HW 4. 1) The star Gliese 33 = HD 4628 has radial velocity variations due to an exoplanet:

$$RV(t) = K \cos\left(\frac{2\pi(t-T)}{P}\right) + v_{sys}$$

where

$T = 2457083.15992$

$P = 5.2616$ days

$K = 69$ m/s

$V_{sys} = -10.4$ km/s [use SIMBAD]

b) mass of star can be estimated from spectral type

sp type = K2.5 $\underline{\text{V}}$

mass $\approx 0.7\ M_\odot$ [Stellar Mass Catalog Belikov, 1995]

c) make a graph showing the radial velocity predicted by formula over 3 cycles.

(see Fig 1)

d) Contributions to measured radial velocity from Earth's spin and Earth's orbit:

spin $V_{max} = \left(465\ \frac{m}{s}\right) \cos(\text{Dec})$

$= \left(465\ \frac{m}{s}\right) \cos(5.^\circ28)$

$= 463\ \frac{m}{s}$

HW 4 p2

Spin $V(t) = V_{max} \cos(\omega t + \phi)$

$$\omega_E = \frac{2\pi \text{ rad}}{86164 \text{ s}} = 7.29212 \times 10^{-5} \frac{\text{rad}}{\text{s}}$$

ϕ = phase, set by position in HA of Gliese 33 at t_0

if t_0 = midnight Sep 30/Oct 1 local

HA = 22:38:37.84 = 339°.65

if t_0 = JD 2457083.15992

HA = 20:27:13.91 = 306°.8

At t_0 = JD 2457083.15992

$$V = \left(465 \frac{m}{s}\right) \cos(\text{Dec}) \sin(306°.8)$$

$$= -370.7 \frac{m}{s}$$

$$= V_{max} \cos(\omega [t - t_0] + \phi)$$

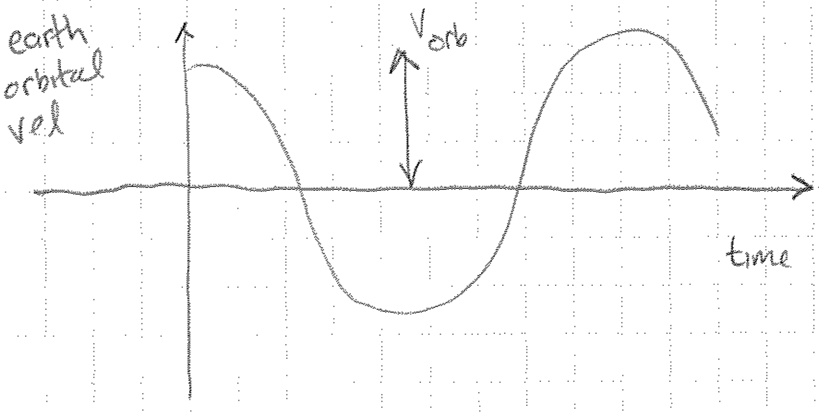
$$\rightarrow \phi = 143° \text{ or } 217°$$

So, if we measure time at $t - t_0$ away from $t_0 = 2457083.15992$, the Earth's contribution to radial velocity is

$$V_{Earth} = 463 \frac{m}{s} \cos\left(\omega_E (t - t_0) + \phi\right)$$

See Fig 2 for comparison of RV due to Gliese 33 alone and due to Earth spin.

We can compute the contribution of Earth's orbital speed to the measured radial velocity:



Sinusoid of amplitude

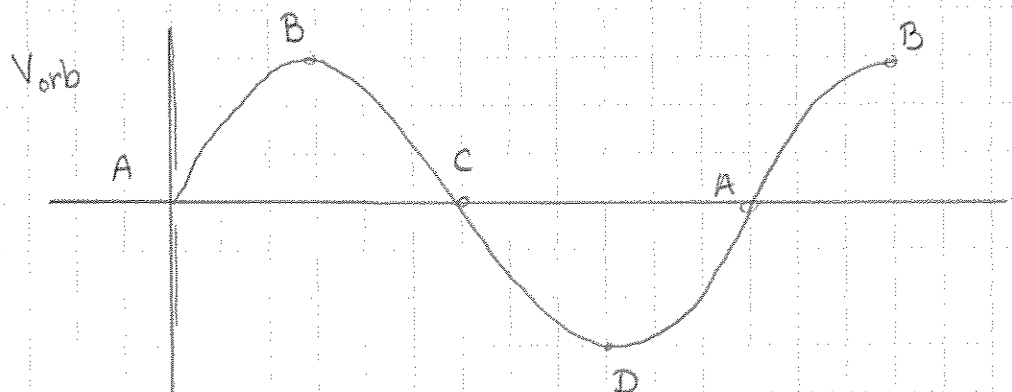
$$V_{orb} = \left(29{,}800\ \tfrac{m}{s}\right) \cos b$$

where b = ecliptic latitude of Gliese 33

$= 0\overset{\circ}{.}076$

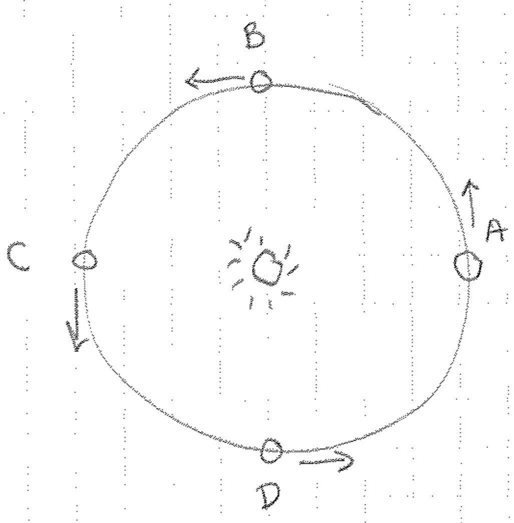
$\cong 0$

$\rightarrow$ amplitude of V_{orb} term will be $\approx 29{,}800$ m/s, and the only question is getting the phase correct.



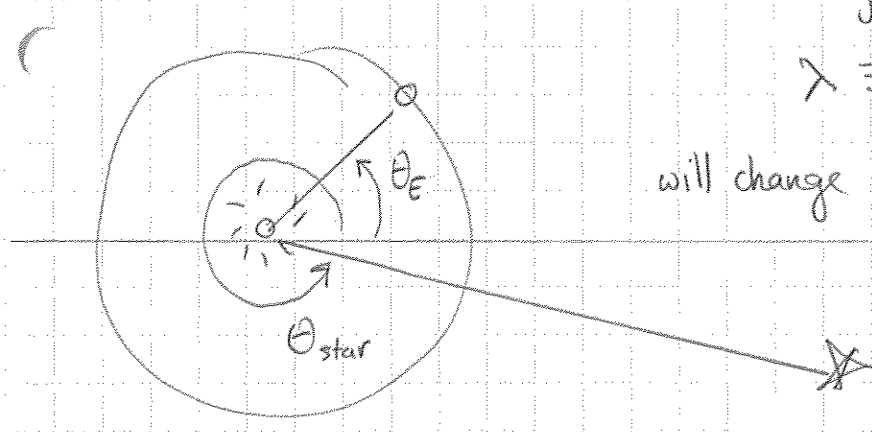
The radial component of V_{orb} will be

zero at opposition
pos max at 1st quad
zero at conjunction
neg max at 3rd quad



☆

By coincidence, the time of observation [Oct 2015] is close to opposition, point A.



The angle

$$\lambda \equiv \theta_{star} - \theta_{Earth}$$

will change with time

$$\lambda(t) = \lambda_0 + \left(\frac{360^\circ}{365.25\,d}\right)(t - t_0)$$

in days

Pick λ_0 so that Earth has the proper angular separation from Gliese 33.

on JD 2457083.15922 $\lambda = 147^\circ$

on Oct 1, 2015 10 PM local $\lambda = -5^\circ$ (near opposition)

Then

$$RV_{orb}(t) = (29{,}800\ m/s)\,\sin(\lambda)$$

See Fig 3 for comparison of orbital and spin and stellar reflex contributions to measured RV.

Adding all 3 contributions, we find overall RV as shown in Fig 4. Note the value grows in positive dir within each night's set. Values at the requested dates + times tabulated in Fig 5.

HW 4

#1

e. To find the distance between Gliese 33 and its planet, we use Kepler's Law #3:

$$P^2 = \frac{4\pi^2}{GM} a^3$$

If we use solar system units, we find

$$P^2_{(years)} = \frac{1}{M_{(solar)}} a^3_{(AU)}$$

Plug in

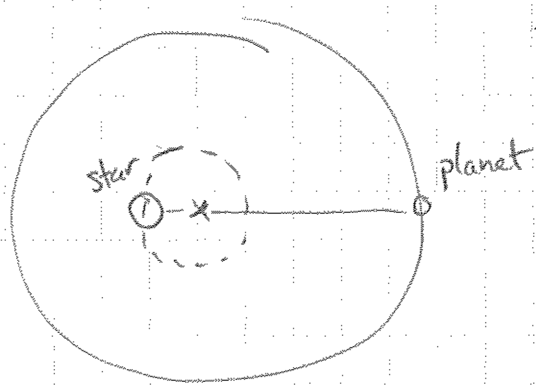
$M = 0.70\ M_\odot$

$P = 5.2616$ day $= 0.0144$ year

$\rightarrow \quad a = 0.0526$ AU $= 7.86 \times 10^9$ m

If we assume

- circular orbit (good, since RV curve sinusoidal)
- $M_s \gg M_p$



then center-of-mass implies

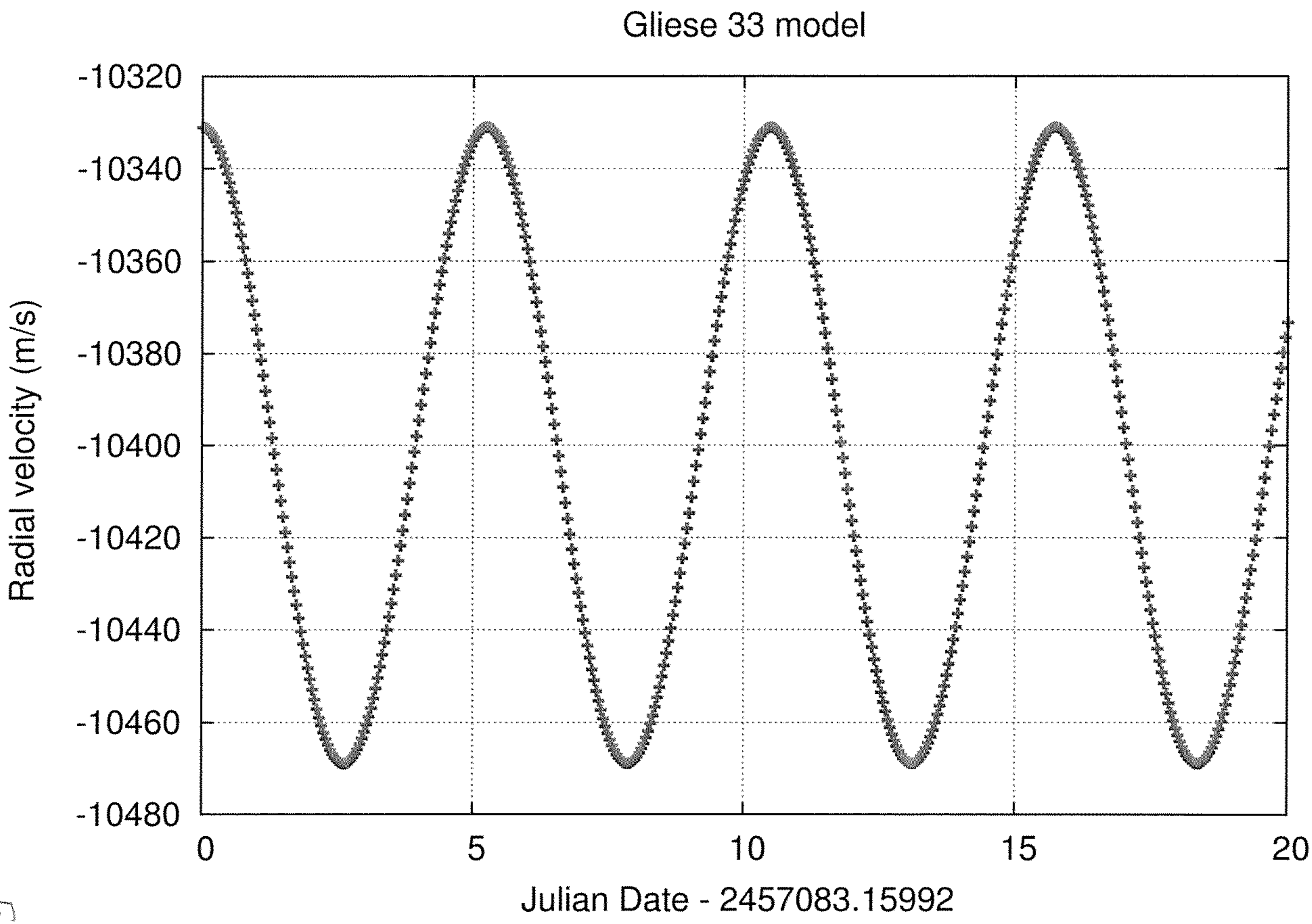
$$\frac{M_p}{M_s} = \frac{r_s}{r_p}$$

and $\quad 2\pi r_s = K P = \left(69\ \frac{m}{s}\right)(5.2616\ \text{day})$

$\Rightarrow \quad r_s = 4.99 \times 10^6$ m

$\Rightarrow \quad r_p = a - r_s \cong a$

$\Rightarrow \quad M_p = 6.3 \times 10^{-4}\ M_s = 0.47\ M_J$



Gliese 33 model
Radial velocity (m/s)
-10320
-10340
-10360
-10380
-10400
-10420
-10440
-10460
-10480
0
5
10
15
20
Julian Date - 2457083.15992

Fig 1

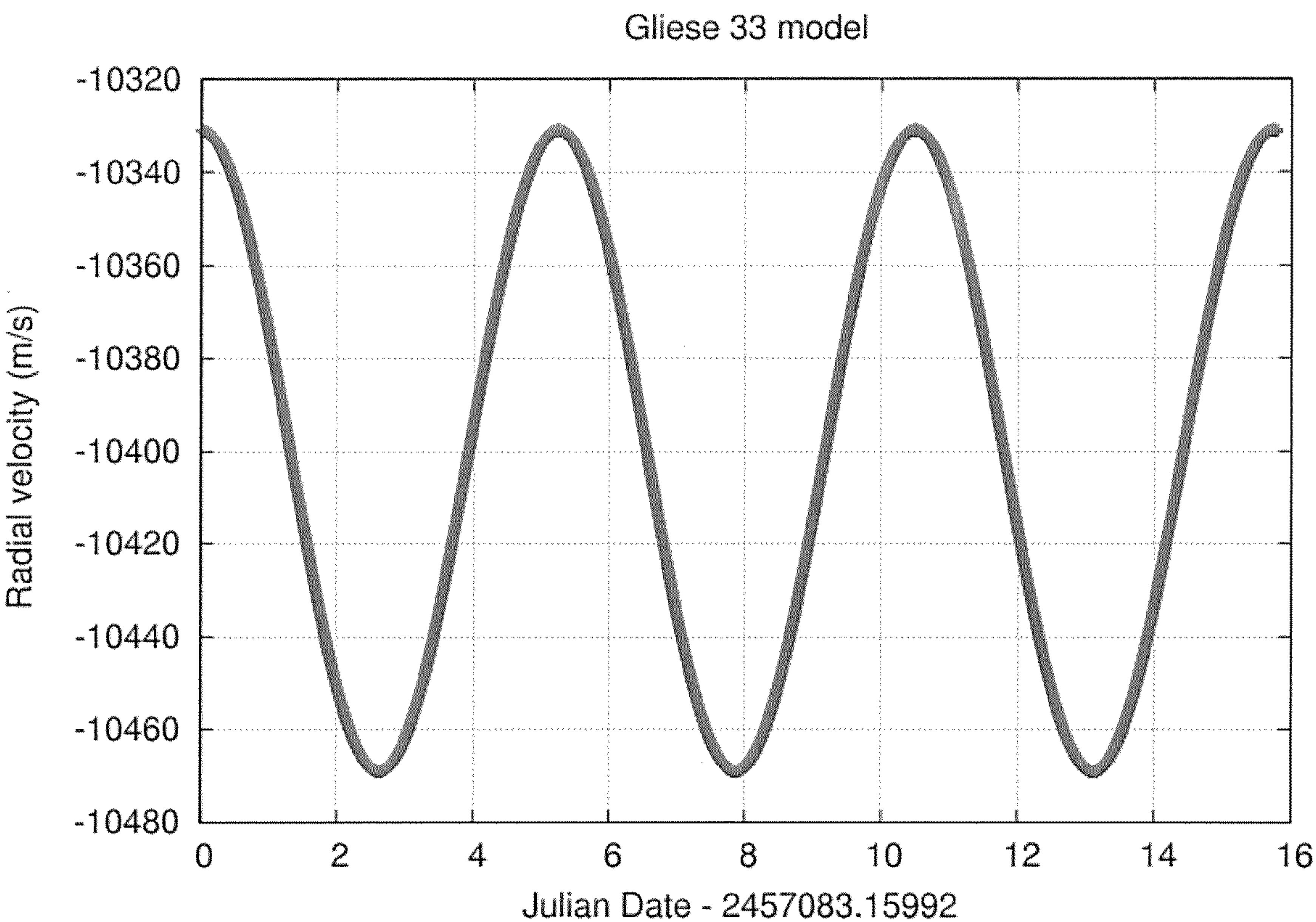

Gliese 33 model
Radial velocity (m/s)
-10320
-10340
-10360
-10380
-10400
-10420
-10440
-10460
-10480
0
2
4
6
8
10
12
14
16
Julian Date - 2457083.15992

Fig 1
4/3/2015

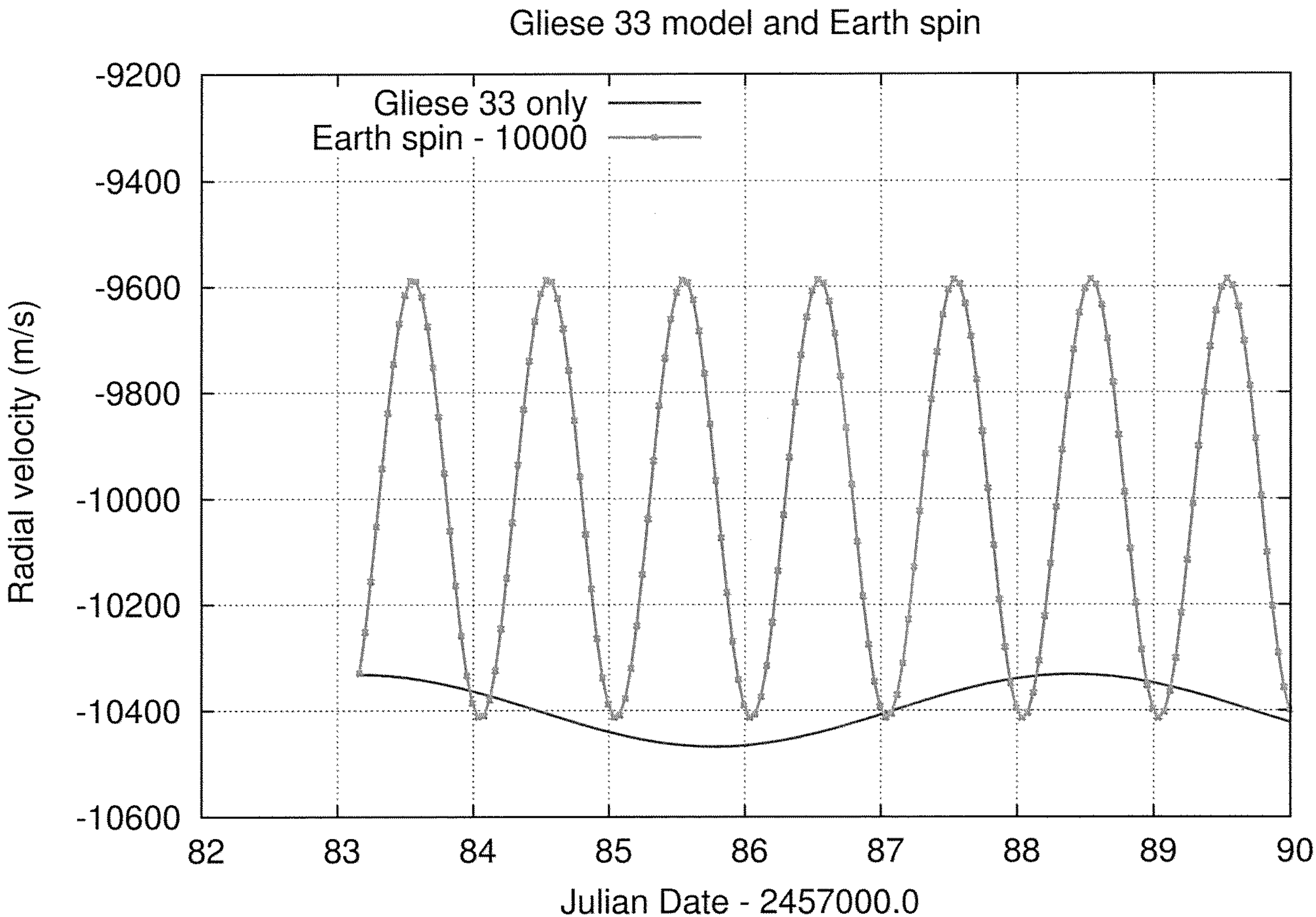

Gliese 33 model and Earth spin
Gliese 33 only
Earth spin - 10000
Radial velocity (m/s)
-9200
-9400
-9600
-9800
-10000
-10200
-10400
-10600
82
83
84
85
86
87
88
89
90
Julian Date - 2457000.0

Fig 2

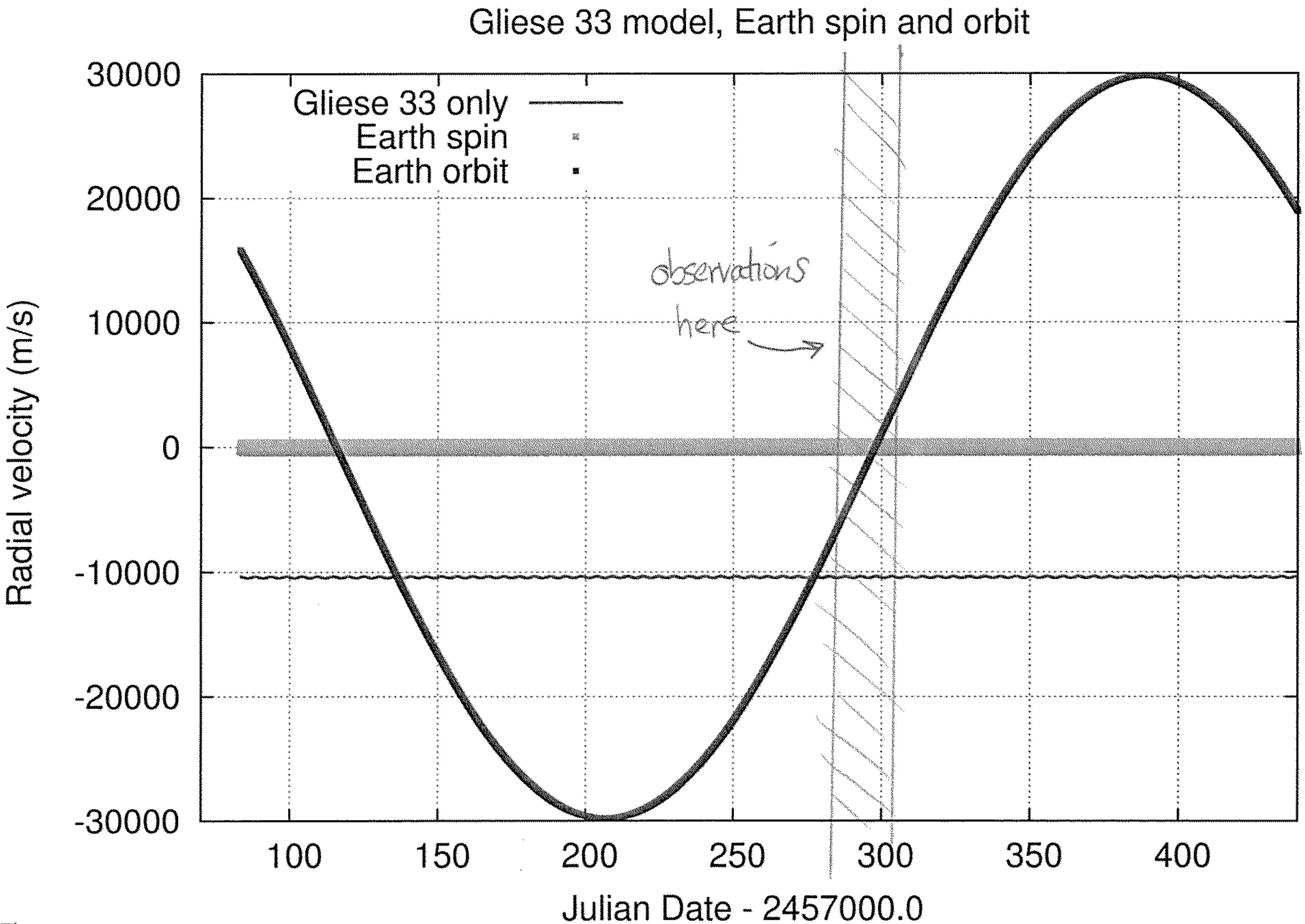

Gliese 33 model, Earth spin and orbit
Gliese 33 only
Earth spin
Earth orbit
observations here
30000
20000
10000
0
-10000
-20000
-30000
Radial velocity (m/s)
100
150
200
250
300
350
400
Julian Date - 2457000.0

Fig 3
4/4/2015

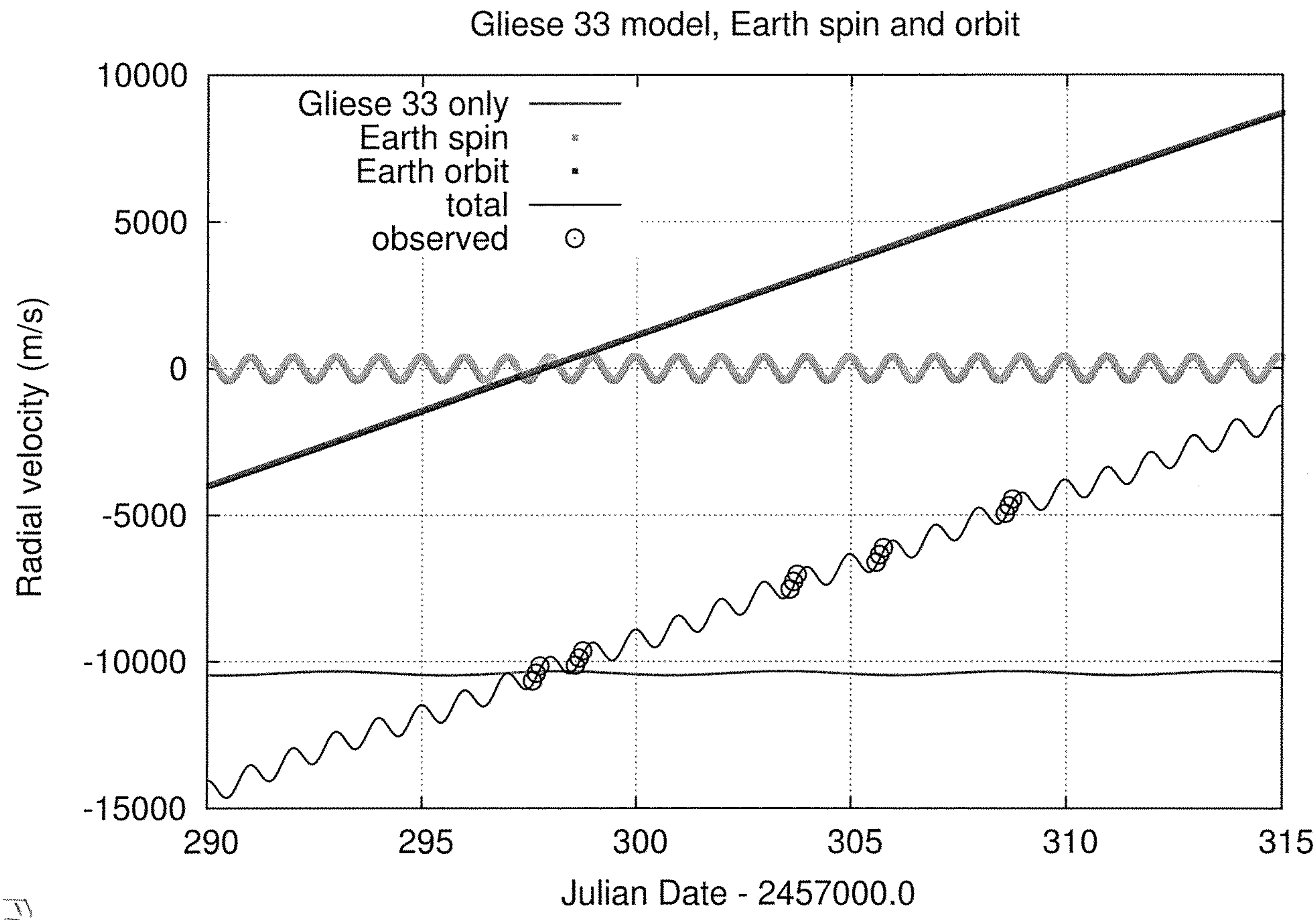

Gliese 33 model, Earth spin and orbit
10000
5000
0
-5000
-10000
-15000
Radial velocity (m/s)
Gliese 33 only
Earth spin
Earth orbit
total
observed
290
295
300
305
310
315
Julian Date - 2457000.0

Fig 4
4/4/2015

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JD 2457297.58353   rv  -10349.03    rv_spin  -196.863   rv_orbit   -112.79  tot -10658.69
JD 2457297.66686   rv  -10344.65    rv_spin    12.489   rv_orbit    -69.94  tot -10402.10
 ) 2457297.75020   rv  -10340.82    rv_spin   218.477   rv_orbit    -27.08  tot -10149.43

JD 2457298.58353   rv  -10338.12    rv_spin  -190.558   rv_orbit    401.46  tot -10127.22
JD 2457298.66686   rv  -10341.47    rv_spin    19.616   rv_orbit    444.31  tot  -9877.54
JD 2457298.75020   rv  -10345.40    rv_spin   224.505   rv_orbit    487.16  tot  -9633.74

JD 2457303.58353   rv  -10331.94    rv_spin  -158.224   rv_orbit   2967.86  tot  -7522.30
JD 2457303.66686   rv  -10333.40    rv_spin    55.119   rv_orbit   3010.50  tot  -7267.78
JD 2457303.75020   rv  -10335.53    rv_spin   253.615   rv_orbit   3053.13  tot  -7028.79

JD 2457305.58353   rv  -10457.68    rv_spin  -144.949   rv_orbit   3989.30  tot  -6613.33
JD 2457305.66686   rv  -10461.17    rv_spin    69.221   rv_orbit   4031.77  tot  -6360.18
JD 2457305.75020   rv  -10464.05    rv_spin   264.745   rv_orbit   4074.23  tot  -6125.08

JD 2457308.58353   rv  -10331.57    rv_spin  -124.720   rv_orbit   5512.18  tot  -4944.11
JD 2457308.66686   rv  -10331.03    rv_spin    90.216   rv_orbit   5554.29  tot  -4686.52
JD 2457308.75020   rv  -10331.17    rv_spin   280.848   rv_orbit   5596.39  tot  -4453.94
```

Fig 5
4/4/2015