Chapter 1

Neutrino Properties and Its Interactions

1.1 Historical Introduction to Neutrinos

1.1.1 Neutrino hypothesis

Beginning with the neutrino hypothesis proposed by Pauli in 1930, the story of the neutrino has been an amazing one [1]. It all started with a letter written by Pauli to the participants of a nuclear physics conference in Tubingen, Germany, on December 4, 1930 [1], in which he proposed the existence of a new neutral weakly interacting particle of spin $\frac{1}{2}$ and called it "neutron" as a "verzweifelten Ausweg" (desperate remedy), to explain the two outstanding problems in contemporary nuclear physics which posed major difficulties with respect to the scientists' theoretical interpretations. These two problems were related with the puzzle of energy conservation in β-decays of nuclei [2, 3], discovered by Chadwick in 1914 [4], and anomalies in understanding the spin–statistics relation in the case of ^{14}N and ^{6}Li nuclei within the context of the nuclear structure model that was prevalent in the early decades of the twentieth century [5, 6] in which electrons and protons were considered to be nuclear constituents.

This proposed 'verzweifelten Ausweg' was considered so tentative by Pauli himself that he postponed its scientific publication by almost three years. Today, neutrinos, starting from being a mere theoretical idea of an undetectable particle, are known to be the most abundant particles in the universe after photons, being present almost everywhere with a number density of approximately 330/cm^3 pan universe. The history of the progress of our understanding of the physics of neutrinos is full of surprises; neutrinos continue to challenge our expectations regarding the validity of certain symmetry principles and conservation laws in particle physics. The study of neutrinos and their interaction with matter has made many important contributions to our present knowledge of physics, which are highlighted by the fact that ten Nobel Prizes have been awarded for physics discoveries in topics either directly in the field of neutrino physics or in the topics in which the role of neutrino physics has been very crucial.

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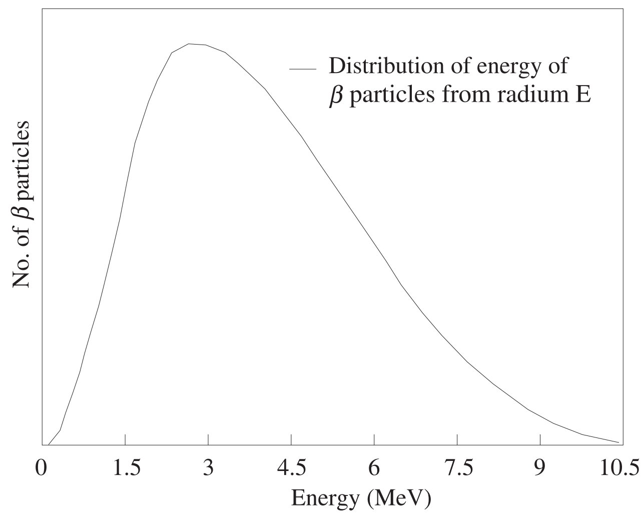

Figure 1.1 Continuous β-decay spectrum of RaE [15].

In this chapter, we will provide a historical introduction to the development of our understanding of neutrinos and their properties as they have emerged from the theoretical and experimental studies made over the last 90 years.

1.1.2 The problem of energy conservation in β-decays of nuclei

Almost three years after the discovery of nuclear radioactivity by Becquerel in 1896 [7], certain new radiation were discovered and studied by Curie, Rutherford, and others [8]. Further studies by Rutherford revealed that these radiations are of two types; Rutherford named them as α-radiation, which are readily absorbed and β-radiation which are more penetrative [9]. One more type of radiation, the γ-radiation was discovered a year later in 1900 by Villard [10]. It was realized quite early that β-radiation were electrons as identified from the study of cathode rays [11]. After the discovery of the nucleus by Rutherford in 1911 [12], it was Bohr [13] who established that β-radioactivity is a nuclear process like α- and γ-radioactivity and β-ray electrons originated from the nuclei. Further investigations by Chadwick [4] established in 1914, that the energy spectrum of the β-rays coming from nuclear β-decay was continuous. A typical β-ray spectrum of electrons from the nuclear β-decay of RaE is shown in Figure 1.1. The continuous energy spectrum of β-electrons is in complete contrast with the energy spectrum of α-decays and γ-decays of atomic nuclei which appear as discrete spectra. The discreteness of the energy spectra of α- and γ-radiation was quite consistent with the quantum description of nuclei which predicted discrete nuclear energy levels that would emit radiation of a fixed energy when de-excited to lower energy levels by the emission of nuclear radiation like α- and γ-rays. In this scenario of the nuclear energy levels being discrete, the continuous energy spectrum of β-electrons was argued by Meitner [14] to be due to the broadening of the discrete energy of primary electrons emitted in the β-decay, caused by secondary processes

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leading to continuous energy loss of the primary electrons as they travel through the nucleus. The other explanation given by Ellis [15] was to assume that the primary electrons emitted in the β-decay have an intrinsically continuous spectrum. This explanation of the phenomenon of primary electrons being emitted with a continuous energy spectrum posed difficulties with respect to its theoretical interpretation in the context of contemporary knowledge of the nuclear structure which, as explained earlier, described nuclear energy levels to be discrete according to quantum mechanics; the phenomenon seemed to violate the law of conservation of energy. A theoretical understanding of the nuclear β-decay depends crucially on whether the continuous energy spectrum of β-electrons is of the primary electrons or a result of the secondary processes suffered by the primary electrons, emitted in the β-decay, during their passage through the nuclear medium. This dilemma was resolved by making calorimetric measurements of the absolute heat (energy) in the absorption of β-electrons coming from the decay of RaE (^{210}Bi-nucleus) in an experiment performed by Ellis and Wooster [15]. The calorimetric measurement of the energy should result in the average energy of β-electrons if the observed spectrum was due to primary electrons according to Ellis, or to the maximum energy of the electron, if it was due to secondary processes according to Meitner.

Ellis and Wooster [15] reported the heat measurement equivalent to be $344\pm10\%$ keV, confirmed by the latter measurement of $332\pm6\%$ keV by Meitner and Orthmann [16] which corresponds to the average energy of the electrons and not to the maximum energy of the electrons corresponding to the spectrum as shown in Figure 1.1. This confirmed the primary origin of the continuous spectrum of β-electrons. After these experiments, it was established that the continuous energy spectrum of the electrons corresponds to the primary electrons emitted in the β-decays of nuclei. There were two very unconventional theoretical interpretations proposed to explain the continuous energy spectrum of β-decay electrons by Pauli [1] and Bohr [2], respectively. They are as follows:

1. Pauli [1] proposed that the conservation of energy holds exactly in the β-decay processes but a very penetrating neutral spin $\frac{1}{2}$ particle was emitted together with the electron.
2. Bohr [2] proposed that the conservation of energy is not exact but only statistical in interactions responsible for β-decays.

The idea of nonconservation of energy was not supported by the further developments in the study of β-decays of nuclei; therefore, Pauli's proposal was accepted by the physics community as the appropriate solution of the continuous energy problem in nuclear β-decays.

1.1.3 Anomalies in the spin–statistics relation for nuclei

In the first few decades of the twentieth century, it was generally assumed that the nuclei were made up of protons and electrons which were the only known particles at the time. In this picture of the nuclear structure, ^{14}N with charge number 7 and mass number 14 should have 14 protons and 7 electrons leading to a half integral spin and obey Fermi statistics for ^{14}N. However, Kronig [5], and Heitler and Herzberg [6] showed, using the molecular band spectra of ^{14}N, that it has spin 1 and satisfies Bose statistics.

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Similar examples were found later; for example, ^{6}Li with 6 protons and 3 electrons and deuteron ^{2}H with 2 protons and 1 electron both should have a half integral spin and follow Fermi statistics according to the proton–electron model of the nucleus but were found to have spin 1 with Bose statistics. This anomalous situation in describing the nuclear structure of ^{14}N, ^{6}Li, and ^{2}H was resolved with the presence of another nuclear constituent with neutral charge and spin $\frac{1}{2}$ in Pauli's proposal. Moreover, the observation in nuclear β-decays that if the initial nucleus had integer/half integer spin then the final nucleus also had integer/half integer spin, could also be explained with the presence of two spin $\frac{1}{2}$ particles in addition to the proton in the β-decay processes, which was otherwise not possible in the proton–electron model of the nucleus.

1.1.4 Pauli's neutron/neutrino vs. Fermi's neutrino

A closer reading of Pauli's letter [1] proposing the new particle makes it clear that Pauli's neutral particle was not exactly the neutrino as we know it today. For this purpose, the original letter translated by Riesselmann has been reproduced here:

Dear Radioactive Ladies and Gentlemen[,]

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the 'wrong statistics of the N and Li-6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the 'exchange theorem' (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin $\frac{1}{2}$ *and obey the exclusion principle and that further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton mass. The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant.*
Now it is also a question of which forces act upon neutrons. For me, the most likely model for the neutron seems to be, for wave-mechanical reasons (the bearer of these lines knows more), that the neutron at rest is a magnetic dipole with a certain moment μ*. The experiments seem to require that the ionizing effect of such a neutron can not be bigger than the one of a gamma-ray, and then* μ *is probably not allowed to be larger than* $e \bullet (10^{-13} cm)$*. But so far I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive people, with the question of how likely it is to find experimental evidence for such a neutron if it would have the same or perhaps a 10 times larger ability to get through [material] than a gamma-ray. I admit that my remedy may seem almost improbable because one probably would have seen those neutrons, if they exist, for a long time. But nothing ventured, nothing gained, and the seriousness of the situation, due to the continuous structure of the beta spectrum, is illuminated by a remark of my honored predecessor, Mr Debye, who told me recently in Bruxelles: 'Oh, It's better not to think about this at all, like new taxes.' Therefore one should seriously discuss every way of rescue. Thus, dear radioactive people, scrutinize and judge. - Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a*

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ball on the night from December 6 to 7. With my best regards to you, and also to Mr Back, your humble servant

W. Pauli

It is evident from the contents of this letter that Pauli's neutral particle had the following properties:

1. The proposed neutral spin $\frac{1}{2}$ particles are called 'neutrons' and are constituents of nuclei.
2. They do not travel with the velocity of light.
3. Their mass is similar to the electron mass but not larger than 0.01 times the proton mass.
4. The new particle (neutron) has a magnetic moment which is of the order of $e \times 10^{-13}$cm and is bound in the nucleus by magnetic forces.
5. The neutral spin $\frac{1}{2}$ particle shares the available energy with the electron leading to the continuous energy spectrum of β-electrons.

Six months later, Pauli himself first talked about the idea of the new particle in June, 1931 in the meeting of the American Physical Society in Pasadena [3, 17]; here, he abandoned the idea of the new particle being a constituent of the nuclei due to considerations of empirical masses. However, he still talked about neutrons and later in the summer of 1931, lectured in the University of Michigan, Ann Arbor about the magnetic properties of the new particle [18]. In October 1931, Pauli attended a conference on nuclear physics in Rome and participated in the discussions on deliberations where Fermi was also present. Fermi was impressed by Pauli's idea of a new particle. In fact, in the words of Pauli, '(Fermi) at once showed a lively interest in my new idea and a very positive attitude towards my new particle'. The very next year, in 1932, Chadwick [19] discovered a new neutral particle with a mass similar to the proton. This particle was named neutron, and Pauli's 'neutron' was rechristened by Fermi as 'neutrino' "little neutral one" [20]. With the discovery of the neutron by Chadwick, a clearer picture of the nuclear structure in terms of the protons and the neutrons emerged as elaborated by Heisenberg [21] and Iwanenko [22]. The theoretical interpretation of the nuclear β-decay was given by Fermi [23] and Perrin [24] in terms of the proton–neutron model of the nucleus in which neutrinos ("neutrons" as proposed by Pauli) were emitted along with the electrons. However, at this time, the interaction of neutrinos with the other material particles remained to be understood as stated by Pauli himself in the Seventh Solvay Conference in October 1933, in Brussels [25]. After the discovery of the neutron and study of its properties, it seems, in hindsight, that Pauli's 'neutron' is more like a hybrid of Chadwick's neutron and Fermi's neutrino.

1.2 Neutrino Interactions

The theory of neutrino interactions with matter has passed through many milestones before the standard model of electroweak interactions was formulated which describes the interaction of neutrinos with leptons and quarks considered to be the fundamental constituents of matter. The first attempt at a description of the nature of neutrino interactions is present in the original

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proposal of Pauli, where he postulated that neutrinos have a penetrating power larger than the photons implying an interaction weaker than the electromagnetic interaction and an electromagnetic interaction of neutrinos through its magnetic moment. However, in the Solvay Conference in 1933, there was no discussion on neutrino interactions except the general acceptability of the idea of the neutrino and its properties as proposed by Pauli. Soon after the Solvay Conference, Fermi [23] and Perrin [24] independently proposed the theory of β-decay, which was the first milestone in the theory of neutrino interactions with matter. The Fermi theory of β-decay, as it is known today, describes the β-decays of nuclei in which no change of angular momentum and parity of the nucleus is observed. The theory was extended by Gamow and Teller [26] and Bethe and Bacher [27] to describe the observation of nuclear β-decays with a change of one unit of angular momentum and no change in parity. A more general phenomenological theory of nuclear β-decays and other weak interaction processes was subsequently developed as more experimental data were accumulated on the weak decays of leptons, hadrons, and nuclei. In this section, we give a historical introduction of the theory of β-decay which led to the phenomenological $V - A$ (vector – axial vector) theory of weak interactions and later to the standard model of electroweak interactions.

1.2.1 Fermi theory of β-decay

The theory of β-decays by Fermi [23] and Perrin [24], in 1933, assumes that an electron–neutrino pair is created in the basic transitions of β-decay, in which neutron is converted into proton that is,

$$n \longrightarrow p + e^- + \bar{\nu}. \tag{1.1}$$

Its interaction Hamiltonian density was written in analogy with quantum electrodynamics (QED). The Hamiltonian density of the electromagnetic (EM) interactions is written as a scalar product of the electromagnetic current of the electron $j_\mu^{EM}(x)$ and the electromagnetic field $A_\mu(x)$, that is,

$$\mathcal{H}^{EM}(x) = e j_\mu^{EM}(x) A^\mu(x), \tag{1.2}$$

where $j_\mu^{EM}(x) = \bar{\psi}_e(x)\gamma_\mu\psi_e(x)$ is the electromagnetic current of the electron defined in terms of $\psi_e(x)$. In the Fermi theory of β-decay, the Hamiltonian density involving the charged fermion fields of electrons is written as:

$$\mathcal{H}(x) = G\bar{\psi}_p(x)\gamma_\mu\psi_n(x)\ \ \bar{\psi}_e(x)\gamma^\mu\psi_\nu(x) + \text{Hermitian conjugate (h.c.)}, \tag{1.3}$$

where G is the strength of the new interaction and $\psi_p(x)$, $\psi_n(x)$, $\psi_e(x)$, and $\psi_\nu(x)$ are the spin $\frac{1}{2}$ Dirac fields of proton, neutron, electron, and neutrino, respectively and γ_μ is the Dirac gamma matrix [28].

The β-decay Hamiltonian proposed by Fermi in Eq. (1.3) represents a point interaction of four fermions (neutron, proton, electron, neutrino) and its strength G has the dimension of M^{-2}. This makes the theory nonrenormalizable and effectively a low energy theory. This is in contrast with the QED Lagrangian, which depicts the interaction of two charged fermions with the electromagnetic field; the coupling strength is $e = \sqrt{4\pi\alpha}$, where $\alpha = \frac{1}{137}$ is dimensionless, making the theory renormalizable.

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The β-decay interaction is shown diagrammatically in Figure 1.2(a) to be contrasted with electromagnetic interaction shown in Figure 1.2(b). In field theoretical language, the β-decay of neutrons described by the Hamiltonian density in Eq. (1.3) occurs at a point x where a neutron field annihilates and creates a proton field, an electron field, and a neutrino field as shown in Figure 1.2(a). It should be noted that in the Fermi theory, the neutrino is created along with the electron and is not emitted being a constituent of the nuclei as proposed by Pauli in his famous letter. The electron created is not the electron of the electron–proton model of the nucleus which was discarded in favor of the nuclear model proposed by Heisenberg [21] and Iwanenko [22], after the discovery of the neutron by Chadwick [19]. The model proposed by Fermi [23] and Perrin [24] was the first successful application of quantum field theory (QFT) beyond QED processes.

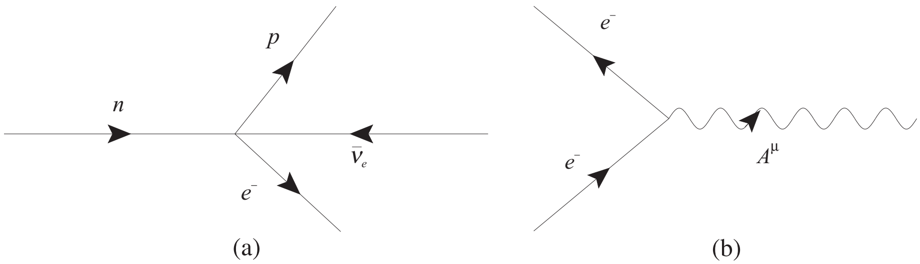

Figure 1.2 (a) Four fermions interact for β-decay; (b) Interaction of two electrons with a photon field (QED).

In nuclei, the neutrons and protons are nonrelativistic and in the nonrelativistic limit of the Dirac spinors, one may write (as shown in Appendix A):

$$\begin{aligned}\bar{\psi}_p(x)\gamma_\mu\psi_n(x) &\rightarrow \chi_p{}^{+}\mathbb{1}\chi_n \qquad \text{for } \mu = 0\\ &\rightarrow 0 \qquad\qquad \text{for } \mu = i,\end{aligned}$$

where χ_n and χ_p are the Pauli spinors and $\mathbb{1}$ is the unit operator in nuclear coordinates and spin space. In the nonrelativistic limit,

$$[\bar{\psi}_p(x)\gamma_\mu\psi_n(x)][\bar{\psi}_e(x)\gamma^\mu\psi_\nu(x)] \propto \chi_p^\dagger\mathbb{1}\chi_n e^{-i(\vec{p}_e+\vec{p}_\nu)\cdot\vec{x}}. \tag{1.4}$$

Moreover, in the case of nuclear β-decay, the total energy available to the electron and the neutrino is a few MeV such that $|(\vec{p}_e + \vec{p}_\nu)| \cdot |\vec{x}|$ (approximated by pR, where p is the total momentum of the electron–neutrino pair and R is the nuclear radius) is of the order of 10^{-2}. Therefore, the exponential term in Eq. (1.4) can be approximated to unity such that:

$$[\bar{\psi}_p(x)\gamma_\mu\psi_n(x)][\bar{\psi}_e(x)\gamma^\mu\psi_\nu(x)] \propto \chi_p^{+}\mathbb{1}\chi_n. \tag{1.5}$$

Thus, the nuclear operator in the Fermi theory is a unit operator $\mathbb{1}$ in the nuclear space of coordinate and spin, and cannot induce any change in the quantum numbers of the nuclei, depending upon the space coordinates $\vec{x}$, that is, orbital angular momentum$(\vec{L})$, or spin $(\vec{S})$ and the total angular momentum $(\vec{J} = \vec{L} + \vec{S})$. Therefore, the Fermi theory describes the nuclear β-decays in which there is no change in the total angular momentum J $(\Delta J = 0)$ and parity.

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The matrix element becomes independent of the electron momentum because pR is very small and is negligible compared to unity. Therefore, the decay rate depends only upon the phase space available to the electron and neutrino, and is proportional to $d\vec{p}_e d\vec{p}_\nu$ subject to energy conservation, that is, $E_e + E_\nu = \Delta = E_i - E_f$, where $(\vec{p}_e, E_e)$ and $(\vec{p}_\nu, E_\nu)$ are the momentum and energy of the electron and neutrino, respectively, in this process. After integrating over the neutrino momentum, we get the the energy spectrum of the electron, that is, $\frac{d\Gamma}{dE_e}$, given in the limit $m_e \to 0$ by:

$$\frac{d\Gamma}{dE_e} \propto p_e E_e (\Delta - E_e)^2. \tag{1.6}$$

This energy spectrum describes very well the continuous energy of nuclear β-decays corresponding to Fermi transitions. The spectrum is further modified in the presence of the Coulomb effect of the electron moving in the electromagnetic field of nuclei due to its charge but these modifications are small (they will be discussed in detail in Chapter 5). The total decay rate Γ and lifetime τ of the nuclear β-decay is given by:

$$\Gamma = \frac{1}{\tau} = \frac{G^2 E_o^5}{32\pi^3}, \tag{1.7}$$

where E_o is the end point energy of the electron. Using τ and E_o, the strength of the coupling G is found to be

$$G \approx \frac{1.0 \times 10^{-5}}{M_p^2}, \text{ where } M_p \text{ is the proton mass.} \tag{1.8}$$

1.2.2 Gamow–Teller theory

The Fermi Hamiltonian discussed in Section 1.2.1 is a scalar product of two vector currents in the lepton and nucleon sectors. This Hamiltonian does not describe the β-decays when the total angular momentum carried by the electron–neutrino pair is one unit (corresponding to $|\Delta J| = 1$) and there is no change in parity. Such transitions were observed and described by Gamow–Teller(GT) [26] who proposed that the Hamiltonian density responsible for these transitions is a scalar product of axial vector–axial vector current in nucleon and lepton sectors in order to conserve parity, that is,

$$\mathcal{H}^{GT}(x) = G_\beta^A [\bar{\psi}_p(x)\gamma_\mu\gamma_5\psi_n(x)][\bar{\psi}_e(x)\gamma^\mu\gamma_5\psi_\nu(x)] + h.c. \tag{1.9}$$

which in the nonrelativistic limit of the nucleon kinematics reduces to

$$\mathcal{H}_{NR}^{GT}(x) = G_\beta^A [\chi_p^\dagger \sigma_i \chi_n][\bar{\psi}_e(x)\gamma^i\gamma_5\psi_\nu(x)], \tag{1.10}$$

where σ_i $(i = 1-3)$ are the Pauli spin operators. The Pauli spin operators σ_i can change the total angular momentum $\vec{J}$ of the initial nucleus by one unit without changing the orbital angular momentum, implying no change in the parity. This leads to the GT transitions in β-decays corresponding to $\Delta J = 1$ transitions. The matrix element for the nuclear β-decay

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corresponding to the Gamow–Teller transformations is also independent of momentum as argued in Section 1.2.1 and yields the similar spectrum as predicted by Eq. (1.6). A typical electron spectrum corresponding to GT transformation is shown in Figure 1.3, which is the same as the β spectrum for Fermi transition. The total decay rate determines the coupling constant G_β^A, that is, the strength of β-decays in GT transitions. A comparison of the lifetimes of the nuclei undergoing GT transitions and Fermi transitions yields a value of $G_\beta^A(0)/G_\beta^V(0) \approx 1.2$. We see that the strength of β-decays in the axial vector sector is larger than that in the vector current sector. This was theoretically explained much later in the phenomenological studies of the $V-A$ currents in the weak transitions (discussed in Chapter 5).

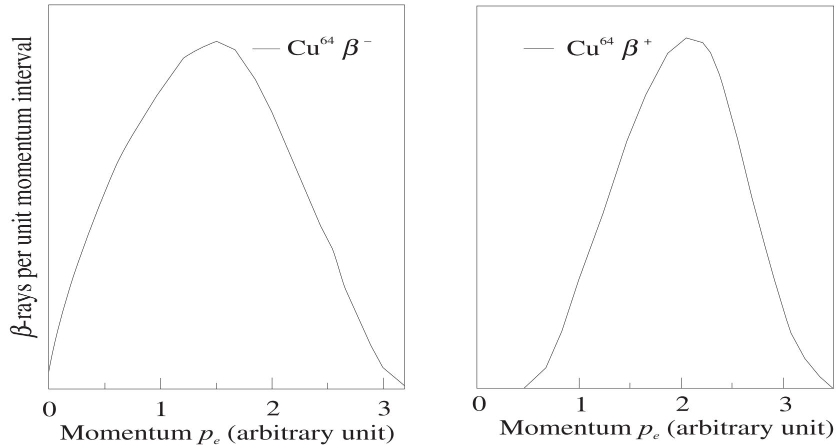

Figure 1.3 β-decay spectrum for GT transitions vs. momentum $|\vec{p}_e|$ (in arbitrary units). [29]

At the time, when the GT Hamiltonian was proposed, it was realized that following Fermi's idea of current–current interactions, the most general form of the four fermion interaction need not be limited to only two terms corresponding to the scalar product of vector–vector and axial vector–axial vector currents but could be a sum of five possible terms constructed from the scalar products of vector (V), scalar (S), pseudoscalar (P), axial vector (A), and tensor (T) currents in nucleonic and leptonic sectors assuming parity conservation [26, 27], that is,

$$\mathcal{H}_I^\beta(x) = \sum_{i=S,V,T,A,P} G_i \bar{\psi}_p(x) O_i \psi_n(x) \bar{\psi}_e(x) O^i \psi_\nu(x) + h.c. \tag{1.11}$$

with five coupling constants G_i which may be complex, implying ten real parameters to determine $\mathcal{H}_I^\beta(x)$. The operators O^i in Eq. (1.11) are constructed from the Dirac matrices as:

$$O^i(i = 1-5) = 1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu\gamma_5, \gamma_5.$$

With the help of these operators O_i, the bilinear covariants $\bar{\psi} O_i \psi$ are constructed in the leptonic and nucleonic sectors (discussed in Chapter 5).

The number of fundamental parameters describing the weak interaction Hamiltonian density $\mathcal{H}_I^\beta(x)$ seemed too many to provide a useful phenomenological theory. However, these parameters were determined using the symmetry properties of the weak interaction Hamiltonian

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and the numerous experimental observations made and interpreted theoretically over the next 20 years. This resulted in a successful description of most of the weak processes at low energies in terms of a very few nonvanishing parameters of $H_I^\beta(x)$. The experimental observation of many observables and their theoretical interpretation have played a decisive role in arriving at a phenomenological theory of weak interactions. We will discuss them in detail in Chapter 5, but here would like to highlight the discovery of parity violation and its interpretation in terms of the two-component theory of neutrinos.

1.2.3 Parity violation and the two-component neutrino

For the next 20 years after the general Hamiltonian density in Eq. (1.3) was proposed, many attempts were made to experimentally study the β-decay processes of nuclei and weak processes of the newly discovered mesons and baryons, and to determine the essential parameters of the phenomenological theory. A major breakthrough in formulating the theory of neutrino interactions came around 1956 when hints were made that parity may not be conserved in the weak interactions while trying to resolve the $\tau - \theta$ puzzle [30]. There were two particles τ and θ discovered experimentally with almost the same mass and lifetime decaying respectively into two and three pions in the S-state ($L = 0$). This implied parity violation if τ and θ were the same particles, because two pions and three pions in the S-state will have opposite parities due to the pseudoscalar nature of pions. In view of this suggestion, Lee and Yang [31, 32] analyzed many weak processes and concluded that there was no experimental evidence to contradict the assumption of parity violation in weak interactions and suggested specific experiments to test this possibility by measuring specific correlation observables in β-decays of nuclei and other weak decays of elementary particles which were parity violating, that is, observables that change sign under the transformation $\vec{r} \rightarrow -\vec{r}$ like $\vec{\sigma}_N \cdot \hat{p}_e$ or $\vec{\sigma}_e \cdot \hat{p}_e$, where $\hat{p}_e$ is the unit vector along the electron momentum and $\vec{\sigma}_N$ and $\vec{\sigma}_e$ are, respectively, the nucleon and electron spin operators. The first experiment to test the parity violation in β-decay was performed by Wu et al. [33] with polarized cobalt nucleus where a large asymmetry of β-electrons with respect to the spin direction of the polarized ^{60}Co was observed. Later on, many experiments on the longitudinal polarization of electrons in nuclear β^--decays and positrons in nuclear β^+-decays were conducted and the violation of parity in weak interactions was firmly established [34, 35]. Therefore, it was established by 1957, from the studies of various β-decays that parity is violated in weak interactions. The fact that neutrinos have almost zero mass, was confirmed from the study of the energy spectrum of β-decay electrons; it can especially be confirmed from the shape of the spectrum near the end point that neutrino is almost massless (see Figure 1.4 [36]) by comparing the energy spectrum with $m_\nu \neq 0$ and $m_\nu = 0$.

The evidence of the presence of parity violation led to the revival of the two-component theory for massless neutrinos by many authors [37, 38, 39]. The theory was first formulated by Weyl [40] but was not pursued further as it violated parity conservation and was not considered appropriate for application to physical processes. In a parallel development, during the time parity violation was being established experimentally, the two-component theory of neutrinos and its implications in physical processes were studied in detail. This led to the formulation of a phenomenological theory of weak interactions in terms of two-component neutrinos.

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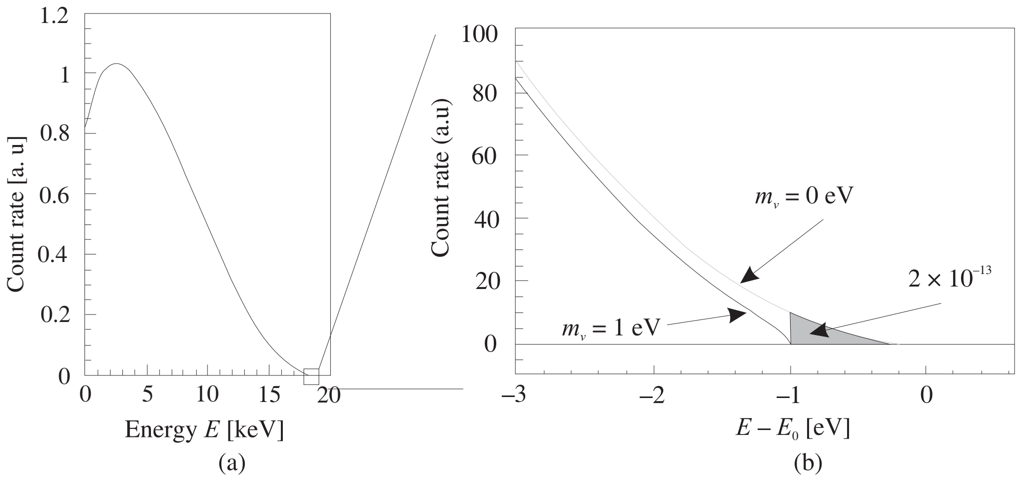

Figure 1.4 β^--decay spectrum with $m_\nu \neq 0$ and $m_\nu = 0$ [36].

1.2.4 Chiral (γ_5) invariance and $V - A$ theory of β-decays

Some major developments were made in the experimental analysis of β-decays of nuclei and many elementary particles were discovered over the next 20 years after the proposed theory by Fermi [23]. The experimental confirmation that the neutrino mass is almost negligible and the discovery of parity violation in weak interactions, along with the observation that the longitudinal polarization of the electron (positron) is $-v$ $(+v)$ and the helicity of the neutrino (antineutrino) is -1 $(+1)$ led to major developments in the phenomenological theory of weak interactions. Once parity violation is allowed, the weak interaction Hamiltonian density can have a pseudoscalar term in addition to a scalar term to accommodate the parity violating effects. The additional pseudoscalar term requires that the weak interaction Hamiltonian density should have five more coupling constants G_i' associated with the pseudoscalar term in analogy with Eq. (1.11). A vast amount of extensive work on various observables of β-decays was done to determine the various coupling constants G_i and G_i' which established that the phenomenological theory of weak interactions is of the current–current form where the currents have a $V - A$ structure in the leptonic and hadronic sectors; they do not have any other variants like scalar, tensor, or pseudoscalar as proposed in the general Hamiltonian. All the fermions, that is leptons and baryons participated in the weak interaction through their left-handed component, that is, $\psi_L = (1 - \gamma_5)\psi$, instead of ψ so that the interacting currents for the leptonic (l^μ) and the hadronic (J^μ) currents can be written as:

$$l^\mu = \bar{\psi}_e \gamma^\mu (1 - \gamma_5) \psi_\nu \tag{1.12}$$

$$J^\mu = \bar{\psi}_p \gamma^\mu (1 - \lambda\gamma_5) \psi_n, \tag{1.13}$$

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where λ is the relative strength of the axial current coupling compared to the vector current in the hadronic sector.

An elegant $V - A$ theory of weak interactions was formulated almost simultaneously by Sudarshan and Marshak [41], Feynman and Gell-Mann [42], and Sakurai [43], though some historical discussions suggest that it was Sudarshan who first proposed and discussed the idea of $V - A$ interaction theory with Marshak.

The basic idea was based on the concept of chiral (γ_5) invariance of the theory of the massless spin $\frac{1}{2}$ neutrinos, in which the equation of motion is invariant under the transformation $\psi \rightarrow \psi' = \gamma_5\psi$. Therefore, a linear combination of ψ and $\gamma_5\psi$ is also a solution. Thus, replacing ψ by the linear combination of $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ can be used to describe the neutrino participating in the weak interaction which leads to the structure of the leptonic current shown in Eq. (1.12).

The structure of the vector and axial vector currents in Eq. (1.13), and their properties like the conservation of vector current, partial conservation of axial current, and the relative strength of the axial vector and vector currents, that is, λ, were established later by many authors [44, 45, 46, 47, 48]. Thus, the interaction Hamiltonian H_{int} is written as:

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} l_\mu J^{\mu\dagger} + \text{h.c.} \tag{1.14}$$

where a factor $\frac{1}{\sqrt{2}}$ is introduced in the definition of H_{int} so that the constant G introduced by Fermi (in Eq. (1.3)) is consistent with G_F.

1.2.5 Intermediate vector boson (IVB)

One of the significant implications of the $V - A$ theory, in addition to explaining the parity violation and observed helicities of the leptons in a natural way in weak interactions was to give credence to the theory of weak interactions mediated by spin 1 intermediate vector bosons (IVB) in analogy with quantum electrodynamics in which the electromagnetic interaction is mediated by spin 1 photons. In the IVB theory, the basic weak interactions between the $e\nu$ pair and the np pair is mediated by a vector field W^μ and the interaction Hamiltonian is given by:

$$H_{\text{int}}^{IVB} = g\left[\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu + \bar{\psi}_n\gamma_\mu(1-\lambda\gamma_5)\psi_p\right] W^\mu. \tag{1.15}$$

The β-decay process $n \rightarrow p + e^- + \bar{\nu}_e$ is then a second order process as shown in Figure 1.5 and the strength g of the weak interaction of the $e\nu$ pair with the vector boson W^μ is related to the Fermi coupling constant $\frac{G_F}{\sqrt{2}} = \frac{g^2}{M_W^2}$, where M_W is the mass of the W boson, being very high compared to $q^2 (M_W^2 >> q^2)$ in these processes. One of the reasons for the introduction of an IVB to mediate weak interactions was to avoid the divergences encountered in the phenomenological $V - A$ theory while extending the theory to higher energies. For example, the total cross sections for the $\nu_e - e^-$ scattering is found to increase with square of the center of mass (CM) energy (s), that is, $\sigma(\nu_e e^- \rightarrow \nu_e e^-) = \frac{G_F^2}{\pi}s$. The cross section would

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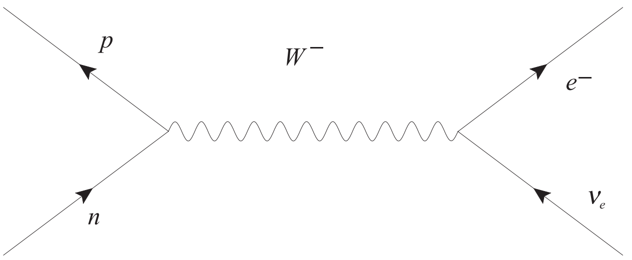

Figure 1.5 β^--decay process mediated through a W^- boson.

diverge at higher energies and violate the unitarity limit which is given by $\sigma \leq \frac{4\pi}{s}$ for $\nu_e e^- \to \nu_e e^-$ scattering as shown in Figure (1.6). In fact, the presence of such divergence problems in the Fermi theory was realized quite early by many authors. It was hoped that the $V - A$ theory mediated by a massive intermediate boson may help to remove this divergence; such a theory was theoretically proposed very early by Schwinger [49], Bludman [50], and Leite Lopes [51] in the hope that it will solve the divergence problem of Fermi theory in higher orders [52, 53, 54, 55] due to the presence of the momentum dependence of the spin-1 W^μ propagator but it does not happen. Moreover, the short range of weak interactions implying a large mass of the mediating vector boson created more problems than the theory of intermediate vector bosons was supposed to solve. Even though the intermediate vector boson of Schwinger [49], Leite Lopes [51] and others, did not solve the divergence problem, it led to other developments which contributed to the formulation of the standard model of electroweak interactions mediated by the intermediate vector bosons.

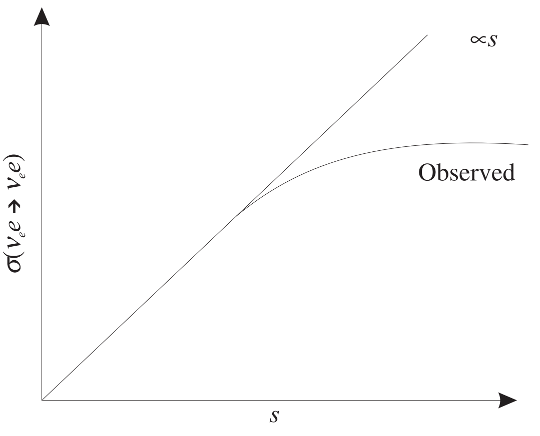

Figure 1.6 Divergence of cross section at high energies ($\sigma \propto s$) which violates unitarity, while saturation at high energies is observed experimentally.

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1.2.6 Weak interactions in strangeness sector: The Cabibbo theory, and the GIM mechanism

With the advent of particle accelerators in the early 1950s, many new particles and their decay modes were discovered. In the context of neutrino interactions, special mention may be made of strange particles, which were so named as they were produced in strong interactions and decayed through weak interactions. A new quantum number, 'strangeness' (S) was assigned to them and a scheme for their classification was proposed by Nishijima [56] and Gell Mann [57] in 1956. The first strange mesons observed were K^+, K^0, K^-, and $\bar{K}^0$ and were assigned strangeness S of $+1$ for $\left(K^+ \quad K^0\right)$ and strangeness of -1 for $\left(\bar{K}^0 \quad K^-\right)$. The first strange baryons observed were named hyperons and assigned strangeness -1 for $\Lambda, \Sigma^{\pm,0}$, strangeness -2 for $\Xi^{-,0}$, and -3 for Ω^-. Accordingly, the already known mesons like pions $(\pi^{\pm,0})$ and baryons like nucleons (p,n) were assigned strangeness quantum number $S = 0$. The strange mesons and hyperons were found to decay into lighter particles including leptons and hadrons and the decays were classified as semileptonic and nonleptonic decays, depending upon the presence or absence of leptons. Some specific examples of purely leptonic as well as semileptonic and nonleptonic decays of strange mesons and hyperons are as follows:

i) Purely leptonic decays:

$$\begin{aligned}
\mu^\mp &\longrightarrow e^\mp + \nu_\mu(\bar{\nu}_\mu) + \bar{\nu}_e(\nu_e), \\
\tau^\mp &\longrightarrow e^\mp + \nu_\tau(\bar{\nu}_\tau) + \bar{\nu}_e(\nu_e), \\
\tau^\mp &\longrightarrow \mu^\mp + \nu_\tau(\bar{\nu}_\tau) + \bar{\nu}_\mu(\nu_\mu).
\end{aligned}$$

ii) Semileptonic decays:

$$\begin{aligned}
K^\pm &\longrightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu), & K^\pm &\longrightarrow e^\pm + \nu_e(\bar{\nu}_e), \\
K^\pm &\longrightarrow \pi^0 + \mu^\pm + \nu_\mu(\bar{\nu}_\mu), & K_L^0 &\longrightarrow \pi^+ + \mu^- + \bar{\nu}_\mu, \\
\Lambda &\longrightarrow p + e^- + \bar{\nu}_e, & \Sigma^- &\longrightarrow n + e^- + \bar{\nu}_e, \\
\Xi^- &\longrightarrow \Lambda + \mu^- + \bar{\nu}_\mu.
\end{aligned}$$

iii) Nonleptonic decays:

$$\begin{aligned}
\Lambda &\longrightarrow p + \pi^-, & \Sigma^+ &\longrightarrow p + \pi^0, \\
\Sigma^- &\longrightarrow n + \pi^-, & \Omega^- &\longrightarrow \Lambda + K^-.
\end{aligned}$$

A phenomenological study of strange particle decays leads to the following observations:

(i) Strange particle decays are suppressed when compared to the decays of nonstrange particles. For example, $K^+ \to \mu^+\nu_\mu$ is suppressed by a factor of $\approx 1/5$ as compared to the $\pi^+ \to \mu^+\nu_\mu$ transition.

(ii) Strange particle decays follow the $|\Delta S| = 1$ and $\Delta S = \Delta Q$ rule, where ΔQ and ΔS are respectively, the changes in hadronic charge and strangeness.

(iii) The strangeness changing decays in which $\Delta Q = 0$, that is, $K_L \longrightarrow \mu^+\mu^-$ or $K^\pm \longrightarrow \pi^\pm\bar{\nu}\nu$, $K^\pm \longrightarrow \pi^\pm e^+e^-$, $K^\pm \longrightarrow \pi^\pm\mu^+\mu^-$, $\Sigma^+ \to pe^+e^-$, $\Sigma^+ \to p\mu^+\mu^-$ are highly suppressed.

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(iv) The strangeness changing $|\Delta S| = 1$ currents follow the $|\Delta I| = \frac{1}{2}$ rule in contrast to the $\Delta S = 0$ currents, which follow the $\Delta I = 1$ rule, where I is the isospin of the hadrons.

In order to explain phenomenologically, the suppression of the strength of $|\Delta S| = 1$ currents as compared to $\Delta S = 0$ currents, Gell-Mann and Levy [58] and Cabibbo [59] proposed that the strength of the $|\Delta S| = 1$ weak current in the hadronic sector is suppressed as compared to the $\Delta S = 0$ currents by a factor described by a parameter to be determined experimentally from the β-decays of hyperons and strange mesons like $\Sigma^0(\Lambda) \to pe^-\bar{\nu}_e$ and $K^\pm \to \pi^0 l^\pm \nu_l(\bar{\nu}_l)$, and the leptonic decays like $K^\pm \to l^\pm \nu_l(\bar{\nu}_l)$, where $l = e, \mu$. The Gell-Mann and Levy [58] proposal was formulated in terms of the physical particles p, n, and Λ following the Sakata model [60] of elementary particles. In this model, the hadronic current is written as:

$$J_\mu^{\text{hadron}} = \frac{1}{\sqrt{1+\epsilon^2}} \left[\bar{\psi}_p \gamma_\mu (1-\gamma_5) \psi_n + \epsilon \bar{\psi}_p \gamma_\mu (1-\gamma_5) \psi_\Lambda \right], \tag{1.16}$$

with the parameter ϵ describing the suppression of $|\Delta S| = 1$ currents. The Cabibbo model [59] was formulated in terms of the quark model of the hadrons which was proposed by Gell-Mann and Pais [61] and Zweig [62, 63]. In the quark model, the proton, neutron, and lambda particles are considered to be the bound states of quarks. The quark contents of proton, neutron, and lambda being uud, udd and uds, respectively, the weak transitions of β-decay correspond to $d \to u$ ($s \to u$) transitions in case of $\Delta S = 0$ ($|\Delta S| = 1$) transitions; the transitions are shown in Figure 1.7. Therefore, in the quark model, the weak hadronic current J_μ was written by Cabibbo as:

$$J_\mu^{\text{Cabibbo}}(x) = \cos\theta_C \bar{\psi}_u(x) \gamma_\mu (1-\gamma_5) \psi_d(x) + \sin\theta_C \bar{\psi}_u(x) \gamma_\mu (1-\gamma_5) \psi_s(x), \tag{1.17}$$

in which the $|\Delta S| = 1$ currents are suppressed by a factor $\tan\theta_C$. The phenomenological value of $\tan\theta_C$ was determined to be $\tan\theta_C = 0.2327$ at that time.

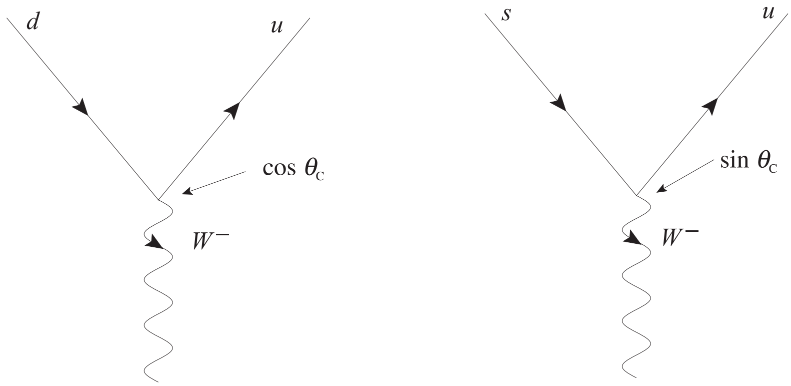

Figure 1.7 Interaction of quark with W boson field.

Equation (1.17) can be rewritten as:

$$J_\mu^{\text{Cabibbo}} = \bar{\psi}_u \gamma_\mu (1-\gamma_5)(\cos\theta_C \psi_d + \sin\theta_C \psi_s), \tag{1.18}$$

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which implies that a linear combination of d and s quarks defined as $d' = d\cos\theta_C + s\sin\theta_C$ participates in weak interactions. Thus, u and d' can be considered to form a doublet under 'weak isospin' represented by a column matrix, that is, $\begin{pmatrix} u \\ d' \end{pmatrix}$ like the isospin $\begin{pmatrix} u \\ d \end{pmatrix}$ doublet which takes part in strong interactions. This raises the question of the physical significance of the orthogonal component of d' defined by $s' = -d\sin\theta_C + s\cos\theta_C$. It was proposed by Glashow, Iliopoulos, and Maiani (GIM) [64], following the earlier suggestion of Bjorken and Glashow [65], that there exists a fourth quark 'c' named charm quark, which forms another 'weak isospin doublet' with s', that is, $\begin{pmatrix} c \\ s' \end{pmatrix}$. Its weak interaction is described by the weak quark current,

$$J_\mu^{GIM} = \bar{\psi}_c(x)\gamma_\mu(1-\gamma_5)(-\sin\theta_C\psi_d(x) + \cos\theta_C\psi_s(x)). \tag{1.19}$$

This also implies that the neutral current now defined as:

$$\bar{u}O_\mu u + \bar{c}O_\mu c + \bar{d}'O_\mu d' + \bar{s}'O_\mu s' = \bar{u}O_\mu u + \bar{c}O_\mu c + \bar{d}O_\mu d + \bar{s}O_\mu s; \quad O_\mu = \gamma_\mu(1-\gamma_5)$$

does not have terms like $\bar{d}s$ and $\bar{s}d$ which change strangeness. Thus, the Cabibbo theory extended by Glashow, Iliopoulos, and Maiani (GIM) explained the absence of flavor changing neutral current (FCNC) and provided the concept of quark mixing, a physical interpretation. Hence, while the d and s quarks participate in strong interactions, d' and s', that is, the mixed state of d and s participate in weak interactions. This mixing of two quarks is described, in general, by a 2×2 unitary mixing matrix U such that:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = U\begin{pmatrix} d \\ s \end{pmatrix}, \text{where} \qquad U = \begin{pmatrix} U_{ud} & U_{us} \\ U_{cd} & U_{cs} \end{pmatrix}. \tag{1.20}$$

The most general unitary 2×2 matrix is parameterized in terms of one parameter. Therefore, the matrix elements U_{ij} are described in terms of only one parameter, the Cabibbo angle θ_C and are given as:

$$\begin{aligned} U_{ud} &= U_{cs} = \cos\theta_C, \\ U_{us} &= -U_{cd} = \sin\theta_C. \end{aligned}$$

1.2.7 Quark flavors and the CKM matrix

Soon after the establishment of the charmed quark with the discovery of J/ψ particles in 1974 [66, 67], the existence of additional quarks called b-quarks was anticipated through the discovery of heavier mesons of the Υ series in 1977 [68] and the B series in 1983 [69], which were interpreted as the bound states of $b\bar{b}$ or $b\bar{q}$ ($q = u, d, s, c$) quarks. Around this time, the existence of five quarks u, d, s, c, and b was considered to be established as many baryon states with b as one of the constituents were also observed. The concept of quark–lepton symmetry was invoked to propose the existence of a sixth quark t (top quark) to form a doublet with b-quark like the doublets of $(u,\ d)$ and $(c,\ s)$ quarks in 4-quarks, the two-flavor doublet model. In analogy with the three-flavor doublet $(\nu_e\ e^-)$, $(\nu_\mu\ \mu^-)$, and $(\nu_\tau\ \tau^-)$ model of six leptons, the

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three-flavor doublet quark model of 6-quarks $(u\ d)$, $(c\ s)$, and $(t\ b)$ was considered in order to maintain symmetry between quarks and leptons. A quark mixing of three-flavor quarks, that is, d, s, and b was formulated by Kobayashi and Maskawa [70] like the three-flavor mixing of neutrinos formulated earlier in 1962 by Maki et al. [71].

In a three-flavor mixing scheme, the quark flavor states participating in the weak interaction, say $q' = d', s', b'$ are assumed to be a mixture of three quark states $q = d, s, b$ participating in the strong interactions and the mixing is described by a unitary 3×3 matrix U called the CKM (Cabibbo–Kobayashi–Maskawa) matrix,

$$q'_i = \sum_{ij} U_{ij} q_j, \tag{1.21}$$

where $q'_i(= d', s', b')$ are the weak interaction eigenstates of quarks and $q_i(= d, s, b)$ are the strong interaction eigenstates. These nine matrix elements of the unitary 3×3 matrix, that is, U_{ij} are described in terms of four independent parameters. There are quite a few parameterizations of this matrix but the most popular parameterization is given by CKM in which these parameters are chosen to be three rotation angles θ_{12}, θ_{13}, and θ_{23} like Euler angles describing rotation in real three-dimensional space and a phase angle δ. Explicitly, the U matrix in this parameterization is written as:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.22}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and δ is the phase angle. For the three-flavor mixing described here, the weak interaction Lagrangian is written in terms of weak quark current as:

$$J^{CKM}_{\mu} = \bar{\psi}_u \gamma_\mu (1 - \gamma_5) \psi_{d'} + \begin{pmatrix} u \to c \\ d' \to s' \end{pmatrix} + \begin{pmatrix} u \to t \\ d' \to b' \end{pmatrix} + \text{h.c.} \tag{1.23}$$

The presence of the phase angle δ in the Lagrangian makes it complex which violates the time reversal invariance (T invariance). The invariance of the weak interaction under the combined CPT (charge conjugation, parity, time reversal) symmetry implies that the T violation is equivalent to CP (charge conjugation, parity) violation; therefore, the phase angle δ is used to describe the phenomenon of CP violation in weak processes.

1.2.8 Nonleptonic weak interaction and CP violation

Strange mesons and baryons (hyperons) also decay through modes which do not involve neutrinos. These decays are called nonleptonic weak decays; some examples are as follows:

$$\begin{array}{lll} K^{\pm} \to \pi^{\pm}\pi^0, & K^{\pm} \to \pi\pi\pi & \\ K^0_L \to \pi^+\pi^-\pi^0,\ \pi^0\pi^0\pi^0, & K^0_S \to \pi^+\pi^-,\ \pi^0\pi^0 & \\ \Lambda \to p\pi^-, n\pi^0, & & \\ \Sigma^+ \to n\pi^+,\ p\pi^0 & \Sigma^0 \to p\pi^-, n\pi^0 & \Sigma^- \to n\pi^-. \end{array} \tag{1.24}$$

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Many nonleptonic decays of strange mesons and hyperons were discovered in the emulsion experiments done with cosmic rays and particle beams in early accelerators where these particles were produced.

The following conclusions were drawn from the analysis of these decays.

1. Nonleptonic decays violated strangeness with $|\Delta S| = 1$ and exhibited dominance of the $\Delta I = \frac{1}{2}$ rule like the semileptonic decays.
2. Parity violation was also established in nonleptonic decays of hyperons like $\Lambda \to n\pi^0$, $\Lambda \to p\pi^-$ by a measurement of the asymmetry in the angular distribution of pions. Historically, this was one of the early observations of parity violation in particle physics which went unnoticed [72].
3. CP violation was discovered in the comparative study of nonleptonic decays of neutral kaons K_L^0 and K_S^0 in two and three pion modes. K_L^0 and K_S^0 are the neutral kaon states defined to be the eigenstates of CP corresponding to eigenvalues of -1 and +1. Therefore, the experimental observation of $K_L^0 \to \pi^0\pi^0$ and $K_L^0 \to \pi^+\pi^-$ would violate CP [73].

1.3 Neutrino Flavors and Universality of Neutrino Interactions

In the earlier sections, we have described the progress in the understanding of neutrinos and their interactions mainly from the study of β-decay of nuclei and nucleons. Simultaneous, with the developments in the experimental and theoretical understanding of β-decay of nuclei and nucleons, many other particles like muons, pions, kaons, and hyperons were discovered in cosmic rays and accelerator experiments at CERN, ANL, BNL, Serpukhov, etc. and their weak decays were studied, which contributed to the study of weak interactions. Neutrinos (antineutrinos) emitted in the β-decay of nucleons and nuclei were identified as electron neutrinos (antineutrinos) because they were always accompanied with positrons (electrons). Later, other heavy leptons like muons and tauons were discovered, which were found to decay weakly into lighter particles involving one or more neutrino (antineutrino). The additional neutrinos (antineutrinos) associated with these particles were later identified to be different from the neutrinos associated with electrons. Three types of neutrinos are known today; they are the electron neutrino (ν_e), muon neutrino (ν_μ), and tau neutrino (ν_τ), and they have antiparticles $\bar{\nu}_e$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$ corresponding to the three charged leptons, e^-, μ^-, τ^- and their antiparticles e^+, μ^+, τ^+. The weak interactions of all these neutrinos have a $V - A$ structure and the same strength, leading to the universality of weak interactions. In this section, we give the historical introduction to our present understanding of neutrino flavors ν_e, ν_μ, and ν_τ and their antiparticles, and the universality of weak interactions.

1.3.1 Experimental discovery of $\bar{\nu}_e$ and $\bar{\nu}_e \neq \nu_e$

The attempts to make direct observation of neutrinos and antineutrinos possible took a very long time to succeed experimentally because of the theoretical calculations by Bethe and

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Peierls [74] as well as by Fierz [55], who found the neutrino nucleus cross section to be very small, of the order of $\approx 10^{-44}\text{cm}^2$, for MeV neutrinos available at that time from β-decay sources. This led them to conclude that there was no possibility to observe neutrinos in the near future. Suggestions were made to observe them indirectly by measuring the recoil of the daughter nucleus in the emission of the $e\nu$ pair in nuclear β-decay [3] and experimental attempts were made early by Rodeback and Allen [75], Leipunski [54], Snell and Pleasonton [76], Jaeobsen et al. [77], Sherwin [78] and Crane and Halpern [79] with clear evidence of the existence of neutrinos. With the development of nuclear reactors where a very high flux of antineutrinos was produced from the fusion reactions of nuclei in the nuclear pile sites, it was argued by Pontecorvo [80], Alvarez [81] and Fermi [82] that direct neutrino–nucleus reactions with antineutrinos being generated as a result of fusion reactions could be observed due to the high flux of $\bar{\nu}_e$ despite small neutrino cross sections. The neutrino event rates could be made still larger by using huge targets of the order of tons of material, thus increasing the number of nucleon targets, so that the number of events could be significant enough to be observed. The attempts to observe neutrinos through the nuclear reactions induced by antineutrinos from the reactors finally succeeded when the group led by Reines and Cowan [83, 84] used a 300 L liquid scintillator target detector to observe neutrinos at the Hanford reactor in 1953 and later with a 4200 L liquid scintillator target detector at the Savannah River reactor in 1956 [85, 86]. They observed the reaction

$$\bar{\nu}_e + p \rightarrow e^+ + n \tag{1.25}$$

by making a coincidence measurement of the photons from particle annihilation $e^- + e^+ \rightarrow \gamma + \gamma$ and a neutron capture $n +^{108}\text{Cd} \rightarrow^{109}\text{Cd} + \gamma$ reaction a few microseconds later as illustrated in Figure 1.8.

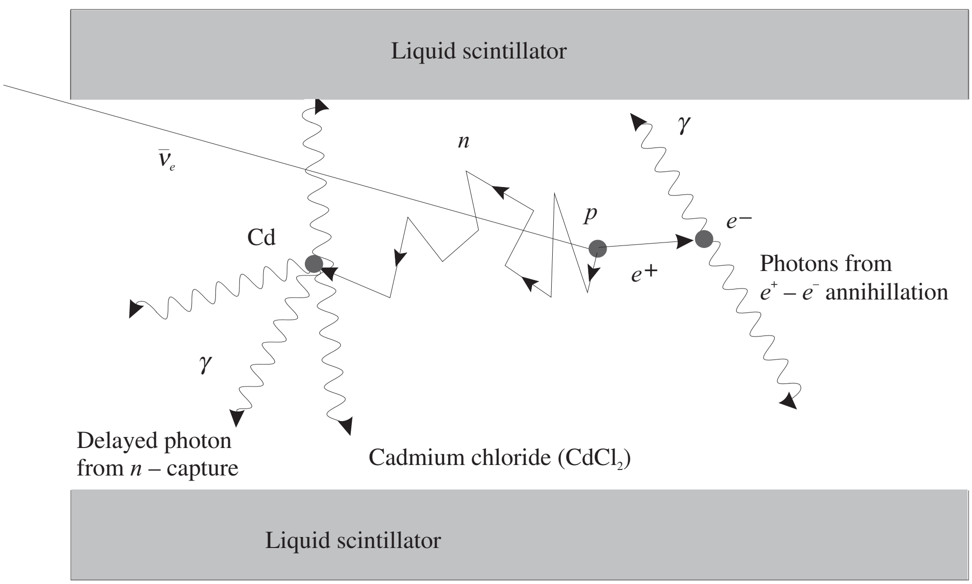

Figure 1.8 Two instant and one delayed photon in Reines and Cowan's experiment [83, 84].

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They observed a cross section of

$$\bar{\sigma} = (11 \pm 2.6) \times 10^{-44} \mathrm{cm}^2/\mathrm{nucleon} \quad (1.26)$$

for the reaction in Eq. (1.25) averaged over the spectrum of $\bar{\nu}_e$ from fusion reactions, which was in good agreement with the theory.

The original proposal of Pontecorvo [80] and Alvarez [81] to use ^{37}Cl as target was followed up by Davis et al. [87, 88] who looked at the reaction $\nu_e + ^{37}\mathrm{Cl} \rightarrow e^- + ^{37}\mathrm{Ar}$ using the Brookhaven reactor with 4000 L of liquid $\mathrm{CCl_4}$ and tried to observe the ^{37}Ar produced in the reaction. No events were observed but a limit of $\bar{\sigma}(\bar{\nu} + ^{37}\mathrm{Cl} \rightarrow e^- + ^{37}\mathrm{Ar}) < 0.9 \times 10^{-45} \mathrm{cm}^2$ was obtained while the prediction was $\approx 2.6 \times 10^{-45} \mathrm{cm}^2$. This negative result was very important as it showed that antineutrinos from reactors do not produce electrons hinting that $\bar{\nu}_e \neq \nu_e$. Moreover, the efforts made by Davis opened up a detection principle which was later used in observing solar neutrinos and played a major role in the development of neutrino physics.

1.3.2 Discovery of muons and muon neutrinos

Muons were discovered in cosmic rays by Neddermeyer and Anderson [89], Street and Stevenson [90], and Nishina et al. [91] in 1937. They were found to have mass between electrons and protons and were very penetrating with long lifetime and therefore, not to be confused with Yukawa's proposed π-mesons [92]. π-mesons were later discovered to decay into muons [93, 94, 95], that is, $\pi \rightarrow \mu + \nu$ as predicted by Tanikawa [96] and Marshak and Bethe [97]. Muons were later confirmed to decay through $\mu^- \rightarrow e^- + \nu + \nu$, that is, into an electron and two neutral leptons in cosmic ray experiments by Conversi et al. [98] as theoretically predicted earlier by Sakata and Inoue [99] and later by many others [100, 101, 102].

The experimental observation of the upper limit of the total energy of neutral leptons emitted in μ-decay, the magnitude of the Michel parameter ρ (Chapter 5), and other theoretical considerations concluded that the two neutral particles have very small mass (like ν_e) and need not be identical particles. Assuming the two neutral leptons in the $\mu^- \rightarrow e^- + \nu + \nu$ to be identical, Feinberg [103] calculated the rate for $\mu^- \rightarrow e^- + \gamma$ in the IVB model which should have been seen in the experiment. However, this decay was not observed which indicated that the neutral leptons are not identical. They could not be particle and antiparticle of each other because no decays of type $\mu^- \rightarrow e^- + \gamma$, $\mu^- \rightarrow e^- + e^- + e^+$ were observed. Moreover, the structure of the charged current $V - A$ interaction suggests that the leptonic currents in $\beta^-(\beta^+)$ decays involve a lepton pair consisting of one charged and one neutral particle as partners like $(e^-, \bar{\nu}_e)$, (e^+, ν_e), etc. This further suggests that the two neutral leptons in the decay $\mu^- \rightarrow e^- + \nu + \nu$ are different as only one of them could be of electron type; and the other neutrino has to be of a different type. It was known that in the case of the electron neutrino, ν_e and $\bar{\nu}_e$ are distinct and an e^- is emitted along with $\bar{\nu}_e$ (and not ν_e) in β^--decay and a ν_e (not $\bar{\nu}_e$) is emitted in nuclear β^+-decay, implying that in $\mu^- \rightarrow e^- + \nu + \nu$, one of the neutral leptons is $\bar{\nu}_e$. Therefore, if the other neutral particle is a neutrino and different from the electron type, it should be associated with a muon and so it was identified as ν_μ, 'the muon neutrino'. The possibility of two pairs of neutral leptons was

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earlier discussed theoretically by Oneda et al. [104]. Depending on the analogy of the emission of $(e^-, \bar{\nu}_e)$ and (e^+, ν_e) pairs in nuclear β-decays, it is clear that the $\pi \rightarrow \mu$ decays should proceed as $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. Motivated by these developments, it was suggested by Pontecorvo [105] and Schwartz [106] to use high energy neutrino beams from pion decays to perform experiments like:

$$\nu + n \longrightarrow \mu^- + p \qquad \nu + n \longrightarrow e^- + p \tag{1.27}$$

$$\bar{\nu} + n \longrightarrow \mu^+ + p \qquad \bar{\nu} + n \longrightarrow e^+ + p \tag{1.28}$$

to test whether the neutrinos from pion decays produce muons or electrons. Theoretical calculations for the aforementioned processes were done by Lee and Yang [107], Cabibbo and Gatto [108], and Yamaguchi [109] using the phenomenological $V - A$ theory. The experiments performed at Brookhaven National Laboratory (BNL) by Danby et al. [110] and later by Bienlein et al. at the European Organisation for Nuclear Research (CERN) [111] observed that neutrinos from the pion decays, which were accompanied by muons, produce only muons in these reactions. This confirmed that these neutrinos are different from electron neutrinos, that is, $\nu_\mu \neq \nu_e$.

1.3.3 Lepton number conservation and $e - \mu$ universality

When it was established from the reactor experiments by Reines and Cowan [85] and Davis [87, 88], that the antineutrinos from reactors produce only positrons (and not electrons), a new quantum number called the lepton number (L_e) was proposed to phenomenologically explain this observation. Particles like electrons (e^-) and electron neutrinos (ν_e) were assigned $L_e = +1$ and their antiparticles, that is, positrons (e^+) and antineutrinos ($\bar{\nu}_e$) were assigned $L_e = -1$ while baryons and mesons like nucleons and pions were assigned L_e= 0. Therefore, the conservation of the lepton number would explain the observed results of the reactor experiments. Accordingly, an electron is produced along with an antineutrino in β^--decays while a positron is produced in β^+-decays. Similarly, in later experiments at Brookhaven, the ν_μ neutrinos from π^+-decays did not produce an e^- but only μ^-, demonstrating that ν_μ is different from ν_e; experiments at CERN demonstrated that ν_μ is different from $\bar{\nu}_\mu$ (as in the case of ν_e and $\bar{\nu}_e$). Therefore, a separate lepton number for muon L_μ (other than L_e) and its conservation was suggested. Separate conservation laws for L_e and L_μ explain the absence of $\mu^- \rightarrow e^- + \gamma$, $\mu^- \rightarrow e^- + e^- + e^+$, or $\mu^- + {}^{32}\mathrm{S} \rightarrow e^- + {}^{32}\mathrm{S}$ reactions. It is natural to extend this scheme of classification of leptons to tauons (τ^-, ν_τ) and its antiparticles which were discovered later.

During this time, various developments in the experimental and theoretical study of β-decays were taking place; other weak processes involving the muon and its decay modes, that is, $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$ and $\mu^- \rightarrow e^- + \nu + \bar{\nu}$ as well as the weak muon and electron capture processes from nuclei, that is, $\mu^- + (A, Z) \rightarrow \nu + (A, Z - 1)$ and $e^- + (A, Z) \rightarrow \nu + (A, Z - 1)$ were also discovered and studied in detail. Pontecorvo [112] compared the probability of μ^- capture with electron capture on nuclei and suggested the hypothesis of $\mu - e$ universality of weak current. According to this hypothesis, the strength of the weak interaction involving muon and nucleon is the same as that of the electron and nucleon. This was later elaborated by Puppi [113], Klein [114], and Tiomno and Wheeler [115].

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The idea of the universality of weak interactions is extended to the hadronic sector where the strength of the weak interactions in the vector and axial vector sectors in the case of $n \to p$ transitions in β-decay is compared with the strength of vector and axial vector currents in μ-decays. While this universality is valid for vector currents as supported by the comparison of the strengths of weak interactions in the e and μ sectors with the strength of vector interactions in the hadronic sector, it is only approximately valid in the case of axial vector current. The analysis of the GT transitions in nuclear β-decay shows that the strength in the axial vector sector is about 20% greater than the strength in the vector sector. This was later understood in terms of the renormalization of the axial strength due to strong interactions and elaborated by Goldberger and Treiman [45] and later by Adler [47] and Weisberger [48]. The idea of the universality of weak interactions was later extended to strange particles by Gell-Mann and Levy [58] and Cabibbo [59] where a very small difference in the strength of vector interactions in the leptonic (μ and e) sector and the hadronic (n and p) sector and a large suppression in the case of the strange sector ($\Lambda \to p$ or $K \to \pi$) as compared to the strength in the $\Delta S = 0$ sector ($n \to p, \Sigma \to \Lambda$) was explained phenomenologically. Later, this universality was understood by introducing the concept of quark mixing proposed by Cabibbo [59] (discussed in Chapter 6).

In summary, the weak interaction Hamiltonian incorporating parity violation and the two-component theory of neutrino which evolved into the $V - A$ is now expressed as follows:

$$\begin{aligned}
\mathcal{H}_I &= \frac{G_F}{\sqrt{2}} \cos\theta_C \left(J_\mu J^{\mu^\dagger} + h.c. \right), && (1.29)\\
\text{where } J_\mu &= J^l_\mu + J^h_\mu, && (1.30)\\
\text{with } J^l_\mu &= \bar{\psi}_e \gamma_\mu (1-\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu \gamma_\mu (1-\gamma_5)\psi_{\nu_\mu} + \bar{\psi}_\tau \gamma_\mu (1-\gamma_5)\psi_{\nu_\tau}, && (1.31)\\
J^h_\mu &= \cos\theta_C \bar{\psi}_u \gamma_\mu (1-\lambda\gamma_5)\psi_d + \sin\theta_C \bar{\psi}_u \gamma_\mu (1-\lambda\gamma_5)\psi_s. && (1.32)
\end{aligned}$$

1.3.4 Discovery of tau neutrino and $e - \mu - \tau$ universality

In 1975, Perl et al. [116] discovered a heavy lepton through the scattering process $e^+ + e^- \to \tau^+ + \tau^-$ of mass around 1.776 GeV. Soon, it was established by many experimental and theoretical analysis that the heavy lepton was a spin $\frac{1}{2}$ fermion and consistent with being a point particle like electrons and muons. $\tau^\mp$ leptons undergo weak decay in all the modes of the leptonic and semileptonic decays. Like muons, it was also found to decay into leptonic modes by emitting a muon or an electron accompanied by two neutral leptons, that is,

$$\begin{aligned}
\tau^- &\longrightarrow \mu^- + \nu + \nu,\\
\tau^- &\longrightarrow e^- + \nu + \nu.
\end{aligned}$$

Being a heavier lepton of mass > 1 GeV, it can also decay into two-particle and three-particle semileptonic modes like:

$$\begin{aligned}
\tau^\mp &\longrightarrow \pi^\mp \nu, && \tau^\mp \longrightarrow K^\mp \nu,\\
\tau^\mp &\longrightarrow \pi^\mp \pi^0 \nu, && \tau^\mp \longrightarrow K^\mp \pi^0 \nu,
\end{aligned}$$

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which were also observed [117]. In analogy with the muon case, it was conjectured that τ has its own neutrino ν_τ which is emitted in the τ-decay. Since there was no possibility to produce a $\nu_\tau(\bar{\nu}_\tau)$ beam in the laboratory, it was not possible to directly confirm its existence; it was observed indirectly from two body decay modes. However, it has now been observed directly in experiments with the accelerator and atmospheric neutrinos by DONUT [118], OPERA [119], and SuperK [120]. A separate leptonic number L_τ for the ν_τ was defined with the conservation of L_τ to explain phenomenologically all the leptonic and semileptonic decays of τ lepton. Thus, neutrinos are found to exist in three flavors (also known as generations) described as electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ). Along with their corresponding leptons, they are grouped into three doublets under the quantum number I_W called the weak isospin, that is,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

with similar assignments for their antiparticles. Later, it was established from $e^+ - e^-$ scattering experiments performed at very high energy that in the low energy region of $E < 46$ GeV, there are only three flavors of neutrinos. This result is also supported by analysis of the relevant cosmological data. The strength of the weak interaction of τ is found to be the same as the strength of μ particles confirming the $\mu - \tau$ universality and thus leading to $e - \mu - \tau$ universality.

1.4 Properties of Neutrinos

1.4.1 Weyl, Dirac and Majorana neutrinos

For massless neutrinos, the Dirac equation may be decoupled as discussed in Chapter 2; all neutrinos are left-handed and antineutrinos are right-handed particles in any frame of reference. This is because these neutrinos travel with the speed of light and an observer would not be able to choose a frame which can move faster than the speed of light. The operation of CPT (charge conjugation, parity, and time reversal taken together) will change a particle with left-handed helicity, say described by the spinor ψ_L^ν, to an antiparticle with right-handed helicity, described by the spinor $\bar{\psi_R}^{\bar{\nu}}$ or vice versa. In the case of Weyl neutrinos/antineutrinos, the scenario would be like that shown in Figure 1.9. Thus, according to Weyl [40], neutrinos and antineutrinos are two different particles, with opposite lepton number for a particular flavor of neutrino(say ν_e and $\bar{\nu}_e$) and would have definite helicity states, that is, a left-handed neutrino will always be left-handed and a right-handed antineutrino will always be right-handed.

However, if the neutrinos are massive, then its speed would be lesser than the speed of light and there would always be a possibility of finding a frame(say II frame) which travels faster than the I frame. For an observer in the I frame, the neutrino is left-handed described by ψ_L^ν, while for an observer in the II frame, the neutrino is right-handed ψ_R^ν, as Lorentz transformation will not change the spin of the particle. Similarly, this would be true for the antineutrino, that is, starting with $\bar{\psi}_R^{\bar{\nu}}$, there is always a possibility of finding an antiparticle with left-handed helicity which is defined by $\bar{\psi}_L^{\bar{\nu}}$. Thus, there are four spinors, viz., ψ_L^ν, ψ_R^ν, $\bar{\psi}_L^{\bar{\nu}}$, and $\bar{\psi}_R^{\bar{\nu}}$.

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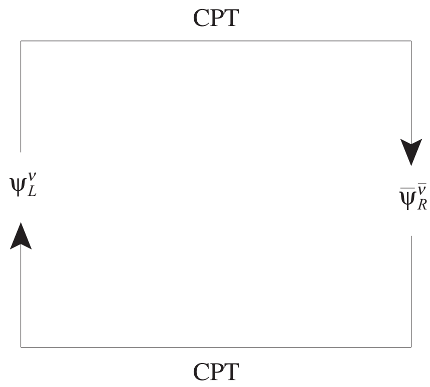

Figure 1.9 Weyl neutrinos.

Therefore, if we assume that $\psi_R^{\bar{\nu}}$ is not the same as $\bar{\psi}_R{}^{\bar{\nu}}$, then it implies that ψ_R^{ν} has its own CPT mirror image $\bar{\psi}_L{}^{\bar{\nu}}$, and there exist four states with the same mass; these four states are called Dirac neutrinos ν^D [28]. This scenario has been shown in Figure 1.10. A Dirac particle is one which is different from its antiparticle. The Dirac particle and antiparticle of the same helicity are two different objects and these two interact differently with matter. If the neutrinos happen to be Dirac neutrinos, then they will have finite magnetic and electric dipole moments.

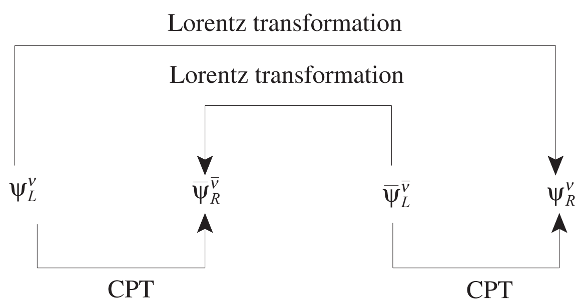

Figure 1.10 Dirac neutrinos.

The concept of a particle and its antiparticle being identical was introduced by Majorana in 1937 [121]. Consider the case of massive left-handed neutrinos (ψ_L^{ν}). If the neutrino happens to be a Majorana neutrino, then after applying Lorentz transformation to a moving reference frame, the right-handed particle ψ_R^{ν}, which is obtained is the same as the particle obtained by CPT operation as shown in Figure 1.11. Thus, unlike the case of the electron and positron which have different charges, neutrinos and antineutrinos are neutral; therefore, if we consider that ψ_R is the same as $\bar{\psi}_R$, then there are only two possible states with the same mass:

$$\begin{aligned} \psi_R &\equiv \bar{\psi}_R, \\ \psi_L &\equiv \bar{\psi}_L. \end{aligned}$$

These sets of states are called Majorana neutrinos ν^M. If the neutrino happens to be a Majorana neutrino, then the particle is its own antiparticle, and it will have identical interactions with

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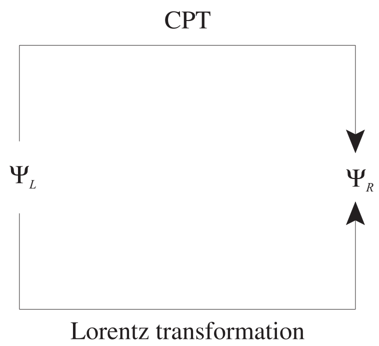

Figure 1.11 Majorana neutrinos.

matter. A Majorana neutrino has neither electric dipole moment nor magnetic dipole moment. It has no charge radius. The lepton number is violated if neutrinos are Majorana particles.

There is a very important consequence of neutrinos being Majorana neutrinos. Some nuclei can undergo neutrinoless double β-decay, that is,

$$^{A}_{Z}X'' \quad \rightarrow \quad ^{A}_{Z+2}Y'' + 2e^-,$$

which may be understood in the following way.

In general, β^- and β^+-decay are described by the following basic processes:

$$\begin{aligned} & n && \rightarrow && p + e^- + \overline{\nu}_e \\ \text{and} \quad & p && \rightarrow && n + e^+ + \nu_e, \end{aligned}$$

or, in general, nuclear β-decay is given by the following reactions:

$$\begin{aligned} ^{A}_{Z}X & \rightarrow & ^{A}_{Z+1}Y + e^- + \overline{\nu}_e, \\ ^{A}_{Z}X' & \rightarrow & ^{A}_{Z-1}Y' + e^+ + \nu_e, \end{aligned}$$

In a neutrinoless double β-decay, virtual neutrinos $(\nu_e = \bar{\nu}_e)$ being emitted at one vertex are

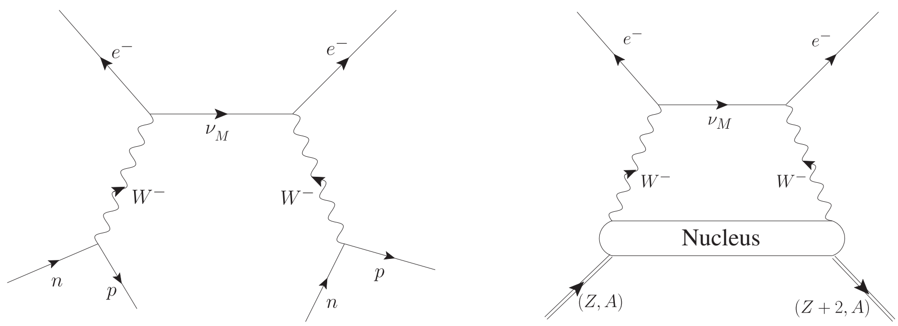

Figure 1.12 Neutrinoless double beta decay ($0\nu\beta\beta$).

absorbed at the other vertex as shown in Figure 1.12, giving electrons and positrons that violate lepton number conservation.

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1.4.2 Neutrino mass

Neutrinos have been assumed to be massless even though Pauli [1] in his original letter proposed a 'tiny mass' for them. Experimentally, neutrino masses are determined by the following ways:

(i) direct determination of the neutrino mass by kinematics of weak decays,

(ii) indirect determination from astrophysics, cosmology, and neutrinoless double β-decay ($0\nu\beta\beta$).

The determination of neutrino mass by using the kinematics of weak decay processes is model independent as it is obtained using energy and momentum conservation while the determination of neutrino mass from astrophysics, cosmology and neutrinoless double β-decay are model dependent and suffer from the uncertainties of the parameters in the theoretical analyses of the cosmological data and the double β-decay.

(i) Direct determination

For many years, neutrino mass for ν_e was determined mainly from the end point of the energy spectrum of β-electron as proposed by Fermi [23] in the β-decay of ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$. In the presence of neutrino mass, the shape factor $\frac{d\Gamma}{dE_e}$ of the electron energy spectrum is given by:

$$S(E_e) \propto (Q - E_e)^2 [(Q - E_e)^2 - m_{\nu_e}^2]^{\frac{1}{2}},$$

where Q= 18.6 keV for β-decay of ^{3}H. This is subject to small atomic corrections due to the excitation of the ^{3}He atom [122] during the β-emission process. In modern times, the Institute of Theoretical and Experimental Physics (ITEP), Moscow group reported a nonzero neutrino mass ($m_{\nu_e} < 35$eV) [123] using β-spectrum of tritium. The ITEP group repeated the measurements and reported in 1980 [124], a neutrino mass in the range 14 eV $< m_{\nu_e} <$ 45 eV. Later, many more measurements were performed by this group.

Another method to determine the mass is to get the neutrino mass from the study of the end point in the energy spectrum of photon in the internal bremsstrahlung of the electron capture (IBEC) of nuclei, that is, $e^- + (Z, A) \rightarrow (Z-1, A) + \nu_e + \gamma$ [125, 126, 127]. However, the best limits come from the ^{3}H beta decay and is given as [117]:

$$m_{\nu_e} < 2.5 \text{ eV} \qquad \text{at} \qquad 95\% \text{ confidence level (C.L.)}$$

The mass of ν_e is also determined by making a comparative study of the e^- capture rates from various orbits in nuclei [128]. Recently, KATRIN [129] experiment has obtained an upper limit on $m_{\nu_e} \leq 1$ eV at 90% C.L.

The limit on the ν_μ mass is obtained from the two-body kinematics of $\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay at rest (DAR). Since the pion decay is a two-body decay and takes place when the pion is at rest, the momentum and energy conservation leads to

$$m_{\nu_\mu}^2 = m_\pi^2 + m_\mu^2 - 2m_\pi \sqrt{m_\mu^2 + |\vec{p}_\mu|^2}. \tag{1.33}$$

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In an experiment performed by Assamagan et al. [130], the momentum of the muon is found to be

$$|\vec{p}_\mu| = 29.792 \pm 0.00011 \text{ MeV}.$$

Using the value of $|\vec{p}_\mu|$ as quoted here and the masses of the muon and pion, the value of $m^2_{\nu_\mu}$ is found to be

$$m^2_{\nu_\mu} = -(0.016 \pm 0.023) \text{ MeV}^2.$$

In many cases, the decaying pion is not completely at rest and may carry a small momentum before decaying. Therefore, a measurement of pion momentum and muon momentum is required to determine the ν_μ mass $m^2_{\nu_\mu} = \left[\sqrt{|\vec{p}_\mu|^2 + m^2_\mu} - \sqrt{|\vec{p}_\pi|^2 + m^2_\pi}\right]^2 - |\vec{p}_\nu|^2$. Experiments of this kind lead to a mass limit of $\nu_\mu < 500$ keV [131]. The best limit on ν_μ mass is [117]

$$m_{\nu_\mu} < 190 \text{ keV} \quad \text{at} \quad 90\% \text{ C.L.}$$

The limit on m_{ν_τ} is obtained from the decay kinematics of $\tau^- \to 3\pi + \nu_\tau$ and $\tau^- \to 5\pi + \nu_\tau$, assuming it to be a two-body decay of τ^- at rest, that is,

$$\tau^- \longrightarrow h^- + \nu_\tau,$$

where h^- represents the hadronic system composed of three or five pions. The aforementioned decay kinematics is similar to what we have obtained in the case of m_{ν_μ} determination from the pion decay in Eq. (1.33). In the case of pion decay, we have only one particle, that is, μ^- in the final state whose momentum is measured experimentally. However, in the case of τ^-, we have a system consisting of three or five particles; therefore, the measurement of the momentum of particles is not easy and hence, E_h is determined from the invariant mass distribution of the hadronic system [132]. From energy and momentum conservation, the energy of the hadronic system is given by:

$$E_h = \frac{m^2_\tau + m^2_h - m^2_{\nu_\tau}}{2m_\tau}. \tag{1.34}$$

ν_τ mass is also determined from the distribution of missing energy in the three charged particle decays of τ^+ and τ^- [132, 133, 134, 135].

The best limit on m_{ν_τ} is [117]:

$$m_{\nu_\tau} < 18.2 \text{ MeV} \quad \text{at} \quad 95\% \text{ C.L.} \tag{1.35}$$

(ii) Cosmological and astrophysical observations

The limits on neutrino masses have also been obtained from cosmological observations. The presence of cosmic neutrinos, which are relics of the Big Bang, like the cosmic microwave background radiation(CMBR) is a prediction of the standard cosmological model. These

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cosmic neutrinos affect cosmological evolution and may be used to get information on neutrino properties, specially, the neutrino mass. The limits on neutrino masses have been obtained by measuring the anisotropy of CMBR using Planck satellite. Some cosmological observations are mainly sensitive to the sum of neutrino masses $\sum_i m_i$. In the standard model of Big Bang cosmology, the three active neutrinos of the standard model of particle physics are assumed to be massless. However, extensions of the standard cosmological model with varying neutrino masses have been used to put limits on neutrino masses. The present bounds using different combinations of current cosmological data give a range of neutrino masses, 0.14 eV$< \sum m_\nu <$0.72 eV.

In a supernova burst, almost 10^{53} neutrinos and antineutrinos of all flavors are released in a few seconds. The neutrinos released during a few seconds of supernova explosion contain mostly electron type neutrinos; the later part of the burst contains neutrino of all flavors. The observations made on the neutrinos from the supernova event (SN1987A) that took place in 1987 in the Large Magellanic Cloud just outside our Milky Way galaxy has also been used to put a limit on neutrino masses. For example, the latest analyses suggest $m_{\bar{\nu}_e} < 5.7$ eV.

(iii) Neutrinoless double β-decay

An alternative way to measure the mass of a neutrino is to look for neutrinoless double β-decay. Many experiments are being conducted to observe $0\nu\beta\beta$ decay of heavy nuclei on ^{76}Ge, ^{136}Xe, etc. The lifetime of the heavy nuclei undergoing double β-decays depends upon the mass of the neutrinos which have to be of the Majorana type ($\nu = \bar{\nu}$). In the past, the Heidelberg-Moscow experiment claims to have observed $0\nu\beta\beta$ decay, but the results are considered to be controversial and inconclusive.

The double β-decay rates depend upon the effective mass of neutrino which is defined as:

$$\langle m_\nu \rangle = \sum_{i=1}^{3} m_i U_{ei}^2 , \tag{1.36}$$

where U_{ei} are the mixing matrix elements and m_i are the masses of the mass eigenstates. The electron type neutrino is represented by a mixed state, described by:

$$|\nu_e\rangle = \sum_i U_{ei} |\nu_i\rangle .$$

The present limits for the effective Majorana mass of the electron neutrino ($\langle m_{\nu_e} \rangle$), which is a coherent sum of the mass eigenvalues weighted with the square of the elements of the mixing matrix, are in the range 0.1 eV$< m_{\nu_e} <$0.4 eV.

However, these limits are model dependent due to the uncertainties in the parameters of neutrino mixing as well as the uncertainties in the knowledge of the nuclear matrix element of heavy nuclei undergoing neutrinoless double β-decay.

1.4.3 Neutrino charge and charge radius

(i) Neutrinos when proposed were assumed to be electrically neutral. However, there are attempts to measure the charge of the neutrino in β-decays by measuring the charge of

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the neutron Q_n and the total charge of the proton and electron, that is, $|Q_p + Q_{e^-}|$ in the decay $n \to p + e^- + \bar{\nu}_e$ [136, 137]. This gives a limit $Q_{\bar{\nu}} < (0.5 \pm 2.9) \times 10^{-21} e$. The astrophysical limit derived from the SN1987A supernova observation is [138]:

$$Q_{\bar{\nu}} < 2 \times 10^{-15} e.$$

(ii) The charge of neutrino is consistent with zero to a very high degree of precision; however, it may have a charge distribution like a neutron even though it is considered a point particle in the field theory. Attempts to determine the charge radius have been made [139] for ν_e and ν_μ from $\nu_e e$ [140], $\bar{\nu}_e e$ [141], and $\nu_\mu e$ [142] scattering. Like hadron, the mean square charge radius is deduced from a measurement of the vector form factor in the $\nu_e e$ and $\nu_\mu e$ elastic scattering using the relation

$$\langle r^2 \rangle = -6 \frac{d}{dq^2} f(q^2)|_{q^2=0}, \tag{1.37}$$

where $f(q^2)$ is the form factor corresponding to the matrix element of the vector current. In the case of neutral particles, the value of $\langle r^2 \rangle$ could be negative or positive and the following limits [143, 144, 145] are obtained in the case of ν_e and ν_μ:

$$\begin{aligned}
-5.3 \times 10^{-32} < & \left[\langle r^2 \rangle_{\nu_\mu}\right] < 1.3 \times 10^{-32}\ \text{cm}^2, \\
-0.77 \times 10^{-32} < & \left[\langle r^2 \rangle_{\nu_\mu}\right] < 2.5 \times 10^{-32}\ \text{cm}^2, \\
-5.0 \times 10^{-32} < & \left[\langle r^2 \rangle_{\nu_e}\right] < 10.2 \times 10^{-32}\ \text{cm}^2.
\end{aligned}$$

1.4.4 Magnetic and electric dipole moments of neutrinos

In general, the electroweak properties of a spin $\frac{1}{2}$ Dirac particle are described in terms of two vector form factors called electric and magnetic form factors which in the static limit define the charge and magnetic moment, and two axial vector form factors called axial and tensor form factors which in the static limit define the axial charge and electric dipole moment. If the neutrinos are considered to be Dirac neutrinos with nonzero mass, it could have these form factors to be nonvanishing and experimental attempts can be made to study them. The electromagnetic properties like electric and magnetic dipole moments of neutrinos depend upon the nature of the neutrinos. Dirac neutrinos can have electric and magnetic moments like neutrons. It is to be noted that a magnetic moment of the order $e \times 10^{-13}$ cm^2 was proposed by Pauli [1]. However, if the neutrinos are Majorana neutrinos, their electric and magnetic moments are zero. The existence of electric dipole moment depends on the validity of CP invariance which forbids a nonzero electric dipole moment for elementary particles. Since neutrinos participate in weak interactions which violate CP invariance, they may have an electric dipole moment; on the other hand, there is no symmetry principle which forbids the existence of magnetic dipole moment for neutral particles. Theoretically, even massless particles can have magnetic dipole moment.

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The standard model calculations for the magnetic moment of a neutrino with mass m_ν yield a very small magnetic moment of the order $3\times10^{-19}\frac{m_\nu}{eV}\mu_B$. In general, the neutrino magnetic moment need not be proportional to the neutrino mass; there are models constructed to give larger magnetic moments. Experimentally, the laboratory limits on the neutrino magnetic moments are obtained by performing elastic $\nu_e e, \bar{\nu}_e e$, and $\nu_\mu e$ scattering. The magnetic moment of the neutrino additionally contributes to the weak cross section due to the electromagnetic scattering of neutrinos. For a neutrino of magnetic moment μ_ν, the additional differential cross section is given by:

$$\frac{d\sigma^{EM}}{dE'_e} = \frac{\pi\alpha^2\mu_\nu^2}{m_e^2}\Big(\frac{1}{T} - \frac{1}{E_\nu}\Big), \tag{1.38}$$

where T is the recoil kinetic energy of the electron. Therefore, to see the maximum effect of magnetic moment on the cross section, very low energy scattering processes are favored. Antineutrinos from reactors and neutrino beams from pions decay at rest (DAR) at the accelerators are favored. Earlier experiments were done by Reines and Cowan, but now many experiments have been performed at various reactors and accelerators around the world to determine the magnetic moment of neutrinos. A stronger limit is obtained from the astrophysical and cosmological considerations but they are model dependent. A summary of these results can be found in Refs. [143, 146, 147, 148]; the present limits are:

$$\mu_{\nu_e} < 1.8 \times 10^{-10}\mu_B.$$

In case of ν_μ, the limit on the magnetic moment μ_{ν_μ} from accelerator experiments is [117]:

$$\mu_{\nu_\mu} < 7.4 \times 10^{-10}\mu_B.$$

The limits on the magnetic moment of ν_τ are weaker and come from the study of $e^+e^- \to \nu\bar{\nu}\gamma$ processes at accelerators [149, 150, 151] and also from $\nu_\tau e^- \to \nu_\tau e^-$ scattering from the bubble chamber experiments at CERN and BEBC [152]. The limits are [117]:

$$\mu_{\nu_\tau} \approx 5.4 \times 10^{-7}\mu_B.$$

1.4.5 Helicity of neutrino

The discovery of parity violation and the revival of the two-component theory of neutrinos in the study of the nuclear β-decays implied that the neutrinos are left-handed and antineutrino are right-handed. Indirect evidence of this property of the neutrino and antineutrino was available from observations made on the polarization of electrons and other spin momentum correlation measurements of the emitted electrons and positrons in many weak decays of elementary particles. The direct measurement of the helicity of ν_e was made in an excellent experiment performed by Goldhaber et al. [153]. They measured the polarization of photons in a weak electron capture experiment by ^{152}Eu nuclei leading to neutrino (ν_e)

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and ^{152}Sm* nucleus which decays to ^{152}Sm by photon emission. From the polarization measurements of photon, it was inferred that the neutrinos emitted in such decays are left-handed. Later, experiments done on the muon capture process on ^{12}C and many other experiments confirmed that all neutrinos are left-handed and antineutrinos are right-handed. This will be discussed in some detail in Chapter 5.

1.5 New Developments

1.5.1 Standard model of electroweak interactions and neutral currents

The standard model of electroweak interactions is one of the most important milestones in our understanding of fundamental interactions. It unifies electromagnetic (EM) and weak interactions and is based on the principle of local gauge invariance of basic interactions. The Fermi theory of β-decay was formulated in analogy with electromagnetic interaction in quantum electrodynamics, which is generated from invariance under the local gauge transformations.

The Fermi theory evolved phenomenologically into the $V - A$ theory which was considered to be a low energy manifestation of a theory considered to be mediated by vector bosons W as suggested earlier by Klein [154], Bludman [50], Schwinger [49], and Leite Lopes [51]. Since weak interactions are of very short range, the mediating W bson has to be of very high mass. Some experiments were suggested by Pontecorvo [105] and Lee and Yang [107] and carried out at BNL [155] and CERN [156] laboratories to search for W bosons in the range of a few GeV without any success. The requirement for W bosons to be massive prevented the formulation of weak interactions as a local gauge theory with W bosons as gauge bosons as the principle of local gauge invariance required that gauge bosons be massless. Therefore, the theories proposed earlier to unify electromagnetic and weak interaction did not progress until Weinberg [157] and Salam [37] in 1967 used the Higgs mechanism of spontaneous breaking of symmetry to generate the masses of the gauge bosons by introducing a scalar field. The mechanism of mass generation by spontaneous breaking of the local gauge symmetry named after Higgs was developed earlier independently by Higgs [158], Englert and Brout [159], Guralnik et al. [160] and Kibble [161] in the 1960s following the works of Nambu [46], Goldstone [162], and Nambu and Jona-Lasinio [163].

The introduction of the scalar field popularly known as Higgs boson (also God particle) facilitates the generation of masses for the massless gauge bosons while preserving the local gauge invariance of the weak interaction Lagrangian. The standard model applied to the leptons reproduces the essential features of the phenomenological weak interactions like parity violation, two-component neutrino, and the massive $W^{\pm}$ bosons mediating the charged current interactions, as well as the massless photon (γ) mediating the electromagnetic interaction. In addition, it also predicts the existence of a new massive neutral gauge boson Z^0 implying neutral currents in the leptonic sector (electron and neutrino) which were discussed quite early in the history of weak interactions but were never observed. The masses of the gauge bosons $W^{\pm}$ and Z^0 and their couplings to the leptons were predicted in terms of g, the weak coupling of

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$W^{\pm}$, e the electromagnetic coupling of the photon to the lepton, and a free parameter θ_W called the weak mixing angle to be determined from the experiments. The mass and the coupling of the Higgs boson remain undetermined in the model. The standard model of the leptons was later extended to the quark sector including strange quarks following the GIM mechanism and is applied, in general, to describe the electroweak interactions of leptons and hadrons.

Specifically, the standard model of the electroweak interactions is based on the local gauge invariance of the Dirac Lagrangian under the group $SU(2)_L \times U(1)_Y$ in which the left-handed leptons and hadrons are assigned to the doublets of the group $SU(2)_L$ and their right-handed partners are assigned to a singlet for each flavor as follows:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L.$$

These left-handed doublets form the basic representation of $SU(2)$ and their right-handed partners like

$$e_R, \ \mu_R, \ \tau_R, \ u_R, \ d'_R, \ c_R, \ s'_R, \ t_R, \ b'_R$$

form the singlet representation. The requirement of the gauge invariance introduces three massless gauge fields $W^{\pm}, W^3_\mu$ corresponding to $SU(2)_L$ with a coupling g and one massless gauge field B_μ corresponding to U(1) with a coupling g'. The $W^{\pm}$ fields are charged, while W^3_μ and B_μ are neutral. The new scalar Higgs field ϕ is introduced in such a way that the linear combinations of B_μ and W^3_μ fields, that is,

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W^3_\mu + g B_\mu \right) \tag{1.39}$$

remains massless and is to be identified with the photon field; the orthogonal combination of B_μ and W^3_μ, that is,

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g W^3_\mu - g' B_\mu \right) \tag{1.40}$$

along with W^+ and W^- become massive and mediate the neutral and charged current weak interactions, respectively (see Chapter 8).

The experimental observation of neutral currents (NC) was a major triumph of the standard model when they were discovered at CERN in experiments with $\nu_\mu / \bar{\nu}_\mu$ beams [164] in the reactions

$$\nu_\mu + N \longrightarrow \nu_\mu + N, \tag{1.41}$$

$$\bar{\nu}_\mu + N \longrightarrow \bar{\nu}_\mu + N. \tag{1.42}$$

The existence of NC events was later observed in many other experiments [165, 166, 167]. Purely leptonic NC events in reactions like $\nu_\mu + e^- \to \nu_\mu + e^-$ and $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$

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were also observed soon afterward [168]. Later at SLAC, NC reactions with electron beams were observed in 1978 through the observation of parity violating effects in polarized electron scattering from nucleons and nuclear targets due to the interference of photon and Z^0 exchange in $e^- p$ scattering [169].

The neutrino experiments induced by neutral currents in the leptonic as well as the hadronic sector played an important role in determining the value of the weak mixing angle θ_W.

1.5.2 Discovery of $W^\pm$, Z^0, and Higgs boson

(i) $W^\pm$ and Z^0 boson:

The weak gauge bosons $W^\pm$ and Z^0 predicted by the standard model of Glashow–Weinberg–Salam (G–W–S) are produced in hadronic collisions like the $\bar{p}p$ collisions in which one of the antiquarks in $\bar{p}$ collides with the quark in p to produce W^+ or W^- which decays through the weak processes, for example,

$$\begin{array}{l} \bar{d} + u \longrightarrow W^+ \longrightarrow \mu^+ + \nu_\mu \; (e^+ + \nu_e), \\ \bar{u} + d \longrightarrow W^- \longrightarrow \mu^- + \bar{\nu}_\mu \; (e^- + \bar{\nu}_e), \\ \bar{d} + d \longrightarrow Z^0 \longrightarrow e^- + e^+ \; (\mu^- + \mu^+), \\ \bar{u} + u \longrightarrow Z^0 \longrightarrow e^- + e^+ \; (\mu^- + \mu^+). \end{array} \tag{1.43}$$

$W^\pm$ and Z^0 bosons were first discovered in the $\bar{p}p$ collision experiments performed at CERN in 1983 by two groups which used the beams of protons with center of mass energy $E_{CM} = 540$ GeV [170, 171]. We see from Eq. (1.43) that the signature of $W^\pm$ bosons are single charged leptons $\mu^\pm$ with large missing transverse momenta, while the signature of Z^0 are two-charged leptons coming out at an angle θ such that

$$M_Z^2 = 2E_+E_-(1 - \cos\theta).$$

After the discovery of $W^\pm$ and Z^0 at CERN, other measurements of their mass have been performed at hadron colliders at CERN and Fermilab as well as at the e^-e^+ colliders at LEP in CERN and SLAC [172, 173, 174]. The masses of $W^\pm$ and Z^0 bosons are measured to be [117]:

$$\begin{aligned} M_W &= 80.379 \pm 0.012 \text{ GeV}, \\ M_Z &= 91.1876 \pm 0.0021 \text{ GeV}. \end{aligned}$$

(ii) Higgs boson:

The search for Higgs boson as formulated in the standard model started quite early first at LEP in CERN and then at Tevatron in Fermilab without any success. However, the experiments at Fermilab indicated that Higgs boson, if existed, could have a mass between 115 GeV and 140 GeV [175]. In July 2012, using the large hadron collider (LHC) at CERN operating at a center of mass energy of 7 TeV in the $\bar{p}p$ scattering, the ATLAS (a toroidal LHC apparatus) and CMS (compact muon solenoid) collaborations succeeded

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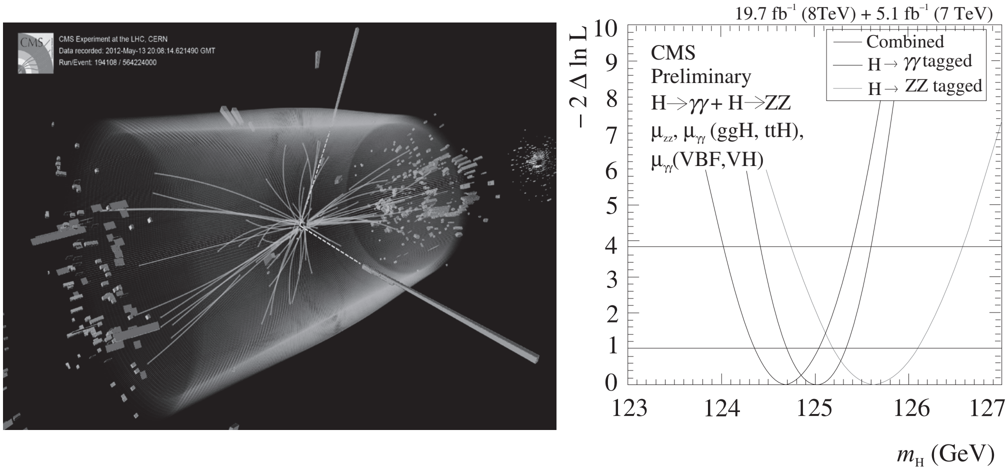
in observing for the first time the Higgs boson through its decays in W^+W^- modes and two photon modes. A typical Higgs event observed by the CMS collaboration has been shown in Figure 1.13 [176, 177]. A combined analysis of both data at ATLAS and CMS gives [117]:

$$M_H \quad = \quad 125.18 \pm 0.16 \text{ GeV}.$$

Later, its mass, properties, and decay modes were measured and confirmed by the collaborations in many other experiments [178].



(a) (b)

Figure 1.13 (a) $p\bar{p}$ collisions inside the CMS detector at CERN. This event was recorded in 2012 by the compact muon solenoid (CMS) at the large hadron collider (LHC). (b) Experimental confirmation of the Higgs boson [176, 177].

1.5.3 Neutrino mass, mixing, and oscillations

Most of the weak interaction phenomenology after the discovery of nuclear β-decays was done assuming neutrinos to be massless but the question of neutrino mass has been a subject of great interest ever since Pauli's proposal of neutrinos and the possibility of its nonzero mass [1]. A proper understanding of neutrino mass and its origin could solve many problems being faced today in neutrino physics like the nature of neutrinos, that is, Dirac or Majorana, mixing of neutrinos and neutrino oscillations, that is, three or four flavors, mass hierarchy, CP violation in lepton sector, and sterile neutrinos. We have given the experimental status of the search for neutrino masses in Section 1.4.2; a theoretical and experimental description of neutrino oscillations is deferred to Chapter 18. We present here a brief discussion on the existence of neutrino mass, mixing, and oscillations and their implications.

Pontecorvo [179] suggested that if a neutrino has nonzero mass, then it may oscillate into an antineutrino $\nu_e \rightleftarrows \bar{\nu}_e$ in analogy with the $K^0 - \bar{K}^0$ oscillation [61]. The possibility that if $\nu_e \neq \nu_\mu$; then ν_e could oscillate into ν_μ and vice versa was suggested by Maki et al. [71]. This

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possibility, called the flavor oscillation, was later formulated by Gribov and Pontecorvo [180] and Bahcall and Frautschi [181]. The possibility of oscillation requires neutrinos to have nonzero mass. In the case of massive neutrinos, other possibilities like the decay of a heavier neutrino into lighter neutrinos by emitting a photon or other lighter particles are also possible.

Interest in neutrino oscillation physics started to grow after the early results of the solar neutrino experiments reported by Davis et al. from the Homestake mines lab in USA [87, 88]. Through this experiment, they claimed to have observed the ν_e flux from the sun which was smaller than the flux predicted by contemporary solar models [182]. The phenomenon of neutrino oscillations in which a ν_e oscillates to other flavors offers a natural explanation for the reduction in the solar neutrino flux. However, this explanation required a large mixing of the two flavors of neutrinos and the fine tuning of neutrino oscillation parameters to explain the experimental results obtained by Davis et al. Moreover, all these conclusions were subject to the uncertainties in the parameters of the standard solar model. In view of this, many experiments to observe the neutrino oscillations using neutrino beams from other sources like nuclear reactors and accelerators were done without any positive result. However, during the 1990s the deep underground detectors at Kamiokande [183] in Japan and the IMB collaboration [184] in USA, initially planned to observe the proton decay predicted by the grand unified theories, succeeded in detecting a depletion in the ν_μ flux in atmospheric neutrinos compared to the theoretical calculations.

The Kamiokande collaboration further studied the zenith angle dependence of the atmospheric ν_μ flux and found that the up going ν_μ flux, where the distance neutrinos travel through the earth, is smaller than the down going ν_μ flux observed in detectors placed in the underground observatory. This angle dependence could be explained on the basis of neutrino oscillations. However, the statistics of the experimental data was poor and the low energy ν_μ data was not as conclusive as the higher energy data on ν_μ flux. Finally, the Super-Kamiokande experiment with a much larger detector mass, of about 15 times the original Kamiokande detector, confirmed these findings and established the phenomenon of neutrino oscillation consistent with the parameters of ν oscillation phenomenology.

After the early indications of neutrino oscillations, in the case of solar neutrinos, with the experiments of Davis and its model dependence on solar model parameters, neutrino oscillations was finally confirmed in a model independent way at the SNO Lab in Sudbury, Canada by an observation of CC (charged current) and NC (neutral current) reactions on deuterium and elastic scattering (ES) on electron targets in three independent experiments, that is,

$$\nu_e + d \longrightarrow e^- + p + p \quad \text{(CC)}, \qquad \nu_{e,\mu,\tau} + d \longrightarrow \nu_{e,\mu,\tau} + n + p \quad \text{(NC)},$$

$$\nu_e + e^- \longrightarrow \nu_e + e^- \qquad \text{(ES)}.$$

It was observed that the flux of the CC reaction [185] which is affected by the $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillations was almost three times smaller than the NC reaction [185] which is not affected by the neutrino oscillation. These observations used the higher energy neutrinos from the decays of ^{8}B produced in the solar core. The ν_e flux of these neutrinos was smaller than ν_e flux produced in $pp \to de^+\nu_e$ reaction by a factor of 10^{-4} which are of lower

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energy. Experiments with these low energy ν_e sources were done at GALLEX and SAGE with the detectors using gallium with a very low Q value of 233 keV. The observed event rates showed that the ν_e flux were smaller by a factor of two compared with the predicted fluxes, thus, confirming the phenomenon of neutrino oscillations for solar neutrinos in the region of very low energy. The phenomenon of neutrino oscillations in the lower energy region was later confirmed with the reactor antineutrinos at KAMLAND, RENO, and other experiments. Finally, in many long baseline neutrino experiments performed with accelerator neutrinos at LSND, MiniBooNE, SciBooNE etc., the phenomenon of neutrino oscillations was also confirmed in the region of intermediate and high energies. A detailed description of these experiments is given in Chapter 17.

The physics of neutrino mass, mixing, and oscillations can be demonstrated by a simple example of two flavor mixing of ν_e and ν_μ in analogy with quark mixing [70]. A pure ν_e beam described by a wave function while traveling in space may develop a component of ν_μ in this beam; the mixture of the ν_μ wave function will describe the probability of finding the ν_μ component in the ν_e beam after a time t as illustrated in Figure 1.14. We assume that the flavor state ν_e and ν_μ participating in the weak interactions are mixtures of the mass eigenstates ν_1 and ν_2 and the mixing is described by a unitary mixing matrix U such that:

$$\nu_{l=e,\mu} = \sum_{i=1,2} U_{li}\nu_i. \tag{1.44}$$

The unitarity of the U matrix requires that in two-dimensional space it is described by one parameter which is generally chosen to be θ such that:

$$U = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix} \tag{1.45}$$

where $c_{12} = \cos\theta$ and $s_{12} = \sin\theta$. As a pure beam of ν_e at $t = 0$ propagates, the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$, occurring in Eq. (1.44), would evolve according to

$$|\nu_1\rangle = \nu_1(0)e^{-iE_1 t} \tag{1.46}$$

$$|\nu_2\rangle = \nu_2(0)e^{-iE_2 t} \tag{1.47}$$

where $E_1 = \sqrt{|\vec{p}|^2 + m_1^2} \approx |\vec{p}| + \frac{m_1^2}{2|\vec{p}|}$ and $E_2 = \sqrt{|\vec{p}|^2 + m_2^2} \approx |\vec{p}| + \frac{m_2^2}{2|\vec{p}|}$, $|\vec{p}| \approx E$ being the common momentum of neutrinos with energy E_1 and E$_2$; m_1 and m_2 are the mass of $|\nu_1\rangle$ and $|\nu_2\rangle$ states, respectively. After a time t, the $|\nu_e(t)\rangle$ will be a different admixture of $|\nu_1\rangle$ and $|\nu_2\rangle$. The probability of finding ν_μ in the beam of ν_e at a later time t is given by (see Chapter 18)

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}L\right) = \sin^2\theta \sin^2\left(1.27\frac{\Delta m^2}{E}L\frac{[\text{eV}^2][\text{km}]}{[\text{GeV}]}\right), \tag{1.48}$$

Thus, we see that for $P(\nu_e \to \nu_\mu) \neq 0$, we need $\Delta m^2 \neq 0$ and $\theta \neq 0$, that is, we need the mass difference between the neutrino mass eigenstates to be nonzero, implying that at least one

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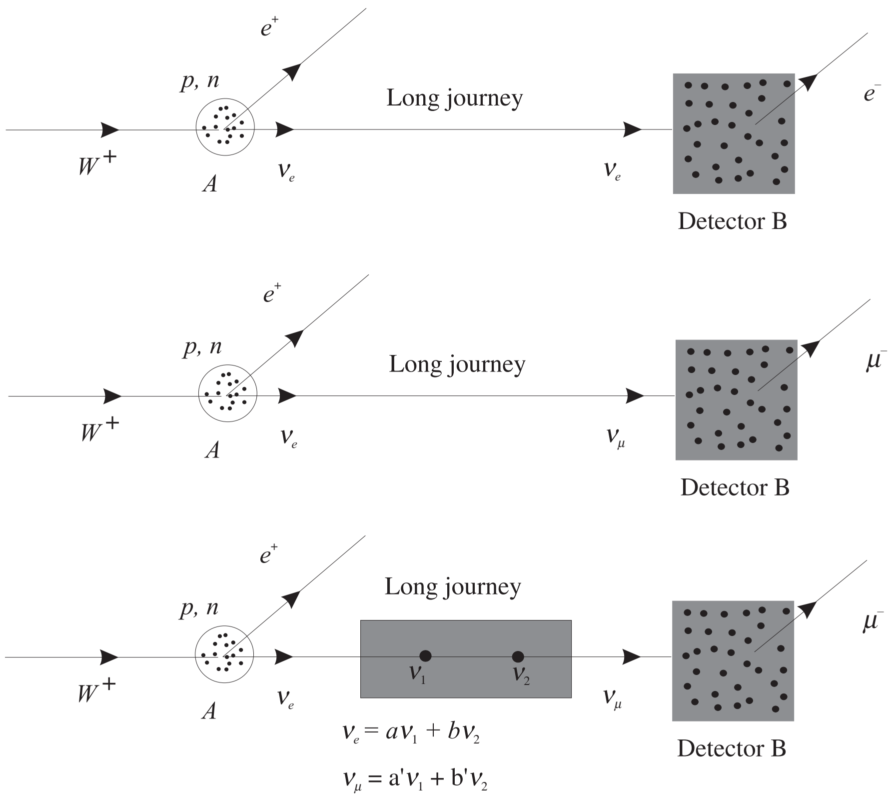

Figure 1.14 A W^+-boson interacts at the point A and produces e^+ and ν_e. The ν_e travels a long distance; it interacts with the detector B and produces a lepton e^- as shown in the top panel. If the flavor of the initial and final lepton is different, that is, $e^- \neq \mu^-$, it can be concluded that during its long journey, a neutrino which was produced initially with a flavor e gets converted into another flavor μ as shown in the central panel. This is only possible when the neutrino propagates as a linear combination of mass eigenstates ν_1 and ν_2, which are different from the flavor eigenstates ν_e and ν_μ as shown in the bottom panel.

of them is massive, and the mixing angle θ to be nonzero. Thus, if the explanation of the solar neutrino flux deficit and other deficits observed in the atmospheric, reactor, and accelerator neutrinos are explained to be due to neutrino oscillations, neutrinos should have nonzero mass and the neutrino flavors should mix.

1.5.4 Matter enhancement of neutrino oscillations and the MSW (Mikheyev–Smirnov–Wolfenstein) effect

One of the major developments in the physics of the neutrino oscillation is the inclusion of matter effects in the study of the solar neutrino problem first discussed by Wolfenstein [186] and later elaborated by Mikheyev and Smirnov [187] and Bethe [188]. Wolfenstein [186] demonstrated that solar neutrinos while propagating from the center of the sun, where they are created in nuclear reactions, to terrestrial detectors may undergo coherent scattering with matter

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which is different for neutrinos with different flavors. For example, while $\nu_e(\bar{\nu}_e)$ can interact with matter through CC as well as NC weak interactions, $\nu_\mu(\bar{\nu}_\mu)$ and $\nu_\tau(\bar{\nu}_\tau)$ can interact only through NC interactions. The coherent scattering of $\nu_e(\bar{\nu}_e)$, with electrons by CC interactions is very important as it affects the free propagation of neutrinos by offering a potential $V(x) = \sqrt{2}G_F N_e(x)$, where $N_e(x)$ is the electron number density at point x and G_F is the Fermi coupling constant. Thus, the motion of the neutrinos is described by the equation of motion determined by the Hamiltonian given by:

$$H = H_0 + \sqrt{2}G_F N_e(x),$$

where H_0 is the Hamiltonian for free propagation and $V(x)$ is the potential generated by the presence of the matter. The additional term in the Hamiltonian affects the propagation of the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$, and we get different equations describing the propagation of ν_1 and ν_2 as given in Eqs. (1.46) and (1.47), respectively, which affects the propagation of ν_e and ν_μ and leads to an expression for the probability of finding ν_μ in a beam of ν_e with energy E after traveling a distance L as:

$$P(\nu_e \longrightarrow \nu_\mu) = \frac{1}{2}\sin^2 2\theta_m \sin^2\left(\frac{\Delta m_m^2 L}{4E}\right), \tag{1.49}$$

where

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}, \tag{1.50}$$

with

$$A = 2\sqrt{2}G_F N_e E, \tag{1.51}$$

$$\Delta m_m^2 = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m \sin 2\theta)^2}. \tag{1.52}$$

The mixing angle θ_m can be maximal (i.e., $\theta_m = \frac{\pi}{4}$) even for very small values of θ, if

$$\Delta m^2 \cos^2 2\theta = A = 2\sqrt{2}G_F N_e E.$$

This condition is called the resonance condition and it explained the solar neutrino problem which needed a large mixing angle. Since $N_e(x)$ is a varying function of number density $((N_e(x) = \frac{Y_e}{m_N}\rho(x)$, where Y_e is the number of electrons per nucleon, m_N is the nucleon mass, and $\rho(x)$ is the density of the sun), there might be an energy for which at a certain point x, the condition (A) in Eq. (1.51) may be satisfied by making ν_e oscillate into ν_μ at that point x. This oscillation may be an adiabatic process or otherwise. The enhancement of neutrino oscillations is called the MSW (Mikheyev–Smirnov–Wolfenstein) effect. The details are discussed in Chapter 18.

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1.5.5 Three-flavor neutrino oscillations and mass hierarchy

In the three-flavor oscillation scenario (ν_e, ν_μ, ν_τ), the mass eigenstates ν_1, ν_2, and ν_3 are related to the flavor eigenstates ν_e, ν_μ, ν_τ such that:

$$\begin{aligned} |\nu_e\rangle &= a_1|\nu_1\rangle + b_1|\nu_2\rangle + c_1|\nu_3\rangle \\ |\nu_\mu\rangle &= a_2|\nu_1\rangle + b_2|\nu_2\rangle + c_2|\nu_3\rangle \\ |\nu_\tau\rangle &= a_3|\nu_1\rangle + b_3|\nu_2\rangle + c_3|\nu_3\rangle \end{aligned} \tag{1.53}$$

where a, b, and c are the different weight factors. If one assumes neutrinos to be Dirac particles, then in a three-flavor oscillation, the flavor and mass eigenstates are related through a unitary 3×3 unitary matrix given by:

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}. \tag{1.54}$$

The oscillation matrix is chosen to be unitary such that the total probability of oscillation (including all possible cases) is unity, that is,

$$P(\nu_e \rightleftharpoons \nu_\mu) \; + \; P(\nu_\mu \rightleftharpoons \nu_\tau) \; + \; P(\nu_\tau \rightleftharpoons \nu_e) = 1. \tag{1.55}$$

The 3×3 unitary matrix is known as the PMNS (Pontecorvo, Maki, Nagakava and Sakata) matrix, whose parameters are three mixing angles θ_{12}, θ_{13}, θ_{23}, and a CP violating phase δ given by the U matrix in Eq. (1.22). Worldwide, many experiments are being performed, using neutrinos from different sources like accelerators, solar, reactor, atmospheric, etc., to determine the parameters $U_{ij}(i = e, \mu, \tau, j = 1, 2, 3, i \neq j)$ very precisely. The experiments like DUNE, T2K, NOνA, HyperK, etc. are expected to give some information about δ.

Corresponding to the three neutrino flavors, there are three mass eigenstates in which two of the neutrino mass eigenstates are nearly degenerate, that is,

$$|\Delta m^2_{21}| \equiv \Delta m^2_{\text{small}} \ll |\Delta m^2_{31}| \cong |\Delta m^2_{23}| \equiv \Delta m^2_{\text{big}} \, . \tag{1.56}$$

The present limits on Δm^2_{ij} $(i \neq j = 1, 2, 3)$ show that $\Delta m^2_{12} = m^2_1 - m^2_2$ is the smallest of all the $\Delta m^2_{ij}(i \neq j)$, while Δm^2_{13} is of the order of Δm^2_{23}. The analysis of the neutrino oscillation experiments does not give any information about the absolute masses corresponding to the three mass eigenstates. Therefore, there are two possibilities:

(i) $m^2_3 > m^2_1$ $(\approx m^2_2)$, or

(ii) $m^2_3 < m^2_1$ $(\approx m^2_2)$

as shown Figure 1.15 corresponding to the normal and inverted mass hierarchy of neutrinos. The two mass hierarchy scenarios affect many aspect of the analysis of the neutrino oscillations. The current analyses of the neutrino oscillation experiments suggest a preference for the normal mass hierarchy.

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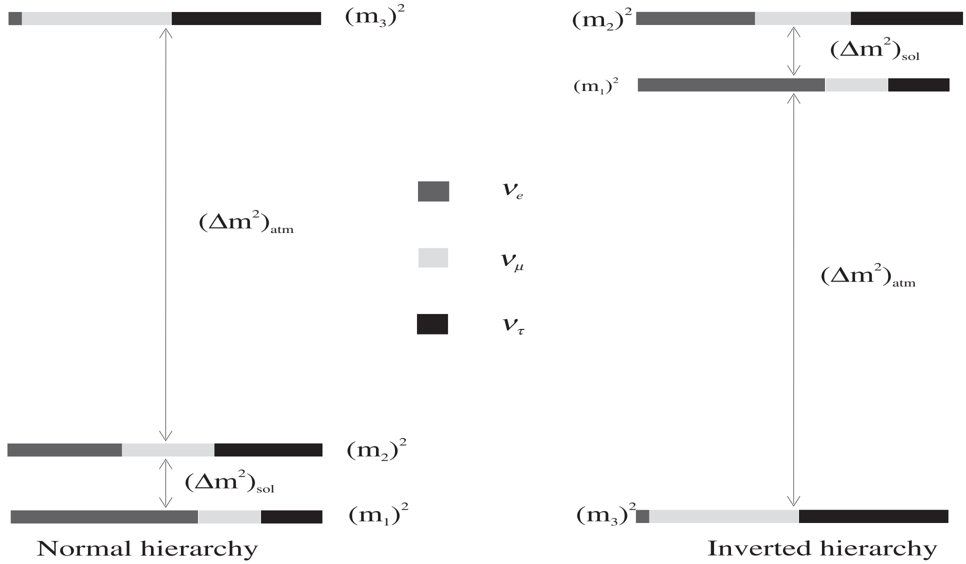

Figure 1.15 Neutrino mass hierarchy.

1.5.6 Sterile neutrinos and 3+1 flavor mixing

The phenomenology of the standard model of the electroweak interactions with three-flavor mixing is successful in describing most of the available experimental data from the various neutrino experiments done with solar, atmospheric, reactor, and accelerator (anti)neutrinos in terms of three mixing angles θ_{ij} $(i \neq j = 1,2,3)$, a phase angle δ, and the mass squared differences Δm^2_{ij} $(i \neq j = 1,2,3)$. However, there are some 'anomalous' results from few oscillation experiments in the low energy regime from LSND, GALLEX, and reactor (anti)neutrinos which are not explained within the three-flavor scenario of the standard model and suggestions have been made for the existence of an additional flavor of neutrinos. Since there is strong experimental evidence from LEP experiments by measuring the total width of Z^0-decays into $q\bar{q}$ and $l\bar{l}$ pairs as well as from the cosmological data on the ^{4}He abundance that there are only three flavors of neutrinos in the energy region of $E < 46$ GeV, the additional neutrino must be 'sterile', that is, having no interaction with matter in this energy region even if they exist.

The simplest and most popular extension of the three-flavor mixing phenomenology within the standard model is to consider the 3+1 flavor neutrino mixing in which three active neutrinos ν_e, ν_μ, ν_τ are mixed states of the three massive neutrino states ν_1, ν_2, ν_3 and a fourth mass neutrino state ν_4. However, the fourth weak interaction state is a sterile neutrino ν_s which is mainly composed of a fourth neutrino mass state (ν_4) at the eV energy scale; the coupling of ν_s to other mass states ν_1, ν_2, ν_3 is negligible. The $3+1$ flavor mixing is described by a 4×4 unitary matrix which is parameterized in terms of 9 parameters, 6 of which are chosen to be the rotation angles, that is, θ_{12}, θ_{13}, θ_{14}, θ_{23}, θ_{24}, and θ_{34} and three phases, that is, δ_1, δ_2, and δ_3. Thus, there are three additional mixing angles and two additional phases making

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the interpretation of experimental data quite difficult and model dependent. There have been many experiments in recent years to determine these parameters. For the latest review on sterile neutrinos, please see Ref. [189, 190, 191]. Due to the presence of increased number of independent parameters, various assumptions have been made about the additional mass squared differences, mixing parameters, and phase angles for analyzing and interpreting the available data.

In the framework of 3+1 active–sterile neutrino mixing, the anomalies in the earlier experiments of LSND, GALLEX, and reactor antineutrinos are found to be sensitive to the oscillations generated by the mixing of the fourth sterile neutrino, that is, ν_4 through $\Delta m^2_{41} \approx \Delta m^2_{42} \approx \Delta m^2_{43} \geq 1$ eV^2 which is much larger than the scale of Δm^2_{ij} in solar and atmospheric neutrino mass squared differences, that is, $\Delta m^2_{\mathrm{sol}} = \Delta m^2_{21} \approx 7.4 \times 10^{-5}$ eV^2 and $\Delta m^2_{\mathrm{atm}} = |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}$ eV^2 which generate the oscillations in the case of solar, atmospheric, and accelerator neutrinos. Moreover, in most of the analyses, the matrix elements of the 3+1 flavor mixing matrix corresponding to the mixing of the active neutrinos to the sterile neutrinos denoted by U_{e4}, $U_{\mu 4}$, and $U_{\tau 4}$ are considered as a perturbation on the 3×3 active neutrino mixing matrix and are taken to be very small, that is, $|U_{e4}|^2 \simeq |U_{\mu 4}|^2 \approx |U_{\tau 4}|^2 \ll 1$. The task of determining the parameters describing the phenomenology of 3+1 flavor mixing of neutrinos would be one of the main objectives of future experiments being done with reactor, solar, atmospheric, and accelerator neutrinos in the low and high energy region of neutrinos. There are already some attempts in this direction, but the current results reported for the values of these parameters, determined from the analyses of present experiments, are likely to change in future when experimental results with improved statistics become available.

1.6 Summary

Thus, starting from 1930, with Pauli's conjecture, neutrinos have played very important role in the understanding of weak interactions. Neutrinos are both puzzles and solution to many puzzles. With the development of precision neutrino experiments, the years to come are expected to be quite exciting as other properties associated with neutrinos are expected to be revealed. We have, in this chapter, introduced the various topics to be discussed in detail in the book. In the next three chapters, we present mathematical and quantum field theoretic preliminaries followed by the chapters on the phenomenological description of the weak interactions of leptons and hadrons leading to the standard model of electroweak interactions. These chapters are the building blocks to understand the physics of neutrino interactions. We discuss the various neutrino scattering phenomenon involving point particles as well as hadrons in separate chapters on the quasielastic, inelastic and deep inelastic scattering processes, followed by the chapters on neutrino-nucleus scattering focussing on the nuclear medium effects and their importance when scattering takes place with a bound nucleon inside a nucleus. The last few chapters deal with neutrino sources and its detection; neutrino oscillation; neutrinos in astrophysics and finally a chapter on the physics beyond the standard model.

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