CSci 435: Formal Languages and Automata

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**Home Assignment 5: 100 points + 15 points (optional)**

In any (N/D)PDA, assume that a start stack symbol z is already in the stack; so, you don’t have to insert z into the stack at the beginning of transition.

Q1. [20] For a given language L = { *w* | *na*(*w*) + *nb*(*w*) = *nc*(*w*) } where Σ = Γ = {*a*, *b, c*}

1. [10] Construct a PDA M that accepts L with Σ = Γ = {*a*, *b, c*}

a,Z0 / aZ0

b,Z0 / bZ0

c,Z0 / cZ0

a,a / aa

b,b / bb

a,b /ab

b,a / ba

a,c / E

b,c / E

c,a / E

c,b / E

c,c / cc

Diagram

Description automatically generated

1. [10] Show the sequence of instantaneous descriptions for the acceptance of *acacbcbc* by M in 1).

Initial stack symbol is Z0

\*a\* come push to stack

\*c\* come delete \*a\*

\*a\* come push to stack

\*c\* come delete \*a\*

\*b\* come push to stack

\*c\* come delete \*b\*

\*b\* come push to stack

\*c\* come delete \*b\*

\*b\* come push to stack

\*c\* come delete \*b\*

\*e\* come above Z0 -> go to accept state

1. [10, optional] Give a CFG G that generates L, L(G) = L.

S → aX/ bX / e

X → S A / A S

A → c

Q2. [20] Construct an NPDA for the following languages.

1. [10] L1 = {*bba*\**bab*\*}

A close up of a clock

Description automatically generated

1. [10] L2 = {*bbb\*aba*}

A picture containing clock, drawing

Description automatically generated

1. [5, optional] L4 = L2 – L1.

L2-L1 = L2-(L1 ∩ L2)

but {L1∩L2}= {bbaba}

so L2 – L1 = L2 – {bbaba}

This means that L2 should have b+ instead of b\*, because then only bbaba is not accepted

So, nfa is

Diagram

Description automatically generated

Q3. [10] Give the language that is accepted by the NPDA M in a formal expression (including a regular expression) where M = ({*q0, q1, q2*}, {*a, b*}, {*a, b*, z}, δ, *q0*, z, { *q0* , *q1*, *q2*}), with transitions

♦ δ(*q0*, *a*, z) = {(*q1*, *a*), (*q2*, λ)},

♦ δ(*q1*, *b*, *a*) = {(*q1*, *b*)},

♦ δ(*q1*, *b*, *b*) = {(*q1*, *b*)},

♦ δ(*q1*, *a*, *b*) = {(*q2*, λ)},

Diagram

Description automatically generated

q0, q1, q2 all are final states

So, q0 will have 2 transitions to q1, q2. q1 will consume any number of b’s.

q2 is the state in which it accepts ending with a.

So, the regular expression: a+abma where m >= 0

Also, we can write it as: a(1+abm)

States with a and can have any number of b1(lambda) and ends with a “is the language accepted by M.

Q4. [20] (A) Construct a NPDA that accepts the language defined by the given grammar and (B) give the language in a formal expression (including a regular expression).

1. S → *ab*S*b* | λ.
2. S → AA | *a*, A → SA | *ab*.

Hint: Convert the grammar into Greibach Normal Form, then apply Thm. 7.1.

1). S → *ab*S*b* | λ.

Language = {b,abb,aabbb,aaabbb}

L = (anbn+1|n>=0 over Σ =(a+b) k}

PDA:

(a,a|aa)

(a,z0|az0)

Diagram

Description automatically generated

2). S → AA | *a*

A → SA | *ab*.

A close up of a clock

Description automatically generated

Both are not regular languages so we can’t give regular expressions.

Q5. [20] Find a (minimal) Context-Free Grammar that generates the language accepted by the NPDA M where M = ({*q0, q1*}, {*a, b*}, {*A*, z}, δ, *q0*, z, {*q1*}), with the transitions

♦ δ(*q0*, *a*, z) = {(*q0*, *Az*)},

♦ δ(*q0*, *b*, *A*) = {(*q0*, *AA*)},

♦ δ(*q0*, *a*, *A*) = (*q1*, λ).

Simplify the production rules by eliminating the useless variables and productions.

Diagram

Description automatically generated

ab\*a

S → aA

A → bA | *a*

Q6. [10] Construct a Deterministic-PDA that accepts L= { *anbm* | 0 ≤ *m* < *n* } to show L is a Deterministic-CFL.

Diagram

Description automatically generated

Transition functions

δ (q0 ,a, z) → (q1 ,az)

δ (q1 ,a, a) → (q1 ,aa)

δ (q1 ,b, a) → (q2, 0λ)

δ (q2 ,b, a) → (q2 , λ)

δ (q2 ,b, z) → (q3 , z)

δ (q3 ,b, z) → (q3 , z)

A close up of a clock

Description automatically generated

L = {anbm | 0 <= m < n}

Q = {1, 2 , 3 ,4 }

E = {a, b }

F = {1, 4 }