Appendix B

Cabibbo Theory

B.1 Cabibbo Theory, SU(3) Symmetry, and Weak N–Y Transition Form Factors

For the $\Delta S = 0$ processes,

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p'); \tag{B.1}$$

$$\bar{\nu}_l(k) + p(p) \longrightarrow l^+(k') + n(p'); \qquad l = e, \mu, \tau, \tag{B.2}$$

and for the $|\Delta S| = 1$ processes,

$$\bar{\nu}_l(k) + p(p) \longrightarrow l^+(k') + \Lambda(p'), \tag{B.3}$$

$$\bar{\nu}_l(k) + p(p) \longrightarrow l^+(k') + \Sigma^0(p'), \tag{B.4}$$

$$\bar{\nu}_l(k) + n(p) \longrightarrow l^+(k') + \Sigma^-(p'); \qquad l = e, \mu, \tau, \tag{B.5}$$

the matrix elements of the vector (V_μ) and the axial vector (A_μ) currents between a nucleon $N'(= p, n)$ or a hyperon $Y(= \Lambda, \Sigma^0$ and $\Sigma^-)$ and a nucleon $N = n, p$ are written as:

$$\begin{aligned}\langle Y(p')|V_\mu|N(p)\rangle &= \bar{u}(p')\left[\gamma_\mu f_1^{NY}(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M+M'}f_2^{NY}(q^2)\right.\\ &+ \left.\frac{2\,q_\mu}{M+M'}f_3^{NY}(q^2)\right]u(p),\end{aligned} \tag{B.6}$$

and

$$\begin{aligned}\langle Y(p')|A_\mu|N(p)\rangle &= \bar{u}(p')\left[\gamma_\mu\gamma_5 g_1^{NY}(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M+M'}\gamma_5 g_2^{NY}(q^2)\right.\\ &+ \left.\frac{2\,q_\mu}{M+M'}\gamma_5 g_3^{NY}(q^2)\right]u(p),\end{aligned} \tag{B.7}$$

where M and M' are the masses of the nucleon and hyperon, respectively. $f_1^{NY}(q^2)$, $f_2^{NY}(q^2)$, and $f_3^{NY}(q^2)$ are the vector, weak magnetic and induced scalar $N - Y$ transition form factors

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and $g_1^{NY}(q^2)$, $g_2^{NY}(q^2)$ and $g_3^{NY}(q^2)$ are the axial vector, induced tensor (or weak electric), and induced pseudoscalar form factors, respectively.

In the Cabibbo theory, the weak vector (V_μ) and the axial vector (A_μ) currents corresponding to the $\Delta S = 0$ and $\Delta S = 1$ hadronic currents whose matrix elements are defined between the states $|N\rangle$ and $|N'\rangle$ or $|Y\rangle$ are assumed to belong to the octet representation of SU(3).

Accordingly, they are defined as:

$$\begin{aligned} V_i^\mu &= \bar{q} F_i \gamma^\mu q, \\ A_i^\mu &= \bar{q} F_i \gamma^\mu \gamma^5 q, \end{aligned} \tag{B.8}$$

where $F_i = \frac{\lambda_i}{2}(i = 1-8)$ are the generators of flavor SU(3) and λ_is are the well-known Gell–Mann matrices written as

$$\begin{aligned} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{aligned} \tag{B.9}$$

The generators obey the following algebra of SU(3) generators

$$\begin{aligned} [F_i, F_j] &= i f_{ijk} F_k, \\ \{F_i, F_j\} &= \frac{1}{3}\delta_{ij} + d_{ijk} F_k, \quad i,j,k = 1-8. \end{aligned} \tag{B.10}$$

$F_{i,j,k} = \frac{1}{2}\lambda_{i,j,k}$, f_{ijk} and d_{ijk} are the structure constants, and are antisymmetric and symmetric, respectively, under the interchange of any two indices. These are obtained using the λ_i given in Eq. (B.9) and have been tabulated in Table B.1.

From the property of the SU(3) group, it follows that there are three corresponding SU(2) subgroups of SU(3) which must be invariant under the interchange of quark pairs ud, ds, and us respectively, if the group is invariant under the interchange of u, d, and s quarks. Each of these SU(2) subgroups has raising and lowering operators. One of them is $SU(2)_I$, generated by the generators $(\lambda_1, \lambda_2, \lambda_3)$ to be identified with the isospin operators (I_1, I_2, I_3) in the isospin space. For example, $I_\pm$ of isospin space is given by

$$I_\pm = I_1 \pm i I_2 = F_1 \pm i F_2 = \frac{1}{2}(\lambda_1 \pm i\lambda_2). \tag{B.11}$$

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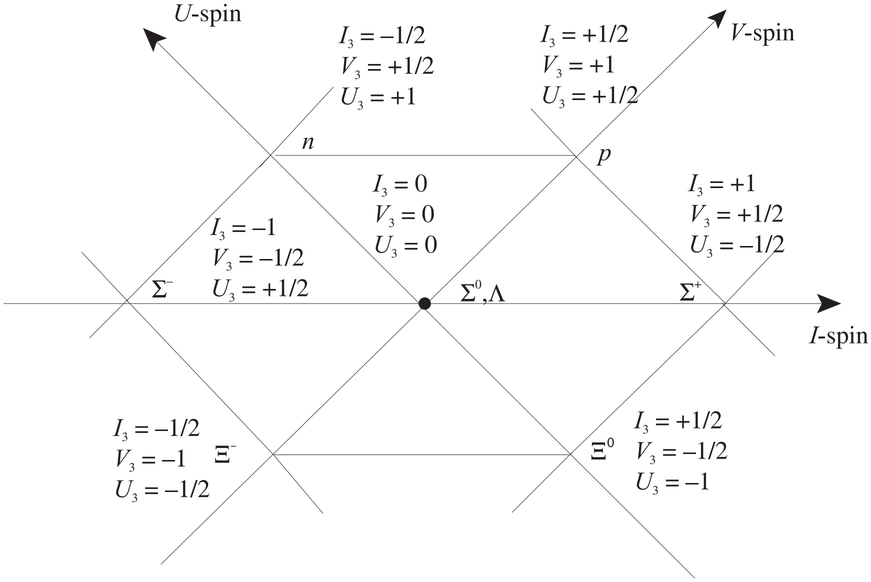

Figure B.1 Baryon octet

The other two are defined as $SU(2)_U$ and $SU(2)_V$ generated by the generators $(\lambda_6, \lambda_7, \frac{1}{2}(\sqrt{3}\lambda_8 - \lambda_3))$ and $(\lambda_4, \lambda_5, \frac{1}{2}(\sqrt{3}\lambda_8 + \lambda_3))$, respectively, in the U-spin and V-spin space with $(d \ \ s)$ and $(u \ \ s)$ forming the basic doublet representation of $SU(2)_U$ and $SU(2)_V$.

The $(I \ \ I_3)$, $(U \ \ U_3)$, and $(V \ \ V_3)$ quantum numbers of $(u \ \ d \ \ s)$ quarks are assigned as:

$$
\begin{aligned}
u \text{ quark:} \quad (I, I_3) &= \left(\frac{1}{2}, +\frac{1}{2}\right), \quad (V, V_3) = \left(\frac{1}{2}, +\frac{1}{2}\right),\\
d \text{ quark:} \quad (I, I_3) &= \left(\frac{1}{2}, -\frac{1}{2}\right), \quad (U, U_3) = \left(\frac{1}{2}, +\frac{1}{2}\right),\\
s \text{ quark:} \quad (U, U_3) &= \left(\frac{1}{2}, -\frac{1}{2}\right), \quad (V, V_3) = \left(\frac{1}{2}, -\frac{1}{2}\right).
\end{aligned}
$$

From the Gell–Mann matrices λ_i, one may obtain the raising and lowering operators with U-spin and V-spin in analogy with I-spin as:

$$
\begin{aligned}
U_\pm &= U_1 \pm iU_2 = F_6 \pm iF_7,\\
V_\pm &= V_1 \pm iV_2 = F_4 \pm iF_5.
\end{aligned}
$$

Accordingly, the I-spin, U-spin, and V-spin for the baryon octet are represented diagrammatically in Figure B.1; for the pseudoscalar $(J^P = 0^-)$ and vector $(J^P = 1^-)$ meson nonet are shown in Figure B.2.

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Table B.1 Values of the structure constants f_{ijk} and d_{ijk}.

ijk	f_{ijk}	ijk	d_{ijk}
123	1	118	$\frac{1}{\sqrt{3}}$
147	$\frac{1}{2}$	146	$\frac{1}{2}$
156	$-\frac{1}{2}$	157	$\frac{1}{2}$
246	$\frac{1}{2}$	228	$\frac{1}{\sqrt{3}}$
257	$\frac{1}{2}$	247	$-\frac{1}{2}$
345	$\frac{1}{2}$	256	$\frac{1}{2}$
367	$-\frac{1}{2}$	338	$\frac{1}{\sqrt{3}}$
458	$\frac{\sqrt{3}}{2}$	344	$\frac{1}{2}$
678	$\frac{\sqrt{3}}{2}$	355	$\frac{1}{2}$
		366	$-\frac{1}{2}$
		377	$-\frac{1}{2}$
		448	$-\frac{1}{2\sqrt{3}}$
		558	$-\frac{1}{2\sqrt{3}}$
		668	$-\frac{1}{2\sqrt{3}}$
		778	$-\frac{1}{2\sqrt{3}}$
		888	$-\frac{1}{\sqrt{3}}$

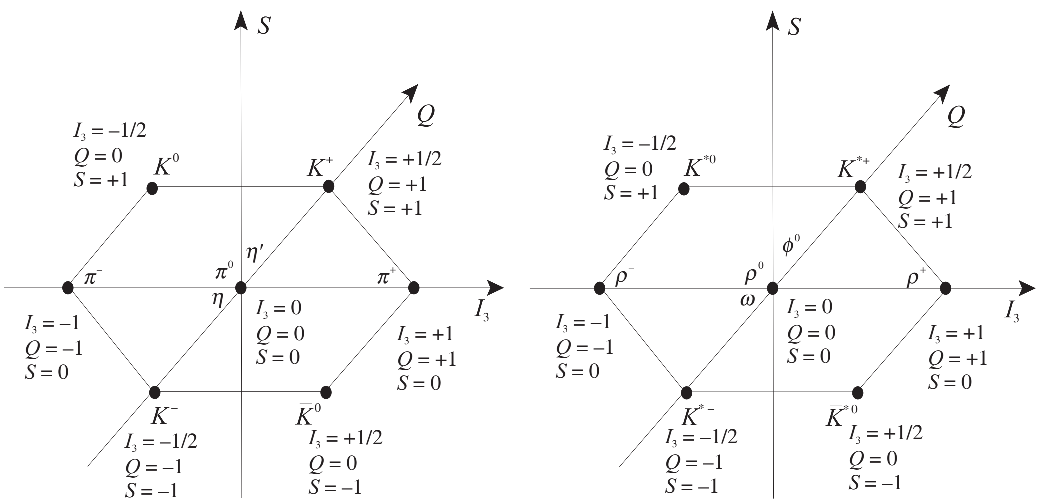

Figure B.2 Pseudoscalar ($J^P = 0^-$) and vector ($J^P = 1^-$) meson nonet.

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In neutron β-decay, a d quark is transformed into a u quark, the vector and the axial vector currents for this transition can be written as

$$\begin{aligned}
\bar{\psi}_u \gamma_\mu \psi_d &= \bar{q}\gamma_\mu \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} q = \bar{q}\gamma_\mu \left(\frac{\lambda_1 + i\lambda_2}{2}\right) q = V_\mu^{1+i2}, \\
\bar{\psi}_u \gamma_\mu \gamma_5 \psi_d &= \bar{q}\gamma_\mu \gamma_5 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} q = \bar{q}\gamma_\mu \gamma_5 \left(\frac{\lambda_1 + i\lambda_2}{2}\right) q = A_\mu^{1+i2}.
\end{aligned}$$

Similarly, the vector and axial vector currents for the $u \to d$ transformation can be written as:

$$\begin{aligned}
\bar{\psi}_d \gamma_\mu \psi_u &= V_\mu^{1-i2}, \\
\bar{\psi}_d \gamma_\mu \gamma_5 \psi_u &= A_\mu^{1-i2},
\end{aligned}$$

and the $s \to u$ and $u \to s$ transformations are written as

$$\begin{aligned}
\bar{\psi}_u \gamma_\mu \psi_s &= \bar{q}\gamma_\mu \left(\frac{\lambda_4 + i\lambda_5}{2}\right) q = V_\mu^{4+i5}, \\
\bar{\psi}_u \gamma_\mu \gamma_5 \psi_s &= \bar{q}\gamma_\mu \gamma_5 \left(\frac{\lambda_4 + i\lambda_5}{2}\right) q = A_\mu^{4+i5}, \\
\bar{\psi}_s \gamma_\mu \psi_u &= \bar{q}\gamma_\mu \left(\frac{\lambda_4 - i\lambda_5}{2}\right) q = V_\mu^{4-i5}, \qquad \text{(B.12)} \\
\bar{\psi}_s \gamma_\mu \gamma_5 \psi_u &= \bar{q}\gamma_\mu \gamma_5 \left(\frac{\lambda_4 - i\lambda_5}{2}\right) q = A_\mu^{4-i5}. \qquad \text{(B.13)}
\end{aligned}$$

The electromagnetic current which is a vector current is written using the charge operator $e = I_3 + \frac{Y}{2} = \lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8$ as:

$$J_\mu^{\text{em}} = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8. \qquad \text{(B.14)}$$

Therefore, the charge changing weak vector and axial vector currents are written, in Cabibbo theory, as:

$$\begin{aligned}
V_\mu^\pm &= \left[V_\mu^1 \pm iV_\mu^2\right] \cos\theta_c + \left[V_\mu^4 \pm iV_\mu^5\right] \sin\theta_c, \\
A_\mu^\pm &= \left[A_\mu^1 \pm iA_\mu^2\right] \cos\theta_c + \left[A_\mu^4 \pm iA_\mu^5\right] \sin\theta_c. \qquad \text{(B.15)}
\end{aligned}$$

In the Cabibbo theory, the isovector part of the electromagnetic current J^μ_{em}, that is, V_μ^3 along with the weak vector currents $V^\mu_\pm$ are assumed to transform as a triplet under SU(2) of isospin. Similarly, the axial vector currents are also assumed to transform as an octet under SU(3). The weak transition form factors $f_i(q^2)$ and $g_i(q^2)$; $i = 1 - 3$ are determined using the Cabibbo theory of V–A interaction extended to the strange sector.

The form factors defined in the matrix element of an octet of the vector (axial vector) currents taken between the octets of the initial and the final baryon states as defined in Eqs. (B.6)

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and (B.7) can, therefore, be expressed in terms of the two couplings of the vector (axial vector) currents corresponding to the symmetric and antisymmetric octets according to the decomposition:

$$8 \times 8 = 1 + 8^S + 8^A + 10 + \overline{10} + 27, \tag{B.16}$$

and the corresponding SU(3) Clebsch–Gordan coefficients. In general, the expression for the matrix element of the transition between the two states of baryons (say B_i and B_k), through the SU(3) octet (V_j or A_j) of currents can be written as:

$$< B_i|V_j|B_k > \;=\; i f_{ijk} F^V + d_{ijk} D^V, \tag{B.17}$$
$$< B_i|A_j|B_k > \;=\; i f_{ijk} F^A + d_{ijk} D^A. \tag{B.18}$$

F^V and D^V are determined from the experimental data on the electromagnetic form factors, and F^A and D^A are determined from the experimental data on semileptonic decays of neutron and hyperons. The physical baryon octet states are written in terms of their octet state B_i as:

$$\begin{aligned}
p &= \frac{1}{\sqrt{2}}(B_4 + iB_5), & n &= \frac{1}{\sqrt{2}}(B_6 + iB_7),\\
\Sigma^{\pm} &= \frac{1}{\sqrt{2}}(B_1 \pm iB_2), & \Xi^- &= \frac{1}{\sqrt{2}}(B_4 - iB_5),\\
\Xi^0 &= \frac{1}{\sqrt{2}}(B_6 - iB_7), & \Sigma^0 &= B_3, \qquad \Lambda^0 = B_8.
\end{aligned} \tag{B.19}$$

The matrix element for the electromagnetic transition in Eq. (A.14) between two octet states B_i and B_k is defined as:

$$\langle B_i|V_3 + \frac{1}{\sqrt{3}}V_8|B_k\rangle = i\left[f_{i3k} + \frac{1}{\sqrt{3}}f_{i8k}\right]F^V + \left[d_{i3k} + \frac{1}{\sqrt{3}}d_{i8k}\right]D^V. \tag{B.20}$$

Applying Eq. (B.20) to the matrix element of electromagnetic current between proton states, we get

$$\begin{aligned}
\langle p|J^{\text{em}}|p\rangle &= \frac{1}{2}\langle B_4 + iB_5|J^{\text{em}}|B_4 + iB_5\rangle\\
&= \frac{1}{2}\left\{\langle B_4|J^{\text{em}}|B_4\rangle + i\langle B_4|J^{\text{em}}|B_5\rangle - i\langle B_5|J^{\text{em}}|B_4\rangle + \langle B_5|J^{\text{em}}|B_5\rangle\right\}\\
&= \frac{1}{2}\left[\left\{i(f_{434} + \frac{1}{\sqrt{3}}f_{484})F^V + (d_{434} + \frac{1}{\sqrt{3}}d_{484})D^V\right\}\right.\\
&+ i\left\{i(f_{435} + \frac{1}{\sqrt{3}}f_{485})F^V + (d_{435} + \frac{1}{\sqrt{3}}d_{485})D^V\right\}\\
&- i\left\{i(f_{534} + \frac{1}{\sqrt{3}}f_{584})F + (d_{534} + \frac{1}{\sqrt{3}}d_{584})D\right\}\\
&+ \left.\left\{i(f_{535} + \frac{1}{\sqrt{3}}f_{585})F^V + (d_{535} + \frac{1}{\sqrt{3}}d_{585})D^V\right\}\right]\\
&= \frac{1}{2}\left[i(0+0)F^V + \left(\frac{1}{2} + \frac{1}{\sqrt{3}}.\frac{-1}{2\sqrt{3}}\right)D^V + i\left\{i\left(-\frac{1}{2} - \frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}\right)F^V + (0+0)D^V\right\}\right.
\end{aligned}$$

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$$
\begin{aligned}
&- \; i\left\{i\left(\frac{1}{2}+\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}\right)F^V+(0+0)D^V\right\}+\left\{i(0+0)F^V+\left(\frac{1}{2}+\frac{1}{\sqrt{3}}.\frac{-1}{2\sqrt{3}}\right)D^V\right\}\Bigg] \\
&= \frac{1}{2}\left[\frac{1}{3}D^V+F^V+\frac{1}{3}D^V+F^V\right] \\
&= \frac{D^V}{3}+F^V.
\end{aligned}
\tag{B.21}
$$

Similarly, for the $n \to n$ transition, the electromagnetic current is calculated as

$$
\begin{aligned}
\langle n|J_{\text{em}}|n\rangle &= \frac{1}{2}\langle B_6+iB_7|J_{\text{em}}|B_6+iB_7\rangle \\
&= \frac{1}{2}\left\{\langle B_6|J_{\text{em}}|B_6\rangle+i\langle B_6|J_{\text{em}}|B_7\rangle-i\langle B_7|J_{\text{em}}|B_6\rangle+\langle B_7|J_{\text{em}}|B_7\rangle\right\} \\
&= \frac{1}{2}\Bigg[\left\{i(f_{636}+\frac{1}{\sqrt{3}}f_{686})F^V+(d_{636}+\frac{1}{\sqrt{3}}d_{686})D^V\right\} \\
&+ i\left\{i(f_{637}+\frac{1}{\sqrt{3}}f_{687})F^V+(d_{637}+\frac{1}{\sqrt{3}}d_{687})D^V\right\} \\
&- i\times\left\{i(f_{736}+\frac{1}{\sqrt{3}}f_{786})F^V+(d_{736}+\frac{1}{\sqrt{3}}d_{786})D^V\right\} \\
&+ \left\{i(f_{737}+\frac{1}{\sqrt{3}}f_{787})F^V+(d_{737}+\frac{1}{\sqrt{3}}d_{787})D^V\right\}\Bigg] \\
&= \frac{1}{2}\Bigg[i(0+0)F^V+\left(-\frac{1}{2}+\frac{1}{\sqrt{3}}.\frac{-1}{2\sqrt{3}}\right)D^V+i\left\{i\left(\frac{1}{2}-\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}\right)F^V+(0+0)D^V\right\} \\
&- i\left\{i\left(-\frac{1}{2}+\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}\right)F^V+(0+0)D^V\right\}+\left\{i(0+0)F^V+\left(-\frac{1}{2}+\frac{1}{\sqrt{3}}.\frac{-1}{2\sqrt{3}}\right)D^V\right\}\Bigg] \\
&= \frac{1}{2}\left[-\frac{2}{3}D^V-\frac{2}{3}D^V\right]=-\frac{2D^V}{3}.
\end{aligned}
\tag{B.22}
$$

The matrix elements of the electromagnetic currents for the protons and neutrons are described in terms of the electromagnetic form factors as:

$$
\begin{aligned}
\langle p(p')|J_\mu^{\text{em}}|p(p)\rangle &= \bar{u}(p')\left[\gamma_\mu F_1^p(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{2M}F_2^p(q^2)\right]u(p), \\
\langle n(p')|J_\mu^{\text{em}}|n(p)\rangle &= \bar{u}(p')\left[\gamma_\mu F_1^n(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{2M}F_2^n(q^2)\right]u(p).
\end{aligned}
$$

Therefore, each of the form factors defined earlier, that is, $F_i^p(q^2)$ and $F_i^n(q^2)$; $(i=1,2)$ can be written in terms of their SU(3) coupling constants as

$$ f_i(q^2) = aF_i^V(q^2)+bD_i^V(q^2) \qquad i=1,2,3 \tag{B.23} $$
$$ g_i(q^2) = aF_i^A(q^2)+bD_i^A(q^2) \qquad i=1,2,3. \tag{B.24} $$

Comparing Eqs. (B.21) and (B.22) with Eq. (B.23), we obtain the Clebsch–Gordan coefficients for the electromagnetic $p \to p$ and $n \to n$ transitions as

$$ p \to p, \qquad a = 1 \qquad b=1/3, \tag{B.25} $$
$$ n \to n, \qquad a = 0 \qquad b=-2/3. \tag{B.26} $$

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Using Eqs. (B.25) and (B.26) in Eq. (B.23), we obtain

$$F_i^{p\to p}(q^2) = F_i^p(q^2) = F_i^V(q^2) + \frac{1}{3}D_i^V(q^2), \tag{B.27}$$

$$F_i^{n\to n}(q^2) = F_i^n(q^2) = -\frac{2}{3}D_i^V(q^2). \tag{B.28}$$

Solving Eqs. (B.27) and (B.28), we get

$$F_i^V(q^2) = F_i^p(q^2) - F_i^n(q^2), \tag{B.29}$$

$$D_i^V(q^2) = -\frac{3}{2}F_i^n(q^2). \tag{B.30}$$

Once $F_i^V(q^2)$ and $D_i^V(q^2)$ are determined in terms of the electromagnetic form factors of the nucleon, they can be used to determine all the form factors in the case of the matrix element of the weak vector current for the various $\Delta S = 0, 1$ transitions.

The Clebsch–Gordan coefficients (a and b) for the various transitions can be obtained as follows. For example, let us obtain the values of a and b for $p(uud) \to \Lambda(uds)$ transition. Using Eq. (B.12), the current for the $u \to s$ transition can be written as

$$\begin{aligned}
\langle \Lambda | j_4 - i j_5 | p \rangle &= \langle \Lambda | j_4 | p \rangle - i \langle \Lambda | j_5 | p \rangle \\
&= \frac{1}{\sqrt{2}}\left[\langle B_8 | j_4 | B_4 \rangle + i \langle B_8 | j_4 | B_5 \rangle\right] - \frac{i}{\sqrt{2}}\left[\langle B_8 | j_5 | B_4 \rangle + i \langle B_8 | j_5 | B_5 \rangle\right] \\
&= \frac{1}{\sqrt{2}}\left[i f_{844} F + d_{844} D + i(i f_{845} F + d_{845} D)\right] \\
&- \frac{i}{\sqrt{2}}\left[i f_{854} F + d_{854} D + i(i f_{855} F + d_{855} D)\right] \\
&= -\sqrt{\frac{3}{2}}F - \frac{1}{\sqrt{6}}D.
\end{aligned} \tag{B.31}$$

Similarly, the coefficients for the various transitions can be obtained and are presented in Table B.2.

B.2 Octet Representation of Mesons

In the SU(3) symmetry, the quark states can be represented as

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{B.32}$$

and the corresponding antiquark states can be represented as

$$\bar{q} = \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix}. \tag{B.33}$$

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Table B.2 Values of the coefficients a and b given in Eqs. (B.23) and (B.24).

Interaction	Transition	a	b
Electromagnetic interaction	$p \to p$	1	$\frac{1}{3}$
	$n \to n$	0	$-\frac{2}{3}$
Weak vector and axial vector	$n \to p$	1	1
	$\Lambda \to p$	$-\sqrt{\frac{3}{2}}$	$-\frac{1}{\sqrt{6}}$
	$\Sigma^0 \to p$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
	$\Sigma^- \to n$	-1	1
	$\Sigma^\pm \to \Lambda$	0	$\sqrt{\frac{2}{3}}$
	$\Sigma^- \to \Sigma^0$	$\sqrt{2}$	0
	$\Xi^- \to \Lambda$	$\sqrt{\frac{3}{2}}$	$-\frac{1}{\sqrt{6}}$
	$\Xi^- \to \Sigma^0$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
	$\Xi^0 \to \Sigma^+$	1	1
	$\Xi^- \to \Xi^0$	1	-1

Mesons are the bound state of $q\bar{q}$, which can be represented as

$$q_i\bar{q}_j = \underbrace{q_i\bar{q}_j - \frac{1}{3}\delta_{ij}\sum_k q_k\bar{q}_k}_{\text{octet of the pseudoscalar mesons}} + \underbrace{\frac{1}{3}\delta_{ij}\sum_k q_k\bar{q}_k}_{\text{singlet of the pseudoscalar mesons}}, \tag{B.34}$$

$i, j, k = 1-3$. In the notation of group theory, Eq. (B.34) can be written as

$$3 \otimes \bar{3} = 8 \oplus 1. \tag{B.35}$$

The operator for the octet of the pseudoscalar mesons can be defined as

$$P_{ji}|0>=|P_{ji}>= q_i\bar{q}_j - \frac{1}{3}\delta_{ij}\sum_k q_k\bar{q}_k. \tag{B.36}$$

Using Eq. (B.36), the elements of P_{ji} can be obtained as follows

$$\begin{aligned} P_{21} &= q_1\bar{q}_2 - \frac{1}{3}\delta_{12}\sum_k q_k\bar{q}_k \\ &= u\bar{d} = \pi^+, \end{aligned} \tag{B.37}$$

$$\begin{aligned} P_{31} &= q_1\bar{q}_3 - \frac{1}{3}\delta_{13}\sum_k q_k\bar{q}_k \\ &= u\bar{s} = K^+, \end{aligned} \tag{B.38}$$

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$$\begin{aligned} P_{33} &= q_3\bar{q}_3 - \frac{1}{3}\delta_{33}\sum_k q_k\bar{q}_k \\ &= s\bar{s} - \frac{1}{3}(u\bar{u} + d\bar{d} + s\bar{s}) \\ &= -\frac{1}{3}(u\bar{u} + d\bar{d} - 2s\bar{s}) \;=\; -\frac{\sqrt{6}}{3}\eta. \end{aligned} \tag{B.39}$$

In the matrix form, P_{ji} can be written as

$$P = \begin{pmatrix} \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{2}}\pi^0 & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{6}}\eta - \frac{1}{\sqrt{2}}\pi^0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{B.40}$$

This matrix can be rewritten in terms of the generators of the SU(3) symmetry as

$$\begin{aligned} P_{ji} &= \frac{1}{\sqrt{2}}\sum_{A=1}^{8}(\lambda_A)_{ji}\,\pi_A, \\ P &= \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_3 + \frac{\pi_8}{\sqrt{3}} & \pi_1 + i\pi_2 & \pi_4 + i\pi_5 \\ \pi_1 - i\pi_2 & -\pi_3 + \frac{\pi_8}{\sqrt{3}} & \pi_6 + i\pi_7 \\ \pi_4 - i\pi_5 & \pi_6 - i\pi_7 & -\frac{2}{\sqrt{3}}\pi_8 \end{pmatrix}. \end{aligned} \tag{B.41}$$

B.3 Octet Representation of Baryons

Baryons are the bound state of three quarks. In the notation of group theory, the bound state of baryons can be represented as follows:

First, we write for the two bound quarks

$$3 \otimes 3 = 6 \oplus \bar{3}, \tag{B.42}$$

where 6 represents the symmetric part with 6 components and $\bar{3}$ represents the antisymmetric part with 3 components. Adding the third quark, we have

$$\begin{aligned} 3 \otimes 3 \otimes 3 &= (6 \otimes 3) \oplus (\bar{3} \otimes 3) \\ &= \underbrace{10 \oplus 8'}_{\text{symmetric}} \oplus \underbrace{8 \oplus 1}_{\text{antisymmetric}}. \end{aligned} \tag{B.43}$$

The wavefunction of the lowest lying baryons ($J^P = \frac{1}{2}^+$) is antisymmetric in nature. Therefore, to obtain the antisymmetric part in Eq. (B.43) for the octet representation of baryons, we proceed in the following way. The bound state for two quarks can be written as

$$q_i q_j = \frac{1}{2}\underbrace{(q_i q_j + q_j q_i)}_{\text{symmetric part}} + \frac{1}{2}\underbrace{(q_i q_j - q_j q_i)}_{\text{antisymmetric part}}, \tag{B.44}$$

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or

$$q_i q_j = \frac{1}{\sqrt{2}} S_{ij} + \frac{1}{\sqrt{2}} A_{ij}, \tag{B.45}$$

where the symmetric and the antisymmetric tensors with 6 and 3 components respectively, are defined as

$$S_{ij} = \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i), \tag{B.46}$$

$$A_{ij} = \frac{1}{\sqrt{2}}(q_i q_j - q_j q_i). \tag{B.47}$$

We define T_k which can be written in terms of A_{ij} as

$$T_k = \epsilon_{klm} A^{lm}, \tag{B.48}$$

or

$$A^{ij} = \frac{1}{2}\epsilon_{ijk} T_k. \tag{B.49}$$

Now, considering the antisymmetric tensor, we have

$$T^k q_j = \underbrace{(T^k q_j - \frac{1}{3}\delta^k_j \sum_i T^i q_i)}_{\text{octet of the baryons}} + \underbrace{\frac{1}{3}\delta^k_j \sum_i T^i q_i}_{\text{singlet state}}. \tag{B.50}$$

The operator for the octet of baryons can be written as

$$\bar{B}_{jk} = T^k q_j - \frac{1}{3}\delta^k_j \sum_i T^i q_i. \tag{B.51}$$

Using Eqs. (B.49) and (B.47) in Eq. (B.51), we get

$$\bar{B}_{jk}|0>=|B_{jk}>= \frac{1}{2\sqrt{2}}\left[\epsilon_{klm}(q_l q_m - q_m q_l)q_j - \frac{1}{3}\delta^k_j \epsilon_{ilm}((q_l q_m - q_m q_l)q_i\right]|0> . \tag{B.52}$$

The elements of B_{jk} can be obtained, using Eq. (B.52), as:

$$\begin{aligned} B_{13} &= \frac{1}{2\sqrt{2}}\left[\epsilon_{3lm}(q_l q_m - q_m q_l)q_1 - \frac{1}{3}\delta^3_1 \epsilon_{ilm}((q_l q_m - q_m q_l)q_i\right], \\ &= \frac{1}{2\sqrt{2}}\left[\epsilon_{312}(q_1 q_2 - q_2 q_1)q_1 + \epsilon_{321}(q_2 q_1 - q_1 q_2)q_1\right], \\ &= \frac{1}{\sqrt{2}}\left[q_1 q_2 q_1 - q_2 q_1 q_1\right], \end{aligned}$$

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$$= \frac{1}{\sqrt{2}}[udu - duu] = p. \tag{B.53}$$

Similarly, the element B_{32} is obtained as

$$\begin{aligned} B_{32} &= \frac{1}{2\sqrt{2}}\left[\epsilon_{2lm}(q_l q_m - q_m q_l)q_3 - \frac{1}{3}\delta^2_3 \epsilon_{ilm}((q_l q_m - q_m q_l)q_i\right], \\ &= \frac{1}{2\sqrt{2}}\left[\epsilon_{213}(q_1 q_3 - q_3 q_1)q_3 + \epsilon_{231}(q_3 q_1 - q_1 q_3)q_3\right], \\ &= \frac{1}{\sqrt{2}}\left[q_3 q_1 q_3 - q_1 q_3 q_3\right], \\ &= \frac{1}{\sqrt{2}}[sus - uss] = \Xi^0. \end{aligned} \tag{B.54}$$

The other elements of B_{jk} can be determined in a similar manner and can be written in matrix form as:

$$B = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \tag{B.55}$$

In terms of SU(3) generators, the matrix can be written, in a similar manner as written for the mesons in Eq. (B.41) as

$$B = \frac{1}{\sqrt{2}}\begin{pmatrix} B_3 + \frac{B_8}{\sqrt{3}} & B_1 + iB_2 & B_4 + iB_5 \\ B_1 - iB_2 & -B_3 + \frac{B_8}{\sqrt{3}} & B_6 + iB_7 \\ B_4 - iB_5 & B_6 - iB_7 & -\frac{2}{\sqrt{3}}B_8 \end{pmatrix}. \tag{B.56}$$

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