

Project 6: Quantum Hall effect

April 2019

1 Introduction

In this project we are going to analyze the phenomenon of Hall conductivity quantization in the so-called Hall bar structure. This effect is a clear manifestation of the quantized Landau levels creation for electrons in the presence of the external magnetic field as well as the appearance of the so-called edge states.

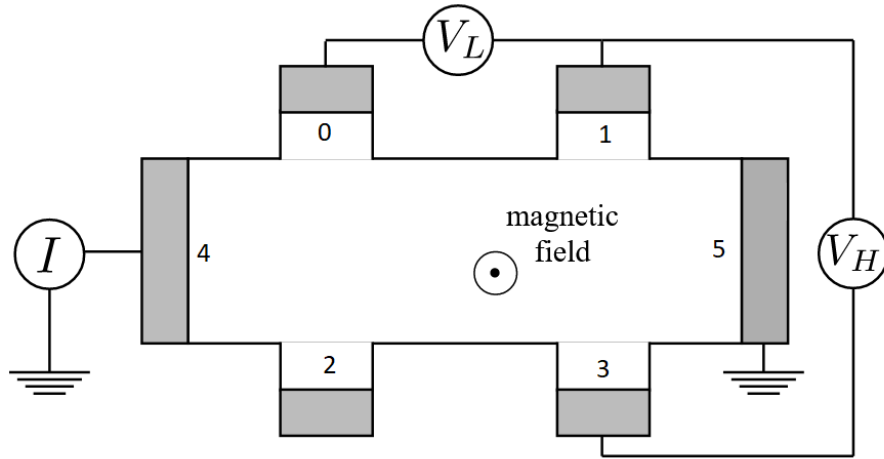


Figure 1: The six terminal Hall bar system in perpendicular magnetic field.

The system that we are going to consider is shown in Fig. 1. As you can see the system contains six leads enumerated from 0 to 5. Our main goal is going to be the calculation of the longitudinal (σ_{xx}) and transverse (σ_{xy}) conductivity as a function of the external magnetic field perpendicular to our two-dimensional system. The following formulas define σ_{xx} and σ_{xy}

$$\sigma_{xx} = \frac{R_L}{R_L^2 + R_H^2}, \quad \sigma_{xy} = \frac{R_H}{R_L^2 + R_H^2}, \quad (1)$$

where R_L and R_H are the longitudinal and Hall resistivity, which are given by

$$R_L = \frac{V_0 - V_1}{I_4}, \quad R_H = \frac{V_1 - V_3}{I_4}. \quad (2)$$

In the above equations V_0, V_1, V_1, V_3 are the voltages in the subsequent leads defined by the index and I_4 is the current in the lead 4.

In order to calculate the values of the voltages at the leads lets first consider the equation for the current flowing in lead p

$$I_p = \sum_{q=0}^5 G_{pq}(V_p - V_q), \quad (3)$$

where I_p and V_q are the currents and voltages corresponding to the p -th contact and G_{pq} is the conductance between contact p and q given by the formula

$$G_{pq} = \frac{2e^2}{h} T_{pq}. \quad (4)$$

In the above equation T_{pq} is the transmission coefficient for electrons to be transferred between lead p and q . Equation (3) can be rewritten in a matrix form

$$\mathbf{I} = \mathcal{G}\mathbf{V}, \quad (5)$$

where \mathbf{I} is the vector of currents corresponding to all the six leads of the system, \mathbf{V} is the vector of voltages at the six leads, and \mathcal{G} is the conductance matrix. Since the currents in various leads depend only on voltage differences among them, we can set one of the voltages to zero without loss of generality. Here we set $V_5 = 0$. As an effect we can erase the last column and the last row of matrix \mathcal{G} in Eq. (5).

We need to be able to calculate the vector of voltages for a given vector of currents, therefore we have to transform the matrix equation into the following form

$$\mathbf{V} = \mathcal{G}^{-1}\mathbf{I}. \quad (6)$$

As you can see in Fig. 1, we attach an electric power source to lead 4 for which we define what current flows into the system. Also, leads 0, 1, 2, and 3 are only attached to the system in order to be able to measure the longitudinal and transversal voltage difference V_L and V_H and actually during the experiment no current is going to flow in those leads. Therefore all the current flowing into the system from lead 4 needs to flow out of the system through lead 5. In such case we can set the current vector to $\mathbf{V} = [0, 0, 0, 0, -1]$ and substitute it to Eq. (6) in order to calculate the potentials. Than, by using Eqs. 2 we can calculate the longitudinal and Hall resistivities and use those to calculate the longitudinal and transversal conductivities with the use of Eq. 1. By carrying out this procedure for different magnetic fields we can determine the functions $\sigma_{xx}(B)$ and $\sigma_{xy}(B)$ what is our main goal here.

2 Description and research tasks

Create the Hall bar system in Kwant according to the scheme shown in Fig. 2 with the lattice constant set to $a = 10 \text{ nm}$. Consider the situation in which the

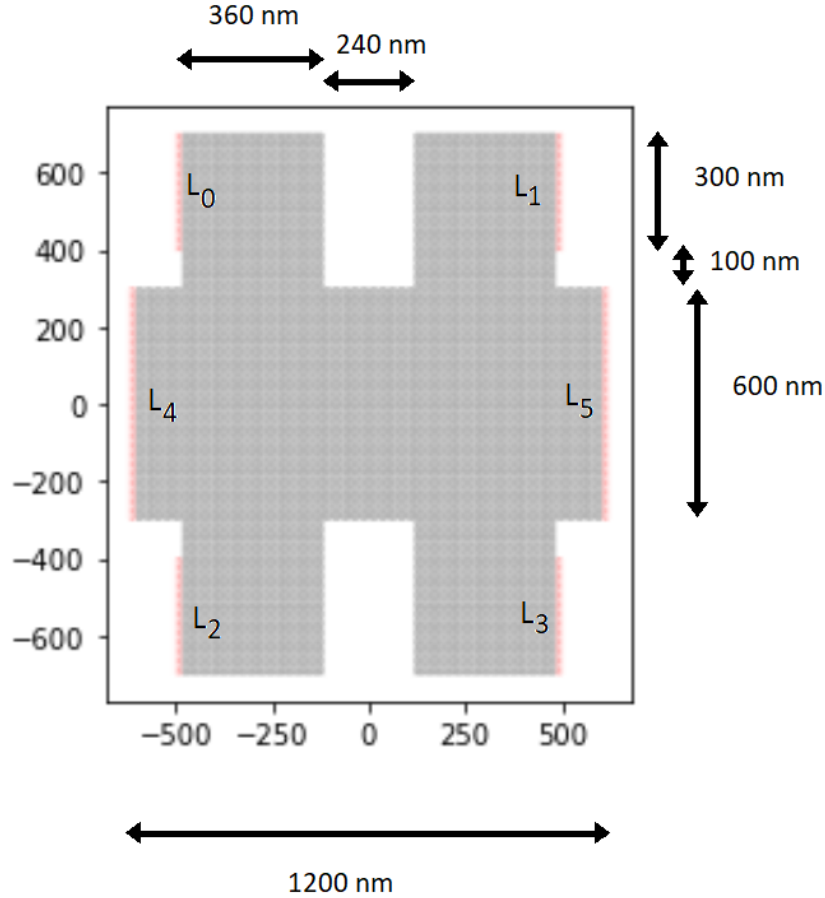


Figure 2: Hall bar system defined in kwant.

magnetic field is directed perpendicular to the system (along the z direction) by including the so-called Peiers phase in the hopping energies:

$$t_{ij} = -t \exp \left(-i \frac{\phi}{2} (y_i + y_j)(x_i - x_j) \right), \quad (7)$$

where $\phi = (2\pi Be/h) \cdot 10^{-18}$, and x_i, x_j, y_i, y_j are the coordinates expressed in nanometers. As usual $t = \hbar^2/(2ma^2)$ is the standard hopping energy which results from the discretization of the Schrödinger equation on our lattice.

Please carry out the following tasks:

- Calculate σ_{xx} and σ_{xy} as a function of external magnetic field in the range $B \in [0.2T, 0.5T]$ for the Fermi energy set to $E_F = 0.004 \text{ eV}$. In order to extract the conductance matrix for a given energy and magnetic field use the following code lines

```
smatrix = kwant.smatrix(sysf, ene, params=dict(B=B))
Gmat = np.transpose(smatrix.conductance_matrix())
```

where ene is the Fermi energy and B is the external magnetic field.

- Analyze the current density of electrons flowing into the system from lead 4 as the magnetic field is increasing.
- Do the same as in point 1 but introduce impurities into the system. In order to do that determine the onsite energies of the system with the use of the following function

```
U0 = 0.1 * t # the amplitude of the disorder potential
salt = 5 # how "dense" the disorder is
def onsite(site):
    return U0 * (uniform(repr(site), repr(salt)) - 0.5) + 4 * t
```

Remember to import the uniform function in the first cell of the code

```
from kwant.digest import uniform
```

Analyze how the results change as one changes the U0 parameter which determines the impact of the impurities

- analyze the current density of electrons flowing into the system from lead 4 as the magnetic field is increasing for the case of the system with impurities for one selected U0 value.

At the end try to summarize the obtained results and conclude your project.