

Patch Modeling and JPEG

Mathematical Models and Methods for Image Processing

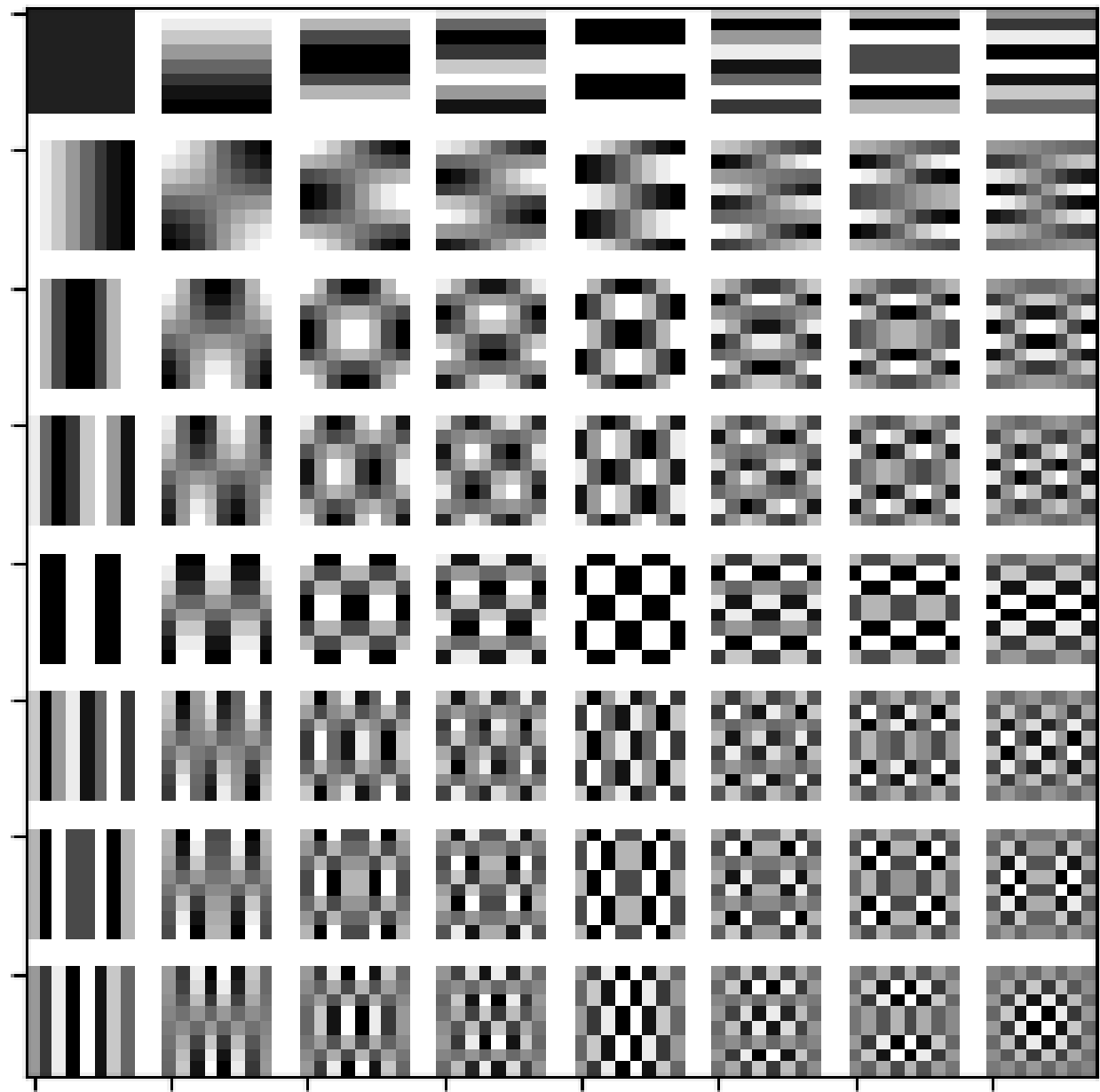
Edoardo Peretti

<https://boracchi.faculty.polimi.it/teaching/MMMIP.htm>

February 20th 2025

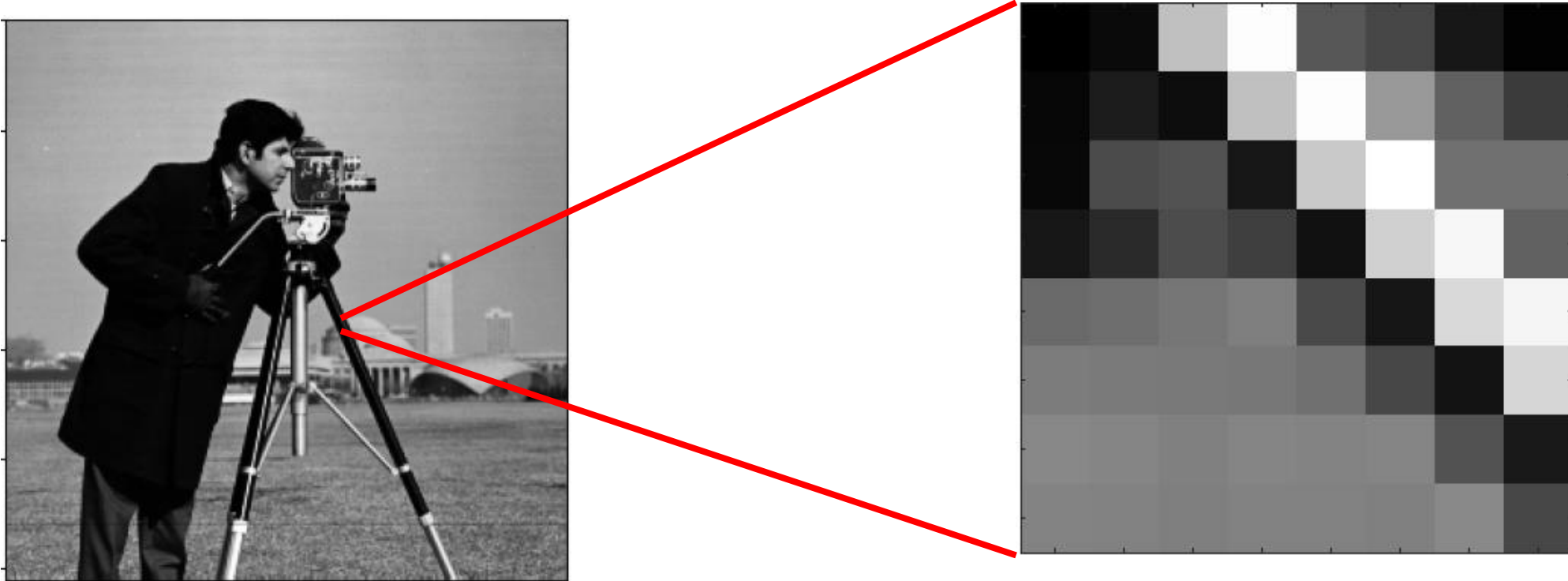
2D DCT

How the atoms in the 2D DCT dictionary look like

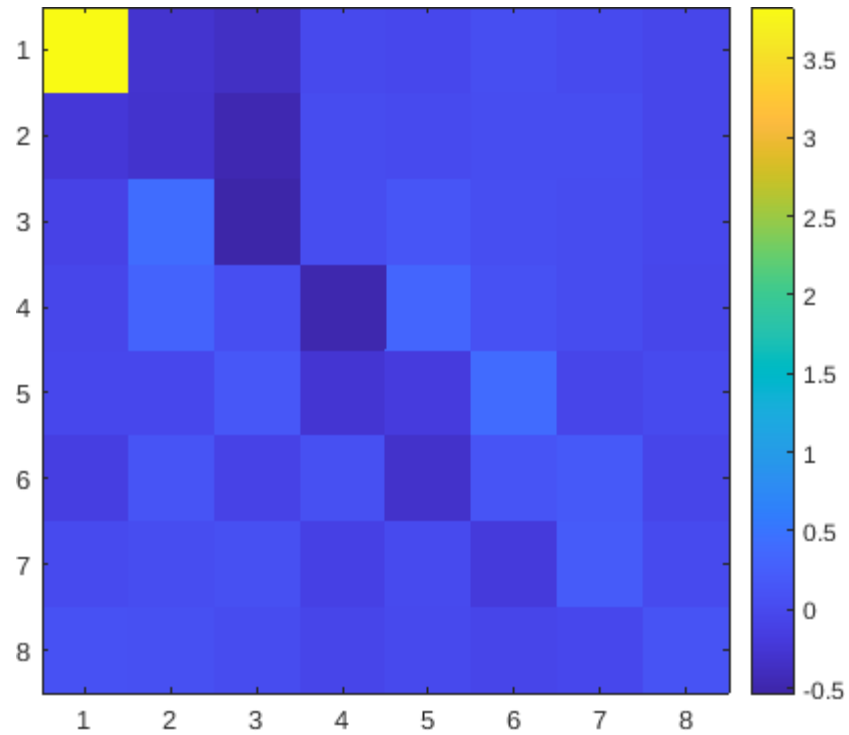


The JPEG Compression

Let's extract a 8x8 patch from an image

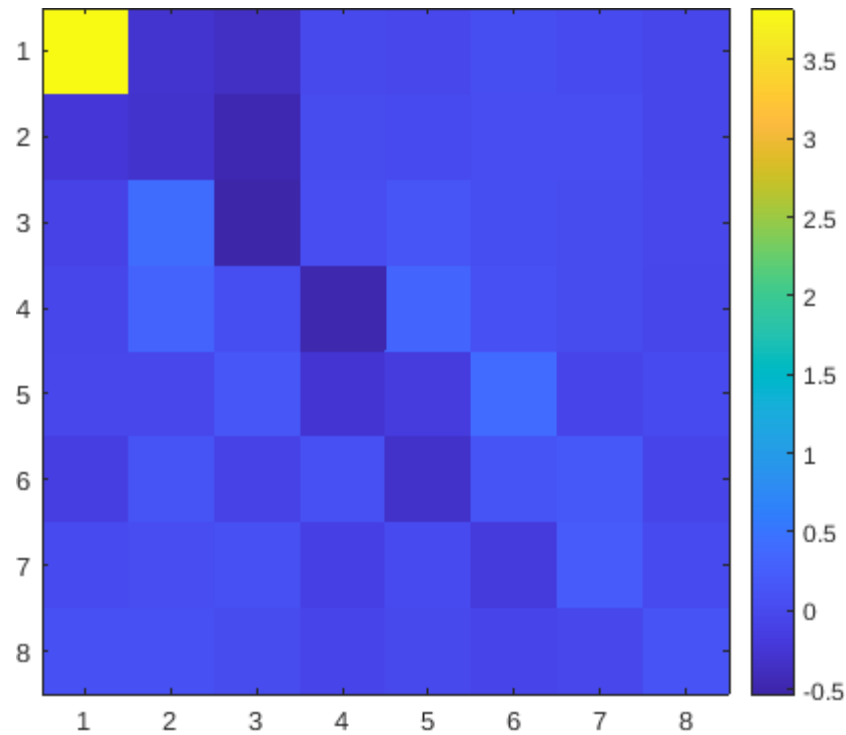


2D DCT of the patch

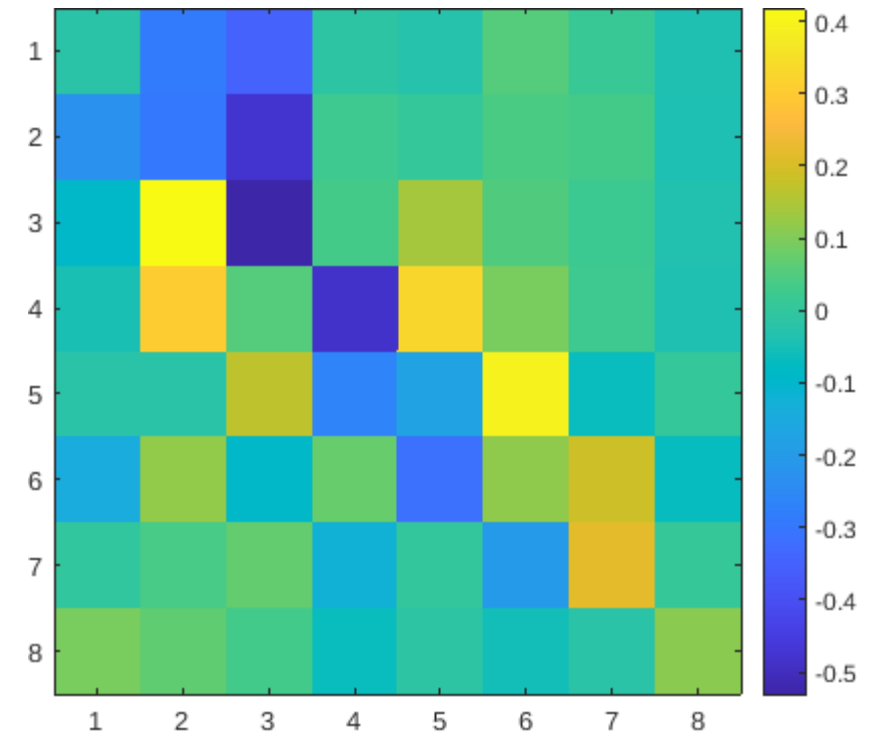


2D DCT of the patch

With DC



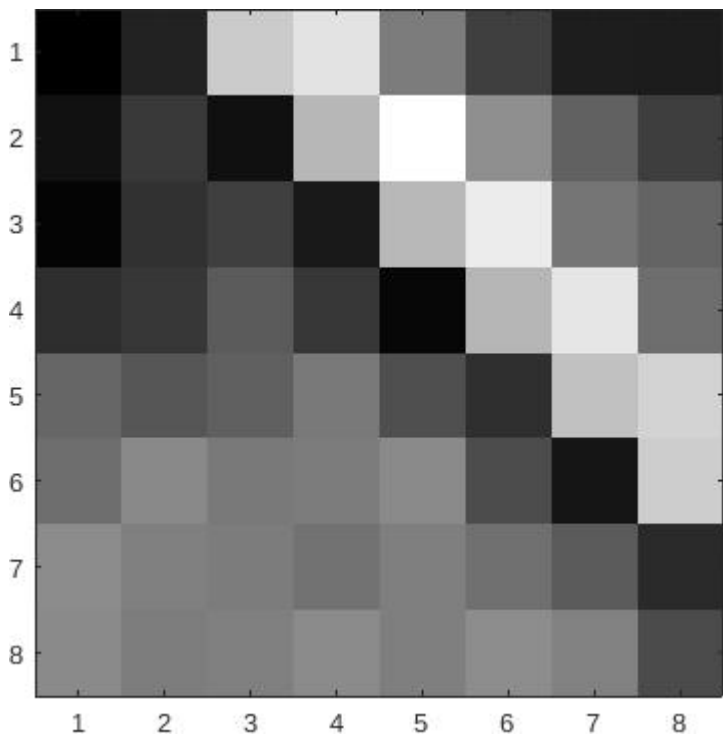
Without DC



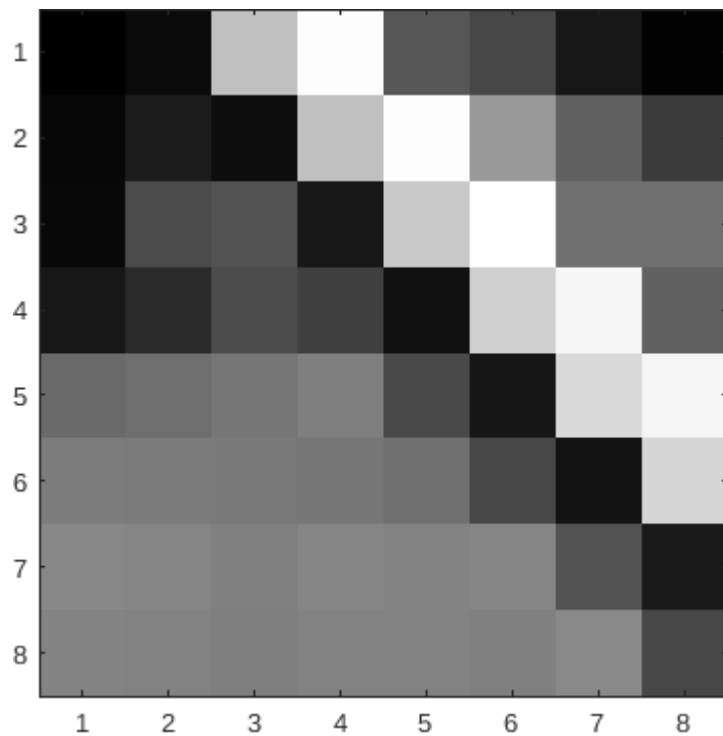
A lot of coefficients are closed to 0!

Reconstructed patch

Original Patch

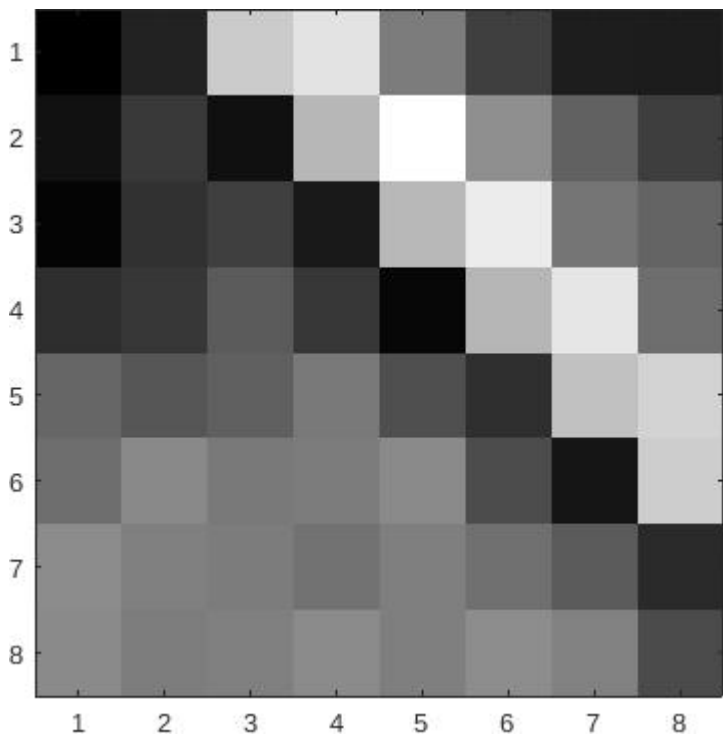


Reconstructed Patch

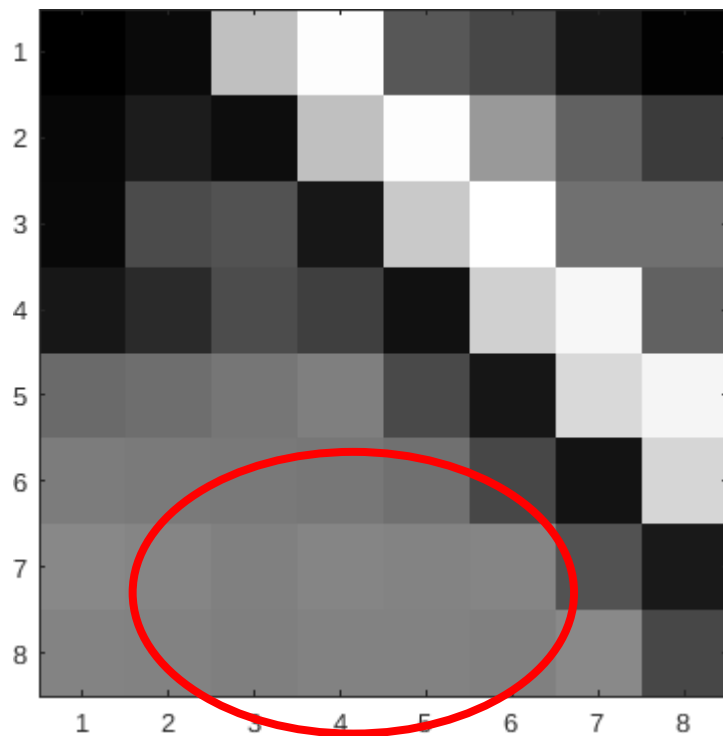


Reconstructed patch

Original Patch



Reconstructed Patch



Smooting in this area

Assignments

Last Time Assignment: Generate the Basis

- Generate the DCT basis according to the following formula (DCT type II) the k -th atom of the DCT basis in dimension M is defined as

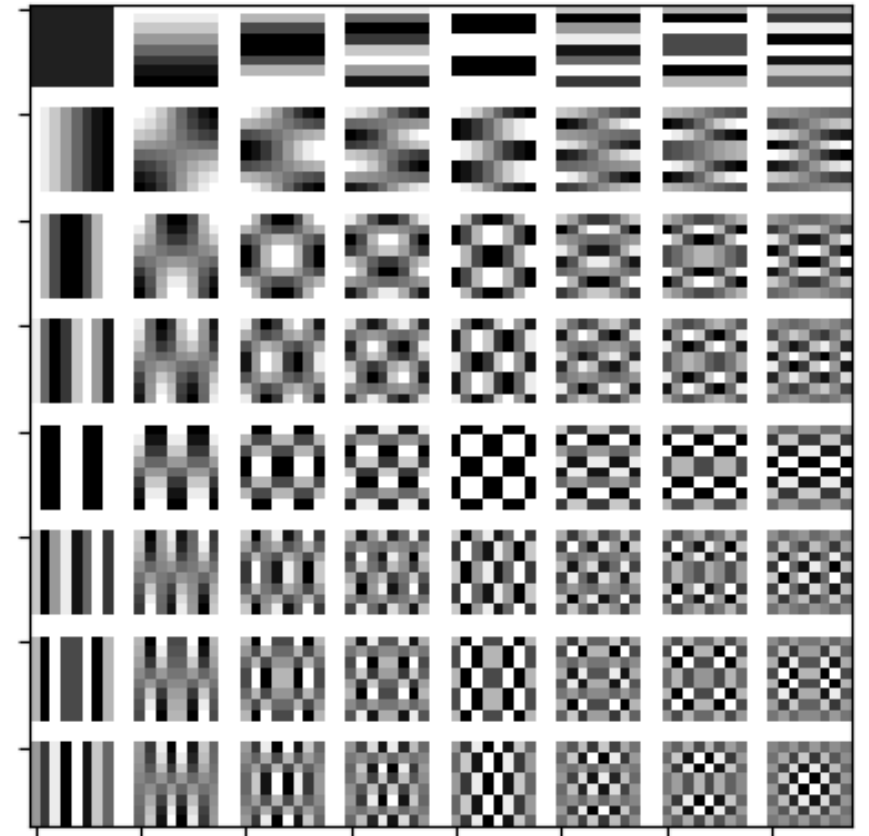
$$DCT_k(n) = c_k \cos\left(k\pi \frac{2n+1}{2M}\right) \quad n, k = 0, \dots, M-1$$

where $c_0 = \sqrt{1/M}$ and $c_k = \sqrt{2/M}$ for $k \neq 0$.

- How can you use the function `dct` and its inverse `idct` to define the DCT matrix?

First Assignment: 2D DCT dictionary

- Generate the 2D DCT dictionary using the `dct2` and `idct2` functions
 - Use this dictionary to compute the representation of a patch
- Generate the 1D DCT dictionary using the `dct` and `idct` functions
 - Use this dictionary to compute the separable 2D DCT of the same patch
- Verify that the coefficients of the two representations are the same

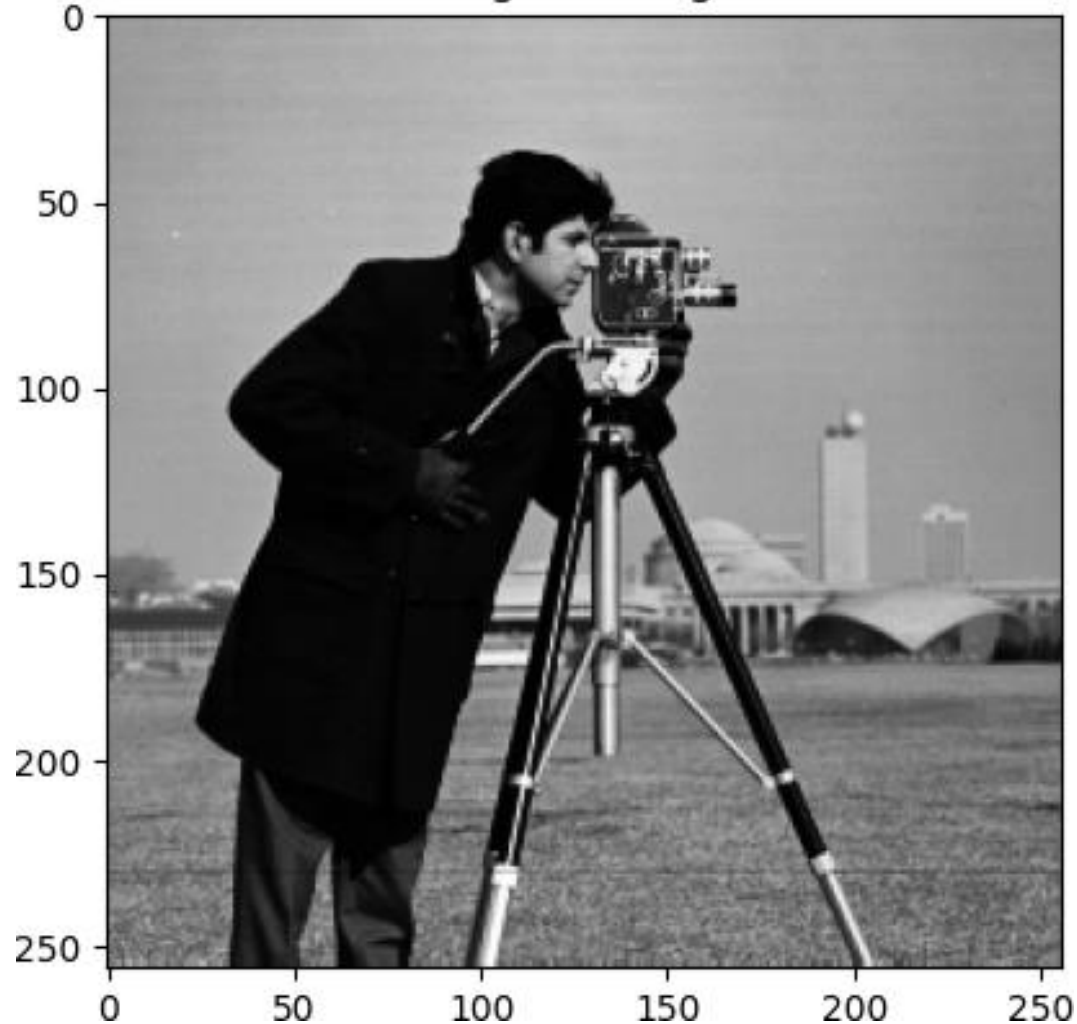


Second Assignment: JPEG Compression

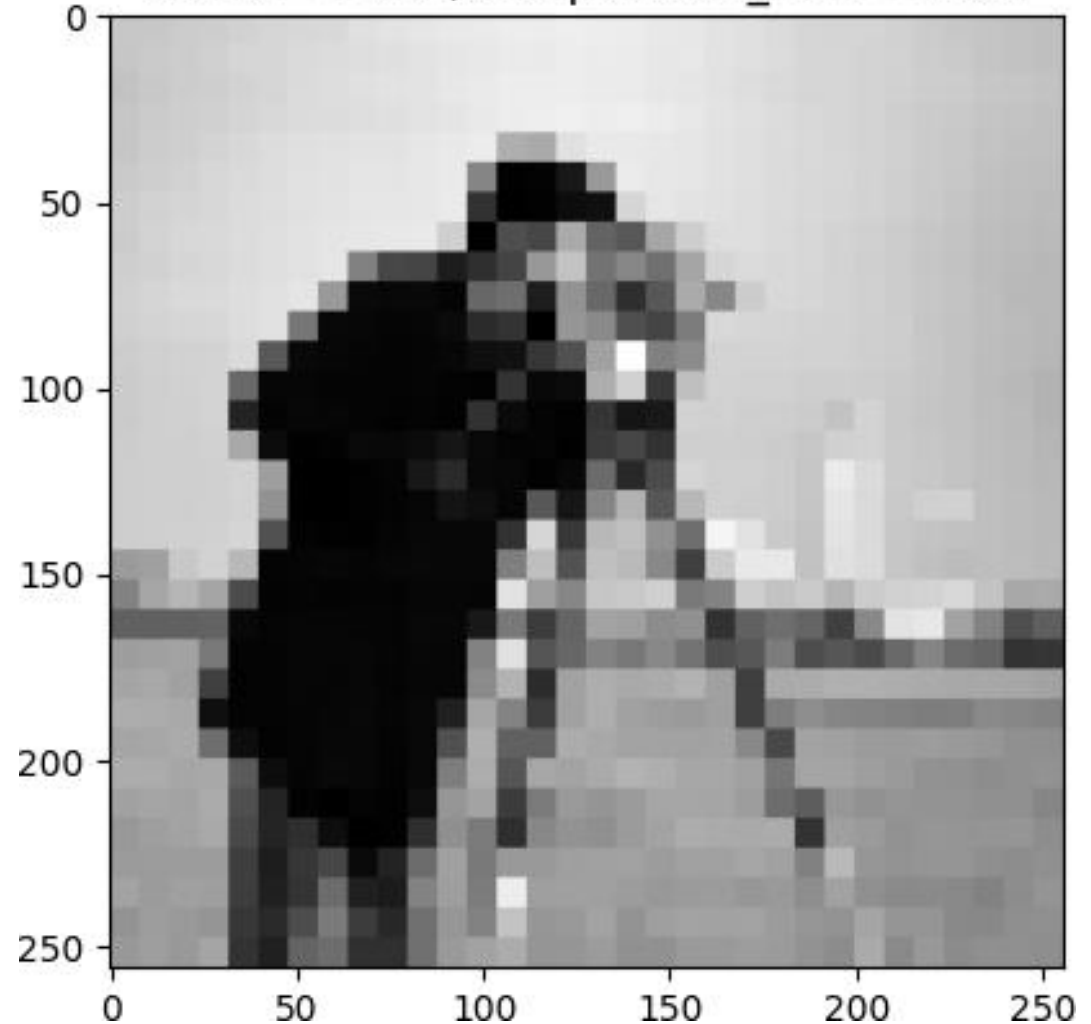
- Implement the JPEG compression algorithm
- Compute the PSNR of the compressed image
- Compute the compression ratio
- Try different thresholds: how the thresholds affect the compression?
- Run the JPEG compression on the test image and try different threshold. What can you observe?

Second Assignment: JPEG Compression

Original Image

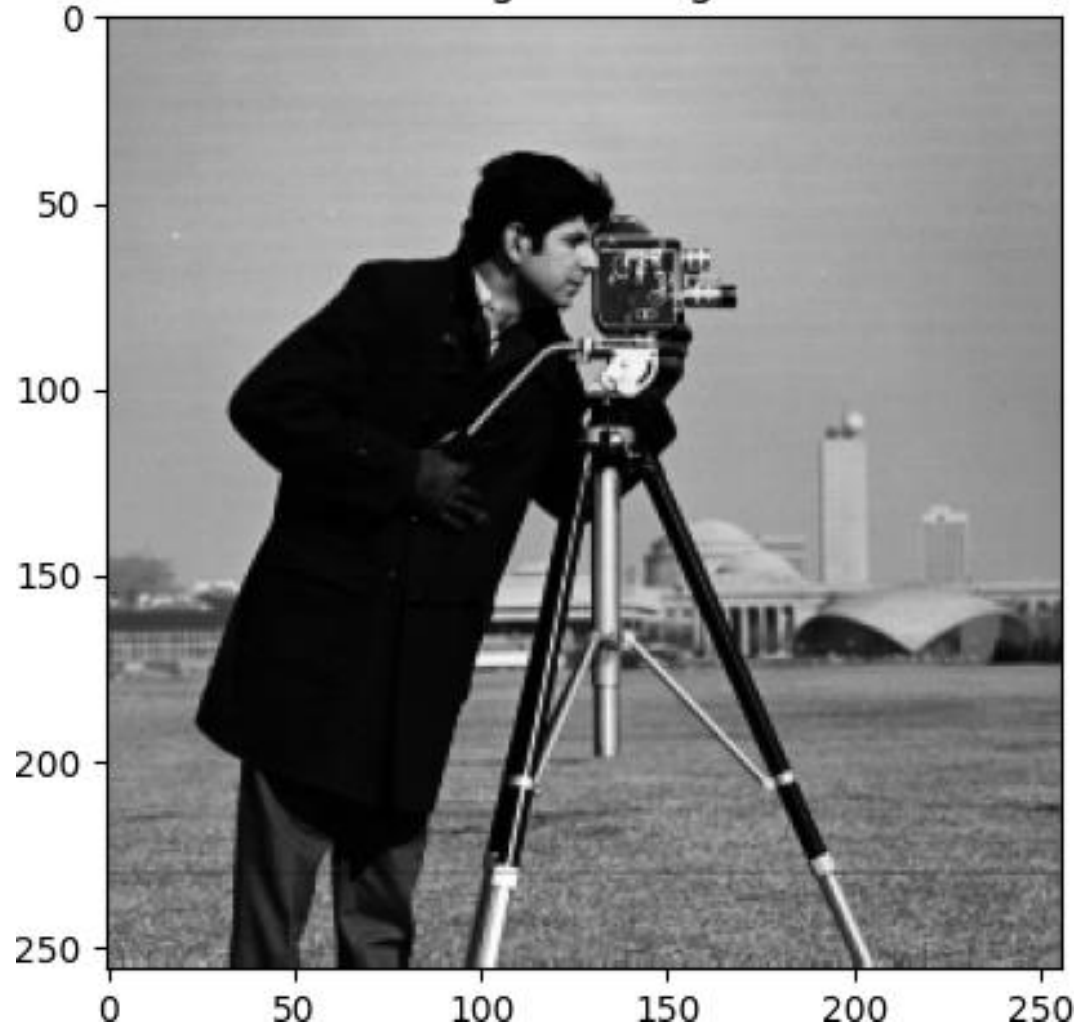


Compressed Image,
PSNR = 19.48, compression_ratio = 0.02

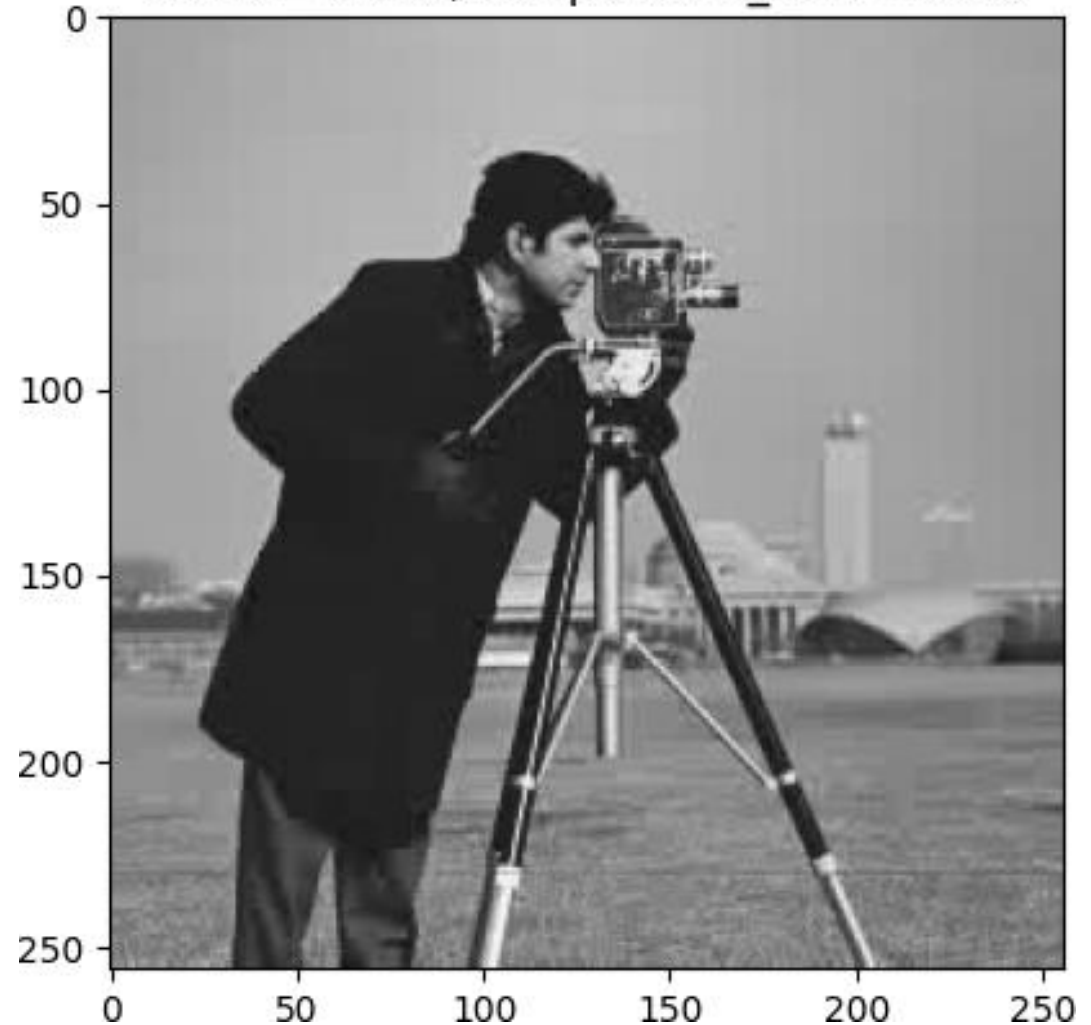


Second Assignment: JPEG Compression

Original Image



Compressed Image,
PSNR = 31.94, compression_ratio = 0.10



Second Assignment: JPEG Compression

Compressed Image,
PSNR = 31.94, compression_ratio = 0.10

