Chapter 8

Unified Theory of Electroweak Interactions

8.1 Introduction

The unified theory of weak and electromagnetic interactions was formulated independently by Weinberg [157] and Salam [37] for leptons. It was later extended to the quark sector using the GIM mechanism of quark mixing proposed by Glashow, Iliopoulos, and Maiani (GIM) [64]. The theory is popularly known as the "standard model of electroweak interactions". S. L. Glashow, A. Salam, and S. Weinberg were awarded the Nobel Prize in Physics in 1979 for the formulation of this theory. In this chapter, we describe the formulation of the standard model for leptons and quarks and its applications to other interactions. The predictions of the model and their experimental confirmations are also presented. The limitations of the model and some processes suggesting the need for the physics beyond the standard model are discussed later in Chapter 20.

In this section, the essential features of the electromagnetic interactions and the phenomenological $V-A$ theory of electroweak interactions, well-known from the experimental and theoretical studies of the various weak processes are summarized. They are used as inputs in formulating the standard model. They are as follows:

1. Both electromagnetic and weak interactions involve, in general, all the elementary particles, that is, leptons and quarks, unlike the strong interaction, which affect only the quarks and the hadrons built from these quarks.

2. Both electromagnetic and weak interactions are mediated by vector fields. Electromagnetic interaction is known to be mediated by photons described by the electromagnetic field A_μ, which is massless. The observed weak interactions are presumed to be mediated by charged intermediate vector bosons (IVB), $W_\mu^\pm$, which are massive with mass M_W.

3. Weak interactions involve a pair of leptons, like $(\nu_e,\ e^-)$ or $(\overline{\nu}_e,\ e^+)$, in which the charged vector bosons, $W^{\pm}_{\mu}$, interact with the charged lepton currents $l^{\pm}_{\mu}$, where only the left-handed leptons participate, that is, $l^-_{\mu} = \overline{\psi}_{e_L}\gamma_\mu\psi_{\nu_L}$, with $\psi_{e_L(\nu_L)} = \frac{1-\gamma_5}{2}\psi_{e(\nu)}$. The right-handed leptons, that is, $\psi_{e_R(\nu_R)} = \frac{1+\gamma_5}{2}\psi_{e(\nu)}$ are not involved in the interaction. The interaction Lagrangian is written as:

$$\mathcal{L}^W = g l^{\mp}_{\mu} W^{\pm\mu} + \text{h.c.}$$

4. Electromagnetic interactions involve the electromagnetic field A_μ, which interacts with the electromagnetic current $l^{\text{em}}_{\mu} = e\overline{\psi}_e\gamma_\mu\psi_e$, implying that both the left- and the right-handed components of the electron participate, because $\overline{\psi}_e\gamma_\mu\psi_e = \overline{\psi}_{e_L}\gamma_\mu\psi_{e_L} + \overline{\psi}_{e_R}\gamma_\mu\psi_{e_R}$. The interaction Lagrangian is written as:

$$\mathcal{L}^{\text{em}} = e j^{\text{em}}_{\mu} A^{\mu}(x).$$

5. Both electromagnetic and weak interactions are universal interactions, that is, they have the same coupling strength for leptons and quarks. However, the coupling strengths of the electromagnetic and weak interactions to the quarks (leptons) are different from each other.

6. The weak mediating fields $W^{\pm}_{\mu}$ couple to $l^{\pm}_{\mu}$, involving vector (V) and axial vector (A) leptonic currents, because $\overline{\psi}_{e_L}\gamma_\mu\psi_{\nu_L} \approx \overline{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu = V_\mu - A_\mu$, implying parity violation through their interference, as well as parity conserving coupling to $W^{\pm}_{\mu}$, while the electromagnetic field A_μ couples to the electromagnetic current l^{em}_{μ} only with the parity conserving coupling.

7. The theory of electromagnetic interactions is renormalizable, being a gauge field theory, with massless gauge field, while the weak interaction theory is not renormalizable due to the mediating fields $W^{\pm}_{\mu}$ being massive; this gives divergent results in the higher orders of the perturbation theory.

The major obstacles in formulating a unified theory of electromagnetic and weak interactions arise due to the following reasons:

i) The different masses of the mediating fields, A_μ and $W^{\pm}_{\mu}$ in the two interactions.
ii) The difference in the nature and strength of the couplings of the mediating fields to lepton and quark currents.
iii) The differences in the renormalizability properties of the two theories, that is, the quantum electrodynamical theory of electromagnetic interaction (QED) is renormalizable, while the $V - A$ theory of the weak interaction with IVB is not renormalizable.

These obstacles are removed in the theory of weak and electromagnetic (electroweak) interactions proposed by Weinberg [157] and Salam [37], in which the following assumptions are made:

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1. The left-handed and the right-handed components of leptons, $(e_L,\ \nu_L)$ and $(e_R,\ \nu_R)$, are used to write the Lagrangian in such a way that the invariance under local gauge symmetry generates the interaction Lagrangian for the interaction of leptons with the massless gauge fields corresponding to the local symmetry. In this Lagrangian, only the left-handed components $(e_L,\ \nu_L)$ participate in the weak interaction; while both the components of the electrons participate in the electromagnetic interaction and ν_R has no interaction with matter.

2. The concept of a spontaneously broken local gauge field theory, introducing Higgs mechanism, is used to generate masses for the gauge fields corresponding to the IVB mediating the weak interactions while keeping the gauge field corresponding to photons mediating the electromagnetic interaction, massless.

3. The renormalizability of the spontaneously broken gauge theories was only speculated in the model; it was proved later by t' Hooft and Veltman [338] and Lee and Zinn- Justin [339].

We will discuss the model in detail in the following sections.

8.2 Description of the Weinberg–Salam Model for Leptons

The model uses the local gauge field theory based on SU(2)$_I\times$ U(1)$_Y$ symmetry, where I is the weak isospin and Y is the weak hypercharge. It is, in fact, a minimal extension of the local gauge field theory (LGFT) based on SU(2)$_I$ symmetry, which was found to be inadequate to unify weak and electromagnetic interactions. The model was similar to the earlier works of Glashow [340], Salam, and Ward [341] in which the limitations of the local gauge field theory based on SU(2)$_I$ for achieving a unified theory were discussed and need for a higher symmetry was emphasized. We begin by listing the fermions and other particles considered in the model and limit ourselves, in this section, to leptons only (shown in Table 8.1). We also show the weak isospin (I and I_3) and weak hypercharge assignments of leptons and charge ($Q = I_3 + \frac{Y}{2}$) defined in analogy with the Gell–Mann–Nishijima relation in strong interactions. In weak

Table 8.1 Weak isospin (I), its third component (I_3), charge ($Q(|e|)$) and hypercharge ($Y = 2(Q - I_3)$) of leptons and scalar mesons in the W–S model with $l = e, \mu, \tau$.

Name	I	I_3	$Q(\|e\|)$	Y
(Leptons) ν_L	$\frac{1}{2}$	$+\frac{1}{2}$	0	-1
l_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
ν_R	0	0	0	0
l_R	0	0	-1	-2
(Scalar ϕ^+	$\frac{1}{2}$	$+\frac{1}{2}$	1	1
mesons) ϕ^0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1

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interactions, electrons(e) and neutrinos (ν_e) always interact in pairs through their left-handed components, that is, $(\nu_L,\ e_L)$ or $(\bar{\nu}_L,\ e_L^+)$; the right-handed components $\nu_R,\ e_R,\ \bar{\nu}_R,\ \bar{e}_R$ are not involved in weak interactions. Therefore, the left-handed components are assigned to a weak isospin doublet ψ_L, that is,

$$\psi_L \quad = \quad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

and the right-handed components are assigned to a singlet representation, that is,

$$e_R \quad \text{and} \quad \nu_R.$$

The weak hypercharge Y is assigned accordingly to reproduce the correct charge Q of leptons, through the relation:

$$Q = I_3 + \frac{Y}{2} \tag{8.1}$$

as shown in Table 8.1.

The Weinberg–Salam (W–S) Lagrangian for free leptons is written considering only the electron type leptons, that is, e^- and ν_e, for the simplicity of presentation. It can be extended to other flavors of leptons like μ^- and ν_μ, and τ^- and ν_τ in a straightforward manner. Using the concepts developed in Chapter 7, the W–S Lagrangian for interacting leptons is derived in the following four steps.

i) A Lagrangian for the free electrons and neutrinos for the left-handed doublet ψ_L and the right-handed singlets e_R and ν_R is written, which is invariant under the global symmetry group SU(2)$_I\times$ U(1)$_Y$, as:

$$\mathcal{L} \quad = \quad i\overline{\psi}_L \partial\!\!\!/ \psi_L + i\bar{e}_R \partial\!\!\!/ e_R + i\bar{\nu}_R \partial\!\!\!/ \nu_R. \tag{8.2}$$

There is no mass term like $m^2\overline{\psi}_e\psi_e$ as it violates the invariance of the Lagrangian under SU(2)$_I\times$ U(1)$_Y$, because $\bar{\psi}_e\psi_e = \bar{e}_L e_R + \bar{e}_R e_L$, where e_L is a member of an isospin doublet and e_R is a singlet.

ii) To make the Lagrangian invariant under the local symmetry group SU(2)$_I\times$ U(1)$_Y$, defined by the unitary transformation $U = U_1U_2$, where $U_1 = e^{\frac{ig}{2}\vec{\tau}.\vec{\alpha}(x)}$ and $U_2 = e^{i\frac{g'}{2}Y.B}$, $\vec{\tau}$ are the Pauli matrices and Y is a unit operator, the ordinary derivatives $\partial^\mu = \frac{\partial}{\partial x_\mu}$ are replaced by the covariant derivatives D^μ defined as:

$$D^\mu = \partial^\mu + i\frac{g}{2}\vec{\tau}.\vec{W}^\mu + i\frac{g'}{2}YB^\mu, \tag{8.3}$$

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where

$$
\begin{aligned}
\vec{\tau}.\vec{W}^{\mu} &= \sum_{i=1}^{3} \tau^i W^{i\mu} \\
&= \tau^1 W^{1\mu} + \tau^2 W^{2\mu} + \tau^3 W^{3\mu} \\
&= \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix}.
\end{aligned} \tag{8.4}
$$

We have introduced a triplet of gauge fields W^i_μ corresponding to the generators of $SU(2)_I$ and B_μ, the gauge fields corresponding to the generator Y of $U(1)_Y$. The constants g and g' are the coupling strengths of the SU(2) gauge fields W^i_μ and U(1) gauge field B_μ, respectively. A factor of $\frac{1}{2}$ is introduced with the B_μ field for later convenience and in analogy with the coupling of W^i_μ fields, in the expression for D^μ in Eq. (8.3). The Lagrangian is thus written as:

$$
\mathcal{L} = i\overline{\psi}_L \not{D} \psi_L + i\bar{e}_R \not{D} e_R + i\overline{\nu}_R \not{D} \nu_R - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu}_i, \tag{8.5}
$$

where

$$
\begin{aligned}
D^\mu \psi_L &= \left(\partial^\mu + \frac{ig}{2} \vec{\tau} \cdot \vec{W}^\mu - \frac{ig'}{2} B^\mu \right) \psi_L, \\
D^\mu e_R &= (\partial^\mu - ig' B^\mu) e_R, \\
D^\mu \nu_R &= \partial^\mu \nu_R,
\end{aligned} \tag{8.6}
$$

using the values of Y for ψ_L, e_R, and ν_R from Table 8.1 and

$$
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{8.7}
$$

is the kinetic energy of the massless gauge field B^μ.

$$
G^i_{\mu\nu} = (\partial_\mu W^i_\nu - \partial_\nu W^i_\mu) + g\epsilon_{ijk} W^j_\mu W^k_\nu, \tag{8.8}
$$

which describes the kinetic energy and the self coupling of the massless $W^{\mu i}$ fields.

Hence, the Lagrangian in Eq. (8.5) can be written as:

$$
\begin{aligned}
\mathcal{L} &= i\overline{\psi}_L \not{\partial} \psi_L - \frac{g}{2} \overline{\psi}_L \gamma_\mu \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix} \psi_L + \frac{g'}{2} \bar{\psi}_L \gamma_\mu B^\mu \psi_L \\
&+ i\bar{e}_R \not{\partial} e_R + i\overline{\nu}_R \not{\partial} \nu_R + g' \bar{e}_R \gamma_\mu B^\mu e_R \\
&= i\overline{\psi}_L \not{\partial} \psi_L + i\bar{e}_R \not{\partial} e_R + i\overline{\nu}_R \not{\partial} \nu_R \\
&- \frac{g}{\sqrt{2}} (\overline{\nu}_L \gamma_\mu W^{\mu+} e_L + \bar{e}_L \gamma_\mu W^{\mu-} \nu_L) \\
&- \frac{1}{2} \overline{\nu}_L \gamma_\mu (g W^{3\mu} - g' B^\mu) \nu_L + \frac{1}{2} \bar{e}_L \gamma_\mu (g W^{3\mu} + g' B^\mu) e_L + g' \bar{e}_R \gamma_\mu B^\mu e_R.
\end{aligned} \tag{8.9}
$$

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iii) The terms in the first line of Eq. (8.9) are the kinetic energy terms of the fields ψ_L, e_R, and ν_R and the terms in the second and third lines describe the interaction of these fields with the gauge fields W^i_μ and B_μ, which are massless. In order to generate the masses of some of the gauge fields corresponding to the vector bosons mediating the weak interactions, a set of scalar fields are introduced in the model. This is done by introducing a doublet of self interacting complex scalar fields $\phi(x)$ given by:

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{pmatrix}, \tag{8.10}$$

where $\phi_i(x)$ $(i = 1-4)$ are real fields. The weak isospin I and hypercharge Y quantum numbers of the $\phi(x)$ fields are shown in Table 8.1, for which the interaction Lagrangian is written as:

$$\mathcal{L}_{\text{scalar}}(\phi) = D_\mu\phi^* D^\mu\phi - V(\phi), \tag{8.11}$$

where
$$D^\mu\phi(x) = \left(\partial^\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}^\mu + \frac{ig'}{2}B^\mu\right)\phi(x) \tag{8.12}$$

and
$$V(\phi) = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2, \ \ \mu^2 < 0. \tag{8.13}$$

The minimum of the potential $V(\phi)$ occurs at $\langle\phi^*\phi\rangle_0 = -\frac{\mu^2}{2\lambda}$ corresponding to the physical ground state of the scalar fields $\phi(x)$, which is infinitely degenerate for $\mu^2 < 0$. The SU(2)$_I\times$ U(1)$_Y$ symmetry is spontaneously broken through the Higgs mechanism [346] by choosing a particular ground state in such a way that one of the four generators, that is, $\frac{1}{2}\tau_i$ and Y of the SU(2)$_I\times$ U(1)$_Y$ symmetry, corresponding to the electromagnetic gauge fields A_μ, that is, the charge operator $Q = \frac{1}{2}\tau_3 + \frac{Y}{2}$ leaves the ground state invariant, keeping the field A_μ massless. Thus, the $SU(2)_I \times U(1)_Y$ symmetry is broken to a lower symmetry $U(1)_Q$. The other generators like $\frac{1}{2}\tau_1$, $\frac{1}{2}\tau_2$ and $\frac{1}{2}\tau_3 - \frac{Y}{2}$ break the symmetry spontaneously, generating masses for the corresponding gauge fields. This is done by choosing a particular ground state from the infinitely degenerate states to be:

$$\phi_1 = \phi_2 = \phi_4 = 0 \text{ and } \phi_3 \neq 0$$

such that

$$\langle\phi^*\phi\rangle_0 = \frac{1}{2}\langle 0|(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)|0\rangle = -\frac{\mu^2}{2\lambda}, \tag{8.14}$$

implying

$$\langle 0|\phi(x)|0\rangle = \frac{1}{\sqrt{2}}\langle 0|\phi_3(x)|0\rangle = \sqrt{\frac{-\mu^2}{2\lambda}}. \tag{8.15}$$

Using the doublet notation, we can write

$$\langle 0|\phi(x)|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \tag{8.16}$$

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where $v = \sqrt{\frac{\mu^2}{\lambda}}$, $\mu^2 > 0$.

Using this choice of the physical ground state, it can be shown that while τ_1, τ_2, τ_3 break the invariance of the vacuum, the operator $Q = T_3 + \frac{Y}{2} = \frac{1}{2}(\tau_3 + Y)$ leaves the vacuum invariant, that is, $Q\langle\phi\rangle_0 = \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 = 0$, by calculating:

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \neq 0, \tag{8.17}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{iv}{\sqrt{2}} \\ 0 \end{pmatrix} \neq 0, \tag{8.18}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{v}{\sqrt{2}} \end{pmatrix} \neq 0, \tag{8.19}$$

$$Y\langle\phi\rangle_0 = +1\langle\phi_0\rangle \neq 0, \tag{8.20}$$

and

$$\frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 = 0, \tag{8.21}$$

$$\frac{1}{2}(\tau_3 - Y)\langle\phi\rangle_0 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{v}{\sqrt{2}} \end{pmatrix} \neq 0. \tag{8.22}$$

Therefore, by giving $\langle\phi\rangle_0$ a non-zero expectation value, only the generator $Q = \tau_3 + \frac{Y}{2}$ leaves the ground state invariant. This leads to the breaking of the symmetry $SU(2)_I \times U(1)_Y$, leaving the gauge field corresponding to the generator of the symmetry $U(1)_Q$, that is, the electromagnetic field A_μ massless and giving mass to the other three gauge fields, corresponding to the generators τ_1, τ_2, and $\frac{1}{2}(\tau_3 - Y)$.

iv) Now, we write the interaction Lagrangian by expanding the scalar field $\phi(x)$ around the minimum (v) of the Higgs potential by writing

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{8.23}$$

where $H(x)$ is called the Higgs field. In fact, the expression for the scalar field $\phi(x)$ given in Eq. (8.10) can be shown to be equivalent to the field $\phi(x)$ given in Eq. (8.23) using gauge invariance and working in the unitary gauge. In order to show this, using gauge freedom, let us consider:

$$\phi(x) = e^{\frac{i\vec{\alpha}(x).\vec{\tau}}{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{8.24}$$

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and choose $\vec{\alpha}(x) = \frac{2}{v}\vec{\theta}(x)$. For $\theta(x) \leq v$ and $H(x) \leq v$, corresponding to the small perturbation around vacuum results in

$$\begin{aligned} \phi(x) &\approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + \frac{i\theta_3}{v} & \frac{i(\theta_1 - i\theta_2)}{v} \\ \frac{i(\theta_1 + i\theta_2)}{v} & 1 - \frac{i\theta_3}{v} \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\ &\simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_2 + i\theta_1 \\ v + H(x) - i\theta_3 \end{pmatrix}, \end{aligned} \tag{8.25}$$

which is the same as Eq. (8.10) for $\theta_1 = \phi_2$, $\theta_2 = \phi_1$, $v + H(x) = \phi_3$, and $\theta_3 = -\phi_4$. Hence, using the gauge freedom, we can choose a gauge transformation U, such that, under this gauge:

$$\phi(x) \to \phi'(x) = U\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}. \tag{8.26}$$

This gauge is called the unitary gauge in which the fields $\theta_i(x)(i = 1, 2, 3)$ are gauged away, except the Higgs field. Strictly speaking, in order to gauge away the gauge fields corresponding to the generators which break the symmetry, we should use τ_1, τ_2, and $\tau_3' = \frac{1}{2}(\tau_3 - Y)$, which is orthogonal to Q, instead of τ_3. However, since τ_3' and τ_3 both break the symmetry, the effect will be the same. Moreover, the situation of the gauge transformation U and the relation between the fields $\vec{\alpha}(x)$ and $\vec{\theta}(x)$ would be different.

v) The interaction Lagrangian is, therefore, written in terms of the scalar field $\phi'(x)$ and the lepton fields ψ_L', e_R', and ν_R' calculated in the unitary gauge, given by:

$$\phi \;\to\; \phi' = U\phi, \qquad \psi_L \to \psi_L' = U\psi_L, \tag{8.27}$$

$$\nu_R \;\to\; \nu_R' = U\nu_R, \qquad e_R \to e_R' = Ue_R, \tag{8.28}$$

$$W_\mu \;\to\; W_\mu' = UW_\mu U^{-1} + \frac{1}{ig}(\partial_\mu U)U^{-1}, \tag{8.29}$$

$$B_\mu \;\to\; B_\mu' = B_\mu - \frac{1}{g}\partial_\mu \alpha_3(x). \tag{8.30}$$

However, we use the earlier notation ϕ, ψ, B_μ etc. for the new fields ϕ', ψ', B_μ' and write the Lagrangian as

$$\mathcal{L}_{\text{int}}^{\text{WS}} = D^\mu \phi^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda(\phi^*\phi)^2 + i\overline{\psi}_L \not{D}\psi_L + i\bar{e}_R \not{D} e_R. \tag{8.31}$$

This Lagrangian describes the interaction of the scalar field ϕ and the lepton fields ψ_L, e_R, and ν_R with the gauge fields. We add to this the interaction of the lepton fields ψ_L and e_R with the scalar field $\phi(x)$ assuming Yukawa interaction, that is,

$$\mathcal{L}_{\text{leptons}} = -f_e(\bar{e}_R \phi^\dagger \psi_L + \overline{\psi}_L \phi e_R), \tag{8.32}$$

since ν_R is non-interacting. The full W–S Lagrangian is thus written as:

$$\begin{aligned}
\mathcal{L}^{WS} &= i\bar{\psi}_L \not{D} \psi_L + i\bar{e}_R \not{D} e_R - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} \\
&\quad + D^\mu \phi^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \\
&\quad - f_e \left(\bar{e}_L \phi e_R + \bar{e}_R \phi^\dagger e_L \right). \qquad (8.33)
\end{aligned}$$

8.3 Predictions of the Weinberg–Salam Model

8.3.1 Masses of gauge bosons

Consider the term $(D^\mu \phi)^* D_\mu \phi$ of the Lagrangian given in Eq. (8.31). Using Eq. (8.12) for $D^\mu(\phi)$ and Eqs. (8.4) and (8.23), we write:

$$\begin{aligned}
D_\mu \phi &= \begin{pmatrix} \partial_\mu + \frac{igW^3_\mu + ig' B_\mu}{2} & ig\frac{W^1_\mu - iW^2_\mu}{2} \\ ig\frac{W^1_\mu + iW^2_\mu}{2} & \partial_\mu - \left(\frac{igW^3_\mu - ig' B_\mu}{2}\right) \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu & 0 \\ 0 & \partial_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} igW^3_\mu + ig' B_\mu & ig(W^1_\mu - iW^2_\mu) \\ ig(W^1_\mu + iW^2_\mu) & -(igW^3_\mu - ig' B_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\mu H(x) \end{pmatrix} + \frac{i}{2} \begin{pmatrix} g(W^1_\mu - iW^2_\mu)(\frac{v+H(x)}{\sqrt{2}}) \\ (g' B_\mu - gW^3_\mu)(\frac{v+H(x)}{\sqrt{2}}) \end{pmatrix} \qquad (8.34)
\end{aligned}$$

and

$$(D^\mu \phi)^* = \frac{1}{\sqrt{2}} \left(0 \quad \partial^\mu H(x)\right) - \frac{i}{2} \left(g(W^{1\mu} + iW^{2\mu})(\tfrac{v+H(x)}{\sqrt{2}}) \quad (g' B^\mu - gW^{3\mu}) \left(\tfrac{v+H(x)}{\sqrt{2}}\right) \right). \qquad (8.35)$$

The kinetic energy term, $(D^\mu \phi)^*(D_\mu \phi)$ is therefore given by:

$$\begin{aligned}
(D^\mu \phi)^*(D_\mu \phi) &= \frac{1}{2} \partial^\mu H(x) \partial_\mu H(x) + \frac{v^2 g^2}{8} (W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}) \\
&\quad + \frac{v^2}{8} (gW^{3\mu} - g' B^\mu)(gW^3_\mu - g' B_\mu) + \frac{g^2}{8} (H^2 + 2Hv) \\
&\qquad (W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}) \\
&\quad + \frac{g^2}{8} W^3_\mu W^{3\mu} (H^2 + 2Hv) - \frac{gg'}{8} (B_\mu W^{3\mu})(H^2 + 2Hv) \\
&\quad - \frac{gg'}{8} (B^\mu W^3_\mu)(H^2 + 2Hv) + \frac{g'^2}{8} B_\mu B^\mu (H^2 + 2Hv). \qquad (8.36)
\end{aligned}$$

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We define,

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}} \tag{8.37}$$

such that

$$W_\mu^1 W^{\mu 1} + W_\mu^2 W^{\mu 2} = |W_\mu^+|^2 + |W_\mu^-|^2 \quad (\because (W_\mu^+)^\dagger = (W_\mu^-))$$

and Eq. (8.36) may be rewritten as:

$$\begin{aligned}(D^\mu\phi)^*(D_\mu\phi) &= \frac{1}{2}\partial^\mu H(x)\partial_\mu H(x) + \frac{v^2 g^2}{8}(|W_\mu^+|^2 + |W_\mu^-|^2) \\ &+\frac{v^2}{8}(gW^{3\mu} - g'B^\mu)(gW_\mu^3 - g'B_\mu) + \frac{g^2}{8}(H^2 + 2Hv) \\ &\quad (|W_\mu^+|^2 + |W_\mu^-|^2) \\ &+\frac{g^2}{8}W_\mu^3 W^{3\mu}(H^2 + 2Hv) - \frac{gg'}{8}(B_\mu W^{3\mu})(H^2 + 2Hv) \\ &-\frac{gg'}{8}(B^\mu W_\mu^3)(H^2 + 2Hv) + \frac{g'^2}{8}B_\mu B^\mu (H^2 + 2Hv).\end{aligned} \tag{8.38}$$

We see that W^+ and W^- have acquired a mass $M_{W^\pm} = \frac{vg}{2}$. Defining the normalized orthogonal combinations of W_μ^3 and B_μ as:

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}}, \tag{8.39}$$

$$A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}}, \tag{8.40}$$

we rewrite Eq. (8.38) as:

$$\begin{aligned}(D^\mu\phi)^*(D_\mu\phi) &= \frac{1}{2}\partial^\mu H(x)\partial_\mu H(x) + \frac{v^2 g^2}{8}(|W_\mu^+|^2 + |W_\mu^-|^2) \\ &+ \frac{g^2}{8}(H^2 + 2Hv)(|W_\mu^+|^2 + |W_\mu^-|^2) + \left(\frac{g^2 + g'^2}{4}\right) \\ &\quad \left(\frac{H^2 + 2Hv + v^2}{2}\right) Z_\mu Z^\mu \\ &+ \left[\frac{g^2 g'^2}{4(g^2 + g'^2)}(H^2 + 2Hv) - \frac{g^2 g'^2}{4(g^2 + g'^2)}(H^2 + 2Hv)\right] A_\mu A^\mu.\end{aligned} \tag{8.41}$$

We see from Eq. (8.41) that the field A_μ remains massless, which is identified with the electromagnetic field of the photon, while Z_μ acquires a mass $M_Z = \frac{\sqrt{g^2+g'^2}}{2}v$, such that $\frac{M_Z}{M_W} = \sqrt{1 + \frac{g'^2}{g^2}} \geq 1$. The model predicts the relative magnitude of M_W and M_Z, but to obtain their absolute values, one needs to know the values of g and g' and the vacuum expectation value (VEV) of the scalar field ($\langle\phi\rangle_0 = \langle v\rangle$) from the phenomenology of the weak and electromagnetic processes.

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8.3.2 Charged current weak interactions and electromagnetic interactions

In order to determine the couplings g and g' of the gauge bosons W_μ^i ($i = 1, 2, 3$) and B_μ, we need to derive the electromagnetic and weak interactions of these fields with weak and electromagnetic lepton currents. For this purpose, consider the last two terms of the Lagrangian given in Eq. (8.31), that is,

$$\mathcal{L} \;=\; i\overline{\psi}_L\gamma^\mu(\partial_\mu + \frac{ig}{2}\vec{\tau}.\vec{W}_\mu - \frac{ig'}{2}B_\mu)\psi_L + i\overline{e}_R\gamma^\mu(\partial_\mu - ig'B_\mu)e_R + i\overline{\nu}_R\gamma_\mu\partial^\mu\nu_R.$$

The terms describing the interaction of leptons with the gauge fields are:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= \overline{\psi}_L\gamma^\mu\frac{1}{2}(g'B_\mu - g\vec{\tau}.\vec{W}_\mu)\psi_L + g'\overline{e}_R\gamma^\mu B_\mu e_R \\
&= \begin{pmatrix}\overline{\nu}_L & \overline{e}_L\end{pmatrix}\gamma_\mu\frac{1}{2}\begin{pmatrix} g'B_\mu - gW_\mu^3 & g(-W_\mu^1 + iW_\mu^2) \\ -g(W_\mu^1 + iW_\mu^2) & g'B_\mu + gW_\mu^3\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix} + g'\overline{e}_R\gamma_\mu B_\mu e_R \\
&= -\frac{g}{\sqrt{2}}\overline{\nu}_L\gamma_\mu W_\mu^+ e_L + \frac{1}{2}\overline{\nu}_L\gamma^\mu(g'B_\mu - gW_\mu^3)\nu_L - \frac{g}{\sqrt{2}}\overline{e}_L\gamma_\mu W_\mu^-\nu_L \\
&\quad + \frac{1}{2}\overline{e}_L\gamma^\mu(g'B_\mu + gW_\mu^3)e_L + g'\overline{e}_R\gamma_\mu B_\mu e_R \\
&= -\frac{g}{\sqrt{2}}\overline{\nu}_L\gamma_\mu W_\mu^+ e_L - \frac{g}{\sqrt{2}}\overline{e}_L\gamma_\mu W_\mu^-\nu_L + \frac{g'}{2}(\overline{\nu}_L\gamma_\mu B_\mu\nu_L + \overline{e}_L\gamma_\mu B_\mu e_L + 2\overline{e}_R\gamma_\mu B_\mu e_R) \\
&\quad + \frac{g}{2}(\overline{e}_L\gamma_\mu W_\mu^3 e_L - \overline{\nu}_L\gamma_\mu W_\mu^3\nu_L).
\end{aligned}$$

Since,

$$\begin{aligned}
W_\mu^3 &= \frac{gZ_\mu + g'A_\mu}{\sqrt{g^2 + g'^2}}, \\
B_\mu &= \frac{gA_\mu - g'Z_\mu}{\sqrt{g^2 + g'^2}}.
\end{aligned} \tag{8.42}$$

We obtain:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= -\frac{g}{2\sqrt{2}}\left(\overline{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \overline{e}\gamma^\mu(1-\gamma_5)\nu_e W_\mu^-\right) - \frac{\sqrt{g^2+g'^2}}{2}\overline{\nu}_L\gamma^\mu\nu_L Z_\mu \\
&\quad + \frac{gg'}{\sqrt{g^2+g'^2}}\overline{e}\gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2+g'^2}}[-g'^2\overline{e}_R\gamma^\mu e_R + \frac{g^2 - g'^2}{2}\overline{e}_L\gamma^\mu e_L].
\end{aligned} \tag{8.43}$$

From Eq. (8.43), we see that the charged current weak interactions in which the charged raising (lowering) lepton current couples to the W_μ^+ and W_μ^- fields, is given by the Lagrangian:

$$\mathcal{L}_{\text{int}}^{\text{CC}} = -\frac{g}{2\sqrt{2}}\overline{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \text{h.c.}, \tag{8.44}$$

where

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}} \qquad \text{i.e.} \quad g^2 = 4\sqrt{2}M_W^2 G_F = \sqrt{2}v^2 g^2 G_F. \tag{8.45}$$

This leads to the determination of the vacuum expectation value v in terms of the weak Fermi coupling constant G_F, that is, $v = (\sqrt{2}G_F)^{-\frac{1}{2}} \simeq 246$ GeV using $G_F \approx 10^{-5}$ GeV^{-2}. The electromagnetic current $\bar{e}\gamma^\mu e$ couples with the electromagnetic field A_μ through the interaction Lagrangian given by:

$$\mathcal{L}_{\text{int}} = \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e A_\mu \tag{8.46}$$

implying that $\frac{gg'}{\sqrt{g^2+g'^2}} = e$ (electronic charge), leading to a relation between e, g, and g', that is,

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}. \tag{8.47}$$

8.3.3 Neutral current interaction and weak mixing angle

We see from the Lagrangian given by Eq (8.43), that the W–S model predicts the existence of a neutral heavy boson Z^μ which interacts via the neutral current carried by neutrinos and electrons. The neutral current interaction Lagrangians for neutrinos and electrons are given by:

$$\mathcal{L}^{\nu}_{\text{NC}} = -\frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu, \qquad \text{(for neutrinos)} \tag{8.48}$$

$$\mathcal{L}^{e}_{\text{NC}} = \frac{Z_\mu}{\sqrt{g^2+g'^2}}[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L], \qquad \text{(for electrons)} \tag{8.49}$$

in which the strength of the coupling Z_μ is different for ν_L, e_L, and e_R. Obviously, there is no interaction of ν_R with any of the gauge fields or electrons.

It is convenient to parameterize the orthogonal and normalized mixing of the neutral gauge bosons in terms of an angle θ_W, known as the weak mixing angle, such that:

$$Z_\mu = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu \quad \text{and} \tag{8.50}$$

$$A_\mu = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu, \tag{8.51}$$

where

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \tag{8.52}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}. \tag{8.53}$$

Since $e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$, we obtain:

$$g = \frac{e}{\sin\theta_W}, \qquad g' = \frac{e}{\cos\theta_W}, \qquad \text{and} \qquad M_W = M_Z\cos\theta_W, \tag{8.54}$$

implying that $g > e$ and $g' > e$. This implies that the gauge field couplings of the physical $W^{\pm}$ and Z bosons to the lepton currents are $\frac{g}{2\sqrt{2}}$ and $\frac{g}{2\cos\theta_W}$, respectively. Therefore, the intrinsic weak couplings of gauge bosons are not small as compared to the electromagnetic coupling; however, the effective couplings are small because of the large mass of W and Z bosons. Moreover, the masses of the W and Z bosons can be predicted in terms of the weak mixing angle using Eqs. (8.45) and (8.54),

$$
\begin{aligned}
M_W^2 &= \frac{g^2}{4\sqrt{2}G_F} = \frac{e^2}{4\sqrt{2}G_F\sin^2\theta_W} = \frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta_W} \\
\Rightarrow \quad M_W &= \frac{37.3}{\sin\theta_W}\,\text{GeV}, && (8.55)\\
\text{and} \quad M_Z &= \frac{M_W}{\cos\theta_W} = \frac{37.3}{\sin\theta_W\cos\theta_W}\,\text{GeV}. && (8.56)
\end{aligned}
$$

It is convenient to express the W–S Lagrangian in terms of the electromagnetic and weak currents and their couplings to the vector boson fields $W_\mu^{\pm}, Z_\mu$, and A_μ, that is, e, g, and θ_W as:

$$
\begin{aligned}
\mathcal{L}^{\text{WS}} &= -\frac{g}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L W_\mu^+ + \bar{e}_L\gamma^\mu\nu_L W_\mu^-\right) + e\,\bar{e}\gamma^\mu e A_\mu - \frac{g}{2\cos\theta_W}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu \\
&- \frac{g}{2\cos\theta_W}\left(2\sin^2\theta_W\bar{e}_R\gamma^\mu e_R + (2\sin^2\theta_W - 1)\bar{e}_L\gamma^\mu e_L\right) Z_\mu, \\
&= -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu_e W_\mu^-\right] - \frac{g}{4\cos\theta_W}\bar{\nu}_e\gamma^\mu \\
&\qquad (1-\gamma_5)\nu_e Z_\mu \\
&+ e\,\bar{e}\gamma^\mu e A_\mu - \frac{g}{4\cos\theta_W}\Big[2\sin^2\theta_W\bar{e}\gamma^\mu(1+\gamma_5)e + (2\sin^2\theta_W - 1) \\
&\qquad \bar{e}\gamma^\mu(1-\gamma_5)e\Big] Z_\mu.
\end{aligned}
$$

Therefore, in the W–S model

$$
\begin{aligned}
\mathcal{L}^{\text{WS}} &= \mathcal{L}^{\text{em}} + \mathcal{L}^{\text{weak}}, \\
\text{where } \mathcal{L}^{\text{weak}} &= \mathcal{L}^{\text{CC}} + \mathcal{L}^{\text{NC}}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{L}^{\text{em}} &= e\,\bar{e}\gamma^\mu e A_\mu, && (8.57)\\
\mathcal{L}^{\text{CC}} &= -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)eW_\mu^+ + \text{h.c.}\right], && (8.58)\\
\mathcal{L}^{\text{NC}} &= -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e + \bar{e}\gamma^\mu(g_V^e - g_A^e\gamma_5)e\right] Z_\mu, && (8.59)
\end{aligned}
$$

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with

$$g_V^e \;=\; 4\sin^2\theta_W - 1, \qquad g_A^e = -1, \tag{8.60}$$

$$\frac{g}{2\sqrt{2}} \;=\; \left(\frac{G_F {M_W}^2}{\sqrt{2}}\right)^{\frac{1}{2}}, \tag{8.61}$$

$$\frac{g}{4\cos\theta_W} \;=\; \frac{1}{\sqrt{2}}\left(\frac{G_F {M_Z}^2}{\sqrt{2}}\right)^{\frac{1}{2}}. \tag{8.62}$$

8.3.4 Mass of the electron

The mass of the electron is generated through the Yukawa coupling of the electron with the scalar field (ϕ) by introducing an interaction Lagrangian, with a coupling strength f_e, given by:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= -f_e\overline{\psi}_L\phi\psi_R + \text{h.c.} && (8.63)\\
&= -f_e\left(\overline{\nu}_L \;\; \overline{e}_L\right)\begin{pmatrix} 0, \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix} e_R + \text{h.c.}, \\
&= -f_e\frac{v+H(x)}{\sqrt{2}}\overline{e}_L e_R + \text{h.c.}, \\
&= -\frac{f_e v}{\sqrt{2}}(\overline{e}_L e_R + \overline{e}_R e_L) - \frac{f_e H(x)}{\sqrt{2}}(\overline{e}_L e_R + \overline{e}_R e_L), \\
&= -\frac{f_e v}{\sqrt{2}}\overline{\psi}_e\psi_e - \frac{f_e H(x)}{\sqrt{2}}\overline{\psi}_e\psi_e. && (8.64)
\end{aligned}$$

The mass of the electron is predicted to be:

$$m_e = \frac{f_e v}{\sqrt{2}} = \frac{f_e}{\sqrt{2}(G_F\sqrt{2})^{\frac{1}{2}}}. \tag{8.65}$$

The strength of the coupling f_e can be experimentally determined from the coupling of the Higgs field($H(x)$) to $e^-\ e^+$ pairs, that is, Higgs $\to e^-e^+$. Theoretically, f_e can be determined from the charged weak decays of $W^\pm$, that is,

$$f_e \;=\; \frac{\sqrt{2}m_e}{v} \;=\; 2^{\frac{3}{4}}m_e(G_F)^{\frac{1}{2}} \;\simeq\; 3\times 10^{-6}. \tag{8.66}$$

8.4 Extension to the Leptons of Other Flavors

In preceding sections, the essentials of the unified gauge theory in the W–S model are described for electrons and neutrinos. This can be extended to other flavors of the leptons. In this section, we consider the extension of the W–S model to other flavors of the leptons in a formalism which can then be applied to the quark sector in a straightforward way.

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We write the Lagrangian for the interaction of leptons $(e^- \;\; \nu_e)$, $(\mu^- \;\; \nu_\mu)$, and $(\tau^- \;\; \nu_\tau)$ with the gauge fields in terms of the covariant derivatives D^μ, making the Lagrangian invariant under the local $SU(2)_I \times U(1)_Y$ gauge symmetry as:

$$\mathcal{L}^{\text{int}}_{\text{leptons}} = \sum_{f=e,\mu,\tau} \bar{l}_{f_L} i \not{D} l_{f_L} + \sum_{f=e,\mu,\tau} \bar{l}_{f_R} \not{D} l_{f_R}, \tag{8.67}$$

where l_{f_L} is the left-handed SU(2) doublet of leptons of flavor f, that is,

$$l_{f_L} = \begin{pmatrix} \nu_{f_L} \\ f_L \end{pmatrix} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \text{ and } \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \tag{8.68}$$

and l_{f_R} is the right-handed SU(2) singlet of leptons of flavor f, that is,

$$l_{f_R} = \nu_{e_R}, \, e^-_R, \, \nu_{\mu_R}, \, \mu^-_R, \, \nu_{\tau_R}, \, \tau^-_R. \tag{8.69}$$

The $\sum_{f=e,\mu,\tau}$ implies the sum over all the doublets and singlets corresponding to the flavor $f(= e, \mu, \tau)$. The weak isospin, weak hypercharge, and electric charge of all the leptons discussed in this section are given in Table 8.1. The covariant derivatives D^μ for the doublet and singlet fields l_{f_L} and l_{f_R} are defined as:

$$D^\mu \;\; = \;\; \left(\partial^\mu + ig \frac{\vec{\tau}.\vec{W}^\mu}{2} + ig' \frac{Y_L}{2} B^\mu \right) \tag{8.70}$$

for all the SU(2) doublets L of flavor f and

$$D^\mu \;\; = \;\; \partial^\mu + ig' \frac{Y_R}{2} B^\mu \tag{8.71}$$

for all the SU(2) singlets R of flavor f, where $\vec{W}^\mu$ and B^μ are the isovector and isoscalar gauge fields corresponding to $SU(2)_I$ and $U(1)_Y$, respectively. From Eqs. (8.67), (8.70), and (8.71), we obtain the following Lagrangian for the interaction of all the lepton flavors f with the gauge fields W^μ and B^μ as:

$$\mathcal{L}^{\text{int}}_{\text{leptons}} = - \sum_{f=e,\mu,\tau} \left[\bar{l}_{f_L} \left(\frac{g}{2} \vec{\tau}.\vec{W}^\mu + g' \frac{Y^L}{2} B^\mu \right) \gamma_\mu f_{f_L} + \bar{l}_{f_R} g' \frac{Y^R}{2} B^\mu \gamma_\mu l_{f_R} \right]. \tag{8.72}$$

Using

$$\frac{\vec{\tau}.\vec{W}^\mu}{2} \;\; = \;\; \frac{1}{\sqrt{2}} \left[\tau_+ \frac{W_1^\mu - iW_2^\mu}{\sqrt{2}} + \tau_- \frac{W_1^\mu + iW_2^\mu}{\sqrt{2}} \right] + \frac{\tau_3}{2} W_3^\mu, \tag{8.73}$$

where $\tau_+ = \frac{\tau_1 + i\tau_2}{2}$ and $\tau_- = \frac{\tau_1 - i\tau_2}{2}$ are the isospin raising and lowering operators and $\frac{W_1^\mu \mp iW_2^\mu}{\sqrt{2}}$ are the vector field components which create $W^\pm$ vector bosons.

The charge current weak interaction Lagrangian using Eq. (8.4) is given by:

$$\mathcal{L}^{\text{int}}_{\text{cc}} \;\; = \;\; -\frac{g}{\sqrt{2}} \sum_{f=e,\mu,\tau} \left[\bar{l}_{f_L} \gamma^\mu \tau^+ l_{f_L} W^+_\mu + \bar{l}_{f_L} \gamma^\mu \tau^- l_{f_L} W^-_\mu \right]. \tag{8.74}$$

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Using $f_L = \frac{1-\gamma_5}{2} f$ and $\overline{f}_L = \frac{1+\gamma_5}{2}\overline{f}$, we find

$$\mathcal{L}_{\text{cc}}^{\text{int}} = -\frac{g}{2\sqrt{2}} \sum_{f=e,\mu,\tau} \left[\overline{\nu}_f \gamma^\mu (1-\gamma_5) f W_\mu^+ + \overline{f}\gamma^\mu (1-\gamma_5)\nu_f W_\mu^-\right] \tag{8.75}$$

$$= -\frac{g}{2\sqrt{2}} \sum_{f=e,\mu,\tau} \left[j_f^\mu W_\mu^+ + \text{h.c.}\right], \tag{8.76}$$

where $j_\mu^f = \bar{\nu}_f \gamma^\mu (1-\gamma_5) f$ and $\frac{g}{2\sqrt{2}}$ is the magnitude of the coupling of the vector and the axial vector charged lepton currents for each flavor e, ν, τ, giving in a natural way, the lepton universality. It also reproduces the relative sign between the vector and the axial vector couplings. The strength of the coupling g is determined in terms of the universal Fermi constant G_F as:

$$\frac{g^2}{8{M_W}^2} = \frac{G_F}{\sqrt{2}}, \quad \text{that is,} \quad g = 2\sqrt{G_F (M_W)^2\sqrt{2}}. \tag{8.77}$$

The lepton universality of the charged current vector interaction is therefore inbuilt in the model.

Similarly from Eq. (8.4), the neutral current interaction Lagrangian is given by:

$$\mathcal{L}_{\text{NC}}^{\text{leptons}} = -\sum_{f=e,\mu,\tau} \left[\bar{l}_{f_L} \left(g\frac{\tau_3}{2}\not{W}^3 + g'\frac{Y^L}{2}\not{B}\right) l_{f_L} + \bar{l}_{f_R} g' \frac{Y^R}{2}\not{B} l_{f_R}\right] \tag{8.78}$$

Since $\tau_3 l_{f_R} = 0$ and τ_3 acts only on the doublet L, we can write:

$$\mathcal{L}_{\text{NC}}^{\text{leptons}} = -\sum_{f=e,\mu,\tau} \sum_{i=L,R} \bar{l}_{f_i} \left[g\frac{\tau_3^{if}}{2}\not{W}^3 + g'\frac{Y^{if}}{2}\not{B}\right] l_{f_i}.$$

We have

$$Q^{if} = \frac{1}{2}\tau_3^{if} + \frac{Y^{if}}{2}. \tag{8.79}$$

Here,

$$\begin{aligned} Q^{if} &= Q^{f_i} \text{ for example, if } f = e, \mu, \tau, \ \ i = L,\ R; \ \ Q^{f_i} = -1 \\ \text{and } Y^{if} &= Y^{f_i} \ \ \text{(see Table 8.1).} \end{aligned} \tag{8.80}$$

We now express $W^{3\mu}$ and B^μ in terms of the neutral bosons Z^μ and A^μ using Eqs. (8.42), (8.52) and (8.53) and write:

$$\begin{aligned} \mathcal{L}_{\text{NC}}^{\text{leptons}} &= -\sum_{f,i} \bar{l}_{f_i}\gamma_\mu \left[g\frac{\tau_3^{if}}{2}(\cos\theta_W Z^\mu + \sin\theta_W A^\mu) + g'(Q^{if} - \frac{\tau_3^{if}}{2})\right. \\ &\qquad (-\sin\theta_W Z^\mu + \cos\theta_W A^\mu)]\, l_{f_i} \\ &= -\sum_{f,i} \bar{l}_{f_i}\gamma_\mu \left[A^\mu (g\sin\theta_W \frac{\tau_3^{if}}{2} - g'\cos\theta_W \frac{\tau_3^{if}}{2} + g' Q^{if}\cos\theta_W)\right. \end{aligned}$$

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$$+Z^{\mu}(g\cos\theta_W\frac{\tau_3^{if}}{2} - g'\sin\theta_W(Q^{if} - \frac{\tau_3^{if}}{2}))\Big] l_{f_i}. \quad (8.81)$$

Since $g\sin\theta_W = g'\cos\theta_W = e$, we obtain:

$$\begin{aligned}\mathcal{L}_{\mathrm{NC}}^{\mathrm{leptons}} &= -\sum_{f,i}\bar{l}_{f_i}\gamma_\mu\Big[A^\mu g'\cos\theta_W Q^{if} + Z^\mu \\ &\quad \left(g\cos\theta_W\frac{\tau_3^{if}}{2} - g'\sin\theta_W Q^{if} + g'\sin\theta_W\frac{\tau_3^{if}}{2}\right)\Big] l_{f_i}.\end{aligned}$$

The first term on the right-hand side is the interaction of the electromagnetic field A^μ with a lepton,

$$\begin{aligned}\mathcal{L}_{\mathrm{em}}^{\mathrm{leptons}} &= -ej_\mu A^\mu, \\ \text{where} \quad j_\mu &= \sum_{f,i}\bar{l}_{f_i}\gamma_\mu Q^{if} l_{f_i} = -\sum_f \bar{l}_f\gamma_\mu l_f \end{aligned} \quad (8.82)$$

for $Q^i = Q = -1$, with $g'\cos\theta_W = e = g\sin\theta_W$. The second term in Eq. (8.81) given by:

$$\mathcal{L}_{\mathrm{NC}}^{\mathrm{leptons}} = -\sum_{f,i} Z^\mu\bar{l}_{f_i}\gamma_\mu\left((g\cos\theta_W + g'\sin\theta_W)\frac{\tau_3^{if}}{2} - Q^{if}g'\sin\theta_W\right) l_{f_i}, \quad (8.83)$$

can be simplified using $g\sin\theta_W = g'\cos\theta_W = e$, to give:

$$\begin{aligned}\mathcal{L}_{\mathrm{NC}}^{\mathrm{leptons}} &= -\frac{e}{2\sin\theta_W\cos\theta_W}\sum_{f,i}Z^\mu\bar{l}_{f_i}\gamma^\mu\left(\tau_3^{if} - 2Q^{if}\sin^2\theta_W\right) l_{f_i} \\ &= -\frac{e}{2\sin\theta_W\cos\theta_W}j_\mu^Z Z^\mu,\end{aligned} \quad (8.84)$$

where

$$\begin{aligned} j_\mu^Z &= \sum_{f,i}\bar{l}_{f_i}\gamma_\mu(\tau_3^{if} - 2Q^{if}\sin^2\theta_W) l_{f_i} \\ &= \sum_f \bar{l}_f(\tau_3^f - 2Q^f\sin^2\theta_W)\gamma_\mu\frac{1-\gamma_5}{2}l_f + \bar{l}_f(-2Q^f sin^2\theta_W)\gamma_\mu\frac{1+\gamma_5}{2}l_f \\ &= \sum_f \bar{l}_f\gamma_\mu\left[\left(\frac{1}{2}\tau_3^f - 2Q^f\sin^2\theta_W\right) - \frac{1}{2}\tau_3^f\gamma_5\right] l_f \\ &= \sum_f \bar{l}_f\gamma_\mu(g_V^f - g_A^f\gamma_5) l_f. \end{aligned} \quad (8.85)$$

Using

$$g_V^f = \frac{1}{2}\tau_3^f - 2Q^f\sin^2\theta_W, \quad (8.86)$$

$$g_A^f = \frac{1}{2}\tau_3^f, \quad (8.87)$$

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we may write Eq. (8.85) as

$$\begin{aligned} j_\mu^Z &= \sum_f \bar{l}_f \gamma_\mu \left[\frac{1}{2}(1-\gamma_5)\tau_3^f - 2Q^f \sin^2\theta_W\right] l_f, \\ &= j_\mu^3 - 2\sin^2\theta_W j_\mu^{\text{em}}. \end{aligned} \tag{8.88}$$

Now writing explicitly in terms of the fermions, for neutrinos with $Q = 0,\ \tau_3 = +1$ and for the charged leptons $e^-,\ \mu^-$,and τ^- with $Q = -1,\ \tau_3 = -1$, we get:

$$\begin{aligned} j_\mu^Z(\nu_f) &= \sum_{f=e,\,\mu,\,\tau} \bar{\nu}_f \gamma_\mu \frac{(1-\gamma_5)}{2}\nu_f \\ j_\mu^Z(\text{charged leptons}) &= \sum_{f=e,\,\mu,\,\tau} \bar{f}\gamma_\mu \left[C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2}\right] f, \\ \text{where } C_L^f &= -1 + 2\sin^2\theta_W, \\ C_R^f &= +2\sin^2\theta_W. \end{aligned}$$

Equivalently, we can write:

$$\begin{aligned} j_\mu^Z(\text{charged leptons}) &= \sum_{f=e,\,\mu,\,\tau} \frac{1}{2}\bar{f}\gamma_\mu \left[(-1+4\sin^2\theta_W) + \gamma_5\right] f \\ &= \sum_{f=e,\,\mu,\,\tau} \bar{f}\left[\gamma_\mu(g_V^f - g_A^f\gamma_5)\right] f, \end{aligned}$$

where $g_V^f = \frac{-1+4\sin^2\theta_W}{2}$ and $g_A^f = -\frac{1}{2}$. The values of g_V and g_A for the charged leptons and neutrinos are given in Table 8.2.

Again, the lepton universality of neutral current weak interaction is inbuilt in the model.

Table 8.2 Couplings of the leptons to Z_μ field.

States	g_V	g_A
ν_l	$1/2$	$1/2$
l	$-\frac{1}{2} + 2\sin^2\theta_W$	$-1/2$

8.4.1 Extension to the quark sector using GIM mechanism

We have seen in Chapter 6 that the weak interaction of $(u,\ d)$ quarks is described in terms of the quark doublet with the Cabibbo rotated quark states, that is, $(u,\ d')$, with $d' = \cos\theta_C d + \sin\theta_C s$, where θ_C is the Cabibbo angle and s is the strange quark. The weak charged currents are, therefore, written as:

$$\begin{aligned} j^\mu(u,\ d,\ s) &= \frac{1}{2}\bar{u}\gamma^\mu(1-\gamma_5)d' \\ &= \frac{1}{2}\left[\cos\theta_C \bar{u}\gamma^\mu(1-\gamma_5)d + \sin\theta_C \bar{u}\gamma^\mu(1-\gamma_5)s\right], \end{aligned}$$

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which describes the weak charged currents in the $\Delta S = 0$ sector as well as in the $\Delta S \neq 0$ sector satisfying $|\Delta S| = 1$ and the $\Delta S = \Delta Q$ rule. It also predicts the existence of neutral current processes like $K_L^0 \rightarrow l^+ l^-$ and $K^\pm \rightarrow \pi^\pm \nu \bar{\nu}$. In order to address the problem of stringent limits on the non-existence of neutral currents in the strangeness sector and maintain the quark–lepton symmetry proposed earlier by Bjorken and Glashow [65], Glashow, Ilioupolis, and Maiani [64] proposed the existence of a fourth quark c, as discussed in Chapter 6. Accordingly, the weak interaction of quarks can be described in terms of three quark doublets with left-handed quarks:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{8.89}$$

with

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{8.90}$$

In this section, we apply the formalism developed in the last section to the quark sector. The weak isospin, weak hypercharge, and electric charge of all the quarks discussed in this section are given in Table 8.3.

Table 8.3 Isospin (I), its third component (I_3), charge ($Q(|e|)$), and hypercharge ($Y = 2(Q - I_3)$) of the first and second generation of quarks, that is, (u, d) and (c, s) quarks.

Name	I	I_3	$Q(\|e\|)$	Y
$u_L(c_L)$	1/2	+1/2	+2/3	1/3
$d_L(s_L)$	1/2	−1/2	−1/3	1/3
$u_R(c_R)$	0	0	+2/3	4/3
$d_R(s_R)$	0	0	−1/3	−2/3

We first write the weak charged current interaction Lagrangian, in the four quark sector, in analogy with the Lagrangian for the weak interaction for leptons (Eq. (8.74)),

$$\mathcal{L}_{\text{cc}}^{\text{int}}(\text{quarks}) = -\frac{g}{\sqrt{2}} \sum_q \left(\bar{q}_L \gamma^\mu \tau^+ q_L W_\mu^+ + \bar{q}_L \gamma^\mu \tau^- q_L W_\mu^- \right) \tag{8.91}$$

$$= -\frac{g}{2\sqrt{2}} \left[\left(\bar{u} \gamma^\mu (1 - \gamma^5) d' + \bar{c} \gamma^\mu (1 - \gamma^5) s' \right) W_\mu^+ + \text{h.c.} \right], \tag{8.92}$$

The neutral current weak interaction Lagrangian for the quarks is written in analogy with the neutral weak interaction Lagrangian for the charged leptons (Eq. (8.84))as:

$$\mathcal{L}_{\text{int}}^{\text{NC}} = -j_\mu^{\text{NC}}(\text{quark}) Z^\mu, \tag{8.93}$$

$$\text{with} \quad j_\mu^{\text{NC}}(\text{quark}) = \frac{e}{2 \sin\theta_W \cos\theta_W} \sum_q \bar{q} \gamma^\mu (g_V^q - g_A^q \gamma_5) q, \tag{8.94}$$

$$\text{where} \quad g_V^q = \frac{1}{2} \tau_3^q - 2 \sin^2\theta_W Q_q \quad \text{and} \tag{8.95}$$

$$g_A^q = \frac{1}{2} \tau_3^q. \tag{8.96}$$

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For example, the values of g_V and g_A for u and d quarks are given in Table 8.4.

Table 8.4 Couplings of the quarks (u, d) to Z_μ field

States	g_V	g_A
u	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$1/2$
d	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-1/2$

Explicitly for the case of two flavors, to demonstrate the application of the GIM mechanism and to explain the absence of neutral currents in the $\Delta S = 1$ sector, let us consider the neutral current in the case of two flavors,

$$\begin{aligned} J^\mu_{\mathrm{NC}}(u, c, d', s') &= \sum_{q=u,c,d',s'} \bar{q} O^q q; \;\; \text{where} \;\; O^q = \gamma^\mu (g_V^q - g_A^q \gamma_5), \\ &= \bar{u}\, O^u u + \bar{c}\, O^c c + \bar{d}\, O^{d'} d + \bar{s}\, O^{s'} s. \end{aligned} \tag{8.97}$$

Since τ_3^q and Q^q are the same for d' and s', both corresponding to $\tau_3 = 2I_3 = -1,\; Q = -\frac{1}{3}$, we can write $O^{d'} = O^{s'}$. Therefore, $\bar{d}\, O^{d'} d + \bar{s}\, O^{s'} s = \bar{d}\, O^d d + \bar{s}\, O^s s$ and the neutral current

$$J^\mu_{\mathrm{NC}}(u, c, d', s') = \bar{u}\, O^u u + \bar{c}\, O^c c + \bar{d}\, O^d d + \bar{s}\, O^s s \tag{8.98}$$

is diagonal in the quark mass states; no terms like $\bar{d}s$ or $\bar{s}d$ corresponding to $\Delta S = 1$ neutral currents appear explaining the absence of FCNC(flavor changing neutral current). This can be extended to six quark mixing using the CKM (Cabibbo–Kobayashi–Maskawa) matrix given in Eq. (8.90).

8.4.2 Triumphs of the Weinberg–Salam–Glashow Model

The standard model of electroweak interactions formulated by Weinberg and Salam is generally known as the Weinberg–Salam–Glashow (WSG) model after its extension to the quark sector. The model makes definite predictions for neutral currents in the neutrino and electron sectors and has implications in all areas of physics like particle physics, nuclear physics, atomic physics, astrophysics, and geophysics. While the neutral current of neutrinos is independent of the weak mixing angle, the neutral weak current carried by the charged leptons and the quarks has explicit dependence on the weak mixing angle θ_W. Therefore, the scattering processes like $\nu_l l^- \to \nu_l l^-$ and $\nu_l q \to \nu_l q$, involving leptons or quarks induced by the neutral current will give the value of the weak mixing angle θ_W. This in turn will give information about the masses and couplings of $W^\pm$, Z, and the Higgs boson. With a meticulous choice of physical processes involving neutrinos (antineutrinos) and the charged leptons, nucleons and nuclei, the masses of W^+, W^-, Z, and Higgs (and their decay modes) have been determined and compared with the predictions of the standard model. The history of neutrino physics over the last 50 years is a story of the quest for a unified theory of electroweak interactions leading to the standard model, its formulations, and its triumphs. In the following section, we will describe the major triumphs of the standard model during this period.

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8.4.3 Discovery of the neutral currents in ν-scattering

Historically, weak neutral currents were postulated in the mid-1930s, but experimentally, their search started in the mid-1960s at CERN; these early searches were inconclusive. The first evidences for neutral currents were reported from the experiments done at CERN [168] on neutrino scattering in 1973 and at SLAC, on the electron scattering in 1978 [347], which provided the limits on the value of the weak mixing angle θ_W, confirming the standard model. Since then, many experiments have been done on neutrino scattering using leptons, nucleons, and nuclear targets; polarized electron scattering from the nucleons and nuclear targets; as well as parity violating experiments in atomic physics, in which the effects of the weak neutral currents have been observed.

i) **Neutrino scattering from leptons**
The existence of neutral currents in the lepton sector implies the following scattering processes with ν_e and ν_μ beam on electrons.

$$\begin{aligned} \nu_\mu e^- &\rightarrow \nu_\mu e^-, \qquad & \bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-, \\ \nu_e e^- &\rightarrow \nu_e e^-, \qquad & \bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-. \end{aligned}$$

It should be noted that $\nu_\mu(\bar{\nu}_\mu)e^- \rightarrow \nu_\mu(\bar{\nu}_\mu)e^-$ scattering is possible only due to the weak neutral currents through the Z exchange, while the $\nu_e e^-$ process is also possible with the weak neutral, as shown in Figure 8.1 (See Chapter 9 for details). The experiments with

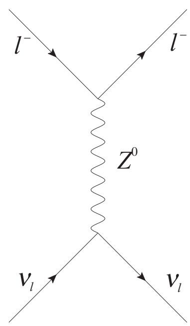

Figure 8.1 $\nu_l - l^-$ neutral current scattering.

ν_μ and $\bar{\nu}_\mu$ have been done at CERN [168] as well as at BNL [348, 349] and FNAL [350]. The statistically most significant data from CHARM II collaboration in CERN [351] gives:

$$\sin^2\theta_W = 0.2324 \pm 0.0058(\text{stat}) \pm 0.0059(\text{syst}).$$

Scattering experiments with ν_e beams on electron targets have been done at Los Alamos while experiments with the $\bar{\nu}_e$ beam have been done at various nuclear reactors around the world. The value of the mixing angle has been determined from these experiments.

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ii) **Neutrino experiments with nucleons and nuclei**
The elastic ν scattering processes like

$$\begin{aligned} \bar{\nu}_l p &\rightarrow \bar{\nu}_l p, \qquad & \nu_l n \rightarrow \nu_l n, \\ \nu_l p &\rightarrow \nu_l p, \qquad & \bar{\nu}_l n \rightarrow \bar{\nu}_l n, \end{aligned}$$

are possible in the W–S model through the interaction Lagrangian $\mathcal{L}_{\mathrm{NC}}$ given in Eqs. (8.59) and (8.93). These processes have been studied earlier at BNL [352] and recently at MiniBooNE [353]. The quoted values from the BNL experiments are $\sin^2\theta_W = 0.220 \pm 0.016(\mathrm{stat})^{+0.023}_{-0.031}(\mathrm{syst})$ [352].

The inelastic antineutrino reactions in the very low energy region have been done at various nuclear reactors [354] with reactor antineutrinos and at the Sudbury Neutrino Observatory (SNO) with solar neutrinos [355] for the neutral current induced processes on the deuterium target, that is,

$$\begin{aligned} \bar{\nu}_l + D &\rightarrow \bar{\nu}_l + n + p\,, \\ \nu_l + D &\rightarrow \nu_l + n + p. \end{aligned}$$

These processes at very low energies correspond to a $D(^3S,\ I=0) \rightarrow np(^1S,\ I=1)$ transition and are possible only through the axial vector current as they involve change in spin. Therefore, they are independent of the weak mixing angle in the standard model. However, these experiments confirmed the existence of neutral currents which led towards the confirmation of the W–S model. At higher energies, inelastic processes of pion (π^0, π^+, π^-) production from nucleon targets like:

$$\begin{aligned} \nu_l + p &\rightarrow \nu_l + p + \pi^0, \qquad & \nu_l + n \rightarrow \nu_l + n + \pi^0, \\ \nu_l + p &\rightarrow \nu_l + n + \pi^+, \qquad & \nu_l + n \rightarrow \nu_l + p + \pi^-, \end{aligned}$$

and coherent and incoherent production of π^0 from nuclear targets like:

$$\nu_l + A \rightarrow \nu_l + A + \pi^0, \qquad \nu_l + A \rightarrow \nu_l + X + \pi^i,$$

where $i = \pm 1, 0$, have been observed at various laboratories. The first experiments were done at ANL and BNL during the 1980s, using hydrogen and deuterium targets but in recent times, experiments have been done at FNAL and JPARC by many collaborative groups like SciBooNE, K2K, MiniBooNE, T2K, etc. to study the neutrino (antineutrino) reactions on the nuclear targets induced by neutral currents. These experiments are analyzed using the value of $\sin^2\theta_W$ obtained from purely leptonic processes. The main emphasis here is to do an isospin analysis and understand the nuclear effects in the context of modeling the $\nu(\bar{\nu})$ nucleus cross section for analyzing the neutrino oscillation experiments as most of these experiments are done on nuclear targets [356].

iii) **Deep inelastic neutrino scattering experiments**

The deep inelastic scattering (DIS) of neutrinos on the nucleon target induced by neutral currents, that is,

$$\begin{array}{lll} \nu_\mu + p & \to & \nu_\mu + X, \qquad \bar{\nu}_\mu + p \to \bar{\nu}_\mu + X, \\ \nu_\mu + n & \to & \nu_\mu + X, \qquad \bar{\nu}_\mu + n \to \bar{\nu}_\mu + X, \end{array}$$

where X is a jet of hadrons, is characterized by the absence of a charged lepton in the final state, unlike the charged current processes in which a $\mu^\pm$ is present,

$$\begin{array}{lll} \nu_\mu + p & \to & \mu^- + X, \qquad \bar{\nu}_\mu \to \mu^+ + X, \\ \nu_\mu + n & \to & \mu^- + X, \qquad \bar{\nu}_\mu \to \mu^+ + X. \end{array}$$

The first evidence of the presence of neutral current induced DIS processes induced by neutrinos was reported at CERN by the Gargemelle collaboration in 1973 [168]. Since then, many experiments have been done using the DIS process induced by ν_μ and $\bar{\nu}_\mu$ on nucleon and nuclear targets at CERN, FNAL, and BNL by collaborations like CCFR [357], BEBC [358], and others. Most DIS experiments with neutrino (antineutrinos) induced by neutral currents are analyzed by studying the following ratios of the cross section:

$$\begin{array}{lll} R^*_{\nu p} & = & \dfrac{\sigma(\nu_\mu p \to \nu_\mu X)}{\sigma(\nu_\mu p \to \mu^- X)} \quad ; \quad R_{\bar{\nu} p} = \dfrac{\sigma(\bar{\nu}_\mu p \to \bar{\nu}_\mu X)}{\sigma(\bar{\nu}_\mu p \to \mu^+ X)}; \\ R^*_{\nu n} & = & \dfrac{\sigma(\nu_\mu n \to \nu_\mu X)}{\sigma(\nu_\mu n \to \mu^- X)} \quad ; \quad R_{\bar{\nu} n} = \dfrac{\sigma(\bar{\nu}_\mu n \to \bar{\nu}_\mu X)}{\sigma(\bar{\nu}_\mu n \to \mu^+ X)}.; \end{array}$$

on nucleons as well as on the isoscalar targets like deuterium and ^{12}C. For nonisoscalar targets like ^{56}Fe and ^{208}Pb, non-isoscalarity corrections are applied. The data on nucleon and nuclear targets are then used to determine the neutral current couplings of the u and d quarks. Neglecting the contribution of strange and heavier quarks, the Paschos–Wolfenstein relation [359] obtained as

$$R^{PW} = \frac{\sigma(\nu_\mu N \to \nu_\mu X) - \sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \to \mu^- X) - \sigma(\bar{\nu}_\mu N \to \mu^+ X)} = \left(\frac{1}{2} - \sin^2\theta_W\right), \tag{8.99}$$

is used to obtained the value of $\sin^2\theta_W$, where θ_W is the Weinberg angle. The values of the weak mixing angle $\sin^2\theta_W$, obtained from various experiments are given in Table 8.5.

With the availability of more data on cross sections with improved precision, the contribution of sea quarks like $\bar{u}, \bar{d}, s, \bar{s}, c, \bar{c}$ are also determined.

Table 8.5 Values of $\sin^2\theta_W$ calculated from various data sets from different experiments [117].

Data	$\sin^2\theta_W$
All data	0.22332(7)
M_H, M_Z, Γ_Z, m_t	0.22351(13)
LHC	0.22332(12)
Tevatron+M_Z	0.22295(30)
LEP	0.22343(47)
SLD+ M_Z, Γ_Z, m_t	0.22228(54)
$\mathcal{A}_{FB}, M_Z, \Gamma_Z, m_t$	0.22503(69)
$\mathcal{A}_{LR}$	0.2220(5)
$\nu_\mu(\bar{\nu}_\mu)p \to \nu_\mu(\bar{\nu}_\mu)p$	0.203(32)
$\nu_\mu(\bar{\nu}_\mu)e \to \nu_\mu(\bar{\nu}_\mu)e$	0.221(8)
Atomic parity violation	0.220(3)

8.5 Discovery of Neutral Currents in Electron Scattering

The W–S model predicts the neutral current coupling of electrons to various quarks through the Z exchange. Therefore, the electron nucleon scattering which is mediated by the photon exchange gets extra contribution from the Z exchange as shown in Figure 8.2. Since the Z exchange diagram involves both vector and axial vector currents due to the structure of the weak neutral current in the lepton and quark sectors, the interference between the photon and Z exchange diagrams would lead to parity violating effects. This can be observed as an asymmetry in the differential cross section $\frac{d\sigma}{dq^2}$ in the elastic scattering of polarized electrons from nucleons and nuclear targets. In principle, one observes the asymmetry A defined by:

$$A(q^2) = \frac{\sigma_R(q^2) - \sigma_L(q^2)}{\sigma_L(q^2) + \sigma_R(q^2)}, \tag{8.100}$$

where $\sigma_{L,R}(q^2)$ are the differential scattering cross sections $\left(\frac{d\sigma}{dq^2}\right)_{L,R}$ for the left-handed (L) and the right-handed (R) polarized electrons. The weak neutral currents in the electron sector were first discovered in the deep inelastic scattering of polarized electrons from deuterons at SLAC in 1978 [347]. Since then, experiments, which observe parity violating asymmetry in the elastic, inelastic, and deep inelastic scattering of polarized electrons from nucleon and nuclear targets like hydrogen(^{1}H), deuterium(^{2}D), helium(^{4}He), carbon(^{12}C), and lead(^{208}Pb), have been done at various electron accelerators like MIT-BATES [360, 361], MAINZ [362, 363], and JLab [364, 365, 366], in the region of low, medium, and high energies [367]. A special feature of the physics of the scattering of polarized electrons from nuclei and asymmetry measurements is that the neutral current coupling of an electron to the neutron is much stronger than its coupling to the proton. This is used to determine, independently, the neutron density of nuclei in polarized electron scattering from heavy nuclei like ^{208}Pb [368]. Thus, weak neutral currents provide a new tool to study the neutron distribution in nuclei. Moreover, experiments on electron annihilation from positrons also provide evidence for neutral currents in the $\mu^-\mu^+$ and $\tau^-\tau^+$ sectors through the various asymmetry measurements in processes like $e^+e^- \to$

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$\mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$. These experiments have decisively confirmed the effect of the weak neutral currents in the charged lepton sector as predicted by the W–S model. In the following section, we briefly present these effects.

Elastic scattering of polarized electrons

The parity violating asymmetries in the elastic scattering of polarized electrons arise due to the interference between the contributions coming from the photon (γ) and the Z exchange diagrams shown in Figure 8.2, for which the matrix elements are written as:

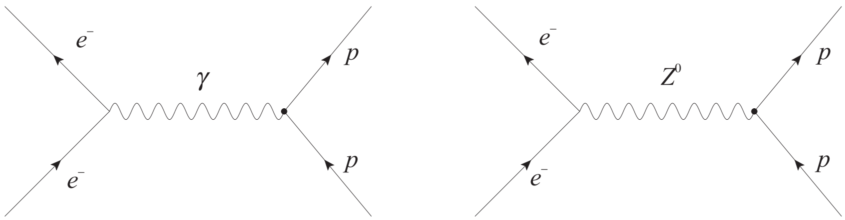

Figure 8.2 Electrons (point particles) getting scattered from a nucleon through electromagnetic (left) and weak (right) interactions. The blob depicts the fact that the nucleon has got a structure.

$$M_\gamma = -\frac{e^2}{q^2}\bar{u}_e(k')\gamma^\mu u_e(k)\bar{u}(p')J^\gamma_\mu u(p), \tag{8.101}$$

$$M_Z = -\frac{G_F}{\sqrt{2}}\bar{u}_e(k')\gamma^\mu(g^e_V - \gamma_5 g^e_A)\bar{u}(p')J^Z_\mu u(p), \tag{8.102}$$

where $g^e_V = -\frac{1}{2} + 2\sin^2\theta_W$, $g^e_A = -\frac{1}{2}$ (as given in Table 8.2), and J^γ_μ and J^Z_μ are given as:

$$\langle p'|J^\gamma_\mu|p\rangle = \bar{u}(p')\left[F^\gamma_1(q^2)\gamma_\mu + i\sigma_{\mu\nu}\frac{q^\nu}{2M}F^\gamma_2(q^2)\right]u(p), \tag{8.103}$$

$$\langle p'|J^Z_\mu|p\rangle = \bar{u}(p')\left[F^Z_1(q^2)\gamma_\mu + i\sigma_{\mu\nu}\frac{q^\nu}{2M}F^Z_2(q^2) + G^Z_A(q^2)\gamma_\mu\gamma_5\right]u(p). \tag{8.104}$$

In writing the aforementioned matrix elements, CVC and G-invariance have been assumed and the pseudoscalar form factor term is neglected as its contribution is negligible. The form factors $F^{\gamma,Z}_{1,2}(q^2)$ are the Pauli–Dirac electromagnetic (γ) and weak (Z) neutral current form factors discussed in detail in Chapter 10. They are expressed in terms of the Sachs electric and magnetic form factors, $G^{\gamma,Z}_{E,M}(q^2)$:

$$G^{\gamma Z}_E = F^{\gamma,Z}_1 + \frac{q^2}{4M^2}F^{\gamma,Z}_2(q^2), \tag{8.105}$$

$$G^{\gamma,Z}_M = F^{\gamma,Z}_1(q^2) + F^{\gamma,Z}_2(q^2). \tag{8.106}$$

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$G_A^Z(q^2)$ is the weak neutral axial vector form factor. Using the isotriplet hypothesis of weak current (Chapter 5), we obtain $F_{1,2}^{\gamma,Z}$ and G_A^Z of the nucleons in terms of the electromagnetic and weak charged current form factors of the nucleons, that is,

$$G_{E,M}^{Z(p,n)} = (1 - 4\sin^2\theta_W)G_E^{\gamma(p,n)}(q^2) - G_{E,M}^{\gamma(n,p)}(q^2), \tag{8.107}$$

$$G_A^{Z,p}(q^2) = -G_A^{Z,n}(q^2) = \frac{1}{2}G_A(q^2). \tag{8.108}$$

The parity violating asymmetry $\mathcal{A}_{LR}$ is then proportional to

$$\begin{aligned} \mathcal{A}_{LR} &\propto \frac{\mathrm{Re}|M^\gamma M^{Z*}|}{|M^\gamma|^2 + |M^Z|^2} \simeq \frac{\mathrm{Re}|M^\gamma M^{Z*}|}{|M^\gamma|^2} \\ &= \frac{G_F q^2}{4\sqrt{2}\pi\alpha}(A_E^p(q^2) + A_M^p(q^2) + A_A^p(q^2)), \end{aligned} \tag{8.109}$$

where $A_E^p(q^2)$ and $A_M^p(q^2)$ are due to the interference of the electron axial vector and nucleon vector currents. The term $A_A^p(q^2)$ is due to the interference of the electron vector and nucleon axial vector currents and is small, being proportional to $(1 - 4\sin^2\theta_W)$. The expression for the asymmetry becomes very simple in the case of ^{4}He, which is a spin zero isoscalar target, so that only spin independent isoscalar hadronic currents, which exist only for the vector current in the G–W–S model contribute through the $F_1^{I=0}(q^2)$ form factor, resulting in

$$\mathcal{A}_{LR}^{^4He} = -\frac{G_F q^2}{\pi\sqrt{2}\alpha}\sin^2\theta_W. \tag{8.110}$$

The experimental results are summarized in Table 8.6; these results are consistent with the predictions of the W–S–G model. In 2005, an experiment for fixed target Møller scattering, that is, $e^-e^- \to e^-e^-$ measured $\mathcal{A} = -0.131 \pm 13\%$ ppm [369]. The values of asymmetries obtained from different experiments are listed in Table 8.6.

Table 8.6 Results from selected parity violating (PV) experiments. Asymmetries are given in parts per million (ppm).

Experiment	$\mathcal{A}_{LR}$(Elastic)
Mainz [362]	$-9.4 \pm 20\%$
Mainz–A4 [363]	$-17.2 \pm 5\%$
MIT–Bates [360]	$1.62 \pm 24\%$
SAMPLE [361]	$-5.61 \pm 20\%$
HAPPEX [364]	$-15.05 \pm 7.5\%$
GØ [365]	$-2 \pm 13\%$
HAPPEX–He [366]	$6.40 \pm 4.1\%$

Inelastic electron scattering

The presence of neutral currents have been observed in the inelastic scattering of electrons from the proton target in the reaction $\vec{e} + p \to \vec{e} + \Delta^+$ at JLab [370]. In this case, the matrix

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elements corresponding to γ and Z exchange, shown in Figure 8.3, will be similar to those given in Eqs. (8.101) and (8.102), respectively, with the matrix elements of hadronic currents $J_\mu^{\gamma,Z}$ defined as $\langle\Delta(p')|J_\mu^\gamma|p(p)\rangle$ and $\langle\Delta(p')|J_\mu^Z|p(p)\rangle$, which are parameterized in terms of four vector and four axial vector form factors $C_i^V(q^2)$ and $C_i^A(q^2), (i = 3-6)$ defined and discussed in detail in Chapter 12.

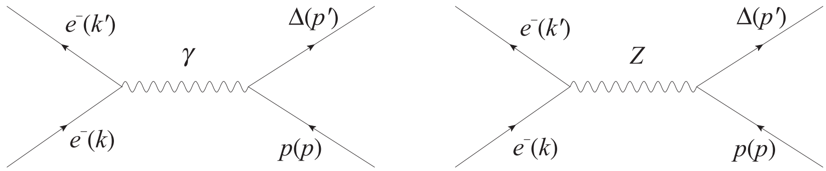

Figure 8.3 Inelastic electron–proton scattering ($e^- p \longrightarrow e^-\Delta$) through the electromagnetic(left) and weak(right) interactions.

Since the transition $p \rightarrow \Delta^+$ can take place only through $\Delta I = 1$ transitions, only the isovector parts of the currents J_μ^γ and J_μ^Z contribute to the Δ excitations for which the isovector form factors are summarized as (Chapter 11):

i) The form factors $C_i^V(q^2)$ $(i = 3, 4, 5)$ are given in terms of $C_i^\gamma(q^2)$, assuming the isotriplet properties of weak currents.

ii) $C_6^\gamma(q^2)$ and $C_6^Z(q^2)$ vanish due to CVC.

iii) $C_6^A(q^2)$ is negligible being proportional to the electron mass.

iv) Using $M1$ dominance of $\Delta \rightarrow p\gamma$ decay and $ep \rightarrow e\Delta$ scattering, it may be shown that $C_5^\gamma(0) = 0$, $C_4^\gamma(0) = -\frac{M}{M+M_\Delta}C_3^\gamma(0)$.

v) Using the data available in the weak excitation of Δ in ν-scattering, we obtain, $C_3^A(0) = 0$, $C_4^A(0) = -0.35$, $C_5^A(0) = 1.20$, $C_3^\gamma(0) = 1.85$, and $C_4^\gamma(0) = -0.89$.

The asymmetry is calculated to be:

$$\mathcal{A}_{\text{inel}} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}[\Delta^V(q^2) + \Delta^A(q^2)], \tag{8.111}$$

where $\Delta^V(q^2)$ includes the contribution arising due to the interference of the electron axial vector (g_A^e) and hadron vector terms and $\Delta^A(q^2)$ is the contribution arising from the interference of the electron vector (g_V^e) and hadron axial vector terms. Neglecting the contribution of the non-resonant terms, which are found to be small ($\approx$ 1.5%), $\Delta^V(q^2)$ becomes independent of the hadronic structure.

The $\Delta^A(q^2)$ term, on the other hand, which involves the hadronic form factors $C_i^V(q^2)$ corresponding to the photon exchange and $C_i^A(q^2)$ corresponding to the Z exchange, is complicated and its discussion is given in Ref. [371, 372, 373]. Fortunately, the contribution of $\Delta^A(q^2)$ is small ($\approx$ 5%) because it is proportional to $(1 - 4\sin^2\theta_W)$. Therefore, the resonance

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contribution to $\Delta^V(q^2)$ is dominant, leading to $\mathcal{A} = -32.2 \times 10^{-6}$ using $\sin^2\theta_W = 0.2353$. The total contribution of $\Delta^V(q^2)$ and $\Delta^A(q^2)$ (including the non-resonant contributions) gives a theoretical value of $\mathcal{A} = (34.6 \pm 1.0) \times 10^{-6}$.

The experimental analysis of the JLab experiment [370] reports a value of the asymmetry as:

$$\begin{aligned}
\mathcal{A} &= (-33.4 \pm 5.3 \pm 5.1) \times 10^{-6} \quad \text{for the proton target,} \\
\mathcal{A} &= (-43.6 \pm 14.6 \pm 6.2) \times 10^{-6} \quad \text{for the deuteron target,}
\end{aligned}$$

which is in reasonable agreement with the predictions of the W–S–G model.

Deep inelastic scattering

In the case of the deep inelastic scattering of the polarized electrons from the nucleons, the scattering takes place from the point-like constituents of the nucleons, that is, quarks as shown in Figure 8.4, through the interaction Lagrangian described in Eqs. (8.84) and (8.93) for the electromagnetic and weak neutral current.

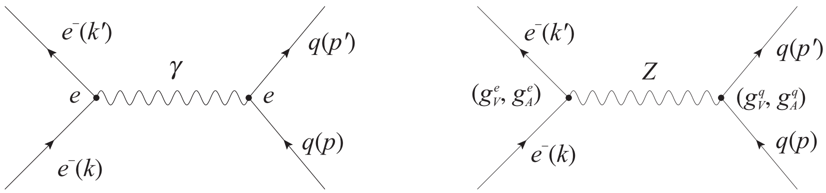

Figure 8.4 Deep inelastic scattering of an electron from a quark inside a nucleon through electromagnetic interaction (left) with a coupling strength e and weak interaction (right) with vector and axial vector coupling strengths g_V^e and g_A^e, respectively.

The strength of the vertex is e for the $ee\gamma$ and $qq\gamma$ interaction, $g_V^e(g_A^e)$ for the eeZ, and $g_V^q(g_A^q)$ for the qqZ interactions with the vector (axial vector) couplings, where:

$$g_V^i = \frac{1}{2}\tau_3^i - 2Q^i \sin^2\theta_W, \qquad g_A^i = \frac{1}{2}\tau_3^i,$$

and Q^i is the charge (in units of $|e|$), for the ith fermion. The parity violating part of the interaction Lagrangian, coming from the interference of the vector and axial vector currents in the Z exchange term, that is, VA and AV terms is given by:

$$\mathcal{L}^{PV} = \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma^\mu\gamma_5 e (C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d) + \bar{e}\gamma^\mu e (C_{2u}\bar{u}\gamma_\mu\gamma_5 u + C_{2d}\bar{d}\gamma_\mu\gamma_5 d) \right], \quad (8.112)$$

where

$$\begin{aligned}
C_{1u} &= -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W, \\
C_{1d} &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_W, \\
C_{2u} &= -\frac{1}{2} + 2\sin^2\theta_W,
\end{aligned}$$

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$$C_{2d} = \frac{1}{2} - 2\sin^2\theta_W = -C_{2u}. \tag{8.113}$$

The asymmetry $(\mathcal{A}_{LR})$ for the deep inelastic scattering of polarized electrons with right-handed and left-handed electrons from the quarks is then given by:

$$\mathcal{A}_{\text{DIS}} \simeq \frac{G_F q^2}{4\sqrt{2}\pi\alpha}(a_1 + f(y)a_3)$$

$$\text{with } a_1 = \frac{2\sum_q e_q C_{1q}(q(x)+\bar{q}(x))}{\sum_q e_q^2(q(x)+\bar{q}(x))},$$
$$a_3 = \frac{2\sum_q e_q C_{2q}(q(x)-\bar{q}(x))}{\sum_q e_q^2(q(x)+\bar{q}(x))}, \tag{8.114}$$

where $q(x)$ and $\bar{q}(x)$ are the parton distribution functions (PDFs) for quarks and $f(y)$ is the kinematic factor given by:

$$f(y) = \frac{1-(1-y)^2}{1+(1-y)^2}, \tag{8.115}$$

with $y = \frac{\Delta E}{E}$, where E and ΔE are respectively the energy and the energy loss of the incident electron. Using these formulae, we obtain the following equations for the parity violating asymmetry in $\mathcal{A}^p_{\text{DIS}}$ in $\vec{e}p \to \vec{e}p$ and $\mathcal{A}^D_{\text{DIS}}$ in $\vec{e}D \to \vec{e}D$ scatterings:

$$\mathcal{A}^p_{\text{DIS}} = \frac{3G_F q^2}{4\sqrt{2}\pi\alpha}\left[\frac{C_{1u} - \frac{1}{2}C_{1d}\cdot\frac{d(x)}{u(x)}}{1+\frac{1}{4}\frac{d(x)}{u(x)}} + \frac{C_{2u} - \frac{1}{2}C_{2d}\cdot\frac{d(x)}{u(x)}}{1+\frac{1}{4}\frac{d(x)}{u(x)}}\cdot f(y)\right],$$
$$\mathcal{A}^D_{\text{DIS}} = \frac{3G_F q^2}{5\sqrt{2}\pi\alpha}\left[(C_{1u} - \frac{1}{2}C_{1d}) + (C_{2u} - \frac{1}{2}C_{2d})\cdot f(y)\right]. \tag{8.116}$$

We see that while $\mathcal{A}^p_{\text{DIS}}$ depends upon the PDFs through the ratio $\frac{d(x)}{u(x)}$ for the d and u quarks, $\mathcal{A}^D_{\text{DIS}}$ is independent of the hadronic structure and depends only upon the parameters $C_{1q,2q}(q = u, d)$ of the G-W-S model which depend on $\sin^2\theta_W$. It should be noted that, in general, C_{2q} is small as compared to C_{1q} for $q = u, d$ in the valence quark model. Therefore, an experimental determination of C_{2q} would be important to test the validity of the valence quark description and to study the sea quark contributions. The first experiment in the deep inelastic scattering region was done at SLAC in 1978 [347] and then at JLab in 2014 [374]. The experimental results are given in Table 8.7.

Table 8.7 Results from selected PV experiments. Asymmetries are given in ppm.

Experiment	$-\mathcal{A}_{LR}(DIS) \times 10^{-6}$ ppm
SLAC–E122 [347]	$-120 \pm 8\%$
JLab–Hall A [374]	$-160 \pm 4.4\%$

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In most recent times, experiments have been done in the region extending from the higher resonance region, that is, beyond the Δ resonance to the DIS region, in a wide range of W, the centre of mass energy. The results are shown in Figure 8.5, where the theoretical and experimental results are taken from Ref. [375, 376, 377, 378]

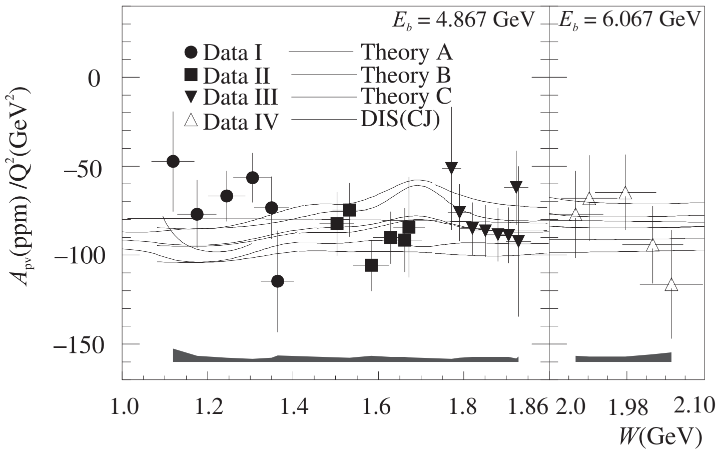

Figure 8.5 A_{pv} vs. W studied in $\vec{e}$–^{2}H scattering experiment. The physics asymmetry results, A_{PV}^{phys}, for the four kinematics I, II, III, and IV (solid circles, solid squares, solid triangles, and open triangles, respectively), in ppm, are scaled by $1/Q^2$ and compared with calculations from Ref. [375] (theory A, dashed lines), Ref. [376] (theory B, dotted lines), Ref. [377] (theory C, solid lines), and the DIS estimation (dash-double-dotted lines) with the extrapolated CJ PDF [378].

Weak neutral current in atomic physics

The presence of the weak neutral boson Z and its interaction with electrons and nucleons through eeZ and NNZ couplings having a $V - A$ structure as shown in Figure 8.6 gives rise to the parity violating term in the interaction Hamiltonian for $e - e$, $e - N$, and $N - N$ systems, arising through its interference with the photon exchange diagrams. The parity violating interactions between $e - N$ and $e - e$ systems are relevant to atomic physics.

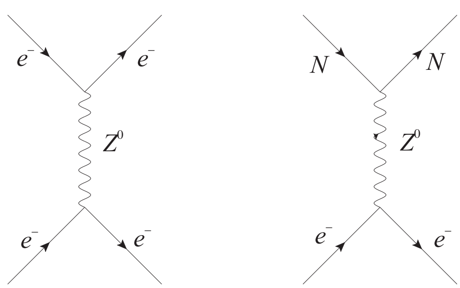

Figure 8.6 Z exchange in electron–electron (left) and electron–nucleon(right) scattering.

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The parity violating part of the interaction Lagrangian for an orbital electron with other electrons and nucleons in the nucleus is written as (suppressing the $\vec{x}$ dependence):

$$\begin{aligned}
\mathcal{L}^{\text{int}}_{PV} &= \mathcal{L}^{eN}_{PV} + \mathcal{L}^{ee}_{PV}, &(8.117)\\
\text{where}\quad \mathcal{L}^{eN}_{PV} &= \frac{G}{\sqrt{2}}(A_e V_N + V_e A_N) &(8.118)\\
\mathcal{L}^{ee}_{PV} &= \frac{G}{\sqrt{2}}(A_e V_e + V_e A_e), &(8.119)\\
\text{where}\quad A_e &= \sum_{i=N} g^e_A \overline{\psi}_e \gamma_\mu \gamma_5 \psi_e,\\
V_N &= \sum (g^p_V \overline{\psi}_p \gamma^\mu \psi_p + g^n_V \overline{\psi}_n \gamma^\mu \psi_n),\\
V_e &= \sum_{i=N} g^e_V \overline{\psi}_e \gamma_\mu \psi_e,\\
A_N &= \sum (g^p_A \overline{\psi}_p \gamma^\mu \gamma_5 \psi_p + g^n_A \overline{\psi}_n \gamma^\mu \gamma_5 \psi_n).
\end{aligned}$$

Since the nucleons in the nucleus are non-relativistic, we can calculate V_N and A_N in the limit $\vec{p} \to 0$; using the values of g^i_V and g^i_A $(i = e, p, n)$ from Eqs. (8.95) and (8.96), $\mathcal{L}^{eN}_{PV}$ and $\mathcal{L}^{ee}_{PV}$ are calculated. The following result is obtained for $\mathcal{L}^{eN}_{PV}$, in the low energy limit [239]

$$\begin{aligned}
\mathcal{L}^{eN}_{PV} = \frac{G_F}{4\sqrt{2}m_e}\Big[& Q_{W\rho_N}(x)\left(\vec{\sigma}\cdot\vec{p}\,\delta^3(\vec{x}\,) + \delta^3(\vec{x}\,)\,\vec{\sigma}\cdot\vec{p}\,\right)\\
& +(1-4\sin^2\theta_W) g_A\, \vec{\sigma}_N \cdot \vec{\sigma}\left(\vec{\sigma}\cdot\vec{p}\,\delta^3(\vec{x}) + \delta^3(\vec{x})\,\vec{\sigma}\cdot\vec{p}\right)\Big],
\end{aligned}$$

where $Q_{W\rho_N} = (1-4\sin^2\theta_W)Z - N$ is called the weak charge of the nucleus. It should be noted that:

i) In view of the smallness of $1-4\sin^2\theta_W$ in the W–S model, the weak charge is mainly determined by the number of neutrons, a fact also emphasized in the scattering of polarized electrons from the heavy nuclei.

ii) The first term dominates over the second term in $\mathcal{L}^{eN}_{PV}$ (Eq. (8.118)) as there is no enhancement due to the large number of nucleons in a heavy nucleus in the second term; this is because it is proportional to the spin of the nucleon $\vec{\sigma}_N$ in the non-relativistic limit.

iii) Similarly, it is straightforward to see that $\mathcal{L}^{ee}_{PV}$ is of the same order as the second term and is therefore small as compared to the first term in Eq. (8.119).

The presence of $\mathcal{L}^{eN}_{PV}$ leads to the mixing of states of opposite parity in the atomic states which can be calculated in the perturbation theory.

In general, any state $|\psi\rangle$ with a given parity will be mixed with the states of opposite parity $|\chi_n\rangle$ given by:

$$|\psi\rangle = |\psi\rangle + \sum_n |\chi_n\rangle \frac{\langle\chi_n|H_{PV}|\psi\rangle}{E(\psi) - E(\chi_n)}, H_{PV} = -\mathcal{L}_{PV} \tag{8.120}$$

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It is found that the mixing depends upon Z^3, where Z is the nuclear charge; therefore, the effect is enhanced for heavier atoms [379].

The atomic parity violation experiments involve observation of magnetic dipole ($M1$) transitions between the initial and final states which are now accompanied by a small amplitude of electric dipole ($E1$) transitions due to the admixture of the opposite parity states in the initial and final states. If $M1$ transition between the states $|\psi_i\rangle$ and $|\psi_f\rangle$ is written as

$$\begin{aligned} M1 &= \langle\psi_f|M1|\psi_i\rangle, \\ \text{then} \qquad E1 &= \sum_n \frac{\langle\psi_f|E1|\chi_n\rangle\langle\chi_n|H^{PV}|\psi_i\rangle}{E_{\psi_i} - E_{\chi_n}} + \frac{\langle\psi_f|H^{PV}|\chi_n\rangle\langle\chi_n|E1|\psi_i\rangle}{E_{\psi_f} - E_{\chi_n}}, \end{aligned}$$

such that initially, a pure $M1$ amplitude is then given by $M1'$ where

$$M1' = M1 \pm E1$$

leading to the parity violating effects proportional to the interference term between $M1$ and $E1$ terms, that is,

$$\frac{2\,\mathrm{Im}(M1E1^*)}{|M1+E1|^2} \simeq \frac{2\mathrm{Im}(M_1E_1^*)}{|M_1|^2}. \tag{8.121}$$

Therefore, in the atomic transitions where the strength of $M1$ is small, the parity violating effects could be large and may lead to observable effects. The transitions studied are therefore $6\,^2S_{\frac{1}{2}} \to 7\,^2S_{\frac{1}{2}}$ in Cs with $Z = 55$ and $6\,^2P_{\frac{1}{2}} \to 7\,^2P_{\frac{1}{2}}$ in Tl with $Z = 81$ which are single valence electron atoms with minimum theoretical uncertainties in describing the atomic wave functions.

The first, experiments were performed on the circular polarization of photons in atomic transitions in ^{133}Cs [380] and later on the optical rotation of photons in ^{209}Bi [381]. The interpretation of experimental results are subject to the theoretical uncertainities of the atomic structure calculations of heavy atoms but the results for the weak mixing angles are consistent with the results obtained from other parity violating experiments as shown in Table 8.5. Some recent reviews of parity violating effects in atoms and molecules are given in [382, 383, 384].

Weak neutral currents in e^-e^+ annihilation experiments

Weak neutral currents in the lepton sector have also been observed in e^-e^+ colliders at LEP and SLAC, where $e^-e^+ \to f\bar{f}$, $f = \mu, \tau, u, d$, etc., are produced. In addition to the photon exchange, there is an additional contribution due to the Z exchange, in the $e^-e^+ \to f\bar{f}$ processes, as shown in Figure 8.7. The presence of the Z exchange leads to the forward–backward asymmetry $\mathcal{A}$ in the production of fermion pairs as well as their polarization P due to the interference between the two diagrams shown in Figure 8.7. The differential scattering cross section in the center of mass (CM) system is given by [279]

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[(1+\cos^2\theta)A + (\cos\theta)B\right], \tag{8.122}$$

where s is the CM energy of the electron and the $\cos\theta$ dependent term gives the forward–backward asymmetry for $0 \le \cos\theta \le 1$ and $-1 \le \cos\theta \le 0$ regions. The coefficients A and

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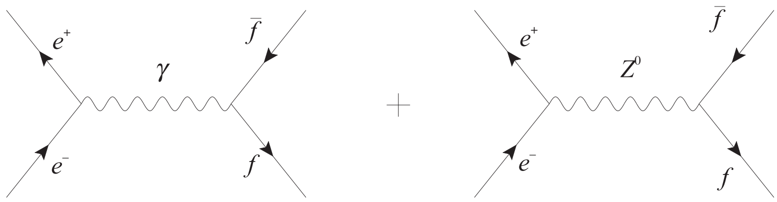

Figure 8.7 Electron–positron annihilation to give $f\bar{f}$ through electromagnetic (left) and weak (right) interactions. The net contribution is a combination of both.

B depend upon the vector and axial vector couplings of the electrons and the final fermions to the Z bosons, that is, g_V^e, g_A^e, g_V^f, and g_A^f and are given as:

$$\begin{aligned} A &= 1 + 2g_V^e g_V^f \mathrm{Re}(F(s)) + [(g_V^e)^2 + (g_A^e)^2][(g_V^f)^2 + (g_A^f)^2]|F(s)|^2, \\ B &= 4g_A^e g_A^f \mathrm{Re}(F(s)) + 8g_A^e g_V^e g_A^f g_V^f |F(s)|^2, \\ \text{where} \quad F(s) &= \frac{s}{s - M_Z^2 + is\Gamma_Z/M_Z}. \end{aligned} \tag{8.123}$$

The cross section in Eq. (8.122) yields a forward–backward asymmetry $\mathcal{A}_{FB} = \frac{N_F - N_B}{N_F + N_B}$, where N_F is the number scattered in the region $0 \le \cos\theta \le 1$ and N_B is the number scattered in the region $-1 \le \cos\theta \le 0$. It is easy to see that $\mathcal{A}_{FB} = \frac{3B}{8A}$. Since, the second term in B involves $g_V^e = 1 - 4\sin^2\theta_W$, which is very small for $\sin^2\theta_W \approx 0.23$, it is the first term which dominates the asymmetry.

Sizeable asymmetries of the order of $\approx 10^{-2}$ are predicted by the W–S model and have been observed in many experiments done at LEP and SLAC. The asymmetries could be large at energies E corresponding to the Z peak, that is, $s \approx M_Z^2$. These observations confirm the presence of weak neutral currents in the lepton sector, involving muons and taouns in addition to electrons. It should be noted that a non-zero forward–backward asymmetry also appears in purely electromagnetic processes due to the interference between the one photon and two photon exchange diagrams; however, this effect is very small.

8.6 Discovery of $W^\pm$ and Z Bosons

8.6.1 Properties of $W^\pm$ and Z boson

The W–S–G model predicts the vector bosons $W^\pm$ and Z with spin 1 and

$$\begin{aligned} M_W^2 &= \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W}, \\ M_Z^2 &= \frac{M_W^2}{\cos^2\theta_W}. \end{aligned}$$

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Their interaction Lagrangian with leptons $(l = \nu, e)$ and quarks $(q = u, d)$ are given by:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= -\left(\frac{G_F M_W{}^2}{\sqrt{2}}\right)^{\frac{1}{2}} \Big[\bar{\nu}_e \gamma^\mu (1-\gamma_5) e\, W_\mu^+ + \bar{e}\gamma^\mu (1-\gamma_5)\nu_e W_\mu^- \\
&\qquad + \bar{q}\gamma^\mu (1-\gamma_5) q W_\mu^+ + \text{h.c.}\Big] \\
&\quad - \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z{}^2}{\sqrt{2}}\right)^{\frac{1}{2}} \left[\bar{f}\gamma^\mu (g_V^f - g_A^f \gamma_5) f\right] Z_\mu,
\end{aligned} \tag{8.124}$$

where $f = e, \nu$ for leptons and $f = q$ for quarks with:

$$g_V^f = \frac{\tau_3^f}{2} - 2Q^f \sin^2\theta_W, \qquad \text{and} \qquad g_A^f = \frac{\tau_3^f}{2}.$$

Diagrammatically, the couplings are shown in Figure 8.8.

We see from Eq. (8.124) and Figure 8.8 that the properties and the phenomenology of weak interactions of $W^\pm$ and Z bosons are defined in terms of M_W, M_Z, $\sin^2\theta_W$, and G_F, which can be determined in their interactions with leptons and quarks through weak processes involving their production and decay. They can be produced in e^+e^- collisions or in $p\bar{p}$ collisions through processes like $e^+e^- \to W^+W^-, ZZ$ or $q\bar{q} \to W^+W^-, ZZ$ and can be observed through their decay processes like $W^\pm \to e^\pm \nu_e(\bar{\nu}_e), Z \to \nu\bar{\nu},\ l^+l^-\ (l = e, \mu)$.

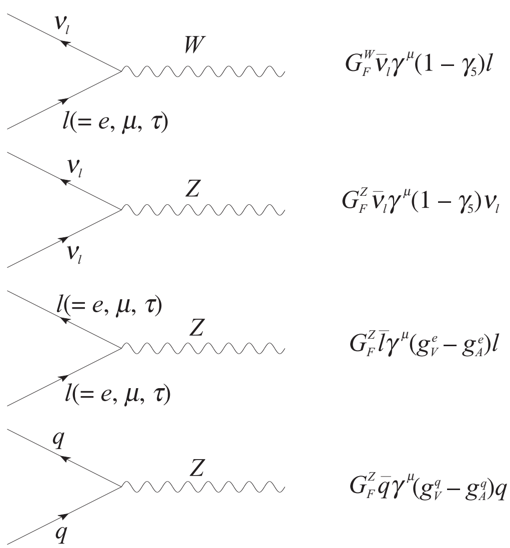

Figure 8.8 Coupling of W, Z bosons with leptons and quarks, where $G_F^W = \left(\frac{G_F M_W{}^2}{\sqrt{2}}\right)^{\frac{1}{2}}$ and $G_F^Z = \frac{1}{\sqrt{2}}\left(\frac{G_F M_Z{}^2}{\sqrt{2}}\right)^{\frac{1}{2}}$.

The first attempts to search for intermediate vector bosons like $W^\pm$ were made at CERN in the mid-1960s [385] in high energy neutrino experiments motivated by the theoretical speculations that weak interactions are mediated by IVB. However, with the success of the

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W–S model, which made definite predictions for $W^\pm$, with mass M_W and also the existence of a neutral boson Z with mass M_Z in terms of G_F and $\sin^2\theta_W$, new efforts were made to search for $W^\pm$ and Z bosons, first at CERN and later at other accelerators.

The weak IVB bosons $W^\pm$ and Z were first discovered experimentally in 1983 in the hadron collider at CERN [170, 171, 386, 387]. Later, at the high energy electron–positron collider machines like SLC at SLAC and LEP at CERN, the direct production of $W^\pm$ and Z through $e^+e^- \to W^+W^-$ and $e^+e^- \to ZZ$ reactions, were studied to make a precise measurement of the masses of $W^\pm$ and Z bosons. The important features of the properties of the $W^\pm$ and Z bosons are summarized here:

i) **Masses of $W^\pm$ and Z bosons**

In hadron colliders with $p\bar{p}$ collision, the production of $W^\pm, Z$ bosons is thought to be due to the Drell–Yan mechanism, in which an antiquark from $\bar{p}$ collides with a quark in p to produce lepton pairs through $W^\pm$ and Z productions, as shown in Figure 8.9.

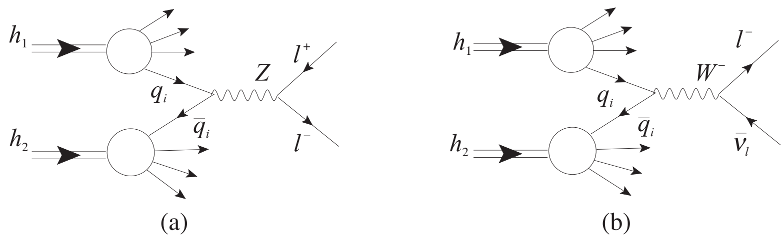

Figure 8.9 The Drell–Yan mechanism. A quark from hadron 1 and an antiquark from hadron 2 combine to give (a) a lepton–antilepton pair, through a Z exchange; (b) a lepton and an antineutrino through a W^- exchange.

The specific reactions studied are:

$$\bar{p}p \to W^\pm X \to e^\pm \nu(\bar{\nu})X, \tag{8.125}$$

$$\bar{p}p \to ZX \to e^+e^- X. \tag{8.126}$$

The reaction in Eq. (8.125) has higher cross section than the cross section for the reaction in Eq. (8.126) and was observed first; however, the reaction in Eq. (8.126) is easier to analyze due to the presence of two charged leptons in the final state, almost in a back to back configuration. By plotting the invariant mass distribution in which both the electrons have well-defined energy in the reaction given in Eq. (8.126) (shown in Figure 8.10), mass M_Z of Z bosons was determined to be:

$$M_Z = 93.0 \pm 1.4(\text{stat}) \pm 3.2(\text{syst})\ \text{GeV} \quad \text{UA1 [386]}, \tag{8.127}$$

$$= 92.5 \pm 1.3(\text{stat}) \pm 1.5(\text{syst})\ \text{GeV} \quad \text{UA2 [387]}. \tag{8.128}$$

In the case of $e^+e^- \to$ hadrons, the invariant mass distribution looks like Figure 8.11 ([388]), corresponding to $M_Z = 91.19$ GeV. The determination of the mass of the W

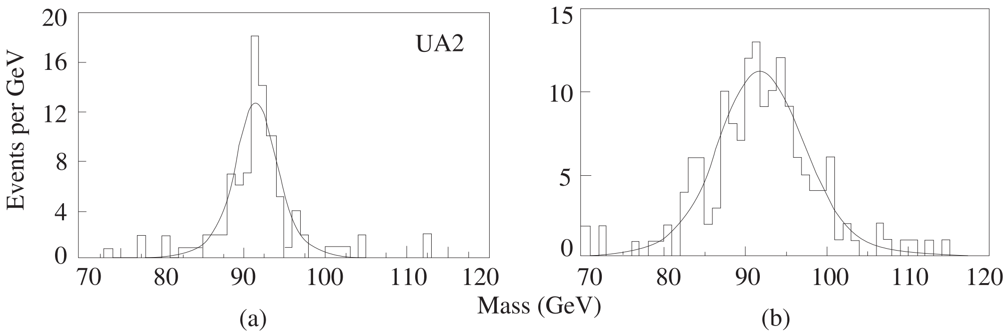

Figure 8.10 Invariant mass spectra for two $Z \to e^+e^-$ event samples, as measured by UA2 [387]. The curves are best fit to the data, using m_Z as a free parameter.

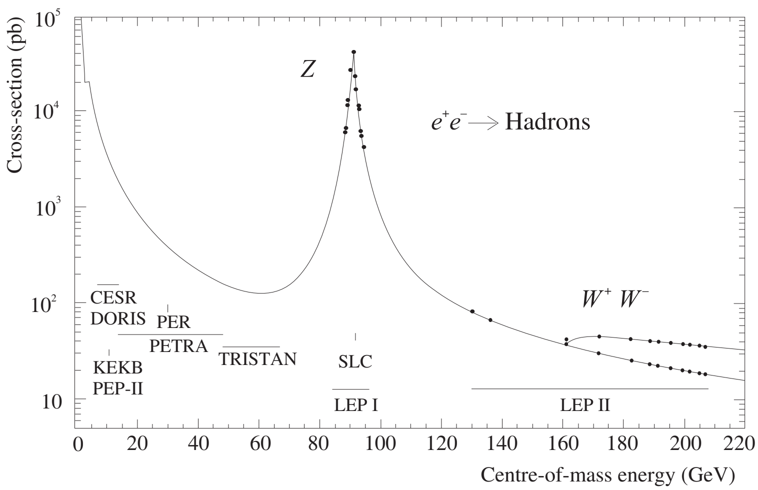

Figure 8.11 Evidence for the existence of Z boson at LEP-I [388]. Energy regions of other experiments have also been shown.

boson is not as straightforward as that of the Z boson. Since $W \to l\nu$ and the neutrino is not observed, the invariant mass distribution of $l\nu$ pair cannot be constructed. Therefore, the W mass determination requires an indirect measurement of the mass using variables which are related to M_W, like the momenta of electron and neutrino. The kinematical variable which is used in the analysis is the transverse mass:

$$M_T{}^2 = (E_{e_T} + E_{\nu_T})^2 - (\vec{P}_{e_T} + \vec{P}_{\nu_T})^2 \tag{8.129}$$

$$\approx 2P_{e_T}P_{\nu_T}(1 - \cos\theta_{e\nu}), \tag{8.130}$$

where $\theta_{e\nu}$ is the angle between the momenta of the electron and neutrino, that is, $\vec{P}_{e_T}$ and $\vec{P}_{\nu_T}$ in the transverse plane. A simulation procedure is used to generate the M_T distribution corresponding to the different values of M_W. The observed M_T distribution from the CDF II experiment [389], is shown in Figure 8.12, from which a value of M_W is obtained to be:

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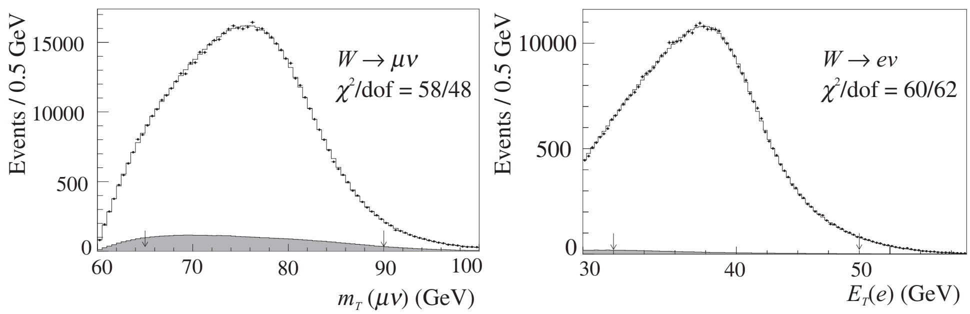

Figure 8.12 Transverse mass distribution for $W \to \mu\nu$(left) and $W \to e\nu$(right) from the CDF II experiment [389].

$$M_W = 80.387 \pm 12(\text{stat}) \pm 15(\text{syst}) \text{ GeV}. \tag{8.131}$$

In the case of $e^+e^- \to q\bar{q}$ at the LEP, the W mass is determined from the reaction

$$e^+e^- \to W^+W^-. \tag{8.132}$$

The event selection is made on the basis of $W^\pm$ decays into leptonic and hadronic modes, that is, $e^+e^- \to ll\nu\nu(11\%)$, $q\bar{q}\nu\nu(43\%)$, or $qql\nu$, having either two coplanar leptons and missing energy or one lepton, two hadronic jets and missing energy. There are many groups which have performed these experiments at LEP [390] and report a value of $M_W = 80.412 \pm 0.042$ GeV.

The average value of the masses of M_W and M_Z determined from these experiments is quoted as:

$$M_W = 80.385 \pm 0.015 \text{ GeV},$$
$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}.$$

Since $M_W = M_Z \cos\theta_W$, it leads to a value of $\sin^2\theta_W = 0.23122 \pm 0.00003$ [117].

ii) **Decay width of $W^\pm$ and Z bosons and neutrino flavors**
The decay width of $W^\pm$ and Z bosons can be calculated at the tree level using the Lagrangians for the charged current and neutral current weak interactions of $W^\pm(Z)$ given in Eqs. (8.58), (8.59), (8.91), and (8.93). The partial width for W^- to the leptonic and hadronic decays are obtained as:

$$\Gamma(W^- \to l^-\bar{\nu}_l) = \frac{G_F M_W^3}{6\pi\sqrt{2}}, \tag{8.133}$$

$$\Gamma(W^- \to \bar{u}_i d_j) = N_c V_{ij}{}^2 \frac{G_F M_W^3}{6\pi\sqrt{2}}. \tag{8.134}$$

We see that all the leptonic modes have the same width Γ, in the limit of the lepton mass $m_l \to 0$. The width for the quark modes depend upon N_c, the number of color and U_{ij}, the matrix element of the quark mixing matrix.

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The decay width for Z decays are given as:

$$\Gamma(Z \to f\bar{f}) = N_f \frac{G_F M_Z^3}{6\pi\sqrt{2}}(|g_V^f|^2 + |g_A^f|^2), \tag{8.135}$$

where g_V^f and g_A^f are the strengths of the vector and axial vector couplings that depend upon f, the type of fermion and therefore, are different for each fermion; $N_f = 1$ for $f = l$ and $N_f = 3$ for $f = q$. Substituting the numerical values, in the appropriate equations we obtain the following:

$$\Gamma(W^- \to e^-\bar{\nu}_e) = 205 \text{ MeV}, \tag{8.136}$$
$$\Gamma(Z \to \nu\bar{\nu}) = 85 \text{ MeV}, \tag{8.137}$$

using $\sin^2\theta_W = 0.2312$.

Assuming the lepton universality as predicted by the W–S model and neglecting the quark masses and the decay of the top quark as it is not allowed due to its high mass, we get the following results:

$$\frac{Br(W^- \to l^-\nu_l)}{(W^- \to \text{all})} = \frac{1}{3+2N_c} = 11.1\% \tag{8.138}$$
$$\text{and } \Gamma_N^{\text{total}} = 1845 \text{ MeV}. \tag{8.139}$$

Experimentally, we find the decay widths as shown in Table 8.8 ([391, 392, 393, 394]), which is in good agreement with theoretical predictions and provide further support for the $e - \mu - \tau$ universality of the weak interaction. Moreover, if Γ_{inv} is an Z decay into invisible modes, that is, $\nu\bar{\nu}$, and N_ν is the number of the neutrino flavor, then:

$$\frac{\Gamma_{\text{inv}}}{\Gamma_l} = \frac{N_\nu\Gamma(\nu\bar{\nu})}{\Gamma_l} = N_\nu\frac{|g_V^\nu|^2 + |g_A^\nu|^2}{|g_V^l|^2 + |g_A^l|^2} = \frac{2N_\nu}{(1-4\sin^2\theta_W)^2 + 1}. \tag{8.140}$$

The experimental value for this quantity is 5.943 ± 0.016 and provides strong support for $N_\nu = 3$.

8.7 Higgs Boson

The most significant aspect of introducing the phenomenon of spontaneous symmetry breaking through the Higgs mechanism, by introducing a doublet of complex scalar fields (or four real

Table 8.8 Experimental determinations of the ratios g_l/g_v [391, 392, 393, 394].

	e	μ	τ	l
$\text{Br}(W^- \to \bar{\nu}_l l^-)$ (%)	10.75 ± 0.13	10.57 ± 0.15	11.25 ± 0.20	10.80 ± 0.09
$\Gamma(Z \to l^+l^-)$ (MeV)	83.91 ± 0.12	83.99 ± 0.18	84.08 ± 0.22	83.984 ± 0.086

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fields) in the Weinberg–Salam–Glashow theory was to generate the masses for the three gauge bosons $W^{\pm}_{\mu}$ and Z. Three of the real scalar fields are absorbed by the massless gauge fields corresponding to $SU(2)_L \times U(1)_I$ generators, to make the gauge fields massive. However, one of the four scalar fields ϕ, acquires mass and is called the Higgs boson, with spin zero and mass:

$$M_H = \sqrt{2}\mu = \sqrt{2\lambda}v. \tag{8.141}$$

μ and λ parameterize the Higgs potential and v is defined by the vacuum expectation value of the scalar field ϕ (see Eq. (8.16)), which is determined by the phenomenology of the charged weak current to be:

$$v = (\sqrt{2}G_F)^{-\frac{1}{2}}. \tag{8.142}$$

$$\lambda = \frac{M_H^2}{2v^2} = \frac{M_H^2 G_F}{\sqrt{2}}. \tag{8.143}$$

Therefore, the mass of the Higgs boson is undetermined unless λ (or μ) is determined or estimated by some other theoretical or experimental physics considerations.

The early experiments at LEP collaborations ALEPH, DELPHI, L3, and OPAL [395], to search for M_H at the center of mass energies between 189 and 209 GeV, indicated that the Higgs boson is of very high mass, most likely to be $M_H > 114.4$ GeV at the 95% confidence level. A very high mass for the Higgs boson would imply very high values of λ, the strength of the quartic self coupling of Higgs which may create problems with the unitarity and renormalizability of the theory. It is, therefore, important to theoretically study the implications of λ being very high and obtain physical constraints on λ so that the theory remains perturbative, making it relevant and applicable to the physical phenomenon. We have discussed earlier that the production of the longitudinal components of the vector boson W_L is responsible for the increase in cross section with energy, leading to violation of unitarity. In a similar way, a systematic analysis of the production of the longitudinal components of W in e^+e^- and $p\bar{p}$ collisions, that is, production of the vector bosons $W_L^+W_L^-, Z_LZ_L$, leads to a condition on the parameter λ, which implies that [396]

$$M_H < \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{\frac{1}{2}} \approx 1 \text{ TeV}. \tag{8.144}$$

The value of λ in terms of the vacuum expectation value of the field v and M_H determined in Eq. (8.143) is obtained in classical field theory. In renormalizable quantum field theory, this value is subject to the quantum field theoretic one loop and higher order loop corrections, which make λ energy dependent, given by [279]

$$\lambda(E) = \frac{\lambda(v)}{1 - \frac{3\lambda(v)}{8\pi^2}\ln\frac{E}{v}}, \tag{8.145}$$

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which implies that λ increases with energy. In fact, the theory becomes non-perturbative at the energy scale of $E > \Lambda$, with $\Lambda \approx v\ \exp(\frac{8\pi^2}{3\lambda(v)})$, which implies that:

$$M_H < v\left[\frac{4\pi^2}{3\ln(\frac{\Lambda}{v})}\right]^{\frac{1}{2}}, \tag{8.146}$$

for $\Lambda = 10^{16}$ GeV, $M_H \lesssim 160$ GeV. Since $\lambda \propto M_H^2$, a higher M_H will decrease the value of Λ, setting the non-perturbative region much earlier in the energy scale than 10^{16} GeV. These theoretical considerations provide a limit on the mass of Higgs boson so that the weak interaction remains weak and perturbative. In view of the high mass of Higgs, $M_H \approx 160$ GeV, by theoretical considerations and the LEP limit of $M_H \geq 114.4$ GeV, very high energy accelerators are required to produce them in laboratories. It is possible to study the Higgs boson and its properties through its decays in which leptons, hadrons, and gauge bosons $W^\pm$ and Z are produced. The interaction Lagrangian, $\mathcal{L}_S$ involving the scalar Higgs boson can be derived using Eq. (8.33),

$$\mathcal{L}_S = \frac{1}{4}\lambda v^4 + \mathcal{L}_H + \mathcal{L}_{HG^2} + \mathcal{L}_{fH}, \tag{8.147}$$

where

$$\mathcal{L}_{fH} = -\frac{f_e}{\sqrt{2}}\bar{e}eH = -\frac{m_e}{v}\bar{e}eH \tag{8.148}$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{1}{2}M_H^2H^2 - \frac{M_H^2}{2v}H^3 - \frac{M_H^2}{8v^2}H^4, \tag{8.149}$$

$$\mathcal{L}_{HG^2} = M_W^2 W_\mu^\dagger W^\mu\left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}, \tag{8.150}$$

using the values of v, λ and μ^2 in terms of $M_H,\ M_W,\ M_Z$ and m_e. All Higgs coupling are, therefore, determined in terms of the masses M_H, M_W, M_Z, m_e and the vacuum expectation value, $v = (\sqrt{2}G_F)^{\frac{-1}{2}}$. The couplings are shown in Figure 8.13. Since all the coupling parameters except M_H are known from experiments, the coupling of H with physical particles through their decay and production can be determined to give its mass.

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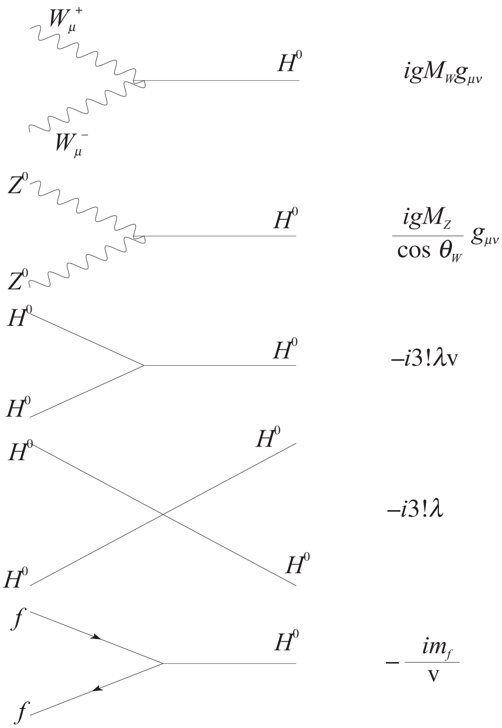

Figure 8.13 The couplings of the Higgs boson to the vector bosons, fermions, and self coupling.

8.7.1 Discovery of Higgs boson

The Higgs boson can be produced at very high energy in the $p\bar{p}$ and e^-e^+ collisions. In e^-e^+ collisions, it can be produced through scattering and annihilation processes like

$$\begin{aligned} e^+e^- &\longrightarrow e^+e^-H, \\ e^+e^- &\longrightarrow ZH, \\ e^+e^- &\longrightarrow \nu\bar{\nu}\,W^+W^- \to \nu\bar{\nu}H. \end{aligned}$$

In the hadronic collisions like pp scattering, the Higgs bosons are produced in the quark– quark interaction or the gluon–gluon interactions through processes like (Figure 8.14):

$$\begin{aligned} gg &\longrightarrow HX, \qquad gg \longrightarrow HWX, \\ qq &\longrightarrow qqH, \qquad qq \longrightarrow HX, \qquad qq \longrightarrow HWX. \end{aligned}$$

The dominant production is through the gluon fusion reaction, that is, $gg \to HX$ in which the production cross section could be as large as $\sigma \approx 10^2\ pb$ at the CM energy of 40 GeV for $M_H \sim 130 - 150$ GeV and is an order of magnitude larger than other processes.

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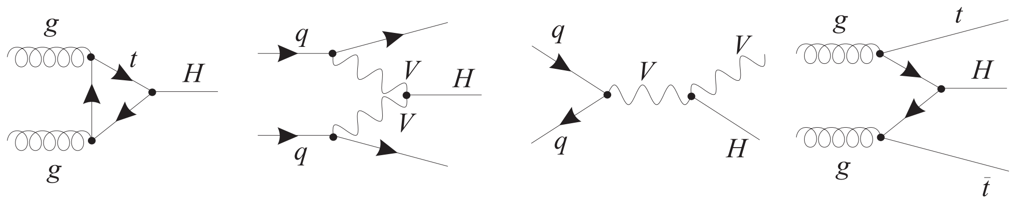

Figure 8.14 Higgs production mechanisms: ggF, VBF, VH, and $t\bar{t}H$.

Once produced, the Higgs boson will decay through processes like (Figure 8.15 and 8.16):

$$
\begin{aligned}
H &\longrightarrow f\bar{f}, \qquad H \longrightarrow f\bar{f}\gamma, \qquad H \longrightarrow gg, \\
H &\longrightarrow VV, \qquad H \longrightarrow W^+W^-, \qquad H \longrightarrow ZZ,
\end{aligned}
$$

with $f =$ leptons, quarks and $V = \gamma, W^\pm, Z$ bosons.

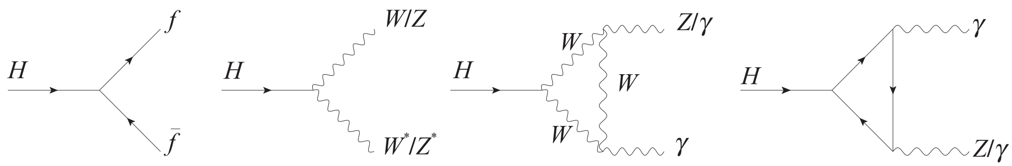

Figure 8.15 Feynman diagrams for dominant Higgs decay processes.

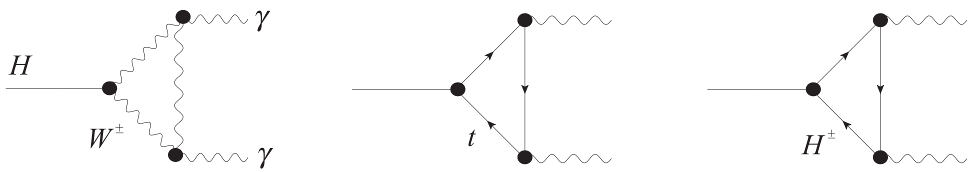

Figure 8.16 One-loop contributions to $H \to \gamma\gamma$. The third diagram shows a possible non-SM contribution from a charged scalar.

The branching ratios of the various decay modes are shown in Table 8.9 where the decay rates are given by [397]

$$
\Gamma(H \to f\bar{f}) = N_f \frac{G_F m_f^2 M_H}{4\sqrt{2}} \left(1 - \frac{4m_f^2}{M_H^2}\right)^{\frac{3}{2}},
$$

with $N_f = 3$ for the quarks and $N_f = 1$ for the leptons,

$$
\begin{aligned}
\Gamma(H \longrightarrow W^+W^-) &= \frac{G_F^2 M_H^3}{8\pi\sqrt{2}} \left(1 - \frac{4M_W^2}{M_H^2} + 12\frac{M_W^4}{M_H^4}\right)^{\frac{3}{2}} \\
\Gamma(H \longrightarrow ZZ) &= \frac{1}{2}\Gamma(H \to W^+W^-)_{M_W = M_Z}.
\end{aligned}
$$

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Table 8.9 Branching ratios of the Higgs boson decays.

Channels	W^+W^-	ZZ	$t\bar{t}$
4 jets	24.6	12.6	
2 jets + 6ν	23.7		
$l^+\nu l^-\bar{\nu}$	5.7		
2 jets + l^-l^+		3.6	
2 jets + $\nu\bar{\nu}$		7.3	
$l^-l^+l^-l^+$		0.3	
$l^-l^+\nu\bar{\nu}$		1.1	
$\nu\bar{\nu}\nu\bar{\nu}$		1.1	
6 jets			9.1
4 jets + $l\nu$			8.1
2 jets + $l^-\bar{\nu}l^+\nu$			2.1

In July 2012, the ATLAS [177] and CMS [176] collaborations at CERN Large Hadron Collider announced the discovery of a "Higgs like particle"(initially named) with mass around 125 GeV in the study of the invariant mass distribution of diphoton events in $p\bar{p}$ collisions at $\sqrt{s} = 7 - 8$ TeV as shown in Figure 8.17 ([398]).

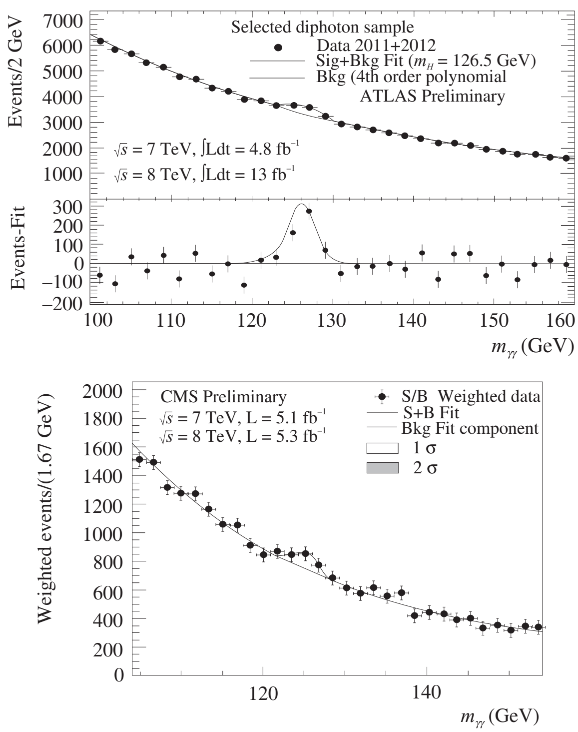

Figure 8.17 The invariant mass distribution of diphoton candidates, with each event weighted by the signal-to-background ratio in each event category, observed by ATLAS [398] at Run 2.

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Later with better statistics, the discovery of Higgs boson was confirmed.

The masses measured by the two experiments are in good agreement, giving the average value [399]:

$$M_H = (125.09 \pm 0.21 \pm 0.11)\ \text{GeV} = (125.09 \pm 0.24)\ \text{GeV}. \tag{8.151}$$