101
	Phs or Grnd	Ex: Cooper	Ex: Form 4C	-	RecElecPhOrGrnd	Ex: 101
Hydraulic Recloser	Phs Curve	Ex: Cooper	Ex: V4H ME	Coil Rat.	RecHydPh	Ex: A
	Grnd Curve	Ex: Cooper	Ex: V4H ME	Coil Rat.	RecHydGrnd	Ex: A
	Phs or Grnd	Ex: Cooper	Ex: V4H ME	Coil Rat.	RecHydPhOrGrnd	Ex: A
Electronic Relay	Phs Curve	Ex: GEC	Ex: MCGG	-	RelElecPh	Family
	Grnd Curve	Ex: GEC	Ex: MCGG	-	RelElecGrnd	Family
	Phs or Grnd	Ex: GEC	Ex: MCGG	-	RelElecPhOrGrnd	Family
	Inst Curve	Ex: GEC	Ex: MCGG	-	RelElecInst	Ex: 3Is
Mechanical Relay	Phs Curve	Ex: Westinghouse	Ex: CO-11	-	RelMechPh	Time Dial
	Grnd Curve	Ex: Westinghouse	Ex: CO-11	-	RelMechGrnd	Time Dial
	Phs or Grnd	Ex: Westinghouse	Ex: CO-11	-	RelMechPhOrGrnd	Time Dial
	Inst Curve	Ex: Westinghouse	Ex: CO-11	-	RelMechInst	Inst4

SynerGEE loads the enabled curve definitions into memory when loading the protection database (CurveDefinitions table). These definitions are used to drive the logic in many of the SynerGEE editors. However, these definitions do not contain the actual mathematical points needed to plot a curve on the graph.

To generate a curve on the TCC graph, SynerGEE uses data in the CurvePoints table of the protection database. This table is not loaded into memory. Rather, SynerGEE uses a special key in the definitions table, called the Internal Curve Number (ICN), to reference the points table as needed. When a curve is to be plotted, the ICN of the curve is established using the definitions table, and then points are queried from the points table based on that ICN.

If a protective device fails to display in the TCC, check to make sure the protection database has been loaded. If so, view the "rap sheet" for that device and check the ICN, listed as the database ID. With the CurveDefinitions table loaded, SynerGEE should at least be able to produce the ICN. If not, you may have more serious data problems. Once you have the ICN, you can open the CurvePoints table in Microsoft® Access in an attempt to track down points associated with the ICN and locate the problem.

Remember that points are only loaded from the database on demand. However, the lookup of the ICN using the loaded definitions table occurs in memory only. Also, keep in mind that individual curve definitions can be enabled or disabled within the CurveDefinitions table using Access. If a curve does not display, ensure that the appropriate record(s) in the definitions table have not been disabled. If a definition is disabled, it will not be loaded, and any device requiring that definition will not display on the TCC (since the SynerGEE will not be able to find the ICN to reference the points table).

Mechanical response times

Often, a device with a particular set of contacts or breakers has a response time listed in standard tables. SynerGEE has a table in the protection database for storing mechanical response times by device name. This data cannot be edited with SynerGEE. New entries must be made to the table in the protection database through Access or an ADO-, DAO-, or ODBC-enabled tool that can access the table (MechResponseTimes).

The device response curves that are stored in the protection database represent the operation time of the controller for electronic reclosers, electronic relays, and mechanical relays. Mechanical response times represent the delay between the controller trip signal and the actual tripping of the mechanical contacts. Mechanical response times are used to shift the curves of electronic reclosers and all breaker relay models by a constant time value (adder). In TCC mode, you have the option of displaying response curves, control curves, or both for these devices.

Mechanical response times typically range from .015 to .055 seconds.

Sectionalizers

Like other protective devices, sectionalizers have no equipment database records. A sectionalizer is specified through an actuating current and a count number. Since sectionalizers have no curves in the protection database either, coordination analysis checks these settings against reclosers paired to the sectionalizer.

Typically, the sectionalizer should have at least one recloser upstream (SynerGEE does not yet support reclosing relays). Also, the minimum fault current through the sectionalizer should exceed the actuating current.

The amp rating of the sectionalizer is used to calculate a value for "percent loading" and to find overload exceptions.

Sectionalizers can be put into one of two "modes" within SynerGEE. Both modes rely on an amp rating for the device.

- "Sectionalizer" mode uses an actuating current and a count number, which is used in check coordination analysis. The sectionalizer curves are drawn as straight lines at the actuating current.
- "Fault indicator" mode uses a trip value and a trip response time. The device is displayed on a TCC graph as a vertical line at the trip value and a horizontal line at the trip response time.

Fuses

Fuses are specified by manufacturer, model, and amp rating. Current limiting fuses also require a voltage rating value. This information allows SynerGEE to find the minimum-melt and the maximum-clear curves from the protection database. These curves are used for TCC graphs and for coordination checking.

Fuses have a time multiplier available, if you want to try accounting for older, derated fuses. Any multiplier less than 1 shifts the curve downward on the graph; that is, requires less time for the fuse to blow at any given amperage. For example, a multiplier of .9 would reduce the time axis to 90 percent of the original on the graph.

Naturally, the default value is 1.0, but you can set it lower to make coordination more conservative, perhaps to compensate for older equipment. You should take care when derating fuses in this fashion, as the process will likely involve a significant amount of personal experience with your system and perhaps some guesswork.

Reclosers

SynerGEE distinguishes between hydraulic and electronic reclosers. Hydraulic reclosers are defined by manufacturer, model, and coil rating. Electronic reclosers are defined by manufacturer, model, and pickup. Electronic reclosers typically have a phase and ground unit, whereas hydraulic reclosers have a phase <u>or</u> ground unit.

Phase and ground units each have two available response curves.

- Fast
- Time delay (or slow)

The operation count or number of "shots" for each response curve can be specified. The sum of the fast and time delay shots must be four or less.

Interrupt and amp ratings are used in fault and load-flow analysis to detect overload exceptions.

Adders and multipliers can be used to independently modify the fast and time delay response curves for electronic reclosers. Mechanical recloser response curves cannot be modified with adders and multipliers.

For more on the mechanical response time and K-factors, see the *Protection Coordination Analysis* chapter.

Mechanical response times

Mechanical response times can be added to the fast, slow, and accumulated response time. For the accumulated response, the fast and slow control curves are determined first, then the mechanical delays are added. Thus, the accumulated time has the delay added only once.

SynerGEE uses K-factor values to account for cumulative heating of fuse links associated with coordination of reclosers and fuses.

Minimum response time

Electronic reclosers are equipped with minimum response times. The recloser's response "locks on" to the specified time if that time exceeds the recloser's curve at a given amp value. If the recloser's minimum response time lies below the recloser's curve, the minimum response time is ignored.

Breakers

Breaker models are broken into three areas. The mechanical aspects of the breaker are specified with data regarding amp rating, interrupt rating, and mechanical response times. The "phase relay" has curve and setting information for the breaker's phase unit. The "ground relay" has corresponding information about the ground unit.

Breaker - relay terminology

In product versions prior to SynerGEE version 3.1, the generic term relay was used to refer to a device that now is classified strictly as a breaker. Now, breakers can be considered as having phase relays and ground relays.

The term "breaker" is used in most places in the SynerGEE program interface. Some applications, however, respond directly to the relays within the breaker. Check

Coordination reports, for example, use the terms phase relay and ground relay in the context of coordination with other devices.

Mechanical response time is added to the phase, ground, and either instantaneous curve for breaker relay curves.

Equation Based Curves in the Protection Database

SynerGEE supports equation based time coordination curves. New fields have been added to the Curve Definitions table of Protection database to support the equation based time coordination curves of modern relays.

Most of the equation based curves follow IEC or IEEE Standard. Some manufacturers define their own equation based curves. We have defined TCC curve equation that satisfies IEC, IEEE and some manufacturer based curves. TCC curves for devices that do not follow the equation like electromechanical relays are still available in SynerGEE protection database as digitized curve points.

The following nomenclature is used in the Equations for Time current coordination curves (TCC curves):

A	Equation based constant
B	Equation based constant
C	Equation based constant
D	Equation based constant
N	Equation based constant
E	Equation based constant
K	Equation based constant
P	Equation based constant
n	Time dial setting
I	Measured current
$I>$	Set start current
M	Multiples of pickup ($I / I>$)

IEC equation for time current characteristic for trip is listed below:

$$T = \left(\frac{A}{M^{p} - 1}\right)n \tag{4-1}$$

IEEE Equation for time current characteristic for trip is listed below. This equation comply with IEEE Standard C37.112-1996

$$T = \left(\frac{A}{M^{p} - C}\right)n + Bn + K \tag{4-2}$$

Some of the manufacturers provide additional time current characteristic curves that do not follow the IEC or IEEE Standard. We have tried to accommodate these TCC curves as well and have defined the time current characteristic equation that satisfies IEC, IEEE and some of the non-standard curve equations. SynerGEE equation based time current characteristic is based on the following equation:

$$T = \left(\frac{A}{M^{p} - C}\right)n + Bn + K + \left(\frac{D}{M^{p} - E}\right) \tag{4-3}$$

The above equation is an extension of IEEE equation with two new equation constants (constant D and constant E).

To define an equation in the protection database, there needs to be a record in the Curve Definitions table of the protection database specifying the manufacturer, model, curve family, device type and the equation parameters. Equation parameters specify the minimum and maximum multiple of pickup (CurrentRangeMin, CurrentRangeMax), minimum and maximum time multiplier (TimeMultMin, TimeMultMax) and the equation constants. Database fields in the CurveDefinitions table of SynerGEE Protection database for the equation parameters are listed below:

Database field	**Equation parameter**
Param_A	A
Param_B	B
Param_C	C
Param_D	D
Param_E	E
Param_N	P
Param_K	K
CurrentRange_Min	Minimum multiple of pickup
CurrentRange_Max	Maximum multiple of pickup

TimeMult_Min	Minimum time multiplier
TimeMult_Max	Maximum time multiplier

Parameters for manufacturers

Parameters of equations are listed in the table below for the equation based time coordination curves added to the protection database.

Manufacturer: **Schweitzer**

Model: **351, 351R, 351S, 500**

Curve Family	***Equation constants***						
	A	***B***	***C***	***P***	***K***	***D***	***E***
U1	0.01040	0.02260	1.0	0.02	0.0	0.0	0.0
U2	5.95000	0.18000	1.0	2.00	0.0	0.0	0.0
U3	3.88000	0.09630	1.0	2.00	0.0	0.0	0.0
U4	5.67000	0.03520	1.0	2.00	0.0	0.0	0.0
U5	0.00342	0.00262	1.0	0.02	0.0	0.0	0.0
C1	0.14000	0.00000	1.0	0.02	0.0	0.0	0.0
C2	13.5000	0.00000	1.0	0.02	0.0	0.0	0.0
C3	80.0000	0.00000	1.0	2.00	0.0	0.0	0.0
C4	120.000	0.00000	1.0	1.00	0.0	0.0	0.0
C5	0.05000	0.00000	1.0	0.04	0.0	0.0	0.0

Manufacturer: **Siemens**

Model: **7SJ512, 7SJ600, 7SJ602, 7SJ61, 7SJ62, 7SJ63**

Curve Family	*Curve constants*						
	A	***B***	***C***	***P***	***K***	***D***	***E***
Inverse	8.9341	0.17966	1.0	2.0938	0.00	0.00	0.00
Short Inverse	0.2663	0.03393	1.0	1.2969	0.00	0.00	0.00
Long Inverse	5.6143	2.18592	1.0	1.0000	0.00	0.00	0.00
Moderately Inverse	0.0103	0.02280	1.0	0.0200	0.00	0.00	0.00
Very Inverse	3.9220	0.09820	1.0	2.0000	0.00	0.00	0.00
Extremely Inverse	5.6400	0.02434	1.0	2.0000	0.00	0.00	0.00
Definite Inverse	0.4797	0.21359	1.0	1.5625	0.00	0.00	0.00
I2t	50.7	0.00000	0.0	2.0000	10.14	0.00	0.00

Manufacturer: **ABB**

Model: **DPU 2000**

Curve Family	*Curve constants*						
	A	***B***	***C***	***P***	***K***	***D***	***E***
Extremely Inverse	9.96644	0.038889	1.0	2.0	-0.01389	-3.55944	1.0
Very Inverse	4.441111	0.110756	1.0	2.0	-0.03956	-1.58611	1.0
Inverse	0.013378	0.028778	1.0	0.02	-0.01028	-0.00478	1.0
Short Time Inverse	0.002676	0.005756	1.0	0.02	-0.00206	-0.00096	1.0
Short Time Ext. Inv.	1.992667	0.007778	1.0	2.0	-0.00278	-0.71167	1.0
Long Time Ext. Inv.	99.66444	0.388889	1.0	2.0	-0.13889	-35.5944	1.0
Long Time Very Inv.	44.41111	1.107556	1.0	2.0	-0.39556	-15.8611	1.0
Long Time Inverse	0.133778	0.287778	1.0	0.02	-0.10278	-0.04778	1.0
Recloser Curve #8	6.550444	0.020222	0.35	1.8	-0.00722	-2.33944	0.35

Manufacturer: **Basler**

Model: **BE1**

Curve Family	*Equation constants*						
	A	*B*	*C*	*P*	*K*	*D*	*E*
S, Short Inverse	0.26630	0.03393	1.0	1.2969	0.028	0.0	0.0
S2,Short Inverse	0.02860	0.02080	1.0	0.9844	0.028	0.0	0.0
L, Long Inverse	5.61430	2.18592	1.0	1.0	0.028	0.0	0.0
L2, Long Inverse	2.39550	0.00000	1.0	1.0	0.028	0.0	0.0
D, Definite Time	0.47970	0.21359	1.0	1.5625	0.028	0.0	0.0
M, Moderately Inverse	0.30220	0.12840	1.0	0.5000	0.028	0.0	0.0
I1, Inverse Time	8.93410	0.17966	1.0	2.0938	0.028	0.0	0.0
I2, Inverse Time	0.27470	0.10426	1.0	0.4375	0.028	0.0	0.0
V, Very Inverse	5.46780	0.10814	1.0	2.0469	0.028	0.0	0.0
V2, Very Inverse	4.43090	0.09910	1.0	1.9531	0.028	0.0	0.0
E, Extremely Inverse	7.76240	0.02758	1.0	2.0938	0.028	0.0	0.0
E2, Extremely Inverse	4.98830	0.01290	1.0	2.0469	0.028	0.0	0.0
BS/IEC Standard Inverse	0.01414	0.00000	1.0	0.0200	0.028	0.0	0.0
BS/IEC Very Inverse	1.46360	0.00000	1.0	1.0469	0.028	0.0	0.0
BS/IEC Extremely Inverse	8.25060	0.00000	1.0	2.0469	0.028	0.0	0.0
Long Time Inverse	12.1212	0.00000	1.0	1.0000	0.028	0.0	0.0

Manufacturer: **ABB**

Model: **SPAA 120C / SPAA 121C / SPAA 341C / SPAA 342C**

Curve Family	***Curve constants***						
	A	***B***	***C***	***P***	***K***	***D***	***E***
Normal inverse	0.14	0.0	1.0	0.02	0.00	0.00	0.00
Very inverse	13.5	0.0	1.0	1.0	0.00	0.00	0.00
Extremely inverse	80.0	0.0	1.0	2.0	0.00	0.00	0.00
Long time inverse	120.0	0.0	1.0	1.0	0.00	0.00	0.00

Manufacturer: **ABB**

Model: **REF 610 relay**

Curve Family	***Curve constants***						
	A	***B***	***C***	***P***	***K***	***D***	***E***
IEC Normal inverse	0.14	0.0	1.0	0.02	0.00	0.00	0.00
IEC Very inverse	13.5	0.0	1.0	1.0	0.00	0.00	0.00
IEC Extremely inverse	80.0	0.0	1.0	2.0	0.00	0.00	0.00
IEC Long time inverse	120.0	0.0	1.0	1.0	0.00	0.00	0.00
IEEE Extremely inverse	6.407	0.025	1.0	2.0	0.00	0.00	0.00
IEEE Very inverse	2.855	0.0712	1.0	2.0	0.00	0.00	0.00
IEEE Moderately inverse	0.0086	0.0185	1.0	0.02	0.00	0.00	0.00

Rules for transformer network protector operation

Protectors will open and close under the following conditions if they are set in automatic mode:

Open when feeder breaker opens	When primary of transformer is no longer fed from the feeder breaker. If the protector is fed from its secondary then the protector will open. This operation can occur from model manipulation or from analysis being performed that would operate a feeder breaker.
Open on reverse S	If the protector sees reverse power flow during a load-flow run, it will open.
Close on voltage	If the voltage and angle difference across the open protector is within limits then it will close.

Switching in DPA/G and other systems

Switch models in SynerGEE are significantly different than those in DPA/G (Distribution Primary Analysis/Graphics, SynerGEE's predecessor). The models in the latter product were used to connect and disconnect a section from a node. As you have seen previously in this chapter, SynerGEE switches are used to break a section's connectivity at either end without affecting the section's node connection.

For the benefit of those upgrading from DPA/G, this section describing the DPA/G switch mode is included for reference.

It is helpful to see a brief description of what a switch does before explaining how to enter switch information. It is probably easier to picture DPA/G switches as ways to connect (close) or disconnect (open) sections to each other. The following figure represents this concept.

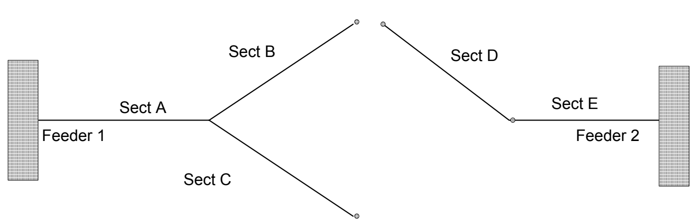


Fig. 4-8 *Switch example using two feeders*

You are going to look at methods for placing a switch between sections B and D of the two feeders shown above. There are four possible configurations for a single switch to connect these sections. <u>All four</u> configurations are shown in the following figure.

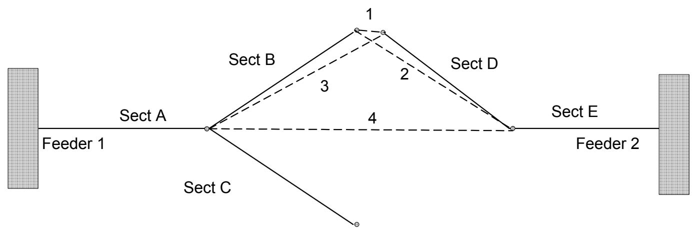


Fig. 4-9 *Feeders showing possible switch configurations for sections B and D*

The table below describes the four configuration possibilities for placing a switch on section B and connecting it to section D.

Configuration	**From**	**To**
1	Load End of "B"	Load End of "D"
2	Load End of "B"	Load End of "E"
	Load End of "B"	Source End of "D"
3	Load End of "A"	Load End of "D"
	Source End of "B"	Load End of "D"
4	Load End of "A"	Source End of "D"
	Load End of "A"	Load end of "E"
	Source End of "B"	Source End of "D"
	Source End of "B"	Load end of "E"

Although this example is based on switches placed between two feeders, switching can be performed on a single feeder or between substation transformers in a substation model.

This section describes how switching works and how this affects the way that you should set up switches within DPA/G. The software operation is a bit different from a real-world switch operation because not all the ties and jumpers are displayed with the switches. When a switch is toggled in DPA/G, the end of the section with the switch to be opened is disconnected from all the sections connected to it.

Consider the following switch scenario.

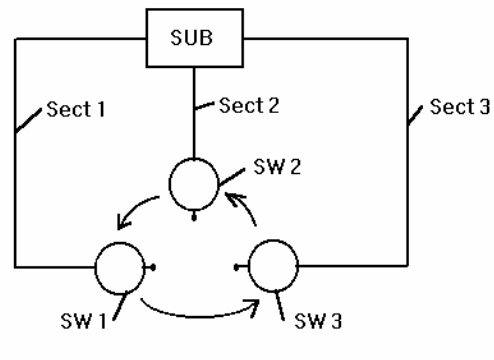


Fig. 4-10 Switch scenario

	SW 1	SW 2	SW 3
State	Open	Open	Open
Closes to	Sect 3 - Load	Sect 1 Load	Sect 2 Load

The operations that can cause a looping problem are as follows:

- Close SW 2
- Open SW 2
- Close SW 3

After operation 3, a loop error occurs.

The figure below shows the action when SW 3 is closed from the previous figure.

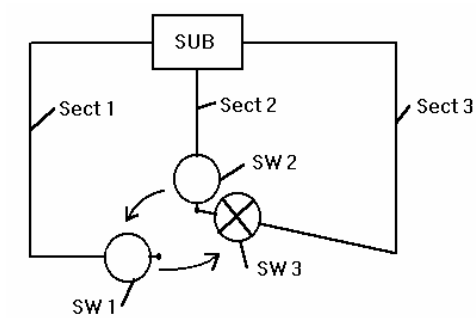


Fig. 4-11 Closed SW 3

As you can see, Sect 2 and Sect 3 are now connected even though SW 2 is an open symbol. Since SW 2 closes to Sect 1, SW 2 is indeed open (open to Sect 1). Since SW 3 closes to Sect 2 and it is closed, it is connected to Sect 2.

Given the previous scenario based on open and closed operations, the loop problem can be corrected. The following figure shows a solution based on the insertion of three small sections.

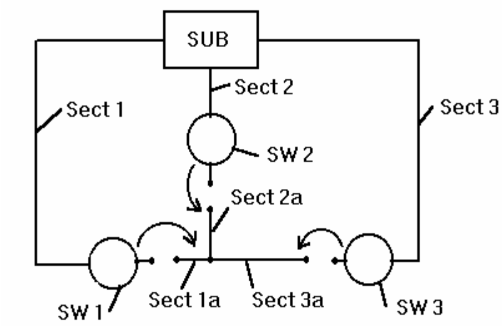


Fig. 4-12 Insertion of small sections

	SW 1	SW 2	SW 3
Closes to	Sect 1a Load	Sect 2a Source	Sect 3a Load

In this scenario, when SW 3 is closed, a loop is not created. The three connected sections will appear if all the switches are open. This also works with four-way switches (a plus sign represents four switches). The extra sections represent the jumpers, ties, or bus bar present in the field.

5
Loads

Introduction

In SynerGEE, load models are placed in the center of the line portion of the section model. This is done for two reasons.

- Placing loads in the center of the section avoids ambiguity during switching. SynerGEE allows extensive switching actions. Since switches are placed at the ends of sections, load modeling at nodes would make it difficult to determine how loads would be fed during switching operations. The assignment of loads is clear with the loads in the center and the switches at the ends of sections.
- It can be shown that the modeling of a load in the center of the section more accurately simulates a distributed load with respect to voltage drop. Losses are also closely modeled in this manner. Studies and simulations show that the SynerGEE approach to load placement and modeling is very well-suited for distribution system simulation.

Each section may contain a by-phase spot and by-phase distributed load. Spot loads are not altered by SynerGEE load allocation. They are considered to be known loads with metered demands, with constant values. Distributed loads can be entered directly, or their values can be calculated from a load allocation analysis. (The section kVA or kWh and possibly customer counts are necessary to run load allocation.)

Loads can only be entered into valid phases of a line section. Consider a line section containing wire A, B, and a neutral. Loads can be entered for phase A and phase B. If the load were connected line-line then only the phase AB field could contain load. Both kW and kvar values must be entered for loads. Entering only kW implies that the load has a power factor of 100%.

Only the load values for kW and kvar are used during load-flow analysis. Values for kWh, kVA, and customers are not utilized during load-flow calculations.

Constant load models

SynerGEE supports the mixed modeling of three load types, including:

- Constant real and reactive power (PQ)
- Constant current (I)
- Constant impedance (Z)

Spot and distributed loads on each section may be specified as a percentage mix of these types.

Defining load types

There are three general load models that are very commonly used. Each of these models is addressed independently. Each situation should determine a function that relates current to voltage in terms of the nominal value of power and the nominal voltage of the load. That is, you are given the following:

$$\begin{aligned} S_{Nom} &= \text{Real \& reactive portion of load at nominal voltage} \\ kV_{Nom} &= \text{Nominal voltage of load} \end{aligned} \tag{5-1}$$

and you wish to determine S_{LT} in the following.

$$\begin{aligned} S_{LT} &= f(kV, S_{Nom}, kV_{Nom}) \\ I &= \frac{S_{LT}^*}{kV^*} \end{aligned} \tag{5-2}$$

where:

LT indicates the load type

I is the load current with applied kV

You will determine the definition of S_{LT} for constant PQ, I, and Z loads.

Constant real and reactive power load model

This model maintains independent constant values of the real and the reactive portions of a load. The model should not be referred to as a constant load or constant power model because these imply that the vector sum of the real and imaginary portions of a load is held constant. In the constant PQ model, the nominal real and reactive load power never change. Thus:

$$S_{PQ} = S_{Nom} \tag{5-3}$$

Constant current load model

In this model, the value of current is independent of the voltage applied to the load. The load current at nominal voltage in amps is as follows.

$$I_{Nom} = \frac{S_{Nom}^*}{kV_{Nom}} \tag{5-4}$$

As the load voltage changes, the load power needed to maintain this value of current is as follows.

$$S_I = kV * I_{Nom}^* = kV * \frac{S_{Nom}}{kV_{Nom}} = S_{Nom}\left(\frac{kV}{kV_{Nom}}\right) \tag{5-5}$$

The expression of S_I based on the loads applied and nominal voltage is sufficient to represent the constant current load model.

Constant impedance load model

The expression for load current at nominal voltage was found in the previous section. The load impedance must, therefore, be the following.

$$Z_{Nom} = \frac{10^3 * kV_{Nom}}{I_{Nom}} = \frac{10^3 * kV_{Nom}^2}{S_{Nom}^*} \tag{5-6}$$

As the load voltage changes, the current consumed by the constant impedance model is as follows.

$$I_Z = \frac{10^3 kV}{Z_{Nom}} = \frac{kV * S_{Nom}^*}{kV_{Nom}^2} \tag{5-7}$$

And, the value of load power needed to maintain this load impedance as the load voltage changes is as follows.

$$S_Z = kV * I_Z^* = \frac{kV^2 * S_{Nom}}{kV_{Nom}^2} = S_{Nom}\left(\frac{kV}{kV_{Nom}}\right)^2 \tag{5-8}$$

Once again, a factor is derived based on load voltage that modifies the nominal load to a corresponding value, so the constant impedance load model is maintained.

Current/voltage relationship

At nominal section voltage, these loads are indistinguishable. However, the loads differ by the current/voltage relationship that they exhibit. The plot below shows this relationship for the three types.

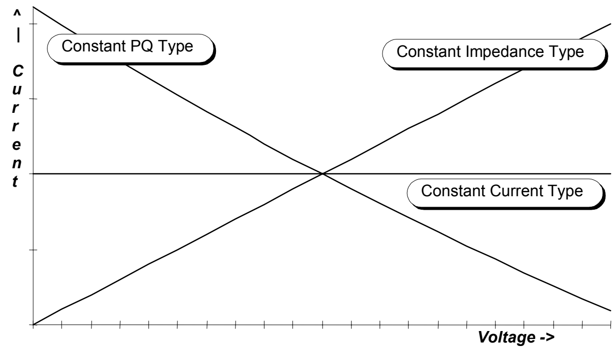


Fig. 5-1 V/I curves for load models

The curves have been normalized and the scale of the independent axis represents a 20% variation in voltage. Although all of the curves appear linear, the constant PQ load curve is actually hyperbolic. It is nearly linear over the practical voltage range used here. Notice how all of the curves intersect at the nominal voltage point. This shows that the nominal voltage and nominal current correspond to the nominal load power that has been specified as the same for all three loads in this plot. This intersection is also important to the creation of the combination model.

Power/voltage relationship

The combination load model allows a nominal load to be specified in kW and kvar (the intersection point).

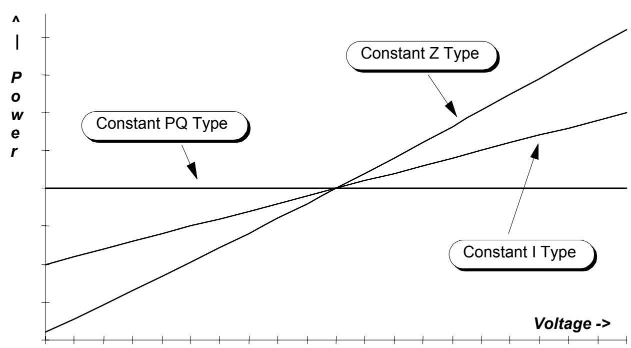


Fig. 5-2 V/S curves for the different load types

The combination load model

The three ideal load models can be combined to represent a single load. A percentage weight is given to each of the three components of that load as follows:

$$S_{CM} = S_{Nom}\left[\%PQ + \%I\left(\frac{kV}{kV_{Nom}}\right) + \%Z\left(\frac{kV}{kV_{Nom}}\right)^2\right] * 0.01 \tag{5-9}$$
$$where: \%PQ + \%I + \%Z = 100\%$$

The factors %PQ, %I, and %Z represent the portion of load that is treated as each of the basic types. The equation requires that the sum of the three factors be 100%.

This equation shows how the effective load is determined from the section's nominal spot or distributed loads, the section's center voltage, the section's center nominal voltage, and the load weighting factors. The load becomes a quadratic function of section voltage. The three load types are directly observable in the following equation.

$$I_{Spot} = \frac{f\left(S_{Spot-Nom}, kV, kV_{Nom}, \%I_{Spot}, \%Z_{Spot}\right)^*}{kV^*} \tag{5-10}$$
$$I_{Dist} = \frac{f\left(S_{Dist-Nom}, kV, kV_{Nom}, \%I_{Dist}, \%Z_{Dist}\right)^*}{kV^*}$$

The load current for spot and distributed loads is determined during each iteration of the SynerGEE load-flows. Spot and distributed loads are averaged in balanced analysis and applied per-phase in by-phase analysis. Both types of analysis acknowledge the line-line or line-ground connectivity of the load.

Example

A feeder with an 8MVA load was modeled using various combinations of load model parameters. The results for a single feeder are listed in the table below, with all loads modeled using parameters listed in the left column.

Load Model	Amps	kVA Demand	Loss	Min. Volts	Max. Ld
100% PQ	367	8115	2.6%	118.3	69%
100% Z	356	7877	2.5%	119.1	67%
100% I	361	7988	2.5%	118.6	68%
40%,40%,20%	360	7965	2.5%	118.7	68%

It can be seen that the impact on losses and loading for this particular feeder is not heavily dependent on the load models used. The minimum voltage on the feeder does vary by nearly a volt over the load models tested. Other feeders with longer lines (this model is dense suburban) may see a much greater impact from load models.

Load connection example

The way that a load is connected can make a very big difference to the power flow, current flow, and voltage levels in a system. There are two options for connecting loads in SynerGEE:

- Line-to-line
- Line-to-ground

Load is affected in a number of ways by its connection. First, there is typically around a 75% difference between line-to-ground and line-to-line voltage. A constant impedance load specified in ohms could see a swing of nearly 300% in load between a line-to-ground and line-to-line connection. (This is one reason that constant impedance loads are specified in kW and kvar.) A second affect is in voltage drop due to different current flows in the phase wires for different load connections. The final affect is in voltage drop (or rise) due to imbalance and neutral / earth current flow in a line-to-ground connected load.

In these examples, we will focus on a line with two phase wires. We will consider the case with and without a neutral wire. The following data will be used for a long line feeding a very large load:

$$\begin{aligned} \text{Source} &= 13.2\,kV/120V \\ \text{Line length} &= 10\,miles \\ \text{Primary} &= 4/0\,ACSR \\ \text{Neutral} &= 1/0\,ACSR \\ \text{Spacing} &= 48"/36" \\ \text{Height} &= 30' \\ \text{Load} &= 1MW\ \ (or\,2x500kW) \\ & \quad\ \text{constant current} \end{aligned} \tag{5-11}$$

If the load is connected line-to-line we will get the following load current:

$$I_{LL} = \frac{1000kW}{13.2kV} = 76A \tag{5-12}$$

We will build to feeders as specified above. One feeder will have a YGnd feeder connection and a neutral to the load. The other will be connected delta. In both cases, a 1000 kW load will be added and connected line-to-line. We run a load-flow and get the following results:

	Wye-Gnd	Delta
Amps	76	76
kW Loss	68	68
Load Volts	113.5	111.4

Notice that the load current and losses are the same for both feeders. Everything should be the same since the load is connected line-to-line in both examples, and both feeders are 13.2. The neutral conductor in the first example has no bearing on voltage drop because there is no neutral or earth current from the delta connected load.

Why is there a difference in voltage? In the wye-gnd example, voltage is measured phase-ground. Even though there is no neutral current, the neutral is still present and used for voltage measurement. In the delta example, voltage is measured line-line because there is no neutral or earth reference. In fact, the by-phase results for the wye-gnd analysis are:

$$\begin{aligned} V_A &= 117.9\angle -5.41 \\ V_B &= 109.2\angle -121.73 \end{aligned} \tag{5-13}$$

The line-line voltage would be $kV_A - kV_B$ or:

$$\begin{aligned} kV_{LL} &= \left(\frac{7.62}{120}\right)(117.9\angle -5.41) - \\ &\left(\frac{7.62}{120}\right)(109.2\angle -121.73) \\ &= 12.25\angle 25.07 \\ V_{LL} &= \left(\frac{120}{13.2}\right)12.25\angle 25.07 \\ &= 111.4\angle 25 \end{aligned} \tag{5-14}$$

This voltage matches the delta case.

The substantial difference in phase 'A' and phase 'B' voltages in the wye-gnd case demonstrates the need for engineering care when mixing single phase line-line and line-ground loads in a feeder.

What happens if we split the load and connect it line-ground in the wye-gnd case? We expect load current like:

$$I_{LG} = \frac{\sqrt{3}\cdot 1000kW}{2\cdot 13.2kV} = 66A \tag{5-15}$$

If we run a load-flow under these conditions we get:

	Wye-Gnd	Delta
Amps	66	76
kW Loss	60	68
Load Volts	112.3	111.4

The 'Delta' column has been copied from above for comparison.

We expect a drop in losses due to the 10A reduction in load current. The drop in losses is partially offset by an increase in neutral / earth current. From SynerGEE, we can see a neutral current of 64A listed. For the line-ground connected load, the neutral current results from the vector sum of the load currents.

There are lots more items that could be discussed for a load connected at the end of a simple two wire line. SynerGEE deals with many details of line construction, earthing, and load connection to deliver solid and accurate simulation results. If you see numbers in SynerGEE that need an explanation, please send us an e-mail or give us a call with your question.

Diurnal customer load curves

SynerGEE supports the modeling of loads using 24-hour load curves. Traditionally, planning studies have been performed with peak loading. However, looking at a system on- and off-peak provides a more comprehensive view. With daily load curves, you can model a system at a particular hour on a day type and month. For example, you can study a system at peak loading or at minimum loading for an upcoming season. Or, you can study the trends in load movement across or between feeders during the daylight hours of a weekday in July, for example.

The use of load curves is not required. SynerGEE can still be used for "peak load" planning, which is the default mode of operation. Load curves are not used unless you configure SynerGEE as such. If you do use load curves, be aware that they integrate at a fundamental level. Most SynerGEE applications can consider load curves and the "time-of-day" that you select for analysis.

There are many ways that load curves can fuel SynerGEE simulations and engineering studies. Some primary ways include:

- Reconfiguration studies can be more complete by looking at loading on- and off-peak.
- Load curves are built from historical load data. Their use gives some degree of foresight into feeder and system demand.
- Running analysis over time allows the consideration of <u>energy</u> delivery and loss.
- SynerGEE can find peak and minimum loading times on given months or over the year. It can present information about loading or voltage trends over that period.

Load curves are a basis for powerful and detailed simulations. It should be noted that the intention of the feature is not to facilitate exact replication of the distribution system. Rather, the goal is to support a consistent and predictable model that supports time-of-day and time range simulation. This type of modeling is essential for making decisions that are most beneficial to customers and the utility throughout daily, monthly, and yearly operation.

Data organization

Load curves are tied into the system model in three stages as demonstrated in the following diagram.

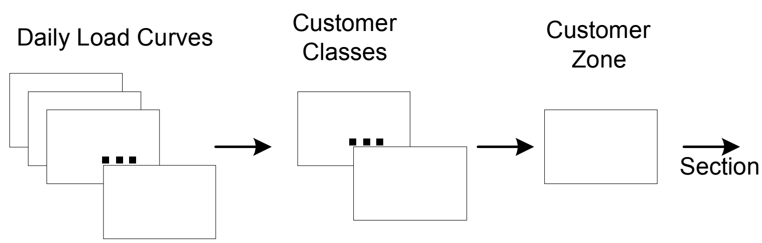


Fig. 5-3 *Relation between load curve data and the section*

Load curves are first organized into customer classes. Afterwards, the classes are organized into customer zones and assigned to sections. Each of these stages will be explained in detail.

Customer Class curves

Basic load curve data is managed within customer classes. Each class contains at least 36 curves, including a percent kW curve for a weekday, weekend, and peak day for each of the twelve months in a year. In addition, you can optionally specify another 36 curves for percent kvar.

SynerGEE has a powerful editor to help you create, view, copy, and modify curves in a customer class.

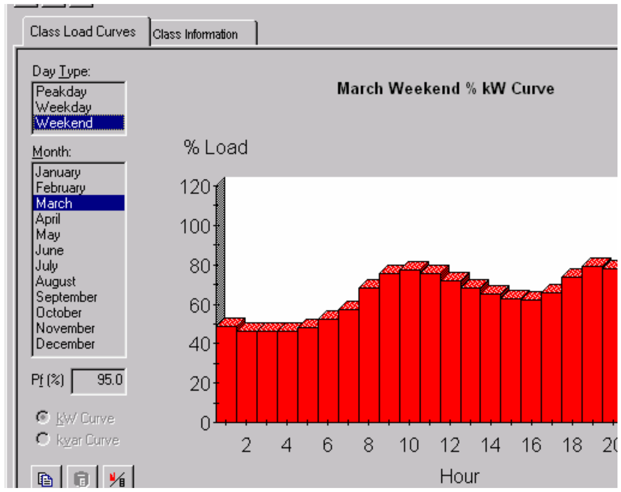


Fig. 5-4 SynerGEE customer class editor

All data for customer class curves resides in the SynerGEE open database.

Customer zones

Once customer classes are established, you can define customer zones. Customer zones can then be applied to model sections using the SynerGEE editors. Customer zones make it simple to associate load curves with general regions on your model, since proximal distribution loads tend to have a similar customer makeup.

A customer zone can be a mix of one, two, or three customer classes, each of which is assigned a percentage. These percentages determine which portion of a section's spot or distributed load pertains to each particular class. Consider the following suburban model, divided into three different hypothetical zones.

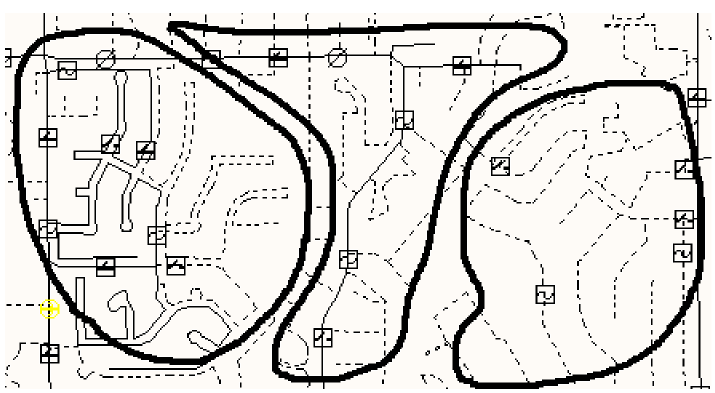


Fig. 5-5 Zones sketched onto a feeder model

Ideally, the system should be divided into zones based on the approximate demand or usage of various customer types. For example, the zone on the left may be a mixture of approximately 30% all-electric customers and 70% gas heat customers. The zone on the right may be 100% all-electric customers.

Assignment of customer zones is optional and independent for spot and distributed loads. If a customer zone is not specified on a section for either or both loads, nominal kW and kvar is used, as applicable.

Time of Day

Once load curves are fully implemented, SynerGEE has a variety of options for selecting the time of day for analysis.

- **Ignore time of day—**Customer load curves are not used. Section load is determined from 100% of the load values (along with growth rates). You should use this option for standard "peak day" analysis.
- **Day type and hour**—You select the specific day type and hour.
- **Peak or minimum load on given day type—**You select the day type, and SynerGEE automatically uses the peak or minimum load hour.
- **Peak load on peak day or minimum load on minimum load day—** SynerGEE automatically finds and uses the yearly peak or minimum load point. You do not need to specify a day or time.
- **Specified day and time—**With an on-screen calendar, you select the day and hour. SynerGEE determines the day type.

Application of load curves

For the purpose of the following discussion, the following abbreviations apply.

$$\begin{aligned} C_r^k(h) &= \text{Customer class curve} \\ \text{k} &= \text{kW or kvar} \\ \text{r} &= \text{class name} \\ \text{h} &= \text{hour of a particular day type} \end{aligned} \quad \textbf{(5-16)}$$

Customer class curves have no units. They are simply used to multiply the kW and kvar load values on sections. The following represents the application of a load curve for customer class "r" at hour "h" to a section's distributed load.

$$kW_{Applied} + jkvar_{Applied} = C_r^{kw}(h) * kW_{Dist} + jC_r^{kvar}(h) * kvar_{Dist} \quad \textbf{(5-17)}$$

Note that the kvar curve is applied directly to the kvar portion of the load.

Kvar load curves

There are two options for the handling of kvar curves. The kvar curves can be specified as independent curves with hourly factors. Or, monthly power factors can be used so that the kvar curve is:

$$C_r^{kvar}\left(h\right)=\frac{C_r^{kw}\left(h\right)}{pf_h}\sqrt{1-pf_h^2} \tag{5-18}$$

The kvar curve is applied to the kvar portion of the load. It is important to note that monthly power factor values are used in the generation of the kvar curve and are not used directly on model loads.

As an example, consider a load of 100+j50 kVA as a section's distributed load. There is also a load curve having a kW factor of 50.0% and monthly power factor of 90.0% at time point "h." The factors for this time point are:

$$\begin{aligned} C^{kw} &= 0.5 \\ C^{kvar} &= \frac{0.5}{0.9}\sqrt{1-0.9^2} = 0.242 \end{aligned} \tag{5-19}$$

The section load would be:

$$load=(0.5)(100)+j(0.242)(50)=50+j12.1kVA \tag{5-20}$$

The power factor of the original section load was 89%. The load after the application of the customer class has a power factor of 97%.

Zones

As discussed earlier, a zone is composed of up to three customer classes (36-72 load curves each). Each class has an associated percentage for application to the section load. Once the customer classes, and thus the load curves, are combined, zones can be represented by a single unit-less factor. The same nomenclature for customer load curves will be used for zones.

$$Z^k\left(h\right)=\frac{\sum_{j=1}^{3}W_j * C_{r(j)}^k\left(h\right)}{\sum_{j=1}^{3}W_j} \tag{5-21}$$

k = kW or kvar

r = zone name

h = hour of a particular day type

W = weight for customer load curve

Zone curves are unit-less. They are used to multiply the kW and kvar load values on sections. The following represents the application of a zone at hour "h" to a section's distributed load.

$$kW_{Applied}+jkvar_{Applied}=Z^{kw}\left(h\right)*kW_{Dist}+jZ^{kvar}\left(h\right)*kvar_{Dist} \tag{5-22}$$

Note that the kvar curve is applied directly to the kvar portion of the load.

Example

Consider these three customer classes, combined into a customer zone as follows.

$$\begin{aligned} re &= \text{Residential all electric - 55\%} \\ rg &= \text{Residential with gas - 35\%} \\ ml &= \text{Municipal lighting and street lighting - 10\%} \end{aligned} \tag{5-23}$$

A newer suburban feeder named Peyton Landing (PL) uses this zone. As such, the zone "curve" can be expressed as:

$$Z_{PL}^{k}\left(h\right)=\frac{0.55*C_{re}^{k}+0.35*C_{rg}^{k}+0.10*C_{ml}^{k}}{0.55+0.35+0.10} \tag{5-24}$$

Applying this equation further, assume that you want the model set for analysis on a March weekday at 4:00 pm, and the following factors are found in the customer class curves.

March Weekday – 4:00 pm		
Class	**kW Factor**	**kvar factor**
re – residential electric	0.6	0.8
rg – residential gas	0.7	0.7
ml – municipal lighting	0.3	0.8

If a section has a distributed load of 300kW and 100kvar, the load used for analysis is:

$$\begin{aligned} &300*\frac{0.55*0.6+0.35*0.7+0.10*0.3}{0.55+0.35+0.10}+ \\ &j100*\frac{0.55*0.8+0.35*0.7+0.10*0.8}{0.55+0.35+0.10} \\ &=181.5kW + j76.5kvar \end{aligned} \tag{5-25}$$

Temperature & weather

Loads in a distribution system are impacted by outdoor temperature, wind, and humidity. SynerGEE has the capacity to model these effects. There are three ways that weather impacts a model in SyneGEE:

- Load increase or decrease due to temperature, wind speed, or relative humidity
- Overhead conductor resistance due to temperature
- Use of summer or winter ratings due to season

In this chapter, we will focus on the weather impact on loads. Two general sets of information need to be supplied for this modeling to work. First, the weather conditions for the model need to be specified. As we will see, those conditions can be set by simply specifying them in the model options or by setting up and applying a more detailed weather profile. The second set of information that is needed is load sensitivity to temperature, wind speed, and relative humidity. Again, these settings can be specified globally or as a part of the customer class definition.

There are a variety of ways to take advantage of weather impacts on load models. A few parameters can be entered to capture general effects. Alternatively detailed information can be supplied by month and customer class to support monthly variations in weather, and years of unusually hot or cold temperatures.

The fundamental load model reacts to temperature, wind speed, and relative humidity in the same way regardless of the extent for considering weather. In other words, an analysis is run at a particular time with particular conditions in the modeling world. Even single-year and mutli-year analysis are cumulative runs from discrete times. At the analysis time, the model is setup with a specific temperature over normal, wind speed above normal, and relative humidity above normal. “Normal” weather conditions are those that were effective at the time that meter flows and load values were collected for the establishment of the SynerGEE model.

With or without explicit weather modeling, weather conditions are always a part of the establishment and calibration of the base load values in a model. The power through a meter, for example, is recorded at a certain time of day (that is where diurnal customer load modeling comes in) and under certain weather conditions. There are many reasons that a power flow at 10:00 a.m. on Tuesday in January would be different than the flow at the same time in February. One reason is the difference in average temperature between January and February. Customer class curves are used to deal with differences between loads through the months. Weather modeling is used to deal with the weather differences between the establishment of load curves and load values and the simulation.

Weather impacts on SynerGEE load models is based on the difference in temperature, wind speed, and humidity between the conditions used to establish the base load models and the conditions at simulation time. In general:

$$\text{Analysis Loads} = f\left(\text{Calculated Loads, } \Delta\text{Weather}\right) \tag{5-26}$$

For example, the impact of temperature is:

$$S_{T2} = kW_{Base} k_{11} \Delta C + jkvar_{Base} k_{21} \Delta C \tag{5-27}$$

The temperature difference between the conditions used to establish the base model and the conditions at simulation time is ΔC . Two factors are used to relate changes in kW and kvar load and degree changes in temperature (temperature is always in Celsius).

$$k_{11} \quad \%\,kw\,change \big/ \Delta C$$
$$k_{21} \quad \%\,kvar\,change \big/ \Delta C \tag{5-28}$$

These factors relate percent change in load for degree change in temperature. Consider the following load and factors:

$$\begin{aligned} S_{20C} &= 150 + j80kVA \\ k_{11} &= 1.5\ \%\big/ \Delta C \\ k_{21} &= 2.3\ \%\big/ \Delta C \end{aligned} \tag{5-29}$$

Note that the load was measured at 20C. Since the factors represent percent changes we have a load that varies as:

$$S_{20C} = 150(1 + 0.015\Delta C) + j80(1 + 0.023\Delta C)kVA \tag{5-30}$$

If the temperature is 20C then the load will have a value of $150 + j80kVA$ since the change in temperature over the base model, ΔC, is zero. If the temperature rises to 25C then the load will be:

$$\begin{aligned} S_{20C} &= 150(1 + 0.015 \cdot 5.0) + j80(1 + 0.023 \cdot 5.0)\ kVA \\ &= 161.25 + j89.2\ kVA \end{aligned} \tag{5-31}$$

The load has increased due to the weather change. There has also been a slight reduction in power factor. The reduction in power factor could be a result of compressor loads for air conditioning equipment. The capability to model separate impacts on kW and kvar is important. However, the engineer can certainly set factors for kW and kvar to the same value.

As we have seen, temperature effects are calibrated to a difference in Celsius. Relative humidity is calibrated as percent change in load for each percent change in relative humidity. Wind speed is change in ft/sec or m/sec. Our overall weather dependent load model is:

$$\begin{aligned} kW' &= \left(1 + \frac{k_{11}\Delta C}{100}\right)\left(1 + \frac{k_{12}\Delta\, ft/sec}{100}\right)\left(1 + \frac{k_{13}\Delta\%RH}{100}\right)kW \\ kvar' &= \left(1 + \frac{k_{21}\Delta C}{100}\right)\left(1 + \frac{k_{22}\Delta\, ft/sec}{100}\right)\left(1 + \frac{k_{23}\Delta\%RH}{100}\right)kvar \end{aligned} \tag{5-32}$$

A change of zero in temperature, wind speed, or relative humidity will cause the respective term to drop to 1.0 in the above equations.

Latency

The final component to weather based load modeling is termed “latency”. It deals with the impact of weather conditions that might persist day after day. Again, the base load model should take into account typical load patterns. If the utility typically experiences weeks of daily high humidity and high temperatures then the load values should be determined as appropriate. If the engineer would like to model the effects of an extended heat wave then latency factors may help.

The “latency” concept is that successive days of unusual weather will result in higher loads than single days of such weather. The factors are available for temperature, wind speed, and relative humidity and are expressed in % / day. A temperature latency factor of 1.5% / day, for example, will increase the impact of temperature variation on load by 1.5% for each latent day. Lets say that an extended heat wave kept temperatures above normal for 7 days. Since 7 x 1.5% is 10.5% then the temperature impact on load would increase by 10.5%.

It is very important to note that this is NOT a 10.5% increase in load. It is a 10.5% increase in the temperature factor applied to the load.

Here is a numerical example:

$$\begin{aligned} S &= 500kW \\ k_{11} &= -1.5 \,{}^{\%}\!/_{\Delta C} \\ l_c &= 5.0 \,{}^{\%}\!/_{day} \\ \Delta C &= -10C \end{aligned} \tag{5-33}$$

The unusually cold temperature results in an increase in kW load. If we have no latent days then the load used in analysis would be:

$$\begin{aligned} S' &= 500\left[1-0.015\cdot(-10)\right]kW \\ &= 500\left[1+0.15\right]kW \\ &= 575kW \end{aligned} \tag{5-34}$$

Now, consider two latent days of unusually cold weather:

$$\begin{aligned} S' &= 500\left[1-0.015\cdot(-10)\left(1+0.05\cdot 2\right)\right]kW \\ &= 500\left[1+0.15\cdot 1.1\right]kW \\ &= 500\left[1.165\right]kW \\ &= 582.5kW \end{aligned} \tag{5-35}$$

If there were five latent days the load would be:

$$S' = 500\left[1-0.015\cdot(-10)\left(1+0.05\cdot 5\right)\right]kW \tag{5-36}$$
$$= 500\left[1+0.15\cdot 1.25\right]kW$$
$$= 500\left[1.1875\right]kW$$
$$= 593.8kW$$

It can be seen that the latent days play a secondary role in the load calculations. They represent a percent increase in the percent impact of temperature, wind speed, and relative humidity. Here is the final formulation of weather's role in load modeling:

$$kW' = \left(1+\frac{k_{11}l_C\Delta C}{100}\right)\left(1+\frac{k_{12}l_W\Delta\, {}^{ft}\!/_{sec}}{100}\right)\left(1+\frac{k_{13}l_{RH}\Delta\%RH}{100}\right)kW \tag{5-37}$$
$$kvar' = \left(1+\frac{k_{21}l_C\Delta C}{100}\right)\left(1+\frac{k_{22}l_W\Delta\, {}^{ft}\!/_{sec}}{100}\right)\left(1+\frac{k_{23}l_{RH}\Delta\%RH}{100}\right)kvar$$

$where:$

$$l_C = 1+\frac{\%latent_C}{100}\cdot days_{latent}$$
$$l_W = 1+\frac{\%latent_W}{100}\cdot days_{latent}$$
$$l_{RH} = 1+\frac{\%latent_{RH}}{100}\cdot days_{latent}$$

Weather data

Weather conditions can be captured in two ways. Values that apply under all conditions can be supplied in the model settings area:

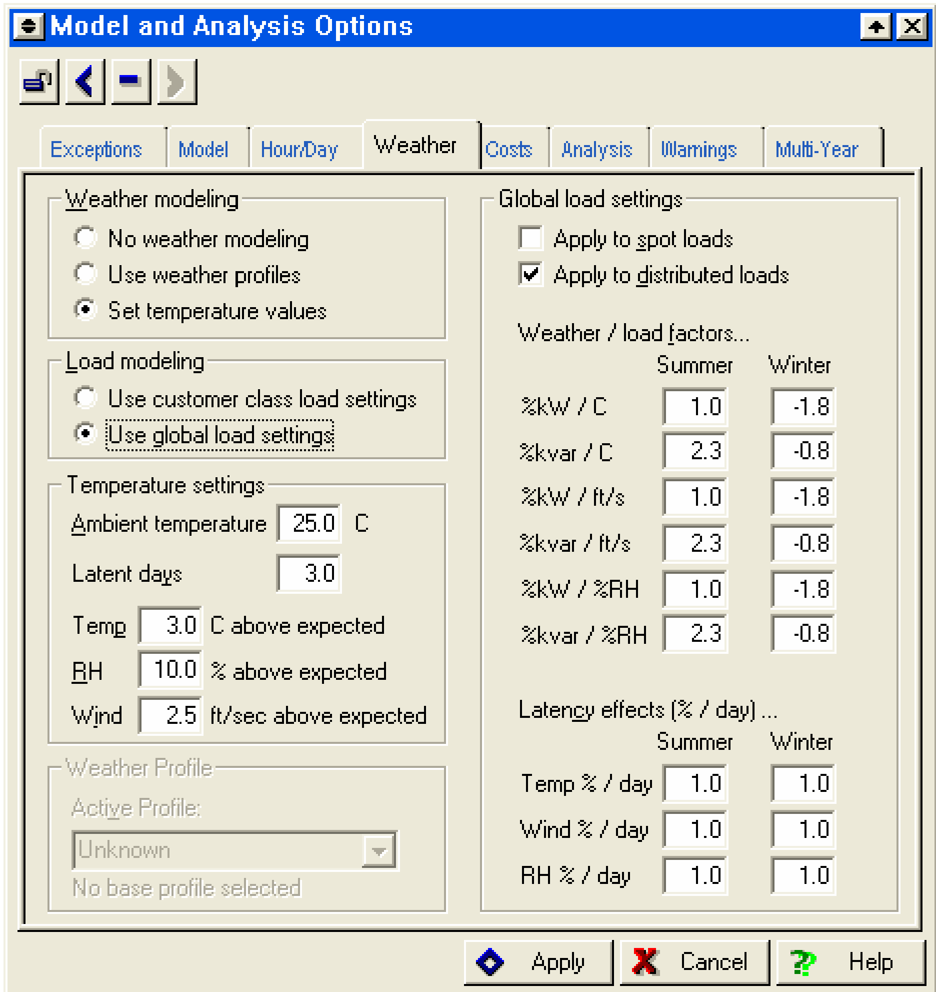


Fig. 5-6 *Model settings for weather*

The weather 'k' values and the latency data are all entered into the right side of the editor and are applied to all spot or distributed loads. The values for ΔC, $\Delta \,^{ft}\!/_{sec}$, and $\Delta\%RH$ are entered in the left column along with the number of latent days. If values are entered this way then the weather effects are the same for any simulated time of analysis.

The 'temperature modeling' group has three options:

- No temperature modeling – loads will be based directly from values entered into kW and kvar fields. If a winter threshold is used for conductor ratings then 'summer' ratings will be used.
- Use weather profiles – Pick a profile from the list below
- Set temperature values – Specify temperature information in the 'temperature settings' group. Temperature is entered for conductor impedance modeling. Other values are differences over nominal for load modeling.

There are two options for load modeling:

- Use information in the customer classes – this is the recommended approach.
- Use the settings in the group box to the right. Summer / winter is defined on the 'exceptions' tab.

An alternative and more granular approach to entering weather information is with the "Weather Profile" in the warehouse. These profiles contain temperature patterns, average wind speed values, average relative humidity values, and latent days for each of twelve months. An advantage of weather profiles is the ability to quickly change from one to another. With just a few mouse clicks, contingency studies, for example, could be run on a 'Mild Summer' profile and then again on a 'Severe Summer' profile. Here is a look at February in a 'Severe Winter' profile:

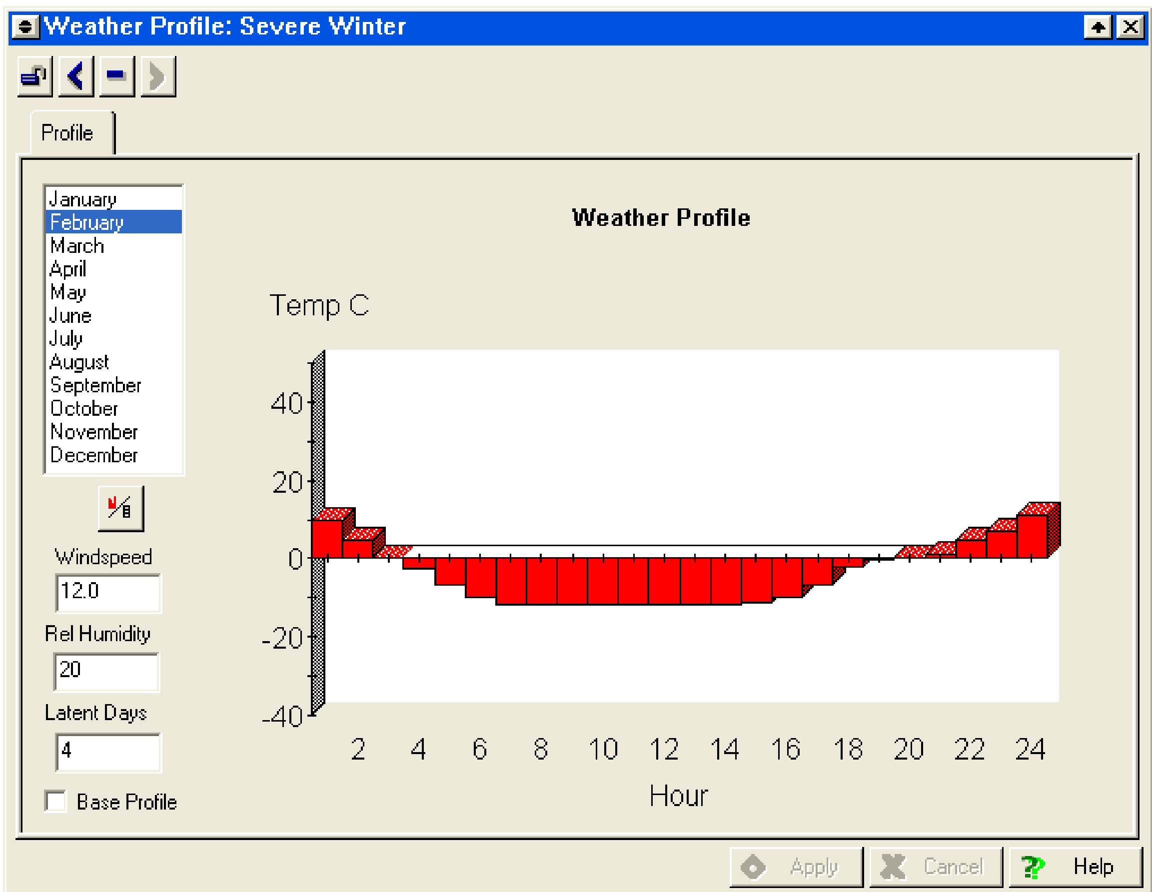


Fig. 5-7 Model settings for weather

If temperature profiles are to be used then one (and only one) profile needs to be established as the 'base profile'. The base profile is the temperature corresponding to the base loads in the model. The differences in temperature, wind speed, and relative humidity between the base profile and the selected profile are used to determine the weather impact on loads. If analysis were to be run using the base profile as the active one then there would be no weather related impact on loads because ΔC, $\Delta \, ^{ft}\!/_{sec}$, and $\Delta\%RH$ would all be zero.

The impact of weather on loads can be captured in customer classes. Each class has by-month values for temperature, wind-speed, and relative humidity load factors as well as factors for latent days. These factors are applied according to the month of analysis that is established in the model settings.

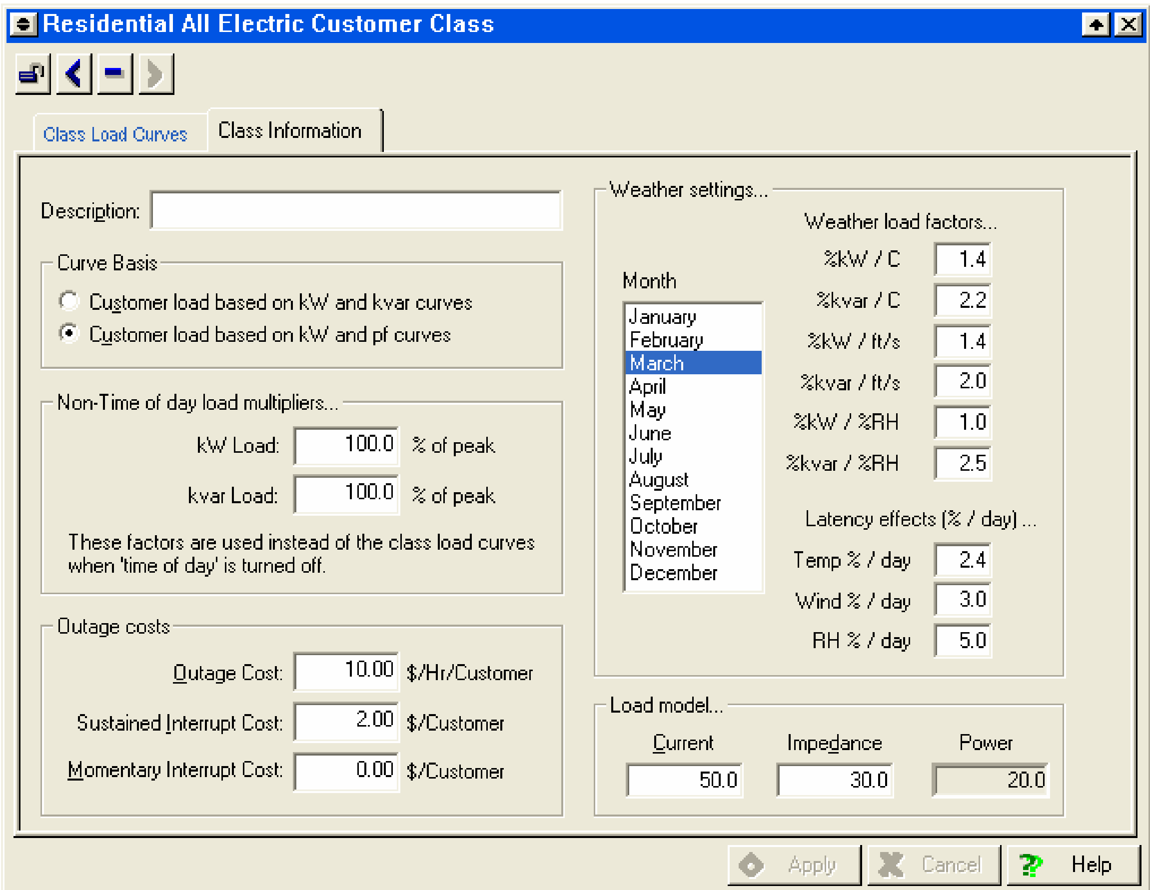


Fig. 5-8 Customer class weather related load factors

Load growth

SynerGEE can simulate load growth over time for analysis purposes. When this type of simulation is used, no permanent changes are made to the load values in your model. Load growth can be applied to spot loads and distributed loads. SynerGEE accepts positive and negative growth rates, simulating increasing or decreasing demand.

You can specify and apply a "growth factor" in a variety of different fashions, including:

- **Feeder Multiplier**—You can specify an annual "Feeder Multiplier" value for each feeder, in the feeder editor. Using this growth option, all loads fed by the feeder are multiplied by that value.
- **Load Multiplier**—This value, entered directly in the growth options editor, applies globally to all loads.
- **Section Growth Rate**—Each section has a growth rate value, which defaults to zero unless otherwise altered. If you set your growth options to consider section growth rates, you must also enter the number of growth years in the growth options editor. Using section growth rates, section loads are multiplied as follows, where "j" is a sample section.

$$S_j^{Grown} = S_j^{Specified} * \left(1 + \frac{r_j}{100}\right)^{\#Years} \tag{5-38}$$

- **Customer Zone Growth Rate**—You can assign growth rates to individual customer zones using the customer zones editor. Using this growth option, all loads on all sections within the zone are grown according to the specified rate, as follows.

$$S_j^{Grown} = S_j^{Specified} * \left(1 + \frac{r_{Zone-j}}{100}\right)^{\#Years} \tag{5-39}$$

If you choose this growth option, sections without a customer zone assigned experience no growth.

I, Z, PQ Calibration

Loads in SynerGEE are modeled with constant impedance, constant current, and constant power components. As such, I,Z,PQ calibration is a calculation tool designed to help you find the appropriate percentages of these components, depending on the load you are modeling. Realistic percentages of load components contribute to the accuracy of your models and the reliability of analyses.

The I,Z,PQ calibration tool also simplifies the process of modeling complex loads. It can aid in finding basic values for constant current and constant impedance loads to make the feeder demand versus feeder voltage response more realistic.

Power mathematics

As detailed this chapter, the actual kW and kvar load at a load point is expressed with:

$$S_{Actual} = \tag{5-40}$$

$$k_{PQ} \cdot S_{Nom} + k_I \left(\frac{kV}{kV_{Nom}} \right) \cdot S_{Nom} + k_Z \left(\frac{kV}{kV_{Nom}} \right)^2 \cdot S_{Nom}$$

$$S_{Nom} = \text{Nominal Voltage}$$

$$k_{PQ} + k_I + k_Z = 1$$

From this equation, you can see that as the feeder voltage drops:

- The nominal part of the load modeled as constant PQ does not change. However, since the feeder voltages are down, line current increases and losses increase somewhat. The net result is that power into the feeder goes up or remain virtually the same.
- The part of the load modeled as constant current goes down proportionally with the voltage. The load current does not change. The result is that power into the feeder goes down.
- The part of the load modeled as constant impedance drops and load current goes down. Power into the feeder goes down significantly.

Therefore, you can see that a change in voltage has a significant impact on feeder loading. If you have a feeder with loads modeled as constant power, you can try the following to demonstrate this.

- Run a load-flow and record the power into the feeder.
- Reduce the feeder voltage by a volt and run load-flow again.
- Compare the two values of power.

The power into the feeder with the reduced voltage should have gone down (you may need to turn off or lock regulators). Distribution systems with residential and commercial loading should have a demand reduction with a drop in feeder voltage.

The I,Z,PQ tool changes the load model on the feeder loads. Different values for the percentage constant current, constant impedance, and constant power are used. All loads use the same load model.

Using the calibrator

Running the tool on a feeder results in a graph like the following.

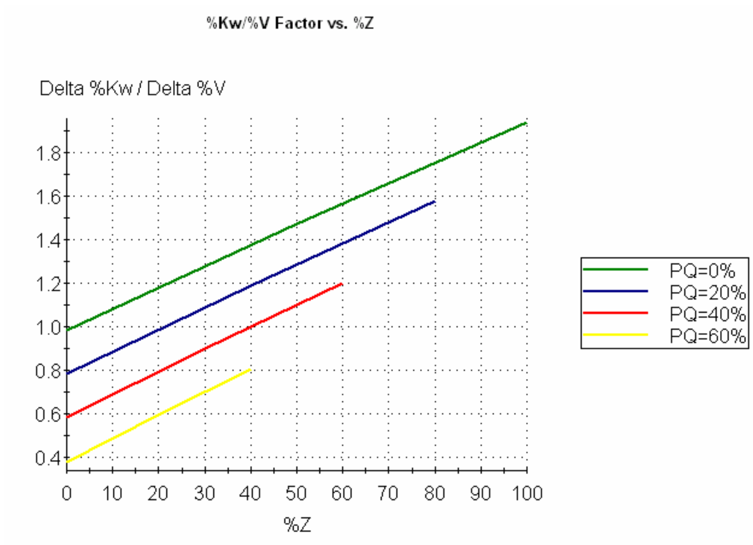


Fig. 5-9 *Graph produced by I,Z,PQ calibrator*

The graph shows lines for four values of constant PQ. Curves for other PQ values can be found by interpolating the graph. To use the graph, find the desired $\Delta\%kW / \Delta\%V$ value on the Y axis. Move across until a line is intersected. The line intersected represents the percent constant PQ value. Next, move down from the intersection point to find the percent constant Z value.

For example, if the $\Delta\%kW / \Delta\%V$ value is 1.1, you could use PQ = 0%, Z=15%, I=85%. Or, you could also use PQ=20%, Z=30%, I=50%. Likewise, PQ=40%, Z=55%, I=5% also works. Smaller values of %PQ are recommended.

Calculating Delta %kW / Delta %V

To use the graphs, an accurate measurement of delta $\Delta\%kW / \Delta\%V$ is important. To find this value, the best approach is to measure feeder load before and after an LTC tap change. For example:

Before tap change

Volts	12.886 kV
kW	5760 kW

After tap change

Volts	12.782 kV
kW	5724 kW

If there are downstream voltage regulators, it is important to make the readings quickly after the LTC tap change. The goal is to see the impact of the voltage change on loads, so the readings need to be made before downstream voltage regulators can respond.

From the data, you can calculate delta %kW.

$$\Delta\%kW = \frac{5760-5724}{5760} * 100\% \tag{5-41}$$
$$= 0.625\%$$

You can also calculate the delta %V.

$$\Delta\%V = \frac{12886-12782}{12886} * 100\% \tag{5-42}$$
$$= 0.807\%$$

You can then calculate delta $\Delta\%kW / \Delta\%V$ as:

$$\Delta\%kW / \Delta\%V = \frac{0.625}{0.807} = 0.77 \tag{5-43}$$

Using the calculated Delta %kW / Delta %V

Once you have the $\Delta\%kW / \Delta\%V$ you can use the graph (assuming it was generated from a valid model of the actual feeder) to determine load model values. As an example, consider the following chart and assume a %PQ of 40.

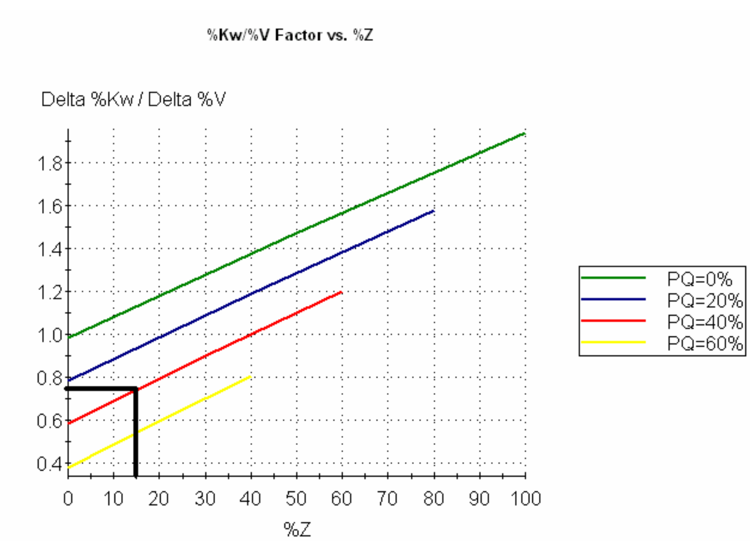


Fig. 5-10 Graph showing usage of delta %kW / Delta %V

The resulting values are as follows.

$$PQ = 40\%$$
$$Z = 15\%$$
$$I = 100\% - 40\% - 15\% = 45\% \quad \textbf{(5-44)}$$

As another example, you could interpolate a 35% PQ curve:

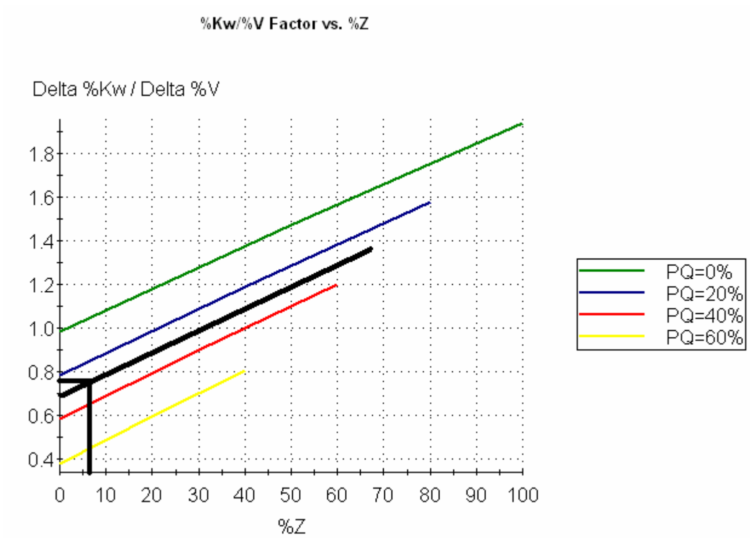


Fig. 5-11 *Graph interpolating a %PQ of 35*

...and get the following values:

$$PQ = 35\%$$
$$Z = 8\%$$
$$I = 100\% - 35\% - 8\% = 57\% \quad \textbf{(5-45)}$$

The graphs generated by the I,Z,PQ calibration tool are produced by adding and subtracting a very small change to the voltage, running a load-flow, and calculating the impact on the kW. The change is then added and the load-flow process is repeated. Three values for $\Delta\%kW / \Delta\%V$ are found from this data. The values are averaged and used on the chart.

The tool uses the following steps.

1. Set all load model I,Z,PQ values based on %PQ line and %Z axis value
2. Calculate $\Delta\%kW / \Delta\%V$ for the feeder
3. Plot the values

The tool has provisions to ignore spot loads and to ignore customer class-based I,Z,PQ values. If either of these selections is made, the load model I,Z,PQ values remain as they are set up in the spot load or customer class, and are not changed to the values used to generate the chart. Care should be taken to not replace these load models if the multiple editor is used to globally change load model I,Z,PQ values.

Sample %I, %Z, %PQ values

Listed below are sample values for load %I, %Z, and %PQ. These values represent typical percentages, and a section load may be a complex mix of loads like these and others.

Load Type	**PF%**	**%PQ**	**%Z**	**%I**
Resistance heaters, water heaters, ranges	100	0	50	50
Heat pumps, air cond., refrigeration	80	15-35	20-40	45
Clothes dryers	99	0	0	100
Televisions	77	0	0	100
Incandescent lighting	100	45	35	20
Fluorescent lighting	90	0	50	50
Pumps, fans, motors	87	40	40	20
Arc furnace	72	0	30	70
Large industrial motors	90	60	40	0
Large agricultural water pumps	85	0	75	25
Power plant auxiliaries	80	40	40	20

6
Feeders & Substations

Introduction

In SynerGEE, sources are swing busses or voltage / degree busses. There are two types of sources in SynerGEE: feeders and substations. In each case, the voltage is specified in nominal kV and volts. Values can be single values resulting in a balanced 120 degree rotation or they can be specified by phase. Feeders and substations also have source impedance values that become important during fault, motor start, and harmonic studies.

Feeders

Oftentimes, feeders are the only source modeled in SynerGEE due to the unavailability of substation data. Feeders are usually setup to represent some point near the secondary of the substation transformer.

Substations

Substations have gone through several changes throughout the life of SynerGEE. Initially, the items were called “substation transformers”. They held information about the source to the substation transformer and also about the transformer itself. In the current version of SynerGEE, the transformer information has been removed from the “substation transformer”. The data held by the substation and the data held by the feeder are nearly identical. Now, the transformer in the substation is represented by a primary transformer with a pad mounted transformer type.

A real substation oftentimes has multiple transmission feeds. In SynerGEE, that substation would be represented by multiple “substation transformer” facilities.

An advantage to substation modeling is that the effects due to the substation transformers to harmonics, faults, flicker, and voltage drop can be simulated. Transfer options are available in contingency studies when the substation is modeled. Also, whenever a substation is analyzed, all connected feeders are also analyzed; even if they are not selected.

7
Capacitors

Introduction

Capacitors are used widely in distribution systems for voltage regulation and power factor correction. Banks are usually placed near a load to provide reactive power locally so the current for the reactive load does not have to be sent through the distribution system. The reduced amount of reactive power flowing on the distribution lines between the source and capacitor bank allows a lower current flow and improves the power factor. Line losses are smaller and thus the voltage becomes higher at the load.

Capacitor compensation is useful for fixed loads. Switched capacitors can be used to meet the demand of time-varying loads. The switching of banks may cause considerable system transients. The level and duration of these transients is dependent on the time constant associated with the line and capacitor. It should also be noted that an operating capacitor bank makes a power system more sensitive to transients since a large *di/dt* through the capacitor causes a large voltage spike.

SynerGEE provides support for section capacitor installations that consist of fixed units and up to three switched modules. The installations can be connected in Wye, Delta, or Wye-Gnd configurations. The capacitors are modeled as fixed impedance devices in the middle of sections. They are specified by their nominal kvar value per phase. You can turn capacitors on or off using various operations in SynerGEE. You can place switched units into manual or automatic modes.

Unlike most other equipment models in SynerGEE, specifications for capacitor installations are made entirely within the Capacitor dialog box and stored in the model database. No device table exists in the equipment database for capacitors.

Capacitor compensation details

Capacitors are used widely in distribution systems for var support, voltage regulation, and power factor correction. Capacitor banks are usually placed near a load to provide vars. The reduced amount of vars that the substation has to provide to the load through the feeder allows a lower current flow and improves the power factor. Line losses are smaller and thus the voltage becomes higher at the load. This capacitor effect can be seen through a simple line model.

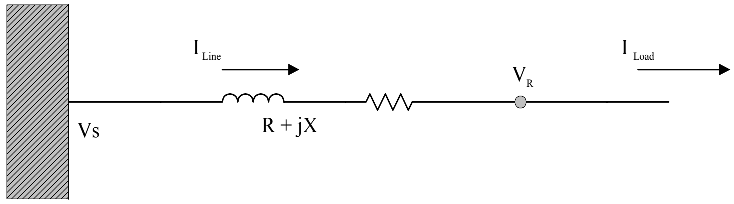


Fig. 7-1 Feeder without capacitor compensation

The diagram shows a feeder without capacitor compensation. The load voltage is:

$$V_{Load} = V_{Source} - I_{Load} * \left(R_{Line} + jX_{Line}\right) \tag{7-1}$$

and can be re-arranged as:

$$V_{Source} = V_{Load} + I_{Load} * \left(R_{Line} + jX_{Line}\right) \tag{7-2}$$

A lagging load is being fed. Therefore, the vector diagram of the system voltages and currents is drawn as follows.

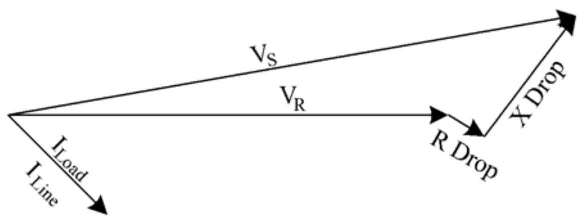


Fig. 7-2 Vector diagram of feeder without capacitor

Now, consider the addition of a slightly oversized capacitor to the system, as follows.

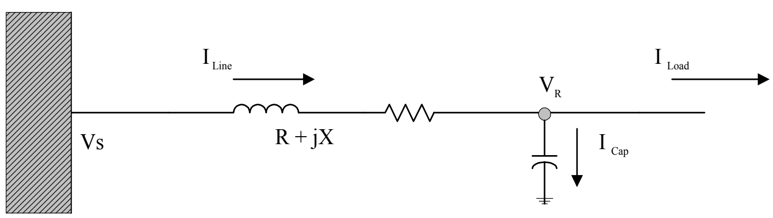


Fig. 7-3 Feeder with capacitor

The expression relating source and load end voltage becomes:

$$V_{Source} = V_{Load} + \left(I_{Load} + I_{Cap}\right) * \left(R_{Line} + jX_{Line}\right) \tag{7-3}$$

The capacitor current leads the voltage by 90 degrees, and is added to the load current. A new vector diagram can be drawn.

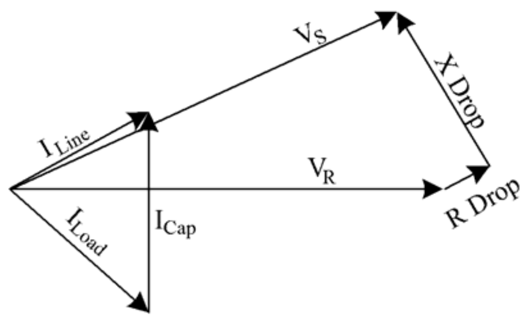


Fig. 7-4 Vector diagram of system with capacitor

This case is very simple, but it does show some of the benefits of capacitor compensation. For example:

- Notice in the vector diagram that a much smaller source voltage is now required to produce the same load voltage. Capacitors generally improve voltage at a load with a given source voltage.
- The power factor is increased. Capacitors can be used to rotate the line current to match the load voltage. To the source, the load appears to have a greater power factor. Less vars are needed from the source, hence why capacitors are said to produce vars.
- The magnitude of line current is decreased with the application of the capacitor. The lower line current should bring about smaller line losses.

There are two very important things to note about this analysis. First, the capacitor current had to flow through the line impedance to get the desired voltage improvement. Second, the capacitor current reduces the load current magnitude to reduce losses. These observations imply that capacitors should be placed toward the outer edges of a feeder, but inward enough to serve adequate load current.

The SynerGEE capacitor model

The SynerGEE capacitor model consists of 4 "modules:" one fixed unit and three switched units, as follows.

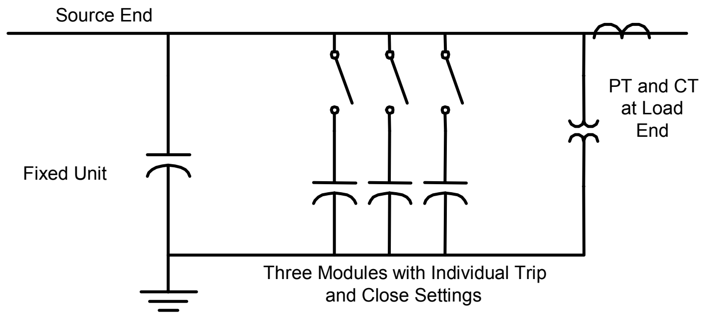


Fig. 7-5 *Diagram of switched capacitor model*

A module refers to a by-phase bank of capacitor units. The kvar values for the fixed module are specified per phase. For the switched modules, you can specify an overall kvar value for a set of selected phases. You can also activate or deactivate the individual switched modules as needed. Therefore, to model a fixed capacitor only, you can simply leave the three switched modules deactivated, essentially making them non-existent.

Capacitor installations can be connected in Wye, Delta, or Wye-Gnd configurations. The diagram above shows a Wye-Gnd configuration.

The three switched modules are composed of *identically* rated capacitors, connected to each phase you specify. Switches connect and disconnect the switched modules from the line section (close and trip, respectively). You can control the switching yourself if the installation is in manual mode. Or you can choose automatic mode, in which SynerGEE controls the switching based on the results of balanced or by-phase analysis, or time of day. For analysis results, automatic mode can consider a variety of different criteria, such as current, voltage, and kvar limits.

In summary:

- The fixed kvar module is always connected to all of the phases of a section. If you do not want to have fixed kvar attached to a particular phase, a value of zero can be selected for that phase.
- All phases of a switched module are connected when the module's switch is closed. You may choose the phases that are attached to switched modules. The phasing selection applies to all three modules.

Modeling capacitors

Capacitor connections

Capacitors are passive linear devices and modeled as constant admittance values within SynerGEE. Sometimes this is referred to as constant susceptance, since the conductance portion of the constant admittance load is zero. Capacitors are always located in the center of the conductor model portion of a section. The selected connection for the installation applies for both the fixed and switched portion. These connections are displayed below.

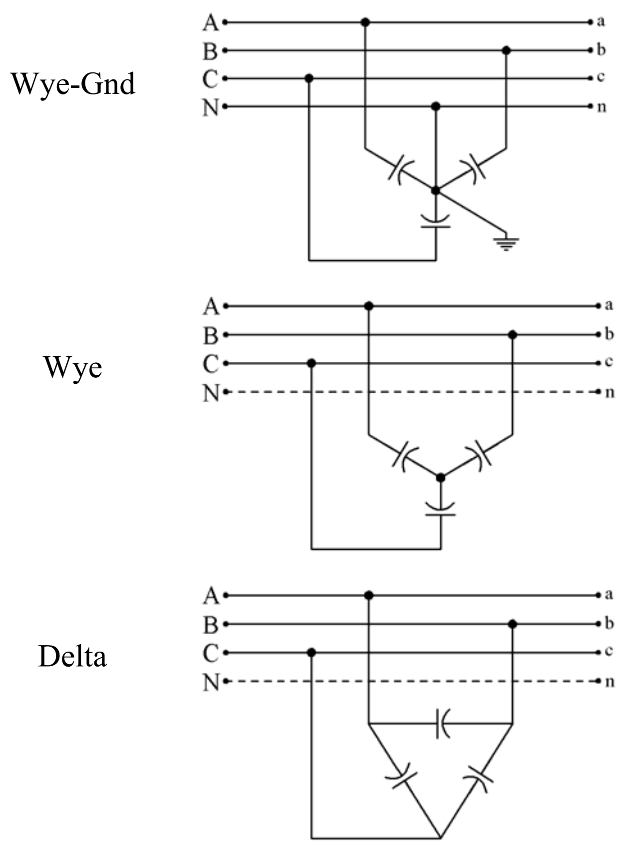


Fig. 7-6 *Capacitor connections*

The dashed lines for the neutral conductor on the Wye and Delta connections indicate that the connections can be used for both grounded and ungrounded sections. Phases of capacitor modules can be removed to simulate open connections.

Delta and Wye-Gnd banks are simulated directly within SynerGEE. Wye banks are converted to Delta equivalents during load-flow simulations using a derivation of the fundamental Wye-to-Delta transformation of passive linear devices. The derivation and application of this conversion are presented later in this chapter.

Fixed and switched units

Capacitor installation models in SynerGEE consist of two parts – the fixed or base unit and three switched modules. The figure below shows a Wye-Gnd installation with the fixed unit and one switched module:

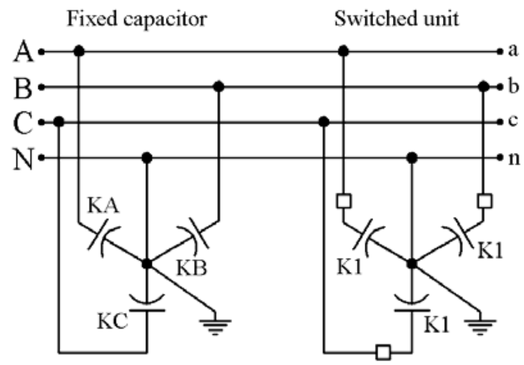


Fig. 7-7 Fixed and switched units in an installation

Notice that three distinct kvar values can be specified for the fixed portion of the installation. On the other hand, each of the switched modules is made up of identical capacitor units. Open connections can be achieved by putting zero values in for the fixed module or by deselecting switched phases for the switched modules.

Positioning relative to load

Capacitor installations are always placed at the center of a section with the distributed and possibly spot loads. The following is a diagram of a model section with a capacitor.

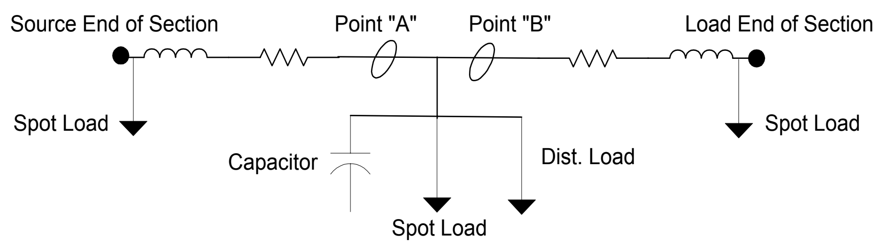


Fig. 7-8 Section model with a capacitor

Note that this figure shows three spot loads, for demonstration purposes only. SynerGEE allows only one spot load per section, at the source end, load end, or center of a section.

The capacitor kvar injection occurs at the same location as the distributed load and a spot load placed at the center of the section. Capacitor metering is always at the center of the section. However, for flows metered with the CT, metering is either right before or right after the capacitor and load terminals.

Type of control	Metering location
Kvar	Point "A" - before capacitor installation
Amp	Point "B" - after capacitor installation
Pf	Point "A" - before capacitor installation
Voltage	Does not matter. An LDC meters after the installation

Type of control	Metering location
Time	Does not matter. Time is used.

kV rating

Normally, you should assume that the voltage rating of capacitor units corresponds to the nominal section voltage. If you are applying a capacitor that is rated at a voltage other than the section voltage to which it is being applied, the kvar value of the capacitor should be de-rated as follows:

$$kvar_{DPA} = kvar_{Rated} * \frac{kV_{Section\,Nominal}}{kV_{Cap\,Rated}} \tag{7-4}$$

Controlling switched modules

About module switching

Each of the three available modules of a switched capacitor has uniformly-sized kvar/phase values. For example, selecting 100 kvar/phase for module 1 of a switched installation indicates that when that module is *on*, each phase nominally injects 100 kvar into the system. Remember that the kvar value of the fixed part of the installation is specified by phase and always connected (unless the capacitor is turned off).

The following represents a sample capacitor.

Fixed kvar	A=150, B=150, C=150
Metering phase	A
Active switch phases	A, C
Primary control	Voltage, Automatic
Module 1 (on)	75 kvar/ph, Close 118V, Trip 125V
PT Ratio	208.0

As such, the following values represent the capacitor kvar layout for the section, between 118 and 125 volts.

Phase A	Phase B	Phase C
150+75 = 225 kvar	150 kvar	150+75 = 225 kvar

Phase B does not get the 75 kvar of switched capacitance because the B is not checked in the Active Switch Phase area. The Active Switch Phase check boxes affect all switched modules in the same way. The value of fixed kvar is always applied regardless of the

Active Switch Phase settings. A value of zero can be entered into the fixed kvar field for those phases not having fixed capacitors.

Tripping and closing rules

Balanced and by-phase load-flow analyses opens or closes switched units based on the section conditions and the settings of the switched capacitor. This action occurs for switched capacitors that are both on and in automatic mode. SynerGEE never alters the switch state of switched capacitors in manual mode.

A module is turned on (the close setting is satisfied) by balanced or by-phase analysis if the following criteria have been met.

- The Primary Control option is set to Var or Current, and the metered value of kvar or current exceeds the close setting
- The Primary Control option is set to Voltage or Power Factor, and the metered value of voltage or power factor is less than the close setting
- Installation is in Automatic Mode
- Voltage Override settings are not violated

A module is turned off (trip setting is satisfied) by balanced or by-phase analysis if the following criteria have been met.

- The Primary Control option is set to Var or Current, and the metered value of kvar or current is less than the trip setting
- The Primary Control option is set to Voltage or Power Factor, and the metered value of voltage or power factor exceeds trip setting
- Installation is in Automatic Mode
- Voltage Override is set to be used and voltage is outside of bandwidth

The following table summarizes the rules and operation ranges for a switched unit.

Switched capacitor operation rules (g = metered value; ts = trip setting; cs = close setting)					
Control	Metering	Verify	Close	No operation	Trip
kvar	g = kvar	cs > ts	g > cs	ts < g < cs	g < ts
amp	g = amps	cs > ts	g > cs	ts < g < cs	g < ts
voltage	g = volts	ts < cs	g < cs	cs < g < ts	g > ts
power factor	g = pf	ts < cs	g < cs	cs < g < ts	g > ts
Time	g = time	cs > ts	g > cs	g < cs	g > ts

Metering phase

You should select metering phase used by capacitor installation when you select switch modules for use. This metering applies to module switch controls and to the voltage override mechanism. The chart below gives the meter phase options for various capacitor installation connections.

Connection	Voltage control	Current control	PF or kvar control
Delta or Wye	A/B, B/C, or C/A	A, B, or C	Total of all phases.
Wye-Gnd	A, B, or C	A, B, or C	A, B, or C

Voltage override

The voltage override controller uses the voltage directly on the low-side of the installation's PT. The voltage override mechanism does not use the CT and R and X settings. If the capacitor installation is connected as a Wye or Delta, the phase voltage is measured between the lines specified by the metering phase options and passed through the PT. Voltage is measured from line to ground if the installation is connected Wye-Gnd. If voltage override is used, the installation voltage is within its voltage override bandwidth when the following relationship holds.

$$\frac{V_{Term}}{PT_{Ratio}} \leq V_{Set} + \frac{1}{2} BW \qquad \textbf{(7-5)}$$

If the installation is outside of its bandwidth, modules are tripped, one at a time, in the order of module 3 to module 1.

Note that the voltage override switches modules on or off. If an installation's voltage is below bandwidth, modules are switched on by the voltage override mechanism (if the installation is in automatic mode). It should also be noted that the line-drop compensations R and X are not contained in the voltage override mechanism.

Line-drop compensator

SynerGEE uses the line-drop compensator (LDC) mechanism to alter the line voltage (as seen by the module switch controllers) by dropping the line voltage through a user-specified impedance by the line current. The LDC option is only available for voltage-controlled units. Voltage override does not utilize the LDC mechanism. The controller for module switches is 120V based. The voltage seen by the voltage controller when the LDC option is selected is as follows.

$$V_{Cont} = \frac{1000 * kV_{Line}}{PT} - \frac{I_{Line}}{CT} * \left(R_{LDC} + jX_{LDC}\right) \quad (7\text{-}6)$$

$$\begin{aligned} PT &= \text{PT Ratio} \\ CT &= \text{CT Rating} \\ R, X_{LDC} &= \text{User specified LDC drop values} \\ kV_{Line} &= \text{Line voltage} \\ I_{Line} &= \text{Line current} \\ V_{Cont} &= \text{Controller voltage} \end{aligned}$$

Manually turning on a module

The capacitance of a switched module is applied to a section when the following criteria are satisfied.

- The section capacitor's Turn Off check box is not checked.
- The module 1,2, or 3 check box is checked.
- The module 1,2, or 3 On check box is checked.

Converting Wye-connected banks to a Delta equivalent

The three-phase terminals with both Delta- and Wye-connected impedances are shown below.

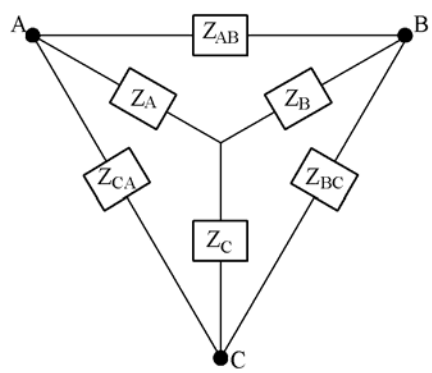


Fig. 7-9 *Delta and wye impedances*

If the sets of impedances are to produce the same terminal conditions, a relationship exists between them. The Wye impedances are functions of the Delta impedances as follows.

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$$
$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$
$$Z_{CA} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B} \tag{7-7}$$

The impedance of capacitors connected in a Wye can be determined from their ratings as follows.

$$Z = \frac{kV_{Rat}^2}{-jkvar} \tag{7-8}$$

These Wye-connected impedances can be converted to Delta impedances as follows.

$$Z_{ij} = \frac{\dfrac{kV_{Rat}^2}{jkvar_A\,jkvar_B} + \dfrac{kV_{Rat}^2}{jkvar_B\,jkvar_C} + \dfrac{kV_{Rat}^2}{jkvar_C\,jkvar_A}}{\dfrac{1}{-jkvar_k}} \qquad \begin{matrix} ij = AB,\ BC,\ CA \\ k \neq i \quad k \neq j \end{matrix} \tag{7-9}$$

This can be simplified as follows.

$$Z_{ij} = \frac{-kV_{Rat}^2}{jkvar_i} + \frac{-kV_{Rat}^2}{jkvar_j} + \frac{-kV_{Rat}^2 kVar_k}{jkvar_i kvar_j} \quad \begin{matrix} ij = AB,BC,CA \\ k \neq i\, k \neq j \end{matrix} \tag{7-10}$$

The nominal power consumption due to these impedances connected in Delta is due to the Delta nominal voltage, as follows.

$$S = \frac{kV_{LL}^2}{Z^*} \tag{7-11}$$

The capacitor impedances can be substituted to get the following.

$$S_i = \frac{kV_{LL}^2}{\dfrac{kV_{Rat}^2}{jkvar_i} + \dfrac{kV_{Rat}^2}{jkvar_j} + \dfrac{kvar_k kV_{Rat}^2}{jkvar_i kvar_j}} \quad \begin{matrix} ij = AB,BC,CA \\ k \neq i\, k \neq j \end{matrix} \tag{7-12}$$

This can be simplified if you multiply by $jkVar_i kVar_j$ as follows.

$$S_i = \left(\frac{kV_{LL}}{kV_{Rat}}\right)^2 \frac{jkvar_i kvar_j}{kvar_j + kvar_i + kvar_k} \quad \begin{matrix} ij = AB,BC,CA \\ k \neq i\, k \neq j \end{matrix} \tag{7-13}$$

Therefore:

$$kvar_i = \left(\frac{kV_{LL}}{kV_{Rat}}\right)^2 \frac{kvar_i kvar_j}{kvar_j + kvar_i + kvar_k} \quad \begin{matrix} ij = AB, BC, CA \\ k \neq i\, k \neq j \end{matrix} \tag{7-14}$$

Motor analysis and capacitors

SynerGEE never switches switched modules in manual mode. Automatic modules are set up in the base run of motor start analysis (MSA) or locked rotor analysis (LRA). In MSA, the state of switched capacitors is not changed until the 100% speed point. In LRA, the state of switched capacitors is not changed until its Running case.

Capacitor placement analysis

Capacitor placement analysis places capacitors having only fixed kvar values. If capacitor placement places new capacitors on a section already having an installation, the kvar values of the new units are simply added to the existing fixed kvar values. SynerGEE only allows one capacitor per line section. More information on capacitor placement can be found in the "design optimization" section of this reference.

Additional notes

Single bank short circuit current

A symmetrical short circuit current is shown as follows.

$$I_{SC} = \frac{1000 * MVA_{SC}}{\sqrt{3} * kV_{LL}} \tag{7-15}$$

The following represents source reactance.

$$X_L = \frac{{kV_{ll}}^2}{MVA_{SS}} \tag{7-16}$$

Capacitor reactance from phase to neutral is as follows.

$$X_C = \frac{{kV_{ll}}^2}{MVar_{3\phi}} \tag{7-17}$$

Maximum value of inrush current is as follows.

$$I_{RMS_{\max}} = \frac{V_{l-n}}{X_C - X_L}\left[1 + \sqrt{\frac{X_C}{X_L}}\right] \tag{7-18}$$

Peak inrush current is always less than the short circuit current at the bank.

Parallel capacitor banks

Capacitor banks are placed in parallel to provide larger Mvar support. When capacitors in a bank are already energized, the maximum inrush into a newly switched capacitor is determined by the momentary discharge of the capacitors already in service. A small impedance between banks allows a high inrush. The worst case is when all banks except one have been switched in.

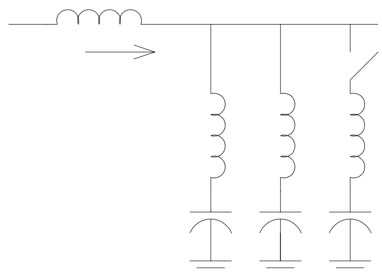


Fig. 7-10 *Switched bank*

If no charge is on the step being energized, the peak inrush is as follows.

$$I_{Peak_{Max}} = \sqrt{2} * V_{L-N} * \sqrt{\frac{C}{L}} \tag{7-19}$$

Where:

- C is the equivalent of all capacitors in circuit in µF.
- L is inductance in µH between energized steps and the step being switched in. (Two parallel + one above). This inrush current applies to uncharged capacitor steps. If restrike occurs, the inrush current may be twice that found above.

Using the same units as before, the inrush current frequency is as follows.

$$f = \frac{10^6}{2\pi\sqrt{LC}} \tag{7-20}$$

Breaker rating

Although inrush current is of a very high frequency, the breaker size should still be set to handle the current. This requires choosing a larger breaker than would otherwise be necessary. You could also add more reactance to limit the inrush current. Otherwise, risk is taken that the breaker may not be able to handle the high inrush.

In the field, to avoid voltage doubling, switched-out capacitors should be given time to discharge internally or through local transformers. Internal discharge takes about 5 minutes.

Switching device criteria

- The voltage and BIL ratings of the system should be maintained by the breaker
- Breakers should be able to handle 135% of capacitor bank rated current
- Momentary current rating should meet system fault current ratings and inrush current of capacitor bank
- Frequency design to withstand repeated switching

8

Voltage Regulators

Introduction

Regulators are used to step-up or step-down the voltage at or downstream of the voltage regulator so that the customer voltage levels are within limits. They are designed to raise or lower the voltage magnitude at and beyond some remote location. Regulators are common and useful devices within power distribution systems. This chapter discusses the basic regulator model and the more detailed model with shunt and series impedance values. It also describes how regulator tap positions are analytically changed and how voltage drop and losses are determined following the load-flow convergence.

Regulator conceptualization

Voltage regulators are constructed from autotransformers. Autotransformers can be best understood by looking back to the operation of a simple transformer model. For example,

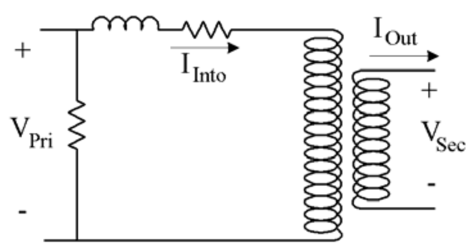


Fig. 8-1 *Ideal transformer*

In the case of voltage regulators, the turns ratio of the ideal transformer is often 1:10. The turns ratio corresponds to the regulator's range of regulation. Consider 120V and 1A being applied to the primary windings of the following transformer.

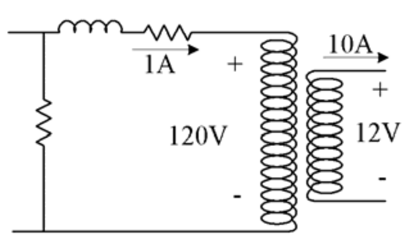


Fig. 8-2 Apply 120V and 1A to primary ideal winding

The power into the primary winding of 120VA must be the same as the power out of the secondary. The voltage and current relationships across the windings must vary inversely as the turns ratio. Thus, the current out of the transformer increases and the voltage is reduced. The autotransformer connection allows the primary line voltage to be increased or decreased by the secondary voltage. The resulting output voltage has been bucked or boosted. The autotransformer connection is achieved through a series connection of the primary and secondary windings.

There are two ways to connect the previous transformer as an autotransformer. Shown below is a connection used to step down or buck voltage. The voltage across the secondary (or series) winding is subtracted from the voltage across the primary (or shunt) winding as follows.

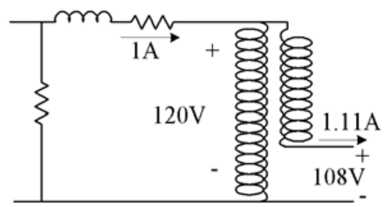


Fig. 8-3 *Autotransformer buck connection*

Notice that there is 120V across the series winding. However, because of the series connection, there is only 1.11A in the winding. This current corresponds to a power flow of 13.3VA through the secondary winding. This is a small power flow in the winding compared to the 120VA still transferred through the entire autotransformer.

The transformer could also be connected to step up the primary winding voltage as follows.

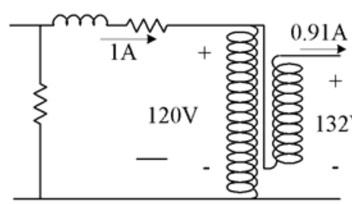


Fig. 8-4 *Autotransformer boost connection*

Assume that this autotransformer has negligible impedance, operates in a 13.8kV system, and is supplied with a nominal line voltage. The power into and out of the autotransformer is as follows.

(8-1)

$$S_{into} = 1A * \frac{13.8kV}{\sqrt{3}} = 7.97kVA$$

$$S_{out} = 0.91A * \frac{13.8kV}{\sqrt{3}} * \frac{132}{120} = 7.97kVA$$

The power through the autotransformer windings is very small, less than 10% of the above. For example:

$$S_{prim} = \frac{0.91A}{10} * \frac{13.8kV}{\sqrt{3}} = 0.73kVA \tag{8-2}$$

$$S_{ser} = 0.91A * \frac{13.8kV}{\sqrt{3}} * \frac{12}{120} = 0.73kVA$$

Thus, an autotransformer is a very useful tool for regulating voltages because only a small portion of the power flowing through the regulator is required to flow through its windings. Therefore, regulators can be built compactly and inexpensively.

The regulator model

The main distinction between an autotransformer and a voltage regulator is created with the application of mechanisms to control the series winding tap and current flow direction. The following model represents a schematic view of the voltage regulator model. This model shows the series winding connected through a switch. The switch allows current flow in either direction through the winding to produce a bucking or boosting. A variable tap is associated with the series winding. This mechanism, called a load tap changer (LTC), allows a higher degree of precision in regulator voltage control since only a portion of the voltage across the series winding will buck or boost the regulator input voltage.

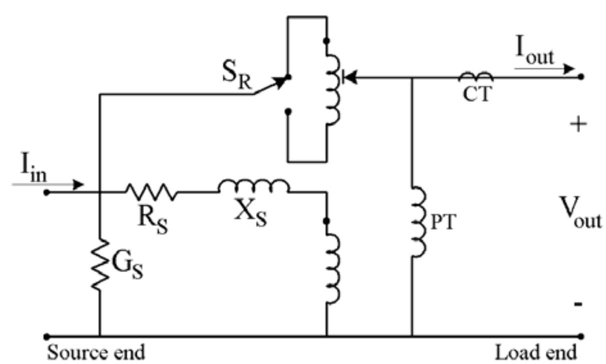


Fig. 8-5 *Voltage regulator model*

The regulator is composed of three components: an autotransformer, an LTC, and a line-drop compensator (LDC).

The autotransformer within the regulator model is characterized by the two coupled windings. The turns ratio between the windings is typically 10:1 for a 10% regulator. All winding impedances and losses are reflected to the inside of the shunt winding. These impedances are often determined from tests performed on regulators at full lower or raise positions. The effect of the series impedance, $R_S + jX_S$, varies with the tap position and its current flow. No current passes through the series impedance when the regulator is at its neutral or zero tap. The shunt conductance, G_S, is derived from the no-load losses and the shunt winding's voltage rating.

The LTC is the mechanical device that moves the tap of the series winding of the autotransformer. The current through and the voltage across the series winding is either increased or reduced by the motion of the LTC respective to the position of the reversing switch, S_R. This reversing switch allows the regulator to either buck or boost its output voltage. It allows the series winding to have a rating of K% of the shunt winding and still provide a +/-K% range of regulation.

The LDC is a controller that monitors its input voltage and current and sends a raise tap or lower tap command signal to the LTC. The input signals for the LDC come from CT and PT attached to the output of the voltage regulator. In the previous figure, notice that PT and CT are always connected to the load end of the regulator. The line-drop compensator is discussed in more detail later in this chapter.

Like most device models in SynerGEE, regulator models are created from a regulator instance that references a section record in the sections table and a specific regulator type in the regulator type table (DevRegulators).

Mathematical representation

The following figure shows the regulator model with some added voltage annotations.

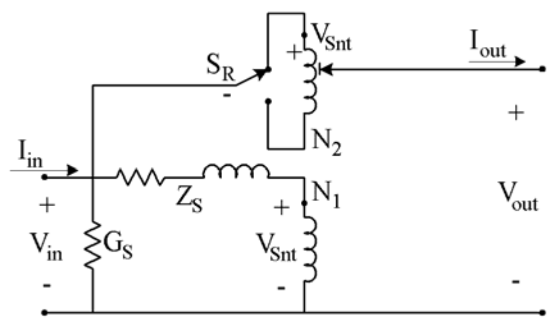


Fig. 8-6 *Model for mathematical representation*

A reversing switch, S_R, facilitates the bucking and boosting of output voltage. Mathematically, S_R has two states based on the following.

$$S_R = [-1,1] \tag{8-3}$$

The tap position of the regulator is represented with K_t as follows.

$$0 \le K_t \le MaxTap \quad \text{where } MaxTap = \text{Number of Taps} \tag{8-4}$$

At a given tap position, the ratio of voltage across the shunt winding to the voltage across the series winding can be expressed as follows.

$$\frac{V_{Ser}}{V_{Snt}} = \frac{N_2}{N_1} * \frac{K_t}{MaxTap} \tag{8-5}$$

The reversing switch factor, K_s, is now applied to the previous expression so that the polarity of voltage and current associated with the series winding may be related to the regulator's input voltage in subsequent equations. The following equation represents this scenario.

$$N = S_R * \frac{N_2}{N_1} * \frac{K_t}{MaxTap} \tag{8-6}$$

The voltage out of the regulator is the sum of the voltage at the input terminals and the voltage across the regulator series winding. In the previous equation, K_s determines the additive nature of the windings as follows.

$$V_{out} = V_{in} + NV_{Snt} \tag{8-7}$$

This equation can be used to find the output voltage in terms of the current in the shunt winding and the input voltage.

$$\begin{aligned} V_{out} &= V_{in} + N(V_{in} - I_s Z_s) \\ &= V_{in}(1+N) - NI_s Z_s \end{aligned} \tag{8-8}$$

By standard dot convention, the current into the shunt winding is related to the current out of the series winding as follows.

$$I_s = NI_{out} \tag{8-9}$$

Equation **8-10** can now be combined with the above equation to express the following.

$$V_{out} = V_{in}(1+N) - N^2 I_{out} Z_s \tag{8-10}$$

This equation shows how to find the voltage at the output of the regulator given the voltage at the input and the current demand at the output of the regulator. The current into the regulator is the sum of winding currents and the current into the shunt impedance branch, as shown below.

$$I_{in} = I_s + I_{out} + G_S V_{in} \tag{8-11}$$

Equation **8-11** can now be used to simplify this expression. For example:

$$I_{in} = (1+N) I_{out} + G_S V_{in} \tag{8-12}$$

Thus, the current into the transformer can be found from the load current.

Comparison to transmission and distribution model

Chapter 5 of the *Transmission and Distribution Reference Book* discusses an autotransformer model that is useful for integration into the bus admittance matrix of classic network load-flow algorithms. The model has been slightly modified to represent a voltage regulator.

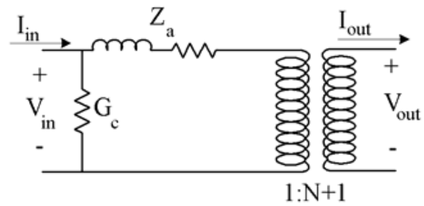


Fig. 8-7 *The classic model from T&D*

The series impedance of the transformer is based on the tap position of the regulator as follows.

$$Z_a = \left(\frac{N}{N+1}\right)^2 Z_s \tag{8-13}$$

The voltage out of the regulator in the previous figure is expressed as follows.

$$V_{out} = \left(V_{in} - I_{in} Z_s \left(\frac{N}{N+1}\right)^2\right)(1+N) \tag{8-14}$$

The input and output current of the regulator is related by the transformer turns ratio expressed in the following equation.

$$I_{in} = (N+1) I_{out} \tag{8-15}$$

The last two equations can be combined.

$$V_{out} = V_{in}(1+N) - I_{out} Z_s N^2 \tag{8-16}$$

The current into the transformer is found using the following equation.

$$I_{in} = (N+1) I_{out} + V_{in} G_c \tag{8-17}$$

This current matches the previous result and the current into the other regulator model.

Regulator kVA ratings

Voltage regulators are rating according to the kVA capacity of the series winding. This capacity is expressed as follows for a single-phase regulator.

$$kVA_{Rated} = \frac{\%Regulation * I_{Rated} kV_{Rated}}{100} \tag{8-18}$$

Three regulators make up the core of a three-phase unit, and the rated voltage is line-line. In this case, the rated kVA is expressed as follows.

$$kVA_{Rated} = 3 * \frac{\%Regulation * I_{Rated}}{100} \frac{kV_{Rated}}{\sqrt{3}} = \sqrt{3} \frac{\%Regulation * I_{Rated} kV_{Rated}}{100} \tag{8-19}$$

Regulator impedance

Regulator losses are modeled as constant impedance values. The no-load losses of the regulator are therefore proportional to the square of its input voltage. Series impedances are determined at full buck or boost conditions. The impedance remains the same because the regulator's autotransformer is a passive device. However, the *effect* of this impedance varies with regulator tap position. When the regulator is in neutral, there is no current through the autotransformer and the series impedance plays no role. However, the shunt impedance is still across the input voltage.

Regulator impedances typically vary under 20%.

Impedance calculation example

Given:

Type:	1Ph 14.4kV regulator
Rating	667 kVA / 463A
No load (core) loss	1.792kW
Load loss (series) at full raise	6.245 kW 133.0 kvar
Total loss at full raise	8.037 kW 133.0 kvar

As a demonstration, we will calculate the impedance of the classic regulator model:

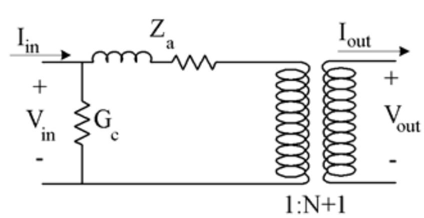


Fig. 8-8 *Classic autotransformer model*

At full load, the current out of the regulator is:

$$I_{Out} = 463A \tag{8-20}$$

The current through the primary winding and impedance is:

$$I_a = (1.1)(463) = 509A \tag{8-21}$$

To get the expected losses, we need:

$$\frac{(509)^2 R_a}{1000} = 6.245\,kW \tag{8-22}$$
$$R_a = 0.0241\Omega$$
$$and$$
$$\frac{(509)^2 X_a}{1000} = 133\,kW$$
$$X_a = 0.513\Omega$$

The base impedance of the regulator is:

$$Z_{Base} = \frac{14.4^2}{0.667} = 310.9 \tag{8-23}$$

And so the impedance and X/R ratio are:

$$Z_{\%} = 100\% \cdot \frac{\sqrt{0.0241^2 + 0.513^2}}{310.9} = 0.1652\% \tag{8-24}$$
$$X/R = \frac{0.513}{0.0241} = 21.3$$

The problem with this model is the continual estimation of losses and recalculation of impedance values required with each tap change.

SynerGEE uses a detailed autotransformer model to represent the regulator. Impedances in the model represent the actual impedances within the regulator. The values do not change with tap changes.

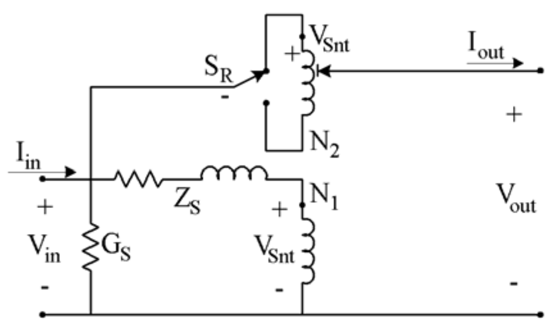


Fig. 8-9 The SynerGEE regulator model

At full raise and full load, we have 463A out of the transformer and 46.3A in the shunt winding.

From the shunt winding we have:

$$\frac{(46.3)^2 R_s}{1000} = 6.245\,kW \tag{8-25}$$
$$R_s = 2.913\,\Omega$$
$$and$$
$$\frac{(46.3)^2 X_a}{1000} = 133\,kW$$
$$X_a = 62.04\,\Omega$$

The base impedance is:

$$Z_{Base} = \frac{14.4^2}{0.667} = 310.9 \tag{8-26}$$

The leads to the impedance and X/R ratio:

$$Z_{\%} = 100\% \cdot \frac{\sqrt{2.913^2 + 62.04^2}}{310.9} = 19.98\% \tag{8-27}$$
$$X/R = \frac{62.04}{2.913} = 21.3$$

Putting the following values into the regulator model will result in the desired losses from the device:

$$Z_{\%} = 19.98\% \tag{8-28}$$
$$X/R = 21.3$$
$$No - load\ losses = 1.792\text{kW}$$

The load tap changer and controller model

The LTC is a mechanical device connected to the series winding tap mechanism of a regulator or transformer. A motor moves the LTC over taps and the output voltage of the regulator changes by small intervals. The LTC is driven by a control system using the regulator output current and voltage to determine if a raise tap or lower tap signal should be sent to the LTC motor. A regulator model with the LTC controller is shown below.

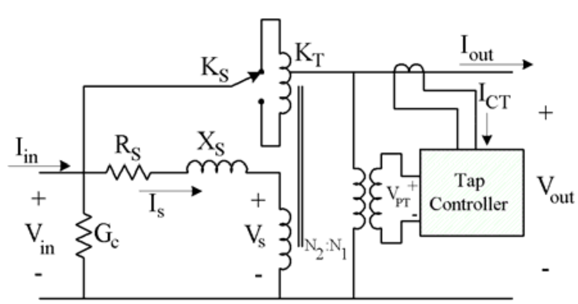


Fig. 8-10 PT's, CT's, and the LDC

IEEE Standard C57.15-1986 9.2.1.2 states that the regulator control circuit should use a 0.2A and 120V base. It turns out that the 0.2A controller current is irrelevant. However, all regulator settings in SynerGEE are based on 120V. This avoids ambiguity between the base of regulator settings and the user-specified base used for displaying SynerGEE analysis results.

Current and potential transformer models

The current transformer (CT) is specified by the rated coil amperage. It has an output current that is proportional to the regulator output current as follows.

(8-29)

$$I_{CT} = I_{out} * \frac{I_{Sec}}{CT}$$

CT = CT Rated Primary Winding Current

I_{Sec} = Secondary Current From Rated Primary Current

I_{out} = Metered Line Current

The potential transformer (PT) voltage is proportional to the regulator output voltage as follows.

(8-30)

$$V_{PT} = V_{out} * \frac{1}{PT_{Ratio}}$$

PT_{Ratio} = PT Turns Ratio

Odd CTs and some additional variable transformers are occasionally used to modify the behavior of the regulator's controller and its line-drop compensator.

The line-drop compensator model

The LDC controller has manually set parameters and receives input from the regulator PTs and CTs. The controller model is pictured below.

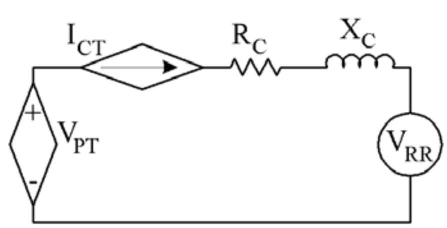


Fig. 8-11 Line-drop compensator model

V_{RR} is the voltage across the regulating relay. It is used, along with the bandwidth setting, to send the raise and lower signals to the load tap changer. If V_{RR} is greater than the voltage set point and outside of the bandwidth, then a raise-tap signal is sent to the LTC.

Correspondingly, if the voltage is below the voltage set point and outside of the regulator bandwidth, then a lower-tap signal is sent.

It is important to understand that the voltage across this relay is what the controller is attempting to regulate. That voltage is very sensitive to the values of the series real and reactive values in the control circuit. It is also very sensitive to the selection of PT and CT values.

The voltage on the PT secondary is determined from the regulator output line voltage. For example:

$$V_{PT} = \frac{1000 * kV_{Out}}{PT\,Ratio} \quad \text{where} \quad \begin{array}{l} kV_{Out} \text{ is L - N for Wye - Gnd or Open - Wye Connections} \\ kV_{Out} \text{ is L - L for Delta, Wye, or Open - Delta Connections} \end{array} \tag{8-31}$$

The LDC current is determined from the regulator's CT as follows.

$$I_{CT} = \frac{I_{Sec} * I_{Out}}{CT\,Rating} \quad \text{where} \quad I_{Out} \text{ is line current outside of connection} \tag{8-32}$$

The series real and reactive values R_c and X_c of the LDC circuit can be dialed directly into the regulator controller. The current from the regulator CT drops the voltage from the PT through R_c and X_c. If a regulator controller does not have a line-drop compensator, the values of R_c and X_c are zero and the voltage across the voltage-regulating relay is simply the voltage on the secondary of the PT.

Setting the line-drop compensator

The purpose of the line-drop compensator is to make the voltage across the regulator relay match the voltage at some point down the line fed by the regulator. However, the voltages, currents, and impedances inside of the LDC are on a different base than the actual line values seen by the primary side of the PT and CT.

The following equation shows how to convert a line impedance between a regulator and its regulation point to corresponding values in volts for use in the LDC.

$$R_c + jX_c = Z_L \frac{CT\,Rating}{PT\,Ratio} \tag{8-33}$$

Rather than deriving this equation, you can verify it with this simple example. Consider this regulator feeding a downstream load. The impedance between the regulator and the load is Z_L.

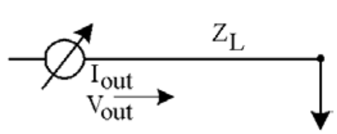


Fig. 8-12 *Regulator feeding a load*

The voltage at the load is as follows.

$$V_{Load} = V_{Out} - I_{Out} Z_L \tag{8-34}$$

The voltage at the regulator relay has a similar form.

$$V_{RR} = V_{PT} - (R_C + jX_C) I_{CT} \tag{8-35}$$

Regulator output voltage and current are computed as follows.

$$V_{RR} = \frac{V_{Out}}{PT\ Ratio} - (R_C + jX_C) \frac{I_{Out}}{CT\ Rating} \tag{8-36}$$

This equation can be manipulated to find the following.

$$V_{RR} = \frac{V_{Out}}{PT\ Ratio} - (R_C + jX_C) \frac{PT\ Ratio}{CT\ Rating} * \frac{I_{Out}}{PT\ Ratio} \tag{8-37}$$

You are looking for a match between the regulator relay voltage and the load voltage expressed on the controller voltage base. Therefore, you want to find the following.

$$V_{RR} = \frac{V_{Load}}{PT\ Ratio} \tag{8-38}$$

Substitution yields the following.

$$\frac{V_{Out} - I_{Out} Z_L}{PT\ Ratio} = \frac{V_{Out}}{PT\ Ratio} - (R_C + jX_C) \frac{PT\ Ratio}{CT\ Rating} * \frac{I_{Out}}{PT\ Ratio} \tag{8-39}$$

You can now solve for the line-drop compensator controller impedance that will provide a match between the regulator relay voltage and the load voltage.

$$R_c + jX_c = Z_L \frac{CT\ Rating}{PT\ Ratio} \tag{8-40}$$

As such, the voltage that the regulator controller sees remotely matches the load voltage expressed on the controller base. The LDC can be used as an effective tool to remotely control the load voltage.

The PT and CT do not have to be sized to regulator ratings. They are often sized to allow the regulator to see farther. Likewise, the LDC R and X values do not have to precisely match the line impedance. Often these values are set to averages of branching lines fed by the LDC.

Regulator example

Shown below is a simple 13.8kV feeder serving a variety of customers and a large commercial motor. The feeder contains a Wye-Gnd bank of three voltage regulators.

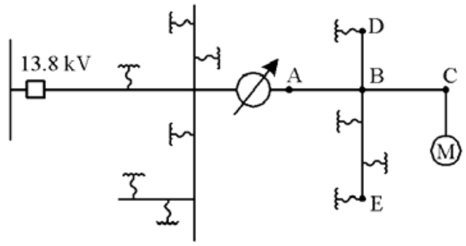


Fig. 8-13 Feeder with regulator

The regulator is described in the table below.

Regulator Data	
kV rating	7.62kV
Current rating	219A
Steps	32
Range	+/- 10%

The line impedances of the sections between the regulator and the motor are given below.

$$Z_{AB} = 0.21 + j0.63\Omega$$
$$Z_{BC} = 0.1 + j0.3\Omega \tag{8-41}$$

The kVA rating for each regulator can be found using the following.

$$7.62kV * 10\% * 219A = 167kVA \quad \text{each} \tag{8-42}$$

Regulator impedance values are neglected in this example. They should be specified on a 167kVA base when available. The regulator windings are rated at 167kVA. Note that the regulator bank could serve nearly 5MW of downstream load.

Potential transformers are chosen for each regulator to provide 120V to the regulator controller at nominal line voltage.

$$PT_{Ratio} = \frac{13.8 * 1000}{\sqrt{3} * 120} \approx 66.5 \tag{8-43}$$

A PT ratio of 66.5 corresponds to a 60:1 PT with its taps appropriately set. Ideally, a CT with primary rating of 219A would be used. You can assume that the closest available CT is a 300A one.

Suppose that there are no loads downstream of the regulator bank except for the motor. The R and X settings for the line-drop compensators can be found using the following.

$$R_c + jX_c = Z_L \frac{CT_{Prim}}{PT_{Ratio}} = (0.21 + j0.63\Omega + 0.1 + j0.3\Omega)\frac{300A}{66.5} = (1.4 + j4.2)Volts \tag{8-44}$$

You can set all of the voltage set points to 123V and the bandwidths to 2.0V. The motor load is modeled as a large balanced load. The following are the results of a load-flow run.

Load-Flow Results	
Voltage on source side of regulator	117.2V
Voltage on load side of regulator	124.5V
Regulator tap position	10
Current out of regulator	177.4A lagging by 35 deg.
Motor terminal voltage	122.5V

You can first check that the motor terminal voltage is within the bandwidth that you selected, as shown below.

$$|122.5V - 123V| \overset{?}{\leq} \frac{2.0V}{2} \tag{8-45}$$

This is true, so the regulators and their line-drop compensators operated correctly. You can verify regulator operation by investigating the tap positions first. Tap number 10 on a 32-step regulator would correspond to a voltage boost at the output given using the following equation.

$$117.2V * \left(1.0 + \frac{20\%}{100} * \frac{10}{32}\right) = 124.5V \tag{8-46}$$

You can see that the regulator's low-side voltage was correctly determined by the load-flow. Finally, you can look at the internal values of the line-drop compensators. The voltage and current on the secondary sides of the PT and CT are found as follows.

$$V_{PT} = \frac{124.5}{120} * \frac{13.8}{\sqrt{3}} * \frac{1}{66.5} = 124.3V \tag{8-47}$$

$$I_{CT} = \frac{177.4}{300} = 0.591\angle -35° A$$

The voltage at the regulator tap changer controlling relay is found by dropping the PT voltage through the R and X values you selected. For example:

$$V_{RR} = |124.3 - (1.4 + j4.2) * 0.591\angle -35° A| = 122.2V \tag{8-48}$$

The small difference between the voltage at the voltage regulator relay and the motor is due to rounding off the above calculations. This example demonstrates the correspondence between the voltage at the voltage regulator relay and the point in the feeder to be regulated.

If loads are now considered in the laterals B-D and B-E, the calculations for determining compensator R and X values to regulate the motor load can no longer be directly determined. Approximations and averages are used to determine settings that allow the regulator to give the best voltage profile to all downstream connections.

Values do not have to be identical for all regulators in a bank. Different R and X settings as well as CT values allow each regulator to look out different distances into the feeder.

Line-drop compensator values for Open-Delta and Delta banks

The LDC meters voltage across the line through its connected PT. Since the CT meters line current, a phase shift exists between the voltage and current in the LDC. This phase shift can be compensated for using modified R and X settings.

The following example is an Open-AC connected regulator bank.

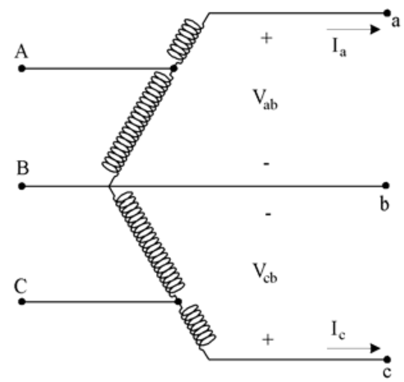


Fig. 8-14 *Open-AC connected regulator bank*

This regulator serves a load-connected line-line through a line having a conductor impedance of Z_L. From the output terminals of the regulator to the load you have the following.

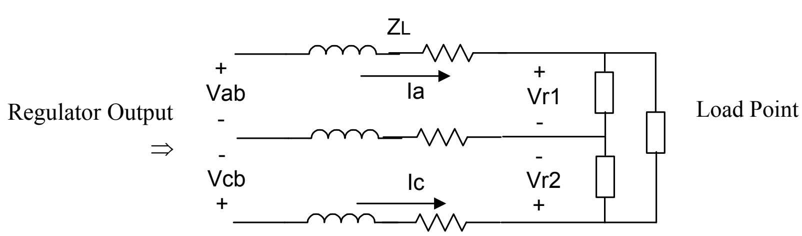


Fig. 8-15 *Circuit from regulator load end to load point*

Since this is a Delta-connected circuit, you have to consider the voltage drop through two-phase wires to get the load voltage. For example:

$$V_{ab} - I_a Z_L - V_{r1} + I_b Z_L = 0 \quad \textbf{(8-49)}$$

This equation can be arranged to isolate the load voltage as follows.

$$V_{r1} = V_{ab} - (I_a - I_b) Z_L \quad \textbf{(8-50)}$$

When calculating the LDC values, you should assume that the load currents are balanced. You can use the phase shift operator *a* to express the current on wire *b* in terms of the current on wire *a*. For example:

$$V_{r1} = V_{ab} - \left(1 - a^2\right) I_a Z_L \tag{8-51}$$

This can be simplified as follows.

$$V_{r1} = V_{ab} - \left(\sqrt{3}\angle 30°\right) I_a Z_L \tag{8-52}$$

The effective line impedance can be moved from the primary of the PT and CT to the secondary to get the following.

$$Z_c(leading) = \left(\sqrt{3}\angle 30°\right) Z_L \frac{CT_{Rating}}{PT_{Ratio}} \tag{8-53}$$

The setting for the other regulator can be handled in a similar fashion. The load voltage can be expressed as follows.

$$V_{r2} = V_{cb} - \left(I_c - I_b\right) Z_L \tag{8-54}$$

This time, the current on wire *b* leads, as shown below.

$$V_{r2} = V_{cb} - \left(1 - a\right) I_c Z_L \tag{8-55}$$

This simplifies as follows.

$$V_{r2} = V_{cb} - \left(\sqrt{3}\angle -30°\right) I_c Z_L \tag{8-56}$$

So, the other regulator's settings are as follows.

$$Z_c(lagging) = \left(\sqrt{3}\angle -30°\right) Z_L \frac{CT_{Rating}}{PT_{Ratio}} \tag{8-57}$$

These calculations can be repeated for Open-AB and Open-BC connections. Notice that the polarity of the PT and the connection of the PT and CT play a critical role in the modified R and X settings.

Reverse power flow modes

Several modes are available to govern the behavior of the regulator while experiencing reverse power flow. The modes are listed below and followed by a table that summarizes their behavior.

A	No Reverse Mode	Regulator responds to voltage at terminals with no regard for power flow direction.
B	Locked Forward	Regulator operates normally when power flow direction is forward. Any reverse power flow causes regulator to lock at last tap.
C	Reverse Idle	This mode is the same as B, except that reverse power flow has no effect when regulator current is below threshold value. This mode allows small (typically 2-5%) reverse flow current.
D	Neutral Idle	Forward or reverse power flow through the regulator with a current magnitude less than the threshold value locks the

		regulator. Forward current above the threshold allows regulator to operate normally. Reverse current above the threshold causes the regulator to park (move to neutral).
E	Bi-Directional	Forward or reverse power flow through the regulator with a current magnitude less than the threshold value locks the regulator. Otherwise, forward power flow puts the regulator into forward operation and reverse power flow puts the regulator into reverse operation.
F	Locked Reverse	Regulator operates in reverse when power flow direction is reverse. Any forward power flow causes regulator to lock at last tap.
G	Co-Generation	Forward or reverse power flow through the regulator with a current magnitude less than the threshold value locks the regulator. Otherwise, the regulator operates normally except that reverse LDC values are used with reverse power flow.

	Reverse Mode						
	A	**B**	**C**	**D**	**E**	**F**	**G**
	Forward Power Flow						
Threshold	No	No	Yes	Yes	Yes	No	Yes
LDC Values	Fwd	Fwd	Fwd	Fwd	Fwd	-	Fwd
V & BW Setting	Fwd	Fwd	Fwd	Fwd	Fwd	-	Fwd
Tap Operation	Fwd	Fwd	Fwd	Fwd	Fwd	Lock	Fwd
	Reverse Power Flow						
Threshold	No	No	No	Yes	Yes	No	Yes
LDC Values	Fwd	-	-	-	Rev	Rev	Rev
V & BW Setting	Fwd	-	-	-	Rev	Rev	Fwd
Tap Operation	Fwd	Lock	Lock	Neutral	Rev	Rev	Fwd

Potential and current transformer connections

The placement of the PTs and CTs is crucial for Open-Delta banks.

Open-Delta Metering						
Open Phase	**CT Metering Amps Out of Load End Node**		**PT Metering Across Load End Nodes**		**Leading Unit**	**Lagging Unit**
	CT #1	CT #2	PT #1	PT #2		
"CA"	a	c	ab	cb	AB	BC
"BC"	c	b	ca	ba	CA	AB

"AB"	b	a	bc	ac	BC	CA

The leading and lagging status of a regulator unit in a bank determines which type of shift should be used to modify the line impedance for LDC settings.

Voltages and power for regulator connections

Regulators can be connected in either grounded or ungrounded configurations. The voltage and power results from by-phase analysis for regulators depend on this connection. The following table summarizes this dependency.

	Wye-Gnd or Open-Wye	**Delta, Wye, or Open-Delta**
Voltage	Line-Neutral	Line-Line
Power	Individual Phase Value	Sum of all Phases

With respect to connection type, SynerGEE distinguishes between three-phase factory-connected units and single-phase regulators that are connected in banks. The connection type for three-phase units is specified in the regulator equipment table when a regulator is selected as the three-phase type. Single-phase regulators have their bank connection specified in the Regulator dialog box. Three-phase units may be connected in Wye-Gnd, Wye, and Delta configurations. Banks may be connected in those three configurations plus Open-Delta and Open-Wye configurations. The general connection scheme of autotransformer windings is the same for both three-phase units and single-phase banks. The models for each of these connections follow.

Model details

Wye-Gnd connected regulators

A bank of Wye-Gnd connected regulators has a common node that is grounded. The node may be grounded through an impedance on the actual implementation. SynerGEE does not currently support a grounding impedance for voltage regulators.

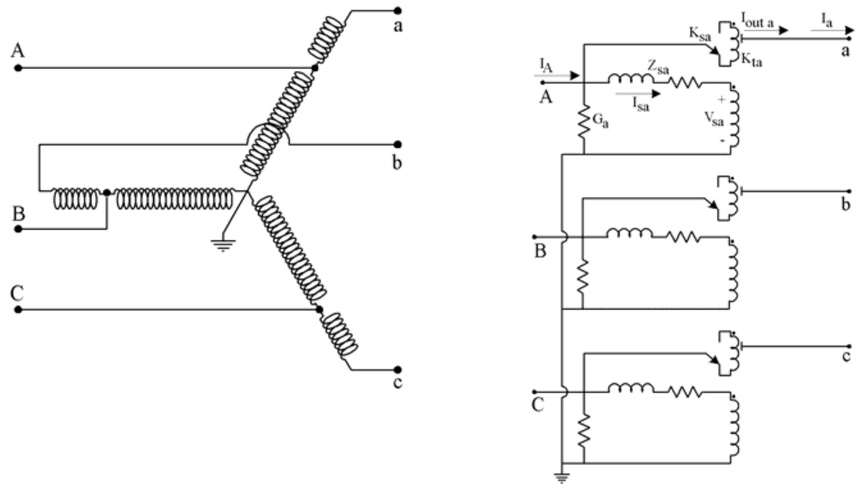


Fig. 8-16 *Wye-gnd bank of regulators*

The model for an Open-Wye connection is created by eliminating one of the regulators from the bank above. This can be achieved by specifying a Wye-Gnd connection for a regulator bank on a two-phase line or by deactivating one of the regulators.

The following equation represents the current relationship between the current into the bank and the current out of the bank.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} N_a+1 & 0 & 0 \\ 0 & N_b+1 & 0 \\ 0 & 0 & N_c+1 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} + \begin{Bmatrix} G_a V_{AN} \\ G_b V_{BN} \\ G_c V_{CN} \end{Bmatrix} \tag{8-58}$$

The following equation expresses the relationship between the voltage at the load end of the bank in terms of the voltage at the source end.

$$\begin{Bmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{Bmatrix} = \begin{bmatrix} N_a+1 & 0 & 0 \\ 0 & N_b+1 & 0 \\ 0 & 0 & N_c+1 \end{bmatrix} \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} - \begin{Bmatrix} N_a^2 I_a Z_{sa} \\ N_b^2 I_b Z_{sb} \\ N_c^2 I_c Z_{sc} \end{Bmatrix} \tag{8-59}$$

Wye-connected regulators (Wye ungrounded)

A bank of Wye-connected regulators has a common node like the Wye-Gnd bank. However, the node is not grounded. These types of banks are not good candidates for use in highly unbalanced systems because of floating problems. SynerGEE models regulator floating.

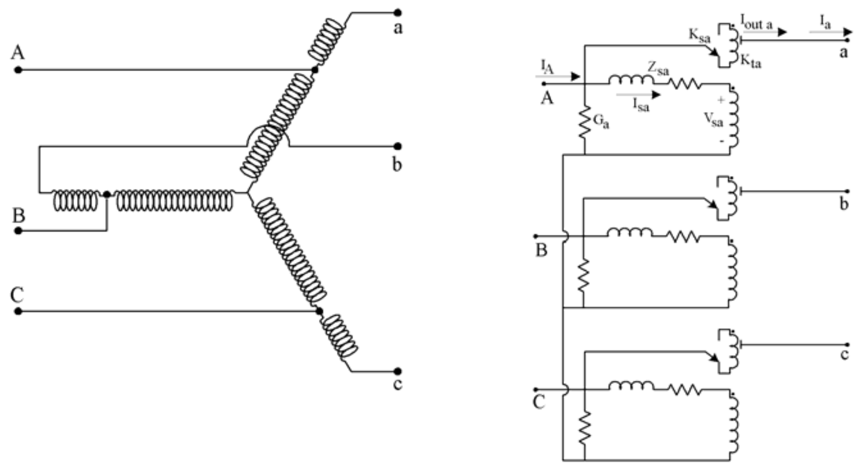


Fig. 8-17 Wye bank of regulators

Delta-connected regulators (closed Delta)

Voltage regulators can be connected in a Delta configuration to achieve a larger range of regulation than can be achieved with Open-Delta connections. For example, □10% regulators connected in a closed Delta have a bank regulation of about □15%. The figure that follows shows the leading Delta connection.

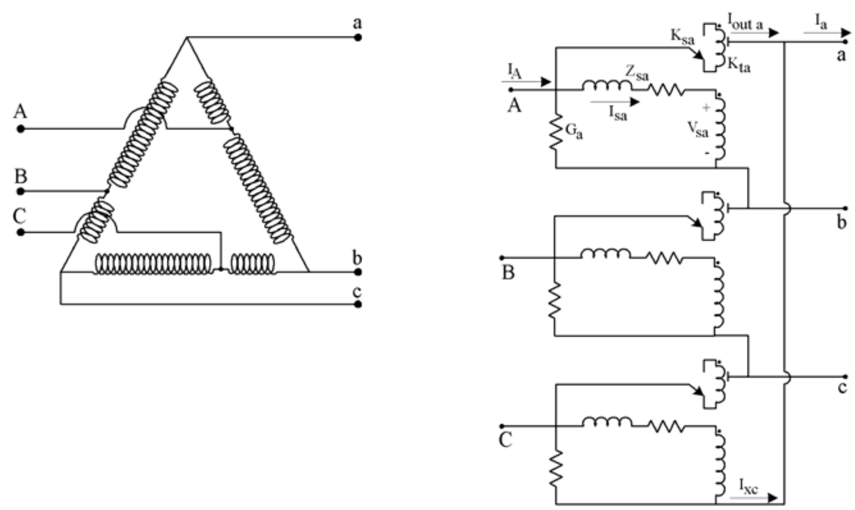


Fig. 8-18 Delta connected bank

This connection is referred to as leading since current out of the regulator leads the line-line voltage by 30 degrees when serving a load with a unity power factor.

Inward propagation of current

The first step in describing the behavior of this regulator connection is to determine the current into the device given the current out of it, and the voltages on the inward and outward side. Although line-line voltages are characteristic to the Delta connection, it is assumed that the voltage between terminals "A", "a", and some reference point have been tallied or determined.

The current out of each outward terminal of the regulator is the sum of two currents, as indicated below.

$$\begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} = \begin{Bmatrix} I_{out\,a} \\ I_{out\,b} \\ I_{out\,c} \end{Bmatrix} + \begin{Bmatrix} I_{xc} \\ I_{xa} \\ I_{xb} \end{Bmatrix} \tag{8-60}$$

The I_x currents are the sum of currents through the shunt winding and shunt resistance of each regulator.

$$\begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} = \begin{Bmatrix} I_{out\,a} \\ I_{out\,b} \\ I_{out\,c} \end{Bmatrix} + \begin{Bmatrix} N_c I_{out\,c} + G_c(V_{CN} - V_{aN}) \\ N_a I_{out\,a} + G_a(V_{AN} - V_{bN}) \\ N_b I_{out\,b} + G_b(V_{BN} - V_{cN}) \end{Bmatrix} \tag{8-61}$$

These equations can be placed into a more suitable matrix arrangement as follows.

$$\begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} = \begin{bmatrix} 1 & 0 & N_c \\ N_a & 1 & 0 \\ 0 & N_b & 1 \end{bmatrix} \begin{Bmatrix} I_{out\,a} \\ I_{out\,b} \\ I_{out\,c} \end{Bmatrix} + \begin{Bmatrix} G_c(V_{CN} - V_{aN}) \\ G_a(V_{AN} - V_{bN}) \\ G_b(V_{BN} - V_{cN}) \end{Bmatrix} \tag{8-62}$$

The current out of each regulator can then be expressed in terms of the current out of the Delta bank. For example:

$$\begin{Bmatrix} I_{out\,a} \\ I_{out\,b} \\ I_{out\,c} \end{Bmatrix} = \frac{1}{K} \begin{bmatrix} 1 & N_b N_c & -N_c \\ -N_a & 1 & N_a N_c \\ N_a N_b & -N_b & 1 \end{bmatrix} \begin{Bmatrix} I_a - G_c(V_{CN} - V_{aN}) \\ I_b - G_a(V_{AN} - V_{bN}) \\ I_c - G_b(V_{BN} - V_{cN}) \end{Bmatrix} \tag{8-63}$$

$$K = 1 + N_a N_b N_c$$

The current into each regulator is the sum of three currents as shown below.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{Bmatrix} G_a(V_{AN} - V_{bN}) \\ G_b(V_{BN} - V_{cN}) \\ G_c(V_{CN} - V_{aN}) \end{Bmatrix} + \begin{Bmatrix} I_{out\,a} \\ I_{out\,b} \\ I_{out\,c} \end{Bmatrix} + \begin{Bmatrix} N_a I_{out\,a} \\ N_b I_{out\,b} \\ N_c I_{out\,c} \end{Bmatrix} = \begin{Bmatrix} G_a(V_{AN} - V_{bN}) \\ G_b(V_{BN} - V_{cN}) \\ G_c(V_{CN} - V_{aN}) \end{Bmatrix} + \begin{Bmatrix} (N_a + 1) I_{out\,a} \\ (N_b + 1) I_{out\,b} \\ (N_c + 1) I_{out\,c} \end{Bmatrix} \tag{8-64}$$

The last two equations can now be combined as shown below.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \frac{1}{K} \begin{bmatrix} N_a + 1 & (N_a + 1) N_b N_c & -(N_a + 1) N_c \\ -(N_b + 1) N_a & N_b + 1 & (N_b + 1) N_a N_c \\ (N_c + 1) N_a N_b & -(N_c + 1) N_b & N_c + 1 \end{bmatrix} \begin{Bmatrix} I_a - G_c(V_{CN} - V_{aN}) \\ I_b - G_a(V_{AN} - V_{bN}) \\ I_c - G_b(V_{BN} - V_{cN}) \end{Bmatrix} + \begin{Bmatrix} G_a(V_{AN} - V_{bN}) \\ G_b(V_{BN} - V_{cN}) \\ G_c(V_{CN} - V_{aN}) \end{Bmatrix} \tag{8-65}$$

The following equations hold in the equation listed above.

$$I_A + I_B + I_C = I_a + I_b + I_c \tag{8-66}$$

So, if proper Delta currents ($I_a + I_b + I_c = \vec{0}$) are demanded out of the bank, proper Delta currents are required to enter the regulator bank.

Outward propagation of voltage

Voltage traces can be made around each of the regulators in the Delta connection as follows.

$$\begin{Bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{Bmatrix} = \begin{Bmatrix} I_{sa}Z_{sa} \\ I_{sb}Z_{sb} \\ I_{sc}Z_{sc} \end{Bmatrix} + \begin{Bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{Bmatrix} + \begin{Bmatrix} V_{sb}N_b \\ V_{sc}N_c \\ V_{sa}N_a \end{Bmatrix} \tag{8-67}$$

The equations can be solved for the voltage across the shunt winding for each transformer, as indicated by the following example.

$$\begin{Bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{Bmatrix} = \frac{1}{K} \begin{bmatrix} 1 & -N_b & N_bN_c \\ N_aN_c & 1 & -N_c \\ -N_a & N_aN_b & 1 \end{bmatrix} \begin{Bmatrix} V_{AB} - I_{sa}Z_{sa} \\ V_{BC} - I_{sb}Z_{sb} \\ V_{CA} - I_{sc}Z_{sc} \end{Bmatrix} \tag{8-68}$$

The outward voltage of the regulator bank is related to the inward voltage as follows.

$$\begin{Bmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{Bmatrix} = \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} + \begin{Bmatrix} V_{sa}N_a \\ V_{sb}N_b \\ V_{sc}N_c \end{Bmatrix} \tag{8-69}$$

The preceding two equations can be combined to result in the following.

$$\begin{Bmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{Bmatrix} = \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} + \frac{1}{K} \begin{bmatrix} N_a & -N_aN_b & N_aN_bN_c \\ N_aN_bN_c & N_b & -N_bN_c \\ -N_aN_c & N_aN_bN_c & N_c \end{bmatrix} \begin{Bmatrix} V_{AB} - N_aI_{out\,a}Z_{sa} \\ V_{BC} - N_bI_{out\,b}Z_{sb} \\ V_{CA} - N_cI_{out\,c}Z_{sc} \end{Bmatrix} \tag{8-70}$$

This equation implies that the current out of each regulator must be stored during inward traces.

Open-Delta CA connected regulators

Open-Delta connections supply rated regulator regulation across the bank. The connection is useful because two regulators can regulate a three-phase line. Shown below is an Open-Delta CA connection. The connection is so named because there is no regulator between wires A and C.

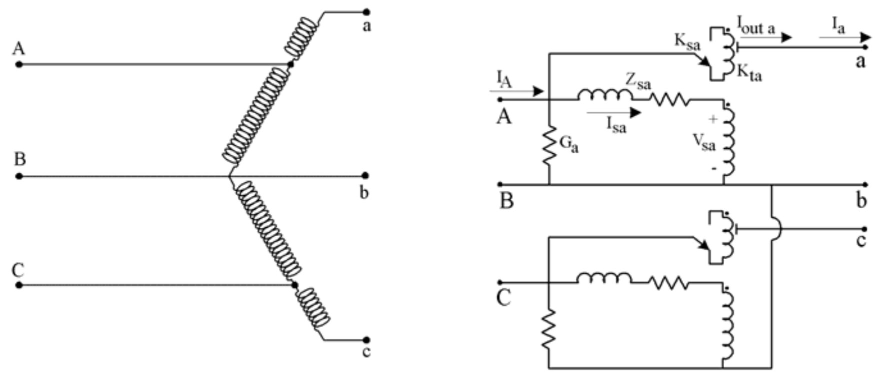


Fig. 8-19 Open-Delta CA Bank

Corresponding diagrams are used for Open-Delta AB and Open-Delta BC connections.

Current summations can be made at the insertion of each phase into the regulator bank to achieve the following relationship between the current into the regulator bank and the current out of the bank.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} N_{ab}+1 & 0 & 0 \\ -N_{ab} & 1 & -N_{bc} \\ 0 & 0 & N_{bc}+1 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} + \begin{Bmatrix} G_{ab}V_{AB} \\ G_{bc}V_{BC} - G_{ab}V_{AB} \\ -G_{bc}V_{BC} \end{Bmatrix} \tag{8-71}$$

The expression for the current into phase B is a complicated function of the current out of all three phases and the voltage across the two regulators.

Voltage can be propagated outward through a regulator, resulting in the equation that follows.

$$\begin{Bmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{Bmatrix} = \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} + \begin{bmatrix} N_{ab} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -N_{bc} & 0 \end{bmatrix} \begin{Bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{Bmatrix} - \begin{Bmatrix} N_{ab}^2 I_a Z_{sab} \\ 0 \\ N_{bc}^2 I_c Z_{sbc} \end{Bmatrix} \tag{8-72}$$

Open-Delta BC connection

The port equations for the Open-Delta regulator bank with phases B and C open are very similar to those described for the Open-Delta AC bank. For example:

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} 1 & -N_{ab} & -N_{ca} \\ 0 & N_{ab}+1 & 0 \\ 0 & 0 & N_{ca}+1 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} + \begin{Bmatrix} G_{ab}V_{AB} - G_{ca}V_{CA} \\ -G_{ab}V_{AB} \\ G_{ca}V_{CA} \end{Bmatrix} \tag{8-73}$$

$$\begin{Bmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{Bmatrix} = \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -N_{ab} & 0 & 0 \\ 0 & 0 & N_{ca} \end{bmatrix} \begin{Bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{Bmatrix} - \begin{Bmatrix} 0 \\ N_{ab}^2 I_b Z_{sab} \\ N_{ca}^2 I_c Z_{sca} \end{Bmatrix} \tag{8-74}$$

Open-Delta AB connection

The port equations for the Open-Delta regulator bank with phases B and C open are very similar to those described for the Open-Delta AC bank. For example:

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} N_{ca}+1 & 0 & 0 \\ 0 & N_{bc}+1 & 0 \\ -N_{ca} & -N_{bc} & 1 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} + \begin{Bmatrix} -G_{ca}V_{CA} \\ G_{bc}V_{BC} \\ G_{ca}V_{CA} - G_{bc}V_{BC} \end{Bmatrix} \tag{8-75}$$

$$\begin{Bmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{Bmatrix} = \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & -N_{ca} \\ 0 & N_{bc} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{Bmatrix} - \begin{Bmatrix} N_{ca}^2 I_a Z_{sca} \\ N_{bc}^2 I_b Z_{sbc} \\ 0 \end{Bmatrix} \tag{8-76}$$

Tables

The following tables are provided for your convenience.

Typical regulator voltage ratings

The following table lists typical ratings for regulators along with the system voltage for application. The table is derived from Table 6 in *ANSI/IEEE C57.15-1996*.

Nominal System Voltage (kV)	Single-Phage kV Rating	Three-Phase kV Rating	PT Ratio
2.4	2.5	2.5	20
2.4/4.16Y	2.5	_______	20
2.4/4.16Y	_______	4.33	34.6
4.8	5	5	40
7.2	7.62	_______	60
7.2	_______	8.66	60
4.8/8.32	5	_______	40
8.32	_______	8.66	69.3
12.47	13.8	13.8	104
7.2/12.47Y	7.62	_______	60
7.2/12.47Y	_______	13.8	104
7.62/13.2Y	7.62	_______	63.5, 66.3
7.62/13.2Y	_______	13.2	110

7.96/13.8Y	_______	13.8	115
7.96/13.8Y	7.96	_______	66.3
13.2	13.8	13.8	110
14.4	13.8	13.8	120
14.4/24.94Y	14.4	_______	120
19.92/34.5Y	19.92	_______	166
14.4/24.94Y	_______	24.94	208
19.92/34.5Y	_______	34.5	287.5
26.56/46Y	_______	46	373.3
39.84/69	_______	69	575

Typical PT ratios

Following is a table of typical values for the PT ratio entry in the regulator table. This table is taken from Table 7 of *C57.15-1986*.

Voltage Rating (kV)		PT Ratio(s)
Single-Phase	**Three-Phase**	
2.5	2.5	20, 20.8
_______	4.33	34.6, 36.1
5	5	40, 41.7
7.62	_______	60, 63.5
7.96	_______	66.3
_______	8.66	69.3, 72.2
_______	13.2	110, 104
13.8	13.8	115, 110
14.4	_______	120
19.92	_______	166
_______	24.94	208
_______	34.5	287.5
_______	46	383.3
_______	69	575

The PT ratio value can be calculated directly using the following equation.

$$PT = \frac{1000 * kV_{Rated}}{120} \tag{8-77}$$

Ratings for single-phase step-voltage regulators

This table is generated from Table 3 (Oil-Immersed) *ANSI/IEEE C57.15-1986*.

Nominal System kV	Rated kVA	Line Amps
2.4/4.16Y	50	200
	75	300
	100	400
	125	500
	167	668
	250	1000
	333	1332
4.8/8.32Y	50	100
	75	150
	100	200
	125	250
	167	334
	250	500
	333	668
7.62/13.2Y	38.1	50
	57.2	75
	76.2	100
	114.3	150
	167	219
	250	328
	333	438
	416	546
	509	668
	667	875
	833	1093
13.8	69	50
	138	100
	207	150
	276	200
	414	300
	552	400
14.4/24.94Y	72	50
	144	100
	216	150
	288	200
	333	231
	432	300
	576	400
	667	463
	833	578
19.92/34.5Y	100	50
	200	100
	333	167
	400	201
	667	334
	833	418

Ratings for three-phase step-voltage regulators

This table is generated from Table 4 (Oil-Immersed) *ANSI/IEEE C57.15-1986*.

Nominal System kV	Self Cooled Type		Self/Forced Cooled Type	
	Rated kVA	Line Amps	Rated kVA	Line Amps
2.4	500	1155	625	1443
	750	1732	937	2165
	1000	2309	1250	2887
2.4/4.16Y	500	667	625	833
	750	1000	937	1250
	1000	1334	1250	1667
4.8	500	577	625	721
	750	866	937	1082
	1000	1155	1250	1443
7.62/13.2Y	500	219	625	274
	750	328	937	410
	1000	437	1250	546
	1500	656	2000	874
	2000	874	2667	1166
7.97/13.8Y	500	209	625	261
	750	313	937	391
	1000	418	1250	523
	1500	628	2000	837
	2000	837	2667	1116
	2500	1046	3333	1394
14.4/24.94Y	500	125.5	625	156.8
	750	188.3	937	235.4
	1000	251	1250	314
	1500	377	2000	502
	2000	502	2667	669
	2500	628	3333	837
19.92/34.5Y	500	83.7	625	104.6
	750	125.5	937	156.8
	1000	167	1250	209
	1500	251	2000	335
	2000	335	2667	447
	2500	418	3333	557
26.56/46Y	500	62.8	625	78.5
	750	94.1	937	117.6
	1000	126	1250	157
	1500	188	2000	251
	2000	251	2667	335
	2500	314	3333	419
39.84/69Y	500	41.8	625	52.5
	750	62.8	937	78.5
	1000	83.7	1250	105
	1500	126	2000	167
	2000	167	2667	223
	2500	209	3333	278

9
Transformers

Introduction

Primary transformers are modeled within SynerGEE to be an accurate representation of real transformer banks and units operating in a three-phase unbalanced distribution system.

The transformer model

Transformers can be modeled in detail within an unbalanced load-flow package by considering the winding-to-winding relationships between those transformer windings on the primary and secondary sides of the bank or unit. The method used in this modeling schema breaks up the transformer bank into three parts as shown below.

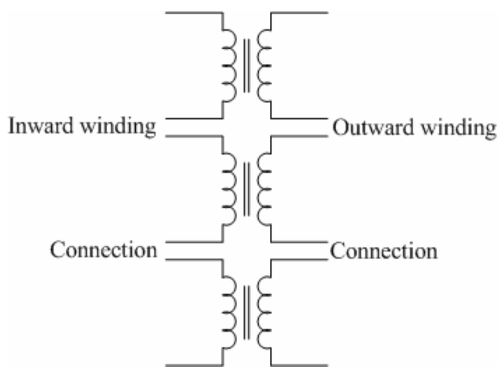


Fig. 9-1 *Transformer bank model*

The center portion of the model consists of three ideal transformer models. The relationship between the voltage and current across these windings is academic, as indicated below.

$$I_{In,\,j} = \frac{I_{Out,\,j}}{a_{ww}} \qquad V_{Out,\,j} = \frac{V_{In,\,j}}{a_{ww}} \tag{9-1}$$

$I_{In}, I_{Out}, V_{Out}, V_{In}$ are inward and outward currents and voltages.

a_{ww} is winding-winding turns ratio of transformers.

The point of using this type of modeling is to modularize the very complicated behavior of transformers associated with the various transformer connections. The effects of transformer impedance, grounding, and no-load losses upon the unbalanced voltages and currents of the bank are modeled. These effects, as well as the essential effects of the transformer connection are included in the Inward Winding Connection and Outward Winding Connection portion of the model. The method for handling these transformer connections is described in general below and then in more detail throughout the remainder of this chapter.

Distributing transformer impedance between windings

Typically, transformers are modeled with their series impedance lumped at either end. In order to properly model transformer behavior in an unbalanced system, it is important to have the series impedance modeled in both windings. This section contains a method for separating the transformer impedance based on the assumption that the winding impedance varies as the number of wire turns. This rather simple assumption neglects the fact that the primary and secondary transformers are often wound with different materials and different wire sizes. It does provide realistic voltage drops in unbalanced situations and a model that is consistent with the sequence domain models used for fault analysis.

If you are given the following:

$$\begin{aligned} Z_T &= \text{Balanced } 3\phi \text{ Impedance Seen Looking Into Primary Winding} \\ a &= \text{Turns ratio (kVIn / kVOut)} \end{aligned} \tag{9-2}$$

and you wish to find Z_P, Z_S in the figure below:

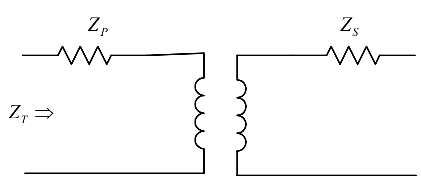


Fig. 9-2 *Transformer having primary and secondary impedances*

By assuming that the distribution of impedance between the primary and secondary varies with the number of turns, you conclude the following.

$$Z_S = aZ_P \tag{9-3}$$

Now, you can reflect the secondary impedance to the primary side as follows.

$$Z_S' = a^2 Z_S = a^3 Z_P \tag{9-4}$$

On the primary side, you have the following.

$$Z_T = \left(a^3 + 1\right) Z_P \tag{9-5}$$

You can now find Z_P and solve for Z_S as follows.

$$Z_P = \left(\frac{1}{a^3+1}\right)Z_T \qquad Z_S = \left(\frac{a}{a^3+1}\right)Z_T \tag{9-6}$$

These equations balance, in that $Z_T = Z_P + a^2 Z_S$.

1Ph / 3Ph transformer impedances

Transformer modeling within SynerGEE differs from traditional analysis packages based on single-line equivalents or sequence domain analysis. Each transformer in a bank of single-phase transformers is modeled. To illustrate the concept, you can compare a three-phase Wye-Gnd/Wye-Gnd transformer to a compatible bank made up of single-phase transformers.

Wye-Gnd/Wye-Gnd case

The three-phase transformer has the following ratings.

$$\begin{aligned} \text{Rated kVA} &= kVA_{3\phi} \\ \text{Rated Primary L - L Voltage} &= kV_{3\phi} \\ \text{Rated Percent Impedance} &= Z_{3\phi} \end{aligned} \tag{9-7}$$

In a single-line equivalent representation with an impedance behind an ideal transformer, the model appears as follows.

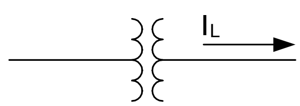


Fig. 9-3 *Three-phase transformer*

Impedance of three-phase transformer

The % voltage drop of the transformer under rated conditions is:

$$\%Drop = \frac{\sqrt{3}\, I_{Rated}\, Z_{Ohm}}{10\, kV_{3\phi}} \tag{9-8}$$

The rated current is:

$$I_{Rated} = \frac{kVA_{3\phi}}{\sqrt{3}\, kV_{3\phi}} \tag{9-9}$$

The impedance is converted from % to Ohms as follows.

$$Z_{Ohm} = \frac{Z_{3\phi}}{100} * \frac{1000kV_{3\phi}^2}{kVA_{3\phi}} = \frac{10kV_{3\phi}^2}{kVA_{3\phi}} Z_{3\phi} \tag{9-10}$$

Substitution yields the following.

$$\%Drop = \frac{\sqrt{3}\frac{kVA_{3\phi}}{\sqrt{3}\,kV_{3\phi}} * \frac{10kV_{3\phi}^2}{kVA_{3\phi}} Z_{3\phi}}{10kV_{3\phi}} = Z_{3\phi} \tag{9-11}$$

Under rated loading, therefore, the transformer's percentage voltage drop will numerically match its percent rated impedance.

Impedance of single-phase bank

Consider a single-phase transformer with compatible transformers having the compatible ratings and the same rated impedance. For example:

$$\text{Rated kVA} = kVA_{1\phi} = \frac{kVA_{3\phi}}{3} \tag{9-12}$$

$$\text{Rated Primary L - L Voltage} = kV_{1\phi} = \frac{kV_{3\phi}}{\sqrt{3}}$$

$$\text{Rated Percent Impedance} = Z_{3\phi}$$

When connected into a Wye-Gnd/Wye-Gnd bank, these transformers work for a model identical to the three-phase transformer.

The single-phase transformer impedance is:

$$Z_{Ohm} = \frac{Z_{3\phi}}{100} * \frac{1000 * \text{Rated kV}^2}{\text{Rated kVA}} = \frac{10kV_{3\phi}^2}{kVA_{3\phi}} Z_{3\phi} \tag{9-13}$$

This matches the ohmic impedance of the three-phase transformer.

Now you can begin finding the voltage drop. The rated current of the single-phase transformer is as follows.

$$I_{Rated} = \frac{kVA_{1\phi}}{kV_{1\phi}} = \frac{kVA_{3\phi}}{\sqrt{3}kV_{3\phi}} \tag{9-14}$$

Thus, the voltage drop is as follows.

$$\%Drop = \frac{I_{Rated}\ Z_{Ohm}}{10\,kV_{1\phi}} = \frac{\sqrt{3}\,I_{Rated}\ Z_{Ohm}}{10\,kV_{3\phi}} = Z_{3\phi} \tag{9-15}$$

The derivations check out. The bank of single-phase transformers has the same characteristics as the three-phase transformer. Also, the percentage drop of the single-phase transformer numerically matches the single-phase percent impedance under the rated load of the single-phase unit.

Delta/Wye-Gnd case

This case requires a different approach, starting with the single-phase transformer. For example:

$$\begin{aligned}\text{Rated kVA} &= kVA_{1\phi} \\ \text{Rated Primary Voltage} &= kV_{1\phi} \\ \text{Rated Percent Impedance} &= Z_{1\phi}\end{aligned} \tag{9-16}$$

You would like to compare a three-phase Delta/Wye-Gnd transformer with a Delta/Wye-Gnd bank made up of these single-phase transformers. For the three-phase transformer:

$$\begin{aligned}\text{Rated kVA} &= kVA_{3\phi} = 3kVA_{1\phi} \\ \text{Rated Primary Voltage} &= kV_{3\phi} = kV_{1\phi} \\ \text{Rated Percent Impedance} &= Z_{3\phi}\end{aligned} \tag{9-17}$$

Determine the relationship between $Z_{3\phi}$ and $Z_{1\phi}$.

The voltage drop under rated load is the Delta winding current (under rated transformer load) times the impedance within the winding. Since you are forming a bank, the winding impedance is the single-phase transformer impedance.

$$\%Drop = \frac{\frac{I_{Rated}}{\sqrt{3}} * Z_{Delta}}{1000 * kV_{3\phi}} * 100\% \tag{9-18}$$

The rated current of the three-phase transformer is as follows.

$$I_{Rated} = \frac{kVA_{3\phi}}{\sqrt{3}kV_{3\phi}} \tag{9-19}$$

The impedance of the single-phase transformer is as follows.

$$Z_{Delta} = \frac{10kV_{1\phi}^2}{kVA_{1\phi}} Z_{1\phi} \tag{9-20}$$

Substitution yields the following.

$$\%Drop = \frac{\frac{kVA_{3\phi}}{3kV_{3\phi}} * \frac{3 * 10kV_{3\phi}^2}{kVA_{3\phi}} Z_{1\phi}}{1000 * kV_{3\phi}} * 100 = Z_{1\phi} \tag{9-21}$$

However, under rated load the numerical values of drop and transformer impedance must match. For example:

$$\%Drop = Z_{3\phi} \tag{9-22}$$

These lead to the following.

$$Z_{3\phi} = Z_{1\phi} \tag{9-23}$$

Therefore, a three-phase Delta/Wye-Gnd transformer sized compatibly with a single-phase transformer forming a Delta/Wye-Gnd bank should have the same voltage drop if it has the same percent impedance.

Inrush and damage curves

Inrush curves are associated with the source side of a transformer. They may be shifted through the transformer to the load side.

The following table lists points for inrush curves.

I_{RS} = Rated Source-Side Amps

Time (s)	Amps	
	Type I	Type II, III, IV
1800	2 * I_{RS}	2 * I_{RS}
300	2 * I_{RS}	2 * I_{RS}
100	2 * I_{RS}	2 * I_{RS}
30	2 * I_{RS}	2 * I_{RS}
10	3 * I_{RS}	3 * I_{RS}
1	6 * I_{RS}	6 * I_{RS}
.1	8 * I_{RS}	12 * I_{RS}
.01	15 * I_{RS}	25 * I_{RS}

Systems with resistive load heating may need to consider using 2 * I_{fl} for 30 minutes and 3 * I_{fl} for 30 seconds. SynerGEE does not currently allow user customization of inrush curves.

Damage Curves

SynerGEE considers infrequent fault damage curves and frequent fault damage curves. The damage curves are associated with the source-side of the transformer and may be shifted to the load side.

$$\begin{aligned} I_{RL} &= \text{ Rated Load-Side Amps} \\ I_{MF} &= \text{ Per-Unit Maximum Fault Amps} \\ K &= 2I_{MF}^2 \\ S &= 0.7 \text{ for Category II} \\ S &= 0.5 \text{ for Category III and IV} \end{aligned} \quad \textbf{(9-24)}$$

Infrequent Fault Damage Curve			Frequent Fault Damage Curve*	
Time (s)	**Category I**	**Category >I**	**Time (Sec)**	**Category >I**
1500	2.0 * I_{RL}	2.0 * I_{RL}	1500	2.0 * I_{RL}
1000	2.3 * I_{RL}	2.3 * I_{RL}	1000	2.3 * I_{RL}
300	3.0 * I_{RL}	3.0 * I_{RL}	300	3.0 * I_{RL}
100	4.0 * I_{RL}	4.0 * I_{RL}	100	4.0 * I_{RL}
50	5.0 * I_{RL}	5.0 * I_{RL}	50	5.0 * I_{RL}
20	7.91 * I_{RL}	7.91 * I_{RL}	$\frac{1250}{\left(S * I_{MF}\right)^2}$	$S * I_{MF} * I_{RL}$
10	11.2 * I_{RL}	11.2 * I_{RL}	$\frac{K}{\left(S * I_{MF}\right)^2} = \frac{2}{S^2}$	$S * I_{MF} * I_{RL}$
4	17.7 * I_{RL}	17.7 * I_{RL}	$\frac{K}{\left(\left(S + 0.2\right) * I_{MF}\right)^2}$	$\left(S + 0.2\right) * I_{MF} * I_{RL}$
2	25.0 * I_{RL}	25.0 * I_{RL}	$\frac{K}{I_{MF}^2} = 2.0$	$I_{MF} * I_{RL}$
.5	50.0 * I_{RL}	-	-	-

* Class I transformers do not have frequent fault damage curves

Connection models

SynerGEE models transformers and their connections in great detail. Three-phase transformer units and banks of single-phase transformers are constructed from connections of the single winding transformer model shown below.

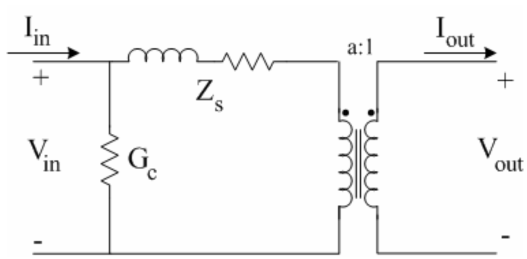


Fig. 9-4 Single winding transformer

The model has a series or winding impedance and a conductance representing the core losses. In SynerGEE, the transformer impedance values are actually spread across on the source and load side windings. The source side can be connected into one of the four general schemes shown below.

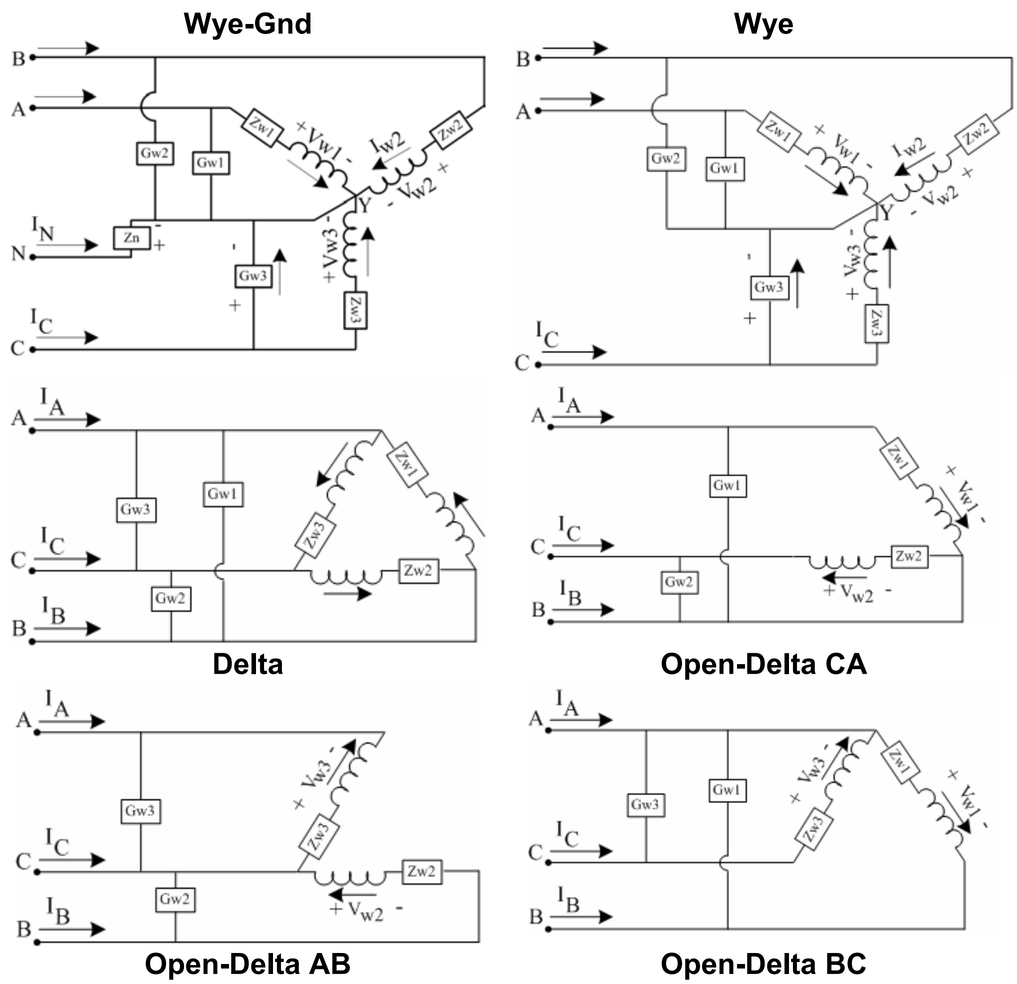


Fig. 9-5 Primary connections

There are a number of other connections that are related to these. Open-Delta AB and CA connections are available. Open-Wye connections can also be formed by eliminating windings from the Wye-Gnd connection.

These same connections are available for the load side of transformer banks.

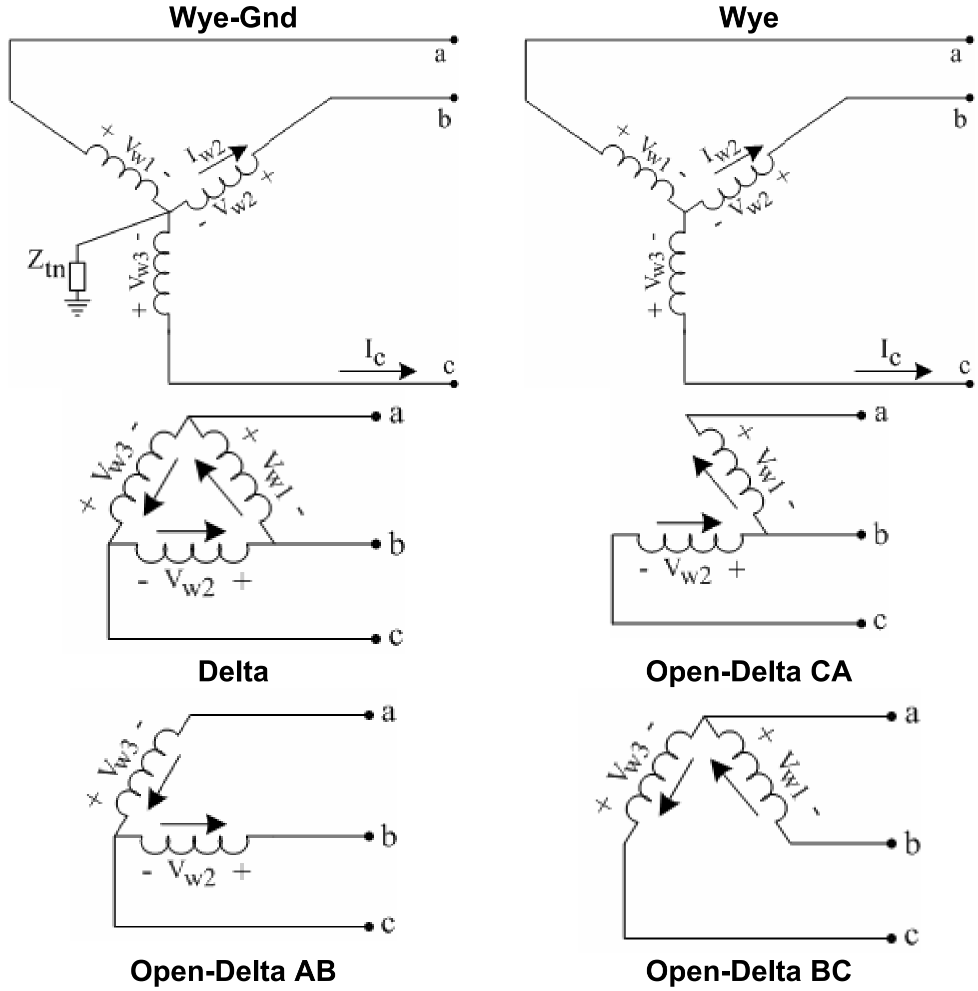


Fig. 9-6 Secondary connections

The source and load side connections are used together to form the high- and low-side connections of the transformer bank or three-phase unit. For example, choosing a Wye-Gnd connection for the source side and a Delta connection for the load side of a transformer bank or unit would result in the following model.

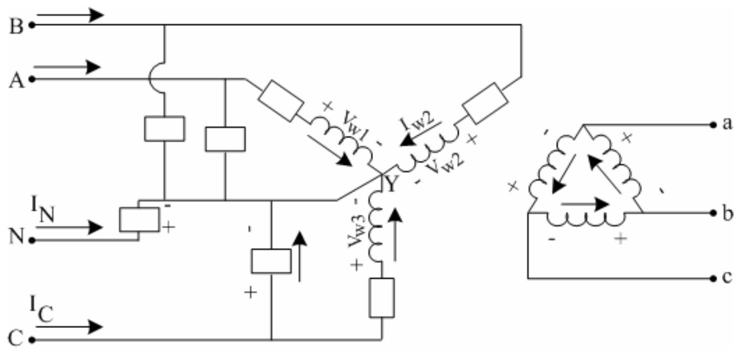


Fig. 9-7 *Model resulting from wye-gnd/delta connection*

Currents and voltages within the windings and at the transformer terminals are consistent with the transformer model presented above and Kirchhoff's laws. These banks can be built using the equipment tables and the section records.

Valid Connection Combinations

Primary	Secondary
Ygnd	Ygnd, Delta, Wye
Delta	Ygnd, Delta, Wye
Wye	Ygnd, Delta, Wye
Open Delta AB	Open Delta AB, Open Y A
Open Delta BC	Open Delta BC, Open Y B
Open Delta CA	Open Delta CA, Open Y C
Open Y A	Open Y A, Open Delta AB
Open Y B	Open Y B, Open Delta BC
Open Y C	Open Y C, Open Delta CA

Method for handling connections

Transformer banks must behave correctly during the inward and outward propagations used to solve the load-flow problem. These propagations are discussed in detail in *Chapter 2 - Lines and Cables*. Both the inward and outward propagation are three-step processes within the transformer model.

During the outward propagation, voltage information is transmitted from the source of the feeder out to the ends. During this process, the transformer receives information about its source voltage. Calculations are then performed which take into account the primary connection type, no-load losses, and series impedance. These calculations produce the voltage across the inward side of the ideal transformers depicted in the figure above. These voltages are transformed across the ideal transformer to the outward windings. Voltage drops through the outward windings are taken into account to get the terminal voltages. Connections and grounding information are accounted for in the outward propagation.

This procedure is performed in reverse and with values of current during the inward propagation. It is very important that the calculations performed during the inward and outward trace are consistent. The next section presents the derivations to be used during these propagations. They are broken up into calculations associated with the inward winding and then the outward windings. Voltage calculations are presented first, followed by current calculations for each connection type.

Wye-Gnd connections

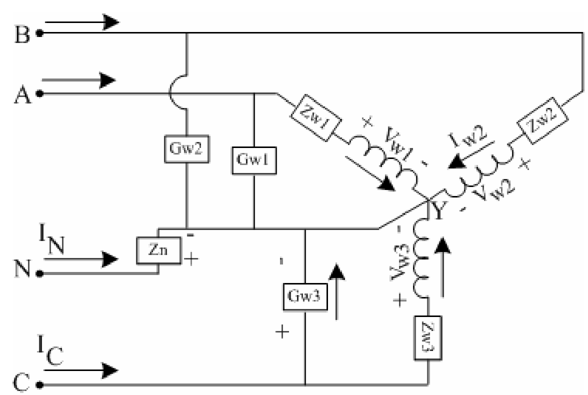


Fig. 9-8 Wye-gnd primary connection

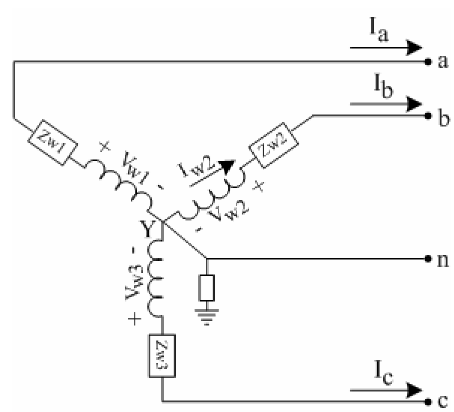


Fig. 9-9 *Wye-grnd secondary connection*

It is important to establish the voltage drop through the neutral (or grounding) impedance. Notice that the impedance resides between the neutral or reference of line voltages and the connection point of the transformer windings (the Y point).

Wye-Gnd connected inward winding

The discussion of Wye-Gnd connected inward winding discusses both voltage and current trace.

Voltage trace

A vertical line can slice the three conductors on the far left of the diagram so that a current summation yields the following.

$$I_N = -(I_A + I_B + I_C) \tag{9-25}$$

The relationship between the voltages referenced to the neutral and those referenced to the Y point can now be expressed as follows.

$$V_{AY} = V_{AN} - V_{ZN} \quad V_{BY} = V_{BN} - V_{ZN} \quad V_{CY} = V_{CN} - V_{ZN} \tag{9-26}$$

You also know the relationship between the winding voltages and the line voltages with respect to the Y point. This relationship is shown below.

$$V_{W1} = V_{AY} - I_{W1}Z_{W1} \qquad V_{W2} = V_{BY} - I_{W2}Z_{W2} \qquad V_{W3} = V_{CY} - I_{W3}Z_{W3} \tag{9-27}$$

The last two sets of equations can now be combined to get a representation of winding voltage with respect to line voltage. For example:

$$\begin{Bmatrix} V_{W1} \\ V_{W2} \\ V_{W3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{Bmatrix} - \begin{Bmatrix} I_{W1}Z_{W1} + I_N Z_N \\ I_{W2}Z_{W2} + I_N Z_N \\ I_{W3}Z_{W3} + I_N Z_N \end{Bmatrix} \tag{9-28}$$

Current trace

The following relationships are evident from the previous figure.

$$\begin{aligned} I_A &= I_{W1} + V_{AY}G_{W1} \\ I_B &= I_{W2} + V_{BY}G_{W2} \\ I_C &= I_{W3} + V_{CY}G_{W3} \end{aligned} \tag{9-29}$$

You can use these equations to get the following:

$$\begin{aligned} I_A &= I_{W1} + (V_{AN} + (I_A + I_B + I_C)Z_N)G_{W1} \\ I_B &= I_{W2} + (V_{BN} + (I_A + I_B + I_C)Z_N)G_{W2} \\ I_C &= I_{W3} + (V_{CN} + (I_A + I_B + I_C)Z_N)G_{W3} \end{aligned} \tag{9-30}$$

These equations can be arranged into matrix form. For example:

$$\begin{Bmatrix} I_{W1} \\ I_{W2} \\ I_{W3} \end{Bmatrix} = \begin{bmatrix} 1-Z_N G_{W1} & -Z_N G_{W1} & -Z_N G_{W1} \\ -Z_N G_{W2} & 1-Z_N G_{W2} & -Z_N G_{W2} \\ -Z_N G_{W3} & -Z_N G_{W3} & 1-Z_N G_{W3} \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} - \begin{Bmatrix} V_{AN} G_{W1} \\ V_{BN} G_{W2} \\ V_{CN} G_{W3} \end{Bmatrix} \tag{9-31}$$

This system can be inverted to get the results shown below.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \frac{1}{K} \begin{bmatrix} 1-Z_N(G_{W2}+G_{W3}) & Z_N G_{W1} & Z_N G_{W1} \\ Z_N G_{W2} & 1-Z_N(G_{W1}+G_{W3}) & Z_N G_{W2} \\ Z_N G_{W3} & Z_N G_{W3} & 1-Z_N(G_{W1}+G_{W2}) \end{bmatrix} \begin{Bmatrix} I_{W1}+V_{AN} G_{W1} \\ I_{W2}+V_{BN} G_{W2} \\ I_{W3}+V_{CN} G_{W3} \end{Bmatrix} \tag{9-32}$$

$$K = 1 - Z_N(G_{W1}+G_{W2}+G_{W3})$$

Wye-Gnd connected outward winding

The discussion of Wye-Gnd connected outward winding provides examples of voltage and current trace.

Voltage trace

$$\begin{Bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \end{Bmatrix} - \begin{bmatrix} Z_{w1} & 0 & 0 \\ 0 & Z_{w2} & 0 \\ 0 & 0 & Z_{w3} \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} + (I_a + I_b + I_c) Z_{gn} \tag{9-33}$$

Current trace

$$\begin{Bmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} \tag{9-34}$$

Delta connections

Shown below are Delta-connected transformer banks.

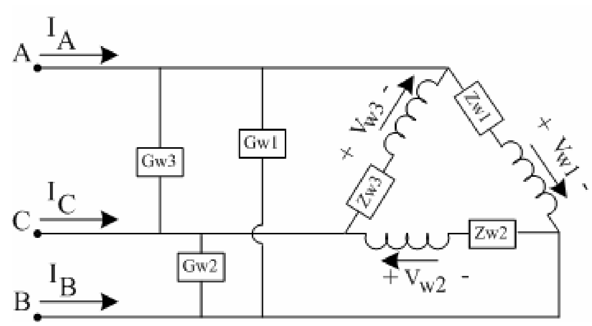


Fig. 9-10 Delta inward winding

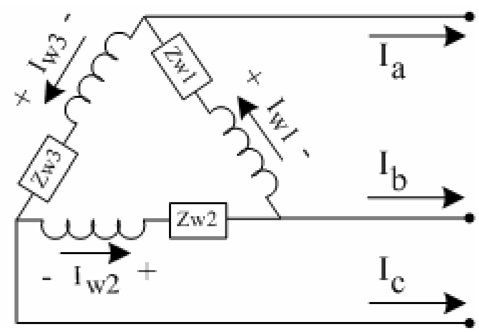


Fig. 9-11 Delta outward winding

Delta-connected inward winding

This discussion of Delta-connected inward winding includes current and voltage trace.

Current trace

The line currents can be found from the winding currents with the following expression.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ I_{W3} \end{Bmatrix} + \begin{Bmatrix} V_{AB}G_1 - V_{CA}G_3 \\ V_{BC}G_2 - V_{AB}G_1 \\ V_{CA}G_3 - V_{BC}G_2 \end{Bmatrix} \tag{9-35}$$

Voltage trace

The expression to find the voltage across the windings is equally straightforward.

$$\begin{Bmatrix} V_{W1} \\ V_{W2} \\ V_{W3} \end{Bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} - \begin{Bmatrix} I_{W1}Z_{W1} \\ I_{W2}Z_{W2} \\ I_{W3}Z_{W3} \end{Bmatrix} \tag{9-36}$$

Note that this expression implies that the current through the Delta windings needs to be maintained throughout the inward and outward propagations of the load-flow.

Delta-connected outward winding

The discussion of Delta-connected outward winding also includes current and voltage trace.

Current trace

Shown above is the circuit diagram for the Delta connection on the secondary portion of a transformer bank. The currents leaving the bank are known. For the propagation of currents from the secondary to the primary side of the transformer to occur, the current flowing through each of the secondary windings must be known. Although impedances are always reflected to the primary windings of transformers, winding impedances have been included in this diagram to show how the division of current is obtained.

Kirchhoff's current law (KCL) can be applied in the above figure. For example:

$$\begin{aligned} I_a &= I_{w1} - I_{w3} \\ I_b &= I_{w2} - I_{w1} \\ I_c &= I_{w3} - I_{w2} \end{aligned} \tag{9-37}$$

This is an under-determined system of equations. The three secondary winding currents are unknown and there are three equations. Any two equations, however, can be combined to form the third equation. Therefore, there are not enough equations to describe the relationship between the winding currents and the line currents.

You can cut the lines out of the transformer and use KCL to get the following results.

$$I_a + I_b + I_c = 0 \tag{9-38}$$

This equation is also implicit in the previous equation.

You can paraphrase the first law of thermodynamics by stating that the current will flow within the Delta to minimize losses within the windings. The loss in the secondary is represented below.

$$loss = I_{w1}^2 Z_{w1} + I_{w2}^2 Z_{w2} + I_{w3}^2 Z_{w3} \tag{9-39}$$

You can represent the losses with the current through a single winding with Equation this equation as follows.

$$loss = I_{w1}^2 Z_{w1} + (I_b + I_{w1})^2 Z_{w2} + (I_{w1} - I_a)^2 Z_{w3} \tag{9-40}$$

To minimize the losses due to current circulating within the Delta, the above equation is differentiated and set to zero as follows.

$$\frac{dloss}{dI_{w1}} = 2I_{w1}Z_{w1} + 2(I_b + I_{w1})Z_{w2} + 2(I_{w1} - I_a)Z_{w3} = 0 \tag{9-41}$$

The current in winding w1 can now be found using the following equation.

$$I_{w1} = \frac{I_a Z_{w3} - I_b Z_{w2}}{Z_{w1} + Z_{w2} + Z_{w3}} \tag{9-42}$$

The loss equation can now be used to determine the currents in the remaining windings of the secondary.

If the impedances for all windings of the transformer are identical or if the impedances approach a left limit, then reduce as follows:

$$I_{w1} = \frac{I_a - I_b}{3} \tag{9-43}$$

This equation can be used to determine the current flows through all transformer windings. For example:

$$\begin{Bmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \end{Bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} \tag{9-44}$$

Voltage trace

An impedance voltage drop should not be considered on the outward winding calculations since the drop is accounted for on the inward side. To begin to find the line voltages, you need to establish a relationship between the winding voltages and the outward terminal voltages to some arbitrary reference point. There are three winding voltages and three terminal voltages. However, the winding voltages are related to the differences in the line voltages. This means that there is insufficient information to determine terminal voltage from the winding voltages. However, as stated above, the terminal voltages are maintained with respect to an arbitrary reference voltage. Since this connection is in the Delta family, your only requirement is to maintain the proper relationship of voltage between terminals. So, you will select the terminal voltage for terminal *a* and determine the other two terminal voltages.

To make a smart selection of terminal *a* voltage, assume for a moment that you have balanced voltages at the terminals. This would be represented as follows.

$$V_{ar} = V\angle\alpha \quad V_{br} = V\angle\alpha - 120 \quad V_{cr} = V\angle\alpha + 120 \tag{9-45}$$

The voltage across winding one would then be as follows.

$$V_{w1} = V_{ar} - V_{br} = V\angle\alpha - V\angle(\alpha - 120) = V\angle\alpha\left(\sqrt{3}\angle 30\right) \tag{9-46}$$

Now you can solve for the voltage at terminal *a* during this special balanced case. For example:

$$V_{ar} = \frac{V_{w1}\angle -30}{\sqrt{3}} \tag{9-47}$$

The voltage at terminal *b* and *c* are related to that of *a* as shown below.

$$\begin{aligned} V_{br} &= V_{ar} - V_{ab} = \frac{V_{w1}\angle -150}{\sqrt{3}} \\ V_{cr} &= V_{ca} - V_{ar} = V_{w3} + \frac{V_{w1}\angle -30}{\sqrt{3}} \end{aligned} \tag{9-48}$$

So, in matrix form, you have the following relationship.

(9-49)

$$\begin{Bmatrix} V_{ar} \\ V_{br} \\ V_{cr} \end{Bmatrix} = \begin{bmatrix} \frac{1\angle -30}{\sqrt{3}} & 0 & 0 \\ \frac{1\angle -150}{\sqrt{3}} & 0 & 0 \\ \frac{1\angle -30}{\sqrt{3}} & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \end{Bmatrix}$$

Wye connections

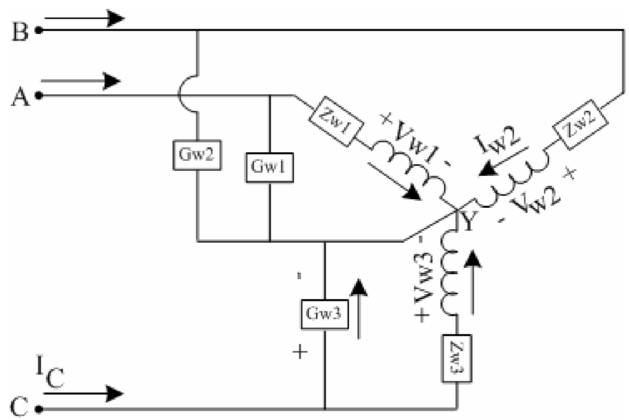


Fig. 9-12 Wye connected inward winding

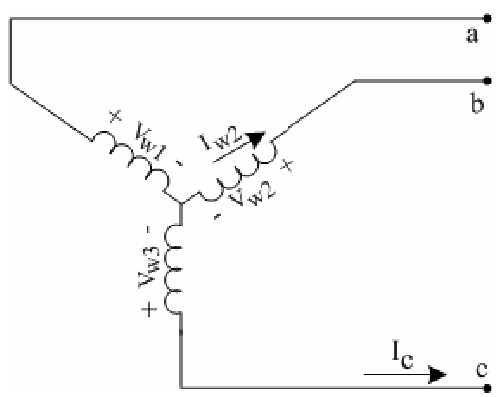


Fig. 9-13 *Wye connected outward winding*

Wye-connected inward winding

The discussion of Wye-connected inward winding includes an explanation of voltage trace and current propagation.

Voltage trace

An outward propagation of voltage can be invoked on an ungrounded, Wye transformer connection as described in this section. Shown above is a diagram of a Wye-connected transformer bank.

The line voltages serving the primary side of this transformer are given at V_{AR}, V_{BR}, and V_{CR}. The current into the transformer windings is given as I_A, I_B, and I_C.

The voltages past the winding impedances in the transformer bank model are given as follows.

$$\begin{Bmatrix} V_{AT} \\ V_{BT} \\ V_{CT} \end{Bmatrix} = \begin{Bmatrix} V_{W1} \\ V_{W2} \\ V_{W3} \end{Bmatrix} + \begin{Bmatrix} I_{w1}Z_{W1} \\ I_{w2}Z_{W2} \\ I_{w3}Z_{W3} \end{Bmatrix} \qquad \textbf{(9-50)}$$

Shown below is a function related proportionally to losses assuming that the impedance of each winding is similar.

$$f = V_{AT}^2 + V_{BT}^2 + V_{CT}^2 \qquad \textbf{(9-51)}$$

You can write this function in terms of V_{AT} as follows.

$$f = V_{AT}^2 + \left(V_{AT} - V_{AB}\right)^2 + \left(V_{CA} - V_{AT}\right)^2 \qquad \textbf{(9-52)}$$

This function can be minimized to find a suitable voltage across the winding and winding impedance. The previous equation can be used to find expressions for all phases. For example:

$$V_{AT} = \frac{V_{AB} - V_{CA}}{3} \quad V_{BT} = \frac{V_{BC} - V_{AB}}{3} \quad V_{CT} = \frac{V_{CA} - V_{BC}}{3} \qquad \textbf{(9-53)}$$

This represents the creation of an expression that relates the voltage across the windings in a Wye-connected bank to the line voltages. The matrix form of this expression is as follows.

$$\begin{Bmatrix} V_{AT} \\ V_{BT} \\ V_{CT} \end{Bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} \qquad \textbf{(9-54)}$$

Since you want the voltages across the transformer windings, you solve for the following.

$$\begin{Bmatrix} V_{W1} \\ V_{W2} \\ V_{W3} \end{Bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} - \begin{Bmatrix} I_{w1}Z_{W1} \\ I_{w2}Z_{W2} \\ I_{w3}Z_{W3} \end{Bmatrix} \qquad \textbf{(9-55)}$$

Current propagation

The winding currents should correspond to the line currents with the Wye-connected windings. Corrective measures should be taken, however, using the fact that $I_{w1}+I_{w2}+I_{w3} = 0$. Current equations can be written for each phase as follows.

$$I_A = I_{W1} + V_{AY}G_{W1}$$
$$I_B = I_{W2} + V_{BY}G_{W2}$$
$$I_C = I_{W3} + V_{CY}G_{W3} \quad \textbf{(9-56)}$$

You should maintain a zero sum of current into the transformer windings. This, therefore, breaks the connection between the node formed by the winding conductances and the Y point of the transformer bank. For example:

$$I_{W1} + I_{W2} + I_{W3} = 0 \quad \textbf{(9-57)}$$

You can minimize the voltage across the winding conductances as shown below.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ I_{W3} \end{Bmatrix} + \begin{Bmatrix} \dfrac{V_{AB} - V_{CA}}{3} G_{W1} \\ \dfrac{V_{BC} - V_{AB}}{3} G_{W2} \\ \dfrac{V_{CA} - V_{BC}}{3} G_{W3} \end{Bmatrix} \quad \textbf{(9-58)}$$

The explicit current through winding 3 is ignored.

Wye-connected outward winding

The discussion of Wye-connected inward winding includes an explanation of voltage trace and current trace.

Voltage trace

An outward propagation of voltage can be invoked on a Wye transformer connection as described in this section.

The winding voltage can be used directly to determine line voltages. The reference voltage of the line voltages thus becomes the center of the Wye bank.

$$\begin{Bmatrix} V_{ar} \\ V_{br} \\ V_{cr} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \end{Bmatrix} \quad \textbf{(9-59)}$$

Current trace

All loading fed from the Wye-connected windings should be line-line type. Thus, the current at the terminals of the windings should sum to zero. To insure that the currents sum properly, only two of them are used to determine winding currents. For example:

$$\begin{Bmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} \quad \textbf{(9-60)}$$

Open-Delta CA connections

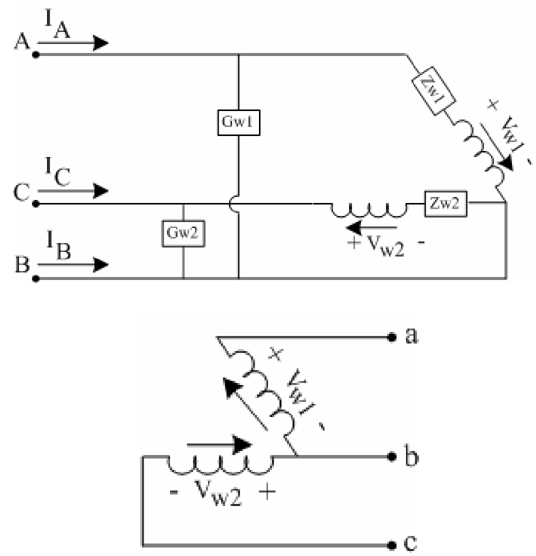


Fig. 9-14 *Open-delta CA connection*

Open-Delta CA connected inward winding

This discussion of Open-Delta CA connected inward winding includes examples of voltage and current trace.

Voltage trace

$$\begin{Bmatrix} V_{W1} \\ V_{W3} \\ - \end{Bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} + \begin{Bmatrix} -I_A Z_{W1} \\ I_C Z_{W2} \\ 0 \end{Bmatrix} \tag{9-61}$$

Current trace

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ I_{W3} \end{Bmatrix} + \begin{Bmatrix} V_{AB} G_1 \\ V_{BC} G_{W2} - V_{AB} G_{W1} \\ -V_{BC} G_{W2} \end{Bmatrix} \tag{9-62}$$

Open-Delta CA connected outward winding

This discussion of Open-Delta CA connected outward winding includes explanations of voltage and current trace.

Voltage trace

For an Open-Delta CA connected inward winding, you desire to determine the voltages V_{ar}, V_{br}, V_{cr} from the winding voltages on an outward trace through the transformer. The voltage across the windings is found from the Inward Winding - Voltage Trace calculations.

To begin, you need to establish a relationship between the winding voltages and the outward terminal voltages to an arbitrary reference point. There are two winding voltages and three terminal voltages. This means that there is insufficient information to determine terminal voltage from the winding voltages. However, as stated above, the terminal voltages are maintained with respect to an arbitrary reference voltage. Since this connection is in the Delta family, your only requirement is to maintain the proper relationship of voltage between terminals. So, you will select the terminal voltage for terminal *a* and determine the other two terminal voltages.

To make a smart selection of terminal *a* voltage, assume for a moment that you have balanced voltages at the terminals. For example:

$$V_{ar} = V\angle\alpha \quad V_{br} = V\angle\alpha - 120 \quad V_{cr} = V\angle\alpha + 120 \tag{9-63}$$

The voltage across winding one would then be as follows.

$$V_{w1} = V_{ar} - V_{br} = V\angle\alpha - V\angle(\alpha - 120) = V\angle\alpha\left(\sqrt{3}\angle 30\right) \tag{9-64}$$

Now you can solve for the voltage at terminal *a* during this special balanced case, as shown below.

$$V_{ar} = \frac{V_{w1}\angle -30}{\sqrt{3}} \tag{9-65}$$

The voltage at terminals *b* and *c* are related to that of *c*. For example:

$$\begin{aligned} V_{br} &= V_{ar} - V_{ab} = \frac{V_{w1}\angle -150}{\sqrt{3}} \\ V_{cr} &= V_{br} - V_{bc} = \frac{V_{w1}\angle -150}{\sqrt{3}} - V_{w2} \end{aligned} \tag{9-66}$$

So in matrix form, you have the following relationship.

$$\begin{Bmatrix} V_{ar} \\ V_{br} \\ V_{cr} \end{Bmatrix} = \begin{bmatrix} \frac{1\angle -30}{\sqrt{3}} & 0 & 0 \\ \frac{1\angle -150}{\sqrt{3}} & 0 & 0 \\ \frac{1\angle -150}{\sqrt{3}} & -1 & 0 \end{bmatrix} \begin{Bmatrix} V_{w1} \\ V_{w2} \\ 0 \end{Bmatrix} \tag{9-67}$$

Current trace

From inspection, you learn the following.

$$\begin{Bmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} \tag{9-68}$$

Open-Delta AB connections

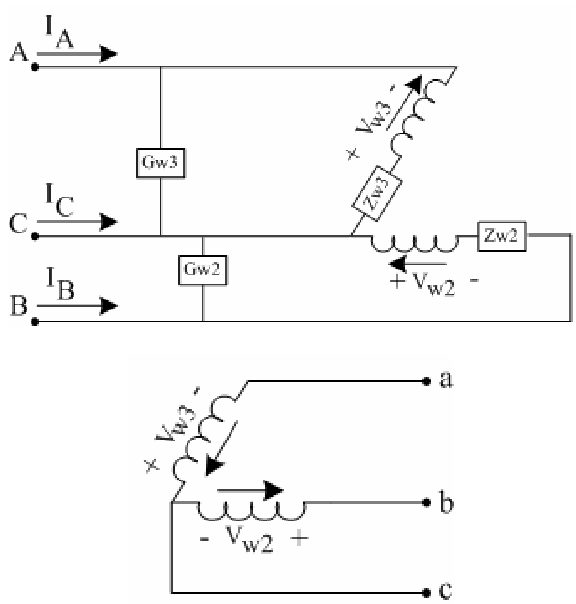


Fig. 9-15 *Open-delta AB connection*

Open-Delta AB connected inward winding

The brief discussion of Open-Delta AB connected inward winding includes examples of voltage and current trace.

Voltage trace

$$\begin{Bmatrix} - \\ V_{W2} \\ V_{W3} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -I_{W2}Z_{W2} \\ I_{W3}Z_{W3} \end{Bmatrix} \tag{9-69}$$

Current trace

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ I_{W3} \end{Bmatrix} + \begin{Bmatrix} -V_{CA}G_3 \\ V_{BC}G_2 \\ V_{CA}G_3 - V_{BC}G_2 \end{Bmatrix} \tag{9-70}$$

Open-Delta AB connected outward winding

The discussion of Open-Delta AB connected inward winding includes examples of voltage and current trace.

Voltage trace

Finally, you have the connection for an Open-Delta AB connection. Following a similar procedure, the terminal voltages can be found from the winding voltages as follows.

(9-71)

$$\begin{Bmatrix} V_{ar} \\ V_{br} \\ V_{cr} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \dfrac{1\angle -150}{\sqrt{3}} \\ 0 & 1 & \dfrac{1\angle -30}{\sqrt{3}} \\ 0 & 0 & \dfrac{1\angle -30}{\sqrt{3}} \end{bmatrix} \begin{Bmatrix} 0 \\ V_{w2} \\ V_{w3} \end{Bmatrix}$$

Current trace

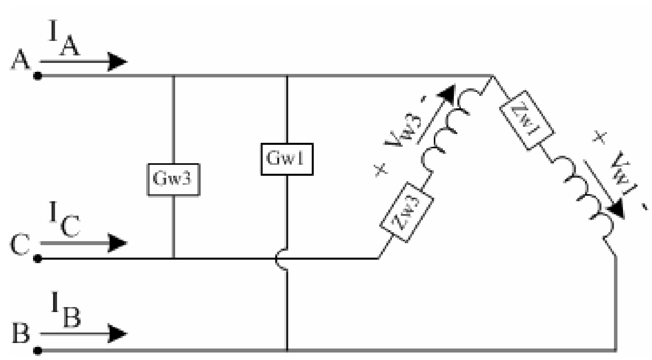
From inspection, you learn the following.

(9-72)

$$\begin{Bmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix}$$

Open-Delta BC connections

Connection for an Open-Delta BC connection is as follows.



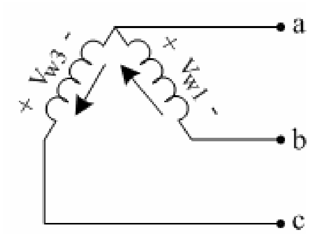


Fig. 9-16 *Open-delta BC connection*

Open-Delta BC connected inward winding

The discussion of Open-Delta BC connected inward winding includes examples of voltage and current trace.

Voltage trace

$$\begin{Bmatrix} V_{W1} \\ - \\ V_{W3} \end{Bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} + \begin{Bmatrix} I_{W1}Z_{W1} \\ 0 \\ -I_{W3}Z_{W3} \end{Bmatrix} \tag{9-73}$$

Current trace

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ I_{W3} \end{Bmatrix} + \begin{Bmatrix} V_{AB}G_1 - V_{CA}G_3 \\ -V_{AB}G_{W1} \\ V_{CA}G_{W3} \end{Bmatrix} \tag{9-74}$$

Open-Delta BC connected outward winding

This discussion of Open-Delta BC connected outward winding includes examples of voltage and current trace.

Voltage trace

Following a similar procedure, the terminal voltages can be found from the winding voltages as follows.

$$\begin{Bmatrix} V_{ar} \\ V_{br} \\ V_{cr} \end{Bmatrix} = \begin{bmatrix} \frac{1\angle -30}{\sqrt{3}} & 0 & 0 \\ \frac{1\angle -150}{\sqrt{3}} & 0 & 0 \\ \frac{1\angle -30}{\sqrt{3}} & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_{w1} \\ 0 \\ V_{w3} \end{Bmatrix} \tag{9-75}$$

Current trace

From inspection, you learn the following.

$$\begin{Bmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \end{Bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} \tag{9-76}$$

Example voltage drop calculations

This section demonstrates the calculation of secondary voltage given secondary current values. The following information is given:

Voltage	230 / 22.9
Connection	Delta / YGnd
Source Volts	124V
Rated MVA	33 MVA
Z%	13.4%
R%	0.33%
No-load Losses	107.1kW
Secondary Amps	736.5 @ 12.11 deg 919.05 @ -109.97 deg 884.98 @ 129.94 deg

The primary or high side connection is:

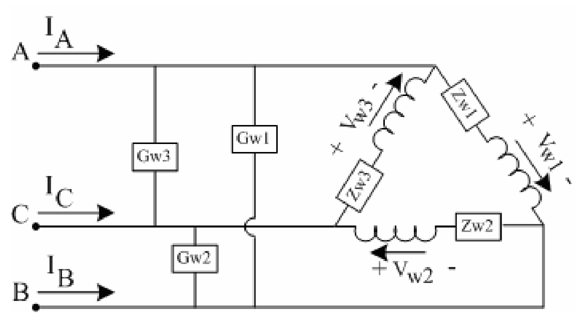


Fig. 9-17 *Primary connection*

Here is the secondary connection:

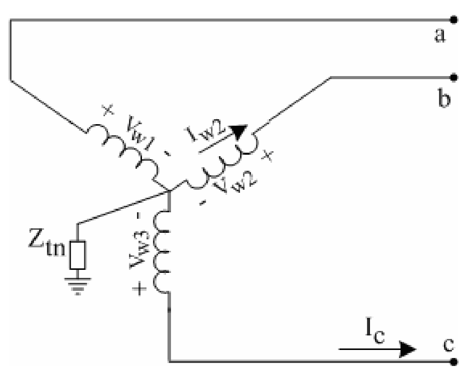


Fig. 9-18 Secondary connection

Simplified calculations

The base impedance on the secondary side is needed:

$$Z_B = \frac{22.9^2}{33} = 15.89 \tag{9-77}$$

The transformer impedance on the secondary of an ideal delta / wye transformer would be:

$$Z_S = (15.89)(j0.134) = j2.129\,\Omega \tag{9-78}$$

Calculate the secondary voltage drop on phase 'A':

$$\begin{aligned} V_A^{'} &= \frac{(736.5\angle 12.11°)(j2.129)}{1000} \\ &= 1.568\angle 102°\ kV \end{aligned} \tag{9-79}$$

Calculate the secondary voltage drop on phase 'B':

$$\begin{aligned} V_B^{'} &= \frac{(919.05\angle -109.97°)(j2.129)}{1000} \\ &= 1.957\angle -19.97°\ kV \end{aligned} \tag{9-80}$$

On phase 'C':

$$\begin{aligned} V_C^{'} &= \frac{(884.98\angle 129.94°)(j2.129)}{1000} \\ &= 1.88\angle -140°\ kV \end{aligned} \tag{9-81}$$

The balanced secondary voltage is the rated secondary voltage running at 124V:

$$V_S = \frac{22.9}{\sqrt{3}}\frac{124}{120} = 13.66\,kV \tag{9-82}$$

Given balanced positive rotation and the shift through the transformer, the following secondary voltages are calculated:

$$\begin{aligned} V_A &= 13.66\angle 30° - 1.568\angle 102° \\ &= 13.26\angle 23.54°\ kV \end{aligned} \tag{9-83}$$

$$\begin{aligned} V_B &= 13.66\angle -90° - 1.957\angle -19.97° \\ &= 13.12\angle -98.06°\ kV \end{aligned}$$

$$\begin{aligned} V_C &= 13.66\angle 150° - 1.884\angle -140° \\ &= 13.14\angle 142°\ kV \end{aligned}$$

In volts, these voltages are running 120.4, 119.1, and 119.1V.

The effect of the grounding reactance is now considered.

Neutral current is:

$$\begin{aligned} I_G = &\left(736.5\angle 12.11°\right) + \\ &\left(919.05\angle -109.97°\right) + \\ &\left(884.98\angle 129.94°\right) \\ = &\,164.8\angle -169.2°\ A \end{aligned} \tag{9-84}$$

The voltage 'rise' across the reactance is:

$$\begin{aligned} V_G &= \frac{\left(164.8\angle -169.2°\right)\left(j1.33\right)}{1000} \\ &= 0.219\angle -79.2°\ kV \end{aligned} \tag{9-85}$$

This rise is added to each secondary voltage to get the following:

$$V_A = 13.21\angle 22°\ kV \tag{9-86}$$

$$V_B = 13.33\angle -97°\ kV$$

$$V_C = 12.98\angle 142°\ kV$$

These secondary voltages in volts are now:

120, 121, and 117.8V. The grounding reactance causes a substantial change in the voltage balance and drip

Transformers in network analysis

Network analysis is performed in the pu system. Therefore, a transformer with ratings matching the nominal voltage for the zones on each side of the transformer results in a diagonal impedance matrix. If the transformer has taps that create an off-nominal voltage, the transformer admittance matrix is no longer diagonal.

Simple Wye-Gnd/Wye-Gnd transformer models are used in network analysis. YBus elements are calculated with the result of this derivation. You can start with a circuit diagram for a Wye-Gnd/Wye-Gnd bank.

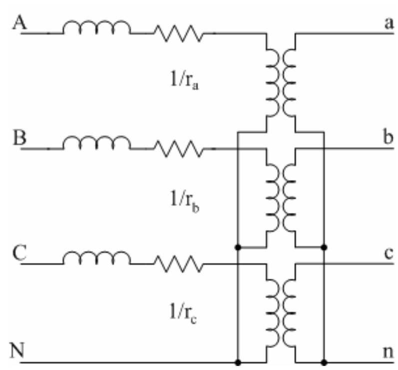


Fig. 9-19 *Wye-grnd connection*

The tap value for the transformer is the PU secondary divided by the primary.

$$r_{\psi} = \frac{n_{\sec - pu}}{n_{pri-pu}} \tag{9-87}$$

The tap value for a transformer set to 5 percent raise would be 1.05.

Assuming a fairly balanced loading, the power into the primary is:

$$S_{\Psi} = \frac{V_{\psi}}{r_{\psi}} I_{\Psi}^{*} \tag{9-88}$$

Power into secondary is:

$$S_{\psi} = V_{\psi} I_{\psi}^{*} \tag{9-89}$$

Assuming an ideal transformer:

$$S_{\Psi} = -S_{\psi} \tag{9-90}$$

or:

$$\frac{V_\psi}{r_\psi} I_\Psi^* = -V_\psi I_\psi^* \tag{9-91}$$

You can solve for the primary current.

$$I_\Psi = -r_a^* I_\psi \tag{9-92}$$

From the circuit, you can write the following relationship.

$$I_\Psi = \left(V_\Psi - \frac{V_\psi}{r_a} \right) Y \tag{9-93}$$

Now, substitute primary current in the secondary expression and solve for secondary current.

$$V_\Psi Y - V_\psi \frac{Y}{r_\psi} = -r_\psi^* I_\psi \tag{9-94}$$

This solution leads to the following.

$$I_\psi = -V_\Psi \frac{Y}{r_\psi^*} Y + V_\psi \frac{Y}{r_\psi r_\psi^*} \tag{9-95}$$

You can write the current/voltage relationship in matrix form.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \\ I_a \\ I_b \\ I_c \end{Bmatrix} = \begin{bmatrix} Y & 0 & 0 & -\frac{Y}{r_a} & 0 & 0 \\ 0 & Y & 0 & 0 & -\frac{Y}{r_b} & 0 \\ 0 & 0 & Y & 0 & 0 & -\frac{Y}{r_c} \\ -\frac{Y}{r_a^*} & 0 & 0 & \frac{Y}{|r_a|^2} & 0 & 0 \\ 0 & -\frac{Y}{r_b^*} & 0 & 0 & \frac{Y}{|r_b|^2} & 0 \\ 0 & 0 & -\frac{Y}{r_c^*} & 0 & 0 & \frac{Y}{|r_c|^2} \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \\ V_C \\ V_a \\ V_b \\ V_c \end{Bmatrix} \tag{9-96}$$

Example secondary impedance reflection

We are given the following information for a transformer installation:

Voltage	69 / 12.47

MVA	11.16
%Z	7.59
Source Z	$Z_1 = 0.15 + j2.33\ \Omega$ $Z_0 = 0.20 + j1.20\ \Omega$

The first thing to do is calculate the base impedance for the secondary of the transformer. We will use that base value to determine the transformer's impedance as seen from the secondary windings:

(9-97)

$$Z_{Base} = \frac{kV_{Low}^2}{MVA_{Tran}} = \frac{12.47^2}{11.16} = 13.934$$

The impedance of the transformer from the secondary is:

(9-98)

$$Z_{Tran} = j\frac{\%Z_{Tran}}{100} \cdot Z_{Base} = j1.0576\,\Omega$$

Now let's reflect the transformer source impedance to the secondary and add the transformer's impedance:

(9-99)

$$Z_{0Sec} = Z_0 \frac{kV_{Low}^2}{kV_{High}^2} + Z_{Tran} = 0.00653 + j1.097\,\Omega$$

$$Z_{1Sec} = Z_1 \frac{kV_{Low}^2}{kV_{High}^2} + Z_{Tran} = 0.0049 + j1.134\,\Omega$$

10
Motors

Introduction

SynerGEE supports a detailed motor model, which is fully considered by analyses such as load-flow and any load-flow based applications. In addition, if you have motors modeled, SynerGEE provides two motor-specific analyses, locked rotor analysis (LRA) and motor start analysis (MSA).

Like all model equipment, motors are attached to sections. The data structure for a modeled motor is also similar to most equipment. Placing a motor on a model creates an instance record in the model database (InstMotor table). This record contains a reference to another record in the equipment database (DevMotors table), which provides nameplate data about the particular motor type needed for analysis. In addition, the instance record contains a reference to the section to which the motor is connected.

Other general facts about motors include the following.

- Once modeled, the motor and motor service loads are considered to be at the center of a section, similar to an equivalent constant impedance spot load.
- Only one motor may be placed per section, since the section name is the unique identifier for a motor.
- Motors can be modeled on substation buses.

For detailed information on motor start analysis and locked rotor analysis, see Chapter 14 of this *Technical Reference*.

Complete motor and service

A "motor" in SynerGEE is made up of the motor service connection, motor starter, and motor model. All three of these components are recognized by locked rotor, motor start, and load-flow based analyses. The diagram below represents the complete motor model.

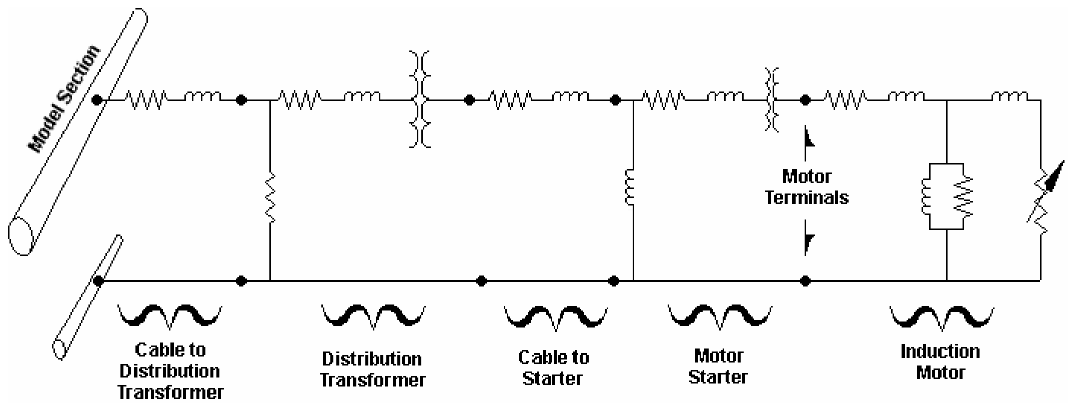


Fig. 10-1 *Section to cable, transformer, cable, starter, and motor*

A detailed by-phase representation of the entire model is used during analysis. The diagram above is simplified to emphasize the starter and motor network. SynerGEE section and transformer models are used to construct the service. These are the same models that are used in the distribution model.

SynerGEE also uses detailed by-phase starter models and motor models.

Motor model

Induction motors are the most widely used AC motors. They have a stationary stator and a rotating rotor. These components are separated by a small air gap to prevent mechanical contact. The primary poly-phase insulated winding on the stator sets up a synchronously rotating magnetic field in the air gap. This rotating field induces currents in the rotor or secondary winding, which causes it to rotate at a speed slightly less than the synchronously rotating air gap magnetic field. Torque is developed to turn the rotor and to drive its connected load.

The classical model of an induction motor is shown below. It consists of a series impedance representing the stator impedance. The impedance is followed by another series impedance representing the rotor. A shunt branch representing the motor's core losses separates the impedances.

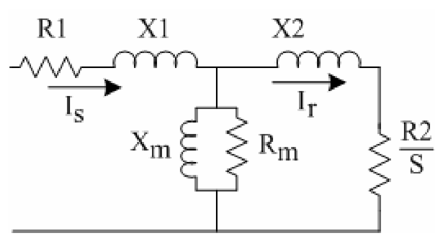


Fig. 10-2 *Induction motor*

The series impedance Z1 = R1+jX1 is the stator resistance and reactance. The shunt impedances *Xm* and *Rm* represent the magnetizing reactance and resistance associated with friction, windage, and iron losses. The reactance *X2* represents the leakage reactance of the rotor. *R2* is the rotor winding resistance.

During its starting process, the motor is modeled as an incrementally changing constant impedance load. At a particular speed point, the full model described above can be reduced to an equivalent circuit consisting of a single impedance.

All parameters are fixed in this model except for the air-gap resistance. For a particular speed point along the start, however, the air-gap resistance is also fixed. This is because the slip, *S*, which determines the air-gap resistance, is directly related to the motor's speed.

Output power

The power transferred across the air-gap of the induction motor can be found from rotor quantities and the machine slip, *S*, as follows.

$$P_{Air-Gap} = I_{Rotor}^2 * R2/S \tag{10-1}$$

Rotor losses are eliminated from this quantity to determine the motor's output power. For example:

$$P_{out} = I_{Rotor}^2 \frac{R2(1-S)}{S} \tag{10-2}$$

The output power is used with the value of slip, to determine the value of instantaneous torque.

Impedance model

A motor draws a current directly proportional to its terminal voltage. A device behaving in this manner can be modeled as a constant impedance load.

Shown below are current and power factor curves for an induction motor. These curves are used for load-flow analysis at each speed point.

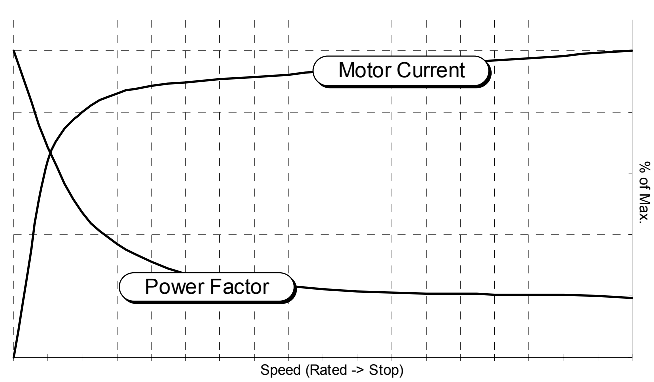


Fig. 10-3 Example rated current and power factor curves

The motor manufacturer typically creates the above curves with rated voltage applied to the terminals. For a given motor speed, the terminal voltage as well as the input current and power factor are known. The equivalent impedance of the induction motor for a particular speed can, therefore, be found from the power factor and current curves, $Pf_{Curve}(s)$, $I_{Curve}(s)$, as follows.

$$Z_{Motor}(s) = \frac{1000 * kV_{Rated}}{\sqrt{3} * I_{Curve}(s) * \left(Pf_{Curve}(s) - j\sqrt{1 - Pf_{Curve}^{2}(s)} \right)} \tag{10-3}$$

The impedance from the above equation refers to a reduced model of an induction motor shown below.

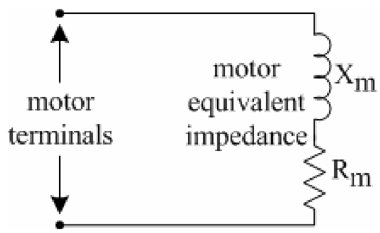


Fig. 10-4 Motor equivalent impedance for given speed

During each of the speed points, the induction motor equivalent impedance calculated for the particular speed being evaluated is applied as a constant impedance load to its respective section. A load-flow analysis is then run on the system to determine the motor's terminal voltage and input current as well as its overall power demand. Since voltages all over the distribution system are found from this analysis, the section having the worst voltage in the system is also located.

Motor states

Motors have three states: Off, Starting, and Running. These states are handled differently by MSA, LRA, and load-flow based applications.

	Off	**Starting**	**Running**
Motor Start Analysis	Ignored	Dynamic motor model	Dynamic model initialized to full speed
Locked Rotor Analysis	Ignored	Constant Z load	Constant PQ load
Load-Flow Analysis	Ignored	Ignored	Load

All applications ignore a motor that is off. However, the motor service (cables, distribution transformer, etc.) is still considered active, though the effect on a load-flow run is insignificant. After a load-flow run, the terminal voltage for a motor that is off can be found.

Motor starters

The electric motor starter is the important connecting link between the motor and the electric-supply system. Properly applied, the motor starter can reduce the effect of motor starting on the system and the driven load. Starting can be accomplished by full voltage, reduced voltage, or reduced inrush methods. Reduced-voltage starters include autotransformer and primary-resistor or primary-reactor types. Reduced inrush starters include part winding and Wye-Delta.

In SynerGEE, a general circuit is used to simulate the variety of possible motor starters that are available. For example:

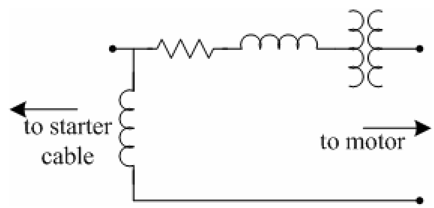


Fig. 10-5 *Starter circuit*

The circuit consists of a two-port device having series impedance and a series connected transformer. For particular starters, various portions of this circuit are ignored. An ideal transformer is used to simulate reduced voltage starters such as autotransformers and Wye-Delta transformers. The series impedance in the general starter above is used to simulate resistive and reactive reduced current starters. Each of the supported motor starters is discussed below.

Full voltage

The starter circuit is removed from the motor starting analysis. Full voltage starting produces maximum starting torque and minimum acceleration time. Because of its simplicity and low cost, full voltage starting should always be the first choice. However, power system disturbance may require a reduction in current inrush.

Capacitor

Values for two three-phase starting capacitors are supplied. The capacitors are assumed to have voltage ratings that match the ratings of the motor. The first capacitor is used from the initial point to the speed you indicated. At that point, the first capacitor is switched out and the second one is switched in. At a second specified motor speed, all capacitors are switched out and the motor is tied directly to the service. Zero values for capacitor kvar are allowed for a single capacitor starting sequence.

Autotransformer

The principal advantage of this starting method is the high value of torque produced per-unit of starting current. Motor current is reduced in proportion to the voltage applied to motor terminals. Line current, however, is reduced in proportion to the square of the motor-terminal voltage because of the autotransformer action. It has the advantage of torque and inrush current adjustment in the field by simple tap selection.

The turns ratio of the ideal transformer in the starter circuit is set to match the tap of the autotransformer starter for the appropriate speed point of the motor. The tap of the autotransformer may change one or two times as the motor speeds from stop through its full-load speed.

Resistance

This method employs series resistors (reactors) in each phase of the motor primary circuit. The value of the resistance is reduced in one or more steps to meet inrush requirements, until full voltage is applied to motor terminals. Starting torque magnitudes are high, but torque efficiency is lower than with autotransformer starting.

One, two, or three resistors may be applied to a motor during its startup. The value of these resistors is chosen such that at the speed of application, the motor terminal voltage would be reduced to the prescribed percentage of rated motor voltage if rated motor voltage were to be applied to the input terminals of the starter.

The effects of resistor starting are evident with a motor connected to a zero impedance feeder with no distribution transformer. In that case, the feeder voltage can be set so that rated motor voltage can be applied to the starter throughout the motor's acceleration. At each of the speed points when a starter resistance is switched in and out, the prescribed voltage reduction is seen at the motor terminals.

The same values of starter resistance used in the case just described will be used when the motor is started in a realistic feeder with reasonable losses and drops. The values of voltage drop specified within the reduced voltage start may not, therefore, be immediately evident in the motor start output because rated motor voltage is usually not applied to the terminals of the starter.

Consider a resistance, R, needed to reduce a motor's voltage by a factor, f, at some particular speed. The equivalent impedance of the starting motor at the particular speed can be found from the rated motor voltage and its current and power factor curves.

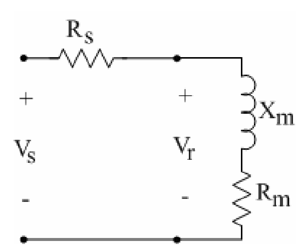


Fig. 10-6 Motor with resistor starter

When the resistive starter is in series with the motor, it is desired that the voltage at the terminals of the motor be a factor of the rated voltage at a given speed when motor rated voltage is applied to the starter.

$$|V_T| = f * V_{Rated} = V_{Rated} * \left| \frac{Z_M}{R_S + Z_M} \right| \tag{10-4}$$

The above equation can be arranged to solve for f as follows.

$$f = \frac{|R_M + jX_M|}{|R_S + R_M + jX_M|} \tag{10-5}$$

The value of starter resistance can be derived as follows.

$$R_S = -R_M + \sqrt{\left(\frac{|R_M + jX_M|}{f} \right)^2 - X_M^2} \tag{10-6}$$

Reactance

Reactance starters follow the same principles for resistance starters. The value of reactance applied at one, two, or three appropriate speeds along the motor start is found as follows.

$$X_S = -X_M + \sqrt{\left(\frac{|R_M + jX_M|}{f} \right)^2 - R_M^2} \tag{10-7}$$

Part-winding

Reduce inrush, closed transition starting by connecting the sectionalized, parallel stator winding to the line in one or more steps. This type of starting requires no auxiliary current reducing device and uses simple switching. Torque efficiency is low for high-speed motors.

In SynerGEE, the equivalent impedance of the motor is increased by the factor specified for part-winding starting. This provides the correct effect of reduced current starting with full starting motor voltage. The derived torque of the motor is appropriately reduced by the part-winding factor.

Delta-Wye

This connection reduces inrush by switching the windings on a motor designed for Wye-Delta connection. Starting torque is only one-third of the value at rated voltage. This method is desirable only where low starting torque is acceptable. The motor must run with Delta connection.

Although actual $\mathrm{Y}-\Delta$ starters manipulate the windings of six-lead motors, the same effects are achieved in MSA with the turns-ratio modification of the starter's ideal transformer. During the open transition state of the starter, 57% voltage $\left(1/\sqrt{3}\right)$ is applied to the motor windings. This effect, along with the reduction of starting current and starting torque by 1/3, is achieved with a turns ratio of $1/\sqrt{3}$ in the ideal transformer of the starter model.

Solid-state reduced voltage

This starter operates like an autotransformer with a turns ratio that ramps down with time. The initial value for the reduction percent is specified along with the time in seconds to reach full voltage. The starter ramps up the voltage at the terminals of the motor until the motor is being fed full line voltage.

Motor data

An actual motor installation or instance references the motor type. The type encapsulates basic nameplate information about the motor.

Fundamental parameters

The basic motor model is specified by three parameters:

- Rated kV
- Rated HP
- Single-/three-phase construction

These parameters are used to model the motor in all applications.

Full load amps

We know that there are 746.0 watts / HP. The kVA load of an induction motor running at full load and rated voltage is:

$$kVA_{Motor} = \frac{0.746 \cdot HP}{pf_{rated} \cdot Efficiency} \tag{10-8}$$

The efficiency is a factor.
We can now calculate the rated full load amps with the rated voltage:

$$I_{Rated} = \frac{0.746 \cdot HP}{\sqrt{3} \cdot kV_{LL} \cdot pf_{rated} \cdot Efficiency} \tag{10-9}$$

Consider a motor with the following ratings:

HP	500
Rated volts	4kV
Full load pf	87%
Full load efficiency	95%

The rated full load current of this motor is:

$$I_{Rated} = \frac{0.746 \cdot 500}{\sqrt{3} \cdot 4 \cdot 0.87 \cdot 0.95} = 65.1A \tag{10-10}$$

Under full voltage and full load running conditions, we would expect 65A of load current from this load. Variations in voltage and load will result in variations in load current.

Full load power factor

In load-flow runs, the full load power factor is used along with the above parameters to determine the equivalent load value for a running motor.

$$S_{Motor} = \frac{HP*(0.746)}{Efficiency}\left(1 + j\sqrt{\frac{1}{pf^2} - 1}\right) \quad (kVA) \tag{10-11}$$

This load is treated as a constant power load during a load-flow run.

This parameter is not used for motor start analysis runs since the motor characteristic curves supply power factor information throughout the start.

Motor RPM

There are two RPM values on a motor type record.

Synchronous RPM - the angular frequency of the stator field. The motor rated frequency and the number of motor poles determine the synchronous RPM:

$$RPM_S = \frac{f_{Hz} * 120}{\# Poles} \tag{10-12}$$

Full Load (Rated) RPM - the mechanical speed of the rotor under rated conditions. The full load RPM must always be less than the synchronous RPM. Motor torque is proportional to this relationship between rated and synchronous RPM:

$$\tau_m \sim \frac{\omega_s - \omega_m}{\omega_m} \tag{10-13}$$

It can be seen that a mechanical speed equal to or greater than the synchronous speed results in loss of torque.

These values are only used in motor start analysis runs.

Rated torque

If you do not have nameplate data for the motor's rated torque (lb-ft), the value can be closely approximated using the equation below. The relationship between a motor's horsepower and its torque (lb-ft) is expressed as follows.

$$\tau = \frac{H.P.*5250}{RPM_{Rated}} \qquad (lb*ft) \tag{10-14}$$

If you are uncertain about the speed (RPM) of the motor, see the tables at the end of this chapter for typical speeds.

Motor and load inertia

The inertia (Wk^2) of the load and the motor are supplied in the motor table. The larger the inertia, the more time it takes for the motor to reach operating speed. *NEMA Publication MG1-20.42* lists normal inertia capabilities for certain motors. See the tables at the end of this chapter for these load inertia values.

Under certain circumstances, even with across-the-line starting, the motor may not be able to break away from standstill or it may stall at some speed before acceleration is completed. The motor/load inertia and torque data for the motor can allow a speed-torque analysis to be performed. A speed-torque analysis can pinpoint problems and permits you to determine the system changes that will eliminate them.

Motor inertia

Motor inertia is the "weight" of the motor and has a direct bearing on the starting time calculated within MSA. A value for motor inertia can be entered directly or SynerGEE can be instructed to calculate a value. SynerGEE uses the following equation to calculate motor inertia.

$$Motor\ Wk^2 = 0.02*2^{0.5*\#Poles}*HP^{1.35-0.025*\#Poles}\ (\text{lb-ft}^2) \tag{10-15}$$

Motor inertia is used only in motor start analysis runs.

Load inertia

Load inertia is the "weight" of the load driven by the motor. It is combined with the motor inertia during an MSA run. The load inertia is modeled independently of the motor inertia so that common motors can drive various loads like pumps, impellers, or fans.

Load inertia can be entered directly or SynerGEE can calculate an approximate value using the following equation.

$$Load\ \ Motor\ Wk^2\ (\text{lb-ft}^2) \tag{10-16}$$

$$= \frac{24 * HP^{0.95}}{\left(0.001 * RPM\right)^{2.4}} - \frac{0.0685 * HP^{1.5}}{\left(0.001 * RPM\right)^{1.8}} \ \text{for 300-1800 RPM}$$

$$= \frac{27 * HP^{0.95}}{\left(0.001 * RPM\right)^{2.4}} - \frac{0.0685 * HP^{1.5}}{\left(0.001 * RPM\right)^{1.8}} \ \text{for 3600 RPM}$$

Inrush current

A conservative multiplier for motor starting inrush currents is obtained by assuming the motor to have a code G characteristic with a locked rotor current equal to approximately six times the full-load current with full voltage applied at the motor terminals (*Transmission and Distribution Reference Book*, 1964). More information is shown about the locked rotor indicating code letters in the tables at the end of this chapter.

The inrush current is not used in SynerGEE calculations.

Supplemental motor data

The Standard Motor Sizes table, the Locked Rotor Letter Codes table, and the Starting Power Factor graph are included in this manual as a reference for working with motors in a locked rotor condition. If you are uncertain of the specifics of a locked rotor condition, you may have to assume values for the locked rotor kVA and starting power factor. Some reasonable assumptions are 6.0 for the locked rotor kVA and 35% for the starting power factor (*Transmission and Distribution Reference Book, 1964, page 723*). As always, company practice or specific data should be used when available.

The Standard Motor Sizes table below shows standard motors sizes in horsepower. No fractional motors (less than 1 HP) are included (*Electrical Engineering Handbook*).

Standard Motor Sizes (Horsepower)						
5	7.5	10	15	20	25	30
40	50	60	75	100	125	150
200	250	300	350	400	450	500
600	700	800	900	1000	1250	1500
1750	2000	2250	2500	3000	3500	4000
4500	5000	6000	7000	8000	9000	

Locked rotor codes

The locked rotor code is shown on the motor's nameplate and indicates the motor input required with a locked rotor condition. These values are a range and a specific number in that range is entered for a specific motor. The high end of each range could allow for higher breakdown torque or higher-speed-type drive for a given motor code. Remember that motors with the higher code and higher end of a code range have higher inrush under rated conditions. The Locked Rotor Code table below contains the letter codes and their associated kVA per HP range. This information is taken from the *National Electrical Safety Code*, 1990.

NEMA Motor Codes	
Letter Code	**kVA per HP**
A	0.0 - 3.14
B	3.15 - 3.54
C	3.55 - 3.99
D	4.0 - 4.49
E	4.5 - 4.99
F	5.0 - 5.59
G	5.6 - 6.29
H	6.3 - 7.09
J	7.1 - 7.99
K	8.0 - 8.99
L	9.0 - 9.99
M	10.0 - 11.19
N	11.2 - 12.49
P	12.5 - 13.99
R	14.0 - 15.99
S	16.0 - 17.99
T	18.0 - 19.99
U	20.0 - 22.39
V	22.4 - and up

Starting power factors

The motor starting power factor determines the amount of reactive current drawn from the system in a locked rotor condition. Motors operate at different power factors, depending what task the motor is performing. Approximate starting power factors are shown in Figure 10.7 as published in a GE industrial power systems reference.

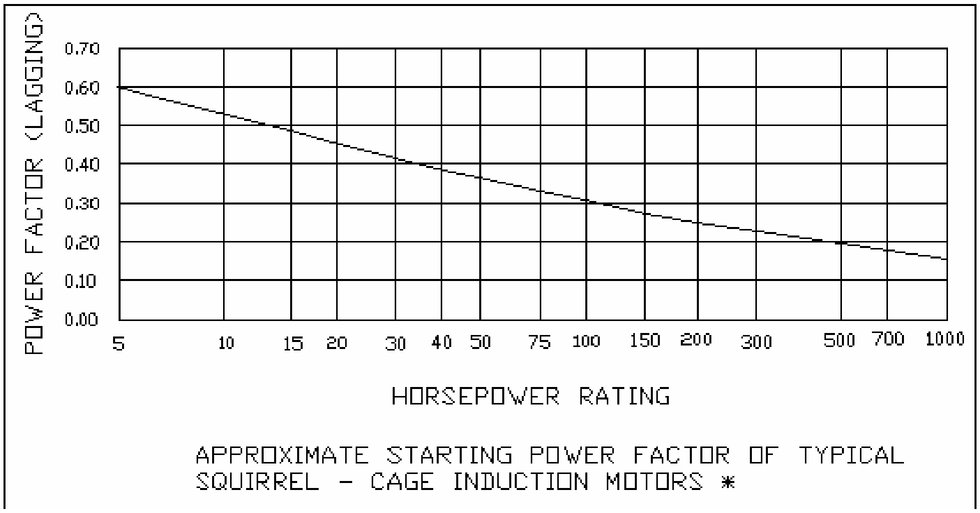


Fig. 10-7 *Approximate motor starting power factor values*

Terms

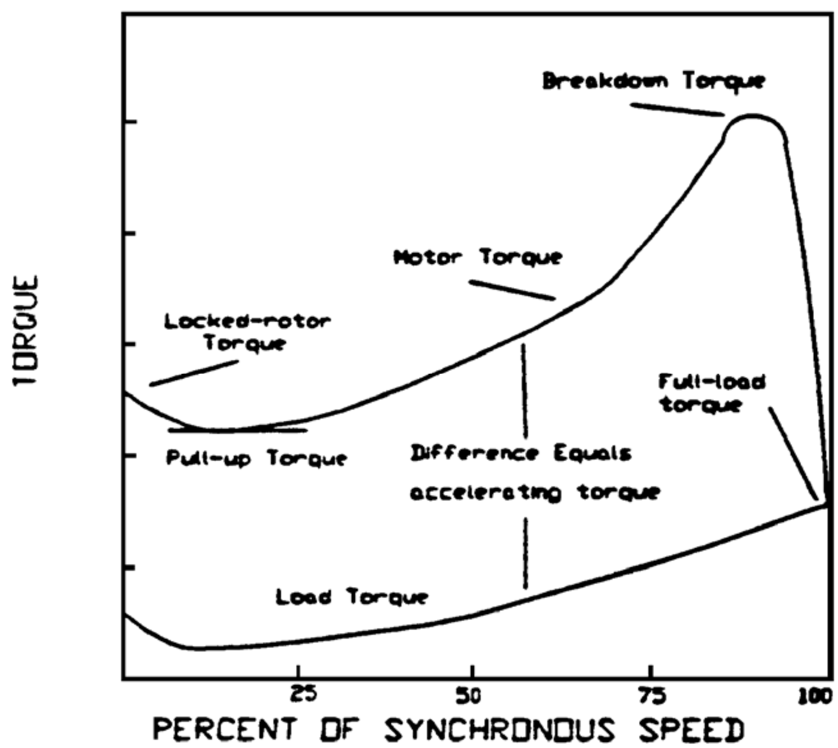


Fig. 10-8 *Terms associated with torque curve*

Term	Definition
torque	Force that tends to produce rotation, the turning moment of tangential effort. Torque in electric motors is the turning force developed by the motor. Torque is expressed in units of force and distance to represent the turning moment. The units normally used are pound-feet, ounce-inches, and gram-centimeters. A preferred method of expression is percent of rated full-load torque. The torque of induction motors can be described in the terms that follow.
full-load torque	Torque necessary to produce rated output with rated power input at rated full speed.
locked rotor torque	Minimum torque that the motor develops at rest for various angular positions of the rotor with the application of rated voltage at rated frequency. This torque is sometimes referred to as breakaway or starting torque.
pull-up torque	Minimum torque developed by the motor during the acceleration period from rest to the speed at which breakdown torque occurs. For motors that do not have definite breakdown torque, the pull-up torque is the minimum torque developed up to rated speed.
breakdown torque	Maximum torque a motor develops at rated voltage and frequency without an abrupt drop in speed. Also referred to as *maximum torque*.
accelerating torque	Torque developed with rated power input during the period from standstill (or zero speed) to full rated speed. The term accelerating torque is frequently referred to and used as the net accelerating torque, which is that positive torque available beyond the torque required by the load.

NEMA codes

Common NEMA codes for various motors[1]

	Type-NEMA		Type-NEMA
Air, gas compressors		**Pumps**	
Axial	1-F	Centrifugal	1-F
Centrifugal	1-F	Charging	1-F
Reciprocating	1-E,G	Boiler feed	1-F
Rotary, vane	1-E	Circulating water	1-D,E
Rotary, (vacuum)	1-F	Condensate	1-F
Coal		Fire	1-F
Mill, attrition	1-F,G	Pipeline	1-E,F
Mill, bowl	1-F,G	Cooling tower	1-G
Conditioner	1-F,G	Sewage	2

[1] Typical applications of induction motors (1=squirrel, 2=wound, 3 = multispeed).

Common NEMA codes for various motors[1]

	Type-NEMA		Type-NEMA
Pulverizer (see mill)		Axial flow	1-E
Crusher	1-F,G	Mixed flow	1-F,2
Fans		Hot well	1-F
Forced draft	1-F,G	Descaling	1-F,G
Induced draft	1-E,F,G	Reciprocating	1-F
Gas recirculating	1-E,F,G	Dredge	1-F,G
Preheat	1-F,G	Sand	1-F
Sintering	1-G	High lift	1-F,2
Mine	1-G	**Rubber**	
Waste gas	1-F,G	Banbury	1-F,G,2,3
Exhauster (kiln)	1-F,G	Extruder	1-F,G,3
Scrubber	1-F,G	Mills	1-F,G
Cooling	1-F,G	Rolls	1-F,G,3
Blower	1-F,G	Strainer	1-F,G,3
Purge	1-F,G	**Sugar**	
Primary air	3-G	Cane knife	1-F,2
General		Centrifugal	1-F,2
Ball, rod mill	1,2	Mill	2
Crusher stone	1-F,G	Shredder	1-E
High-inertia drive start	1,2	**Wood**	
Kiln	1-F,G	Beater	1-D
Mine hoist	2	Chipper	1-F, G,2
Metals		Hydropulper	E,F
Roughing rolls	2	Jordan	E,F
Piercing mill	2	Refiner	1-E,F
Drawbench	3	Shredder	1-E,F
Rolling mill	2	Scalper	1-F

Horsepower versus load torque

	Maximum NEMA Load Inertia (lb-Ft^2)						
hp / RPM	**3,600**	**1,800**	**1,200**	**900**	**720**	**600**	**514**

hp / RPM	Maximum NEMA Load Inertia (lb-Ft²) 3,600	1,800	1,200	900	720	600	514
1	----	5	15	31	53	82	118
1-1/2	1.8	8	23	45	77	120	174
2	2.4	11	30	60	102	158	228
3	3.5	17	44	87	149	231	356
5	5.7	27	71	142	242	375	544
7-1/2	8.3	39	104	208	356	551	798
10	11.0	51	137	273	467	723	1048
15	16.0	75	200	400	685	1061	1538
20	21.0	99	262	526	898	1393	2018
25	26.0	122	324	647	1108	1719	2491
30	31.0	144	384	769	1316	2042	2959
40	40.0	189	503	1007	1725	2677	3861
50	49.0	232	620	1241	2127	3302	4788
60	58.0	275	735	1473	2524	3519	5660
75	71.0	338	904	1814	3111	4831	7010
100	92.0	441	1181	2372	4070	6320	9180
125	113.0	542	1452	2919	5010	7790	11310
150	133.0	640	1719	3458	5940	9230	-------
200	172.0	831	2238	4508	7750	12080	-------
250	210	1,017	2,744	5,540	9,530	14,830	21,560
300	246	1,197	3,239	6,740	11,270	17,550	25,530
350	281	1,373	3,723	7,530	12,980	20,230	29,430
400	315	1,546	4,199	8,500	14,670	22,870	33,280
450	349	1,714	4,666	9,460	16,320	25,470	37,090
500	381	1,880	5,130	10,400	17,970	28,050	40,850
600	443	2,202	6,030	12,250	21,190	33,100	48,260
700	503	2,514	6,900	14,060	24,340	38,080	55,500
800	560	2,815	7,760	15,830	27,440	42,950	62,700
900	615	3,108	8,590	17,560	30,480	47,740	69,700
1,000	668	3,393	9,410	19,260	33,470	52,500	76,600
1,250	790	4,073	11,380	23,390	40,740	64,000	93,600

hp / RPM	Maximum NEMA Load Inertia ($lb\text{-}Ft^2$)						
	3,600	**1,800**	**1,200**	**900**	**720**	**600**	**514**
1,500	902	4,712	13,260	27,350	47,750	75,100	110,000
1,750	1,004	5,310	15,060	31,170	54,500	85,900	126,000
2,000	1,096	5,880	16,780	34,860	61,100	96,500	141,600
2,250	1,180	6,420	18,440	38,430	67,600	106,800	156,900
2,500	1,256	6,930	20,030	41,900	73,800	116,800	171,800
3,000	1,387	7,860	23,040	48,520	85,800	136,200	200,700
3,500	1,491	8,700	25,850	54,800	97,300	154,800	228,600
4,000	1,570	9,460	28,460	60,700	108,200	172,600	255,400
4,500	1,627	10,120	30,890	66,300	118,700	189,800	281,400
5,000	1,662	10,720	33,160	71,700	128,700	206,400	306,500
5,500	1,677	11,240	35,280	76,700	138,300	222,300	330,800
6,000	1,673	11,690	37,250	81,500	147,500	237,800	354,400
7,000	1,612	12,400	40,770	90,500	164,900	267,100	399,500
8,000	1,484	12,870	43,790	98,500	181,000	294,500	442,100
9,000	1,294	13,120	46,330	105,700	195,800	320,200	482,300
10,000	1,046	13,170	48,430	112,200	209,400	344,200	520,000

This table applies to standard poly-phase squirrel-cage motors having locked rotor torque equal to 60% of full-load torque and a rated temperature rise of 40^{o}C. Motors can accelerate without injurious temperature rise under the following conditions.

1. Rated voltage and frequency applied.
2. During the acceleration period, the connected load torque should be equal to or less than motor torque.

Table starter characteristics[2]

Type of Starter	Motor terminal voltage	Motor starting torque	Motor starting current	Line starting current	Torque efficiency	Approx. relative cost 50 hp 440 volt
Full voltage	1.0	1.0	1.0	1.0	1.0	1.0
Reduced Voltage Auto transformer:						

[2] Comparison of motor starting methods for squirrel-cage induction motors

Type of Starter	Motor terminal voltage	Motor starting torque	Motor starting current	Line starting current	Torque efficiency	Approx. relative cost 50 hp 440 volt
80% voltage tap	0.8	0.64	0.8	0.64*	1.0	5.5
65% voltage tap	0.65	0.42	0.65	0.42*	1.0	
50% voltage tap	0.5	0.25	0.5	0.25*	1.0	
Primary resistor:						
80% terminal voltage	0.8	0.64	0.8	0.8	0.8	5.0
65% terminal voltage	0.65	0.42	0.65	0.65	0.65	
50% terminal voltage	0.5	0.25	0.5	0.5	0.5	
Primary reactor	Values the same as for primary resistor starter as listed above **					
Reduced inrush: Part-winding 50%:						
Low-speed motors	1.0	0.5	0.5	0.5	0.5	3.2
High-speed motors	1.0	0.5	0.7	0.7	0.7	
Part-winding 75%:						
Low-speed motors	1.0	0.75	0.75	0.75	1.0	
Wye-Delta	1.0	0.33	0.33	0.33	1.0	4.0-6.0

* Does not include transformer magnetizing current, which is usually 25% of motor full-load current.

** This method is generally more applicable to larger motors and at voltages above 600 volts.

Low-speed motors are considered to be 514 rpm and below. All values shown are per unit. One per-unit is equal to motor rated terminal voltage, rated starting current, and rated starting torque. Torque efficiency is motor-starting torque divided by motor-starting current on a per-unit basis.

Closed transition starting is now practically a standard requirement. Closed transition refers to a method of starting that will not open the circuit between the motor and the line at any time during the starting cycle.

NEMA designs

NEMA classifies the squirrel-cage induction motors into four standard designs to meet various starting and running requirements. A brief description of the characteristic features for these designs follows.

Design A Normal starting torque, normal starting current, low slip.
Basic design for motors in sizes below 7.5 and above 200 HP.

It emphasizes good running performance at the expense of starting. The full-load slip is low and the full-load efficiency is high. The maximum torque is usually well over 200% of full-load torque. The starting torque at full voltage varies from about 200% of full-load torque in small motors to about 100% in large motors. The high starting current of 500-800% of full-load current when started at full voltage is the principle disadvantage of this design. Design A motors are usually used for applications where high efficiency and high full-load rpm are required. The field of application for a Design A motor is about the same as that of Design B described below.

Design B Normal starting torque, low starting current.
Basic design for motors 7.5 through 200 hp.

This design has approximately the same starting torque as Design A, but 75% of the starting current. Therefore, full voltage starting may be used with larger sizes than with Design A in a given situation. The full-load slip and efficiency are good – about the same as for Design A. Design B motors are the standard general-purpose motors used when low locked rotor current and moderate locked rotor torque are required along with high full-load rpm and efficiency. Typical applications are fans, blowers, pumps, and machine tools.

Design C High starting torque, low starting current.

This design has higher starting torque compared to designs A and B, low starting current but somewhat lower running efficiency and higher slip than designs A or B. Typical applications are compressors and conveyers.

Design D High starting torque, high slip.

This design has higher starting torque at low starting current, high maximum torque at 50 - 100% slip, but runs at a high slip at full-load (7 - 11%) and consequently has low running efficiency. Typical applications are intermittent loads involving high accelerating duty and for driving high-impact loads such as punch presses and shears.

Speed torque, current curves for NEMA codes

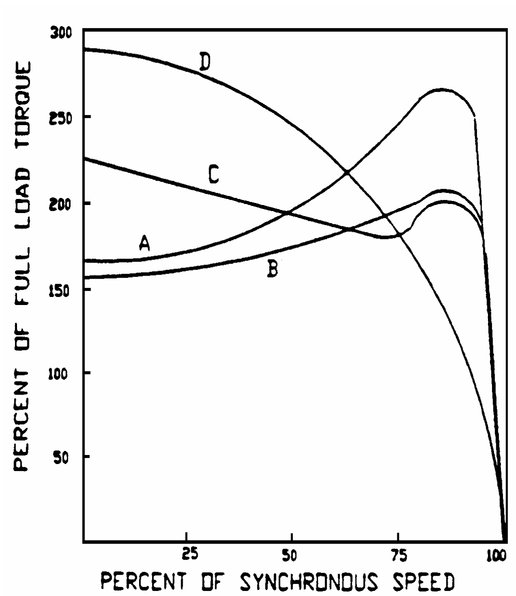


Fig. 10-9 Speed / Torque curves for NEMA A, B, C, D

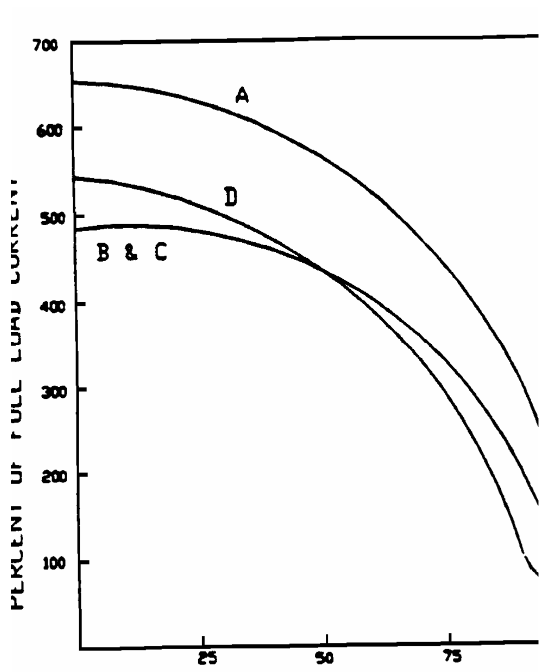


Fig. 10-10 Speed / Current curves for NEMA A, B, C, D

11
Generators

Introduction

SynerGEE supports detailed by-phase models of generators. There are three types of models.

- **Induction**—The machine is modeled with passive components. A variable resistance represents the electro-mechanical coupling through the notion of slip. The output power in kW can be specified.
- **Synchronous**——Series winding impedances and a back EMF are used to represent the machine's behavior. The machine responds to the voltage and output power settings.
- **PQ**—The PQ model behaves just like a negative constant power load.

The synchronous and induction models are referred to as active models because they respond to the by-phase conditions of the distribution system as seen through their terminals. Depending on the distribution system and other active devices, they may or may not be able to achieve the output values you specified. Since the generator models were designed with real machines in mind, attention should be paid to the operated or planned machine in cases when the SynerGEE model does not meet your kW or voltage settings.

PQ models do not actively respond to distribution system conditions. Their output power value will be the user-specified value following a successful load-flow run.

Generators are always placed at the center of a section. Their terminals are driven from the voltage at the center of the line and they inject current into the center of the line.

Data

Like most devices within SynerGEE, a generator model is specified with an instance record in the model database, with a reference to a record in the equipment database (DevGenerators table). The following tables show the data requirements for both.

Equipment database (DevGenerators)

	Synchronous	Induction	PQ
Three-, Single-Phase	✓	✓	✓
Rated kW	✓	✓	✓
Rated kV	✓	✓	✓
Rated pf	✓	✓	✓
X''_d	✓		
X''		✓	
Winding Z0 and Z1	✓	✓	
PT Ratio	✓		

The rated kW, kV, and pf are nameplate values for the generator. The rated pf is the power factor output of the machine at rated kW and kV.

X''_d and X'' are the subtransient reactances of synchronous and induction machines respectively. These are the impedances seen looking into the machine at the instance of a fault.

The winding impedances are used in unbalanced load-flow calculations. They are specified in percent, based on the machine's rating. The conversion from percent to Ohms is as follows.

$$Z_{Ohms} = Z_{Pct} \frac{kV_{Rated}^2 * pf_{Rated}}{10 * kW_{Rated}} \quad \textbf{(11-1)}$$

The PT ratio is required for synchronous machines. The PT ratio is used to tie the generator's voltage settings to the actual terminal voltage seen during an analysis.

Model database (InstGenerators)

	Synchronous	Induction	PQ
Output Power	✓	✓	✓
Voltage Setting	✓		
Min/Max Field	✓		
Metering Phase	✓		
Pos. Seq. Metering	✓		
Grounding Ohms	✓	✓	

Treat as PQ Load	✓	✓	

The output power is entered as a percentage of the rated power of the generator. The slip or rotor angle of the machine is adjusted to achieve a total output power that matches this value. The output power may be unbalanced during a by-phase analysis.

The voltage setting is the desired terminal voltage for a synchronous machine. During analysis, the field current is adjusted within the field limits to attempt to reach this value. The metering phase specifies the phase to which the PT is connected for voltage metering.

The grounding impedance is used for active generator models in both load-flow and fault studies. Residue from unbalanced load current and/or fault current has a direct effect on generator terminal voltage. The grounding impedance also has the effect of reducing this current.

A synchronous or induction generator can be treated as a PQ load.

Enabling generators

The following table lists all of the requirements to have generators modeled as active generators or as negative PQ type generators.

State	PQ	Active
Turned "On" in instance dialog box	✓	✓
Specified as Induction or Synchronous in equipment table		✓
"Treat as PQ" NOT marked in instance dialog box		✓
PQ type in equipment table or "Treat as PQ" marked in instance dialog box	✓	
"Treat Generators as Negative Load" marked in analysis dialog box	✓	
"Treat Generators as Negative Load" NOT marked in analysis dialog box		✓

Synchronous machines

Under steady-state, the speed of a synchronous machine is proportional to the frequency of the armature current. As DC excitation is applied to the field winding, AC current flows through the armature winding. The armature is typically wound on the stator and is usually three-phase.

The exciter is a DC generator typically on the same shaft as the motor. Usually, the voltage and frequency at the armature terminals are fixed by the connected system. Typically, the generator cannot affect terminal voltage or frequency. The rotor must turn at a precisely synchronous speed.

Induction motor action must be used to get the synchronous machine started since the machine has no starting torque. To make a synchronous machine self-starting, a squirrel-cage winding called an amortisseur or damper winding is inserted in the rotor. The rotor then comes up almost to synchronism speed by induction-motor action with the field winding unexcited. If the load and inertia are not too great, the motor pulls into synchronism when the field winding is energized from a DC source (Fitzgerald, *Electric Machinery*, 325). SynerGEE does not currently model generator startup.

Shown below are the Wye and Wye-Gnd models of the synchronous generator.

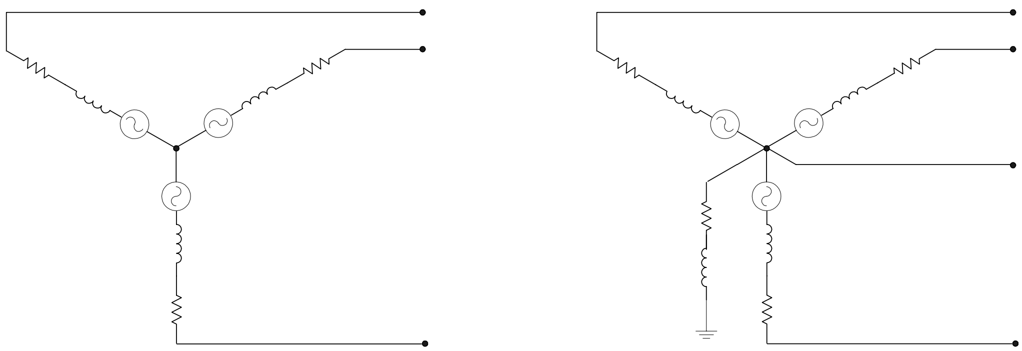


Fig. 11-1 *Three-phase synchronous generator models*

The back EMF of the synchronous generator is a three-phase balanced voltage vector. The magnitude of this voltage is adjusted by the exciter to achieve the desired terminal voltage. The angle (corresponding to the rotor angle) is adjusted by the governor to achieve the desired output power. The following is an expanded view of a grounded model.

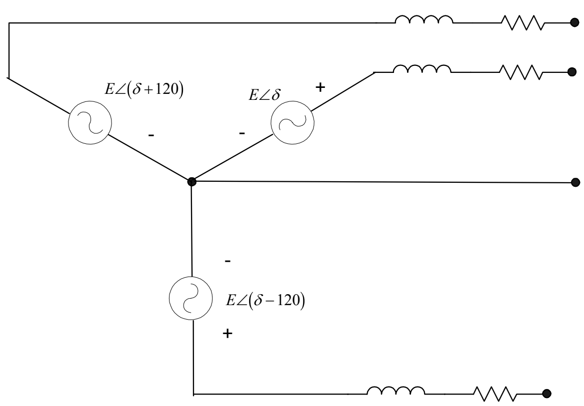


Fig. 11-2 *Synchronous generator model showing back EMF voltages*

The governor and exciter controls on the synchronous machine models have simple data requirements—their settings. However, the behavior of these controls during the load-flow analysis is complex. This behavior can be qualified as follows.

Metered Output	Response
Output power is below kW setting	$\delta\uparrow$
Output power is above kW setting	$\delta\downarrow$
Output voltage is below voltage setting	$E\uparrow$
Output voltage is above voltage setting	$E\downarrow$

Power metering on three-phase synchronous generators is always on all phases. Voltage metering is on a particular phase or on the positive sequence. The metering of a particular phase causes adjustments in magnitude and angle of the balanced three-phase back EMF of the machine. This phenomenon can aggravate an already unbalanced distribution system.

Induction machines

The induction machine model is complex. The shunt branch and explicit modeling of slip are necessary for motor starting studies. Slip is not necessary for static load-flow modeling of an induction generator and the shunt conductance and susceptance representing the core losses can be neglected. Thus, you have the following model.

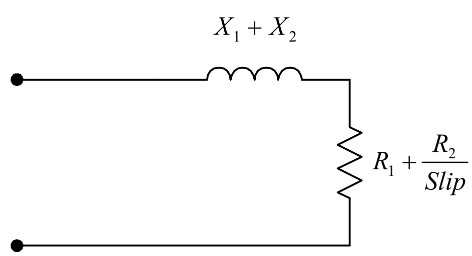


Fig. 11-3 *Single-phase equivalent of induction machine*

The values of the parameters in this figure are of no consequence. From the perspective of the load-flow, the generator model appears as follows.

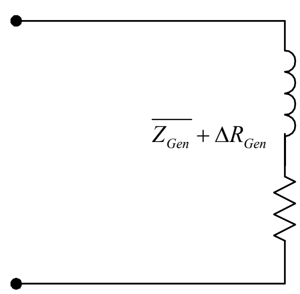


Fig. 11-4 *Reduced single-phase equivalent*

You can determine the impedance Z_{Gen} from the ratings of the generator. During the load-flow, you compare the output of the generator with its specified output and adjust the value of ΔR_{Gen}. This adjustment, in effect, changes the value of slip and is proper because the generator's reactance is held constant while the $R_2 / Slip$ portion of the resistance is changed. Once the load-flow is converged, the value of slip could be determined if you calculated R1 and R2 (based on rated speed, etc.).

The following parameters are required to describe the generator type.

(11-2)

kV_{Gen}	Rated Voltage
kW_{Gen}	Rated Output Power
pf_{Gen}	Rated Power Factor

It is also necessary to know whether the generator is a three-phase or single-phase unit.

Assuming that you have a three-phase unit, you can determine the rated current of the machine as follows.

$$I_{Gen} = \frac{kW_{Gen}}{\sqrt{3} * kV_{Gen} * pf_{Gen}} \angle -\theta_{pf} \qquad \left(\theta_{pf} = \cos^{-1} pf_{Gen}\right) \tag{11-3}$$

This current is out of the generator, therefore:

$$Z_{Gen} = -\frac{kV_{Gen}}{I_{Gen}} = \frac{-1000 * kV_{Gen}^2 * pf_{Gen}}{kW_{Gen}} \angle \theta_{pf} \tag{11-4}$$

Constant real and reactive power models

The nominal output of a generator is found from the ratings of the generator table record (equipment database) and the settings of the instance record (model database).

$$\begin{aligned} kW_{Output} &= \alpha * kW\,Contribution \\ kVAr_{Output} &= \alpha * kVAr\,Contribution \\ 100 * \alpha &= \text{Generator \% Contribution from Section/Generator} \end{aligned} \tag{11-5}$$

The generator output is divided over those phases selected in the Generator dialog box.

Generators in network load-flow

Generators are now recognized as negative constant power loads by network load-flow analysis. The generators are modeled at the center of the section with an artificial bus. If the generators are set up as synchronous or induction machines, they are viewed as PQ machines by network load-flow.

Additional notes

Fault analysis

During fault analysis, induction and synchronous machines are seen as reactances tied to ground through their grounding impedances. PQ generators do not contribute fault current during fault analysis.

Multiple generators

SynerGEE is a steady-state simulation tool. Because of this, there are no limits to the number of active generators that can be modeled. Only one generator can be placed on a section. You should remember that SynerGEE does not support frequency controls and complex exciter models.

Delta-connected generators

Although it is theoretically possible to build a generator with Delta-connected windings, it is not practical. This type of generator would promote a high level of circulating current. Also, the windings would have to be insulated to a much higher level with respect to the Wye-connected equivalent. This insulation would lead to cooling problems.

SynerGEE does support ungrounded Wye connections.

12
Load-Flow

Introduction

Balanced and by-phase load-flow analyses calculate current flows, voltage drops, losses, and loading of lines, equipment, switches, and protective devices. You can perform either type of load-flow analysis for a substation or selected feeders. If a substation is selected, all feeders of the substation are analyzed as well as the substation transformers, buses, equipment, and switches. There are no limits on the number of sections that can be included in a load-flow analysis. The calculated results for each section or substation bus can be saved to a database, or a detailed report can be produced. These reports can be viewed, saved, and printed. You can also create customized reports from results using the SSQL program and customized queries.

Overview of SynerGEE technology

The load-flow engine is object-oriented, driven by a robust algorithm used to enforce Kirchhoff's laws between objects representing power system devices. The objects respond to various messages supplied by the load-flow algorithm. They contain the complicated and highly nonlinear models for various types of distribution equipment such as voltage regulators, transformers, and switched capacitors.

The solution of radial power systems is achieved by using methods of network flows based on walks along directed-in and directed-out trees. These trees have arcs composed of objects designed to simulate various distribution system devices. Each object is connected radially so that it has one predecessor and possibly multiple descendants. The collection of independent objects reaches a state representative of the load-flow solution through an interaction of objects based on Kirchhoff's laws. This interaction is facilitated through the load-flow algorithm by two types of walks or object propagations.

The load-flow engine

The inward walk is used to accumulate current from the ends of the radial system to the source. The load-flow algorithm sends messages to device objects signaling them to collect the demand current from all of the devices connected to their downstream end. An object receiving this message takes this downstream current and propagates it through its particular by-phase model to form its own demand current. This demand current is eventually accumulated by the object's source. By sending these messages to objects in a proper inward order, the total current demand of the feeder can be accumulated. At the conclusion of this propagation, each device object has current flows for its source and load terminals.

The outward walk enforces Kirchhoff's voltage law. During this walk, the load-flow algorithm signals each device object to take the output voltage of its source object, use it for a source voltage, and determine a load or output voltage of its own. Each device is responsible for calculating its own internal voltage drop. At the conclusion of this walk, voltage drops have been calculated from the source of the feeder through all devices.

A load-flow iteration includes one inward and outward propagation along with a linearization step and a convergence query. During the linearization step, various loads are evaluated at their terminal voltage to develop a load current. Regulators are allowed to change taps and switched capacitors are allowed to trip and close. By separating these operations into a single step, the most complicated and nonlinear behavior of the distribution system model is isolated from the linear inward and outward propagations. This allows for a much more robust and stable load-flow engine.

The final step in the load-flow iteration is the convergence query. Devices that have undergone fluctuations in voltage or power above some threshold (or tolerance) value may signal the load-flow algorithm to continue with another iteration. Devices with controllers may send a similar signal if they change taps or states.

The load-flow algorithm is designed so that values of current stay fixed during voltage propagations and voltages stay fixed during current propagations. Thus during any one inward or outward propagation, either the current or voltage of a particular device object is one iteration old. As the load-flow progresses, however, the changes in device current and voltage become smaller and convergence is reached. It is essential to note that the load-flow engine simply directs inward and outward propagations of general arbitrary objects. By design, the engine is not capable of making a distinction between a transformer object and a generator object. All modeling associated with a particular device is completely contained within that device's object model. This allows for a detailed, robust, and realistic modeling of power distribution devices. For example, an object representing an Open-Delta bank of voltage regulators can be constructed with a detailed model of an autotransformer, a load tap-changer, a line-drop compensator, and metering equipment. This object is constructed with objects within SynerGEE much the same way that the actual regulator is built at the manufacturer and installed onto the distribution system. Device models can be added or modified without altering the load-flow engine.

The SynerGEE advantage

The radial load-flow engine of SynerGEE has many advantages over the traditional Gauss-Seidel and Newton-Raphson methods for two reasons. First, SynerGEE does not rely on a matrix representation or an abstract mathematical representation of the power system. This allows the model of a power system to simply be a collection of device models. Furthermore, there is no limit to the size of the system SynerGEE can model. Analysis can also be run directly from the database. The second advantage is that you can make a coupled three-, two-, or one-phase representation of power lines and power equipment. Unbalanced loading, long single-phase laterals, ungrounded systems, non-symmetrical transformer banks, mutual coupling, earth return, and device controller actions can all be modeled in a manner consistent with the actual construction and physical behavior of these devices. These advantages make SynerGEE a powerful tool for the simulation and analysis of power distribution systems. Traditionally, however, there have been two prohibitive aspects of the radial load-flow described above. These are the handling of loops and generators. Fortunately, Advantica Stoner's research has produced new mathematical theory, algorithms, and software to effectively model these items.

The current version of SynerGEE contains a robust algorithm for modeling looped distribution systems. This algorithm is built directly on top of the radial load-flow engine of SynerGEE so that all of the modeling detail associated with radial analysis is maintained. In the looped analysis, current flows are determined through switches that form loops. These flows are determined using a driving point admittance matrix as seen at the terminals of the loop tie switches. The matrix is used to determine the amount of current needed to inject into one side and extract from the other side of each switch so that the voltage drop across the switch is zero. At this point, the state of the looped distribution system can be determined from the state of the radial system imposed with these switch flows.

SynerGEE can also accurately model both synchronous and induction generators. The synchronous generators have excitor limits and a balanced three-phase back EMF. Induction generators are modeled with a variable resistive element representing slip. Both of these models are based on carefully designed models and are powerful tools for representing the behavior of these machines in unbalanced distribution systems. It can be important for you to simulate cases in which synchronous cogeneration facilities may aggravate the pre-existing imbalance of their distribution system.

The entire analysis engine within SynerGEE is by-phase. Modifications can be made to the data going into the engine and the data coming out to represent a balanced system.

By-phase load-flow

SynerGEE includes a very detailed by-phase analysis engine. During by-phase load-flow the engine accounts for the following:

- One-, two-, and three-phase lines and laterals
- Grounded and ungrounded areas
- Unbalanced loading
- Various V/I load characteristics

- Electric and magnetic field coupling
- Transformer, regulator, and capacitor connections
- Regulator and capacitor controls
- Active generators

Every device within SynerGEE is represented with a detailed by-phase model.

Balanced analysis

Balanced analysis is designed to produce useful information for the occasions when by-phase loading and phasing information is unknown, or for the occasions when you are not interested in seeing by-phase results. The analysis uses the same by-phase engine and detailed by-phase models as by-phase analysis.

In most situations, balanced analysis produces results representing a single-line model of the distribution system. Loads are averaged over the phases of the associated line section. Currents are averaged at section intersections.

To achieve the balanced analysis, SynerGEE follows the following steps.

- Loads are averaged over the phases present on sections
- The SynerGEE by-phase engine is used to analyze the system
- The resulting flows and voltages are vectorially averaged over valid phases

This approach allows SynerGEE to have one very sophisticated analysis engine and one set of models. The models account for coupling and line admittance for a three-, two- or single-phase grounded or ungrounded lines. The effects of long laterals are captured.

Convergence

After each iteration of the balanced or by-phase analyses, a check is made for convergence. The analysis is considered converged if these criteria have been met.

- The percent change in voltage at the end of every section during the last iteration is less than the convergence factor specified in model preferences. This criterion must be met for every phase in by-phase analysis.
- No regulator taps have changed during the last iteration.
- No capacitor modules have tripped or closed during the last iteration.
- Generators have met settings or reached field limits.

The analysis stops if the number of iterations exceeds the maximum number of iterations set in model preferences. In this case, a report is still generated with a note stating that the maximum number of iterations has been reached. Further information about voltages, currents, and power values during the last iteration is listed for each section.

The convergence and iteration limits are set in model preferences. These values are parts of a common group of default values. Please refer to the *Online Help* for more information about how to set default preferences.

Voltage drop calculations in balanced and by-phase analysis

In a balanced system, the voltage drop calculation can be expressed vectorially as follows.

$$V_{Ctr} = V_{Src} - \tfrac{1}{2} Z I_{Into} \tag{12-1}$$

This equation shows how the voltage in the center of the section is found from a line's source voltage, its impedance, and the current entering the line section.

By-phase analysis uses a three-phase impedance matrix to model distribution lines. Line voltage drops are calculated with the following vector equation.

$$\begin{Bmatrix} V_{Out}^{A} \\ V_{Out}^{B} \\ V_{Out}^{C} \end{Bmatrix} = \begin{Bmatrix} V_{Ctr}^{A} \\ V_{Ctr}^{B} \\ V_{Ctr}^{C} \end{Bmatrix} - \tfrac{1}{2} \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \begin{Bmatrix} I_{Out}^{A} \\ I_{Out}^{B} \\ I_{Out}^{C} \end{Bmatrix} \tag{12-2}$$

This equation represents the calculation used to determine the voltage at the outward or load end of a section given its center voltage, demand or output current, and the line series and mutual impedances. Some impedance values may be zero, based on the rules above for one-, two- and three-phase wire line sections.

Line loss calculations

Losses are calculated for sections and devices after the balanced or by-phase analysis has converged. These losses are determined from the difference in the complex power into the device and the complex power out of the device. The contribution of loads, capacitors, and generators are removed from this difference in the case of the line or cable portion of a section. Losses and the power into and out of ungrounded devices are always summed.

Losses for all devices are calculated by subtracting the real and reactive power out of the device from the power into the device. The line connection and the analysis being run have an impact on the loss calculation.

Calculations for losses

Line in balanced analysis	$S_{Loss} = S_{Into} - S_{Load} + jCap + S_{Gen} - S_{Out}$
Gnd Line in by-phase analysis	$S^k_{Loss} = S^k_{Into} - S^k_{Load} + jCap^k + S^k_{Gen} - S^k_{Out} \;\; k = Phase$
Ungnd Line in by-phase analysis	$Total\, S_{Loss} = \Sigma S_{Into} - \Sigma S_{Load} + j\Sigma Cap + \Sigma S_{Gen} - \Sigma S_{Out}$

Notice that the losses represent the total losses of the line for ungrounded cases. The concept of losses per-phase is ambiguous for ungrounded devices. SynerGEE always averages by-phase values of power in these cases over the number of phases.

Values of complex power *S* are calculated using the necessary voltage and current values. The power into a phase of a grounded section, for example, is calculated as follows:

$$S_{Into} = kV_{Into} I^*_{Into} \tag{12-3}$$

Values for the various voltages and currents associated with a line, its loads, and attached devices are maintained in the temporary database during analysis. Values such as the voltage in the center of a line or the current into a load can be queried out using SSQL.

SynerGEE load-flow analysis is performed using only voltages and currents. Nonlinear loads are linearized during each iteration. The values of power flow are calculated after the load-flow solution has been reached. This supports a very robust and precise calculating engine within SynerGEE.

Ratings and exception flagging

The continuous and emergency ratings of conductors are used to calculate percent loading and exception coloring. These values can be taken from the manufacturer's data, your company standards, or from criteria used in cable selection. The values are used for comparison of conductor loading for the analytical outputs.

Radial load-flow algorithms

The next several sections describe the load-flow algorithms in a general fashion. The solution of radial power systems is found using methods of network flows based on walks along directed-in and directed-out trees having arcs composed of the devices on sections such as power lines, regulators, and transformers. Each section is connected in a radial fashion so that it has one predecessor and possibly multiple descendants. There are two types of walks, an inward walk and an outward walk. An inward walk is achieved by starting at the end component and passing inward so that every device is passed over before its predecessor. An outward walk is achieved by starting at the source that is either the substation or feeder and passing outward so that every section's predecessor is passed over before the section itself. These walks can be shown in the following simple feeder. Sections on the feeder are labeled with letters.

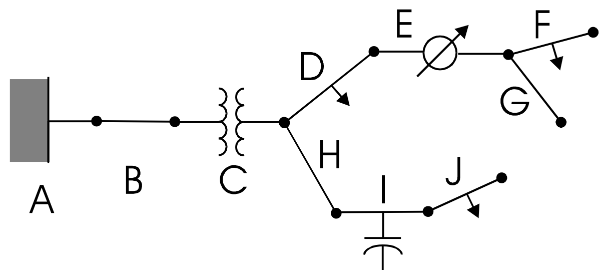


Fig. 12-1 *Sample Feeder*

An outward walk is made by starting at the feeder's source A and alphabetically descending B, C, through J. An inward walk is similarly made by starting at J and alphabetically ascending to A.

A few things should be noted. First, inward and outward walks are not unique. There are a number of ways to order the propagation from section to section so that the walk definition described earlier is satisfied. Second, the feeder source (labeled A in this case) is always unique. The source component is the only component not having a predecessor. Finally, the end component (labeled J, in this case) is not unique. It is convenient to make the end component of the inward walk match the last component passed over along the outward walk.

The load-flow solution is achieved by iterating over a process composed of three steps. The steps that involve the different types of walks previously described are discussed below.

Load-flow outward propagation

The first iteration step involves load-flow outward propagation. The purpose of the outward propagation is to transmit the fixed voltage specified at the feeder or substation source through each of the components of the feeder. When a device is recognized along the outward walk, it subtracts its voltage drop from its predecessor's voltage to get a voltage at its outward end. This voltage is then available to be used as a source voltage by the device's descendants when they are recognized during the remainder of the walk.

Shown below is a line model recognized during an outward propagation.

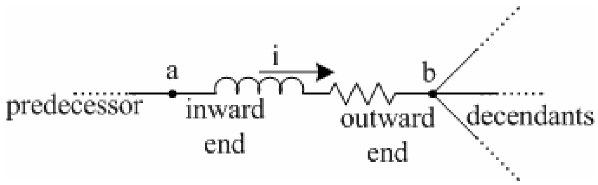


Fig. 12-2 Section during outward propagation

During the propagation, the device's predecessor is recognized first and it determines a voltage at its outward end that would be node *a* in the above figure. When the above device is recognized, it uses the voltage supplied by the predecessor and the voltage drop due to the current *i* to determine the outward voltage at node *b*. This voltage is available for the descendants of the device to use when they are recognized later in the propagation.

Notice that the current flow through the device is constant during outward propagations. The value of current is determined during inward propagations. It follows that during the first outward propagation, component currents have not been established.

This outward propagation process is used to emulate Kirchhoff's voltage law. At the conclusion of the propagation, the sum of the voltages and voltage drops around any path of the feeder is zero.

Load-flow inward propagation

The second step of the iteration process involves load-flow inward propagation. During the outward propagation, values of current were held constant and information about voltage was transmitted outward along the feeder. During the inward propagation, voltage values are held constant and information about current is transmitted inward along the feeder using an inward walk. Your sample device is used below to exemplify this process.

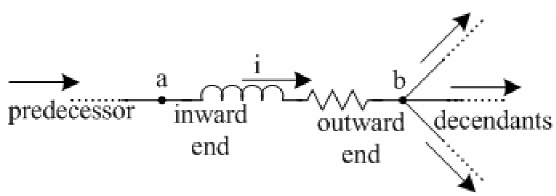


Fig. 12-3 Section during inward propagation

During the inward walk, each of the descendants of the device are recognized. They convert their total demand current along with any internal load or injections to determine their value of inward current. The current is then passed to the device *ab* and cumulatively forms the basis of the device's outward or demand current. When the above device is recognized, this current is stored for use in the next outward propagation and is combined with any internal load current and passed to the device's predecessor.

This process of propagating load current inward toward the feeder's source is used to satisfy Kirchhoff's current law. At the completion of the propagation, the total current into either the inward or outward end of any device is zero.

Linearization

The third step of the iteration process includes linearization. The processes outlined in the previous two steps are linear processes. That is, nonlinear operations on or between the state variables of current and voltage are never performed during the inward or outward propagations. During the linearization step, however, currents and voltages obtained from the most recent inward and outward propagation are used to perform a number of operations. Some of these operations are listed below.

- Component loads are evaluated with the most recent component voltage. The load current achieved from the linearization step is maintained and used to represent the load during the inward propagation of the feeder. The current remains constant until the next linearization step.
- Active devices such as switched capacitor controllers and regulator LTC's are activated. Regulators may change taps and switched capacitor modules may be tripped or closed.
- Devices may report back the change-in-state from the last linearization step due to the last iteration of inward and outward propagations. This determines if the state of the feeder is satisfactorily near the load-flow solution.

Putting the steps together

The steps outlined in the previous section are invoked during each iteration of the load-flow analysis. The diagram below demonstrates how the load-flow steps fit together within iterations to determine the solution of a feeder.

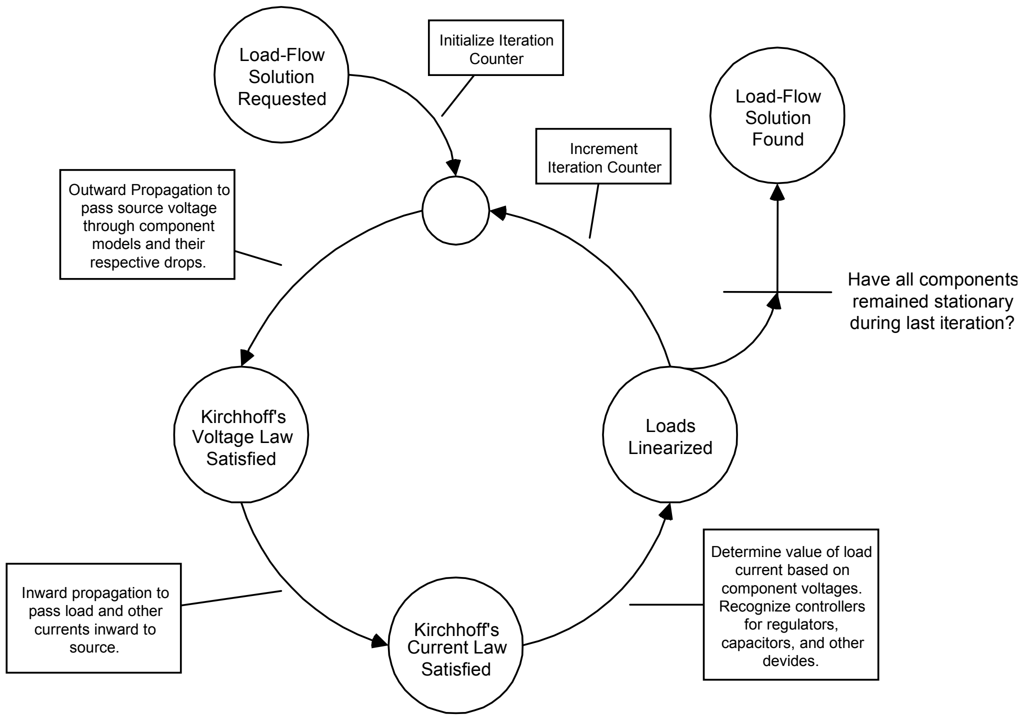


Fig. 12-4 *Load-flow diagram*

Characteristics of detailed by-phase analysis

Analysis results are accurate reflections of actual system physics with a detailed analysis tool like SynerGEE.

Voltage rise on lightly loaded phases

The line models in SynerGEE account for conductor characteristics, mutual magnetic and electric coupling, and line capacitance among the phase conductors, the neutral conductor, and the earth. This detailed modeling can have interesting effects on very unbalanced multi-phase, grounded lines. Typically, the effects of coupling on line voltage are slight in these situations. However, the modeling of the earth return can cause the voltage on lightly loaded phases to rise significantly. The following is a simplified diagram of a long three-phase line.

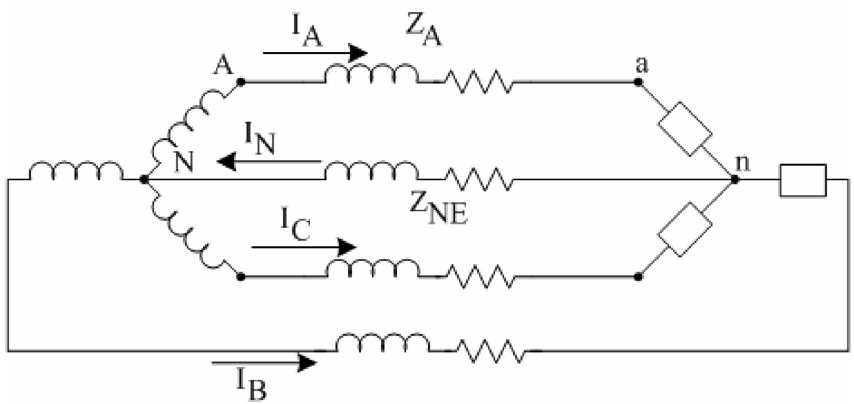


Fig. 12-5 A long three-phase line with stiff source

The impedance Z_{NE} represents the effective impedance of the neutral conductor and the earth. If this diagram represents a SynerGEE line section, the value V_{AN} is the inward or source voltage and V_{an} is the outward or load end voltage. Notice that the voltages on grounded sections are referenced to the local neutral voltage.

The voltages around the loop on phase A can be tabulated as follows.

$$V_{AN} = Z_A I_A + V_{an} + Z_{NE} I_N \tag{12-4}$$

You can use vector diagrams to show how the phase voltage rises in the case when phase A is unloaded and the others are loaded. Here is a vector diagram showing the source voltage for phase A and the load current in the other two phases.

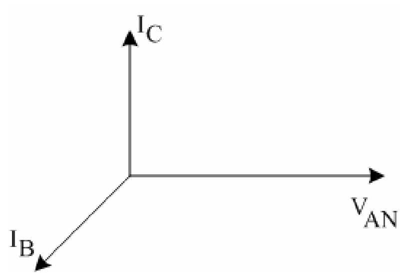


Fig. 12-6 Currents and voltages in the simple circuit

The neutral current is the sum of the phase currents.

$$I_N = I_A + I_B + I_C \tag{12-5}$$

Here is the vector addition for neutral current in this example.

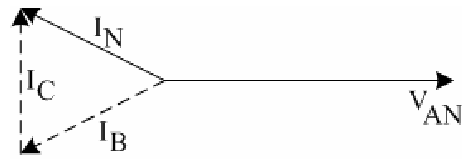


Fig. 12-7 Vector diagram used to get neutral current

You can now determine the voltage at the end of the line on phase A.

$$V_{an} = V_{AN} - Z_A I_A - Z_{NE} I_N \quad \textbf{(12-6)}$$

The vector version of the equation is as follows.

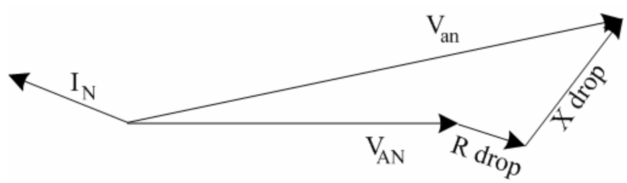


Fig. 12-8 Vector diagram to get outward voltage

As such, it is evident that the voltage on phase A at the end of the line is higher than at the source. There are three reasons for this.

- Load currents are shifted in time (120 degrees here)
- The neutral and earth impedance are modeled
- Voltages are measured from phase to remote ground just like a meter reads

There are many more ramifications of this phenomenon. For example, a long lateral with a moderate load on phase A, a light load on B, and a heavy load on C may behave in an unexpected manner. If the line is balanced by shifting load from phase C to phase B, the voltage at the phase A end will probably get worse.

Negative losses on individual phases

It is possible to see negative real losses on an individual phase after running a by-phase load-flow. The following are the results from a feeder analyzed by SynerGEE.

Feeder kW Losses		
Phase A	**Phase B**	**Phase C**
34 kW	89 kW	-1 kW

The loss value is determined by subtracting the total load on each phase from the power into the feeder. The negative value on phase C is a result of such a simple calculation being used on a complex system.

Certainly it is impossible for a passive device, such as a line, to produce negative losses. The reason that you can see negative values, such as the one in the table above, is in the way that results are reported.

In a power distribution system model, phases are not independent. Phases are coupled through:

- Magnetic and electric field coupling
- Line-to-line loads, devices, and windings
- Common return path through neutral and earth

While by-phase results are sometimes considered as if each phase were an independent model, the truth is that the distribution system phases are a complex network of dependent voltage drops and current coupling. The next few sections show how power loss is calculated and why the I^2R loss concept is not valid in by-phase modeling.

The roots of I^2R

The following is a very simple representation of a three-phase line.

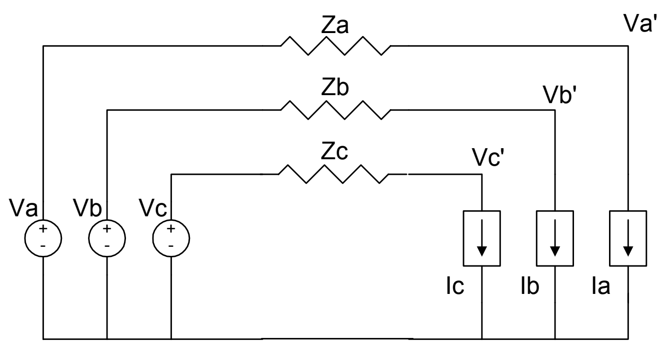


Fig. 12-9 *Simple three-phase line*

Looking at phase A:

$$V_a^{'} = V_a - I_a Z_a \tag{12-7}$$

The power into and out of the section can be expressed as follows.

$$S_{a,Into} = V_a I_a^* \tag{12-8}$$
$$S_{a,Out} = V_a^{'} I_a^* = \left(V_a - I_a Z_a\right) I_a^* = V_a I_a^* - \left|I_a\right|^2 Z_a$$

Power loss is the difference between power into the section and power out of it. For example,

$$S_{a,Loss} = V_a I_a^* - \left(V_a I_a^* - \left|I_a\right|^2 Z_a\right) = \left|I_a\right|^2 Z_a \tag{12-9}$$

As such, the loss is the square of the current magnitude multiplied by the line impedance. Similar derivations result for the other two phases since all three phases operate independently.

Loss on a section with neutral/earth return impedance

Now, take the previous section and make it more realistic with an impedance in the return path. This impedance represents the impedance of the neutral conductor and the impedance through any earth return path. The model is as follows.

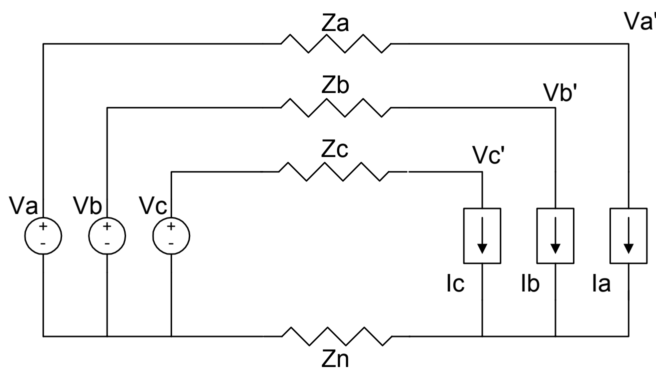


Fig. 12-10 *Three-phase line with neutral / earth return impedance*

The voltage at the end of the line is measured locally. This means that the drop through the neutral must be accounted for. For example:

$$V_a' = V_a - I_a Z_a - (I_a + I_b + I_c) Z_n \tag{12-10}$$

The power into and out of the section is as follows.

$$\begin{aligned} S_{a,Into} &= V_a I_a^* \\ S_{a,Out} &= (V_a - I_a Z_a - (I_a + I_b + I_c) Z_n) I_a^* \\ &= V_a I_a^* - |I_a|^2 Z_a - (I_a + I_b + I_c) I_a^* Z_n \end{aligned} \tag{12-11}$$

And, the power loss is as follows.

$$S_{a,Loss} = |I_a|^2 Z_a + (I_a + I_b + I_c) I_a^* Z_n \tag{12-12}$$

You can re-form the equation to get the following.

$$S_{a,Loss} = |I_a|^2 (Z_a + Z_n) + (I_b + I_c) I_a^* Z_n \tag{12-13}$$

Consider $I_\alpha = I_b + I_c$. The second term in the above equation will have a negative value when $\delta_\alpha - \delta_a + \theta_n < 0$. Under certain conditions of high neutral path impedance and large imbalance, the second term may cause the overall $S_{a,Loss}$ to be negative.

Complete this exercise by determining the total loss on all phases in your example. Replacing $I_a + I_b + I_c$ with I_n you found the following.

$$S_{a,Loss} = |I_a|^2 Z_a + I_a^* I_n Z_n$$
$$S_{b,Loss} = |I_b|^2 Z_b + I_b^* I_n Z_n$$
$$S_{c,Loss} = |I_c|^2 Z_c + I_c^* I_n Z_n \tag{12-14}$$

You can add the losses on all three phases to get the following.

$$S_{Total} = |I_a|^2 Z_a + |I_b|^2 Z_b + |I_c|^2 Z_c + I_n Z_n \left(I_a^* + I_b^* + I_c^*\right) \tag{12-15}$$

You know that $I_a^* + I_b^* + I_c^* = I_n^*$, so:

$$S_{Total} = |I_a|^2 Z_a + |I_b|^2 Z_b + |I_c|^2 Z_c + |I_n|^2 Z_n \tag{12-16}$$

This equation indicates that the total of the real losses will always be positive.

This example demonstrated the effects of the neutral return path on the calculation of by-phase losses. There are many more modeling effects like mutual coupling, transformer windings, line-to-line load models, and line admittances that also affect losses.

Due to the complexity of the power distribution model under by-phase analysis, loss should never be calculated using the I^2R formula. Loss should always be determined as the difference between the power into a device and the power coming out of the device. In some cases, the real loss value for some phases may be negative. The total of the by-phase real loss values must always be positive.

Conservation voltage reduction analysis

Conservation voltage reduction (CVR) helps you evaluate feeder performance as a function of feeder voltage. It makes successive load-flow analysis runs from the lowest specified feeder voltage to the highest. You can specify the step size or voltage interval. You can also list results by the feeder voltage for each feeder. Report results can easily be copied to a spreadsheet for plotting and manipulation.

For each feeder analyzed reported information may include:

- Feeder volts – The range specified when the application was launched. Values are listed in volts.
- Drop – The largest drop in the feeder is listed in the report. The voltage drop is calculated by comparing the feeder voltage to a section voltage, as follows.

$$V_{Drop} = |V_{Fdr}^1| - |V_{Sect}^1| \tag{12-17}$$

- Low volts – The lowest voltage in the feeder. The drop value and the low volts value should add up to match the feeder voltage value.
- High volts – The highest section voltage in the feeder.
- kVA – Feeder kVA demand at a specified feeder voltage.
- kW – Feeder real power demand at a specified feeder voltage.
- Amps – Feeder load current.
- pf – Power factor of feeder demand at a specified feeder voltage.
- Pct loss – Feeder loss as a percentage of feeder kW demand.
- VLT exceptions – Instances where one or more voltage exceptions exists at the specified feeder voltage.
- OLD exceptions – Instances where one or more overloaded lines or devices exist.
- CTL exceptions – Instances where at least one regulator is sitting at its minimum or maximum tap.

Example

An example of the benefits of CVR analysis follows. It shows a study to justify the installation of one or two regulators on a feeder.

As with all SynerGEE analyses, you can copy and plot values from the report. The following image contains the results from the CVR runs that were copied and plotted using a spreadsheet package. This plot shows the range of low voltages as a function of feeder voltage.

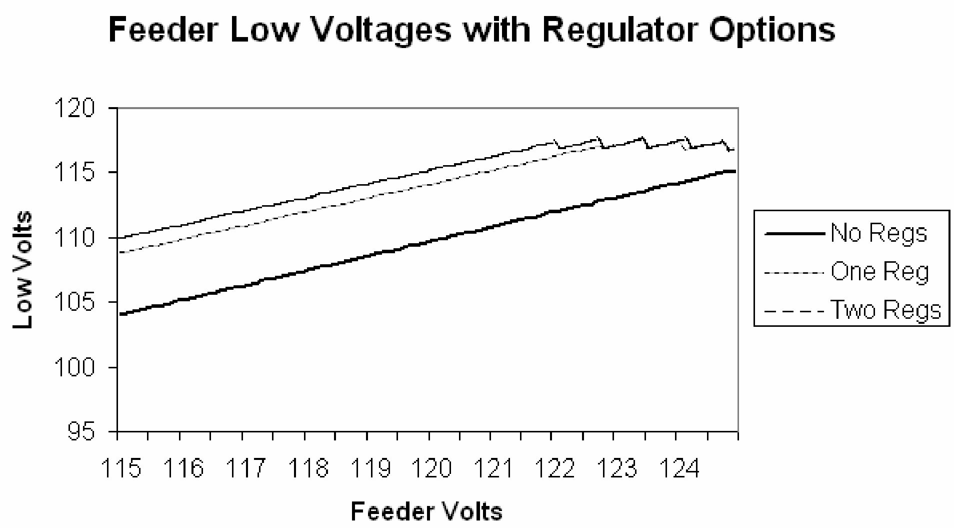


Fig. 12-11 Plot of feeder low voltage

You can see the improvement in low voltage due to regulation. You can also see the effect of voltage settings on the regulator and the drop from the regulator to the end of the feeder at higher feeder voltages.

The following plot addresses high voltage problems.

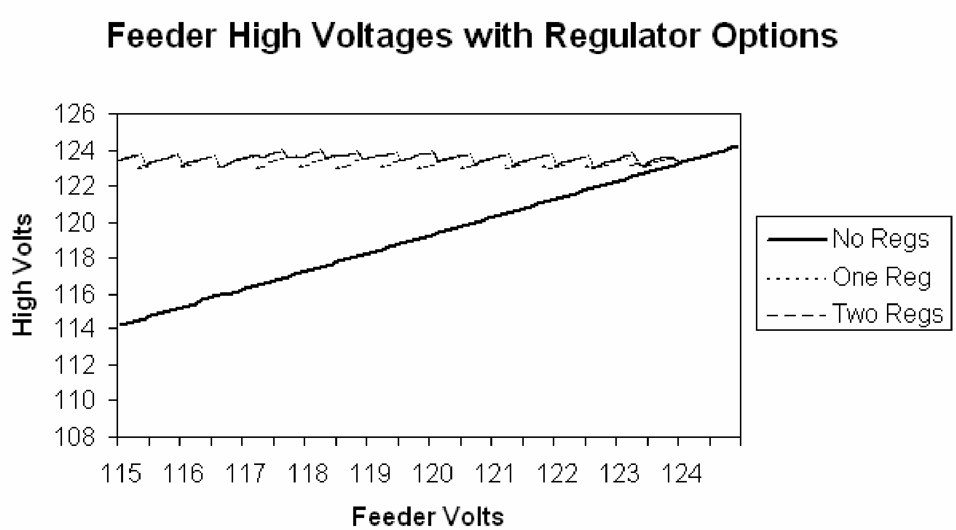


Fig. 12-12 Plot of feeder high voltage

Again, the effect of the regulator voltage settings is apparent. This study may be useful to justify the installation of one regulator, while demonstrating that the benefits of a second regulator would not be worth the cost.

Looped load-flow analysis

The modular SynerGEE analysis system is built on top of a powerful load-flow engine. One component of this system is looped analysis, which enables SynerGEE users to model distribution networks or distribution systems with loops. The following section discusses the SynerGEE load-flow engine and the implementation of looped analysis. Some results from an example analysis are also given.

Load-flow foundation

The purpose of load-flow is to determine the state of a power system model. SynerGEE is a distribution analysis tool and the model is composed of substations, feeders, lines, and various devices such as transformers, capacitors, and generators. The model description consists of parameters for these devices as well as descriptions of loads and values for feeder or substation transformer voltages.

The state of the load-flow model resulting from load-flow analysis is described by voltages, current flow, and positions of controllers for regulators, switched capacitors, and other devices. Once obtaining the load-flow solution, SynerGEE also determines the values of power flow, losses, and other parameters throughout the model.

The foundation of SynerGEE is constructed with a very sophisticated by-phase power distribution simulation engine. Within this engine are the detailed by-phase models of distribution system devices. These devices are modeled using an object-oriented scheme of components that are constructed in computer code like the actual devices are manufactured at the factory. The step voltage regulators, for example, within SynerGEE are modeled with a detailed representation of an autotransformer, a load tap-changer, PT and CT, a voltage regulating relay, and a line-drop compensator. These regulators can be put into banks to form Deltas, Open-Deltas, Wyes, and other connections. Each device model in SynerGEE is constructed so that current flows, voltage drops, and control operation closely represent the by-phase behavior of these devices in your distribution system.

SynerGEE also contains a collection of powerful algorithms for finding load-flow, fault, and other analysis solutions. These algorithms are modular and are used to enforce Kirchhoff's current and voltage laws between the object-oriented device models. The most fundamental of these algorithms is the one dedicated to radial load-flow analysis. This algorithm accommodates specific methods of handling Kirchhoff's laws and ensures that loads and device models are properly evaluated. The load-flow engine has been the focal point of SynerGEE development. It entails the most accurate and robust by-phase load-flow algorithm available. The radial load-flow forms the backbone to other important analysis algorithms within SynerGEE such as looped analysis.

SynerGEE load-flow and looped analysis

Looped analysis is an algorithm driven by the radial load-flow engine of SynerGEE. When using looped analysis, you are able to select a set of switches as tie switches. If these switches were closed, loops would be formed in the system. Looped analysis uses radial analysis to solve the system with the tie switches open. The algorithm then determines the current flow through each tie switch so that the voltage on each side of the switch is exactly the same. Once the voltage drops across the tie switches are zero, the solution of the set of radial systems is the same as the solution of the looped system or network. The benefit of this type of analysis is that all of the detailed by-phase models supported by SynerGEE's radial analysis are supported in the looped analysis. For example, the self- and mutual-electric and magnetic fields of one-, two-, and three-phase lines are evaluated. Controllers for individual regulators connected in Open-Delta banks are handled. By-phase loads with various connections and V/I curves are analyzed. And, the controller for switched capacitor banks connected to a single phase can be tripped or closed.

Example

SynerGEE still analyzes feeders as radial when looped analysis is selected. However, the program does determine a current flow through the tie switch of the loop so that the voltage on both sides of each switch is the same. The current representing the switch flow is directed out of the section on one side of the switch and into the section on the other side. When the switch current is determined so that the voltages on both sides of the switch match, the radial load-flow solution matches the network solution.

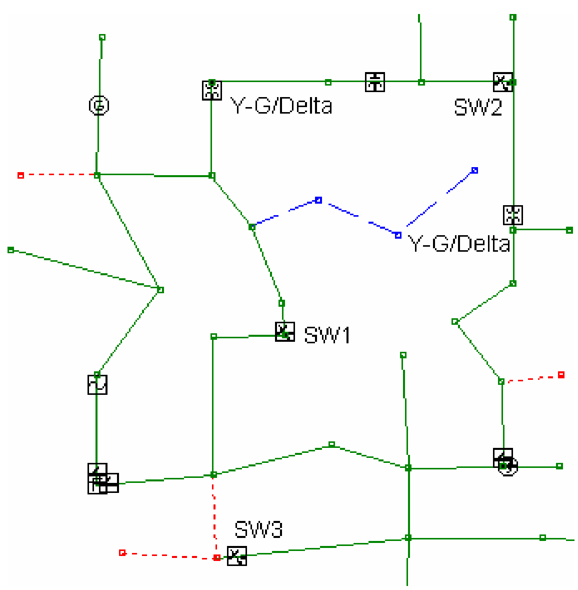


Fig. 12-13 *Example system*

The example system demonstrates advantages to SynerGEE's by-phase looped analysis that cannot be found in any other distribution analysis software. The system has loop tie switches forming loops between two feeders. Each 12.47kV feeder contains a Yg/Δ transformer bank that produces a 24.9 kV ungrounded secondary voltage. A switch ties the two feeders forming a loop through the 12.47 kV grounded and 24.9kV ungrounded systems. A second switch ties the feeders at a second location in the 12.47 kV portion. Finally, a third switch forms a loop between single-phase laterals of the second feeder.

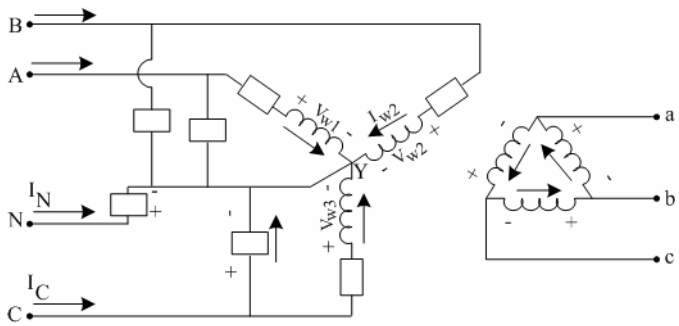


Fig. 12-14 *Wye-Gnd / Delta transformer*

The analysis of these two feeders and tie switches is complex for a number of reasons. First, the system contains a three-phase trunk and many one-, two-, and three-phase laterals, some of which are ungrounded. Second, there are many complex devices in these feeders such as an Open-Delta voltage regulator bank, switched capacitors connected in Delta, and two transformer banks. Finally, the load-flow solution is found with the tie switches placed simultaneously within three distinct areas.

The first switch, SW1, connects two grounded line sections. The algorithm must determine current flow through the switch and also neutral or earth return flows through the sections forming the loop. The second switch ties two ungrounded sections. Even though there are three blades to the switch connecting three wires, there are only two independent phases. (The voltages are measured line-line on ungrounded systems and any voltage can be found from the other two.) Finally, the last switch connects a fairly long single-phase lateral.

SynerGEE uses a unique approach to form a non-singular electrical equivalent that can be seen by looking into the tie switches. This allows the powerful by-phase load-flow and the looped analysis to form a modular and complete method for solving looped systems.

The table below summarizes the voltages on each side of the switch following a SynerGEE by-phase load-flow analysis without analyzing loops.

Voltages on each side of a switch with SynerGEE by-phase radial load-flow			
		One side	**Other side**
SW1	A	118.3	120.7
	B	116.9	118.7
	C	116.4	122.9
SW2	A/B	117.3	124.3
	B/C	117.6	124.7
	C/A	115.4	124.8
SW3	C	122.3	120.9

These numbers indicate a significant voltage difference across each phase of each tie switch.

You can now analyze the system with looped analysis to account for the loops formed by the tie switches. The next table summarizes the voltages on each side of the switches after this analysis.

Voltages on each side of a switch with SynerGEE by-phase looped analysis			
		One side	**Other side**
SW1	A	122.0	122.0
	B	122.2	122.2
	C	121.9	121.9
SW2	A/B	120.9	120.9
	B/C	119.5	119.5
	C/A	122.2	122.2
SW3	C	121.5	121.5

The controller for the switched capacitor and the line-drop compensator of the regulator both responded to their line voltage and current values in the looped analysis. The switched capacitor closed in both cases. The voltage regulator bucked one tap on one of the two units forming the Open-Delta.

Benefits of SynerGEE load-flow

Classical matrix methods such as Gauss-Seidel or Newton-Raphson are not utilized within SynerGEE load-flow or looped analysis. This allows SynerGEE to take advantage of object-oriented modeling and provide a series of benefits, including:

- The incorporation of coupled by-phase models so that lines, laterals, unbalanced loads, transformer windings, and other devices can be properly modeled.

- Analysis with no limit to the number of sections, devices, or loop tie switches that can be analyzed.

The attention to detail, ease-of-use, and reliable results represented by the SynerGEE load-flow engine are characteristic of Advantica Stoner's effort to provide the best modeling and analysis software available.

Wandering laterals

Often, a three-phase line drops off a phase to serve the single phase loading of a subdivision. After serving the subdivision load, the single-phase line connects back to the main line. The following is an example of a line that feeds a subdivision with the center phase.

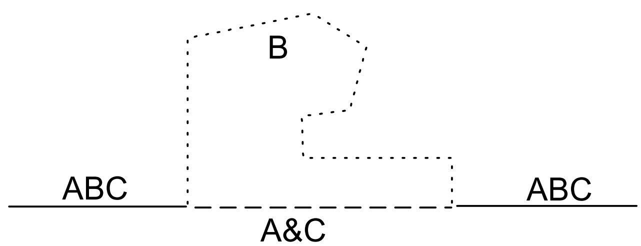


Fig. 12-15 *Wandering lateral*

SynerGEE is equipped to model wandering laterals, using a special switch called a wandering lateral tie switch. These switches are similar to loop tie switches, and must be used since the SynerGEE model topology is based on sections, not phases. Topologically, the system above looks like the following to SynerGEE:

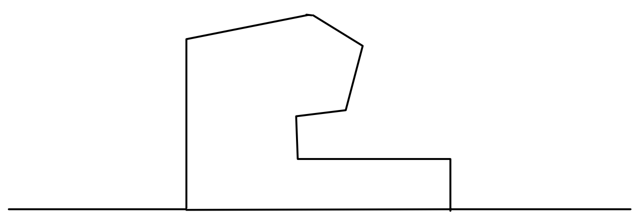


Fig. 12-16 *Topology of wandering lateral*

As you can see, there is a loop, which is prohibited under standard radial system modeling. Wandering laterals must be modeled using the tie switches. Like loop tie switches, the wandering lateral tie switches are open switches that are treated as closed when running a fault or load-flow analysis with the wandering lateral option enabled. The switch is generally placed on the last section of the lateral, where it reconnects with the trunk, as follows.

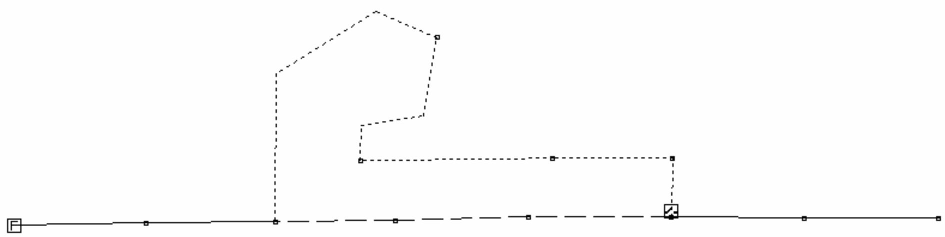


Fig. 12-17 Wandering lateral tie switch in SynerGEE

For the purpose of modeling, the switch avoids the formation of a loop. However, if you have the wandering lateral option selected in your model preferences, this switch is treated as closed during analyses.

Wandering lateral tie switches must always be placed at the outward connecting section of the wandering lateral. If a tie switch is not placed on a wandering lateral, loop errors result in SynerGEE. Also, if the tie switch is improperly placed, phasing errors show up when running load-flow or fault analysis with wandering laterals.

To demonstrate the analysis of wandering laterals, consider the same feeder with the following kW flow values, after a load-flow run. Initially, the model started with a 1500kW three-phase load at the end of the feeder, and a 500kW load about halfway on the wandering lateral.

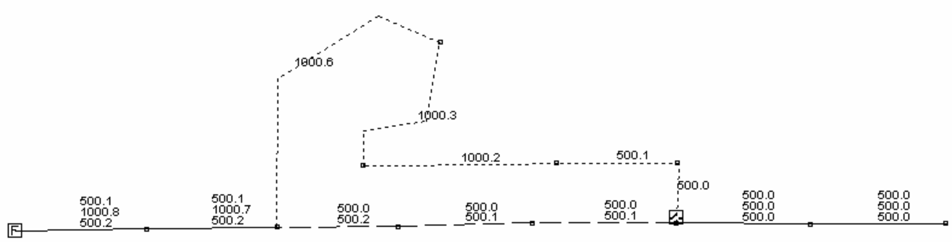


Fig. 12-18 kW flows with wandering lateral

Notice the 1000kW on the center phase. It flows from the source, along the wandering lateral to serve the 500kW load there. The remaining 500kW flows around and back to the trunk where it serves the three-phase load. The 500kW on the outside phases flows right from the feeder to the three-phase load.

To further demonstrate the capabilities of the software and the placement of wandering lateral tie switches, you could add a second wandering lateral.

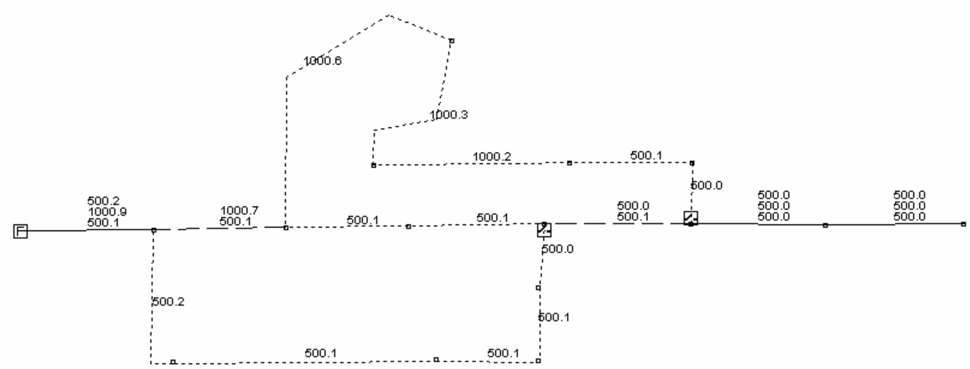


Fig. 12-19 *A second wandering lateral*

No load was added to the model. Notice the power flow into the first wandering lateral. The second wandering lateral takes another single-phase flow. The center of the diagram shows power flow on three single-phase lines. The flows join back up to serve the three-phase load.

Analysis using wandering laterals uses the fully detailed by-phase load-flow model and engine.

Network load-flow

Powerful methods for analyzing electric power transmission networks have grown from their inception in the early 1960's. Newton-Raphson and Gauss-Seidel methods are taught in most power engineering programs and are familiar to most engineers. These methods have been applied to simplified distribution system models.

Methods for analyzing electric power distribution systems began in the mid 1970's. Over the years, the efforts of Advantica Stoner have contributed significantly to this technology through the SynerGEE simulation engine. The engine is based on a class of distribution analysis methods referred to as the "network flow methods." In the following section, Advantica Stoner's method will be contrasted with the Newton-Raphson load-flow method.

The SynerGEE approach

The SynerGEE simulation engine facilitates the detailed by-phase modeling of distribution systems and their equipment. The engine is built from an innovated object-oriented design which focuses on the best ways to model distribution system devices, not on matrix-based modeling limitations.

This design allows SynerGEE to wrap up very extensive device models. Every model has terminals that respond to voltage and current conditions just like the real devices. When the load-flow application is run, it does not see lines, regulators, and capacitors. It sees distribution power system devices with interconnected terminals. When the load-flow applies voltages or demand current to a device like a regulator, that device handles its own internal workings.

As a model example, the regulator model is constructed internally like a real voltage regulator. Models exist for the autotransformer, the load tap-changer, the tap controller, and the line-drop compensator. These components are all "wired" together within the software like a real step voltage regulator is built at the factory. The regulator is able to look at its terminal conditions and its settings and possibly raise or lower a tap. The regulator model ratchets up and down during the load-flow like a real regulator in a distribution system.

By building realistic device models, Advantica Stoner produces more effective software and provides a more realistic tool. The SynerGEE approach decouples the engineering analysis routines from the device models. This allows a more focused concentration on analysis results, rather than specific device models. For example, when Advantica Stoner designed fault voltage analysis, it could concern itself primarily with fault voltages, rather than the proper LDC modeling when working on the voltage regulator model. Designers of Newton-Raphson based programs cannot make this separation because they need all of their equations in the Jacobian in order to reach the correct solution.

Advantica Stoner's detailed models are based on manufacturer schematics, and are not limited to passive admittance values like models in a Newton-Raphson environment. Designers of SynerGEE are proud to acknowledge that many users with extensive models have reported model voltages within one volt of measured values, field regulator taps matching the SynerGEE values to within one tap, and imbalance within a percent of that predicted by SynerGEE.

General Gauss-Seidel approach

From the Ybus matrix, you can find the current into any bus as:

$$I_k = \sum_{j=1}^{NBus} Y_{kj} V_j \tag{12-18}$$

But the power into the bus can also be expressed fundamentally as:

$$I_k = \frac{S_k^*}{V_k^*} \tag{12-19}$$

You can combine these equations through a substitution of the current variable. You can also pull one element out of the summation so that you can isolate the voltage on bus "k".

$$\frac{S_k^*}{V_k^*} = Y_{kk} V_k + \sum_{\substack{j=1 \\ j \neq k}}^{NBus} Y_{kj} V_j \tag{12-20}$$

Solving for the voltage:

$$V_k = \frac{1}{Y_{kk}} \left(\frac{S_k^*}{V_k^*} - \sum_{\substack{j=1 \\ j \neq k}}^{NBus} Y_{kj} V_j \right) \tag{12-21}$$

By-phase Gauss-Seidel

Again, start from the Ybus matrix to establish the current into bus "k".

$$\{I\}_k = \sum_{j=1}^{NBus} [Y]_{kj} \{V\}_j \tag{12-22}$$

You can write a matrix expression for the power into the bus.

$$\{I\}_k = \left(\underline{U} \{V_k^*\} \right)^{-1} \{S_k^*\} \tag{12-23}$$

You can combine these equations through a substitution of the current variable. You can also pull one element out of the summation so that the voltage on bus "k" is isolated.

$$\left(\underline{U} \{V_k^*\} \right)^{-1} \{S_k^*\} = [Y]_{kk} \{V\}_k + \sum_{\substack{j=1 \\ j \neq k}}^{NBus} [Y]_{kj} \{V\}_j \tag{12-24}$$

Solving for the voltage:

$$\{V\}_k = [Y]_{kk}^{-1} \left(\left(\underline{U} \{V_k^*\} \right)^{-1} \{S_k^*\} - \sum_{\substack{j=1 \\ j \neq k}}^{NBus} [Y]_{kj} \{V\}_j \right) \tag{12-25}$$

General Newton-Raphson approach

The Newton-Raphson method can be used to analyze a distribution system that is modeled with transmission system-style components. The reason for this can be seen in the stages of development of the Newton-Raphson matrices for a system.

Power flow relationships are expressed as follows.

$$P_i = \sum_{n=1}^{NBus} \left| Y_{in} V_i V_n \right| \cos(\theta_{in} + \delta_n - \delta_i) \quad Q_i = \sum_{n=1}^{NBus} \left| Y_{in} V_i V_n \right| \sin(\theta_{in} + \delta_n - \delta_i) \tag{12-26}$$

Next, the linear admittance relationships are expressed in matrix form.

$$\{\underline{I}\} = [YBus]\{\underline{V}\} \tag{12-27}$$

Partial differential equations are formed from the power flow relationships and the resulting equations are used to express the approximated linear relationship between power and voltage for the given iteration.

$$
\begin{Bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_N \\ \Delta Q_2 \\ \vdots \\ \Delta Q_N \end{Bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_N} & |V_2|\frac{\partial P_2}{\partial |V_2|} & \cdots & |V_N|\frac{\partial P_2}{\partial |V_N|} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial P_N}{\partial \delta_2} & \cdots & \frac{\partial P_N}{\partial \delta_N} & |V_2|\frac{\partial P_N}{\partial |V_2|} & \cdots & |V_N|\frac{\partial P_N}{\partial |V_N|} \\ \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_N} & |V_2|\frac{\partial Q_2}{\partial |V_2|} & \cdots & |V_N|\frac{\partial Q_2}{\partial |V_N|} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_N}{\partial \delta_2} & \cdots & \frac{\partial Q_N}{\partial \delta_N} & |V_2|\frac{\partial Q_N}{\partial |V_2|} & \cdots & |V_N|\frac{\partial Q_N}{\partial |V_N|} \end{bmatrix} \begin{Bmatrix} \Delta \delta_2 \\ \vdots \\ \Delta \delta_N \\ \Delta |V_2| / |V_2| \\ \vdots \\ \Delta |V_N| / |V_N| \end{Bmatrix} \tag{12-28}
$$

After some iterations and all the voltages are known, the system is solved and the current flows can be found from the admittance matrix.

Newton-Raphson and large distribution systems

The Newton-Raphson method is a powerful tool that is well suited for solving large, balanced, and symmetrical transmission networks. However, there are many reasons why this method is not suited for solving large distribution networks.

- The power flow equations are based on a by-phase power flow. On many occasions in distribution systems, by-phase power flow is unknown due to a lack of a local ground. This occurs within the Delta-connected windings of transformer or regulator. It also occurs with open connections and Delta- or Wye-connected loads. Voltages and currents are fundamental quantities that can always be determined. The SynerGEE load-flow engine is based entirely on voltages, currents, and Kirchoff's laws.
- Matrix-based approaches assume to some degree that nonlinear loading is applied to a linear network. In order to analyze a system with matrix-based methods, one of two things must hold true. The system has to either have balanced voltages and currents throughout or it has to be made of grounded four wire devices. These criteria are necessary to form a single-line equivalent or to use three sequence domain single-line models. Neither of the criteria holds true for distribution systems.
- Voltages on transmission systems are typically close to 1.0 per-unit. This allows for a good initial guess to start a Newton-Raphson load flow or some other matrix-based solution method. However, voltages in distribution systems vary widely. During a motor start, voltages can differ by over 30 percent from one end of a feeder to another.
- Most matrix-based approaches require the Jacobian to be diagonally dominant. Long, small conductored lines which are common to distribution systems typically violate this requirement. Each year, a

number of papers are published in technical journals suggesting new ways to solve systems with high R/X ratios using a matrix-based algorithm. SynerGEE has no trouble modeling long laterals. It has never failed to solve a realistically modeled system. Advancement beyond Newton-Raphson allows SynerGEE to make this claim.

- Matrix-based approaches force models to be represented in an admittance matrix. Advantica Stoner has found that realistic models of most distribution devices cannot be represented in closed form, much less in a matrix imposing a linear continuous relationship between state variables.
- Work has been done to some degree of success in retrofitting Newton-Raphson to model unbalanced distribution networks. Some methods can handle line coupling, others can handle unbalanced loads, and others can handle simple regulator models. Unlike the SynerGEE method, no matrix-based method has been found that can model a power distribution system to the level of detail and realism necessary to support proper engineering decision-making and troubleshooting.
- The derivatives in the power flow equations do not support distributed load models or mixed load types. All loads and devices must be nodal PQ loads. In SynerGEE, loads and various other devices can be placed in the center of a section or anywhere along the section. Loads can also have various V/I curves.

Which method is more accurate?

It is clear that the quality of a power distribution system simulation is only as good as the line and device models. If a Newton-Raphson algorithm and SynerGEE model the same system and they both solve it, they will produce the same numerical solution. However, aspects of a basic model for realistic modeling of a distribution system are very difficult to model in the Newton-Raphson domain. Furthermore, models for lines, regulators, transformers, and switched capacitors that make SynerGEE most suitable for distribution system simulation are simply impossible to model in the Newton-Raphson domain.

The SynerGEE simulation engine is based on real power system equipment. To this end, object-oriented technology and the SynerGEE load-flow methods have allowed the meticulous design of the device models. By alleviating the need to fit them into admittance or impedance matrices, full attention has been devoted to building accurate and realistic models. Using manufacturer guides and technical references, device models are constructed with a focus on the power engineering aspects of the simulation, providing you with the most valuable tool possible.

By-phase Newton-Raphson

This section presents a derivation of a by-phase Netwton-Raphson algorithm. It will start with the fundamental power flow equation that expresses the total nodal power injection into the system based on the YBus matrix.

$$\{P_k\} + j\{Q_k\} = \underline{U}\{V_k\}\left(\sum_{n=1}^{NBus}[Y_{kn}]\{V_n\}\right)^* \tag{12-29}$$

You can expand this for an expression of power injected into each phase, i, on bus k.

$$P_k^i + jQ_k^i = V_k^i\left(\sum_{n=1}^{NBus}\sum_{j}^{A,B,C} Y_{kn}^{ij}V_n^j\right)^* \tag{12-30}$$

Expressing the right side of the equation in phasor form:

$$P_k^i + jQ_k^i = \left|V_k^i\right|\sum_{n=1}^{NBus}\sum_{j}^{A,B,C}\left|Y_{kn}^{ij}\right|\left|V_n^j\right|\angle\left(\delta_k^i - \delta_n^j - \theta_{kn}^{ij}\right) \tag{12-31}$$

So real power injected into phase i at bus k is:

$$P_k^i = \left|V_k^i\right|\sum_{n=1}^{NBus}\sum_{j}^{A,B,C}\left|Y_{kn}^{ij}\right|\left|V_n^j\right|\cos\left(\delta_k^i - \delta_n^j - \theta_{kn}^{ij}\right) \tag{12-32}$$

And the reactive power injection is:

$$Q_k^i = \left|V_k^i\right|\sum_{n=1}^{NBus}\sum_{j}^{A,B,C}\left|Y_{kn}^{ij}\right|\left|V_n^j\right|\sin\left(\delta_k^i - \delta_n^j - \theta_{kn}^{ij}\right) \tag{12-33}$$

The next step is to use these power flow equations to determine voltage changes, using the following form.

$$\begin{bmatrix}[J1_\alpha] & [J2_\alpha] \\ [J3_\alpha] & [J4_\alpha]\end{bmatrix}\begin{Bmatrix}\Delta\delta_\alpha \\ \Delta V_\alpha\end{Bmatrix} = \begin{Bmatrix}\Delta P_\alpha \\ \Delta Q_\alpha\end{Bmatrix} \tag{12-34}$$

The delta power varies based on the difference in load power and calculated power injection. Therefore:

$$\begin{Bmatrix}\Delta P_\alpha \\ \Delta Q_\alpha\end{Bmatrix} = \begin{Bmatrix}P_{Inj,\alpha} - P\left(\delta_\alpha, V_\alpha\right) \\ Q_{Inj,\alpha} - Q\left(\delta_\alpha, V_\alpha\right)\end{Bmatrix} \tag{12-35}$$

You can apply this to the power flow equations to get:

$$\begin{Bmatrix} \Delta P_k^i \\ \Delta Q_k^i \end{Bmatrix} = \tag{12-36}$$

$$\begin{Bmatrix} P_{Gen,k}^i - P_{Load,k}^i - \left|V_k^i\right| \sum_{n=1}^{NBus} \sum_{j}^{A,B,C} \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \cos\left(\delta_k^i - \delta_n^j - \theta_{kn}^{ij}\right) \\ Q_{Gen,k}^i - Q_{Load,k}^i - \left|V_k^i\right| \sum_{n=1}^{NBus} \sum_{j}^{A,B,C} \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \sin\left(\delta_k^i - \delta_n^j - \theta_{kn}^{ij}\right) \end{Bmatrix}$$

For convenience, you can make the following substitution.

$$Let \quad \gamma_{k,n}^{i,j} = \delta_k^i - \delta_n^j - \theta_{kn}^{ij} \tag{12-37}$$

For off-diagonal terms, you get the following Jacobian elements:

$$k \neq n \ or \ \ i \neq j \tag{12-38}$$

$$J1_{k,n}^{i,j} = \frac{\partial P_k^i}{\partial \delta_n^j} = \left|V_k^i\right| \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \sin\left(\gamma_{k,n}^{i,j}\right)$$

$$J2_{k,n}^{i,j} = \frac{\partial P_k^i}{\partial V_n^j} = \left|V_k^i\right| \left|Y_{kn}^{ij}\right| \cos\left(\gamma_{k,n}^{i,j}\right)$$

$$J3_{k,n}^{i,j} = \frac{\partial Q_k^i}{\partial \delta_n^j} = -\left|V_k^i\right| \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \cos\left(\gamma_{k,n}^{i,j}\right)$$

$$J4_{k,n}^{i,j} = \frac{\partial Q_k^i}{\partial V_n^j} = \left|V_k^i\right| \left|Y_{kn}^{ij}\right| \sin\left(\gamma_{k,n}^{i,j}\right)$$

For diagonal terms, you get:

$$k = n \; and \;\; i = j \tag{12-39}$$

$$J1_{kk}^{i,i} = \frac{\partial P_k^i}{\partial \delta_k^i} = -\left|V_k^i\right| \sum_{\substack{n=1 \\ n \neq k}}^{NBus} \sum_{j}^{A,B,C} \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \sin\left(\gamma_{k,n}^{i,j}\right) - \left|V_k^i\right| \sum_{\substack{j \\ j \neq i}}^{A,B,C} \left|Y_{kk}^{ij}\right| \left|V_k^j\right| \sin\left(\gamma_{k,n}^{i,j}\right)$$

$$J2_{kk}^{i,i} = \frac{\partial P_k^i}{\partial V_k^i} = \left|V_k^i\right| \left|Y_{kk}^{ii}\right| \cos(\theta_{kk}^{ii}) + \sum_{n=1}^{NBus} \sum_{j}^{A,B,C} \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \cos\left(\gamma_{k,n}^{i,j}\right)$$

$$J3_{kk}^{i,i} = \frac{\partial Q_k^i}{\partial \delta_k^i} = \left|V_k^i\right| \sum_{\substack{n=1 \\ n \neq k}}^{NBus} \sum_{j}^{A,B,C} \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \cos\left(\gamma_{k,n}^{i,j}\right) + \left|V_k^i\right| \sum_{\substack{j \\ j \neq i}}^{A,B,C} \left|Y_{kk}^{ij}\right| \left|V_k^j\right| \cos\left(\gamma_{k,n}^{i,j}\right)$$

$$J4_{kk}^{i,i} = \frac{\partial Q_k^i}{\partial V_k^i} = -\left|V_k^i\right| \left|Y_{kk}^{ii}\right| \sin(\theta_{kk}^{ii}) + \sum_{n=1}^{NBus} \sum_{j}^{A,B,C} \left|Y_{kn}^{ij}\right| \left|V_n^j\right| \sin\left(\gamma_{k,n}^{i,j}\right)$$

The Jacobian can now be used to determine numerical relations between power and voltage.

$$\begin{Bmatrix} \Delta\delta_k^i \\ \Delta V_k^i \end{Bmatrix} = \left[J\right]^{-1} \begin{Bmatrix} \Delta P_k^i \\ \Delta Q_k^i \end{Bmatrix} \tag{12-40}$$

Any swing busses are left out of the Jacobian.

LDU Decomposition

The Zbus matrix is a valuable tool for by-phase network analysis. By definition, the Zbus matrix is the inverse of the Ybus matrix. Zbus is a full matrix of complex submatrices. You clearly do not want to store or directly calculate the Zbus elements. A powerful alternative is matrix decomposition. Instead of storing the Zbus matrix, you can store three specially formed matrices that can be used in the place of Zbus.

Starting with the linear system:

$$\left\{I\right\} = \left[YBus\right]\left\{V\right\} \tag{12-41}$$

And factoring the Ybus matrix into three matrices:

$$\left\{I\right\} = \left[L\right]\left[D\right]\left[U\right]\left\{V\right\} \tag{12-42}$$

[L] is defined as a lower diagonal matrix. [U] is upper diagonal. [D] is a diagonal matrix. Therefore:

$$[YBus] = [L][D][U] \tag{12-43}$$

$$\begin{gathered} Where\ \ i > j\ \ and \\ L_{ij} = [0] \quad L_{ii} = [\Upsilon] \\ U_{ij} = [0] \quad U_{ii} = [\Upsilon] \\ D_{ij} = [0] \quad D_{ji} = [0] \end{gathered}$$

Matrix multiplication yields:

$$YBus_{i,k} = \sum_{j=1}^{N} L_{ij} D_{jj} U_{jk} \tag{12-44}$$

You can look at the diagonal term of Ybus:

$$YBus_{i,i} = \sum_{j=1}^{i} L_{ij} D_{jj} U_{ji} \tag{12-45}$$

Pull out the corresponding element in the [D] matrix and note that the elements from [L] and [U] are identity matrices.

$$YBus_{i,i} = D_{ii} + \sum_{j=1}^{i-1} L_{ij} D_{jj} U_{ji} \tag{12-46}$$

You can now solve for the diagonal element of the LDU matrix.

$$D_{ii} = YBus_{i,i} - \sum_{j=1}^{i-1} L_{ij} D_{jj} U_{ji} \tag{12-47}$$

Notice that the [L] and [U] values are in rows and columns behind the diagonal element i.

Now, revert to the equation that relates Ybus to the [L][D][U] product. You can find an expression for elements of the [U] matrix by looking at rows of Ybus.

$$YBus_{\substack{i,k \\ k>i}} = \sum_{j=1}^{i} L_{ij} D_{jj} U_{jk} \tag{12-48}$$

You can pull out the term from the summation where j=i.

$$YBus_{\substack{i,k \\ k>i}} = D_{ii} U_{ik} + \sum_{j=1}^{i-1} L_{ij} D_{jj} U_{jk} \tag{12-49}$$

Then you can solve for the element in [U].

$$U_{\substack{i,k \\ k>i}} = D_{ii}^{-1} \left(YBus_{i,k} - \sum_{j=1}^{i-1} L_{ij} D_{jj} U_{jk} \right) \tag{12-50}$$

Follow the same procedure to find [L] matrix elements:

$$YBus_{\substack{k,i\\k>i}} = \sum_{j=1}^{i} L_{kj} D_{jj} U_{ji} \tag{12-51}$$

Again, pull out the summation term where j=i:

$$YBus_{\substack{k,i\\k>i}} = L_{ki} D_{ii} + \sum_{j=1}^{i-1} L_{kj} D_{jj} U_{ji} \tag{12-52}$$

And solve for the [L] matrix value:

$$L_{\substack{k,i\\k>i}} = D_{ii}^{-1} \left(YBus_{k,i} - \sum_{j=1}^{i-1} L_{kj} D_{jj} U_{ji} \right) \tag{12-53}$$

In summary, you have the following expressions for the LDU matrix.

$$\begin{aligned} D_{ii} &= YBus_{i,i} - \sum_{j=1}^{i-1} L_{ij} D_{jj} U_{ji} \\ U_{\substack{i,k\\k>i}} &= D_{ii}^{-1} \left(YBus_{i,k} - \sum_{j=1}^{i-1} L_{ij} D_{jj} U_{jk} \right) \\ L_{\substack{k,i\\k>i}} &= D_{ii}^{-1} \left(YBus_{k,i} - \sum_{j=1}^{i-1} L_{kj} D_{jj} U_{ji} \right) \end{aligned} \tag{12-54}$$

Solving the equations

Once the LDU matrix has been formed, the next task is to solve the original set of equations.

$$\{I\} = [L][D][U]\{V\} \tag{12-55}$$

You can make a substitution:

$$\{\alpha\} = [D][U]\{V\} \tag{12-56}$$

To solve the following:

$$\{I\} = [L]\{\alpha\} \tag{12-57}$$

With forward substitution yielding:

$$\underline{\alpha}_k = \underline{I}_k - \sum_{j=1}^{k-1} L_{kj} \alpha_j \tag{12-58}$$

You can make a second substitution:

$$\{\beta\} = [U]\{V\} \tag{12-59}$$

Resulting in:

$$\{\alpha\} = [D]\{\beta\} \tag{12-60}$$

Since [D] is a diagonal matrix, you can easily find the elements of $\{\beta\}$ as:

$$\underline{\beta}_k = \underline{D_{kk}^{-1}} * \underline{\alpha}_k \tag{12-61}$$

Finally, back substitution allows you to solve for the voltage values.

$$\underline{V}_k = \underline{\beta}_k - \sum_{j=k+1}^{N} U_{kj} V_j \tag{12-62}$$

Note that the vector does not need to be explicitly stored. You could write the above expression as:

$$\underline{V}_k = \underline{D_{kk}^{-1}} * \underline{\alpha}_k - \sum_{j=k+1}^{N} U_{kj} V_j \tag{12-63}$$

Radial/looped and network modeling differences

If the SynerGEE radial or looped analysis engine were analyzing the exact same model as the SynerGEE network engine, the results would match. As was discussed in this chapter, the models that are analyzed by each engine differ based on the strengths and weaknesses of the fundamental mathematics behind each approach.

Radial and looped models are analyzed by trace methods. This allows very robust and detailed models for power system devices like regulators and capacitors. Loops have to be formed with loop tie switches. Network analysis handles dense networks. Loads have to be placed on nodes and device models are simplified.

Switches in a network

In the distribution model, loads are placed inside of all devices. The network model places loads at the nodes. When an open switch is put into a network, amp flows and power flow values may look peculiar if the section also has load. The peculiarities come from translating network results into the radial distribution context. The network load-flow report should properly represent the model and its results. As such, it is recommended that loads not be placed on sections with open switches.

13
Fault

Introduction

SynerGEE fault analysis is able to determine the balanced or by-phase fault currents flowing to and into a fault. It can also determine the balanced or by-phase fault voltages. SynerGEE supports three types of fault analysis: basic fault, fault flow, and fault voltage. Each type allows you to select options for making the analysis appropriate for your needs.

This section discusses some common issues, then briefly presents each type of analysis. The second portion of this chapter covers SynerGEE fault calculation methods in detail.

Line impedances

Fault calculations are discussed in this chapter with impedances expressed in the sequence domain and in the phase domain. For a grounded four-wire line, the relationship between self- and mutual-impedances in the phase domain and sequence impedances in the sequence domain is straightforward. For example:

$$\begin{aligned} Z_S &= (Z_0 + 2Z_1)/3 \qquad & Z_0 &= Z_S + 2Z_M \\ Z_M &= (Z_0 - Z_1)/3 \qquad & Z_{1,2} &= Z_S - Z_M \end{aligned} \tag{13-1}$$

Notice that the positive and negative sequence impedances are the same because SynerGEE averages the self-impedances and averages the mutual impedances of lines.

Pre-fault voltages

All of the derivations in this chapter are based on a linear model of the distribution system. The voltage used for the fault calculations can be selected arbitrarily based on engineering judgment. SynerGEE allows the voltage to be either the nominal voltage at the fault or the pre-fault voltage. The pre-fault voltage is calculated by a load-flow run.

By-phase fault flow analysis utilizes either by-phase nominal voltages or by-phase pre-fault voltages from a load-flow run. Imbalance in voltage from this latter case can have a tremendous impact on fault current and fault flows.

Types of analysis

SynerGEE supports three types of fault analysis.

Fault analysis

Fault analysis places a fault on each section of the model, one by one, and then computes the fault current in amps for the load end of each section. The fault values include minimum and maximum line-ground, phase-phase, and three-phase faults. The distances from the substation and the positive and zero sequence impedance are accumulated along the feeder. The analysis checks the minimum fault current from the selected protective device down-line to the next section with a protective device. Fault analysis reports can be directed to a screen, a printer, or a database file. Fault analysis is available for substation models and in feeders or substations with looped configurations.

Fault values are calculated using the symmetrical components method of solution. This classical method is outlined in the *Fault Current Calculations* section below. If a system has loops or active generators, the impedance seen by the fault is calculated in the phase domain, then transformed to the sequence domain model.

Pre-fault load current

Fault analysis determines the fault current into the fault. SynerGEE lists the results for all sections in a feeder by faulting the lines one at a time. Including the pre-fault load current into these results does not affect the current into the fault. It only affects the current into the faulted section and the fault current on other sections.

Fault flow analysis

Fault flow analysis places a fault at one location in the distribution system model and calculates the flows throughout. By-phase models of regulators, transformers, loads, and other devices are utilized. The analysis results in a report listing the by-phase or maximum current flows on each section for line-ground, line-line, and three-phase faults at the given fault location. A bolted maximum line-ground fault and a minimum line-ground fault through a fault impedance are used. If the faulted section has a neutral, a line-line-ground fault type is used. Otherwise, a line-line type is used.

Pre-fault load current

Pre-fault load current can be included in the results for fault flow analysis. In this case, a load-flow is run before the fault analysis and the flows are stored. After the fault flow analysis is completed, the pre-fault flows are combined with the fault flows as follows.

$$\left|I_{Total}\right| = \left|I_{From\,Fault}\right| + \left|I_{Prefault}\right| \tag{13-2}$$

The values of $|I_{Total}|$ for affected lines are listed in the fault flow analysis report.

Fault voltage analysis

Fault voltage analysis calculates voltages throughout a system due to a fault at a single location. This application combines the SynerGEE fault current calculations, fault flow analysis, and the load-flow calculations to simulate worst-case voltage conditions near the time of the fault. The calculations are performed in the order that follows.

1. Fault flow analysis is used to determine fault current into the fault, any generators, and through tie switches for each type of fault.
2. A load-flow analysis is used to calculate load currents and the source voltage behind the feeder or substation source impedance.
3. For each fault type, the fault currents and the load currents are accumulated.
4. For each fault type, voltage drops are calculated throughout the feeder.

The fault voltage analysis report lists balanced or by-phase voltages and current flows for each section and for each fault type. The report also lists the percent dip, and percent loading of conductors and other devices. The kVA flow is also given.

Sequence domain fault calculations

All three types of fault analysis within SynerGEE depend on a common set of fault current calculation methods. In some cases, these methods are applied to the sequence domain and in others they are applied to the phase domain. In the following sections, derivations for each fault type are given in each domain.

SynerGEE has a sequence domain fault analysis derived from classical methods. Used directly, the classical methods can model distribution systems that are balanced, symmetrical, and three-phase up to the point or area of imbalance (Blackburn, *Symmetrical Components for Power Systems Engineering*, 1993). Faults can be unbalanced as can the portion of the distribution system local to the fault.

Stoner has made other advancements that allow the incorporation of long lateral, one- and two- phase, grounded or ungrounded lines that are not local to the fault. Fault calculations can be performed completely within the sequence domain. This section discusses the classical symmetrical component approach to fault analysis. The next section discusses some of the advances that are specific to detailed distribution system modeling.

Equation summary

Fault currents are determined from the impedance looking into the fault. The parameters that determine fault current values are defined as follows.

kV_{LL} = Pre-Fault Line-Line kV **(13-3)**

Z_1 = Cumulative Positive Sequence Impedance From Fault Through Source

Z_0 = Cumulative Negative Sequence

Z_f = Fault Resistance ($R_f + j0$)

The calculations for determining fault current for the five types of fault are shown below. The expressions are used to determine the magnitude of phase fault current during the particular fault. Expressions are given for determining the fault current using both sequence and phase domain impedance values. Both expressions in each case supply the same result. The total sequence impedance seen by a fault is typically viewed in fault analysis reports.

Sequence network for line-to-Gnd fault

Fault current values based on phase or sequence domain impedances		
	$I_f = f\left(Z_{Seq}\right)$	$I_f = g\left(Z_{Phase}\right)$
Phase-Gnd Min.	$I_f = \dfrac{1000*\sqrt{3}*kV_{LL}}{\left\|Z_0 + 2Z_1 + 3Z_f\right\|}$	$I_f = \dfrac{1000*kV_{LL}}{\sqrt{3}*\left\|Z_S + Z_f\right\|}$
Phase-Gnd Max.	$I_f = \dfrac{1000*\sqrt{3}*kV_{LL}}{\left\|Z_0 + 2Z_1\right\|}$	$I_f = \dfrac{1000*kV_{LL}}{\sqrt{3}*\left\|Z_S\right\|}$
Phase-Phase	$I_f = \dfrac{1000*kV_{LL}}{2*\left\|Z_1\right\|}$	$I_f = \dfrac{1000*kV_{LL}}{2*\left\|Z_S - Z_M\right\|}$
Phase-Phase-Gnd	**See Below**	**See Below**
Three-Phase	$I_f = \dfrac{1000*kV_{LL}}{\sqrt{3}Z_1}$	$I_f = \dfrac{kV_{LL}}{\sqrt{3}\left(Z_S - Z_M\right)}$

Although these expressions could be generated from either a phase or sequence domain model, the sequence domain models will be presented herein with each fault type.

Phase-ground faults

The following is the sequence network for a line-ground fault that includes a fault impedance.

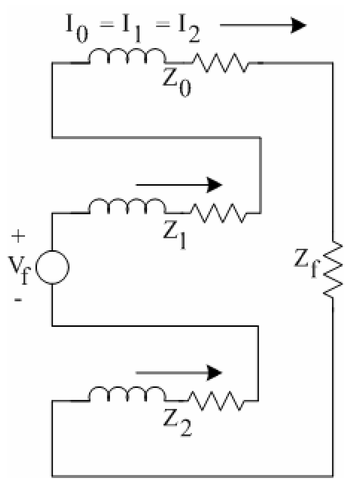


Fig. 13-1 *Sequence network for line-grnd fault*

All sequence currents are equal for this fault type. The phase domain current corresponding to three equal sequence currents is as follows.

$$I_{PhGnd} = 3\frac{V_f}{\left|Z_0 + Z_1 + Z_2 + 3Z_f\right|} = \frac{1000 * \sqrt{3} * kV_{LL}}{\left|Z_0 + 2Z_1 + 3Z_f\right|} = \frac{1000 * kV_{LL}}{\sqrt{3} * \left|Z_S + Z_f\right|} \tag{13-4}$$

The minimum fault current occurs when Z_f is some positive value and the maximum fault current occurs when $Z_f = 0$.

Phase-phase faults

The sequence network for a phase-phase fault with a fault impedance is shown below.

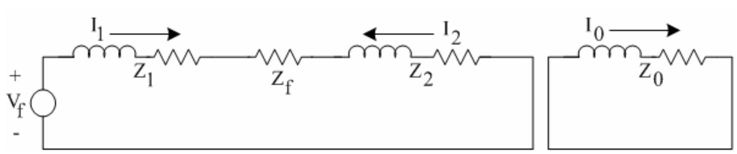


Fig. 13-2 *Sequence network for line-line fault*

In this fault, the positive and negative sequence currents are equal and opposite. The zero sequence current is zero. After a transformation to the phase domain, the magnitude of fault current can be found as follows.

$$I_{Ph-Ph} = \sqrt{3}I_1 = \frac{1000 * kV_{LL}}{2 * \left|Z_1\right|} = \frac{1000 * kV_{LL}}{2 * \left|Z_S - Z_M\right|} \tag{13-5}$$

Phase-phase-ground faults

The sequence network for this type of fault is shown below.

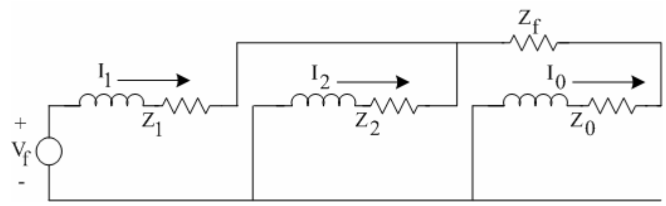


Fig. 13-3 *Sequence network for line-line-gnd fault*

Due to the complexity of this sequence network, you cannot formulate a simple expression for the phase-domain fault current. A process for finding the fault current is outlined below.

1. Apply positive sequence pre-fault voltage to sequence network. Determine all sequence currents.
2. Use sequence/phase transformation to get phase currents.
3. Average magnitudes of phase B and phase C currents to get fault current.

Three-phase faults

A three-phase fault can be represented in the sequence domain with the following network.

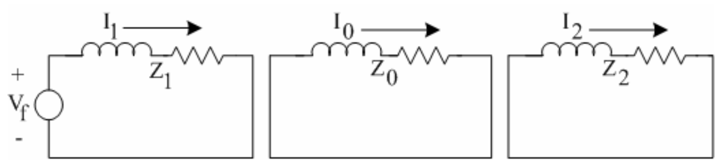


Fig. 13-4 *Sequence network for three-phase fault*

The positive sequence fault current is the pre-fault voltage divided by the positive sequence impedance to the fault location. The zero and negative sequence currents are always zero. If you transform the sequence currents back to the phase domain, you find that the magnitude of fault current on any of the three phases is as follows.

$$I_{3\phi} = \frac{1000 * kV_{LL}}{\sqrt{3} * |Z_1|} = \frac{1000 * kV_{LL}}{\sqrt{3} * |Z_S - Z_M|} \qquad \textbf{(13-6)}$$

The expression for three-phase fault current can also be determined directly from the cumulative impedance matrix.

Asymmetrical three-phase and line-to-ground fault current

An actual fault can be broken into AC and DC components.

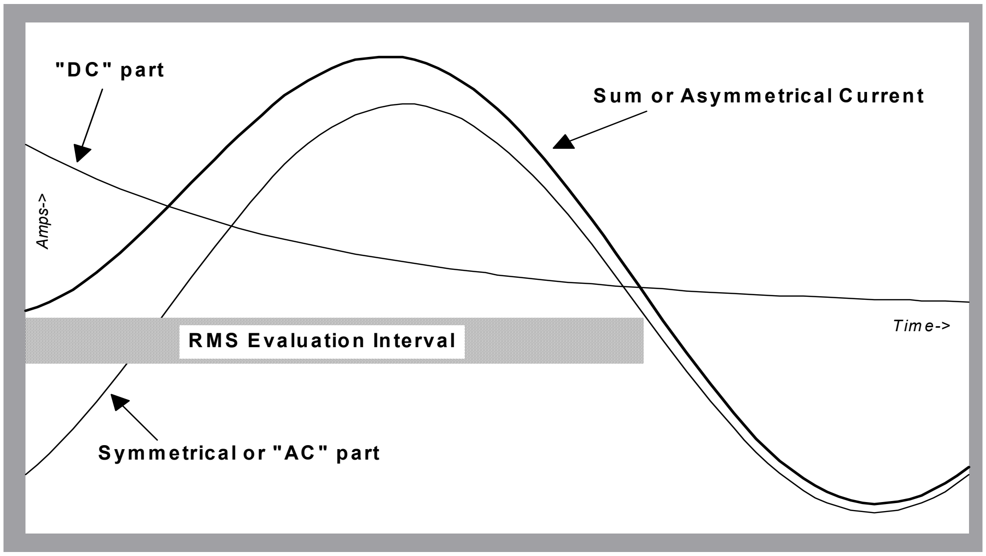


Fig. 13-5 *Fault current components*

The following circuit represents a simplified distribution feeder during a fault.

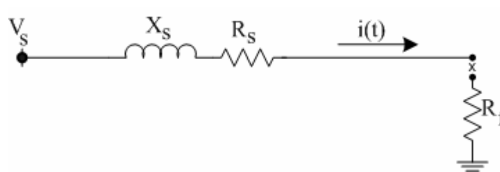


Fig. 13-6 *Equivalent model of system at time, t*

R_s and X_s are the line resistance and reactance between the source and the fault location. R_f represents the fault resistance. Many simplifying assumptions are made about the feeder. First, pre-fault load current is negligible. The circuit is unloaded until the initiation of the fault at t=0. Second, the voltage source is assumed to be balanced.

$$e_i(t) = \sqrt{2}V_s \sin(\omega t + \alpha + \delta_i) \qquad i = a, b, c$$
$$\delta_a \equiv 0 \qquad \delta_b \equiv -120 \qquad \delta_c \equiv 120 \tag{13-7}$$

Consider the figure above as a representation of phase *i* in your feeder. The following differential equation can be written while summing the voltages around the circuit.

$$\sqrt{2}V_s \sin(\omega t + \alpha + \delta_i) - \left(\frac{X_L}{\omega}\right)\frac{di(t)}{dt} - \left(R_L + R_f\right)i(t) = 0 \qquad t \geq 0 \tag{13-8}$$

Your equation should be arranged into a standard form. For example:

$$\frac{di(t)}{dt} + \frac{1}{\tau}i(t) = \frac{\omega\sqrt{2}V_s}{X_L}\sin(\omega t + \alpha + \delta_i) \qquad \text{where } \tau = \frac{X_L}{\omega\left(R_L + R_f\right)} \tag{13-9}$$

This form is similar to the following family of differential equations.

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{13-10}$$

This equation has the integrating factor shown below.

$$e^{\int P(x)dx} \tag{13-11}$$

It has the following solution.

$$ye^{\int P(x)dx} = \int e^{\int P(x)dx} Q(x)dx + c \tag{13-12}$$

The solution for your equation is as follows.

$$i(t)e^{\frac{t}{\tau}} = \int e^{\frac{t}{\tau}} \frac{\omega\sqrt{2}V_s}{X_L} \sin(\omega t + \alpha + \delta_i)dt + c \tag{13-13}$$

Evaluating the integral, you conclude the following.

$$i(t) = \frac{\omega\sqrt{2}\tau * V_s}{\left(1+\omega^2\tau^2\right)X_L}\left[\sin\left(\omega t + \alpha + \delta_i\right) - \omega\tau\cos\left(\omega t + \alpha + \delta_i\right)\right] + ce^{-\frac{t}{\tau}} \tag{13-14}$$

The two sinusoids are at the same frequency and can thus be combined using phasors. You will want to convert back to a sine-based function.

$$i(t) = \frac{\omega\sqrt{2}\tau * V_s}{\left(1+\omega^2\tau^2\right)X_L}\sqrt{1+\omega^2\tau^2}\,\sin\left(\omega t + \alpha + \delta_i - \theta\right) + ce^{-\frac{t}{\tau}} \tag{13-15}$$

$$\theta = \tan^{-1}\left(\omega\tau\right) = \tan^{-1}\left(\frac{X_L}{R_L + R_f}\right)$$

Common terms can be factored out of the numerator and denominator.

$$i(t) = \frac{\omega\sqrt{2}\tau * V_s * \left(R_L + R_f\right)}{X_L\sqrt{\left(R_L + R_f\right)^2 + X_L^2}}\sin\left(\omega t + \alpha + \delta_i - \theta\right) + ce^{-\frac{t}{\tau}} \tag{13-16}$$

The circuit time constant is evident and a simplification can be made.

$$i(t) = \frac{\sqrt{2} * V_s}{Z}\sin\left(\omega t + \alpha + \delta_i - \theta\right) + ce^{-\frac{t}{\tau}} \tag{13-17}$$

$$Z = \sqrt{\left(R_L + R_f\right)^2 + X_L^2}$$

You can now determine the integration constant since $I(0) = 0$.

$$c = -\frac{\sqrt{2}V_s}{Z}\sin\left(\alpha + \delta_i - \theta\right) \tag{13-18}$$

The RMS AC fault current is a function of the RMS source voltage and impedance from the fault to ground.

$$I_{ac} = \frac{V_s}{Z} \tag{13-19}$$

In conclusion:

$$i(t) = \sqrt{2} * I_{ac} \left(\sin\left(\omega t + \alpha + \delta_i - \theta\right) - \sin\left(\alpha + \delta_i - \theta\right) e^{-\frac{t}{\tau}} \right) \tag{13-20}$$

It is clear that the fault current is composed of an AC symmetrical term and a DC asymmetrical term.

The RMS asymmetrical fault current is calculated as the root mean square of the AC and DC fault components.

$$I_{RMS}(t) = \sqrt{I_{ac}^2 + I_{dc}^2} \tag{13-21}$$

I_{ac} is found identically.

$$I_{ac} = I_{ac} \sqrt{\frac{1}{\pi} \int_0^{2\pi} \sin^2\left(\omega t + \alpha + \delta_i - \theta\right) dt} \tag{13-22}$$

The RMS value of the DC component is found in a similar manner except that the exponential term is approximated as a constant.

$$I_{dc} = \sqrt{2} * I_{ac} * \sin\left(\alpha + \delta_i - \theta\right) e^{-\frac{t}{\tau}} \tag{13-23}$$

The RMS asymmetrical fault current can now be expressed as follows.

$$I_{RMS}(t) = I_{ac} \sqrt{1 + 2 * \sin^2\left(\alpha + \delta_i - \theta\right) e^{-\frac{2t}{\tau}}} \tag{13-24}$$

If you consider phase A to be the reference phase, the maximum asymmetrical fault current clearly occurs when α and θ differ by 90 degrees.

$$I_{RMS}(t) = I_{ac} \sqrt{1 + 2 * \sin^2\left(\pm\frac{\pi}{2}\right) e^{-\frac{2t}{\tau}}} \tag{13-25}$$

You can make the above substitution and the substitutions that follow:

$$\begin{aligned} \omega &= 2\pi f \qquad f = \text{Frequency in Hertz} \\ t &= \frac{N}{2\pi} \qquad N = \text{Number of Cycles} \\ K &= \frac{X_L}{R_L + R_f} \end{aligned} \tag{13-26}$$

The result is the maximum single-phase RMS asymmetrical fault current shown below.

$$Max\left[I_{RMS}(t)\right]_{1\phi} = I_{ac} \sqrt{1 + 2e^{-\frac{4N\pi}{K}}} \tag{13-27}$$

The average three-phase RMS current can also be found with the phase shifts defined within δ_i. For example:

$$Avg[I_{RMS}(t)]_{3\phi} = I_{ac}\frac{\sqrt{1+2e^{-\frac{4N\pi}{K}}}+2\sqrt{1+\frac{1}{2}e^{-\frac{4N\pi}{K}}}}{3} \quad \textbf{(13-28)}$$

Typically, RMS asymmetrical fault currents are calculated at the half cycle. Therefore, $N = 0.5$:

$$Max\left[I_{RMS}(t)\right]^{N=0.5} = I_{ac}\sqrt{1+2e^{-\frac{2\pi}{K}}} \quad \textbf{(13-29)}$$

$$Avg\left[I_{RMS}(t)\right]_{3\phi}^{N=0.5} = I_{ac}\frac{\sqrt{1+2e^{-\frac{2\pi}{K}}}+2\sqrt{1+\frac{1}{2}e^{-\frac{2\pi}{K}}}}{3}$$

The maximum instantaneous asymmetrical fault current can be found by numerically maximizing the fault current expressed in equation the equation.

Phase domain calculations

Fault calculations can be performed directly in the phase domain. This approach is more suitable for the by-phase fault analysis of distribution systems. Unlike the symmetrical component approach, no assumptions are made about balance, grounding, or phasing at or outside of the fault locality.

The phase domain calculations are used for handling loops and generators. They are also used for determining the by-phase or balanced fault flows.

Fault calculations from 3 by 3 cumulative impedance matrix

Conceptually, the 3 by 3 impedance matrices calculated from Carson's equations and used to represent line sections can be propagated from the source of a feeder out through the feeder. The impedance matrix can be propagated across capacitors and through transformer and regulator banks. As a result, an impedance is visible from a fault location looking back into the feeder.

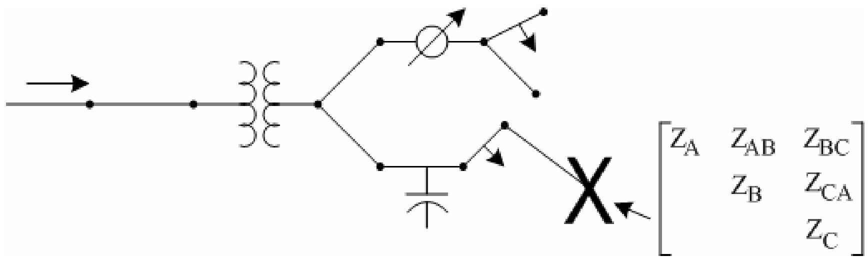


Fig. 13-7 *Feeder with a fault*

In practice, some transformer and regulator connections, as well as generators, other devices, and loops, make the propagation of phase domain impedances impossible in the distribution model. This problem is overcome by using the load-flow engine to calculate the impedance looking into the fault.

Three-phase fault current from phase domain impedance matrix

The following exercise demonstrates that the three-phase fault current calculated in the phase domain matches the fault current calculated in the sequence domain if the line model is balanced. Assume that you have a balanced three-phase line represented with the following impedance matrix.

$$Z_X = \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \tag{13-30}$$

This matrix is used directly in the relationship between the voltage drop along the line and the current through the line as shown below.

$$\begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} = \begin{bmatrix} Z_S & Z_M & Z_M \\ & Z_S & Z_M \\ & & Z_S \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} \tag{13-31}$$

The system can be inverted to get the following:

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \frac{1}{(Z_S + 2Z_M)(Z_S - Z_M)} \begin{bmatrix} Z_S + Z_M & -Z_M & -Z_M \\ & Z_S + Z_M & -Z_M \\ & & Z_S + Z_M \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} \tag{13-32}$$

To simulate a fault at the end of the line, you allow the voltage to be the fault voltage and solve for the line current as follows.

$$I_A = \frac{1}{(Z_S + 2Z_M)(Z_S - Z_M)} \left[(Z_S + Z_M) - a^2 Z_M - aZ_M \right] V_{Fault} \tag{13-33}$$

Vector rotated terms can be combined to get the following.

$$I_A = \frac{1}{(Z_S + 2Z_M)(Z_S - Z_M)}[Z_S + 2Z_M]V_{Fault} \tag{13-34}$$

Recall that the positive and negative sequence values can be expressed in terms of the self- and mutual-impedances as follows.

$$Z_{1,2} = Z_S - Z_M \qquad Z_0 = Z_S + 2Z_M \tag{13-35}$$

You can use this relationship to reduce the expression of fault current on wire A to the familiar form expressed with sequence impedance. For example:

$$I_A = \frac{V_{Fault}}{(Z_S - Z_M)} = \frac{V_{Fault}}{Z_1} \tag{13-36}$$

Phase domain faults on grounded sections

This subsection lists the derivations for fault currents for various types of faults occurring on grounded sections.

Three-phase fault

Shown below is the fault model for a three-phase fault.

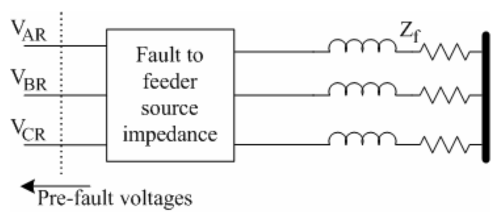


Fig. 13-8 *Three-phase fault*

The following equations can be written during the fault.

$$\begin{aligned} V_f &= V_{AR} - (Z_{AA} + Z_f)I_A - Z_{AB}I_B - Z_{AC}I_C \\ V_f &= V_{BR} - Z_{AB}I_A - (Z_{BB} + Z_f)I_B - Z_{BC}I_C \\ V_f &= V_{CR} - Z_{CA}I_A - Z_{CB}I_B - (Z_{CC} + Z_f)I_C \end{aligned} \tag{13-37}$$

There is no ground connection in this fault case. Therefore:

$$I_A + I_B + I_C = 0 \tag{13-38}$$

This yields the following set of equations.

$$\begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} = \begin{bmatrix} Z_{AA} - Z_{AC} + Z_f & Z_{AB} - Z_{AC} & 1 \\ Z_{BA} - Z_{BC} & Z_{BB} - Z_{BC} + Z_f & 1 \\ Z_{CA} - Z_{CC} - Z_f & Z_{CB} - Z_{CC} - Z_f & 1 \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ V_f \end{Bmatrix} \tag{13-39}$$

You can now solve for two of the three fault currents and the fault voltage. For example:

$$\begin{Bmatrix} I_A \\ I_B \\ V_f \end{Bmatrix} = \begin{bmatrix} Z_{AA} - Z_{AC} + Z_f & Z_{AB} - Z_{AC} & 1 \\ Z_{BA} - Z_{BC} & Z_{BB} - Z_{BC} + Z_f & 1 \\ Z_{CA} - Z_{CC} - Z_f & Z_{CB} - Z_{CC} - Z_f & 1 \end{bmatrix}^{-1} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} \tag{13-40}$$

The fault current on wire C can be found from the current summation above.

Three-phase-Gnd through single-fault impedance

This fault assumes that the three phases are bolted for a fault and that the single fault point is connected to ground through a single fault impedance. For example:

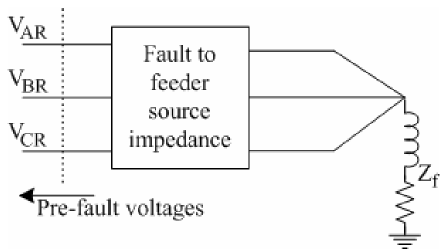


Fig. 13-9 *Three-phase fault through ground impedance*

The faulted system can be described by the following equations.

$$\begin{Bmatrix} V_f \\ V_f \\ V_f \end{Bmatrix} = \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} - \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ & Z_{BB} & Z_{BC} \\ & & Z_{CC} \end{bmatrix} \begin{Bmatrix} I_A \\ I_B \\ V_f \end{Bmatrix} \tag{13-41}$$

The fault voltage is a function of the fault impedance and the sum of the fault current on each conductor. For example:

$$V_f = (I_A + I_B + I_C) Z_f \tag{13-42}$$

This equation can be substituted to get the following.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \left[\begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ & Z_{BB} & Z_{BC} \\ & & Z_{CC} \end{bmatrix} + \begin{bmatrix} Z_f & Z_f & Z_f \\ & Z_f & Z_f \\ & & Z_f \end{bmatrix} \right]^{-1} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} \tag{13-43}$$

So in this case, all three-fault currents are found from the calculations.

Three-phase-Gnd through separate fault impedances

SynerGEE does not perform this calculation. It is included here for reference only.

This fault assumes that each of the three phases has its own fault impedance. For example:

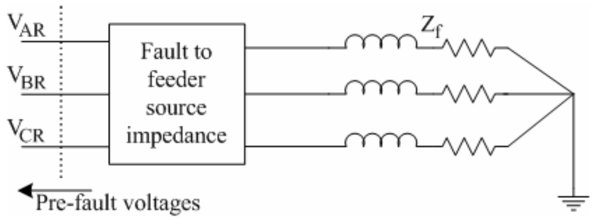


Fig. 13-10 Three-phase gnd type fault

The faulted system can be described from inspection as follows.

(13-44)

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \left[\begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ & Z_{BB} & Z_{BC} \\ & & Z_{CC} \end{bmatrix} + \begin{bmatrix} Z_f & 0 & 0 \\ & Z_f & 0 \\ & & Z_f \end{bmatrix} \right]^{-1} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix}$$

Line-Gnd fault

Shown below is a diagram representing a line-ground fault.

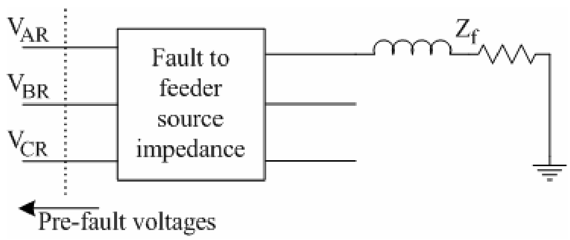


Fig. 13-11 Line-gnd fault on phase A

The fault is shown on phase A but it could be placed on any of the three phases. Fault current on those non-faulted phases is zero.

(13-45)

$$I_A = \frac{V_{AR}}{Z_{AA} + Z_f} \qquad I_B = \frac{V_{BR}}{Z_{BB} + Z_f} \qquad I_C = \frac{V_{CR}}{Z_{CC} + Z_f}$$

Line-line fault

The feeder diagram is shown below.

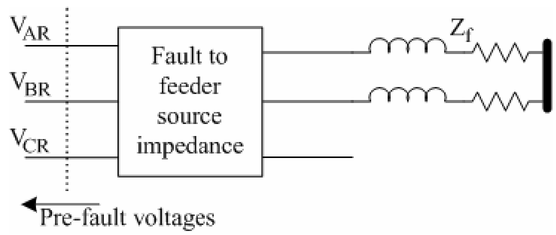


Fig. 13-12 *Line-line fault*

These equations can be written around the fault.

$$\begin{aligned} V_f &= V_{AR} - (Z_{AA} + Z_f)I_A - Z_{AB}I_B \\ V_f &= V_{BR} - Z_{AB}I_A - (Z_{BB} + Z_f)I_B \end{aligned} \tag{13-46}$$

This equation indicates the relationship between the current on the faulted conductors.

$$I_A + I_B = 0 \tag{13-47}$$

This equation can be substituted to get the following.

$$\begin{aligned} V_{AR} &= (Z_{AA} + Z_f)I_A - Z_{AB}I_A + V_f \\ V_{BR} &= Z_{AB}I_A - (Z_{BB} + Z_f)I_A + V_f \end{aligned} \tag{13-48}$$

For each of the fault combinations, you have the following results.

$$\begin{Bmatrix} I_X \\ V_f \end{Bmatrix} = \begin{bmatrix} Z_{XX} - Z_{XY} + Z_f & 1 \\ -Z_{YY} + Z_{XY} - Z_f & 1 \end{bmatrix}^{-1} \begin{Bmatrix} V_{XR} \\ V_{YR} \end{Bmatrix} \quad \text{Fault on X-Y} \tag{13-49}$$

$$\text{X,Y=A,B,C} \qquad I_X + I_Y = 0$$

Line-line fault through common fault impedance

In this fault, two phases are tied and the tie point is grounded through a single-fault impedance.

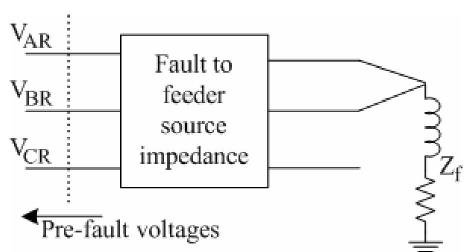


Fig. 13-13 *Line-line-gnd fault through a common fault impedance*

The voltage at the fault point is as follows.

$$V_f = (I_A + I_B)Z_f \tag{13-50}$$

The following equations can now be made.

$$V_{AR} = Z_{AA} I_A + Z_{AB} I_B + \left(I_A + I_B\right) Z_f$$
$$V_{BR} = Z_{AB} I_A + Z_{BB} I_B + \left(I_A + I_B\right) Z_f \quad \textbf{(13-51)}$$

This corresponds to the following system.

$$\begin{Bmatrix} I_X \\ I_Y \end{Bmatrix} = \begin{bmatrix} Z_{XX} + Z_f & Z_{XY} + Z_f \\ Z_{XY} + Z_f & Z_{YY} + Z_f \end{bmatrix}^{-1} \begin{Bmatrix} V_{XR} \\ V_{YR} \end{Bmatrix} \quad \text{Fault on X-Y} \quad \textbf{(13-52)}$$

X,Y=A,B,C

Line-line fault through separate fault impedance

SynerGEE does not perform this calculation. It is included here for reference only.

In this case, each phase has its own fault impedance.

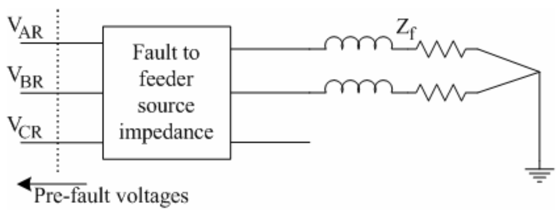


Fig. 13-14 *Line-line-gnd fault through a separate fault impedance*

The following equations can be constructed for the above figure.

$$V_{AR} = (Z_{AA} + Z_f) I_A - Z_{AB} I_A$$
$$V_{BR} = Z_{AB} I_A - (Z_{BB} + Z_f) I_A \quad \textbf{(13-53)}$$

This corresponds to the following system.

$$\begin{Bmatrix} I_X \\ I_Y \end{Bmatrix} = \begin{bmatrix} Z_{XX} + Z_f & Z_{XY} \\ Z_{XY} & Z_{YY} + Z_f \end{bmatrix}^{-1} \begin{Bmatrix} V_{XR} \\ V_{YR} \end{Bmatrix} \quad \text{Fault on X-Y} \quad \textbf{(13-54)}$$

X,Y=A,B,C $\qquad I_X + I_Y = 0$

Phase domain faults on ungrounded sections

This subsection points out the relationship between the grounded and ungrounded model of a fault. The derivations for faults on various types of ungrounded lines are then given.

Conversion of grounded Z model to ungrounded model

Fig. 13-15 *Ungrounded three-wire section*

You can determine the following impedance relationship for this system.

$$\begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} = \begin{bmatrix} Z_{ab-ab} & Z_{ab-bc} \\ Z_{ab-bc} & Z_{bc-bc} \end{bmatrix} \begin{Bmatrix} I_{ab} \\ I_{bc} \end{Bmatrix} \tag{13-55}$$

The goal is to start with a three-wire system with a neutral and derive a similar form. You can start with the following system of equations.

$$\begin{Bmatrix} V_a \\ V_b \\ V_c \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ & Z_{22} & Z_{23} \\ & & Z_{33} \end{bmatrix} \begin{Bmatrix} I_a \\ I_b \\ I_c \end{Bmatrix} \tag{13-56}$$

You can subtract equations to reduce the order of the system to line-line voltages. For example:

$$\begin{aligned} V_{ab} &= (Z_{11} - Z_{21}) I_a + (Z_{12} - Z_{22}) I_b + (Z_{13} - Z_{23}) I_c \\ V_{bc} &= (Z_{21} - Z_{31}) I_a + (Z_{22} - Z_{32}) I_b + (Z_{23} - Z_{33}) I_c \end{aligned} \tag{13-57}$$

If you look at the flows in the ungrounded line model, you can relate the line-line current flows to line flows as follows.

$$\begin{aligned} I_a &= I_{ab} \\ -I_c &= I_{bc} \end{aligned} \tag{13-58}$$

From this definition and the fact that $I_a = I_b = I_c$ in the ungrounded model, you can formulate the following.

$$I_b = I_{bc} - I_{ab} \tag{13-59}$$

You can now eliminate I_b from your system of equations as follows.

$$\begin{aligned} V_{ab} &= (Z_{11} - Z_{21}) I_{ab} + (Z_{12} - Z_{22})(I_{bc} - I_{ab}) - (Z_{13} - Z_{23}) I_{bc} \\ V_{bc} &= (Z_{21} - Z_{31}) I_{ab} + (Z_{22} - Z_{32})(I_{bc} - I_{ab}) - (Z_{23} - Z_{33}) I_{bc} \end{aligned} \tag{13-60}$$

This can now be simplified and written in matrix form. For example:

$$\begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} = \begin{bmatrix} Z_{11} - Z_{21} - Z_{12} + Z_{22} & Z_{12} - Z_{22} - Z_{13} + Z_{23} \\ Z_{21} - Z_{31} - Z_{22} + Z_{32} & Z_{22} - Z_{32} - Z_{23} + Z_{33} \end{bmatrix} \begin{Bmatrix} I_{ab} \\ I_{bc} \end{Bmatrix} \tag{13-61}$$

If you have or assume a balanced construction by using the same value for all diagonal terms and the same value for all off-diagonal terms in the original description of the grounded system, you can use your notation for series and mutual impedance values as shown below.

$$\begin{aligned} Z_{11} &= Z_{22} = Z_S \\ Z_{12} &= Z_{13} = Z_{23} = Z_M \end{aligned} \tag{13-62}$$

You now have the following.

$$\begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} = \begin{bmatrix} 2(Z_S - Z_M) & -(Z_S - Z_M) \\ -(Z_S - Z_M) & 2(Z_S - Z_M) \end{bmatrix} \begin{Bmatrix} I_{ab} \\ I_{bc} \end{Bmatrix} \tag{13-63}$$

Now, the impedance of the ungrounded line can be expressed in terms of sequence quantities. For example:

$$\begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} = \begin{bmatrix} 2Z^+ & -Z^+ \\ -Z^+ & 2Z^+ \end{bmatrix} \begin{Bmatrix} I_{ab} \\ I_{bc} \end{Bmatrix} \tag{13-64}$$

Three-phase fault calculations can be performed with these positive sequence values.

L-L-L fault on ungrounded three-wire sections

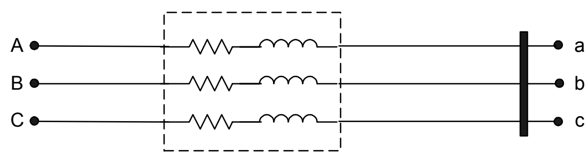


Fig. 13-16 *Ungrounded section with a line-line fault*

Since the fault occurs on an ungrounded section, the relationship between the line-line voltages and the current flowing between phases can be found.

$$\begin{Bmatrix} \Delta V_{ab} \\ \Delta V_{bc} \end{Bmatrix} = \begin{bmatrix} Z_{ab-ab} & Z_{ab-bc} \\ Z_{ab-bc} & Z_{bc-bc} \end{bmatrix} \begin{Bmatrix} \Delta I_{ab} \\ \Delta I_{bc} \end{Bmatrix} \tag{13-65}$$

A current quantity such as I_{xy} refers to a current out of phase x and into phase y. Given this definition along with the equation above, it can be seen that the following relationship holds for the line current referenced in both of the equations in the above matrix representation.

$$\Delta I_b = \Delta I_{bc} - \Delta I_{ab} \tag{13-66}$$

During a fault, the voltage between the faulted wires goes to zero. Therefore:

$$-\begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} = \begin{bmatrix} Z_{ab-ab} & Z_{ab-bc} \\ Z_{ab-bc} & Z_{bc-bc} \end{bmatrix} \begin{Bmatrix} I_{ab} \\ I_{bc} \end{Bmatrix} \tag{13-67}$$

where V_{ab} and V_{bc} are the pre-fault voltages.

You can now invert the system to get the following results.

$$\begin{Bmatrix} I_{ab} \\ I_{bc} \end{Bmatrix} = -\begin{bmatrix} Z_{ab-ab} & Z_{ab-bc} \\ Z_{ab-bc} & Z_{bc-bc} \end{bmatrix}^{-1} \begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} \tag{13-68}$$

This equation is valid for determining the by-phase fault current for an ungrounded section. You can make some assumptions to simplify things. For example:

$$\begin{aligned} Z_{ab-ab} &= Z_{bc-bc} = Z_1 \\ Z_{ab-bc} &= Z_2 \\ V_{ab} - V_{bc} &= \sqrt{3}V_{LL} \end{aligned} \tag{13-69}$$

If you use the previous equations along with the above assumptions you get the following results.

$$I_b = \frac{(Z_1 + Z_2)V_{ab} - (Z_1 + Z_2)V_{bc}}{Z_1^2 - Z_2^2} = \frac{V_{ab} - V_{bc}}{(Z_1 - Z_2)} = \frac{\sqrt{3}V_{LL}}{(Z_1 - Z_2)} \tag{13-70}$$

It is interesting to note that if you pad the matrix in the previous equation and apply the phase-to-sequence domain transformation, you find the following.

$$Z_1 - Z_2 = Z^+ \tag{13-71}$$

L-L faults on ungrounded sections

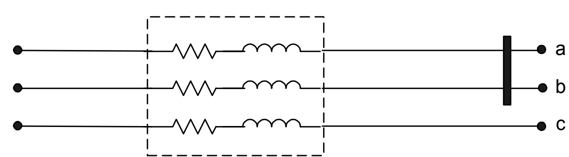


Fig. 13-17 *Ungrounded section with line-line fault*

You have the same system description from the fault terminals as you did in the previous section, except the current from wire b to wire c is zero since there is no connection. Therefore:

$$-\begin{Bmatrix} V_{ab} \\ V_{bc} \end{Bmatrix} = \begin{bmatrix} Z_{ab-ab} & Z_{ab-bc} \\ Z_{ab-bc} & Z_{bc-bc} \end{bmatrix} \begin{Bmatrix} I_{ab} \\ 0 \end{Bmatrix} \tag{13-72}$$

You can use the top row of this equation and solve for the current flow in wires a and b. For example:

$$I_{ab} = -\frac{V_{ab}}{Z_{ab-ab}} \tag{13-73}$$

If you use a nominal voltage for pre-fault, the line-line fault current is as follows.

$$I_{LL} = \frac{V_{LL}}{\left|Z_{ab-ab}\right|} \tag{13-74}$$

Performing the same type of derivation on a system with the fault across wires b and c yields the following.

$$-\begin{Bmatrix} V_{bc} \\ V_{ca} \end{Bmatrix} = \begin{bmatrix} Z_{bc-bc} & Z_{bc-ca} \\ Z_{bc-ca} & Z_{ca-ca} \end{bmatrix} \begin{Bmatrix} I_{ab} \\ 0 \end{Bmatrix} \tag{13-75}$$

The rest of the derivation to find the line-line fault current is similar.

Performing fault analysis on systems with loops

The distribution model in SynerGEE is radial and loops are indicated with open switches marked as tie switches. During the analysis, the switches are treated as if they were closed.

Consider a system with *n* loops. A fault is to be simulated on section X. The distribution system is considered linear during the fault and you can therefore calculate the impedance looking into the fault by observing the relationship between voltages and currents under small perturbations. The following matrix equation corresponds to the system.

$$\begin{Bmatrix} \Delta_X V_A \\ \Delta_X V_B \\ \Delta_X V_C \\ \Delta_1 V_A \\ \cdots \\ \Delta_{N-1} V_C \\ \Delta_N V_A \\ \Delta_N V_B \\ \Delta_N V_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Tie/Fault} & \\ & & \end{bmatrix} \begin{Bmatrix} \Delta_X I_A \\ \Delta_X I_B \\ \Delta_X I_C \\ \Delta_1 I_A \\ \cdots \\ \Delta_{N-1} I_C \\ \Delta_N I_A \\ \Delta_N I_B \\ \Delta_N I_C \end{Bmatrix} \tag{13-76}$$

You are observing the following in this system.

$$\begin{aligned} \text{Change in voltage at fault} &= \Delta_X V_{A,B,C} \\ \text{Change in voltage across tie switch} &= \Delta_{1\ldots N} V_{A,B,C} \\ \text{Change in current into fault} &= \Delta_X I_{A,B,C} \\ \text{Change in current across tie switch} &= \Delta_{1\ldots N} I_{A,B,C} \end{aligned} \tag{13-77}$$

During a fault, the voltages across tie switches are unchanged. Therefore:

$$\Delta_{1\ldots N} V_{A,B,C} = 0 \tag{13-78}$$

The matrix equation transforms to the following.

$$\begin{Bmatrix} \Delta_X V_A \\ \Delta_X V_B \\ \Delta_X V_C \\ 0 \\ 0 \\ 0 \\ \ldots \\ \ldots \\ 0 \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Tie/Fault} & \\ & & \end{bmatrix} \begin{Bmatrix} \Delta_X I_A \\ \Delta_X I_B \\ \Delta_X I_C \\ \Delta_1 I_A \\ \ldots \\ \Delta_{N-1} I_C \\ \Delta_N I_A \\ \Delta_N I_B \\ \Delta_N I_C \end{Bmatrix} \tag{13-79}$$

A Kron reduction can now be performed to get a reduced impedance matrix. The matrix is the distribution system as seen from the fault point.

$$\begin{Bmatrix} \Delta_X V_A \\ \Delta_X V_B \\ \Delta_X V_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Red} & \\ & & \end{bmatrix} \begin{Bmatrix} \Delta_X I_A \\ \Delta_X I_B \\ \Delta_X I_C \end{Bmatrix} \tag{13-80}$$

The rank of $Z_{Tie/Fault}$ is dependent upon the phasing of the switches. A grounded three-phase switch increases the rank by 3. An ungrounded three-phase switch increases it by only 2. A switch across grounded single-phase lines increases the rank by 1. Regardless of the rank of $Z_{Tie/Fault}$, the rank of Z_{Red} matches the phasing of the faulted line. Thus, if a single-phase line is being faulted, Z_{Red} has a rank of 1.

The Z_{Red} impedance matrix includes the effects of loops, generators, and other devices. It corresponds to the system model used in the development of the calculations from 3 by 3 cumulative impedance matrix. The matrix may need to be padded if the faulted line is anything other than a grounded three-phase line. For a particular fault type, the fault currents can be determined from a system similar to the following.

$$\begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Fault} = f(Z_{\mathrm{Red}}) & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} \tag{13-81}$$

Since the fault currents are performed on an unloaded system, it follows that the fault current calculated above corresponds to the change in current shown in the previous equation. Therefore:

$$\begin{Bmatrix} \Delta_1 I_A \\ \Delta_1 I_B \\ \Delta_1 I_C \end{Bmatrix} = \begin{Bmatrix} I_A \\ I_B \\ I_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Fault} = f(Z_{\mathrm{Red}}) & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} V_{AR} \\ V_{BR} \\ V_{CR} \end{Bmatrix} \tag{13-82}$$

This is the expression used to determine fault currents. The phase domain derivations for the various fault types are used for $Z_{Fault} = f(Z_{\mathrm{Red}})$ and the by-phase fault currents are determined.

Fault flows on looped systems

The previous equation is used to find the current into the fault given the pre-fault voltages and the reduced $Z_{Tie/Fault}$ matrix. The discussion of faults in a looped system starts with the relationship between fault current, fault voltage, tie current, and tie voltage. You can write the system in inverted form as shown below.

(13-83)

$$\begin{Bmatrix} \Delta_X I_A \\ \Delta_X I_B \\ \Delta_X I_C \\ \Delta_1 I_A \\ \dots \\ \Delta_{N-1} I_C \\ \Delta_N I_A \\ \Delta_N I_B \\ \Delta_N I_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Tie/Fault} & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} \Delta_X V_A \\ \Delta_X V_B \\ \Delta_X V_C \\ 0 \\ 0 \\ 0 \\ \dots \\ \dots \\ 0 \end{Bmatrix}$$

Since you know the fault current from before and $f(Z_{\mathrm{Red}})$, you can use the previous equation to calculate the changes in voltage at the fault, due to the fault current. For example:

(13-84)

$$\begin{Bmatrix} V_{F-A} \\ V_{F-B} \\ V_{F-C} \end{Bmatrix} = \begin{Bmatrix} \Delta_X V_A \\ \Delta_X V_B \\ \Delta_X V_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Red} & \\ & & \end{bmatrix} \begin{Bmatrix} \Delta_X I_A \\ \Delta_X I_B \\ \Delta_X I_C \end{Bmatrix}$$

The change in voltage can be substituted as follows:

(13-85)

$$\begin{Bmatrix} \Delta_X I_A \\ \Delta_X I_B \\ \Delta_X I_C \\ \Delta_1 I_A \\ \dots \\ \Delta_{N-1} I_C \\ \Delta_N I_A \\ \Delta_N I_B \\ \Delta_N I_C \end{Bmatrix} = \begin{bmatrix} & & \\ & Z_{Tie/Fault} & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} V_{F-A} \\ V_{F-B} \\ V_{F-C} \\ 0 \\ \dots \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

It is important to note that only the first three columns of the inverse of the $Z_{Tie/Fault}$ impedance matrix are needed for these calculations. They result in the fault current that should match your previous calculation. They also determine the current flows through each of the tie switches. These current flows along with the fault current are the only injections into the distribution system. Therefore, all of the flows in the system can be found.

Including generators

During a fault, active synchronous and induction generators behave like a voltage source behind a subtransient reactance tied to their grounding impedance. Since they are effectively a source, generators contribute to the system of equations used to handle loops. The rank of the Tie/Fault impedance matrix is adjusted according to the characteristics of each generator included in the analysis. Flows out of the generator during a fault are handled with the Tie/Fault and Red matrices discussed above.

Fault analysis with wandering laterals

Chapter 11, *Load-flow Analysis*, contains a discussion on wandering laterals, including the following diagram.

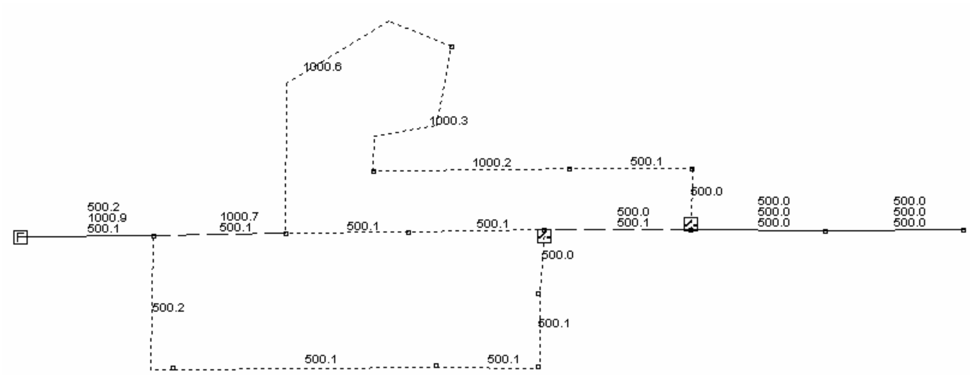


Fig. 13-18 *Feeder with two wandering laterals*

The feeder serves a 500kW load on the upper wandering lateral and a three-phase 1500kW load at the end of the feeder. The map shows the kW flows from the feeder and along the laterals and back to the main line to serve the three-phase load.

fault, fault flow, and fault voltage can also be run on this feeder. If fault analysis is run, fault impedances are found using looped fault analysis. Fault currents and flows are found from the fault impedances. Fault flow and fault voltage analysis use their load-flow based engine that is fully compatible with wandering laterals.

Example 1: Transformers in sequence fault analysis

Given:

Feeder	13.8kV. Source Z0 = 2.2 +j 3.2, Source Z1 = 1.2 +j 2.3 Ohms
Transformer	1000 kVA 13.8 / 4.16 kV Ygnd / Ygnd Z = 4.8%. R = 0.8%. Tap = 1.0
Line 1	7000 feet of 4/0 ACSR. #1 ACSR neutral. Dp = 5 feet. Dn = 3.5 feet

Line 2	15000 feet of 4/0 ACSR. #1 ACSR neutral. Dp = 5 feet. Dn = 3.5 feet

Desired:

Fault currents at end of line 2.

Calculations:

First, find the impedance for the transformer on the high side.

$$Z_{Tran} = (0.8 + j\sqrt{4.8^2 - 0.8^2})\% = (0.8 + j4.7329)\% \tag{13-86}$$

The high side base impedance is as follows.

$$ZBase_{Tran} = \frac{1000 * 13.8^2}{1000} = 190.44 \tag{13-87}$$

So the transformer impedance is:

$$Z_{Tran} = 1.5235 + j9.01\Omega \tag{13-88}$$

You previously calculated line impedance values, so you are ready to get the impedance at the fault point.

$$\begin{aligned} ZHigh_0 &= (2.2 + j3.2) + \frac{7}{5.28}(1.344 + j2.324) + (1.5235 + j9.01) \\ &= 5.505 + j15.291\Omega \\ ZHigh_1 &= (1.2 + j2.3) + \frac{7}{5.28}(0.592 + j0.779) + (1.5235 + j9.01) \\ &= 3.508 + j12.343\Omega \end{aligned} \tag{13-89}$$

Next, find the impedance looking into the secondary of the transformer.

$$\begin{aligned} ZLow_0 &= (5.505 + j15.291) * \left(\frac{4.16}{13.8}\right)^2 = 0.5 + j1.390\Omega \\ ZLow_1 &= (3.508 + j12.343) * \left(\frac{4.16}{13.8}\right)^2 = 0.319 + j1.122\Omega \end{aligned} \tag{13-90}$$

Finally, you can add the impedance from the second line and come up with the impedance seen from the fault.

$$ZFlt_0 = (0.5 + j1.390) + \frac{15}{5.28}(1.344 + j2.324) = 4.318 + j7.992\Omega \tag{13-91}$$

$$ZFlt_1 = (0.319 + j1.122) + \frac{15}{5.28}(0.592 + j0.779) = 2.0 + j3.335\Omega$$

Fault current values can now be calculated.

$$I_{PhG} = \frac{1000 * \sqrt{3} * 4.16}{|(4.318 + j7.992) + 2(2.0 + j3.335)|} = 427A \tag{13-92}$$

$$I_{3Ph} = \frac{1000 * 4.16}{\sqrt{3} * |2.0 + j3.335|} = 617A$$

Example 2: Off-nominal transformers in sequence fault

Given:

Data from "Transformers in sequence fault analysis" with the transformer at 1.10 tap. (This is an unrealistic tap. However, the calculations will not match the SynerGEE numbers exactly, so the tap was chosen to contrast the effect of the tap.)

Desired:

Fault currents at end of line 2.

Calculations:

The impedances are reflected differently through the transformer with the tap setting.

$$ZLow_0 = (5.505 + j15.291) * \left(\frac{4.16}{13.8 * 1.1}\right)^2 = 0.413 + j1.14\Omega \tag{13-93}$$

$$ZLow_1 = (3.508 + j12.343) * \left(\frac{4.16}{13.8 * 1.1}\right)^2 = 0.263 + j0.927\Omega$$

Add the second line impedance:

$$ZFlt_0 = (0.413 + j1.14) + \frac{15}{5.28}(1.344 + j2.324) = 4.232 + j7.751\Omega \tag{13-94}$$

$$ZFlt_1 = (0.263 + j0.927) + \frac{15}{5.28}(0.592 + j0.779) = 1.945 + j3.14\Omega$$

Calculate fault currents with nominal voltage:

(13-95)

$$I_{PhG} = \frac{1000 * \sqrt{3} * 4.16}{|(4.232 + j7.751) + 2(1.945 + j3.14)|} = 444A$$

$$I_{3Ph} = \frac{1000 * 4.16}{\sqrt{3} * |1.945 + j3.14|} = 650A$$

Calculate fault currents with off-nominal voltage:

(13-96)

$$I_{PhG} = \frac{1000 * \sqrt{3} * 4.16/1.1}{|(4.232 + j7.751) + 2(1.945 + j3.14)|} = 404A$$

$$I_{3Ph} = \frac{1000 * 4.16/1.1}{\sqrt{3} * |1.945 + j3.14|} = 591A$$

Example 3: Faulting single phase lines

Sequence domain process

Fault analysis in the sequence domain consists of these steps:

1. Calculate all phase domain impedance values
2. Start at feeder source and outwardly accumulate sequence domain impedances.
3. For each line and device, convert phase domain impedance to sequence domain.
4. For each device, accumulate sequence impedance from source end to load end.
5. Calculate fault levels with standard sequence domain approach using each device's sequence domain source impedance.

This approach generates fault values consistent with sequence network based fault values.

L-G fault in phase domain

A single-phase line is fed by a source and a three-phase line. In matrix form, the single phase line has the following impedance:

$$Z_{1\phi} = \begin{bmatrix} Z_{AA}^{1\phi} & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{bmatrix} \tag{13-97}$$

Here, we are looking at phase 'A'. The three phase system feeding the single-phase line has this impedance matrix:

$$Z_{3\phi} = \begin{bmatrix} Z_{AA}^{3\phi} & Z_{AB}^{3\phi} & Z_{AC}^{3\phi} \\ & Z_{BB}^{3\phi} & Z_{BC}^{3\phi} \\ & & Z_{CC}^{3\phi} \end{bmatrix} \tag{13-98}$$

Finally the source has these impedances:

$$Z_{Src} = \begin{bmatrix} Z_S^{Src} & Z_M^{Src} & Z_M^{Src} \\ & Z_S^{Src} & Z_M^{M} \\ & & Z_S^{Src} \end{bmatrix} \tag{13-99}$$

So the source impedance seen from the end of the single-phase line is:

$$Z_{AA}^{Fault} = Z_{AA}^{1\phi} + Z_{AA}^{3\phi} + Z_S^{Src} \tag{13-100}$$

The fault current for a L-G fault on phase 'A' can be calculated by using superposition:

$$\begin{Bmatrix} -V_f \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} Z_{AA}^{Fault} & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{bmatrix} \begin{Bmatrix} I_A \\ 0 \\ 0 \end{Bmatrix} \tag{13-101}$$

The fault current is:

$$I_A = -\frac{V_f}{Z_{AA}^{Fault}} \tag{13-102}$$

Example in phase domain

We have a feeder source impedance of:

$$\begin{aligned} Z_s^0 &= 0.01 + j1 \ \ \Omega \\ Z_s^{1,2} &= 0.01 + j1 \ \ \Omega \end{aligned} \tag{13-103}$$

The feeder source impedance in the phase domain is therefore:

$$Z_S = \begin{bmatrix} 0.01 + j1 & 0 & 0 \\ & 0.01 + j1 & 0 \\ & & 0.01 + j1 \end{bmatrix} \Omega \tag{13-104}$$

A three-phase line with the following phase domain impedance is fed:

$$Z_{Line1} = \begin{bmatrix} 0.311 + j\,1.028 & 0.099 + j\,0.424 & 0.100 + j\,0.349 \\ & 0.309 + j\,1.041 & 0.099 + j\,0.426 \\ & & 0.311 + j\,1.027 \end{bmatrix} \Omega \tag{13-105}$$

That line ties to our faulted single phase line:

$$Z_{Line2} = \begin{bmatrix} 1.735 + j\,1.369 & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{bmatrix} \Omega \tag{13-106}$$

The impedance seen at the fault is the total along phase A:

$$Z_{Line2} = \begin{bmatrix} 2.056 + j3.397 & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{bmatrix} \Omega \tag{13-107}$$

The line is on a 12.47kV system so the fault current is:

$$I_f = \left| \frac{12.47}{\sqrt{3}\left(2.056 + j3.397\right)} \right| = 1.813\,kA \tag{13-108}$$

L-G fault in sequence domain

Impedances in the sequence domain can only be found for three phase (grounded) lines. To use sequence impedances for calculating L-G faults on a single phase line, we will set all diagonal elements in the phase domain impedance matrix to the value of the single phase:

$$Z_{1\phi} = \begin{bmatrix} Z_{AA}^{1\phi} & 0 & 0 \\ & Z_{AA}^{1\phi} & 0 \\ & & Z_{AA}^{1\phi} \end{bmatrix} \tag{13-109}$$

This in effect models three non-coupled single phase lines. The modeling is valid as long as fault calculations on it are limited to L-G types.

Because there is no coupling, the sequence domain impedance numerically matches the phase domain value:

$$Z_0^{1\phi} = Z_{AA}^{1\phi}$$
$$Z_{1,2}^{1\phi} = Z_{AA}^{1\phi} \quad \textbf{(13-110)}$$

The sequence domain for the three-phase line and source can also be converted to the sequence domain. The total sequence domain impedance seen from the fault location is added up and the fault current is calculated as:

$$I_f = \frac{1000 \cdot \sqrt{3} \cdot kV_{LL}}{\left| Z_0^{Fault} + 2Z_1^{Fault} \right|} \quad \textbf{(13-111)}$$

Example in sequence domain

Again, we have a feeder source impedance of:

$$Z_0^{Src} = 0.01 + j1 \ \ \Omega$$
$$Z_{1,2}^{Src} = 0.01 + j1 \ \ \Omega \quad \textbf{(13-112)}$$

The three phase line impedance is the sequence domain is:

$$Z_0^{Src} = 0.509 + \text{j } 1.832 \ \ \Omega$$
$$Z_{1,2}^{Src} = 0.211 + \text{j } 0.632 \ \ \Omega \quad \textbf{(13-113)}$$

And the single phase line's "sequence domain" impedance is:

$$Z_0^{1\phi} = 1.735 + \text{j } 1.369 \ \ \Omega$$
$$Z_{1,2}^{1\phi} = 1.735 + \text{j } 1.369 \ \ \Omega \quad \textbf{(13-114)}$$

The total sequence domain impedance at the fault is:

$$Z_0^{Fault} = 2.245 + \text{j } 4.201 \ \ \Omega$$
$$Z_{1,2}^{Fault} = 1.956 + \text{j } 3.001 \ \ \Omega \quad \textbf{(13-115)}$$

The fault current is:

$$I_f = \frac{1000 \cdot \sqrt{3} \cdot 12.47}{\left| (2.245 + \text{j } 4.201) + 2(1.956 + \text{j } 3.001) \right|}$$
$$= 1813 \ \ kA \quad \textbf{(13-116)}$$

Example 4: Faulting two-phase lines

The sequence domain is defined for three-wire systems with a reference or ground. Because decoupled positive, negative, and zero domain networks are used, sequence domain modeling is restricted to impedance balanced three-phase models. With special considerations, sequence domain values can still be used for distribution analysis.

Phase domain impedance matrix

We will explore the handling of a two-phase line in phase domain fault analysis and sequence domain analysis. A two-phase line tied to a 4.16kV infinite bus has the following impedance:

(13-117)

$$Z_{Phase} = \begin{bmatrix} 1.229 + j\,1.209 & 0 & 0.203 + j\,0.422 \\ 0 & 0 & 0 \\ 0.203 + j\,0.422 & 0 & 1.229 + j\,1.209 \end{bmatrix} \Omega$$

This phase domain impedance matrix shows series impedances along the diagonal for phase 'A' and 'C'. There is also an impedance coupling 'A' to 'C' at the off-diagonal corners of the matrix.

Here is the matrix put into the context of voltages generated from current flow:

(13-118)

$$\begin{Bmatrix} V_A \\ - \\ V_C \end{Bmatrix} = \begin{bmatrix} 1.229 + j\,1.209 & 0 & 0.203 + j\,0.422 \\ 0 & 0 & 0 \\ 0.203 + j\,0.422 & 0 & 1.229 + j\,1.209 \end{bmatrix} \begin{Bmatrix} I_A \\ 0 \\ I_C \end{Bmatrix}$$

The voltage drop for phase 'A' is a function of the current flows on phase 'A' and 'C':

(13-119)

$$V_A = (1.229 + j1.209)I_A + (0.203 + j0.422)I_C$$

The voltage effects of earth / neutral return are accounted for in the Carson based calculations used to generate the impedance matrix. In a distribution system, I_A would not match I_C unless the line were delta connected (which would lead do a whole other discussion).

L-G Fault value

For a line-ground fault on phase A, we neglect load current and set I_C to zero. We use superposition to deal with the 0V fault voltage. Fault current on our 4.16kV line is therefore:

(13-120)

$$I_{LG} = \frac{4.16}{\sqrt{3}\,|(1.229 + j1.209)|}$$
$$= 1393A$$

All calculations to this point have been phase domain based on a simple circuit.

The <u>wrong</u> sequence conversion

The phase domain impedance matrix cannot be converted to the sequence domain in its present form. The lack of a 'B' phase or more specifically, the zero in the diagonal invalidates the basis of sequence domain theory (not necessary symmetrical components).

Let's try the fault calculations anyway. If we do the phase->sequence domain conversion:

(13-121)

$$Z_S = A^{-1} Z_P A$$

on the matrix with a zero in the Z_{BB} spot we get:

(13-122)

$$Z_0 = (0.955 + j1.087)$$
$$Z_1 = (0.752 + j0.665)$$

The sequence domain model leads to the usual equation for LG fault amps:

(13-123)

$$I_f = \frac{1000 * \sqrt{3} * kV_{LL}}{|Z_0 + 2Z_1|}$$

Plug in our numbers:

(13-124)

$$I_f =$$
$$\frac{1000 * \sqrt{3} * 4.16}{|(0.955 + j1.087) + 2(0.752 + j0.665)|}$$
$$= 2090A$$

Something is wrong since we get a different answer than we did for the phase domain calculation.

The right sequence conversion

We will make our phase domain impedance matrix valid for sequence domain conversion by copying the phase 'A' and 'C' values into the phase 'B' diagonal spot:

(13-125)

$$Z'_{Phase} =$$

$$\begin{bmatrix} 1.229 + j\,1.209 & 0 & 0.203 + j\,0.422 \\ 0 & 1.229 + j\,1.209 & 0 \\ 0.203 + j\,0.422 & 0 & 1.229 + j\,1.209 \end{bmatrix} \Omega$$

Now we can convert to the sequence domain to get:

(13-126)

$$Z_0 = (1.364 + j1.49)$$
$$Z_1 = (1.161 + j1.068)$$

We can proceed with the fault calculations in the sequence domain:

(13-127)

$$I_f = \frac{1000 * \sqrt{3} * 4.16}{\left|(1.364 + j1.49) + 2(1.161 + j1.068)\right|}$$
$$= 1393A$$

This calculated fault current exactly matches the phase domain value.

A similar procedure could be used to verify the consistency of L-L and L-L-G fault values between the phase and sequence domain.

Also, it was shown in an earlier chapter that the series phase domain impedance is related to the sequence domain impedances by:

(13-128)

$$Z_S = \left(Z_0 + 2Z_1\right)/3$$

For our example, we get:

(13-129)

$$Z_S = \frac{(1.364 + j1.49) + 2(1.161 + j1.068)}{3}$$
$$= (1.229 + j1.209)$$

This value matches our original phase domain series impedance values:

(13-130)

$$Z_{Phase} =$$

$$\begin{bmatrix} 1.229 + j\,1.209 & 0 & 0.203 + j\,0.422 \\ 0 & 0 & 0 \\ 0.203 + j\,0.422 & 0 & 1.229 + j\,1.209 \end{bmatrix} \Omega$$

Example 5: Parallel / dual feed substation transformers

Given:

Subtransmission	34.4kV
Source Z (pu, 100 MVA)	$Z_0 = 0.49528 + j1.51723\Omega$ $Z_1 = 0.32665 + j1.08843\Omega$
Transformers	Two units tied at secondary bus. Each: 7.5MVA and 5.95% 34.4kV Delta / 13.09kV Wye-Gnd

Desired:

Impedance at secondary.

Calculations:

There are two ways to model this situation in SynerGEE. Both approaches will yield valid fault values. However, the handling of the high-side source impedance is very different in each case.

The first situation would be a dual feed from different sources and look like this in SynerGEE:

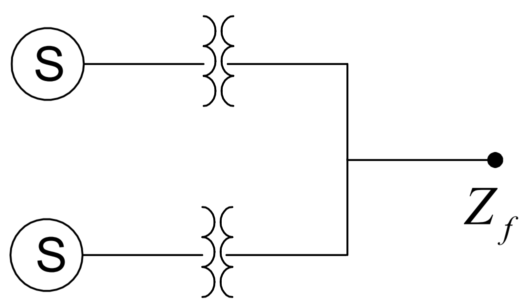


Fig. 13-19 *Fault fed from dual transformers*

The p.u. impedance of each of the transformers on the 100MVA system base is:

$$Z_{Tran-pu} = j0.0595\left(\frac{100}{7.5}\right) = j0.7933\,pu \tag{13-131}$$

The impedance on each of the upper and lower legs will be the sum of the source and transformer impedances:

$$Z_{0,Leg} = 0 + (0, 0.7933) = j0.7933\,pu$$
$$Z_{1,Leg} = (0.32665, 1.08843) + (0, 0.7933) = 0.32665 + j1.8817\,pu$$
(13-132)

Both of these legs serve the fault location in parallel. So,

$$Z_{0,F} = \frac{Z_{0,Leg}}{2} = j0.3967\,pu$$
$$Z_{1,Leg} = \frac{Z_{1,Leg}}{2} = 0.1633 + j0.9409\,pu$$
(13-133)

The base impedance on the transformer secondary is:

$$Z_{Base} = \frac{13.09^2}{100} = 1.71348$$
(13-134)

And the fault impedance is:

$$Z_{0,F} = j0.3967 \cdot Z_{Base} = 0.6797\Omega$$
$$Z_{1,Leg} = (0.1633, 0.9409) \cdot Z_{Base} = 0.279 + j1.612\Omega$$
(13-135)

The second way to model this situation is with parallel transformers. To make the transformers parallel and not dual feed, the model needs to be setup as follows:

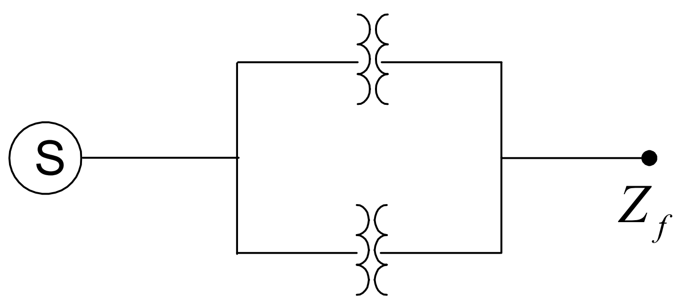


Fig. 13-20 *Fault fed from parallel transformers*

Now, the parallel equivalent impedance of the transformers is calculated and added to the source impedance:

$$Z_0 = \frac{j0.7933}{2} = j0.3967\,pu$$
$$Z_1 = (0.32665, 1.08843) + \frac{j0.7933}{2} = 0.32665 + j1.485\,pu$$
(13-136)

In Ohms, we have:

$$Z_{0,F} = j0.3967 \cdot Z_{Base} = 0.6797\Omega$$
$$Z_{1,Leg} = (0.32665, 1.485) \cdot Z_{Base} = 0.5597 + j2.545\Omega$$
(13-137)

14
Arc Flashover Analysis

Introduction

Electric Arc Flash is the passage of current between two electrodes through ionized gases and vapors. The electrical energy supplied to the arc is converted into other forms of energy and results in intense heat. The extremely high temperatures of these arcs can cause fatal burns at about 5 ft and major burns at up to 10 ft distance from the arc. The intensity of arc depends on voltage, fault current, clearing time of protective device, enclosure space etc. Arc flash is primarily found in circuits with the operating voltage of 208V and higher in commercial and industrial facilities.

Electric arc burns make up a major portion of injuries related to electric malfunctions. Arc Flash standards are issued by the Institute of Electrical and Electronic Engineers (IEEE) and National Fire Protection Agency (NFPA). These are a result of the industry efforts to protect employees from arc flash hazards. Most of the incidents could be prevented or the intensity significantly reduced if the employees wore proper type of clothing or were forewarned of safety hazards. IEEE and NFPA standards now require switchboards, panel boards and motor control centers to be individually field marked with arc flash warning labels that also specify the protective clothing and arc flash boundary distance.

Arc Flashover calculations form the basis to develop strategies to minimize burn injuries. These strategies can include specifying the rating of personal protective equipment (PPE), working de-energized, using arc-resistant switchgear, or applying other engineering techniques and work practices.

The calculations for Arc Flashover in SynerGEE are based on IEEE standard 1584-2002 and NFPA 70E-2004. They are completely integrated into SynerGEE Electric and are available as an analysis tool. Arc fault calculations are performed for all sections and devices in the selection set. Labels can be generated for the devices.

IEEE and NFPA standards specify that flash-protection boundary, working distance, and incident energy should be prominently displayed on every piece of electrical equipment where arc flash hazard exists.

Disclaimer

The data and the information are believed to be correct in the program and the documentation. However, any and all liability, for the content and any omissions including any inaccuracies, errors or misstatements in data, calculations or information is expressly disclaimed. Advantica disclaims any liability for the use of software, calculations or other information.

Exceptions

IEEE standards do not cover single-phase ac systems.

For a three-phase line, only three-phase faults are considered for arc flashover calculations as the severity of three-phase faults is the highest. This is according to the practice followed by IEEE standard.

Analysis models

Arc Flash analysis is based on two models.

1. Empirically derived model based on statistical analysis and the curve fitting programs described in IEEE standard 1584-2002.
2. Physical model developed by Ralph Lee.

The empirically derived model is based on several arc flashover tests that were performed or witnessed by the IEEE committee. Data was gathered and equations were developed after performing statistical analysis and running curve fitting programs.

Test data showed that arc current primarily depends on available fault current, incident energy is directly proportional to the arc time, and flash boundary distance has an inverse exponential effect on the incident energy. It is to be noted that three-phase arcs produce the greatest possible arc-flash hazard.

Empirically derived model is used for following cases:

- Voltage is in the range of 208 V – 15000 V,
- equipment enclosures of commonly available sizes,
- cables and conductors in air with gaps between conductors of 13 mm – 152 mm

The empirical model is developed for three-phase faults. However, we are extending the model to include single-phase faults so that analysis can be performed on single-phase equipment and conductors.

For cases where the voltage is over 15 kV, the theoretically derived Lee method is applied. This method is also included in the IEEE Std. 1584-2002 for incident energy calculations.

Methodology

Using the above analysis models, arc fault currents are determined from the bolted three-phase fault current on the line. In case the line is single-phase, the program uses single-phase line to ground fault instead of bolted three-phase fault current. The calculated arc fault current is lower than the bolted fault current due to arc impedance, especially for voltage lower than 1 kV. The arc current is then used to determine the protective device operating time. It is difficult to accurately determine arcing current which makes it difficult to predict the clearing time. Arc current releases energy and the energy impressed on a surface at a certain distance (working distance) is called incident energy and is calculated by arc flashover analysis. Incident energy varies directly with the clearing time and it is possible that incident energy is higher at lower arcing current. Therefore, incident energy is also calculated at reduced arcing current (typically 85% of the calculated arcing current). Incident energy is then calculated for both sets of arc currents and operating times and the larger incident energy is taken as the result. Using this method provides results for arcing current with more than 95% confidence level for the tests performed for the empirical model. Incident energy then helps to establish the approach limit at a distance from the live parts that are not insulated (where a person could receive a second degree burn). This approach limit is called flash protection boundary. Safety grade for Personal protective equipment (PPE) is also suggested based on the incident energy. If the incident energy is higher, then higher safety grade is suggested for PPE to reduce the impact of arc flashover.

Approach boundaries are also defined based on the voltage level of the exposed live parts, to warn the unqualified persons to stay outside of the approach boundaries. NFPA defines three approach boundaries that are:

- Limited Approach Boundary.
- Restricted Approach Boundary.
- Prohibited Approach Boundary.

A safety label that lists the results of arc flashover analysis is then available for printing and can be displayed on the equipment. The label forewarns the user about the danger and the necessary safety precautions that need to be taken before working on the equipment.

Model Requirements

Arc Flash analysis requires similar model data requirements as the load flow and fault analysis. Source resistance and reactance should be accurately specified in the source, line impedances and lengths should be correctly modeled, and protective devices (fuses, reclosers, breakers) should be correctly modeled with the necessary settings. Once this is done, you are ready to specify the settings for arc flash analysis and then run the arc flash analysis.

Basic Nomenclature

The following basic nomenclature is used in the Arc Flashover:

log	Log_{10}
I_a	Arcing Current (kA)
K	Constant based on the open or box configuration (See table 2)
I_{bf}	Fault Current
V	System Voltage (kV)
G	Gap between conductors
E_n	Incident energy (J/cm2) normalized for time and distance
K_1	constant value (see table 3)
K_2	constant value (see table 4)
C_f	calculation factor based on voltage (see table 5)
T	arcing time (seconds)
D	distance from the possible arc point to the person (mm)
x	distance exponent from table (see table 1)
I_{bf}	Fault Current

Arc Current Calculations

Arc current or arc fault current is the fault current flowing through electrical arc plasma. It can be less than the fault current due to arc impedance or it can be the same as the fault current when voltages are higher.

For voltages in the range of 208V – 15000V, the calculations are based on empirically derived models. For voltages higher than 15000V, the calculations are based on theoretically derived models based on the work done by Ralph Lee.

The arc current calculations vary based on the operating voltage and the protective device.

Formulas are listed below for different variations:

Voltages less than 1 kV:

$$\log(I_a) = K + 0.662\log(I_{bf}) + 0.0966V + 0.000526G + 0.5588V\log(I_{bf}) - 0.00304G\log(I_{bf}) \quad \textbf{(14-1)}$$

K varies with

G varies with voltage and equipment type. Please see Table 1.

Voltage higher than 1 kV and less than 15 kV:

$$\log(I_a) = 0.00402 + 0.983\log(I_{bf}) \quad \textbf{(14-2)}$$

And then convert from log

$$I_a = 10^{\log(I_a)} \quad \textbf{(14-3)}$$

Reduced Arc current is calculated at 85% of the Arcing Current (Ia), so that a second duration can be determined.

$$I_{a\_reduced} = 0.85\ I_a \quad \textbf{(14-4)}$$

Voltage higher than 15 kV:

For Voltages higher than 15 kV, arc current is the same as the fault current.

$$I_a = I_{bf} \quad \textbf{(14-5)}$$

Incident Energy

According to IEEE Std. 1584, Incident energy is the amount of energy impressed on a surface at a certain distance from the source generated during an electric event. Incident energy is measured in joules per centimeter squared (J / cm^2) or calories per centimeter squared (cal / cm^2). Arc Flashover analysis in SynerGEE calculates the incident energy from the electric arc at a working distance specified by the user that helps to establish the arc flash boundary (discussed in the section for flash boundary) and suggests protective personal equipment (PPE).

Incident energy varies directly with the arc current and the fault clearing time. It is prudent to calculate incident energy for both the arc current and the reduced arc current (85% of the arc current). Fault clearing time can be higher at reduced arc current and sometimes it may result in higher incident energy than the incident energy at full arc current.

First, incident energy is calculated based on arc time of 0.2 seconds and a distance from the possible arc point to the person of 610 mm. This is called normal incident energy. Then incident energy is calculated based on the protecting device clearing time and the actual distance from the possible arc point to the person.

Voltage less than 15 kV:

Normalized incident energy is calculated as follows for the cases where the nominal voltage is less than 15 kV:

The log normal incident energy is calculated as given by the equation as follows:

$$\log(E_n) = K_1 + K_2 + 1.081 \log(I_a) + 0.0011\, G \tag{14-6}$$

K_1 varies with configuration of equipment. See Table 3.

K_2 varies with configuration of equipment. See Table 4.

G varies with voltage and equipment. See Table 1.

Normal incident energy is calculated from log10 of normal incident energy as shown below.

$$E_n = 10^{\log(E_n)} \tag{14-7}$$

Incident energy is then calculated from normal incident energy as follows :

$$E = 4.184 C_f\, E_n \left(\frac{t}{0.2}\right)\left(\frac{610^x}{D^x}\right) \tag{14-8}$$

C_f varies with the voltage. Please see Table 4.

x varies with the voltage and equipment type. Please see Table 5.

Voltage higher than 15 kV:

For cases where the voltage is more than 15 kV, Lee model is used instead of empirical model. Incident energy is calculated as follows:

$$E = 2.142 \; x \; 10^6 \; V \; I_{bf} \left(\frac{t}{D^2} \right) \tag{14-9}$$

Flash Protection Boundary Calculations

Flash protection boundary is an approach limit at a distance from live parts that are not insulated within which a person could receive a second degree burn. It is necessary to specify flash protection boundary for each device to forewarn the person approaching the equipment.

Flash boundary can be calculated when the incident energy is known.

Voltage less than 15 kV:

For voltages less than 15 kV, we use the empirically derived model equations:

$$D_B = \left[4.184 \; C_f \; E_n \left(\frac{t}{0.2} \right) \left(\frac{610^x}{E_B} \right) \right]^{\frac{1}{x}} \tag{14-10}$$

C_f varies with the voltage. Please see Table 4.

x varies with the voltage and equipment type. Please see Table 5.

Voltage higher than 15 kV:

For voltages higher than 15 kV, Lee model is used to calculate flash boundary

$$D_B = \sqrt{2.142 \; x \; 10^6 V I_{bf} \left(\frac{t}{E_B} \right)} \tag{14-11}$$

Effect of Current Limiting Fuses

Current limiting fuses reduce arc time and limit the let-through current. This makes it difficult to calculate incident energy in circuits protected by current-limiting fuses. Tests were done to determine the effect of current-limiting fuses on incident energy. All the tests were performed for circuits operating at voltages 600V or under. The tests cover class L and class RK1 fuses. These fuses will subsequently be added to the SynerGEE protection database and will be covered in the arc flash analysis.

For voltages higher than 600V, no test data is available, and available equations for incident energy are used. This would give more conservative results than the actual.

Working Distances

Working distance for arc flashover analysis is defined as the sum of the distance between the worker standing in front of the equipment, and from the front of the equipment to the potential arc source inside the equipment. Arc flash protection is always based on the incident energy level on the person's face and body at the working distance and not the incident energy on the hands or arms. Since the head and body are the large percentage of the total skin surface area, injury to those areas (head and body) is much more life threatening than burns on hands or arms. Typical values of working distances are given in IEEE standard. SynerGEE Arc Flashover analysis allows the user to enter the working distances at different voltage class. Working distance may vary not only with the voltage, but also with the class of equipment (switchgear, motor control centers, cable, others) and from a case to case basis. Therefore it is necessary that SynerGEE user choose working distance carefully in SynerGEE, depending on the equipment being studied for arc flashover analysis.

Selection of PPE rating for the Clothing

Incident energy at the working distance calculated as part of arc flashover analysis is used to suggest the rating of personal protective equipment (PPE) and to also help develop strategy for safe working conditions. PPE for the arc flash hazard is the last line of defense and is intended to mitigate the damage from arc flashover impact on the individual in case it occurs. The selection of PPE is an attempt to balance between the calculated incident energy exposure and the work activity being performed. The selection of PPE should provide enough protection to prevent a second degree burn, while at the same time avoid providing too much protection that may cause heat stress, poor visibility and limited body movement. Description of clothing for different grades of PPE is listed in Table 6.

IEEE standard emphasizes that it is not intended to imply that workers be allowed to perform work on exposed energized equipment or circuit parts. However, even making the equipment unenergized may expose a person to hazard and proper PPE will help reduce the impact in case arc flashover occurs.

Approach Boundaries

NFPA 70E_2004 defines the shock protection boundaries (or approach boundaries) that are applicable to the situation in which approaching personnel are exposed to live parts. Approach boundaries are identified as:

- Limited Approach Boundary

- Restricted Approach Boundary
- Prohibited Approach Boundary

Table 8 lists the approach boundaries to live parts for shock protection for various system voltages. Appropriate safety precautions need to be taken when working at or close to the approach boundaries. Safety precautions vary based on the type of approach boundary and are discussed below.

Limited Approach Boundary

The designated person in charge of the work shall advise the unqualified person of the electrical hazard and warn him to stay outside of the limited approach boundary. When there is a need for unqualified person to cross the limited approach boundary, a qualified person shall advise him or her of the potential hazards and continuously escort the unqualified person while inside the limited approach boundary.

Restricted Approach Boundary

Under no circumstances shall the escorted unqualified person be permitted across the Restricted Approach Boundary.

No qualified person shall approach or take conductive object closer to exposed live parts operating at 50 Volts or more than the Restricted Approach Boundary unless any of the conditions listed below apply:

- The qualified person is appropriately insulated or guarded from the live parts operating at 50 volts or more and no non-insulated part of the qualified person's body should cross the prohibited approach boundary.
- The live part operating at 50V or more is insulated from the qualified person and also insulted from any other conductive object.
- The qualified person is insulated from any other conductive object.

Prohibited Approach Boundary

No non-insulated part of the qualified person's body should cross the prohibited approach boundary.

Reference Tables

Table 1: Gap and distance

Gap (G) and Distance Exponent (x) table (from IEEE Standard)

System voltage (kV)	Equipment type	Typical gap between conductors (G) (mm)	Distance x factor (x)
0.208 – 1.0	Open air	10 – 40	2.000
	Switchgear	32	1.473
	MCC and panels	25	1.641
	Cable	13	2.000
1.0 – 5.0	Open air	102	2.000
	Switchgear	13 – 102	0.973
	Cable	13	2.000
5.0 – 15.0	Open air	13 – 153	2.000
	Switchgear	153	0.973
	Cable	13	2.000

Table 2: Constant K

Configuration	K
Open	-0.153
Enclosed equipment	-0.097

Table 3: Constant K_1

Configuration	K_1
Open	-0.792
Enclosed equipment	-0.555

Table 4: Constant K_2

Grounding	K_2
Grounded systems	-0.113
Ungrounded or high resistance grounding	0

Table 5: C_f

Voltage	C_f
< 1 kV	1.5
> 1 kV	1.0

Table 6: PPE Rating Table (from NFPA 70E)

Min Rating of PPE (cal / cm^2)	Max Rating of PPE (cal / cm^2)	Risk Category
0	1.2	0
1.2001	5.0	1
5.001	8.0	2
8.001	25.0	3
25.001	40.0	4
40.001	No Maximum	X

Table 7: Clothing Required Table

Risk Category	Range of Incident Energy Levels (cal / cm^2)	Minimum PPE rating (cal / cm^2)	Clothing Required
0	0 - 1.2	N/A	4.5 to 14.0 oz / yd2 untreated cotton
1	1.2 – 4	4	FR shirt and pants or overalls
2	4 - 8	8	Cotton underclothing plus FR shirt and pants
3	8 – 25	25	Cotton underclothing plus FR shirt, pants, overalls or equivalent
4	25 – 40	40	Cotton underclothing plus FR shirt, pants, plus multilayer flash suit
FR = Fire resistance fabric			

Table 8: Approach Boundaries to Live Parts for Shock Protection

Nominal Voltage	Limited Approach Boundary		Restricted Approach Boundary	Prohibited Approach Boundary
	Exposed movable conductor	**Exposed Fixed part**		
Less than 50 V	Not specified	Not specified	Not specified	Not specified
50 V to 300 V	10 ft 0 in.	3 ft 6 in.	Avoid contact	Avoid contact
301 V to 750 V	10 ft 0 in.	3 ft 6 in.	1 ft 0 in.	0 ft 1 in.
751 V to 15 kV	10 ft 0 in.	5 ft 0 in.	2 ft 2 in.	0 ft 7 in.
15.1 kV to 36 kV	10 ft 0 in.	6 ft 0 in.	2 ft 7 in.	0 ft 10 in.
36.1 kV to 46 kV	10 ft 0 in.	8 ft 0 in.	2 ft 9 in.	1 ft 5 in.
46.1 kV to 72.5 kV	10 ft 0 in.	8 ft 0 in.	3 ft 2 in.	2 ft 1 in.
72.6 kV to 121 kV	10 ft 8 in.	8 ft 0 in.	3 ft 3 in.	2 ft 8 in.
138 kV to 145 kV	11 ft 0 in.	10 ft 0 in.	3 ft 7 in.	3 ft 1 in.
161 kV to 169 kV	11 ft 8 in.	11 ft 8 in.	4 ft 0 in.	3 ft 6 in.
230 kV to 242 kV	13 ft 0 in.	13 ft 0 in.	5 ft 3 in.	4 ft 9 in.
345 kV to 362 kV	15 ft 4 in.	15 ft 4 in.	8 ft 6 in.	8 ft 0 in.
500 kV to 550 kV	19 ft 0 in.	19 ft 0 in.	11 ft 3 in.	10 ft 9 in.
765 kV to 800 kV	23 ft 9 in.	23 ft 9 in.	14 ft 11 in.	14 ft 5 in.

15 Allocation

Introduction

Load allocation is used to distribute load throughout a model, based on demands specified at the feeder source. To accomplish this, it adjusts all distributed loads on a feeder so that the total load into a feeder after a balanced or by-phase analysis matches the demands specified in the feeder dialog box. The value of these distributed loads is determined by running a load-flow analysis, then looking at the difference between the specified feeder demand values and the actual power into the feeder. This mismatch is divided among the sections and phases.

The following items should be noted.

- Load allocation applies to section distributed loads only. Spot loads are not affected, since they are known loads at specific points and do not need allocation.
- Sections without kVA values (or kWh if you select allocation based on kWh) do not receive portions of the allocated loads.
- Losses are included in the allocation since load-flow analysis is used during each iteration of the allocation.
- Capacitors and generators increase the amount of load that is allocated, since they supply real and reactive values of power to the feeder.
- Following convergence and allocation completion, you have the option of making the loads permanent. If you choose not to, a report is still generated, but no changes are made to the model.

Allocation steps

During allocation, SynerGEE performs multiple iterations of the following steps.

1. Remove all distributed loads from feeder.
2. Run balanced or by-phase load-flow.
3. $\Delta S^i = S_{Specified} - S^i_{Into\,Feeder}$.

4. Add ΔS^i to feeder distributed loads.

5. If $\left|\Delta S^i\right| > \text{Tolerance}$, repeat to Step 2.

If balanced allocation is selected, balanced analysis is used. Otherwise, by-phase analysis is used. The distribution of ΔS^i among sections is dependent on allocation factors for each section.

Types of allocation

SynerGEE supports three overall types of allocation.

- By-phase allocation using by-phase demands - Using a by-phase load-flow, SynerGEE allocates loads by phase and uses the by-phase demands to proportion values.
- By-phase allocation using total demands - Using a by-phase load-flow, by-phase loads are determined from the ratio of by-phase information to the feeder total. Feeder or substation demands are shifted to be proportional to the total by-phase allocation parameters such as kWh or kVA.
- Balanced allocation - Using a balanced load-flow, SynerGEE allocates evenly across all phases of a line. It also totals feeder and substation transformer demands.

These options are designed to facilitate the various levels of detail in feeder models, phasing, and feeder load meter readings.

Demands format

In the feeder and subtransformer editors, SynerGEE allows demands to be specified in the following formats.

- Amps and power factor
- kW and kvar
- kVA and power factor
- kW and power factor

In the database, demands are stored as kW and kvar values, and conversion is done as necessary to store or display information from the dialog box. This conversion uses the nominal voltage of the feeder or substation transformer. Therefore, if the nominal voltage is changed, demands displayed in the dialog box may change, particularly if you have specified demands based on amps or a power factor.

Allocation by specified demands

As an option, you can enter system demands directly into your load allocation settings. These kW and kvar demands can be used in place of demands in the feeder or substation records. This option is useful primarily for allocation of substation demands on systems without substation models. If you choose this option, the specified demands are used for allocating load on all selected feeders.

Consider the following system, which does have a substation model. Assume that the substation has demands of 5 MW and 2 Mvar.

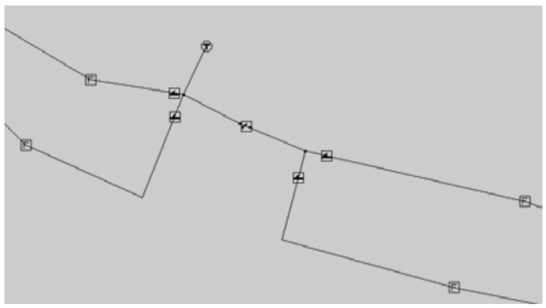


Fig. 15-1 *Model with a substation and feeders*

To allocate by substation demands, you could simply specify the demands in the substation editor and run the analysis as such.

Alternatively, consider the same model without the substation.

Fig. 15-2 *Model without substation*

In this case, you could still allocate based on the 5 MW and 2 Mvar substation demands by entering them directly in the load allocation setting editor.

Substation transformer demand is considered to be a secondary measurement. Therefore, the results from each approach will be the same except for the impact of busbar losses (negligible) and voltage. The voltage impact may be significant depending on the V/I characteristics of the load models and the drop through the substation transformer.

Initial load estimate

Power systems sometimes have multiple solutions that are numerically achievable. To avoid reaching an unrealistic solution, the initial load-flow used on the feeder model is simplified. An initial estimate of the total load to allocate is found using the following.

$$\begin{array}{c} \text{Specified FeederDemand} \\ -\ \text{Estimated SpotLoad} \\ -\ \text{Estimated Losses} \\ +\ \text{Estimated Generator Injection} \\ +\ \text{Estimated Capacitor Injection} \\ \hline \text{Initial Distributed Load to Allocate} \end{array} \tag{15-1}$$

This initial estimate of distributed load is spread out using the allocation factors discussed above.

Suggested approaches to load allocation

There are many approaches to allocating load. It is recommended that you try various settings and run different allocations first without making the loads permanent. This way, you can look at the resulting reports and find an approach to allocation that produces the most reasonable numbers based on your system and experience.

The following are some suggested approaches.

Scenario	**Recommendation**
You have total value for feeder demand but your section by-phase information is unreliable.	Use balanced allocation.
You have a total value for feeder demand and your section phasing is accurate.	Use by-phase allocation with total demand.
You have by-phase feeder demands and accurate section phasing.	Use by-phase allocation with by-phase demands.
You have a total value for feeder demand, accurate section phasing, and a fairly balanced loading.	Use by-phase allocation with by-phase demands. Average the demands in the feeder record before allocating.

Example

Consider the following simple feeder, showing feeder demand in kW and connected kVA for the three sections.

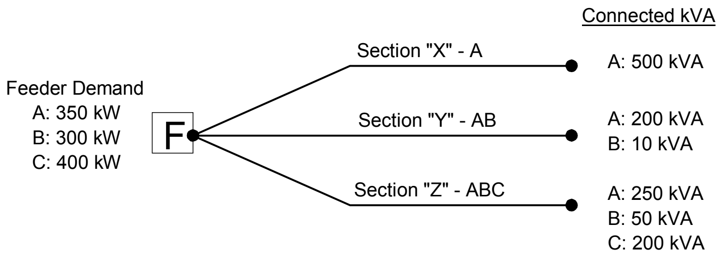


Fig. 15-3 *Sample feeder*

The feeder serves three sections phased A, AB, and ABC. All sections are short, so the effects of losses, charging, and coupling are negligible. There are no controlled devices like regulators or capacitors. All load models are 100 percent constant power for a Wye-ground system. To simplify further, only kW demands and the connected kVA will be used as the allocation parameter.

The following table shows the kW results for each type of load allocation.

Sect	**Phase**	**Connected kVA**	**Balanced Allocate**	**By-Phase Allocate with Total Demand**	**By-phase Allocate with By-Phase Demand**
X	A	500 kVA	186 kW	186 kW	79 kW
	- - -	- - -	- - -	- - -	- - -
	- - -	- - -	- - -	- - -	- - -
Y	A	200 kVA	39 kW	74 kW	32 kW
	B	10 kVA	39 kW	4 kW	17 kW
	- - -	- - -	- - -	- - -	- - -
Z	A	250 kVA	62 kW	93 kW	39 kW
	B	50 kVA	62 kW	19 kW	83 kW
	C	200 kVA	62 kW	74 kW	200 kW

Notice that for balanced allocation, the resulting distributed loads have the same value for all phases of a section. When by-phase allocation with total demand was used, you can see that the total load for each section is the same as the balanced results, though the values are spread across the phases proportionally to the by-phase, connected kVA.

The following table displays data and results for the entire feeder for each type of allocation. The values used for convergence comparison are underlined.

	A	B	C	Total
Initial data				
Connected kVA	950	60	200	1210
Spot Loads (kW)	200	200	200	600
Specified Demand (kW)	350	300	400	1050
Balanced allocation				
Distributed Loads	287	101	62	450
Dist. and Spot Loads	487	301	262	1050
By-Phase allocation with by-phase demands				
Distributed Loads	150	100	200	450
Dist. and Spot Loads	350	300	400	1050
By-phase allocation with total demand				
Distributed Loads	353	23	74	450
Dist. and Spot Loads	553	223	274	1050

You can see the diversity of results that can appear over the range of allocation options. The large spot load in this example contributed significantly. You can compare the distributed loads to the connected kVA values to gain a feel for the relationship between phasing, allocation parameter, and resulting load.

Numbers for balanced allocation

A section's load is determined by the ratio of the section's total allocation parameter to the total for the feeder. In the following example, kWh or connected kVA are used. The load for a section is spread over the phases that have positive values for the allocation parameter.

(15-2)

$$X_A = \frac{500kVA}{1210kVA} * \left(1050kW - 600kW\right) = 186kW$$
$$Y_A = Y_B = \frac{210kVA}{1210kVA} * \left(1050kW - 600kW\right) * \tfrac{1}{2} = 39kW$$
$$Z_A = Z_B = Z_C = \frac{500kVA}{1210kVA} * \left(1050kW - 600kW\right) * \tfrac{1}{3} = 62kW$$

Numbers for by-phase allocation using by-phase demands

The load for a particular phase is determined by the ratio of the phase allocation parameter to the total feeder allocation parameter for that phase.

$$X_A = \frac{500kVA}{950kVA} * (350kW - 200kW) = 79kW$$
$$Y_A = \frac{200kVA}{950kVA} * (350kW - 200kW) = 32kW$$
$$Y_B = \frac{10kVA}{60kVA} * (300kW - 200kW) = 17kW$$
$$Z_A = \frac{250kVA}{950kVA} * (350kW - 200kW) = 39kW$$
$$Z_B = \frac{50kVA}{60kVA} * (300kW - 200kW) = 83kW$$
$$Z_C = \frac{200kVA}{200kVA} * (400kW - 200kW) = 200kW \tag{15-3}$$

Numbers for by-phase allocation using total demand

The load for a particular phase on a section's distributed load is determined by the ratio of the phase allocation parameter to the total feeder allocation parameter for all phases.

$$X_A = \frac{500kVA}{1210kVA} * (1050kW - 600kW) = 186kW$$
$$Y_A = \frac{200kVA}{1210kVA} * (1050kW - 600kW) = 74kW$$
$$Y_B = \frac{10kVA}{1210kVA} * (1050kW - 600kW) = 4kW$$
$$Z_A = \frac{250kVA}{1210kVA} * (1050kW - 600kW) = 93kW$$
$$Z_B = \frac{50kVA}{1210kVA} * (1050kW - 600kW) = 19kW$$
$$Z_C = \frac{200kVA}{1210kVA} * (1050kW - 600kW) = 74kW \tag{15-4}$$

Allocation factors

Scaler allocation factors are calculated for each section by phase. If a balanced allocation is desired, the factors for a section are the same for all phases. (This section presents calculations for by-phase allocation). These factors are used to apportion ΔS_p^i among the sections as follows.

$$SectDistLoad_{k,p}^{i+1} = SectDistLoad_{k,p}^{i} + f_{k,p} * \Delta S_p^i \tag{15-5}$$
$$i = Iteration, \quad k = Section, \quad p = Phase$$

Notice that the total load to apportion ΔS_p^i is a by-phase quantity.

The allocation factors are calculated based on the allocation method selected. The by-phase factors, $f_{k,p}$, for section k can be found with the following expressions. (Factors used in balanced allocation have similar expressions.)

Allocate by connected kVA

This factor corresponds to the ratio of the section's by-phase kVA values to the by-phase total kVA values for the entire feeder. A section must have connected kVA values in order to pick up allocated load. For example:

$$f_{k,p} = \frac{kVA_{k,p}}{\sum_{Fdr,k \in Fdr} kVA_{j,p}} \tag{15-6}$$

Allocate by total kWh

This factor corresponds to the ratio of the section's by-phase kWh values to the by-phase total kWh values for the entire feeder. A section must have kWh values in order to pick up allocated load. For example:

$$f_{k,p} = \frac{kWh_{k,p}}{\sum_{Fdr,k \in Fdr} kWh_{j,p}} \tag{15-7}$$

Allocate by RUS method

This factor corresponds to the ratio of the section's by-phase RUS (formerly REA) values to the by-phase total RUS values for the entire feeder. A section must have kWh and customer values in order to pick up allocated load, as in the following.

$$A_{k,p} = Cust_{k,p} * \left(2.5 - Cust_{k,p} + \sqrt{Cust_{k,p}^2 + 40}\right) * 0.4 \tag{15-8}$$

$$B_{k,p} = 0.005925 * \left(\frac{kWh_{k,p}}{Cust_{k,p}}\right)^{0.885}$$

$$RUS_{k,p} = A_{k,p} * B_{k,p}$$

$$f_{k,p} = \frac{RUS_{k,p}}{\sum_{Fdr,k \in Fdr} RUS_{j,p}}$$

Example showing convergence process

The preceding sections contain calculations used to determine distributed load, based on feeder demands. The allocation process uses these calculations in an iterative fashion, running in conjunction with load-flow analyses. Mismatch values between the feeder demand and the actual power flow into the feeder are used along with the allocation parameters and the allocation method.

By-phase allocation using by-phase demands and the kVA method will be outlined on the sample feeder shown below.

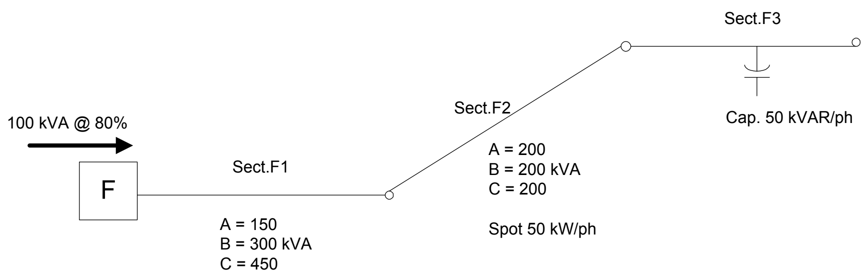


Fig. 15-4 *Sample feeder*

The allocation factors are calculated as follows.

$$f_{F1,A} = {}^{150}\!/_{350} = 0.429 \qquad f_{F2,A} = {}^{200}\!/_{350} = 0.571 \qquad f_{F3,A} = {}^{0}\!/_{350} = 0.0$$

$$f_{F1,B} = {}^{300}\!/_{500} = 0.600 \qquad f_{F2,B} = {}^{200}\!/_{500} = 0.400 \qquad f_{F3,B} = {}^{0}\!/_{500} = 0.0$$

$$f_{F1,C} = {}^{450}\!/_{650} = 0.692 \qquad f_{F2,C} = {}^{200}\!/_{650} = 0.308 \qquad f_{F3,C} = {}^{0}\!/_{650} = 0.0$$

After the load-flow run, the power into each phase of the feeder was found to be 62 + j48 kVA. The by-phase load to apportion can be found as follows.

	$S_{Specified}$	-	$S^{i}_{Into\ Feeder}$	=	ΔS^{i}
A	80 + j60 kVA		62 - j48 kVA		18 + j12 kVA
B	80 + j60 kVA	-	62 - j48 kVA	=	18 + j12 kVA
C	80 + j60 kVA		62 - j48 kVA		18 + j12 kVA

The by-phase distributed loads are now modified by adding in the apportioned load factored to each section. For example:

$$SectDistLoad^{i}_{k,p} + f_{k,p} * \Delta S^{i}_{p} = \qquad SectDistLoad^{i+1}_{k,p}$$

Section F1

0 +j 0 kVA + 0.429 * (18 + j12 kVA) = 7.7 + j5.2 kVA

0 +j 0 kVA + 0.600 * (18 + j12 kVA) = 10.8 + j7.2 kVA

0 +j 0 kVA + 0.692 * (18 + j12 kVA) = 12.5 + j8.3 kVA

Section F2

0 +j 0 kVA + 0.571 * (18 + j12 kVA) = 10.3 + j6.8 kVA

0 +j 0 kVA + 0.400 * (18 + j12 kVA) = 7.2 + j4.8 kVA

0 +j 0 kVA + 0.308 * (18 + j12 kVA) = 5.5 + j3.7 kVA

Notice that the $SectDistLoad^{i}_{k,p}$ value for each section is zero since the entire distributed load was remodeled at the start of the analysis. The loads are apportioned, and a new load-flow is run. After the second load-flow run, the calculated ΔS^{i} load to apportion on the next iteration of allocate will be very small.

	$S_{Specified}$	-	$S^{i+1}_{Into\ Feeder}$	=	ΔS^{i+1}
A	80 + j60 kVA		82 - j59 kVA		-2 + j1 kVA
B	80 + j60 kVA	-	80 - j61 kVA	=	- j1 kVA
C	80 + j60 kVA		80 - j60 kVA		0

The apportionment process is repeated until ΔS is acceptably small.

After each iteration, losses, voltages, and currents change. Also, regulators and capacitors may operate.

Device options

The handling of switched capacitors and regulators can be set up before starting the allocation process. The following options are available for each type of device.

Regulators[3]
- Maximum tap
- Minimum tap
- Neutral tap
- Controlled by SynerGEE load-flow
- Hold current tap

Capacitors[4]
- Trip modules
- Close models
- Controlled by SynerGEE load-flow
- Hold current switch state

3 Only regulators in automatic mode are affected by these selections. SynerGEE never adjusts manual mode regulators.

4 Only the switched portions of capacitor installations in automatic mode are affected by these selections. SynerGEE never adjusts manual mode capacitors.

Allocating and meter adjustment

Examples will be presented in this section to demonstrate general concepts of load allocation. The examples should also help the reader determine when allocation should be run and when metered values should be adjusted.

The Load-Allocation application deals with complex distribution networks, loads, phasing, grounding and configurations. Even with the added complexities the basic concepts are similar to the ones presented in this document.

Load allocation is driven by the difference between meter specified values and flow through the meters after a load-flow run. These are the steps taken by load allocation:

1. Zero all distributed loads
2. Run load-flow
3. Find difference between meter demands and actual meter flow
4. Spread difference to distributed load on downstream dependent sections.
5. Go to #2 until #3 is zero

In this document, we will fix losses and use a simple model to that load values can be directly calculated.

Example system

All of the examples will use variations of this system:

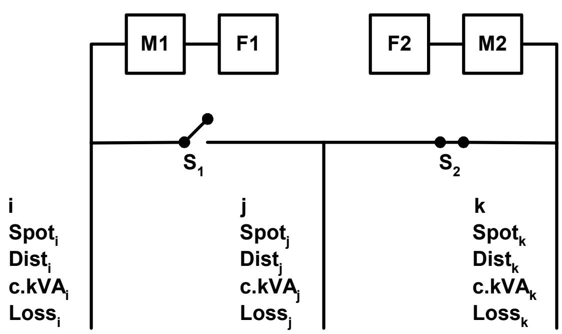


Fig. 15-5 *Example System*

There are three laterals, i, j, and k. To keep things simple, all loads, demands and losses will be evaluated as kW values. Losses on the backbone will be ignored and all loads will be considered constant kW. Finally, losses in each lateral will be estimated at 50 kW regardless of load on the lateral. Allocation in these examples will be done using connected kVA values.

The goal of load allocation is to setup distributed load values so that the power flow through meters matches the specified power demands. Here are some examples.

Allocating load

Actual metered demands:

	Meter 1	Meter 2
Metered kW	1000 kW	3000 kW

Connected kVA:

	Lat i	Lat j	Lat k
c.kVA	350 kVA	500 kVA	750 kVA
Spot	0	0	0
Loss	50 kW	50 kW	50 kW

The distributed load for lateral i is:

$$Dist_i = 1000kW - 50kW = 950kW \tag{15-9}$$

The 950kW of load along with the 50kW of losses will draw 1000kW through meter 1.

Load on the second feeder has to be divided between the laterals. The metered demand is 3000 kW and we are assuming 100kW in losses for both laterals. That leaves 2900kW to be allocated. The load will be divided proportionally to the laterals:

$$Dist_j = \frac{500}{500+750} 2900kW = 1160kW \tag{15-10}$$

And

$$Dist_k = \frac{750}{500+750} 2900kW = 1740kW \tag{15-11}$$

The results can be checked by tallying the total load through meter 2:

$$\begin{array}{rr} & 1160kW \\ + & 1740kW \\ & 100kW \\ \hline & 3000kW \end{array} \tag{15-12}$$

Switching after peak demand allocation

Yearly peak demands on feeders are usually recorded by utilities. Those demands can certainly be used for allocating load. If this is done, care should be taken when switching is performed. There are a couple of issues to keep in mind.

Non-coincident peaks

In the above example, consider the meter 1 peak of 1000 kW to have occurred in July and the Meter 2 peak of 3000 kW to have occurred in September. If the switches, S_1 & S_2, are operated in the above example then feeder 1 will have a total load of 950kW + 1160kW. This value may be unrealistically large since the loads are not coincident.

Switchable area contributes to multiple peaks

The 3000 kW peak for feeder 2 was field recorded with the system in the pictured configuration. At some other time, the switches were toggled. The 1000 kW peak on feeder 1 was recorded with S1 and S2 in the opposite configuration. If this has happened and we have allocated as above then there is a problem. The load on lateral j has been counted twice.

A good way around these issues is to record all feeder demand values at the same time (like at the system peak). This approach will maintain a consistent model when switching is done after allocation. It is possible that allocated load on feeders may not be large enough because a particular feeder was not near its peak when the system hit its peak.

The impact of connected kVA changes

Lets go to the initial calculations and double the amount of connected transformer kVA on laterals i and j. The feeder demands will be left unchanged.

Actual metered demands:

	Meter 1	Meter 2
Metered kW	1000 kW	3000 kW

Connected kVA:

	Lat i	Lat j	Lat k
c.kVA	700 kVA	1000 kVA	750 kVA
Spot	0	0	0
Loss	50 kW	50 kW	50 kW

For lateral i:

$$Dist_i = 1000kW - 50kW = 950kW \tag{15-13}$$

There is no change to the allocated distributed load.

For lateral j we have:

$$Dist_j = \frac{1000}{1000 + 750} 2900kW = 1657kW \tag{15-14}$$

And for k:

$$Dist_k = \frac{750}{1000 + 750} 2900kW = 1243kW \tag{15-15}$$

Here are the values before and after the c.kVA change:

	Before	After
Lateral i	950	950
Lateral j	1160	1657
Lateral k	1740	1243

Doubling the connected kVA on lateral j did not double its load. That lateral did pickup a greater portion of the load allocated to the feeder.

The impact of spot loads

Now we will return to the initial example and add spot loads to lateral i and lateral j without changing the metered demands.

	Meter 1	Meter 2
Metered kW	1000 kW	3000 kW

Connected kVA:

	Lat i	Lat j	Lat k
c.kVA	350 kVA	500 kVA	750 kVA
Spot	500 kW	500 kW	0
Loss	50 kW	50 kW	50 kW

The distributed load for lateral i is:

$$\begin{aligned} Dist_i &= 1000kW - 50kW - 500kW \\ &= 450kW \end{aligned} \tag{15-16}$$

The load to be allocated to feeder 2 is

$$\begin{aligned} Load &= 3000kW - 500kW - 100kW \\ &= 2400kW \end{aligned} \tag{15-17}$$

The laterals for feeder 2 are:

$$Dist_j = \frac{500}{500+750}2400kW = 960kW \tag{15-18}$$

And

$$Dist_k = \frac{750}{500+750}2400kW = 1440kW \tag{15-19}$$

Allocating the switched circuit

In this example, we will use the parameters from the first example but will toggle the switches:

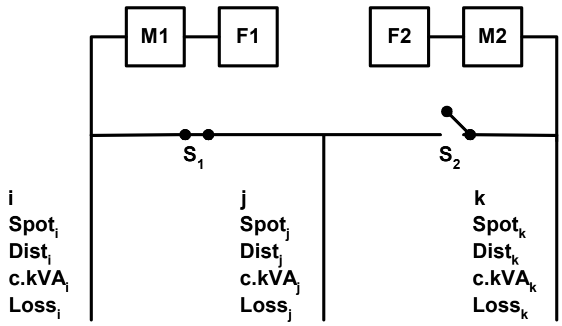


Fig. 15-6 *Example system*

Feeder 1 now serves lateral i and j.

We will keep the same metered demands:

	Meter 1	Meter 2
Metered kW	1000 kW	3000 kW

and connected kVA values:

	Lat i	Lat j	Lat k
c.kVA	350 kVA	500 kVA	750 kVA
Spot	0	0	0
Loss	50 kW	50 kW	50 kW

The distributed load for lateral i is:

$$Dist_i = \frac{350}{350+500}900kW = 371kW \tag{15-20}$$

For j:

$$Dist_j = \frac{500}{350+500} 900kW = 529kW \tag{15-21}$$

The distributed load for lateral k is simply the demand less the 50kW in losses:

$$Dist_k = 3000kW - 50kW = 2950kW \tag{15-22}$$

The configuration of the feeders has a profound impact on load allocation results.

Anticipating new spot loads

Again we return to the initial example and its switching configuration. It is shown again to avoid confusion with the previous example:

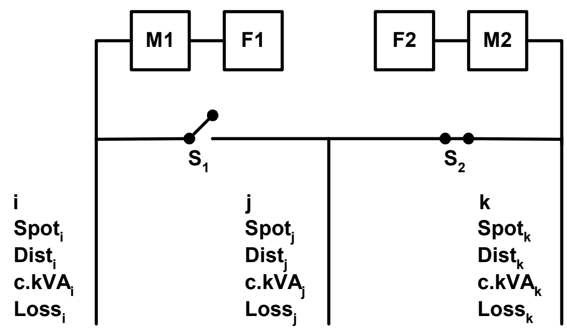


Fig. 15-7 Example system

We have actual metered demands and realistic connected kVA values. Now, we are going to consider a new 1 MW spot load that WILL BE connected to lateral j in the coming months. When that load is connected, our current demand on meter 2 will jump up from 3MW to 4MW. We will alter our data to account for the expected load:

	Meter 1	Meter 2
Metered kW	1000 kW	3000 + 1000kW

Connected kVA:

	Lat i	Lat j	Lat k
c.kVA	350 kVA	500 kVA	750 kVA
Spot	0	1000 kW	0
Loss	50 kW	50 kW	50 kW

We adjusted to demand on meter 2 to reflect our expected feeder demand. We also added 1000kW in spot load to lateral j.

Obviously, these changes have no impact on feeder 1. The flow through feeder 2 with no distributed load values would be the spot load and the losses:

$$Flow = 1000kW + 100kW = 1100kW \tag{15-23}$$

Our load to allocate is the difference between that initial flow and the meter demands:

$$Load = 4000kW - 1100kW = 2900kW \tag{15-24}$$

Our calculations from this point on will remain exactly like the initial example:

$$Dist_j = \frac{500}{500 + 750} 2900kW = 1160kW \tag{15-25}$$

And

$$Dist_k = \frac{750}{500 + 750} 2900kW = 1740kW \tag{15-26}$$

We could have gotten the same overall result by simply adding the 1MW spot load to the lateral after running the load allocation in the first example.

Anticipating switching changes

How can the parameters be adjusted so that the metered values for the initial configuration of the system can be reliably applied to the switched configuration? Adjusting connected kVA values is not important. As we saw earlier, connected kVA only affects load disbursement within a given feeder. If the system is reconfigured away from the configuration used for metering then demands will need to be adjusted before running allocation.

One approach is to allocate the initial configuration and then run a load-flow to see how much load is flowing into the lateral. That flow would be subtracted from the demands of meter 2 and added to the demands of meter 1 since that is how the load will be transferred.

From the first example, we would see a flow into lateral j of 1160kW + 50kW if a load flow were to be run. Let's adjust the meter demands by this amount:

	Meter 1	Meter 2
Metered kW	2210 kW	1790 kW

Now, we allocate with the reconfigured system:

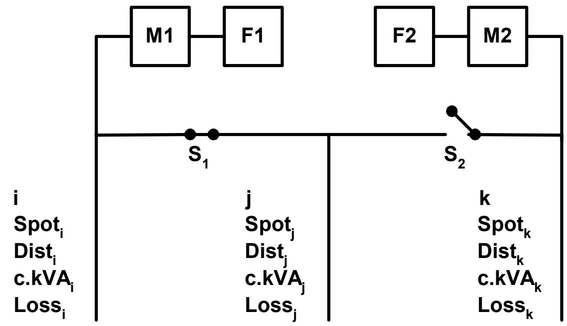


Fig. 15-8 *Example system*

We get (with the 100kW of losses):

$$Dist_i = \frac{350}{350+500} 2110kW = 869kW \tag{15-27}$$

For j:

$$Dist_j = \frac{500}{350+500} 2110kW = 1241kW \tag{15-28}$$

The distributed load for lateral k is simply the demand less the 50kW in losses:

$$Dist_k = 1790kW - 50kW = 1740kW \tag{15-29}$$

Another approach is to find the proportion of total demand associated with the lateral. In our example that would be:

$$\begin{aligned} Load_j &= \frac{500}{500+350+750}(1000+3000)kW \\ &= 1250kW \end{aligned} \tag{15-30}$$

That load would be used to adjust the meter demands:

	Meter 1	Meter 2
Metered kW	2250 kW	1750 kW

Now, when we allocate with the reconfigured system:

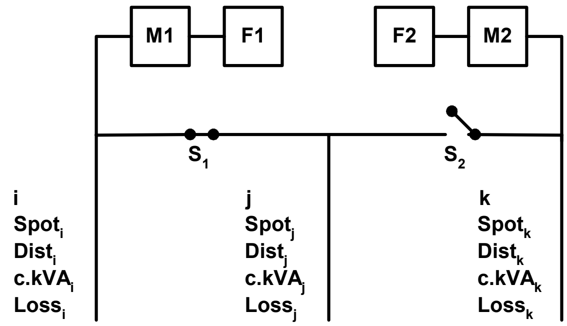


Fig. 15-9 *Example system*

Remembering the 50kW of loss per lateral we get:

$$Dist_i = \frac{350}{350+500} 2250kW = 885kW \tag{15-31}$$

For j:

$$Dist_j = \frac{500}{350+500} 2150kW = 1265kW \tag{15-32}$$

And for k:

$$Dist_k = 1750kW - 50kW = 1700kW \tag{15-33}$$

These values are close to the ones calculated in the initial example.

Here is a summary table:

	Base	Try 1	Try 2
i	950	869	885
j	1160	1241	1265
k	1740	1740	1700

Load can be allocated after a feeder reconfiguration if the metered values are adjusted.

Customer load curves

SynerGEE supports daily load shape curves for customer classes. These classes are consolidated into customer zones and associated with sections. For detailed information on load curves and the nomenclature that follows, see *Chapter 3 - Loads*.

Skipping the by-phase issues and other details, the following equation represents the load to be placed on a single section, "i", during an allocation iteration without load curves.

$$L_i = \Delta D_f \frac{kVA_i}{\sum_{k \in Fdr} kVA_k} \tag{15-34}$$

In this equation, ΔD_f is the mismatch between the power into the feeder and the specified feeder demand at the current iteration.

If you want to use load curves, the feeder demand must be specified for the hour at which the loads are to be allocated. So, if you are allocating at a particular hour, h, which is an hour within a day type, then the mismatch is the difference between the feeder demand at the current iteration and the specified demand for the time point. That is:

$$\Delta D_f(h) = \text{Specified Demand at h - LoadFlow Demand} \tag{15-35}$$

Customer zones affect section loads as discussed in Chapter 3. That effect has to be accounted for in the ratio of the previous equation. Also, the effect of the zones on the resulting load must be accounted for. To simplify, consider a zone without the different kW and kvar curves. You can then write the following expression for the per-iteration allocated load using customer load curves.

$$\begin{aligned} L_i &= \Delta D_f(h) \frac{kVA_i * Z_i(h)}{\sum\limits_{k \in Fdr} kVA_k * Z_k(h)} * \frac{1}{Z_i(h)} \\ &= \Delta D_f(h) \frac{kVA_i}{\sum\limits_{k \in Fdr} kVA_k * Z_k(h)} \end{aligned} \tag{15-36}$$

Example

The following is a simple two-section feeder.

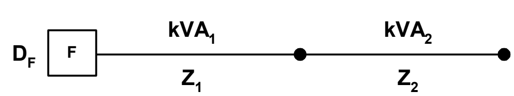


Fig. 15-10 *Simple feeder*

Each section has a connected kVA value and a zone assignment. This example assumes a 1000kW feeder demand and will step through a few allocation scenarios.

For the first case, consider the following values.

$$\begin{aligned} kVA_1 &= 200kVA \\ kVA_2 &= 200kVA \\ Z_1(h) &= 0.5 \\ Z_2(h) &= 0.25 \end{aligned} \tag{15-37}$$

You can determine the load values as follows.

$$L_1 = 1000 \cdot \frac{200(0.5)}{200(0.5)+200(0.25)} \cdot \frac{1}{0.5} = 1333kW \tag{15-38}$$

$$L_2 = 1000 \cdot \frac{200(0.25)}{200(0.5)+200(0.25)} \cdot \frac{1}{0.25} = 1333kW$$

The equations above show the zone values in the numerator even though it was shown earlier that they cancel out. It is interesting to note that since the zone values cancel out of the numerator of the allocation expression, loads with identical connected kVA values (or kWh or REA factors) have the same kW and kvar values allocated.

You can check the validity of the results. When a load-flow is run on the simple feeder, the zone factors are used as multipliers on the allocated load. So, if you run a load-flow at the time point "h," you end up with a feeder loading of:

$$F_L(h) = 1333(0.5) + 1333(0.25) = 1000kW \tag{15-39}$$

As a different example, consider a case with differing connected kVA values but the same zone factors at the time point being simulated.

$$\begin{aligned} kVA_1 &= 200kVA \\ kVA_2 &= 400kVA \\ Z_1(h) &= 0.5 \\ Z_2(h) &= 0.5 \end{aligned} \tag{15-40}$$

You can determine the load values as follows.

$$L_1 = 1000 \cdot \frac{200}{200(0.5)+400(0.5)} = 667kW \tag{15-41}$$

$$L_2 = 1000 \cdot \frac{400}{200(0.5)+400(0.5)} = 1333kW$$

If you used these loads in a load-flow run, you would get:

$$F_L(h) = 667(0.5) + 1333(0.5) = 1000kW \tag{15-42}$$

The section with the larger connected kVA contributes a larger load at the time point being analyzed. After allocating at a specific time point, other time points could be used for load-flow. Variation in zone factors could result in section "1" contributing more load.

Additional notes

Customer load curves, zones, etc. are analytically straightforward in SynerGEE. Their handling and the data used to generate them can have tremendous impacts on engineering studies. Their use can provide valuable insight into the best overall operation of a distribution system. As customer load modeling becomes increasingly important in modern distribution systems, Stoner looks forward to working with you and receiving your continued input on this topic.

In summary:

- The use of load curves is optional. By default, they are not used.
- Loads should be allocated once using a known demand at a known time point. After allocating, applications such as load-flow can be run at different time points. The quality of the load models at other time points depends on the quality of the customer load curves and the formulation of customer zones.
- Billing data is important to the formulation of load curves and the formulation of zones. It is possible to construct load data completely outside of SynerGEE without the use of load allocation.

Metering points

Load allocation can consider metering points, which are assigned directly to sections. Currently, SynerGEE accepts metering point values in amps only. During load allocation, SynerGEE adjusts the downstream load to meet the specified amp flow on metered sections. Loads on all sections directly fed through a metering point are adjusted. If a metering point feeds a second metering point, the load between the two points is adjusted to meet the flow through the first point, and all downstream load is adjusted to meet the flow through the second point.

Since metering point values are assigned at the section level, it is possible to put metered values on every section. However, too many metering points may result in non-convergence of load allocation due to the granular load groups between metering points. For best allocation results, you should use a limited number of metering points in conjunction with realistic feeder demands.

Metering point amps are specified as by-phase current magnitude values. Therefore, they cannot be used to directly adjust the downstream power factor. Also, metering points are considered by load allocation only if you have your analysis options set up as such.

Other considerations

Substation transformers

When allocating on substation transformers, the substation transformer demands are recognized as occurring on the secondary of the transformer. After allocating and viewing a load-flow report, the power into the primary side will not match the demands. Instead, the power out of the substation transformer should match the specified demands.

Loops

Load allocation only handles radial models. If tie switches are modeled, they are treated as open switches during allocation.

Generators

Synchronous and induction generators are reverted to their corresponding constant power (PQ) model during allocation. They produce their specified output power (scheduled percent times rated power) at rated power factor.

16
Multi-year Studies

Various tools

Ten year growth studies

This tool analysis the model and summarizes the year-by-year impact of ten years of growth. The application results in a report listing demands, low voltage, and maximum loading values for each feeder over a ten-year period.

Growth Summary Report

Feeder /	Feeder Load				Max Pct	Min	Exceptions	
Year	kVA	PF	Amps	kW Loss	Loading	Volts	Volt	Ldng
Feeder Ship - West Martin								
Year 0	7566	93.9%	320.2	97	60%	119.6	0	0
Year 1	8017	93.9%	339.3	109	64%	119.4	0	0
Year 2	8496	93.8%	359.6	122	68%	119.1	0	4
Year 3	9004	93.7%	381.1	137	72%	118.8	0	4
Year 4	9543	93.7%	403.9	154	76%	118.5	0	4
Year 5	10116	93.6%	428.2	172	81%	118.2	0	4
Year 6	10723	93.6%	453.9	193	86%	117.8	1	6
Year 7	11368	93.5%	481.2	217	91%	117.5	3	8
Year 8	12053	93.4%	510.2	243	96%	117.1	14	10
Year 9	12779	93.3%	540.9	273	102%	116.7	14	10
Year 10	13551	93.3%	573.6	307	108%	116.3	14	10

Fig. 16-1 *Ten year growth study report*

This application uses customer zone or section growth rates, set up in the growth settings editor of SynerGEE.

17
Forecasting

Introduction

A utility needs to forecast for two critical reasons:

First, the utility must determine if it has the capacity to provide or buy and deliver the power and energy that customers will be demanding in future years. Transmission infrastructure must be built to support future demands. Contracts need to be put into place for future bulk load transfers. Finally, regulating bodies need to be informed about the utilities plans to support its customers in upcoming years.

Second, the utility needs to evaluate its future revenue to ensure the solvency of the company and financial performance to its stake or stock holders. Investor owned utilities may look for ways to attract load. If forecasts look bad then the utility may more aggressively look for new load.

Engineers at the utility must ensure that the distribution infrastructure can support future load. For them, the loading along critical paths during the upcoming years is important. Forecasting individual load growth within an engineering model is important to doing this. The above two reasons for forecasting do not typically involved detailed feeder analysis. Engineering analysis is needed for evaluating the ability of the distribution system to deal with future loads.

SynerGEE forecasting is a tool to apply expected growth trends to the distribution model. The goal is to go from growth rates categorized in various ways to actual kW and kvar values in the model. Forecasting is specifically applied to the five load models in SynerGEE:

- Distributed
- Spot
- Large customer
- Project
- Speculative

Growth expectations can be applied to these load types or combinations of these load types in a spatial or logical fashion.

SynerGEE utilizes growth percentages that have been determined from land use, regression, or other types of analysis. These analyses are generated from trends in historic and expected metrics and demographics.

Regional Peak

The business development or marketing group at a utility generally oversees the generation of a peak forecast study. The results of this study are the anticipated yearly growth in system peak. The growth values for the distribution model should result in an overall total demand growth that is consistent with the regional peak study.

The regional peak forecast is not based on an analytical feeder analysis. Here are some elements that are oftentimes used for regional studies:

- Population growth
- Housing stocks
- New large customers
- Energy prices
- Economic growth
- Zoning changes
- Legislation
- Contracts
- Demographics
- Road / highway / railway construction
- Electric heating trends
- Market research

The result of studies using this information is a list of growth values by year such as this:

05	06	07	08	09	10	11
2.5%	3.6%	4.2%	3.0%	2.6%	2.5%	2.0%

Fig. 17-1 *Example system growth rates*

The growth values are cumulative. In the example, growth rates rise and fall. This represents an acceleration and deceleration in load growth.

Areas

Regional peak studies may be performed so that only system wide growth values are determined. The utility service region may be broken up into various 'areas'.

It can be very helpful to distribution engineers if the regional studies break growth down into planning areas. The planning areas consist of 50-300 feeders. A utility typically defines 5-25 planning areas. Oftentimes, planning areas are assigned to specific utility engineers. An engineer with growth rates for her planning area can be more effective at load forecasting.

Another breakdown that can be useful is one of peak forecast areas. Feeders and substations with similar trends in peak and coincidence can be grouped into 'peak forecast areas'. Growth rates for these areas can be used effectively to forecast load growth.

Customer Classes

The general forecasting procedure is to use customer class based growth values to grow loads. The resulting growth is iteratively reconciled with broader district or system growth values.

Residential

Load growth estimates by residential customer class are useful for forecasting overall load growth. Shown below are some example customer classes:

- Large Single Family Electric (LSFE)
- Large Single Family Non-Electric (LSFN)
- Single Family Electric (SFE)
- Single Family Non-Electric (SFN)
- Apartments 1 (A1)
- Apartments 2 (A2)
- Row Homes Electric (RHE)
- Row Homes Non-Electric (RHN)

The goal is to use billing, utilization and demographic studies to generate growth rates for each customer class. Here is an example of results:

	05	06	07	08
LSFE	1.6%	1.4%	1.2%	1.0%

LSFN	1.8%	1.9%	1.1%	1.0%
SFE	2.1%	2.6%	2.3%	2.1%
SFN	2.5%	2.2%	1.2%	1.0%
A1	3.6%	3.6%	3.6%	3.6%
A2	4.6%	4.6%	4.6%	4.6%
RHE	1.0%	1.0%	1.0%	1.0%
RHN	0.5%	0.7%	0.6%	0.3%

Fig. 17-2 *Growth rates for residential customer classes*

A general energy usage value for a single-family home is 10 – 20 MWh / year. The energy consumption, however, is a very different value than the growth in peak demand; and especially different than the growth in peak demand coincident with the system peak.

An approach to dealing with peak load growth is to use the growth in consumption to represent the growth in system coincident peak demand. Therefore, when dealing with long term load-forecasting, it is valid to discuss the growth in residential consumption as a model for peak load growth contribution from that customer class.

The numbers in the previous chart can be generated in a number of ways. One approach is to look at residential energy usage from an appliance perspective. Below is a breakdown of energy usage at a typical household:

- 20-30% - Space heating
- 10-15% - Water heating
- 10-15% - Refrigeration
- 5-10% - Cooking
- 30-50% - Other

Market and retail trends can be evaluated along with the capabilities of modern appliances to develop consumption growth (or decline) in these categories. This type of analysis is performed for hundreds of thousands of customers and the resulting trends at the total usage by customer class can be quite relevant.

Commercial

The following is a list of commercial customer classes used by a large utility:

- Transportation, communications, & utilities (TCU)
- Wholesale & retail trade (Trade)

- Finance, insurance & real estate (FIRE)
- Community, business & personal services (Services)
- Government services (Gov)
- Other

These categories are comprehensive to the various types of commercial loads. The breakdown may not be the best for utilities in other geographies or economies. This particular utility saw commercial energy consumption broken down like this:

- TCU – 14%
- Trade – 20%
- FIRE – 20%
- Service – 37%
- Gov – 7%

Like the residential customer classes, commercial classes need to have growth values predicted. There are a number of approaches to estimating growth based on floor space, metering, etc.

Industrial

Growth in these classes may be less difficult to determine because of metering, contracts, and marketing efforts. The major correlated industries served by the utilities should be formed into industrial customer classes. Growth rates per year by customer class should be determined.

Driving forces

Regression analysis and other methods can be used to estimate growth by customer class. Here are some likely driving factors:

For residential

- Electricity price
- Cooling / heating days
- Price of heating oil
- Price of natural gas
- Disposable income
- Appliance price index

- People / household

For commercial

- Electricity price
- Occupied office space
- Commercial employment index
- Number of residential customers
- Price of gas
- School-age population
- Government spending
- Personal consumption
- Business investment
- Heating / cooling days

For industrial

- Industry output
- Electricity price
- Industrial employment
- Output / worker
- Manufacturer earnings
- Price of gas
- Price of oil
- Environmental laws
- Heating / cooling days

There are many other metrics that correlate to the growth of electricity demand.

SynerGEE Forecasting

SynerGEE Forecasting is a tool that manages load growth or load loss to meet the engineer’s expectations for future loads. Future load expectations can be based on a variety of data. Detailed growth studies may have been performed to identify pockets of growth in a system. Regression values at metered values may give insight for the load growth downstream from a meter. Regional growth values may be supplied as broad targets by the utility. Or, the engineer may simply have a gut-feeling for the manner that loads in an area will grow over the coming years.

SynerGEE helps the engineer setup and manage load growth from sections, feeders, areas, and regions. The forecasting application will resolve growth inputs together so that the overall forecast is consistent with high and low level expectations.

Growth can be specified at various levels. Look at this system:

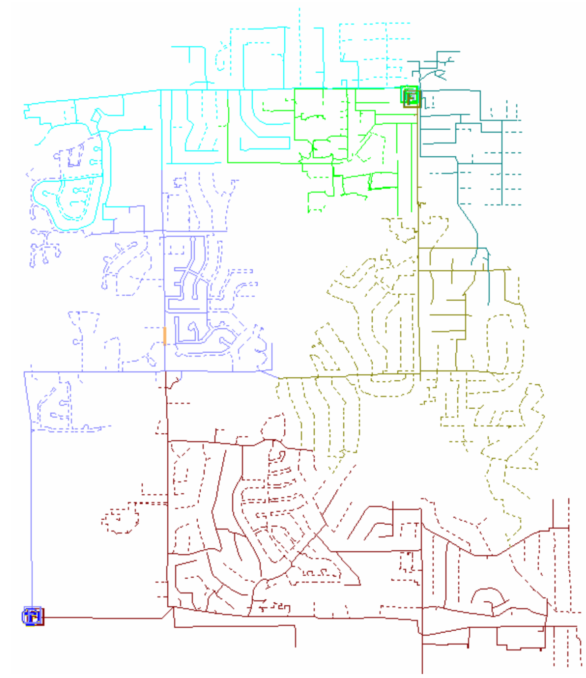


Fig. 17-3 *Distribution system*

The system has two substations and six feeders. An engineer may have information about growth in a particular area:

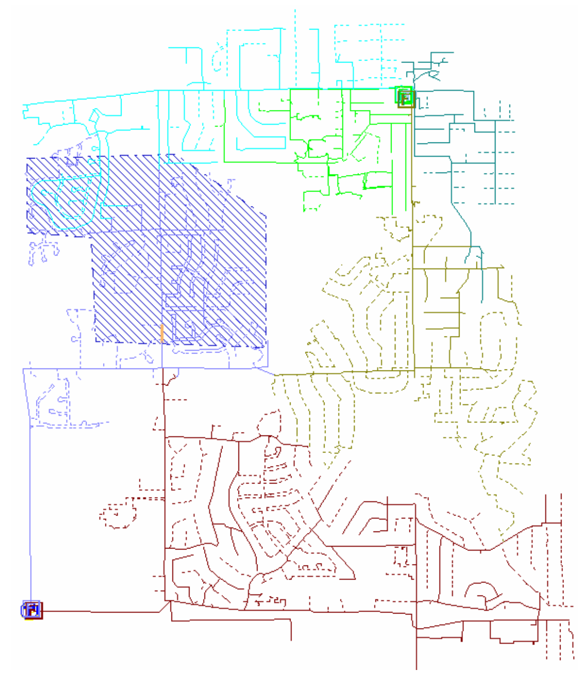


Fig. 17-4 *Area of growth*

SynerGEE supports a spatial area or polygon of growth that allows the engineer to supply growth data that will be applied to sections lying beneath the polygon.

The engineer may also have predictions about growth on a particular feeder:

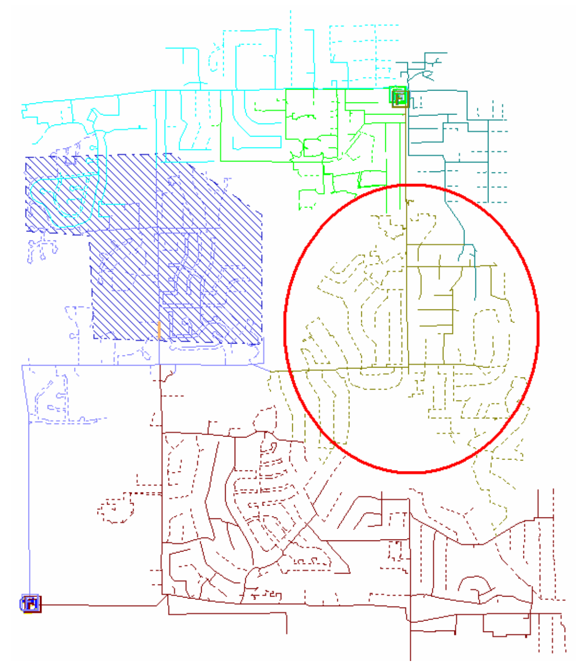


Fig. 17-5 Growth for a feeder

Information may also be available about the growth of the entire region:

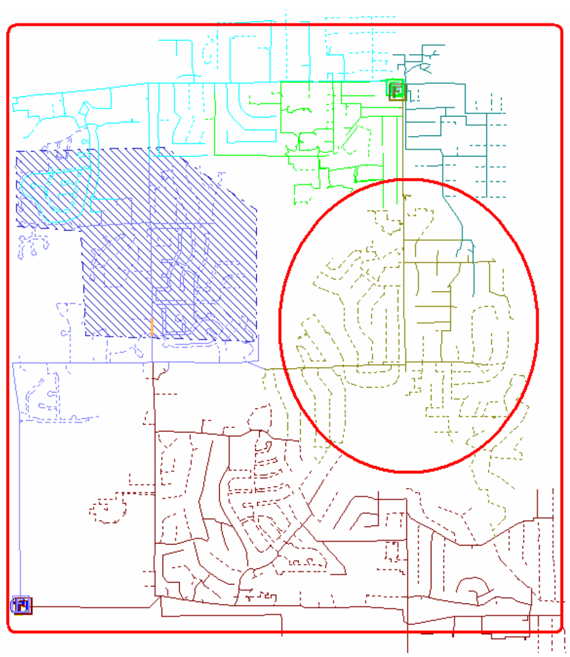


Fig. 17-6 Regional growth

SynerGEE forecasting resolves growth at different levels affecting sections so that the resulting future loads match well with the expectations by feeder, area, and region.

Agents

An agent is one person acting to represent another's interests. In SynerGEE Forecasting, an "agent" is used to represent the load growth or decline expectations for a spatially or logically selected set of sections.

Forecasts are achieved by managing growth. Loads are grouped by location, feed, or characteristics. These groups are managed by an entity called an "agent".

An agent includes a set of specific growth objectives. During the forecasting process, the agent adjusts loads on the loads within its group until the objectives are met. Loads may be within the groups of several agents. A load adjusted by one agent may impact the loading in another group. Agents operate in an iterative process to meet their constraints.

Agents are created for the various areas where the engineer expects to see load growing over time. For example, if a system is predicted to have 5% overall load growth and 8% growth on spot loads in one of the feeders then two agents would be created. The first agent would manage the overall system growth. The second agent would focus on the spot load growth for the given feeder. If things work well during the forecast, the engineer would see the desired system growth as well as the expected spot load growth on the feeder.

There are many types of agents supported SynerGEE Forecasting.

<u>Growth Area</u> – Sections lying within a polygon region on the map are managed. The polygon is defined by creating and setting up an "Area of Interest" object. Create the area of interest and then set is as a 'Growth area'. Growth values by year can be specified like this:

Year	%/yr	Kw/yr	Kvar/yr
'06	3.0	0	0
'07	3.5	300	100
'08	2.0	400	150
'09	1.5	500	200
…			

Spot and distributed loads have independent sets of growth values. Scripts can be used for creating more detailed growth behaviors for agents.

<u>Section growth curves</u> – Data wise, a growth set is just like a growth area. The difference is that a growth area applies to everything underneath its polygon. Growth curves are applied on a section-by-section basis.

A section can belong to at most one growth curve. The same section can belong to many growth areas if it lies within overlapping polygons.

Regional Growth – factors for these types of agents apply to all selected feeders. The settings dialog for forecasting can be used to setup a few regional agents for spot and distributed loads. Scripts can be used to develop more detailed agents.

Customer zone growth – a section can be assigned to a customer zone. Zones are typically assigned spatially or logically downstream from certain points. The customer zone contains weighted references to up to three customer class daily load curves. The zone also has growth information. An agent setup on a customer zone will manage growth for the sections assigned to the zone. When setup through the SynerGEE user interface, the growth values from the customer zone editor are used. Scripts enable more customization of agents to customer zones.

Customer class growth – Customer class growth is not yet supported by SynerGEE. When implemented an agent would manage growth for all loads assigned to a customer class. All “gas heat .ge. 2000ft^2” customers could be set to grow at 3%, for example.

Feeder / subtran growth – All loads on a feeder are handled by this agent. The growth curves are specified in the feeder or substation record.

An agent’s scope refers to the loads that it manages and grows (or reduces). The scope is typically fixed for the agents generated through the SynerGEE forecasting editor. The scope can be more directly manipulated when agents are created with a forecasting script.

Agents may have scope over these load types:

- Distributed loads
- Spot loads
- Large customers
- Projects
- Speculative load (area of interest)

Some example agents

I’ve been told the system growth will be 5% / year for the next five years.

- Create a regional agent with 5% / year on spot, distributed, and large customer loads.

A new commercial area is being planned between Canal and Main streets.

- Create a growth area agent over the commercial area. Set yearly growth rates (maybe initially declining) and values for new load. Setup values for spot and distributed loads.

We’ve reconductored our backbone for a substation to 266 ACSR. We expect to see about 3% organic growth on customers served in that sub.

- Create a growth set with 3% growth on distributed and spot loads. Create a query set for all 1ph sections or for all sections not 266. Assign the new growth set to those sections. Create an agent for the growth set.

Studies have show that we should be seeing a 3% growth in all electric household consumption. It will accelerate up to a 6% rate in eight years.

- Create an agent for the “all electric” customer zones.

We used regression to develop a growth curve for a feeder demand.

- Setup an agent on the feeder with the year-by-year forecast.

Settings for load growth

SynerGEE Forecasting sets up loads on the multi-year model. It may also utilize multi-year load values beyond the base year for its analysis. There are three settings that affect the use of post base year load values.

Forecast period

In one run, forecasts may be run from the base year up through the tenth year. Forecasts over shorter runs may be desired for a few reasons.

- The availability of growth information and quality of load estimates may not be consistent over a ten year period. It might make sense to run a forecast using short and medium term information and then running a second forecast using long term information.
- Forecasting can be used to simply bump up values to the next year or grow a particular type of load through a series of events spanning a small number of years.

A selectable forecast period will help tailor the forecast based on the available data and the analytical goals.

Agent selection

A model may contain dozens of agents. A particular forecast can be performed by picking the agents to be activated. This allows the engineer to formulate the best set of agents to use in the forecast. It also allows what-if type activities by deselecting agents and reviewing the impact.

The following objectives will be measured to meet the growth goals.

Objective	Measure
Regional growth	Total loads within the selected feeders.
Section curves	Loads on sections assigned to growth curves
Meter growth	Load downstream from meter.
Customer zone	Loads associated with sections assigned to a customer zone.
Feeder growth	Total load growth on feeder.
Section growth	Load growth on each section.

Hitting growth targets

This is an important setting if the multi-year model has been populated with non base year loads.

Sometimes loads that have been setup after the base year in a multi-year model correspond to estimated load growth that should be accommodated by the growth agents. In this case the growth targets should be hit even with new load being energized in future years.

Sometimes loads are project related and supplemental to the forecast. Maybe a system has an expected growth of 5% but we know that in 2008 there will be a large factory added to the system. SynerGEE needs to account for the 5% growth and also apply the new factor load.

If the 'hit growth targets' option is not set then growth is managed from each year. If new load is setup in future years then that load simply adds to the forecast. If the option is set then forecast load as well as newly energized multi-year loads will be worked together to get the growth expectations.

Consider a simple system having a growth of 10% / year with 1000kW of load in 2006 and 500kW of load on sections that are to be energized in 2007.

	Existing load	2007 load
2006	1000kW	---
2007		500kW

If we forecast without hitting growth targets then the 1000kW load will grow by 10% and the new 500kW load will be added in 2007. This leads to:

	Newly energized load	2007 load
2006	1000kW	---
2007	1100 kW	500kW

The total load in 2007 is 1600kW, a growth of 60% over the 2006 load.

If we forecast to hit the growth targets then the forecast may look something like:

	Newly energized load	2007 load
2006	1000kW	---
2007	733 kW	367 kW

Here, the 2006 load is 1100kW, a growth of 10%.

Block reduction in load

During a forecast, load need to be adjusted to meet the agent requirements. If multiple agents are involved, load growth in one area may need to be offset by load reduction in another area. This setting can be used to prevent the reduction of load that was in place at the start of the analysis year.

Forecast method

The forecast should typically be run from year to year. Two other options are made available. Load can be grown from the base year to the target year. This prevents growth compounding and may be desired to deal with new load estimates. The other option allows the forecast to be run from the base year to the final year. All intermediate year loads are calculated from interpolation. Here is a graph of growth expectations for each setting:

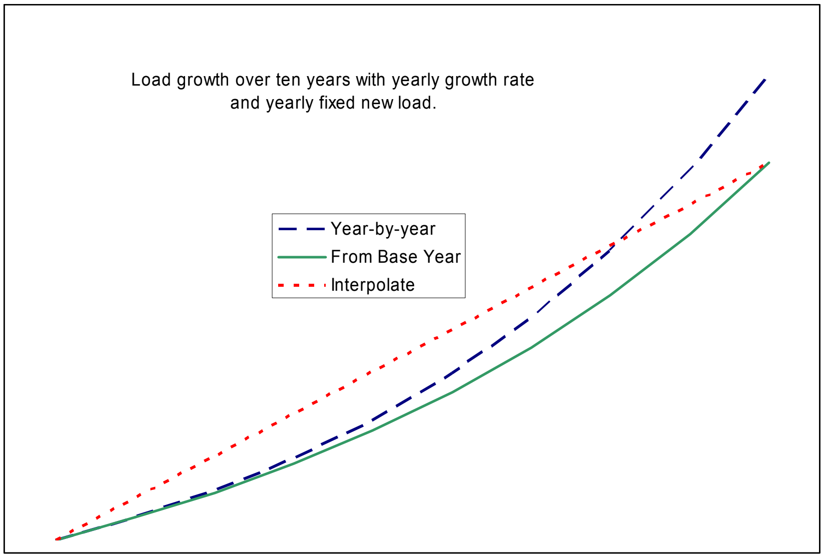


Fig. 17-7 Growth methods

Note that in the final year, forecasting from the base year is the same as the interpolation method. The interpolation method may give large load values for intermediate years. The method is made available because of the significant volume of books and technical papers using this approach.

Beginning, middle, end of year loads

Loads can be assumed at the beginning, mid, or end of year. All loads are represented at the same point in the year. Consider the load inputs below:

Year	% Growth	Input Load kW
2006 – Base	5%	1000
2007	7%	1200
2008	4%	1400
2009	2%	1600

We expect 5% load growth on this system and choose to not initialize load values (the load initialization option will be covered next). So this is a case where the engineer has elected to put in 200kW / year of new load. This may or may not be meaningful information. Perhaps a new large customer is ramping up its manufacturing capacity. On the other hand, the engineer may have been manually experimenting with load increases and now wishes to update with SynerGEE Forecasting.

Beginning year loads

First, let's consider the loads to be beginning year values.

Growth occurs throughout 2006. This load will supplement the load entered for 2007. The 2007 load. Growth of 5% on the 1000kW load for 2006 gives 50kW. The 50kW is added to the specified load for all future loads.

Year	Input kW	Forecast
Jan 1, 2006	1000	1000
Jan 1, 2007	1200	1250
Jan 1, 2008	1400	1450
Jan 1, 2009	1600	1650

We have completed the forecast to January 2007.

The growth in 2007 is 7% of 1250kw or 87.5 kW. This is added to the 2008 and 2009 loads:

Year	Input kW	Forecast
Jan 1, 2006	1000	1000
Jan 1, 2007	1200	1250
Jan 1, 2008	1400	1538
Jan 1, 2009	1600	1738

To get to January 2009, we grow the 1538kW through 2008 with the 4% growth to get 61.52 kW. This is added to the 2009 load:

Year	Input kW	Forecast
2006 – Base	1000	1000 kW
2007	1200	1250 kW
2008	1400	1538 kW
2009	1600	1799 kW

The fact that we are using "beginning year loads" dictates that we apply the growth from the previous year to the "January 1" values for all future loads.

End year loads

Now, we assume that loads represent end of year values. Begin with the same settings:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	
Dec 31, 2008	1400	
Dec 31, 2009	1600	

The 1000 kW at January 1, 2007 will grow to the end of the year with the 7% growth rate. Add the 70kW to the 1200 kW for end of year 2007 and all future years.

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	1270
Dec 31, 2008	1400	1470
Dec 31, 2009	1600	1670

We have 4% growth in 2008 generating 50.8kW from the beginning of year 1270kW:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	1270
Dec 31, 2008	1400	1521
Dec 31, 2009	1600	1721

Finally, we have 2% growth in 2009. Two percent of 1721 kW is 34.4kW:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000 kW
Dec 31, 2007	1200	1270 kW
Dec 31, 2008	1400	1521 kW
Dec 31, 2009	1600	1755 kW

End of year loads with constant input kW

We will use a constant 5% growth value for all years and these settings:

Year	Input kW	Forecast

Dec 31, 2006	1000	1000
Dec 31, 2007	1000	
Dec 31, 2008	1000	
Dec 31, 2009	1000	

The 5% growth in 2007 yields:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1000	1050
Dec 31, 2008	1000	1050
Dec 31, 2009	1000	1050

In 2008, we get a growth of (5%)(1050) or 52.5kW. Add this to future years:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1000	1050
Dec 31, 2008	1000	1103
Dec 31, 2009	1000	1103

55.2kW growth through 2009 gets us to the final numbers:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000 kW
Dec 31, 2007	1000	1050 kW
Dec 31, 2008	1000	1103 kW
Dec 31, 2009	1000	1158 kW

If we look at 5% growth over three years on the base year 1000kW we would get:

$$\begin{aligned} S_{2009} &= 1000\left(1+\frac{5}{100}\right)^3 \\ &= 1158kW \end{aligned} \tag{17-1}$$

Mid-year loads

Growth rates are always assumed to apply to calendar years. Loads can be specified as mid-year loads. In this case, the average of the previous and current years will be used to calculate an effective growth rate. The very small error introduced by this manipulation can be evaluated. Consider growth rates on adjacent years as r_1 and r_2. The growth of load through half of the first year and half of the second would be:

$$S_2 = S_1\left(1+\frac{r_1}{2\cdot 100}\right)\left(1+\frac{r_2}{2\cdot 100}\right) \tag{17-2}$$

$$= S_1\left(1+\frac{r_1+r_2}{2}+\frac{r_1 r_2}{4\cdot 10^4}\right)$$

For yearly growth rates as high as 20%, the last term in the above expression would only be 1%. So clearly we can assume:

$$\frac{r_1 r_2}{4\cdot 10^4} \approx 0 \tag{17-3}$$

Giving us a rate that is the average of each year's rate:

$$S_2 = S_1\left(1+\frac{r'}{100}\right) \tag{17-4}$$

$$\textit{where}$$

$$r' = \frac{r_1+r_2}{2}$$

Load initialization

In the examples above, the load value supplied by the engineer was used as the starting point for each year. Loads can optionally be initialized by copying forward the previous year's load. We will go back to the setup from the previous section to look at initialized loads:

Year	% Growth	Input Load kW
2006 – Base	5%	1000
2007	7%	1200
2008	4%	1400
2009	2%	1600

We will use beginning year load assumptions. Since loads are initialized by copying the previous year's load, the 2007 load will be 1000kW plus the 5% growth that occurred during 2006:

Year	Input kW	Forecast
Jan 1, 2006	1000	1000
Jan 1, 2007	1200	1050
Jan 1, 2008	1400	1450
Jan 1, 2009	1600	1650

To try and remain consistent, we are showing the propagation of the 50kW growth to future years. That load addition is insignificant because the 1050kW is copied forward and combined with the 2007 7% growth to get:

Year	Input kW	Forecast
Jan 1, 2006	1000	1000
Jan 1, 2007	1200	1050
Jan 1, 2008	1400	1124
Jan 1, 2009	1600	Don’t care

The 4% growth in 2008 is used on the 1124kW copied forward into 2009:

Year	Input kW	Forecast
Jan 1, 2006	1000	1000 kW
Jan 1, 2007	1200	1050 kW
Jan 1, 2008	1400	1124 kW
Jan 1, 2009	1600	1168 kW

If we look at compounding 5%, 7%, and 4% growth onto our starting kW we get:

$$\begin{aligned} S_{2009} &= 1000(1.05)(1.07)(1.04) \\ &= 1168kW \end{aligned} \tag{17-5}$$

This makes sense.

Initializing loads by copying values forward eliminates any load change data beyond the base year. This may be desired for some categories of loads and not desired by others. The load initialization function is therefore broken down by load type:

- Distributed loads
- Spot loads
- Large customers
- Projects

- Speculative loads

Be careful with copy forward

A ‘copy forward’ setting on a load type affects all loads of that type. If an agent is not assigned to loads the copy forward setting may still affect them. The result is to initialize all future loads to the base year.

New load agents

Values for new load can be typed directly into the multi-year model. The load initialization settings discussed above govern if multi-year changes are considered as new loads or not. Some agents can be setup to apply new load to the forecast model. Particular agents designed to handle new load are the growth areas and growth sets. These constructs support growth of existing load and addition of new load.

Settings in SynerGEE Forecasting govern the way in which new load is applied to the model. There are three options:

- New load before growth
- New load after growth
- New load with growth

In the first case, new load will be applied to the model at the beginning of the year and then growth rates will be used:

$$S_{i+1} = \left(S_i + S_{New}\right)\left(\frac{r_i}{100}\right) \tag{17-6}$$

In the second case, the load will be growth with the growth rate and then new load will be applied:

$$S_{i+1} = S_i\left(\frac{r_i}{100}\right) + S_{New} \tag{17-7}$$

In the final case, the forecasted load is iterated to be the expected value based on growth and new load values:

$$S_{i+1} = f\left(S_i + S_{New}, \frac{r_i}{100}\right) \tag{17-8}$$

Here is new data to demonstrate these settings:

Year	% Grw	New Load	Input kW

2006 – Base	5%	100	1000
2007	7%	150	1200
2008	4%	50	1400
2009	2%	50	1600

For this example, we will use end of year load assumption and no load initialization.

New load before growth

We get the 2007 end of year load by starting with the 1000 kW for the beginning of 2007, adding the 150kW new load, and the 7% growth for 2007. Seven percent of 1150kW is 80.5kW. Summing 150kW and 80.5kW gives 230.5kW. We add this to all future loads:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	1431
Dec 31, 2008	1400	1631
Dec 31, 2009	1600	1831

Now we will do 2008:

$$0.04(1431+50)=59.2kW \tag{17-9}$$

And the additional load for the year will be 109.2kW:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	1431
Dec 31, 2008	1400	1740
Dec 31, 2009	1600	1940

Finally for 2009:

$$0.02(1740+50)=35.8kW \tag{17-10}$$

And we will add 85.8 kW:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000 kW

Dec 31, 2007	1200	1431 kW
Dec 31, 2008	1400	1690 kW
Dec 31, 2009	1600	2026 kW

New load after growth

These numbers are very similar to the previous example. New load is added in after the yearly growth values are considered.

Again, we get the 2007 end of year load by starting with the 1000 kW for the beginning of 2007, adding 7% growth and then adding the 150kW new load. The growth is 70kW. The total additional load to propagate is therefore 220kW:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	1420
Dec 31, 2008	1400	1631
Dec 31, 2009	1600	1831

Look at 2008 next:

$$0.04 \cdot 1420 + 50 = 106.8kW \tag{17-11}$$

We will add 107kW forward:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000
Dec 31, 2007	1200	1420
Dec 31, 2008	1400	1738
Dec 31, 2009	1600	1938

Finally for 2009:

$$0.02 \cdot 1738 + 50 = 84.8kW \tag{17-12}$$

We will add 85kW:

Year	Input kW	Forecast
Dec 31, 2006	1000	1000 kW

Dec 31, 2007	1200	1420 kW
Dec 31, 2008	1400	1690 kW
Dec 31, 2009	1600	2023 kW

There is very little impact on the model with these two options.

New load summary

The following chart summarizes the handling of new loads when loads are setup to represent start and end of year values. The following variables are used in the table:

$$\begin{aligned} S_i' &= \text{New load to add in year i} \\ S_i &= \text{Load in year i} \\ \mathrm{r_i} &= \text{Year i growth rate} \\ \mathrm{N_i} &= \text{New load for year i} \end{aligned} \tag{17-13}$$

Entries for mid-year load values are not listed.

	Loads are start of year	Loads are end of year
New load before	$S'_i = (S_{i-1} + N_{i-1})r_{i-1}$	$S'_i = (S_{i-1} + N_i)r_i$
New load after	$S'_i = S_{i-1}r_{i-1} + N_{i-1}$	$S'_i = S_{i-1}r_i + N_i$

Mixed load types

An agent can apply to various combinations of these load types:

- Distributed
- Spot
- Large Customer
- Project
- Speculative

An agent assigned with a 5% yearly growth for speculative and large customer loads will work to keep the total load in all three of those categories at 5% / year.

	Large Customer	Speculative
2006	3000	1000
2007	3600	1200

Let's consider the loads as 'beginning year' and choose to initialize forecast loads. In 2006, the agent covers 4000kW of load. We expect 4200kW in 2007. The growth will be spread proportionally to the two categories of load:

(17-14)

$$S_{LargeCust}^{New} = 200\frac{3000}{4000} = 150kW$$

$$S_{Speculative}^{New} = 200\frac{1000}{4000} = 50kW$$

The results of the forecast are:

	Large Customer	Speculative
2006	3000	1000
2007	3150	1050

If we had chosen to not initialize the 2007 loads then we would have:

	Large Customer	Speculative
2006	3000	1000
2007	3750	1250

18
Reliability

Introduction

SynerGEE Reliability is a comprehensive package to aid in the simulation and analysis of distribution system reliability. Delivered on the SynerGEE platform, it is a powerful tool for investigating root-cause and configuration effects on system and customer level reliability. SynerGEE Reliability has the following characteristics.

- Zone-based failure rates, repair times, and repair costs with provisions for single- or three-phase lines.
- Use of failure rates based on historical outages
- In depth root-cause analysis
- Comprehensive and detailed switching models
- By-phase analysis
- By-cause analysis
- Sectionalizing, reclosing, pickup
- Capacity evaluation
- Unlimited and customizable causes
- Failure rates by category and subcategory.
- Mitigation over multiple subcategories
- Comprehensive contingency-based interruption, switching, and pickup plans
- By-phase analysis and results reporting
- Handling of automatic switches and auto-transfer switches

Importance of reliability

Reliability metrics indicate how well a utility serves its customers. More specifically, they indicate the value that customers realize through their current service. Since quality of service is basic to the long-term health of any utility, reliability metrics are a fundamental concern of engineers, managers, and executives alike. These metrics often affect financial decisions related to long-range and business planning. In addition, as movements toward deregulation and open competition continue, issues of distribution system reliability become even more important.

The SynerGEE approach

SynerGEE Reliability provides an intuitive and simple user interface to analyze system reliability. Though reliability simulation results tend to be vast and complex, SynerGEE presents them in a manner which is easily understood and manipulated. In many utilities, reliability calculations are performed by select individuals with specialized knowledge and complicated software, and often the crucial information is not understood by those who need it. Now, through powerful reporting and display capabilities, SynerGEE helps shield you from the complexity of system-wide metrics and gives information that many parties can use, particularly those in a decision-making capacity. SynerGEE results do not require advanced knowledge in the technical aspects of modeling and simulation.

Reliability scenario evaluations should not be independent studies. As such, SynerGEE provides reliability simulation based on the same data model as other engineering simulations and applications. SynerGEE considers security, capacity, efficiency, and power quality. using the powerful SynerGEE platform which already provides load-flow, contingency, protection coordination, and other analyses. In this manner, all SynerGEE analyses, including reliability, are fully compatible and performed on the same model.

Our approach to representing the analytical calculation of reliability indices is to find expressions for some "base factors" and then use those factors to calculate our indices. Here are some general concepts that will be discussed later in this chapter:

Every device can fail. Each series device and section in SynerGEE has the following associated data:

$$\begin{aligned} &\lambda(i,j)_d - \text{Failure rate, by cause, of series device 'd' (fl/yr)} \\ &r(i,j)_d - \text{Repair time, by cause, of device 'd'} \\ &Cr(i,j)_d - \text{Repair cost, by cause, of device 'd'} \end{aligned} \qquad \textbf{(18-1)}$$

A failure on a device ‘d’ is referred to as an ‘event’.

$$E(i,j)_d - \text{Event on 'd' associated with caused by K(i,j)} \qquad \textbf{(18-2)}$$

The probability of ‘n’ events occurring during a year due to a single cause is:

$$P(n)_d = \frac{\lambda(i,j)_d^n}{n!} e^{-\lambda(i,j)_d} \tag{18-3}$$

How do failure rates combine? Here is the general rule for overlapping exposures:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{18-4}$$

It states that the probability of event 'A' or event 'B' occurring is the sum of the event probabilities for each of them less the probability that they both occur at the same time. We will assume that:

- No two devices fail concurrently
- Any failure can be tracked to precisely one cause

Challenges

By nature, reliability simulation is a complex statistical analysis of a very large and highly non-linear system. The randomness of failures does not allow precise predictions, no matter how good the data or calculations. At best, reliability simulation can generate overall expectations of system's performance. It must be understood that these expectations may or may not be realized. For example, even if a simulation indicates that a substation reconfiguration could drop SAIFI from 2.5 to 2.2, it may be unlikely that the improvement will be realized immediately even if the reconfiguration were implemented. Changes in conditions and hazards, cyclic behaviors, and simple bad luck can potentially prevent the change in SAIFI. However, with SynerGEE and a well-built model, the predicted trends should generally be seen over the upcoming years.

It is vitally important to use reliability simulation with care. Knowledge of the basis of the reliability model and analysis should be available. Assumptions should be recognized. A process for building distribution system models, validating them, and analyzing them with simulation software should be formalized.

Keep in mind that results are significantly affected by the quality of the model. And, even a good model is still a simplification of the complicated power distribution system. Therefore, reliability simulation should be considered as one aspect of a utility's overall process for maintaining and improving reliability.

Aspects of SynerGEE Reliability

Reliability simulation is a statistical analysis yielding expectations of customer, feeder, and system level reliability metrics. It is used to predict reliability problems by geographic region, electrical zone, and root-cause. SynerGEE provides analytical insight into the complex realm of system performance under various conditions of failure, loading, and configuration.

With a properly calibrated model, SynerGEE Reliability can provide realistic predictions of expected reliability. Though random events may cause unexpected fluctuation, long-term trends should tend to match these predictions.

Supported indices

Indices that result from a SynerGEE analysis are described below, along with the typical equation-based representation for each index.

Term	Represented	Definition
System average interruption frequency index	SAIFI	Rate indicating the number of customer interruptions per customer served. $SAIFI = \frac{\text{Total Number of Customer Interruptions per Year}}{\text{Total Number of Customers Served}}$ The units for SAIFI are interruptions/feeder customer/year.
Customer average interruption frequency index	CAIFI	Rate used for number of customer interruptions per number of customers affected. $CAIFI = \frac{\text{Total Number of Customer Interruptions per Year}}{\text{Total Number of Customers Affected}}$ The units for CAIFI are interruptions/customer affected/year.
System average interruption duration index	SAIDI	Rate used for average number of customer interruption. $SAIDI = \frac{\text{Total Customer Interruption Duration Hours per Year}}{\text{Total Number of Customers}}$ The units for SAIDI are hours/feeder customer/year.
Customer average interruption duration index	CAIDI	Rate used for number of customer interruption duration per customer experiencing interruptions. $CAIDI = \frac{\text{Total Customer Interruption Duration Hours per Year}}{\text{Total Number of Affected Customers}}$ The units for CAIDI are hours/customer interruption.
Average service availability index	ASAI	Rate used for number of customer interruption duration per customer experiencing interruptions. $ASAI = \frac{\text{Hours of Available Service}}{\text{Hours of Customer Demand}}$

Publishable results

SynerGEE presents simulation results in a simple and comprehensive format. Since an enormous amount of data is typically available from a simulation, it is best presented in charts and maps. The chart below, for example, depicts the correlation between a reliability index and a billing code typically used for load modeling in load-flow analysis.

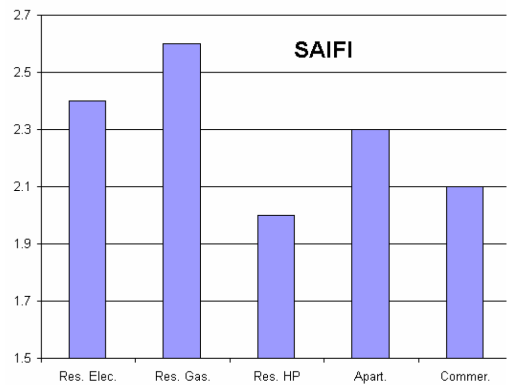


Fig. 18-1 *SAIFI vs. Customer class*

This type of chart provides valuable insight, demonstrating the importance of customer- or zone-based index and standard deviation values.

Scenario comparison

SynerGEE provides the ability to create and compare multiple scenarios within the same model. You can evaluate any number of scenarios and select one based on supporting evidence, cost/benefit ratio, utility planning guidelines, variance of results, and basic engineering judgment.

Switching plan models

SynerGEE Reliability generates and evaluates reasonable switching plans that are consistent with actual utility operation. It carefully considers which devices operate during a failure, how they operate, and whether fault-related failures can be isolated. In addition, it examines issues such as alternate feed sources for outaged customers and the operation of automatic switches. Clearly, it is impossible to mimic real world operations within a simulation for every type of failure. However, SynerGEE can provide a reasonable switching plan for a failure at any point in the system.

Capacity considerations

Since capacity constraints can affect a switching operation, detailed by-phase load-flow analyses are integral to SynerGEE Reliability simulations. The load-flow analysis performed by SynerGEE Reliability represents the same proven calculations used for planning studies elsewhere within SynerGEE.

Metric variance

Since reliability simulation is a statistical analysis, and metrics such as SAIDI and SAIFI are based on random variables, SynerGEE results can include values of variance. For example:

Feeder	Cust Count	SAIFI	SAIDI	MAIFI
System	10920	4.1 +/- 1.0	10.3 +/- 2.8	6.1 +/- 1.4
New - Big Spring	1616	6.8 +/- 2.6	20.5 +/- 8.5	0.0 +/- 0.0
New - Cove	860	3.6 +/- 2.4	13.9 +/- 10.6	0.0 +/- 0.0
New - Liberty	1213	1.3 +/- 1.1	2.7 +/- 2.8	2.2 +/- 1.8
New - Springfield	1704	2.0 +/- 1.5	5.1 +/- 4.0	6.0 +/- 2.3
Ship - South Penn	1758	4.5 +/- 2.3	10.5 +/- 6.1	9.0 +/- 3.4
Ship - West Martin	3769	4.8 +/- 2.3	9.8 +/- 5.9	10.0 +/- 3.6

Fig. 18-2 *Table with index and variance values*

Variance is very sensitive to feeder configuration and device placement. For example, a feeder reconfiguration may change a SAIFI value from 2.5 to 2.3. However, if the SAIFI was originally 2.5 +/- 1.0 and the reconfiguration resulted in a value of 2.3 +/- 1.5, the original configuration may be considered superior from a strict metric perspective. The necessity of the variance calculation becomes even more evident when considering cost functions associated with performance-based rates.

Root-cause

In SynerGEE, reliability results are calculated and can be presented on the basis of root-cause. Therefore, an understanding of root-cause is essential for evaluating reliability issues. As an example, the following pie chart shows apportionment of SAIDI values for a sample system, by root-cause.

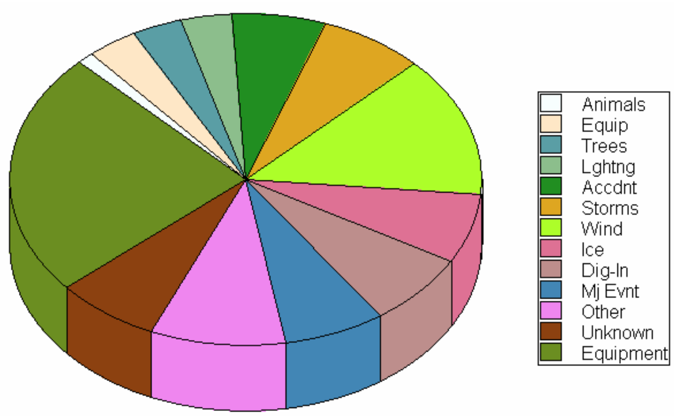


Fig. 18-3 SAIDI break-down by root-cause

Correlation between root-cause exposure, feeder configuration, and protective device placement is high. Diagrams such as this help evaluate the effectiveness of a particular mitigation effort. Root-cause categories may be extensive and there may be considerable variation between utilities.

Spatial mitigation

Since mitigation efforts are often dispatched in regions or zones, SynerGEE Reliability supports the spatial application of mitigation. You can create mitigation zones and apply them to geographic regions of your model. Zones should be constructed with data that represents the average effects of mitigation for a geographic area.

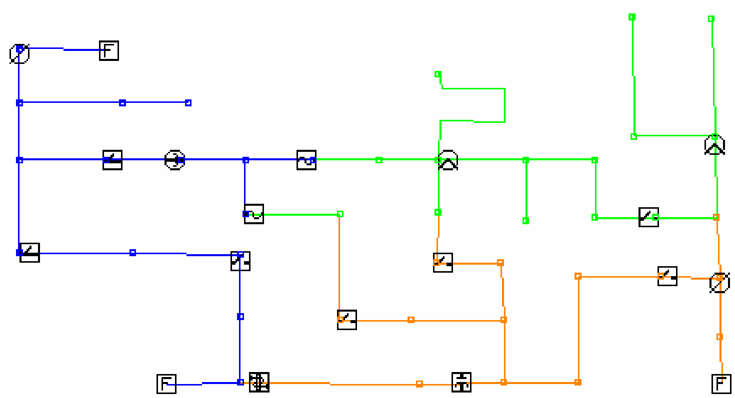


Fig. 18-4 Map colored by mitigation zones

Supporting applications

Reliability cannot be an isolated simulation. Voltage support, contingency, power quality, capacity, and efficiency are just a few of the serious considerations that accompany it. With SynerGEE, you have the support of many other analysis features, including:

- Load-flow

- Economic analysis
- Fault flow and fault voltage
- Contingency
- Optimal switching
- Phase balancing
- Harmonic analysis
- Capacitor analysis
- Coordination checking
- Time range load analysis

The collaboration of these applications enables you to fully evaluate system performance while making reliability decisions. You do not have to base decisions on any single type of engineering analysis. Also, since these applications are used in other planning, protection, and operational roles, you can take advantage of the consistency and compatibility of all SynerGEE modeling and analysis features.

Reliability Model

Reliability analysis is based on an engineering model of the distribution system. The model includes sub-models of event causes, interruptions, failures, and switching devices. In SynerGEE, this reliability model is the same model that is used for other engineering simulations.

Failures

Distribution reliability simulation is based on the probability of device failure. With a device in continuous operation, you ultimately wish to capture the probability that the device will fail. You can start with the following assumptions.

- A device either fails or it operates. A regulator, for example, cannot operate at reduced capacity.
- The probability that a device will fail remains consistent over time. For example, the probability that a regulator will fail is the same regardless of the time-of-day.
- The reliability evaluation period is constant for each and every device.
- The probability of failure for each device is independent. If a regulator fails, the probability of the downstream line failing is unaffected. The probability of the regulator failing again is unaffected.

Probability calculations

The assumption above can be applied to any repeated trial or continual process, and they lead to the binomial distribution.

$$P(j) = \frac{n!}{j!(n-j)!} p^j (1-p)^{n-j} \tag{18-5}$$

$P(j) =$ Probability of j failures in n trials

$p =$ Probability of failure in a single trial

This equation indicates the probability of failure over a set of trials. The equation can be applied to failures over a period of operation.

Note that the mean of the binomial distribution is derived as:

$$E(J) = np \tag{18-6}$$

You can now define a "failure rate" to be the average number of failures over a time period.

$$\lambda = np \tag{18-7}$$

Now, the number of slices in the period are grown to infinity and the corresponding probability of a failure within a time slice is reduced to zero based on the equation above. That is:

$$n \to \infty \tag{18-8}$$
$$p \to 0$$

The failure rate per slice is expressed as:

$$p = \frac{\lambda}{n} \tag{18-9}$$

...and the probability of "j" failures over a time period is:

$$p(j) = \lim_{n\to\infty} \frac{n!}{j!(n-j)!} \left(\frac{\lambda}{n}\right)^j \left(1-\frac{\lambda}{n}\right)^{n-j} \tag{18-10}$$

...noting the following:

$$\lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda} \tag{18-11}$$

The probability of "j" failures can be reduced to:

$$p(j) = \frac{\lambda^j}{j!} e^{-\lambda} \tag{18-12}$$

This probability distribution is known as a Poisson distribution and is the backbone of failure modeling in reliability studies in process control, power transmission, manufacturing, and power distribution. SynerGEE uses the Poisson distribution for calculating the probability of device failures.

Numerical example

To demonstrate how this distribution works, consider a device that has a failure rate of 1.0 failures per year. The probability that the device will not fail in a given year is:

(18-13)

$$p(0) = \frac{1^0}{0!}e^{-\lambda} = \frac{1}{e} = 0.367$$

Therefore, the device is expected to operate without failure one year out of every three. The probability that the device will have exactly one failure in a given year is:

(18-14)

$$p(1) = \frac{1^1}{1!}e^{-1} = \frac{1}{e} = 0.367$$

This indicates that every third year would see exactly one failure.

With these figures, it is clear that the device will probably fail more than once during some years, in order to satisfy the overall failure rate of 1.0 per year. The probability of two failures in a given year is:

(18-15)

$$p(2) = \frac{1^2}{2!}e^{-1} = \frac{1}{2e} = 0.183$$

Likewise, the probability of three and four failures per year is:

(18-16)

$$p(3) = \frac{1}{6e} = 0.06$$

$$p(4) = \frac{1}{24e} = 0.02$$

Using these numbers, you can look at the average number of annual failures for the device over 100 years.

(18-17)

$$\bar{f} = \frac{37 + 18(2) + 6(3) + 2(4)}{100} \cong 1.0$$

You can see that the original failure rate of 1.0 per year is validated. For any year, however, the device can have no failures or any number of failures.

Interruptions

Customers can experience momentary or sustained interruptions. SynerGEE considers any failure that can be cleared by reclosing equipment to be a momentary interruption.

Your organization may use different standards, since the criteria for distinguishing the two can vary between utilities.

Consider the example of a tree branch striking a line, creating an arc. The branch disintegrates or is pulled away, but the arc remains until its current is interrupted by an interrupting device.

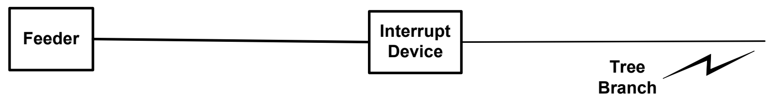


Fig. 18-5 *Failure*

If the interrupting device has reclosing capability, the interruption caused by the tree branch failure will be momentary. However, if the interrupting device is a fuse or a breaker without a reclosing relay, the interruption will be permanent.

Some failures cannot be cleared by reclosing equipment. If a tree branch falls on a line and it does not disintegrate or fall away, reclosing equipment will operate to lockout. Therefore, SynerGEE considers the following failure rates.

- **Sustained fault failure rate**—Expected yearly rate for failures that remain after reclosing equipment actions.
- **Momentary fault failure rate**—Expected yearly rate for failures that can be cleared by reclosing equipment.

If no reclosing equipment exists to protect against a momentary event, a sustained interruption results. Consider the following situation, which will result in a sustained interruption.

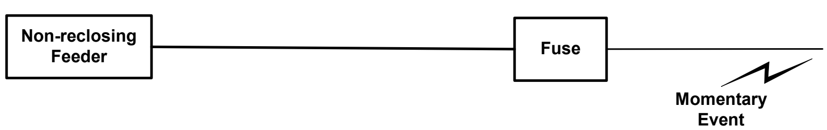


Fig. 18-6 *Sustained interruption resulting from momentary event*

However, in the following case, only a momentary interruption results because of the upstream reclosing device.

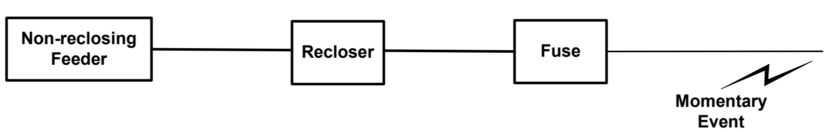


Fig. 18-7 *Momentary interruption from momentary event*

If a permanent event occurred on the system above, SynerGEE assumes that the recloser and fuse are coordinated so that the fuse blows.

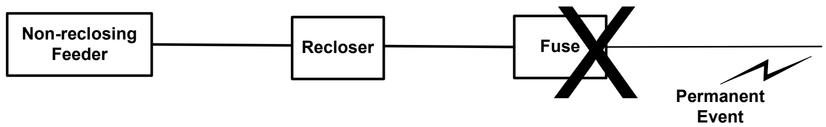


Fig. 18-8 Sustained interruption resulting from permanent event

Even with the assumption, overall performance of the model should be reasonable. Consideration of time-coordination settings of devices and the breadth of available fault levels from a failure would be very complex, and would not likely be of value to the simulation.

Causes

Failure analysis is based on root-cause in SynerGEE. These root-causes are completely customizable to match the conditions in your system. All cause data is stored in a text file, ReliCauseMeta.txt, found in your SynerGEE or Solver install folder. The following is a portion of a sample file.

10, -1, Animals

10, 5, Squirrels

10, 7, Raccoons

10, 8, Snakes

10, 22, Chimps

15, -1, Trees

15, 5, Deciduous

15, 9, Evergreen

20, -1, Accident

20, 5, Driver

20, 8, Dig-in

20, 15, Vandalism

32, -1, Birds

32, 10, Nesting

32, 18, Pigeon

32, 14, Condor

32, 22, Gull

42, -1, Weather

42, 16, Rain

42, 18, Ice / Snow

42, 22, Hail

42, 24, Hurricane

…

Figure **18-18** – Cause category and subcategory examples

Each entry in the file has the following format:

{Category}, {Sub-category}, {Description}

For example:

10, -1, Animals

10, 5, Squirrels-Grey

The numbers used for categories and sub-categories may be arbitrarily chosen. The only significant subcategory value is -1, which indicates that the entry contains the category name. Otherwise, you can create subcategories as needed, using the appropriate category number, and choosing a subcategory number. The category number can be any integer, so long as all subcategories use that same number. In addition, you can place any entry anywhere in the file, since it has no ordering requirements. However, you may wish to keep all entries for a particular category together, for convenience.

The category and subcategory numbers that you assign are referenced by SynerGEE during analysis. Although the text descriptions appear in SynerGEE editors, the numbers are vital to internal functions. Therefore, you should use caution if changing or removing items. If inconsistencies arise between different files used by different users, data problems could result.

Ideally, you should establish one ReliCauseMeta.txt file for your entire organization. If you make changes afterwards, you should:

1. Make sure all users get and apply the updated file
2. Review the impact of changes on existing model databases

Generally, adding new items causes less impact than removing or changing items.

Devices

Most devices, such as regulators and transformers, have failure rates. The failure of these devices can cause a sustained or momentary event. Switchable devices also have failure rates, but they play an even larger role in reliability simulation because they govern reconfiguration. In SynerGEE, devices are grouped in four ways.

Fail device—Device that can fail with a momentary or sustained type of event. Upstream protection devices are used to interrupt the fault.

Interrupting device—Device that can interrupt fault current. The interruption can be sustained if the fault is sustained or the device cannot reclose. The interruption can be momentary if the device has reclosing capabilities.

Isolating device—Device that can isolate the faulted area to allow resetting of an upstream interrupting device. An isolating device is used to quickly pick up customers between itself and the upstream interrupting device, while the fail device is being repaired.

Pickup device—Device used to quickly restore service to customers isolated from a failure by isolating devices.

Motors, capacitors, and generators do not have failure rates. However, these devices may still contribute to the reliability characteristics of a set of feeders if capacity constraints are an issue.

Exposure zones and failure rates

Sections with similar failure causes, geography, foliage, age, and construction can be grouped into an exposure zone. Sections assigned to a particular exposure zone are given common values for:

- Sustained failure rate per set length of line
- Momentary failure rate per set length of line
- Repair cost
- Repair time

The values in an exposure zone are itemized by cause category and subcategory. When you define exposure zones, SynerGEE draws categories and subcategories from the ReliCauseMeta.txt file, and you apply failure information to them as needed. This information is grouped according to single-phase and two/three-phase lines.

The following is an example layout of some of the data associated with an exposure zone.

	Two- or Three- Phase-Lines				**Single-Phase Lines**			
	Sust. Fail Rate	Momt. Fail Rate	Repair Time	Repair Cost	Sust. Fail Rate	Momt. Fail Rate	Repair Time	Repair Cost
Animals								
Squirrels								
Snakes								
Birds	{Customized values}							
Raptors								

Nesting								
Weather								
Lightning								
Wind > 25MPH								
Wind < 25 MPH								

Once your exposure zones are defined, each section can be assigned to any zone. Zones can be set up geographically to account for similarities in construction, location, history, and terrain.

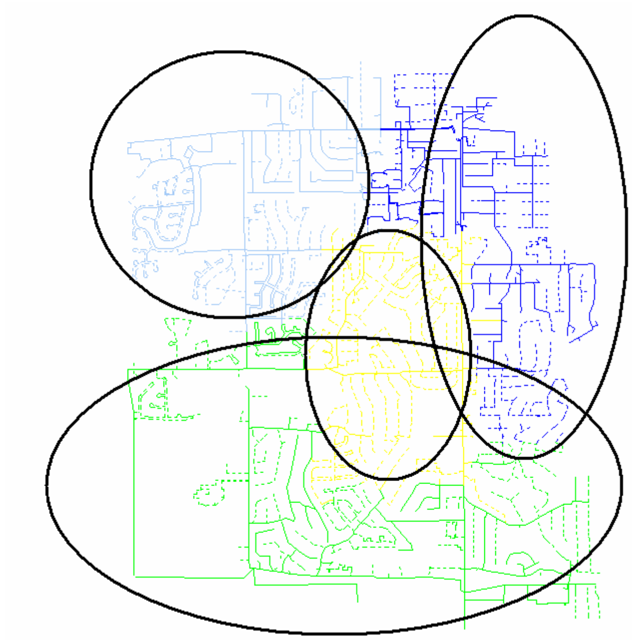


Fig. 18-9 *Exposure zones by geography*

Exposure zones can also be set up to correspond to interrupting device, which allows a more direct application of historical outage data. You would probably construct zones in this manner if your outage data is assigned by interrupting device.

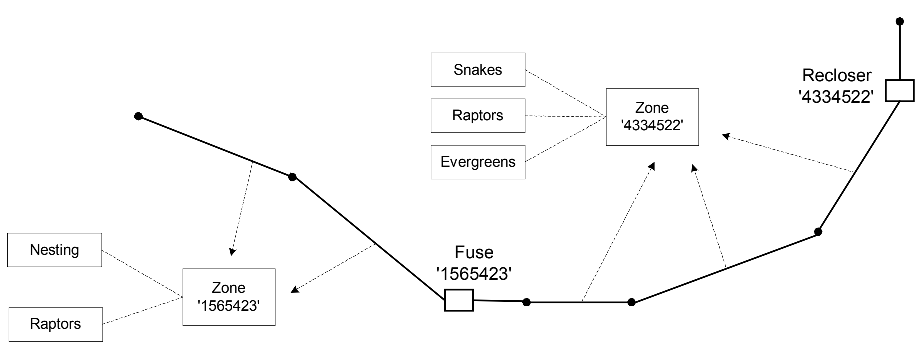


Fig. 18-10 *Exposure zones by interrupting device*

The SynerGEE exposure zone provides a means of applying multiple causes to whole regions of your system, and varying degrees of each. It also allows the correlation of lines with similar construction, in like areas, and with common isolation devices. For example, you may have a region in which squirrels, snakes, and deciduous trees are a problem. In SynerGEE, you can create an exposure zone that represents these causes, and apply it to sections in your model as appropriate.

Device failure data

Devices have failure data similar to exposure zones. Since the data is the actual failure data for the particular device, it is not broken up into the single-phase/three-phase groups. It is also not characterized by category. Switchable devices like switches and protective devices also have a "crew operation time" value. This value corresponds to the typical time it takes for a crew to reach and operate the device.

In reliability reports, indices resulting from device failures are listed under an "Equipment" category.

Mitigation zones

From a data perspective, mitigation is managed in a manner similar to exposure zones. Every section can be assigned to a mitigation zone. Mitigation zones are composed of causes and the impact of the mitigation on those causes. The following table represents a sample layout of mitigation zone data.

Cause	Mitigation Percent
Animals	
Squirrels	
Snakes	{Customized
Birds	values}
Raptors	
Nesting	
Weather	
Lightning	
Wind > 25MPH	
Wind < 25 MPH	

You can assign a percentage of mitigation effectiveness to each cause in the zone. Like exposure zones, categories and subcategories for causes are drawn from the ReliCauseMeta.txt file. Generally, the mitigation on most causes will be zero for a particular zone. However, keep in mind that some mitigation efforts can affect multiple causes. For example, squirrels, birds, trees, and other causes may be partially mitigated through tree trimming.

Mitigation zones and failure zones intersect in various ways, since they should share the same cause categories. If not, and mitigation is applied to a cause not in the failure zone, the mitigation has no impact. If the failure zone has a cause that is not mitigated, again there is no impact.

Analysis

Reliability indices are calculated based upon customer exposure to outage, isolation, and repair time. A simple outage process model is used to make use of failure rates and repair times. The process is applied to failures at every device. The aggregation of results from the process results in SynerGEE's reliability indices.

Process

During analysis, faults are simulated at every device. The resulting effects of these faults are combined based on the failure rates of relevant devices. For each fault, steps are taken to:

1. Interrupt the fault with the closest feeding protective device.
2. Isolate the faulted/damaged device with the closest feeding switch or protective device.
3. Pick up customers that were outaged in step 1 and separated from the fault in step 2.
4. Repair the fault.
5. Close the switch or protective device used in step 2.

The following sections present conceptual and mathematical representations of this process.

Examples

As an example, consider the following feeder. The components are designated by letters A through G. There is a feeder, fuse, switch, and four lines.

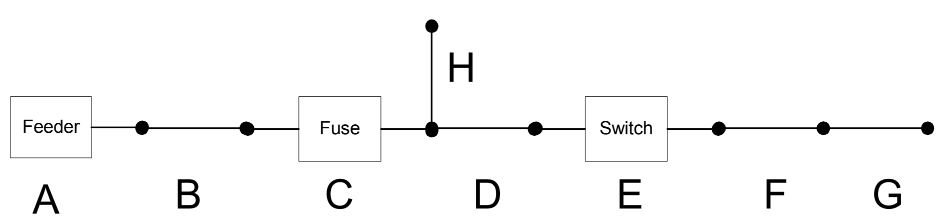


Fig. 18-11 *Simple feeder*

Consider a fault at line F. In this case, the analysis process would assume the following sequence.

1. Fault occurs at F.
2. Fuse at C blows.
3. Customers on D, F, G, and H experience an outage.

4. After a specified sectionalizing interval, switch E is opened to isolate the fault.
5. Fuse C is reset to restore service to customers on D and H.
6. After a repair time based on the line type for F, switch E is closed to restore service to customers on F and G.

Now, consider a fault at D. The process would then assume this sequence.

1. Fault occurs at D.
2. Fuse at C blows.
3. Customers on D, F, G, and H experience an outage.
4. There is no isolating switch, so C remains open throughout the repair time of D.
5. D is repaired and service is restored to customers on D, F, G, and H.

Results

Reliability analysis calculates different levels of SAIFI, SAIDI, and MAIFI values within the analysis set. Since the indices are calculated by-phase, by-phase reclosers or feeders with by-phase reclosing capabilities will have an impact. Also, the variation of customers attached to each phase contributes to by-phase index results.

Results are collected at the following levels.

- **System**—Index values for all feeders together are generated.
- **Feeders and Subtrans**—Indices for each feeder are generated. The feeders are analyzed independently with respect to indices. Switching may be performed between feeders to pick up customers during the analysis.
- **Protection Zone**—Unique zones can be defined by an interrupting/isolation device pair. Every customer directly fed by these two devices will share the same outage characteristics. Indices are generated for these zones.
- **Customer Zone**—If customer classes are assigned to sections, indices are collected for the customer zones.
- **Mitigation Zone**—Index values are reported for the areas receiving mitigation.
- **Section**—Indices are calculated for each section.

Results presentation

You can view results in reports or by special map coloring. The map can be colored based on a wide variety of results, mostly corresponding to data in the reports. There are some values available for coloring that are not found in the reports.

- **Cause In Zone**—Reliability indices calculated using protection zone and only those failures occurring within the protection zone.
- **Percent Cause In Zone**—Percent of index caused by failures within a section's zone.

Charts are also available to summarize and distinguish reliability characteristics.

Trial and study

The SynerGEE Reliability environment is well-equipped for a pattern of multiple trials and searching for improvement. You have a variety of aspects available to help perform a well-rounded study of your system.

- New model saving
- On-screen results and comprehensive reporting
- Scenarios
- Undo/Redo
- Contingency analysis
- Fault analysis
- SynerGEE protection

Assumptions

Outage and failure modeling can be vast and complicated. SynerGEE makes certain assumptions to simplify the problem into a manageable process. These assumptions include the following.

- Reliability analysis isolates all outage devices as if they were faulted. In actuality, some device failures may just result in loss of service to downstream customers and not require the operation of upstream protective devices. SynerGEE always isolates an outage with an upstream protective device.
- SynerGEE assumes that a feeder device is capable of interrupting service even if a breaker is not modeled directly after the feeder. An event on a

device having no protective devices along its feeder path results in an outage of the entire feeder.

No parallel components are considered. A regulator bypass, for example, is not modeled within SynerGEE.

Analysis settings

SynerGEE Reliability has several settings which affect how calculations are performed and how switching is conducted.

- **Calculate confidence intervals**—SynerGEE can run a simulation to generate a population of simulated years. This population can be used to determine the variance in indices and a confidence interval to indicate the range around the mean that each index is expected to lie within 90% of the time.
- **Use mitigation**—If mitigation is set up on the model, it can be globally turned on and off with this option.
- **Allow feeder reclosing**—If this option is selected, momentary faults fed directly by the feeder result in momentary interruptions for all customers on the feeder. If the option is not selected, unprotected momentary faults result in an outage for all feeder customers. Some customers will be picked up with adjacent feeders if possible.
- **By-phase feeder operation**—This option allows feeders to reclose in a by-phase manner on momentary faults.
- **Percent of failures that are single-phase**—Since most faults are single-phase, this value indicates the percent of faults to be treated as single-phase faults. Reclosers and feeders with by-phase operation capabilities can isolate single-phase faults with fewer customers experiencing an outage.
- **Pickup constraint**—This option controls how customers are picked up with adjacent feeders or adjacent feeds during an outage. You can choose to always allow pickup, never allow pickup, or allow pickup within a specified percentage of emergency loading.
- **Allow pickup with protective devices**—With this option, protective devices such as fuses, reclosers, sectionalizers, and breakers are used to pick up customers.
- **Automatic switching time**—As an option, this global value can be used in place of operation times assigned to individual automatic switches. This option can be used to generalize the SCADA control capabilities of automatic switches.

- **Auto-transfer switching time**—This option allows loads tied to auto-transfer switches to see only a momentary interruption with the loss of the primary feed. If this option is not chosen, crew operation times assigned to the switches are used instead.

Analysis options can have a significant impact on simulation results. You should configure them in a manner that matches the characteristics of your system in general. The goal of simulation is to calculate the general trends of system reliability, since each failure in a real system has unique and special characteristics that cannot be precisely captured in a model or simulation.

Isolating/switching/pickup example

The handling of failure contingencies is a critical aspect of reliability analysis. Outage and realistic pickup schemes are evaluated for every possible failure in the SynerGEE model. Consider an outage on section 1 in the following system.

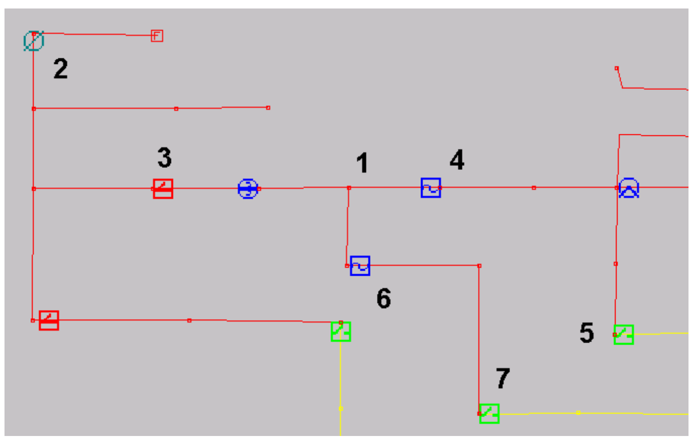


Fig. 18-12 *Failure zones by interrupting device*

Recloser 2 will operate to interrupt the outage resulting in an outage for all customers fed by the recloser. Switch 3 will then be opened to isolate the outage. The recloser will be reset. Fuse 4 will be opened and switch 5 will be closed to pick up those customers. Fuse 6 and switch 7 will be handled in a similar manner.

Thresholds for momentary & disregarded events

If customers can be switched to an alternate feed quickly enough, SynerGEE will consider the outage as momentary or even as disregarded for those customers. Here is a very simple sample model to demonstrate:

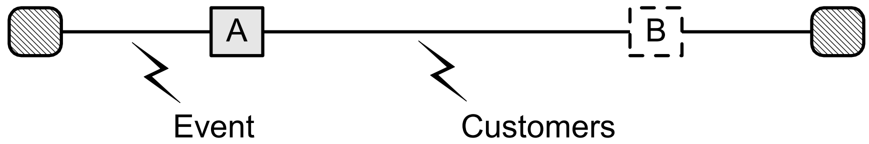


Fig. 18-13 *Customers and outage on very simple circuit*

An outage of :

- under 7 minutes is disregarded
- under 14 minutes is considered a momentary

Failure rates at the event point are:

- Momentary 2 failures / year
- Sustained 1 failure / year

A repair time of 5 hours is assumed.

The transfer time for switches A and B is altered in the table below as well as the ability of the feeder to reclose.

Reclosing	Switch Time	SAIFI	MAIFI	SAIDI
No	8 hours	3.0	0.0	5.0
No	1 hours	3.0	0.0	1.0
Yes	1 hours	1.0	2.0	1.0
Yes	12 minutes	0.0	3.0	0.0
No	12 minutes	0.0	3.0	0.0
Yes	6 minutes	0.0	0.0	0.0
No	6 minutes	0.0	0.0	0.0

Data management

Evaluating reliability involves complex procedures, large data sets, and comprehensive paradigms. Overall, reliability evaluation can be done in three ways.

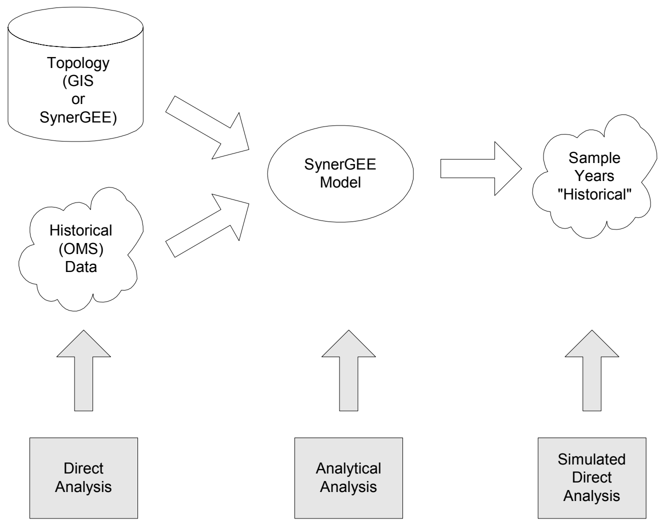


Fig. 18-14 *Three types of distribution reliability analysis*

SynerGEE handles two of the approaches, "analytical analysis" and "simulated direct analysis." The third type, "direct analysis," involves near-raw outage data and specific utility customs. You should have some means of conducting direct analysis, since it is an important part of evaluating system reliability performance as well as expected performance. Often, spreadsheets or database procedures can adequately evaluate reliability performance.

Importing events

Events that resulted in momentary or sustained interruptions can be imported into SynerGEE with a text file. The following information is needed for each event:

- Event location (section or device) or interrupt device that operated
- Year and month of event
- Cause numerical category and subcategory
- Outage duration (hours)
- Outage cost (optional)

Outage events are imported with the Reliability import tool into SynerGEE. SynerGEE can:

- Create exposure zones.
- Filter and load events
- Calculate zone based failure rates and repair times based on events.

Lets look at the particulars of the records in the outage data file.

Outage records

The following record types are used to bring in specific outage information:

- 6050 – Outage occurred on the specified section (or in the same protection zone)
- 6051 – Outage occurred on the specified device
- 6052 – Downstream outage was interrupted by specified device

By default, an outage is considered permanent if its duration is greater than 0.01 hours (3.6 seconds). This cutoff value can be set (in hours) using the 6006 record.

Zone creation

SynerGEE can establish 'exposure zones'. When outages (records 6050-6052) are loaded into SynerGEE, they are combined with other events in the same 'exposure zone' to generate correlated failure rates and repair times for the entire zone.

If zones are created based on interrupt device then any events downstream from a particular device and upstream from any other protective device will be used to calculate the cumulative failure rate for that zone. If data of a desired confidence is available at that level of granularity then this approach might be useful.

Some utilities track events by conductor (particularly cable) type. In that case, SynerGEE could be used to generate zones by conductor type. All events, for example, associated with 500 MCM cable could then be correlated to generate a failure rate for this type of conductor.

SynerGEE can create zones in one of four ways:

- 6030 – Create zones by interrupt device. A selection of fuse saving vs. fuse blowing can be made.
- 6031 – Zones are created by type of conductor.
- 6032 – Zones are created by number of conductors. Up to three zones will be created
- 6033 – Zones are created by feeder. One zone for each feeder is established.

If a zone creation record is not included in the MiddleLink file, SynerGEE will not create zones and outage events will be loaded onto any existing zones.

Filtering of Events

Outage events utilized by SynerGEE can be limited with 'filter records' in the MiddleLink file.

- 6003 – Ignore all events of specified category and subcategory. Specifying a subcategory of –1 will result in the exclusion of all records of the given category.
- 6004 – Ignore all events before specified year.
- 6005 – Ignore all events after specified year.

A discussion of the data format used by the SynerGEE outage importer is given later.

Other Settings

These records can be used to setup the import of outage events:

- 6001 – Number of years in study. Default is 1 if this record not supplied.
- 6002 – Prefix for creation of new zones. If this record is not supplied then SynerGEE will use a prefix based on the type of zone creation. This may be of use if outage data is being applied to different combinations of feeders.
- 6006 – Cutoff (in hours) for momentary / sustained outage. The default for this value is 0.01 hours. Any event with an outage duration less than the cutoff will be considered a momentary event and contribute only to the temporary outage failure rate.

Dates

The SynerGEE Reliability import tool uses a double value to specify the year. The decimal portion of the value is for the month and day within the given year. Data can be established and filters can be applied based on just years like 2003.

Outages by month can be managed with the decimal of the year number representing the month in twelfths.

Month	Value for year 2003
January	2003.000
February	2003.083
March	2003.167
April	2003.250

May	2003.333
June	2003.417
July	2003.500
August	2003.583
September	2003.667
October	2003.750
November	2003.833
December	2003.917

A seven hour outage occurring on section '21365' in June of 2003 may be represented with:

```
6050,21365,L,2003.417,40,6,7,10000
```

All events after August of 2003 can be ignored with:

```
6005,2003.583
```

This decimal approach to representing months can be extended to days if that type of granularity is desired.

Notable

SynerGEE does not necessarily read MiddleLink files sequentially. Event records, filter records, and settings records can be placed in any order with the MiddleLink file. SynerGEE will look for and retrieve the necessary records from the file when they are needed.

There is no limit to the number of event records that can be loaded by SynerGEE. Only the first fifty '6003', category based filter records, will be handled by SynerGEE.

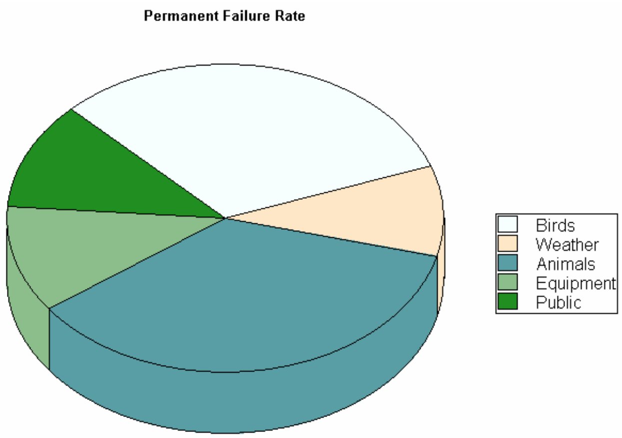
All events that can be associated with the model in memory are read by SynerGEE. There can be events in the file that do not correspond to the model.

SynerGEE creates zones for and associated events to just the feeders selected for analysis. Different combinations of feeders can therefore be selected for outage data import.

Output

Successful outage data importing results in a report with several tables and charts describing the zones, categories, and zone assignments. Failure rates are listed in failures / year and failures / year / mile based on the context of the table or chart.

Here is a chart showing failure rate by category for a set of feeders after an outage data import:



The same report also contains a table breaking down failures by category and subcategory:

Category	Subcategory	# Zones	Miles	Customers	Sus. FR (Total)	Mom.
Animals	Squirrels	5	8.60	1282	1.40	
Animals	Unknown	10	15.60	1972	0.40	
Animals	Other	2	6.00	1037	0.60	
Animals	Rodent	5	5.50	583	1.00	
Animals	Raccoons	2	0.80	91	0.40	
Birds	Migratory	7	16.80	2533	1.40	
Birds	Nesting	5	6.40	367	1.20	
Birds	Other	2	3.10	119	0.40	
Birds	Woodpecker	2	1.80	117	0.40	
Equipment	Arrester	1	0.70	0	0.20	
Equipment	Elbow	1	2.50	181	0.20	
Equipment	Cable	2	4.50	341	0.60	
Equipment	Splice	1	1.70	125	0.20	
Public	Other	1	1.00	66	0.20	
Public	Vehicle	1	0.80	78	0.20	
Public	Fire	1	0.40	55	0.20	
Public	Vandalism	1	0.40	51	0.40	
Public	Overhead Contact	1	2.30	41	0.20	
Weather	Wind <25 mph	2	5.40	637	0.20	
Weather	Ice/Snow	2	1.10	0	0.40	
Weather	Storm	4	6.70	325	0.40	
Weather	Unknown	1	2.30	41	0.00	

This table shows the number of zones impacted by each category as well as the number of miles and customers.

Example MiddleLink file

```
' ** Outage data from L. Trussell - 10/03
```

```
' ** I've got 5 years worth of data in this file

6001, 5
' ** We'll create zones based on interrupt devices
6030, 0
'6031   (Unused records can be commented out with ')
'6032
'6033

6002, MyPrefix_
' ** For next load, ignore everything before Feb, 2001
6004, 2001.1
' ** Ignore everything after April, 2003
6005, 2003.3

' ** Ignore records for the following cat / subcat
6003,40,6
6003,60,3

' ** Ignore all of category 15
6003,15,-1

' ** List outage data
6050,21365,L,2001.4,40,6,7,10000
6050,21364,L,2002.4,40,6,7,10000
6050,21364,L,2003.4,40,4,6,8500
6050,21365,L,2003.4,40,8,8,10000
6050,21365,L,2002.4,60,2,6,9000
6050,50471,L,2001,15,5,0,0
6050,21364,L,2000.4,60,2,6,9000
6051,21101,Recloser,2002.4,0,0,8,40000
6051,22125,Fuse,2002.4,0,0,6,2000
6050,21365,L,2002.4,60,3,5,5000
6050,21364,L,2001.5,60,3,6,8500
6050,52224,L,2002.4,10,6,0,0
6050,21365,L,2000.4,60,3,5,5000
6053,21365,Fuse,2002.4,40,2,6,12000
6050,21364,L,2002.4,40,5,7,8500
6050,51981,L,2000.6,20,2,8,3500
6050,50471,L,2002.4,15,5,0,0
6050,51928,L,2002.4,20,5,5,1000
6050,51915,L,2003.4,40,3,7,10000
6050,51941,L,2000.4,40,3,7,6000
6050,52534,L,2002,15,2,6,1000
6050,52534,L,2002.4,15,2,5,1000
6050,52534,L,2000.4,15,2,7,2000
6050,50471,L,2002.4,15,5,0,0
6050,52540,L,2003,30,1,5,1000
6050,52540,L,2000.4,30,1,6,1000
6050,52534,L,2002.4,40,3,0,0
6050,52540,L,2002.4,30,4,7,2500
6050,51697,L,2001.6,15,1,5,1000
```

etc…

Data calibration

Unlike typical model data, calibration of reliability data requires a more insightful approach. Your model was probably constructed with a focus on more consistent features, such as lines and equipment, which allow a precise calibration of parameters such as voltage and flow. Calibration of models using historical outage data is more difficult, since failures are random events. Even under the same conditions, outages in a distribution system in any given year are different. As such, failure rate calibration must be done with consideration of the known system history so that predicted trends reflect the performance of the actual system.

Historical data calibration

In many cases, existing historical outage data may be incomplete or not fully compatible with your GIS or SynerGEE model. For example, call records typically do not fully capture the interruption extent of an event. Customers may be away from home or asleep when an interruption occurs. Customers may not be inclined to phone when their service is interrupted. Phasing information may be inaccurate or other data issues may be present.

Therefore, historical data calibration involves validating recorded outage information against a reliable network model. As an example, assume that you have recorded outage information for a recloser lockout event. The outage record indicates that 120 customers were out of service. However, your GIS model shows that the recloser serves 155 customers. Therefore, you should update your outage record to reflect 155 affected customers for reliability analysis. This type of validation should be applied to any applicable correlations between outage and network information.

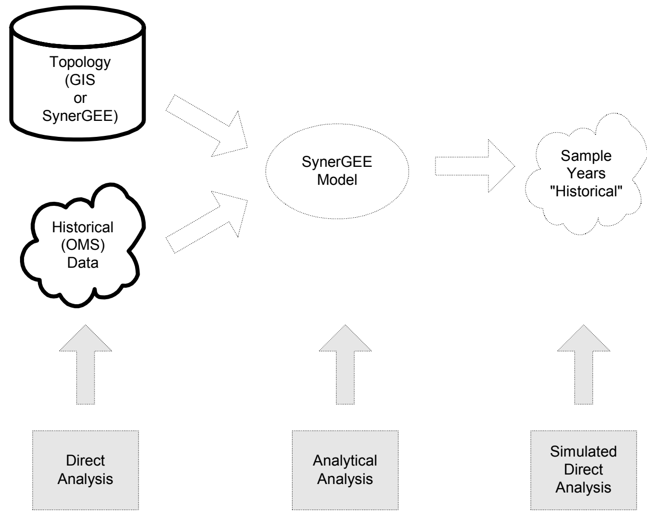


Fig. 18-15 *Historical data calibration*

Analytical model calibration

This type of calibration involves running reliability studies within the analytical engine and comparing the results with known indices calculated from quality outage data.

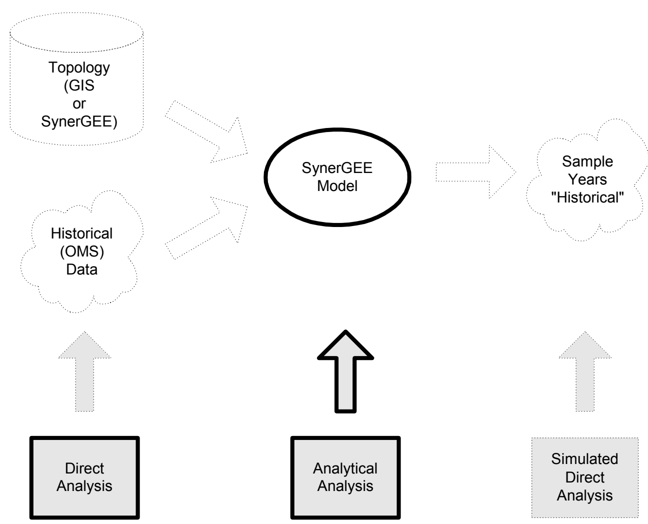


Fig. 18-16 *Analytical data calibration*

Failure rates, repair times, and other information should be adjusted in the model until analysis results are consistent with historical values.

Variation analysis

You should give consideration to the variation or variance in indices predicted by reliability simulation. An extremely low variance might indicate that data is unrealistic or that configuration options are limited in the feeders being analyzed. An extremely large variance might indicate poor failure data or unresolved correlations in failure data.

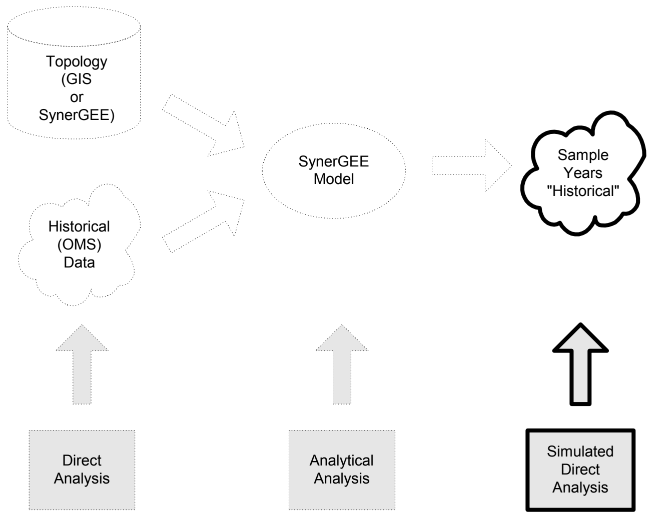


Fig. 18-17 *Variation data calibration*

It may be a challenge to identify and resolve problems with large variance values.

Example calculations

This section reviews the basic concepts of reliability. In this section, the following is assumed.

- No pickup switching
- Balanced modeling
- No automatic switching
- Non-specific failure causes

The approach to analytical calculation of reliability indices is to find expressions for some base factors, and then to use those factors to calculate the indices.

Variable definitions

The following variables are used in the equations.

λ_i – Failure rate of series device i (failures / year) **(18-19)**
χ_i – Customers served through device i
r_i – Repair time for device i (hours)
η_i – Event (fault) at device i
Ψ_i – Set of devices along the source path of device i
Z_i – Set of devices in same protection zone(s) as devices in Ψ_i
Γ_i – Set of devices fed by device i
Isd(i) - Nearest upstream isolating device for device i
Pd(i) - Nearest upstream interrupting device for device i
t_{sctz} – Sectionalizing time to isolate a fault and reset protective device

Failure probability

The probability of a device experiencing an outage during a year is found with the failure rate.

$$P(\eta_i) = \lambda_i \qquad \textbf{(18-20)}$$

You would like to find the probability of a failure anywhere on a feeder. The general probability addition rule states:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad \textbf{(18-21)}$$

Since you are not considering multiple events within an outage interval, you can eliminate the last term in the above rule and apply it to all devices in a model to get:

$$\lambda_{Fdr} = \sum_{\Gamma_{Fdr}} \lambda_i \qquad \textbf{(18-22)}$$

Base factor calculations

Total number of feeder customers

The feeder customer count is the number of customers fed by the feeder device. Therefore:

$$FeederCustomers = \chi_{Fdr} \qquad \textbf{(18-23)}$$

Annual customer interruptions (ACI)

Indices like SAIFI and CAIFI use the number of customer interruptions per year for a feeder. You can construct an expression for ACI by considering the event process model discussed earlier. If device "i" has a failure, the nearest upstream protective device, $Pd(i)$, will operate. All customers fed by that protective device experience an outage. Those customers are represented as $\chi_{Pd(i)}$. You can find the exposure of those customers to interruption due to failure on "i" through the use of the failure rate of the failing device, λ_i. The customer exposure to interruption can be summed for failures on all devices in the feeder.

$$ACI_{Fdr} = \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Pd(i)} \tag{18-24}$$

Customer interruption duration (CID)

This factor accounts for the total interruption time for all customers on a feeder for a year. The factor is used in SAIDI and CAIDI calculations.

Interruption duration values for a customer can be repair duration times or sectionalizing duration times based on the relationship of the customer to the failed device, "i," the nearest isolating device to "i," $\chi_{Isd(i)}$, and the nearest protecting device to "i," $\chi_{Pd(i)}$.

Customers experience repair duration times if they are fed by $\chi_{Isd(i)}$ because according to the failure process model, their service cannot be restored until the failed device is repaired. So:

$$\text{Total of Repair Durations} = \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Isd(i)} r_i \tag{18-25}$$

Customers experience sectionalizing duration times if they are located between the protective device that blows for a failed device and the isolating device that isolates it. Those customers will be out of service for the sectionalizing time.

$$\text{Total of Sectionalizing Durations} = \sum_{\Gamma_{Fdr}} \lambda_i \left(\chi_{Pd(i)} - \chi_{Isd(i)} \right) t_{sctz} \tag{18-26}$$

You can combine these totals to get the customer interruption duration for a feeder.

$$\text{CID} = \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Isd(i)} r_i + \sum_{\Gamma_{Fdr}} \lambda_i \left(\chi_{Pd(i)} - \chi_{Isd(i)} \right) t_{sctz} \tag{18-27}$$

Annual customers affected (ACA)

This factor is a count of the expected number of customers who experience an outage during a year. It is used in finding the CAIFI index. To find it, you must determine the likelihood that a customer will experience an outage in a year. If that likelihood is greater than (or equal to) one, you count the customer's single outage. If the likelihood is less than one, you add the customer's exposure to the cumulative total.

To determine the likelihood that a customer will experience an outage, go to each customer in the feeder and find all other devices in its protection "zone." The term "zone" is used loosely. The desire is to find all devices that by failing would cause their protecting device to isolate the customer. Devices in the zone for customer "i" are noted as Z_i. You can find Z_i by tracing a path from customer "i" to the feeder device. Add the devices in that set to Z_i. Next, find all devices fed by a device in Z_i but not through a protective device. Add these new devices to Z_i.

Therefore, the failure of any device in Z_i would directly or indirectly cause an outage for the customer, through protection operation.

The likelihood of a failure in Z_i is:

$$\lambda_Z = \sum_Z \lambda_i \tag{18-28}$$

And you can express ACA as:

$$ACA_{Fdr} = \sum_{i=\Gamma_{Fdr}} c_i * \begin{Bmatrix} \sum_{j=Z_i} \lambda_j \text{ for } \sum_{j=Z_i} \lambda_j < 1.0 \\ 1.0 \text{ for } \sum_{j=Z_i} \lambda_j \geq 1.0 \end{Bmatrix} \tag{18-29}$$

Index calculation

With the base factors, you can calculate the indices.

$$SAIFI_{Fdr} = \frac{ACI_{Fdr}}{NumberCustomers} \tag{18-30}$$

$$SAIDI_{Fdr} = \frac{CID_{Fdr}}{NumberCustomers}$$

$$CAIDI_{Fdr} = \frac{CID_{Fdr}}{ACI_{Fdr}}$$

$$ASAI_{Fdr} = \frac{8760 * NumberCustomers - CID_{Fdr}}{8760 * NumberCustomers}$$

$$CAIFI_{Fdr} = \frac{ACI_{Fdr}}{ACA_{Fdr}}$$

The following relationship also exists between the indices.

$$CAIDI = \frac{SAIDI}{SAIFI} \tag{18-31}$$

Example 1

This example demonstrates the calculation of reliability indices for the following simple feeder, starting with the base factors.

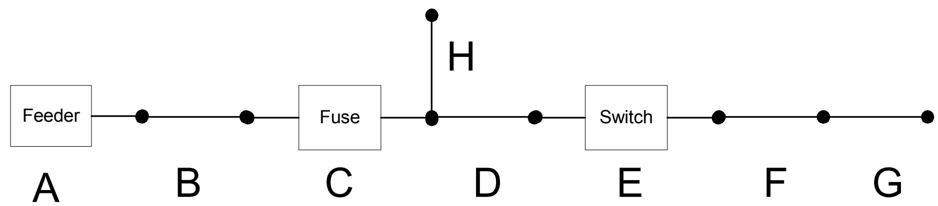


Fig. 18-18 *Simple feeder*

The following data is relevant.

Device	Connected Customers	Failure Rate (f/yr)	Trees (f/yr)	Animals (f/yr)	Lghtng (f/yr)	Repair Time (hr)
B	30	0.2	0.05	0.1	0.05	3.0
C	0	0.05	0	0	0.05	2.0
D	40	0.2	0.1	0.1	0	3.0
E	0	0.01	0	0	0.01	4.0
F	20	0.1	0.05	0.05	0	6.0
G	35	0.1	0.05	0.05	0	6.0
H	50	0.1	0	0.05	0.05	6.0
Sectionalizing Time						0.5 hr

Feeder or system level calculations

Annual customer interruptions

Applying the general equation:

$$ACI_{Fdr} = \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Pd(i)} \tag{18-32}$$

$$\begin{aligned}
&= 0.2*(30+40+20+35+50)+\\
&\ 0.05*(40+20+35+50)+\\
&\ 0.2*(40+20+35+50)+\\
&\ 0.1*(40+20+35+50)+\\
&\ 0.01*(40+20+35+50)+\\
&\ 0.1*(40+20+35+50)+\\
&\ 0.1*(40+20+35+50)\\
&= 116.2 \text{ interruptions / yr}
\end{aligned}$$

Customer interruption duration

Again, viewing the feeder topology and applying the general equation:

(18-33)

$$
\begin{aligned}
CID_{Fdr} &= \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Isd(i)} r_i + \lambda_i \left(\chi_{Pd(i)} - \chi_{Isd(i)} \right) t_{sctz} \\
&= 0.2*(30+50+40+20+35)*3.0+ \\
&\quad 0.05*(50+40+20+35)*2.0+ \\
&\quad 0.2*(50+40+20+35)*3.0+ \\
&\quad 0.1*(50+40+20+35)*6.0+ \\
&\quad 0.01*(20+35)*4.0+ \\
&\quad 0.01*(40+50)*0.5+ \\
&\quad 0.1*(20+35)*6.0+ \\
&\quad 0.1*(40+50)*0.5+ \\
&\quad 0.1*(20+35)*6.0+ \\
&\quad 0.1*(40+50)*0.5 \\
&= 371.15 \text{ interruption - hr / yr}
\end{aligned}
$$

Annual customers affected

The final base factor is found with:

(18-34)

$$
\begin{aligned}
ACA_{Fdr} &= \sum_{i=\Gamma_{Fdr}} c_i * \begin{cases} \sum_{j=Z_i} \lambda_j \text{ for } \sum_{j=Z_i} \lambda_j < 1.0 \\ 1.0 \text{ for } \sum_{j=Z_i} \lambda_j \geq 1.0 \end{cases} \\
&= 30*(0.2)+ \\
&\quad 40*(0.2+0.05+0.2+.01+.1+.1+.1)+ \\
&\quad 20*(0.2+0.05+0.2+.01+.1+.1+.1)+ \\
&\quad 35*(0.2+0.05+0.2+.01+.1+.1+.1)+ \\
&\quad 50*(0.1+0.2+0.05) \\
&= 95.7 \text{ affected customers / yr}
\end{aligned}
$$

Reliability indices

You can now calculate indices:

$$SAIFI_{Fdr} = \frac{ACI_{Fdr}}{NumberCustomers} = \frac{116.2}{175} = 0.664$$ **(18-35)**

$$SAIDI_{Fdr} = \frac{CID_{Fdr}}{NumberCustomers} = \frac{371.15}{175} = 2.120$$

$$CAIDI_{Fdr} = \frac{CID_{Fdr}}{ACI_{Fdr}} = \frac{371.15}{116.2} = 3.194$$

$$ASAI_{Fdr} = \frac{8760 * NumberCustomers - CID_{Fdr}}{8760 * NumberCustomers}$$

$$= \frac{8760*175 - 371.15}{8760*175} = 0.99976$$

$$CAIFI_{Fdr} = \frac{ACI_{Fdr}}{ACA_{Fdr}} = \frac{116.2}{95.7} = 1.214$$

Device indices

You can also look at reliability from a section and device level.

CID Calculations

The calculations for the device level CID values have already been done.

(18-36)

$$\begin{aligned}
CID_B &= 0.2*(30+50+40+20+35)*3.0 = 105 \\
CID_C &= 0.05*(50+40+20+35)*2.0 = 14.5 \\
CID_D &= 0.2*(50+40+20+35)*3.0 = 87 \\
CID_E &= 0.01*(20+35)*4.0 + 0.01*(40+50)*0.5 = 2.65 \\
CID_F &= 0.1*(20+35)*6.0 + 0.1*(40+50)*0.5 = 37.5 \\
CID_G &= 0.1*(20+35)*6.0 + 0.1*(40+50)*0.5 = 37.5 \\
CID_H &= 0.1*(50+40+20+35)*6.0 = 87 \ int\text{-}hr/yr
\end{aligned}$$

ACI Calculations

Annual customer interruptions for failures on each device are calculated as:

(18-37)

$$\begin{aligned}
ACI_B &= 0.2*(30+40+20+35+50) = 35 \\
ACI_C &= 0.05*(40+20+35+50) = 7.25 \\
ACI_D &= 0.2*(40+20+35+50) = 29 \\
ACI_E &= 0.01*(40+20+35+50) = 1.45 \\
ACI_F &= 0.1*(40+20+35+50) = 14.5 \\
ACI_G &= 0.1*(40+20+35+50) = 14.5 \\
ACI_H &= 0.1*(40+20+35+50) = 14.5 \\
\sum ACI_j &= 116.2 \text{ interruptions / yr}
\end{aligned}$$

SAIDI

Now you can determine SAIDI values by dividing the CID values by the total number of customers.

$$\begin{aligned}
&SAIDI_B = 105/175 = 0.600\\
&SAIDI_C = 14.5/175 = 0.083\\
&SAIDI_D = 87/175 = 0.497\\
&SAIDI_E = 2.65/175 = 0.015\\
&SAIDI_F = 37.5/175 = 0.214\\
&SAIDI_G = 37.5/175 = 0.214\\
&SAIDI_H = 87/175 = 0.497\\
&\sum SAIDI_j = 2.12 \text{ hours}
\end{aligned} \tag{18-38}$$

The device level values for SAIDI allow you to see the sections and devices that contribute the most to customer outage hours. Failures on sections above the switch (E) cause the most problems. Sections B, D, and H have the largest impact on SAIDI. Notice that section H contributes twice as many outage hours to SAIDI as do F and G even though all three sections have the same failure rates and repair times.

SAIFI

Annual outage frequency due to each device can now be calculated with ACI values and customer totals.

$$\begin{aligned}
&SAIFI_B = 35/175 = 0.2\\
&SAIFI_C = 7.25/175 = 0.041\\
&SAIFI_D = 29/175 = 0.165\\
&SAIFI_E = 1.45/175 = 0.008\\
&SAIFI_F = 14.5/175 = 0.083\\
&SAIFI_G = 14.5/175 = 0.083\\
&SAIFI_H = 14.5/175 = 0.083
\end{aligned} \tag{18-39}$$

Root cause

Next, you can analyze the feeder with the cause-based failure rates. Assume that the repair time is the same for all failure types. The failure causes are trees, animals, and lightning.

CID

First, calculate the CID values. For trees:

$$
\begin{aligned}
CID_T &= 0.05*(30+50+40+20+35)*3.0+ \\
&0*(50+40+20+35)*2.0+ \\
&0.1*(50+40+20+35)*3.0+ \\
&0*(20+35)*4.0+0*(40+50)*0.5+ \\
&0.05*(20+35)*6.0+0.05*(40+50)*0.5+ \\
&0.05*(20+35)*6.0+0.05*(40+50)*0.5+ \\
&0*(50+40+20+35)*6.0 \\
&=107.25
\end{aligned} \tag{18-40}
$$

For animals:

$$
\begin{aligned}
CID_A &= 0.1*(30+50+40+20+35)*3.0+ \\
&0*(50+40+20+35)*2.0+ \\
&0.1*(50+40+20+35)*3.0+ \\
&0*(20+35)*4.0+0*(40+50)*0.5+ \\
&0.05*(20+35)*6.0+0.05*(40+50)*0.5+ \\
&0.05*(20+35)*6.0+0.05*(40+50)*0.5+ \\
&0.05*(50+40+20+35)*6.0 \\
&=177
\end{aligned} \tag{18-41}
$$

For lightning:

$$
\begin{aligned}
CID_L &= 0.05*(30+50+40+20+35)*3.0+ \\
&0.05*(50+40+20+35)*2.0+ \\
&0*(50+40+20+35)*3.0+ \\
&0.01*(20+35)*4.0+0.01*(40+50)*0.5+ \\
&0*(20+35)*6.0+0*(40+50)*0.5+ \\
&0*(20+35)*6.0+0*(40+50)*0.5+ \\
&0.05*(50+40+20+35)*6.0 \\
&=86.9
\end{aligned} \tag{18-42}
$$

ACI Calculations

Annual customer interruptions for failures due to trees are:

$$\begin{aligned} ACI_T &= 0.05*(30+40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0.1*(40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0.05*(40+20+35+50)+ \\ &0.05*(40+20+35+50)+ \\ &0*(40+20+35+50) \\ &= 37.75 \end{aligned} \tag{18-43}$$

Due to animals:

$$\begin{aligned} ACI_A &= 0.1*(30+40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0.1*(40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0.05*(40+20+35+50)+ \\ &0.05*(40+20+35+50)+ \\ &0.05*(40+20+35+50) \\ &= 53.75 \end{aligned} \tag{18-44}$$

Due to lightning:

$$\begin{aligned} ACI_L &= 0.05*(30+40+20+35+50)+ \\ &0.05*(40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0.01*(40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0*(40+20+35+50)+ \\ &0.05*(40+20+35+50) \\ &= 24.7 \end{aligned} \tag{18-45}$$

SAIDI

Next, you can determine SAIDI contributions from each failure cause.

$$\begin{aligned} SAIDI_T &= 107.25/175 = 0.613 \\ SAIDI_A &= 177/175 = 1.011 \\ SAIDI_L &= 86.9/175 = 0.497 \\ \sum SAIDI_j &= 2.12 \text{ hours} \end{aligned} \tag{18-46}$$

It appears that mitigation of animal-caused faults may substantially benefit the reliability of this feeder.

SAIFI

Next, you can determine SAIFI contributions from each failure cause:

$$
\begin{aligned}
SAIFI_T &= 37.75/175 = 0.216 \\
SAIFI_A &= 53.75/175 = 0.307 \\
SAIFI_L &= 24.7/175 = 0.141 \\
\sum SAIFI_j &= 0.664 \text{ hours}
\end{aligned}
\tag{18-47}
$$

Again, it appears that mitigation of animal-caused faults may substantially benefit the reliability of this feeder.

CAIDI

You can determine CAIDI contributions from each failure cause:

$$
\begin{aligned}
CAIDI_T &= 107.25/37.75 = 2.84 \\
CAIDI_A &= 177/53.75 = 3.29 \\
CAIDI_L &= 86.9/24.7 = 3.51
\end{aligned}
\tag{18-48}
$$

Example 2

The following is a feeder model from SynerGEE.

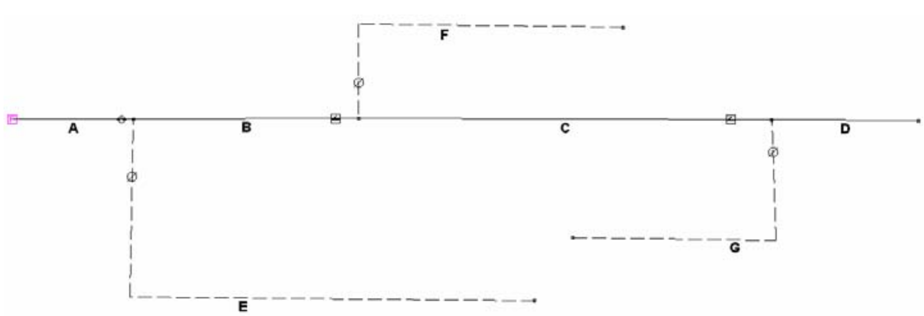


Fig. 18-19 *Simple feeder*

In this example, the possibility of failure for the breakers, reclosers, and switches is ignored. Only line outages are considered. Also, section A is a zero-length section used to provide a clearer diagram. Failures on A are ignored.

The following data is relevant.

Device	Connected Customers	Failure Rate (f/yr)	Repair Time (hr)
B	0	0.2	3.0
C	0	0.3	3.0
D	0	0.1	3.0
E	250	0.75	1.0
F	100	0.5	1.0
G	50	0.25	1.0
Sectionalizing Time			0.5 hr

Base factors

Annual customer interruptions

You can apply the general equation:

(18-49)

$$\begin{aligned} ACI_{Fdr} &= \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Pd(i)} \\ &= 0.2*(250+100+50)+ \\ &\ \ 0.3*(250+100+50)+ \\ &\ \ 0.1*(250+100+50)+ \\ &\ \ 0.75*250+ \\ &\ \ 0.5*100+ \\ &\ \ 0.25*50 \\ &= 490 \text{ interruptions / yr} \end{aligned}$$

Customer interruption duration

Again, applying the general equation:

$$CID_{Fdr} = \sum_{\Gamma_{Fdr}} \lambda_i \chi_{Isd(i)} r_i + \lambda_i \left(\chi_{Pd(i)} - \chi_{Isd(i)} \right) t_{sctz} \tag{18-50}$$

$$\begin{aligned}
&= 0.2 * (250 + 100 + 50) * 3.0 + \\
&\quad 0.3 * 250 * 0.5 + \\
&\quad 0.3 * (100 + 50) * 3.0 + \\
&\quad 0.1 * (250 + 100) * 0.5 + \\
&\quad 0.1 * 50 * 3.0 + \\
&\quad 0.75 * 250 * 1.0 + \\
&\quad 0.5 * 100 * 1.0 + \\
&\quad 0.25 * 50 * 1.0 \\
&= 695 \text{ interruption - hr / yr}
\end{aligned}$$

Annual customers affected

The final base factor is found with:

$$ACA_{Fdr} = \sum_{i=\Gamma_{Fdr}} c_i * \left\{ \begin{array}{c} \sum_{j=Z_i} \lambda_j \text{ for } \sum_{j=Z_i} \lambda_j < 1.0 \\ 1.0 \text{ for } \sum_{j=Z_i} \lambda_j \geq 1.0 \end{array} \right\} \tag{18-51}$$

$$\begin{aligned}
&= 100 * (0.5 + 0.2 + 0.3 + 0.1) + \text{ use 100*1.0} \\
&\quad 250 * (0.75 + 0.2 + 0.3 + 0.1) + \text{ use 250*1.0} \\
&\quad 50 * (0.25 + 0.1 + 0.3 + 0.2) \\
&= 392.5 \text{ affected customers / yr}
\end{aligned}$$

Reliability indices

You can now calculate the indices:

$$\begin{aligned}
SAIFI_{Fdr} &= \frac{ACI_{Fdr}}{NumberCustomers} = \frac{490}{400} = 1.225 \\
SAIDI_{Fdr} &= \frac{CID_{Fdr}}{NumberCustomers} = \frac{695}{400} = 1.738 \\
CAIDI_{Fdr} &= \frac{CID_{Fdr}}{ACI_{Fdr}} = \frac{695}{490} = 1.418 \\
ASAI_{Fdr} &= \frac{8760 * NumberCustomers - CID_{Fdr}}{8760 * NumberCustomers} \\
&= \frac{8760 * 400 - 695}{8760 * 400} = 0.99980 \\
CAIFI_{Fdr} &= \frac{ACI_{Fdr}}{ACA_{Fdr}} = \frac{490}{392.5} = 1.248
\end{aligned} \tag{18-52}$$

Example 3

Here is an example feeder with a backup:

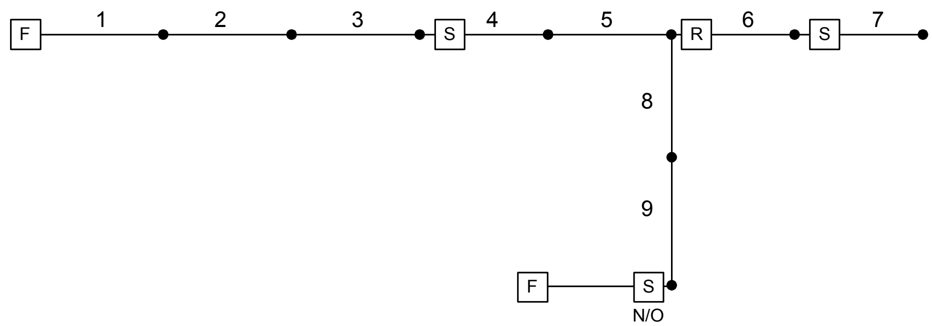


Fig. 18-20 *Feeder with backup*

Here is failure data for various conductor types:

Conductor	Failure Rate
336 ACSR	0.1 f/yr/mi
PECN	0.404 f/yr/mi
XLPEJ	0.051 f/yr/mi

We will assume that switches and reclosers in this model do not fail. The crew reset time for the switches is 1 hour.

The table below lists section data:

#	Conductor feet 336 ACSR	PECN	XLPEJ	Repair Time Hours	Cust
1	9,240			3.5	80
2		4,555		3.5	80
3			496	3.5	88
4	755			3.5	300
5			849	3.5	210
6	3,220			3.5	656
7			598	2.5	328
8	6,682				100
9			3,607	2.5	99

We can generate a reduced model by grouping sections, or outage zones, that will respond similarly to outages.

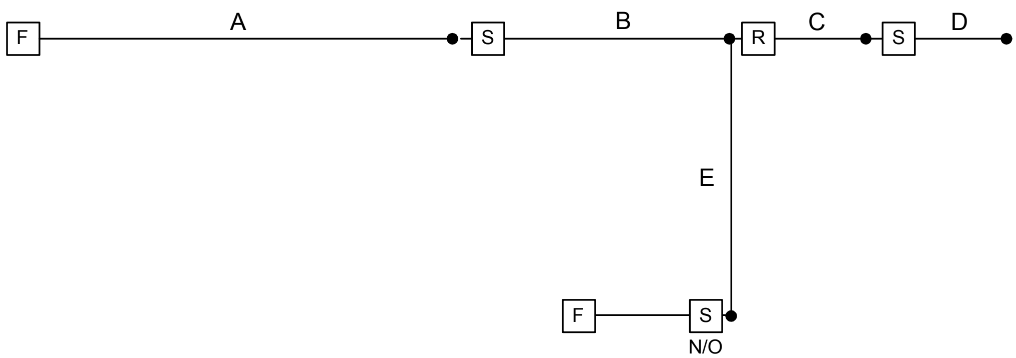


***Fig. 18-21** Reduced feeder with backup*

We can now calculate failure rates and customer counts within these outage zones:

Zone	Failure Rate	Repair	Cust
A	$\lambda_A = \frac{(9240)(0.1)+(4555)(0.404)+(496)(0.051)}{5280} = 0.5283$	3.5 h	248
B	$\lambda_B = \frac{(755)(0.1)+(849)(0.051)}{5280} = 0.0225$	3.5 h	510
C	$\lambda_C = \frac{(3220)(0.1)}{5280} = 0.0610$	3.5 h	656
D	$\lambda_D = \frac{(598)(0.051)}{5280} = 0.00578$	3.5 h	328
E	$\lambda_E = \frac{(6682)(0.1)+(3607)(0.051)}{5280} = 0.1614$	2.5 h	199

We can calculate SAIFI contributions due to failures in each outage zone as follows:

$$SAIFI_A = \frac{\lambda_A(C_A+C_B+C_C+C_D+C_E)}{(C_A+C_B+C_C+C_D+C_E)} = 0.5283 \tag{18-53}$$

$$SAIFI_B = \frac{\lambda_B(C_A+C_B+C_C+C_D+C_E)}{(C_A+C_B+C_C+C_D+C_E)} = 0.0225 \tag{18-54}$$

$$SAIFI_C = \frac{\lambda_C(C_C+C_D)}{(C_A+C_B+C_C+C_D+C_E)} = 0.0309 \tag{18-55}$$

$$SAIFI_D = \frac{\lambda_D(C_C+C_D)}{(C_A+C_B+C_C+C_D+C_E)} = 0.00293 \tag{18-56}$$

$$SAIFI_E = \frac{\lambda_E(C_A + C_B + C_C + C_D + C_E)}{(C_A + C_B + C_C + C_D + C_E)} = 0.1614 \tag{18-57}$$

The SAIFI for the system is the sum of the outage zone SAIFI values:

$$SAIFI = \sum SAIFI_Z = \tag{18-58}$$

Now we can evaluate SAIDI by outage zone in a similar manner. When a failure occurs in zone 'A', the switch on section 'B' and the n/o switch on the feeder tie can be used to pick up customers. The one hour switching delay on each switch will be accumulated to the outage time of the picked up customers. So our SAIDI is:

$$SAIDI_A = \frac{\lambda_A\left[(C_A)(R_A) + (2T_S)(C_B + C_C + C_D + C_E)\right]}{(C_A + C_B + C_C + C_D + C_E)} = 1.158 \tag{18-59}$$

If a failure occurs on section 'B', section 'A' can be isolated after a 1 hour delay:

$$SAIDI_B = \frac{\lambda_B\left[(C_A)(T_S) + (R_B)(C_B + C_C + C_D + C_E)\right]}{(C_A + C_B + C_C + C_D + C_E)} = 0.0716 \tag{18-60}$$

A failure on section 'C' is isolated by the recloser:

$$SAIDI_C = \frac{\lambda_C(R_C)(C_C + C_D)}{(C_A + C_B + C_C + C_D + C_E)} = 0.1082 \tag{18-61}$$

The switch on section 'D' can be used to isolate a failure on that section:

$$SAIDI_D = \frac{\lambda_D\left[T_S C_C + R_C C_D\right]}{(C_A + C_B + C_C + C_D + C_E)} = 0.0054 \tag{18-62}$$

A failure on 'E' behaves like the failure on 'B':

$$SAIDI_E = \frac{\lambda_E\left[(C_A)(T_S) + (R_E)(C_B + C_C + C_D + C_E)\right]}{(C_A + C_B + C_C + C_D + C_E)} = 0.372 \tag{18-63}$$

The SAIDI for the system is the sum of the outage zone SAIDI values:

$$SAIDI = \sum SAIDI_Z = 1.715 \quad h/yr \tag{18-64}$$

Example 4

This example contains two feeders, an automatic transfer switch, and a normally open tie switch:

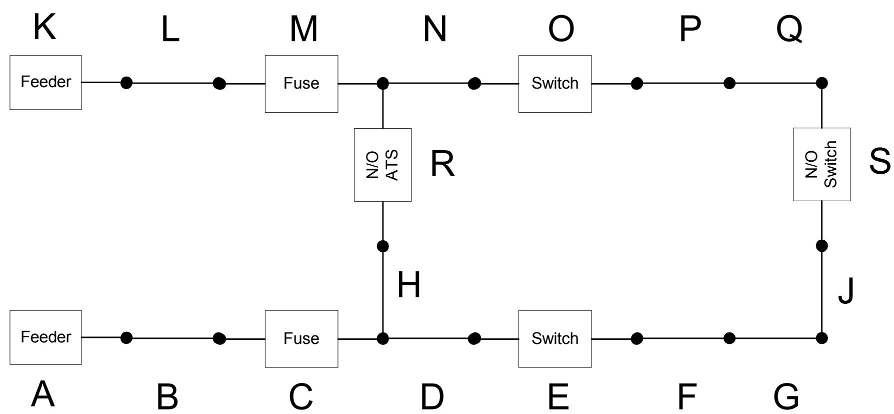


Fig. 18-22 *More complex feeder*

Here is some relevant data:

Device	Connected Customers	Failure Rate (f/yr)	Trees (f/yr)	Animals (f/yr)	Lghtng (f/yr)	Repair Time (hr)
B	30	0.2	0.05	0.1	0.05	3.0
C	0	0.05	0	0	0.05	2.0
D	40	0.2	0.1	0.1	0	3.0
E	0	0.01	0	0	0.01	4.0
F	20	0.1	0.05	0.05	0	6.0
G	35	0.1	0.05	0.05	0	6.0
H	50	0.1	0	0.05	0.05	6.0
J	40	0.2	0.1	0.05	0.05	4.0
L	20	0.4	0.2	0.1	0.1	5.0
M	0	0.05	0	0	0.05	2.0
N	30	0.3	0.2	0.1	0	3.0
O	0	0.01	0	0	0.01	6.0
P	40	0.3	0.1	0.2	0	3.0
Q	40	0.4	0.3	0.1	0	3.0
R	0	0.01	0	0	0.01	12.0
S	0	0.01	0	0	0.01	8.0
Sectionalizing Time						0.5 hr

The calculations for this example are not yet included in this chapter. The problem definition is included for discussion and to lead into the may complex aspects of reliability calculations.

Single section examples

The following examples are based on a feeder with a single section. They are intended to show how single-phase and three-phase failures are handled. Three parameters are varied among the example calculations that follow. In some cases, the feeder will support three-phase operations. In some cases the feeder will support reclosing. Finally, the percent of failures that are single phase will be varied.

Here is the circuit:

Fig. 18-23 Feeder with one section

In this model, the line section has these failure rates:

$$\begin{aligned} \text{Sustained Failure Rate: } & \lambda_S = 1 \ f/yr \\ \text{Temporary Failure Rate: } & \lambda_T = 2 \ f/yr \\ \text{Repair Time: } & R = 1 \ hour \end{aligned} \quad \textbf{(18-65)}$$

No failures occur on the feeder. Customers are arranged on the section as follows:

$$\begin{aligned} C_A &= 100 \text{ customers} \\ C_B &= 200 \\ C_C &= 300 \end{aligned} \quad \textbf{(18-66)}$$

Case 1:

Feeder supports reclosing:	No
Feeder supports by-phase operation:	No
Failures that are single-phase:	Don’t care

Since there is no reclosing, every temporary failure will result in an outage for the section. Also, the limitation to three-phase operation will result in single-phase failures causing an outage for all customers.

$$SAIFI = \frac{(\lambda_S + \lambda_T)\sum C}{\sum C} = 3 \text{ int/yr} \quad \textbf{(18-67)}$$

Although the temporary failures do result in an interruption, SynerGEE considers the outage time to be zero.

$$SAIDI = R\frac{(\lambda_S)\sum C}{\sum C} = 1 \text{ hr/yr} \tag{18-68}$$

No momentary outages are experienced since no devices can recluse:

$$MAIFI = 0 \ \text{ int/yr} \tag{18-69}$$

Case 2:

Feeder supports reclosing:	Yes
Feeder supports by-phase operation:	No
Failures that are single-phase:	Don't care

Temporary failures will now be picked up with feeder reclosing. This will reduce SAIFI values.

$$SAIFI = \frac{\lambda_S \sum C}{\sum C} = 1 \text{ int/yr} \tag{18-70}$$

SAIDI is straightfoward.

$$SAIDI = R\frac{(\lambda_S)\sum C}{\sum C} = 1 \text{ hr/yr} \tag{18-71}$$

Reclosing on the temporary failures gives us MAIFI:

$$MAIFI = \frac{\lambda_T \sum C}{\sum C} = 2 \text{ int/yr} \tag{18-72}$$

Case 3:

Feeder supports reclosing:	Yes
Feeder supports by-phase operation:	Yes
Failures that are single-phase:	70%

Lets consider 'g' to be the factor representing the percent of failures that are single phase. A value of g = 0.7 would correspond to 70% of failures being single phase. Also, the number of phases in a section is given by 'n'.

Customers will see service interruption with a failure on their line or a three-phase failure. Customer interruptions are:

$$CI_j = \left(\frac{g}{n} + (1.0 - g)\right)\lambda_S C_j \text{ cust-int/yr} \tag{18-73}$$

This simplifies to:

$$CI_j = \frac{n - g(n-1)}{n}\lambda_S C_j \text{ cust-int/yr} \tag{18-74}$$

For this case we get:

$$\begin{aligned} CI_A &= \frac{3-0.7(3-1)}{3}(1.0)(100) = 53.3 \text{ cust-int/yr} \\ CI_B &= \frac{3-0.7(3-1)}{3}(1.0)(200) = 106.7 \text{ cust-int/yr} \\ CI_C &= \frac{3-0.7(3-1)}{3}(1.0)(300) = 160.0 \text{ cust-int/yr} \end{aligned} \tag{18-75}$$

The total number of customer interruptions are added up and the total is divided by the number of customers to get SAIFI:

$$SAIFI = \frac{53.3+106.7+160.0}{600} = 0.533 \text{ int/yr} \tag{18-76}$$

By-phase SAIFI values would be the same as the total SAIFI.

$$SAIFI_A = \frac{53.3}{100} = 0.533 \text{ int/yr} \tag{18-77}$$

SAIDI and MAIFI are have similar calculations.

19

Protection Coordination

Introduction

SynerGEE's protection coordination analysis application evaluates the characteristics of fuses, reclosers, breakers, and sectionalizers, and examines the coordination of these devices throughout your system. The analysis uses a large rule base to evaluate these devices independently and relatively.

Check coordination analysis verifies overcurrent coordination among protective devices as well as the protection of transformers. This analysis is performed by breaking a feeder up into protective device pairs and verifying the proper coordination of each pair. Each protective device pair is developed from a combination of the following protective device types.

- Expulsion fuse
- Current limiting fuse
- Hydraulic recloser
- Electronic recloser
- Electromechanical relay
- Electronic relay

The application also considers:

- Transformer inrush curves
- Transformer damage curves

A detailed discussion of these devices and their settings can be found in the *Chapter 10 - Protective Devices.* Please note that the application does not evaluate "classic" protective devices since they lack detailed time-current characteristics.

This application is designed to provide useful engineering information about coordination problems or settings problems that may exist among the protective devices in a model. However, a successful, violation-free run of the application is not a guarantee of proper system coordination. Check coordination analysis is limited to evaluations based on an internal rule-base, and cannot account for nuances which may be unique to your system. The information provided by the analysis is valuable; however, you should use analysis results in conjunction with your own engineering experience and knowledge of your particular system.

Application operation

The operation of check coordination analysis is simple and direct. The application uses the following steps:

1. Determine all coordination pairs from the set of protective devices to be analyzed.
2. Find applicable rules for each type of pair.
3. Check each rule.
4. Mark each check as a "pass" or "fail."
5. Generate a report.

The rule-base used for steps 2 and 3 is extensive, and Advantica Stoner welcomes your ideas for expansion for upcoming releases.

Starting the application

Check coordination analysis can be run from a SynerGEE Map view or from a SynerGEE TCC view with or without devices selected. The protective devices to be included in the analysis are determined as follows.

- **Map view**—All protective devices on the map are considered. All observable combinations of protective devices are evaluated.
- **TCC view with no devices selected**—All protective devices belonging to the selected feeders are considered. All observable combinations of protective devices are evaluated.
- **TCC view with devices selected**—Only combinations of the selected devices are evaluated. Coordination between selected devices and other devices on the model is not evaluated.

The settings for check coordination analysis include a list of device margins. These margins represent limits that are used for checking the coordination of the particular pairs they represent.

Coordination pair categories

There are a number of different combinations of protective devices that can make up protecting and protected pairs. To help manage these combinations and relate them to the particular margins and rules used in the application, a numbering scheme has been developed, as follows.

1	Fuse
2	Recloser
3	Relay
4	Sectionalizer
5	Transformer
6	General

For example, rules and margins for a fuse protecting a recloser would prefixed with a "12-" (1-fuse, 2-recloser). In general, the numbers in the prefixes are in the order of protecting/protected. If a rule pertains to a particular isolated device, the "protected" value would be zero. For example, a rule for an isolated fuse would be prefixed with "10-".

There are general and specific rules that are applied to the different combination of devices making up a coordination pair. The coordination pairs recognized by SynerGEE are as follows.

Protected Device	**Protecting Device**					
	All	**Fuse**	**Recloser**	**Relay**	**Sctzlr**	**Tran**[5]
None	60-	10-	20-	30-	40-	X
Fuse	X	11-	21-	31-	X	51-
Recloser	X	12-	22-	32-	X	52-
Relay	X	13-	23-	33-	X	53-
Sectionalizer	X	X	X	X	X	X
Transformer	X	15-	25-	35-	X	X

Margins

Coordination time margins can be specified for the types of devices in a device pair. Usually, the margins are in a "percent" format. In some cases, the margin is specified in seconds. Some pairs, like relay-fuse pairs, have margins specified in seconds and percent. Fuse minimum fault pickup is specified in amps.

[5] Noted for topological relationship. Transformers do not protect.

Your copy of SynerGEE included default values for each of these margins, as shown below. You can edit these at any time, within the analysis settings editor.

Devices	Default Time Margins		Rule Group
	Percent	Seconds	
Fuse minimum fault pickup (amps)	220%	---	10-
Fuse maximum load margin	90%	---	10-
Fuse protecting fuse	75%	---	11-
Fuse protecting recloser	95%	---	12-
Fuse protecting relay	--	0.3 Sec	13-
Fuse protecting transformer	75%	---	15-
Recloser protecting fuse	90%	---	21-
Recloser protecting recloser	75%	---	22-
Recloser protecting relay	75%	---	23-
Recloser protecting transformer	75%	---	25-
Relay protecting fuse	50%	0.3 Sec	31-
Relay protecting recloser	75%	---	32-
Relay protecting relay	75%	2.0 Sec	33-
Relay protecting transformer	75%	---	35-
Transformer inrush/fuse	75%	---	51-
Transformer inrush/recloser	75%	---	52-
Transformer inrush/relay	75%	---	53-

Coordination range

When SynerGEE evaluates the coordination of two devices, it uses a coordination range. This range extends through the possible fault levels seen at the protecting (downstream) device.

SynerGEE can calculate the maximum fault level of the range using fault analysis, based on sequence domain fault models. Or, if the feeder lacks data to run a valid fault calculation, user-defined fault levels or cutoff amps can be entered into each protective device editor. Within the analysis settings editor, you can choose whether to have calculated fault amps or cutoff amps used.

The minimum of the coordination range is the minimum fault current expected at the protecting device. There are four options for handling minimum fault current.

- **Line-ground minimum fault current**—SynerGEE uses the fault impedance specified in the fault analysis settings or the feeder record to calculate minimum line-ground fault currents. All devices fed by the protecting device and no other downstream protective devices are evaluated. The fault current from each device is propagated and possibly reflected through transformers to the protecting device. The smallest calculated line-ground fault current is selected for both the phase and ground range minimum.

- **Percentage of bolted fault**—Fault analysis is run. Using no fault impedance, phase and ground faults are evaluated. All devices fed by the protecting device and no other downstream protective devices are evaluated. The fault current from each device is propagated and possibly reflected through transformers to the protecting device. The smallest calculated fault current is multiplied by this percentage to obtain the range minimum. For the ground range minimum, the maximum ground current is used because of the bolted fault condition. The phase range minimum uses the smallest phase fault current.
- **Percentage of load current**—The load current through the protecting device is multiplied by this percentage value to obtain the range minimum. The phase range minimum uses phase current and the ground range minimum uses the neutral current.
- **Use fixed current**—A user-defined fixed current is used for both the phase and ground range minimum. The same minimum is used for all protective device pairs regardless of the nominal voltage level.

Protection pairs

SynerGEE uses a concept of protective device pairs when performing coordination checks. An upstream (protected) device will form coordination pairs with any and all downstream devices. Each pair is evaluated by SynerGEE and coordination results are reported.

As an example, consider the following feeder.

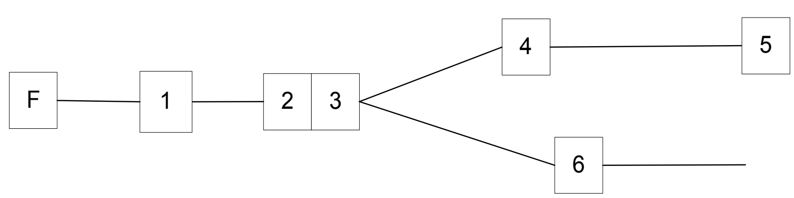


Fig. 19-1 Feeder serving a load through a transformer

The following coordination pairs are generated.

Pair	Protecting device		Protected device
A	2	protecting	1

Pair	Protecting device		Protected device
B	3	protecting	2
C	3	protecting	1
D	6	protecting	3
E	6	protecting	2
F	6	protecting	1
G	4	protecting	3
H	4	protecting	2
I	4	protecting	1
J	5	protecting	4
K	5	protecting	3
L	5	protecting	2
M	5	protecting	1

All pairs look past sectionalizers when considering the fault zone. Sectionalizers are not interrupting devices and should not affect the fault zone of an upstream device. The minimum fault current for a protective device is found from the minimum fault current of all downstream sections (propagated through necessary transformers) that are not fed by other downstream protective devices, unless those downstream protective devices are electrically at the given device.

Check coordination rules

The table below outlines the rule-base used by check coordination analysis.

Rule	Protecting	Protected	Margin	Discussion
10-01	Fuse	---	---	Fuse should not blow under possible load current. Fuse rating must be greater than maximum continuous load current.
10-02	Fuse	---	C% (220%)	Fuse should detect phase fault current in its zone. Minimum phase fault current seen by the fuse should be greater than C% of fuse amp rating.
10-03	Fuse	---	C% (220%)	Fuse should detect ground fault current in its zone. Minimum ground fault current seen by fuse should be greater than C% of fuse amp rating.
10-04	Fuse	---	T% (90%)	Fuse link should not be pre-damaged by load current. The fuse link should not experience load current within T% of its minimum melt curve.

Rule	Protecting	Protected	Margin	Discussion
10-05	Fuse	---	---	Fuse should have minimum melt and maximum clear curves.
11-01	Fuse	Fuse	T% (75%)	Protecting link will interrupt and clear fault before protected link is damaged. Maximum clearing time of protecting link should not exceed T% of the minimum melting time of the protected link. This rule generally accounts for the effects of temperature variation, TCC tolerances, pre-loading, and pre-damage.
11-02	Fuse	Fuse	T% (75%)	Same as rule 11-01. Checks coordination for ground fault range.
11-03	Fuse	Fuse	---	I^2t curves should be used to coordinate current-limiting protecting current-limiting fuse pairs below 0.01 seconds.
12-01	Fuse	Recloser	T% (95%)	Protecting fuse minimum melt curve should lie above the recloser fast curve by a margin of T%.
12-02	Fuse	Recloser	T% (95%)	Rule 12-01 applied to ground faults.
12-03	Fuse	Recloser	T% (95%)	Protecting fuse maximum clear curve should lie below the recloser slow curve by a margin of T%.
12-04	Fuse	Recloser	T% (95%)	Rule 12-03 applied to ground faults
13-01	Fuse	Relay	S (0.3 Sec)	Protecting fuse should blow on downstream faults. Fuse maximum-clear curve should lie below the relay curve over the coordination interval with a margin of S seconds.
13-02	Fuse	Relay	S (0.3 Sec)	Rule 13-01 applied to ground faults.
15-01	Fuse	Tran	T% (75%)	Protecting fuse link will not allow transformer to be damaged. Transformer damage curve should lie above protecting fuse maximum clear curve by a margin of T%.
15-02	Fuse	Tran	T% (75%)	Rule 15-01 applied to ground faults.
20-01	Recloser	---	---	Reclosers should not experience fault levels beyond their interrupt rating.
20-02	Recloser	---	---	Rule 20-01 applied to ground faults.
20-03	Recloser	---	---	Minimum fault level seen by a recloser should exceed its minimum pickup. Minimum fault current must be greater than the recloser minimum pickup.
20-04	Recloser	---	---	Rule 20-03 applied to ground faults.

Rule	Protecting	Protected	Margin	Discussion
20-05	Recloser	---	---	Fast curve should lie below slow curve. A faster curve should be selected for the fast operation than for the slow operation. This check is made for phase and ground curves.
20-06	Recloser	---	---	A recloser should have at least one fast or one time-delay operation selected.
20-07	Recloser	---	---	Largest amp value on recloser phase operation curve should exceed minimum fault current.
20-08	Recloser	---	---	Rule 20-08 applied to ground curves.
20-09	Recloser	---	---	Recloser fast or slow phase operation curves should pickup below maximum fault amps.
20-10	Recloser	---	---	Rule 20-08 applied to ground curves.
20-11	Recloser	---	---	Maximum fault current must not exceed the recloser phase operation curves.
20-12	Recloser	---	---	Rule 20-11 applied to ground curves.
20-13	Recloser	---	---	Recloser should have phase or ground curves.
21-01	Recloser	Fuse	T% (90%)	Protected fuse should not be damaged throughout the entire operating sequence of the recloser. Fuse minimum-melt curve should lie above the sum of the recloser operation curves by a margin of T%. In the case of hydraulic reclosers, coordination is only checked within the capabilities of the recloser curves. Fault levels past the recloser curves are not considered.
21-02	Recloser	Fuse	T% (90%)	Rule 21-01 applied to ground faults.
22-01	Recloser	Recloser	---	The number of fast operations of the protecting recloser should be the same or greater than the protected recloser.
22-02	Recloser	Recloser	T% (75%)	The protecting recloser should operate its slow curve without the protected recloser tripping on its slow curve. The protecting recloser's slow curve should lie below the protected recloser's slow curve by a margin of T%. If 'T' were 75% then the protecting recloser should lie below 75% of the protected recloser slow curve.
22-03	Recloser	Recloser	T% (75%)	Rule 22-02 applied to ground curves.
22-04	Recloser	Recloser	S (0.3 Sec)	The protecting recloser should operate its slow curve without the protected recloser tripping on its slow curve. The protecting recloser's slow curve should lie below the protected recloser's slow curve by a margin of S seconds.

Rule	Protecting	Protected	Margin	Discussion
22-05	Recloser	Recloser	S (0.3 Sec)	Rule 22-04 applied to ground curves.
22-06	Recloser	Recloser	T% (75%)	The protecting recloser should operate to lockout without the protected recloser tripping on its lockout curve. The protecting recloser's total clear curve should lie below the protected recloser's total response time by a margin of T%.
22-07	Recloser	Recloser	T% (75%)	Rule 22-06 applied to ground curves.
22-08	Recloser	Recloser	S (0.3 Sec)	The protecting recloser should operate to lockout without the protected recloser tripping on its lockout curve. The protecting recloser's total clear curve should lie below the protected recloser's total response time by a margin of S seconds.
22-09	Recloser	Recloser	S (0.3 Sec)	Rule 22-08 applied to ground curves.
23-01	Recloser	Relay	T% (75%)	The protecting recloser should operate to lockout without tripping the protected relay. The protecting recloser t.d. curve should lie below the protected relay curve by a margin of T%.
23-02	Recloser	Relay	T% (75%)	Rule 23-01 applied to ground curves.
25-01	Recloser	Tran	T% (75%)	Protecting recloser phase curve will not allow transformer to be damaged. Transformer damage curve should lie above protecting recloser phase curve by a margin of T%.
25-02	Recloser	Tran	T% (75%)	Rule 25-01 applied to ground faults.
30-01	Relay	---	---	Breakers should not experience fault levels beyond their interrupt rating. Maximum fault current must not exceed the breaker interrupt rating.
30-02	Relay	---	---	Rule 30-01 applied to ground faults.
30-03	Relay	---	---	Minimum fault level seen by phase relay should exceed its minimum pickup.
30-04	Relay	---	---	Rule 30-03 applied to ground faults.
30-05	Relay	---	---	Phase relay should not trip under load current.
30-06	Relay	---	---	Ground relay should not trip under neutral load current.
30-07	Relay	---	---	Relay time dial or time multiplier setting must be within the range dictated by available curves. SynerGEE should find a matching curve or should be able to interpolate between two curves to generate the relay characteristic curve.

Rule	Protecting	Protected	Margin	Discussion
30-08	Relay	---	---	Rule 30-07 applied to ground relays.
30-09	Relay	---	---	Breaker should have phase or ground relay curves.
31-01	Relay	Fuse	T% (50%)	Protected fuse should not be damaged by faults beyond the relay. To coordinate, the fuse minimum-melt curve should lie above the relay curve by a margin of T% over the coordination interval.
31-02	Relay	Fuse	T% (50%)	Rule 31-01 applied to ground faults.
31-03	Relay	Fuse	S (0.3 Sec)	Protected fuse should not be damaged by faults beyond the relay. To coordinate, the fuse minimum-melt curve should lie above the relay curve by a margin of S seconds over the coordination interval.
31-04	Relay	Fuse	S (0.3 Sec)	Rule 31-03 applied to ground faults
32-01	Relay	Recloser	T% (75%)	The protecting relay should trip before the protected recloser operates to lockout. The relay phase curve should lie below the recloser total clear curve by a margin of T%.
32-02	Relay	Recloser	T% (75%)	Rule 32-01 applied to ground faults.
33-01	Relay	Relay	T% (75%)	Protecting relay will trip and clear fault before protected relay trips. Protecting relay curve should not exceed the protected relay curve by a margin of T%.
33-02	Relay	Relay	T% (75%)	Rule 33-01 applied to ground faults.
33-03	Relay	Relay	S (2.0 Sec)	Protecting relay will trip and clear fault before protected relay trips. Protecting and protected relays should maintain an S second margin.
33-04	Relay	Relay	S (2.0 Sec)	Rule 33-03 applied to ground faults
35-01	Relay	Tran	T% (75%)	Protecting relay phase curve will not allow transformer to be damaged. Transformer infrequent fault damage curve should lie above protecting relay phase curve by a margin of T%.
35-02	Relay	Tran	T% (75%)	Rule 35-01 applied to ground faults.
40-01	Sctnlzr	---	---	A sectionalizer should have an upstream recloser. Sectionalizer should have recloser along feed path.

Rule	Protecting	Protected	Margin	Discussion
51-01	Tran	Fuse	T% (75%)	Feeding fuse link will not be damaged by transformer inrush current. Transformer inrush curve should lie below the feeding fuse minimum-melt curve by a margin of T%.
51-02	Tran	Fuse	T% (75%)	Feeding fuse link should protect the transformer. Fuse max clear curve should lie below the transformer damage curve by a margin of T%.
52-01	Tran	Recloser	T% (75%)	Feeding recloser will not operate to lockout by transformer inrush current. Transformer inrush curve should lie below the recloser t.d. curve by a margin of T%.
52-02	Tran	Recloser	T% (75%)	Feeding recloser fast operation will not occur from transformer inrush current. Transformer inrush curve should lie below the recloser fast curve by a margin of T%.
52-03	Tran	Recloser	T% (75%)	Feeding recloser should protect the transformer. Recloser phase lockout curve should lie below the transformer damage curve by a margin of T%.
53-01	Tran	Relay	T% (75%)	Feeding relay will not trip from transformer inrush current. Transformer inrush curve should lie below the relay phase curve by a margin of T%.
53-02	Tran	Relay	T% (75%)	Feeding breaker should protect the transformer. Phase relay curve should lie below the transformer damage curve by a margin of T%.
60-01	Any	---	---	Zone fault at a protective device should be detectable. The load current through a protective device should not exceed the minimum fault current of any device fed by that protective device. If this rule is not met, a downstream protective device may be necessary in order to reduce the zone.
60-02	Any	---	---	Zone fault at a protective device should be detectable. The neutral load current through a protective device should not exceed the minimum ground fault current of any device fed by that protective device. If this rule is not met, a downstream protective device may be necessary in order to reduce the zone.

Cases not checked

This section lists known cases that are not checked by SynerGEE. Many of these cases require additional data that is not handled by SynerGEE.

The computer analysis relies solely on the rule-base in the previous section. If you find a need for an expansion of those rules, please contact Advantica Stoner with your suggestions or concerns.

Protective device coordination is complex and in many cases requires human judgment. You should always couple analysis results with your own knowledge and experience.

Ground curve tripping from the inrush of a 1-ph transformer

Single-phase transformers have inrush currents that look like a line-ground fault to upstream protective devices. Reclosers and relays equipped with ground units may trip on a real feeder with sufficient transformer inrush. SynerGEE does not check the coordination of protective device ground curves with transformer inrush curves.

Curve shifting through transformers

A device can be set as a reference point for curve shifting through a transformer. For a device to be a valid reference, any other device on the TCC has to either be in the reference device's feed path or it has to contain the reference device in its feed path.

Time-current curves are reflected through transformers based on the following factors.

$$I_{Src} = I_{Ld} * \left(V_{Ld-NomLL} / V_{Src-NomLL}\right) * k_{Connection} \qquad \textbf{(19-1)}$$

		Values of $k_{Connection}$			
Source Side	**Load Side**	**L-G**	**L-L**	**3-Phase**	**Reflect Ground Curves?**
Delta	Wye-G	$1/\sqrt{3}$	$2/\sqrt{3}$	1.0	Yes
Wye-G	Delta	-	$\sqrt{3}$	1.0	Yes
Delta	Wye	$1/\sqrt{3}$	$\sqrt{3}$	1.0	Yes
Wye	Delta	-	$\sqrt{3}$	1.0	Yes
Delta	Delta	-	1.0	1.0	Yes
Wye	Wye	-	1.0	1.0	Yes
Wye-G	Wye-G	1.0	1.0	1.0	Yes
Wye	Wye-G	1.0	1.0	1.0	Yes
Wye-G	Wye	-	1.0	1.0	Yes

Curve shifting example

Shown below is a simple model with a 34.5 kV source feeding through a Delta/YGnd transformer rated 34.5 / 12.47 kV. A Kearney 65A T-Link fuse is on the high side of the transformer and a Cooper V4H reclosers with a 5A coils is located on the secondary.

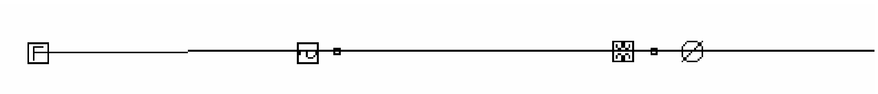


Fig. 19-2 *Feeder with fuse, transformer, and recloser*

The TCC generated from the fuse and reclosers appears as follows.

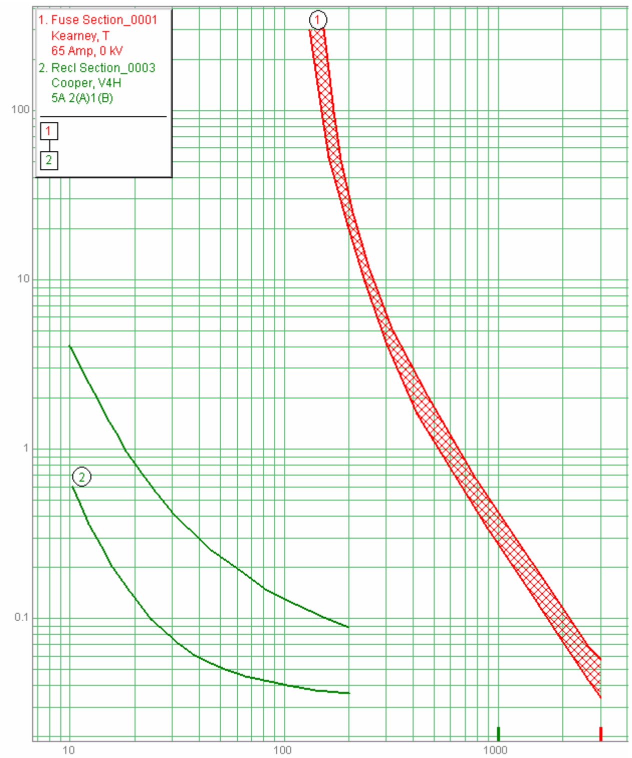


Fig. 19-3 *Fuse and reclosers on the TCC*

This TCC does not account for any shifting. The curves are replicas of published values.

You can look at the devices on either side of the transformer. For example, take a look at the curves on the high side with respect to a line-line fault. The fuse curve remains unshifted since it is placed on the high side. The reclosers are shifted by:

$$
\begin{aligned}
I_{Src} &= I_{Ld} * (12.47/34.5) * \left(2/\sqrt{3}\right) \\
&= 0.4173 I_{Ld}
\end{aligned}
\tag{19-2}
$$

The TCC graph then appears as follows.

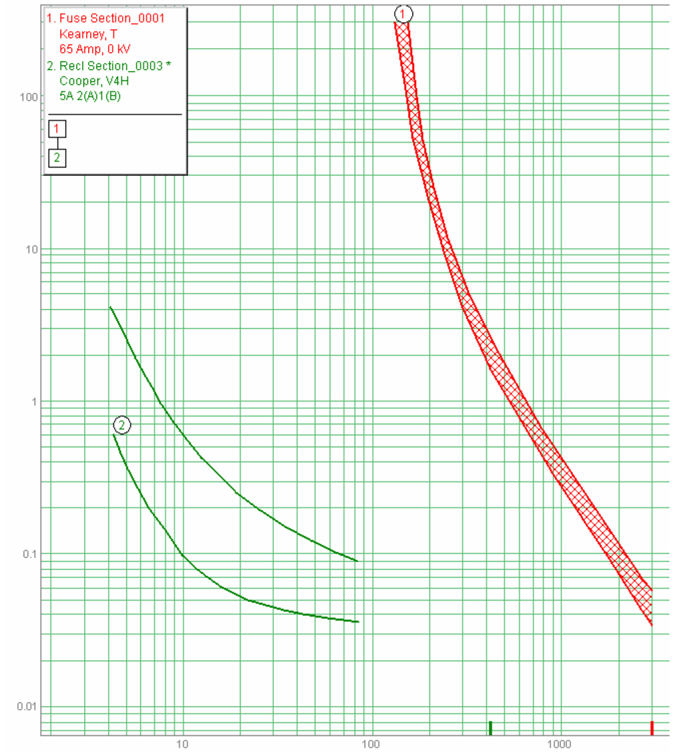


Fig. 19-4 Curves shifted to high side

You can see that the recloser curve is now shifted to the right and the fuse curve remains un-shifted.

As another example, you can shift the curves to the secondary side, thus expecting the recloser curves to move back to their previous positions. You need to shift the fuse curve from the high side of the transformer to the low side. Recalling the equation for reflecting curves through transformers:

$$I_{Src} = I_{Ld} * \left(V_{Ld-NomLL} / V_{Src-NomLL} \right) * k_{Connection} \tag{19-3}$$

This equation provides current values for the source side of the transformer based on load-side or secondary current values. Now, there is a curve on the source side and you need to find load-side values. The equation can be rearranged as follows.

$$I_{Ld} = I_{Src} * \left(V_{Src-NomLL} / V_{Ld-NomLL} \right) * {}^{1}\!/_{k_{Connection}} \tag{19-4}$$

And for the transformer:

$$\begin{aligned} I_{Ld} &= I_{Src} * (34.5/12.47) * {}^{\sqrt{3}}\!/_{2} \\ &= 2.396 I_{Src} \end{aligned} \tag{19-5}$$

The TCC graph now appears as follows.

Fig. 19-5 Curves shifted to high side

Notice that the recloser curve is back to its non-shifted state and the fuse is shifted to the right by a factor of 2.4.

Minimum response and instantaneous settings

Electronic curves typically have minimum response values. These settings are in seconds (or maybe cycles) and represent a guaranteed minimum operation time for the device so that other devices might see high current fault levels. Here is a relay curve without a minimum response value:

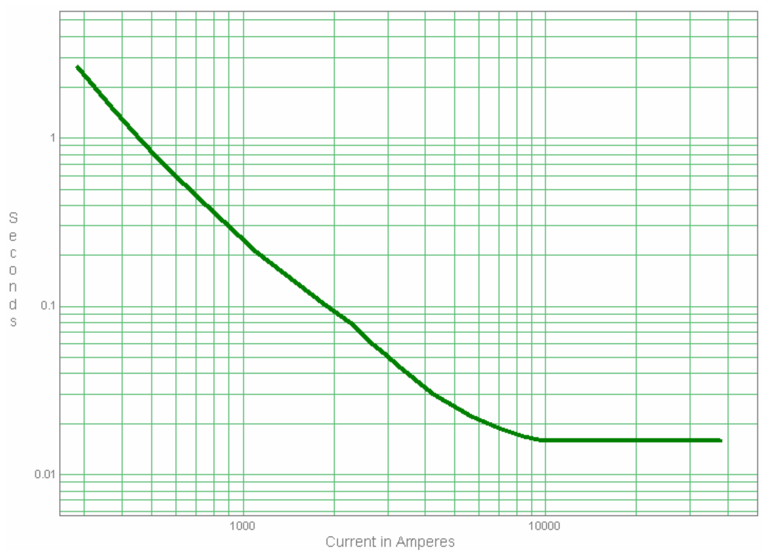


Fig. 19-6 Relay curve

A minimum response setting of 0.1 seconds has been applied. The relay will not operate any faster than 1/10th of a second. This allows better coordination with upstream or downstream equipment.

Fig. 19-7 Curve with minimum response setting

The minimum response setting allows other equipment to respond to fault levels. Contrarily, the instantaneous setting forces the device to operate on any fault over a given level. Here is another relay curve:

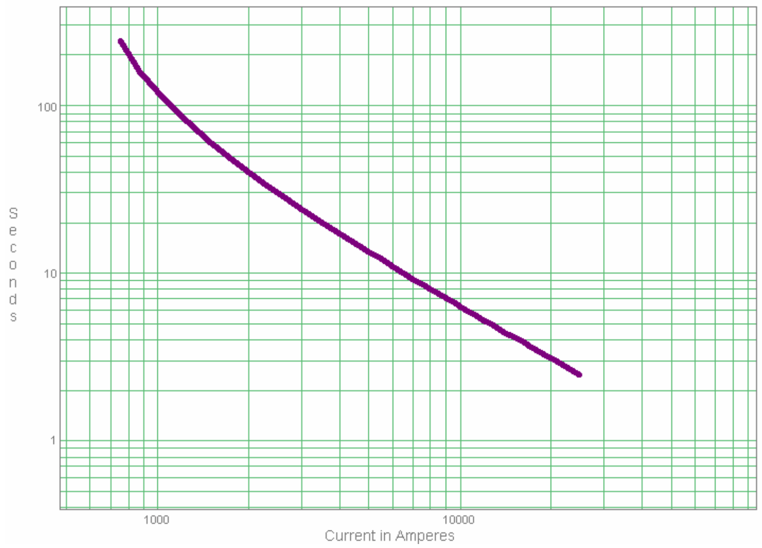


Fig. 19-8 *Fuse curves on secondary side*

Now, we have setup an instantaneous operation time of 2000 amps. Notice the steep drop-off of the curve.

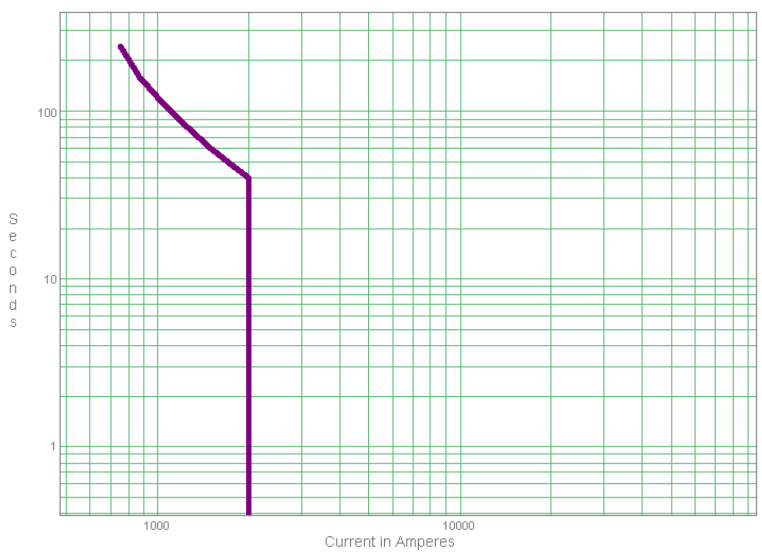


Fig. 19-9 *Fuse curves on secondary side*

Any fault beyond 2000 amps will result in tripping of the relay.

Ignoring load calculations

You have the option to "ignore load calculations" when running check coordination analysis. When this option is selected, coordination rules relating to load current on phase and ground are ignored, including rules 10-01,10-04,30-05,30-06,60-01,60-02 and others relating to load.

Coordination over non-overlapped ranges

In some cases, the coordination range of a device pair may correspond to an amp range in which the devices do not overlap. In the following example, an S&C 65A-Slow SMU-20 fuse serves a transformer and a low side Kearney 65A T-Link fuse. The source, transformer, and line impedances are such that the maximum fault current seen by the secondary fuse is about 1800A. The following is the TCC shown with L-G fault shifting to the secondary fuse.

Fig. 19-10 *Fuse curves on secondary side*

You can see that there is no overlap in the amp ranges of either curve. Currently, SynerGEE treats this as a mis-coordination.

20
Cable Ampacity

Introduction

Cables should be loaded so that their operating temperature is well below levels that would damage the insulation. SynerGEE supports the modeling of direct buried cables in and out of ducts and cables within encased duct banks. Modeling is circuit based and circuit amp values can be specified. SynerGEE calculates the maximum current allowed in the remaining circuits to avoid thermal overloading of cables with non-specified current values.

Thermodynamic modeling of cables and cable ampacity calculations involve many parameters. The goal of ampacity analysis is to determine the safe current limit for cable conductors. The approach to this analysis is to:

- Determine maximum temperature rise of cables
- Calculate maximum thermal losses for temperature rise
- Calculate allowable current flow to reach calculated thermal losses

The analysis is circuit based. Circuits are created. Then the cable and phases making up the circuit are specified. Cables can be placed in ducts within a duct bank as shown below:

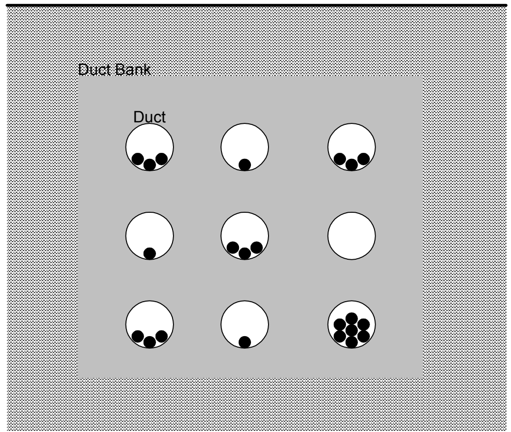


Fig. 20-1 Duct Bank

They can also be buried directly in the ground as shown below:

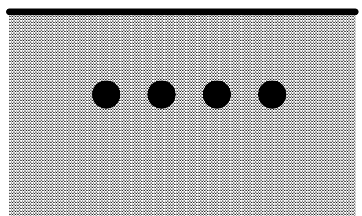


Fig. 20-2 Direct Buried

Ampacity Calculations

The following procedure should be applied to cables and banks with the following conditions:

- Ducts buried in thermal backfill like sand
- Once cable per duct
- Multiple cables per circuit
- Equal current in all cables of a given circuit

Concentric neutral cables will be assumed to be composed of these layers:

- Conductor
- Conductor shield
- Insulation (for example XLPE)
- Insulation shield
- Concentric neutral strands and bedding

- Cable jacket

Tape shield cables will be assumed to be composed of these layers:

- Conductor
- Conductor shield
- Insulation (for example XLPE)
- Insulation shield
- Metallic shield
- Cable jacket

The following basic nomenclature is used in cable thermal analysis:

$$\begin{aligned} W &= \text{Thermal heat transfer rate W/m} \\ T &= \text{Thermal resistance } (°\text{C} \cdot \text{m/W}) \\ \theta &= \text{Temperature } (°\text{C}) \\ \overline{\text{v}} &= \text{Given value - not calculated} \end{aligned} \tag{20-1}$$

Derivations start with the thermal equation describing the temperature rise in a conductor due to the losses in the conductor and the thermal resistance in and around the conductor.

$$\begin{aligned} &\Delta\theta = W \cdot T \\ \Delta\theta &= \text{Temperature rise } (°\text{C}) \\ W &= \text{Losses developed in the conductor (W/m)} \\ \text{T} &= \text{Thermal resistance } (°\text{C} \cdot \text{m/W}) \end{aligned} \tag{20-2}$$

Basic Equations

In general, the rate of heat flow through a cylindrical wall can be derived as:

$$W = \frac{2\pi}{\rho \ln \frac{r_{out}}{r_{in}}} \left(\theta_{in} - \theta_{out} \right) \tag{20-3}$$

The thermal resistance of a cylindrical wall can also be derived as:

$$T = \frac{\rho}{2\pi} \ln \frac{r_{out}}{r_{in}} \tag{20-4}$$

These concepts are used in the remainder of the chapter with an electrical model of a cable. If we look at the cable, the electrical resistance and current flow create losses in the conductor:

$$W = I^2 R \tag{20-5}$$

The second equation can be substituted into the first and the current can be isolated to get:

$$I = \sqrt{\frac{\Delta\theta}{R \cdot T}}\ kA \tag{20-6}$$

The equation accounts for the allowable temperature rise of the conductor. That allowable rise is reduced because of dielectric losses, duct and duct bank materials and temperature rise due to other conductors. These contributions lead to this general form equation:

$$I = \sqrt{\frac{\Delta\theta - \Delta\theta_d - \Delta\theta_{ext}}{R \cdot T}}\ kA \tag{20-7}$$

There are many sub-components to each of the factors in this equation. Those things will be presented in the details of this chapter. The final form of the ampacity equation is listed below.

$$I = \sqrt{\frac{\Delta\theta - \Delta\theta_d - Z_{BT}\Delta\theta_{ext} - (1 - Z_{BT})\Delta\theta_{sol} - W_d\left[0.5T_1 + n(T_2 + T_3 + T_4)\right]}{R\left[T_1 + n(1+\lambda_1)T_2 + n(1+\lambda_1+\lambda_2)\left(T_3 + Z_{BT}T_{4\mu} + (1 - Z_{BT})T_4\right)\right]}}\ A \tag{20-8}$$

This general form representation of cable ampacity can be applied to many styles of manufactured cables installed in many types of conditions. The equation can be used for high voltage oil-filled pipe type conductors and low voltage sector shaped cables. The advanced reader is encouraged to gain an understanding of cable thermal analysis through Anders. The material in his book is applicable to transmission and distribution cables.

SynerGEE cable deals with:

- Underground cables
- Distribution voltage (480V – 69kV)
- Concentric neutral cables
- Tape shield cables
- Separate neutral cables

Because of our focus on these things, some of the factors in the previous equation can be reduced. We end up with the final form of the ampacity equation:

$$I = \sqrt{\frac{\Delta\theta_{\max} - \Delta\theta_{amb} - \Delta\theta_{ext}}{RT_1 + nR(1+\lambda_1)(T_2 + T_3 + T_4)}}\ A \tag{20-9}$$

In some situation, the W_d value may be calculated and applied. The impact of the factor is minor despite its contribution to the complexity of the ampacity equation.

The table below indicates some of the values that will be calculated based on cable type:

Value and description		Units	Tape Shield	Concentric Neutral	Separate Neutral
R	Conductor resistance	Ω/m	✓	✓	✓
W_d	Dielectric Loss	W/m	✓	✓	✓
T_1	Insulation Thermal Resistance	$K \cdot m / W$	✓	✓	✓
T_2	Sheath Thermal Resistance	$K \cdot m / W$	✓	✓	✓
T_3	Jacket Thermal Resistance	$K \cdot m / W$	✓	✓	✓
T_4	External Thermal Resistance	$K \cdot m / W$	✓	✓	✓
λ_1	Shield loss factor	-		✓	
λ_2	Armor loss factor	-			

Cables in duct bank

The external thermal resistance is made of three parts:

- Resistance between cable and duct
- Resistance of non-metal duct
- External resistance of duct

$$T_4 = T_{4A} + T_{4B} + T_{4C} \quad \textbf{(20-10)}$$

The thermal resistance of the duct wall will be calculated next.

$$T_{4B}^{j} = \frac{\rho_{Duct}}{2\pi}\ln\frac{D_o}{D_o - 2t_{duct}}\left(K \cdot m / W\right) \tag{20-11}$$

where:

ρ_d = Thermal resistivity of duct

D_o = Outside diameter

t_{duct} = Duct thickness

Finally,

$$T_4^{"} = \frac{\rho_{Duct}}{2\pi}\ln\frac{D_o}{D_o - 2t_{duct}}\left(K \cdot m / W\right) \tag{20-12}$$

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi} - \frac{x}{y}\right)\ln\left(1 + \frac{y^2}{x^2}\right) + \ln\frac{x}{2} \tag{20-13}$$

$$u = \frac{L_G}{r_b} \tag{20-14}$$

$$G_b = \ln\left[u + \sqrt{u^2 - 1}\right] \tag{20-15}$$

$$D_X = \frac{205}{\sqrt{\varpi \cdot \rho_{fill}^{0.4}}} \tag{20-16}$$

where:

ϖ = Number of load cycles each day

$\rho_{fill}^{0.4}$ = Fill resistivity

$$T_{4u}^{j} = \frac{\rho_f}{2\pi}\left(\ln\frac{D_x}{D_o} + \mu\ln\frac{4L \cdot F}{D_x}\right) + \mu\frac{N}{2\pi}\left(\rho_e - \rho_f\right)G_b\ (K \cdot m / W) \tag{20-17}$$

ρ_f = Thermal resistivity of fill

ρ_e = Thermal resistivity of earth

$$T_{4c}^{j} = \frac{\rho_{fill/concrete}}{2\pi}\ln\frac{4L\cdot F}{D_o} + \frac{N}{2\pi}\left(\rho_{earth} - \rho_{fill/concrete}\right)G_b \quad (K\cdot m/W) \tag{20-18}$$

$$T_4 = T_{4a} + T_{4b} + T_{4c} \tag{20-19}$$

$$T_{4\mu} = T_{4a} + T_{4b} + T_{4\mu} \tag{20-20}$$

$\Delta\theta_{\text{int}}$ - Temperature effect of other cables

The temperature rise on a cable due to other cables in the duct bank needs to be calculated. Cables from other circuits impact the circuit being studied. For each circuit, the hottest cable will be considered. Each neutral cable will also be considered. The heat produced by a cable is:

$$W_k = n\left[I_k^{\ 2}R_k\mu_k\left(1+\lambda_1+\lambda_2\right)+W_{dk}\right] \tag{20-21}$$

Again, the loss factor is:

$$\mu = 0.3\cdot LF + 0.7\cdot LF^2 \tag{20-22}$$

Direct buried cables

$$T_{ij} = \frac{\rho_{soil}}{2\pi}\ln\frac{d_{ij}^{'}}{d_{ij}}(K\cdot m/W) \tag{20-23}$$

Cables in duct bank or backfill

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi}-\frac{x}{y}\right)\ln\left(1+\frac{y^2}{x^2}\right)+\ln\frac{x}{2} \tag{20-24}$$

$$u = \frac{L_G}{r_b} \tag{20-25}$$

$$G_b = \ln\left[u+\sqrt{u^2-1}\right] \tag{20-26}$$

$$T_{ij} = \frac{\rho_{fill/concrete}}{2\pi} \ln \frac{d'_{ij}}{d_{ij}} + \frac{N \cdot G_b}{2\pi}\left(\rho_{earth} - \rho_{fill/concrete}\right) \; (K \cdot m / W) \quad \textbf{(20-27)}$$

N = Number of cables in duct bank

ρ_c = Thermal resistivity of fill

ρ_e = Thermal resistivity of earth

d_{jk} = Conductor ducts

d'_{jk} = Duct j and reflection of k

Total rise on cable

Temperature rise at the surface of cable i due to cable j is found with:

$$\Delta\theta_{jk} = W_k T_{jk} \; (C) \quad \textbf{(20-28)}$$

Temperature rise due to external cables is:

$$\Delta\theta^{j}_{\text{int}} = \sum_{k \neq j} \Delta\theta_{jk} \quad \textbf{(20-29)}$$

Example 1: 750 MCM_CN in duct

Three 1000 MCM cables are buried in 6” PVC conduit. The table below lists the various data that will be used in the calculations.

Data

Cable:	
Size	750 MCM
Core	Aluminum
Insulation	XLPE $\rho = 3.5$
Jacket	PE $\rho = 3.5$
Conductor diameter	24.7 mm
Conductor shield	1 mm
Insulation diameter	35.3 mm
Shield diameter	37.3 mm
Jacket outside	45.9 mm

diameter	
Neutral strands	Copper #12
Neutral strand count	24
Duct:	
Material	PVC $\rho = 6.0$
Duct outside diameter	102 mm (4”)
Duct inside diameter	88.9 mm (3.5”)
Bank	
Burial depth	914mm (36”)
Spacing	191mm (7.5”)
Duct width	762 mm (30”)
Earth resistivity	$\rho_e = 1.0$
Backfill resistivity	$\rho_e = 0.9$
Circuit(s)	
Load Factor	100%

Calculations

Resistance

The MCM cable size is given. The size of the cable in m^2 needs to be determined.

The cross sectional area of this found with:

$$S = 161.29 \cdot \pi \cdot AWG_{MCM} \cdot 10^{-9} \quad \text{m}^2 \qquad \textbf{(20-30)}$$

The 750 MCM cable size is put in:

$$S = 161.29 \cdot \pi \cdot 750 \cdot 10^{-9} \quad \text{m}^2$$
$$= 380mm^2 \tag{20-31}$$

The additional length due to stranding could be taken into account. Since layering information is not available, we will not consider stranding issues beyond the use of the concentric neutral lay factor (Anders, 117).

A mistake can be made with conductor diameter values. In this example, the conductor has a diameter of 0.974 in. Using that diameter to calculate the area would lead to 480 mm^2. That is an increase of over 25% that can result in over 10% higher calculated ampacity.

The MCM value of cable conductor represents the cumulative area of cable strands. It is the effective area. Some conductor areas are given as equivalent mm^2 values.

The DC resistance of the core is now calculated. The resistivity is for copper or aluminum at 20C. At higher temperatures, the dimensions and resistivity of the cable core changes. A linear factor is used to get to the cable operating temperature

$$R^{DC} = \frac{\rho_{20}}{S}\left[1+\alpha_{20}\left(\theta_{\max}-\theta_{20}\right)\right] \quad \Omega/\text{m} \tag{20-32}$$

For aluminum:

$$\rho_{20} = 2.8264 \cdot 10^{-8} \ \ \Omega \cdot m$$
$$\alpha_{20} = 4.03 \cdot 10^{-3} \tag{20-33}$$

Substitutions are made:

$$R^{DC} = \frac{2.8264 \cdot 10^{-8}}{3.80 \cdot 10^{-4}}\left[1+4.03 \cdot 10^{-3}\left(90-20\right)\right]$$
$$= 9.536 \cdot 10^{-5} \ \Omega / m \tag{20-34}$$

Next, the skin effect needs to be considered. The following Bessel function is evaluated:

$$\chi_s^2 = k_s \frac{8\pi \cdot 60}{R^{DC}} \cdot 10^{-7} \tag{20-35}$$

Evaluating:

$$\chi_s^2 = \frac{8\pi \cdot 60}{9.536 \cdot 10^{-5}} \cdot 10^{-7}$$
$$= 1.5813 \tag{20-36}$$

The skin effect factor is determined with:

$$y_s = \frac{\chi_s^4}{192 + 0.8\chi_s^4} \tag{20-37}$$

Or

$$\begin{aligned} y_s &= \frac{1.5813^2}{192 + 0.8\left(1.5813^2\right)} \\ &= 1.289 \cdot 10^{-2} \end{aligned} \tag{20-38}$$

The AC resistance is finally calculated with:

$$\begin{aligned} R^{AC} &= R^{DC}\left(1 + y_s\right) \\ &= 9.536 \cdot 10^{-5}\left(1 + 1.289 \cdot 10^{-2}\right) \\ &= 9.659 \cdot 10^{-5}\,\Omega / m \end{aligned} \tag{20-39}$$

Internal Thermal Resistance

The overall thermal resistance up to the ring of concentric neutral strands will be calculated as:

$$T_1 = T_{CSh} + T_{Insul} + T_{Shld} \tag{20-40}$$

First, the conductor shield:

$$\begin{aligned} T_{CSh} &= \frac{\rho_{CSh}}{2\pi}\ln\left(1 + \frac{2t_{CSh}}{d_{core}}\right) \\ &= \frac{3.5}{2\pi}\ln\left(1 + \frac{2 \cdot 1.0}{24.73}\right) = .04332\ \mathrm{K \cdot m/W} \end{aligned} \tag{20-41}$$

For the insulation:

$$\begin{aligned} T_{Insul} &= \frac{\rho_{Insul}}{2\pi}\ln\left(1 + \frac{2t_{Insul}}{d_{Insul}}\right) \\ &= \frac{3.5}{2\pi}\ln\left(1 + \frac{2 \cdot 4.288}{24.73}\right) = 0.166\ \mathrm{K \cdot m/W} \end{aligned} \tag{20-42}$$

For the shield:

$$\begin{aligned} T_{Shld} &= \frac{\rho_{Shld}}{2\pi}\ln\left(1 + \frac{2t_{Shld}}{d_{Shld}}\right) \\ &= \frac{3.5}{2\pi}\ln\left(1 + \frac{2 \cdot 1.0}{37.34}\right) = 0.029\ \mathrm{K \cdot m/W} \end{aligned} \tag{20-43}$$

The resulting thermal resistance is:

$$T_1 = 0.04332 + 0.166 + 0.029 \qquad \textbf{(20-44)}$$
$$= 0.238 \text{ K} \cdot \text{m/W}$$

Jacket Thermal Resistance

The thermal resistance of the jacket is calculated as:

$$T_3 = \frac{\rho_{Jk}}{2\pi} \ln\left(1 + \frac{2t_{Jk}}{d_{Jk}}\right) \qquad \textbf{(20-45)}$$
$$= \frac{3.5}{2\pi} \ln\left(1 + \frac{2 \cdot 4.318}{37.338}\right) = .1159 \text{ K} \cdot \text{m/W}$$

Sheath Loss Factor

In this case, the 'sheath' is the ring of concentric neutral strands. In other cases, it may be a tape shield.

The sheath resistance at the maximum temperature is:

$$R_S = \frac{10^6 \rho_{20}}{\pi \cdot n_s \cdot r^2} \left[1 + \alpha_{20}\left(\theta_{\max} - \theta_{20}\right)\right] \ \Omega/\text{m} \qquad \textbf{(20-46)}$$

The cable in this example has 24 strands of #12 copper.

For copper:

$$\rho_{20} = 1.7241 \cdot 10^{-8} \ \Omega \cdot m \qquad \textbf{(20-47)}$$
$$\alpha_{20} = 3.93 \cdot 10^{-3}$$

Substitutions are made:

$$R_S = \frac{1.7241 \cdot 10^{-2}}{24\pi\left(1.03\right)^2} \left[1 + 3.93 \cdot 10^{-3}\left(90 - 20\right)\right] \qquad \textbf{(20-48)}$$
$$= 2.748 \cdot 10^{-4} \ \Omega / m$$

The sheath reactance is determined with:

$$X_1 = 4\pi f \cdot 10^{-7} \ln\left[2\sqrt[3]{2}\left(\frac{s}{d}\right)\right] \ \Omega/\text{m} \qquad \textbf{(20-49)}$$

S is the spacing between cables of the same circuit and d is the diameter to the ring of concentric neutral strands. The ring diameter will be approximated as the average of insulation and jacket diameters.

(20-50)

$$X_1 = 240\pi \cdot 10^{-7} \ln\left[2\sqrt[3]{2}\left(\frac{191}{41.6}\right)\right]$$

$$=1.846 \cdot 10^{-4} \ \Omega/\text{m}$$

The sheath loss factor is determined as:

(20-51)

$$\lambda_1^{'} = \frac{R_S}{R}\frac{1}{1+\left(\frac{R_S}{X_1}\right)^2}$$

Putting in numbers for this situation yields:

(20-52)

$$\lambda_1^{'} = \frac{2.748 \cdot 10^{-4}}{9.659 \cdot 10^{-5}}\frac{1}{1+\left(\frac{2.748 \cdot 10^{-4}}{1.846 \cdot 10^{-4}}\right)^2}$$

$$= 0.8846 \ \ \Omega/\text{m}$$

External Thermal Resistance

This calculation is complex. The cable system in this example is made of cables buried within ducts. The system will be treated similarly to ducts within concrete.

The external thermal resistance is made of three parts:

Resistance between cable and duct

Resistance of non-metal duct

External resistance of duct

(20-53)

$$T_4 = T_4^{'} + T_4^{''} + T_4^{'''}$$

First, the resistance between the cable and duct will be accounted for with the 1950 linearization of the more comprehensive equations accounting for duct fill. The linearization assumes a temperature around the duct of 20C drop between the duct surface and wall (Anders, 227) and a cable having diameter from 1-4".

(20-54)

$$T_4^{'} = \frac{U}{1+0.1\left(V+Y\theta_m\right)D_e}$$

In these calculations, the average temperature within the duct will be assumed 10C less than the maximum cable temperature. The constants U, V, Y are gathered from table 9.6 in Anders. The table is limited. The “fiber duct in concrete” numbers will be selected.

$$T_4^{'} = \frac{5.2}{1+0.1\left(0.91+0.01\cdot 80\right)\cdot 45.9} \tag{20-55}$$
$$= 0.588 \text{ K}\cdot\text{m/W}$$

The thermal resistance of the duct will be calculated next.

$$T_4^{"} = \frac{\rho_{Duct}}{2\pi}\ln\left(\frac{D_o}{D_i}\right) \tag{20-56}$$
$$= \frac{6.0}{2\pi}\ln\left(\frac{101.6}{88.9}\right) = 0.128 \text{ K}\cdot\text{m/W}$$

The external thermal resistance will now be determined. The load factor is used to get:

$$\mu = 0.3\cdot LF + 0.7\cdot LF^2 \tag{20-57}$$

The loss factor diameter is determined from the load factor.

If the load factor is 100%

$$D_x = D_e \tag{20-58}$$

If the load is cyclic sinusoidal:

$$D_x = \frac{205}{\rho_s^{0.4}\sqrt{\varpi}} \tag{20-59}$$

If it is rectangular:

$$D_x = \frac{493\sqrt{\mu}}{\rho_s^{0.4}\sqrt{\varpi}} \tag{20-60}$$

Otherwise it is:

$$D_x = \frac{103+246\sqrt{\mu}}{\rho_s^{0.4}\sqrt{\varpi}} \tag{20-61}$$

The factor ϖ is the number of load cycles / 24 hours.

The load factor for this example is 100% so:

$$D_x = D_e \tag{20-62}$$

The mutual heating factor is calculated with the conductor reflections. The factor is calculated from the hottest cable. The hottest cable is assumed to be the center one in this example.

$$F = \left(\frac{d'_{21}}{d_{21}}\right)\left(\frac{d'_{23}}{d_{23}}\right) \tag{20-63}$$

Using 36" depth

$$F = \left(\frac{1838}{191}\right)\left(\frac{1838}{191}\right) = 92.6 \tag{20-64}$$

Next, the size and shape of the bank is used to calculate the following factor:

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi} - \frac{x}{y}\right)\ln\left(1 + \frac{y^2}{x^2}\right) + \ln\frac{x}{2} \tag{20-65}$$

Substituting:

$$\ln r_b = \frac{762}{2 \cdot 914}\left(\frac{4}{\pi} - \frac{762}{914}\right)\ln\left(1 + \frac{914^2}{762^2}\right) + \ln\frac{762}{2} = 6.106 \tag{20-66}$$

Solving the logarithm:

$$r_b = 448.6 \tag{20-67}$$

This allows the calculation of the geometric factor as:

$$G_b = \ln\left(\frac{L_G}{r_b} + \sqrt{\frac{L_G^2}{r_b^2} - 1}\right) \tag{20-68}$$

The factor L_G is the distance to the center of the bank.

$$G_b = \ln\left(\frac{457}{448.6} + \sqrt{\frac{457^2}{448.6^2} - 1}\right) = 0.193 \tag{20-69}$$

Finally,

$$T_4^{'''} = \frac{\rho_c}{2\pi}\left(\ln\frac{D_x}{D_o} + \mu\ln\frac{4L\cdot F}{D_x}\right) + \mu\frac{N}{2\pi}(\rho_e - \rho_c)G_b \tag{20-70}$$

The 100% load factor reduces this to:

$$T_4^{'''} = \frac{\rho_c}{2\pi}\left(\ln\frac{4L\cdot F}{D_o}\right) + \frac{N}{2\pi}(\rho_e - \rho_c)G_b \tag{20-71}$$

Values are substituted:

$$\frac{0.9}{2\pi}\left(\ln\frac{4\cdot 914\cdot 92.6}{102}\right) + \frac{3}{2\pi}(1.0-0.9)0.193 \\ = 1.17\ \text{K}\cdot\text{m/W} \tag{20-72}$$

The overall thermal resistivity is:

$$T_4 = 0.588 + 0.176 + 1.17 \\ = 1.93\ \text{K}\cdot\text{m/W} \tag{20-73}$$

Ampacity

$$I = \sqrt{\frac{\theta_C - \theta_A}{R^{AC}\cdot T_1 + R^{AC}(1+\lambda_1)(T_3+T_4)}} \tag{20-74}$$

Substituting:

$$\sqrt{\frac{(90-20)\cdot 10^5}{9.659(0.238+(1+0.885)(.1159+1.93))}} \\ = 420A \tag{20-75}$$

Example 2: 500 MCM direct buried

Given:

Cable size	500 MCM
Cable diameter	0.794” aluminum
Insulation thickness	0.175”

Concentric neutral	16 strands #12 copper
Insulation diameter	1.20”
Diameter outside jacket	1.56” (39.62 mm)
Diameter of insulation shield	1.29 (32.77 mm)
Maximum temp	90C
Burial depth	36” deep (914.4 mm)
Spacing between cables	7.5” (190.5 mm)
Earth rho	0.90 C*m/W
Load factor	100%
Spacing	7.5” horizontal
Grounding	Short circuited shields

The following values were also given in the tables.

Max amps	451
Z1	68+j67 uohm/ft

Filling in SynerGEE numbers

Conductor Diam	0.794” (20.17mm)
Insulation Diam	1.20 (30.48mm)
Jacket Diam	1.56 (39.62 mm)
Jacket Thickness	(1.56 – 1.29) / 2 = 0.17
CN Strand diameter	0.0808”
Diam to strand ring	1.29 – 0.0808 = 1.21
# Conductor strands	19 (T&D)
Core strand diam	0.1622” (T&D) 4.12mm

Preparation

Core ρ_{20} $(\Omega \cdot m)$	2.8264 x10^{-8}
$XLPE\ dielectric\ \rho$	2.5
$XLPE\ insulation\ \rho$	3.5
$PVC\ insulation\ \rho$	5.0

Soil ρ	0.9

Check the area

First, lets verify that the conductor strand count and diameter are consistent with the 500MCM value of the cable. The area of the strands is:

$$Size = 1000 \cdot 19 \cdot 0.1622^2 = 500\,MCM \tag{20-76}$$

The area is consistent with the 500 MCM name of the conductor.

AC Resistance

The resistance of the entire conductor, with stranding factor, is:

$$R_{cs,20}^{dc} = \frac{4 \cdot 10^6 \cdot 2.8264\text{x}10^{-8}}{\pi \cdot (4.12)^2 \cdot 19} = 111.6x10^{-6}\Omega / m \tag{20-77}$$

In ohms/mile, the value is:

$$R_{cs,20}^{dc} = 111.6x10^{-6}\Omega / m \cdot 1609m / mi = 0.1795\Omega / mi \tag{20-78}$$

It looks reasonable. Next, we need to determine the AC factors and the AC resistance. First:

$$F_k = \frac{8\pi \cdot 60 \cdot 10^{-7}}{111.6x10^{-6}} = 1.351 \tag{20-79}$$

A factor for skin effect is given by:

$$y_s = \frac{(1.351)^2}{192 + 0.8(1.351)^2} = 6.999x10^{-3} \tag{20-80}$$

Because the cables do not share a duct:

$$y_p = 0 \tag{20-81}$$

The ac resistance of the cable is:

$$R^{'} = 111.6x10^{-6}\left[1 + 6.999x10^{-3}\right] = 112.37x10^{-6}\,\Omega / m \tag{20-82}$$

Note that this value is at 20C. Llets compare it to the value given with the manufacturer data. Shift the resistance to 50C and ohms/mile:

$$R'_{Table} = 112.37x10^{-6} \cdot \frac{228+50}{228+20} \cdot 1609 = 0.2027 \ \ \Omega / mi \quad \textbf{(20-83)}$$

This compares to within 2% of the 0.206 Ω / mi value found in tables for 500MCM aluminum conductor at 50C.

Now, we need the resistance at the maximum temperature of the cable:

$$R = R'\left[1+4.03x10^{-3}\left(90-20\right)\right] = 143.068x10^{-6}\,\Omega / m \quad \textbf{(20-84)}$$

T_1 - Internal thermal resistance

For the conductor shield:

$$T_{Shield} = \frac{2.5}{2\pi}\ln\left(1+\frac{2\cdot 1}{20.17}\right) = 37.622x10^{-3} \ \ ^{\circ}C \cdot m / W \quad \textbf{(20-85)}$$

For the insulation:

$$T_{Insul} = \frac{3.5}{2\pi}\ln\left(\frac{30.48}{20.17}\right) = 230.055x10^{-3} \ \ ^{\circ}C \cdot m / W \quad \textbf{(20-86)}$$

For the insulation shield:

$$T_{InsulShld} = \frac{2.5}{2\pi}\ln\left(\frac{32.77}{30.48}\right) = 28.775x10^{-3} \ \ ^{\circ}C \cdot m / W \quad \textbf{(20-87)}$$

We can add up the resistance of the components to get the total internal resistance:

$$T_1 = T_{Shield} + T_{Insul} + T_{InsuShld} = 296.452x10^{-3} \ \ ^{\circ}C \cdot m / W \quad \textbf{(20-88)}$$

T_3- Jacket thermal resistance

For the conductor shield:

For the jacket:

$$T_3 = \frac{5.0}{2\pi}\ln\left(\frac{39.62}{32.77}\right) = 151.232x10^{-3} \ \ ^{\circ}C \cdot m / W \quad \textbf{(20-89)}$$

T_4- External thermal resistance

The distance to the center of the "bank" is 36". We can calculate the mutual heating factor as:

(20-90)

$$F_{center} = 1 + 4\left(\frac{36}{7.5}\right)^2 = 93.16$$

The thermal resistance is:

(20-91)

$$T_4 = \frac{0.9}{2\pi}\ln\left(\frac{4\cdot 93.16\cdot 914.4}{39.62}\right) = 1.298\ {}^{\circ}C\cdot m/W$$

λ_1 - Loss Factor

The overall resistance of the concentric neutral is:

(20-92)

$$R_{cs,20}^{dc} = \frac{4\cdot 10^6\cdot 2.8264\text{x}10^{-8}}{\pi\cdot(4.12)^2\cdot 19} = 415.435x10^{-6}\,\Omega/m$$

The reactance is:

(20-93)

$$X = 4\pi 60\cdot 10^{-7}\ln\frac{190.5}{32.77} = 170.8x10^{-6}\,\Omega/m$$

The loss factor is:

(20-94)

$$\lambda_1 = 1.25\cdot\frac{415.435x10^{-6}}{143.068x10^{-6}}\cdot\frac{1}{1+\left(\frac{415.435x10^{-6}}{170.8x10^{-6}}\right)^2} = 521.832x10^{-3}$$

Maximum Amps

Finally, the amp limit can be calculated:

(20-95)

$$I = \sqrt{\frac{90-20}{143.068x10^{-6}\cdot 296.452x10^{-3} + 143.068x10^{-6}(1.52183)(151.232x10^{-3} + 1.298)}} = 441A$$

This value is within 2% of the 451A published value from the manufacturer.

Comparison to T&D Numbers

In this example, we will compare results from SynerGEE to values derived from the tables in the Transmission and Distribution Reference Book.

Given:

250 MCM cable direct buried with 9 inches between conductors.

20 C ambient earth temperature

15 kV nominal voltage

From T&D:

Table 18 in Chapter 4 is used.

	30% LF	50% LF	75% LF	100% LF
3 Cables	440	423	396	367

With SynerGEE:

Table 10 in the T&D Reference Book is used for cable details.

Insulation thickness = 0.175 inches

Conductor diameter = 0.575 inches

Assume 50 strands

Strand diameter = $2 \cdot \sqrt{\dfrac{0.250}{50\pi}} = 0.0798\,\text{inches}$

	30% LF	50% LF	75% LF	100% LF
3 Cables	440	423	398	369

Example 3: 300 mm^2 direct buried

Three single-phase copper cables are directly buried. Here is data for the cables:

Conductor	20.5mm diameter, 37 strands and 300 mm^2
Insulation	XLPE with 28.5mm diameter
Jacket	PVC with 35.8 mm diameter
CN strands	66 copper wires. 0.70 mm in 30.5 mm ring
Burial depth	1m

Spacing	Flat with 71.6 mm between cable cores
Ambient temperature	15C
Soil	1.0K m/W

We need the diameter of the core strands.

$$37 \cdot \pi r^2 = 300mm^2 \tag{20-96}$$

So:

$$r = \sqrt{\frac{300}{37\pi}} = 1.607mm \tag{20-97}$$

$$d = 3.213mm$$

T_1 - Internal thermal resistance

The thickness of the insulation is the distance from the concentric neutral ring to the conductor:

$$t_i = \frac{30.5 - 0.70 - 20.5}{2.0} = 4.65mm \tag{20-98}$$

XLPE has a thermal resistivity of 3.5. The resistance of the insulation is therefore:

$$T_1 = \frac{3.5}{2\pi} \ln\left(1 + \frac{4.65 \cdot 2}{20.5}\right) = 208.4x10^{-3} \ {}^{\circ}C \cdot m / W \tag{20-99}$$

T_3 - Jacket thermal resistance

The thickness of the jacket is

$$t_i = \frac{35.8 - (30.5 + 0.70)}{2.0} = 2.3mm \tag{20-100}$$

PVC has a thermal resistivity of 5.0. The resistance of the jacket is therefore:

$$T_3 = \frac{5.0}{2\pi} \ln\left(1 + \frac{2 \cdot 2.3}{30.5 + .7}\right) = 109.4x10^{-3} \ {}^{\circ}C \cdot m / W \tag{20-101}$$

Thermal Network Parameters

T - Thermal resistance

The thermal resistances of cables are found as the thermal resistances of cylindrical bands. In general, the thermal resistance of a cylinder is:

$$T = \frac{\rho}{2\pi} \ln\left(\frac{r_{out}}{r_{in}}\right) \tag{20-102}$$

This can be factored to get:

$$T = \frac{\rho}{2\pi} \ln\left(1 + \frac{2t}{d_{in}}\right) \quad \text{where } t = r_{out} - r_{in} \tag{20-103}$$

Like resistances in an electrical circuit, these values can be added when in series. The equation above will be used many times below as the various materials in the cables, their ducts, and their banks are thermally modeled as cylindrical shells.

w_c - Core conductor heat

A number of values need to be calculated to determine the heat generated at the core of a cable. Fundamentally, we have:

$$w_c = R_{AC} I_C^2 \;\; \text{w/m} \tag{20-104}$$

In a duct being studied, a cable will either:

1. be a phase cable with given amp value
2. be a neutral cable with amps given as percentage of phase cable
3. be in the set of cables with <u>unknown</u> but equal amp values

The following procedure is used to calculate the AC resistance of the cable.

AC Resistance

The AC resistance is determined from the DC resistance at the operating temperature of the cable. The fact that AC voltage and AC current are applied to the cable has an impact on the resistance. These are called the skin effect, and the proximity effect (if any) of the cable.

$$R_j^{AC} = R_j^{DC}\left(1 + y_s + y_p\right) \;\; \Omega/\text{m} \tag{20-105}$$

There are a few ways to calculate the DC resistance of the core. The two approaches will give very similar results for consistent cable data. Cable data is oftentimes hard to find and not necessarily consistent.

DC resistance using cable strands

The dc resistance of a conductor is given by:

$$R_{20}^{dc} = \frac{1.02 \cdot 10^6 \rho_{20}}{A_c} \tag{20-106}$$

This equation corresponds to 20C and includes an empirical value of 1.02 to account for stranding. Cables change their resistance and dimensions with temperature.

The resistivity of copper or aluminum along with the cable core strand count and strand diameter can be used to determine the dc resistance of the cable core at 20C. The resistance of an individual strand can be determined by:

$$R_{cs,20}^{dc} = \frac{4 \cdot 10^6 \rho_{20}}{\pi \cdot D_{cs}^2} \tag{20-107}$$

The resistance of the entire conductor, with stranding factor, is:

$$R_{cs,20}^{dc} = \frac{4 \cdot 10^6 \rho_{20}}{\pi \cdot D_{cs}^2 \cdot N_{cs}} \tag{20-108}$$

Example:

Calculate the dc resistance of a 91 strand aluminum cable conductor with strands having diameters of 132 thousandths of an inch:

First, calculate the strand diameter in meters:

$$D_{cs-m} = 0.132 \cdot 25.4\left(mm/in\right) = 3.3528 \; mm \tag{20-109}$$

Now, calculate the resistance:

$$\begin{aligned} R_{cs,20}^{dc} &= \frac{4 \cdot 1.02 \cdot 10^6 \cdot 2.8264 \cdot 10^{-8}}{\pi \cdot (3.3528)^2 \cdot 91} = 35.88 \; \Omega / m \\ &= .05773 \;\; \Omega / mi \end{aligned} \tag{20-110}$$

We could move this impedance to 25C with:

$$R_{cs,25}^{dc} = .05773 \frac{25+278}{20+278} = 0.0587 \ \ \Omega / mi \tag{20-111}$$

which matches the value from the electric T&D book.

DC resistance using conductor area

The DC resistance is calculated as:

$$R_j^{20} = \frac{\rho_{20}}{S_j} \ \ \Omega/\text{m} \tag{20-112}$$

$$\rho_{20} = \text{Resistivity of core @ 20C}$$

This equation requires the area of the conductor (in square meters) which is calculated from the AWG wire size in MCM:

$$S_j = 161.29 \cdot \pi \cdot AWG_{MCM} \cdot 10^{-9} \ \ \text{m}^2 \tag{20-113}$$

$$where$$

$$AWG_{MCM} = \text{Wire size in MCM}$$

The additional length due to stranding could be taken into account. Since layering information is not available, we will not consider stranding issues beyond the use of the concentric neutral lay factor (Anders, 117).

A mistake can be made with conductor diameter values. Consider a conductor with a diameter of 0.974 in. Using that diameter to calculate the area would lead to 480 mm^2. That is an increase of over 25% that can result in over 10% higher calculated ampacity.

The MCM value of cable conductor represents the cumulative area of cable strands. It is the effective area. Some conductor areas are given as equivalent mm^2 values.

AC Factors

The DC resistance of the core is now calculated. The resistivity is for copper or aluminum at 20C. At higher temperatures, the dimensions and resistivity of the cable core changes. A linear factor is used to get to the cable operating temperature. Anders indicates that the following linear relationship holds over typical temperature ranges:

$$R_j^{DC} = R_j^{20}\left[1+\alpha_{20}\left(\theta_{\max}-\theta_{\rho}\right)\right] \ \ \Omega/\text{m} \tag{20-114}$$

$$\theta_{\max} = \text{ Maximum conductor C}$$

$$\theta_{\rho} = \text{ Temperature used for DC resistance}$$

$$\alpha_{20} = \text{Temperature coefficient from 20C}$$

Next, the skin effect needs to be considered. Conductor resistance is increased when carrying ac current due to skin and proximity effect and ferromagnetic losses.

A Bessel function is used to evaluate skin effect. References for this work are given in [Anders]. We will calculate this factor first:

$$F_k = \frac{8\pi f \cdot 10^{-7}}{R^{dc}} \tag{20-115}$$

A factor for skin effect is given by:

$$y_s = \frac{\left(F_k k_s\right)^2}{192 + 0.8\left(F_k k_s\right)^2} \tag{20-116}$$

Alternatively, the permeability of the conductor at the given frequency is captured with this Bessel function:

$$\chi_s^2 = k_s \frac{8\pi \cdot f}{R^{DC}} \cdot 10^{-7} \tag{20-117}$$

The constant k_s comes from tables and the conductor material.

The skin effect factor is found with:

$$y_s = \frac{\chi_s^4}{192 + 0.8\chi_s^4} \tag{20-118}$$

If conductors are in separate ducts:

$$y_p = 0 \tag{20-119}$$

However, three phase cables are pulled through the same duct then the following series of equations are used to yield the proximity factor:

$$\chi_p^2 = k_p \frac{8\pi \cdot 60}{R_j^{DC}} \cdot 10^{-7} \tag{20-120}$$

Again, k_p comes from tables and the conductor material.

Next:

$$y_p^{'} = \frac{\chi_p^4}{192 + 0.8\chi_p^4} \tag{20-121}$$

And:

(20-122)

$$y_p = y_p^{'}\left(\frac{d_c}{s}\right)^2\left[0.312\left(\frac{d_c}{s}\right)^2 + \frac{1.18}{y_p^{'} + 0.27}\right]$$

d_c = conductor diameter (mm)

s = distance between conductors (mm)

A factor for cables in magnetic pipe and conduit is needed:

(20-123)

$$f_{pipe} = 1.5 \text{ for } Z_{um} = 6 \text{ (metal pipe or conduit)}$$
$$= 1 \text{ otherwise}$$

Conductor resistance

Finally, the ac resistance of the cable can be found:

(20-124)

$$R_{AC} = R^{dc}\left[1 + f_{pipe}\left(y_s + y_p\right)\right] \Omega / m$$

The heat generated by the conductor core is:

(20-125)

$$w_c = R_{AC} I_C^2 \text{ w/m}$$

Mutual Heating Factor

This factor is calculated and applied to a duct bank of identical cables with identical loading.

(20-126)

$$F_i = \prod_{j \neq i} \frac{d_{ij}^{'}}{d_{ij}}$$

Concentric neutral AC resistance

The resistance of the concentric neutral at 20C is:

(20-127)

$$R_{20}^{dc} = \frac{10^6 \rho_{20-cn}}{A_{cn}} \quad \Omega/\text{m}$$

A_{cn} = Area of all strands - mm^2

ρ_{20-cn} = Strand resistivity

The strand count and strand diameter are used to get:

$$R_{20}^{dc} = \frac{4\rho_{20-cn} \cdot 10^6}{\pi N_{cn} d_{cn}^2} \tag{20-128}$$

$$N_{cn} = \text{Number of strands}$$

$$d_{cn} = \text{Strand diameter}$$

$$\rho_{20-cn} = \text{Strand resistivity}$$

AC resistance for the concentric neutral is found by accounting for maximum cable temperature and lay factor. The sheath resistance at the maximum temperature is:

$$R_S = \frac{10^6 \rho_{20}}{\pi \cdot n_s \cdot r^2}\left[1+\alpha_{20}\left(\theta_{\max} - \theta_{20}\right)\right] \ \ \Omega/\text{m} \tag{20-129}$$

Tape shield AC resistance

In this case, the 'sheath' is the metallic tape shield of the conductor. The resistance is:

$$R_S = \frac{10^6 \rho_{20}}{\pi \cdot d \cdot t_s}\left[1+\alpha_{20}\left(\theta_{\max} - \theta_{20}\right)\right] \ \ \Omega/\text{m} \tag{20-130}$$

$$t_s = \text{ Tape thickness}$$

$$d = \text{ Tape diameter}$$

The resistance of the tape or the concentric neutral in the previous section is used to calculate the sheath loss factor.

Loss Factor

This factor is only calculated and used for buried sets of cables.

$$\mu = 0.3 \cdot LF + 0.7 \cdot LF^2 \tag{20-131}$$

$$LF = \text{load factor}$$

Neutral Amps

Neutral loading is expressed as a percentage of circuit phase current.

$$I_N = F_N \cdot I_{Ckt-Max} \tag{20-132}$$

Heating due to neutral current does have an impact on the ampacity of circuits. Furthermore, an undersized separate neutral may limit its circuit current if it approaches its maximum temperature. Neutral current results in heat generation. If the neutral is a separate conductor then that conductor will be a heat source within various layers of insulation. If the neutral return path is through concentric neutral strands or a tape shield then heat will be added to cables between the insulation and jacket.

Separate neutral

The dc resistance of the neutral core is:

$$R_{20}^{dc} = \frac{1.02 \cdot 10^{6} \rho_{20}}{A_c} \tag{20-133}$$

The resistance of the entire conductor at 20C, with stranding factor, is:

$$R_{cs,20}^{dc} = \frac{4 \cdot 10^{6} \rho_{20}}{\pi \cdot D_{cs}^{2} \cdot N_{cs}} \tag{20-134}$$

A linear factor is used to get the cable from 20C to operating temperature. The following relationship is used:

$$R_j^{DC} = R_j^{20}\left[1 + \alpha_{20}\left(\theta_{\max} - \theta_{\rho}\right)\right] \;\; \Omega/\text{m} \tag{20-135}$$

$\theta_{\max}$ = Maximum conductor C

θ_{ρ} = Temperature used for DC resistance

α_{20} = Temperature coefficient from 20C

Skin effect is accounted for with this factor:

$$F_k = \frac{8\pi f \cdot 10^{-7}}{R^{dc}} \tag{20-136}$$

And the skin effect Bessel function:

$$y_s = \frac{\left(F_k k_s\right)^2}{192 + 0.8\left(F_k k_s\right)^2} \tag{20-137}$$

Alternatively, the permeability of the conductor at the given frequency is captured with this Bessel function:

$$\chi_s^2 = k_s \frac{8\pi \cdot f}{R^{DC}} \cdot 10^{-7} \tag{20-138}$$

The constant k_s comes from tables and the conductor material.

The skin effect factor is found with:

$$y_s = \frac{\chi_s^4}{192 + 0.8\chi_s^4} \tag{20-139}$$

If conductors are in separate ducts:

$$y_p = 0 \tag{20-140}$$

However, three phase cables are pulled through the same duct then the following series of equations are used to yield the proximity factor:

$$\chi_p^2 = k_p \frac{8\pi \cdot 60}{R_j^{DC}} \cdot 10^{-7} \tag{20-141}$$

Again, k_p comes from tables and the conductor material.

Next:

$$y_p^{'} = \frac{\chi_p^4}{192 + 0.8\chi_p^4} \tag{20-142}$$

And:

$$y_p = y_p^{'} \left(\frac{d_c}{s}\right)^2 \left[0.312\left(\frac{d_c}{s}\right)^2 + \frac{1.18}{y_p^{'} + 0.27}\right] \tag{20-143}$$

d_c = conductor diameter (mm)

s = distance between conductors (mm)

Finally, the ac resistance of the cable can be found:

$$R_N = R^{dc}\left[1 + y_s + y_p\right] \Omega / m \tag{20-144}$$

The heat generated by the neutral conductor core is:

$$w_n = R_N I_N^2 \quad \text{w/m} \tag{20-145}$$

Tape shield

In this case, the 'sheath' is the metallic tape shield of the conductor. The resistance is:

$$R_S = \frac{10^6 \rho_{20}}{\pi \cdot d \cdot t_s}\left[1 + \alpha_{20}\left(\theta_{\max} - \theta_{20}\right)\right] \quad \Omega/\text{m} \tag{20-146}$$

t_s = Tape thickness

d = Tape diameter

The heat generated by the tape shield is.

$$w_n = R_S I_N^2 \quad \text{w/m} \tag{20-147}$$

Concentric neutral

The resistance of the concentric neutral at 20C is:

$$R_{20}^{dc} = \frac{10^6 \rho_{20-cn}}{A_{cn}} \quad \Omega/\text{m} \tag{20-148}$$

$$A_{cn} = \text{Area of all strands - mm}^2$$

$$\rho_{20-cn} = \text{Strand resistivity}$$

The strand count and strand diameter are used to get:

$$R_{20}^{dc} = \frac{4\rho_{20-cn} \cdot 10^6}{\pi N_{cn} d_{cn}^2} \tag{20-149}$$

$$N_{cn} = \text{Number of strands}$$

$$d_{cn} = \text{Strand diameter}$$

$$\rho_{20-cn} = \text{Strand resistivity}$$

AC resistance for the concentric neutral is found by accounting for maximum cable temperature and lay factor. The sheath resistance at the maximum temperature is:

$$R_S = \frac{10^6 \rho_{20}}{\pi \cdot n_s \cdot r^2}\left[1+\alpha_{20}\left(\theta_{\max} - \theta_{20}\right)\right] \quad \Omega/\text{m} \tag{20-150}$$

The heat produced by the concentric neutral is:

$$w_n = R_S I_N^2 \quad \text{w/m} \tag{20-151}$$

External thermal resistance factors

A number of factors will need to be calculated before the external thermal resistance can be determined.

The load factor is used to get:

$$\mu = 0.3 \cdot LF + 0.7 \cdot LF^2 \tag{20-152}$$

The loss factor diameter is determined from the load factor.

Cyclic loads are accounted for with a fictitious ‘loss factor’ conductor diameter. The diameter is calculated as:

If the load factor is 100%

$$D_x = D_e \tag{20-153}$$

If the load is cyclic sinusoidal:

(20-154)

$$D_x = \frac{205}{\rho_s^{0.4}\sqrt{\varpi}}$$

If it is rectangular:

(20-155)

$$D_x = \frac{493\sqrt{\mu}}{\rho_s^{0.4}\sqrt{\varpi}}$$

Otherwise it is:

(20-156)

$$D_x = \frac{103 + 246\sqrt{\mu}}{\rho_s^{0.4}\sqrt{\varpi}}$$

The factor ϖ is the number of load cycles / 24 hours.

Ducts buried in a thermal backfill require the following factors to account for thermal flow within the duct bank and between the bank and earth:

(20-157)

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi} - \frac{x}{y}\right)\ln\left(1 + \frac{y^2}{x^2}\right) + \ln\frac{x}{2}$$

x = bank width in mm

y = bank height in mm

Once has been calculated, we use the depth of the bank center to get:

(20-158)

$$u = \frac{L_G}{r_b}$$

L_G = Depth to center of bank (mm)

Finally, the factor for the rectangular geometry of the bank is calculated:

(20-159)

$$G_b = \ln\left[u + \sqrt{u^2 - 1}\right]$$

The concept of reflection about the thermal breakpoint of the earth's surface is used in ampacity calculations. The mutual heating factor accounts for the geometry of cables and their depth. It is:

(20-160)

$$F_j = \prod_{j \neq k}\left(\frac{d_{jk'}}{d_{jk}}\right)$$

If we have three cables buried flat with equal spacing then we can simplify. Assume the spacing between cables:

(20-161)

$$s = \text{distance between cables}$$

For the center cable:

$$F_{center} = \frac{\sqrt{s^2 + 4L_G^2}}{s} \cdot \frac{\sqrt{s^2 + 4L_G^2}}{s} = 1 + 4\left(\frac{L_G}{s}\right)^2 \tag{20-162}$$

For the edge cable:

$$F_{edge} = \frac{\sqrt{s^2 + 4L_G^2}}{s} \cdot \frac{\sqrt{4s^2 + 4L_G^2}}{2s} \tag{20-163}$$

T_1 – Insulation thermal resistance

In general, the thermal resistance of a cylinder is:

$$T = \frac{\rho}{2\pi} \ln\left(\frac{r_{out}}{r_{in}}\right) \tag{20-164}$$

This can be factored to get:

$$T = \frac{\rho}{2\pi} \ln\left(1 + \frac{2t}{d_{in}}\right) \quad \text{where } t = r_{out} - r_{in} \tag{20-165}$$

Like resistances in an electrical circuit, these values can be added when in series.

$$T_1 = T_{CSh} + T_{Insul} + T_{InsulSh} \tag{20-166}$$

The thermal resistance of the conductor shield is:

$$T_{CSh} = \frac{\rho_{CSh}}{2\pi} \ln\left(1 + \frac{2t_{CSh}}{d_{core}}\right) \quad \mathrm{K \cdot m/W} \tag{20-167}$$

$$t_{CSh} = \text{Thickness of conductor shield - mm}$$
$$d_{core} = \text{Core diameter - mm}$$
$$\rho_{CSh} = \text{Shield resistivity}$$

The resistance of the insulation is:

$$T_{Insul} = \frac{\rho_{Insul}}{2\pi} \ln\left(1 + \frac{2t_{Insul}}{d_{core} + 2t_{CSh}}\right) \quad \mathrm{K \cdot m/W} \tag{20-168}$$

$$t_{Insul} = \text{Thickness of insulation - mm}$$
$$\rho_{Insul} = \text{Shield resistivity}$$

The resistance of the insulation shield is:

(20-169)

$$T_{InsulSh} = \frac{\rho_{InsulSh}}{2\pi}\ln\left(1+\frac{2t_{InsulSh}}{d_{core}+2t_{CSh}+2t_{Insul}}\right)\ \mathrm{K\cdot m/W}$$

$$t_{InsulSh} = \text{Thickness of insulation shield - mm}$$

$$\rho_{InsulSh} = \text{Insulation shield resistivity}$$

The insulation thermal resistance lies between the conductor and the armor and jacket thermal resistance.

T_2 - Armor Thermal Resistance

Armor may lie between the concentric neutral strands and the jacket. The thermal insulation of the armor is expressed as:

(20-170)

$$T_2 = \frac{\rho_{Arm}}{2\pi}\ln\left(1+\frac{2t_{Arm}}{d_{cn}}\right)\ \mathrm{K\cdot m/W}$$

$$t_{Jkt} = \text{Thickness of armor - mm}$$

$$d_{cn} = \text{Diameter of concentric neutral ring - mm}$$

$$\rho_{Jkt} = \text{Armor resistivity}$$

Armor is not currently supported by SynerGEE.

T_3 - Jacket Thermal Resistance

The jacket covers the concentric neutral / bedding portion of the cable. It has a thermal insulation of:

(20-171)

$$T_3 = \frac{\rho_{Jkt}}{2\pi}\ln\left(1+\frac{2t_{Jkt}}{d_{cn}}\right)\ \mathrm{K\cdot m/W}$$

$$t_{Jkt} = \text{Thickness of jacket - mm}$$

$$d_{cn} = \text{Inside diameter jacket - mm}$$

$$\rho_{Jkt} = \text{Jacket resistivity}$$

T_{ij} - Thermal resistance between direct buried cables

The distance between cables and reflections along with the soil resistivity are used to calculate the thermal resistance for direct buried cables. In the first case, we consider cables buried directly in native soil:

(20-172)

$$T_{ij} = \frac{\rho_{soil}}{2\pi}\ln\frac{d_{ij}^{'}}{d_{ij}}(K\cdot m/W)$$

In the second case, we consider cables in a thermal backfill:

$$T_{ij} = \frac{\rho_{fill}}{2\pi} \ln \frac{d_{ij}^{'}}{d_{ij}} + \frac{N}{2\pi}\left(\rho_{earth} - \rho_{fill}\right) G_b \quad (K \cdot m / W) \tag{20-173}$$

If cables are in ducts then we will calculate the thermal resistance between the cables within the same duct. The thermal resistance between cables in separate ducts will be accounted for indirectly through the network made up of duct wall and duct bank thermal resistance values.

T_{ij} - Thermal resistance between cables in same duct

The resistance between the cable and duct will be accounted for with the 1950 linearization of the more comprehensive equations accounting for duct fill. The linearization assumes a temperature around the duct of 20C drop between the duct surface and wall (Anders, 227) and a cable having diameter from 1-4".

$$\mathrm{T_{ij}} = \frac{U}{1 + 0.1\left(V + Y\theta_{Max}\right) f_{cnt} D_e} \tag{20-174}$$

where:

$$\begin{aligned}
U, V, Y &= \text{given} \\
\theta_{Avg} &= \text{Average of cable and duct wall} \\
D_e &= \text{Outside diameter of cable} \\
f_{cnt} &= 1 \text{ for one cable in duct} \\
&= 1.65 \text{ for two cables in duct} \\
&= 2.15 \text{ for three cables in duct} \\
&= 2.50 \text{ for four cables in duct}
\end{aligned}$$

In these calculations, the average temperature within the duct will be assumed 10C less than the maximum cable temperature. The constants U, V, Y are gathered from table 9.6 in Anders. The table is limited. The "fiber duct in concrete" numbers are oftentimes selected.

The constant f_{cnt} is 1.0 for these calculations. The other values were listed for reference in other sections.

Ideally the value θ_{Avg}, is the temperature in the space between the cable and the wall of the duct bank. An iterative technique can be used to determine this temperature based on the heat rate flow through the cable jacket and the duct wall. A conservative approach would be to set $\theta_{DuctMax}$ equal to the maximum temperature of any cable within the duct. We will use 2/3 of the temperature spanning from the ambient around the duct bank to the average maximum temperature of the cables in the duct.

$$\theta_{DuctMax} = \tfrac{1}{3}\left[2\frac{\sum_{\substack{Duct\\Cables}} \theta_{i,\max}}{N_{\substack{Cable\\Duct}}} + \theta_{Amb} \right] = \frac{2\sum_{\substack{Duct\\Cables}} \theta_{i,\max} + N_{\substack{Cable\\Duct}}\theta_{Amb}}{3N_{\substack{Cable\\Duct}}} \quad \textbf{(20-175)}$$

T_A - Thermal resistance between cables and ambient

This thermal resistance accounts for the soil or engineered material between the cable and ambient earth.

First, we need the mutual heating factor:

$$F_i = \prod_{j \neq i} \frac{d_{ij}'}{d_{ij}} \quad \textbf{(20-176)}$$

For direct buried cables:

$$T_A = \frac{\rho_{soil}}{2\pi} \ln \frac{4L \cdot F}{D_e} \quad (K \cdot m / W) \quad \textbf{(20-177)}$$

Considering load factor, we have:

$$T_{A\mu} = \frac{\rho_{soil}}{2\pi}\left[\ln\frac{D_x}{D_e} + \mu \ln\frac{4L \cdot F}{D_x} \right] \quad (K \cdot m / W) \quad \textbf{(20-178)}$$

If the cables are buried in a thermal backfill then we need several steps. First:

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi} - \frac{x}{y} \right) \ln\left(1 + \frac{y^2}{x^2} \right) + \ln\frac{x}{2} \quad \textbf{(20-179)}$$

In this equation, the duct bank or thermal backfill dimensions are expressed as x and y so that the larger dimension is y. That is:

$$y > x \quad \textbf{(20-180)}$$

Also, the relationship is only valid for:

$$\frac{y}{x} < 3.0 \quad \textbf{(20-181)}$$

A rectangular duct bank or backfill must have a longer dimension within three times the shorter dimension. If this rule is not met, the shorter dimension should be assumed to be 1/3 the longer dimension.

(20-182)

$$u = \frac{L_G}{r_b}$$

(20-183)

$$G_b = \ln\left[u + \sqrt{u^2 - 1}\right]$$

(20-184)

$$T_A = \frac{\rho_{fill}}{2\pi} \ln \frac{4L \cdot F}{D_e} + \frac{N}{2\pi}\left(\rho_{earth} - \rho_{fill}\right) G_b \ \ (K \cdot m / W)$$

Using load factor we have:

(20-185)

$$T_{A\mu} = \frac{\rho_{fill}}{2\pi}\left(\ln \frac{D_x}{D_e} + \mu \ln \frac{4L \cdot F}{D_x}\right) + \mu \frac{N}{2\pi}\left(\rho_{earth} - \rho_{fill}\right) G_b \ \ (K \cdot m / W)$$

Again, these thermal resistance values are for cables that are not in ducts.

TD_{ij} - Thermal resistance between ducts

If the ducts are direct buried, the thermal resistance is:

(20-186)

$$TD_{ij} = \frac{\rho_{soil}}{2\pi} \ln \frac{d'_{ij}}{d_{ij}} (K \cdot m / W)$$

If the ducts are in a backfill or duct bank then several steps are needed to determine the thermal resistance. First, the dimensions of the envelope are used to get:

(20-187)

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi} - \frac{x}{y}\right) \ln\left(1 + \frac{y^2}{x^2}\right) + \ln \frac{x}{2}$$

And then:

(20-188)

$$u = \frac{L_G}{r_b}$$

This allows the calculation of the geometry factor:

(20-189)

$$G_b = \ln\left[u + \sqrt{u^2 - 1}\right]$$

Finally, the thermal resistance between two ducts is:

$$TD_{ij} = \frac{\rho_{fill/concrete}}{2\pi} \ln \frac{d'_{ij}}{d_{ij}} + \frac{G_b}{2\pi}\left(\rho_{earth} - \rho_{fill/concrete}\right)\ (K \cdot m / W) \tag{20-190}$$

$\rho_{fill/concrete}$ = Thermal resistivity of fill

ρ_{earth} = Thermal resistivity of earth

d_{jk} = Distance between duct 'j' and 'k'

d'_{jk} = Distance between duct 'j' and reflection of 'k'

We will account for load factor with:

$$TD_{ij} = \frac{\rho_{fill/concrete}}{2\pi}\left(\ln \frac{D_x}{d_{ij}} + \mu \ln \frac{d'_{ij}}{D_x}\right) + \mu \frac{G_b}{2\pi}\left(\rho_{earth} - \rho_{fill/concrete}\right)\ (K \cdot m / W) \tag{20-191}$$

TD$_A$ - Thermal resistance between duct and ambient

A fictional conductor diameter representing load cycles and soil heat transfer is given as:

$$D_X = 211mm \tag{20-192}$$

Again, the concept of reflection about the thermal breakpoint of the earth's surface is used in ampacity calculations. The mutual heating factor accounts for the geometry of cables and their depth. It is:

$$F_j = \prod_{j \neq k}\left(\frac{d_{jk'}}{d_{jk}}\right) \tag{20-193}$$

The loss factor is calculated from the load factor:

$$\mu = 0.3 \cdot LF + 0.7 \cdot LF^2 \tag{20-194}$$

LF = load factor

For direct buried ducts:

$$T_i = \frac{\rho_{earth}}{2\pi}\left(\ln \frac{D_X}{D_O} + \mu \ln \frac{4L_i \cdot F}{D_X}\right)(K \cdot m / W) \tag{20-195}$$

For ducts in a backfill or concrete we use the envelope dimensions go get:

$$\ln r_b = \frac{x}{2y}\left(\frac{4}{\pi} - \frac{x}{y}\right)\ln\left(1 + \frac{y^2}{x^2}\right) + \ln \frac{x}{2} \tag{20-196}$$

And then:

(20-197)

$$u = \frac{L_G}{r_b}$$

The geometry factor is:

(20-198)

$$G_b = \ln\left[u + \sqrt{u^2 - 1}\right]$$

Finally, the thermal resistance to ambient earth is:

(20-199)

$$T_i = \frac{\rho_{fill}}{2\pi}\left(\ln\frac{D_x}{D_e} + \mu \ln\frac{4L \cdot F}{D_x}\right) + \mu\frac{1}{2\pi}\left(\rho_{earth} - \rho_{fill}\right) G_b \ \ (K \cdot m / W)$$

W_d - Dielectric heat loss

Two parameters are based on cable insulation:

(20-200)

$$\varepsilon = f\left(Z_{IT}\right)$$
$$\tan\delta = f\left(Z_{IT}\right)$$

The capacitance of the cable is calculated as:

(20-201)

$$C = \frac{\varepsilon}{18\ln\left(\frac{D_i}{d_c}\right)} \cdot 10^{-9} \ \ F/m$$

And dielectric loss is calculated as:

(20-202)

$$W_d = 2\pi f \cdot \tan\delta \cdot \frac{kV_{LL}^2 \cdot C}{3} \cdot 10^6$$

λ_1 - Concentric neutral loss factor

Concentric Neutral

Concentric neutral resistance at 20C is:

(20-203)

$$R_{20}^{dc} = \frac{10^6 \rho_{20-cn}}{A_{cn}} \ \ \Omega/\text{m}$$
$$A_{cn} = \text{Area of all concentric neutral strands - mm}^2$$
$$\rho_{20-cn} = \text{Strand resistivity}$$

We can now apply the area of a strand and the number of strands to get the total resistance. The strand count and strand diameter are used to get:

$$R_{20}^{dc} = \frac{4\rho_{20-cn} \cdot 10^6}{\pi N_{cn} d_{cn}^2} \tag{20-204}$$

$$N_{cn} = \text{Number of strands}$$

$$d_{cn} = \text{Strand diameter}$$

$$\rho_{20-cn} = \text{Strand resistivity}$$

The effects of strand lay, AC, and temperature are accounted for to get:

$$R_{cn} = \frac{4\rho_{20-cn} \cdot 10^6}{\pi N_{cn} d_{cn}^2}\left[1+\alpha_{20}\left(\theta_{Max}-20\right)\right]\sqrt{1+\left(\frac{\pi D_{cn}}{l_{cn}}\right)^2} \tag{20-205}$$

The reactance of a cable is estimated as:

$$X = 4\pi f \cdot 10^{-7} \ln\frac{GMS}{d_{cn}} \tag{20-206}$$

$$d_{cn} = \text{Strand diameter - mm}$$

$$\text{GMS} = \text{Geometric average distance between circuit cables - mm}$$

The loss factor is calculated as:

$$\lambda_1 = 1.25 \cdot \frac{R_{cn}}{R} \cdot \frac{1}{1+\left(\frac{R_{cn}}{X}\right)^2} \tag{20-207}$$

λ_1 - Tape shield loss factor

Tape shield resistance at 20C is:

$$R_{20}^{dc} = \frac{10^6 \rho_{20-cn}}{A_t} \quad \Omega/\text{m} \tag{20-208}$$

$$A_t = \text{Area of tape - mm}^2$$

$$\rho_{20-cn} = \text{Tape resistivity}$$

The area of the tape is:

$$A_t = \frac{\pi}{4}\left[d_{tape}^2 - \left(d_{tape} - 2t_{tape} \right)^2 \right] \tag{20-209}$$

$$\begin{aligned} A_t &= \text{Area of tape - mm}^2 \\ d_{tape} &= \text{Diameter to outside of tape} \\ t_{tape} &= \text{Tape thickness} \end{aligned}$$

This reduces to:

$$A_t = \frac{\pi}{2}\left(d_{tape} t_{tape} - 2t_{tape}^2 \right) \tag{20-210}$$

Now the dc resistance can be determined as:

$$R_{20}^{dc} = \frac{2 \cdot 10^6 \rho_{20-cn}}{\pi \left(d_{tape} t_{tape} - 2t_{tape}^2 \right)} \quad \Omega/\text{m} \tag{20-211}$$

AC, and temperature effects are accounted for to get:

$$R_{cn} = \frac{2 \cdot 10^6 \rho_{20-cn}}{\pi \left(d_{tape} t_{tape} - 2t_{tape}^2 \right)} \left[1 + \alpha_{20} \left(\theta_{Max} - 20 \right) \right] \tag{20-212}$$

The reactance of a cable is estimated as:

$$X = 4\pi f \cdot 10^{-7} \ln \frac{GMS}{d_{cn}} \tag{20-213}$$

$$\begin{aligned} d_{tape} &= \text{Diameter to tape - mm} \\ \text{GMS} &= \text{Geometric average distance between circuit cables - mm} \end{aligned}$$

The loss factor is calculated as:

$$\lambda_1 = 1.25 \cdot \frac{R_{cn}}{R} \cdot \frac{1}{1 + \left(\frac{R_{cn}}{X} \right)^2} \tag{20-214}$$

Categories & parameters

Cable calculations have many variations. The categories listed in this section are used to differentiate parameters, models, and calculations associated with a particular cable installation.

Duct Bank Type

Direct buried (with or without backfill)
Duct bank (with concrete or fill)

Cable Types

Single Conductor
Concentric Neutral
Tape shield

Conductor Material

Conductor Material	Resistivity ρ_{20}	Temp. Coeff. α_{20}
	$\Omega \cdot m$	$1/°C$
Copper	1.7241x10^{-8}	3.93x10^{-3}
Aluminum	2.8264 x10^{-8}	4.03 x10^{-3}

Sheath material coefficients at 20C:

Sheath Material	Resistivity ρ_{20}	Temp. Coeff. α_{20}
	$\Omega \cdot m$	$1/°C$
Copper	1.7241x10^{-8}	3.93x10^{-3}
Aluminum	2.84 x10^{-8}	4.03 x10^{-3}
Lead	21.4 x10^{-8}	4.0 x10^{-3}
Steel	13.8 x10^{-8}	4.5 x10^{-3}
Bronze	3.5 x10^{-8}	3.0 x10^{-3}
Stainless	70. x10^{-8}	0.00

Insulation Thermal Resistances

Insulation Type	Thermal Resistivity $\rho\ (K \cdot m / W)$
Paper	6.0
Butyl rubber	5.0
EPR	4.0
PVC	5.5
PE	3.5
XLPE	3.5
PPL	6.5

Jacket Thermal Resistances

Jacket Type		Thermal Resistivity $\rho\ (K \cdot m / W)$	Capacity *10^{-6} $J / m^3 K$
Z_{JM} - Fibre	Fibre	6.0	2.0
Z_{JM} - Rubber	Rubber	6.0	2.0
Z_{JM} - PlyClPr	Polychloroprene	5.5	2.0
Z_{JM} - PVC	PVC	5.0	1.7
Z_{IT} - PE	PE	3.5	2.4

Dielectric constants

Insulation Type	ε	$\tan\delta$
Paper	4.0	0.01
Butyl rubber	4.0	0.05
EPR	3.0	0.02
PVC	8	0.1
PE	2.3	0.001
XLPE	2.5	0.004
PPL	2.8	0.001

AC factors:

Conductor type	k_s	k_p

Copper	1	1
Aluminum	1	1

Duct thermal constants

Duct Material	Thermal Resistivity $\rho \quad (K \cdot m / W)$
Concrete	1.0
Fibre	4.8
PE	3.5
PVC	6.0
Steel / Metal	0.0
Transite	6.0
Asbestos	2.0
Earthenware	1.2

Duct Bank thermal constants

Duct Material	Thermal Resistivity $\rho \quad (K \cdot m / W)$
Native Ground	Soil Resistivity Table
Concrete	1.0
Loam	1.2
Control Density Fill	1.5
Clay	2.0
Sand	3.0
Gravel	3.5

Soil Resistivity

Soil Type	Thermal Resistivity ρ_{soil} $(K \cdot m / W)$
Wet	0.25
Moist	0.5
Normal1	0.7
Normal	0.9
Normal2	1.0
Normal3	1.2
Dry	1.5
Arid	2.5
Sandy	3.5
Rocky	4.5

Backfill Resistivity

Backfill Type	Thermal Resistivity ρ_{soil} $(K \cdot m / W)$
Spongy soil	0.5
Soil	0.75
Normal	0.9
Loam	1.0
Sandy loam	1.5
Sand / Clay	2.0
Gravel / Clay	2.5
Sand	3.0
Gravel	3.5
Crushed quarry	4.0

Duct installation constants

Z_{UM}	Z_{DT}	U	V	Y
Fibre, Asbestos	Concrete, earthen	5.2	0.91	0.010
Fibre, Asbestos	Tunnel	5.2	0.83	0.006
Steel	All	5.2	1.4	0.011
Earthenware	All	1.87	0.28	0.0036
None, Concrete, PE, PVC, Transite	Concrete, earthen	5.2	1.1	0.011
None, Concrete, PE, PVC, Transite	Tunnel	5.2	1.1	0.011

Study Parameters

Parameter and description		Units
f	Frequency	Hz

Duct Parameters

Parameter and description		Units
X_d	X coordinate of duct	m
Y_d	Y coordinate of duct	m
D_o	External diameter of duct	mm
t_{duct}	Internal diameter of duct	mm

Cable Parameters

			Cable Types - Z_{CT} =		
Parameter and description		Units	Single Conductor	Concentric Neutral	Tape Shield
Z_{cm}	Conductor material	-	✓	✓	✓
Z_{ct}	Type of conductor	-	✓	✓	✓
$\theta_{\max}$	Maximum conductor temperature	C	✓	✓	✓
R^{dc}	DC Resistance at 20 C	Ω/m	✓	✓	✓
d_c	Diameter of conductor	mm	✓	✓	✓
t	Insulation thickness between conductors	mm			
t_1	Insulation thickness between conductors and sheath or conductor and jacket.	mm	✓	✓	✓
t_s	Thickness of sheath	mm	✓	✓	✓
t_j	Thickness of jacket	mm	✓	✓	✓
d_x	Diameter of equivalent circular conductor having same area and compactness as shaped one	mm			
s	Average spacing between conductors of same circuit	mm	✓	✓	✓
D_i	Diameter over insulation	mm	✓	✓	✓
D_s	Diameter of sheath	mm	✓	✓	✓
D_e	External diameter of cable	mm	✓	✓	✓
D_{cn}	Diameter of concentric neutral ring	mm		✓	
d_{cn}	Diameter of concentric neutral strand			✓	

Circuit Parameters

Parameter and description		Units
kV_{LL}^2	Circuit line-line voltage	kV
Lf	Circuit load factor	-

Derived Values

Parameter and description		Description
D_x	External loss diameter	$D_x =$ $61200\sqrt{\text{soil diffusivity} \cdot \text{load cycle hours}}$ $= 211mm$ for 24 hr cycle and $\delta = 0.5x10^{-6}\, m^2/s$
$\theta_{DuctMax}$	Duct internal temperature	Maximum temperature over all cables within a duct.

21

Switching Analysis

Contingency analysis

Contingency analysis simulates the loss or fault of a section(s) or a bus, and then searches for a switching recovery plan based on an objective you specify. The application is intended to provide useful information to support a contingency switching plan.

Application operation

The following basic process is used in the analysis.

1. Analyze the base model without a contingency.
2. Create an outage with a contingency or multiple contingencies. The previously fed sections involved in the outage are known as the "outage set."
3. Isolate the contingency, if it is treated as a "fault type."
4. Open all closed switches in outage set.
5. This action allows smaller increments of load to be picked up when switch closing is begun. Without this action, many switch closings to pick up unfed sections would result in overloading and possibly models with infeasible load-flow solutions.
6. Close switches to pick up the outage, according to the specified objective.
7. The objective has a significant impact on the recommended switching sequence.

The application only considers contingencies within the selected feeders, but uses adjacent non-selected feeders in the analysis for a switching recovery plan. Any switch in an adjacent feeder may be used to pick up unfed sections.

Objectives

When generating the switching recovery plan, the application considers one of the following objectives.

- **Pick up most sections**—Switches are selected so that closing them results in a feasible model with the most sections being fed.
- **Pick up most load**—Switches are selected so that closing them results in a feasible model with the most kVA load being fed.
- **Pick up most customers**—Switches are selected so that closing them results in a feasible model with the most customers being fed. This option requires customer information in the section records.
- **Pick up to keep exceptions minimal**—Switches are selected so that closing them results in a feasible model with the least total number of exceptions.
- **Pickup to maximize low voltage**—Switches are selected so that the lowest voltage of a previously outaged section is as high as possible.
- **Pickup to minimize losses**—Keep overall losses as low as possible during pickup.

Contingency analysis finds switching solutions in a cumulative manner. Beginning with the model "as is," it finds one switching operation to improve the objective. It then continues with more switching operations until the objective can no longer be improved. This process is generally known as local optimization because the objective is improved with each step. The alternative, called global optimization, can produce different results and require a degradation in the objective for one or more switching operations. It can also result in a solution that is not practical for a line crew and not achievable without imposing even more outages than were seen by the original contingency.

Contingency types

SynerGEE can create three different types of contingency situations.

- **All Together**—All sections marked as contingency sections are "taken out" at once, and a switching recovery plan is sought.
- **One at a Time**—Each section marked as a contingency section is taken out, one at a time. With each outage, contingency analysis looks for a separate switching plan and all contingencies are summarized in a report.
- **Selected Section**—Only the section that is set for analysis is outaged, and sections marked as contingency sections are treated normally.

Remember that marking a section as a contingency section is different than setting a section for analysis. You can mark a section as a contingency section using the section editor, and once applied, that setting is stored in the database (InstSections table). Any sections marked in this fashion apply to the first two types of analysis listed above.

To set a section for analysis, you should right-click on that section on the map, and select Set for Analysis from the popup menu. In order to use the Selected Section type of contingency analysis, you must set a section for analysis in this manner.

Outage types

Before you run the analysis, you can choose the type of outage you want to result when the contingency section(s) is taken out.

- **Node (buss) outage**—The bus or source node of the contingency section is considered out. Any other sections fed by that node are also considered out.
- **Branch (section) outage**—The contingency section is considered out, but the source node remains fed.

In addition, you can have the contingency simulated as a fault, in which case SynerGEE first isolates the contingency area by opening all necessary switches. In this case, the switching recovery plan will not include the closure of any switches that would feed power to the contingency area.

Examples of outage types

Consider this simple model, in which <u>section "4" only</u> is marked for contingency.

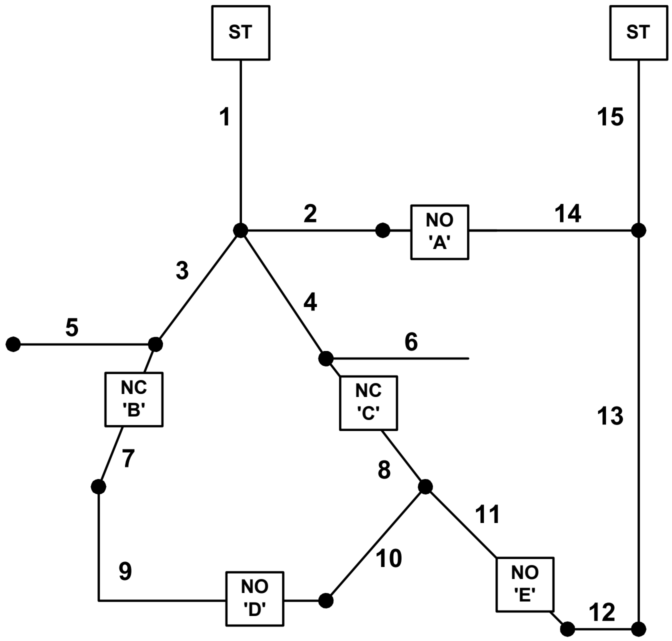


Fig. 21-1 *Example model*

The following would be the analysis results for the different outage types.

- Branch contingency – Don't isolate
- Section 4 is taken out, but the source node is still fed. Sections 4, 6, 8, 10, and 11 are in the outage. Switch E or switch D could be used to pickup the outage sections. Sections 4, 6, and 8 will be picked up with switch C if possible.
- Branch contingency – Isolate as a fault
- Section 4 is considered faulted, and the area is isolated by opening switches B and C, and locking open switch A. The only section that can be recovered is section 9, by closing switches E and D.
- Node contingency – Don't isolate
- The source node of section 4 is taken out, and sections 3, 4, 2, and all downstream sections are outaged. Switches A, B, and C can be used to pick up the outage.
- Bus contingency – Isolate as a fault
- A fault is assumed at the source node of section 4, and the area is isolated by opening switches B and C, and locking open switch A. The only section that can be recovered is section 9, by closing switches E and D.

Constraints

The application is constrained by the requirement to improve a single objective. In addition, voltage and loading constraints prohibit any switching operation which would cause:

- A voltage below the specified minimum.
- A section, transformer, or regulator loading above the maximum.
- A fuse or protective device loading above the maximum.

A number of switching options are available at each iteration. A switch is selected if:

- It is an automatic switch and preference is given to automatic switches.
- The switch was closed in the pre-outage model and "minimize switch operations" was selected.
- It produces the maximum progress towards the selected objective.

Protective devices

Contingency analysis can operate fuses, reclosers, sectionalizers, and breakers as switchable devices. These devices can be individually excluded from consideration during contingency analysis, like switches. This exclusion is done through a check box on the respective device editor. Or, you can globally exclude all protective devices from analysis with an option in your contingency analysis settings.

Whether used for switching or not, protective devices are evaluated for emergency loading constraints during switching operations. Device loading exceeding the load limit in the analysis will disallow a switching operation.

Automatic and auto-transfer switches

If a switch is marked as an automatic switch and contingency analysis is set up to Prefer Automatic Switches, automatic switches are closed to pick up load at the earliest opportunity, provided a constraint is not violated. The selected objective for contingency analysis is not considered, and other switching operations may have resulted in a larger pickup. If multiple automatic switches can be operated, the specified objective is used to select among them.

Likewise, auto-transfer switches are toggled to pick up respective load at the earliest opportunity during contingency analysis. Loads in the contingency area served by auto-transfer switches are switched to an unaffected feeder immediately, if possible. Auto-transfer switches are evaluated after each contingency selection to determine if their unfed load can be switched to another feeder.

Switch position files

Contingency analysis can produce SSP (Stoner Switch Position) files of suggested recovery plans. These XML files contain all the switch positions recommended by the analysis, and can be loaded directly into a SynerGEE model. When loaded, all switchable devices in the model are configured according to the switch positions file.

Switch position files from contingency analysis allow you to load the suggested recovery plan into the model after the analysis, and evaluate it directly. For example, consider the following useful procedure.

1. Save the base switch positions to a separate switch positions file before the analysis (File > Switch Positions > Save). This step preserves the original model switching configuration.
2. Run contingency analysis with the option to save switch position files enabled.
3. After the analysis, load the switch positions file from the analysis into your model.
4. Evaluate the solution.
5. Load the switch positions file from Step 1 back into the model to restore the original switching configuration.

This procedure allows you to study the recovery plan in more detail than the report might allow, or if you want to use the plan with other applications.

Optimal switching

SynerGEE's optimal switching application is a powerful tool that helps you find the best operating state for feeder switches. The application accounts for exceptions, low voltages, demands, and other objectives. It is easy to use and produces a clear and understandable summary report with suggested switching operations.

Application operation

The optimal switching tool finds the local extreme for a single objective. The analysis starts with a base load-flow run with switches in their pre-analysis state. From all available switching pairs it finds the switching pair that would result in the best evaluation of the objective. A switching pair is one open switch and one closed switch combination. The tool performs the switching operation, runs a new load-flow, then repeats the process until no switching pairs result in an improvement of the objective. As such, the operations are cumulative.

The SynerGEE approach, called local optimization, reaches a solution through successively realizable states. SynerGEE avoids a second approach, known as global optimization, in which the objective may be found by initially opening all switches and then reconnecting the system. Global optimization tends to lead to theoretical results that are not very practical, for the following reasons.

- Getting from the base state to a global optimum state may require cumbersome switching plans and contingency studies.
- A global state may be unachievable without causing an outage.
- Reaching a global state can be expensive in terms of line crew time.

A clear benefit of the local method used by optimal switching is the ease of actual implementation of the results. You can apply results in the model or in the actual system in a step-by-step fashion, without losing any loads or customers. You may implement as many recommendations as desired. For instance, if a report lists eight switching operations, you may look at the various items within the report and decide to apply only the first three operations. Keep in mind that the results are generated in a cumulative manner according to performance, and should be implemented in order. If you had eight recommended operations, you could choose to implement operations 1, 2, and 3, but implementing 1, 3, and 7, for instance, would produce unpredictable results, irrelevant to the analysis you performed.

Optimal switching does not make changes to the network model; it simply reports recommendations. All switching operations involve all phases of the relative sections. The analysis includes all selected feeders. The program ignores switches tied to feeders that are not selected in the current view.

A feeder "owns" all sections to which it is electrically tied. Therefore, a section's feeder may change many times during an analysis.

Objectives

Optimal switching may be set up to toggle switches to achieve one of the following objectives. These objectives are independent and evaluated for all feeders selected within the view used to invoke the application.

- **Minimize losses**—Switches are toggled to reduce the total kW loss for all selected feeders.
- **Improve lowest voltage**—Switches are toggled in order to obtain the best <u>low</u> voltage for all sections in all feeders being analyzed. The section having the lowest voltage may change from the original run and the summary as successive switching operations are evaluated and performed.

- **Minimize number of exceptions**—Loading and voltage exception counts are minimized through switching operations. The total count from all feeders being analyzed is considered.
- **Minimize feeder demand**—This selection allows optimal switching to find the lowest total kVA demand for all feeders being analyzed. Since the total for all feeders is considered, the demand for particular feeders could conceivably increase and be offset by a reduction in the demand on other feeders.
- **Minimize feeder kVA imbalance**—Percent kVA imbalance calculations are made for each feeder. Switching operations are performed to reduce the average of all feeder imbalance values.
- **Minimize substation transformer loading**—Switching operations are made to reduce the total loading on all substation transformers. If no substation transformers are in the selected feeders, loading on all primary transformers is considered.
- **Equate loading**—Switches are toggled to move load so that the total load is distributed more evenly among the feeders.

The analysis results table lists values for all of these options, regardless of which objective was selected for optimization.

Constraints

The application is constrained by the requirement to improve the single objective. It also recognizes a maximum number of switching operations, which you can specify.

Voltage and loading constraints are in place to disallow any switching operation that would result in a device loading exceeding the loading limit or a section voltage falling under the voltage limit.

Application output

SynerGEE's optimal switching tool produces a report with the components of a load-flow analysis, including a standard heading and message summary. It also produces a detailed load-flow report and switch summary.

Switching Plan to Reduce Losses

Step	Open Switch		Close Switch		Low	Total	Total	Average	SubTran	Number	Loading
Num	Id	Section	Id	Section	Volts (120V)	Loading	Losses	Imbalance	Loading	Exceptions	Variance
		Base Run			104.2	16876	643.01	3.3%	44.6%	25	17.2
1	SW 217 N-1	217 N-1	SW P7	P7	104.1	16866	631.34	12.2%	44.7%	25	14.4
2	Switch 23LL	23LL	SW CC9	CC9	104.3	16856	624.03	12.4%	44.7%	19	14.9

Fig. 21-2 *Switching plan sub-report*

In this part of the report, you can review the improvements made by the switching operations, as well as the performance of other items like total loading, exceptions, low voltage, and transformer loading.

Application speed

Constraints and objectives evaluated by optimal switching are very sensitive. Switching cases are brought to full convergence if possible, which could take some time. Therefore, selection of iteration limits and convergence tolerance are important to application performance. To keep analysis time to a minimum, the following are some suggested settings.

Suggested optimal switching settings

Iteration Limit	12 to 20 (higher values will slow the application)
Convergence Tolerance	0.5 to 0.1 (lower values will slow the application)

Auto-transfer switch analysis

This power tool allows you to see the impact that the toggling of auto-transfer switches will have on your system. Essentially, it analyzes a set of feeders tied to critical loads through auto-transfer switches. The resulting report has two tables. The first presents the feeder loading, low voltage, and maximum loading of all feeders affected by auto-transfer switches under normal conditions, then with all auto-transfer loads connected, and finally with all loads disconnected.

Feeder Minimun / Maximum Loading Report

Load Condition	kVA	Amps A	B	C	Bal	Min Volts	Max PctLd
Feeder New - Big Spring (1 NC and 1 NO ATS)							
Normal	11623	488.4	504.1	483.4	492.0	118.2	92.8%
Minimum	10077	423.1	438.7	417.9	426.6	118.5	80.5%
Maximum	12500	526.5	540.8	520.0	529.1	118.1	99.8%
Feeder New - Cove (1 NC and 0 NO ATS)							
Normal	6214	259.5	259.3	270.3	263.0	122.1	49.6%
Minimum	5217	217.4	216.2	229.0	220.8	122.2	41.7%
Maximum	6214	259.5	259.3	270.3	263.0	122.1	49.6%
Feeder New - Liberty (0 NC and 1 NO ATS)							
Normal	7754	332.2	331.7	320.7	328.2	122.0	61.9%
Minimum	7754	332.2	331.7	320.7	328.2	122.0	61.9%
Maximum	9306	397.9	397.4	386.4	393.9	121.9	74.3%

Fig. 21-3 Feeder information

The second table in the report lists each auto-transfer switch pair. The load, low voltage, and maximum loading on the feeders associated with each switch are given for the normal switch positions and the toggled switch positions.

Automatic Transfer Switch Report

Status	NO Switch Feeder							NC Switch Feeder						
NC: Switch 00191 on 20245 - NO: Switch 00190 on 00190														
	Feeder New - Big Spring							Feeder New - Liberty						
		Amps				Min	Max	Fdr	Amps				Min	Max
	kVA	A	B	C	N	Volts	PctLdg	kVA	A	B	C	N	Volts	PctLdg
Normal	11623	488.4	504.1	483.4	492.0	118.2	92.8%	7754	332.2	331.7	320.7	328.2	122.0	61.9%
Switched	10077	423.1	438.7	417.9	426.6	118.5	80.5%	9306	397.9	397.4	386.4	393.9	121.9	74.3%
NC: Switch 33907 on 33907 - NO: Switch 76213 on 76213														
	Feeder New - Cove							Feeder New - Big Spring						
		Amps				Min	Max	Fdr	Amps				Min	Max
	kVA	A	B	C	N	Volts	PctLdg	kVA	A	B	C	N	Volts	PctLdg
Normal	6214	259.5	259.3	270.3	263.0	122.1	49.6%	11623	488.4	504.1	483.4	492.0	118.2	92.8%
Switched	5217	217.4	216.2	229.0	220.8	122.2	41.7%	12500	526.5	540.8	520.0	529.1	118.1	99.8%

Fig. 21-4 Auto-transfer switch information

These two tables provide a feeder level and switch level review of the impact of auto-transfer switches on feeder loading

22 Harmonics

Introduction

Harmonic analysis is designed to aid in distribution planning by helping you spot potential problems with resonant frequencies. The application looks at a particular section and presents the driving point impedance as a function of frequency or harmonic order.

Harmonic analysis performs a full network analysis of the feeder. Loads, capacitors, generators, transformers, regulators, and line models contribute to the harmonic driving point impedance of a section. These devices may be upstream or downstream for the section being analyzed, and they may be on or off the feeder path.

SynerGEE's harmonic analysis is a powerful and unique tool for distribution system planning and operations studies because:

- It requires no additional data beyond that needed for SynerGEE load-flow and fault analysis.
- It can handle very large distribution system models.
- It uses fully detailed models. At fundamental frequency, these models correspond to the models used in load-flow analysis.
- It performs by-phase analysis. It also takes into account complex interconnections and coupling.
- Analysis is performed at a high resolution (intervals of 0.25 fundamental frequency).
- You can choose to present results summarized as positive and zero sequence values in the sequence domain or on a phase domain phase-by-phase basis.
- It is robust and simple to use.

Harmonic analysis is a tool to help you find possible harmonic problems through the presentation of driving point impedance values. The application does not model filters, harmonic bus voltages, or harmonic load currents.

All devices, with the exception of switches and protective devices, have a possible impact on the harmonic impedance scan on a section. Consider the following feeder.

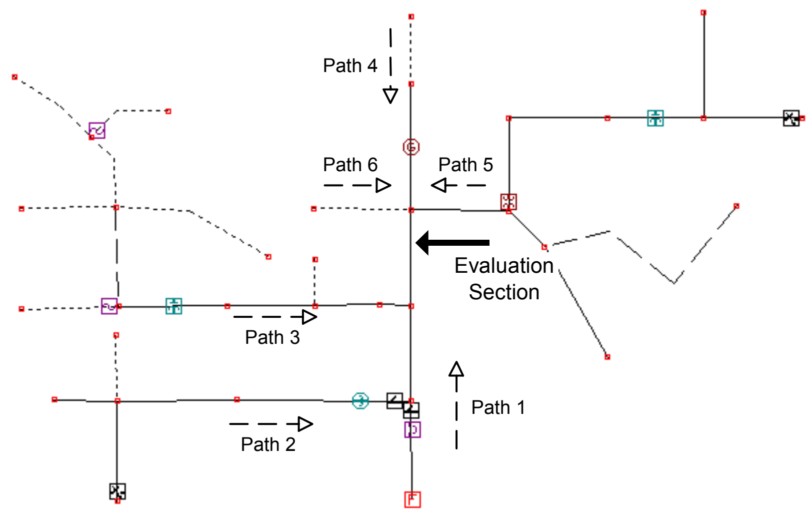


Fig. 22-1 *Feeder with a section marked for analysis*

In a radial fault analysis, Path 1 or the path between the evaluation section and the feeder source would be used along with the feeder source impedance to determine the driving point impedance for fault analysis. This is valid because a balanced model is assumed and loads do not contribute to fault current levels.

Harmonic analysis is different. In the figure, each of the five general paths has to be evaluated in detail. Loads, capacitors, charging admittances, and transformer winding conductances all represent a path to ground and a harmonic impedance path for the section being evaluated. The feeder source impedance, the capacitors on paths 3 and 5, the regulator on path 2, the transformer on path 5, the generator on path 4, and any large loads on path 6 all play a significant role in the resonant frequency at the evaluation section. The calculation of the driving point impedance is performed in detail and in the phase domain.

Example use of harmonic analysis

In this example, imagine that you have been informed of a growing number of complaints about power line noise. You find that the calls refer to a general area on a particular feeder. You load the feeder model into SynerGEE and run harmonic analysis, which produces the following graph.

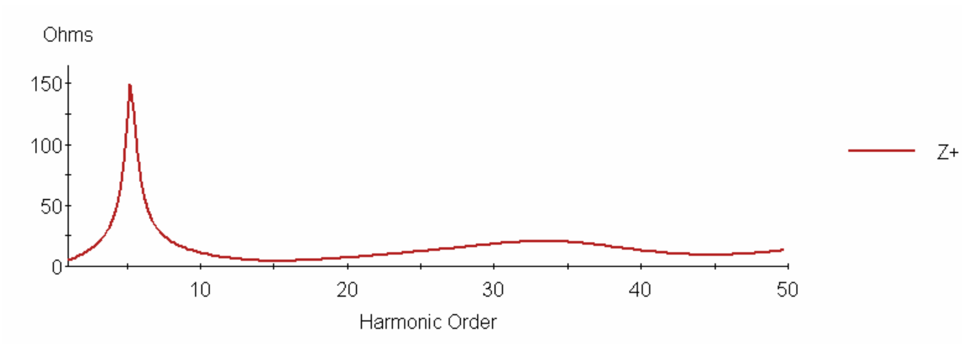


Fig. 22-2 *Analysis of existing feeder*

The graph suggests the possibility of 5th harmonic problems on the circuit. After further investigation into the feeder, large customers, and field measurements, you decide to relocate a capacitor bank serving the feeder.

After you relocate the capacitor bank in SynerGEE, harmonic analysis produces the following graph on the same section.

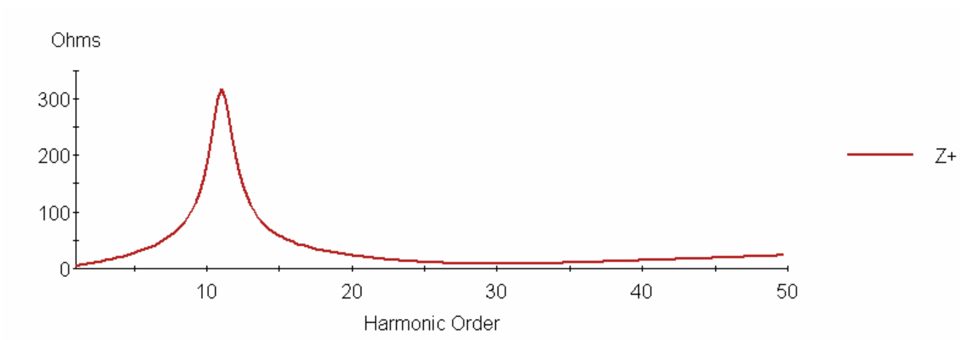


Fig. 22-3 *Analysis after relocation of capacitor bank*

This action has shifted the resonant frequency for that area of the feeder. When applied to your actual feeder, line noise complaints should no longer be received following the physical relocation of the capacitor.

Comparison to resonant harmonic from fault studies

SynerGEE fault analysis studies produce the cumulative impedance from a line section to the feeder source. This value is used to approximate the resonant frequency seen from the terminals of a capacitor. Therefore, near a capacitor, resonant frequency is approximately:

$$h_r \cong \sqrt{\frac{1000 * kV_{ll}^2}{kvar_{cap} X_{sc}}} \quad \textbf{(22-1)}$$

The harmonic predicted by this formula should match the results of harmonic analysis fairly closely.

The example that follows shows a fault analysis on a sample feeder. The following values were used in the above equation.

$$h_r \cong \sqrt{\frac{1000*(13.2)^2}{1200*10.87}} = 3.7 \tag{22-2}$$

As an alternative to the plot, you could verify the results by running harmonic analysis and having SynerGEE generate an HTML report. For example:

Harmonic Analysis of Section 1A01

	Zero Sequence			Pos Sequence		
h	R	X	|Z|	R	X	|Z|
1.00	13.91	19.63	24.06	8.9	11.4	14.5
1.25	18.51	26.20	32.08	11.0	14.9	18.5
1.50	25.61	34.12	42.66	13.8	18.8	23.3
1.75	37.64	43.55	57.56	17.7	23.4	29.3
2.00	59.86	52.83	79.84	23.4	28.8	37.1
2.25	101.16	50.46	113.04	32.5	34.8	47.6
2.50	147.37	-0.49	147.37	47.7	40.3	62.4
2.75	120.95	-76.37	143.05	73.1	39.7	83.2
3.00	65.37	-90.59	111.71	105.9	16.8	107.2
3.25	34.81	-78.32	85.70	112.0	-35.6	117.5
3.50	20.45	-65.02	68.16	77.3	-68.8	103.5
3.75	13.36	-54.54	56.15	44.9	-70.0	83.2
4.00	9.67	-46.52	47.51	26.6	-61.5	67.0

Fig. 22-4 *Portion of harmonic analysis report showing resonant point*

The analysis calculates harmonic impedance values at increments of one-fourth the fundamental frequency. According to this report, the resonant frequency lies somewhere between 3.00 and 3.5. In this case, you may need to be concerned about 3rd harmonic problems.

If you reduce the size of your capacitor to 300 kvar, you would expect a resonance at:

$$h_r \cong \sqrt{\frac{1000*(13.2)^2}{300*10.87}} = 7.3 \tag{22-3}$$

The plot from SynerGEE appears as follows:

Fig. 22-5 *Graph showing predicted harmonic problem*

The plot and the calculation indicate that a resonant frequency exists close to the 7^{th} harmonic. The graph also shows a second resonant point due to another capacitor located on the same feeder.

Harmonic analysis can analyze any three-phase section with or without a capacitor. However, when you analyze a section with a capacitor, you must recognize the line impedance between the capacitor and the end of the section.

Harmonic analysis at capacitor terminals

Harmonic impedance values are calculated as driving point values looking into the load end of three-phase sections. This can create some confusion regarding the asymptotic behavior of the point of interest.

Two examples are shown below. The approximate variation of line impedance with frequency can be given as a function of harmonic order. For example:

$$Z_{Line}(h) = R_{Line}\sqrt{h} + jX_{Line}h \quad \textbf{(22-4)}$$

Example looking at capacitor terminals

The picture below represents a feeder tied to a capacitor.

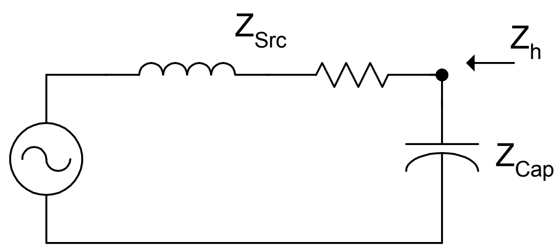


Fig. 22-6 Driving point impedance at capacitor terminals

The impedance looking into the terminals of the capacitor is as follows.

$$Z = \frac{Z_{Src} Z_{Cap}}{Z_{Src} + Z_{Cap}} \tag{22-5}$$

The line impedance varies in a direct manner to the harmonic frequency. The capacitor impedance varies indirectly. Therefore:

$$Z_h \approx \frac{(hZ_{Src})\left(\dfrac{Z_{Cap}}{h}\right)}{hZ_{Src} + \dfrac{Z_{Cap}}{h}} = \frac{Z_{Src} Z_{Cap}}{hZ_{Src} + \dfrac{Z_{Cap}}{h}} \tag{22-6}$$

As the harmonic frequency increases:

$$\lim_{h \to \infty} Z_h = \vec{0} \tag{22-7}$$

You would expect the plot of harmonic impedance to appear similar to the following.

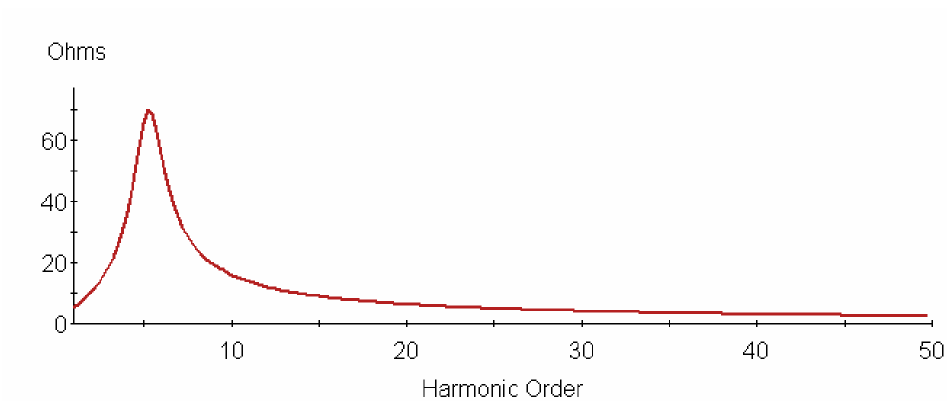


Fig. 22-7 Driving point impedance at capacitor terminals

This holds true in cases when a capacitor is located on a fairly short or low impedance section. For a long or high impedance section, see the next example.

Example looking at end of section

In SynerGEE, capacitors are located in the center of line sections and the harmonic impedance values are calculated at the ends of lines. For example:

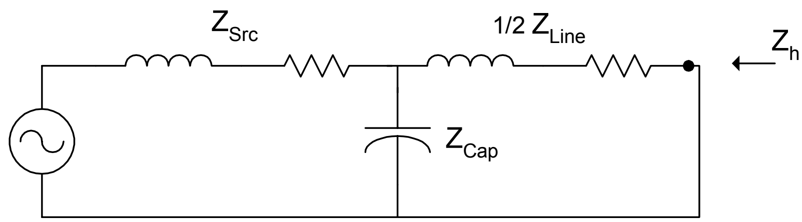


Fig. 22-8 Driving point impedance at capacitor terminals

The harmonic impedance is as follows.

$$Z = \frac{Z_{Src} Z_{Cap}}{Z_{Src} + Z_{Cap}} + \frac{1}{2} Z_{Line} \tag{22-8}$$

And, the driving point impedance gets larger as the harmonic order increases. For example:

$$\lim_{h \to \infty} Z_h \approx \overrightarrow{\infty} \tag{22-9}$$

If Z_{Line} is large because the capacitor is installed on a long line section, the harmonic impedance plot may look like the following.

Fig. 22-9 Driving point impedance at end of capacitor's section

The resonant frequency is the same in both cases. They react differently for damping after resonance.

Effect of loads on resonance point

Harmonic analysis accounts for loads. As discussed previously, loads are represented by series resistance and reactance values connected line-to-ground or line-to-line. Larger loads clearly play a major role in the location of resonant harmonics, as proven in the sample that follows.

The graph below is a frequency scan at the load end of a section containing a capacitor. Notice that the loads are not modified.

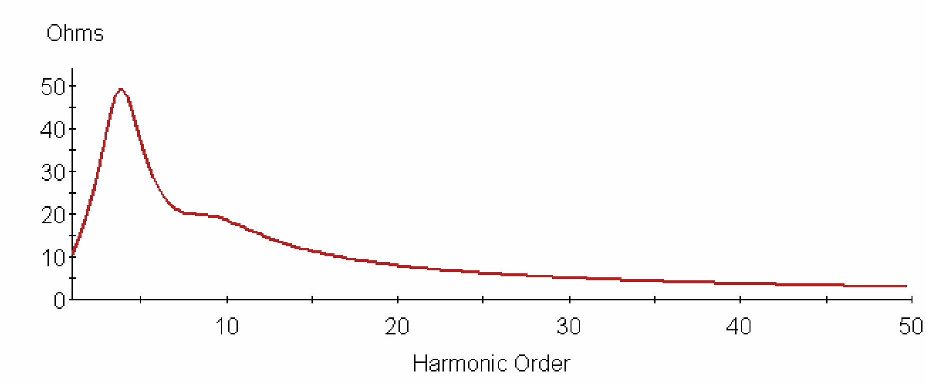


Fig. 22-10 Harmonic impedance looking into section with capacitor

The following is a frequency scan performed in the same manner. In this case, a load multiplier of zero is applied to eliminate the effect of loads.

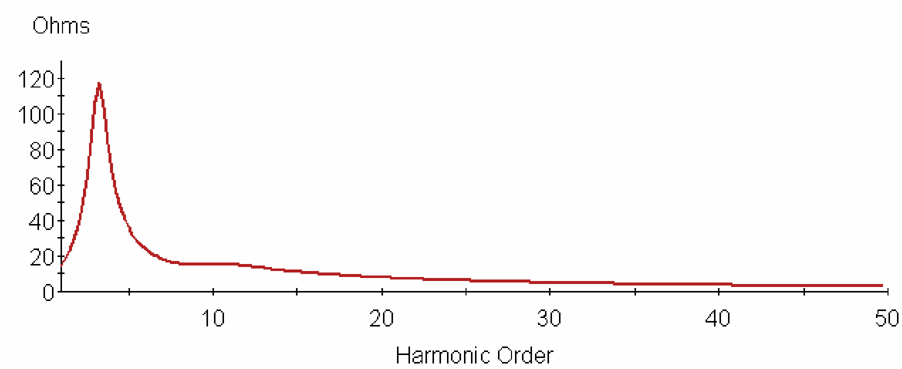


Fig. 22-11 Harmonic impedance with loads reduced

You can copy the data out of SynerGEE and plot the two frequency scans together to get the following.

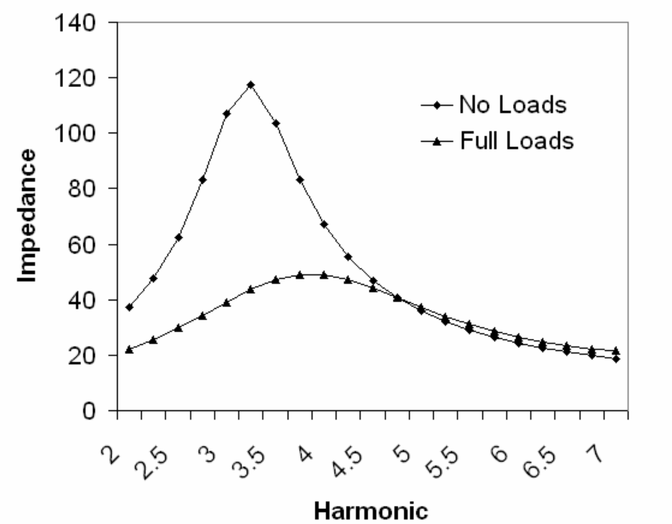


Fig. 22-12 *Comparison of Z_h with full loads and without loads*

Notice that the peak of the impedance magnitude is significantly increased without load models. More importantly, the resonant frequency is increased.

Load modeling is a very complicated aspect of harmonic analysis. Although there is no universally accepted model, SynerGEE uses one of the most commonly used models.

Device models

Highly detailed device models are examined during a harmonic analysis. The models generally correspond to those presented in earlier chapters of this *Technical Reference*.

Lines

The *Lines and Cables* chapter presents a detailed discussion of the line model within SynerGEE that was developed using Carson's equations.

These equations can be summarized as:

(22-10)

$$Z_{ii} = r_i + k_1 f + jk_2 f\left(k_3 - \ln GMR_i + \ln\sqrt{\frac{\rho}{f}}\right) \Omega/_{mi}$$

$$Z_{ij} = k_1 f + jk_2 f\left(k_3 - \ln D_{ij} + \ln\sqrt{\frac{\rho}{f}}\right) \Omega/_{mi} \quad i \neq j$$

The effect of frequency on line impedance is complicated. These primitive impedance values are further reduced to form the 3 by 3 impedance matrix used within SynerGEE.

Line admittance values are capacitive and treated as:

$$Y_{Shnt}(h) = \frac{Y_{Shnt}}{h} \tag{22-11}$$

User-defined conductor impedance values are varied as:

$$\begin{aligned}
&\text{If } X_{Line} < 0: Z_L(h) = R_{Line}\sqrt{h} + j\frac{X_{Line}}{h} \\
&\text{otherwise}: Z_L(h) = R_{Line}\sqrt{h} + jX_{Line}h
\end{aligned} \tag{22-12}$$

This indicates that a negative line reactance is assumed to be associated with a series capacitor.

Loads

Loads are modeled as a series of connected resistance and reactance values tied line-line or line-ground. The value of the series impedance is determined from a solved load-flow case. For example:

$$Z_L = \frac{\left|V_{Load-Flow}\right|^2}{S^*_{Load-Flow}} \tag{22-13}$$

During load-flow analysis, the behavior of the load model can be represented by a second order quadratic equation that represents the constant current, constant impedance, and constant power aspects of the load model. In harmonic analysis, load models are always represented by constant impedance values corresponding to the power consumption at the state of load-flow conversion.

SynerGEE assumes negative reactive load values to be capacitive loads. Therefore, the variation of load impedance with harmonic frequency is expressed as:

$$\begin{aligned}
&\text{If } im\{Z_L\} < 0: Z_L(h) = re\{Z_L\}\sqrt{h} + j\frac{im\{Z_L\}}{h} \\
&\text{otherwise}: \quad Z_L(h) = re\{Z_L\}\sqrt{h} + j * im\{Z_L\}h
\end{aligned} \tag{22-14}$$

Regulators

Impedance values within the regulator model vary with the harmonic being studied. The following is the single-phase regulator model from the *Regulators* chapter:

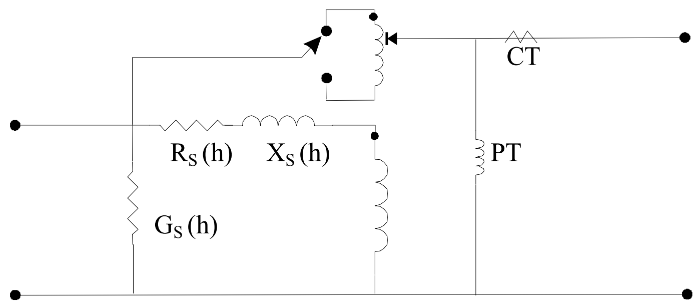


Fig. 22-13 *Regulator model for harmonic analysis*

The impedance values within this model vary as follows:

$$R_S(h) = R_S\sqrt{h}$$
$$X_S(h) = X_S h$$
$$G_S(h) = \frac{G_S}{\sqrt{h}} \tag{22-15}$$

The *Regulators* chapter presents various ways that the regulator model may be connected in SynerGEE regulator banks. These connections have a significant impact on the overall effect of a single regulator's shunt and series harmonic impedance.

Capacitors

Capacitors are modeled as shunt admittance values tied line-line or line-ground. Rated kvar and voltage values are used to determine the capacitor admittance. The capacitor admittance varies as:

$$Y_{Cap}(h) = \frac{Y_{Cap}}{h} \tag{22-16}$$

As the frequency increases, the capacitor admittance decreases, and the capacitive impedance decreases.

Transformers

In SynerGEE, transformer models contain values for series impedance and a shunt conductive value.

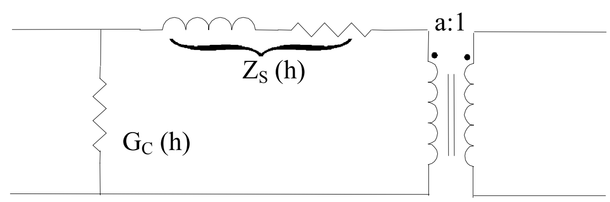


Fig. 22-14 *Basic transformer harmonic model*

The series impedance is shown on the primary winding in this figure. The value is actually split between the primary and secondary winding in SynerGEE. See the *Transformers* chapter for a more detailed discussion of the distribution of impedance between windings.

During a harmonic study, transformer impedance values vary as follows.

$$Z_S(h) = R_S\sqrt{h} + jX_S h \qquad \textbf{(22-17)}$$

$$G_C(h) = \frac{G_C}{\sqrt{h}}$$

Again, refer to the *Transformers* chapter. The transformer model presented here is typically used in more complex connections. The effects and significance of transformer impedance values vary with the type of connection used.

Generators

During harmonic analysis, generators are modeled using an equivalent impedance value. The value is determined from the base load-flow run. Generator impedance values vary as follows.

$$Z_{Gen}(h) = re\{Z_{Gen}\}\sqrt{h} + j * im\{Z_{Gen}\}h \qquad \textbf{(22-18)}$$

Note that the generator windings are modeled as inductive, regardless of the direction of reactive power flow from the load-flow analysis.

Feeder and substation transformer source impedance

During harmonic analysis, source impedance values vary as follows.

$$Z_{Src}(h) = re\{Z_{Src}\}\sqrt{h} + j * im\{Z_{Src}\}h \qquad \textbf{(22-19)}$$

The application does not recognize a capacitive source impedance.

23
Motor Analysis

Introduction

Locked Rotor Analysis (LRA) and Motor Start Analysis (MSA) are two SynerGEE applications that model the starting, startup, and running conditions of a motor. LRA is typically used to determine starting or worst case effects of a starting motor on the entire feeder. MSA is designed to study a specific motor or motors in detail. You can use MSA to set up a motor's starter or to set up the minimum delay time between the sequenced starting of two motors.

Locked rotor analysis

Locked rotor analysis (LRA) uses the SynerGEE motor model and the motor service to simulate the initial starting conditions of one or more motors. In "locked rotor mode," motors are modeled as constant impedance loads. They are served through cables, distribution transformers, and starters specified in the motor editor. The service and motor are attached to the center of their parent section. For more information about the motor models and the service, please refer to the *Motors* chapter of this *Technical Reference*. The methods used to represent locked rotor motors comply with those discussed in *IEEE Std. 399*.

The highest current draw for a motor typically occurs during its locked rotor state. A motor is in its locked rotor state when starting voltage is applied to its terminals and the rotor has not yet begun to turn. No coupling exists between the stator and rotor, and the source sees a fault across the motor's stator impedance. As a result, the large starting current from a locked rotor can cause a serious voltage drop. SynerGEE uses the motor's starting kVA/HP, the section voltage, and a starting power factor to calculate locked rotor effects. Motor power factor and amp curves can also be used to set up the locked rotor condition. Locked rotor analysis lets you evaluate the feeder's response at various sections. This is a tremendous advantage for design, reliability, and system integrity. Using LRA, the kW and kvar contribution of all the motors can be seen along with the impact to their respective feeder or feeders, and substation.

LRA is available for substation analysis and can examine the voltage drop at each substation bus and the connected feeders. Motors can be placed on substation buses; however, this would be unusual. You can see the effect of motors placed on feeders by running LRA with a substation model displayed. If LRA is run from a substation model, the feeders associated with the appropriate substation transformer that feeds the starting motor are listed in the output. The drop through the substation transformers and the substation source impedance is accounted for.

If you want a more detailed analysis, use motor start analysis (MSA) for analysis of motors along their complete starting cycle. MSA accounts for motor and load torque curves and determines time curves and voltage profiles for the complete motor start. MSA also analyzes possible stalling due to lack of accelerating torque.

Motor equivalent Z

Motor locked rotor impedance can be determined two ways in SynerGEE. The first approach uses curve data. If the motor amp and pf curves are provided, the motor amps and power factor at the locked rotor point are used along with the motor's rated voltage to determine the motor impedance.

In the second approach, nameplate voltage rating, HP, and starting pf are used. You can determine rated locked rotor amps as:

$$\mathrm{I_{LR}} = \frac{\mathrm{HP*kVA}/HP}{\sqrt{3}kV_{rated}} \angle -\delta_{pf}^{start} \tag{23-1}$$

The locked rotor impedance can then be found from the rated locked rotor amps and the rated voltage of the motor.

$$Z_{\mathrm{LR}} = \frac{kV_{rated}^{2}}{HP*kVA/HP} \angle \delta_{pf}^{start} \tag{23-2}$$

Running motor model

SynerGEE uses a constant PQ load within the motor model to simulate the running motor. The rated full load current and rated voltage are used to calculate the rated full load motor load. Full load power factor is used in this calculation. That power factor can be specified with other nameplate data or the last point of the motor's power factor curve can be used.

Rated motor current can be found with:

$$I_{Running} = \frac{10*HP*746.0}{\sqrt{3}*Eff*Pf_{Run}*kV_{Rated}} \angle -\delta_{pf}^{running} \tag{23-3}$$

And the running power is:

$$S_{Running} = \frac{10 * HP * 746.0}{Eff * Pf_{Run}} \angle \delta_{pf}^{running} \tag{23-4}$$

Locked rotor calculations

The motors analyzed in LRA are placed as constant impedance loads at the end of their service fed by their respective sections. The following is the model for an induction motor.

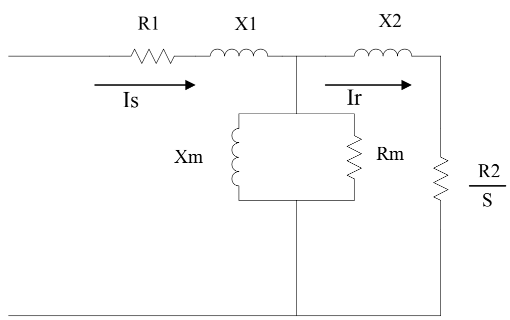


Fig. 23-1 *Model for induction motor*

The moment a motor is served with a starting voltage, the slip, *S*, represents unity and the impedance values in the above figure represent stator quantities. The model can be reduced to a single impedance with simple circuit reduction techniques. For example:

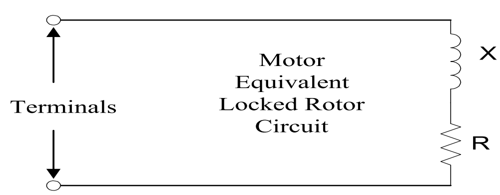


Fig. 23-2 *Locked rotor equivalent circuit*

The locked rotor impedance of this model can be determined directly from the data specified in the motor types database. When this impedance is used to represent the motor during a load-flow analysis, the motor load consumes a power given by the following.

$$S = HP * R \angle \theta_{PF} \left(\frac{kV_{Term}}{kV_{Rated}} \right)^2 \tag{23-5}$$

$HP =$ Rated HP $\qquad \theta_{PF} = \cos^{-1} pf$

$R =$ Starting kVA/HP Factor $\qquad kV_{Rated} =$ Rated Voltage

$kV_{Term} =$ Terminal Voltage from Load-Flow

You can see that as the voltage at the terminals of the motor drops, the kVA load of the motor drops exponentially.

Using source voltage

The voltage drop through the feeder or substation source voltage can be significant when starting large motors. LRA accounts for the source voltage in its calculations. To do this, LRA performs a base run to calculate the nominal current entering the feeder or substation transformer prior to the application of the locked rotor or rotors, using the voltage specified in the feeder record as the feeder's fixed starting voltage, V_F. This current is notated as follows.

$$\mathrm{I_F} = \text{Current into feeder} \tag{23-6}$$

LRA uses the feeder source impedance, Z_F, along with these values to get the system voltage. The system voltage, V_S, must be determined so that the voltage drop due to the feeder impedance results in a feeder voltage represented by the following.

$$\mathrm{V}_S = V_F + I_F Z_F \tag{23-7}$$

This voltage is held constant as a stiff system voltage. In the next load-flow run or the actual locked rotor run, current from the locked rotor load and source impedance results in a feeder voltage lower than the base run.

Converting a motor to a load

Motors are permanent components in the model and the database. There may be occasions in which a spot load is preferable to a motor model. A motor can be converted to a kW and kvar load and be manually combined with the other spot loads of a section by using the rated HP efficiency and running power factor of the motor as follows.

$$S_{Motor} = \frac{HP*(0.746)}{Efficiency}\left(1 + j\sqrt{\frac{1}{pf^2} - 1}\right) \quad (kVA) \tag{23-8}$$

Starters and service

Motor starters are used in locked rotor calculations in the initial state. The service cable, autotransformer (if any), and starter are all used to model each motor installation during a run. Series R and X starters, autotransformer starters, and reduced voltage starters have an effect on a motor during LRA.

Useful expressions

The following parameters are typically given for a particular motor.

HP	Horsepower of motor
KV_{Rat}	Rated motor voltage
R	Ratio of kVA during locked rotor to horsepower of motor
Pf	Locked rotor power factor

During locked rotor, the motor slip is zero and no power is transferred across the motor's air gap. The power consumed by the motor can be given as follows.

$$S_{LR} = (HP * R)\angle \cos^{-1} pf \tag{23-9}$$

This power is consumed when rated voltage is applied to the motor terminals. Since the motor is a constant impedance load, the power consumed by the motor at voltages other than nominal is given by the following.

$$S = S_{LR}\left(\frac{kV}{kV_{Rat}}\right)^2 \tag{23-10}$$

Data substitution

Sometimes, you may be missing precise manufacturer's data for motors. Some common assumptions to use in these incomplete cases are stated below. If you have any part of these items with specific data, use the most detailed items available.

Horsepower rating

The horsepower rating of a motor can be approximated from its running kVA. The relationship between kVA and HP for a motor is given as follows.

$$kVA = \frac{HP * (0.746)}{Efficiency * pf} \tag{23-11}$$

If you approximate a running power factor of 80 percent and an efficiency of 93 percent, you can rearrange this equation to determine the approximate value of HP/kVA. For example:

$$HP/_{kVA} = \frac{Efficiency * pf}{(0.746)} \cong \frac{(0.93)*(0.80)}{(0.746)} \cong 1.0\, HP/_{kVA} \tag{23-12}$$

As such, the horsepower can be approximated as the same numerical value as the rated running kVA of the motor. Note that this calculation is not related to the kVA/HP factor that is required in the motor table.

Starting power factor

The power factor of a motor during starting determines the amount of reactive current that is drawn from the system. This directly affects the maximum voltage drop.

Typical data from the *Transmission and Distribution Reference Book* suggests the following starting power factors ($PF_{starting}$) if other information is not available. If you have more detailed information for your case, use those specific values.

Motor Size	$PF_{starting}$
Motors under 1,000 hp	20%
Motors 1,000 hp and up	15%

If you are uncertain about the specifics of a locked rotor condition, you may have to assume values for the locked rotor kVA and starting power factor. Some reasonable assumptions are 6 for the locked rotor kVA and 35% for the starting power factor (*Transmission and Distribution Reference Book,* 1964*,* 723).

As always, company practice or specific data should be used when available.

Rated LR Amps from actual LR Amps

Since the motor is modeled as a constant impedance during locked rotor conditions, locked rotor current at the motor voltage can be used to determine locked rotor current at rated terminal voltage.

(23-13)

$$I_{Rat\ kV} = I_{Act\ kV}\left(\frac{kV_{Rat}}{kV_{Act}}\right)$$

Motor start analysis

Starting large motors can cause severe disturbances to the motor, nearby loads, and loads throughout the feeder. SynerGEE's motor start analysis tool is designed for evaluating the effects of starting large motors on distribution systems. It enables you to examine voltage drop at the feeder, service, and terminals of the motor after it is switched onto the system and throughout its start. The analysis is a dynamic analysis resulting in time-domain based results.

Motor start analysis permits you to determine and analyze the following:

- Motor speed and torque vs. acceleration time
- Motor terminal voltage and current draw versus time
- kW and kvar into service versus time
- Service drop versus time

- Effects of various types of starters

The speed-torque/accelerating time study can help to verify that starting times are within acceptable limits. It can also help determine the best sequence for starting motors and the minimum delay required between the successive starting of multiple motors.

Starting, running, off

During a motor start analysis run, motors marked as "starting" or "running" are analyzed. Starting motors begin at standstill and run up to full speed. Running motors begin in the analysis at full speed. Voltage dips from other starting motors may cause a running motor to drop in speed or even stall.

Motor start calculations

During successive load-flow runs within the motor starting analysis, the total equivalent impedance of the motor for a particular motor speed is applied to the end of the service consisting of cables, distribution transformers, and starters. The load-flow includes detailed models for all service components. After the load-flow is completed, the voltage at and the current into the motor are known. The torque derived from the motor can be found. Acceleration can be determined. Finally, the motor speed for the next time point can be calculated through numerical integration.

Use of source voltage

The voltage drop through the feeder or substation source voltage can be significant when starting large motors. SynerGEE's MSA tool accounts for the source voltage in its analysis. To do this, LRA performs a base run to calculate the nominal current entering the feeder or substation transformer prior to the application of the locked rotor or rotors, using the voltage specified in the feeder record as the feeder's fixed starting voltage, V_F. This current is notated as follows.

$$I_F = \text{Current into feeder} \tag{23-14}$$

The MSA tool uses the feeder source voltage, Z_F, along with these values to get the system voltage. The system voltage, V_S, must be determined so that the voltage drop due to the feeder impedance results in a feeder voltage represented by the following.

$$V_S = V_F + I_F Z_F \tag{23-15}$$

This voltage is now held constant as a stiff system voltage. Subsequent feeder current with the load from the starting motor will drop through the feeder's source impedance and produce a feeder voltage that is lower than the feeder voltage of the base run.

Torque and starting time

Just as the current and power factor curves for a motor are required for motor start analysis, so is the torque curve. This curve is usually generated by the motor manufacturer and is produced when the motor is started under rated voltage. For example:

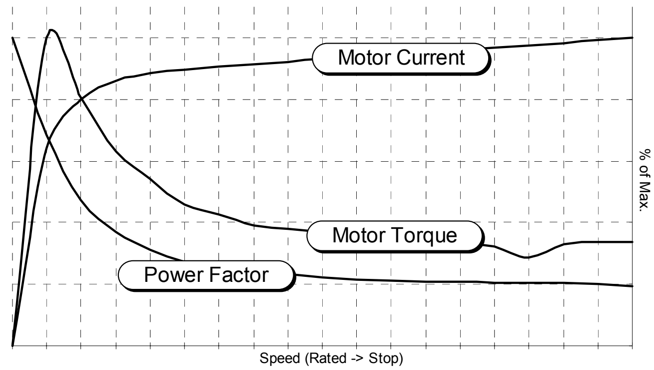


Fig. 23-3 *Complete set of motor rated curves*

From motor theory, it is evident that the torque developed by a starting induction motor is related to the air-gap resistance. However, *IEEE Standard 300* states that the torque developed in an induction motor varies by the square of the terminal voltage. Therefore:

(23-16)

$$\mathrm{T}_{Motor}\left(s\right)=T_{Rated}\left(s\right)*\left(\frac{kV_{Terminal}}{kV_{Rated}}\right)^{2}$$

The net torque produced by the motor is the motor's mechanical torque minus the load torque value at the given speed of the motor.

(23-17)

$$\mathrm{T}_{Net}\left(s\right)=T_{Rated}\left(s\right)*\left(\frac{kV_{Terminal}}{kV_{Rated}}\right)^{2}-T_{Load}\left(s\right)$$

The angular acceleration of the motor shaft due to the motor's torque is:

(23-18)

$$\alpha_{Motor}=\frac{2g\mathrm{T}_{Net}}{Wk_{Motor}^{2}+Wk_{Load}^{2}}$$

$$g=\text{ acceleration due to gravity}$$

$$\mathrm{Wk}^{2}=\text{inertia}$$

Motor speed can be found by integrating acceleration.

(23-19)

$$\omega_{Motor}=\int\alpha_{Motor}dt$$

SynerGEE runs successive load-flow runs with motor models updated from their last state and curve information. Motor speed is found by numerically integrating acceleration.

Motor and load inertia

Load inertia has a tremendous impact on the starting time of a motor. Notice in the above equations that for a fixed torque, the acceleration is inversely proportional to the sum of motor and load inertia. Altering the inertia values reduces or extends the starting time of the motor. Changes do not, however, alter the stalling state of a motor or change any low voltage levels seen during the start.

SynerGEE calculates load inertia using NEMA equations. It also allows you to enter your own values. If the starting time seems too long, try reducing the load inertia.

Full load or running state

During an MSA run, a motor reaches its running state when its generated torque matches the load torque. That is, when:

(23-20)

$$T_{Rated}\left(s_{full\,speed}\right)*\left(\frac{kV_{Terminal}}{kV_{Rated}}\right)^2 - T_{Load}\left(s_{full\,speed}\right)=0$$

The motor current and power factor are determined from the current and power factor curves at full speed. The values at the full speed point may not be realistic. If this is the case and a full speed state is important to your study, try modifying the points of the load torque curve or the motor torque curve so that they intersect at around 99 percent of the motor's synchronous speed. The motor current and pf should be close to rated values at 99 percent of synchronous speed.

Generated amp and pf curves

Amp and pf curves are sometimes difficult to find. SynerGEE is capable of generating curves to approximate the amp and pf characteristics of a motor. When setting up a motor type, the amp and pf curves can be specified, or the starting and full load pf along with the kVA/HP values can be specified. If the latter approach is taken, SynerGEE scales the amp and pf curves for an internal well-defined motor to have starting pf, running pf, starting amps, and full load amps to match the supplied characteristics for the new motor.

SynerGEE contains internal values for a motor circuit model similar to the following.

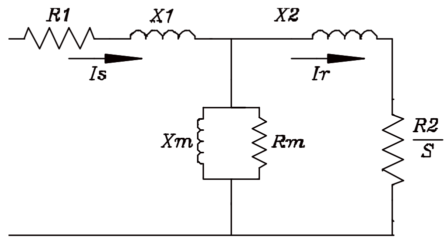


Fig. 23-4 *Built-in motor model*

The amp and pf curves for this motor are scaled to meet the locked rotor and full speed characteristics of a motor type.

Torque curves must always be supplied for motors to be run with motor start analysis.

Approximate method

IEEE Standard 300 lists an approximation for determining the time needed for the motor to accelerate from one speed point to another.

$$t(s) = \frac{Wk^2 (RPM_1 - RPM_2) * 2\pi}{60 g T_n} \quad \textbf{(23-21)}$$

where: Wk^2 = Motor Inertia $RPM_{1,2}$ = Motor Speed

g = Acceleration due to gravity T_n = Average Interval Accelerating Torque

The acceleration times through a few intervals of the motor start can be accumulated to determine a total start time for the motor.

If the accelerating torque, $\tau_{Actual}(s) - \tau_{Load}(s)$, becomes negative, the motor stalls.

24
Time & Time Range Analysis

Introduction

Time range analyses performs a variety of different functions, mostly involving a load-flow analysis over a specified period of time. The reports produced are intended to summarize the trends in energy, loading, or voltage that are seen on the model throughout a time period. Time range works in conjunction with customer load curves, which are explained in *Chapter 3 - Loads*.

Analysis is run for every hour of every day type for the days falling within the selected range. Minimum, maximum, and average values are found and the data is used to generate various types of plots.

Real-time data interface

Semi real-time data can be brought into the simulation environment. This data typically comes from database systems or data warehouses tied to real-time data hubs. In the SynerGEE environment, the information can affect switch status, metered demands, load values and more.

Interfacing with PI Historian

SynerGEE can run a messaging script as requested by the user. The script can also be run on a timer for automation. The messaging script connects to a PI Historian database through a PI OleDb driver configured on the current machine. Once connected to the database, SynerGEE retrieves values for the tags specified in the script. The values are then associated with parameters for various facilities within the SynerGEE model. Here is a diagram showing the interface between SyneGEE and Pi Historian through the PI OleDb provider:

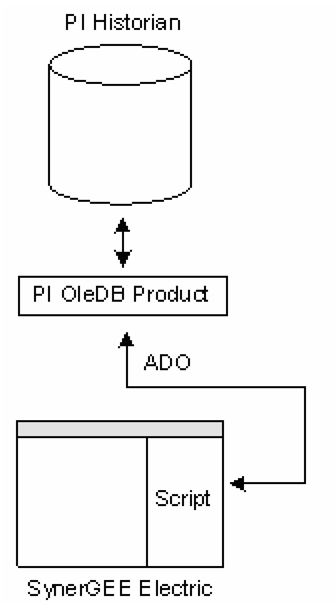


Fig. 24-1 SynerGEE / PI interface

The script can also be configured to have SynerGEE write calculated values into the historian. This results in tags with calculated values or virtual SCADA points.

The messaging script

SynerGEE's scripting format is used to specify connection information as well as the desired relationships between tags in the PI system and parameters within SynerGEE facilities. The '15205' record is used to connect to a PI Historian data source through ADO.

```
15205, Server, Username, Password
```

(Note that the PI OleDB component must be installed.) The status of the connection can be checked with a '15206' record. (Testing the connection is recommended but the '15206' should not be a part of the final working script.)

Tags from the PI system are mapped to SynerGEE facilities with the following records:

Facility	Script record number
Capacitor on / off	15210
Switch open / closed	15215
Metered kW	15220
Metered amps	15221
Recloser open / closed	15225

Here is a working script

```
' ***** Start of code sample
'Log into the Pi server
15205,localhost,piadmin,

'Indicate connection succeeded
15206

'Retrieve 3Ph kW demands for meters
15220,79520,7,MT-001
15220,79521,7,MT-002

'Get switch open / closed status
15215,22630,SW-011
15215,79530,SW-012
15215,33564,SW-021

'Get capacitor on/off status
15210,Dkn_00053,CAP-001
15210,53515,CAP-002

'Connect the model
15002

'Run load allocation
15003,alloc

'Write voltage
15250,52536,VOLT-001

'Refresh the map
15001


' ***** End of code sample
```

Notice that this script is writing calculated voltage values back into the PI Historian with the '15250' record.

There are some script commands that are general to all of SynerGEE's messaging platforms.

15001 – Refresh SynerGEE map after retrieval

15002 – Reconnect the model after switching

15003 – Run an engineering analysis application

15004 – Post a message in a message box

These commands can be mixed into a messaging script to support user interaction.

Automating scripts

Messaging scripts can be run at the engineers request or SynerGEE can be configured to run a script on a time interval. The messaging wizard is used to run or configure automatic running of scripts:

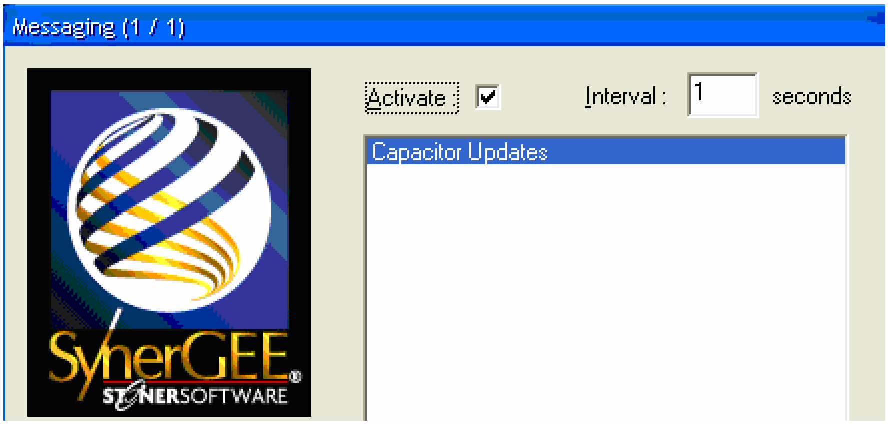


Fig. 24-2 *Messaging wizard*

Above, the wizard is configured to run the "Capacitor Updates" script once a second. Every second, SynerGEE will check the PI Historian for updated values.

All feeders at peak study

This time range analysis tool focuses on individual feeders at their respective peak loading. The analysis first finds individual feeder peaks, then configures each feeder to its respective peak load value. It then runs a load-flow with these configured loads. As such, the load-flow and the resulting report correspond to each feeder in the model being at its specific peak.

There is a distinct difference between this application and the model settings for running the model at peak load. Running at peak load runs all feeders at the same time point. That time point corresponds to the peak load of the combined feeders. However, the feeder peak study report analyzes feeders with each feeder at its particular peak time point. The load-flow report is analogous to a load-flow run on a model allocated with peak demands, without load curves implemented.

Feeder peak study

Time range analysis includes a feeder peak study report. The analysis runs a load-flow over all time points within the selected range, and summarizes the conditions on each feeder at its individual peak. The following is an excerpt from a sample report.

Feeder Peak Summary Report

Feeder	Peak	Feeder Load				Max Pct	Min	Exceptions	
Id	Time	kVA	PF	Amps	kW Loss	Loading	Volts	Volt	Ldng
System	Dec Wd 5 p.m.	36922							
Feeder New - Big Spring	Dec Wd 5 p.m.	5714	99.2%	241.9	47	46%	121.9	0	0
Feeder New - Cove	Dec Wd 5 p.m.	3651	99.3%	154.5	22	40%	122.1	0	0
Feeder New - Liberty	Dec Wd 5 p.m.	6229	99.2%	263.6	26	50%	123.1	0	0
Feeder New - Springfield	Dec Wd 5 p.m.	3089	98.9%	130.8	13	25%	123.2	0	0
Feeder Ship - South Penn	Dec Wd 5 p.m.	9332	99.6%	395.0	230	75%	119.1	0	8
Feeder Ship - West Martin	Dec Wd 5 p.m.	8917	99.0%	377.4	134	71%	120.0	0	4

Fig. 24-3 *Feeder peak study report*

In the report, you can see when each feeder peaks and information about loading and voltage on each feeder at its peak. Customer load curves must be implemented in the model for the report to have value.

25
Design & Economy

Balance improvement

Imbalance in loading leads to imbalance in current flow, resulting in higher losses, overloading, reduced capacity, and voltage problems. Therefore, SynerGEE provides this balance improvement tool to help find loads or laterals that can be rephased to achieve the greatest benefit for a predefined objective.

Overall application operation

Balance improvement has two primary modes of operation.

- **Load balancing**—The application looks for spot and/or distributed loads that can be reconnected to improve feeder balance.
- **Phase balancing**—The application looks for single- and two-phase laterals which can be rephased to improve feeder balance.

Objectives

Phase or load balancing can consider one of the following objectives.

- **Improve feeder amp imbalance**—Reduce the current imbalance at the feeder.
- **Reduce worst voltage imbalance**—Reduce the worst voltage imbalance on any section in the feeder. The section with the worst imbalance may vary as improvements are accumulated.
- **Reduce feeder loss**—Reduce total feeder loss value.
- **Improve lowest voltage**—Increase the lowest voltage on any phase. The section or phase with the worst voltage may vary as improvements are accumulated.
- **Improve feeder customer imbalance**—Minimize customer imbalance.

- **Reduce exception count**—Reduce the number of exceptions.

The application attempts to find the best laterals or loads to rephase in order to obtain the greatest gain in any one of these objectives. All objectives are listed on the balance improvement report so that the best all-around decision can be made.

Implementing the results

Balance improvement analysis never makes permanent changes to a model. It only makes suggestions, in the form of a report. In your actual system, it will often be infeasible or impractical to implement all the suggestions. As such, the report is provided so that you can make an educated judgment about how many of the changes should be made.

Constraints

You can specify the maximum number of rephase operations allowed. For load balancing, this value is the maximum number of loads that will be rephased. For phase balancing, this value is the maximum number of lateral rephase suggestions that will be presented. Rephasing a lateral includes changing the phase of all sections fed from that lateral.

You can also specify the "depth" factor, which allows a reduction in the scope of the analysis to the more heavily loaded laterals or larger loads. A depth of 10 percent in load balancing, for example, only looks at largest 10 percent of the loads.

Optimization process

With each successive step, the application focuses on the change that results in the greatest benefit. Like optimal switching and contingency analysis, it relies on an optimization routine that results in a local optimum. The application only suggests changes that improve the performance of the feeder with respect to the selected objective.

Phase balancing specifics

SynerGEE suggests single- and two-phase laterals that can be rephased. The point to be rephased is always the point at which the lateral connects to its trunk.

Lateral Option	Laterals to Consider
Single-phase laterals	1-phase lateral fed from 2-phase trunk 1-phase lateral fed from 3-phase trunk
Two-phase laterals	2-phase lateral fed from 3-phase trunk

When balance improvement is run, a report is generated with the suggested phase changes. The following is an example of a phase balancing report.

Phase Balance Summary

First Section On Lateral	Lateral Amps	Phase Change to Lateral & All Fed Sections	Src Amp %Imb	Max Volt %Imb	kW Loss	Low Volts (120V)	Cust %Imb	Excpt Cnt
*** Base Run ***			7.5%	5.2%	307.8 (4316.3)	104.2	25.8%	28
NC3	15.5	C->A	8.0%	2.0%	301.0 (4315.6)	107.6	33.4%	31
247	9.0	A->B	6.0%	2.2%	294.6 (4311.0)	107.8	31.0%	30

Fig. 25-1 ***Summary table from balance improvement / phase balance run***

This analysis was run to improve the lowest voltage. The first change improved the lowest voltage significantly, from 104.2 volts to 107.6 volts. The second change made a small, additional improvement, from 107.6 volts to 107.8 volts. As you can see, you might consider implementing the first change, but the second is probably not worth the time or cost.

Looking at the report, you should notice that the first change indicates a phase change "C->A." To implement this change, section "NC3" and all sections fed by it would need to be rephased. In the real system, the change may be as simple as moving some jumpers. On your model, however, this change must include switching all loads and equipment from phase C to phase A, on all sections fed by NC3. In the modeling environment, the phase changing effort must be applied to all sections, since each section holds independent phase data.

Load balancing specifics

The following options are available for a load balance run.

- Consider distributed and/or spot loads
- Consider one-phase, three-phase, or all spot loads

When the application is run, it produces a report similar to the following.

Load Balance Summary

Section Id	Load Type	Nom. kVA	Phase Change	Src Amp %Imb	Max Volt %Imb	kW Loss	Low Volts (120V)	Cust %Imb	Excpt Cnt
*** Base Run ***				7.5%	5.2%	307.8	104.2	25.8%	23
226	Dist	231	B->C, C->B	5.5%	4.4%	299.7	105.5	25.8%	23
KCK	Dist	295	A->C, C->A	4.2%	3.8%	298.6	106.0	30.5%	23
229 N-1	Dist	90	A->C, C->A	4.6%	3.4%	297.5	106.5	31.0%	25
24F	Dist	310	A->C, C->A	7.2%	3.2%	298.4	106.7	34.8%	26
235 N-1	Dist	172	A->C, B->A, C->B	8.3%	3.1%	299.8	106.8	34.8%	27

Fig. 25-2 ***Summary table***

The objective in this case was to improve the lowest voltage on the feeder. If all the changes were implemented, the system low voltage would improve by 2.6 volts. However, the first two changes would accomplish 1.8 volts alone, so you might choose to implement them only. Also, you may notice that while voltage continues to rise, so does the number of exceptions, but only after the third suggestion.

On the report, you can see that the first change is listed as "B->C, C->B." This indicates that the load on "B" should be shifted to "C" and the load that was on "C" should be shifted to "B."

Loads are never combined by balance improvement, only shifted.

Capacitor placement

SynerGEE Electric supports a comprehensive tool for aiding you in finding locations for new capacitor installations. The application is designed to operate under a variety of placement objectives and to provide results in a clear and concise manner.

Capacitor placement operates in a cumulative manner starting from a base case. First, it finds the most advantageous location for a capacitor based on constraints that you specify, and offers that as the first recommendation. It then assumes the placement of that capacitor, and looks for the next best location to improve performance further. This process is continued per feeder until one of the following occur.

- The application cannot improve system performance with another capacitor.
- The application has placed the maximum number of capacitors, based on your settings.

Once finished, the application presents the recommendations in a report, listed in order of increasing performance, as they were found. Capacitor placement makes no changes to your model. It simply reports its recommendations, and you may choose to implement them as desired.

Application operation

The capacitor placement tool finds optimum placement for capacitors of fixed size in a cumulative manner. This form of optimization is referred to as local optimization. It does not find a theoretical global optimum by considering all capacitor sizes and all locations simultaneously. Instead, it operates by moving from the base case to the next best state, then to the next best state, and so on. This approach provides results that are more realistic and useful than the global optimization method.

As mentioned previously, capacitor placement does not make changes to the network model; it merely suggests locations. The application places capacitors from the largest kvar value to the smallest. It finds the best location for a capacitor, places it, analyzes the system, and attempts to place another capacitor of the same size. If a placement cannot be found, the application considers the next smaller size of capacitor. All capacitor placements are three-phase units.

Capacitor placement places capacitors on a feeder-by-feeder basis. It completely analyzes one feeder before moving to the next, if multiple feeders are selected.

Placement objectives

Capacitor placement can evaluate capacitor placement for one of five objectives. These objectives are independent and evaluated on a feeder-by-feeder basis. They include:

- **Improve losses**—Total feeder kW losses are evaluated, and capacitors are placed to reduce these loss values.
- **Improve lowest voltage**—Capacitors are placed to improve the lowest voltage of the feeder. The section with the lowest voltage may change as capacitors are added.
- **Reduce demand**—Capacitor locations supporting the lowest total feeder kVA demand are selected.
- **Improve power factor**—Placements are made to improve the power factor of the feeder total demand.
- **Reduce exceptions**—Capacitors are placed to reduce the number of voltage and overload exceptions. All exceptions are weighted equally. Exception limits are set in your model options.

Values for all of these items are listed in the analysis results table, regardless of which item is selected for optimization.

Constraints

A capacitor of a given size will be placed on a section only if:

- The placement gives the best performance of the objective.
- All sections and devices operate within the given power factor limits.
- The feeder is operating within the power factor limits.
- The maximum number of placements on the feeder has not been reached.

You can adjust some of these constraints, such as the maximum number of placements. You can also configure the application to not place capacitors on cables sections, with a non-bare conductor type.

Application output

Capacitor placement produces a report designed to summarize placement locations in a format that allows you to make decisions on a number of complex and interrelated issues. The report may include:

- Standard heading information, warnings, messages, and analysis options.
- A placement summary for all feeders analyzed. It lists each available capacitor size along with the number placed from that category.
- A summary for each feeder being analyzed. The feeder summary lists each recommended placement along with capacitor size, load-flow results, and metric values for the various objective functions.
- SynerGEE capacitor placement optimizes based on a single objective. The simple feeder summary allows you to view other metrics and determine the number of suggested placements to accept. It also lists the total capacitor kvar suggested for each section and the effective kvar after load-flow analysis. If multiple placements are suggested for a section, the values are added and listed as a single row in the report.

A complete load-flow listing on a feeder-by-feeder basis. The load-flow listing includes all suggested placements.

Regulator setting application

SynerGEE is equipped with a tool to aid in regulator settings. Using the regulator voltage setting and the minimum downstream voltage as input, the application attempts to find R and X settings for the regulator that will keep the lowest voltage downstream above the specified minimum.

Peak load conditions are used to determine the regulator R and X settings, and these conditions are reported. In addition, minimum load conditions are evaluated and reported with the regulator set up with the proposed settings.

Example 1

The following is a report from a sample run.

Parameter	A/AB	B/BC	C/CA
VSet (Volts)	120.0	120.0	120.0
RSet (Volts)	2.0	1.4	-0.5
XSet (Volts)	1.7	2.3	-0.6
BW (Volts)	2.0	2.0	2.0
Results at peak feeder loading			
Taps	1	0	-2
Vrr	119.5	120.8	120.9
Minimum downstream volts	118.6	118.7	118.6
Maximum downstream volts	121.7	121.8	120.4
Regulator output volts	121.7	121.9	120.5
Regulator demand Amps	160.8	120.3	132.7
Results at minimum feeder loading			
Taps	-5	-4	-5
Vrr	120.2	120.8	120.2
Minimum downstream volts	120.3	120.2	120.0
Maximum downstream volts	121.0	120.9	120.3
Regulator output volts	120.4	120.8	120.2
Regulator demand Amps	56.5	39.0	45.0

Fig. 25-3 ***Sample report***

Reading the report

The voltage setting, R and X settings, and bandwidth are listed at the top. The voltage setting is a reflection of the user input. The bandwidth is always 2V. Keep in mind that the settings are always in volts on a 120V base.

The next section of the report lists the results for the regulator and feeder under peak load conditions, as follows.

- The expected taps are given. "Vrr" is the voltage on the regulating relay within the regulator's controller.

  $$V_{rr} = \frac{1000 * kV_{out}}{PT} - \frac{I_{out}}{CT} * Z_{RX} \quad \textbf{(25-1)}$$

  The voltage of the regulating relay is used to determine if a tap change is needed. If the voltage is more than one-half the bandwidth away from the voltage setting, a tap change will occur. The voltage is included on the reports so the effects of the R and X settings can be seen. For more information on voltage regulators, see *Chapter 6 - Regulators.*
- The minimum and maximum downstream volts are determined by looking at all line sections downstream from the regulator. Sections fed by the regulator under study and a second regulator are not considered.
- The regulator output volts and amps are listed to help with the evaluation of first house settings and tap limiter values.

The same results are then listed for minimum feeder loading.

Example 2

In the previous example, the voltage setting was 120V. If the voltage setting were 122V, you would expect that the tap positions under peak load would not change because the regulator needs to be at the same tap to meet the minimum voltage constraint. The following is a report from another sample run, with the 122V setting.

Parameter	A/AB	B/BC	C/CA
VSet (Volts)	122.0	122.0	122.0
RSet (Volts)	0.3	-1.2	-2.5
XSet (Volts)	0.3	-2.1	-3.2
BW (Volts)	2.0	2.0	2.0
Results at peak feeder loading			
Taps	1	0	-2
Vrr	121.4	122.9	122.9
Minimum downstream volts	118.6	118.7	118.6
Maximum downstream volts	121.7	121.8	120.4
Regulator output volts	121.7	121.9	120.5
Regulator demand Amps	160.8	120.3	132.7
Results at minimum feeder loading			
Taps	-2	-2	-2
Vrr	122.8	122.3	122.6
Minimum downstream volts	122.7	121.7	122.4
Maximum downstream volts	123.4	122.5	122.6
Regulator output volts	122.8	122.4	122.5
Regulator demand Amps	57.2	39.2	45.5

Fig. 25-4 ***Sample report using 122V setting***

The R and X settings are different because the voltage setting is different. Notice that the peak load tap settings are the same. The minimum and maximum downstream voltages at peak are also the same.

The minimum load values are very different. The regulator taps are higher and the downstream voltages are higher with the 122V setting. Comparing the two examples, therefore, you might decide that the low regulator voltage setting is better. As with this situation, multiple runs of the regulator settings tool may often be needed to get a good picture of the possible settings.

Loading

The tool has two options for handling peak and minimum loading.

- Customer load curves can be used. The peak and minimum load times are found and used in the analysis for this tool.
- Feeder load can be used directly. In this case, a factor (like 20%) is used to multiply the loads in order to get minimum loading on the feeder.

Regulator handling

The regulator settings tool evaluates one regulator at a time. The regulator must be connected in a Wye-Gnd connection. First house, tap limiter, gang operation, and reverse flow settings are disabled.

Operation

During an operation, the tool uses the following steps.

1. Determine R/X ratio
2. At peak load, find R and X values
3. Evaluate regulator at peak load and minimum load

The R/X ratio is determined from a weighted average of downstream section impedances.

$$R/X_{\psi} = \frac{\sum_{j=Downstream} \left|I_{j,\psi}\right| * R_j}{\sum_{j=Downstream} \left|I_{j,\psi}\right| * X_j} \tag{25-2}$$

The factor is weighted by the current flow through the downstream line. Long sections are going to be weighted higher. Sections with a larger current flow are going to be weighted higher.

The R/X ratio is not listed in the report. However, the ratio between the R setting and the X setting is the same as the R/X ratio.

Performance Comparison

Model performance comparison is an application designed to present operational and cost comparisons of various modeling alternatives by comparing the model in memory to variations of that model stored in the Gallery. The memory model is referred to as the 'base' model. When the model performance comparison application is run, a report is generated. Each model from the Gallery that was compared to the base model is listed. Comparisons to the base are made in the following areas:

- Low voltage and total kw demand at minimum load
- Hi voltage and total kw demand peak load
- Losses at peak
- Reliability SAIFI, MAIFI, and SAIDI
- Capital cost difference between base model and gallery model
- Operating cost difference between base model and gallery model

Model loads are assumed to be allocated at peak demand. Peak performance for a model is determined by running load-flow analysis directly on the model. The minimum loading on all models is calculated with direct application of a light run factor. All loads are multiplied by the same factor to determine minimum loading conditions. This is a very simple approach to light-load modeling but one that should give useful numbers within the context of the assumptions made.

Costs

Model costs are generated from capital costs and yearly costs. Capital costs should reflect one time costs associated with the differences between a model and the base model. It is based on four components:

	Capital costs for new equipment
+	Labor cost to install new equipment
+	Cost to move equipment
-	Salvage recovery for retired equipment
	Total capital costs

Yearly costs are those recurring costs over the recurring costs of the base model:

	Peak charge per kW on total model
+	Cost for primary kW losses
+	Cost for secondary kW losses
+	Yearly maintenance on new equipment
-	Yearly maintenance on retired equipment
	Total yearly costs

Secondary losses

SynerGEE is a typically used for primary system analysis. Secondary losses may be 50% or more of distribution losses. These secondary losses may need to be considered if planning decisions are to be made based on loss improvement.

A simple model for looking at general trends in secondary losses is used by SynerGEE. The final equation for secondary losses on a per-load basis is:

$$kW_{Loss} \approx \frac{k_1}{100\%} \cdot \left| \frac{kV_{Nom}}{kV_{Act}} \right|^2 \frac{kW_{Act}^2}{kW_{Nom}} \qquad \textbf{(25-3)}$$

The factor k_1 is the “secondary loss factor” expressed as a percent. It is a general factor that represents the percentage of load that will be attributed to secondary losses. In other words:

$$kW_{\substack{Nom-Loss\\Secondary}} = \frac{k_1}{100\%} \cdot S_{Nom} \tag{25-4}$$

Secondary load has been aggregated into the distributed load used by SynerGEE. The number of loads or 'taps' will be 't'.

We can determine secondary losses from the nominal load current, secondary resistance, and number of taps:

$$kW_{\substack{Nom-Loss\\Secondary}} = \left[\left|\frac{I_{Nom-Load}}{t}\right|^2 \cdot R_{Secondary}\right] \cdot t \tag{25-5}$$

This equation can be expressed in terms of load if we assume equal loading on all taps:

$$kW_{\substack{Nom-Loss\\Secondary}} = \left|\frac{S^*_{Nom}}{t \cdot kV^*_{Nom}}\right|^2 \cdot R_{Secondary} \cdot t \tag{25-6}$$

We can now equate our two expressions for $kW_{\substack{Nom-Loss\\Secondary}}$:

$$\frac{k_1}{100\%} \cdot S_{Nom} = \left|\frac{S^*_{Nom}}{t \cdot kV^*_{Nom}}\right|^2 \cdot R_{Secondary} \cdot t \tag{25-7}$$

Secondary resistance can be solved for as:

$$R_{Secondary} = \frac{kV^2_{Nom} \cdot t \cdot k_1}{S^*_{Nom}} \tag{25-8}$$

Actual secondary losses based on load divided equally among taps is:

$$kW_{\substack{Act-Loss\\Secondary}} = \left|\frac{S^*_{Act}}{t \cdot kV^*_{Act}}\right|^2 \cdot R_{Secondary} \cdot t \tag{25-9}$$

The expression for secondary resistance is substituted in to get:

$$kW_{\substack{Act-Loss\\Secondary}} = \frac{S^2_{Act}}{t^2 \cdot kV^2_{Act}} \cdot \frac{kV^2_{Nom} \cdot t^2 \cdot k_1}{S^*_{Nom}} \cdot t \tag{25-10}$$

this reduces to:

$$kW_{\substack{Act-Loss\\Secondary}} = \left|\frac{kV_{Nom}}{kV_{Act}}\right|^2 \cdot \frac{S^2_{Act}}{S^*_{Nom}} \cdot k_1 \tag{25-11}$$

Lets assume that the kW ratio between actual and nominal load matches the kVA ratio. This leads to the final form of our secondary loss expression:

$$kW_{\substack{Act-Loss\\Secondary}} = \left| \frac{kV_{Nom}}{kV_{Act}} \right|^2 \cdot \frac{kW_{Act}^2}{kW_{Nom}} \cdot k_1 \quad \text{(25-12)}$$

We have made assumptions that:

- The relationship between secondary losses and total secondary load is linear and independent of voltage.
- kW and kvar losses can be decoupled
- The single k_1 factor can be applied to all distributed loads
- Loads are distributed evenly among taps on a distributed load

Clearly, these assumptions and this derivation indicate that the expression for secondary losses is not valid on a per-load basis. In other words, secondary losses at a particular load in a system should not be evaluated with the above expression.

The intention of the secondary losses expression is to provide a system wide representation of the trend for secondary losses. The k_1 factor can be set in SynerGEE. A typical value might be 3%, which indicates that a typical primary distributed load would have a secondary kW loss component of 3% at nominal load voltage. If half of distribution losses can be attributed to secondary losses then we would expect total feeder losses to run at around 6%.

Secondary losses are only evaluated by the Model Performance Comparison application. Secondary losses are also ignored for spot load since the split of losses between the customer and the utility may be complex.

A cost for secondary losses is determined by multiplying secondary kW losses by the utility cost per kW lost.

Cost of Losses

Losses for an entire year are multiplied by the utility cost for a kWh.

$$Cost_{Losses} = kW_{Loss}^{Avg} \cdot 8760(hr) \cdot Cost_{kWh} \quad \text{(25-13)}$$

Model performance comparison determines the average losses on a model by assuming a linear relationship between losses and loading. As we stated before, the model is assumed to be calibrated for peak load conditions. A load-flow is run to determine the losses at peak. Another load-flow is run with the minimum load factor to determine the losses at minimum load. The losses at the average loading are determined from the average load factor.

Model loading	Losses
Model at peak load	Found from load-flow run

Model at minimum load	Found from load-flow run
Model at average load	Extrapolated

A general extrapolation can be determined for $x_a = f(y_a)$ from a line consisting of (x_1, y_1) and (x_2, y_2).

In this system, we know that:

$$y_2 = \frac{y_2 - y_1}{x_2 - x_1} x_2 + b \tag{25-14}$$

so that the intercept is:

$$b = y_2 - = \frac{y_2 - y_1}{x_2 - x_1} x_2 \tag{25-15}$$

and in general:

$$y_a = \frac{y_2 - y_1}{x_2 - x_1} x_a + y_2 - \frac{y_2 - y_1}{x_2 - x_1} x_2 \tag{25-16}$$

which can be simplified to get:

$$y_a - y_2 = \frac{y_2 - y_1}{x_2 - x_1} (x_a - x_2) \tag{25-17}$$

Now we will make these substitutions:

$$\begin{aligned} (x_1, y_1) &= \left(kW_{Loss}^{Min}, LF_{Min}\right) \\ (x_a, y_a) &= \left(kW_{Loss}^{Avg}, LF_{Avg}\right) \\ (x_2, y_2) &= \left(kW_{Loss}^{Peak}, 1\right) \end{aligned} \tag{25-18}$$

to get:

$$\begin{aligned} & kW_{Loss}^{Avg} = \\ & \frac{\left(LF_{Avg} - 1\right)\left(kW_{Loss}^{Peak} - kW_{Loss}^{Min}\right)}{1 - LF_{Min}} + kW_{Loss}^{Peak} \end{aligned} \tag{25-19}$$

We know that the average load-factor should be less than 1. Things can be re-arranged to get our equation in a more intuitive form:

$$kW_{Loss}^{Avg} = kW_{Loss}^{Peak} - \frac{\left(1 - LF_{Avg}\right)\left(kW_{Loss}^{Peak} - kW_{Loss}^{Min}\right)}{1 - LF_{Min}} \quad (25\text{-}20)$$

Equipment Data

This section lists model data and settings that are managed within the settings for the Performance Comparison Application.

General Model Information

Conductor Data	
$Cond_{1\phi Cost}$	Material (wire, poles, cross-arms, arresters, etc.) cost per mile for a typical single-phase line having this phase conductor.
$Cond_{1\phi Inst}$	Install cost per mile for a typical single-phase line having this phase conductor.
$Cond_{1\phi Maint}$	Yearly maintenance cost per mile for a typical single-phase line having this phase conductor.
$Cond_{1\phi Salv}$	Salvage value per mile for a typical single-phase line having this phase conductor.
$Cond_{3\phi Cost}$	Material (wire, poles, cross-arms, arresters, etc.) cost per mile for a typical three-phase line having this phase conductor.
$Cond_{3\phi Inst}$	Install cost per mile for a typical three-phase line having this phase conductor.
$Cond_{3\phi Maint}$	Yearly maintenance cost per mile for a typical three-phase line having this phase conductor.
$Cond_{3\phi Salv}$	Salvage value per mile for a typical three-phase line having this phase conductor.
Voltage Regulator Data	
Reg_{Equip}	Cost of a new regulator and materials needed for installation.
Reg_{Inst}	Installation cost of regulator.
Reg_{Maint}	Yearly maintenance cost.
Reg_{Salv}	Salvage value of typical regulator.
Switch Data	
$Switch_{Equip}$	Cost of a new switch and materials needed for installation.
$Switch_{Inst}$	Installation cost of switch.
$Switch_{Maint}$	Yearly maintenance cost.
$Switch_{Salv}$	Salvage value of typical switch.

Generator Data	
Gen_{Equip}	Cost of a new generator and materials needed for installation.
Gen_{Inst}	Installation cost of generator.
Gen_{Maint}	Yearly maintenance cost.
Gen_{Salv}	Salvage value of typical generator.
Transformer Data	
$Tran_{Equip}$	Cost of a new transformer and materials needed for installation.
$Tran_{Inst}$	Installation cost of transformer.
$Tran_{Maint}$	Yearly maintenance cost.
$Tran_{Salv}$	Salvage value of typical transformer.

Settings

$Set_{2\phi\,Line\,C}$	Multiplier to determine material cost for a two-phase line from the three-phase line data contained within the conductor table.
$Set_{2\phi\,Line\,I}$	Multiplier to determine install cost for a two-phase line from the three-phase line data contained within the conductor table.
$Set_{2\phi\,Line\,M}$	Multiplier to determine maintenance cost for a three-phase line from the single-phase line data contained within the conductor table.
$Set_{Line\,Remove}$	Factor relating cost to remove a line to the cost of installing a line.
$Set_{Equip\,Remove}$	Factor relating cost to remove equipment to the cost of installing equipment.
$Set_{Reg\,Move}$	Multiplier to determine cost to move a regulator.
$Set_{Switch\,Move}$	Multiplier to determine cost to move a switch, sectionalizer or fuse.
$Set_{Tran\,Move}$	Multiplier to determine cost to move a transformer.
$Set_{Gen\,Move}$	Multiplier to determine cost to move a generator.
$Set_{ProtDev\,Move}$	Multiplier to determine cost to move a breaker or recloser.
$Set_{Bank\,I}$	Multiplier to determine installation of bank of three single-phase regulators or transformers from the installation cost of a single-phase unit.
$Set_{Bank\,M}$	Multiplier to determine maintenance cost of bank of three single-phase regulators or transformers from the maintenance cost of a single-phase unit.
$Set_{Cap\,E}$	Equipment and material cost for a typical capacitor installation.
$Set_{Cap\,I}$	Installation cost for capacitor bank.
$Set_{Cap\,M}$	Yearly maintenance cost for capacitor bank.

$Set_{Cap\,S}$	Salvage value for capacitor bank.
$Set_{Cap\,U}$	Cost increase factor per kvar.
$Set_{Cap\,Sw}$	Cost increase factor if installation is switched.
$Set_{ExpFuse\,E}$	Per/phase cost for an expulsion fuse, holder, etc.
$Set_{ExpFuse\,I}$	Per/phase installation cost for an expulsion fuse.
$Set_{ExpFuse\,M}$	Per/phase expulsion fuse maintenance cost.
$Set_{ExpFuse\,S}$	Per/phase expulsion fuse salvage value.
$Set_{CLFuse\,E}$	Per/phase cost for a current limiting fuse, holder, etc.
$Set_{CLFuse\,I}$	Per/phase installation cost for a current limiting fuse.
$Set_{CLFuse\,M}$	Per/phase current limiting fuse maintenance cost.
$Set_{CLFuse\,S}$	Per/phase current limiting fuse salvage value.
$Set_{HydRec\,E}$	Hydraulic recloser cost
$Set_{HydRec\,I}$	Installation cost for hydraulic recloser
$Set_{HydRec\,M}$	Yearly maintenance cost for hydraulic recloser
$Set_{HydRec\,S}$	Salvage value for hydraulic recloser
$Set_{ElecRec\,E}$	Electronic recloser cost
$Set_{ElecRec\,I}$	Installation cost for electronic recloser
$Set_{ElecRec\,M}$	Yearly maintenance cost for electronic recloser
$Set_{ElecRec\,S}$	Salvage value for electronic recloser
$Set_{MechRel\,E}$	Mechanical relay, breaker, hardware cost
$Set_{MechRel\,I}$	Installation cost for mechanical relay and breaker
$Set_{MechRel\,M}$	Yearly maintenance cost for mechanical relay and breaker
$Set_{MechRel\,S}$	Salvage value for mechanical relay
$Set_{ElecRel\,E}$	Electronic relay, breaker, hardware cost
$Set_{ElecRel\,I}$	Installation cost for electronic relay and breaker
$Set_{ElecRel\,M}$	Yearly maintenance cost for mechanical relay and breaker
$Set_{ElecRel\,S}$	Salvage value for electronic relay and breaker
$Set_{Secz\,E}$	Sectionalizer cost.
Set_{Secz}	Installation cost for sectionalizer.
$Set_{Secz\,M}$	Yearly maintenance cost for sectionalizer.
$Set_{Secz\,S}$	Salvage value for sectionalizer.

Cost calculations

Cost calculations are determined based on the detected changes between the Gallery model and the base model. Not all changes are detected or deemed significant to cost of the alternative.

Code	Description	Cost
	Line Section Cost Calculations	
Sect_01	**Line extended**	
	Material	$1\phi\, Line:\ ExtensionLength \cdot Cond_{1\phi Cost}$ $2\phi\, Line: ExtensionLength \cdot Set_{2\phi LineC} \cdot Cond_{3\phi Cost}$ $3\phi\, Line: ExtensionLength \cdot Cond_{3\phi Cost}$
	Installation	$1\phi\, Line:\ ExtensionLength \cdot Cond_{1\phi Inst}$ $2\phi\, Line: ExtensionLength \cdot Set_{2\phi Line I} \cdot Cond_{3\phi Inst}$ $3\phi\, Line: ExtensionLength \cdot Cond_{3\phi Inst}$
	Maintenance	$1\phi\, Line:\ ExtensionLength \cdot Cond_{1\phi Maint}$ $2\phi\, Line: ExtensionLength \cdot Set_{2\phi Line M} \cdot Cond_{3\phi Maint}$ $3\phi\, Line: ExtensionLength \cdot Cond_{3\phi Maint}$
	Salvage	$0
Sect_02	**Shortened line**	
	Material	$0
	Installation	$1\phi\, Line:\ Length \cdot Cond_{1\phi Inst} \cdot Set_{Line Remove}$ $2\phi\, Line: Length \cdot Set_{2\phi Line I} \cdot Cond_{3\phi Inst} \cdot Set_{Line Remove}$ $3\phi\, Line: Length \cdot Cond_{3\phi Inst} \cdot Set_{Line Remove}$
	Maintenance	$1\phi\, Line:\ -Length \cdot Cond_{1\phi Maint}$ $2\phi\, Line: -Length \cdot Set_{2\phi Line M} \cdot Cond_{3\phi Maint}$ $3\phi\, Line: -Length \cdot Cond_{3\phi Maint}$
	Salvage	$1\phi\, Line:\ Length \cdot Cond_{1\phi Salv}$ $2\phi\, Line: 0.5 \cdot Length \cdot \left(Cond_{1\phi Salv} + Cond_{3\phi Salv}\right)$ $3\phi\, Line: Length \cdot Cond_{3\phi Salv}$
Sect_03	**New line added to model**	
	Material	$1\phi\, Line:\ Length \cdot Cond_{1\phi Cost}$ $2\phi\, Line: Length \cdot Set_{2\phi LineC} \cdot Cond_{3\phi Cost}$ $3\phi\, Line: Length \cdot Cond_{3\phi Cost}$

	Installation	$1\phi\, Line:\ Length \cdot Cond_{1\phi\, Inst}$ $2\phi\, Line: Length \cdot Set_{2\phi\, Line\, I} \cdot Cond_{3\phi\, Inst}$ $3\phi\, Line: Length \cdot Cond_{3\phi\, Inst}$
	Maintenance	$1\phi\, Line:\ Length \cdot Cond_{1\phi\, Maint}$ $2\phi\, Line: Length \cdot Set_{2\phi\, Line\, M} \cdot Cond_{3\phi\, Maint}$ $3\phi\, Line: Length \cdot Cond_{3\phi\, Maint}$
	Salvage	$0
Sect_04	**Removed line**	
	Material	$0
	Installation	$0
	Maintenance	$1\phi\, Line:\ -Length \cdot Cond_{1\phi\, Maint}$ $2\phi\, Line: -Length \cdot Set_{2\phi\, Line\, M} \cdot Cond_{3\phi\, Maint}$ $3\phi\, Line: -Length \cdot Cond_{3\phi\, Maint}$
	Salvage	$0
Switch Cost Calculations		
Swch_01	**New Switch**	
	Material	$Switch_{Equip}$
	Installation	$Switch_{Inst}$
	Maintenance	$Switch_{Maint}$
	Salvage	$0
Swch_02	**Removed Switch**	
	Material	$0
	Installation	$Switch_{Inst} \cdot Set_{Equip\, Remove}$
	Maintenance	$-Switch_{Maint}$
	Salvage	$Switch_{Salv}$
Swch_03	**Moved Switch**	
	Material	$0
	Installation	$Switch_{Inst} \cdot Set_{Switch\, Move}$
	Maintenance	$0
	Salvage	$0
Swch_04	**Switch Closed**	
	Material	$0
	Installation	$0
	Maintenance	$0
	Salvage	$0
Swch_05	**Switch Opened**	
	Material	$0
	Installation	$0

	Maintenance	$\$0$
	Salvage	$\$0$
Regulator Cost Calculations		
Reg_01	**New Regulator**	
	Material	$1\phi\, or\, 3\phi\, Unit = Reg_{Inst}$ $2\phi\, Bank = 2 \cdot Reg_{Inst}$ $3\phi\, Bank = 3 \cdot Reg_{Inst}$
	Installation	$1\phi\, or\, 3\phi\, Unit = Reg_{Inst}$ $2\phi\, Bank = \frac{Reg_{Inst}\left(Set_{Bank\,I}+1\right)}{2}$ $3\phi\, Bank = Reg_{Inst} \cdot Set_{Bank\,I}$
	Maintenance	$1\phi\, or\, 3\phi\, Unit = Reg_{Maint}$ $2\phi\, Bank = \frac{Reg_{Maint}\left(Set_{Bank\,M}+1\right)}{2}$ $3\phi\, Bank = Reg_{Maint} \cdot Set_{Bank\,M}$
	Salvage	$\$0$
Reg_02	**Removed Regulator**	
	Material	$\$0$
	Installation	$1\phi\, or\, 3\phi\, Unit = Reg_{Inst} \cdot Set_{Equip\,Remove}$ $2\phi\, Bank = \frac{Reg_{Inst} \cdot Set_{Equip\,Remove}\left(Set_{Bank\,I}+1\right)}{2}$ $3\phi\, Bank = Reg_{Inst} \cdot Set_{Equip\,Remove} \cdot Set_{Bank\,I}$
	Maintenance	$1\phi\, or\, 3\phi\, Unit = -Reg_{Maint}$ $2\phi\, Bank = -\frac{Reg_{Maint}\left(Set_{Bank\,M}+1\right)}{2}$ $3\phi\, Bank = -Reg_{Maint} \cdot Set_{Bank\,M}$
	Salvage	$1\phi\, or\, 3\phi\, Unit = Reg_{Salv}$ $2\phi\, Bank = 2 \cdot Reg_{Salv}$ $3\phi\, Bank = 3 \cdot Reg_{Salv}$
Reg_03	**Moved Regulator**	
	Material	$\$0$

	Installation	$1\phi\ or\ 3\phi\ Unit = Reg_{Inst} \cdot Set_{Reg\ Move}$ $2\phi\ Bank = \frac{Reg_{Inst} \cdot Set_{Reg\ Move}\left(Set_{Bank\ I} + 1\right)}{2}$ $3\phi\ Bank = Reg_{Inst} \cdot Set_{Reg\ Move} \cdot Set_{Bank\ I}$
	Maintenance	$0
	Salvage	$0
Reg_04	**Regulator Settings Change**	
	Material	$0
	Installation	$0
	Maintenance	$0
	Salvage	$0
Generator Cost Calculations		
Gen_01	**New Generator**	
	Material	Gen_{Equip}
	Installation	Gen_{Inst}
	Maintenance	Gen_{Maint}
	Salvage	$0
Gen_02	**Removed Generator**	
	Material	$0
	Installation	$Gen_{Inst} \cdot Set_{Equip\ Remove}$
	Maintenance	$-Gen_{Maint}$
	Salvage	Gen_{Salv}
Gen_03	**Moved Generator**	
	Material	$0
	Installation	$Gen_{Inst} \cdot Set_{Gen\ Move}$
	Maintenance	$0
	Salvage	$0
Gen_04	**Generator Settings Change**	
	Material	$0
	Installation	$0
	Maintenance	$0
	Salvage	$0
Transformer Cost Calculations		
Tran_01	**New Transformer**	
	Material	$1\phi\ or\ 3\phi\ Unit = Tran_{Inst}$ $2\phi\ Bank = 2 \cdot Tran_{Inst}$ $3\phi\ Bank = 3 \cdot Tran_{Inst}$

	Installation	$1\phi\ or\ 3\phi\ Unit = Tran_{Inst}$ $2\phi\ Bank = \frac{Tran_{Inst}\left(Set_{Bank\ I}+1\right)}{2}$ $3\phi\ Bank = Tran_{Inst}\cdot Set_{Bank\ I}$
	Maintenance	$1\phi\ or\ 3\phi\ Unit = Tran_{Maint}$ $2\phi\ Bank = \frac{Tran_{Maint}\left(Set_{Bank\ M}+1\right)}{2}$ $3\phi\ Bank = Tran_{Maint}\cdot Set_{Bank\ M}$
	Salvage	\$0
Tran_02	**Removed Transformer**	
	Material	\$0
	Installation	$1\phi\ or\ 3\phi\ Unit = Tran_{Inst}\cdot Set_{Equip\ Remove}$ $2\phi\ Bank = \frac{Tran_{Inst}\cdot Set_{Equip\ Remove}\left(Set_{Bank\ I}+1\right)}{2}$ $3\phi\ Bank = Tran_{Inst}\cdot Set_{Equip\ Remove}\cdot Set_{Bank\ I}$
	Maintenance	$1\phi\ or\ 3\phi\ Unit = -Tran_{Maint}$ $2\phi\ Bank = -\frac{Tran_{Maint}\left(Set_{Bank\ M}+1\right)}{2}$ $3\phi\ Bank = -Tran_{Maint}\cdot Set_{Bank\ M}$
	Salvage	$1\phi\ or\ 3\phi\ Unit = Tran_{Salv}$ $2\phi\ Bank = 2\cdot Tran_{Salv}$ $3\phi\ Bank = 3\cdot Tran_{Salv}$
Tran_03	**Moved Transformer**	
	Material	\$0
	Installation	$1\phi\ or\ 3\phi\ Unit = Tran_{Inst}\cdot Set_{Tran\ Move}$ $2\phi\ Bank = \frac{Tran_{Inst}\cdot Set_{Tran\ Move}\left(Set_{Bank\ I}+1\right)}{2}$ $3\phi\ Bank = Tran_{Inst}\cdot Set_{Tran\ Move}\cdot Set_{Bank\ I}$
	Maintenance	\$0
	Salvage	\$0
Tran_04	**Transformer Settings Change**	
	Material	\$0
	Installation	\$0
	Maintenance	\$0
	Salvage	\$0
Capacitor Cost Calculations		

Cap_01	**New Capacitor**	
	Material	$Set_{Cap\,E}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}$
	Installation	$Set_{Cap\,I}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}$
	Maintenance	$Set_{Cap\,M}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}$
	Salvage	$0
Cap_02	**Removed Capacitor**	
	Material	$0
	Installation	$Set_{Cap\,I}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}\cdot Set_{Equip\,Remove}$
	Maintenance	$-Set_{Cap\,M}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}$
	Salvage	$Set_{Cap\,S}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}$
Cap_03	**Moved Capacitor**	
	Material	$0
	Installation	$Set_{Cap\,I}\left(1+kvar\cdot Set_{CapU}\right)\cdot Set_{Cap\,Sw}\cdot Set_{Cap\,Move}$
	Maintenance	$0
	Salvage	$0
Fuse Cost Calculations		
Fuse_01	**New Fuse**	
	Material	$Expulsion\ Fuse = Set_{ExpFuse\,E}$ $Current\ Limiting = Set_{CurLimFuse\,E}$
	Installation	$Expulsion\ Fuse = Set_{ExpFuse\,I}$ $Current\ Limiting = Set_{CurLimFuse\,I}$
	Maintenance	$Expulsion\ Fuse = Set_{ExpFuse\,M}$ $Current\ Limiting = Set_{CurLimFuse\,M}$
	Salvage	$0
Fuse_02	**Removed Fuse**	
	Material	$0
	Installation	$Expulsion\ Fuse = Set_{ExpFuse\,I}\cdot Set_{Equip\,Remove}$ $Current\ Limiting = Set_{CurLimFuse\,I}\cdot Set_{Equip\,Remove}$
	Maintenance	$Expulsion\ Fuse = -Set_{ExpFuse\,M}$ $Current\ Limiting = -Set_{CurLimFuse\,M}$
	Salvage	$Expulsion\ Fuse = Set_{ExpFuse\,S}$ $Current\ Limiting = Set_{CurLimFuse\,S}$
Fuse_03	**Moved Fuse**	
	Material	$0

	Installation	$Expulsion\ Fuse = Set_{ExpFuse\,I} \cdot Set_{Switch\,Move}$ $Current\ Limiting = Set_{CurLimFuse\,I} \cdot Set_{Switch\,Move}$
	Maintenance	\$0
	Salvage	\$0
Recloser Cost Calculations		
Rec_01	**New Recloser**	
	Material	$Hydraulic\ Recloser = Set_{HydRec\,E}$ $Electronic\ Recloser = Set_{ElecRec\,E}$
	Installation	$Hydraulic\ Recloser = Set_{HydRec\,I}$ $Electronic\ Recloser = Set_{ElecRec\,I}$
	Maintenance	$Hydraulic\ Recloser = Set_{HydRec\,M}$ $Electronic\ Recloser = Set_{ElecRec\,M}$
	Salvage	\$0
Rec_02	**Removed Recloser**	
	Material	\$0
	Installation	$Hydraulic\ Recloser = Set_{HydRec\,I} \cdot Set_{Equip\,Remove}$ $Electronic\ Recloser = Set_{ElecRec\,I} \cdot Set_{Equip\,Remove}$
	Maintenance	$Hydraulic\ Recloser = -Set_{HydRec\,M}$ $Electronic\ Recloser = -Set_{ElecRec\,M}$
	Salvage	$Hydraulic\ Recloser = Set_{HydRec\,S}$ $Electronic\ Recloser = Set_{ElecRec\,S}$
Rec_03	**Moved Recloser**	
	Material	\$0
	Installation	$Hydraulic\ Recloser = Set_{HydRec\,I} \cdot Set_{ProtDev\,Move}$ $Electronic\ Recloser = Set_{ElecRec\,I} \cdot Set_{ProtDev\,Move}$
	Maintenance	\$0
	Salvage	\$0
Breaker Cost Calculations		
Brkr_01	**New Breaker**	
	Material	$Mechanical\ Relay = Set_{MechRel\,E}$ $Electronic\ Relay = Set_{ElecRel\,E}$
	Installation	$Mechanical\ Relay = Set_{MechRel\,I}$ $Electronic\ Relay = Set_{ElecRel\,I}$
	Maintenance	$Mechanical\ Relay = Set_{MechRel\,M}$ $Electronic\ Relay = Set_{ElecRel\,M}$
	Salvage	\$0

Brkr_02	**Removed Breaker**	
	Material	$0
	Installation	$Mechanical\ Relay = Set_{MechRel\ I} \cdot Set_{Equip\ Remove}$ $Electronic\ Relay = Set_{ElecRel\ I} \cdot Set_{Equip\ Remove}$
	Maintenance	$Mechanical\ Relay = -Set_{MechRel\ M}$ $Electronic\ Relay = -Set_{ElecRel\ M}$
	Salvage	$Mechanical\ Relay = Set_{MechRel\ S}$ $Electronic\ Relay = Set_{ElecRel\ S}$
Brkr_03	**Moved Breaker**	
	Material	$0
	Installation	$Mechanical\ Relay = Set_{MechRel\ I} \cdot Set_{ProtDev\ Move}$ $Electronic\ Relay = Set_{ElecRel\ I} \cdot Set_{ProtDev\ Move}$
	Maintenance	$0
	Salvage	$0
Sectionalizer Cost Calculations		
Sczr_01	**New Sectionalizer**	
	Material	$Set_{Secz\ E}$
	Installation	$Set_{Secz\ I}$
	Maintenance	$Set_{Secz\ M}$
	Salvage	$0
Sczr_02	**Removed Sectionalizer**	
	Material	$0
	Installation	$Set_{Secz\ I} \cdot Set_{Equip\ Remove}$
	Maintenance	$-Set_{Secz\ M}$
	Salvage	$Set_{Secz\ S}$
Sczr_03	**Moved Sectionalizer**	
	Material	$0
	Installation	$Set_{Secz\ I} \cdot Set_{Switch\ Move}$
	Maintenance	$0
	Salvage	$0

Conductor Selection

The analysis is to determine the best conductors to use for a sub-network based on conductor cost and performance. Conductors are selected through a partitioning process. The least expensive conductor to support the entire sub-network is selected. Then, sub-networks are determined based on conductor changes and forks. Evaluation is used to determine if smaller conductors would be suitable for those sub-networks. Those areas are then searched for phase changes and forks and the process is repeated.

Approach

Step 1 – Select an area to be reconductored. This area is called the selection digraph. The selection digraph will be composed of at least one selection out-tree.

Step 2 – Place monitors at the most inward device in each selection out-tree.

Step 3 – Place monitors within the selection out-trees at devices where phasing changes or splits in three phase lines occur.

Step 4 – The out-tree bridged by each monitor will be considered a conductor selection zone. Form an outward trace of conductor selection zones.

Step 5 – Find the most suitable conductor for each conductor selection zone using the outward trace.

Suitability

Conductor selection is made by evaluating
all conductors in the selection digraph for:

- Voltage level
- Loading
- Losses
- Fault level

All conductors dependent on the selection digraph for:

- Voltage level
- Fault level

Conservation voltage reduction

Conservation voltage reduction (CVR) helps you evaluate feeder performance as a function of feeder voltage. It makes successive load-flow analysis runs from the lowest specified feeder voltage to the highest. You can specify the step size or voltage interval. You can also list results by the feeder voltage for each feeder. Report results can easily be copied to a spreadsheet for plotting and manipulation.

For each feeder analyzed reported information may include:

- Feeder volts – The range specified when the application was launched. Values are listed in volts.
- Drop – The largest drop in the feeder is listed in the report. The voltage drop is calculated by comparing the feeder voltage to a section voltage, as follows.

$$V_{Drop} = \left|V_{Fdr}^{1}\right| - \left|V_{Sect}^{1}\right| \tag{25-21}$$

- Low volts – The lowest voltage in the feeder. The drop value and the low volts value should add up to match the feeder voltage value.
- High volts – The highest section voltage in the feeder.
- kVA – Feeder kVA demand at a specified feeder voltage.
- kW – Feeder real power demand at a specified feeder voltage.
- Amps – Feeder load current.
- pf – Power factor of feeder demand at a specified feeder voltage.
- Pct loss – Feeder loss as a percentage of feeder kW demand.
- VLT exceptions – Instances where one or more voltage exceptions exists at the specified feeder voltage.
- OLD exceptions – Instances where one or more overloaded lines or devices exist.
- CTL exceptions – Instances where at least one regulator is sitting at its minimum or maximum tap.

Example

An example of the benefits of CVR analysis follows. It shows a study to justify the installation of one or two regulators on a feeder.

As with all SynerGEE analyses, you can copy and plot values from the report. The following image contains the results from the CVR runs that were copied and plotted using a spreadsheet package. This plot shows the range of low voltages as a function of feeder voltage.

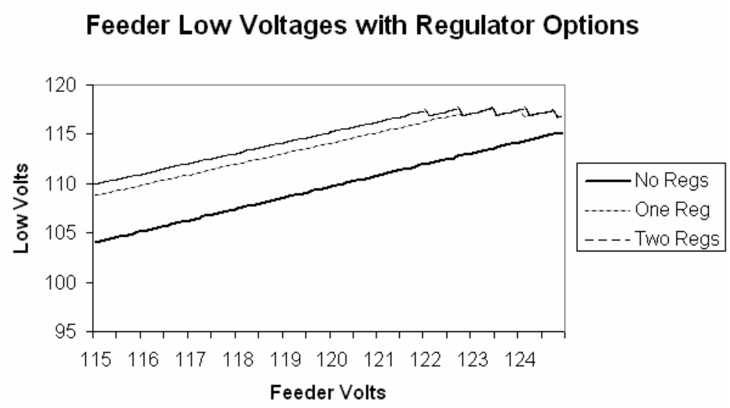


Fig. 25-5 ***Plot of feeder low voltage***

You can see the improvement in low voltage due to regulation. You can also see the effect of voltage settings on the regulator and the drop from the regulator to the end of the feeder at higher feeder voltages.

The following plot addresses high voltage problems.

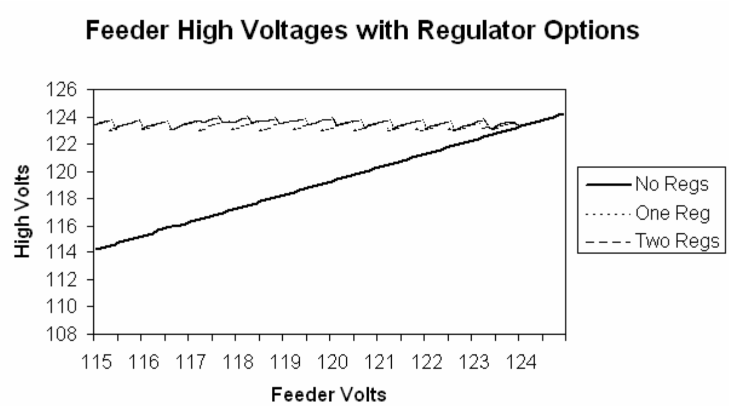


Fig. 25-6 ***Plot of feeder high voltage***

Again, the effect of the regulator voltage settings is apparent. This study may be useful to justify the installation of one regulator, while demonstrating that the benefits of a second regulator would not be worth the cost.

Financial Worksheets

Time range analysis includes a special worksheet for handling engineering economics. You can have SynerGEE generate a worksheet based on the model in memory, or you can create a blank worksheet and work with it as needed.

The worksheet displays present worth, future worth, and uniform amount values based on costs incurred with various models and a cost of money interest rate.

The following cost models are supported.

Model	Description
Present cost	Cost occurring once at the beginning of the project
Fixed cost	Cost occurring at some specified period in a project.
Recurring cost	Fixed costs occurring at regular intervals.
Future cost	Cost occurring at the end of the project.
Interval cost	Fixed cost occurring at regular intervals between a starting and ending period.
Growing cost	Cost growing from zero at the starting period to the specified amount at the ending period.
Declining cost	Cost declining from the specified amount at the starting period to zero at the ending period.

The present worth is determined in all cost models. Once the present worth is known, the future worth and uniform amounts are found.

The "Net APR" in the financial worksheets represents the combination of interest rates, inflation rates, and other factors affecting the cost of money. It is an annual percentage rate. The number of periods in the study, N, is given. The number of periods per year is m. Therefore:

APR	Net APR
N	Number of periods in study
m	Number of periods per year

If you were to compound the periodic rate into an annual percentage rate, you would see:

(25-22)

$$APR = \left(1 + \frac{r_{nom}}{m}\right)^m - 1$$

Therefore, the periodic interest rate is:

(25-23)

$$r_p = \left(APR + 1\right)^{1/m} - 1$$

Again, the input into the worksheet is the APR and m.

The following are some of the fundamental equations used by the worksheet.

$$PW / FW = \frac{1}{\left(1 + r_p\right)^N} \tag{25-24}$$

$$PW / UA = \frac{\left(1 + i\right)^N - 1}{r_p \left(1 + r_p\right)^N} \tag{25-25}$$

If an application generates a financial worksheet, you can manually add new cost items or modify existing cost items.

26

Customer & Distribution Transformer Management

Introduction

The Customer Management Module (CMM) provides a link between SynerGEE and customer information systems (CIS). It offers the ability to utilize the customer information effectively in SynerGEE models. CMM reads customer and distribution transformer information from a comma separated value (CSV) text file, processes it, and updates the SynerGEE model. The CSV file contains data taken out of Graphical Information System (GIS) and Customer Information System (CIS) databases.

CMM assigns customers to distribution transformers and sections in the model, calculates transformer utilization, capacity factor, load values, and creates customer zones and classes. Then it updates the model and saves the information to a CMM database, so that user can generate reports and update the model at any time.

Using this data in a SynerGEE model allows a variety of analysis, such as:

- Calculating the demand on specific distribution transformers to prevent overload damage or inefficient operation
- Determining the customer makeup of a poorly performing area
- Testing the feasibility of system changes, based on the number/type of customers affected
- Validating the accuracy of your current load models and load allocation methods
- Generate customer and transformer reports and find customers within SynerGEE network analysis environment to enhance model effectiveness and system knowledge.

Customer data is never a permanent part of a model, but load data is updated in the model. CMM utilizes tools to generate reports and updates load data from the saved database, as necessary.

Understanding the CMM process

CMM can process five categories of customer information. These are the following:

- Customers
- Distribution Transformers
- Billing Records (kWh consumption, kW and kVAR demand)
- Hourly Load Records
- Factors (Coincidence Factor, Load Factor, Power Factor)
- Poles

This information is gathered from GIS and CIS and exported to the Customers CSV file. GIS contains geospatial information about the network and CIS contains billing, rate class and address information about the customers. From the GIS, information about the connectivity of transformers and customers and sections (electric lines) is exported to the Customers CSV file. Information about billing and customers is exported from CIS database. Next, command scripts are added to the Customers CSV file to assign, validate customers, and perform other functions. Factors from load research and the assignment commands are also incorporated into the customer CSV file.

CMM reads the information from Customers CSV file, performs the script commands, and then saves the data with the assignment information into the CMM database (Access, Oracle). CMM can process the information from GIS, CIS and the command scripts as one Customers CSV file or it can process each file separately.

Finally, once the CMM database has been updated, SynerGEE can also read the data directly from CMM database and update the SynerGEE model and generate reports.

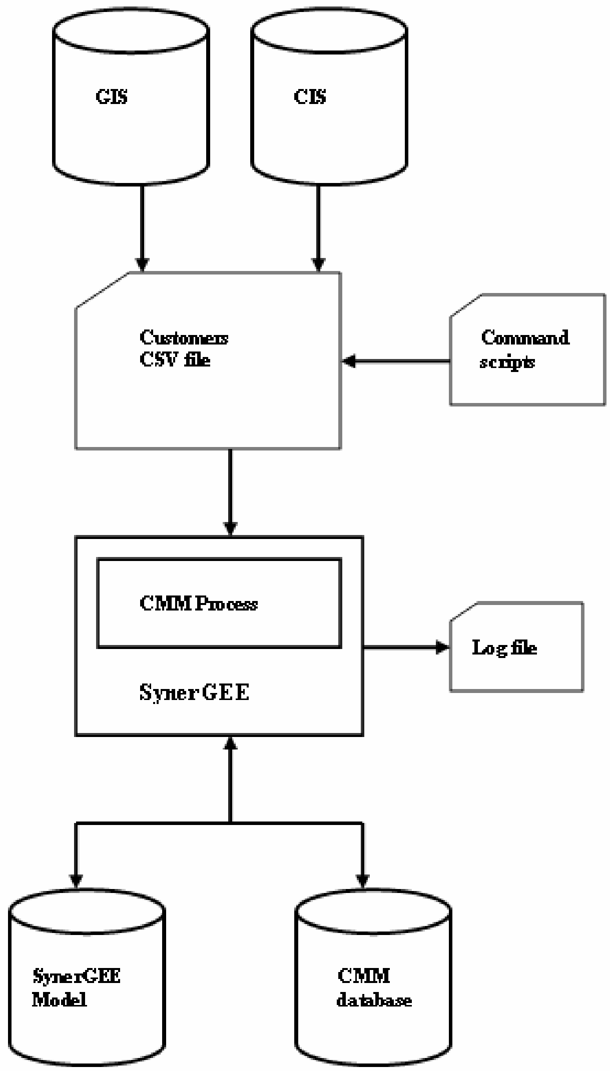


Fig. 26-1 *Data flow with CMM*

In the above process, it is shown that CMM database and model can be updated together with the "Customers CSV file". The model can also be updated in two separate processes. First, the CMM script file is read and CMM database is updated. Next, CMM reads the customer data from the CMM database and updates the model. If the data is already saved in the CMM database or if the model is big, this two step process will be the preferred approach.

Basic Nomenclature

The following basic nomenclature is used in the CMM:

Lf	Load factor

Cf	Coincidence factor
Pf	Power factor
E	Energy usage (kWh)
k	Index for a customer class
j	Index for individual customer
l	Index for individual transformer
h	Index for hour of day 1, 2, … , 24
m	Index for month 1, 2, … , 12
P_{max}	Maximum Power demand (kW)
P_{ave}	Average Power demand (kW)
S	Volt Ampere demand (kVA)
U	Transformer utilization

Electric Demand

Different types of customers use electricity to meet their needs. Customer needs may be unique to the type of customers depending on their uses of electricity and activity patterns. Electric demand on the system is characterized by the number and type of customers using electricity. An electric utility assigns a customer class or a rate class to identify the type of each customer. This helps to analyze electricity usage on a customer class basis by recognizing that customers of different classes show different usage patterns and priorities.

The electricity usage within a customer class depends on a number of aspects. Main aspects are the variation of electricity demand by time of day and season (load curves), variation of peak load with the number of customers, and power factor. These aspects are summarized into three factors:

- Load factor
- Coincidence factor
- Power factor

About Load Factors

Load on a power system will vary from hour to hour, from day to day and from season to season. Typically, the customers belonging to same customer class will show similar load pattern. Planners are most interested in the peak demand that must be delivered. The average value of power during the demand interval is obtained by dividing the kilowatt-hours consumed during the demand interval by the length of the interval. Load factor is the ratio of the average to the peak demand. Figure below shows the load curve for a customer class with peak and average values.

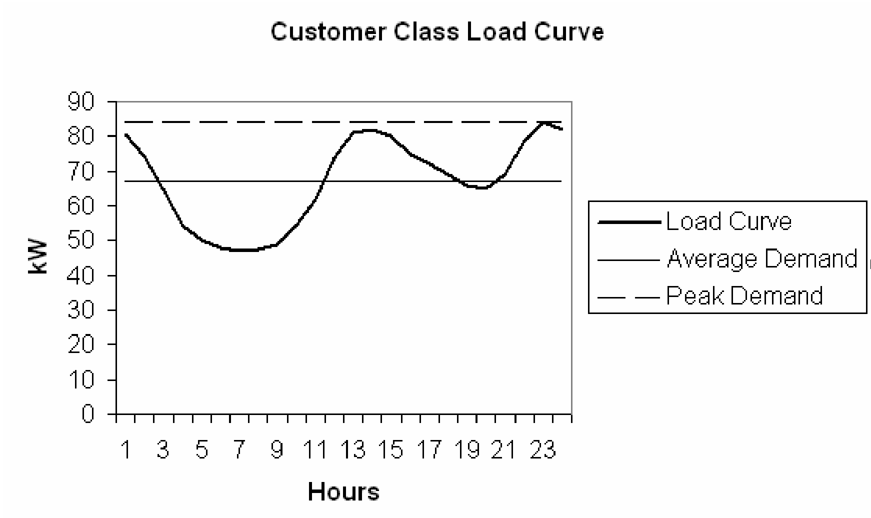


Fig. 26-2 Load curve

The following are definitions:

$$Average\ Demand = \frac{kWh}{hours} \tag{26-1}$$

$$Load\ Factors = \frac{Average\ Demand}{Maximum\ Demand} \tag{26-2}$$

From the individual customer usage information, the average demand for the customers of each customer class on the transformer is determined for the billing period. Then the maximum demand on the transformer is obtained by applying the load factor for each customer class and then summing up the maximum demand for the customers of each customer class.

$$Max\ Demand_{After\ Load\ Factor} = \frac{Average\ Demand_{Customer\ Usage}}{Load\ Factor} \tag{26-3}$$

CMM requires load factor information for each customer class to determine transformer utilization and capacity factor. The load factor is the ratio of average demand during a period of time to the maximum demand during that period. For example, a customer may have a maximum demand of 2 kW during the evening when the lights, television and electrical appliances are in use. The same customer may have 12 kWh during a 24 hour period. The average demand is 12 kWh divided by 24 hours or 0.5 kW. The load factor is 0.5 kW divided by 2 kW or 25%.

About Diversity Factors/Coincidence Factor

Coincidence factor for a customer class is a function of the number of customers in the customer class. The demand created by two or more customers is normally less than the sum of their individual demands because the peak demand for each customer occurs at a different time. For example, one customer's peak demand may occur in the early morning and another customer's peak may occur in the early evening. Peak load (demand) per customer drops as more customers are added to the group.

Group peak occurs when the combination of individual load curves peak at the same time. This is called diversity and is measured by the diversity factor. It has a value greater than 1 and varies with the number of customers in the same manner as the peak load varies. For large group of customers, the diversity factor can be as high as 5.

The diversity factor is the sum of individual peak demands of 2 or more customers divided by the maximum demand created by them.

$$Diversity\ Factor = \frac{Sum\ of\ Peak\ Demands}{Peak\ Demand\ of\ Customers} \tag{26-4}$$

Coincidence factor is the inverse of diversity factor. It has a value between 0 and 1.

$$Coincidence\ Factor = \frac{1}{Diversity\ Factor} \tag{26-5}$$

A diversity or coincidence factor can be used to calculate the actual peak demand given the sum of the individual peak demands.

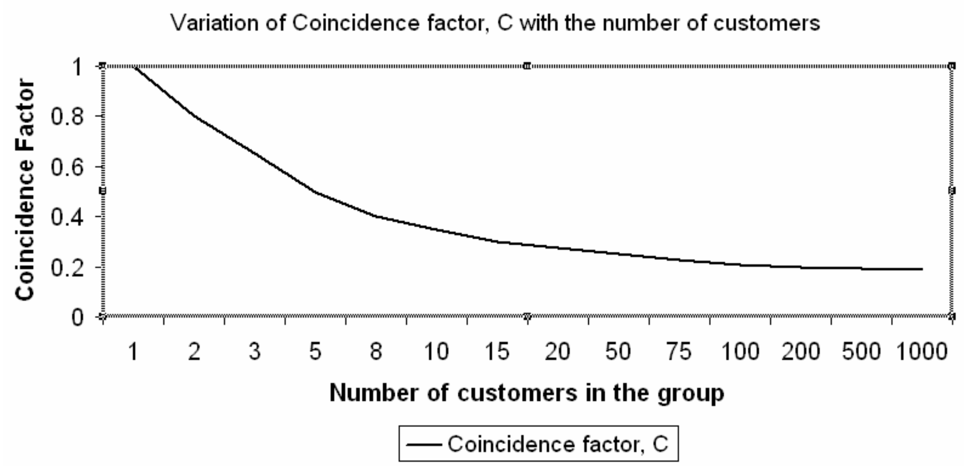


Fig. 26-3 *Variation of coincidence factor with number of customers*

Coincidence factor varies with the customer class and the number of customers in each class as shown in figure above. Therefore coincidence values are specified for the customer class for various customer counts. Coincidence factor values are saved in CMM database.

About Power Factors

Customer electric loads belonging to same customer class will exhibit homogeneous power factor. CMM requires the power factor by customer class to calculate the electric demand at each transformer and section in the model. Electrical loads can be resistive, capacitive, inductive, or a combination of the three. Resistive loads draw only real power, measured in watts. Loads which are capacitive or inductive draw only reactive or imaginary power. These loads are measured in volt-amperes reactive (VAR). Most loads are a combination of resistance and reactance. The resultant load is measured in volt-amperes.

The power factor of a load is the load in watts divided by the load in volt-amperes.

$$pf = \frac{watts}{VA} \tag{26-6}$$

Power factor of a customer class is the ratio of load in Watts for sample of customers within a customer class to the load in Volt Amperes (VA) for the same sample of customers.

Power factors range between negative one hundred percent (-100%) and positive one hundred percent (100%). Resistive loads have a power factor of 100% because the voltage and current are in phase. Capacitive loads have power factors ranging from 0 to -100% and inductive loads have power factors ranging from 0 to 100%. Capacitive power factors are considered leading because the current leads the voltage. Inductive power factors are considered lagging because the current lags the voltage.

Transformer Utilization Calculation

Transformer utilization is a measure of transformer load during the billing period. If the transformer is heavily loaded, it will have a high utilization and is more likely to be replaced / damaged.

Peak demand must be estimated in order to get a more detailed analysis and to calculate the utilization at the individual distribution transformer level. Peak demand is estimated using the individual load characteristics of the load served by the distribution transformer. Maximum demand at each transformer is determined from the billing data for each customer served by the transformer and then applying factors (load factor, coincidence factor, power factor) based on the customer class of the customer.

Typically, transformer load management reports at utilities apply a factor to monthly usage to determine transformer utilization and monthly usage is the addition of billing usage for each of the customers assigned to the transformer. Problems occur when the customer mix on the systems is non-homogeneous.

CMM uses customer class based conversion factors to provide improved load estimates. Coincidence factor, load factor, and power factor are all customer class based factors that are used to calculate transformer load utilization. Customer classes are more homogeneous, which creates much better results than the old way of doing transformer load management reports.

When the load data is imported, customer loads are summed by load class (customer class) for each transformer. Starting and ending dates of the billing period are established. Using the power factor, diversity factor, load factor, and number of customers in each customer class, transformer demand is calculated using the following equations.

$$S_{Calc} = \sum_{\substack{j:1\to Cust \\ k:1\to Classes}} \frac{E_{j,k} C_{j,k}}{H \cdot Lf_{j,k} \cdot pf_{j,k}} \tag{26-7}$$

In this equation, k varies from 1 to number of customer classes served by transformer and j varies from 1 to number of customers served by the transformer.

Once the transformer demand is calculated for each transformer, utilization is calculated as follows:

$$U = \frac{S_{Calc}}{S_{Rated}} \tag{26-8}$$

After the calculations are processed, the transformer utilization report shows the utilization of each transformer.

If a transformer has rated kVA of 100kVA and the calculated demand is 90 kVA, then the transformer is 90% utilized.

Example of Transformer Utilization Calculation:

The Application assigns customers to the transformers and calculates usage based on the billing data. In this example, following customers are assigned to one Transformer (TR_0001)

Transformer ID	**Customer ID**	**Customer Class**	**Usage (kWh)**	**Billing Days**
TR_0001	Customer_01	Residential	5000	119
TR_0001	Customer_02	Residential	6000	119
TR_0001	Customer_03	Residential	4000	119
TR_0001	Customer_04	Residential	3000	119
TR_0001	Customer_05	Residential	7000	119
TR_0001	Customer_06	Residential	8000	119
TR_0001	Customer_07	Commercial	15000	119
TR_0001	Customer_08	Commercial	25000	119

Total usage for 119 days is 73000 kWh

Average kW for transformer = Usage / Hours billed

$= 73000 / 119 * 24 = 25.56$ kW

Average kW for the set of Residential class customers

$= 33000 / 119 * 24 = 11.55$

Average kW for the set of Commercial class customers

$= 40000 / 119 * 24 = 14.01$

Following are the factors for each customer classes:

Load Factor

Apply the Load Factor:

Average kW for the set of Residential customer class

$= \quad 11.55 / 0.5 = 23.1$

Average kW for the set of Commercial customer class

= 14.01 / 0.7 = 20.01

Customer Class	Load Factor
Residential	0.5
Commercial	0.7

Power Factor

Apply the Power Factor:

Average kVA for the set of Residential customer class

= 23.1 / 0.9 = 25.67

Average kVA for the set of Commercial customer class

= 20.01 / 0.8 = 25.01

Customer Class	Power Factor
Residential	0.90
Commercial	0.80

Coincidence Factor

Apply Coincidence Factor:

From the table below, diversity factor for 6 residential customers

= 0.7

Coincidence factor for 2 commercial customers

= 0.9

Average kVA for the set of Residential customers = 25.67 * 0.7

= 17.97

Average kVA for the set of Commercial customers = 25.01 * 0.9

= 22.51

Customer Class	Number of Customers	Coincidence Factor
Residential	2	0.9

Residential	4	0.8
Residential	6	0.7
Residential	8	0.65
Commercial	2	0.9
Commercial	4	0.85
Commercial	6	0.80
Commercial	8	0.75

Calculated average kVA for Transformer = 17.97 + 22.51 = 40.48 kVA

Rated kVA for the transformer = 50 kVA

Transformer Utilization = 40.48 / 50 = 0.81 or 81%

Updating Load Values

CMM sums the electricity usage for the customers supplied by each transformer during the specified time interval and then applies the factors by customer class to the usage to determine the electric demand at the transformer. This electric demand is compared to the rated kVA of the transformer to help determine if the transformer is over-utilized or under-utilized. The utilization factor calculated for each transformer is also saved to the CMM database.

Electricity usage is also summed at the section level to calculate the kWh, kVA, and number of customers served by the section. Spot load and large customer demand information is summed at the section level for updating the SynerGEE model.

After processing the customer data, CMM module updates the SynerGEE model with the new calculated values.

Load values are assigned in the following ways:

- Update section distributed load connected kVA with distribution transformer values
- Update section distributed load customer count
- Update distribution transformer connected kWh with customer total kWh values
- Update spot load demand (kW, kVAR) and connected kVA

- Update Large customer loads kW and kVAR values

The respective values on all distributed, spot, and large customer loads in the target area will be updated. The target includes selected feeders in SynerGEE Map Display. The relationship between customers and distribution transformers and the relationship between distribution transformers and model sections need to be established before assigning load. These relationships are normally established based on the connectivity information available in the GIS.

Validation is the final function of the updating loads process. SynerGEE can check the validity of phasing on customers and/or distribution transformers. It can also check the consistency of the SectionId field between customers and their connected distribution transformer.

Capacity Factor Calculation

Capacity factor is needed to allocate the meter demands accurately for the distributed loads. Capacity factor allows you to simulate under-utilized transformers in your system. The factor, which can be from 0 (0%) to 1 (100%) or greater, is applied as a flat multiplier to connected kVA on all phases during load allocation. If you know that certain transformers are consistently under-utilized, capacity factors can help you achieve a more accurate representation of actual load following allocation. Studies have shown that this can greatly improve the accuracy of the model. Capacity factors are specified by section. Only one factor is permitted per section, which applies to all phases.

Capacity factor on a section is determined by calculating electric demand on the section based on the usage and factors of the customers served by the section. Calculated demand is compared with the connected kVA of the section to determine the capacity factor on the section. CMM calculates the capacity factor of the distributed load on the section as follows:

Capacity factor is calculated in a similar way as the transformer utilization. The difference is that all the customers served by section are considered and these can be served by multiple transformers.

(26-9)

$$S_{Calc} = \sum_{\substack{j:1 \to Cust \\ k:1 \to Classes \\ l:1 \to \#Tran}} \frac{E_{j,k,l} C_{j,k,l}}{H \cdot Lf_{j,k,l} \cdot pf_{j,k,l}}$$

Again, k varies from 1 to number of customer classes served by transformer and j varies from 1 to number of customers served by the transformer and l varies from 1 to the number of transformers on the section.

Once the transformer demand is calculated for each transformer, utilization is calculated as follows:

$$U = \frac{S_{Calc}}{S_{Rated}} \tag{26-10}$$

CMM calculates and updates the model with capacity factor for distributed loads. If a section has only one transformer, then the transformer utilization and capacity factor on a section will be the same. However, if there are several transformers on a section, then the capacity factor may be different than the utilization for each transformer and will be a weighted average of the transformer utilization of all transformers on that section.

Capacity factor is used during the load allocation process to accurately allocate kW and kvar loads for the distributed load.

Customer Zone Setup

Each customer zone can be comprised of three customer classes in different percentages for each customer class. Customer class percentages are determined by the usage of customers on the section for each customer class.

This is shown by an example:

Example: Section 0001 contains some customers from residential and some from commercial. The consumption data:

Customer ID	Customer Class	Usage (kWh)
Customer_01	Residential	5000
Customer_02	Residential	6000
Customer_03	Residential	4000
Customer_04	Residential	3000
Customer_05	Residential	7000
Customer_06	Residential	8000
Customer_07	Commercial	15000
Customer_08	Commercial	25000

Total residential usage is 2200 kWh, and total commercial usage is 5000 kWh. Residential usage percentage is 2160 / 7160 (30%). Commercial usage percentage is 5000 / 7200 (70%).

CMM will create a customer zone "Residential_30_Commercial_70" (if not already present in the warehouse) and will assign the customer zone to section 0001.

Updating Customer Class Curves

SynerGEE supports the modeling of loads using 24-hour daily load curves. CMM can read the hourly load data for a class of customer and create a load curve (customer class curve) in SynerGEE. This combined with the creation and assignment of customer zones to the model provides the basis for detailed simulations involving time of day and time range analysis in SynerGEE.

With the wide availability of electronic demand recorders, utilities can gather hourly load data for diverse classes of customers. The hourly kW and kvar measurements can then be processed by CMM to create a customer class load curve in SynerGEE. Customer class curve is an hourly load profile exhibited by the customer class by the type of day and month.

CMM reads the hourly kW and kvar information (8760 data points) for the customer class and creates weekday and weekend load curves for each month for the customer class. CMM calculates the Hour 1 value of weekday load curve for the month as the average of hour 1 values of all the weekdays in the month. Similar process is done for each of the hours of the weekday and weekend of the 12 months to create customer class curves. The weekday and weekend customer class curves of all months are then normalized by the maximum value for the weekday or weekend customer class curve and added to the SynerGEE warehouse.

By creating and applying customer class curves, you can conduct much more comprehensive analyses than with traditional peak loading alone. With daily load curves, you can study your system on- or off-peak, at a chosen hour, day, and month. For example, you could study your system at peak loading or at minimum loading for an upcoming season. Or, as another example, you could study the trends in load movement across or between feeders during the daylight hours of a weekday in July.

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