Chapter 6

Phenomenological Theory II: Weak Decays of Hadrons

6.1 Introduction

Hadrons are strongly interacting particles which also participate in weak and electromagnetic interactions. They are not elementary particles and are composed of quarks, which come in six flavors; and each flavor comes in three colors (R, B, and G) as shown in Table 6.1. Hadrons are specified by their quantum numbers like spin, parity, isospin, strangeness, baryon number, etc., and are classified as mesons and baryons on the basis of their quark content. Mesons are the bound states of a quark and an antiquark pair ($q\bar{q}$), while baryons are the bound states of three quarks (qqq). They are bound in such a way that the physical states of mesons and baryons

Table 6.1 Quarks are spin $\frac{1}{2}$ fermions with baryon number $\frac{1}{3}$; they have been assigned positive parity. $Q(|e|)$ is the quark's charge in the units of electronic charge, I is the isospin and its 3rd component is I_3. S, C, B, and T stand for the strangeness, charm, bottom, and top quantum numbers, respectively [117].

Quark flavors	$Q(\|e\|)$	I	I_3	S	C	B	T
u	$+2/3$	$1/2$	$+1/2$	0	0	0	0
d	$-1/3$	$1/2$	$-1/2$	0	0	0	0
s	$-1/3$	0	0	-1	0	0	0
c	$+2/3$	0	0	0	$+1$	0	0
b	$-1/3$	0	0	0	0	-1	0
t	$+2/3$	0	0	0	0	0	$+1$

are color singlets. The strong forces which bind the quarks together, in the case of baryons, or quarks and antiquarks together, in the case of mesons, are provided by the exchange of massless vector fields between them called the gluons. The dynamics of the strong forces, that is, the binding of the quarks and antiquarks and their interactions is described by the theory of strong interactions known as quantum chromodynamics (QCD), in a way similar to QED

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

which describes the interactions among charged particles. In Tables 6.2 and 6.3, some of the low lying mesons and baryons, which are considered in this chapter while discussing their weak interactions, are listed along with their quark contents and other quantum numbers.

Table 6.2 Mesons ($J^P = 0^-$) and their properties: quark content, J represents angular momentum, P the parity, I the isospin, I_3 the 3rd component of I, S the strangeness, M the mass and τ the lifetime of a given meson [117].

Particles (quark content)	I	I_3	S	M (MeV)	τ(s)
$\pi^+(u\bar{d})$	1	$+1$	0	139.57018	26×10^{-9}
$\pi^0(u\bar{u}$ or $d\bar{d})$	1	0	0	134.9766	8.4×10^{-17}
$\pi^-(d\bar{u})$	1	-1	0	139.57018	26×10^{-9}
$K^+(u\bar{s})$	1/2	$+1/2$	$+1$	493.677 ± 0.013	$(12.380 \pm 0.021) \times 10^{-9}$
$K^-(s\bar{u})$	1/2	$-1/2$	-1	493.677 ± 0.013	$(12.380 \pm 0.021) \times 10^{-9}$
$K^0(d\bar{s})$	1/2	$+1/2$	$+1$	497.614±0.024	-
$\bar{K}^0(\bar{d}s)$	1/2	$-1/2$	-1	497.614±0.024	-
$K_s(\frac{d\bar{s}-s\bar{d}}{\sqrt{2}})$	1/2	-	-	-	$(89.54 \pm 0.04) \times 10^{-12}$
$K_l(\frac{d\bar{s}+s\bar{d}}{\sqrt{2}})$	1/2	-	-	-	$(51.16 \pm 0.21) \times 10^{-9}$

Table 6.3 Baryons ($J^P = \frac{1}{2}^+$) and their properties: quark content, isospin(I) and its 3rd component (I_3), total angular momentum (J) and parity (P), strangeness (S), mass (M), and lifetime(τ) [117].

Particles (quark content)	I	I_3	S	M (MeV)	τ(s)
$p(uud)$	$\frac{1}{2}$	$+\frac{1}{2}$	0	938.28	stable
$n(udd)$	$\frac{1}{2}$	$-\frac{1}{2}$	0	939.57	8.815×10^2
$\Lambda(uds)$	0	0	-1	1115.683	2.6×10^{-3}
$\Sigma^+(uus)$	1	$+1$	-1	1189.37	$(8.018 \pm 0.026) \times 10^{-11}$
$\Sigma^0(uds)$	1	0	-1	1192.642(24)	$7.4 \pm 0.7 \times 10^{-20}$
$\Sigma^-(dds)$	1	-1	-1	1197.449(30)	$1.479 \pm 0.011 \times 10^{-10}$
$\Xi^0(uss)$	1/2	$+1/2$	-2	1314.83(20)	$(2.90 \pm 0.09) \times 10^{-10}$
$\Xi^-(dss)$	1/2	$-1/2$	-2	1321.31(13)	$1.639 \pm 0.015 \times 10^{-10}$

The theory of the weak interaction of hadrons is not as simple as the weak interaction of leptons, which has been described in the previous chapter. We have seen that the interaction Hamiltonian of the weak interaction, in the case of leptons, is described by

$$H_{\text{int}} = \frac{G}{\sqrt{2}} l_\mu l^{\mu\dagger} + \text{h.c.}, \tag{6.1}$$

where

$$l_\mu = \sum_{l=e,\mu,\tau} \bar{\psi}_{\nu_l} \gamma_\mu (1-\gamma_5) \psi_l \tag{6.2}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

The currents here are of the $V - A$ type. These currents l_μ are constructed from the lepton fields $\psi_l(x)$ described by the Dirac equation for a point spin $\frac{1}{2}$ particle. In the case of hadrons, the interaction is also of the current$\times$current type with $V - A$ currents at the level of quarks which are considered as point particles. However, the hadrons are not point particles. Therefore, the general structure of the matrix element of the vector and axial vector currents has to be determined from the quantum fields describing the hadrons using the principles of Lorentz covariance, parity, time reversal, etc. For example, the nuclear β-decay in which a neutron decays to give a proton, electron, and neutrino, that is, $n \to p + e^- + \bar{\nu}_e$, is a weak interaction process involving hadrons where the currents are of $V - A$ type but with the difference that there would be more terms contributing to the matrix element of the hadronic current, in addition to that appearing in the case of leptons.

The weak interactions of the hadrons are classified into three types of processes:

i) Semileptonic weak processes,

ii) Nonleptonic weak processes,

iii) Radiative weak processes,

which are described briefly in the following sections.

6.2 Semileptonic Weak Decays of Hadrons without Strangeness

Semileptonic processes are defined as those processes in which both the leptons and hadrons are involved either in a decay process like $n \to p + e^- + \bar{\nu}_e$ or in a scattering process like $\nu_\mu + n \to \mu^- + p$. We have discussed the semileptonic decays of spin $\frac{1}{2}$ particles like the neutron decay, that is, $n \to p + e^- + \bar{\nu}_e$ in some detail in the last chapter and will take up neutrino scattering in Chapter 9. There are simpler processes in which a hadron of spin zero like a pion or a kaon decays into two leptons, that is, $\pi^\pm \to e^\pm \nu_e(\bar{\nu}_e)$, $\pi^\pm \to \mu^\pm \nu_\mu(\bar{\nu}_\mu)$, $K^\pm \to e^\pm \nu_e(\bar{\nu}_e)$, and $K^\pm \to \mu^\pm \nu_\mu(\bar{\nu}_\mu)$. There are also decays like $\pi^\pm \to \pi^0 e^\pm \nu_e\ (\bar{\nu}_e)$, $K^\pm \to \pi^0 e^\pm \nu_e(\bar{\nu}_e)$, and $K^\pm \to \pi^+ \pi^- e^\pm \nu_e\ (\bar{\nu}_e)$ in which three or four particles are involved. In Tables 6.4 and 6.5, we have shown the decay modes of some low lying mesons and baryons, respectively. In this section, we describe the semileptonic weak decays of spin zero particles with strangeness $S = 0$ like pions.

6.2.1 Two-body decay of pions: πl_2 decays

Pions are pseudoscalar particles (0^-) with spin zero and negative parity, first predicted by Yukawa [272] in 1935 as the carrier of the strong nuclear force. They were first discovered by Powell and collaborators [291] in 1947 in cosmic ray experiments. Pions exist in three charged states π^+, π^-, and π^0 which form an isospin triplet. Their properties are listed in Table 6.2. The two-body weak decays of π^+ and π^- take place through electron and muon channels,

$$\pi^- \longrightarrow l^- + \bar{\nu}_l, \tag{6.3}$$

$$\pi^+ \longrightarrow l^+ + \nu_l, \quad l = e, \mu. \tag{6.4}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Table 6.4 Decay modes with the corresponding branching ratios of charged as well as neutral pions and kaons [117].

Particle	Decay mode	Branching ratio (%)	Particle	Decay mode	Branching ratio (%)
π^+	$\mu^+\nu_\mu$	99.98770 ± 0.00004	K_s	$\pi^0\pi^0$	30.69 ± 0.05
	$\mu^+\nu_\mu\gamma$	$(2.00\pm0.25)\times10^{-4}$		$\pi^+\pi^-$	69.20 ± 0.05
	$e^+\nu_e$	$(1.230\pm0.004)\times10^{-4}$		$\pi^\pm e^\mp\bar{\nu}_e(\nu_e)$	$(7.04\pm0.08)\times10^{-4}$
				$\pi^\pm\mu^\mp\bar{\nu}_\mu(\nu_\mu)$	$(4.69\pm0.05)\times10^{-4}$
π^0	2γ	98.823 ± 0.034			
	$e^+e^-\gamma$	1.174 ± 0.035			
$K^\pm$	$\mu^+\nu_\mu(\mu^-\bar{\nu}_\mu)$	63.56 ± 0.11	K_l	$\pi^\pm e^\mp\bar{\nu}_e(\nu_e)$	40.55 ± 0.11
	$e^+\nu_e(e^-\bar{\nu}_e)$	$(1.582\pm0.007)\times10^{-5}$		$\pi^\pm\mu^\mp\bar{\nu}_\mu(\nu_\mu)$	27.04 ± 0.07
	$\pi^0\, e^\pm\, \nu_e(\bar{\nu}_e)$	5.07 ± 0.04		$\pi^0\pi^0\pi^0$	19.52 ± 0.12
	$\pi^0\mu^\pm\nu_\mu\ (\bar{\nu}_\mu)$	3.352 ± 0.033		$\pi^+\pi^-\pi^0$	12.54 ± 0.05
	$\pi^\pm\pi^0$	20.67 ± 0.08		$\pi^+\pi^-$	$(1.97\pm0.01)\times10^{-3}$
	$\pi^0\pi^\pm\pi^0$	1.760 ± 0.023		$\pi^0\pi^0$	$(8.64\pm0.06)\times10^{-4}$
	$\pi^\pm\pi^+\pi^-$	5.583 ± 0.024			

Table 6.5 Decay modes of hyperons with branching ratios [117]. Neutron decays to $pe^-\bar{\nu}_e$ with 100% branching ratio.

Particle	Mode	Branching ratio (%)	Particle	Mode	Branching ratio (%)
Λ	$p\pi^-$	63.9 ± 0.5	Ξ^0	$\Lambda\pi^0$	99.524 ± 0.012
	$\mathrm{n}\pi^0$	35.8 ± 0.5		$\Sigma^+e^-\bar{\nu}_e$	$(2.52\pm0.08)\times10^{-4}$
	$n\gamma$	$(1.75\pm0.15)\times10^{-3}$		$\Lambda\gamma$	$(1.17\pm0.07)\times10^{-3}$
	$pe^-\bar{\nu}_e$	$(8.32\pm0.14)\times10^{-4}$			
	$p\mu^-\bar{\nu}_\mu$	$(1.57\pm0.35)\times10^{-4}$			
Σ^+	$p\pi^0$	51.57 ± 0.3	Ξ^-	$\Lambda\pi^-$	99.887 ± 0.035
	$p\gamma$	1.23×10^{-3}		$\Lambda e^-\bar{\nu}_e$	$(5.63\pm0.31)\times10^{-4}$
	$n\pi^+$	48.31 ± 0.3			
	$\Lambda e^+\nu_e$	$(2.0\pm0.5)\times10^{-5}$			
Σ^0	$\Lambda\gamma$	100			
	Λe^+e^-	5×10^{-3}			
Σ^-	$n\pi^-$	99.848	Ω^-	ΛK^-	67.8 ± 0.7
	$ne^-\bar{\nu}_e$	1.02×10^{-3}		$\Xi^0\pi^-$	23.6 ± 0.7
	$n\mu^-\bar{\nu}_\mu$	$(4.5\pm0.4)\times10^{-4}$		$\Xi^-\pi^0$	8.6 ± 0.4
				$\Xi^0e^-\bar{\nu}_e$	$(5.6\pm0.28)\times10^{-3}$

In the $V-A$ theory, the S matrix element for the processes $\pi^-(p) \rightarrow l^-(k') + \bar{\nu}_l(k)$ corresponding to the Feynman diagram shown in Figure 6.1, is written as

$$S = i\frac{G_F}{\sqrt{2}}\int d^4x\langle l^-\bar{\nu}_l|J_h^\mu(x)J_\mu^l(x)|\pi^-\rangle \tag{6.5}$$

$$= i\frac{G_F}{\sqrt{2}}(2\pi)^4\delta^4(p-k-k')\langle 0|V^\mu - A^\mu|\pi^-(p)\rangle\bar{u}(k)\gamma_\mu(1-\gamma_5)v(k') \tag{6.6}$$

and the decay rate is given by

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

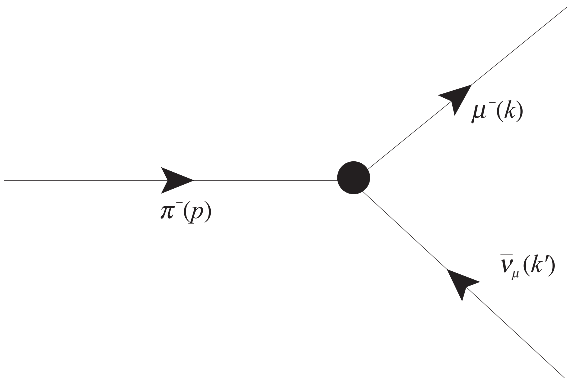

Figure 6.1 Feynman diagram for the pion decay at rest, $\pi^-(p) \longrightarrow l^-(k) + \bar{\nu}_l(k')$ (for $l = \mu$). The quantities in the brackets are the respective momenta of the particles.

$$d\Gamma = \frac{1}{2m_\pi}(2\pi)^4\delta^4(p-k-k')\frac{d\vec{k}'}{2E_l(2\pi)^3}\frac{d\vec{k}}{2E_\nu(2\pi)^3}|\mathcal{M}|^2, \tag{6.7}$$

where

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\langle 0|V^\mu - A^\mu|\pi^-(p)\rangle \bar{u}(k)\gamma_\mu(1-\gamma_5)v(k'). \tag{6.8}$$

The hadronic matrix element is expressed as:

$$\langle 0|V^\mu - A^\mu|\pi^-(p)\rangle = \langle 0|V^\mu|\pi^-(p)\rangle - \langle 0|A^\mu|\pi^-(p)\rangle. \tag{6.9}$$

In order to write the matrix element in Eq. (6.9), it should be noted that the matrix element of the vector current between $|\pi\rangle$ and $|0\rangle$ state vanishes as the Lorentz structure of the matrix element $\langle 0|V^\mu|\pi^-(p)\rangle$ is axial vector in nature: V^μ is a vector and $|\pi^-\rangle$ is pseudoscalar and no axial vector can be constructed with the only momentum available, that is, the momentum of the pion p_μ. Using the same argument, the Lorentz structure of the matrix element $\langle 0|A^\mu|\pi^-(p)\rangle$ is a vector and the matrix element is constructed using the momentum of the pion p_μ. Therefore,

$$\begin{aligned} \langle 0|V^\mu|\pi^-(p)\rangle &= 0, \\ \langle 0|A^\mu|\pi^-(p)\rangle &= if_\pi p^\mu, \end{aligned} \tag{6.10}$$

where f_π is the pion decay constant.

Using the expression for the matrix element given in Eq. (6.8) leads to:

$$\overline{\sum_{s_f}}\sum_{s_i}|\mathcal{M}|^2 = \left(\frac{G_F^2}{2}\right) f_\pi^2 p_\mu p_\nu \sum_{s_i}|\bar{u}(k)\gamma^\mu(1-\gamma^5)v(k')|^2, \tag{6.11}$$

$$\begin{aligned} \sum_{s_i}|\bar{u}(k)\gamma^\mu(1-\gamma^5)v(k')|^2 &= \sum_{s_i}\{\bar{u}(k)\gamma^\mu(1-\gamma^5)v(k')\}\{\bar{u}(k)\gamma^\nu(1-\gamma^5)v(k')\}^\dagger, \\ &= 8(k'^\mu k^\nu - k\cdot k' g^{\mu\nu} + k'^\nu k^\mu) - 8ik'_\alpha k_\beta \epsilon^{\alpha\mu\beta\nu}. \end{aligned} \tag{6.12}$$

where $s_i (= 0)$ and s_f represent the spins of the initial and final states, respectively. Contracting it with $p_\mu p_\nu$ will lead to

$$\overline{\sum_{s_f}}\sum_{s_i}|\mathcal{M}|^2 = 4G_F^2 f_\pi^2\,(2p\cdot k p\cdot k' - p^2 k\cdot k') \qquad (\because p = k + k'). \tag{6.13}$$

The scalar products of the four momenta are given by:

$$\begin{aligned} p\cdot k &= (k+k')\cdot k = k^2 + k\cdot k' = m_l^2 + k\cdot k' \\ p\cdot k' &= (k+k')\cdot k' = k\cdot k' + k'^2 = k\cdot k'\ ; \quad (\because m_\nu = 0) \\ p^2 &= (k+k')^2 = m_l^2 + 2k\cdot k' = m_\pi^2. \end{aligned}$$

Equation (6.13) gives

$$\begin{aligned} \overline{\sum_{s_f}}\sum_{s_i}|\mathcal{M}|^2 &= 4G_F^2 f_\pi^2\,(2[m_l^2 + k\cdot k']k\cdot k' - m_\pi^2 k\cdot k') \\ &= 4G_F^2 f_\pi^2 \left(2\left[m_l^2 + \frac{(m_\pi^2 - m_l^2)}{2}\right]\left[\frac{m_\pi^2 - m_l^2}{2}\right] - m_\pi^2\left[\frac{m_\pi^2 - m_l^2}{2}\right]\right) \\ &= 2G_F^2 f_\pi^2\,(m_\pi^2 - m_l^2)m_l^2. \end{aligned}$$

Using $\overline{\sum}\sum|\mathcal{M}|^2$ as obtained here and performing the momentum integration over $d\vec{k}$ in Eq. (6.7), we obtain

$$\Gamma = \frac{G_F^2\, f_\pi^2}{8\pi\, m_\pi}\,\frac{m_l^2}{m_\pi^2}\,(m_\pi^2 - m_l^2)^2.$$

Using the numerical values of m_π, m_μ and $\Gamma = 38.46\ \mu\text{s}^{-1}$, we find $f_\pi = 131$ MeV.

We note that

i)

$$R_\pi = \frac{\Gamma(\pi^- \longrightarrow e^-\bar{\nu}_e)}{\Gamma(\pi^- \longrightarrow \mu^-\bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\,\frac{(1 - \frac{m_e^2}{m_\pi^2})^2}{(1 - \frac{m_\mu^2}{m_\pi^2})^2} = 1.2834 \times 10^{-4}.$$

This is in good agreement with the experimental value of $R_\pi = 1.23 \pm 0.02 \times 10^{-4}$ [292]. Including the radiative corrections, the value of R_π becomes $R_\pi = 1.233 \times 10^{-4}$.

ii) In the limit $m_e \to 0$, $\Gamma(\pi^- \to e^-\bar{\nu}_e) \to 0$, and $\Gamma(\pi^+ \to e^+\nu_e) \to 0$, πe_2 decay modes are forbidden. This really follows if the neutrinos are left-handed and the weak interactions have $V - A$ structure, contributing through the axial vector part.

iii) There were indications in the pre $V - A$ era that only axial vector interactions could explain these decays in the calculations made by Ruderman and Finkelstein [293], in view of the experimental limit of $R < 5 \times 10^{-3}$ available at that time for these decays.

6.2.2 Three-body decays of pions: πl_3 decays

Pions also decay into three particles:

$$\pi^+/\pi^-(p_1) \longrightarrow \pi^0(p_2) \;+\; e^+/e^-(k) \;+\; \nu_e/\bar{\nu}_e(k') \tag{6.14}$$

and are called pion β-decays in analogy with the neutron β-decay, that is, $n \to p + e^- + \bar{\nu}_e$. The S matrix element in the $V - A$ theory, for the process given in Eq. (6.14) is written as

$$S \;=\; i\frac{G_F}{\sqrt{2}}(2\pi)^4\delta^4(p_1 - p_2 - k - k')\langle\pi^0(p_2)|V^\mu - A^\mu|\pi^-(p_1)\rangle\bar{v}_{\bar{\nu}_e}\gamma_\mu(1-\gamma_5)u_e(k).$$

Using the general expression for $d\Gamma$, the decay rate for three particles in the final state, we write:

$$d\Gamma = (2\pi)^4\delta^4(p_1 - p_2 - k - k')\frac{1}{2E_\pi}\frac{d\vec{k}}{2E_e(2\pi)^3}\frac{d\vec{k}'}{2E_\nu(2\pi)^3}\frac{d\vec{p}_2}{2E_\pi(2\pi)^3}\overline{\sum}\sum|\mathcal{M}|^2, \tag{6.15}$$

where

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\langle\pi^0(p_2)|V^\mu - A^\mu|\pi^-(p_1)\rangle\bar{v}_{\bar{\nu}_e}(k')\gamma_\mu(1-\gamma_5)u_e(k). \tag{6.16}$$

The hadronic matrix element $\langle\pi^0(p_2)|V^\mu - A^\mu|\pi^-(p_1)\rangle$ is written as:

$$\langle\pi^0(p_2)|V^\mu - A^\mu|\pi^-(p_1)\rangle = \langle\pi^0(p_2)|V^\mu|\pi^-(p_1)\rangle - \langle\pi^0(p_2)|A^\mu|\pi^-(p_1)\rangle. \tag{6.17}$$

Again the Lorentz structure of the matrix element of the axial vector current demands that $\langle\pi^0|A^\mu|\pi^-\rangle$ vanishes because no covariant axial vector term can be constructed from the momenta p_1 and p_2, and therefore,

$$\langle\pi^0|A^\mu|\pi^\pm\rangle \;=\; 0. \tag{6.18}$$

Similar arguments show that $\langle\pi^0|V^\mu|\pi^\pm\rangle$ is a vector which is written as

$$\langle\pi^0|V^\mu|\pi^\pm\rangle \;=\; f_1(q^2)p_1^\mu + f_2(q^2)p_2^\mu, \tag{6.19}$$

where $f_1(q^2)$ and $f_2(q^2)$ are the form factors. These form factors depend upon the scalar quantities that can be constructed from p_1^μ and p_2^μ, that is, p_1^2, p_2^2, $p_1 \cdot p_2$. Since $p_1^2 = m_{\pi^+}^2$ and $p_2^2 = m_{\pi^0}^2$, only $p_1 \cdot p_2$ is the independent variable and is generally expressed in terms of $q^2 = m_{\pi^+}^2 + m_{\pi^0}^2 - 2p_1 \cdot p_2$. Defining the aforementioned matrix element in terms of $q = p_1 - p_2$ and $P = p_1 + p_2$, we get

$$\langle\pi^0|V^\mu|\pi^+\rangle = f_+(q^2)P^\mu + f_-(q^2)q^\mu, \tag{6.20}$$

where the form factors $f_+(q^2)$ and $f_-(q^2)$ are defined as

$$f_+(q^2) = \frac{f_1(q^2) + f_2(q^2)}{2} \qquad \text{and} \qquad f_-(q^2) = \frac{f_1(q^2) - f_2(q^2)}{2}. \tag{6.21}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

The matrix element in Eq. (6.16) is then given by

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \bar{v}_{\bar{\nu}_e}(k') \left(f_+(q^2) \not{P} + f_-(q^2) \not{q} \right) (1-\gamma_5) u_e(k). \tag{6.22}$$

Since $\bar{v}_{\bar{\nu}_e}(k') \not{q} u_e(k) = \bar{v}_{\bar{\nu}_e}(k')(\not{k} + \not{k}') u_e(k) = m_e \bar{v}_{\bar{\nu}_e}(k') u_e(k)$, the contribution of the $f_-(q^2)$ term is very small and can be neglected. In fact, the contribution of this term vanishes if one uses the hypothesis of the conserved vector current (CVC) to be discussed later in this chapter.

Using Eq. (6.22), the matrix element for the three-body decay of the pion, is written as:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} f_+(q^2) \left[\bar{v}_{\bar{\nu}_e}(k') \not{P} (1-\gamma_5) u_e(k) \right].$$

Using $P^\mu = p_1^\mu + p_2^\mu = 2p_1^\mu - k^\mu - k'^\mu$ and the Dirac equation for u_e and $\bar{v}_{\bar{\nu}_e}$, we get:

$$\mathcal{M} = \sqrt{2}\, G_F f_+(q^2)\, m_\pi [\bar{v}_{\bar{\nu}_e}(k') \gamma_0 (1-\gamma_5) u_e(k)]. \tag{6.23}$$

Evaluating $\overline{\sum}\sum |\mathcal{M}|^2$ in the rest frame of the pion leads to the decay rate Γ as:

$$\begin{aligned}
\Gamma &= \frac{16 G_F^2 \cos^2\theta_c f_+^2(q^2) m_\pi^2}{(2\pi)^5 2m_\pi} \int \frac{E_e E_{\nu_e} d\vec{p}_2 d\vec{k} d\vec{k}' \delta^4(p_1 - p_2 - k - k')}{8 m_\pi E_e E_{\nu_e}} \\
&= \frac{G_F^2 \cos^2\theta_c f_+^2(q^2)}{(2\pi)^5} \int d\vec{k} d\vec{k}' \delta(\Delta - E_e - E_{\nu_e}), \quad \text{where } \Delta = m_{\pi^\pm} - m_{\pi^0} \\
&= \frac{4\pi G_F^2 \cos^2\theta_c f_+^2(q^2)}{(2\pi)^5} \int_0^\Delta 4\pi E_e^2 dE_e (\Delta - E_e)^2 \\
&= \frac{G_F^2 \cos^2\theta_c f_+^2(q^2) \Delta^5}{60\pi^3}.
\end{aligned} \tag{6.24}$$

Experimentally using the value of the πe_3 decay rate, $\Gamma = 0.39855\ \mathrm{s}^{-1}$ and $\Delta = 4.594$ MeV, and the other numerical constants in Eq. (6.24), $f_+(0)$ is obtained as [292]:

$$|f_+(0)| = 1.37 \pm 0.02.$$

The significance of this value of $|f_+(0)|$ is discussed further in Section 6.3.5.

6.3 Symmetry Properties of the Weak Hadronic Current

The weak hadronic current $J_\mu(x)$ has the vector $V_\mu(x)$ and the axial vector $A_\mu(x)$ terms which under Lorentz transformation are constructed as bilinear covariants from the nucleon fields. These bilinear covariants have certain definite properties under discrete transformation like C, P and T as well as internal symmetries like the isospin and unitary symmetry. These symmetry properties are exploited in writing the matrix elements of these currents between the initial and final states of spin $\frac{1}{2}$ or spin zero particles. In this section, we will discuss these symmetry properties and their role in writing the general structure of the matrix elements.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

6.3.1 Lorentz transformation properties and matrix elements

The weak hadronic current $J_\mu(x)$ is written as

$$J_\mu(x) = V_\mu(x) - A_\mu(x) \tag{6.25}$$

and the general structure of the matrix elements of the vector and the axial vector currents between the neutron and proton states of momentum p and p' is written as

$$\begin{aligned}
\langle u_p(\vec{p}\,')|V_\mu|u_n(\vec{p})\rangle &= \sum_i \bar{u}_p(\vec{p}\,')O^i_\mu u_n(\vec{p}),\\
\langle u_p(\vec{p}\,')|A_\mu|u_n(\vec{p})\rangle &= \sum_i \bar{u}_p(\vec{p}\,')O^i_\mu \gamma_5 u_n(\vec{p}),
\end{aligned}$$

where $u_n(\vec{p})$ and $u_p(\vec{p}\,')$ are the Dirac spinors for the neutrons and protons with four momenta p and p', respectively. O^i_μ are the operators constructed from the four-momenta of the initial and final particles and four Dirac matrices like $\gamma_\mu,\ p_\mu,\ p'_\mu,\ \sigma_{\mu\nu}p^\mu$ and $\sigma_{\mu\nu}p'^\mu$ such that the quantity $\bar{u}_p(p')O^i u_n(p)$ transforms as a vector. In general, there are five terms but they are not all independent; they may be related to each other using the Dirac equation for $u_n(\vec{p})$ and $u_p(\vec{p}\,')$. It can be shown that there are only three independent operators which are generally chosen to be γ_μ, q_μ, and $\sigma_{\mu\nu}q^\nu$. Similarly, the matrix elements of the axial vector currents are also written in terms of the three independent operators. These are explained in detail in Chapter 10 and we use some of those results here for introducing the subject. The three independent operators imply that using the Lorentz invariance, the matrix elements of the vector and axial vector currents are defined in terms of the six form factors, that is, $f_i(q^2)$ and $g_i(q^2)$ with $i = 1, 2, 3$. The matrix elements for the transition $n \to p$ is written as [294]:

$$\langle u_p(p')|V_\mu|u_n(p)\rangle = \bar{u}_p(p')\left[f_1(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M}f_2(q^2) + f_3(q^2)\frac{q^\mu}{M}\right]u_n(p), \tag{6.26}$$

$$\langle u_p(p')|A_\mu|u_n(p)\rangle = \bar{u}_p(p')\left[g_1(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M}g_2(q^2) + g_3(q^2)\frac{q^\mu}{M}\right]\gamma_5 u_n(p), \tag{6.27}$$

where we have taken $M_p = M_n = M$, with M_p and M_n being the masses of the proton and neutron, respectively. $f_i(q^2)$ and $g_i(q^2)$, $(i = 1-3)$ are the vector and axial vector form factors. This is similar to the matrix elements written for the electromagnetic current for the proton($\mathcal{M}^p_{\text{em}}$) and the neutron($\mathcal{M}^n_{\text{em}}$) (see Chapter 10), that is,

$$\mathcal{M}^p_{\text{em}} = \bar{u}_p(p')\left[F^p_1(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F^p_2(q^2)\right]u_p(p), \tag{6.28}$$

$$\mathcal{M}^n_{\text{em}} = \bar{u}_n(p')\left[F^n_1(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F^n_2(q^2)\right]u_n(p), \tag{6.29}$$

where $F^p_{1,2}(q^2)$ are the electromagnetic form factors for the transition $p \to p$ and $F^n_{1,2}(q^2)$ are the electromagnetic form factors for the transition $n \to n$. In the limit $q^2 \to 0$, these form factors are normalized as $F^p_1(0) = 1$, $F^p_2(0) = \mu_p(= 1.7928\mu_N)$, $F^n_1(0) = 0$, $F^n_2(0) = \mu_n(= -1.91\mu_N)$ and the q^2 dependence is given by a dipole form.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

6.3.2 Isospin properties of the weak hadronic current

The weak hadronic currents between the neutron and proton states involve a change of charge $\Delta Q = \pm 1$ in the case of $n \to p + e^- + \bar{\nu}_e$ and $p \to n + e^+ + \nu_e$. Since Q = $I_3 + \frac{B}{2}$, $\Delta Q = \pm 1$ implies $\Delta I_3 = \pm 1$ using baryon number conservation. Since protons and neutrons are assigned to a doublet under isospin representation corresponding to $I = \frac{1}{2}$, $I_3 = +\frac{1}{2}$ and $I_3 = -\frac{1}{2}$, respectively, they can be written as a two-component isospinor under the group of isospin transformation, given by

$$u = \begin{pmatrix} u_p \\ u_n \end{pmatrix}. \tag{6.30}$$

This isospin group of transformations is generated by the three 2×2 Pauli matrices τ_i, defined as $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ with the algebra described by $[\tau_i, \tau_j] = i\epsilon_{ijk}\tau_k$, where ϵ_{ijk} is the antisymmetric Levi–Civita tensor, which is $+1(-1)$ for cyclic (anticyclic) permutations and zero for any two repeated indices, that is, $\epsilon_{ijk} = -\epsilon_{jik}$ and $\epsilon_{iik} = 0$. By defining the isospin raising and lowering operators $\tau^\pm = \frac{\tau_1 \pm i\tau_2}{2}$, we can write

$$\begin{aligned} \bar{u}_p O^\mu_W u_n &= \bar{u} O^\mu_W \tau^+ u, \\ \bar{u}_n O^\mu_W u_p &= \bar{u} O^\mu_W \tau^- u, \end{aligned} \tag{6.31}$$

where u is the isospinor in the isospin space $u = \begin{pmatrix} u_p \\ u_n \end{pmatrix}$. Similarly, in the case of the electromagnetic interaction,

$$\begin{aligned} \bar{u}_p O^\mu_{\text{em}} u_p &= \bar{u} O^\mu_{\text{em}} \frac{\mathbb{1} + \tau_3}{2} u, \\ \bar{u}_n O^\mu_{\text{em}} u_n &= \bar{u} O^\mu_{\text{em}} \frac{\mathbb{1} - \tau_3}{2} u, \end{aligned} \tag{6.32}$$

implying that the isoscalar and isovector current matrix elements are expressed as

$$\begin{aligned} \bar{u}\, \mathbb{1} O^\mu_{\text{em}} u &= \bar{u}_p O^\mu_{\text{em}} u_p + \bar{u}_n O^\mu_{\text{em}} u_n, \\ \bar{u} \tau_3 O^\mu_{\text{em}} u &= \bar{u}_p O^\mu_{\text{em}} u_p - \bar{u}_n O^\mu_{\text{em}} u_n. \end{aligned} \tag{6.33}$$

If we parameterize the matrix element of the isoscalar and isovector components as

$$\begin{aligned} \bar{u}\, \mathbb{1} O^\mu_{\text{em}} u &= \bar{u} \left[F_1^S(q^2)\gamma_\mu + iF_2^S(q^2)\frac{\sigma_{\mu\nu}q^\nu}{2M} \right] u, \\ \bar{u} \tau_3 O^\mu_{\text{em}} u &= \bar{u} \left[F_1^V(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M} F_2^V(q^2) \right] \tau_3 u, \end{aligned} \tag{6.34}$$

and using the electromagnetic matrix element of protons and neutrons as given in Eqs. (6.28) and (6.29), we can write:

$$F^S_{1,2} = F^p_{1,2}(q^2) + F^n_{1,2}(q^2), \tag{6.35}$$

$$F^V_{1,2} = F^p_{1,2}(q^2) - F^n_{1,2}(q^2). \tag{6.36}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

We see that while the weak currents transform as τ^+ and τ^- components of an isovector current, the electromagnetic current transforms as the sum of an isoscalar and isovector current.

6.3.3 T invariance

It may be recalled from Chapter-3 that the time reversal invariance holds if

$$\mathcal{M}' = \mathcal{M}^*, \tag{6.37}$$

where $\mathcal{M}'$ represents the time reversed matrix element and $\mathcal{M}^*$ represents the Hermitian conjugate of the unreversed matrix element. Under time-reversal invariance, the initial and final state particles are interchanged as well as the initial and final state particle's spin and angular momenta are reversed.

Now, we examine what happens, when time reversal invariance [295] is applied on the weak vector and axial vector matrix elements defined in Eqs. (6.26) and (6.27). The transformation of the form factors under T invariance is defined as:

$$\bar{u}_p(p')f(q^2)u_n(p) \quad \longrightarrow \quad \bar{u}_n(p)\gamma_0\gamma_1\gamma_3\tilde{f}(q^2)\gamma_3\gamma_1\gamma_0 u_p(p), \tag{6.38}$$

where f and $\tilde{f}$ represent the unreversed and time reversed form factors, and p and n represent the unreversed initial and the final state particles. Taking all the bilinear covariants used with the form factors in the vector and the axial vector current individually, we obtain the transformation of the vector and axial vector form factors under T invariance as:

$$\begin{aligned}
\bar{u}_p u_n &\xrightarrow{\hat{T}} \bar{u}_n u_p, & \bar{u}_p\gamma_5 u_n &\xrightarrow{\hat{T}} -\bar{u}_n\gamma_5 u_p,\\
\bar{u}_p\gamma^\mu u_n &\xrightarrow{\hat{T}} \bar{u}_n\gamma_\mu u_p, & \bar{u}_p\gamma^\mu\gamma_5 u_n &\xrightarrow{\hat{T}} \bar{u}_n\gamma_\mu\gamma_5 u_p,\\
\bar{u}_p\sigma^{\mu\nu} u_n &\xrightarrow{\hat{T}} \bar{u}_n\sigma_{\mu\nu} u_p, & \bar{u}_p\sigma^{\mu\nu}\gamma_5 u_n &\xrightarrow{\hat{T}} -\bar{u}_n\sigma_{\mu\nu}\gamma_5 u_p.
\end{aligned}$$

The hadronic current J_μ is defined in Eq. (6.25) with V_μ and A_μ defined in Eqs. (6.26) and (6.27), respectively. The time reversed current J'_μ is obtained as

$$\begin{aligned}
J'_\mu &= \bar{u}_n\left[f_1(q^2)\gamma_\mu + (-i)(\sigma_{\mu\nu})\frac{q^\nu}{2M}f_2(q^2) + \frac{q_\mu}{M}f_3(q^2) - g_1(q^2)\gamma_\mu\gamma_5\right.\\
&- \left.(-i)(-\sigma_{\mu\nu}\gamma_5)\frac{q^\nu}{2M}g_2(q^2) - \frac{q_\mu}{M}(-\gamma_5)g_3(q^2)\right]u_p.
\end{aligned} \tag{6.39}$$

Hermitian conjugate of Eq. (6.25) is written as

$$\begin{aligned}
\tilde{J}_\mu &= \bar{u}_p\left[f_1^*(q^2)\gamma_\mu + (-i)\sigma_{\mu\nu}\frac{q^\nu}{2M}f_2^*(q^2) + \frac{q_\mu}{M}f_3^*(q^2) - g_1^*(q^2)\gamma_\mu\gamma_5\right.\\
&- \left.(-i)(-\sigma_{\mu\nu}\gamma_5)\frac{q^\nu}{2M}g_2^*(q^2) - \frac{q_\mu}{M}(-\gamma_5)g_3^*(q^2)\right]u_n.
\end{aligned} \tag{6.40}$$

Comparing Eqs. (6.39) and (6.40), we find that $f_i = f_i^*$ and $g_i = g_i^*$ which implies that if time reversal invariance holds, the form factors must be real.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

6.3.4 Conserved vector current hypothesis

The hypothesis of the conserved vector current was proposed by Gershtein and Zeldovich [44] and Feynman and Gell-Mann [42]. They made an important observation in the study of the nuclear β-decays in Fermi transitions ($\Delta J = 0$) driven by vector currents with no change in parity. They observed that the strength of the weak vector coupling (weak charge) for the muon and neutron decays are the same, just like in the case of the electromagnetic interactions where the strength of the electromagnetic coupling, that is, electric charge (e) remains the same for electrons and protons. Since the equality of the charge coupling, also known as the universality of the electromagnetic interactions follows from the conservation of the electromagnetic current, it was suggested that the weak vector current is also conserved, that is, $\partial_\mu V^\mu(x) = 0$, which leads to the equality of the weak vector coupling for leptons and hadrons.

In fact, they proposed a stronger hypothesis of the isotriplet of the vector currents which goes beyond the hypothesis of CVC and predicts the form factors $f_{1,2}(q^2)$ describing the matrix elements of the weak vector current in terms of the electromagnetic form factors of hadrons. According to this hypothesis, the weak currents V_μ^+, V_μ^-, and the isovector part of the electromagnetic current V_μ^{em} are assumed to form an isotriplet under the isospin symmetry such that f_1 and f_2 are given in terms of the isovector electromagnetic form factors,

$$\begin{aligned} f_1(q^2) &= F_1^p(q^2) - F_1^n(q^2), \\ f_2(q^2) &= F_2^p(q^2) - F_2^n(q^2). \end{aligned}$$

The conservation of the vector current hypothesis, that is, $\partial_\mu V^\mu(x) = 0$ then follows from the conservation of electromagnetic current and implies, using the Dirac equation for $u(p)$ and $\bar{u}(p')$ in Eq. (6.26), that

$$f_3(q^2) = 0.$$

It should be noted that while the isotriplet current hypothesis implies CVC due to the isospin symmetry, the vice versa is not true. In the literature, the term CVC is mostly used to refer to both the isotriplet hypothesis of weak vector currents V_μ^+ and V_μ^- and the conservation of the vector current.

6.3.5 Implications of the CVC hypothesis

(i) **Pion β-decay: πl_3 decays**

The matrix element for the pion β-decays $\pi^\pm \rightarrow \pi^0 e^\pm \nu_e(\bar{\nu}_e)$ was derived in Section 6.2.2. This matrix is rewritten using the isospin notation in which $V_\mu^- = V_\mu I^-$, where I^- is the isospin lowering operator in three-dimensional representation, as:

$$\mathcal{M} = \langle \pi^0 | V_\mu^- | \pi^+ \rangle = \big(f_+(q^2) P_\mu + f_-(q^2) q_\mu \big) \langle \pi^0 | I^- | \pi^+ \rangle. \tag{6.41}$$

The conservation of V_μ implies that $q^\mu \langle \pi^0 | V_\mu^- | \pi^+ \rangle = 0$, that is,

$$f_+(q^2) q.P + f_-(q^2) q^2 = 0. \tag{6.42}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Since $q.P = m_{\pi^+}^2 - m_{\pi^0}^2 \approx 0$, Eq. (6.42) implies that $f_-(q^2)$= 0. Moreover, the non-vanishing form factor $f_+(q^2)$ is given in terms of the electromagnetic form factor appearing in the hadronic matrix element for the process $e^-\pi^+ \to e^-\pi^+$, that is,

$$\langle \pi^+ | V_\mu^{\text{em}} | \pi^+ \rangle = f^{\text{em}}(q^2) P_\mu \langle \chi_\pi^+ | \chi_\pi^+ \rangle,$$

where $f^{\text{em}}(q^2)$ is the charge form factor of the pion with $f^{\text{em}}(0)$=1. Since $\langle \pi^0 | I^- | \pi^+ \rangle = \sqrt{2}\langle \pi^+ | \pi^+ \rangle = \sqrt{2}$, we get $f_+(q^2) = \sqrt{2} f^{\text{em}}(q^2)$. In the limit $q^2 \to 0$, $f_+(0) = \sqrt{2} f^{\text{em}}(0) = \sqrt{2}$. Therefore, the hypothesis of CVC predicts $f_+(0) = \sqrt{2}$ which is in fair agreement with the experimental value $|f_+(0)| = 1.37 \pm 0.02$, obtained from the decay of $\pi^\mp \to \pi^0 + e^\mp + \bar{\nu}_e(\nu_e)$.

(ii) **Weak magnetism**

The conserved vector current hypothesis predicts that in the low $q^2(\to 0)$ limit,

$$\begin{aligned} f_1(0) &= F_1^p(0) - F_1^n(0) = 1, \\ f_2(0) &= F_2^p(0) - F_2^n(0) = \mu_p - \mu_n = 3.7\mu_N. \end{aligned} \tag{6.43}$$

This was first verified in the $\beta^\mp$-decays of the $A = 12$ triplet nuclei ^{12}B, ^{12}N, and ^{12}C*, all having $J^P = 1^+$. These states decay by β^-, β^+, and γ decays of magnetic dipole transitions:

$$\begin{aligned} ^{12}\text{B} &\longrightarrow\ ^{12}\text{C}\, e^-\, \bar{\nu}_e, \qquad\qquad ^{12}\text{N} \longrightarrow ^{12}\text{C}\, e^+\, \nu_e, \\ ^{12}\text{C}^* &\longrightarrow\ ^{12}\text{C}\, \gamma, \end{aligned}$$

as shown in Figure 6.2. The energy spectrum of the $\beta^-(\beta^+)$-decays are given by [239, 296]:

$$d\Gamma \simeq F(Z, E_e) p_e E_e (\Delta - E_e)^2 \left(1 \pm \frac{8}{3} a E_e\right) dE_e, \tag{6.44}$$

where

$$a \approx \frac{\mu_p - \mu_n}{2M} \left|\frac{f_2(0)}{g_1(0)}\right|, \text{using the conserved vector current hypothesis.} \tag{6.45}$$

We see from the non-relativistic limit of the vector and axial vector currents (Appendix A) that in the limit of small q^2, if the term of the order of $\frac{|\vec{q}|}{2M}$ is retained, then there is an additional contribution to the matrix element of $\beta^\mp$-decays due to the vector terms containing the f_2 term. The correction term to the $\beta^\mp$ spectrum, that is, $\pm\frac{8}{3}aE$ comes due to the interference of the vector and the axial vector terms if the CVC value of $f_2(0) = \mu_p - \mu_n$ is assumed. Several research groups have measured the $\beta^\mp$ spectrum of ^{12}B and ^{12}N and studied the correction factor $\frac{8}{3}aE$. The shape of the correction factor is found to be in agreement with the prediction of CVC given in Eq. (6.44). This term is

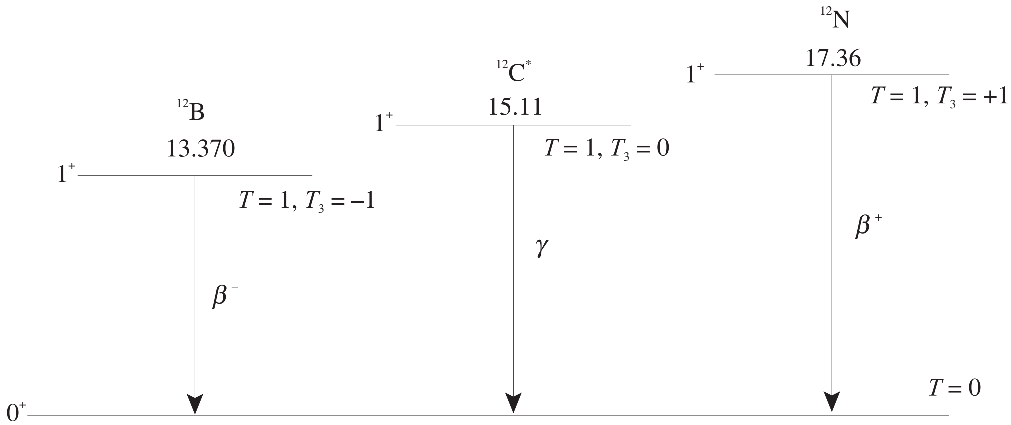

Figure 6.2 β-decay of $A = 12$ nuclei, viz. ^{12}B, ^{12}N, and ^{12}C* [239].

called the weak magnetism because of its structural similarity to the F_2 term in the matrix element of the electromagnetic current (Eq. (6.28)), which describes the electromagnetic interactions due to the magnetic moments of the charged particles. This is one of the major confirmation of the hypothesis of CVC from nuclear β-decays. The hypothesis of CVC has also been confirmed in the case of β-decays of other nuclei and the process of muon capture in nuclei.

6.3.6 Partial conservation of axial vector current (PCAC)

In contrast to the vector current which is conserved, as shown in the previous section, the axial vector current is not conserved. To see this explicitly, we define the matrix element of the axial vector current between one pion state and vacuum which enters in the πl_2 decay of pion (see Section 6.2.1),

$$< 0|A^\mu(x)|\pi^-(q) >= i f_\pi q^\mu e^{-iq.x}, \tag{6.46}$$

where q is the four-momentum of the pion. Taking the divergence of Eq. (6.46),

$$< 0|\partial_\mu A^\mu(x)|\pi^-(q) > \;=\; (-i) i f_\pi q_\mu q^\mu e^{-iq.x} = f_\pi m_\pi^2 e^{-iq.x}, \tag{6.47}$$

since $q^2 = m_\pi^2$. If the axial vector current A^μ is divergenceless, then either $f_\pi = 0$ or $m_\pi = 0$, implying that the pion is massless or that it does not decay. Since $m_\pi \neq 0$, conservation of the axial vector current implies $f_\pi = 0$, which is not true. Therefore, the axial vector current is not conserved. However, since the pion is the lightest hadron, we can work in the limit of $m_\pi \to 0$, and say that the axial vector current is conserved,

$$\lim_{m_\pi \longrightarrow 0} \partial_\mu A^\mu(x) = 0,$$

which is termed as the partial conservation of axial vector current (PCAC). The hypothesis of PCAC has been very useful in calculating many processes in weak interaction physics and

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

deriving relations between various processes in the limit $m_\pi \to 0$. However, the real predictive power of PCAC lies in making further assumptions about the divergence of the axial vector field $\partial_\mu A^\mu(x)$ that establishes a connection between weak and strong interaction physics. The success of PCAC in various applications where physical processes are calculated is based on the following assumptions:

(i) The divergence of the axial vector field is a pseudoscalar; and the pion is also described by a pseudoscalar field. If it is assumed that both are the same, then the physical pion field is described by the divergence of the axial vector, that is

$$\partial_\mu A^\mu(x) \propto \phi_\pi(x), \tag{6.48}$$

such that

$$\partial_\mu A^\mu(x) = C_\pi \phi_\pi(x), \tag{6.49}$$

and

$$\langle 0|\partial_\mu A^\mu|\pi^-\rangle = f_\pi q^2 e^{-iq\cdot x} = C_\pi \langle 0|\phi_\pi(x)|\pi^-(q)\rangle \tag{6.50}$$

This assumption makes it possible to relate the weak interaction processes induced by A^μ to the pion physics in the strong interaction processes through the matrix element of its derivative, that is, $\partial_\mu A^\mu$.

(ii) Taking the limit $m_\pi \to 0$ (corresponding to the conserved axial vector current) in the processes involving pions and nucleons, makes it easier to evaluate the transition amplitude in many weak processes. If further assumption is made that these amplitudes vary smoothly with q^2 and do not change much over the range of q^2 involved in the processes, then the amplitudes evaluated at $q^2 = 0$ can be extrapolated to the physical limit of $q^2 = m_\pi^2$. This is called the soft pion limit widely used in weak interaction physics. However, there remains an ambiguity whether to take the limit as $m_\pi^2 \to 0$ or $m_\pi \to 0$.

6.3.7 Implications of PCAC

There are important implications of PCAC in the evaluation of the matrix elements of the axial vector current between the hadronic states. Some of the most celebrated applications of PCAC are:

(i) Goldberger–Treiman [297] relation and pseudoscalar coupling,

(ii) Adler–Weisberger [47, 48] relation for axial vector coupling,

(iii) Soft pion theorems and their applications.

In this section, we describe each one of them, very briefly.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

(i) Goldberger-Treiman relation and the pseudoscalar coupling

The Goldberger–Treiman relation gives a relation between the axial vector coupling in the weak interaction, $g_1(q^2)$, and the pion–nucleon coupling in the strong interaction, $g_{\pi NN}$ using the hypothesis of PCAC. Consider the matrix element of the axial vector current between $n \to p$ states, that is,

$$< p|A_\mu|n >= \bar{u}(p')\left(g_1(q^2)\gamma_\mu + q_\mu \frac{g_3(q^2)}{M}\right)\gamma_5 u(p). \tag{6.51}$$

The divergence of the current given in this equation in the momentum space is given by:

$$\begin{aligned} q^\mu < p|A_\mu|n > &= \bar{u}(p')\left(g_1(q^2)\not{q} + q^2\frac{g_3(q^2)}{M}\right)\gamma_5 u(p), \\ &= \bar{u}(p')\left(g_1(q^2)(\not{p}' - \not{p}) + q^2\frac{g_3(q^2)}{M}\right)\gamma_5 u(p), \\ &= \left(2Mg_1(q^2) + q^2\frac{g_3(q^2)}{M}\right)\bar{u}(p')\gamma_5 u(p). \end{aligned} \tag{6.52}$$

Assuming that the matrix element of A_μ between the nucleon states is dominated by one pion pole, as shown in Figure 6.3, we can write:

$$< p|A_\mu|n > \;=\; -\sqrt{2}g_{\pi NN}\bar{u}_p(p')\gamma_5 u_n(p)\frac{1}{q^2 - m_\pi^2}f_\pi q_\mu. \tag{6.53}$$

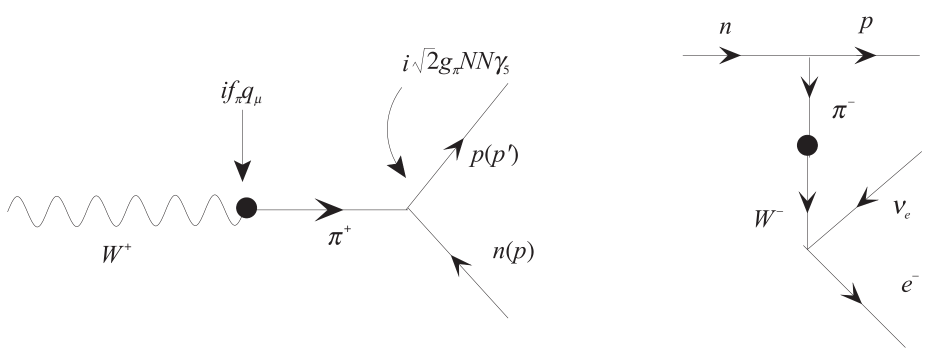

Figure 6.3 Coupling of the axial vector current to a particle of zero mass.

Calculating the divergence of this equation, we obtain

$$q^\mu < p|A_\mu|n >= -\sqrt{2}g_{\pi NN}\bar{u}_p(p')\gamma_5 u_n(p)\frac{1}{q^2 - m_\pi^2}f_\pi q^\mu q_\mu, \tag{6.54}$$

which results in

$$q^\mu < p|A_\mu|n >= -\sqrt{2}g_{\pi NN}\bar{u}_p(p')\gamma_5 u_n(p)\frac{1}{q^2 - m_{\pi^2}}f_\pi m_\pi^2. \tag{6.55}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

In the pion pole dominance, the pseudoscalar form factor $g_3(q^2)$ given in Eq. (6.51) is related to the matrix element given in Eq. (6.53). Comparing the second term of Eq. (6.52) with Eq. (6.55), we obtain

$$-\sqrt{2}g_{\pi NN}\bar{u}(p')\gamma_5 u(p)\frac{1}{q^2-m_\pi^2}f_\pi m_\pi^2 = q^2\frac{g_3(q^2)}{M}m_\pi\bar{u}(p')\gamma_5 u(p), \tag{6.56}$$

which gives

$$\frac{g_3(q^2)}{M} = -\frac{\sqrt{2}g_{\pi NN}f_\pi}{q^2-m_\pi^2}. \tag{6.57}$$

Using this value of $g_3(q^2)$ in Eq. (6.52), we get

$$q^\mu\langle p|A_\mu|n\rangle = \left(2Mg_1(q^2)+q^2\left(-\frac{\sqrt{2}g_{\pi NN}f_\pi}{q^2-m_\pi^2}\right)\right)\bar{u}(p')\gamma_5 u(p). \tag{6.58}$$

In the chiral limit ($m_\pi \to 0$), the axial vector current is conserved. that is

$$\left(2Mg_1(q^2)-\sqrt{2}g_{\pi NN}f_\pi\right)\bar{u}(p')\gamma_5 u(p) = 0. \tag{6.59}$$

The above exprssion yields

$$\sqrt{2}Mg_1(q^2) = g_{\pi NN}f_\pi, \tag{6.60}$$

with $g_1(q^2) = \frac{g_1(0)}{\left(1-\frac{q^2}{M_A^2}\right)}$ and $g_1(0) = 1.267$. Equation (6.60) is known as the Goldberger–Treiman relation. This relation was derived by Goldberger and Treiman [297], Wolfenstein [277], and Leite Lopes [276] around the same time (in 1958) using the dispersion relation and perturbation theory.

(ii) Renormalization of axial vector coupling g_1

Adler [47] and Weisberger [48] used the hypothesis of PCAC and methods of current algebra to express the axial vector coupling $g_1(0)$ in terms of the off shell $(m_\pi^2 \to 0)$ pion–nucleon scattering cross sections. Defining quantities called chirality $\chi^i(t)$ in terms of the axial vector current $A_i^\mu(x)$ as

$$\chi_i(t) = \int d\vec{x}A_i^0(x) \qquad (i = 1,2,3) \tag{6.61}$$

and using the PCAC relation in Eq. (6.49), one can write

$$\frac{d}{dt}\chi_i(t) = C_\pi\int d\vec{x}\,\phi_i^\pi(x), \tag{6.62}$$

relating the axial vector current with the pion fields through its derivative. Assuming that the weak current satisfies a current algebra corresponding to $SU(2)\times SU(2)$ symmetry, we obtain

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

the commutation relation for $A_i(x)$ as

$$[A_i^0(x_1), A_j^0(x_2)]_{x_0=y_0} = \delta(\vec{x}-\vec{y}) i\epsilon_{ijk} V_k^0, \tag{6.63}$$

leading to the following commutation relation for chiralities $\chi_i(t)$.

$$[\chi_+(t), \chi_-(t)] = 2I_3, \tag{6.64}$$

where +, −, and 3 are the raising, lowering, and third components of the isospin. Taking the matrix element of Eq. (6.64) on both sides between the proton states, we get

$$\langle p|[\chi_+(t), \chi_-(t)]|p\rangle = 2\langle p|I_3|p\rangle, \tag{6.65}$$

in the limit of $q^2 \to 0$, the R.H.S. gives 1 while the L.H.S. is evaluated by inserting a complete set of intermediate states. The one-particle nucleon state in the intermediate state gives g_1. The contribution from other intermediate states like Δ resonance and higher resonances are expressed in terms of the cross section for these processes in the pion-nucleon scattering. The following result is obtained

$$1 - \frac{1}{g_A^2} = \frac{4M^2}{g_{\pi NN}^2} \frac{1}{\pi} \int_{M+m_\pi}^{\infty} \frac{WdW}{W^2 - M^2} [\sigma_0^+(W) - \sigma_0^-(W)], \tag{6.66}$$

where W is the center of mass energy and $\sigma_0^{\pm}(W)$ are the cross sections for zero mass pion ($m_\pi^2 \to 0$) for $\pi^{\pm}$ scattering on proton. Using the experimental values of the cross section at $q^2 = m_\pi^2$ and extrapolating it to $m_\pi^2 \to 0$ limit, the integral is performed numerically to evaluate g_A, which is found to be 1.24, in good agreement with the experiments. It demonstrated that the renormalisation of g_A from its quark value of 1 is due to the presence of strong interactions through the πN scattering.

(iii) Soft pion theorems and pion physics

The hypothesis of PCAC and current algebra have been used to obtain various predictions on the production of pions induced by photons, electrons, or pions in the very low energy region, where the extrapolation of the results for $m_\pi^2 \to 0$, that is, the soft pion limit, to the physical pion could be valid. Some of the most important results are as follows [298]:

(a) S wave amplitude in the threshold photoproduction of pions.

(b) Determination of $g_1(q^2)$ in terms of the amplitudes of the threshold electroproduction of pions.

(c) Relations between the threshold one-pion production to the two-pion production induced by photons, electrons, and neutrinos.

(d) Adler's consistency conditions in neutrino nucleon scattering in the $q^2 \to 0$ region.

(e) Various consistency conditions for the pion nucleon scattering amplitudes in the very low energy of pions.

6.3.8 G-parity and second class currents

G-parity is a multiplicative quantum number first used to classify the multipion states in pp and πp collisions [299] and later used by Weinberg [300] to classify weak hadronic currents. It is defined as the product of C, the charge conjugation, and a rotation by 180° about the Y-axis in the isotopic spin (isospin) space,

$$G = Ce^{i\pi I_Y}. \tag{6.67}$$

Since strong interactions are invariant under C and isospin, they are also invariant under G-parity. The G-parity is a very useful concept in the study of pion production in N$\bar{\text{N}}$ collisions. Since weak currents involve bilinear covariants formed out of nucleon fields $\overline{\psi}(p')$ and $\psi(p)$, their transformations can be well defined under G-parity. The weak vector and axial vector currents between a neutron and a proton are defined in Eqs. (6.26) and (6.27). The vector and axial vector terms corresponding to f_1 and g_1 enter in the case of leptons and contain no q^2 dependence because they are point particles and $f_1 = g_1 = 1$. However, in the case of hadrons, which have a composite structure and are strongly interacting particles, there are additional terms in the definition of the matrix elements of the vector and the axial vector currents as defined in Eqs. (6.26) and (6.27). Consequently, all the form factors acquire a q^2 dependence, that is, $f_i(q^2)$ and $g_i(q^2)$ and deviate from unity to get renormalized even at $q^2 = 0$ (except in the case of $f_1(q^2)$, where $f_1(0) = 1$ due to CVC). Thus, the additional terms like $f_2(q^2)$, $f_3(q^2)$, $g_2(q^2)$, and $g_3(q^2)$ which are induced due to the presence of the strong interactions should respect the symmetries of the strong interactions and should have the same G-parity as the original $f_1(q^2)$ and $g_1(q^2)$, respectively, in the vector and axial vector sector, that is, $GV^\mu G^{-1} = V^\mu$ and $GA^\mu G^{-1} = -A^\mu$. Since the currents belong to the triplet representation of the isospin, all the terms have similar transformation properties under the rotation $e^{i\pi I_Y}$. It is their transformation properties under C-parity which defines their relative transformation properties under G-parity. Under G-parity, the bilinear terms in Eqs. (6.26) and (6.27) transform as:

$$\bar{u}_p u_n \xrightarrow{\text{G}} -\bar{u}_p u_n \qquad (\text{associated with } f_3) \tag{6.68}$$

$$\bar{u}_p \gamma_5 u_n \xrightarrow{\text{G}} -\bar{u}_p \gamma_5 u_n \qquad (\text{associated with } g_3) \tag{6.69}$$

$$\bar{u}_p \gamma_\mu \gamma_5 u_n \xrightarrow{\text{G}} -\bar{u}_p \gamma_\mu \gamma_5 u_n \qquad (\text{associated with } g_1) \tag{6.70}$$

while

$$\bar{u}_p \gamma^\mu u_n \xrightarrow{\text{G}} \bar{u}_p \gamma^\mu u_n \qquad (\text{associated with } f_1) \tag{6.71}$$

$$\bar{u}_p \sigma^{\mu\nu} u_n \xrightarrow{\text{G}} \bar{u}_p \sigma^{\mu\nu} u_n \qquad (\text{associated with } f_2) \tag{6.72}$$

$$\bar{u}_p \sigma^{\mu\nu} \gamma_5 u_n \xrightarrow{\text{G}} \bar{u}_p \sigma^{\mu\nu} \gamma_5 u_n. \qquad (\text{associated with } g_2) \tag{6.73}$$

What is observed from Eqs. (6.68)–(6.73) is that the bilinear terms associated with f_2 transforms the same way as f_1 does, while f_3 transforms in the opposite way. Similarly, g_3 transforms the same way as g_1 does while g_2 transforms in a different way. It was Weinberg [300] who

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

first used the G-properties to classify weak currents. He called the currents associated with f_1, f_2, g_1, and g_3, which are invariant under G-parity, as first class currents, and the currents associated with f_3 and g_2, which violate G-parity, as second class currents. Consequently, if the G invariance is valid in weak interactions, only currents with form factors $f_1(q^2)$, $f_2(q^2)$, $g_1(q^2)$, and $g_3(q^2)$ should exist and $f_3(q^2) = g_2(q^2) = 0$. It should be noted that $f_3(q^2) = 0$ is also predicted as a consequence of CVC hypothesis.

6.4 Semileptonic Weak Decays of Hadrons with Strangeness

Around the time pions were discovered, a new class of particles named strange particles were also discovered in the cosmic ray experiments; these particles belonged to both categories of bosons and fermions. Strange bosons are known as K-mesons and strange fermions are known as hyperons. They are copiously produced in strong interactions and decay through weak and electromagnetic processes; they have a long lifetime. It was the study of the production of K-mesons and their decay into pions that gave rise to the $\tau - \theta$ puzzle leading to the discovery of parity violation in weak interactions. With the availability of electron and neutrino beams having high energy and high intensity, these strange particles can also be produced through the electromagnetic and weak interactions in the processes shown in Table 6.6; however, the production cross sections are small. In electromagnetic interactions, the cross sections are of the order of $\sim 10^{-31}$ cm^2 while in weak interactions, the cross sections are extremely small, that is, of the order of $\sim 10^{-41}$ cm^2 as compared to the strong interaction production cross section ($\sim 10^{-27}$ cm^2).

The properties and the quark content of strange mesons and hyperons are given in Tables 6.2 and 6.3 and the dominant decay modes of K mesons and hyperons are given in Tables 6.4 and 6.5, respectively. Strange particles decay through semileptonic, nonleptonic, and radiative

Table 6.6 Production modes of kaons.

Strong interaction	Electromagnetic interaction	Weak interaction
$\pi^- + p \longrightarrow \Lambda + K^0$	$l^- + p \longrightarrow l^- + \Lambda + K^+$	$\nu_l + p \longrightarrow l^- + p + K^+$
$\longrightarrow \Sigma^0 + K^0$	$\longrightarrow l^- + \Sigma^0 + K^+$	$\nu_l + n \longrightarrow l^- + n + K^+$
$\longrightarrow \Sigma^- + K^+$	$\longrightarrow l^- + \Sigma^+ + K^0$	$\longrightarrow l^- + p + K^0$
$\pi^+ + p \longrightarrow \Sigma^+ + K^+$	$l^- + n \longrightarrow l^- + \Lambda + K^0$	
$\longrightarrow K^+ + \bar{K}^0 + p$	$\longrightarrow l^- + \Sigma^0 + K^0$	
$p + \bar{p} \longrightarrow \pi^+ + K^- + K^0$	$l = e, \mu, \tau$	
$\longrightarrow \pi^- + K^+ + \bar{K}^0$		

decay modes. Semileptonic and nonleptonic as well as weak radiative decays of mesons and hyperons have been the focus of experimental studies for a long time. The following observations can be made from the study of weak decays of strange mesons and hyperons:

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

i) The strangeness changing semileptonic decays follow the $|\Delta S| = 1$ and $\Delta S = \Delta Q$ rule, that is, $\Delta Q = +1,\ \Delta S = +1$, or $\Delta Q = -1,\ \Delta S = -1$ like the processes

$$\begin{array}{rclcrcl} \Lambda\ (\Sigma^0) & \longrightarrow & p + l^- + \bar{\nu}_l, & \quad & K^{\pm} & \longrightarrow & \pi^0 + l^{\pm} + \nu_l(\bar{\nu}_l), \\ \Sigma^- & \longrightarrow & n + l^- + \bar{\nu}_l, & \quad & K_L^0 & \longrightarrow & \pi^{\pm} + l^{\mp} + \bar{\nu}_l(\nu_l), \end{array}$$

are allowed, but $\Delta Q = -1,\ \Delta S = +1$ like the processes

$$\Sigma^+ \nrightarrow n + e^+ + \nu_e$$

are not allowed

The selection rule $\Delta Q = \Delta S$ implies the isospin properties of these currents to be $\Delta I = \frac{1}{2}$.

ii) The strangeness changing semileptonic decays with $\Delta S = 1$ are suppressed as compared to the strangeness conserving $\Delta S = 0$ decay by a factor of $20 - 25$. For example:

$$\frac{\Gamma(\Lambda \longrightarrow p + e^- + \bar{\nu}_e)}{\Gamma(n \longrightarrow p + e^- + \bar{\nu}_e)} \approx 0.05 \qquad \text{and} \qquad \frac{\Gamma(K^- \longrightarrow l^- + \bar{\nu}_l)}{\Gamma(\pi^- \longrightarrow l^- + \bar{\nu}_l)} \approx 0.05.$$

iii) The strangeness changing semileptonic decays with $\Delta S = 1$ and $\Delta Q = 0$ are highly suppressed by a factor of $10^{-8} - 10^{-9}$. This is known as the absence of the flavor changing neutral current (FCNC). For example,

$$\frac{\Gamma(K_L \longrightarrow \mu^+\mu^-)}{\Gamma(K^+ \longrightarrow \mu^+\nu)} \approx 10^{-9}.$$

iv) The suppression of the strangeness changing currents in the strange mesons and hyperons are of the same order, suggesting a universal suppression of the strength of the coupling of $\Delta S = 1$ current.

v) The nonleptonic weak decays of the strange mesons and hyperons obey the $\Delta I = \frac{1}{2}$ rule. This means that in decays like $K^{\pm} \to \pi^{\pm}\pi^0, K^0 \to \pi^{\pm}\pi^{\mp}$, or $Y \to N\pi$, the transition amplitude corresponding to $\Delta I = \frac{1}{2}$ transitions dominate over the amplitudes corresponding to $\Delta I \geq \frac{3}{2}$ transitions.

vi) The nonleptonic weak decays of neutral kaons, that is, K_L^0, violate CP invariance; this phenomenon has been observed in the decays $K_L^0 \to \pi^+\pi^-$ and $K_L^0 \to \pi^0\pi^0$.

vii) The hypothesis of CVC is also valid in the case of the strangeness changing decays but the hypothesis of PCAC (partially conserved axial current) and its predictions are not established at the same level of validity as in the case of $\Delta S = 0$ currents.

Based on these observations, Cabibbo [59] in 1963 proposed an extension of the $V - A$ theory which describes the weak interaction of strange particles based on the SU(3) symmetry of weak currents. The theory established the universality of weak interactions involving both $\Delta S = 0$ and $\Delta S = 1$ currents and also led to the concept of quark mixing in weak interactions. In the following sections, we will describe Cabibbo's extension of the phenomenological theory of $V - A$ interaction to the strangeness sector [59].

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

6.4.1 The Cabibbo theory and the universality of weak interactions

The Cabibbo theory was proposed against the backdrop of two outstanding problems of that time. One was the apparent suppression of the $\Delta S = 1$ currents with respect to the $\Delta S = 0$ current couplings. The other problem was the small discrepancy in the coupling strengths of the vector currents in the case of weak decays of muons and neutrons. The coupling strengths G_μ in the muon decay($\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$) and G_β in the neutron decay($n \to p + e^- + \bar{\nu}_e$) were found to be $G_\mu = 1.1663787 \times 10^{-5}$ GeV^{-2} and $G_\beta = G_F \approx 1.136 \pm 0.003010^{-5}$ GeV^{-2}, respectively.

Motivated by the approximate equality of G_μ and G_β, the hypothesis of the conservation of vector current (CVC) was proposed [44, 42] with the hope that the small discrepancy may be explained by the different contributions of the radiative corrections in the case of μ decays and neutron β decays. However, detailed calculations by Berman [263], Kinoshita, and Sirlin [261] showed that the corrections enhanced the discrepancy instead of reducing it, casting doubts about the validity of the CVC hypothesis. The difference in the numerical values of G_β and G_μ seemed to have a deeper origin in the physics of weak interactions and needed to be understood. In this background, Gell-Mann and Levy [58] suggested a mechanism based on the Sakata model [60]. In the Sakata model, the proton, neutron, and lambda hyperon are elementary particles; all the other particles are made up of these three particles. In the Gell-Mann and Levy model, the $\Delta S = 1$ currents appear in the interaction Lagrangian with a smaller strength as compared to the $\Delta S = 0$ currents. The hadronic current $J_h^\mu(x)$ is written as

$$J_h^\mu = \frac{1}{\sqrt{1+\epsilon^2}} \overline{\psi}_p(x)\gamma^\mu(1-\gamma_5)[\psi_n(x) + \epsilon\psi_\Lambda(x)]. \tag{6.74}$$

With $\epsilon \approx 20\%$, one is able to explain the suppression of the strength of the $\Delta S = 1$ currents. Moreover with $\epsilon = 20\%$, the factor $(1+\epsilon^2)^{-\frac{1}{2}} = 0.97$, which is of the right order of magnitude to make $\frac{G_\mu}{\sqrt{1+\epsilon^2}}$ consistent with G_β. This model could explain the suppression of the strength of the $\Delta S = 1$ current coupling as compared to the $\Delta S = 0$ coupling and reduce the strength of G_β to restore the relation $G_\beta \approx G_\mu$. However, the model could not explain the enhancement of nonleptonic weak decays like $\Lambda \to n\pi^0(p\pi^-)$ compared to the strengths of semileptonic weak decays with $\Delta S = 1$ and the $\Delta I = \frac{1}{2}$ rule in the nonleptonic decays. Moreover, the model was based on the Sakata model which was later found to be inadequate for explaining the structure of hadrons. Therefore, the mechanism proposed by Gell-Mann and Levy was not pursued any further.

Cabibbo considered hadronic currents in the context of SU(3) symmetry in which $\Delta S = 0$ and $\Delta S = 1$ currents would appear as members of the octet current. The implementation of the universality between leptonic and hadronic currents would require that the $\Delta S = 0$ and $\Delta S = 1$ hadronic current, which appear in the weak interaction Hamiltonian, may have different strengths allowing the suppression of $\Delta S = 1$ current with respect to $\Delta S = 0$ current; however, the sum of $\Delta S = 0$ and $\Delta S = 1$ currents have be normalized to the normalization of the lepton currents, that is, e and μ currents. This will restore the $e - \mu$ universality as well as

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

the CVC in the case of weak vector currents. Cabibbo, therefore, proposed the hadronic weak vector current entering the weak interaction Hamiltonian to be

$$V_\mu^h = aV_\mu(\Delta S = 0) + bV_\mu(\Delta S = 1), \tag{6.75}$$

such that $|a|^2 + |b|^2 = 1$. A convenient parameterization for a and b, that is, $a = \cos\theta_C$ and $b = \sin\theta_C$ was taken by Cabibbo, where θ_C is known after him as the Cabibbo angle. Cabibbo also extended this analogy to axial vector currents to write the weak hadronic current as

$$J_\mu^h = \cos\theta_C J_\mu(\Delta S = 0) + \sin\theta_C J_\mu(\Delta S = 1), \tag{6.76}$$

with $J_\mu = V_\mu - A_\mu$. This provided a natural suppression of the $\Delta S = 1$ weak currents as compared to the $\Delta S = 0$ weak currents by a factor of $\tan\theta_C$. It also provided a suppression of the $\Delta S = 0$ transition in the hadronic vector current by a factor of $\cos\theta_C$ to bring about universality of weak vector currents making $G_\mu \cos\theta_C = G_F \cos\theta_C \approx G_\beta$. An experimental analysis of the $\Delta S = 1$ β-decays of hyperons and kaons can provide the value of θ_C.

In an earlier work [59], Cabibbo determined the value of θ_C from the ratios of the leptonic and semileptonic decays of kaons and pions; he found $\tan\theta_C = 0.2623$ from leptonic decays and $\tan\theta_C = 0.266$ from semileptonic decays. However, the analysis of the available data on semileptonic decays of Λ and other hyperons yields $\tan\theta_C = 0.2327$, which is the current global value [117]. In formulating the theory, Cabibbo ignored the enhancement of $\Delta I = \frac{1}{2}$ currents in nonleptonic decays and believed it to have a different origin than the physics of the Cabibbo angle and the suppression of the semileptonic $\Delta S = 1$ decays. In fact, the theoretical origin of the Cabibbo angle is still not understood; however, phenomenologically, the theory has been very successful in explaining the experimental data on $\Delta S = 1$ decays of strange mesons and hyperons and has provided important inputs toward formulating the standard model of electroweak interactions. The theory also succeeded in solving the two problems posed at the beginning of the section.

6.4.2 The Cabibbo theory in the quark model and quark mixing

Interestingly, around the time Cabibbo proposed the idea of an SU(3) structure for weak currents, the quark model of the hadrons proposed by Gell-Mann [61] and Zweig [62, 63] with SU(3) symmetry became very successful in explaining the structure of hadrons as well as their decay properties. The Cabibbo theory of $V - A$ currents was then formulated in terms of quarks. The Feynman diagrams for some of the processes at the quark level have been shown in Figure 6.4; the diagrams show that the mesonic and baryonic decays in terms of the quark–quark transitions, are described as:

$$\begin{array}{lcl}
n \longrightarrow p + e^- + \bar{\nu}_e & \Longrightarrow & d \longrightarrow u + e^- + \bar{\nu}_e \\
\Lambda \longrightarrow p + e^- + \bar{\nu}_e & \Longrightarrow & s \longrightarrow u + e^- + \bar{\nu}_e \\
K^+ \longrightarrow \mu^+ + \nu_\mu & \Longrightarrow & \bar{s}u \longrightarrow |0\rangle \Longrightarrow u \to s \\
K^- \longrightarrow \mu^- + \bar{\nu}_\mu & \Longrightarrow & \bar{u}s \longrightarrow |0\rangle \Longrightarrow s \to u \\
\pi^+ \longrightarrow \mu^+ + \nu_\mu & \Longrightarrow & \bar{d}u \longrightarrow |0\rangle \Longrightarrow u \to d
\end{array}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

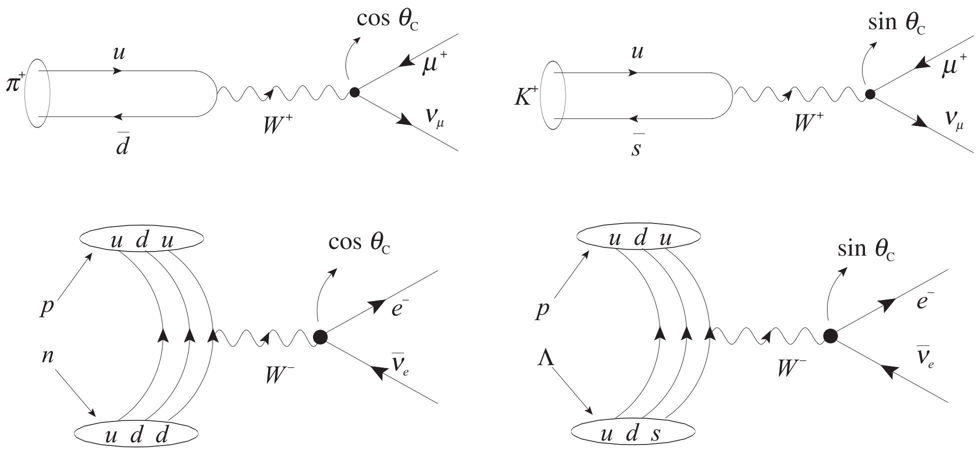

Figure 6.4 Top panel: (left) pion and (right) kaon decay. Bottom panel: (left) neutron and (right) lambda decay.

In the pre-Cabibbo $V - A$ theory, the hadronic vector and axial vector currents in the quark model are written in term of the quark fields $q(= u, d, s)$ as:

$$J^h_\mu \sim g\left[\bar{u}_u(p')\gamma_\mu(1-\gamma_5)u_d(p) + \bar{u}_u(p')\gamma_\mu(1-\gamma_5)u_s(p)\right]. \tag{6.77}$$

In the Cabibbo theory, they are written as

$$J^h_\mu \sim g\left[\bar{u}_u(p')\gamma_\mu(1-\gamma_5)(d\cos\theta_C + s\sin\theta_C)\right]. \tag{6.78}$$

If we define the linear combination of the d and s quark in Eq. (6.78) as d', that is,

$$d' = d\cos\theta_C + s\sin\theta_C,$$

then the hadronic current J^h_μ can be rewritten as

$$J^h_\mu \sim g\left[\bar{u}_u(p')\gamma_\mu(1-\gamma_5)d'\right], \tag{6.79}$$

meaning that d' takes part in the weak interaction and not the d quark. The following observations may be made from the aforementioned discussions:

(i) The $\Delta S = 0$ current in the nuclear β-decays occurs with a reduced strength of $G_F\cos\theta_C$ and G_F becomes comparable to G_μ, the coupling strength in the case of pure leptonic decays like $\mu \longrightarrow e^- + \nu_\mu + \bar{\nu}_e$. This makes the universality of the weak current in the leptonic and hadronic sectors more apparent, that is, $G_\mu = G_F$ in the case of vector currents.

(ii) The Cabibbo theory explains satisfactorily the suppression of $\Delta S = 1$ decay rate as compared to the $\Delta S = 0$ decay rate by a factor of $\tan^2\theta_C$, an observation for which the theory was formulated.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

(iii) In this model, $\Delta S = 0$ currents are due to $d(\bar{d}) \rightleftharpoons u(\bar{u})$ transitions corresponding to $\Delta I = 1$ isovector currents, while $\Delta S = 1$ transitions are due to $s(\bar{s}) \rightarrow u(\bar{u})$ transitions corresponding to $\Delta I = \frac{1}{2}$ both with $\Delta Q = \Delta S$ rules. This provides a natural explanation for the absence of $\Delta Q = -\Delta S$ currents.

6.4.3 Applications of Cabibbo theory: K decays

(i) Kl_2 decays

We see from Table 6.4 that only $K^{\pm}$ decays into two leptons through charged current interactions like the decay $K^{\pm} \rightarrow l^{\pm}\nu_l(\bar{\nu}_l)$, which is suppressed as compared to πl_2 decays (Section 6.2.1); due to a suppression factor of $\tan^2\theta_C$ in the decay rate. The two-body decays of neutral kaons like $K_{L,S} \rightarrow \mu^+\mu^-$, e^+e^- are highly suppressed ($B.R. < 10^{-8}$). The matrix element for the Kl_2 mode of the charged kaon decay is obtained by following the same method as that used for writing the matrix elements for πl_2 decays, that is,

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \langle 0|V^{\mu} - A^{\mu}|K(p)\rangle \left[\bar{u}_l\gamma_{\mu}(1-\gamma_5)u_{\nu}(k)\right]. \tag{6.80}$$

We write the matrix element of $V - A$ current in analogy with the πl_2 decays (Section 6.2.1) as:

$$\langle 0|A^{\mu}|K^{\pm}(p)\rangle = i\sin\theta_C f_K p_{\mu}, \qquad \text{and} \qquad \langle 0|V^{\mu}|K^{\pm}(p)\rangle = 0,$$

leading to the decay rate

$$\Gamma(K^{\pm} \longrightarrow l_{\nu}^{\pm} + \nu_l(\bar{\nu}_l)) = \frac{G_F^2 f_K^2 \sin^2\theta_C}{8\pi} m_l^2 m_K \left(1 - \frac{m_l^2}{m_K^2}\right)^2. \tag{6.81}$$

The experimentally observed value of $\tau = \frac{1}{\Gamma}$ =$1.237\pm0.003\times10^{-8}$ s [117], may be obtained with $\sin\theta_C = 0.221$, $f_K = 160$ MeV.

Moreover, a comparison with the decay rate of electron/muon decay modes yields

$$R^{e\mu} = \frac{\Gamma(K^- \longrightarrow e^-\bar{\nu})}{\Gamma(K^- \longrightarrow \mu^-\bar{\nu})} = \frac{m_e^2}{m_{\mu}^2}\left(\frac{m_K^2 - m_e^2}{m_K^2 - m_{\mu}^2}\right)^2 = 2.58 \times 10^{-5}, \tag{6.82}$$

which is in agreement with the experimentally observed value for this ratio, $R^{\text{exp}} = (2.425 \pm 0.012) \times 10^{-5}$. The branching ratio (BR) is [117]

$$\frac{\Gamma(K^- \longrightarrow \mu^-\bar{\nu}_{\mu})}{\Gamma(K^- \longrightarrow all)} = 0.6356. \tag{6.83}$$

Using the experimental value of $\Gamma(K^- \rightarrow \mu^-\bar{\nu}_{\mu})$ as $5.75 \times 10^7\text{s}^{-1}$ and the present value of $\sin\theta_C$, the ratio $\frac{f_K}{f_{\pi}}$ is obtained as 1.1928 [117]. Theoretically, the value of f_K/f_{π} is calculated using lattice QCD (quantum chromodynamics) and is found to be very close to this value [301].

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

(ii) Kl_3 decays

The three-particle semileptonic decay of charged and neutral mesons are

$$K^+ \longrightarrow \pi^0\mu^+\nu_\mu, \qquad K_l \longrightarrow \pi^-\mu^+\nu_\mu, \tag{6.84}$$

$$\longrightarrow \pi^0 e^+\nu_e, \qquad \longrightarrow \pi^+\mu^-\bar{\nu}_\mu, \tag{6.85}$$

$$K^- \longrightarrow \pi^0\mu^-\bar{\nu}_\mu, \qquad \longrightarrow \pi^- e^+\nu_e, \tag{6.86}$$

$$\longrightarrow \pi^0 e^-\bar{\nu}_e, \qquad \longrightarrow \pi^+ e^-\bar{\nu}_e. \tag{6.87}$$

The amplitude in the Cabibbo $V-A$ theory for the process $K^+(p_K) \to \pi^0(p_\pi)\mu^+(p_l)$ $\nu_\mu(p_{\nu_l})$ is written in analogy with the πl_3 decays and has the general form

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\sin\theta_C[f_+(q^2)P^\mu + f_-(q^2)q^\mu]\bar{u}_l\gamma_\mu(1-\gamma_5)u_{\nu_l}, \text{where } l = e, \mu$$

$$\text{with} \qquad P^\mu = p_K^\mu + p_\pi^\mu, q^\mu = p_K^\mu - p_\pi^\mu = p_l^\mu + p_{\nu_l}^\mu. \tag{6.88}$$

The condition of the conservation of the weak vector current, that is, $q_\mu\langle p_K|V_\mu|p_\pi\rangle = 0$ does not lead to $f_-(q^2) = 0$ because $q.P = m_K^2 - m_\pi^2 \neq 0$. However, we can see that the contribution of $f_-(q^2)$ will be small because

$$f_-(q^2)q^\mu\bar{u}_l\gamma_\mu(1-\gamma_5)u_{\nu_l} = f_-(q^2)m_l\bar{u}_l(1-\gamma_5)u_{\nu_l} \tag{6.89}$$

and is proportional to the lepton mass. Therefore, Ke_3 decays are sensitive only to $f_+(q^2)$ while $K\mu_3$ decays are sensitive to both $f_+(q^2)$ and $f_-(q^2)$. Defining $\xi = \frac{f_-}{f_+}$, we get

$$\begin{aligned}\mathcal{M} &= \frac{G_F}{\sqrt{2}}\sin\theta_C[f_+(q^2)(p_K^\mu + p_\pi^\mu)\bar{u}_l\gamma_\mu(1-\gamma_5)u_{\nu_l} + \xi f_+(q^2)m_l\bar{u}_l(1-\gamma_5)u_{\nu_l}] \\ &= \frac{G_F}{\sqrt{2}}\sin\theta_C f_+(q^2)[2p_K^\mu\bar{u}_l\gamma_\mu(1-\gamma_5)u_{\nu_l} + (\xi-1)m_l\bar{u}_l(1-\gamma_5)u_{\nu_l}].\end{aligned}$$

The form factors are the functions of q^2 and are generally parameterized as

$$f_\pm(q^2) = \left[1 + \lambda_\pm\left(\frac{q^2}{m_\pi^2}\right)\right]f_\pm(0). \tag{6.90}$$

The analysis of $K\mu_3$ decays is done in terms of $f_+(q^2)$ and $f_0(q^2)$, where $f_0(q^2)$ is defined in terms of $f_-(q^2)$ and $f_+(q^2)$ as

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2}f_-(q^2) = f_0(0)\left(1 + \lambda_0\frac{q^2}{m_\pi^2}\right). \tag{6.91}$$

Here $f_0(q^2)$ is called a scalar form factor. If $f_-(q^2)$ is well behaved near $q^2 \approx 0$, that is, it does not have a pole, then $f_0(0) = f_+(0)$ follows from Eq. (6.91) and $K\mu_3$ decays are described in terms of the parameters $f_+(0)$, λ_+, and λ_0.

The energy distribution is given by [63, 238, 239, 302, 303]:

$$\frac{d\Gamma}{dE_\pi dE_\mu} \propto G_F^2\sin^2\theta_C f_+^2(q^2)[A + B\xi(q^2) + C\xi^2(q^2)], \tag{6.92}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

$$
\begin{aligned}
\text{where} \qquad \xi &= \frac{f_-(q^2)}{f_+(q^2)}, \qquad \text{and} \\
A &= m_K\left[2(E_l E_\nu - m_K E'_\pi) + m_l^2\left(\frac{E'_\pi}{4} - E_\nu\right)\right], \\
B &= m_l^2\left(E_\nu - \frac{E'_\pi}{2}\right), \\
C &= \frac{m_l^2 E'_\pi}{4}, \quad E'_\pi = \frac{m_K^2 + m_\pi^2 - m_l^2}{2m_K} - E_\pi.
\end{aligned}
$$

The three parameters $f_+(q^2), \lambda_+$, and λ_0 are determined from the experimental results of $K\mu_3$ and Ke_3 decay of charged and neutral kaons. The parameters are found by analyzing the total decay rate, energy distribution of the charged particles, branching ratio for $K\mu_3$ (Ke_3) decay, and the polarization of muons (electrons) in the final state. It can be shown that in the case of Ke_3 decays $(m_e \to 0)$, the decay rate is given by:

$$
d\Gamma = \frac{G_F^2 \sin^2\theta_C}{12\pi^3} m_K f_+^2(q^2)|\vec{p}_\pi|^3 dE_\pi, \tag{6.93}
$$

which determines $f_+(q^2)$.

The following are the main features of the aforementioned study:

i) The Ke_3 decay determines the value of $f_+(q^2)$ while $K\mu_3$ decays give the values of $f_+(0)$, λ_+, and λ_0. An average value of $\sin\theta_C|f_+(0)|$ from these experiments is found to be 0.2163 within 0.1% experimental uncertainties [301], from which $f_+(0)$ is found to be 0.961 ± 0.008 [304].

ii) Using the expression given in Eq. (6.92), the function $\rho(E_\pi, E_\mu)$ given by

$$
\rho(E_\pi, E_\mu) = \sin^2\theta_C f_+^2(q^2)[A + B\xi + C\xi^2] \tag{6.94}
$$

is studied and the values of f_+ and ξ are determined which give λ_+ and λ_0.

iii) The branching ratio of $\frac{\Gamma(K\mu_3)}{\Gamma(Ke_3)}$ is determined theoretically using the values of $f_+(q^2), \lambda_+$, and λ_0 and is compared with the experimentally observed values.

iv) These parameters are also sensitive to the muon polarization ($\mathcal{P}$), which is given as $\mathcal{P} = \frac{\vec{A}}{|\vec{A}|}$, where

$$
\begin{aligned}
\vec{A} &= a_1(\xi)\vec{p}_\mu - a_2(\xi)\left[\frac{\vec{p}_\mu}{m_\mu}[m_K - E_\pi + \frac{\vec{p}_\pi \cdot \vec{p}_\mu}{|\vec{p}_\mu|^2}(E_\mu - m_\mu)] + \vec{p}_\pi\right] \\
&\quad + m_K \,\mathrm{Im}\xi(\vec{p}_\pi \times \vec{p}_\mu)
\end{aligned} \tag{6.95}
$$

$$
\text{with} \quad a_1(\xi) = \left(\frac{2m_K^2}{m_\mu}\right)\left[E_\nu + E'_\pi \,\mathrm{Re}\, b(q^2)\right],
$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

$$
\begin{aligned}
a_2(\xi) &= m_K^2 + 2\,\mathrm{Re}\,b(q^2)m_K E_\mu + m_\mu^2|b(q^2)|^2, \\
b(\xi) &= \frac{1}{2}[\xi(q^2)-1].
\end{aligned}
$$

v) Any component of the polarization lying in a direction perpendicular to the decay plane is due to Im$\xi \neq 0$ (see the last term in Eq. (6.95)). The presence of any non-zero value of the imaginary component of ξ implies T violation. This result has been used to test time-reversal invariance in the charged kaon sector using $K\mu_3$ decay.

In addition to the dominant decay modes discussed here, kaons also decay into two pions and two leptons known as Kl_4 decays and radiative decays in which a photon is emitted in addition to leptons and hadrons. These decays have branching ratios in the range $10^{-4}-10^{-5}$ (except $K^\pm \to \mu^\pm\nu_\mu(\bar{\nu}_\mu)\gamma$) and are not discussed here.

6.4.4 Semileptonic decays of hyperons (Y)

The study of semileptonic decays has played a very important role in the theory of weak interactions. We have already seen in the earlier sections that the Cabibbo theory explains the weak decays in the case of the strange sector. In this section, we now use the Cabibbo theory of $V-A$ interactions to describe the semileptonic decays of hyperons. The properties and decay modes of hyperons are given in Tables 6.3 and 6.5. These decays obey the $\Delta S = 1$, $\Delta Q = \Delta S$ rule (except for $\Sigma^- \to \Lambda e^-\bar{\nu}_e$, which is a $\Delta S = 0$ decay like $n \to p + e^- + \bar{\nu}_e$). These decays are described in terms of the $N-Y$ transition form factors occurring in the definition of the matrix elements of the vector and axial vector currents in analogy with the nucleon form factors defined in Eqs. (6.26) and (6.27) but with a coupling strength of $\frac{G_F}{\sqrt{2}}\sin\theta_C$. Hyperon beams are also produced with significant polarization leading to the study of these decays with polarized hyperons. The polarization as well as the spin correlation of the final leptons have also been studied in the decays of unpolarized and polarized hyperons. High energy polarized hyperon beams were used to carry out precision measurements of heavy hyperon decays which helped to establish the Cabibbo theory.

The transition matrix element for the generic hyperon decay $Y(p) \to B(p') + l^-(k) + \bar{\nu}_l(k')$, where Y and B are the initial and final baryon states, is written as:

$$
\mathcal{M} = \frac{G_s}{\sqrt{2}}\bar{u}_B(p')[O_\mu^V - O_\mu^A]u_Y(p)\bar{u}_l\gamma^\mu(1-\gamma_5)\nu_l, \tag{6.96}
$$

where $G_s = G_F\cos\theta_C$ for $\Delta S = 0$ transitions, $G_s = G_F\sin\theta_C$ for $|\Delta S| = 1$ transitions, and [294]

$$
O_\mu^V = f_1(q^2)\gamma^\mu + i\frac{f_2(q^2)}{M_B+M_Y}\sigma_{\mu\nu}q^\nu + \frac{2f_3(q^2)}{M_B+M_Y}q_\mu, \tag{6.97}
$$

$$
O_\mu^A = g_1(q^2)\gamma^\mu\gamma_5 + i\frac{g_2(q^2)}{M_B+M_Y}\sigma_{\mu\nu}q^\nu\gamma_5 + \frac{2g_3(q^2)}{M_B+M_Y}q_\mu\gamma_5 \tag{6.98}
$$

are the operators defining the matrix elements of the vector and the axial vector currents, respectively. M_Y and M_B, are, respectively, the masses of the initial hyperon and the final baryon. There are other parameterizations in the literature which are equivalent to each other in the calculation of the physical observables except in the definition of $f_2(q^2)$ [59, 303, 305].

In the Cabibbo $V-A$ theory with quarks, the vector and axial vector currents V_μ^i and A_μ^i $(i=1-8)$ are the members of two octets defined as (Appendix B)

$$V_\mu^i = \bar{q}\frac{\lambda^i}{2}\gamma_\mu q, \qquad A_\mu^i = \bar{q}\frac{\lambda^i}{2}\gamma_\mu\gamma_5 q, \tag{6.99}$$

where $\frac{\lambda^i}{2}$ are the generators of SU(3). In this notation, the weak charged current in the $\Delta S = 0$ sector are given by $J_\mu^{1\pm i2} = V_\mu^{1\pm i2} - A_\mu^{1\pm i2}$ with $V_\mu^{1\pm i2} = \bar{q}\frac{\lambda_1 \pm i\lambda_2}{2}\gamma_\mu q$, $A_\mu^{1\pm i2} = \bar{q}\frac{\lambda_1 \pm i\lambda_2}{2}\gamma_\mu\gamma_5 q$. The electromagnetic current J_μ^{em} is given by

$$J_\mu^{\mathrm{em}} = V_\mu^3 + \frac{1}{\sqrt{3}}V_\mu^8, \tag{6.100}$$

while the $\Delta S = 1$ current corresponding to the $s \to u$ transition is given by

$$J_\mu^{4+i5} = V_\mu^{4+i5} - A_\mu^{4+i5}. \tag{6.101}$$

The weak hadronic current(CC) is written as

$$\begin{aligned} J_\mu^{\pm cc} &= V_{ud}(V_\mu^{1\pm i2} - A_\mu^{1\pm i2}) + V_{us}(V_\mu^{4\pm i5} - A_\mu^{4\pm i5}), \\ \text{where} \quad V_{ud} &= \cos\theta_C,\ V_{us} = \sin\theta_C \end{aligned} \tag{6.102}$$

The Cabibbo theory, assuming SU(3) symmetry, describes the relation among the various form factors of the weak transition, that is, $f_i(q^2)$ and $g_i(q^2)$ $(i=1,2,3)$. It relates the vector form factors $f_1(q^2)$ and $f_2(q^2)$ to the electromagnetic form factors of the proton and the neutron; it also relates the axial vector form factor $g_1(q^2)$ to the axial vector form factor in the nucleon sector. This is because the matrix elements of an octet operator O_μ^i; $i = V, A$, taken between two octets of baryons Y and B can be written as:

$$\langle Y_i | O_j^{V,A} | B_k \rangle = f_{ijk}F^{V,A} + d_{ijk}D^{V,A}, \tag{6.103}$$

where D and F correspond to symmetric and antisymmetric coupling of the two octets to an octet (i.e., octet of B_k and O_j coupling to the octet Y_i) in the decomposition $8\otimes 8 = 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27$. The superscripts V, A in Eq. (6.103) correspond to the vector and axial vector currents and the coefficient f_{ijk} and d_{ijk} are the structure constants of SU(3). Therefore, all the vector form factors corresponding to the vector current operator O^V for all the decays shown in Table 6.7 are expressed in terms of F^V and D^V which are determined in terms of the electromagnetic form factors $F_{i=1,2}^{p,n}(q^2)$. Similarly, all the axial vector form factors are determined in terms of F^A and D^A which are in turn given in terms of the axial vector form factor for the process $n \to p + e^- + \bar{\nu}_e (g_1 = F^A + D^A)$ and one unknown axial vector parameter chosen to be D^A/F^A. This unknown parameter is experimentally determined from the analysis of semileptonic hyperon decays. Following the methods shown in detail in Appendix B, we obtain the form factors for various hyperon decays as shown in Table 6.7, where the superscript A on D and F are dropped for brevity.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Table 6.7 Cabibbo model predictions for the β-decays of baryons [306]. Here, for simplicity, we have dropped the superscript A from F, D in $g_1(0)$ and g_1/f_1. The variables used in the table are defined as: $V_{ud} = \cos\theta_C$, $V_{us} = \sin\theta_C$, $\mu_p = 1.7928\mu_N$, and $\mu_n = -1.9130\mu_N$.

Decay	Scale	$f_1(0)$	$g_1(0)$	g_1/f_1	f_2/f_1
$n \to pe^-\bar{\nu}$	V_{ud}	1	$D+F$	$F+D$	$\mu_p - \mu_n$
$\Xi^- \to \Xi^0 e^-\bar{\nu}$	V_{ud}	-1	$D-F$	$F-D$	$\mu_p + 2\mu_n$
$\Sigma^\pm \to \Lambda e^\pm \nu$	V_{ud}	0^a	$\sqrt{\frac{2}{3}}D$	$\sqrt{\frac{2}{3}}D$	$-\sqrt{\frac{3}{2}}\mu_n$
$\Sigma^- \to \Sigma^0 e^-\bar{\nu}$	V_{ud}	$\sqrt{2}$	$\sqrt{2}F$	F	$\frac{(2\mu_p+\mu_n)}{2}$
$\Sigma^0 \to \Sigma^+ e^-\bar{\nu}$	V_{ud}	$\sqrt{2}$	$-\sqrt{2}F$	$-F$	$\frac{(2\mu_p+\mu_n)}{2}$
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}$	V_{us}	1	$D+F$	$F+D$	$\mu_p - \mu_n$
$\Xi^- \to \Sigma^0 e^-\bar{\nu}$	V_{us}	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}(D+F)$	$F+D$	$\mu_p - \mu_n$
$\Sigma^- \to ne^-\bar{\nu}$	V_{us}	-1	$D-F$	$F-D$	$\mu_p + 2\mu_n$
$\Sigma^0 \to pe^-\bar{\nu}$	V_{us}	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}(D-F)$	$F-D$	$\mu_p + 2\mu_n$
$\Lambda \to pe^-\bar{\nu}$	V_{us}	$-\sqrt{\frac{3}{2}}$	$-\frac{1}{\sqrt{6}}(D+3F)$	$F+\frac{D}{3}$	μ_p
$\Xi^- \to \Lambda e^-\bar{\nu}$	V_{us}	$\sqrt{\frac{3}{2}}$	$-\frac{1}{\sqrt{6}}(D-3F)$	$F-\frac{D}{3}$	$-(\mu_p+\mu_n)$

[a]Since $f_1(0) = 0$ for $\Sigma^\pm \to \Lambda e^\pm \nu$, predictions are given for f_2 and g_1 rather than f_2/f_1 and g_1/f_1.

In the vector sector, the scalar form factor $f_3(q^2)$ vanishes due to the hypothesis of the conserved vector current as well as the G invariance; in the axial vector sector, the weak electric form factor $g_2(q^2)$ violates the G-parity in the limit of exact SU(3) symmetry and is assumed to vanish, that is, $g_2(q^2) = 0$. The G-parity and the absence of the second class currents are well supported in the $\Delta S = 0$ current of the nucleon sector as the isospin symmetry is a very good symmetry. However, the validity of G-parity in the case of $|\Delta S| = 1$ processes is only as good as the validity of SU(3) symmetry and is expected to be at the level of 15–20%. Nevertheless, G invariance is generally assumed in analyzing semileptonic decays of hyperons.

6.4.5 Physical observables in semileptonic hyperon decays

Experimental studies of semileptonic hyperon decays have been made possible by the availability of high intensity polarized beams of hyperons at CERN, BNL, and FNAL. The following precision measurements on the following observables were analyzed using the Cabibbo theory: (i) decay rate, (ii) the lepton–neutrino angular correlations, (iii) the asymmetry coefficients for the decay of polarized hyperons, and (iv) the polarization of the final state baryons in the case of unpolarized hyperon decays. The analysis was made using the following general assumptions:

i) SU(3) symmetry is present but some symmetry breaking effects are included by retaining the different masses of nucleons and hyperons, that is, $\delta = M_Y - M_B$.

ii) A dipole parameterization is used for the $N - Y$ transition form factors $f_i(q^2)$ and $g_i(q^2)$, $(i = 1 - 3)$.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

iii) T invariance is assumed which implies that all the vector and axial vector form factors are real.

iv) The CVC hypothesis requires that $f_3(q^2) = 0$, while the assumption of G invariance, that is, the absence of second class currents, requires $f_3(q^2) = 0$ and $g_2(q^2) = 0$.

v) There are contributions proportional to $\frac{m_e}{M}$ and $\frac{|\vec{q}|}{M}$ appearing in the analysis of the experiments which are expected to be small. While the terms dependent upon $\frac{m_e}{M}$ are neglected, the terms up to second order $O(\frac{q^2}{M^2})$ are included. The muon decay modes are generally analyzed separately by taking the term $O(\frac{m_\mu}{M})$ also into account.

With these assumptions, an effective Hamiltonian for the decay $Y \to Be\bar{\nu}$ is defined as:

$$\mathcal{M} =< Be\bar{\nu}|H_{\text{eff}}|Y >$$

with [306],

$$H_{\text{eff}} = \sqrt{2}G_S \frac{1 - \vec{\sigma}_e.\hat{p}_e}{2} \left[G_V + G_A\vec{\sigma}_e.\vec{\sigma}_B + G_P^e\vec{\sigma}_B.\hat{p}_e + G_p^\nu\vec{\sigma}_B.\hat{p}_\nu\right] \frac{1 - \vec{\sigma}_e.\hat{p}_\nu}{2}, \tag{6.104}$$

where the effective couplings $G_i(i = V, A, P)$ are derived in terms of $f_i(q^2)$ and $g_i(q^2)$ using the Dirac spinors for the baryons $|Y >$ and $|B >$; $\hat{p}_e$, $\hat{p}_\nu$ are the unit vectors along the direction of the electron and antineutrino momenta; and $\vec{\sigma}_e$ and $\vec{\sigma}_B$ are the spins of the electron and the final baryon operating on the two-component spin wave function of the electron and final baryon, respectively. In the rest frame of the decaying hyperon Y, the couplings are given (neglecting g_3 as being proportional to the lepton mass, its contribution is negligible) as:

$$G_V = f_1 - \delta f_2 - \frac{E_\nu + E_e}{2M_Y}f_1', \qquad G_A = -g_1 + \delta g_2 + \frac{E_\nu - E_e}{2M_Y}f_1'$$

$$G_P^e = \frac{E_e}{2M_Y}[-f_1' + g_1 + \Delta g_2], \qquad G_P^\nu = \frac{E_\nu}{2M_Y}[f_1' + g_1 + \Delta g_2],$$

with $f_1' = f_1 + \Delta f_2$, $\delta = \frac{M_Y - M_B}{M_Y}$, $\Delta = \frac{M_Y + M_B}{2M_Y}$. In the rest frame of the initial hyperon with polarization $\vec{\mathcal{P}}_Y$, the decay distribution is given as:

$$d\Gamma = \frac{E_B + M_B}{2M_Y} \frac{{E_e}^2 E_\nu^3}{E_e^{\text{max}} - E_e} \frac{|\mathcal{M}|^2}{(2\pi)^5} dE_e d\Omega_e d\Omega_\nu, \tag{6.105}$$

where

$$|\mathcal{M}|^2 = G_s^2\xi[1 + a\hat{p}_e.\hat{p}_\nu + A\vec{\mathcal{P}}_Y \cdot \hat{p}_e + B\vec{\mathcal{P}}_Y \cdot \hat{p}_\nu + A'(\vec{\mathcal{P}}_Y \cdot \hat{p}_e)(\hat{p}_e.\hat{p}_\nu)$$
$$+B'(\vec{\mathcal{P}}_Y \cdot \hat{p}_\nu)(\hat{p}_e.\hat{p}_\nu) + D\vec{\mathcal{P}}_Y \cdot (\hat{p}_e \times p_\nu)], \tag{6.106}$$

and the coefficients ξ, a, A, B, A', B', and D are given in terms of the couplings G_i $(i = V, A, P)$. Similar expressions can be derived for the decay of the polarized hyperon in which the polarization of the final baryon is observed. The complete expressions for the decay distribution of leptons in the case of the decays of polarized hyperons and for the polarization

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

of the final baryon in the case of the decay of unpolarized hyperons are not reproduced here and can be found in Ref. [306].

Since the vector form factors are given in terms of the electromagnetic form factors, which are already determined from electron scattering, the analysis of semileptonic hyperon decays are done in terms of the Cabibbo angle θ_C and the axial vector form factors. The axial vector form factors are given in terms of the couplings F^A and D^A. The q^2 dependence of the D^A and F^A are taken to be the same as the q^2 dependence of the axial vector form factor $g_1(q^2)$ in neutron decay or in quasielastic scattering processes like $\nu_\mu + n \to \mu^- + p$ because $g_1(q^2) = D^A(q^2) + F^A(q^2)$. Therefore, the following parameterizations are used for the vector and axial vector form factors:

$$\frac{f_1(q^2)}{f_1(0)} = \frac{f_2(q^2)}{f_2(0)} = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2} \quad \text{and} \quad g_1(q^2) = \frac{g_1(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2} \tag{6.107}$$

with $M_V = 0.84 \pm 0.04$ GeV, $M_A = 1.08 \pm 0.08$ GeV; $g_1(0) = D^A + F^A = 1.26$ is taken from the neutron β-decay and the quasielastic (anti)neutrino–nucleon scattering. Many experiments have been done to measure the following observables:

i) The rate of various $\Delta S = 1$ and $\Delta S = 0$ decays.

ii) The electron–neutrino correlations corresponding to the term a in Eq. (6.106).

iii) Electron correlations with respect to the initial polarization of the hyperons corresponding to term A in Eq. (6.106).

iv) The polarization of the final baryons in the decays of unpolarized hyperons.

An analysis of these results gives [306]:

$$\begin{aligned} F^A + D^A &= 1.2670 \pm 0.0030, & \sin\theta_C = 0.2250 \pm 0.0027 \\ F^A - D^A &= -0.314 \pm 0.016. & \end{aligned}$$

These analyses have also been used to test the hypothesis of the G invariance and the presence of second class currents. A non-zero value of the form factor g_2 corresponding to the second class current has been reported in these decays but the results are not conclusive. Table 6.8 presents the values of $\sin\theta_C$ obtained from the semileptonic decays of hyperons. It should be noted that all semileptonic decays of strange mesons and hyperons satisfy the $\Delta I = \frac{1}{2}$ rule experimentally, which is predicted in a natural way in the Cabibbo model through Eq. (6.78) in which an s-quark changes to a u-quark.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Table 6.8 Results from $\sin\theta_C$ analysis using measured g_1/f_1 values. The table has been taken from Cabibbo et al. [306].

Decay process	Rate (μs^{-1})	g_1/f_1	$\sin\theta_C$
$\Lambda \to pe^-\bar{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \to ne^-\bar{\nu}$	6.88(24)	−0.340(17)	0.2282 ± 0.0049
$\Xi^- \to \Lambda e^-\bar{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}$	0.876(71)	1.32(+.22/−.18)	0.209 ± 0.027
Combined	—	—	0.2250 ± 0.0027

6.5 Nonleptonic Decays of Strange Particles

6.5.1 Nonleptonic decays of K-mesons

Nonleptonic decays are those decays in which a heavier meson decays into lighter mesons involving no leptons. Charged kaons like K^+ and K^- as well as neutral kaons like K_L^0 and K_S^0 are both observed to decay by these modes. The dominant nonleptonic decay modes of $K^\pm$ and $K_{L,S}^0$ mesons are listed in Table 6.4. Since $K_L^0 \longrightarrow \pi^+\pi^-$ and $K_L^0 \longrightarrow \pi^0\pi^0$ are CP violating decays, we describe them separately in the next section. In this section, we will discuss two and three particle decay modes assuming CP invariance.

(i) $K\pi_2$ decays

The two pions in the decays of K^+, K^-, and K_S^0 are in zero angular momentum state, which is always symmetric in space. Therefore, the symmetry of the final state is described by their isospin contents. Since the pions are isovector, the pionic states can have total isospin $I = 0, 1, 2$. The pion in $I = 0$ or $I = 2$ states are symmetric; whereas, $I = 1$ state is antisymmetric. Since K^+, K^0, K^-, and $\bar{K}^0$ states belong to isospin $I = \frac{1}{2}$, the interaction for $K\pi_2$ decays should have $\Delta I = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. If we assume that $\Delta I = \frac{1}{2}$ amplitude dominates, then only $I = 0$, $I_3 = 0$ state would be possible in the final state for K^0 decays. Writing

$$|I=0, I_3=0\rangle = \frac{1}{\sqrt{3}}|\pi^+(1)\pi^-(2) + \pi^-(1)\pi^+(2) - \pi^0(1)\pi^0(2)\rangle, \tag{6.108}$$

it is predicted that

$$\frac{\Gamma(K_S^0 \longrightarrow \pi^+\pi^-)}{\Gamma(K_S^0 \longrightarrow \pi^0\pi^0)} = 2, \tag{6.109}$$

apart from the phase factor. In the case of $K^+ \longrightarrow \pi^+\pi^0$, the final state has $I_3 = 1$; therefore, $I = 0$ is excluded. Hence in the final state I should be 2 implying that only $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{5}{2}$ amplitudes will contribute. With the assumption of $\Delta I = \frac{1}{2}$ dominance, the rate of $K^\pm \longrightarrow \pi^\pm\pi^0$ should be very small (forbidden in exact limit) as compared to $K_S^0 \longrightarrow \pi^+\pi^-(\pi^0\pi^0)$.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Denoting the amplitudes in $\Delta I = \frac{1}{2}, \frac{3}{2}$, and $\frac{5}{2}$ channels as a_1, a_3, and a_5, it can be shown that [239]:

$$\frac{\Gamma(K^+ \longrightarrow \pi^+\pi^0)}{\Gamma(K_s \longrightarrow \pi^+\pi^-) + \Gamma(K_s \longrightarrow \pi^0\pi^0)} = \frac{\frac{3}{4}|a_3 - \frac{2}{5}a_5|^2}{A_0^2 + (\mathrm{Re}A_2)^2} \simeq \frac{\frac{3}{4}|a_3 - \frac{2}{5}a_5|^2}{|a_1|^2}, \tag{6.110}$$

where

$$\begin{aligned} A_0 &= \langle \pi\pi \rangle^S_{I=0} |H_{wk}|K^0\rangle, & A_2 &= \langle (\pi\pi)^S_{I=2}|H_{wk}|K^0\rangle, \\ a_1 &= A_0 e^{i\delta_0}, & a_3 + a_5 &= A_2 e^{i\delta_2}. \end{aligned}$$

Here δ_0 and δ_2 are the pion–pion scattering phase shifts in $I = 0$ and $I = 1$ channels. A detailed analysis and comparison with the experimental values of $K\pi_2$ decay rates gives [239]:

$$\left|\frac{a_3}{a_1}\right| = 0.045 \pm 0.005 \qquad \text{and} \qquad \left|\frac{a_5}{a_1}\right| = 0.001 \pm 0.003. \tag{6.111}$$

This supports $\Delta I = \frac{1}{2}$ dominance in $K\pi_2$ decays.

(ii) $K\pi_3$ decays

Three pion decays of charged and neutral kaons are given in Table 6.4. The final state has three pions, so they can have $I = 0, 1, 2, 3$ with

$$1 \oplus 1 \oplus 1 = (0 \oplus 1 \oplus 2) \oplus 1 = \overbrace{1} \oplus \overbrace{0 \oplus 1 \oplus 2} \oplus \overbrace{1 \oplus 2 \oplus 3}. \tag{6.112}$$

This has multiplicities of 1 for $I = 0$; 3 for $I = 1$; 2 for $I = 2$; and 1 for $I = 3$ states. Decays, where two pions are either $\pi^+\pi^+$ or $\pi^-\pi^-$ or $\pi^0\pi^0$, will be in symmetric states under the exchange of the first two particles. Hence, let us consider

$$\begin{aligned} K^+ &\longrightarrow \pi^+\pi^+\pi^-, & K^- &\longrightarrow \pi^-\pi^-\pi^+, \\ K_L^0 &\longrightarrow \pi^+\pi^-\pi^0, & K_L^0 &\longrightarrow \pi^0\pi^0\pi^0. \end{aligned}$$

$I = 0$ state is completely antisymmetric; $I = 3$ state is completely symmetric; while $I = 1$ and $I = 2$ will be of mixed symmetric states. The symmetry considerations in this case are not very simple so let us first make the assumption that $\Delta I = \frac{1}{2}$ rule is followed and compare its predictions with the experimental results. Assuming $\Delta I = \frac{1}{2}$ rule, the final state could be $I = 0$ or $I = 1$. Since $I = 0$ is completely antisymmetric, the dominant contribution will come from the $I = 1$ state, while the $I = 2$ state will contribute only if there is a contribution from the $\Delta I = \frac{3}{2}$ amplitudes. The three pion states can be expressed in terms of the isospin states as:

$$\begin{aligned} |\pi^+\pi^+\pi^-\rangle &= \frac{2}{\sqrt{5}}|I = 1, I_3 = 1\rangle + \frac{1}{\sqrt{5}}|I = 3, I_3 = 1\rangle, \\ |\pi^+\pi^0\pi^0\rangle &= \frac{1}{\sqrt{5}}|I = 1, I_3 = 1\rangle + \frac{2}{\sqrt{5}}|I = 3, I_3 = 1\rangle, \end{aligned}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

$$|\pi^+\pi^-\pi^0\rangle = \sqrt{\frac{2}{5}}|I=1, I_3=0\rangle + \sqrt{\frac{3}{5}}|I=3, I_3=0\rangle,$$

$$|\pi^0\pi^0\pi^0\rangle = -\sqrt{\frac{3}{5}}|I=1, I_3=0\rangle + \sqrt{\frac{2}{5}}|I=3, I_3=0\rangle.$$

If we assume that only $I=1$ state contributes due to the $\Delta I = \frac{1}{2}$ rule, then assuming CP invariance, we get the following relations:

$$\begin{aligned}
\frac{\Gamma(K^+ \longrightarrow \pi^+\pi^+\pi^-)}{\Gamma(K^+ \longrightarrow \pi^+\pi^0\pi^0)} &= 4,\\
\frac{\Gamma(K_L^0 \longrightarrow \pi^+\pi^-\pi^0)}{\Gamma(K_L^0 \longrightarrow \pi^0\pi^0\pi^0)} &= \frac{2}{3},\\
\frac{\Gamma(K_L^0 \longrightarrow \pi^+\pi^-\pi^0)}{\Gamma(K^+ \longrightarrow \pi^+\pi^0\pi^0)} &= 2,\\
\frac{\Gamma(K_L^0 \longrightarrow \pi^0\pi^0\pi^0)}{\Gamma(K^+ \longrightarrow \pi^+\pi^+\pi^-) - \Gamma(K_L^0 \longrightarrow \pi^+\pi^0\pi^0)} &= 1.
\end{aligned}$$

Detailed analysis of $K\pi_3$ decays and comparison with experimental data shows that [239]:

$$\left|\frac{a_3}{a_1}\right| = 0.06 \pm 0.01, \tag{6.113}$$

supporting the dominance of $\Delta I = \frac{1}{2}$ rule.

There is no theoretical explanation of the $\Delta I = \frac{1}{2}$ dominance in the $V-A$ theory.

6.5.2 Nonleptonic decays of hyperons

Nonleptonic decays of hyperons are of the type $Y \to B + M$, where the hyperon (Y) decays into a baryon (B) which can be a nucleon or a hyperon and a meson (M). The list of hyperon two-body decays is given in Table 6.5. All the decays have $\Delta S = 1$; no decay with $\Delta S > 1$ has been observed. In fact, the decay $\Lambda \to p\pi^-$ was historically the first decay to be observed which indicates the existence of parity violation; this violation was not noticed at that time [72].

In the phenomenological theory, these decays as well as the nonleptonic mesonic decays are supposed to be explained by the term of the type $\frac{G}{\sqrt{2}}(J_\mu)^h (J^\mu)^{h^\dagger}$, that is,

$$\frac{G}{\sqrt{2}}\left[\bar{s}\gamma^\mu(1-\gamma_5)u + \text{h.c.}\right]\left[\bar{u}\gamma^\mu(1-\gamma_5)d + \text{h.c.}\right]^\dagger.$$

Since the first term has $\Delta I = \frac{1}{2}$ and the second term has $\Delta I = 1$, these decays in the phenomenological theory could have $\Delta I = \frac{1}{2}, \frac{3}{2}$ in the isospin space. Therefore, the $V-A$ theory does not predict the dominance of $\Delta I = \frac{1}{2}$ transitions.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

We write the general and simplest term of the matrix element for $Y(\frac{1}{2}^+) \to B(\frac{1}{2}^+) + M(0^-)$ transitions allowing for parity violation, as

$$\mathcal{M} = G_F m_M^2 \bar{u}_Y (a - b\gamma_5) u_B. \tag{6.114}$$

Evaluating the matrix element in the rest frame of the initial baryon, using

$$\begin{aligned}
u_i &= \begin{pmatrix} \chi_i \\ 0 \end{pmatrix}, \quad \bar{u}_i u_i = 1 \\
u_f &= \sqrt{\frac{E_B + M_B}{2M_B}} \begin{pmatrix} \chi_f \\ \frac{\vec{\sigma}.\vec{p}}{E_B+M_B}\chi_f \end{pmatrix}, \quad \bar{u}_f u_f = 1 \\
\Rightarrow \qquad \mathcal{M} &= G_F m_M^2 \sqrt{\frac{E_B + M_B}{2M_B}} \chi_f^\dagger \Big(a + b\frac{\vec{\sigma}.\vec{p}}{E_B + M_B} \Big) \chi_i \\
&= G_F m_M^2 \sqrt{\frac{E_B + M_B}{2M_B}} \chi_f^\dagger (S + P\vec{\sigma}.\hat{n}) \chi_i,
\end{aligned}$$

where $S = a$, $P = \frac{b|\vec{p}|}{E_B+M_B}$, $\hat{n} = \frac{\vec{p}}{|\vec{p}|}$, and M_B is the mass of the final baryon. If $\Delta I = \frac{1}{2}$ dominance is assumed, then

$$\frac{\Gamma(\Lambda \longrightarrow p\pi^-)}{\Gamma(\Lambda \longrightarrow n\pi^0)} \simeq 2. \tag{6.115}$$

Moreover, a relation between the amplitudes of the various decay modes of hyperons is obtained as

$$A(\Sigma^+ \longrightarrow n\pi^+) - A(\Sigma^- \longrightarrow n\pi^-) = -\sqrt{2}A(\Sigma^+ \longrightarrow p\pi^0), \tag{6.116}$$

which seems to be satisfied quite well experimentally, once the appropriate corrections are made for the final state interactions in the respective pion nucleon final states.

A careful analysis of nonleptonic hyperons decays gives [239]:

$$\Lambda \longrightarrow N\pi : \quad \left|\frac{a_3}{a_1}\right| \quad = \quad 0.03^{\pm 0.01 \text{ (for s-wave)}}_{\pm 0.03 \text{ (for p-wave)}} \quad \text{and} \quad \Sigma \longrightarrow N\pi : \quad \left|\frac{a_{I\geq\frac{3}{2}}}{a_1}\right| = 0.07 \pm 0.03.$$

It is to be noted that a_1 and a_3 amplitudes are expected to be of the same order of magnitude as that obtained in the processes satisfying $\Delta Q = \Delta S$, $\Delta I = \frac{1}{2}$ rules in the semileptonic decays of strange particles. However, the amplitude a_1 is higher than a_3 by a factor of $15 - 30$. In view of this, Cabibbo formulated a model for the semileptonic weak interactions assuming that the physics of the weak interaction processes in the semileptonic decays are completely different from the nonleptonic $\Delta S = 1$ decays.

In the case of a hyperon decay like $\Lambda \to N\pi, \Sigma \to N\pi$ etc., using the matrix element given in Eq. (6.114), the decay rate is obtained as [239],

$$d\Gamma \propto 1 + \gamma\hat{s}.\hat{s'} + (1-\gamma)\vec{s}.\hat{n}\, \vec{s}'.\hat{n} + \alpha(\vec{s}.\hat{n} + \vec{s}'.\hat{n}) + \beta(\vec{s} \times \vec{s}').\hat{n} \tag{6.117}$$

where $\vec{s}$ and $\vec{s}\,'$ are the spins of the initial and final baryons.

$$\alpha = 2\mathrm{Re}\frac{SP^*}{|S|^2+|P|^2}, \qquad \beta = 2\mathrm{Im}\frac{SP^*}{|S|^2+|P|^2},$$
$$\gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2},$$

with $\alpha^2+\beta^2+\gamma^2=1$. For unpolarized hyperon decays, summing over all the directions of $\vec{s}$, we find:

$$d\Gamma \propto 1+\alpha\vec{s}\,'.\hat{n}, \tag{6.118}$$

where α gives the polarization of the final baryon. For polarized hyperon decays, summing over the directions of $\vec{s}\,'$, we find:

$$d\Gamma \propto 1+\alpha\vec{s}.\hat{n}. \tag{6.119}$$

The parameter α gives the asymmetry of the final baryon with respect to the spin polarization of the initial hyperon. α is used to determine the polarization of the initial hyperon. In the presence of T invariance, the amplitudes S and P are real and imply $\beta = 0$. Therefore, a non-zero value of β gives a measure of T non-invariance in these decays. However, an analysis of the final state interactions (FSI) in the πN system need to be done to study the presence of T non-invariance, if any. Table 6.9 lists the decay rates and asymmetry observed in these decays.

Table 6.9 Nonleptonic decays of hyperons [117].

Decay	Asymmetries	Branching ratios	Decay rate ($10^3\ \mu s^{-1}$)
$\Lambda \to p\pi^-$	0.750 ± 0.010	$(63.9 \pm 0.5)\%$	2.428
$\Lambda \to n\pi^0$	0.692 ± 0.017	$(35.8 \pm 0.5)\%$	1.36
$\Sigma^+ \to p\pi^0$	-0.980 ± 0.015	$(51.57 \pm 0.3)\%$	6.432
$\Sigma^+ \to n\pi^+$	0.068 ± 0.013	$(48.31 \pm 0.3)\%$	6.025
$\Sigma^- \to n\pi^-$	-0.068 ± 0.008	$(99.848 \pm 0.005)\%$	4.571
$\Xi^0 \to \Lambda\pi^0$	-0.347 ± 0.010	$(99.524 \pm 0.012)\%$	3.432
$\Xi^- \to \Lambda\pi^-$	-0.392 ± 0.008	$(99.887 \pm 0.035)\%$	6.094

6.5.3 Radiative weak decays

Radiative weak decays are those in which a photon is emitted in a weak decay process. There are two types of radiative decays:

(i) Decays in which a photon is emitted in an allowed weak decay at the quark level, for example, in the decays like:

$$\begin{aligned}\Lambda &\longrightarrow p\pi^-\gamma; \qquad \pi^\pm \longrightarrow \mu^\pm\nu(\bar{\nu})\gamma\\ \Sigma^\pm &\longrightarrow n\pi^\pm\gamma; \qquad K^\pm \longrightarrow \mu^\pm\nu(\bar{\nu})\gamma.\end{aligned}$$

In these decays, there are two types of processes which contribute as shown in Figure 6.5. The decays shown in Figures 6.5(a) and 6.5(b) are the internal Bremsstrahlung (IB) contributions in which the charged particle radiates after being emitted; the decay shown in Figure 6.5(c) is called a structure dependent (SD) or direct emission (DE) process in which the photon is emitted from the intermediate states in the transition. There

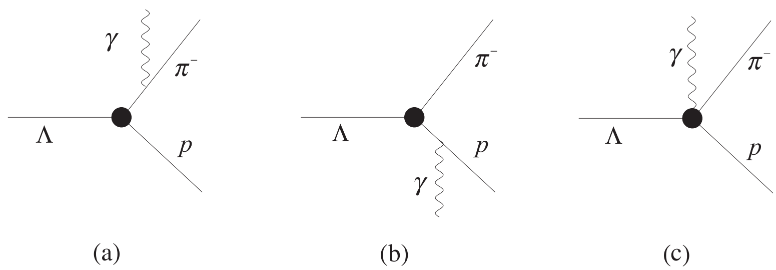

Figure 6.5 Internal Bremsstrahlung and direct emission radiative decays.

could also be contribution from the interference between the two processes. While the weak coupling shown in Figures 6.5(a) and 6.5(b) are known from $V - A$ theory, the form factors and couplings for the decay shown in Figure 6.5(c) are not known and are determined from the requirement of the gauge invariance of the total amplitude in the absence of specific knowledge of the intermediate states.

(ii) Weak radiative decays in which a photon is emitted corresponding to the weak forbidden decays at the quark level like $s \to d\gamma$ or $c \to u\gamma$: Examples of such decays are the radiative decays of hyperons like $\Sigma^+ \to p\gamma$ and $\Lambda \to n\gamma$:

These processes get contributions from three types of decays shown in Figure 6.6(a), (b), and (c) corresponding to one quark, two quark, and three quark transitions. The calculation of the one quark transitions with two spectator quarks is model dependent due to the unknown structure of the direct emission diagram in which many states of intermediate particles can also be excited before radiating. A theoretical model is needed for evaluating all such diagrams. In the simplest model, as shown in Figure 6.6, the photon is either radiated from the virtual W boson or the virtual quark. The two quark transition corresponds to the W exchange between the two quarks in which one of the participating quark radiates with one spectator quark and three quarks transition with no spectator quark in which the third quark not participating in the W exchange radiates. All these contributions are subject to QCD corrections due to the gluon exchange.

There are two types of experimental measurements made on these decays: decay rate as well as the asymmetry parameter of the photon. The amplitude for transitions like

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

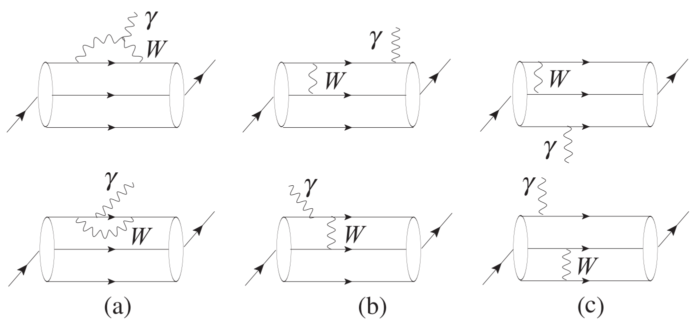

Figure 6.6 Radiative decays corresponding to (a) one quark, (b) two quark, and (c) three quark, transitions.

$B(p) \rightarrow B'(p') + \gamma(k)$ is similar to the structure of nonleptonic decays $B(p) \rightarrow B'(p') + \pi(k)$ and can be written as:

$$\mathcal{M}(B \longrightarrow B'\gamma) = ieG_F\bar{u}(p')\sigma_{\mu\nu}[A_\gamma + B_\gamma\gamma_5]\epsilon^\mu k^\nu, \tag{6.120}$$

where $A_\gamma(B_\gamma)$ are the parity conserving (parity violating) amplitudes. The decay rate (Γ) and the asymmetry (α) is expressed as:

$$\Gamma \quad \propto \quad |A|^2_\gamma + |B|^2_\gamma, \qquad\qquad \alpha \propto \frac{2Re(A^*_\gamma B_\gamma)}{|A|^2_\gamma + |B|^2_\gamma}. \tag{6.121}$$

The theory can be easily extended to weak radiative decays involving heavy flavors, that is, $c \rightarrow u\gamma$ or $b \rightarrow s\gamma$, to describe the radiative decays of charmed and other particles with heavier flavors. The experimental results for the decay rates have been explained satisfactorily in many models by using heavy quarks as shown in Figure 6.6. However, there is no model which describes the decay rate as well as the asymmetry, simultaneously. This remains a problem yet to be understood in the context of the Cabibbo theory.

6.6 CP Violation in the Neutral Kaon Sector

6.6.1 Neutral kaons, CP eigenstates, and $K^0 - \bar{K}^0$ oscillations

Pure K^0 and $\bar{K}^0$ beams are states of definite strangeness and parity, that is, under parity(P) operation,

$$\begin{aligned}\hat{P}|K^0\rangle &= -|K^0\rangle, \\ \hat{P}|\bar{K}^0\rangle &= -|\bar{K}^0\rangle.\end{aligned} \tag{6.122}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Thus, K^0 and $\bar{K}^0$ are definite $\hat{P}$ eigenstates with eigenvalue -1. Under charge conjugation(C) operation,

$$\begin{aligned}\hat{C}|K^0\rangle &= |\bar{K}^0\rangle, \\ \hat{C}|\bar{K}^0\rangle &= |K^0\rangle,\end{aligned} \tag{6.123}$$

which implies K^0 and $\bar{K}^0$ are not definite $\hat{C}$ eigenstates. Under combined CP operation,

$$\begin{aligned}\hat{C}\hat{P}|K^0\rangle &= -|\bar{K}^0\rangle, \\ \hat{C}\hat{P}|\bar{K}^0\rangle &= -|K^0\rangle.\end{aligned} \tag{6.124}$$

Thus, K^0 and $\bar{K}^0$ are not definite eigenstates of combined CP operation. However, their linear combinations are eigenstates of CP. We can define

$$\begin{aligned}|K_1^0\rangle &= \tfrac{1}{\sqrt{2}}\left\{|K^0\rangle + |\bar{K}^0\rangle\right\} \\ |K_2^0\rangle &= \tfrac{1}{\sqrt{2}}\left\{|K^0\rangle - |\bar{K}^0\rangle\right\}\end{aligned} \tag{6.125}$$

such that

$$\begin{aligned}\hat{C}\hat{P}|K_1^0\rangle &= -\frac{1}{\sqrt{2}}\left\{|\bar{K}^0\rangle + |K^0\rangle\right\} = -|K_1^0\rangle, \\ \hat{C}\hat{P}|K_2^0\rangle &= -\frac{1}{\sqrt{2}}\left\{|\bar{K}^0\rangle - |K^0\rangle\right\} = +|K_2^0\rangle.\end{aligned} \tag{6.126}$$

Notice that the $|K_1^0\rangle$ and $|K_2^0\rangle$ states are definite CP eigenstates with eigenvalue –1 and $+1$, respectively. Since $|K_1^0\rangle$ and $|K_2^0\rangle$ are not particle and antiparticle states, their masses as well as lifetimes may differ. K-mesons decay either into two or three pions. Since the pions have negative intrinsic parity and the two pion and three pion states are in S-states for kaons decaying at rest, the 2π states have $(\mathrm{CP})_{2\pi} = +1$ and the three pion states have $(\mathrm{CP})_{3\pi} = -1$. Therefore, CP conservation implies $K_1^0 \to 3\pi$ and $K_2^0 \to 2\pi$. The 2π decay is faster because of the availability of more phase space than the 3π decay; thus, K_2^0 has a shorter lifetime of $\tau_S = 0.892 \times 10^{-10}$s as compared to K_1^0 which has a longer lifetime of $\tau_L = 5.18 \times 10^{-8}$s. For this reason, K_1^0 is called K_L^0 and K_2^0 is called K_S^0. The strangeness eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$ are expressed in terms of $|K_S^0\rangle$ and $|K_L^0\rangle$ as

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left[|K_S^0\rangle + |K_L^0\rangle\right], \tag{6.127}$$

$$|\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left[|K_L^0\rangle - |K_S^0\rangle\right]. \tag{6.128}$$

Since strangeness is conserved in the strong interactions, K^0 and $\bar{K}^0$ can be produced in processes like $\pi^- p \to K\Lambda$ but they decay into 2π and 3π modes through weak interactions which violate strangeness. However, if we assume T invariance or CP invariance (due to the CPT invariance) of the physical laws, then only K_1^0 would decay into pion states with CP=-1;

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

K_2^0 would decay into pion states with CP = $+1$. Since strangeness is not conserved in weak interactions, and both K^0 to $\bar{K}^0$ are neutral particles, it was suggested that K^0 may get converted into $\bar{K}^0$ while propagating and vice versa. However, this transformation of K^0 to $\bar{K}^0$ or $\bar{K}^0$ to K^0 is not possible in the first order at the tree level because of the $\Delta S = 1$ rule. It is possible only in the second order with the exchange of two W bosons as shown in Figure 6.7. This simply means that $|K^0\rangle$ and $|\bar{K}^0\rangle$ are just production states, and they propagate in space as a linear combination of some other states, say $|K_1^0\rangle$ and $|K_2^0\rangle$.

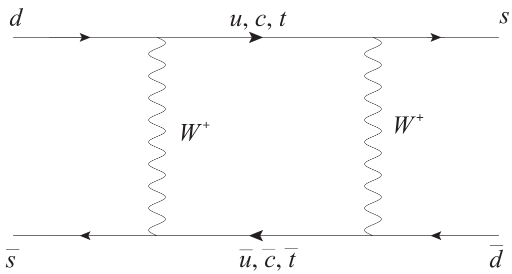

Figure 6.7 $K^0 \longrightarrow \bar{K}^0$ oscillation via two W bosons exchange.

Assuming CP invariance, we describe the phenomena of the evolution of a kaon beam and kaon oscillation ($K^0 \leftrightarrow \bar{K}^0$). We start with a K^0 beam produced at, say, time $t = 0$, represented by a state $|\psi\rangle$ as

$$|\psi(t=0)\rangle = |K^0\rangle = \frac{1}{\sqrt{2}}\left[|K_S^0\rangle + |K_L^0\rangle\right]. \tag{6.129}$$

This state evolves with time such that at a later time t, the time-dependent wave function of $|K^0\rangle$, using the expression for the time evolution of a state is:

$$|\psi(t)\rangle = |\psi(t=0)\rangle e^{-iEt} \quad = \quad |\psi(t=0)\rangle e^{-i\left(m-\frac{i\Gamma}{2}\right)t}, \tag{6.130}$$

where Γ^{-1} is the lifetime. Equation (6.130) may be rewritten as

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{\sqrt{2}}\left[e^{-im_S t}e^{-\frac{\Gamma_S t}{2}}|K_S^0\rangle + e^{-im_L t}e^{-\frac{\Gamma_L t}{2}}|K_L^0\rangle\right] \\ &= \frac{e^{-im_S t}}{\sqrt{2}}\left[e^{-\frac{\Gamma_S t}{2}}|K_S^0\rangle + e^{-i(m_L-m_S)t}e^{-\frac{\Gamma_L t}{2}}|K_L^0\rangle\right] \\ &= \frac{e^{-im_S t}}{\sqrt{2}}\left[e^{-\frac{\Gamma_S t}{2}}|K_S^0\rangle + e^{-i\Delta m t}e^{-\frac{\Gamma_L t}{2}}|K_L^0\rangle\right], \end{aligned} \tag{6.131}$$

where $\Delta m = m_L - m_S$, with m_S and m_L being the masses of shorter and longer lived kaons, respectively. Using Eq. (6.125), we find:

$$|\psi(t)\rangle = \frac{e^{-im_S t}}{2}\left[e^{-\frac{\Gamma_S t}{2}}\left\{|K^0\rangle - |\bar{K}^0\rangle\right\} + e^{-i\Delta m t}e^{-\frac{\Gamma_L t}{2}}\left\{|K^0\rangle + |\bar{K}^0\rangle\right\}\right]. \tag{6.132}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

The probability of observing $|K^0\rangle$ after a time t is

$$\begin{aligned}
P(K^0 \longrightarrow K^0) &= |\langle K^0|\psi(t)\rangle|^2 \\
&= \langle K^0|\psi(t)\rangle \langle K^0|\psi(t)\rangle^* \\
&= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t} e^{i\Delta m t} + e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t} e^{-i\Delta m t}\right]
\end{aligned}$$

$\because \frac{\tau_S}{\tau_L} \sim \frac{1}{570} \Rightarrow \Gamma_L \sim 570\Gamma_S \Rightarrow t\Gamma_S$ would be very small, therefore, $e^{-t\Gamma_S} \approx 1$,

$$P(K^0 \longrightarrow K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\left(\frac{\Gamma_L}{2}\right)t} (\cos \Delta m t)\right]. \tag{6.133}$$

The probability of finding $|\bar{K}^0\rangle$ after a time t, in the initial K^0 beam, is

$$\begin{aligned}
P(K^0 \longrightarrow \bar{K}^0) &= |\langle \bar{K}^0|\psi(t)\rangle|^2 \\
&= \frac{1}{4}\left[e^{-\Gamma_S t} - e^{-\Gamma_L t} - e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t} e^{i\Delta m t} - e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t} e^{-i\Delta m t}\right] \\
&= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_L}{2}\right)t} (\cos \Delta m t)\right].
\end{aligned} \tag{6.134}$$

It can be seen from Eqs. (6.133) and (6.134) that:

$$\begin{aligned}
P(K^0 \longrightarrow K^0) &= P(\bar{K}^0 \longrightarrow \bar{K}^0) \quad \text{and} \\
P(K^0 \longrightarrow \bar{K}^0) &= P(\bar{K}^0 \longrightarrow K^0).
\end{aligned} \tag{6.135}$$

Figure 6.8 plots the probability of the neutral kaons oscillation. The curve (a) represents the surviving probability of K^0, that is, P($K^0 \to K^0$) while the curve (b) represents the oscillating

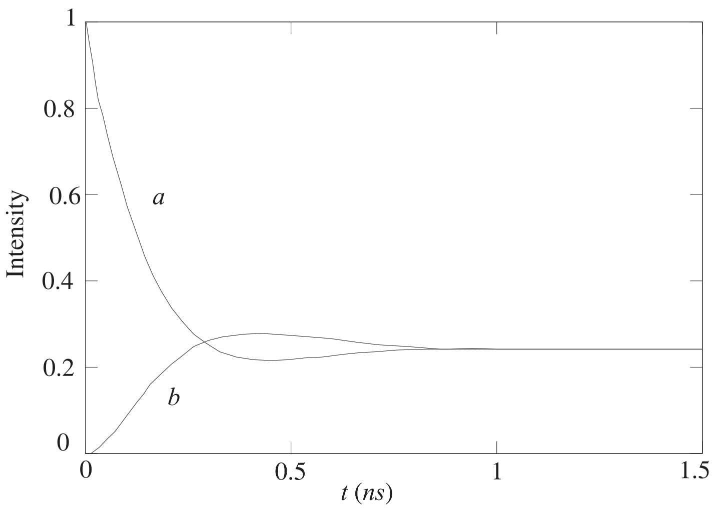

Figure 6.8 Probability of neutral kaon oscillation. (a) represents the survival probability of K^0, that is, $P(K^0 \to K^0)$, while (b) represents the oscillation probability, $P(K^0 \to \bar{K}^0)$, for a fixed $\Delta m \neq 0$.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

probability of K^0 to $\bar{K}^0$, that is, $P(K^0 \to \bar{K}^0)$, for a fixed value of Δm. The intensity of $\bar{K}^0$ rises from zero and shows oscillatory behavior with the same frequency. Using the observed data for oscillations ($K^0 \leftrightarrow \bar{K}^0$), the mass difference $m_L - m_S$ is found to be 3.52×10^{-12} MeV. It has been observed experimentally that the mass difference between K_1^0 and K_2^0 is $3.484 \pm 0.006 \times 10^{-12}$ MeV. $\Delta m = m_L - m_S$ is a positive quantity; this implies that the shorter lived kaon is less massive than the longer lived kaon.

6.6.2 CP violation in the neutral kaon decays

In 1964, Christenson et al. [73] observed that one in a thousand events $|K_L^0\rangle$ also decays to a two pion mode which was a clear evidence of CP violation. The degree of CP violation is generally presented by the ratio of the amplitudes for the processes

$$\begin{aligned} K_L^0 &\longrightarrow \pi^+\pi^- \qquad \text{and} \qquad K_S^0 \longrightarrow \pi^+\pi^-, \\ K_L^0 &\longrightarrow \pi^0\pi^0 \qquad \text{and} \qquad K_S^0 \longrightarrow \pi^0\pi^0. \end{aligned} \tag{6.136}$$

In the presence of CP violation, K_L^0 and K_S^0 would not be the pure CP eigen states as K_1^0 and K_2^0, but would involve a small admixture of the state with opposite CP. We, therefore define [239]:

$$|K_S^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon_1|^2)}}\left[(1+\epsilon_1)|K_0\rangle - (1-\epsilon_1)|\bar{K}^0\rangle\right], \tag{6.137}$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon_2|^2)}}\left[(1+\epsilon_2)|K_0\rangle + (1-\epsilon_2)|\bar{K}^0\rangle\right], \tag{6.138}$$

where ϵ_1, ϵ_2 are complex numbers. If CP violation is accompanied by T violation such that CPT is conserved, then $\epsilon_1 = \epsilon_2 = \epsilon$, while if T is conserved and CPT is violated, then $\epsilon_1 \neq \epsilon_2$. We calculate the decay amplitudes of K_S^0 and K_L^0 into two pions as given in Eq. (6.136), assuming CPT invariance. Since pions are bosons, the wavefunction of the final state should be symmetric. The total isospin of the final state should be $I = 0$ or $I = 2$ with $I_3 = 0$. There are four amplitudes which may correspond to these decays:

$$\begin{aligned} &\langle \pi\pi, I=0|H_{wk}|K_S^0\rangle, \qquad \langle \pi\pi, I=2|H_{wk}|K_S^0\rangle, \\ &\langle \pi\pi, I=0|H_{wk}|K_L^0\rangle, \qquad \langle \pi\pi, I=2|H_{wk}|K_L^0\rangle. \end{aligned} \tag{6.139}$$

Using the Clebsch–Gordan coefficients, we may write

$$|\pi^+\pi^-\rangle = \sqrt{\frac{1}{3}}|\pi\pi, I=2\rangle + \sqrt{\frac{2}{3}}|\pi\pi, I=0\rangle, \tag{6.140}$$

$$|\pi^0\pi^0\rangle = \sqrt{\frac{2}{3}}|\pi\pi, I=2\rangle - \sqrt{\frac{1}{3}}|\pi\pi, I=0\rangle, \tag{6.141}$$

where the charged pion state is defined as $|\pi^+\pi^-\rangle = \frac{1}{\sqrt{2}}(|\pi^{+(1)}\pi^{-(2)}\rangle + |\pi^{+(2)}\pi^{-(1)}\rangle)$. The pions in the final state undergo final state interactions described by the phase shifts δ_0 and

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

δ_2, respectively, for $I = 0$ and $I = 2$ final states. The decay amplitudes for the decay of K^0 in $I = 0$ and $I = 2$ states are defined as:

$$\langle \pi\pi, I = 0|H_{wk}|K^0\rangle = A_0 e^{i\delta_0}, \tag{6.142}$$

$$\langle \pi\pi, I = 2|H_{wk}|K^0\rangle = A_2 e^{i\delta_2}. \tag{6.143}$$

The amplitudes for $\bar{K}^0$ are obtained, using the CPT invariance, as:

$$\langle \pi\pi, I = 0| \longrightarrow |\pi\pi, I = 0\rangle; \qquad \langle \pi\pi, I = 2| \longrightarrow |\pi\pi, I = 2\rangle; \qquad |K^0\rangle \longrightarrow -\langle \bar{K}^0|,$$

which leads to the decay amplitudes for $\bar{K}^0$ becoming

$$\langle \pi\pi, I = 0|H_{wk}|\bar{K}^0\rangle = -A_0^* e^{i\delta_0}, \tag{6.144}$$

$$\langle \pi\pi, I = 2|H_{wk}|\bar{K}^0\rangle = -A_2^* e^{i\delta_2}. \tag{6.145}$$

Taking A_0 to be real, which eliminates the arbitrary phase, the amplitudes for the decays given in Eq. (6.136) are obtained using Eqs. (6.137)–(6.145) assuming $\epsilon_1 = \epsilon_2$ as

$$\begin{aligned}
&\langle \pi^+\pi^-|H_{wk}|K_L^0\rangle \\
&= \frac{1}{\sqrt{3}}\langle \pi\pi, I = 2|H_{wk}|K_L^0\rangle + \sqrt{\frac{2}{3}}\langle \pi\pi, I = 0|H_{wk}|K_L^0\rangle, \\
&= \frac{1}{\sqrt{6(1+|\epsilon|^2)}}\left[(1+\epsilon)\langle \pi\pi, I = 2|H_{wk}|K^0\rangle + (1-\epsilon)\langle \pi\pi, I = 2|H_{wk}|\bar{K}^0\rangle\right] \\
&+ \frac{1}{\sqrt{3(1+|\epsilon|^2)}}\left[(1+\epsilon)\langle \pi\pi, I = 0|H_{wk}|K^0\rangle + (1-\epsilon)\langle \pi\pi, I = 0|H_{wk}|\bar{K}^0\rangle\right], \\
&= \frac{1}{\sqrt{6(1+|\epsilon|^2)}}\left[(A_2 - A_2^*)e^{i\delta_2} + \epsilon(A_2 + A_2^*)e^{i\delta_2} + 2\sqrt{2}\epsilon A_0 e^{i\delta_0}\right].
\end{aligned} \tag{6.146}$$

Similarly, other amplitudes are obtained as

$$\langle \pi^+\pi^-|H_{wk}|K_S^0\rangle = \frac{1}{\sqrt{6(1+|\epsilon|^2)}}\left[(A_2 + A_2^*)e^{i\delta_2} + (A_2 - A_2^*)\epsilon e^{i\delta_2} + 2\sqrt{2}A_0 e^{i\delta_0}\right], \tag{6.147}$$

$$\langle \pi^0\pi^0|H_{wk}|K_L^0\rangle = \frac{1}{\sqrt{3(1+|\epsilon|^2)}}\left[(A_2 - A_2^*)e^{i\delta_2} + (A_2 + A_2^*)\epsilon e^{i\delta_2} - 2\sqrt{2}\epsilon A_0 e^{i\delta_0}\right], \tag{6.148}$$

$$\langle \pi^0\pi^0|H_{wk}|K_S^0\rangle = \frac{1}{\sqrt{3(1+|\epsilon|^2)}}\left[(A_2 + A_2^*)e^{i\delta_2} + (A_2 - A_2^*)\epsilon e^{i\delta_2} - 2\sqrt{2}A_0 e^{i\delta_0}\right]. \tag{6.149}$$

The experimental observations are made on the ratios of the amplitude defined as:

$$\eta_{+-} = \frac{\langle \pi^+\pi^-|H_{wk}|K_L^0\rangle}{\langle \pi^+\pi^-|H_{wk}|K_S^0\rangle}, \qquad \eta_{00} = \frac{\langle \pi^0\pi^0|H_{wk}|K_L^0\rangle}{\langle \pi^0\pi^0|H_{wk}|K_S^0\rangle}. \tag{6.150}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Since the values ϵ and $|A_2|$ are small with the experimental limit $|\frac{A_2}{A_0}| \sim \frac{1}{20}$, neglecting second order terms in ϵ and $|A_2|$, we obtain

$$\begin{aligned}\eta_{+-} &\simeq \frac{(A_2 - A_2^*)e^{i\delta_2} + \epsilon(A_2 + A_2^*)e^{i\delta_2} + 2\sqrt{2}\epsilon A_0 e^{i\delta_0}}{(A_2 + A_2^*)e^{i\delta_2} + 2\sqrt{2}A_0 e^{i\delta_0}} \\ &= \epsilon + \frac{2i\,\mathrm{Im}\,(A_2)e^{i\delta_2}}{2\,\mathrm{Re}(A_2)e^{i\delta_2} + 2\sqrt{2}A_0 e^{i\delta_0}}.\end{aligned}$$

Since $A_2 \ll A_0$, we get

$$\begin{aligned}\eta_{+-} &\simeq \epsilon + \frac{1}{\sqrt{2}}\,\mathrm{Im}\left(\frac{A_2}{A_0}\right) e^{i(\pi/2+\delta_2-\delta_0)}, \\ &= \epsilon + \epsilon',\end{aligned} \tag{6.151}$$

with $\epsilon' = \frac{1}{\sqrt{2}}\,\mathrm{Im}\left(\frac{A_2}{A_0}\right) e^{i(\pi/2+\delta_2-\delta_0)}$. Similarly, one may obtain

$$\eta_{00} \simeq \epsilon - 2\epsilon'. \tag{6.152}$$

The present limit of these parameters are [117]:

$$\begin{aligned}|\eta_{+-}| &= (2.22 \pm 0.011) \times 10^{-3}, \\ |\eta_{00}| &= (2.232 \pm 0.011) \times 10^{-3}.\end{aligned}$$

6.7 Flavour Changing Neutral Currents (FCNC) and GIM Mechanism

The Cabibbo theory was able to explain several decay rates including the decays of strange as well as non-strange particles. However, the theory allowed decays like $K_L^0 \to \mu^+\mu^-$, $K^\pm \to \pi^\pm \nu\bar{\nu}$ which were experimentally found to be highly suppressed. This is because in the Cabibbo model, the hadronic current $J_\mu^\pm$ is given by:

$$J_\mu^\pm = g\bar{q}\gamma_\mu \tau^\pm q_L, \qquad \text{where} \quad q_L = \frac{(1-\gamma_5)}{2}q. \tag{6.153}$$

With $q = \begin{pmatrix} u \\ d' \end{pmatrix}$, we get

$$\begin{aligned}J_\mu^+ &\simeq g\bar{u}\gamma_\mu(1-\gamma_5)(d\cos\theta_C + s\sin\theta_C), \\ J_\mu^- &\simeq g(\bar{d}\cos\theta_C + \bar{s}\sin\theta_C)\gamma_\mu(1-\gamma_5)u.\end{aligned} \tag{6.154}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

If u and d' belong to a doublet representation of SU(2) in weak isospin space, then the symmetry group implies the existence of the third component of the current

$$\begin{aligned} J^0_\mu &\simeq 2g\bar{q}\gamma_\mu(1-\gamma_5)\tau_3 q \qquad\qquad (\because\ [\tau^+,\tau^-]=2\tau_3)\\ &= g\Big(\bar{u}\gamma_\mu(1-\gamma_5)u - \bar{d}'\gamma_\mu(1-\gamma_5)d'\Big)\\ &= g\Big(\bar{u}\gamma_\mu(1-\gamma_5)u - \bar{d}\gamma_\mu(1-\gamma_5)d\cos^2\theta_C - \bar{s}\gamma_\mu(1-\gamma_5)s\sin^2\theta_C\\ &\quad -\Big[\bar{d}\gamma_\mu(1-\gamma_5)s+\bar{s}\gamma_\mu(1-\gamma_5)d\Big]\sin\theta_C\cos\theta_C\Big). \end{aligned} \tag{6.155}$$

The last term predicts the flavor changing neutral current (FCNC) decays with $|\Delta S|=1$ and $\Delta Q=0$ with a strength of $g\sin\theta_C\cos\theta_C$, which is suppressed by a factor $\sin\theta_C$ comparable to the $\Delta S=0$ decays with strength $g\cos\theta_C$ in contrast to the experimental results showing a suppression of the order of $10^{-8}-10^{-9}$.

Glashow, Iliopoulos, and Maiani [64] proposed a mechanism to suppress these decays by postulating the existence of a fourth quark c (known now as the charm quark) following the earlier proposal of Bjorken and Glashow [65] which forms a weak doublet with the quark s', the orthogonal combination of d' proposed earlier by Cabibbo

$$s' = -d\sin\theta_C + s\cos\theta_C \tag{6.156}$$

such that we now have another doublet of quarks, that is, $q'=\begin{pmatrix}c\\ s'\end{pmatrix}$ in addition to $q=\begin{pmatrix}u\\ d'\end{pmatrix}$. Adding this doublet to the quark picture, we get additional weak currents as

$$\begin{aligned} J^{\pm\prime}_\mu &= g\bar{q}'\gamma_\mu(1-\gamma_5)\tau^\pm q',\\ J^{0\prime}_\mu &= 2g\bar{q}'\gamma_\mu(1-\gamma_5)\tau_3 q', \end{aligned}$$

giving

$$J^{+\prime}_\mu = g\bar{c}\gamma_\mu(1-\gamma_5)[-d\sin\theta_C+s\cos\theta_C], \tag{6.157}$$

$$J^{-\prime}_\mu = g[-\bar{d}\sin\theta_C+\bar{s}\cos\theta_C]\gamma_\mu(1-\gamma_5)c, \tag{6.158}$$

$$\begin{aligned} J^{0\prime}_\mu &= g\Big(\bar{c}\gamma_\mu(1-\gamma_5)c+\bar{d}\gamma_\mu(1-\gamma_5)d\sin^2\theta_C+\bar{s}\gamma_\mu(1-\gamma_5)s\cos^2\theta_C\\ &\quad -[\bar{d}\gamma_\mu(1-\gamma_5)s+\bar{s}\gamma_\mu(1-\gamma_5)d]\sin\theta_C\cos\theta_C\Big). \end{aligned} \tag{6.159}$$

In addition to predicting the new weak charged current coupling of c quarks, with d and s quarks, in Eqs. (6.157) and (6.158), the total neutral current now becomes

$$\begin{aligned} J^{\mathrm{NC}}_\mu &= J^0_\mu + J^{0\prime}_\mu\\ &= g\Big(\bar{u}\gamma_\mu(1-\gamma_5)u+\bar{c}\gamma_\mu(1-\gamma_5)c-\bar{d}\gamma_\mu(1-\gamma_5)d-\bar{s}\gamma_\mu(1-\gamma_5)s\Big). \end{aligned} \tag{6.160}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Thus, there are no FCNC in the first order. This is called the Glashow–Iliopoulos–Maiani (GIM) mechanism. In this model, the FCNC could occur in the second order as shown in Figures 6.9(a) and 6.9(b).

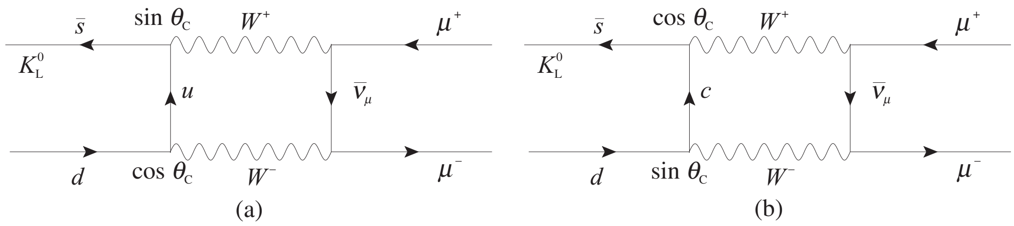

Figure 6.9 Feynman diagrams for the K^0 decay into $\mu^+\mu^-$ with a (a) u-quark exchange and (b) c-quark exchange.

Without the GIM mechanism, the amplitude will be proportional to $\simeq f(m_u)g^4 \sin\theta_C \cos\theta_C$, where $f(m_u)$ is a factor which depends upon the mass m_u obtained from the loop diagram; this mass would give too large a value for the decay rate for FCNC decays. With the GIM mechanism, there will be another term proportional to $\simeq g^4 f(m_c) \sin\theta_C \cos\theta_C$, which will cancel the contribution from the first term. There would be an almost complete cancelation if $m_c \simeq m_u$, and a limit on m_c/m_u can be obtained using the experimental limit of FCNC decays. A calculation by Gaillard and Lee [305] put a limit of $m_c \simeq 1 - 3$ GeV. This extension of the Cabibbo model of quark mixing to four quarks solved the problem of the FCNC and provided a way to estimate the mass of the new quark.

GIM mechanism re-established the symmetry between the leptons and quarks. Therefore, instead of u and d, it is now u and d' which are the counterparts of e^- and ν_e in the leptonic sector; c and s' are the counterparts of μ^- and ν_μ. Figure 6.10 shows the quark mixing between the two generations of quarks with their couplings to W bosons. In 1973, Kobayashi and Maskawa [70] extended the idea to six flavors of quarks and provided a quark mixing model which explains CP violation.

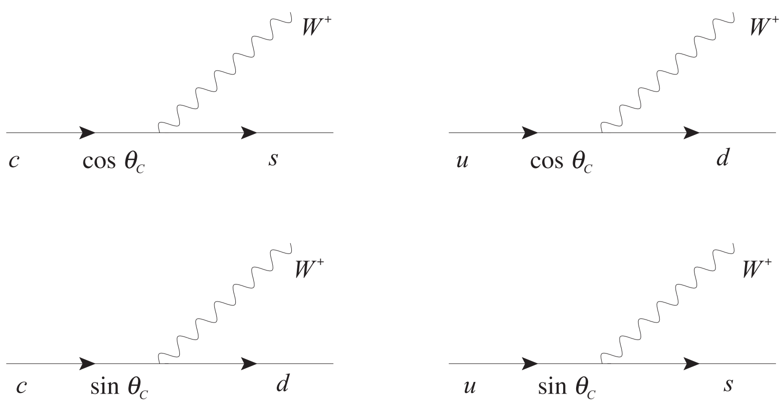

Figure 6.10 Quark mixing.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

6.7.1 Six quark mixing and CKM matrix

In 1975, a new lepton called τ [116] was discovered, which spoiled the symmetry between the leptons and quarks proposed by Bjorken and Glashow [65]. There were now six leptons (i.e., e^-, μ^-, τ^- and their corresponding neutrinos) but only four quarks. Soon after the discovery of τ leptons, a new heavy meson (the upsilon) was discovered [68, 307] and was recognized as the carrier of a fifth quark b called *beauty* or *bottom*. The new meson *upsilon* (Υ) was considered to be a bound state of $b\bar{b}$ with "hidden beauty", that is, with the beauty quantum number zero. The search for mesons (baryons) with non-zero "beauty" quantum numbers in quark–antiquark and three quark bound states, where one of the quarks was b like $b\bar{d}$ and $b\bar{u}$ (udb, cdb) was started; non-zero beauty quantum number mesons were finally discovered in the 1980s [69, 308, 309]. Such mesons are called B-mesons and baryons like Λ_b, Σ_b, etc. Therefore, the extension of the lepton–quark symmetry to six leptons necessitated the existence of a sixth quark t called "top", to make the total number of quarks equal to six. They are called u, d, s, c, b, t in the order they appear in literature. The top quark was also discovered in 1995 at Fermilab [310, 311], with a mass $m_t = 173.0 \pm 0.4$ GeV.

In order to describe the weak interaction of all these quarks and leptons, the Cabibbo– GIM model of the quark mixing was extended to six quarks by Kobayashi and Maskawa [70]. For this purpose, these quarks are classified according to the weak isospin and weak hypercharge as in the case of the Cabibbo theory. In this scheme, the left-handed components of the quarks are assigned to a doublet under SU(2), that is, $\begin{bmatrix} u \\ d \end{bmatrix}_L$, $\begin{bmatrix} c \\ s \end{bmatrix}_L$, $\begin{bmatrix} t \\ b \end{bmatrix}_L$; the right-handed components are assigned to a singlet, that is, $u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$.

The quark mixing theory of Cabibbo was extended to six quarks in which the d, s, b quarks mix to give the states d', s', b'; these states participate in the weak interactions. The states $(d',\ s',\ b')$ are written in terms of $(d,\ s,\ b)$ states by a 3×3 unitary matrix known as the Cabibbo–Kobayashi–Maskawa (CKM) matrix, that is,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \left(U_{\text{CKM}}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{6.161}$$

which can be written as:

$$q_i' = \sum_{i=1}^{3} U_{ij} q_j. \tag{6.162}$$

The CKM matrix U_{ij} has nine complex numbers implying a total of 18 real numbers. However, they are not all independent but are constrained by the conditions of the unitarity of the U matrix and freedom in choosing the phases of the states q_i' and $q_j (i, j = 1, 2, 3)$ which reduces the number of independent parameters to describe the CKM matrix. To illustrate this point, let us consider a general case of the mixing of quarks of n generators $q_i (i = 1, ...N)$ such that:

$$q_i' = \sum U_{ij} q_j, \qquad i, j = 1, n. \tag{6.163}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

The matrix U_{ij} is defined by n^2 complex numbers, that is, $2n^2$ real numbers with the following conditions:

i) The unitarity condition in n dimension, that is, $U^\dagger U = 1$ leads to n^2 conditions reducing the independent parameters to $2n^2 - n^2 = n^2$.

ii) There are $2n$ quark states (n states q_i and n rotated states $q_i{}'$), with $2n$ phases which are not physical. Leaving out one overall phase, there would be $2n - 1$ phases which can be eliminated by redefining the states leaving $n^2 - (2n - 1) = (n - 1)^2$ independent parameters.

iii) In order to choose $(n - 1)^2$ independent parameters to define an $n \times n$ matrix in n dimensions, an orthogonal matrix in n dimension can be chosen which has $n(n - 1)/2$ real parameters leaving $(n-1)^2 - \frac{n(n-1)}{2} = \frac{(n-1)(n-2)}{2}$ independent parameters, chosen to be phases inserted in any of the matrix elements U_{ij}.

Therefore, in n dimensions, an $n \times n$ unitary matrix is specified by $\frac{n(n-1)}{2}$ real angles and $\frac{(n-1)(n-2)}{2}$ phases. We see that in two dimensions, we have one angle, that is, the Cabibbo angle θ_C. In three dimensions, there would be three real angles and one phase. Kobayashi and Maskawa chose the three Euler angles $\theta_{12}, \theta_{23}, \theta_{13}$ and a phase $\delta_{13} (= \delta)$ associated with the third mixing angle θ_{13}. The matrix is explained in Chapter 1.

It can be shown that one may choose all the angles θ_{12}, θ_{23}, and θ_{13} to lie in the range $0 < \theta_{ij} < \frac{\pi}{2}$ so that $c_{ij} \geq 0$ and $s_{ij} \geq 0$ and the phase δ is in the range $0 \leq \delta \leq 2\pi$. The phase δ appearing in the U matrix makes the interaction Hamiltonian, written in terms of $q_i{}'$, non-real, which violates CP invariance. Therefore, the phase δ can be used to describe the CP violation in hadron physics. This was emphasized by Kobayashi and Maskawa in their original work. The current values of the CKM matrix elements have been determined in the experiments using semileptonic decays; they are given in Eq. (6.165). For example, $|U_{ud}|$ is determined while comparing $n \to p + e^- + \bar{\nu}_e$ and $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$ decay rates, while the comparison of the decay rates of $K^- \to \pi^0 + e^- + \bar{\nu}_e$ and $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$ is used to determine U_{us}. $|U_{cs}|$ is obtained by comparing the decay rates of $\bar{D}^0 \to K^- + e^+ + \bar{\nu}_e$ with $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$. Similarly, the other parameters have been determined by comparing the decay rates of other particles involving strange, charm, bottom, and top quarks.

However, the strength of the transition involving the third generation of quarks is quite small except the diagonal element U_{tb}. This can be seen in a transparent way by using Wolfenstein's parameterization of the CKM matrix; Wolfenstein emphasized the hierarchical character of the mixing between the generations of quarks. Wolfenstein's parameterization of the CKM matrix is written as [312]:

$$U_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & \rho\lambda^3 e^{i\phi} \\ -\lambda & 1 - \lambda^2/2 & \lambda^2 \\ \lambda^3(1 - \rho e^{i\phi}) & -\lambda^2 & 1 \end{pmatrix}, \tag{6.164}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

where $\lambda \approx \sin\theta_C \approx 0.22$, $\rho < 1$, and ϕ is the CP-violating phase factor. The structure of matrix elements in Equation (6.164) shows that $b \to u$, $t \to d$, $t \to d$, $b \to c$ transitions are quite small. The experimentally determined values of the CKM matrix at present are given as follows [117]:

$$U_{CKM} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97354 \pm 0.00010 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 0.00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}. \quad (6.165)$$

6.8 Weak Interaction of Hadrons with Charm and Heavy Flavors

6.8.1 Discovery of charm and heavy flavors

One of the reasons that the fourth quark called charm and denoted by c was proposed was to restore the lepton–quark symmetry [65]. There were four leptons, that is, the electron, the muon, and their respective neutrinos but only three quarks, that is, u, d, and s. The existence of a fourth quark would bring a complete symmetry between the quarks and the leptons. The idea of the fourth quark was the basic ingredient to formulate the GIM mechanism which resolved the problem of FCNC. The existence of a fourth quark will extend the flavor symmetry of the strong interactions from $SU(3)_f$ to $SU(4)_f$. The $SU(3)_f$ symmetry is broken due to the large mass of s quark compared to the mass of the u, d quarks. $SU(4)_f$ is also expected to be broken due to the large mass of c quark as compared to mass of the u, d, and s quarks. However, the following multiplets of mesonic $Q\bar{q}$ or $q\bar{Q}$ and baryonic Qqq, qQq, QQq, or QQQ states, where $Q(=c)$ is a heavy quark and $q(=u,d,s)$ is a light quark, are predicted according to the group structure of $SU(4)_f$.

$$4 \otimes \bar{4} = 1 \oplus 15 \quad (6.166)$$

$$\text{and } 4 \otimes 4 \otimes 4 = 20_S + 20_M + 20_M + \bar{4}_A. \quad (6.167)$$

The mesons are predicted to occur in the multiplet of a singlet and 15-plet. This implies that in addition to 9 well-known mesons corresponding to $3 \otimes \bar{3} = 1 + 8$ decomposition under $SU(3)_f$, we would have seven more states corresponding to one singlet states and 6 states belonging to the 15-plet under SU(4). They are identified as $c\bar{c}$, $c\bar{u}$, $c\bar{d}$, $c\bar{s}$, $\bar{c}u$, $\bar{c}d$, $\bar{c}s$ states. The $c\bar{c}$ state is called a charmonium state with total charm quantum number zero; the state has mesons with hidden charm content. This state is like a positronium system which is a bound state of e^-e^+ with lepton number zero. However, the potentials responsible for the binding of the e^-e^+ system and $q\bar{q}$ system are entirely different. The other $q\bar{Q}$ and $Q\bar{q}$ states are mesonic states with charm quantum number $C = \pm 1$ like kaon states with strangeness quantum number $S = \pm 1$.

The discovery of the charmonium states was made as the result of a narrow resonance J/ψ observed simultaneously at BNL and SLAC [313, 314]. Later, many states of charmonium were observed which provided information about the strong interaction force

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

between c and $\bar{c}$ quarks. A list of some $c\bar{c}$ states is given in Table 6.10. The direct evidence of the existence of the charmed particles with $C = +1$ like $c\bar{u},\ c\bar{d},\ c\bar{s}$ called $D^0,\ D^+$, and D_s^+ and with $C = -1$ like $\bar{c}u,\ \bar{c}d,\ \bar{c}s$ called $\bar{D}^0,\ D^-$, and D_s^-, respectively, were observed soon after the discovery of the charmonium state. These mesons decay by weak interactions. The particles with heavier quark content decay into particles with lighter quark content. Soon after the observation of charmed mesons, some baryonic states with a charm content of cqq with $q = u, d, s$ were also observed [315], which decay through weak interactions.

The discovery of the τ lepton in 1975, at the Stanford Positron-Electron Asymmetric Ring (SPEAR) [116] and its decay into other lighter leptons like $e,\ \mu$, and hadrons like $\pi,\ K$, through weak interactions accompanied by its own neutrino ν_τ due to the conservation of the τ lepton number(L_τ), makes the number of leptons six. The lepton–quark symmetry then requires the existence of two more quarks in the list of (u, d), (c, s); the two quarks were named (t, b), the top and bottom. Soon after the discovery of τ leptons, resonances similar to the $b\bar{b}$ states were observed in 1977. A spectrum of $b\bar{b}$ states was observed; they are known as η_b (9.4 GeV), χ_{b_1} (9.9 GeV), η_b (10.0 GeV) (see Table 6.10). Mesonic states like $q\bar{Q}$ or $\bar{q}Q$, where one of the quarks Q is a heavy quark b, were also observed and are called B mesons like the D mesons in the case of the charmed quark and K mesons in the case of the s quark.

Table 6.10 Predicted $c\bar{c}$ and $b\bar{b}$ states with principal quantum numbers n=1 and 2, and radial quantum number $n_r = n - L$, compared with experimentally observed states. Masses are given in MeV/c^2.

$^{2S+1}L_J$	n	n_r	J^{PC}	$c\bar{c}$ state	$b\bar{b}$ state
$^1S_0$	1	1	0^{-+}	$\eta_c(2984)$	$\eta_b(9398)^{(*)}$
$^3S_1$	1	1	1^{--}	$J/\psi(3097)$	$\Upsilon(9460)$
$^3P_0$	2	1	0^{++}	$\chi_{c0}(3415)$	$\chi_{b0}(9859)$
$^3P_1$	2	1	1^{++}	$\chi_{c1}(3511)$	$\chi_{b1}(9893)$
$^3P_2$	2	1	2^{++}	$\chi_{c2}(3556)$	$\chi_{b2}(9912)$
$^1P_1$	2	1	1^{+-}	$h_c(3525)$	$h_b(9899)$
$^1S_0$	2	2	0^{-+}	$\eta_c(3639)$	$\eta_b(9999)$
$^3S_1$	2	2	1^{--}	$\psi(3686)$	$\Upsilon(10023)$

The discovery of the top quark, t, took a much longer time than the bottom quark due to the very high energies involved in producing $t\bar{t}$ pairs or mesonic states like $t\bar{q}$ and baryonic states like tqq, etc. The top quark was discovered in 1995 in the hadron collider at Fermilab. It is produced in pairs through the process $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$. The expected decay modes are:

$$t\bar{t} \longrightarrow W^+ b W^- \bar{b}.$$

The decays of $W^+(W^-)$ in the hadronic and leptonic modes along with the production of jets due to $b\bar{b}$ are the signatures of $t\bar{t}$ production in the high energy hadronic collision. Some recent results combined with the old ones for a few detectors at CDF [310], DØ [311] and Tevatron [316] have measured cross sections in the range of 7.55 to 7.65 pb:

$$\sigma_{t\bar{t}} = 7.63 \pm 0.50\ pb\ \text{(CDF)}, \tag{6.168}$$

$$\sigma_{t\bar{t}} = 7.56 \pm 0.59\ pb\ \text{(DØ)}, \tag{6.169}$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

$$\sigma_{t\bar{t}} = 7.60 \pm 0.41\ pb \text{ (Tevatron)}, \tag{6.170}$$

which are in agreement with the standard model expectation of $7.35^{+0.28}_{-0.33}$ pb in the perturbative QCD for $m_t = 172.5$ GeV/c^2. Therefore, the mass of the top quark is determined to be 172.5 GeV/c^2. It is a very heavy quark and can decay weakly into b quarks and the intermediate vector bosons. Its lifetime is 5×10^{-25} s. At LHC, 90% production of the top quarks is through gg collisions at the energy $\sqrt{s} = 14$ TeV and 80% at energy $\sqrt{s} = 7$ TeV.

6.8.2 Weak decays of particles with charm and heavy flavors

The weak semileptonic and nonleptonic decays of particles with charm and heavy flavors are understood in terms of the quark model in which a quark current is involved in the weak decay through its coupling with the lepton current via the vector meson exchange of W^+ or W^-. This model is subject to QCD corrections involving the gluon exchange, thus, involving more than one quark. However, we will not go into the details of these corrections and describe the single quark model, that is, spectator model in which other quarks do not participate in the weak transition. For simplicity, we first describe the weak decays of particles with charm quark using the four-quark Cabibbo–GIM model of quark mixing and extend it to particles with heavier flavors using the six quark mixing of CKM.

6.8.3 Weak decays of particles with charm

Semileptonic decays of charmed mesons

(i) Two-body decay modes

The two-body semileptonic decay of mesons like $P(q\bar{q} \to l\nu_l)$, where P could be π^+, K^+, D^+, D_s^+ are depicted in Figure 6.11. The decay rates for the two-body decays of a pseudoscalar

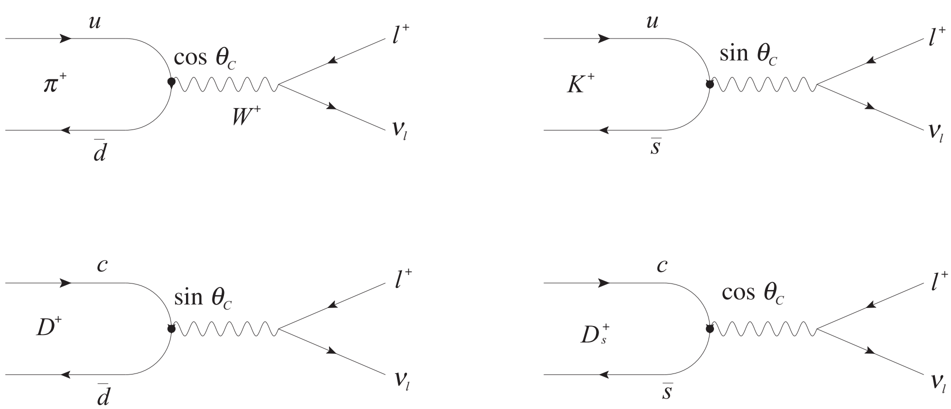

Figure 6.11 Leptonic weak decays of mesons.

meson P are proportional to $U_{ij}^2 f_P^2 m_l^2 \left(1 - \frac{m_l^2}{m_P^2}\right)^2 m_P$ (see Eq. (6.81)), where $U_{ij} = \cos\theta_C$ $(\sin\theta_C)$ for the $\Delta S = 0$ $(\Delta S = 1)$ transitions, m_P and f_P are, respectively, the mass and

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

decay constant of meson P. Therefore, the predictions for charmed particle decays are:

$$\frac{\Gamma(D^+ \longrightarrow \mu^+\nu_\mu)}{\Gamma(K^+ \longrightarrow \mu^+\nu_\mu)} \approx \frac{m_{D^+}}{m_{K^+}}, \qquad \frac{\Gamma(D_s^+ \longrightarrow \mu^+\nu_\mu)}{\Gamma(K^+ \longrightarrow \mu^+\nu_\mu)} = \cot^2\theta_C \frac{m_{D_s^+}}{m_{K^+}},$$

which are in agreement with the experimentally observed rates.

(ii) Three body decay modes

In the spectator model, the semileptonic decays of mesons with a charm quark is depicted in Figure 6.12. Similar diagrams for the semileptonic decays of charmed baryons are depicted in Figure 6.13.

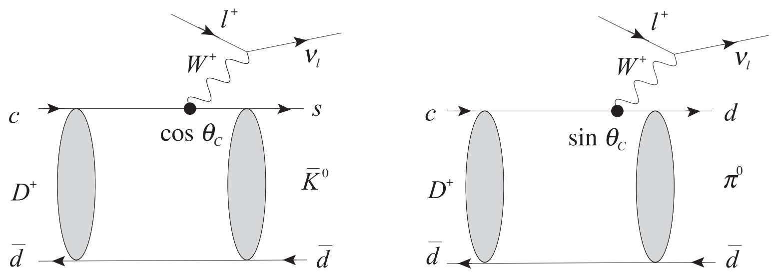

Figure 6.12 Semileptonic weak decays of charmed mesons.

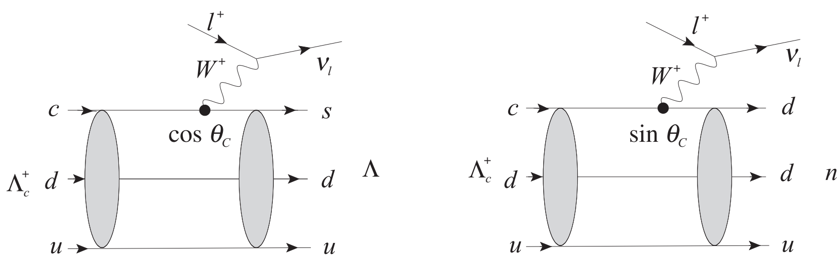

Figure 6.13 Semileptonic weak decays of charmed baryons.

In the case of the charmed mesons, the decays with $\Delta C = \Delta S = \Delta Q$ are Cabibbo allowed with $\Delta I = 0$, while the decays with $\Delta C = \Delta Q = \pm 1$ with $\Delta S = 0$ are Cabibbo suppressed with $\Delta I = 1/2$. Comparing the strength of the $c \leftrightarrows s$ and $c \leftrightarrows d$ transitions. It can be shown that:

$$\frac{\Gamma(D^+ \longrightarrow \bar{K}^0 l^+ \nu)}{\Gamma(K^0 \longrightarrow \pi^- l^+ \nu)} = \left(\frac{M_{D^+}}{M_K}\right)^5 \cot^2\theta_C \frac{f(m_K/m_D)}{f(m_\pi/m_K)},$$

where $f(x)$ is a phase factor given by:

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x.$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

In this case, the agreement of the theoretical calculations in the spectator model with the experimental results is not very good as the QCD corrections due to gluon exchange could be large.

(iii) Nonleptonic decays of charmed mesons

In the spectator model, the nonleptonic decays of the charmed mesons are diagrammatically represented in Figure 6.14. Similar diagrams may be depicted for baryon decays with two

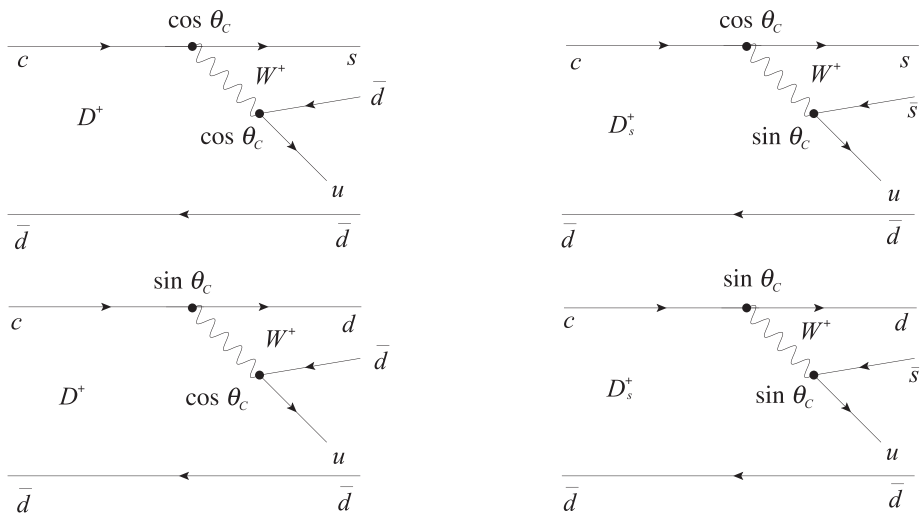

Figure 6.14 Nonleptonic decays of charmed mesons.

spectator quarks. These diagrams may give the order of magnitude of the decay amplitudes. For example, the diagrams shown in Figure 6.14 predict (including a factor three due to the color factor at the $W \to u_i d_j$ vertices in all the diagrams) the rates corresponding to the following transitions

$$c \longrightarrow s\,u\,\bar{d} \propto 3\cos^4\theta_C, \qquad c \longrightarrow s\,u\,\bar{s} \propto 3\cos^2\theta_C\sin^2\theta_C,$$
$$c \longrightarrow d\,u\,\bar{s} \propto 3\sin^4\theta_C, \qquad c \longrightarrow d\,\bar{d}\,u \propto 3\sin^2\theta_C\cos^2\theta_C.$$

These predictions compare very poorly with the experimentally observed values. This is because the QCD effects of hadrons with charm and the heavy flavors due to the quark–quark interactions arising due to kinematical and/or dynamical effects become more prominent as compared to the case of the nonleptonic decays of strange mesons and hyperons. These effects have been shown to play an important role in theoretically explaining the dominance of $\Delta I = \frac{1}{2}$ transition amplitudes in the case of nonleptonic decays of strange mesons and hyperons. Some of these corrections may also arise due to the weak interaction effects between two quarks inside the hadron as a result of the W exchange and W annihilation (shown in Figure 6.15). They are further affected by the final state interactions. However, the major correction arises due to the QCD effects involving gluon exchanges as shown in Figure 6.16. All these corrections improve the agreement with the experimental data [278].

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

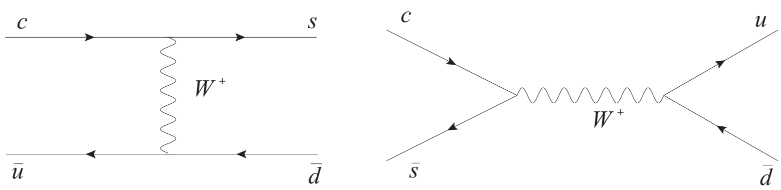

Figure 6.15 W boson exchanges.

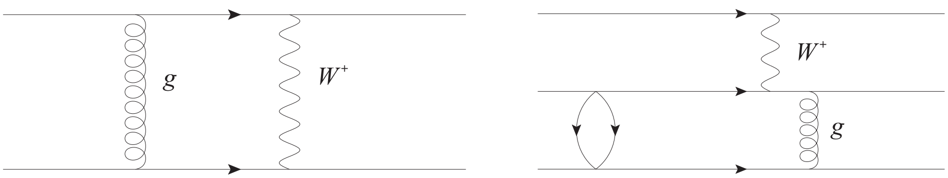

Figure 6.16 QCD corrections.

6.8.4 Weak decays of particles with heavy flavors

In order to describe the weak decays of particles with heavy flavors like b and t quarks, we consider the six quark mixing model of Cabibbo–Kobayashi–Maskawa using the CKM matrix. In this model of weak interactions, the hadronic current J_μ^h is written as:

$$J_\mu^h = q\gamma_\mu(1-\gamma_5)q' = \begin{pmatrix}\bar{u} & \bar{c} & \bar{t}\end{pmatrix} \gamma_\mu(1-\gamma_5) \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad (6.171)$$

where, $U_{ij}(i = u, c, t\, ;\, j = d, s, b)$ are the matrix elements of the CKM matrix, described in terms of the Euler angles θ_1, θ_2, θ_3 and the phase δ as given in Section 6.7.1. In the case of four quarks, the mixing matrix is described in terms of one parameter, the Cabibbo angle θ_C which is determined from the weak decays of strange particles (Section 6.4.1). The other terms in the matrix elements of the matrix U_{ij} are determined from the decays of heavier particles involving $b \to c$, $b \to u$, and $t \to b$ decays. These numerical values are used to test the unitarity relations between these matrix elements, that is,

$$\begin{aligned} |U_{ud}|^2 + |U_{us}|^2 + |U_{ub}|^2 = 1, \\ |U_{cd}|^2 + |U_{cs}|^2 + |U_{cb}|^2 = 1, \\ |U_{td}|^2 + |U_{ts}|^2 + |U_{tb}|^2 = 1. \end{aligned} \qquad (6.172)$$

6.9 Limitations of the Phenomenological Theory from the Hadron Sector

We have discussed the limitations of the phenomenological theory by studying the weak processes in the leptonic sector. The study of the various weak processes in the hadronic sector

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

bring to light some more deficiencies of the phenomenological theory which will be discussed in this section.

(i) Absence of neutral currents and implications for parity violating effects in nuclei
Neutral currents are conspicuous by their absence in the conventional picture of the phenomenology of weak interactions in the leptonic as well as the hadronic sectors. Experimentally, there was no evidence for the existence of such currents until 1973 [164], despite attempts to search for them at CERN in the early 1960s [156]. Theoretically, the existence of neutral currents has been speculated way back in the 1930s by Gamow–Teller [317], Klein [154], and Kemmer [318], and later by Schwinger [49] and others in the late 1950s in the context of the unified theory of weak and electromagnetic interactions, but there was no experimental support for their existence. Consequently, no further progress was made in the study of the weak neutral currents in the development of the phenomenological $V - A$ theory of weak interactions.

It is to be noted that in the conventional picture of the weak interaction theory, the interaction between two nucleons is described in terms of the charged currents mediated by the exchange of charged W bosons between the nucleons as shown in Figure 6.17, leading to a parity violating (PV) pseudoscalar nucleon–nucleon potential $V^{PV}(r)$. Such a mechanism also leads to a parity violating πNN coupling constant $f_{\pi NN}$ giving rise to a PV potential in the one boson exchange model similar to the parity conserving $N - N$ potential as shown in Figure 6.18 [319]. This parity violating potential $V^{PV}(r)$ gives rise to an admixture $\delta\psi_i$ of the opposite parity states in a given nuclear state ψ_i (characterized by definite parity and isospin) and is calculated in the usual way by using the first order perturbation to be

$$\delta\psi_i = \sum_{j\neq i} \frac{\langle\psi_j|V^{PV}|\psi_i\rangle}{E_i - E_j}, \tag{6.173}$$

where ψ_i and ψ_j are the eigenstates of the parity conserving nuclear interaction Hamiltonian. The presence of such an admixture in the nuclear states is responsible for the parity violating effects in some nuclear processes of scattering and decays involving nucleons and nuclei driven by strong and electromagnetic interactions. Some examples of such processes with experimentally observed parity violating observables are given in Table 6.11.

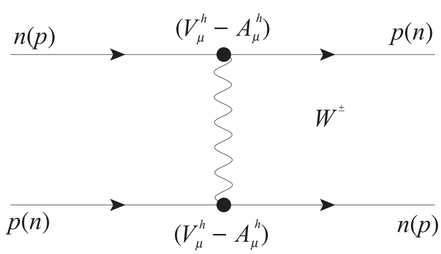

Figure 6.17 Nucleon–nucleon scattering through a W exchange.

The theoretical attempts to calculate $V^{PV}(r)$ using the phenomenological $V - A$ theory with charged currents only and explain the parity violating effects are not satisfactory. Even in

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

the context of the Cabibbo theory with GIM mechanism, in which weak neutral currents are present in the $\Delta S = 0$ sector, the conventional theory is not able to explain these PV effects in a consistent manner. Inspite of the theoretical uncertainties inherent in the calculation of $V^{PV}(r)$ and the experimental uncertainties reported in the parity violating observables shown in Table 6.11, the phenomenological $V - A$ theory is found to be inadequate.

Table 6.11 Some of the observed parity violating effects in nucleons and nuclei [319, 320]. A_L represents asymmetry and P_L represents the polarization of the hadron(photon) in the final state.

	Process	Value of A_L and P_L	Experiment
Scattering	$\vec{p}p \to \vec{p}p$	$A_L = (-0.93 \pm 0.20 \pm 0.05) \times 10^{-7}$	Bonn
		$(-1.7 \pm 0.8) \times 10^{-7}$	LANL
		$(-1.57 \pm 0.23) \times 10^{-7}$	PSI
		$(0.84 \pm 0.34) \times 10^{-7}$	TRIUMP
	$\vec{p}\,{}^4He \to \vec{p}\,{}^4He$	$A_L = -(3.3 \pm 0.9) \times 10^{-9}$	PSI
Decay	${}^{18}F(0^-) \to {}^{18}F(1^+) + \gamma$	$P_\gamma = (-7 \pm 20) \times 10^{-4}$	Caltech/Seattle
		$(-10 \pm 18) \times 10^{-4}$	Mainz
		$(3 \pm 6) \times 10^{-4}$	Florence
		$(2 \pm 6) \times 10^{-4}$	Queens
	${}^{19}F(\frac{1}{2}^-) \to {}^{19}F(\frac{1}{2}^+) + \gamma$	$A_\gamma = (-8.5 \pm 2.6) \times 10^{-5}$	Seattle
		$(-6.8 \pm 1.8) \times 10^{-5}$	Mainz
Capture	$np \to d\vec{\gamma}$	$P_\gamma = (1.8 \pm 1.8) \times 10^{-7}$	LNPI
	$\vec{n}p \to d\gamma$	$A_\gamma = (0.6 \pm 2.1) \times 10^{-7}$	Grenoble
		$(-1.2 \pm 1.9 \pm 0.2) \times 10^{-7}$	LANL

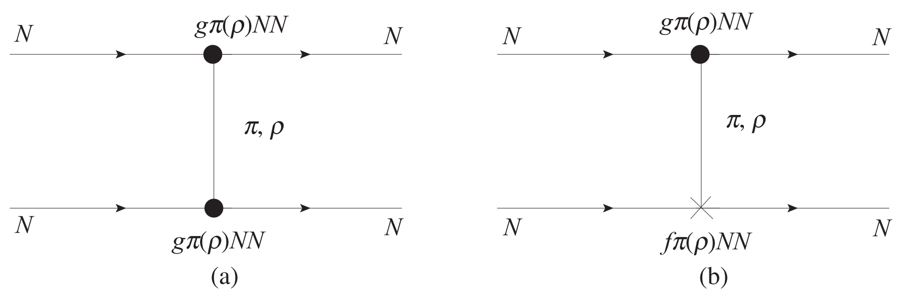

Figure 6.18 Nucleon–nucleon potential in the the meson exchange model. (a) Parity-conserving (PC) potential, (b) PV potential. $g\pi(\rho)NN$ and $f\pi(\rho)NN$ are the parity conserving and parity violating couplings.

(ii) $\Delta I = \frac{1}{2}$ rule in hadronic decays

We have seen that the $\Delta I = \frac{1}{2}$ rule in the semileptonic decays of strange particles like K-mesons and hyperons follows from the structure of the hadronic current appearing in the interaction Lagrangian, that is,

$$\mathcal{L}_{\text{int}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sin\theta_C [\bar{u}\gamma_\mu(1-\gamma_5)s\, l^\mu + h.c.].$$

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

Since (u, d) belong to the isospin doublet $I = \frac{1}{2}$ and s is an isosinglet $I = 0$, the $\Delta I = \frac{1}{2}$ rule for the $\Delta S = 1$ semileptonic decays follows. However, in the case of nonleptonic decays like $K^{\pm} \to \pi^+\pi^-$, $K_s \to \pi^0\pi^0$, $\Lambda \to p\pi^-$, $\Sigma^+ \to p\pi^0$, $\Sigma^+ \to n\pi^+$, etc., the $\mathcal{L}_{\text{int}}^{\text{nonleptonic}}(x)$ is written as:

$$\mathcal{L}_{\text{int}}^{\text{nonleptonic}} = \frac{G_F}{\sqrt{2}} J_\mu^{h\dagger} J^{\mu h} + \text{h.c.}$$

in which, the $\Delta S = 1$ decays are described by the Lagrangian:

$$\mathcal{L}_{\text{int}}^{\Delta S=1}(x) = \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \left\{ [\bar{u}\gamma_\mu(1-\gamma_5)d] \right.$$
$$\left. [\bar{u}\gamma^\mu(1-\gamma_5)s]^\dagger \right\},$$

a product of two currents with $\Delta I = 1$ in the $\Delta S = 0$ sector and $\Delta I = \frac{1}{2}$ in the $\Delta S = 1$ sector

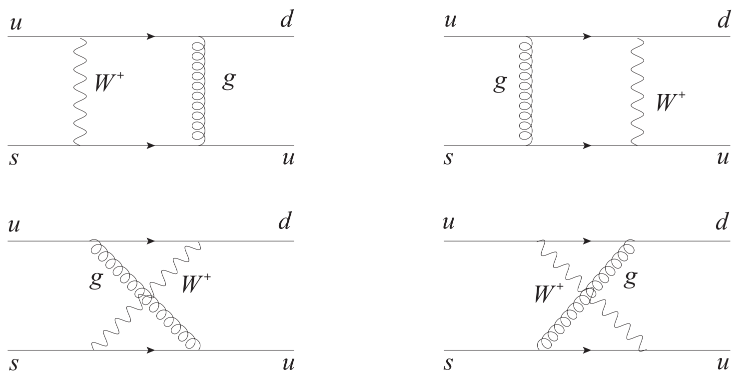

Figure 6.19 Gluon radiative diagrams.

leading to $\Delta I = \frac{1}{2}, \frac{3}{2}$. There is a priori no reason to expect that the $\Delta I = \frac{3}{2}$ transitions will be suppressed as compared to the $\Delta I = \frac{1}{2}$ transitions as they would have the same strength as $\frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C$. Moreover, the strength of the $\Delta I = \frac{1}{2}$ transitions in the nonleptonic decays was experimentally found to be much larger than the strength of the $\Delta I = \frac{1}{2}$ transitions in semileptonic decays, while the $V - A$ theory predicts it to be only slightly smaller by a factor $\cos\theta_C$. A satisfactory explanation of the strength of the nonleptonic weak decays pose quite severe problems to be explained by the $V - A$ theory when applied to the strangeness sector.

It is now believed that the suppression of $\Delta I \geq \frac{3}{2}$ transitions in the nonleptonic decays of mesons and hyperons and the enhancement of $\Delta I = \frac{1}{2}$ transitions in the nonleptonic decays as compared to the semileptonic decays are dynamical effects arising due to the renormalization of the weak couplings in quantum chromodynamics when the effects of gluon exchanges are included. There are two types of gluon exchange diagrams that contribute to the renormalization of the weak couplings. These are shown in Figures 6.19 and 6.20 and are called the gluon

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press

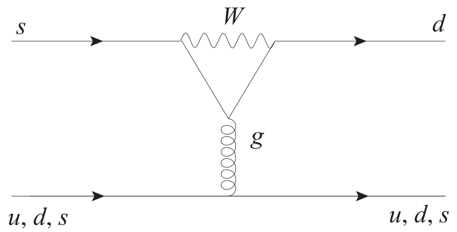

Figure 6.20 Penguin diagrams.

radiative and penguin diagrams, respectively. The details of these calculations are beyond the scope of this book and the readers are referred to textbooks on QCD [321]. In this context, the intuition of Cabibbo that the origin of the physics of the $\Delta I = \frac{1}{2}$ rule in nonleptonic decays is beyond the scope of the phenomenological weak interaction and lies elsewhere was indeed correct.

https://doi.org/10.1017/9781108489065.007 Published online by Cambridge University Press